
Problem 1. *Prove that in homogeneous coordinates, the intersection point* $\hat{\mathbf{x}}$ *of two lines* $\hat{\mathbf{l}}_1$ *and* $\hat{\mathbf{l}}_2$ *is given by* $\hat{\mathbf{x}} = \hat{\mathbf{l}}_1 \times \hat{\mathbf{l}}_2$.

Problem 2. *Prove that the line that joins two points* $\hat{\boldsymbol{x}}_1$ *and* $\hat{\boldsymbol{x}}_2$ *is given by* $\hat{\boldsymbol{l}} = \hat{\boldsymbol{x}}_1 \times \hat{\boldsymbol{x}}_2$.

Problem 3. *Given the following two lines in* $\mathbb{R}^3$*:*

$$l_1 = \{(x, y)^T \in \mathbb{R}^3 \mid x + y + 3 = 0\},$$
$$l_2 = \{(x, y)^T \in \mathbb{R}^3 \mid -x - 2y + 7 = 0\},$$

perform the following tasks:

1. *Find the intersection point of the two lines by solving the corresponding system of linear equations.*
2. *Rewrite the lines using homogeneous coordinates, and calculate their intersection point using the cross product of their homogeneous representations.*
3. *Verify whether the intersection point obtained in homogeneous coordinates is the same as the one obtained from solving the system of equations.*

Problem 4. *Write down the equation of the line whose normal vector is in the direction* $(3, 4)^T$ *and which is at a distance of* 3 *units from the origin.*

Problem 5. *Determine the distance from the origin and the normalized normal vector for the homogeneous line* $\hat{l} = \begin{pmatrix} 2 \\ 5 \\ \frac{\sqrt{29}}{5} \end{pmatrix}$.

Problem 6. *Write down the* 2×3 *translation matrix that maps the point* $(1, 2)^T$ *to* $(0, 3)^T$.

Problem 7. *Assume you are given* N *correspondence pairs in 2D:*

$$(\mathbf{x}_i, \mathbf{y}_i) = \left(\begin{pmatrix} x_1^i \\ x_2^i \end{pmatrix}, \begin{pmatrix} y_1^i \\ y_2^i \end{pmatrix} \right), \quad i = 1, 2, \ldots, N.$$

Find the 2×3 *translation matrix* $\mathbf{T}$ *that maps* $\mathbf{x}_i$ *onto* $\mathbf{y}_i$*, which is optimal in the least-squares sense.*

Hint: *Define a cost function as*

$$E(\mathbf{T}) = \sum_{i=1}^{N} \|\mathbf{T}\mathbf{x}_i - \mathbf{y}_i\|_2^2$$

and find the optimal $\mathbf{T}^*$ *that minimizes* E*:*

$$\mathbf{T}^* = \arg\min_{\mathbf{T}} E(\mathbf{T}).$$

You can find $\mathbf{T}^*$ *by calculating the Jacobian* $\mathbf{J}_E$ *of* E *and setting it to* $\mathbf{0}^T$*:*

$$\mathbf{J}_E = \left[\tfrac{\partial E}{\partial t_1}, \ldots, \tfrac{\partial E}{\partial t_N}\right] = \mathbf{0}^T.$$

Can you provide an intuitive explanation for the equation you derive for $\mathbf{T}^*$*?*

Problem 8. *You are given the following three correspondence pairs:*

$$\left(\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}3\\-5\end{pmatrix}\right), \quad \left(\begin{pmatrix}5\\7\end{pmatrix}, \begin{pmatrix}7\\6\end{pmatrix}\right), \quad \left(\begin{pmatrix}4\\1\end{pmatrix}, \begin{pmatrix}5\\-4\end{pmatrix}\right).$$

Using the equation you derived for $\mathbf{T}^*$*, calculate the optimal* 2×3 *translation matrix* $\mathbf{T}^*$*.*