# Applications of Markov Chains and Brownian Motion in Insect Ecology

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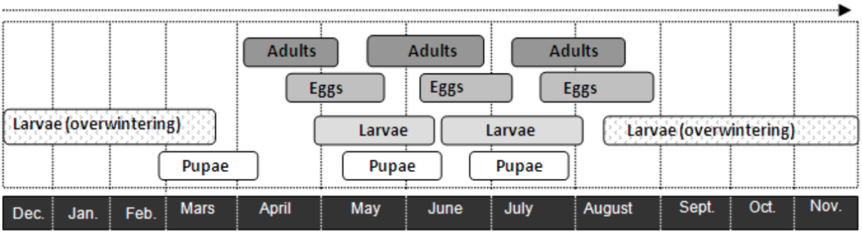
# Context of the study and assumptions

- Temporal evolution of **certain developmental** events during the life of an insect
- *Poikilothermic* organisms
- Diapause and the seasonality of an insect

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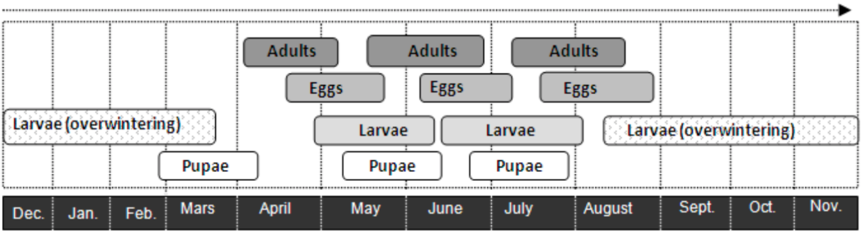


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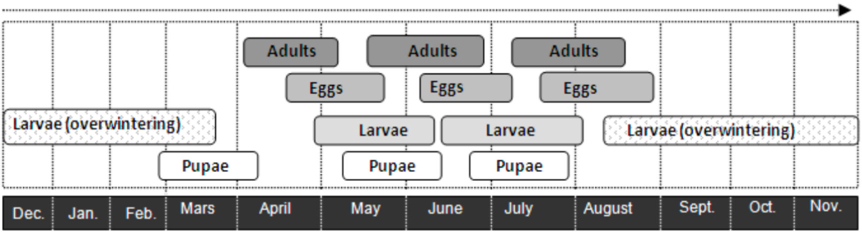


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- **Time independence** A process is said to be independent of time if $\forall t \geq 0, \quad X_{t+h} - X_t \sim X_h - X_0$
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# Birth and death process

## Birth and death process

A birth and death process is a continuous-time Markov chain with state S={0,1,. . . } for which transitions from state $i$ can only go to either state $i-1$ or state $i+1$. Here S stand for **a population size** at a specific time

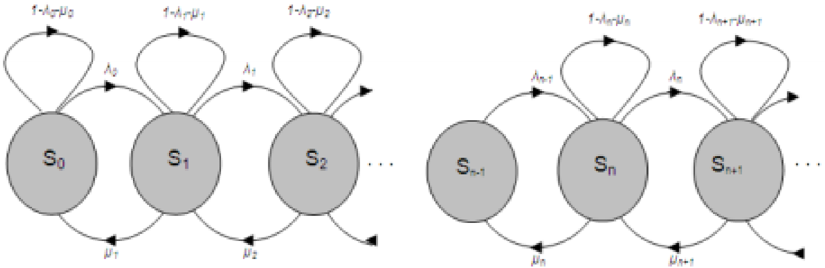



Figure – State transition diagram of a birth-death process

# Birth and death process

## Notion of time

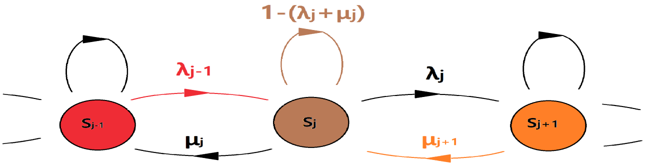
Time interval : $\Delta t$

$$p_{ij} = \begin{cases} \lambda_i \Delta t & j = i+1 \quad \text{an arrival occurs} \\ 1 \text{-} (\lambda_i + \mu_i)\Delta t & j = i \quad \text{nothing occurs} \\ \mu_i \Delta t & j = i-1 \quad \text{a departure occurs} \end{cases}$$

$\mu_i$ and $\lambda_i$ : transitions rates

- The probability of going to State $j$ for the next $\Delta t$

$$p_{ij}(t+\Delta t) = p_{i,j-1}(t)\lambda_{j-1}\Delta t + p_{i,j+1}(t)\mu_{j+1}\Delta t + p_{i,j}\left[1 - (\lambda_j + \mu_j)\Delta t\right]$$



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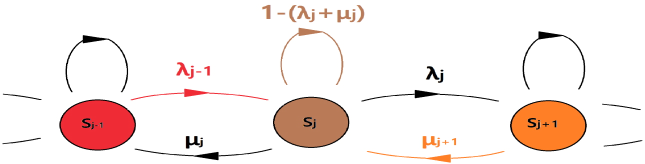

## Equilibrium state

In some conditions, it can show that a state of equilibrium exist for a birth and death process. In case of existence :

$$\lim_{n\to\infty} p_j^{(n)} \to \pi_j \ .$$

$\pi_j$ is the final probability to be at the state $j$

$$\pi_j = \frac{\lambda_0\lambda_1 \ldots \lambda_{j-1}}{\mu_1\mu_2 \ldots \mu_n}\pi_0 = \rho_j\pi_0$$

$$\pi_j = \frac{\rho_j}{\sum_0^{+\infty} \rho_k}$$

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# Description of the Random Walk

## Uni-dimensional random walk

- $\delta$ : Spatial parameter of the random walk.
- $(\varepsilon_i)_{i\in\mathbb{N}^*}$ : Independent random variables associated with the displacement $\delta$ : $\forall n \in \mathbb{N}^*,\ \varepsilon_n = \pm\delta$ with a likelihood $p = \frac{1}{2}$
- $S_n := \sum_{i=1}^n \varepsilon_i$ gives the position of the insect at step $n$.

## Properties of the Random walk

$$\mathbb{E}(S_n) = \sum_{i=1}^{n} \mathbb{E}(\varepsilon_i) = 0$$

Average distance to the origin after $n$ steps $\sqrt{\mathbb{E}(S_n^2)} = \delta\sqrt{n}$

$$\mathbb{E}(S_n^2) = \underbrace{\sum_{i=1}^{n} \mathbb{E}(\varepsilon_i^2)}_{\mathrm{n}\delta^2} + \underbrace{\sum_{i\neq j}^{n} \mathbb{E}(\varepsilon_i\varepsilon_j)}_{0}$$

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# Applications

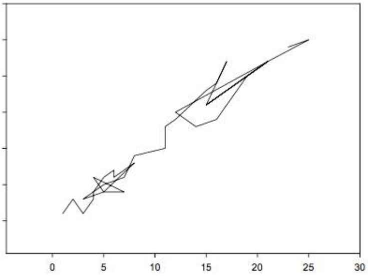


Figure – Random walk of a hypothetical insect movement

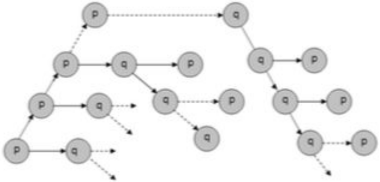


Figure – Diapause state or non diapause state model

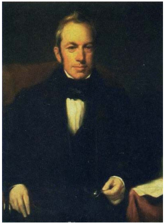

Figure – The Botanist Robert Brown (1773-1858)

- $\mathbb{P}(X_t = x)$ : Likelihood of finding a particle in position $x$ at time $t$ in the (uni-dimensional) random walk.
- $\delta$ and $\tau$ : Spatial and temporal parameters in the discretisation.
- $(p, q) \in [0, 1], \quad p + q = 1$

$$\mathbb{P}(X_{t+\tau} = x) := p\mathbb{P}(X_t = x - \delta) + q\mathbb{P}(X_t = x + \delta)$$

# Brownian Motion

$$\begin{cases} \delta \to 0 \\ \tau \to 0 \end{cases} \Rightarrow \boxed{\frac{\partial f(x,t)}{\partial t} = \frac{\delta}{\tau}(q-p)\frac{\partial f(x,t)}{\partial x} + \frac{\delta^2}{2\tau}\frac{\partial^2 f(x,t)}{\partial x^2}}$$

- $f(x,t)$ : density of probability associated with $\mathbb{P}(x,t) := \mathbb{P}(X_t = x)$
- In the case of **isotropy** $(p = q)$,

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# Brownian Motion

## Definition

A standard **Brownian Motion** is a stochastic process $(B_t)_{t\in\mathbb{R}^+}$ such as

- $B_0 = 0$
- $\forall (t,s) \in \mathbb{R}^+, \quad s < t \Rightarrow B_t - B_s \hookrightarrow \mathcal{N}(0, t-s)$
- $B_v - B_u$ is independent from $B_l - B_m$ if $]u, v[\bigcap]m, l[= \varnothing$
- For $\omega \in \Omega$, the function $t \to B_t(\omega)$ in almost surely continuous on $\mathbb{R}^+$

## Link with the random walk

For the precedent random walk, after $n$ steps $< S_n^2 >= n\delta^2$.
If $t = n\tau$, then $< x^2(t) >= n \times 2\tau D = \frac{t}{\tau} 2\tau D = 2Dt$
What leads us to define $Z_t := \sigma B_t$ where $\sigma^2 = 2D$ ( $\mathbb{V}(Z_t) =< x^2(t) >$)

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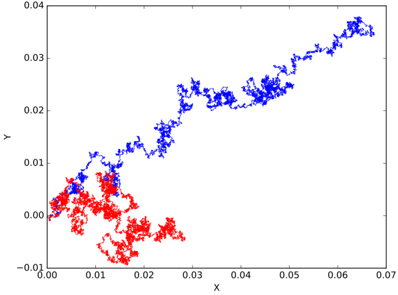
# Process of Wiener (generalization)

## A Wiener process

A stochastic process $(Z_t)_{t\in\mathbb{R}^+}$ is said to be a Wiener process, if its variation during $dt$ can be written as

$$dZ_t = {\color{blue}a}dt + {\color{red}\sigma} dB_t$$

Where $B$ is a **Brownian motion**.



## Memory effect

Autocorrelations for data : $R(k) := \frac{\sum_{i=0}^{N-k}(X_i - \bar{X})(X_{i+k} - \bar{X})}{\sigma^2}$

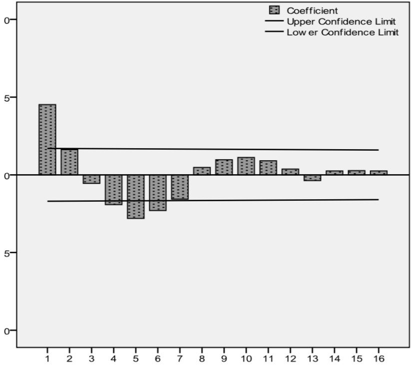



Figure – Autocorrelation for data in a time series

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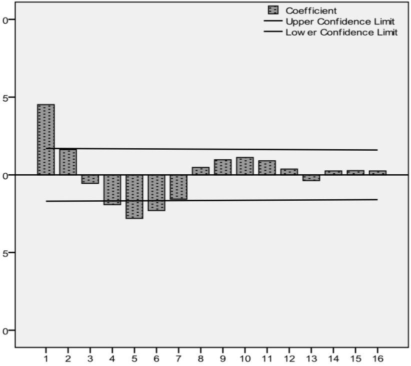


Figure – Autocorrelation for data in a time series

Thank you for you attention

# Sources

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