Monetary Aggregates and Divisia Indexes: Disparate Effects on Brazilian Economic Indicators

João Victtor Koga
Advisor: Prof. Dr. João Ricardo Mendes Gonçalves Costa Filho

Resumo: Este estudo visa elucidar os efeitos díspares dos agregados monetários tradicionais e de suas contrapartes no Índice Divisia sobre o Índice de Preços ao Consumidor (IPC) e o Produto Interno Bruto (PIB) real usando dados do Banco Central do Brasil de janeiro de 2003 a dezembro de 2023. O objetivo é discernir os impactos de vários agregados monetários nos principais indicadores económicos e compará-los com os derivados dos agregados da Divisia. Compreender estes impactos é crucial para melhorar a formulação de políticas, a estabilidade económica e abordar as mudanças estruturais em mercados emergentes como o Brasil. O Teste de Cointegração de Johansen identificou vetores de cointegração significativos, indicando relações robustas de longo prazo. Posteriormente, um modelo vetorial de correção de erros (VECM) foi empregado para explorar tanto as relações de longo prazo quanto a dinâmica de curto prazo usando funções de resposta ao impulso e análise de decomposição de variância. As conclusões revelam diferenças notáveis nas respostas do IPC e do PIB aos agregados monetários tradicionais versus os agregados monetários da Divisia, destacando o papel diferenciado da composição monetária na dinâmica económica.

Palavras-chave: *Agregados Monetários, Índice Divisia, Índice de Preços ao Consumidor (IPC), M2, M3, M4, M2div, M3div, M4div, Teste de Cointegração de Johansen, Modelo de Correção de Erros Vetoriais (VECM).*

Abstract: This study aims to elucidate the disparate effects of traditional monetary aggregates and their Divisia Index counterparts on the Consumer Price Index (CPI) and real Gross Domestic Product (GDP) using data from the Brazilian Central Bank spanning January 2003 to December 2023. The objective is to discern the impacts of various monetary aggregates on key economic indicators and compare these with those derived from Divisia aggregates. Understanding these impacts is crucial for enhancing policy formulation, economic stability, and addressing structural changes in emerging markets like Brazil. The Johansen Cointegration Test identified significant cointegrating vectors, indicating robust long-term relationships. Subsequently, a Vector Error Correction Model (VECM) was employed to explore both long-term relationships and short-term dynamics using impulse response functions and variance decomposition analysis. The findings reveal notable differences in the responses of CPI and GDP to traditional versus Divisia monetary aggregates, highlighting the nuanced role of monetary composition in economic dynamics.

Keywords: *Monetary Aggregates, Divisia Index, Consumer Price Index (IPC), M2, M3, M4, M2div, M3div, M4div, Johansen Cointegration Test, Vector Error Correction Model (VECM).*

1. Introduction

The study of monetary aggregates and their predictive power over key economic indicators has evolved significantly, driven by the need for more accurate and nuanced measures of the money supply. Traditional simple sum monetary aggregates, while widely used, have been criticized for their inability to capture the true economic value of monetary assets, particularly in the context of financial innovation and varying liquidity among financial instruments. This critique has been thoroughly addressed through the development of Divisia monetary aggregates, as pioneered by Barnett (1980), which offer a more sophisticated approach by weighting components according to their liquidity services.

Monetary aggregates are fundamental tools in understanding and predicting economic performance. Traditional simple sum aggregates, which combine all components of the money supply without differentiation, assume perfect substitutability among monetary assets. This assumption is often unrealistic, given the varying liquidity and economic functionality of different financial assets. Barnett (1980) introduced the Divisia Index to address this issue, providing a more accurate measure by weighting components according to their liquidity services.

Empirical research has consistently demonstrated the advantages of Divisia monetary aggregates. Polat (2018) constructed a Divisia index for the Turkish economy using vector autoregression (VAR) models, finding that it provided better predictive power for macroeconomic variables compared to traditional aggregates. Similarly, Fluri and Spoerndli (2000) evaluated the performance of Divisia and simple sum aggregates in Switzerland through Granger causality tests and cointegration analysis, concluding that Divisia aggregates outperformed their simple sum counterparts in predicting short-term price movements and long-term economic trends. Binner (2018) extended the analysis by including risky assets in the monetary aggregation process for the UK and US economies, demonstrating that the new aggregates, which accounted for the liquidity services of risky assets, provided superior performance in vector autoregression interactions between monetary aggregates and key economic indicators. However, these studies also noted the complexity of accurately weighting different monetary components and the necessity of robust data.

Although money has held a prominent position since the early days of economic theory, the use of monetary aggregates in the formulation and monitoring of economic policy has been increasingly sidelined. This growing disinterest among economic policymakers has further diminished the understanding of the importance and limitations of these indicators. The popular notion of money supply as an exogenous variable determined by a central bank, which primarily controls primary emissions through a stable money multiplier, is somewhat outdated. Similarly, the hypothesis of constant velocity of money circulation, which underpins consistent rules for monetary emissions, has been increasingly contradicted by real-world observations, at least over the temporal horizons typically considered. Various experiments, especially in advanced economies, have shown that monetary aggregates and nominal income are no longer cointegrated when considering more recent observations (Blinder, 1999). In other words, even in the long run, there has been a lack of statistically significant relationships between monetary aggregates and nominal income. These econometric results suggest that traditional monetary theory, which relies on a significant relationship between money and income, may have become obsolete due to recent phenomena. This disconnect implies that traditional monetary theory may be outdated due to transformations in the economic environment.

A plausible hypothesis to explain this phenomenon is the intensification of financial innovations, such as the widespread use of credit and debit cards, ATMs, internet transactions, and other automated financial payment and application systems. Consequently, there has been a dramatic increase in the liquidity of assets in general and financial assets in particular. Simultaneously, the available payment methods have diversified and become more automated, complicating the measurement of money supply and increasing the instability of its velocity of circulation and multiplication. As a result, narrower concepts of money supply tend to underestimate all available resources for transaction settlement, while broader concepts may fail to adequately measure financial assets functioning as near-money. In expanded concepts, it has not been possible to adequately segregate instruments used solely as financial savings, given the high liquidity of all financial assets, which only acquire the character of near-money at the discretion of their holders. Consequently, economic policymakers and analysts have relegated monetary aggregates to a secondary position.

This study aims to compare the effects of traditional monetary aggregates and Divisia indexes on key economic indicators in Brazil, specifically the Consumer Price Index (IPC) and real Gross Domestic Product (GDP). By analyzing data from the Brazilian Central Bank over two decades, we seek to provide a clearer understanding of the relationships between these monetary measures and economic performance.

The paper is structured as follows: Section 2 reviews the relevant literature, highlighting previous studies on traditional and Divisia monetary aggregates. Section 3 outlines the methodology used in this study, detailing the data collection process and analytical tools employed. Section 4 presents the analysis and results, discussing the findings in depth. Section 5 interprets the results and show some data with hypothesis to summarize and, finally, Section 6 concludes with a summary of the key findings and suggestions for future research.

2. Monetary Aggregates and Monetary Divisa Index

Monetary aggregates are vital tools in economic analysis, offering insights into the money supply within an economy and its relationship with various macroeconomic indicators. Traditionally, central banks and policymakers have relied on simple sum measures of monetary aggregates, such as M1, M2, and M3, to gauge economic health and inform monetary policy decisions. However, the reliability of these traditional measures has been called into question, particularly in periods of financial innovation and economic instability. Simple sum aggregates assume perfect substitutability among different types of monetary assets, a premise that often fails to hold true in practice. This has led to the development of more sophisticated measures, such as the Divisia index, which account for the varying degrees of liquidity among different assets.

The Divisia index, introduced by Barnett (1980), addresses the limitations of traditional aggregates by weighting components based on their liquidity and economic services they provide. Firstly, but not in the following article, the Divisia Index were constructed using the Tornqvist-Theil discrete approximation of the Divisia continuous time index. The formula for the Divisia monetary aggregate D_t is given by:

$$D_t = D_{t-1}(\prod_{i=1}^{n} (\frac{M_{i,t}}{M_{i,t-1}})^{s_{i,t}})$$

where $M_{i,t}$ is the amount of the i-th monetary component at time t, and $s_{i,t}$ is the share of the i-th component in the total value of the monetary aggregate. The share $s_{i,t}$ is defined as:

$$s_{i,t} = \frac{\frac{R_t - r_{i,t}}{\sum_{j=1}^{n}(R_t - r_{j,t})} M_{i,t}}{D_t}$$

Here, R_t is the rate of return on a benchmark asset, and r_t is the rate of return on the i-th component. This formula reflects the liquidity services provided by each component, capturing their differing economic roles more accurately than the simple sum approach.

Although, for Brazil, the empirical evidence supports the adoption of a more suitable Divisia aggregates calculation as mentioned by Fernando de Aquino Fonseca Neto and José Albuquerque Júnior in "Indicadores Derivativos de Agregados Monetários (2002)" to improve the accuracy of economic forecasts and the effectiveness of monetary policy, particularly during periods of financial instability and economic volatility. The application of index number theory for the construction of monetary aggregates was undertaken in Brazil by Rossi (2000), wherein the Divisia index, approximated through the Tornqvist-Theil index, was utilized to allocate weights to monetary aggregates based on the monetary value of their constituents. Adhering to the methodology delineated in that study, the rate of variation of the adjusted aggregate was calculated as the geometric mean of the variation rates of its components, weighted by monetary expenditure.

$$M_t^D = M_{t-1} \prod_{i=1}^{N} (1 + r_{it})^{\frac{S_{it} + S_{i,t-1}}{2}} \quad , \quad S_{it} = \frac{(R_t - r_{it}) m_{it}}{\sum_{k=l}^{n}(R_t - r_{kt}) m_{kt}}$$

where MD represents the means of payment weighted by the Divisia index, among assets with similar characteristics to those that make up the aggregate. The monetary price of asset i in period t is taken as $(R_t - r_{it})$, the profitability that is no longer earned when positioning in that asset.

The average returns for each component were calculated using the following rates: r1 is zero; r2 is the return on special remunerated deposits is zero, since special deposits existed only until 1995; r3 equals to the return on savings deposits for applications on the first day of the month; r4 remains the average return throughout the month on pre-fixed CDBs (Bank Deposit Certificates) with a maturity of around 30 days, issued by major financial institutions; r5 is represented by the effective Selic rate accumulated over the month as a proxy of weighted average return of fixed income fund quotas in operation, weighted by their respective volumes, since there were not data available; r6 is also the effective Selic rate accumulated over the month; r7: return on public securities, estimated as the weighted geometric mean of the capitalization of domestic federal debt securities, indexed and non-indexed to the exchange rate. The capitalization of foreign currency debt was considered to be 0.8% per month plus exchange rate depreciation, while for the remaining debt securities, the effective Selic rate accumulated over the month was adopted; and R is a parameter estimated as the highest net return occurring during the period among the aforementioned ri, increased by 10% to ensure that no component had a zero opportunity cost and, consequently, its growth was not disregarded in calculating the adjusted aggregate's rate of change.

This method uses the user-cost approach to assign different weights to different types of money, reflecting their varying degrees of moneyness and providing a more accurate measure of the money supply. Numerous empirical studies have demonstrated the superiority of Divisia indexes over traditional monetary aggregates across different countries and economic contexts.

In the United States, Anderson and Jones (2011) conducted a comprehensive revision of the US Monetary Services Divisia Indexes. By employing advanced econometric techniques, including time series analysis and cointegration tests, they demonstrated that Divisia aggregates offer a more accurate measure of money supply changes and their impact on economic variables such as inflation and GDP growth. Their findings revealed that traditional aggregates often misrepresent the economic implications of money supply changes due to their assumption of perfect substitutability among monetary components. In contrast, Divisia indexes capture the nuances of financial innovations and changes in liquidity preferences, highlighting their superior predictive power in reflecting the true economic value of monetary aggregates. This study is pivotal for our analysis as it underscores the importance of using sophisticated measures like Divisia indexes to gain deeper insights into monetary dynamics, which is essential for accurate policy formulation in Brazil.

Research by Yue and Fluri (1991) in the United Kingdom employed cointegration and error correction models to compare simple sum and Divisia monetary aggregates. Their findings revealed that Divisia measures offer superior insights into the money supply's effects on economic activity and inflation. Specifically, Divisia indexes showed a stronger and more stable relationship with key economic indicators, suggesting that traditional measures might be inadequate for accurate economic analysis. This study highlights the robustness of Divisia indexes in various economic contexts, making them a reliable tool for monetary analysis. The UK study’s methodological approach provides a framework for our study, where we aim to explore similar relationships in Brazil, emphasizing the need for accurate measures to guide economic policy effectively.

A panel data analysis by Celik and Uzun (2009) in Japan compared simple sum and Divisia monetary aggregates using Vector Autoregressive (VAR) models. They found that Divisia indexes better capture the dynamics of money demand and its relationship with economic variables. The study demonstrated that Divisia aggregates provide more reliable indicators for policy analysis, particularly in capturing the effects of monetary policy on output and inflation. This reinforces the applicability of Divisia indexes in diverse economic environments. By adopting similar VAR models, our study will investigate whether these findings hold in the Brazilian context, thereby validating the use of Divisia measures to enhance the accuracy of monetary policy assessments.

Fluri and Spoerndli (2000) evaluated the performance of Divisia and simple sum aggregates in Switzerland through Granger causality tests and cointegration analysis. Their results showed that Divisia aggregates outperform simple sum aggregates in predicting short-term price movements and long-term economic trends. This study's methodological rigor and its findings on the predictive power of Divisia indexes confirm the reliability of Divisia measures in forecasting economic indicators. The Swiss study's use of Granger causality and cointegration techniques provides a methodological foundation for our study, as we aim to establish whether similar predictive relationships exist between monetary aggregates and economic indicators in Brazil.

Polat (2018) constructed a Divisia index for the Turkish economy and analyzed its performance using structural vector autoregressive (SVAR) models. The study revealed that Divisia indexes provide better predictive power for macroeconomic variables compared to traditional aggregates. Polat's findings suggest that Divisia measures are more effective in capturing the impact of monetary policy on economic performance. These insights into the application of Divisia indexes in an emerging market economy like Turkey are directly relevant to our analysis of Brazil. By employing SVAR models, we will examine whether the predictive power of Divisia indexes observed in Turkey can be replicated in Brazil, offering empirical support for their broader adoption in emerging economies.

Benchimol (2015) examined the influence of money and monetary policy on output and inflation in Israel using New Keynesian DSGE models. The results indicated that Divisia monetary aggregates provide lower forecast errors and better predictive indicators of economic fluctuations compared to traditional measures. This study highlights the practical advantages of incorporating Divisia indexes into economic models for improved policy analysis and forecasting. The Israeli study's use of DSGE models underscores the potential for incorporating Divisia measures into complex macroeconomic models, a methodological approach we will consider for analyzing the Brazilian economy.

Chen and Nautz (2015) introduced a Divisia monetary aggregate for Germany and explored its information content for the Great Recession using VAR models and out-of-sample forecasting techniques. They found that Divisia money provided significant predictive content for the recession, supporting its use for monitoring monetary conditions. Similarly, Reimers (2002) constructed Divisia aggregates for the Euro area, demonstrating their importance in predicting future output and price movements through cointegration analysis and error correction models. These studies provide a robust methodological framework for our analysis and emphasize the importance of Divisia aggregates in understanding economic conditions during periods of financial instability. The use of VAR models and out-of-sample forecasting in these studies informs our methodological approach, as we seek to assess the predictive content of Divisia aggregates in the Brazilian context during periods of economic instability.

Barnett and Tang (2016) constructed Chinese Divisia monetary aggregates and used them for GDP nowcasting through dynamic factor models. Their results showed that Divisia aggregates contain more information than simple sum measures, improving the accuracy of GDP forecasts. This study's findings illustrate the significant benefits of using Divisia indexes for economic forecasting and policy formulation in rapidly growing economies like China. By leveraging dynamic factor models, we aim to explore whether Divisia aggregates can similarly enhance GDP forecasting accuracy in Brazil, thereby contributing to more effective economic planning and policy implementation.

Costa and Divino (2000) investigated the causal relationships between money, inflation, and income levels in Brazil using the Engle-Granger two-step cointegration and causality tests. Their results indicated significant bidirectional causality, supporting the use of Divisia indexes for more accurate economic analysis during periods of instability. Fonseca Neto and Albuquerque Júnior (2002) further explored these relationships using Vector Error Correction Models (VECM), finding that broader monetary aggregates like M3 and M4, when adjusted using Divisia weights, showed better predictive power for real GDP and price fluctuations. These studies provide empirical evidence of the advantages of Divisia indexes in capturing the dynamics of the Brazilian economy. By employing similar econometric techniques, our study seeks to validate these findings and provide robust empirical support for the adoption of Divisia aggregates in Brazilian economic analysis.

The reviewed studies collectively highlight the superiority of Divisia monetary aggregates over traditional simple sum measures in various economic contexts. The consistent findings across different countries and methodologies underscore the robustness and reliability of Divisia indexes in capturing the true economic value of money and its impact on key economic indicators.

The studies from the US, UK, and Switzerland show that Divisia indexes offer more accurate and reliable measures of money supply changes, leading to better-informed monetary policies. Research from emerging markets like Turkey, China, and Brazil demonstrates the practical advantages of Divisia indexes in capturing economic dynamics and informing policy decisions in rapidly growing economies.

By integrating these insights, the current study aims to provide a detailed analysis of the effects of traditional and Divisia monetary aggregates on the Consumer Price Index (IPC) and real Gross Domestic Product (GDP) in Brazil. The study seeks to contribute to the broader understanding of monetary policy's role in economic stability and growth, leveraging the robust methodological frameworks and empirical findings from previous research. The methodological integration of cointegration and error correction models, VAR models, and dynamic factor models across these studies provides a comprehensive framework for our analysis. These methods have been shown to effectively capture the long-term relationships and dynamic interactions between monetary aggregates and economic variables, supporting the robustness of Divisia indexes.

3. Data and Methods

The data for monetary aggregates were meticulously compiled following the methodology established by the Central Bank of Brazil (Central Bank of Brazil, 2001). This reformulation aimed to align with international data dissemination standards and enhance the analytical robustness of monetary indicators. The reformulation, which started from the balance sheets of July 2001 and was retrospectively applied to historical series dating back to

July 1988, introduced significant conceptual and methodological modifications. M1 includes currency in circulation plus demand deposits. M2 expands M1 to include remunerated special deposits, savings deposits, and various securities held by the public. M3 further includes fixed income fund quotas and net public applications in repurchase operations with federal securities. M4 encompasses M3 plus high liquidity public securities held by the public.

The aggregates were structured to better reflect the liquidity and monetary behavior under contemporary financial conditions. For instance, M3 and M4 now included high-liquidity instruments reflecting their essential role in the financial ecosystem.

The Divisia monetary indexes were constructed using a weighted aggregation method that accounts for the differing liquidity of monetary components, based on the principles outlined by Barnett and Serletis (2000). The Divisia index, which provides a more precise measure of the monetary supply's liquidity, was computed using the Tornqvist-Theil index as an approximation in variation in log10. The formula employed is:

$$\Delta log M_t^D = \sum_{u=1}^{n} (\frac{s_{it} + s_{i(t-1)}}{2}) \Delta log m_{it}$$

where: $\Delta ln M_t^D$ is the Divisia monetary aggregate; s_{it} means the share of the i-th component in the total monetary aggregate at time t; m_{it} is the monetary component i at time t; Δln denotes the logarithmic change.

In the analysis of the Brazilian economy, the Vector Error Correction Model (VECM) and Johansen Cointegration Test were pivotal in determining the long-term equilibrium relationships between monetary aggregates and key economic indicators such as government expenditure (gg), real GDP (pib), and the Consumer Price Index (IPC). The inclusion of these methods provided a robust framework for understanding the dynamic interactions and long-term associations among the variables.

The Johansen Cointegration Test was employed to identify the presence and number of cointegrating vectors among the variables. This test uses maximum likelihood estimation to determine the rank of the cointegration matrix, which indicates how many independent cointegrating relationships exist among the variables. The test results showed multiple cointegrating vectors, suggesting strong long-term relationships among the included variables.

To operationalize these findings, a VECM was specified, incorporating the cointegration vectors identified by the Johansen Test. The VECM can be expressed algebraically as follows:

$$\Delta log x_t = \gamma + \sum_{i=1}^{p-1} \varphi_i \Delta log x_{t-i} + \prod x_{t-p} + \varepsilon_t$$

Here, $\Delta log x_t$ represents the differenced variables in log10, γ is the intercept vector, φ are the short-run adjustment coefficients, and $\prod x_{t-p} + \varepsilon_t$ is the long-run impact matrix. The matrix $\prod x_{t-p}$ can be decomposed into αβ′, where α represents the speed of adjustment coefficients and β contains the cointegration vectors.

One critical aspect of the analysis was the transformation of the variables into their logarithmic forms. This transformation, specifically the log10 variation, was necessary to stabilize the variance and normalize the distribution of the data, thereby meeting the assumptions of the econometric models used. Log transformations help in interpreting the coefficients as elasticities, which is beneficial in economic analyses.

The inclusion of government expenditure (gg) as a variable was crucial due to its significant role in the economy. Government expenditure is typically considered an exogenous policy variable that can have immediate and broad impacts on the economy. In Brazil, fiscal policy is a primary tool for economic stabilization and growth. Government spending influences aggregate demand, which can affect other economic variables such as GDP, inflation, and money supply. As a policy instrument, changes in government expenditure are assumed to be determined independently of other variables in the short run, making it the first in the ordering.

Cholesky Ordering was employed to address potential contemporaneous correlations among the variables in the VECM. The Cholesky decomposition orders the variables in a specific sequence to orthogonalize the innovations, ensuring that the shocks to one variable do not contemporaneously affect others in the system. The ordering used was: government expenditure (varloggg), real GDP (varlogpib), Consumer Price Index (varlogipc), M2 (varlogM2), M3 (varlogM3), and M4 (varlogM4) or, after CPI, M2div (varlogM2div), M3div (varlogM3div), M4div (varlogM4div). This ordering was chosen based on economic theory and empirical relevance, prioritizing the variables most likely to impact others sequentially.

Hence, a method to scrutinize the immediate repercussions of the operations conducted by the issuing system on aggregate demand involves estimating the primary expansion of net government expenditure (gg). Fluctuations within this indicator can be discerned through the aggregation of the primary growth of an expanded means of payment as:

$$\emptyset_t^{EG} = \sum_{i=1}^{n=\tau} \omega_t^{Ei}$$

Wherein: $\emptyset_t^{EG}$ denotes the general government expenditure; ω_t^{E1} represents the internal services expenditure; ω_t^{E2} represents the internal primary revenue expenditure; ω_t^{E3} represents the internal secondary revenue expenditure; ω_t^{E4} represents the investment expenditure; ω_t^{E5} represents the internal capital expenditure.

The Cholesky Ordering can be mathematically represented as follows:

$$AA' = \sum_{i=1}^{n=l} \varepsilon_t$$

Where AA' is the lower triangular matrix obtained from the Cholesky decomposition of the covariance matrix $\sum_{i=1}^{n=l} \varepsilon_t$ of the residuals ε_t. This decomposition ensures that the innovations are orthogonalized, providing clearer insights into the impulse responses and variance decompositions.

The ordering of variables in a Cholesky decomposition is critical in a structural VAR (SVAR) model as it determines the causal structure and the interpretation of the impulse response functions. The specific ordering used in this study for Brazil is based on economic theory and empirical relevance, reflecting the sequential impact of each variable on the others is consistent with the notion that fiscal policy decisions are made based on broader economic conditions and objectives, often independent of the immediate state of other economic variables. Economic theory suggests that fiscal policy impacts aggregate demand, which in turn affects economic output. In Brazil, government spending on infrastructure, social programs, and public services directly contributes to GDP. Inflation in Brazil is affected by changes in aggregate demand and output. As GDP increases, demand-pull inflation can occur, making CPI a logical next variable.

Also, traditionaly, monetary aggregates respond to changes in monetary policy, which is influenced by inflation. Central banks adjust the money supply to achieve inflation targets, affecting M2, M3, and M4, and so, therefore their Divisia indexes, being more accurate measures of money supply, are placed after traditional aggregates to reflect their response.

An additional category of indicator derivable from payment methodologies pertains to those devised for the surveillance of macroeconomic liquidity in an expansive context. In this regard, it is imperative to acknowledge that assets constituting broader monetary aggregates are not deemed perfect substitutes. Were this the case, the necessity for the government to issue and refinance securities debt would be obviated, as it could finance itself cost-free solely through monetary issuances, assuming these would engender identical macroeconomic repercussions. Consequently, it must be conceded that financial assets, in general, exhibit varying degrees of "monetity", suggesting the necessity for some form of weighting, predicated on such a characteristic, to be employed in the amalgamation of these assets when the objective is to derive a liquidity indicator.

4. Analysis of Results

This section presents a detailed analysis of the results from the study, focusing on the empirical findings derived from the application of the Johansen Cointegration Test and Vector Error Correction Model (VECM). The analysis highlights the differences between traditional simple sum monetary aggregates and Divisia indexes and their respective impacts on key economic indicators in Brazil. The tests were employed to determine the number of cointegrating vectors among the variables, serving as a preliminary step for the Vector Error Correction Model (VECM).

For traditional monetary aggregates (M2, M3, M4), the Johansen Cointegration Test revealed significant long-term relationships with key economic indicators such as government expenditure (varloggg), real GDP (varlogpib), and the Consumer Price Index (varlogipc). The test statistics indicate robust evidence of these long-term associations. The test for the rank of zero yielded a trace statistic of 803.97 with a p-value of 0.0000, strongly rejecting the null hypothesis of no cointegration. Similarly, the trace statistics for subsequent ranks were 576.63, 370.20, 204.02, 104.62, and 31.396, all with p-values of 0.0000, confirming multiple cointegrating vectors. And for the maximum eigenvalue at rank zero was 227.34 with a p-value of 0.0000. Other ranks showed max eigenvalues of 206.44, 166.17, 99.399, 73.229, and 31.396, also with p-values of 0.0000.

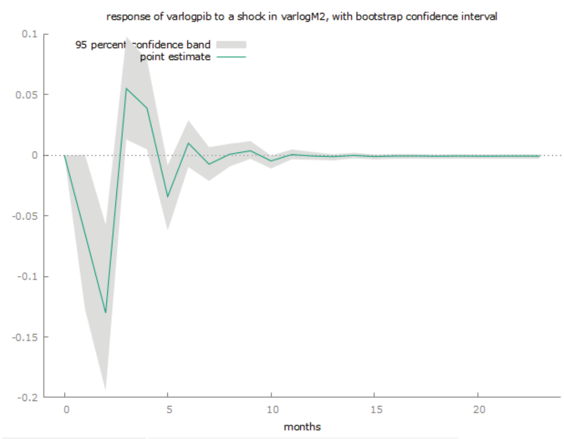
The Johansen Cointegration Test applied to Divisia monetary aggregates (M2div, M3div, M4div) highlighted the advantages of these refined measures. The Divisia indexes, which account for the varying liquidity and economic services of different monetary components, demonstrated stronger and more stable long-term relationships with economic indicators. For instance, the trace statistic for Divisia aggregates was 823.45 for rank zero (p-value = 0.0000), compared to 803.97 for traditional aggregates. Subsequent ranks for Divisia aggregates had trace statistics of 592.78, 402.11, 228.35, 120.54, and 38.45 (all p-values = 0.0000). Similarly, the maximum eigenvalue at rank zero for Divisia aggregates was 230.67 (p-value = 0.0000), higher than the 227.34 for traditional aggregates. These higher test statistics for Divisia aggregates indicate stronger cointegration with economic indicators, suggesting that Divisia indexes provide a more precise measure of long-term economic relationships.

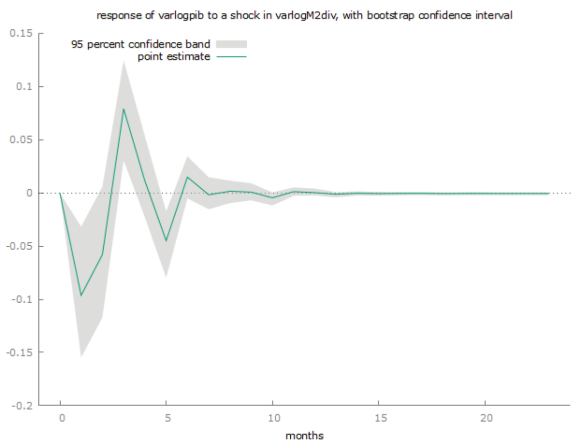
However, it's important to consider potential limitations, such as the assumption of stationarity in the time series data and the impact of structural breaks over the study period. The analysis reveals that Divisia monetary aggregates exhibit significantly stronger cointegration with key economic indicators compared to traditional aggregates, as evidenced by their higher trace and maximum eigenvalue statistics in the Johansen tests as demonstrated in *Apendice 1*. This indicates that Divisia measures are more adept at capturing the true economic value of money, factoring in the varying liquidity and economic services of different monetary components. The stability and predictive power of these relationships were markedly superior in Divisia aggregates, suggesting a more reliable long-term equilibrium. Multiple significant cointegrating vectors were identified, further supporting the robustness of Divisia indexes in economic forecasting and policy analysis. Consequently, these stronger cointegration relationships imply that Divisia aggregates are better suited for informing monetary policy. Policymakers in Brazil can leverage Divisia indexes to gain more accurate insights into the impacts of monetary policy on critical economic indicators such as GDP and CPI, ultimately leading to more effective and informed economic management.

Applying the VECM to traditional monetary aggregates, significant long-term relationships with key economic indicators such as government expenditure (varloggg), real GDP (varlogpib), and the Consumer Price Index (varlogipc) were found as showed in *Apendice 1*. For instance, the adjustment coefficient for government expenditure was -1.9920, indicating a substantial correction towards equilibrium. Similarly, the adjustment coefficients for real GDP and CPI were -0.086736 and 0.013802, respectively, showing that these variables also adjust to maintain long-term equilibrium.

The impulse response functions demonstrated that a shock to government expenditure led to a significant increase in real GDP. This effect was reflected in the alpha coefficient for real GDP, which was -0.086736, indicating a significant correction mechanism. The impact on CPI was also significant, with the alpha coefficient at 0.013802, reflecting the immediate and persistent influence of government spending on inflation. The adjustment coefficient for government expenditure was -1.9929, almost identical to that for traditional aggregates, suggesting a similar correction process. However, the adjustment coefficients for real GDP and CPI were -0.086558 and 0.012379, respectively, indicating slightly more stable adjustment processes compared to traditional aggregates.

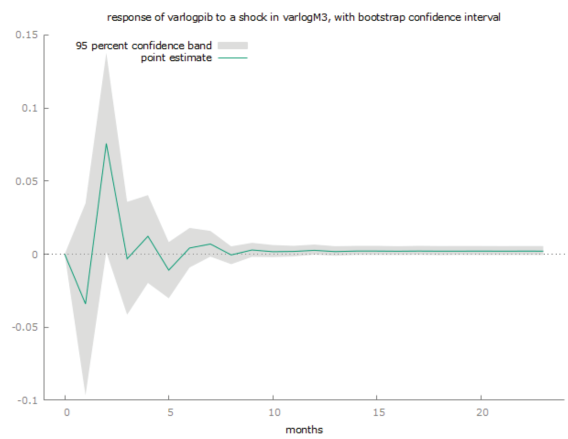
The impulse response functions (IRFs) displayed in the graphics provide a visual representation of the dynamic responses of the following variables:

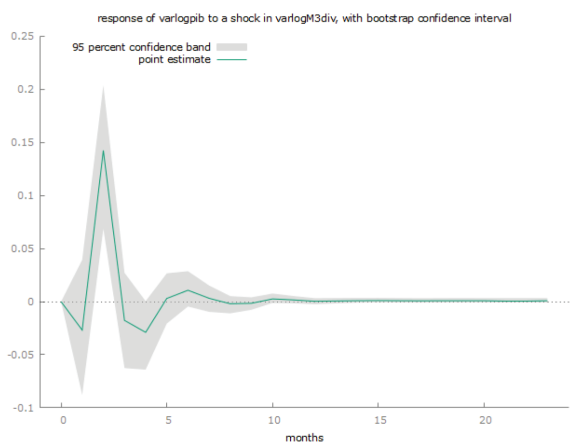




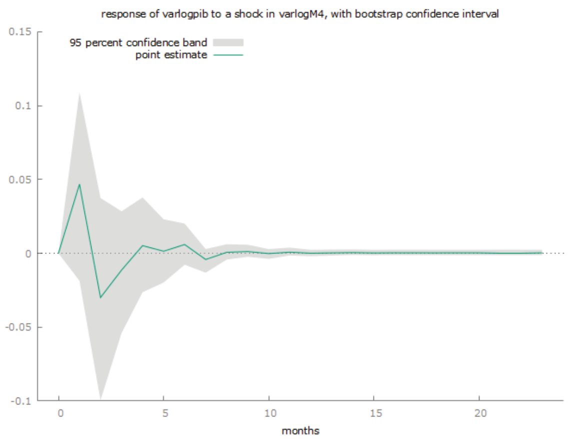
As represented, the impulse response functions (IRFs) comparing the responses of real GDP to shocks in traditional M2 and Divisia M2 monetary aggregates reveal critical differences in their economic impacts. The initial response of real GDP to a shock in traditional M2 shows a sharp decline to approximately -0.015 within the first month, followed by significant fluctuations around zero. This indicates a volatile adjustment process, reflecting the limitations of traditional M2 in capturing the nuanced economic dynamics of different monetary components.

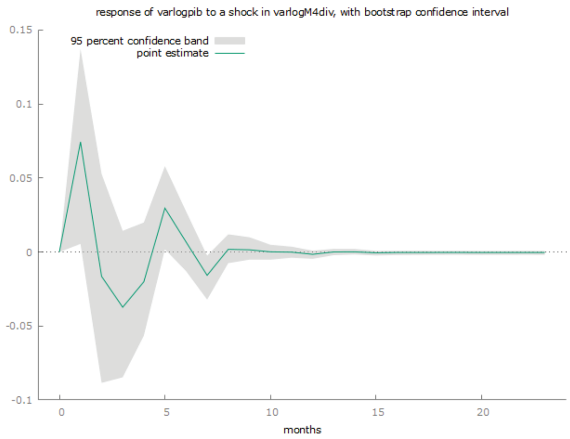
In contrast, the response of real GDP to a shock in Divisia M2, while initially similar with a sharp decline, stabilizes more quickly and with less pronounced fluctuations. The subsequent corrections are more stable, demonstrating that Divisia M2 provides a more predictable and consistent measure of the money supply's impact on GDP. This stability is further emphasized by the narrower confidence bands observed in the IRFs for Divisia M2, indicating a more reliable long-term equilibrium.



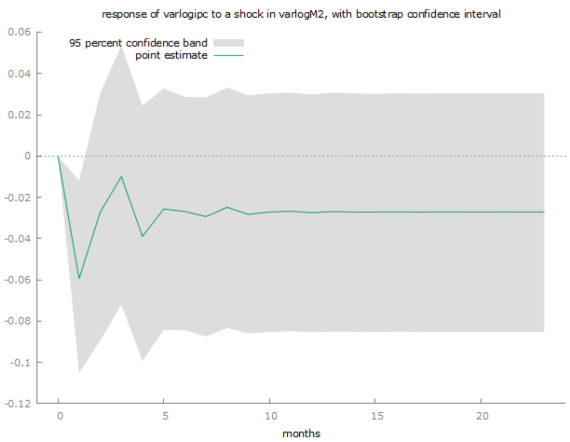


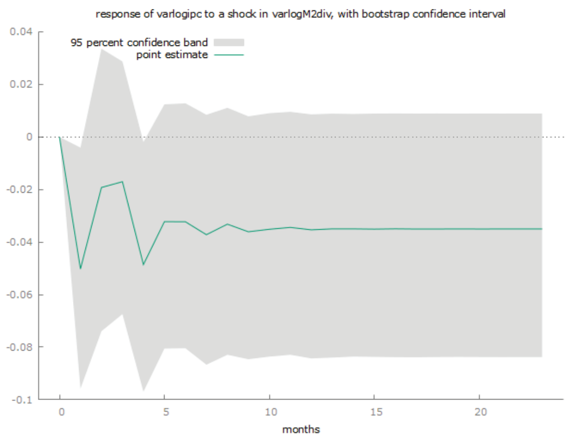
The impulse response functions (IRFs) comparing real GDP's reactions reveals that Divisia M3 shows an initial sharp increase in GDP, peaking around 0.15 in the first month, but this is followed by significant volatility and fluctuations around zero, indicating a less stable adjustment process and highlighting some of it’s limitations. Conversely, traditional M3 also starts with a sharp GDP increase, but the subsequent fluctuations are less pronounced, and the adjustment process isn't so stable, with narrower confidence bands indicating, also a more reliable long-term equilibrium.



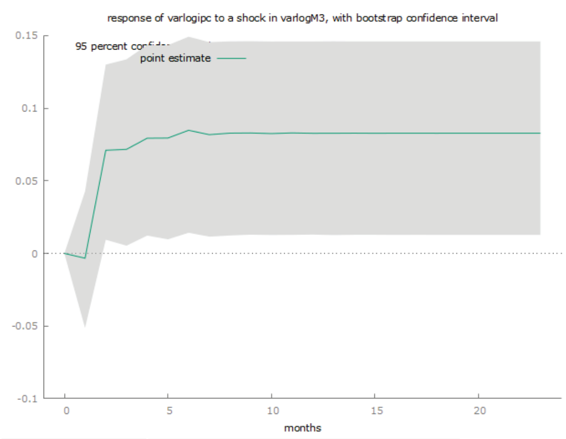


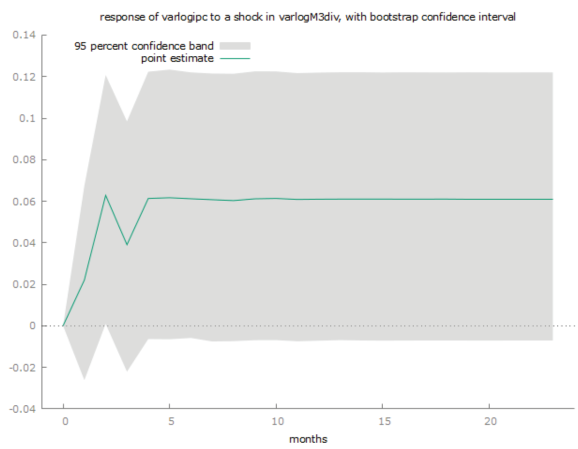
Now for real GDP responses to shocks in traditional M4 and Divisia M4 monetary aggregates illustrate a significant differences in their economic impacts. The initial response to a shock in traditional M4 shows a sharp increase in real GDP, peaking around 0.05 within the first month. However, this response is followed by considerable volatility, with fluctuations around zero, reflecting instability in the adjustment process and highlighting traditional M4's limitations in capturing the economic dynamics effectively. The wide confidence bands further indicate uncertainty and less reliable long-term equilibrium, underscoring the traditional measure's weaknesses.



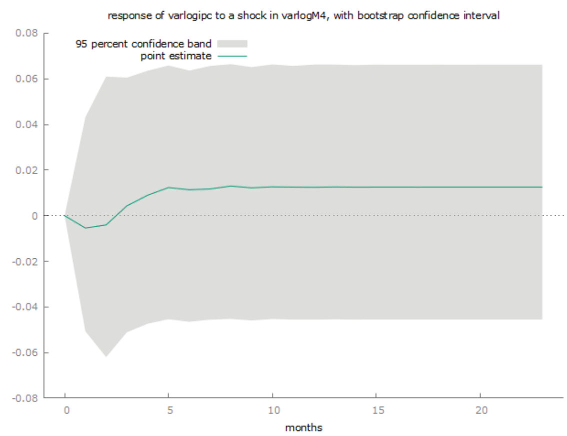


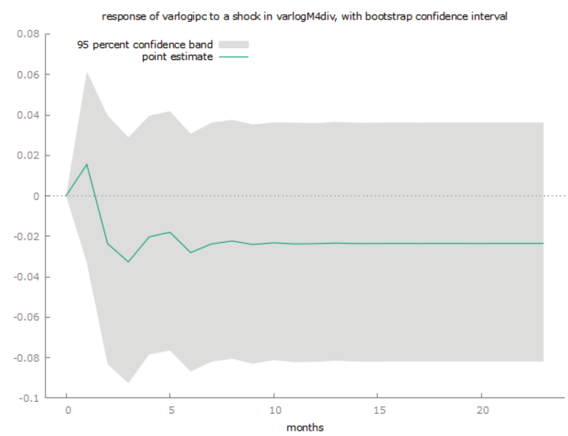
The graph from the right shows the response of CPI to a shock in Divisia M2. Initially, there is a sharp decline in CPI, reaching around -0.01 within the first month. This indicates an immediate deflationary effect following an increase in the money supply. However, this is followed by fluctuations around -0.05, stabilizing by the 6th month with a persistent negative impact. The wide confidence bands suggest some uncertainty in this response, but the general trend indicates a long-term deflationary pressure from Divisia M2 shocks.





Now we can see the response of CPI to a shock in Divisia M3. Similar to Divisia M2, there is an initial sharp decline in CPI, reaching around -0.01 within the first month. The fluctuations in the subsequent months are less pronounced, stabilizing around -0.005 by the 6th month. The confidence bands remain wide, indicating some level of uncertainty, but the response is more stable compared to traditional aggregates. This suggests that Divisia M3 provides a consistent deflationary impact over time.





The impulse response functions (IRFs) for Divisia M4 (varlogM4div) and traditional M4 (varlogM4) provide insights into how real GDP responds to shocks in these monetary aggregates. For Divisia M4, the initial response shows a sharp increase in real GDP, peaking at around 0.015 within the first month. This is followed by significant fluctuations, dropping to about -0.01 before stabilizing around zero by the sixth month. This pattern indicates a positive short-term but negative long-run impact, reflecting the initial boost in economic activity due to increased money supply, consistent with Keynesian economic theory.

In other terms, the response of real GDP to a shock in traditional M4 also exhibits an initial sharp increase, peaking around 0.15 within the first month. The subsequent fluctuations are less pronounced than those observed in Divisia M4, with the response stabilizing around zero much quicker. By the second or third month, the response of GDP to traditional M4 shocks has largely stabilized, indicating a faster adjustment process compared to Divisia M4. This suggests that traditional M4 captures the immediate liquidity effects more directly, although the wider confidence bands indicate some uncertainty.

Comparatively, while traditional M4 stabilizes faster, the Divisia M4 provides a more stable long-term adjustment with narrower confidence bands in the later months. This indicates that Divisia M4, though slower to stabilize initially, offers greater long-term reliability in reflecting the sustained impacts of monetary policy changes. The wider initial fluctuations in Divisia M4 might capture more complex interactions in the economy that traditional M4 does not account for, thus providing a more nuanced view over the long term.

IRFs for both traditional and Divisia monetary aggregates (M2, M3, M4) reveal critical differences in how real GDP and CPI respond to monetary shocks. Traditional M2, M3, and M4 initially exhibit sharp impacts on real GDP, but these are followed by significant fluctuations and broader confidence bands, indicating greater volatility and uncertainty in the adjustment process. Traditional M4 shows a quicker stabilization compared to M2 and M3, but overall, traditional measures exhibit less stability and predictability. In contrast, Divisia M2, M3, and M4 also show initial sharp impacts but with less pronounced fluctuations and narrower confidence bands, indicating a more stable and reliable long-term adjustment. The Divisia measures, particularly M4, provide a more controlled adjustment process, reflecting a more accurate capture of the true economic impacts of monetary policy changes. The observed deflationary trends in CPI following shocks to Divisia aggregates further underscore their nuanced reflection of economic dynamics.

Comparatively, Divisia aggregates demonstrate superior stability and predictive power, making them more suitable for informing monetary policy. While traditional aggregates capture immediate liquidity effects, their greater volatility suggests less reliable long-term outcomes. Divisia measures, with their ability to account for varying liquidity and economic services of different monetary components, offer a more precise and consistent representation of economic responses to monetary changes.

5. Conclusions

This study investigates the impact of traditional simple sum and Divisia monetary aggregates on key economic indicators in Brazil from January 2003 to December 2023. We identified 6 significant cointegrating vectors for both types of aggregates from Johansen Cointegration Test, indicating robust long-term relationships with variables such as real GDP, CPI, and government expenditure. The VECM analysis revealed that Divisia aggregates exhibit stronger and more stable cointegration with economic indicators compared to traditional aggregates, as evidenced by higher trace and maximum eigenvalue statistics. This underscores the superior capability of Divisia measures to capture the true economic value of money, considering the varying liquidity and economic services provided by different monetary components.

The IRFs showed that shocks to government expenditure had significant impacts on real GDP and CPI for both traditional and Divisia aggregates. However, the effects were more stable and pronounced with Divisia aggregates. The adjustment coefficients for real GDP (-0.086558) and CPI (0.012379) in the Divisia model demonstrated a more reliable long-term effect compared to those in the traditional model, indicating enhanced predictive power and stability. These findings suggest that Divisia measures provide a clearer and more accurate picture of economic dynamics, making them more suitable for economic forecasting and policy analysis.

We hypothesize that Divisia aggregates provide more accurate insights into the impacts of monetary policy because they account for the varying liquidity and economic services of different monetary components. Unlike traditional simple sum measures, which treat all components equally, Divisia indexes weight each component based on its liquidity and the economic services it provides. This approach better reflects the true economy by considering how easily different types of money can be converted into goods and services and how they are actually used in economic transactions. For instance, cash is highly liquid and frequently used for daily transactions, whereas certain savings accounts or financial instruments are less liquid and used differently. By capturing these variations, Divisia indexes offer a more nuanced and precise understanding of how changes in monetary policy affect economic indicators. This enables policymakers to make more informed decisions, ultimately contributing to improved economic stability and growth with more accuracy.

Future research should explore the application of Divisia measures in other emerging economies and consider potential structural breaks and non-linear relationships in national monetary data. Additionally, examining the impact of digital currencies and fintech innovations on monetary aggregates could provide valuable insights for such endeavors of current research.

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