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I Mehansko nihanje in valovanje

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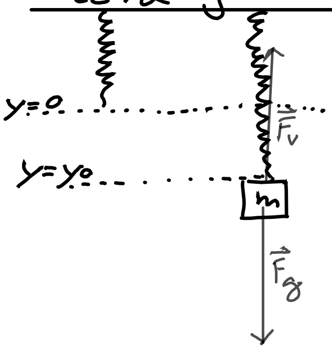
Utež na vijačni vzmeti



$$F_g = \begin{bmatrix} 0 \\ -mg_0 \\ 0 \end{bmatrix}$$ smer navzdol

$g_0 \approx 10 \ m/s^2$

$$F_v = \begin{bmatrix} 0 \\ -ky_0 \\ 0 \end{bmatrix}$$ k... koeficient vzmeti N/m

$y_0 < 0$

$-ky_0 > 0 \Rightarrow$ smer je navzgor

neto sila

$\vec{a} = 0 \Leftrightarrow \vec{F} = m\vec{a} = 0$

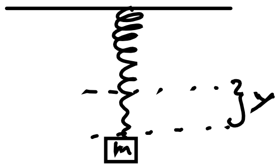
$\vec{F}_g + \vec{F}_v = 0$

$-mg_0 - ky_0 = 0 \Rightarrow$

$mg_0 = -ky_0$

$$y_0 = \frac{mg_0}{k}$$

$\vec{F}_g = -m g_0 \hat{e}_y$



$\vec{F}_v = -k y \hat{e}_y$

$\vec{F}_u$ .... sila upora

$\vec{F}_u = -\vec{c}\,\vec{v}$ (linearna sila upora) $\quad v_y = \dot{y} = \frac{dy}{dt} \neq 0$

$c > 0$ sorazmerna z viskoznostjo tekočine in površino uteži

$\vec{F}_u = -c\dot{y}\hat{e}_y$

$\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$

$\vec{F} = m \cdot \vec{a}$; $\vec{a} = \ddot{y}\,\hat{e}_y$

$$-c\dot{y}\hat{e}_y - k y \hat{e}_y - m g_0 \hat{e}_y = m \ddot{y}\,\hat{e}_y$$

$$(\underbrace{\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m} y + g_0}_{=0})\,\hat{e}_y = 0$$

Vpeljemo $\beta := \frac{c}{m}$ $\quad [\beta] = s^{-1}$

$\omega_0^2 = \frac{k}{m}$ $\quad [\omega_0^2] = s^{-2}$

$$\ddot{y} + \beta\dot{y} + \omega_0^2 y + g_0 = 0$$

$y_0 = \frac{m g_0}{k}$

$g_0 = \frac{y_0 k}{m} = y_0 \omega^2$

$$\ddot{y} + \beta\dot{y} + \omega_0^2 (y - y_0) = 0$$

$$\dot{y} + \beta y + \omega_0^2 y = \omega_0^2 y_0$$

- diferencialna enačba za y 2. reda
- linearna enačba
- konstantni koeficienti
- nehomogena (pogojno, ker jo lahko z lahkoto prevedemo v homogeno)

$y' := y - y_0$

$\dot{y}' = (\dot{y - y_0}) = \dot{y}$

$\ddot{y}' = \ddot{y}$

Dobimo: $\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0$

je homogena :)

$$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y = 0$$

Nastavek: $y' = Ae^{\lambda t}$, A, λ konstanti

$[\lambda] = s^{-1}$

$[A] = m$

$$\dot{y}' = \lambda y$$
$$\ddot{y}' = \lambda^2 y$$

$$(\lambda^2 + \beta\lambda + \omega_0^2)\, \underbrace{Ae^{\lambda t}}_{\neq 0 \text{ ko } A\neq 0} = 0 \quad \text{za } \forall t$$

$$\lambda^2 + \beta\lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 =: -4\omega^2$$

$$(\omega^2 := \omega_0^2 - (\tfrac{\beta}{2})^2)$$

$D < 0 \Rightarrow 4\omega^2 > 0$: podkritično dušenje

$$\sqrt{D} = \sqrt{-4\omega^2} = \sqrt{-1}\, 2\omega \quad ; \omega = +\sqrt{\omega^2}$$

$$\Rightarrow \lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = \frac{-\beta}{2} \pm \omega i$$

$$y_1' = A_1 e^{\lambda_1 t} = A_1 \exp(-\tfrac{\beta}{2} + i\omega)t) =$$
$$= A_1 \exp(-\tfrac{\beta}{2}t) \exp(i\omega t)$$

$$y_2' = A_2 \exp\{-\tfrac{\beta}{2}t) \exp(-i\omega t)$$

$$\left.\begin{array}{l} \ddot{y}_1' + \beta \dot{y}_1' + \omega_0^2 y_1' = 0 \\ \ddot{y}_2' + \beta \dot{y}_2' + \omega_0^2 y' = 0 \end{array}\right\} +$$

$$(\ddot{y}_1' + \ddot{y}_2') + \beta(\dot{y}_1' + \dot{y}_2') + \omega_0(y_1' - y') = 0$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}\left(A_1 \exp(i\omega t) + A_2 \exp\{-i\omega t\}\right)$$

Eulerjeva enačba

$$\exp\{\pm i\omega t\} = \cos\{\omega t\} \pm i \sin(\omega t)$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}\{(A_1 + A_2)\cos(\omega t) + i(A_1 - A_2)\sin\omega t$$

$$= e^{-\frac{\beta}{2}t}\left(B_1 \cos(\omega t) + B_2 \sin(\omega t)\right)$$

$$= B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta)$$

Fazni zamik

$B > 0$; δ = fazni zamik

$$B e^{-\frac{\beta}{2}t}(\sin\omega t \sin\delta + \cos\omega t \sin\delta)$$

$$= e^{-\frac{\beta}{2}t}(B\sin\delta \cos(\omega t) + B\cos\omega t \sin\delta)$$

$$B_1 = B\sin\delta$$

$$B_2 = B\cos\delta$$

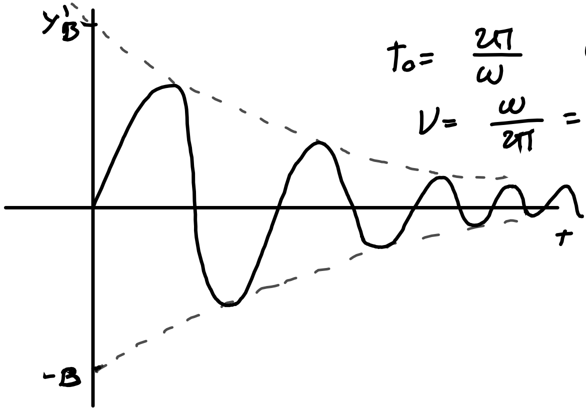
$$\tan\delta = \frac{B_1}{B_2}$$

$$B^2 = B_1^2 + B_2^2$$

$$B = \sqrt{B_1^2 + B_2^2}$$

Primer:

$\delta = 0 \Rightarrow y' = B\, e^{-\frac{\beta}{2}t}\sin(\omega t)$



$t_0 = \frac{2\pi}{\omega}$ $\quad \omega = 2\pi\nu$

$\nu = \frac{\omega}{2\pi} = \frac{1}{t_0}$

$\delta = \frac{\pi}{2}$

$y'(t) = Be^{-\frac{\beta}{2}t}\sin(\omega t + \frac{\pi}{2}) = \sin\omega t\cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2}$

$= Be^{-\frac{\beta}{2}t}\cos(\omega t)$

$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = 0$

$y' = y - y_0$ → odmik obremenjene vzmeti v ravnovesni legi
↳ odmik od konca neobremenjene vzmeti

$\omega_0^2 = \frac{k}{m}$ (>0)

$\beta \propto C \propto \eta$
↑ sorazmerno

Nastavek $y' = A e^{\lambda t}$

$\Longrightarrow \lambda^2 + \beta \lambda + \omega_0^2 = 0$

$D = \beta^2 - 4\omega_0^2 = -4\omega^2$ $\omega^2 = \omega_0^2 - \left(\frac{\beta}{2}\right)^2$

a) $D < 0 \Rightarrow (\omega^2 > 0)$

$y' = B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta)$; $\omega = \sqrt{\omega^2} =$

$= \sqrt{\omega_0^2 - \left(\frac{\beta}{2}\right)^2}$

Zelo šibko dušenje $\left(\frac{\beta}{2}\right)^2 \ll \omega_0 \Rightarrow$

$$t_0 = \frac{2\pi}{\sqrt{\omega_0^2}} = 2\pi\sqrt{\frac{m}{k}}$$

Primer:

$m = 500\,g = 0{,}5\,kg$

$k y_0 = -m g_0$

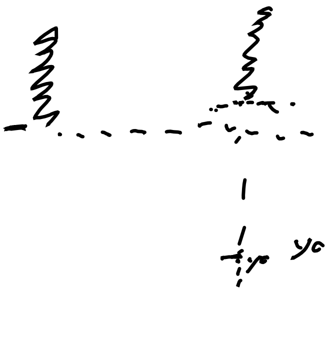
$$k = -\frac{m g_0}{y_0} \Rightarrow \frac{m}{k} = -\frac{y_0}{g_0}$$

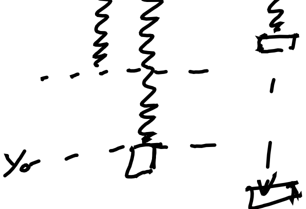
$y_0 = -0{,}4\ m$

$$\frac{m}{k} = \frac{0{,}4\,m}{10\,m/s^2} = 0{,}04\,s^2 =$$

$$\sqrt{\frac{m}{k}} = 2\cdot 10^{-1} = 0{,}2\,s$$

$$\Rightarrow t_0 = 2\pi\cdot 0{,}2\,s \cong 1{,}2\,s$$





B in δ dobimo iz začetnih pogojev

$$y' = Be^{-\frac{\beta}{2}t} \sin(\omega t + \delta)$$

$$\dot{y}' = -\frac{\beta}{2} B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) + \omega B e^{-\frac{\beta}{2}t} \cos(\omega t + \delta)$$

$$\dot{y}'(0) = -\frac{\beta}{2} B \sin\delta + \omega B \cos\delta =$$

$$= B\left(-\frac{\beta}{2}\sin\delta + \omega\cos\delta\right)$$

$$y'(0) = B\sin\delta$$

$$r = \frac{y'(0)}{\dot{y}'(0)} = \frac{\sin\delta}{\omega\cos\delta - \frac{\beta}{2}\sin\delta} = \frac{\tan\delta}{\omega - \frac{\beta}{2}\tan\delta}$$

$$\Rightarrow \delta = \arctan\left(\frac{r\omega}{1 + \frac{r\beta}{2}}\right)$$

$$\Rightarrow B = \frac{y'(0)}{\sin\delta}$$

$$\dot{y}'(0) = B\omega\sin\delta$$

$$\Rightarrow B = \frac{\dot{y}'(0)}{\pm\omega} = \frac{|\dot{y}'(0)|}{\omega}$$

Energija nihala

$$W_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{y}^2$$

$$W_{pr} = \frac{1}{2} k y^2 = \frac{1}{2} k (y' + y_0)^2$$

$$W_p = m g_0 (y' + y_0)$$

Skupna $\boxed{W = W_k + W_p + W_{pr}}$

a) W za zelo šibko dušenje ?

$$\frac{\beta}{2} t \ll 1$$

$$\left(\frac{\beta}{2}\right)^2 \ll \omega_0^2 \Rightarrow \omega \simeq \omega_0 = \sqrt{\frac{k}{m}}$$

$$y' = B \sin(\omega_0 t + \delta)$$

$$\dot{y}' = \omega_0 B \cos(\omega_0 t + \delta)$$

$$W_k = \frac{1}{2} m \omega_0^2 B^2 \cos^2(\omega_0 t + \delta) =$$

$$= \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} B^2 \cos^2(\omega_0 t + \delta)$$

$$W_{pr} = \frac{1}{2} k B^2 \sin^2(\omega_0 t + \delta) + \frac{1}{2} k y_0^2 + k y_0 B \sin(\omega_0 t + \delta)$$

$$W_p = m g_0 B \sin(\omega_0 t + \delta) + m g_0 y_0$$

$$W = \frac{1}{2} k B^2 \underbrace{\left(\sin^2(\omega_0 t + \delta) + \cos^2(\omega_0 t + \delta)\right)}_{1}$$

$$+ (k y_0 + m g_0) B \sin(\omega t + \delta) +$$

$$+ \frac{1}{2} k y_0^2 + m g_0 y_0$$

$$= \frac{1}{2} k B^2 + \frac{1}{2} k y_0^2 + m g_0 y_0 = \text{konst.}$$

b) kritično dušenje

$D=0$ $(\omega=0)$ (kritično dušenje

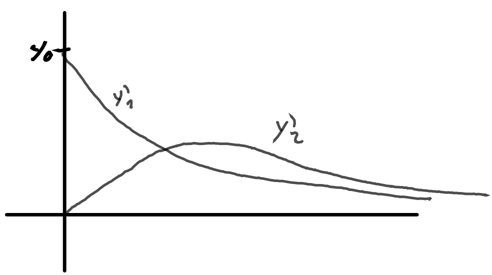
$D=\beta^2-4\omega_0^2 \Rightarrow \omega_0=\frac{\beta}{2}$

$\lambda_{1,2}=\frac{-\beta\pm\sqrt{D}}{2}=-\frac{\beta}{2}$

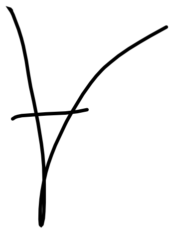
$\Rightarrow y_1'=B_1 e^{-\frac{\beta}{2}t}$

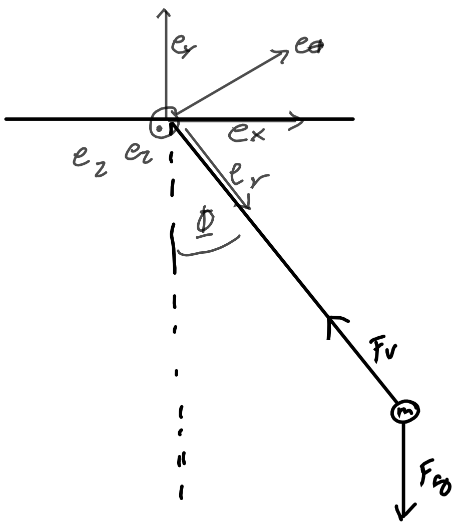
$y_2'=B_2 t e^{-\frac{\beta}{2}t}$ je tudi rešitev (DN)

$\Longrightarrow y'=y_1'+y_2'=(B_1+Bt)e^{-\frac{\beta}{2}t}$



20.2





e_ϕ, e_z, e_r ... cilindrične koordinate

$$\vec{F} = \vec{F}_g + \vec{F}_v = m\vec{a}$$

$$F_g = -m g_0 \hat{e}_y$$

$$F_v = -F_v \hat{e}_r$$

$$\vec{r} = l \cdot \vec{e}_r$$

$$\vec{r} \times \vec{F} = l \vec{e}_r \times (-m g_0 \hat{e}_y - F_v \hat{e}_r) =$$

$$= -m g_0 l \, \hat{e}_r \times \hat{e}_y - \underbrace{l F_v \, \hat{e}_r \times \vec{e}_r}_{=0}$$

$$= -m g_0 \, l \hat{e}_r \times (\hat{e}_r \cos\phi + \hat{e}_\phi \cancel{\cos} \sin\phi)$$

$$= -m g_0 l \sin\Phi \; \hat{e}_r \times \hat{e}_\phi =$$

$$= -m g_0 l \sin\Phi \, \hat{e}_z$$

$$\vec{r} \times \vec{F} = m \cdot \vec{r} \times \vec{a} = m \cdot l \hat{e}_r \times (a \hat{e}_\phi) =$$

$$\vec{a} = a \cdot e_\Phi \qquad = m a l \, \hat{e}_z$$

$$a = a_t = l \ddot{\Phi}$$

$$m g_0 l \sin\phi \, e_z = m l \ddot{\Phi} e_z$$

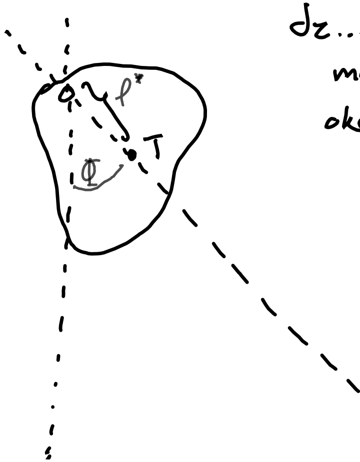
$$m l (l \ddot{\Phi} + g_0 \sin\Phi) \overset{\neq \vec{0}}{e_z} = 0$$

$$-l \ddot{\Phi} = g_0 \sin\Phi$$

$$\ddot{\Phi} + \frac{g_0}{l} \sin\Phi = 0$$

$$\approx \ddot{\Phi} + \underbrace{\frac{g_0}{l}}_{\omega_0^2} \Phi = 0$$

$$\ddot{\Phi} + \omega_0^2 \Phi = 0$$



J_z .... vztrajnostni moment za vrtenje okrog fiksne osi

primer: palica

$J_z = \frac{1}{3} m l^2 \quad l^* = \frac{l}{2}$

$\sin\Phi \approx \Phi$

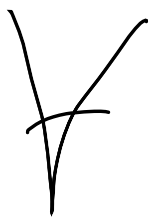
$\Rightarrow \ddot{\Phi} + \omega_0^2 \Phi = 0$

$$\omega_0^2 = \frac{m g l^*}{J_z} = \frac{3 m g l}{2 m l^2} = \frac{3}{2}\frac{g_0}{l}$$

Nitno nihalo:

$J_z = m l^2 \quad l^* = l$

$$\omega_0^2 = \frac{m g_0 l}{m l^2} = \frac{g_0}{l}$$



21.2

Nism të shkele

$F = F_0 \sin(\omega_v t) \qquad \omega_v = 2\pi \nu_v$

$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = \frac{F_0}{m} \sin(\omega_v t)$

$\omega_0^2 = \frac{k}{m} \qquad \beta = \frac{c}{m} \qquad y' = y - y_0$ raztezek ko je obtežena vzmet mirna

$y' = y'_h + y'_p$ ← partikularna rešitev

homogena

$y'_p = B \sin(\omega t - \delta_p)$

$B_p \{ (\omega_0^2 - \omega_v^2)[\cos\delta_p \sin(\omega_v t) - \sin\delta_p \sin(\omega_v t)]$

$+ \omega_v \beta [\cos\delta_p \cos(\omega_v t) + \sin\delta_p \sin(\omega_v t)] \}$

$= \frac{F_0}{m} \sin(\omega_v t) \qquad$ za $\forall t$

a) $t_1 = 0 \Rightarrow \sin\omega_v t = 0, \cos\omega_v t = 1$

$B_p \{ -(\omega_0^2 - \omega_v^2)\sin\delta_p + \omega_v \beta \cos\delta_p \} = 0$

$\Rightarrow \tan\delta_p = \frac{\omega_v \beta}{\omega_0^2 - \omega_v^2}$

$\cos\delta_p = \pm \frac{(\omega_0^2 - \omega_v^2)}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$

$\sin\delta_p = \pm \frac{\omega_v \beta}{\sqrt{\quad}}$

$\omega_v \to 0 \Rightarrow \tan\delta_p \to +0 \Rightarrow \delta_p \to 0$

$\omega_v \to \uparrow \omega_0 \Rightarrow \tan\delta_p \to +\infty \Rightarrow \delta_p \to \uparrow \frac{\pi}{2}$

$\omega_v \to \downarrow \omega_0 \Rightarrow \tan\delta_p \to -\infty \Rightarrow \delta_p \to \downarrow \frac{\pi}{2}$

$\omega_v \to \infty \Rightarrow \tan\delta_p \to -0 \Rightarrow \delta_p \to \pi$

$\omega_v \to 0 \Rightarrow \delta_p \to 0 \Rightarrow \cos\delta_p \to +1$

$\cos\delta_p \to \frac{\pm \omega_0^2}{\sqrt{\omega_0^4}} = \pm 1$ očitno mora biti +

b) $t_2 = \frac{\pi}{2\omega_v} \Rightarrow \omega_v t_2 = \frac{\pi}{2} \Rightarrow \sin(\omega_v t) = 1$

$\cos(\omega_v t) = 0$

$B_p \{ (\omega_0^2 - \omega_v^2)\cos\delta_p + \omega_v \beta \sin\delta_p \} = \frac{F_0}{m}$

$B_p \{ \frac{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta}} \} = \frac{F_0}{m}$

$\Rightarrow B_p = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$

$\omega_v \to 0 \Rightarrow B_p \to \frac{F_0}{m} \cdot \frac{1}{\omega_0^2} = \frac{F_0 \cdot m}{m \cdot k} = \frac{F_0}{k}$

$\omega_v \to \infty \Rightarrow B_p \to 0$

$B_p(\omega_v) = \max$

Kdaj doseže maksimum ?

Torej kdaj je v resonanci ?

Ko je imenovalec najmanjši

$(\omega_0^2-\omega_v^2)^2+\beta^2\omega_v^2$ je najmanjši

$$\frac{d}{d\omega_v}\left((\omega_0^2-\omega_v^2)^2+\beta^2\omega_v^2\right)=0$$

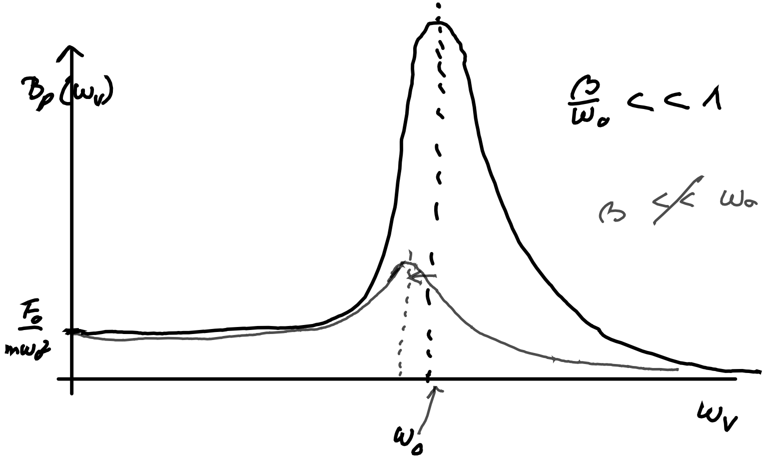
$$2(\omega_0^2-\omega_v^2)(-2\omega_v)+2\omega_v\beta^2=0$$

$$-2(\omega_0^2-\omega_v)+\beta^2=0$$

$$\omega_v^2=\omega_0^2-\frac{\beta^2}{2}$$

$$\omega_v=+\sqrt{\omega_0^2-\frac{\beta^2}{2}}=\omega_0\sqrt{1-\frac{\beta^2}{2\omega_0^2}}$$

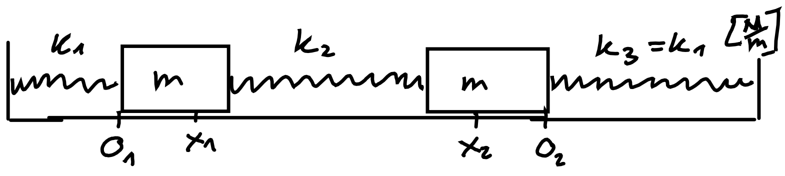
$$\beta<<\omega_0 \Rightarrow \omega_v=\omega_0$$



$B_p(\omega_v)$
$\frac{F_0}{m\omega_0^2}$
$\frac{\beta}{\omega_0} << 1$
$\beta \not\ll \omega_0$
ω_0
ω_v

Sklopljeno nihanje

- $\beta \to 0$ dušenje pošljemo proti 0
- za začetek: simetrični primer



Iz drug vzmetjo, ki ji bomo rekli: tretja vzmet

$\vec F_{1\to1,r}$ ... sila prve vzmeti na prvi voziček ko je v ravnovesni legi

$$\left.\begin{aligned}\vec F_{1\to1,r} &= -k_1\Delta l_{1,r}\hat e_x\\ \vec F_{2\to1,r} &= +k_2\Delta l_{2,r}\hat e_x\end{aligned}\right\} + = 0$$

v ravnovesju

$$\left.\begin{aligned}F_{2\to2,r} &= -k_2\Delta l_{2,r}\hat e_x\\ F_{3\to2,r} &= +\underset{k_3}{k_1}\Delta l_{3,r}\hat e_x\end{aligned}\right\} + = 0$$

$$\vec F_{1\to1} = \vec F_{1\to1,r} - k_1 x_1\hat e_x$$
$$\vec F_{2\to1} = \vec F_{2\to1,r} - k_2(x_1-x_2)\hat e_x$$
$$\vec F_{2\to2} = \vec F_{2\to2,r} + k_2(x_1-x_2)\hat e_x$$
$$\vec F_{3\to2} = \vec F_{3\to2,r} - k_1 x_2\hat e_x$$
$$\vec F_{1\to1} + \vec F_{2\to1} = m\ddot x_1\hat e_x$$
$$\cancel{F_{1\to1,r}} - k_1 x_1\hat e_x + \cancel{F_{2\to1,r}} - k_2(x_1-x_2)\hat e_x = \\ = m\ddot x_1\hat e_x$$
$$m\ddot x_1 + k_1 x_1 + k_2(x_1-x_2) = 0$$
$$\ddot x_1 + \omega_1^2 x_1 + \omega_2^2(x_1-x_2) = 0$$
$$\omega_1^2 = \frac{k_1}{m} \qquad \omega_2^2 = \frac{k_2}{m}$$

Zelo podobno $\ddot x_2 + \omega_1^2 x_2 - \omega_2^2(x_1-x_2) =$

$$\left.\begin{aligned}x_a &= x_1 + x_2\\ x_b &= x_1 - x_2\end{aligned}\right\} \Rightarrow x_1 = \frac{x_a+x_b}{2}$$
$$x_2 = \frac{x_a-x_b}{2}$$

seštejemo enačbi:

$$\ddot x_1 + \ddot x_2 + \omega_1^2 x_1 + \omega_1^2 x_2 = 0$$
$$(\ddot{x_1+x_2}) + \omega_1^2(x_1+x_2) = 0$$
$$\ddot x_a + \omega_a^2 x_a = 0 \qquad \omega_a = \omega_1$$
$$\ddot x_b + \omega_b^2 x_b = 0 \qquad \omega_b = \sqrt{\omega_1^2 + 2\omega_2^2}$$
$$x_a = B_a\sin(\omega_a t + \delta_a)$$
$$x_b = B_B\sin(\omega_b t + \delta_b)$$
$$\Rightarrow x_1 = \underbrace{\frac{B_a}{2}}_{B_1}\sin(\omega_a t + \delta_a) + \underbrace{\frac{B_b}{2}}_{B_2}\sin(\omega_b t + \delta_b)$$
$$x_2 = B_1\sin(\omega_a t + \delta_a) - B_2\sin(\omega_b t + \delta_b)$$
$$B_1, B_2, \delta_a, \delta_b = ?$$

začetni pogoji: $x_1(t=0)$ $\dot x_1(t=0)$
$x_2(t=0)$ $\dot x_2(t=0)$

je treba izvesti

Primer

$x_1(t=0) = x_0 \ (>0)$ $\qquad \dot{x}_1(t=0) = 0$

$x_2(t=0) = 0$ $\qquad \dot{x}_2(t=0) = 0$

$$\dot{x}_1 = B_1 \omega_a \cos(\omega_a t + \delta_a) + B_2 \omega_b \cos(\omega_b t + \delta_b)$$

$$\dot{x}_2 = B_1 \omega_a \cos(\omega_a t + \delta_a) - B_2 \omega_b \cos(\omega_b t + \delta_b)$$

$$\left.\begin{aligned} x_0 &= B_1 \sin\delta_a + B_2 \sin(\delta_b) \\ 0 &= B_1 \sin\delta_a - B_2 \sin\delta_b \end{aligned}\right\} +, -$$

$$\left.\begin{aligned} 0 &= B_1 \omega_a \cos(\delta_a) + B_2 \omega_b \cos(\delta_b) \\ 0 &= B_1 \omega_a \cos(\delta_a) - B_2 \omega_b \cos(\delta_b) \end{aligned}\right\} +, -$$

$$x_0 = 2B_1 \sin\delta_a = 2B_2 \sin\delta_B$$

$$2B_1 \omega_a \cos\delta_a = 0 \Rightarrow \cos\delta_a = 0$$

$$\Rightarrow B_1 = B_2 = \frac{x_0}{2} \qquad \Rightarrow \begin{aligned} \delta_a &= \pm\frac{\pi}{2} \\ \delta_b &= +\frac{\pi}{2} \end{aligned}$$

7.3

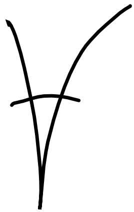
mentalna odsotna

Lastna nihanja sestavljenega nihala

$$x_1 = B\sin(\omega_a t + \delta_a) + B_2 \sin(\omega_b t + \delta_b)$$
$$x_2 = \ldots$$

$$\begin{cases} x_1 = B_1 \sin(\omega_c t + \delta_1) \\ x_2 = B_2 \sin(\omega_c t + \delta_2) \end{cases}$$

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0$$
$$\ddot{x}_2 + \omega_2^2 x_2 - \omega_2^2 (x_1 - x_2) = 0$$



11.3

$u(x,t) = ?$

$$u(x,t) = f(x-ct) + g(x+ct)$$

$\eta := x - ct$

$\chi := x + ct$

ena izmed možnosti:

$$u(x,t) = f(x-ct)$$

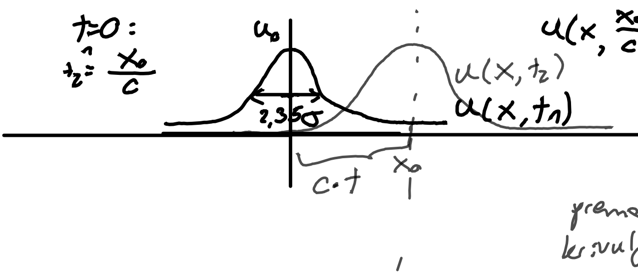
npr: $u(x,t) = u_0 e^{-\frac{(x-ct)^2}{2\sigma^2}}$

$$u(x,0) = u_0 e^{-\frac{x^2}{2\sigma^2}}$$

$$u\left(x, \frac{x_0}{c}\right) = u_0 e^{-\frac{(x-x_0)^2}{2\sigma}}$$

$t_1 = 0$:

$t_2 = \frac{x_0}{c}$



sama premaknjena krivulja

$$\Rightarrow v = \left|\frac{du}{dt}\right| = \left|\frac{\partial u}{\partial t}\right| = \left|\frac{\partial}{\partial t} f(x-ct)\right| = |f'| c$$

$$v \neq c$$

$u(x,t) = f(x-ct)$

a) $\underline{u(x,t) = u(x-ct, 0)}$

$x \longrightarrow x' = x - ct$

$t \rightarrow t' = 0$

$u(x',t') = u(x-ct, 0) = f(x', ct') = f(x-ct) =$

$= u(x,t)$

Če poznamo $u(x',t')$ ob času $t'=0$ za $\forall x'$
Potem poznam $u(x,t)$ za $\forall x$ po $\forall$ času t

b) $u(x,t) = u(0, t - \frac{x}{c})$

$x \rightarrow x' = 0$

$t \rightarrow t' = t - \frac{x}{c}$

$u(x',t') = u(0, t - \frac{x}{c}) =$

$f(0 - c(t - \frac{x}{c})) = f(x-ct) = u(x,t)$

Če poznamo $u(x'=0, t')$ za $\forall t'$
$\Rightarrow$ poznam $u(x,t)$ za $\forall x, \forall t$

Potujoče sinusno valovanje

$u(x,t) = f(x-ct)$

$u(x=0,\ t-\frac{x}{c}) = u_0 \sin(-\omega t + \delta)$; $\quad u_0 > 0$

$u(x,t) = ?$

$$u(x,t) = u(0, t-\frac{x}{c}) = u_0 \sin(-\omega(t-\frac{x}{c})+\delta) =$$

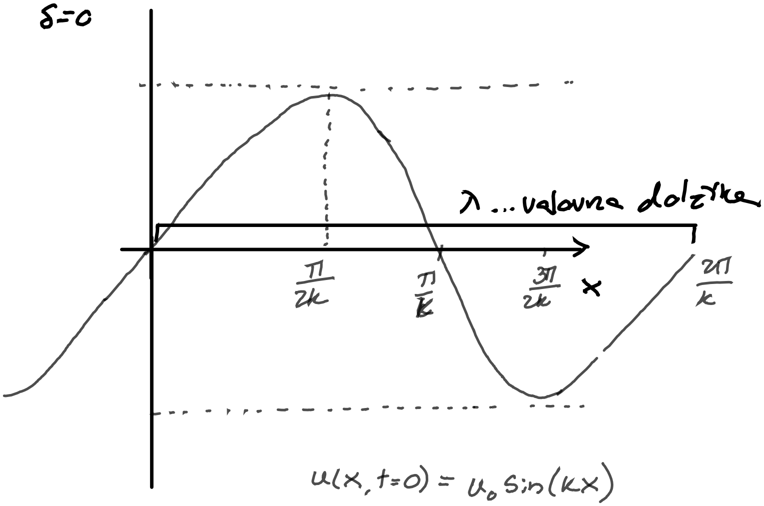
$$= u_0 \sin(kx - \omega t + \delta)\ ; \quad k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$$

$$[k] = \frac{1}{m}$$

$t=0$:

$u(x, t=0)$

$\delta = 0$



$u(x, t=0) = u_0 \sin(kx)$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{2\pi\nu} = \frac{c}{\nu}$$

$$c = \lambda\nu$$

$$u(x,t) = u_{0,1} \sin(kx - \omega t + \delta_1) + u_{02} \sin(kx + \omega t + \delta_2)$$

Valovanje v dve smeri:

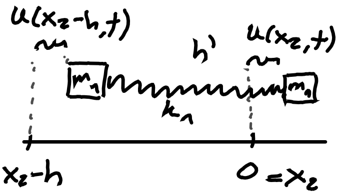
Robni pogoji in stoječe sinusno valovanje

a) togo vpeta vzmet

$u(x_1,t)=u(x_2,t)=0$

b) Vzmet (palica) s prostima koncema, x_1, x_2

Model:



$h'-h = u(x_2)-u(x_2-h)$

$F_{x_2-h \to x_2} = -k_1 \Delta h$

$= -\frac{k\ell}{h}\left(u(x_2,t)-u(x_2-h,t)\right)$

$m_1 \ddot{u}(x_2,t) = -k\ell\left(\frac{u(x_2,t)-u(x_2-h,t)}{h}\right)$

$m_1 = \frac{m}{\ell h}$ ← št vseh uteži

$\frac{m}{\ell h}\frac{\partial^2 u(x_2,t)}{\partial t^2} = -k\ell\left(\frac{u((x_2'+h),t)-u(x^2,t)}{h}\right)$

$x_2 = x_2' + h$

$n \to \infty \Leftrightarrow h \to 0 \quad \Rightarrow \quad h\ddot{u} \to 0$

$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x^2-h} \overset{h\to 0}{=} \left.\frac{\partial u(x,t)}{\partial x}\right|_{x^2}$

$\left.\frac{du(x,t)}{\partial x}\right|_{x=x_2} = \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=1} = 0$