Formule

\# permutacij $= |\{\sigma : A \xrightarrow{\sim} A\}| = n!$

$B \subseteq A \Rightarrow |A - B| = |A| - |B|$

$|A \times B| = |A||B|$

Št neurejenih izborov podmnožice k od $n = \binom{n}{k}$

multinomski: n_1 tipa 1 n_2 tipa 2, ... n_k tipa k

\# permutacij neurejeno $= \dfrac{n!}{n_1! \cdots n_k!}$

① $J = \{$ juha, solata, riba, zelenjava, sladoled, torta $\}$

$\underbrace{\text{juha, solata}}_{\text{predjed}}$, $\underbrace{\text{riba, zelenjava}}_{\text{glavna jed}}$ (meso), $\underbrace{\text{sladoled, torta}}_{\text{sladica}}$

$2 \cdot 3 \cdot 2$ je št možnih kosil

$= 12$

② kup kart 52 kart. karte razdelimo na 4 igralce vsak dobi 13

1. kolko načinov lahko karte tako razdelimo
2. kolko da ima vsak igralec karte samo enega tipa (pik, srce, križ, kara)

$$4! \cdot \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} =$$

$$= \frac{52! \; (52-13)! \cdots}{13!\,(52-13)!\; 13! \cdots \; 13} = \frac{52!}{(13!)^4} 4!$$

2. $1, \checkmark\ 4!$

③ Kostanjevica na Krki ima 702 prebiv.

Dokaži: Najmanj dva prebivalca imata enaki začetnici (25 črk)

$$\underline{25\cdot\overset{2}{25}} = 625$$

50

$\underline{125}$

623

$702-625>0$

④ imamo 10 kart

2 rdeči, 3 zelene, 5 modrih

1). # možnosti da je prva rdeča izvlečena pred prvo zeleno

__ __ __ __ __ __ __ __ __ __

1. 0
2. $\frac{8!}{1!2!5!}$
3. $\frac{7!}{2!5!} + 2\cdot\frac{7!}{4!1!2!}$
4.

_ R _ Z _ Z _ Z _

__ __ __ __ __ __ __ __

$$\frac{\binom{10}{4}\cdot 6}{2} = \frac{10\cdot\cancel{8}^{2}\cdot 9\cdot 7}{\cancel{2}\cancel{3}\cancel{4}\cdot 2}\cdot\cancel{6} = \frac{1260}{2} = 630$$

modre

$$\binom{10}{5}4 = \frac{10!}{5!5!}4 = \frac{10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2}\cdot 4 =$$

$=$

ii) možnosti da bo prva rdeča kr. izvlečena pred zadnjo zeleno

$$\binom{10}{5}\cdot\binom{4}{2} \qquad \frac{4\cdot 3}{2} = 6 \quad \_\ \_\ \_\ \_\ \_$$

zelena

$$\binom{10}{5}6+\binom{10}{5}3$$

$$\frac{10!}{3!5!2!} - \binom{10}{5} = \frac{10!}{5!}\left(\frac{1}{3!2!}-\frac{1}{5!}\right) =$$

$$= \frac{10!}{5!}\left(\frac{5!-\overset{3!2}{12}}{5!2!3!}\right) =$$

$$= \frac{10!}{5!}\,\frac{4\cdot 5-2}{5!2!} = \frac{10!}{5!5!}\,\frac{18}{2}$$

$$= \binom{10}{5}9$$

13.10

1) 3 rdeče
1 zelena
5 belih

izberemo 2

i) $\Omega = ?$

$$\Omega = \binom{V}{2} \qquad V = \{r, r, r, z, b, b, b, b, b\}$$

$$\Omega = \{(r,r), (r,b), (r,z), (z,b), (z,r), (b,b) (b,r), (b,z)\}$$

ii) najprej rdeča: $\frac{3}{9} = \frac{1}{3}$

iii) $R_1 = \frac{1}{3}$ druga zelena: $\frac{8}{9} \cdot \frac{1}{8} = \frac{1}{9}$

$$P(R_1 \cup Z_2) = P(R_1) + P(Z_2) - P(R_1 \cap Z_2)$$

$$P(R_1 \cap Z_2): \frac{3}{9} \cdot \frac{1}{8}$$

$$P(\dots) = \frac{4}{9} - \frac{3}{8 \cdot 9} = \frac{29}{8 \cdot 9}$$

2) 2 orn: n bílých

i) $\bar{C}_{j,k} = \{$nejtem in ktem kroku zvolena koule$\}$

$P(\bar{C}_{j,k}) = ?$ funkce j,k,n

$$\bar{C}_{1,k} = \left(\frac{n}{n+2}\right)^{j-1} \frac{2}{n+2} \left(\frac{n+1}{n+2}\right)^{k-j-1} \left(\frac{1}{n+2}\right) =$$

$$= \frac{2 \cdot n^{j-1}}{(n+2)^{j+k-j-1+1}} (n+1)^{k-j-1} = 2n^{j-1} \frac{(n+1)^{k-j-1}}{(n+2)^k}$$

$X = k$

$$P(x,k) = P\left(\bigcup_{i=1}^{k-1} \bar{C}_{i,k}\right) = \sum_{i=1}^{k-1} P(\bar{C}_{i,k}) = \frac{2}{(n+2)^k} \sum_{k=1}^{k} n^{j-1} (n+1)^{k-j-1}$$

$$n^{j-1} (n+1)^{k-j-1} = \frac{n^j}{n(n+1)^j} (n+1)^{k-1} = \left(\frac{n}{n+1}\right)^j \frac{(n+1)^{k-1}}{n}$$

$$= \frac{2(n+1)^{k-1}}{n(n+2)^k} \frac{1-\left(\frac{n}{n+1}\right)^k}{1-\frac{n}{n+1}} = \frac{2((n+1)^k - n^k)}{n(n+2)^k}$$

ii) $P(\bar{C}_1 | X = k)$

$$P\left(\bigcup_{i=2}^{k} \bar{C}_{i,k}\right) = P(x=k) - P(\bar{C}_{1,k}) = \frac{2(n+1)^{k-1} - n^{k-1})}{(n+2)^k} -$$

$$2 \frac{(n+1)^k}{(n+2)^k} =$$

$$= \frac{2}{n(n+2)^k}\left((n+1)^k (n+1) - n^k\right) = \frac{1}{n(n+2)^k}(n+1)^{k+1} - n^k)$$

cz: $\frac{2}{\cancel{(n+2)}^k}\left((n+1)^{k-1} \cancel{(n+2)}^k - n^{k-1}\right) =$

$$2(n+1)^{k-1} - \frac{n^{k-1}}{(n+2)^k}$$

3)

K ... konzervativec $\rightsquigarrow P(A|K) = 1$

L ... levičar $\rightsquigarrow P(A|L) = r$

$r=0 \Rightarrow P(A)=1$

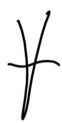
$$P(A) = P(A|K)P(K) + P(A|L)P(L) =$$
$$1 \cdot p + (1-p)(1-r)$$

$$P(V_3|V_2) = \frac{P(V_3 \cap V_2)}{P(V_2)} = \frac{P(V_3)}{P(V_2)}$$

$$P(V_2) = P(V_2|C)P(C) + P(V_2|L)P(L) =$$
$$= 1 \cdot p + (1-p)(1-r)$$

$$P(V_3) = P(V_3|C)p + P(V_3|L)(1-p) =$$
$$1 \cdot p + (1-r)^2(1-p)$$

$$\Rightarrow P(V_3|V_2) = \frac{p + (1+r)^2(1-p)}{p + (1+r)(1-p)}$$



20.10

1) Mečemo kovanec, dokler ne pade najmanj eno štiriko in en grb

$P(\{grb\}) = p \quad 0 < p < 1$

Naj bo $\{X = k\} = \{$ prvič pade najmanj 1 štirika in 1 grb na koraku $k\}$

Meti so sobaj neodvisni

Računajte $P(X \geq k)$

$$P(X \geq k) = \sum_{k=n}^{\infty} P(X=n) = \sum_{\cancel{k=1}\; n=k}^{\infty} \left(p^{n-1}(1-p) + (1-p)^{n-1} p \right) =$$

$$= \frac{(1-p)}{p^k} \sum_{k=0}^{\infty} p^n + \frac{p}{(1-p)^k} \sum_{n=0}^{\infty} (1-p)^n =$$

$$\cancel{\frac{(1-p)}{p^{k-1}}} \frac{1}{\cancel{1-p}} + \cancel{\frac{p}{(1-p)^{k-1}}} \frac{1}{\cancel{1-1+p}} =$$

$$= \frac{1}{p^{k-1}} + \frac{1}{(1-p)^{k-1}}$$

v prvih $n-1$ metih ~~...~~ je samo isto

$\cancel{p(k=)}\; p^{k-1} + (1-p)^{k-1}$

$p = 0 \vee p = 1 \Rightarrow \quad p(X > k) = 0$

$$= \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1}(1-p^{(k-1)})} = \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1} - p^{2k-1}} \quad ???$$

2)

Mečemo kovanec dokler ne dobimo ali

m grbov ali na števil

max metov: m+n min metov: min{m,n}

$P(\{grb\}) = p$

$P(\{X = k\}) =$

$\{X = k\}$ = prič ko dobimo ali m grbov ali n števil

$P(\{X = \text{grb pri } k \text{ ali } m \text{ grbov}\}) = p^m (p-1)^{k-m}$

$+ p^{k-m} (p-1)^m$

~~k metov~~ m jih je neke vrste : $k < 2m$

$p^m (p-1)^{k-m} + p^{k-m} (p-1)^m$

$P(X = \{\ \}) =$

$P(X = \{k-1$ tem mestu neke $m-1$ grbov$\})$

$\{G = k\}$ = { pade $m-1$ grbov v $k-1$ kosadih,
potem pa spet pade grb }

$\{\check{S} = k\}$ = { Na $k-1$ kovanca pade $m-1$ števil potem
pa spet število }

$$\binom{k-1}{m-1}\left(p^m(1-p)^{k-m} + p^{k-m}(1-p)^m\right)$$

3)

$X \sim \mathrm{Bin}(n, \underset{p}{\underset{\shortparallel}{\tfrac{1}{2}}})$

$$P(X=k) = \binom{n}{k}\left(\frac{1}{2}\right)^k\left(1-\frac{1}{2}\right)^{n-k} = \binom{n}{k}\frac{1}{2^n}$$

Definiramo slučajni spremenljivki Y za $k=0,\dots,n$ velja $P(Z) \sim P(X,Y)$

$$P(X=k, Y=k+1) = P(X=k)\frac{n-k}{n} \qquad (a)$$

in $P(X=k, Y=k-1) = P(X=k)\frac{k}{n}$ (b)

in $P(X=k, Y=l) = 0$ če $|k-l|>1$ ali $l=k$

Računaj te porazdelitev slučajne spremenljivke Y

Rešitev:

Popolna vrjetnost

$$P(Y=\cancel{l}) = \sum_{k=0}^{n} P(X=k, Y=\cancel{l}) =$$

$$= P(X=l-1, Y=l) + P(X+l, \cancel{Y} = \cancel{l})$$

$$P(Y=l) = \binom{n}{l-1}\frac{1}{2^n}\frac{n-l+1}{n} + \binom{n}{l+1}\frac{1}{2^n}\frac{l+1}{n} =$$

$$= \frac{1}{2^n}\left(\frac{n!}{(l-1)!\,(n-(\cancel{n}l-1))!}\cdot\frac{n-(l-1)}{n} + \frac{n!}{(l+1)!\,(n-(l+1))!}\cdot\frac{l+1}{n}\right)$$

$$= \frac{1}{2^n}\left(\binom{n-1}{l-1} + \binom{n-1}{l}\right) = \frac{1}{2^n}\binom{n}{l}$$

4)

Naj bodo $X \sim B(\frac{1}{2}) \Rightarrow X=0$ z verjetnostjo $\frac{1}{2}$ in
$X=1$ z verjetnostjo $\frac{1}{2}$

$Y \sim B(\frac{1}{2})$

$Z = Z(X,Y)$

$P(Z) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$ "$\in \mathbb{R}^4$"

$P(X=1, Y=0)$

Velja: $*\ 0 \leq p_{ij} \leq 1$

$*\ \sum_{0 \leq i,j \leq 1} p_{ij} = 1$

Recimo da $P(Z) = \begin{pmatrix} \frac{1}{4}+\delta & \frac{1}{4}-\delta \\ \frac{1}{4}-\delta & \frac{1}{4}+\delta \end{pmatrix}$

kdaj sta X in Y neodvisno od δ

$-\frac{1}{4} \leq \delta \leq \frac{1}{4}$

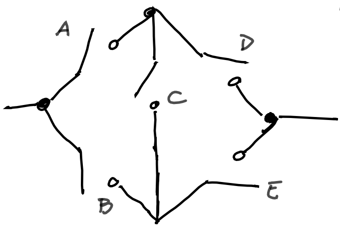
neodvisno

Dejstvo: $X \perp Y \Leftrightarrow \det(p_{ij}) = 0$

$\det(p_{ij}) = \left((\frac{1}{4}-\delta)^2 - (\frac{1}{4}-\delta)^2 \right) = \delta$

$\Rightarrow X \perp Y$ če je $\delta = 0$

5) V vezju



Vsako stikalo prepušča el. tok z verjetnostjo $\frac{1}{3}$ in posamezna stikala so med seboj neodvisna

A = dogodek da tok gre preko A

Kolikšna je verjetnost da vezje prepušča?

A

$$\frac{1}{3}\left(\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3}\right)+\frac{1}{3}\left(\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3}\right) \quad ??$$

D C E B E C D

A

$$\frac{1}{3}\left(\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3}-\frac{1}{3}\cdot\frac{1}{3}\right)+\frac{1}{3}\cdot\frac{1}{3}-\underbrace{\frac{1}{3}\cdot\frac{1}{3}}_{\text{odštejem}}=\frac{1}{9} \quad ??$$

A D

$$\frac{2}{3}\left(\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3}\right)+\frac{1}{3}(\dots)$$

↑

zaprto