

Nihanje

$$V = \frac{1}{T_0} \quad \omega = 2\pi V \quad T_0 = \frac{2\pi}{\omega}$$

$$\sum \vec{F} = m\vec{a} \quad \sum M = J\alpha$$

Enačba: $\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = 0$ $\beta \dots$ koeficient dušenja

$$D = 4\beta^2 - 4\omega_0^2$$

$\omega_0 \dots$ ko zna
frekvenca

$D < 0 \dots$ podkritično dušenje

$$y = B e^{-\beta t} \sin(\omega t + \delta)$$

$$\omega^2 = \omega_0^2 - \beta^2$$

$D = 0 \dots$ kritično dušenje

$$y = (B_1 + B_2 t) e^{-\beta t}$$

$D > 0 \dots$ nadkritično dušenje

$$y = B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t}$$

kjer sta λ_1, λ_2 rešitvi: $\lambda^2 + 2\beta\lambda + \omega_0^2 = 0$

$$\lambda_{1,2} = -\beta \pm i\omega \quad ; \quad \omega^2 = \omega_0^2 - \beta^2$$

Vijarčna vzmet (linearni zakon upora)

$$F_u = C \cdot \dot{y} \dots \text{sila upora}$$

$$\rho = \frac{C}{2m} \quad \omega_0^2 = \frac{k}{m}$$

Matematično nihalo (ni dušenja)

$$\ddot{\varphi} + \frac{g}{l} \varphi = 0$$

$$\rho \ll 1 \Rightarrow \omega_0^2 = \frac{g}{l}$$

Fizično nihalo (ni dušenja)

$$\omega_0^2 = \frac{mgl^*}{J_z}$$

$J_z \dots$ vztrajnostni
moment okoli osi
vrtenja

Greenove funkcije

Nihalo zbujamo s silo $F(t)$

$$x(t) = \int_0^t \frac{F(J)}{m\omega} e^{-\rho(t-J)} \sin(\omega t - \omega J) dJ$$

Vsiljeno nihanje (sinusno)

$$\ddot{y} + 2\beta y + \omega_0^2 y = A_0 \sin(\omega_v t)$$

\uparrow \uparrow frekvenca
amplituda vsiljevanja

Rezultat: $y = y_h + y_p$
 \uparrow \uparrow partikularni del
homogeni del

$$y = B e^{-\alpha t} \sin(\omega t + \delta) \quad (\omega^2 = \omega_0^2 - \beta^2)$$

\rightarrow nastavek $B_p \sin(\omega_v t - \delta_p)$

$$\tan \delta_p = \frac{2\omega_v \beta}{\omega_0^2 - \omega_v^2}$$

$$B_p = \frac{A_0}{\omega_v \sqrt{\frac{(\omega_0^2 - \omega_v^2)^2}{\omega_v^2} + 4\beta^2}}$$

B_p je funkcija ω_v . Ko je B_p maksimalen

$$\omega_m = \omega_0 \sqrt{1 - \frac{2\beta^2}{\omega_0^2}}$$

Sklopljeno nihanje

Zapišemo Newtonove zakone za vsako nihalo posebej zapišemo v obliko

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2 (x_1 - x_2) = 0$$

$$\ddot{x}_2 + \omega_1^2 x_2 + \omega_2 (x_1 - x_2) = 0$$

$x_a = x_1 + x_2$ $x_b = x_1 - x_2$ potem prevedemo

$$\leadsto x_1 = C_1 \sin(\omega_a t + \delta_a) + C_2 \sin(\omega_b t + \delta_b)$$

$$x_2 = C_1 \sin(\omega_a t + \delta_a) - C_2 \sin(\omega_b t + \delta_b)$$

$$\omega_a^2 = \omega_1^2 \quad \omega_b^2 = \omega_1^2 + 2\omega_2^2$$

→ EKVIVALENTNO

$$x_1 = A \sin(\omega_a t) + B \cos(\omega_a t) - C \sin(\omega_b t) - D \cos(\omega_b t)$$

$$x_2 = A \sin(\omega_a t) + B \cos(\omega_a t) + C \sin(\omega_b t) + D \cos(\omega_b t)$$

Valovanje

$$u(x,t) = f(x-ct) + g(x+ct) \quad f, g \in C^2(\mathbb{R})$$

iz začetnih pogojev dobimo

$$u(0, \dots) \text{ in } u(\dots, 0)$$

$$u(x, 0) = A(x) \quad u_t(x, 0) = B(x) \Rightarrow$$

$$u(x,t) = \frac{1}{2} (A(x-ct) + A(x+ct)) + \frac{1}{2c} \int_{x-ct}^{x+ct} B(x) dx$$

Robni pogoji x_0 krajšee palice

• v x_0 palica vpetá v steno $\Rightarrow u(x_0, t) = 0$

• v x_0 prost konec palice $\Rightarrow u_x(x_0, t) = 0$

Hitrost valovanja

palica: $c = \sqrt{\frac{E}{\rho}}$ $E \dots$ prožnostni modul

struna: $c = \sqrt{\frac{F}{\rho S}}$

vijačnavzmet: $c = l \sqrt{\frac{k}{m}}$

kapjevina: $c = \sqrt{\frac{\gamma}{\rho}}$

plin: $c = \sqrt{\frac{\gamma R T}{M}}$

$\gamma \dots$ adiabatška
stisljivost

Sinusno valovanje

če so robni pogoji

$$u(x=0, t) = u_0 \sin(-\omega t + \delta) \Rightarrow$$

$$u(x, t) = u_0 \sin(kx - \omega t + \delta)$$

$$k = \frac{\omega}{c} \dots \text{valovno število}$$

$$\omega = 2\pi\nu \quad \lambda = \frac{v}{\nu} \quad k = \frac{2\pi}{\lambda}$$

Stojno sinusno valovanje

$$u(x, t) = u_0 \sin(kx + \omega t + \delta_1) + u_0 \sin(kx - \omega t + \delta_2)$$

prosta na obeh koncih \Rightarrow

$$k = k_n = \frac{n\pi}{l} \quad \lambda_n = \frac{2l}{n} \quad \nu_n = \frac{nc}{2l}$$

Energija sinusnega valovanja

$$w = w_0 \cos^2(kx - \omega t + \delta) \quad w_0 = \rho u^2 \omega^2$$

$$\bar{w} = \frac{1}{2} \rho u^2 \omega^2$$

$$P = c S \bar{w} \dots \text{energijski tok valovanja [W]}$$

$$j = c \bar{w} \dots \text{gostota energetskega toka}$$

ZNOK

jjakost glasišimo: $[10^{-12} \frac{W}{m^2}, 1 \frac{W}{m^2}]$

$$\text{glasnost} = 10 \cdot \log_{10} \frac{j}{j_0} [db] \quad j_0 = 10^{-12} \frac{W}{m^2}$$

$$\Delta p = - \frac{\omega u_0}{\chi c} \cos(kx - \omega t + \varphi)$$

χstisljivost plina

$$\chi = - \frac{1}{V} \frac{\partial V}{\partial p} > 0$$

izotermno stiskanje: $\chi = \frac{1}{p}$

adiabatsno stiskanje: $\chi = \frac{1}{\gamma p}$

Dopplerjev pojav

$$V_s = \frac{c + v + v_s}{c + v - v_o} \lambda_0$$

v ... hitrost vetra

oddajnik in sprejemnik se gibljeta proti drug drugemu, veter gre v desno

Taylorjevi približki

$$(1+x)^{\alpha} = 1 + \alpha x$$

$$\tan x = x$$

$$\sin x = x$$

$$\cos x = 1 - \frac{1}{2}x^2$$

Prvi lebnik

$$x(t) = x_0 + v_0 t + \frac{a t^2}{2} \longrightarrow t = \sqrt{\frac{2h}{g}}$$

$$v = v_0 + at$$

$$v = \frac{ds}{dt}$$

$$v^2 = v_0^2 + 2a_0 x$$

$$a = \frac{dv}{dt}$$

$$x = \frac{1}{2} a_0 t^2$$

$$p = \omega_0 t + \frac{\alpha t^2}{2}$$

$$\omega = \frac{dp}{dt}$$

$$\omega^2 = \omega_0^2 + 2\alpha p$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$p = \frac{1}{2} \omega t^2$$

$$v = \omega r$$

$$\omega = \frac{2\pi}{T_0} = 2\pi \nu$$

$$a_r = r\alpha$$

$$W_k = \frac{mv^2}{2}$$

$$G = mv$$

$$W_p = mgh$$

$$Ft = \Delta G$$

$$W_e = \frac{kx^2}{2}$$

$$A = F_x \quad F = xk$$

$$\Gamma = J\omega \quad \Gamma = Gr$$

$$M = Fr = \frac{d\Gamma}{dt} = J\alpha = J\frac{a_r}{r}$$

$$F_g = G \frac{m_1 m_2}{r^2} \quad W_p = -G \frac{m_1 m_2}{r}$$

$$n = \frac{pV}{RT} \quad W_n = mc\Delta T \quad W_k = \frac{2}{3} k_B T$$

\uparrow
 $c_p \geq c_v$ erhöhten Teilchen

$$\Delta V = \beta V \Delta T \quad pV = Nk_B T \quad \frac{R}{N_A} = k_B$$

$$A = -\int p dV \quad c_p = c_v + \frac{R}{M}$$

ideale Gase:

$$1: \frac{3}{2} k_B T \rightarrow c_v = \frac{3}{2} \frac{R}{M} \quad c_p = \frac{5}{2} \frac{R}{M}$$

$$2: \frac{5}{2} k_B T \rightarrow c_v = \frac{5}{2} \frac{R}{M} \quad c_p = \frac{7}{2} \frac{R}{M}$$

$$n: \frac{6}{2} k_B T \rightarrow c_v = 3 \frac{R}{M} \quad c_p = 4 \frac{R}{M}$$

$$\kappa = \frac{c_p}{c_v} \quad 1: \frac{5}{3} \quad 2: \frac{7}{5} \quad n: \frac{4}{3}$$

adiabatisch: $dW_n = A \quad Q = 0$

$$TV^{k-1} = \text{konst} \rightarrow pV^k = \text{konst}$$