7.1.

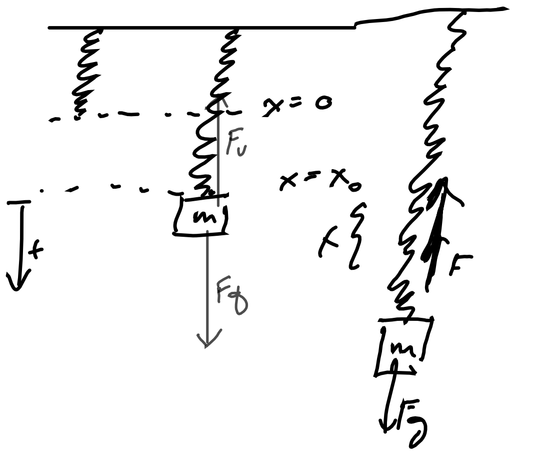
$l = 1\,m$

$k = 500\,N/m$

$m = 8\,kg$

$a_0 = 0{,}1\,m$

$x_0 = ?$



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

$$F_g + F_{vz} = m \cdot \vec{a} \qquad \ddot{x}(t)$$

$$+mg - k(x(t) + x_0) = m\vec{a}(t)$$

$$\underbrace{mg - kx_0}_{=0} - kx = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\ddot{x} = \frac{-kx}{m} \qquad -\frac{k}{m} = \omega_0^2$$

$$x(t) = \sin(\omega t) \cdot A + B\cos(\omega t)$$

začetni pogoji:

$$x(0) = \vec{a}_0$$

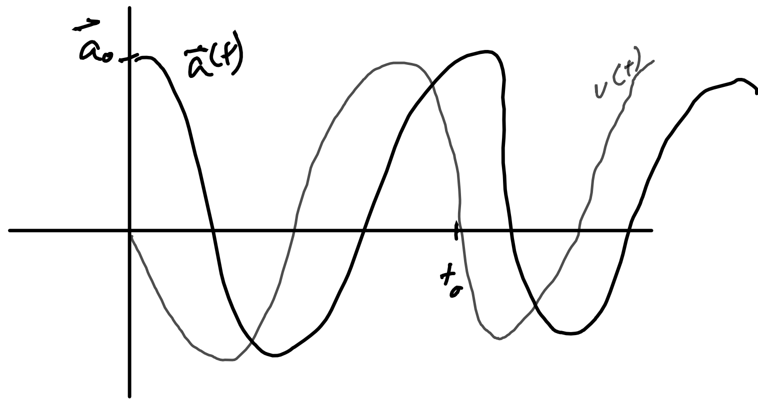
$$\vec{a}_0 = x(0) = B$$

$$v(t) = \dot{x}(t)$$

$$v(0) = \dot{x}(0) = A\omega\cos(\omega t) - B\omega\sin(\omega t)$$

$$0 = A\omega \Rightarrow A = 0$$

$$x(t) = a_0 \cos(\omega t)$$



$$\cos(\omega t_0) = 1$$

$$\omega t_0 = 2\pi \qquad t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{m}}{\sqrt{k}}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$ ... krožna frekvenca

$$\omega_0 = 7{,}9\ \frac{1}{s}$$

Energija nihanja

$$W = \frac{mv^2}{2} + mg(-x) + \frac{k(x+x_0)^2}{2} - \frac{k}{2}x_0^2$$

$$= \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}ka_0^2 = 2{,}5\ \mathrm{J}$$

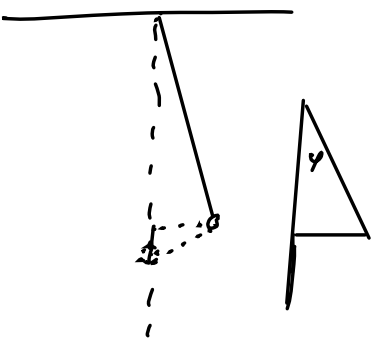
$l = 3m$

$\varphi_0 = 3{,}5°$

$t = 15\,s$

$N = ?$

$v_{max} = ?$

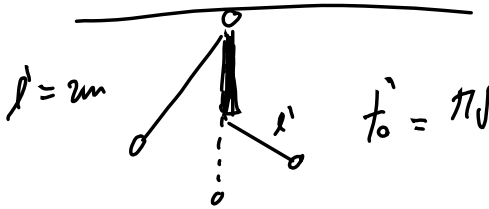


$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$N = \frac{t}{t_0} = 4{,}32$$

$$v = \sqrt{2gh}$$

$$h = l - \cos\varphi \cdot l$$



$l' = 2m$

$$t_0' = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l'}{g}} =$$

$$= \frac{\pi}{\sqrt{g}}\left(\sqrt{l} - \sqrt{l_1}\right) = 3{,}15\,s$$

$$N = \frac{t}{t_0} =$$

$\varphi(t) = \varphi_0 \cos(\omega t)$ ← nazna frekvenca

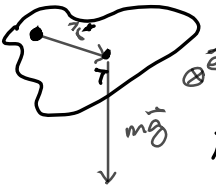
$v_{max} = l \cdot \Omega_{max}$ ← kotna hitrost

$\Omega(t) = \dot{\varphi}(t) = -\underbrace{\varphi_0 \omega}_{\Omega_{max}} \sin(\omega t)$

$v_{max} = l \cdot \varphi_0 \omega$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

Telo nihalo



$$\vec{M} = J\vec{\alpha}$$

$$\vec{r}^{*} \times m\vec{g} = J\vec{\alpha}$$

$$\vec{\varphi} = \varphi \cdot \hat{e}$$

$$r^{*} mg \sin\varphi \,(-\hat{e}) = J \cdot \ddot{\varphi}\,\hat{e}$$

$$\ddot{\varphi} = -\frac{mgr^{*}}{J}\sin\varphi$$

$$\varphi \ll 1$$

$$\ddot{\varphi} \approx -\underbrace{\frac{mgr^{*}}{J}}_{\omega^2}\varphi$$

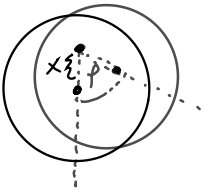
$$t_0 = 2\pi\sqrt{\frac{J}{mgr^{*}}}$$

Matematično nihalo:

$$J = ml^2$$

$$r^{*} = l \qquad t_c = 2\pi\sqrt{\frac{l}{g}}$$

7.7.)



$R = 0{,}5\,m$

t_0 min

$t_0(x) = ?$

okoli osnovne osi (Steinerjev izrek) + translacija

$$J = \frac{1}{2} m R^2 + m x^2$$

$$t_0 = 2\pi \sqrt{\frac{J}{mgx}} =$$

$$t_a = 2\pi \sqrt{\frac{\frac{1}{2}R^2 + x^2}{gx}}$$

odvajamo nex:

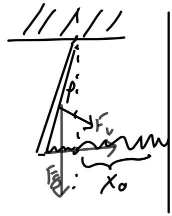
$$\frac{2x(gx) - g\left(\frac{1}{2}R^2 + x^2\right)}{gx^2} = 0$$

$$2gx^2 - gx^2 - \frac{1}{2}R^2 g$$

$$gx^2 = \frac{1}{2}R^2 g$$

$$x = \frac{R}{\sqrt{2}}$$

(7.8)



$l = 1\,m$

$m = 0{,}5\,kg$

$k = 5\,N/m$

$\nu = ?$

$\varphi \ll 1$

$\sum \vec{M} = J \cdot \vec{\alpha}$

$$J\alpha = -F_g \cdot \frac{l}{2} \sin\varphi - kx \cdot l \cdot \cos\varphi$$

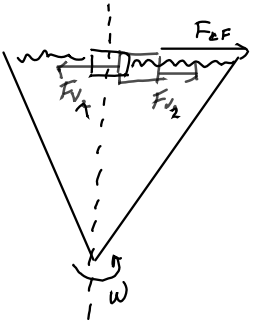
(raćica: $F_v = kx$; $\alpha = \alpha(t) = \ddot{\varphi}(t)$; $\sin\varphi \approx \varphi(t)$; $x(t) = l \cdot \varphi(t)$; $\cos\varphi \approx 1$)

$$\ddot{\varphi}(t) = -\frac{mg \cdot l}{2J}\varphi(t) - \frac{kl^2}{J}\varphi(t) =$$

$$= -\frac{3g}{2l}\varphi(t) - \frac{3k\varphi(t)}{m} = -\left(\frac{3g}{2l} + \frac{3k}{m}\right)\varphi(t)$$

$$\omega_0 = \sqrt{\frac{3g}{2l} + \frac{3k}{m}} = 6{,}7/s$$

$2\pi\nu = \omega_0 \qquad \nu = \frac{\omega_0}{2\pi} = 1{,}06\,Hz$



$m = 20g$

$k_1 = 1\ N/m$

$k_2 = 3\ N/m$

$\omega = 10/s$

$\nu = ?$

$\sum F = ma$ $\qquad \omega^2 x$

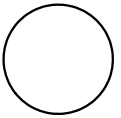
$ma = -k_1 x - k_2 x + m\ddot{a}_r$

$\ddot{x}(t) = -\left(\frac{k_1 + k_2 - m\omega^2}{m}\right)x$

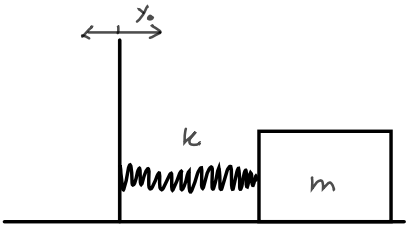
$\omega_0 = \sqrt{\frac{k_1 + k_2 + m\omega^2}{m}} = 10/s$

$\nu = 1{,}6\,Hz$

Foucaultovo nihalo



(7.25)



$y_0 = 5\,cm$

$m = 100\,g = 0{,}1\,kg$

$k = \frac{N}{m}$

$\beta = 0{,}2/s$

$\nu = 12\,Hz$

$x_0 = ?$

$\delta = ?$

$x_{0max} = ?$

$V_{max} = ?$

$m\ddot{x} = -k(x-y) - 2\beta m\dot{x}$

$\ddot{x} + 2\beta\dot{x} + \frac{k}{m}x = \frac{k}{m}y \rightarrow y_0\cos(\omega t)$

$\omega^2 = \frac{k}{m}$

Nehomogena diferencialna enačba

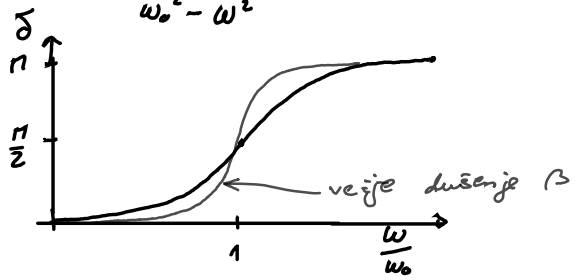
nastavek: $x(t) = X_0 \cdot \cos(\omega t - \delta) + x_0' e^{-\beta t}\cos(\omega' t - \delta')$

$\omega' = \sqrt{\omega^2 - \beta^2}$

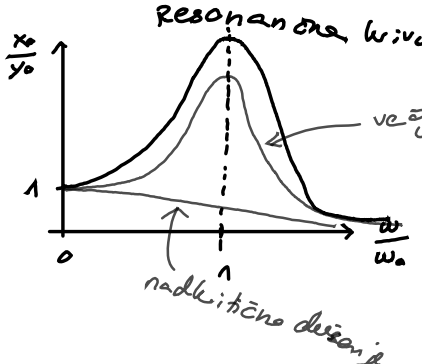
se zaduši, nas pa zanima ko se ustali

Rezultati:

$\tan\delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$



$\frac{x_0}{y_0} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}}$



vrh resonančne krivulje

$\omega_{max} = \sqrt{\omega_0^2 - 2\beta^2}$

Rezultati:

$\omega_0 = \sqrt{\frac{k}{m}} = 10/s$

$\delta = 179{,}7°$ $\Rightarrow \nu = 1{,}6\,Hz$

$x_0 = 0{,}89\,mm$

$V_{max} \approx 1{,}6\,Hz$

$x_{0max} = 1{,}25\,m$

moč pri vsiljenem nihanju

$$P = \frac{dA}{dt} = \frac{F dx}{dt} = Fv = 2\beta m v^2(t) =$$

$$v = \dot{x} = x_0 (-\sin(\omega t - \delta)) \cdot \omega$$

$$\overline{P} = 2\beta m x_0^2 \omega^2 \overline{\sin^2(\omega t - \delta)} = \beta m x_0^2 \omega^2$$

$$\overline{\sin^2} = \frac{1}{2}$$

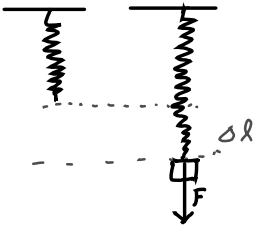
$$\overline{P} \propto \omega^2 \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}$$

$$\frac{dP}{d\omega} = \frac{2\omega\left[(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2\right] - \left[4\omega(\omega_0^2 - \omega^2) + 8\beta^2\omega\right]}{\left((\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2\right)^2} = 0$$

splošno nihanje

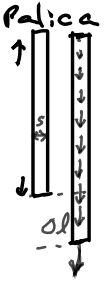
14.3

Elastomehanika: Hookov zakon



$$F = k \cdot \Delta l$$

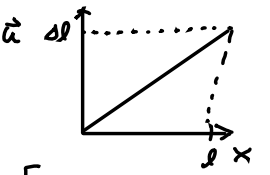
Vpeljava prožnostnega modula



majhna raztezka

$$\frac{F}{S} = \frac{\Delta l}{l} \cdot E \qquad [E] = \frac{N}{m^2} = Pa$$

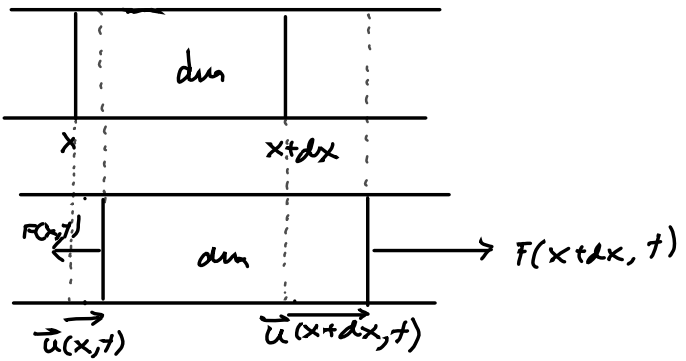
vektorska polje



$$\frac{du}{dx} = \frac{dl}{l}$$

$$\frac{F}{S} = E \frac{du}{dx}$$

Valovanje v elastični palici



$$F(x+dx,t) - F(x,t) = dm \cdot a\left(x+\frac{dx}{2},t\right)$$

$$S E \frac{du}{dx}(x+dx,t) - S E \frac{du}{dx}(x,t) = dm \cdot \ddot{u}\left(x+\frac{dx}{2},t\right)$$

$dx \longrightarrow 0$

$$S E \frac{d^2u}{dx^2}\left(x+\frac{dx}{2},t\right) \cdot dx = \rho S \, dx \, \ddot{u}\left(x+\frac{dx}{2},t\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E}\frac{\partial u^2}{\partial t^2}$$

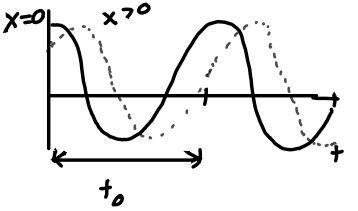
$$\frac{1}{c^2}$$

$$u(x,t) = \overleftarrow{\phi(x+ct)} + \overrightarrow{\psi(x-ct)}$$

$$c = \sqrt{\frac{E}{\rho}}$$

Periodično nihanje

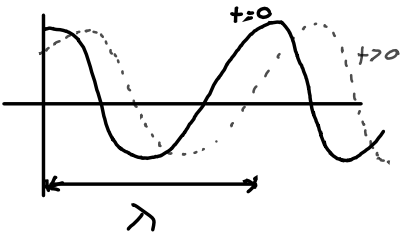
$u(x,t) = u_0 \cdot \cos(\omega t - kx)$



Na danem mestu

$\omega t_0 = 2\pi$

$\omega = \frac{2\pi}{t_0} = 2\pi\nu$



ob danem času

$c = \frac{\delta x}{\delta t} = \frac{\lambda}{t_0} \Rightarrow c = \lambda\nu$

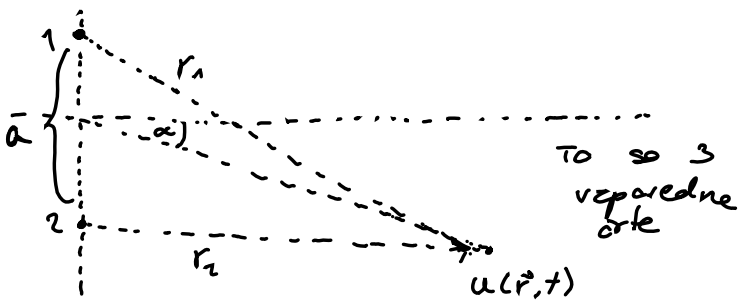
7.49

$a = \frac{3}{10}\lambda$

1. izvor zakasnjen za $\frac{1}{4}$ nihaja $\Rightarrow \delta = \frac{\pi}{2}$

smer i ojačitev = ?

smmo delec od anten



$r_1, r_2 \gg a$

$u = \underbrace{u_0 \sin(\omega t - k r_1 - \delta)}_{\text{prvi izvor}} + \underbrace{u_0 \sin(\omega t - k r_2)}_{\text{2. izvor}}$

$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

$u = \underbrace{u_0 \sin\left(\frac{2\omega t - k(r_1+r_2) - \delta}{2}\right)}_{\text{amplituda}} \underbrace{\cos\left(\frac{k(r_2-r_1) - \delta}{2}\right)}$

točke za katere je $r_1 - r_2$ konstanta tvorej pripada hiperbola

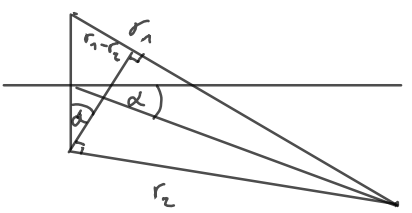
$\frac{k}{2}(r_2 - r_1) - \frac{\delta}{2} = n\pi \quad n \in \mathbb{Z}$

$j \propto u^2$ mora biti maksimalen (ker iščemo oj-čitve)

sorazmerno

odločimo se za neko

n in potem gledamo da je tudi na levi strani konstantno



$r_1 - r_2 = a \sin\alpha$

$-\frac{k}{2}(a\sin\alpha) - \frac{\delta}{2} = n\pi$

$\sin\alpha = \frac{n\pi + \frac{\delta}{2}}{-\frac{ak}{2}} = \frac{2\pi n + \delta}{-ak}$

k ..... valovni vektor

$k = \frac{2\pi}{\lambda} \qquad a = \frac{3}{10}\lambda$

$\sin\alpha = \frac{\lambda(2\pi n + \delta)}{-2\pi a} = \frac{5(2\pi n + \delta)}{-3\pi}$

$\sin\alpha = -\frac{10}{3}\left(n + \frac{1}{4}\right)$

recimo $n = 0$

$\sin\alpha_0 = -\frac{10}{3} \cdot \frac{1}{4} = -\frac{5}{6}$

$\Rightarrow \alpha_0 = -56{,}4$

$n = 1 \Rightarrow \sin\alpha_1 = -\frac{10}{3} \cdot \frac{5}{4} = -\frac{25}{6}$

$\alpha_1 = err$

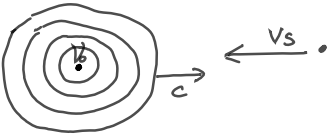
$n = -1 \Rightarrow \sin\alpha_{-1} = -\frac{10}{3} \cdot \left(-\frac{3}{4}\right) = \frac{5}{2}$

$\nu = 1485\ kH$

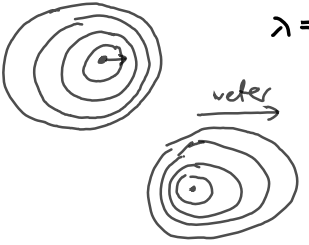
$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^{8}\ \frac{m}{s}}{1485 \cdot 10^{3}\ /s} = 202m$$

Dopplerjev pojav

$$N = \frac{ct}{\lambda} + \frac{v_s t}{\lambda} + \frac{v t}{\lambda}$$



$$\lambda = \frac{c}{\nu_0} + \frac{v}{\nu_0} + \frac{v_v}{\nu_0}$$



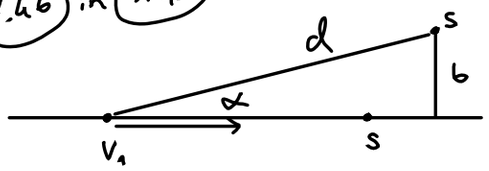
$$\nu_s = \frac{N}{t} = \frac{c + v + v_s}{c + v + v_v} \cdot \nu_0$$

$$= \frac{c + (v + v_s)}{v - (v_0 - v)}$$

hitrost spojemnika glede na vetr

hitrost oddajnika glede na veter

7.46 in 7.47



$v_1 = 100 \frac{km}{h} = 27{,}8 \frac{m}{s}$

$v_0 = 1000$ Hz

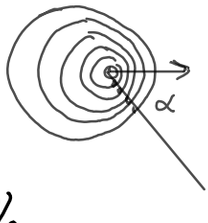
a) $v_s = ?$ $\quad v_s = \frac{c + v_s}{c - v_1} v_0$ $\quad v_s = 0$ $\quad c = 340 \frac{m}{s}$

$v_1 = 27{,}8 \frac{m}{s}$

$v_s = 189$ Hz

b) $d = 1000$ m

$b = 500$ m



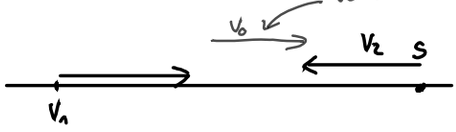
$v_s = \frac{c}{c - v_1 \cos\alpha} v_0$

$\sin\alpha = \frac{b}{d} = \frac{1}{2} \Rightarrow \alpha = 30°$

$\cos\alpha = \frac{\sqrt{3}}{2}$

$v_s = 1076$ Hz

c) veter iz d) nalaga



$v_2 = 80 \frac{km}{h} = 22 \frac{m}{s}$

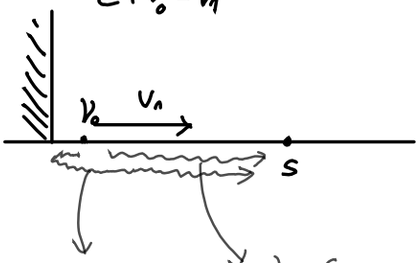
$v_s = \frac{c + v_2}{c - v_1} v_0 = 1160$ Hz

d) $v_0 = 30 \frac{km}{h} = 8{,}3 \frac{m}{s}$

↑ veter

$v_s = \frac{c + v_0 + v_2}{c + v_0 - v_1} = 1156$ Hz

e)



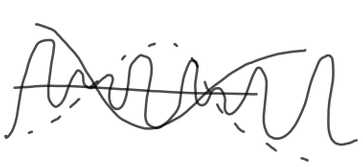
$v_{s_1} = \frac{c}{c + v_1} v_0$ $\qquad v_{s_2} = \frac{c}{c - v_1} v_0$

$v_s = v_{s_1} + v_{s_2} = c v_0 \left(\frac{1}{c + v_1} + \frac{1}{c - v_1} \right) =$

$= c v_0 \left(\frac{c - v_1 + c + v_1}{c^2 - v_1^2} \right) = c v_0 \left(\frac{2c}{c^2 - v_1^2} \right)$

$\cos(\omega' t) + \cos(\omega'' t) =$

$2 \cos\left(\frac{\omega' + \omega''}{2} t\right) \cos\left(\frac{\omega - \omega''}{2} t\right)$



slišimo utripanje

15.4.

Vzamudila 20 minut

(nalogo 10.13)

$$\int_V \operatorname{div} \vec{D}\, dV = \oint_{\partial V} \vec{D}\,\vec{n}\, dS \qquad \vec{D} = \varepsilon_0 \vec{E}$$

$$\int \rho_e dV = \varepsilon_0 \oint \vec{E}\cdot d\vec{S}$$

$$e = \varepsilon_0 \oint \vec{E}\, d\vec{S}$$

← Nalaga $\qquad \vec{E}_\perp = \frac{\lambda}{2\pi\varepsilon_0 \rho}$

$$e = \varepsilon_0 \oint_S \vec{E}\,\vec{n}\, dS$$

plašč, vrh in dno valja

$$\lambda \cdot l = \varepsilon_0 \oint E_\perp dS + 0 + 0$$

$$\lambda \cdot l = \varepsilon E_\perp \oint dS = \varepsilon = \varepsilon E_\perp 2\pi\rho \cdot l$$

$$E_\perp = \frac{\lambda}{\varepsilon_0 2\pi\rho}$$

$W_k + W_{el} = \text{konst}$

$$\frac{m v_a^2}{2} + W_{pe}(a) = \frac{m v_b^2}{2} + W_{pe}(b)$$

$$\frac{m}{2}(v_a^2 - v_b^2) = W_{pe}(b) - W_{pe}(a)$$

$$\int_a^b F\, d\rho = -\int_a^b e \frac{\lambda}{2\pi\varepsilon_0 \rho} d\rho = -\frac{\lambda e}{2\pi\varepsilon_0} \ln\frac{b}{a} = \frac{\lambda e}{2\pi\varepsilon_0} \ln\frac{a}{b}$$

$F = -E_\perp e$

$= \frac{m}{2} \sin^2\varphi \cdot v_a^2$

kot pri pasovnem metu

$\vec{v}_{a\parallel} = \vec{v}_b$

$$\ln\frac{a}{b} = \frac{\pi\varepsilon_0 m \sin^2\varphi}{\lambda e}$$

$$b = 16.8\ \mu m$$

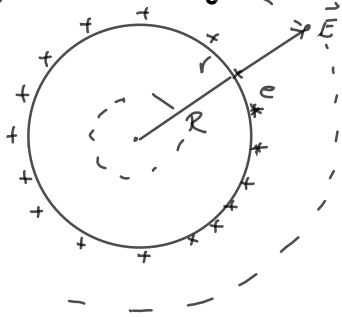
$$W_{ep}(\vec{r}_2) - W_p(\vec{r}_1) = -e \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r} \qquad / : e$$

$$\phi(\vec{r}_2) - \phi(\vec{r}_1) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r}$$

10. 18

10. 20

Kovinska krogla ki nosi naboj e



$r > R$

$\leftarrow S$ površina

$e = \varepsilon_0 \int_S \vec{E}\, d\vec{S}$

$\vec{E}$ je konstanta

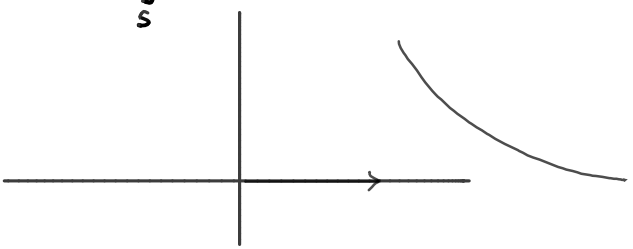
$e = \varepsilon_0 E\, 4\pi r^2$

$\Rightarrow E = \frac{e}{\varepsilon_0 4\pi r^2}$

$r < R$

V kovini znotraj ni nič naboja

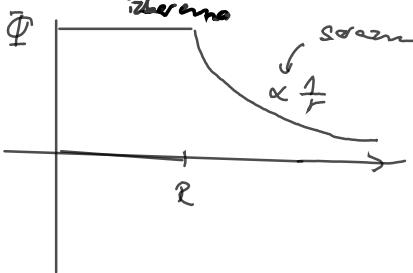
$e = \varepsilon_0 \int_S E\, dS \Rightarrow E = 0$



$$\Phi(r) = \Phi(\infty) - \int_{\infty}^{r} \vec{E}\, d\vec{r} = -\int_{\infty}^{r} \frac{e}{\varepsilon_0 4\pi r^2}\, dr = \frac{e}{\varepsilon 4\pi}\left(\frac{1}{r} - 0\right) =$$

$\Phi(\infty)$ izberemo

$$= \frac{e}{\varepsilon_0 4\pi r}$$



$\Phi(r < R) = \Phi(R) = \frac{e}{\varepsilon_0 4\pi R}$

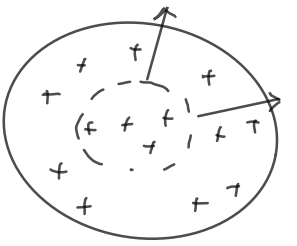
Enakomerno nabita krogla

$r > R \Rightarrow$ isto kot prej

$$E = \frac{e}{\varepsilon_0 4\pi r^2}$$

$$\Phi(r) = \frac{e}{\varepsilon_0 4\pi r}$$

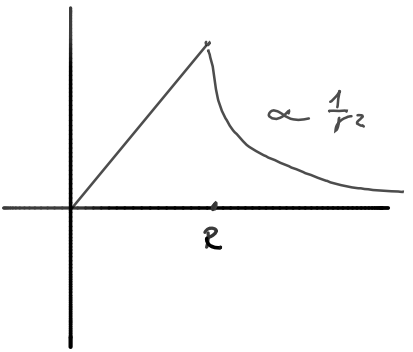
$r < R \Rightarrow$



$$e = \varepsilon_0 \int_S E \, dS$$

$$e' = \varepsilon_0 E 4\pi r^2 \qquad e' = \frac{3e}{4\pi R^3} \cdot \frac{4\pi r^3}{3} = e\frac{r^3}{R^3}$$

$$E = \frac{e r^3}{R^3 \varepsilon_0 4\pi r^2} = \frac{e r}{R^3 \varepsilon_0 4\pi}$$



$$\Phi(r) = \Phi(R) - \int_R^r \frac{e r}{\varepsilon_0 4\pi R^3} dr =$$

$$\Phi(R) - \frac{e r^2}{\varepsilon_0 8\pi R^3} \Big|_R^r = \Phi(R) - \frac{e r^2}{\varepsilon_0 8\pi R^3} + \frac{e}{\varepsilon_0 8\pi R} =$$

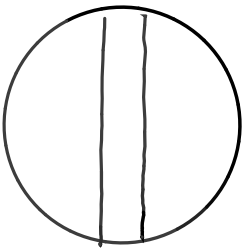
$$= \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{r^2}{2R^3} + \frac{R^2}{2R^3}\right) = \frac{e}{4\pi_0 R \varepsilon_0}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right.$$

$$\vec{E} = \frac{e}{\varepsilon_0 4\pi r^3} \vec{r}$$

7. naloga

$$e_1 \cdot E(\vec{r}) = -e_1 \frac{e\vec{r}}{4\pi\varepsilon_0 R^3} =$$

b)



$M = 6 \cdot 10^{24}\ kg$

$R = 6400\ km$

$e_1 \leftrightarrow m_1$

$$F_e = \frac{e_1 e_2}{4\pi\varepsilon_0 r^2} \qquad F_g = G\frac{m_1 m_2}{r^2} \qquad \frac{1}{4\pi\varepsilon_0} \rightarrow G$$

Lahko zamenjamo ker razlagi

$$ma = F_g = -mG\frac{Mr}{R^3}$$

$$\ddot{r} + G\frac{M}{R^3} r = 0 \qquad \omega^2$$

$$\nu = \frac{\sqrt{G\frac{M}{R^3}}}{2\pi} \Rightarrow t_0 = \frac{1}{\nu} = \frac{2\pi}{\sqrt{G\frac{M}{R^3}}} = 84{,}4\ min$$

10.27

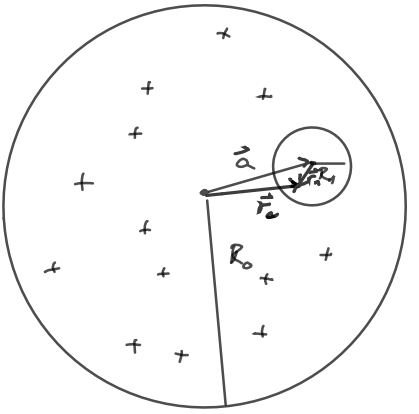
$R_0 = 10\,m$

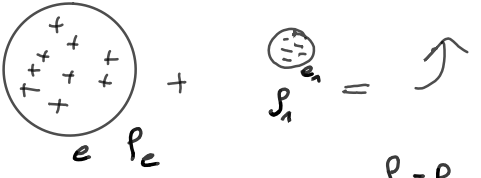
$\rho_e = 2\,\mu As/m^3$

$R_1 = 2\,m$

$a = 7\,m$

$E = ?$ v vrhu





$$\vec{E}_s = \vec{E}_0 + \vec{E}_1$$

celotna velika krogla — manjša na krogla

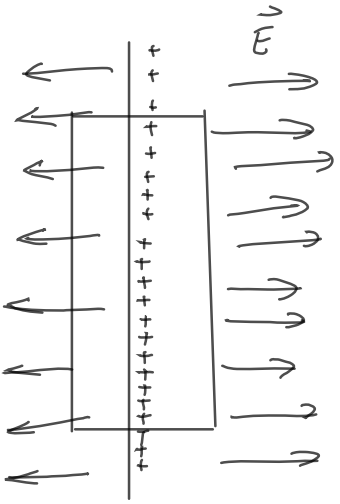
$$\frac{e\,\vec{r}_0}{4\pi\varepsilon_0 R_0^3} + \frac{e_1\vec{r}_1}{4\pi\varepsilon_0 R_1^3} =$$

$$= \frac{\rho_e \frac{4\pi R_0^3}{3}\vec{r}_0}{4\pi\varepsilon_0 R_0^3} - \frac{\rho_e \cdot \frac{4\pi R_1^3}{3}\vec{r}_1}{4\pi\varepsilon_0 R_1^3} =$$

$$= \frac{\rho_e\,(\vec{r}_0 - \vec{r}_1)}{3\varepsilon_0} = \frac{\rho_e}{3\varepsilon_0}\vec{a}$$

$$|\vec{E}_s| = \frac{2\cdot 10^{-6}\,As \cdot 7\,m}{m^3 \cdot 3 \cdot 8.85\cdot 10^{-12}\,As}$$

10.18 ⇝ 10.30



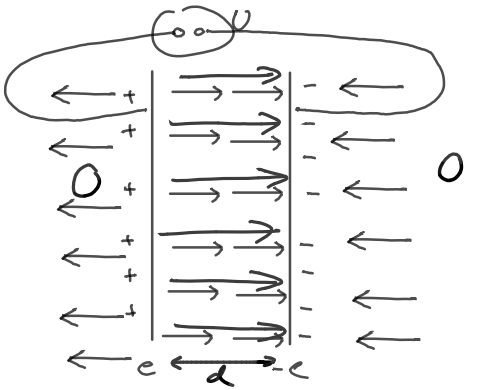
enakomerno nabite plošče

$$e = \varepsilon_0 \oint_S \vec{E}\cdot\vec{n}\,dS = \varepsilon_0 \oint_S \vec{E}\cdot\vec{n}\,dS = \varepsilon_0 E \oint_S dS =$$

(∂K) ← vzporedno ← konstanten

$$= 2\varepsilon_0 E\, P(\text{plošče}) + 0$$

$$E = \frac{e}{2\varepsilon_0 S} = \frac{\sigma}{2\varepsilon_0}$$



$$E = \frac{2\sigma}{\varepsilon_0}$$

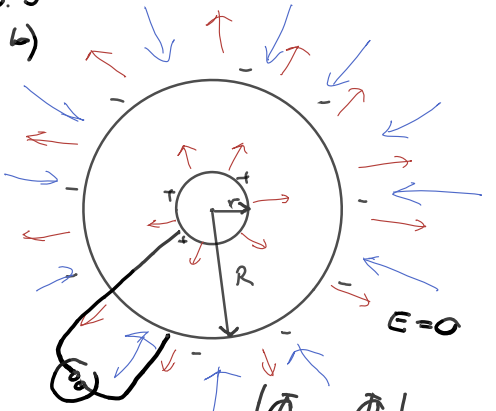
$$\Delta\Phi = -\int \vec{E}\,d\vec{r} = U = E\,d = \frac{\sigma}{\varepsilon_0} d = \frac{ed}{S\varepsilon_0}$$

$$e = \frac{U S \varepsilon_0}{d} = U \cdot C$$

$$C = \frac{e}{U} \quad [F = \frac{As}{V}$$

10.30

б)



$$|\Phi_R - \Phi_r|$$

$$|\Delta\Phi| = U = \left| \frac{e}{4\pi\varepsilon_0 R} - \frac{e}{4\pi\varepsilon_0 r} \right| = \left| \frac{e(r-R)}{4\pi\varepsilon_0 Rr} \right| =$$

$$= \frac{e(R-r)}{4\pi\varepsilon_0 Rr}$$

$$C = \frac{e}{U} = \frac{4\pi\varepsilon_0 Rr}{(R-r)}$$

(10.4.)

Na tabl.:

$$I = \frac{U\ [V]}{R\ [\Omega]}\ [A]$$

6.5

11.1

$$R = \varrho \frac{l}{S}$$

$$A = W_{pe} = e^{I \cdot t} U$$

$$P = IU$$

$S_1 = 0{,}1\ mm^2$ $\quad$ $S_2 = 0{,}2\ mm^2$

$l_1 = 100\ m$ $\quad$ $l_2 = 28 m$

$\varrho_1 = 0{,}1\ \frac{\Omega mm^2}{m}$ $\quad$ $\varrho_2 = 0{,}5\ \frac{\Omega mm^2}{m}$

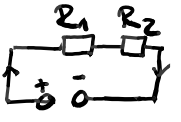
$U = 220 V$

a) zaporedna vezava $\quad \frac{P_a}{P_b} = ?$

b) vzporedna vezava

$$P = \frac{A_e}{t} = UI = U\left(\frac{U}{R}\right) = \frac{U^2}{R} = RI^2$$

a)



$R_1 = 100\Omega$

$R_2 = 70\Omega$

$R = R_1 + R_2$ $\quad$ ker $U - IR = 0$

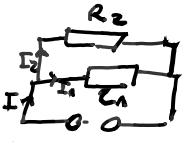
$U - I_a R_1 - I_a R_2 = 0$

$R = R_1 + R_2$

$I_a = \frac{U}{R_1 + R_2} = 1{,}294 A$

$P_a = U \cdot I_a = \underline{284{,}7\ W}$

b)



$U = I_1 R_1 = I_2 R_2$

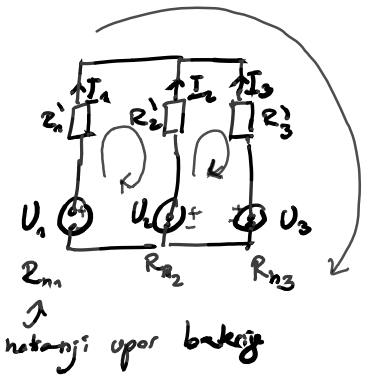
$I = I_1 + I_2 = \frac{U}{R_1} + \frac{U}{R_2}$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} =$

$I = 5{,}34 A$

$P_b = UI_2 = 1175{,}4\ W$

11.11



notranji upor baterije

$U_1 = 1V$
$U_2 = 2V$
$U_3 = 3V$
$R_{n1} = 15\Omega$
$R_{n2} = 20\Omega$
$R_{n3} = 30\Omega$
$R_1' = 85\Omega$
$R_2' = 180\Omega$
$R_3' = 270\Omega$

$R_1 = R_{n1} + R_1'$

$R_1 = 100\Omega$
$R_2 = 200\Omega$
$R_3 = 300\Omega$

$U_1 - I_1R_1 + I_2R_2 - U_2 = 0$ (lev: krog)
$U_2 - I_2R_2 + I_3R_3 - U_3 = 0$ (desn: krog)
$U_1 - I_1R_1 + I_3R_3 - U_3 = 0$ (cel krog)

Linearna odvisnost ↑ (dve enačbi, tri neznanke)

$I_1 + I_2 + I_3 = 0$ (Kirchoffov zakon)

$I_1 = -I_2 - I_3$

$U_1 + I_2R_1 + I_3R_1 + I_2R_2 - U_2 = 0$

$U_2 - I_2R_2 + I_3R_3 - U_3 = 0$

$I_2 = \frac{U_2 - U_3 + I_3R_3}{R_2}$

$U_1 + \frac{R_1}{R_2}(U_2 - U_3 + I_3R_3) + (U_2 - U_3 + I_3R_3) - U_2 = 0$

$\frac{R_1R_3}{R_2}I_3 + R_3I_3 = U_2 - U_1 - \frac{R_1}{R_2}(U_2 - U_3) - U_2 + U_3$

$I_3 = \frac{R_2(U_3 - U_1) - R_1(U_2 - U_3)}{R_1R_3}$

$I_3 = \frac{200\Omega\,(2V) - 100\Omega\,(-1V)}{100\Omega \cdot 300\Omega} =$

$= \frac{500\,\Omega V}{100 \cdot 300\,\Omega} = \frac{5}{300} V = 0{,}016\,A$

To je narobe :(

$I_3 = 4{,}5k\,mA$

$I_2 = 1{,}81\,mA$

$I_1 = -6{,}35\,mA$

11.3

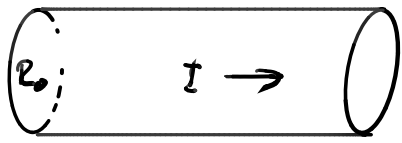
$S_0 = 3\,mm^2$

$S_1 = 1\,mm^2$

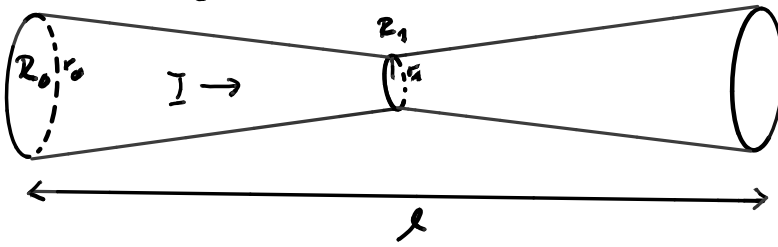
$l = 15\,cm$

$\frac{U}{U_0} = 2$.

U_0:



U:



$$\frac{U}{U_0} = \frac{RI}{R_0 I} = \frac{R}{R_0}$$

$$R_0 = \rho \frac{l}{S_0} \qquad R = \int dR = 2 \int_0^{\frac{l}{2}} \rho \frac{dx}{\pi r(x)^2}$$ (sodoset)

$$r(x) = \frac{2(r_0 - r_1)}{l} x + r_1$$

$u = r(x)$

$du = \frac{2(r_0 - r_1)}{l} dx$

$$R = 2\rho \int_{r_1}^{r_0} \frac{2}{\pi} \frac{(r_0 - r_1)}{l} \frac{du}{u^2} = \frac{4\rho}{\pi l} \frac{r_0 - r_1}{} \left(-\frac{1}{u}\right) \Big|_{r_1}^{r_0} =$$

$$= \frac{4\rho}{l\pi} (r_0 - r_1) \left(\frac{1}{r_1} - \frac{1}{r_0}\right) =$$

$$= \frac{4\rho}{l\pi} \frac{(r_0 - r_1)^2}{r_0 r_1} = \frac{2\rho l}{2\pi} \frac{(r_0 - r_1)^2}{r_0 r_1}$$

$\pi r_0^2 = S_0$

$r_0 = \sqrt{\frac{S_0}{\pi}} \qquad r_1 = \sqrt{\frac{S_1}{\pi}}$

$$R = \frac{4\rho}{l\pi} \frac{\pi \left(\frac{S_0}{\pi} + \frac{2}{\pi}\sqrt{S_0 S_1} + \frac{S_1}{\pi}\right)}{\sqrt{S_0 S_1}} =$$

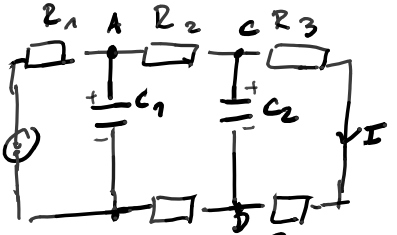
$$= \frac{\rho l}{\pi} \frac{(S_0 + S_1 + 2\sqrt{S_0 S_1})}{\sqrt{S_0 S_1}}$$

$\frac{U}{U_0} = \frac{R}{R_0}$.

mai sus se zmotile

$$R = \frac{l\rho}{\pi r_0 r_1}$$

$$\frac{U}{U_0} = \frac{r_0}{r_1} = \frac{\sqrt{S_0}}{\sqrt{S_1}} = \sqrt{3}$$



$U_0 = 6V$

$R_1 = 1\Omega$

$R_2 = 1\Omega$

$R_3 = 5\Omega$

$R_4 = 5\Omega$

$R_5 = 1\Omega$

$C_1 = 5\mu F$

$C_2 = 0{,}1\mu F$

$e_1 = ?$

$e_2 = ?$

Po dolgem času bez kondenzator ne teče tok

$\Rightarrow R = R_1 + R_2 + R_3 + R_4 + R_5$

$[F = \frac{As}{V}]$

$C = \frac{e}{U}$ (U_{AB})

$e_1 = C_1 U_{AB}$

$e_2 = C_2 U_{CD}$

$U_0 - I R_1 - U_{AB} = 0$

$U_0 - I R_1 - I R_2 - U_{CD} - I R_5 = 0$

$U_0 - I \underbrace{(R_1 + R_2 + R_3 + R_4 + R_5)}_{R_n} = 0$

$I = \frac{U_0}{R_n} = 0{,}5A$

$U_{AB} = U_0 - I R_1 = 6V$

$U_{CD} = U_0 - I R_1 - I R_2 - I R_5 = 5V$

$e_1 = C_1 \cdot U_{AB} = 30\mu As$

$e_2 = 0{,}5\mu As$

Transcendibile per via

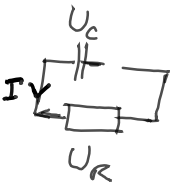
11.13 Praznenje kondenzatorja

$C = \frac{e}{U}$

$R = 1\ M\Omega$

$C = 300\ nF$

$t_{1,2} = ?$



$(U_g +) \quad U_C + U_R = 0$

$(U_g) \quad -\frac{e}{C} - IR = 0$

$I = \frac{de}{dt}$ odvisnood časa

$$-\frac{e(t)}{C} = R\frac{de}{dt}$$

$$de = -\frac{e(t)}{RC}\,dt$$

$$\frac{de}{e} = -\frac{dt}{RC}$$

$$\ln e\Big|_{e'}^{e} = \left(-\frac{t}{RC}\right)\Big|_0^t$$

$$\ln\frac{e}{e'} = -\frac{t}{RC}$$

$$e = e' e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = \frac{1}{2}$$

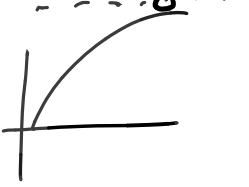
$$-\frac{t}{RC} = \ln\frac{1}{2}$$

$$t = \ln 2 \cdot RC$$

$1 \cdot 10^6\ \Omega \cdot 300 \cdot 10^{-9}\ F \cdot 0{,}69 = 0{,}21\ s$

Polnenje kondenzatorja

$$e(t) = CU_g\left(1 - e^{-\frac{t}{RC}}\right)$$



Magnetna polje

12.1

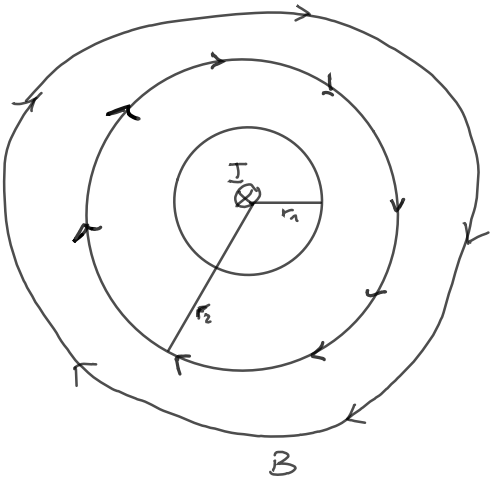
$R = 1\,cm$

$I = 1000\,A$

$r_1 = 0{,}5\,cm$

$r_2 = 2.0\,cm$

$B(r) = ?$



$$\oint \vec{B}\,d\vec{s} = 0$$

Amperci

$$\frac{1}{\mu_0}\oint \vec{B}\,d\vec{s} = \frac{1}{\mu_0}\int_S \mathrm{rot}\,\vec{B}\,d\vec{S} \qquad \frac{1}{\mu_0}\oint B\,ds = I|_{objekt}$$

$\mathrm{rot}\,\vec{B} = \vec{j}$ ... gostota električnega toka

I

$$\vec{B} = \mu_0 \vec{H}$$

$\vec{B}$ ... gostota, $\vec{H}$ ... jakost

$$\mu_0 = 4\pi \cdot 10^{-7}\,\frac{Vs}{Am}$$

$$\frac{1}{\mu_0}\oint B\,ds = I$$

$$\frac{B}{\mu_0}\int ds = \frac{B}{\mu_0}\,2\pi r$$

$$B(r) = \frac{I\mu_0}{2\pi r} = 0{,}01\,\frac{V}{m^2}s$$

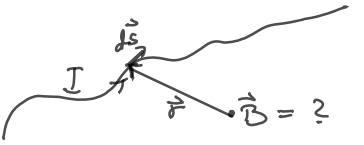
$$\frac{S}{I} = \frac{S'}{I'} = \frac{1}{j} \Rightarrow I = \frac{S'}{S}I = \frac{4\pi r_1^2}{4\pi R^2}I =$$

$$= \frac{B'}{\mu_0}2\pi r_1 \Rightarrow$$

$$\frac{\mu_0}{2\pi R}\frac{r_1}{^2}I = B'$$

$$\Rightarrow B(r_1) = \frac{\mu_0 r_1}{2\pi R^2}I = 0{,}01\,T$$

B : ot - Savar tave eksperimenti

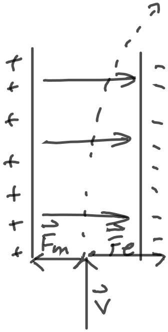


ne nekem denem meşken

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\text{Eğri}} \frac{\vec{r} \times d\vec{s}}{r^3}$$

12.4.

Lorentzova sila $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$



$B = ?$ da se pot delca ne ukrivi

$U = 50V$

$d = 1cm$

$\vec{F}_e = -\vec{F}_m$

$E = v \cdot B$

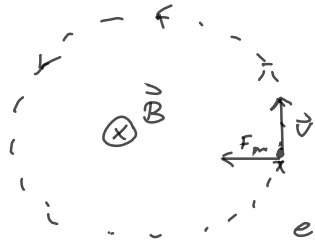
$\frac{U}{d}$

(B za najmanjšo pot ...)

Prof: A je prav?

Matija na tabli: Mel smo dva podatka, da smo porabl

$$B = \frac{U}{dv} = \frac{50V}{0{,}01m \cdot 1040\frac{m}{s}} = \left[\frac{Vs}{m^2}\right] 4{,}8$$



$F_m = F_{cp}$

$evB = m\omega^2 r$

$e\omega r B = m\omega^2 r$

$\omega = \frac{eB}{m} = \omega_c$

ciklotronska frekvenca

$evB = m\frac{v^2}{r}$

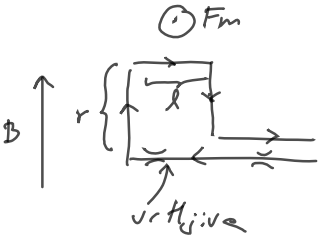
$r = \frac{mv}{eB}$

$dA = \vec{F}_m d\vec{x} = e(\vec{v} \times \vec{B})\vec{v}dt = 0$

pravokotna

12.10

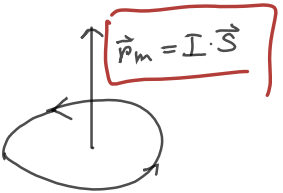
20.5

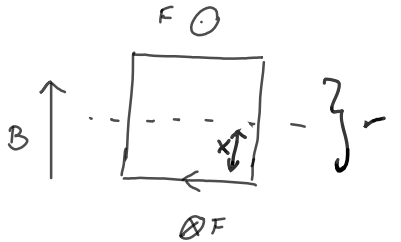


magnetni moment

$$M = r \cdot F_m = r \cdot I l B = \underbrace{I S}_{p_m} \cdot B = p_m B$$

Zanka s tokom



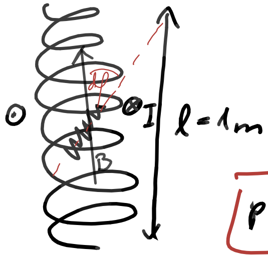


$$M = x F_m + (r - x) F_m = r F_m$$

$$\boxed{M = p_m \vec{B}}$$

$$M = \vec{r} \times F_m$$

$N_1 = 100 \quad l = 1m \quad I_1 = 25A$



$N_2 = 500$

$S_2 = 20\ cm^2$

$I_2 = 0{,}1A$

$A = ?$

$$p_m = N I \vec{S}$$

$$dA = F r \cdot d\varphi = M d\varphi$$

$$M = \vec{p}_m \times \vec{B} = p_m B \sin\varphi = N_2 I_2 \cdot S B \sin\varphi$$

B?

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = \sum I$$

za dolgo ozko tuljavo

$$\frac{1}{\mu_0}(Bl + 0) = N_1 I_1$$

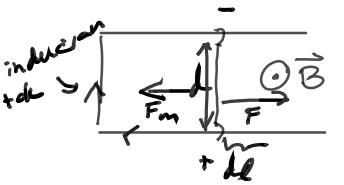
ker je tok tam tako majhen

$$B = \frac{N_1 I_1 \mu_0}{l}$$

$$A = \int_0^{\pi} N_2 S_2 I_2 \frac{\mu_0 N_1 I_1}{l} \sin\varphi d\varphi =$$

$$+2\ N_2 S_2 I_2 \frac{\mu_0 N_1 I_1}{l} = 6{,}82 \cdot 10^{-4} J$$

Elektromagnetna indukcija



$$F_m = I\,\vec{d}\times\vec{B}$$

$$F\,dl = dA_{el} = U_i \cdot I \cdot dt$$

$$I \cdot d \cdot B\,dl = U_i \cdot I \cdot dt$$

$\vec{v}\cdot dt$

magnetni pretok

$$\Phi_m = \vec{B}\cdot\vec{S}$$

$$U_i = B \cdot d \cdot v = B \cdot \frac{d \cdot dl}{dt} = B\frac{dS}{dt} = \frac{d\Phi_m}{dt}$$

$$U_i = (\vec{d}\times\vec{B})\,\vec{v}$$

Φ_m hoče oslabiti:

$\Phi_m = \vec{B}\cdot\vec{S}$ zato se se $\vec{S}$ poveča se mora B zmanjšati

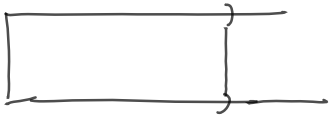
12. 18

$\xi = \text{konst}$

$d = 5\,cm$

$B = 1\,T$

$\vec{a} = \text{konst}$



$l(I = max) = ?$

$l = \frac{1}{2}at^2$

$v = at$

$l = \frac{v^2}{2a}$

$v = \sqrt{2al}$

$$I_i = \frac{U_i}{R} = \frac{B \cdot d \cdot v(l)}{R(l)} =$$

$$= \frac{B \cdot d \cdot \sqrt{2al}}{\frac{\xi \cdot (2d+2l)}{S}} =$$

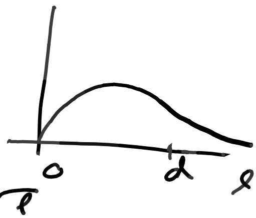
$R = \frac{\xi \cdot l'}{S}$

$$= \frac{SBd}{\xi} \cdot \frac{\sqrt{2al}}{(2d+2l)}$$

išćemo samo le $\propto \frac{\sqrt{l}}{d+l}$

$d \ll l \Rightarrow I_i \propto \frac{1}{\sqrt{l}}$

$l \ll d \Rightarrow I_i \propto \sqrt{l}$



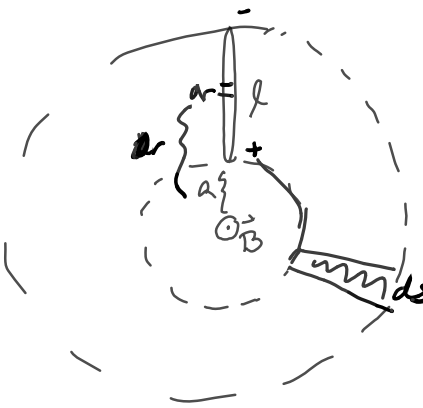
$$\left(\frac{\sqrt{l}}{d+l}\right)' = \frac{\frac{1}{2\sqrt{l}}(d+l) - \sqrt{l}}{(d+l)^2} =$$

$$= \frac{\frac{d}{2\sqrt{l}} + \frac{1}{2}\sqrt{l} - \sqrt{l}}{(d+l)^2} = \frac{\frac{d}{2\sqrt{l}} - \frac{1}{2}\sqrt{l}}{(d+l)^2} = 0$$

$\frac{d}{\sqrt{l}} - \sqrt{l} = 0 \quad \cdot \sqrt{l}$

$d - l = 0$

$d = \sqrt{l}$



$a = 25\ cm$

$l = 0{,}5\ m$

$\nu = 2\ Hz$

$B = 0{,}5\ T$

$U_i = ?$

$U_i = B \cdot d \cdot v$

Lorenzova sila

$F_m = e\, v \times B$

$$dU_i = B \cdot dr \cdot v$$

$$U_i = \int_a^{a+l} B\omega r\, dr = \frac{B\omega}{2}\left((a+l)^2 - a^2\right) =$$

$$= \frac{B\omega}{2}(2al + l^2) = B\omega al + \frac{B\omega}{2} l^2$$

$$= \frac{B\omega}{2}(l + 2a) = B\pi\nu(l + 2a)$$

$\frac{Vs}{m^2}$

$$= 0{,}5\,T \cdot 2\pi \cdot 2/s \cdot (0{,}5 + 0{,}5)\,m = 1{,}57\,V$$

2 način

$$U_i = \frac{d\Phi_m}{dt} = \frac{B\,dS}{dt} =$$

$$dS = \pi\left((a+l)^2 - a^2\right) \cdot \frac{dl}{2\pi} = \frac{dl}{2} l(l + 2a)$$

$$= \frac{B\,dl}{dt \cdot 2} l(l+2a) = \frac{B\omega l}{2}(l+2a)$$

12.20

$\frac{d\Phi_m}{dt} = U_i \quad \Phi_m = \vec{B}\vec{S}$

$\xi = 0,05 \ \frac{\Omega mm^2}{m}$

$r = 5 \ cm$

$S_0 = 1 \ mm^2$

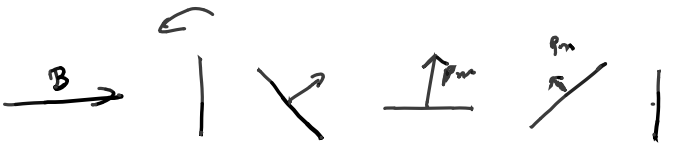
$B = 0,7 \ T$

$\nu = 10 \ Hz$

$\overline{M} = ?$

obrać

$\Phi_m = B \cdot S \cdot \cos\varphi$



$\vec{M} = p_m \times B = p_m \cdot B \cdot \sin\varphi = I_i B S \sin\varphi =$

$p_m = I_i \cdot S \qquad I_i = \frac{U_i}{R} = \frac{U_i}{R} = \frac{d\Phi_m}{dt\,R} =$

$= \frac{B \cdot S \cdot \cos\varphi \, d\varphi}{dt \, R} = \quad \varphi = \omega dt$

$= -\frac{B \cdot S \omega}{R} \sin \omega t$

$= -\frac{B^2 S^2 \omega}{R} \sin^2 \omega t$

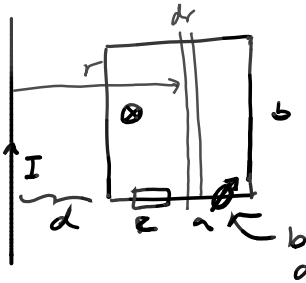
$|M| = \frac{B^2 S^2 \omega}{R} \sin^2 \omega t \qquad \overline{\sin^2} = \frac{1}{2}$

$\overline{M} = \frac{B^2 S^2 \omega}{2R} = \frac{B^2 S^2 S_0 \omega}{4\pi r \xi} = 0,06 \ Nm$

$R = \xi \frac{2\pi r}{S_0}$

12.22

$I = 300\,A$ $\quad a = 5\,cm$ $\quad b = 10\,cm$ $\quad d = 3\,cm$ $\quad R = 10\,\Omega$



$F \longrightarrow \int F\,dt$

$M \longrightarrow \int M\,dt$

$I \longrightarrow \int I\,dt = e$ tokovni sunek = naboj

ko je vključeno

$$\Phi_m = \int_S \vec{B}\cdot d\vec{S} = \int_d^{d+a} B\,b\,dr = \int_d^{d+a} \frac{\mu_0 I}{2\pi r}\,b\,dr =$$

$dS = b\,dr$

$$= \frac{\mu_0 I}{2\pi}\,b\,\ln\frac{d+a}{d}$$ ko je $I \le e$ tekel

$$\frac{d\Phi_m}{dt} = U_i = \frac{I_i}{R} \qquad I_i = \frac{d\Phi_m}{dt}R$$

$$\int_{\text{izklop}} I_i\,dt = \int_{\text{izklop}} \frac{1}{R}\,d\Phi_m = \frac{1}{R}\int_{\text{izklop}} d\Phi_m$$

$0 \to \infty$

$\Phi_m \quad 0$

$$\int I\,dt = \frac{1}{R}(\Phi_{m1} - \Phi_{m2}) = \frac{\mu_0 I}{2\pi R}\ln\frac{d+a}{d}$$

$$= 5{,}88\cdot 10^{-7}\ As$$