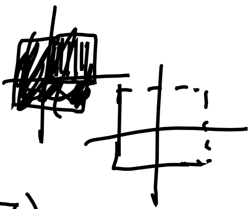
1.1.

$A = [-1,1] \times [-1,1]$

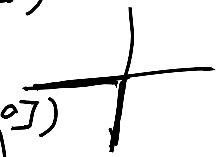
$g : \mathbb{R}^2 \to \mathbb{R}/A$ hoc



a) $g_*((-\infty,0] \times (-\infty,0])$

~~zapte~~ nie odp nic ~~nie otw~~ zap

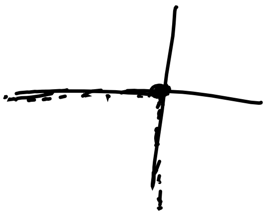
$g^*(g_*((-\infty,0] \times (-\infty,0]))$



b) $g_*(\mathbb{R}^2 - (-\infty,0] \times (-\infty,0])$

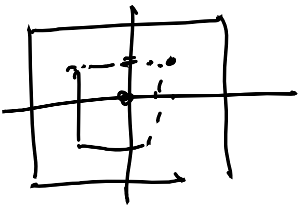
nie otwarte

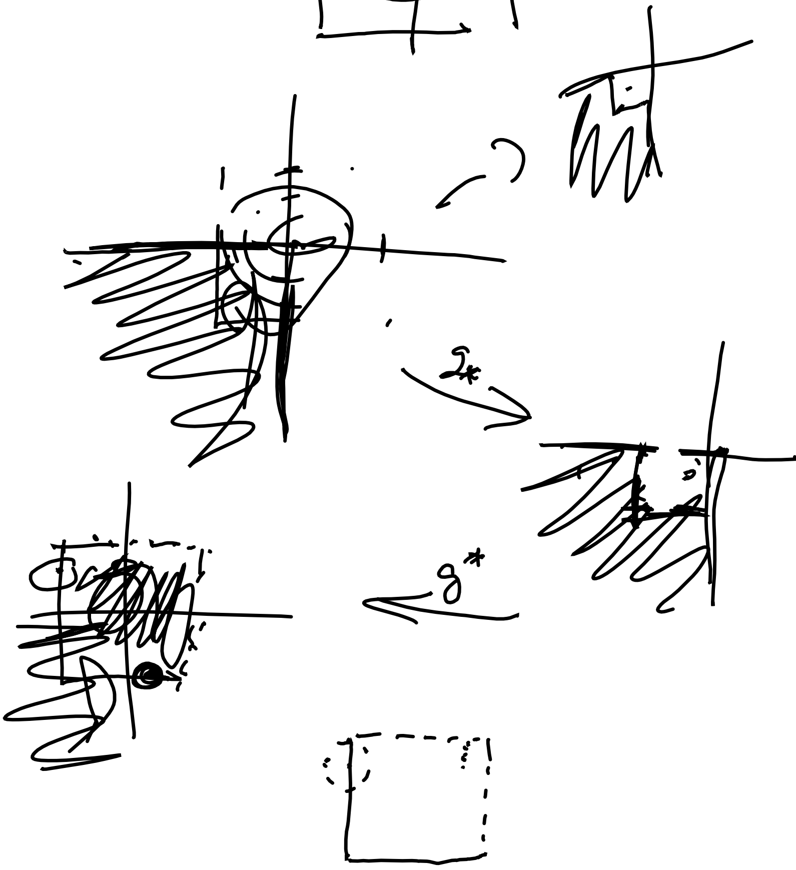
nie zapte



c) $g_*([-2,2] \times [-2,2])$

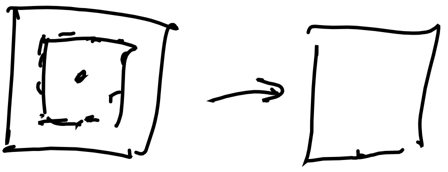
otwarte





5)

$g^*(g([-2,2] \times [-2,2]))$



zapte

nie otw

c) nie dy jezy

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a) $[-2,2]/[-1,1]$ 

$\cong [-1,1]$

b) $[-1,1]/\{-1,0,1\}$ 

$\cong$ 

$\mathbb{R}^2$ 

c) /

$$\mathbb{R}^n / K(0,1) \xrightarrow{\ \cong\ } \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[rr, "f"] \arrow[d] & & \mathbb{R}^n \\
\mathbb{R}^n / K(0,1) & &
\end{tikzcd}$$

$$f(\vec{a}) := \begin{cases} 0 ; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} ; & a \in \mathbb{R}^n - \overline{K}(0,1) \end{cases}$$

d) $\mathbb{R}^n/_\sim$ $\quad x \sim y \Leftrightarrow \|x\| = \|y\|$

$\mathbb{R}^n/_\sim \;\tilde{\approx}\; [0, \infty)$

$f: \mathbb{R}^n \longrightarrow [0, \infty)$

$f(\vec{x}) = \|\vec{x}\|$ je surj

$a \in [0, \infty) \quad (a, 0, 0 \ldots) \mapsto a$

$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$

je zvezna

iščemo preslikavo s, da velja $\quad f \circ s = id_{[0, \infty)}$

$s: a \longmapsto (a, 0 \ldots\ldots)$

Dokažimo da če $r \circ s = id_X$

$\Rightarrow r$ kvocientna, sub...

$S \subseteq Y$ taka da $r^*(S)$ odp v X

$S = s^*(r^*(S)) =$

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S$ $\blacksquare$

$\Longrightarrow$ S je odprta

f)

$S^n \times [-1,1]) / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$

$f: X \longrightarrow S^{n+1}$

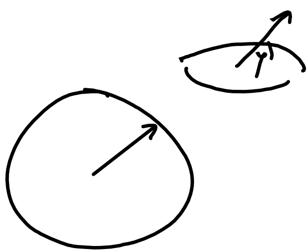
$(x,t) \longmapsto (xb, t) = (\sqrt{1-t^2}\,x, t$

anstatt voller

$\|xb\|^2 + \|t\|^2 = 1$

$\|b\|^2 + \|t\|^2 = 1$

$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$



$G/H = \{ g \cdot H \; ; g \in G \} =$

$= \{ \{ g \cdot h \; ; h \in H \} \; ; g \in G \}$

G top. grupa

$a \in G$

$L_a : G \longrightarrow G$

$x \longmapsto a \cdot x$ leva translacija

$a, b \in G$

$h : G \longrightarrow G$

$h(a) = b$ $h = ?$

možnosti

$L_{ba^{-1}} : x \longmapsto ba^{-1}x$

$ba^{-1}x$

$xa^{-1}b$

bxa^{-1}

$a^{-1}xb$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\Longleftrightarrow$ $ba^{-1}A$ okolica $b \in G$

$\exists\, U^{odp} \subseteq A \,.\, a \in U$

$b \in ba^{-1}A$

$a \in U \Rightarrow \underbrace{ba^{-1}a}_{= b} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \subseteq A$ odprta v G

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ H okolica $1 \Rightarrow H$ odp in zap v G

$$a \in aH \subseteq H$$

$\Rightarrow H$ je okolica vsake svoje točke

$G - H$ je odp.

$$aH \cap H = \emptyset \Rightarrow aH = H$$

$a \in G - H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$ vsak element ima okolico, ki ne seka $H \Rightarrow H$ je zaprta

c) C komponenta $1 \in C$

$\Rightarrow C$ zaprta edinka v G

$C \leq G$

$L_a : x \mapsto ax$ je homeomorfizem za $\forall a \in C$

$\forall a \in C.\ L_a \subseteq C$

L_a ohranja povezanost

Prav tako $L_a \cdot 1 = a \Rightarrow$

$L_{a*}C \cap C \Rightarrow L_{a*}C \subseteq C$

invertiranje: invertiranje je tudi homeo $i : x \mapsto x^{-1}$

$\Rightarrow$ po istih argumentih

Ali je edinka?

$\forall a \in G.\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \mapsto axa^{-1}$ je homeomorfizem

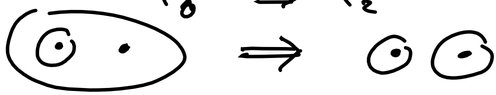
$\Rightarrow$ jo povezano in $a\,id\,a^{-1} = id \in C$

$\Rightarrow aCa^{-1} \subseteq C$

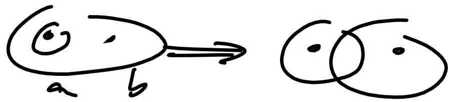
ker je kompozitum dveh translacij (leva in desna)

d) Za G je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$



$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U,\ b \notin U$ BŠZS

$$U^{-1} = \{a^{-1} : a \in U\}$$

$aU^{-1}b \ni b \qquad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U.\ a = ac^{-1}b$

$\Rightarrow b = c \Rightarrow b \in U$ ⨳

$T_1 \Rightarrow T_2$

$\exists U, V \in G$ (odp). $a \in U,\ b \in V.\ a \notin V\ \ b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$\Delta_G = f^*(\{1\})$

$f : (x, y) \mapsto xy^{-1}$

~~za~~ zvezna praslika zaprte je zaprta

② T_{cc} je T_1 in ni T_2

$\Rightarrow$ $(\mathbb{R}, +)$ ni top grupa za T_{cc}

(ker 1.d)

(2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

d $(m,n)(x,y) := (m+x,\ n+y)$

$\Phi: g \longmapsto \underbrace{(a \longmapsto g\cdot a)}_{\text{bijekcija (izomorfizem)}}$

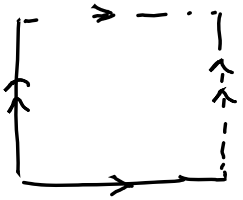
$\Phi: G \longrightarrow \mathrm{Bij}(A)$ ← grupa za komponiranje

Ubistu: $\Phi: G \longrightarrow (\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ)$

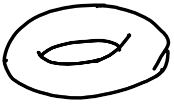
mamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ja

na nivoju topologije ? NE



Dobimo torus



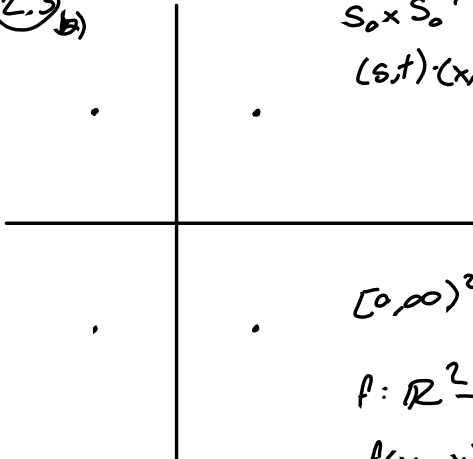
$\cong S^1 \times S^1$

$f: \mathbb{R}^2 \longrightarrow S^1 \times S^1$

$(x,y) \longmapsto (e^{2\pi i x}, e^{2\pi i y})$

Tuk nisem poslušala več :(

(2,3) b)



$S_0 \times S_0 \curvearrowright \mathbb{R}^2$

$(s,t)\cdot(x,y) = (sx, ty)$

$[0,\infty)^2$

$f: \mathbb{R}^2 \to [0,\infty)^2$

$f(x,y) = (|x|, |y|)$

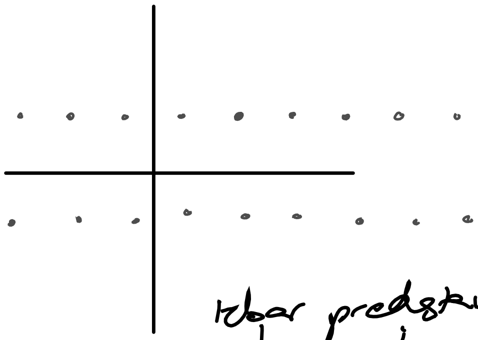
$s: [0,\infty)^2 \to \mathbb{R}^2$

$(x,y) \longmapsto (x,y)$

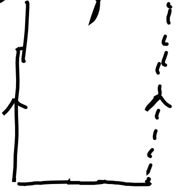
f, s zvezni $\Rightarrow f$ kvocientna c)

$\mathbb{Z} \times S^0 \curvearrowright \mathbb{R}^2$

$(m,t)\cdot(x,y) = (m+x,\ ty)$



Izbor predstavnikov



$S_1 \times [0,\infty)$

$f: \mathbb{R}^2 \longrightarrow \underset{\subseteq \mathbb{C}}{S^1} \times [0,\infty)$

$(x,y) \longmapsto (e^{i2\pi x}, |y|)$

zvezna ✓

surjektivna ✓

Po standardnem postopku tvori identifikacije med delovanje

$((n - \frac{1}{n}, 0))_n$

slike zaporedja zap v $\mathbb{R}^2$

f-slike pa ni zaprta v $S^1 \times [0,\infty)$

$(1,0)$ je v zaprtju ni pa v f-sliki

Produkt dveh odprtih preslikav je odprta

$h: \mathbb{R} \longrightarrow [0,\infty)$

$x \longmapsto |x|$

Dovolj preveriti na bazi

$0 \notin (a,b)$: $h(a,b) = (\min\{|a|,|b|\}, \max\{\dots\})$

Če $0 \in (a,b)$: $h(a,b) = [0, \max\{|a|,|b|\})$

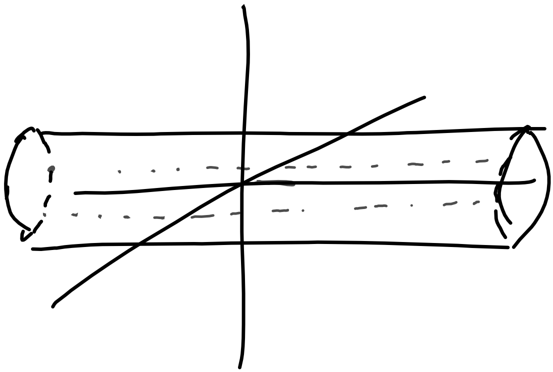
$g: \mathbb{R} \to S^1 \subseteq \mathbb{C}$

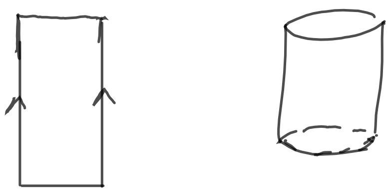
$x \mapsto e^{i2\pi x}$

Baza $\mathbb{R}$ odprti intervali dolžine <1 $\Rightarrow$ slike odprt lok d)

$\mathbb{Z} \times S_0 \curvearrowright \mathbb{R} \times S^1 \subseteq \mathbb{R}^3$

$(m,t).(x,y,z) := (m+x, y, tz)$

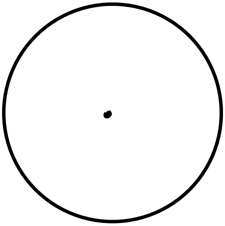




$\mathbb{R} \times S_1 \longrightarrow S_1 \times [-1,1]$

$(t, y, z) \longmapsto (e^{2i\pi t}, y)$

a)

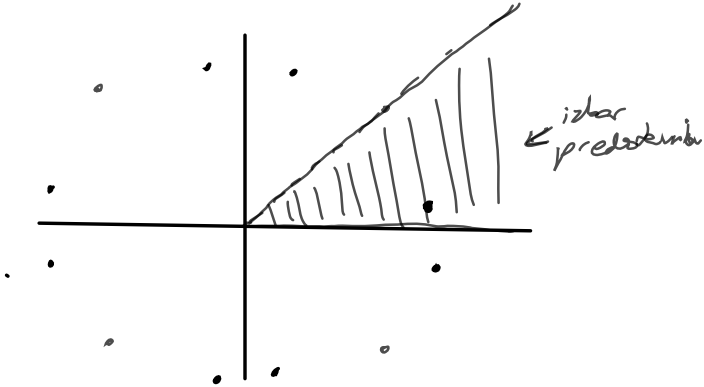


$$\vec{x} \sim \vec{y} \iff \|x\| = \|y\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(a)



$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \cong -1 \in \mathbb{C} \quad \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \cong i$

Topologija ker [illegible]

$Y = \{(x,y) \in \mathbb{R}^2;\ y < x;\ x > 0$

$f: \mathbb{R}^2 \longrightarrow Y$

$(x,y) \longmapsto (\max(|x|,|y|);\ \min(|x|,|y|)$

A retrakcija $\Rightarrow$ kvocientna v ožjem smislu

2(4)

$2x = g + x$

$\mathbb{R}/\mathbb{Q}$ ni mateč vlozna v obeh

← po delovanju grupe

$U^{\circ dg} \subseteq \mathbb{R}/\mathbb{Q}$

Ubit vse