Formule

$\#$ permutacij $= |\{\sigma : A \xrightarrow{\sim} A\}| = n!$

$B \subseteq A \Rightarrow |A - B| = |A| - |B|$

$|A \times B| = |A||B|$

Št neurejenih izborov podmnožice k od $n = \binom{n}{k}$

multinomski: n_1 tipa 1 n_2 tipa 2, … n_k tipa k

$\#$ permutacij neurejeno $= \dfrac{n!}{n_1! \cdot \ldots \cdot n_k!}$

① $J = \{$ $\underbrace{\text{juha, solata}}_{\text{predjed}}$, $\overset{\text{meso}}{\text{riba}}$, zelenjava, sladoled, tortica $\}$

$\underbrace{\text{riba, zelenjava}}_{\text{glavna jed}}$ $\underbrace{\text{sladoled, tortica}}_{\text{sladica}}$

$2\cdot 3\cdot 2$ je št možnih kosil

$= 12$

② kup kart 52 kart. karte razdelimo na 4 igralce

vsak dobi 13

1. kolko načinov lahko karte tako razdelimo
2. kolko je da ima vsak igralec karte samo enega tipa (pik, srce, križ, kara)

$$4!\cdot\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} =$$

$$= \frac{52!\ (52-13)!\ \cdots}{13!\,(52-13)!\ 13!\ \cdots\ 13} = \frac{52!}{(13!)^4}4!$$

2. 1, 4!

③ Kostanjevica na Krki ima 702 prebivalcev

Dokaži: Najmanj dva prebivalca imata enaki začetnici (25 črk)

$$\frac{25\cdot 25}{2} = 625$$

50
125
623

$702-625>0$

④ imamo 10 kart

2 rdeči, 3 zelene, 5 modrih

1). # možnosti da pred rdečo ni izbran ... pred ... modra

— — — — — — — — — —

1. 0
2. $\frac{8!}{1!\,2!\,5!}$
3. $\frac{7!}{2!\,5!} + 2\cdot\frac{7!}{4!\,1!\,2!}$
4.

R Z Z Z

— — — — — — — — — —

$$\frac{\binom{10}{4}\cdot 6}{2} = \frac{10\cdot \cancel{8}\cdot 9\cdot 7}{\cancel{2\cdot 3\cdot 4}\cdot 2}\cdot\cancel{6} = \frac{1260}{2} = 630$$

modre

$$\binom{10}{5}4 = \frac{10!}{5!\,5!}4 = \frac{10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2}\cdot 4 =$$

=

ii) možnosti da bo prva rdeča krogla izvlečena pred zadnjo zeleno

$\binom{10}{5}\cdot\binom{4}{2}$ $\frac{4\cdot 3}{2} = 6$ — — — — — —

zelena

$\binom{10}{5}6 + \binom{10}{5}3$

$$\frac{10!}{3!5!2!} - \binom{10}{5} = \frac{10!}{5!}\left(\frac{1}{3!2!} - \frac{1}{5!}\right) =$$

$$= \frac{10!}{5!}\left(\frac{5! - \overset{3!2}{12}}{5!2!3!}\right) =$$

$$= \frac{10!}{5!}\,\frac{4\cdot 5 - 2}{5!2!} = \frac{10!}{5!5!}\,\frac{18}{2} = \binom{10}{5}9$$

13.10

1) 3 rdeče
1 zelena
5 belih

izberemo 2

i) $\Omega = ?$

$$\Omega = \binom{V}{2} \quad V = \{r, r, r, z, b, b, b, b, b\}$$

$$\Omega = \{(r,r), (r,b), (r,z), (z,b), (z,r), (b,b) (b,r), (b,z)\}$$

ii) najprej rdeča: $\frac{3}{9} = \frac{1}{3}$

iii) $R_1 = \frac{1}{3}$ druga zelena: $\frac{8}{9} \cdot \frac{1}{8} = \frac{1}{9}$

$$P(R_1 \cup Z_2) = P(R_1) + P(Z_2) - P(R_1 \cap Z_2)$$

$$P(R_1 \cap Z_2): \frac{3}{9} \cdot \frac{1}{8}$$

$$P(\dots) = \frac{4}{9} - \frac{3}{8 \cdot 9} = \frac{29}{8 \cdot 9}$$

2) 2 orní n bílých

i) $\bar{C}_{j,k} = \{$ nejprve v k-tém kroku zločené koule $\}$

$P(\bar{C}_{j,k}) = ?$ funkce j,k,n

$$\bar{C}_{1,k} = \left(\frac{n}{n+2}\right)^{j-1} \frac{2}{n+2} \left(\frac{n+1}{n+2}\right)^{k-j-1} \left(\frac{1}{n+2}\right) =$$

$$= \frac{2 \cdot n^{j-1}}{(n+2)^{j+k-j-1+1}} (n+1)^{k-j-1} = 2n^{j-1} \frac{(n+1)^{k-j-1}}{(n+2)^k}$$

$X = k$

$$P(x,k) = P\left(\bigcup_{i=1}^{k-1} \bar{C}_{i,k}\right) = \sum_{i=1}^{k-1} P(\bar{C}_{i,k}) = \frac{2}{(n+2)^k} \sum_{k=1}^{k} n^{j-1} (n+1)^{k-j-1}$$

$$n^{j-1} (n+1)^{k-j-1} = \frac{n^j}{n(n+1)^j} (n+1)^{k-1} = \left(\frac{n}{n+1}\right)^j \frac{(n+1)^{k-1}}{n}$$

$$= \frac{2(n+1)^{k-1}}{n(n+2)^k} \quad \frac{1-\left(\frac{n}{n+1}\right)^k}{1-\frac{n}{n+1}} = \frac{2((n+1)^k - n^k)}{n(n+2)^k}$$

ii) $P(\bar{C}_1 | X=k)$

$$P\left(\bigcup_{i=2}^{k} \bar{C}_{i,k}\right) = P(x=k) - P(\bar{C}_{1,k}) = \frac{2(n+1)^{k-1} - n^{k-1})}{(n+2)^k} -$$

$$2 \frac{(n+1)^k}{(n+2)^k} =$$

$$= \frac{2}{n(n+2)^k} \left((n+1)^k (n+1) - n^k\right) = \frac{1}{n(n+2)^k} (n+1)^{k+1} - n^k)$$

$$\text{cz}: \frac{2}{\cancel{(n+2)}^k} \left((n+1)^{k-1} \cancel{(n+2)}^k - n^{k-1}\right) =$$

$$2(n+1)^{k-1} - \frac{n^{k-1}}{(n+2)^k}$$

3)

K.... konzervativec $\rightsquigarrow P(A|K) = 1$

L.... levičar $\rightsquigarrow P(A|L) = r$

$r=0 \Rightarrow P(A)=1$

$$P(A) = P(A|K)P(K) + P(A|L)P(L) =$$
$$1\cdot p + (1-p)(1-r)$$

$$P(V_3|V_2) = \frac{P(V_3 \cap V_2)}{P(V_2)} = \frac{P(V_3)}{P(V_2)}$$

$$P(V_2) = P(V_2|C)P(C) + P(V_2|L)P(L) =$$
$$= 1\cdot p + (1-p)(1-r)$$

$$P(V_3) = P(V_3|C)p + P(V_3|L)(1-p) =$$
$$1\cdot p + (1-r)^2(1-p)$$

$$\Rightarrow P(V_3|V_2) = \frac{p + (1+r)^2(1-p)}{p + (1+r)(1-p)}$$