Formule

$\#$ permutacij $= |\{\sigma : A \xrightarrow{\sim} A\}| = n!$

$B \subseteq A \Rightarrow |A-B| = |A| - |B|$

$|A \times B| = |A||B|$

Št neurejenih izborov podmnožice k od $n = \binom{n}{k}$

multinomski: n_1 tipa 1 n_2 tipa 2, ... n_k tipa k

$\#$ permutacij neurejeno $= \dfrac{n!}{n_1! \cdot \ldots \cdot n_k!}$

1. $J = \{$ $\underbrace{\text{juha, solata}}_{\text{predjed}}$, $\underbrace{\overset{\text{meso}}{\text{riba}}, \text{zelenjava}}_{\text{glavna jed}}$, $\underbrace{\text{sladoled, torta}}_{\text{sladica}}$ $\}$

$2 \cdot 3 \cdot 2$ je št možnih kosil
$$= 12$$

2. kup kart 52 kart. karte razdelimo na 4 igralce vsak dobi 13

1. kolko načinov lahko karte tako razdelimo
2. kolko ~~verj~~ da ima vsak igralec karte samo enega tipa (pik, srce, križ, karo)

$$4! \cdot \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} =$$

$$= \frac{52! \; (52-13)! \cdots}{13!\,(52-13)!\; 13! \cdots \; 13} = \frac{52!}{(13!)^4} 4!$$

2. $1, \vee 4!$

③ Konstanjevica na Krki ima 702 prebivalcev

Dokaži: Najmanj dva prebivalca imata enaki začetnici (25 črk)

$$\frac{25 \cdot 25^{2}}{50} = 625$$

125

623

$702 - 625 > 0$

④ imamo 10 kart

2 rdeči, 3 zelene, 5 modrih

1). # možnosti da je prva rdeča bila izvlečena

pred prva zelena

1. 0
2. $\frac{8!}{1!\,2!\,5!}$
3. $\frac{7!}{2!\,5!} + 2 \cdot \frac{7!}{4!\,1!\,2!}$
4.

R Z Z Z

$$\frac{\binom{10}{4} \cdot 6}{2} = \frac{10 \cdot \cancel{8}^{2} \cdot 9 \cdot 7}{\cancel{2 \cdot 3 \cdot 4} \cdot 2} \cdot \cancel{6} = \frac{1260}{2} = 630$$

modre

$$\binom{10}{5} 4 = \frac{10!}{5!\,5!} 4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2} \cdot 4 =$$

$=$

ii) možnosti da bo prva rdeča kroglica izvlečena pred zadnjo zeleno

$\binom{10}{5}\cdot\binom{4}{2}$ $\quad \frac{4\cdot 3}{2} = 6$ $\quad$ _ _ _ _ _

zelena

$\binom{10}{5}6 + \binom{10}{5}3$

$$\frac{10!}{3!5!2!} - \binom{10}{5} = \frac{10!}{5!}\left(\frac{1}{3!2!} - \frac{1}{5!}\right) =$$

$$= \frac{10!}{5!}\left(\frac{5! - \overset{3!2}{12}}{5!2!3!}\right) =$$

$$= \frac{10!}{5!}\,\frac{4\cdot 5 - 2}{5!2!} = \frac{10!}{5!5!}\,\frac{18}{2}$$

$$= \binom{10}{5}9$$

13.10

1) 3 rdeče
1 zelena
5 belih

izberemo 2

i) $\Omega = ?$

$$\Omega = \binom{V}{2} \qquad V = \{r, r, r, z, b, b, b, b, b\}$$

$\Omega = \{(r,r), (r,b), (r,z), (z,b), (z,r), (b,b)\ (b,r), (b,z)\}$

ii) najprej rdeča: $\frac{3}{9} = \frac{1}{3}$

iii) $R_1 = \frac{1}{3}$ druga zelena: $\frac{8}{9} \cdot \frac{1}{8} = \frac{1}{9}$

$$P(R_1 \cup Z_2) = P(R_1) + P(Z_2) - P(R_1 \cap Z_2)$$

$$P(R_1 \cap Z_2): \frac{3}{9} \cdot \frac{1}{8}$$

$$P(\ldots) = \frac{4}{9} - \frac{3}{8 \cdot 9} = \frac{29}{8 \cdot 9}$$

2) 2 črni n belih

i) $\bar{C}_{j,k} = \{$ne j-tem in k-tem koraku izvlečena kroglica$\}$

$P(\bar{C}_{j,k}) = ?$ funkcije j, k, n

$$\bar{C}_{1,k} = \left(\frac{n}{n+2}\right)^{j-1} \frac{2}{n+2} \left(\frac{n+1}{n+2}\right)^{k-j-1} \left(\frac{1}{n+2}\right) =$$

$$= \frac{2 \cdot n^{j-1}}{(n+2)^{j+k-j-1+1}} (n+1)^{k-j-1} = 2n^{j-1} \frac{(n+1)^{k-j-1}}{(n+2)^k}$$

$X = k$

$$P(X, k) = P\left(\bigcup_{i=1}^{k-1} \bar{C}_{i,k}\right) = \sum_{i=1}^{k-1} P(\bar{C}_{i,k}) = \frac{2}{(n+2)^k} \sum_{k=1}^{k} n^{j-1} (n+1)^{k-j-1}$$

$$n^{j-1} (n+1)^{k-j-1} = \frac{n^j}{n(n+1)^j} (n+1)^{k-1} = \left(\frac{n}{n+1}\right)^j \frac{(n+1)^{k-1}}{n}$$

$$= \frac{2(n+1)^{k-1}}{n(n+2)^k} \quad \frac{1 - \left(\frac{n}{n+1}\right)^k}{1 - \frac{n}{n+1}} = \frac{2((n+1)^k - n^k)}{n(n+2)^k}$$

ii) $P(\bar{C}_1 | X = k)$

$$P\left(\bigcup_{i=2}^{k} \bar{C}_{i,k}\right) = P(X = k) - P(\bar{C}_{1,k}) = \frac{2(n+1)^{k-1} - n^{k-1})}{(n+2)^k} - 2\frac{(n+1)^k}{(n+2)^k} =$$

$$= \frac{2}{n(n+2)^k}\left((n+1)^k (n+1) - n^k\right) = \frac{1}{n(n+2)^k}(n+1)^{k+1} - n^k)$$

cz: $\frac{2}{\cancel{(n+2)}^k}\left((n+1)^{k-1} \cancel{(n+2)}^k - n^{k-1}\right) =$

$$2(n+1)^{k-1} - \frac{n^{k-1}}{(n+2)^k}$$

3)

$K \ldots$ konzervativec $\rightsquigarrow P(A|K) = 1$

$L \ldots$ levičar $\rightsquigarrow P(A|L) = r$

$r = 0 \Rightarrow P(A) = 1$

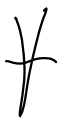
$$P(A) = P(A|K)P(K) + P(A|L)\,P(L) =$$
$$1 \cdot p + (1-p)(1-r)$$

$$P(V_3|V_2) = \frac{P(V_3 \cap V_2)}{P(V_2)} = \frac{P(V_3)}{P(V_2)}$$

$$P(V_2) = P(V_2|C)\,P(C) + P(V_2|L)\,P(L) =$$
$$= 1 \cdot p + (1-p)(1-r)$$

$$P(V_3) = P(V_3|C)\,p + P(V_3|L)\,(1-p) =$$
$$1 \cdot p + (1-r)^2(1-p)$$

$$\Rightarrow P(V_3|V_2) = \frac{p + (1+r)^2(1-p)}{p + (1+r)(1-p)}$$



20.10

1) Mečemo kovanec, dokler ne pade najmanj eno število in en grb

$P(\{grb\}) = p \quad 0<p<1$

Naj bo $\{X=k\} = \{$ prvič pade najmanj 1 število in 1 grb na koraku $k\}$

Meti so seboj neodvisni

Računajte $P(X \geq k)$

$$P(X \geq k) = \sum_{k=n}^{\infty} P(x=n) = \sum_{n=k}^{\infty} \left(p^{n-1}(1-p) + (1-p)^{n-1} p \right) =$$

$$= \frac{(1-p)}{p^k} \sum_{k=0}^{\infty} p^n + \frac{p}{(1-p)^k} \sum_{n=0}^{\infty} (1-p)^n =$$

$$\frac{\cancel{(1-p)}}{p^{k-1}} \frac{1}{\cancel{1-p}} + \frac{\cancel{p}}{(1-p)^{k-1}} \frac{1}{\cancel{1-1+p}} =$$

$$= \frac{1}{p^{k-1}} + \frac{1}{(1-p)^{k-1}}$$

v prvih $n-1$ metih ~~...~~ je samo isto

~~p(k-1)~~ $p^{k-1} + (1-p)^{k-1}$

$p=0 \vee p=1 \Rightarrow \quad p(x>k) = 0$

$$= \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1}(1-p^{(k-1)})} = \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1} - p^{2k-1}} \quad ???$$

2)

Mečemo kovanec dokler ne dobimo ali

m grbov ali na števil

max metov: $m+n$ min metov: $\min\{m,n\}$

$P(\{grb\}) = p$

$P(\{X = k\}) =$

$\{X = k\} = $ prič ko dobimo ali m grbov ali m števil

$P(\{X = \cancel{grb}$ pri čemer m grbov$\}) = p^m (p-1)^{k-m}$

$+ p^{k-m} (p-1)^m$

~~k metov~~ m jih je neke vrste $\therefore$ $k < 2m$

$p^m (p-1)^{k-m} + p^{k-m} (p-1)^m$

$P(X = \{\ \}) =$

$P(X = \{k-1$ tem mestu neke m-1 grbov$\})$

$\{G=k\} = \{$ pade $m-1$ grbov v $k-1$ kovancih, potem pa spet pade grb $\}$

$\{S=k\} = \{$Na $k-1$ kovancih pade $m-1$ številk potem pa spet številka$\}$

$$\binom{k-1}{m-1}\left(p^m(1-p)^{k-m} + p^{k-m}(1-p)^m\right)$$

3)

$X \sim \mathrm{Bin}\,(n, \underset{\overset{\shortparallel}{p}}{\tfrac{1}{2}})$

$$P(X=k) = \binom{n}{k}\left(\frac{1}{2}\right)^k\left(1-\frac{1}{2}\right)^{n-k} = \binom{n}{k}\frac{1}{2^n}$$

Definiramo slučajni spremenljivki

Y za $k=0,\dots,n$ velja $P(Z) \sim P(X,Y)$

$$P(X=k, Y=k+1) = P(X=k)\frac{n-k}{n} \quad (a)$$

in $P(X=k, Y=k-1) = P(X=k)\frac{k}{n}$ (b)

in $P(X=k, Y=l) = 0$ če $|k-l|>1$ ali $l=k$

Računaj te porazdelitev slučajne spremenljivke Y

Rešitev:

Popolna verjetnost

$$P(Y=l) = \sum_{k=0}^{n} P(X=k, Y=l) =$$

$$= P(X=l-1, Y=l) + P(X=l+1, Y=l)$$

$$P(Y=l) = \binom{n}{l-1}\frac{1}{2^n}\frac{n-l+1}{n} + \binom{n}{l+1}\frac{1}{2^n}\frac{l+1}{n} =$$

$$= \frac{1}{2^n}\left(\frac{n!}{(l-1)!\,(n-(l-1))!}\cdot\frac{n-(l-1)}{n} + \frac{n!}{(l+1)!\,(n-(l+1))!}\cdot\frac{l+1}{n}\right)$$

$$= \frac{1}{2^n}\left(\binom{n-1}{l-1} + \binom{n-1}{l}\right) = \frac{1}{2^n}\binom{n}{l}$$

4)

Naj bodo $X \sim B(\frac{1}{2}) \Rightarrow X=0$ z verjetnostjo $\frac{1}{2}$ in
$X=1$ z verjetnostjo $\frac{1}{2}$

$Y \sim B(\frac{1}{2})$

$Z = Z(X,Y)$

$$P(Z) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \quad "\in \mathbb{R}^4"$$

$P(X=1, Y=0)$

Velja: $* \ 0 \leq p_{ij} \leq 1$

$* \ \sum_{0 \leq i,j \leq 1} p_{ij} = 1$

Recimo da $P(Z) = \begin{pmatrix} \frac{1}{4}+\delta & \frac{1}{4}-\delta \\ \frac{1}{4}-\delta & \frac{1}{4}+\delta \end{pmatrix}$

kdaj sta X in Y neodvisno od δ

$-\frac{1}{4} \leq \delta \leq \frac{1}{4}$

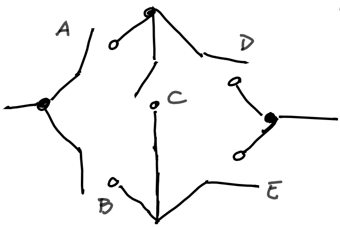
neodvisni

Dejstvo: $X \perp Y \Longleftrightarrow \det(p_{ij}) = 0$

$\det(p_{ij}) = \left((\frac{1}{4}-\delta)^2 - (\frac{1}{4}-\delta)^2 \right) = \delta$

$\Rightarrow X \perp Y$ če je $\delta = 0$

5) V vezju



Vsako stikalo prepušča el. tok z verjetnostjo $\frac{1}{3}$ in posamezna stikala so med seboj neodvisna

A = dogodek, da tok gre preko A

Kolikšna je verjetnost, da vezje prepušča?

$$\overset{A}{\frac{1}{3}}\left(\underset{D}{\frac{1}{3}}+\underset{C}{\frac{1}{3}}\cdot\underset{E}{\frac{1}{3}}\right)+\underset{B}{\frac{1}{3}}\left(\underset{E}{\frac{1}{3}}+\underset{C}{\frac{1}{3}}\cdot\underset{D}{\frac{1}{3}}\right) \quad ??$$

A

$$\underset{A}{\frac{1}{3}}\left(\underset{D}{\frac{1}{3}}+\frac{1}{3}\cdot\frac{1}{3}-\frac{1}{3}\cdot\frac{1}{3}\right)+\frac{1}{3}\cdot\frac{1}{3}-\underbrace{\frac{1}{3}\cdot\frac{1}{3}}_{\text{odštejem}}=\frac{1}{9} \quad ??$$

$$\underset{\uparrow\ \text{zaprto}}{\frac{2}{3}}\left(\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{3}\right)+\frac{1}{3}\left(\dots\right)$$

st gabov p8

$$P(X=k) = P(H_c)\cdot P(X=k|H_c) + P(X=k|H_{gg})P(H_{gg}) + P(H_{gc})P(X=k|H_{g})$$

$$P(X=0|H_{gg})=1$$
$$P(X=k|H_{gg})=0$$
$$P(X=0|H_{gc})=0$$
$$P(X=k|H_{gc})=P(X=k-1)$$
$$P(X=0|H_c)$$

$k=0 \Rightarrow$

$$P(X=0) = (1-p)P(X=0) + 1\cdot p^2 + p(1-p)\cdot 0$$

$$P(X=0)(1-1+p) = p^2$$
$$P(X=0) = p$$

$k\neq 0 \Rightarrow$

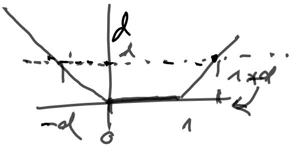
$$P(X=k) = P(X=k)(1-p) + \cancel{P(X=k-1)p^2} + p(p-1)P(X=k-1)$$

V españa

Naj bo $X \in U[3,3]$ in definirano
$$\{D=d\} = \{ m_{erv}(X,[0,1]) = d \}$$

Izračunajte kumulativno porazdelitveno funkcijo na $F_d(D)$ in narišite njen graf

$\boxed{F_D(d) = P(D \le d)}$ $D = g(X)$ za $g(x) = d(X,[0,1])$



$$g(x) = \begin{cases} -x & ; x \le 0 \\ 0 & ; 0 \le x \le 1 \\ x-1 & ; 1 \le 0 \end{cases}$$

gostota:
$$f_X(x) = \begin{cases} \frac{1}{3-(-3)} = \frac{1}{6} & ; x \in [3,3] \\ 0 & ; \text{sicer} \end{cases}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)dx = \int_{-3}^{x} f_X(x)dx = \begin{cases} \frac{x+3}{6} & ; -3 \le x \le 3 \\ 0 & ; x < -3 \\ 1 & ; x > 3 \end{cases}$$

Kaj so dogodki $\{D \le d\}$?

$d < 0 \Rightarrow \{D \le d\} = \{\}$

$d = 0 \Rightarrow \{D \le d\} = [0,1]$ $\quad F_D(0) = P(X \in [0,1]) = F_X(1) - F_X(0)$

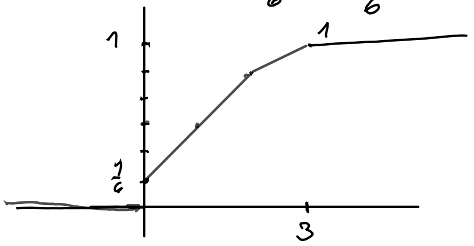
$d \in [0,2] \Rightarrow \{D \le d\} = [-d, 1+d]$ $\quad P_D(d) = F_X(1+d) - F(-d)$

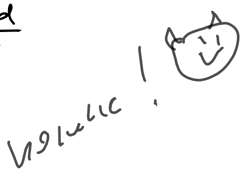
$d \in [2,3] \Rightarrow \{D \le d\} = [-d, 3]$ $\quad P_D(d) = F_X(3) - F(-d)$

$$F_D(0) = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$F_D(d \in [0,2]) = \frac{d+4}{6} - \frac{-d+3}{6} = \frac{2d+1}{6}$$

$$F_D(d \in [2,3]) = \frac{6}{6} - \frac{-d+3}{6} = \frac{3+d}{6}$$





$$f_d = F_D'(d) = \begin{cases} 0 & : d < 0 \\ \frac{1}{3} & d \in [0,2] \\ \frac{d}{6} & d \in [2,3] \\ 0 & d > 0 \end{cases}$$

Definirajmo $X \sim \text{Cauchy}$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

a) Izračunajte $Y = \frac{1}{1+X^2}$

Namig: $F_Y(y) = P(Y \leq y) = P\left(\frac{1}{1+X^2} \leq y\right) =$

$$= P\left(\frac{1}{y} \leq 1+X^2\right) = P\left(\sqrt{\frac{1}{y}-1} \leq X\right) = \int_{-\infty}^{x} f_X(*)\,d*$$

$-X \leq$

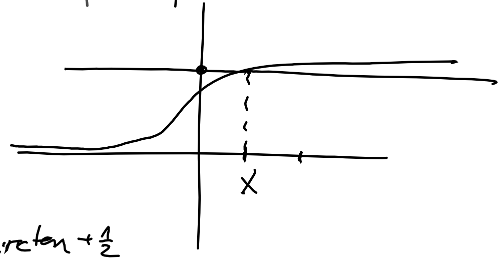
X

$$P\left(\sqrt{\frac{1}{y}-1} \leq X\right) = \frac{1}{\pi}\arctan(x)\Big|_{-\infty}^{x} = \frac{1}{\pi}\arctan x + \frac{1}{2}$$

$-X \leq$

$$P\left(\sqrt{\frac{1}{y}-1} \leq X\right) - P\left(-\sqrt{\frac{1}{y}-1} \leq X\right) =$$

$\cancel{\frac{1}{\pi}\arctan x + \frac{1}{2}}$



$y = \frac{1}{\pi}\arctan + \frac{1}{2}$

$\tan\left(\pi x - \frac{\pi}{2}\right)$

$$P(\cancel{Y} \le y) = P_X\left(X^2 \ge \frac{1}{y} - 1\right) = \qquad y = \frac{1}{1+X^2}$$

$$= \underset{x \ge 0}{P}\left(X \ge \sqrt{\frac{1-y}{y}}\right) + \underset{x<0}{P}\left(X \le -\sqrt{\frac{1-y}{y}}\right) =$$

det

$$= 2\,P\left(X \ge \sqrt{\frac{1-y}{y}}\right) = 2\left(1 - F_X\left(\sqrt{\frac{1-y}{y}}\right)\right) = F_Y(y)$$

simetrija

$$f_X(y) = \frac{d}{dx} F_X(x) \qquad \text{iščemo } f_Y(y)$$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

$$f_Y(y) = F_Y'(y)$$

$$\underline{f_Y(y)} = \left(2 - 2F_X\left(\sqrt{\frac{1-y}{y}}\right)\right)' = \qquad \sqrt{\frac{1}{y}-1}$$

$$= -2 f_X\left(\sqrt{\frac{1-y}{y}}\right) \frac{-\frac{1}{y^2}}{2\sqrt{\frac{1}{y}-1}} =$$

$$= -2 \frac{1}{\pi\left(1 + \frac{1-y}{y}\right)} \cdot \frac{-1}{2y^2\sqrt{\frac{1-y}{y}}} = \frac{1}{\pi} \frac{\sqrt{y}}{y^2\sqrt{1-y}}$$

$$Y = \frac{1}{1+X^2}$$

$$F_Y(y) = P_Y(Y \leq y) = P_X\left(\frac{1}{1+X^2} \leq y\right) =$$

$$\overset{?}{\underset{\checkmark}{=}} P_X\left(X^2 \geq \frac{1}{y} - 1\right) = \quad \text{Velja, ker je } y > 0$$

ker $Y : \Omega \to (0, 1]$

in ker je $1 + x^2 > 0$

$$= P\left(X \geq \sqrt{\frac{1-y}{y}}\right) + P\left(X \leq -\sqrt{\frac{1-y}{y}}\right) \underset{\text{simetrija}}{=} 2P\left(X \geq \sqrt{\frac{1-y}{y}}\right) =$$

$$= 2\left(1 - F_X\left(\sqrt{\frac{1-y}{y}}\right)\right) \qquad f_X(x) = \frac{1}{\pi(1+x^2)}$$

$F_X'(y) = f_X(y)$, če obstaja

$$\Rightarrow f_Y(y) = F_Y'(y) = -2F_X'\left(\sqrt{\frac{1-y}{y}}\right)$$

$$= -2 f_X\left(\sqrt{\frac{1-y}{y}}\right) \frac{1}{2}\sqrt{\frac{y}{1-y}} \cdot \frac{-y-1+y}{y^2} =$$

$$= f_X\left(\sqrt{\frac{1-y}{y}}\right) \sqrt{\frac{y}{1-y}} \, \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \, \frac{1}{1 + \frac{1-y}{y}} \sqrt{\frac{y}{1-y}} \, \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \, \frac{y}{1} \, \frac{1}{y^2} \sqrt{\frac{y}{1-y}} = \frac{1}{\pi y}\sqrt{\frac{y}{1-y}} = \underline{\underline{\frac{1}{\pi\sqrt{y(1-y)}}}}$$

Deth

	No	yes	
White	141	19	160
Black	149	17	166
	290	36	326

Definiramo $R: \{\text{white}, \text{black}\} \longrightarrow \mathbb{R}$

$R(\text{white}) = 0$

$R(\text{black}) = 1$

$D: \{\text{No deth}, \text{yes deth}\}$

$\underline{X} = (R, D)$

$\underline{U} = \{$ število $x = (i, j)$

Vprašanja?

1. $P(D=1 \mid R=i) \quad ; i \in \{0, 1\}$
2. $D \perp R$? (Ali sta neodvisni spremenljivki ?)

Lahko ocenimo $\hat{P}_{ij} = \dfrac{\#(x; x=(i,j))}{N=136}$

$$\hat{P}_{ij} = \begin{pmatrix} \frac{141}{326} & \frac{19}{326} \\ \frac{149}{326} & \frac{17}{326} \end{pmatrix}$$

$\hat{P}_{00} + \hat{P}_{01} =$ vrjetnost da je bel $= P(R=0)$

$= P(R=1)$

$P(D=0) \quad P(D=1)$

1) $P(D=1 \mid R=0) = \dfrac{P_{01}}{P_{00}+P_{01}} = 0{,}11$

$\dfrac{P_{11}}{P_{10}+P_{11}} = 0{,}10$

1. 1.12.

n gledalcev z črnimi jaknami

X... št gledalcev ki dobi nazaj jakno

$E(X) = ?$

$A_k = \{$ k-ti obiskovalec dobi svojo jakno

$P(A_k) = \frac{1}{n}$

Definirajmo $I_k = \begin{cases} 1 & : A_k \text{ se zgodi} \\ 0 & : \neg A_k \end{cases}$

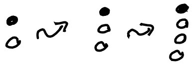
$$X = \sum_{k=1}^{n} I_k$$

$$E(X) = \sum E(I_k) = \sum \frac{1}{n} = n \cdot \frac{1}{n} = 1$$

$$\| \qquad \text{0 ali 1}$$

$$\sum 1 P(I_k = 1)$$

2. naloga



$$P(X=k) = \frac{k}{(k+1)!}$$

$X \ldots$ št korakov igre

$$P(X > k) = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\ldots\cdot\frac{1}{k}\cdot\frac{1}{k+1} = \frac{1}{(k+1)!}$$

št korakov $> k$

$$\cancel{\sum_{k=X}^{\infty} P(X=k)}$$

$$I_k \ldots \begin{cases} 1 & : \text{ne ko se konča} \\ 0 & ; \end{cases}$$

$E(X) = ?$

$P(X=k)$

$$E(X) = \sum k \frac{k}{(k+1)!} = \sum \frac{k^2}{(k+1)!}$$

$$E(X) = E\Big(\sum_{k=} I_k\Big) = \sum E(I_k) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} = e - 1$$

3. naloga

X zvezna slučajna spremenljivka

$$f_X = \begin{cases} \frac{1}{\sqrt{2\pi}\, x^{\frac{3}{2}}} e^{-\frac{1}{2x}} & \text{za } x>0 \\ 0 & \text{; sicer} \end{cases}$$

$Z \sim N(0,1)$ pokažite

$$F_X(x) = 2P_Z\left(Z > \frac{1}{\sqrt{x}}\right) = 2\left(1 - F_Z\left(Z \le \frac{1}{\sqrt{x}}\right)\right)$$

$$= 2\left(1 - \Phi\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$F_X(x) = \int_0^x \frac{1}{\sqrt{2\pi}\, x^{\frac{3}{2}}} e^{-\frac{1}{2x}} dx = -2\int_{\infty}^{\frac{1}{\sqrt{x}}} \frac{1}{\sqrt{2\pi}} \cancel{u} e^{-\frac{u^2}{2}} =$$

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty}$$

$$2\int_{\frac{1}{\sqrt{x}}}^{\infty} \dots \; 2\left(1 - \Phi\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$u^2 = \frac{1}{x}$$

$$2u\,du = -\frac{1}{x^2} dx \quad = dx - \frac{2}{u}$$

$$dx = -2\frac{1}{u^3} du$$

$X \sim U(a,b) \qquad 0<a<b<1$

$Y = -\frac{1}{\lambda} \ln X \qquad \lambda > 0$

$$f_X(x) = \begin{cases} \frac{1}{a-b} & x \in (a,b) \\ 0 & ; \end{cases}$$

$$P(Y \le y) = P(\cancel{x \le} \tfrac{1}{\lambda} \ln (\cancel{X})) = P(-\lambda x \le -\ln X) =$$

$$P(e^{-\lambda x} \ge X) \overset{1-}{=} F_X(e^{\lambda x}) \overset{1-}{=} \int_{-\infty}^{e^{-\lambda x}} \begin{cases} \frac{1}{a-b} & ; \\ 0 & ; \end{cases} =$$

injektivnost

$-\infty = a$

$$= \begin{cases} 0 & ; e^{\lambda x} < a \text{ nikdi} \\ \frac{e^{\lambda x} - a}{b-a} & . \ e^{\lambda x} \in (a,b) \\ 1 & ; e^{\lambda x} \ge b \end{cases}$$

$$\begin{cases} 1 & \\ 1 - \frac{e^{-\lambda x} - a}{b-a} & e^{-\lambda x} \in (a,b) \\ 0 & \end{cases}$$

$e^{-\lambda x} \in (a,b)$

$-\lambda x \in (\ln \cancel{a}, \ln b)$

$\lambda x \in (-\ln b, \ln a)$

$x \in \left(-\frac{\ln b}{\lambda}, -\frac{\ln a}{\lambda}\right)$

$$P(X=k, Y=l) = \begin{cases} \frac{1}{2\log 2}\frac{1}{l 2^k} & k \in \mathbb{N}\ l \in \{1, \dots k\} \\ 0 & \text{sicer} \end{cases}$$

$P(X=k) = ?$

$P(Y=l) = ?$

$$P(X=k) = \sum_l P(X=k, Y=l) = \sum_{l=1}^{k} \frac{1}{2\log 2} \frac{1}{l 2^k} =$$

$$\frac{1}{2^{k+1}\ln 2} \sum_{l=1}^{k} \frac{1}{l}$$

$$P(Y=l) = \sum_k P(X=k, Y=l) = \frac{1}{2\ln 2\, \cancel{l}} \cdot \sum_{k=l}^{\infty} \frac{1}{2^k} =$$

$$\frac{1}{2\ln 2} \cdot \frac{1}{2^l} \frac{1 \cdot 2}{(1-\frac{1}{2})\cdot 2} = \frac{1}{\ln 2\, 2^l}$$

$$P(X=k)P(Y=l) = \frac{1}{2 \cdot (\ln 2)^2 2^l 2^k} \cdot \sum_{i=1}^{k} \frac{1}{i} \neq \frac{1}{\log 2}$$

n. neodvisne