Furierove vrste

$$FV(f) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$\boxed{\begin{array}{l} f \text{ soda} \Rightarrow b_n = 0 \\ f \text{ liha} \Rightarrow a_n = 0 \end{array}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

Parsevalove enakost

$$\frac{1}{\pi}\int_{-\pi}^{\pi} |f(x)|^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$\int x\cos(nx)\,dx = \frac{x\sin(nx)}{n} + \frac{\cos(nx)}{n^2} + C$$

$$\int x\sin(nx)\,dx = -\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2} + C$$

$$\int x^2\cos(nx)\,dx = \frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)$$

$$\int x^2\sin(nx)\,dx = -\frac{x^2}{n}\cos x + \frac{2x}{n^2}\sin(nx) + \frac{2}{n^3}\cos(nx)$$

$$\int \cos(nx)\,dx + i\int \sin(nx)\,dx = \int e^{inx}\,dx$$

Kotne zadeve

$$\sin x \sin y = -\frac{1}{2}\left(\cos(x+y) - \cos(x-y)\right)$$

$$\cos x \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$

$$\sin x \cos y = \frac{1}{2}\left(\cos(x+y) + \sin(x-y)\right)$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

Integral:

$$X_T = \frac{\int_k x\,dm}{m(K)} = \frac{\int_k x\,\rho\,ds}{\int_k \rho\,ds}$$

$$ds = |\dot{\vec{r}}(t)|\,dt = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\,dt$$

$$\int_K u\,ds = \int_a^b u(\vec{r}(t))\,|\dot{\vec{r}}(t)|\,dt$$

$$l(K) = \int_K ds = \int_a^b |\dot{\vec{r}}(t)|\,dt$$

$$\int_K \vec{R}\,d\vec{r} = \int_K \vec{R}\cdot\vec{T}\,ds = \int_a^b \vec{R}(\vec{r}(t))\cdot\dot{\vec{r}}(t)\,dt$$

$$\iint_\Sigma u\,dS = \iint_D u(\vec{r}(s,t))\sqrt{EG - F^2}\,ds\,dt =$$

$$\iint_D u(\vec{r}(s,t))\,|\vec{r}_s \times \vec{r}_t|\,ds\,dt$$

$$\iint_{\vec{\Sigma}} \vec{R}\,d\vec{S} = \iint_D \vec{R}\cdot\vec{N}\,dS = \iint_D \vec{R}(\vec{r})\cdot(\vec{r}_s \times \vec{r}_t)\,ds\,dt$$

$$\int_K X\,dx + Y\,dy + Z\,dz = \int_K (X, Y, Z)\,d\vec{r}$$

$$\iint_\Sigma X\,dz\,dy + Y\,dx\,dz + Z\,dx\,dy = \iint_\Sigma (X, Y, Z)\,d\vec{S}$$

$$\iint_D |\vec{r}_u \times \vec{r}_v|\,du\,dv = \iint_D \sqrt{EG - F^2}\,du\,dv$$

$$E = |\vec{r}_u|^2 \quad F = \vec{r}_u \cdot \vec{r}_v \qquad G = |\vec{r}_v|^2$$

Površina grafa $f \in C^1(D)$

$$\iint_D \sqrt{1 + f_x^2 + f_y^2}$$

Površina torusa $0 < a < R$: $P = 2\pi a\, 2\pi R$

Gauss

D omejena odprta

- rob iz končnega števila odsekoma gladkih ploskev

$\vec{R} \in C^1(\bar{D})$

$$\oiint_{\partial D} \vec{R}\, d\vec{S} = \iiint_D \vec{\nabla} \cdot \vec{R}\, dV$$

Green

D omejena odprta

- končno število odsekoma gladkih krivulj za rob

$X, Y \in C^1(\bar{D})$

$$\int_{\partial D} X\,dx + Y\,dy = \iint_D (Y_x - X_y)\,dx\,dy$$

Stokes

Σ omejena odsekoma gladka,

- rob iz končnega števila odsekoma gladkih krivulj

$\vec{R} \in C^1(\bar{\Sigma})$

$$\int_{\partial \Sigma} \vec{R}\, d\vec{r} = \iint_\Sigma \vec{\nabla} \times \vec{R}\, d\vec{S}$$

Gradient $\vec{\nabla} u = (u_x, u_y, u_z)$

divergenca $\vec{\nabla} \cdot \vec{R} = (X_x, Y_y, Z_z)$

rotor $\vec{\nabla} \times \vec{R} = (Z_y - Y_z, X_z - Z_x, Y_x - X_y)$

$$S \xrightarrow{\text{grad}} V \xrightarrow{\text{rot}} V \xrightarrow{\text{div}} S$$

če naredimo dva zaporedna koraka pride 0

$\text{div} \circ \text{rot} = 0 \qquad \text{rot} \circ \text{grad} = 0$

$\text{div} \circ \text{grad} = \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$

Polje u je harmonično če $\Delta u = 0$

Polje $\vec{R}$ je potencialno če $\vec{R} = \vec{\nabla} u$

Polje $\vec{R}$ ima vektorski potencial če je $\vec{R} = \vec{\nabla} \times \vec{f}$ za nek $\vec{f}$

Polje $\vec{R}$ je irotacionalno če $\vec{\nabla} \times \vec{R} = 0$

Polje $\vec{R}$ je solenoidealno če $\vec{\nabla} \cdot \vec{R} = 0$

Furierove vrsta

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$$S \xrightarrow{\text{grad}} V \xrightarrow{\text{rot}} V \xrightarrow{\text{div}} S$$

če naredimo dva zaporedna koraka pride 0

$\operatorname{div}(\operatorname{rot}(\vec{R})) = 0$ $\qquad$ $\operatorname{rot}(\operatorname{grad}(u)) = 0$

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Gauss

D omejena odprta

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