1.1.

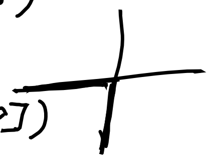
$A = [-1,1] \times [-1,1]$

$g : \mathbb{R}^2 \to \mathbb{R}/A$ hoc

a) $g_*((-\infty, 0] \times (-\infty, 0])$

~~zapte~~ ni odp ni ~~otw~~ zap

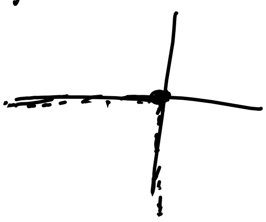
$g^*(g_*((-\infty,0] \times (-\infty,0]))$



b) $g_*(\mathbb{R}^2 - (-\infty,0] \times (-\infty,0])$

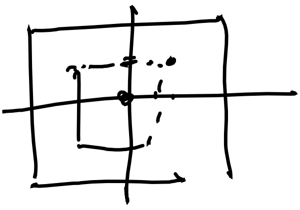
ni: oapta

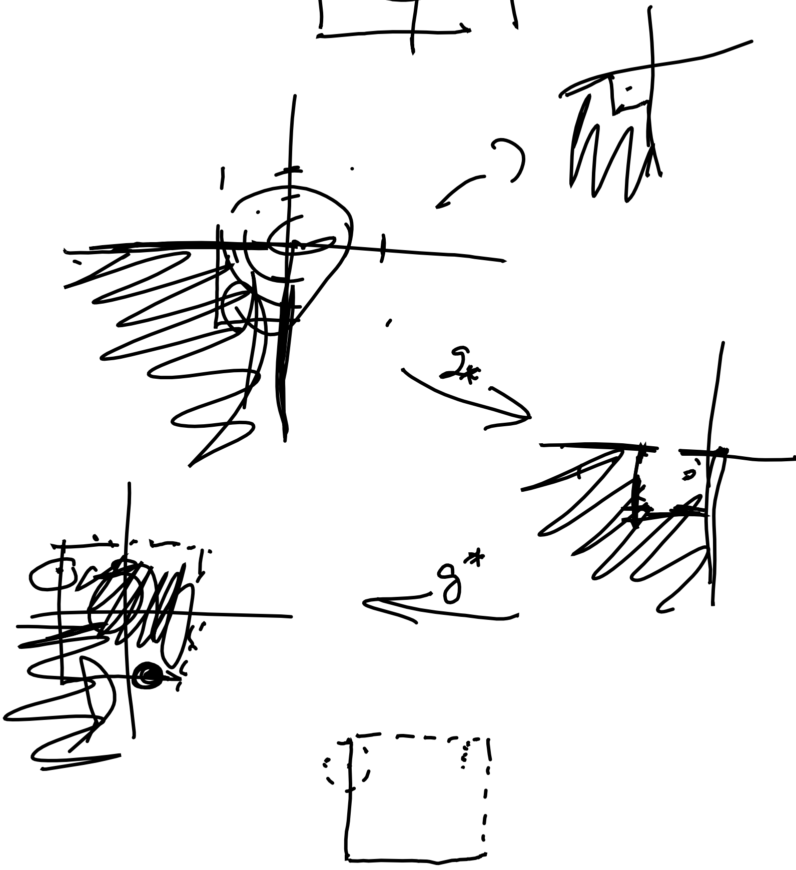
ni: zapta



c) $g_*([-2,2] \times [-2,2])$

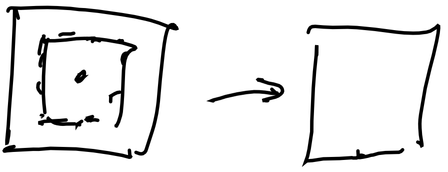
otwarta






5)

$g^*(g([-2,2] \times [-2,2])$



zapta

ni odz

c) niczdy jorzy

74

a) $[-2,2] / [-1,1] \cong [-1,1]$

b) $[-1,1] / \{-1,0,1\}$

$\cong$  $\mathbb{R}^2$

c) /

$$\mathbb{R}^n/K(0,1) \xrightarrow{\;\approx\;} \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[d] \arrow[dr, "f"] & \\
\mathbb{R}^n/K(0,1) & \mathbb{R}^n
\end{tikzcd}$$

$$f(\vec{a}) := \begin{cases} 0 ; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} ; & a \in \mathbb{R}^n - \{\overline{K}(0,1)\} \end{cases}$$

d)

$\mathbb{R}^n/_\sim \qquad x \sim y \iff \|x\| = \|y\|$

$\mathbb{R}^n/_\sim \ \tilde{\approx} \ [0,\infty)$

$f: \mathbb{R}^n \longrightarrow [0,\infty)$

$f(\vec{x}) = \|\vec{x}\|$ je surj

$a \in [0,\infty) \qquad (a,0,0\ldots) \longmapsto a$

$[x]=[y] \iff \|x\| = \|y\| \iff f(x)=f(y)$

je zvezna

iščemo preslikavo s, da velja $\quad f \circ s = id_{[0,\infty)}$

$s: a \longmapsto (a, 0 \ldots\ldots)$

Dokažimo da če $r \circ s = id_X$

$\Rightarrow r$ kvocientna, sub. žikev

$S \subseteq Y$ taka da $r^*(S)$ odp $v\ X$

$S = s^*(r^*(S)) =$

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S$

$\Rightarrow S$ je odprta

f)

$$S^n \times [-1,1] \big/ \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \to S^{n+1}$$

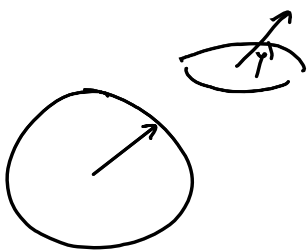
$$(x,t) \longmapsto (xb, t) = (\sqrt{1-t^2}\,x, t$$

ansatz voller

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{ g \cdot H \,;\, g \in G \} =$$

$$= \{ \{ g \cdot h \,;\, h \in H \} \,;\, g \in G \}$$

G top. grupa

$a \in G$

$$L_a : G \longrightarrow G$$
$$x \longmapsto a \cdot x$$ leva translacija

$a, b \in G$

$$h : G \longrightarrow G$$

$h(a) = b$ $\quad h = ?$

možnosti

$$L_{ba^{-1}} : x \longmapsto ba^{-1}x$$

$ba^{-1}x$

$xa^{-1}b$

$bxa^{-1}$

$a^{-1}xb$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\iff$ $ba^{-1}A$ okolica $b \in G$

$\exists U^{odp} \subseteq A \,.\, a \in U$

$b \in \; ba^{-1}A$

$a \in U \Rightarrow \underset{\overset{\parallel}{b}}{ba^{-1}a} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \overset{\subseteq A}{\text{odprta}}$ v $G$

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ $\qquad$ H okolica 1 $\Rightarrow$ H odp in zap v G

$a \in aH \subseteq H$

$\Rightarrow$ H je okolica vsake svoje točke

$G - H$ je odp.

$aH \cap H = \emptyset \Rightarrow aH = H$

$a \in G - H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$

vsak element ima okolico, ki ne seka H $\Rightarrow$ H je zaprta

c) $C$ komponenta $1 \in C$

$\Rightarrow C$ zaprta edinka v $G$

$C \leq G$

$L_a : x \mapsto ax$ je homeomorfizem za $\forall a \in C$

$\forall a \in C.\ L_a \subseteq C$

$L_a$ ohranja povezanost

Prav tako $L_a \cdot 1 = a \Rightarrow$

$L_{a*}C \cap C \Rightarrow L_{a*}C \subseteq C$

inverstiranje: invertiranje je tudi homeo $i : x \mapsto x^{-1}$

$\Rightarrow$ po istih argumentih

Ali je edinka?

$\forall a \in G.\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \mapsto axa^{-1}$ je homeomorfizem

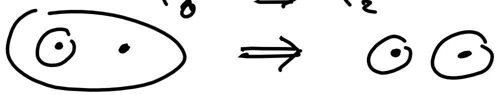
$\Rightarrow$ jo povezano in $a\,id\,a^{-1} = id \in C$

$\Rightarrow aCa^{-1} \subseteq C$

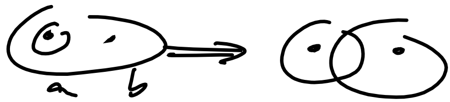
ker je kompozitum dveh translacij (leva in desna)

d) za $G$ je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$




$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G$ $\quad a \in U$, $b \notin U$ $\quad$ BŠZS

$$U^{-1} = \{a^{-1} : a \in U\}$$

$aU^{-1}b \ni b \qquad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U$. $a = ac^{-1}b$

$\Rightarrow b = c \Rightarrow b \in U$ $\times$

$T_1 \Rightarrow T_2$

$\exists U, V \in G$. $a \in U$, $b \in V$. $a \notin V$ $b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$\Delta_G = f^*(\{1\})$

$f : (x, y) \mapsto xy^{-1}$

~~za~~ zvezna praslika ~~zap~~ je zaprta

② $T_{ee}$ je $T_1$ in ni $T_2$

$\Rightarrow$ $(\mathbb{R}, +)$ ni top. grupa za $T_{ee}$

(ker 1.d)

2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

d $(m,n)(x,y) := (m+x,\ n+y)$

$\Phi: g \longmapsto \underbrace{(a \longmapsto g\cdot a)}$

bijekcija (izomorfizem)

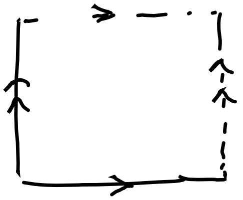
$\Phi: G \longrightarrow Bij(A)$ ← grupa za komponiranje

V širšu: $\Phi: G \longrightarrow (\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ)$

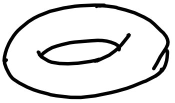

mamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ja

za nivoju topologije ? NE



Dobimo torus



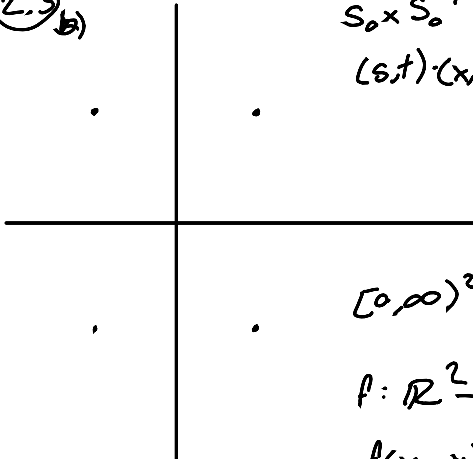
$\cong S^1 \times S^1$

$f: \mathbb{R}^2 \longrightarrow S^1\times S^1$

$(x,y) \longmapsto (e^{2\pi i x}, e^{2\pi i y})$

Tuk nism poslušala več :)

(2.3)(a)



$S_0 \times S_0 \curvearrowright \mathbb{R}^2$

$(s,t)\cdot(x,y) = (sx, ty)$

$[0,\infty)^2$

$f: \mathbb{R}^2 \to [0,\infty)^2$

$f(x,y) = (|x|, |y|)$

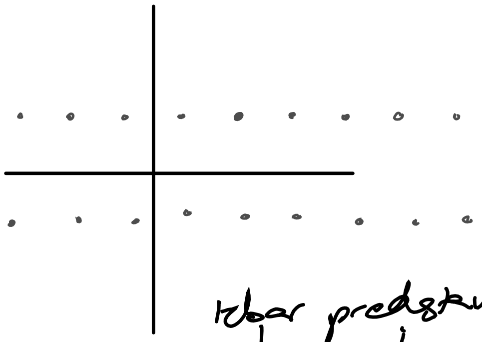
$s: [0,\infty)^2 \to \mathbb{R}^2$

$(x,y) \mapsto (x,y)$

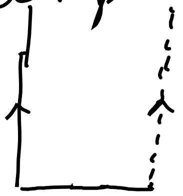
$f, s$ zvezni $\Rightarrow f$ kvocientna

c)
$\mathbb{Z}\times S^0 \curvearrowright \mathbb{R}^2$

$(m,t)\cdot(x,y)=(m+x,\ ty)$



Izbor predstavnikov



$S_1\times[0,\infty)$

$f:\mathbb{R}^2 \longrightarrow \overset{\subseteq\mathbb{C}}{S^1}\times[0,\infty)$

$(x,y)\longmapsto (e^{i2\pi x},|y|)$

zvezna ✓
surjektivna ✓

Po standardnem postopku tudi identifikacije med delovanje

$((n-\frac{1}{n},0))_n$

slika zaporedja zap v $\mathbb{R}^2$
f-slika pa ni zaprta v $S^1\times[0,\infty)$

$(1,0)$ je v zaprtju ni pa v f-sliki

Produkt dveh odprtih preslikav je odprta

$h:\mathbb{R}\longrightarrow[0,\infty)$
$x\longmapsto|x|$

Dovolj preveriti na bazi

$0\notin(a,b)$: $h(a,b)=(\min\{|a|,|b|\},\max\{\dots\})$

če $0\in(a,b)$: $h(a,b)=[0,\max\{|a|,|b|\})$

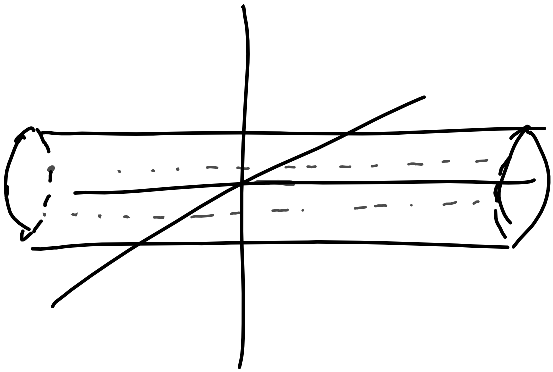
$g:\mathbb{R}\to S^1\subseteq\mathbb{C}$
$x\mapsto e^{i2\pi x}$

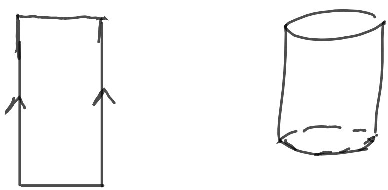
Baza z odprtimi intervali dolžine $<1$ $\Rightarrow$ slike odprte

d)

$\mathbb{Z} \times S_0 \curvearrowright \mathbb{R} \times S^1 \subseteq \mathbb{R}^3$

$(m, t) \cdot (x, y, z) := (m + x, y, tz)$

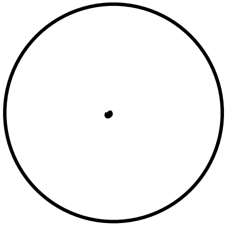




$\mathbb{R} \times S_1 \longrightarrow S_1 \times [-1, 1]$

$(t, y, z) \longmapsto (e^{2i\pi t}, y)$

a)

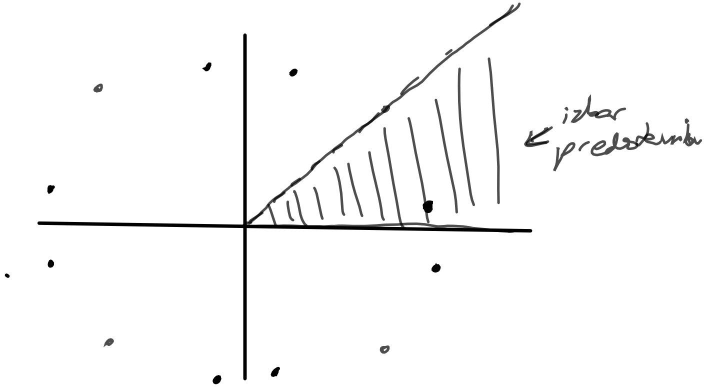


$$\vec{x} \sim \vec{y} \iff \|x\| = \|y\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(a)




$$\begin{bmatrix}1&0\\0&-1\end{bmatrix} \cong -1 \in \mathbb{C} \quad \begin{bmatrix}0&-1\\1&0\end{bmatrix} \cong i$$

Topologija ker [illegible]

$$Y = \{(x,y)\in\mathbb{R}^2;\ y<x;\ x>0$$

$$f:\mathbb{R}^2 \longrightarrow Y$$

$$(x,y) \longmapsto (\max(|x|,|y|);\ \min(|x|,|y|)$$

f retrakcija $\Rightarrow$ kvocientna v ožjem smislu

2 4

$2x = g + x$

$\mathbb{R}/\mathbb{Q}$ ni meter vložen v noben

← po delovanju grupe

$U^{\circ} \cdot g \subseteq \mathbb{R}/\mathbb{Q}$

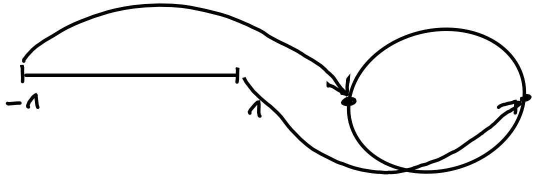
Ubit vse

(3.1) 8.4

a)

$X = [-1, 1]$ $A: \{-1, 1\}$

$Y = S^1$ $f(x) := (x, 0)$



$Z = [-1, 1] \times \{0\} \cup S^1$

$g : X + Y \longrightarrow Z$

$in_1(x) \longmapsto (x, 0)$

$in_2(y) \longmapsto y$

ekvivalenčni razredi: $(in_1(1,0)$, $in_2(1,0)$ in $in_1(-1,0)$ in $in_2(-1,0))$ neenostavno

$g(in_1(1,0)) = 1,0$

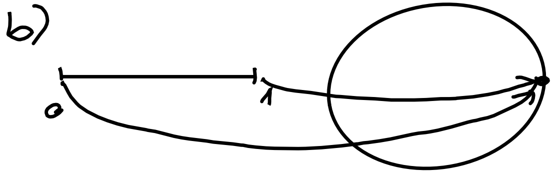
$g(in_2(1,0)) = 1,0$ podatana z drug

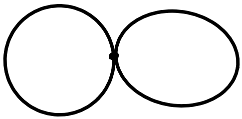
zveznost pa, ker sta posamezni zvezni in ker ~~se~~ se ujemata na preseku

Potrebno še pokazati še da loči ekv. razrede

Slikamo iz kompakta v Hausdorffov

b)



$$Z = Y \cup S((2,0), 1)$$

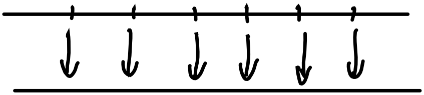
$$g : X + Y \longrightarrow Z$$

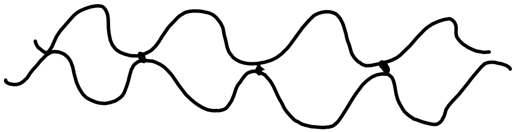
$$in_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$in_2(x,y) \longmapsto (-x+2, y)$$

Preveriti moramo

- loči ekv razr
- konst na ekv razr
- zvez, surj. kvocientna v ožjem smislu

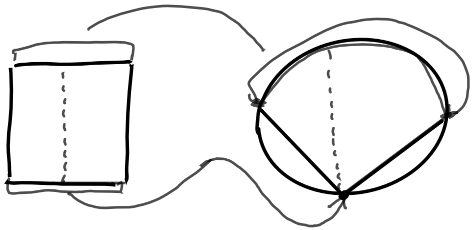




$|\sin x| \cup -|\sin x|$

$x \longmapsto$

d)



$Z = S^1 \cup \{(x,y) \in \mathbb{R}^2 : |x-1| \leq 1 \leq \sqrt{1-x^2}\}$

$g : X + Y \longrightarrow Z$

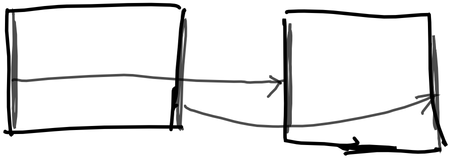
$in_2(z) \longmapsto z$

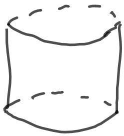
$in_1(x,y) \longmapsto (0,-1) + \frac{y+1}{2}\left(x, \sqrt{1-x^2}+1\right)$

kompaktna v hausdorffov

e)

X:





$Z = S_1 \times [-1, 1]$

$g: X + Y \longrightarrow Z$

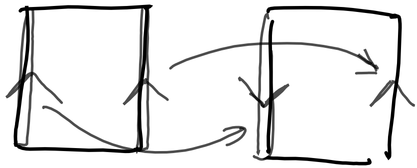
$in_1(x,y) \longmapsto (x, -\sqrt{1-x^2}, y)$

$in_2(x,y) \longmapsto (x, \sqrt{1-x^2}, y)$

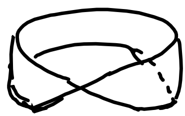
kompakt u hausdorff ✓

stetig ~~die~~ identifizierung

f)



Möbiusov trak

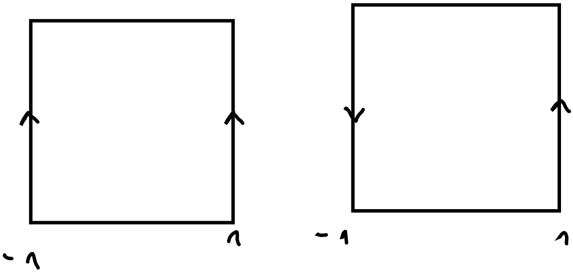


Parametrizacija möbiusovega traku

$$x(u,v) = (1+\frac{v}{2}\ \csc\frac{u}{2}\ os\, u$$

$$y(u,v) =$$

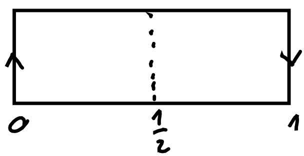
$$z(u,v) =$$



15.4

$X + Y \longrightarrow M$ ... Möbiusov trak

$M = [0,1]\times[0,1]/\sim$



$$\begin{tikzcd}
X+Y \arrow[r, "g'"] \arrow[d, "2"'] \arrow[dr, "g"] & {[0,1]^2} \arrow[d, "2m"] \\
X \cup_f Y \arrow[r, dashed] & M
\end{tikzcd}$$

$$in_1(u,v) \longmapsto \left(\frac{u+1}{4}, \frac{v+1}{2}\right)$$

$$in_2(u,v) \longmapsto \left(\frac{3-u}{4}, \frac{v+1}{2}\right)$$

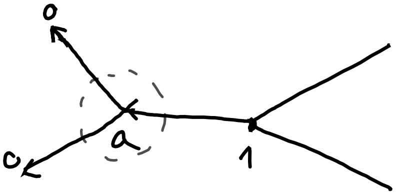
$$x \sim y \iff g(x) = g(x)$$

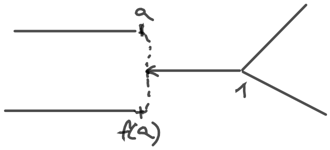
$$x \sim y \iff g'(x) \sim g'(y)$$

$(a,1] \longrightarrow (0,\infty)$

$f(x)=x$

zlepek $(0,\infty) \cup_f (a,\infty)$ je zlepek za katere hausdorfove

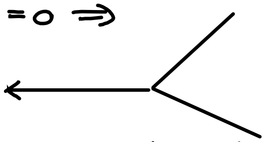



Poglejmo točko a

Vsaka okolica od a in od f(a) seka te del $(0,1]$ za $\forall a \in (0,1)$

za $a=0 \Rightarrow$



Vložimo v evklidski prostor

$(0,\infty) \cup_f (0,\infty) \longrightarrow \mathbb{R}^2$

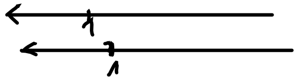
$x \in in_1(0,1] \longmapsto (0,x)$

$x \in in_1(1,\infty) \longmapsto (x, x-1)$

$x \in in_2(1,\infty) \longmapsto (x, -x+1$

je hausdorfov?

lokalna kompaktnost




kompakti: zaprti intervali v $(0,\infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh premicah

kompakti gredo v kompakte torej

3.3

$$X/A \approx X \cup_f \mathbb{1}$$

$$\begin{tikzcd}
X \arrow[d, "f"'] & X + \mathbb{1} \arrow[l, "g'"'] \arrow[dl, "g"'] \arrow[d, "g"] \\
X/A \arrow[r, "\approx"] & X \cup_f \mathbb{1}
\end{tikzcd}$$

$1 \in \mathbb{1}$

$h$:

$[x] \longmapsto x \quad ; x \notin A$

$[a] \longmapsto 1 \quad : a \in A$

je homeomorfizem

$g' : X + \mathbb{1} \longrightarrow X$

$in_1 x \longmapsto x$

$in_2 \ddot\smile \longmapsto a \quad a \in A$

$g = f \circ g'$

$u \sim v \Leftrightarrow g'(u) \sim_A g'(v)$

$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$

$f(g'(u)) = f(g'(v)) \Leftrightarrow$

$[g'(u)] \sim [g'(v)]$

Ekv. razredi v $X \cup_f \mathbb{1}$

$in_1 x = \{x\} \quad x \notin A$

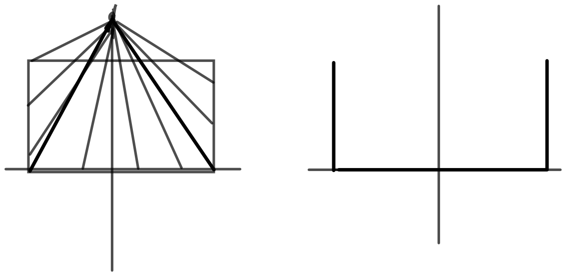
$in_1 a = A \cup \ddot\smile \quad a \in A$

$in_2 \ddot\smile = A \cup \ddot\smile$

$[in_1 x] = \{in_1 x\}$

$(in_1)_*(A) \cup \{in_2 \ddot\smile\}$

# Retrakti, homotopije, ekstenzije [illegible]



Projeciramo iz $(0,2)$ na rob

$y > 2x+2$: $\left(-1, 2-\frac{y-2}{x}\right)$

$2x+2 > y < -2x+2$: $\left(\frac{2x}{2-y}, 0\right)$

$y > -2x+2$: $\left(1, \frac{y-2}{x}+2\right)$

$(a,b) \rightsquigarrow p_1$: $y = \frac{b-2}{a}x+2$

pri $x=-1$

$y = -\frac{b-2}{a}+2$

$p_2$: $y = \frac{b-2}{a}x+2 \quad a \neq 0$

$y=0 \to x = \frac{-2a}{b-2} = \frac{2a}{2-b}$

$a=a$

$x=1$:

$y = \frac{b-2}{a}+2$

Ali je to id na $Y$?

$x=-1$: $\left(-1, 2-\frac{y-2}{-1}\right) = (-1, 2+y-2) = (-1,y)$

$y=0$: $\left(\frac{2x}{2}, 0\right) = (x,0)$

$x=1$: $\left(1, \frac{y-2}{1}+2\right) = (1,y)$ ✓

Ali homotopne id na $X$

$H(x,y,t) = t(x,y) + (1-t)r(x,y)$

zvezna, ker so vsi kosi zvezni

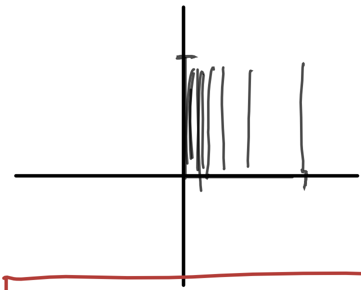
f homotopna g

ko $\exists\ H: X \times [0,1] \longrightarrow Y$

$$(x,0) \longmapsto f(x)$$

$$(x,1) \longmapsto g(x)$$

zvezna



$\boxed{\text{Retrakt } T_2 \text{ prostora je zaprt}}$

Recimo da $\exists r$ retrakt $: X \to Y$

$r|_Y = id$

deljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

mora pokriti točke $(\frac{1}{n}, 0)$

(ker gre vse pt iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1]$ čez točke

Torej

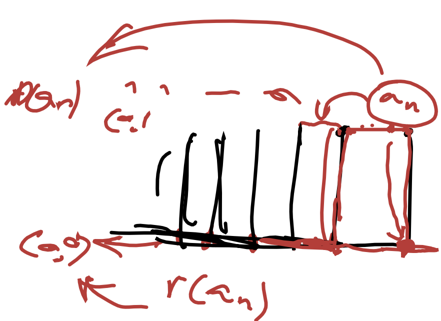
Naj bo zaporedje $a_n$; $r(a_n) = \frac{1}{n}$

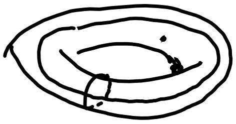
$a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim a_n = (0,1)$

$\lim r(a_n) = (0,0)$

r ni zvezna





$G \curvearrowright X$

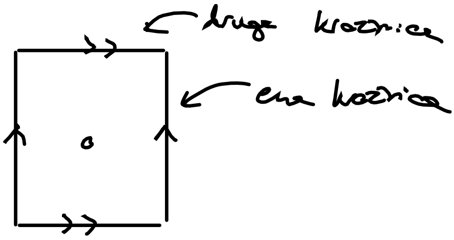
$(X, G) \longrightarrow X$

$(x, g) \longmapsto x \cdot g$

Deformacijska retrakcija "ima homotopijo do id"
je zvezna preslikava $H : X \times [0,1] \to X$.

$H(x,0) = x$ za $\forall x \in X$, $H(a,1) = a$ za $\forall a \in A$ $\wedge$ $H(x,1) \in A$ za $\forall x \in X$

$X \times [0,1] \longrightarrow$




$$\begin{tikzcd}
\left([-1,1]^2 - \{(0,0)\}\right) \times [0,1] \arrow[r, "h"] \arrow[d, "q \times id_{[0,1]}"] \arrow[dr, "g \circ h"] & {[-1,1]^2 - \{(0,0)\}} \arrow[d, "g"] \\
X \times [0,1] \arrow[r, dashed, "\text{zvezna preslikava}"'] & X
\end{tikzcd}$$

$$h : (x, t) \longmapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x}(1-t)$$

$g \circ h$ mora biti konstantna na ekv. razredih

$$x \sim y \Rightarrow g \circ h(x) = g \circ h(y)$$
$$\Longleftrightarrow$$
$$[h(x)] = [h(y)]$$
$$\Longleftrightarrow$$
$$h(x) \sim h(y)$$

$h$ ekvivalentne netrivialne razrede ohrani na meje

$\Rightarrow$

$$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1,1]^2 - \{(0,0)\} \times [0,1]$$

$$\Longleftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$$

$$\Rightarrow \ldots \frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1)(1-t_1)$$

$$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|} t_2 + (x_1, y_1)(1-t_2)$$

Izrek

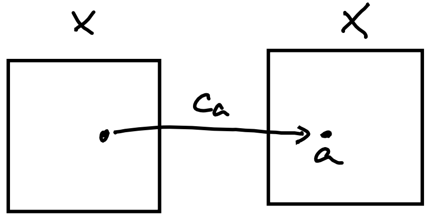
$q$ kvocientna in $X$ kompakten $T_2$ prostor

$\Longrightarrow$ $q \times id_X$ je kvocientna

Bolj splošno:

$q$ kvocientna, $X$ lokalno kompaktna

$\Longrightarrow$ $q \times id$ kvocientna




$\Rightarrow$ naj bo $X$ povezan s potmi:

Naj bo $\gamma$ ta pot med $a$ in $b$ poljubna

$\gamma(0)=a \quad \gamma(1)=b$

$H: X\times[0,1] \longrightarrow X$

$x,t \longmapsto \gamma(t)$

$H=\gamma\circ pr_2$ torej je zvezna

$\Leftarrow$

Recimo ~~da~~ $\forall a,b\in X$ velja $c_a \simeq c_b$

Naj bosta $a,b$ poljubna

$\exists H.\ X\times[0,1] \longrightarrow X$

$H_0 = c_a$

$H_1 = c_b$

$\gamma(t) = H(a,t)$

4.5

a) $f\ S^n \to S^n$ $n$: sur

homotopna konstanti

$a \notin f_*(S^n)$ b je nasprotni tej

$$H\ S^n \times [0,1] \longrightarrow S^n$$

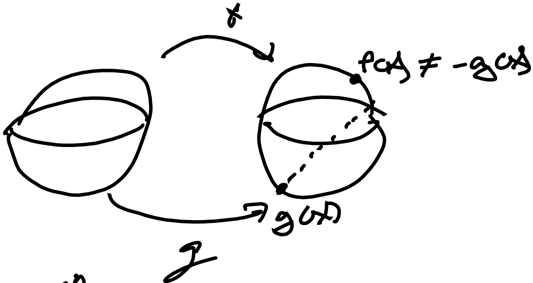
$$(x, t) \longmapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$$

Ali je slučajno $(1-t)f(x) + t \cdot x = 0$

$\Rightarrow$ če gre ta daljica skozi 0 ampak $a$ ni zalogi vrednosti, zato nikoli njena daljica skozi 0,0

b) $f,g : S^n \longrightarrow S^n$

$f(x) \neq -g(x) \quad \forall\, x \in S^n \Longrightarrow f \simeq g$



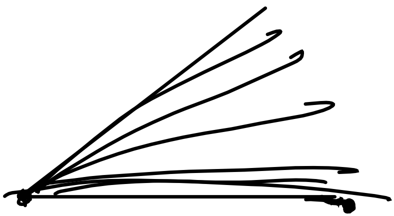

$H : S^n \times [0,1] \longrightarrow S^n$

$(x,t) \longmapsto \dfrac{(1-t)\,(g(x)) - t\,f(x)}{\|(1-t)\,g(x) - t\,f(x)\|}$

4.6 $X = \{[0,1]\times\{0\}\} \cup \{x, \tfrac{x}{n},\ x \in [0,1] \wedge n \in \mathbb{N}\}$


$H: X \times [0,1] \longrightarrow CX$

$(x,y), t \;=\; (1,t)(x,y)$



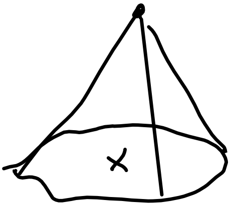
Recimo da je ~~H~~ G poljubna krepka deformacijska retrakcija X-a na tisto točko

$\forall n \in \mathbb{N} .\ \exists\, t_n \in [0,1]$ $G\left(\left(1, \tfrac{1}{n}\right), t_n\right) = 0,0)$

ker je $[0,1]$ kompaktna lahko izberemo konvergentno podzaporedje $t_{i_n}$

?

$$CX = X \times [0,1] / X \times \{1\}$$




$$CX \times [0,1] \longrightarrow CX$$

$$\begin{tikzcd}
X \times [0,1] \times [0,1] \arrow[r, "H"] \arrow[d, "q \times id"] & X \times [0,1] \arrow[d, "q"] \\
CX \times [0,1] \arrow[r, dashed] & CX
\end{tikzcd}$$

ne moremo vzeti seštevat

$H : (x, u, t) \longmapsto \cancel{(x,1)t + (x,u)(1-t)}$

$(x, u(1-t) + t)$

Preveriti moramo samo

$(x_1, 1, t) \sim_H (x_2, 1, t)$

edini netrivialni ekvivalenčni razred ,

$\parallel$ $(x_1, 1)$ $\parallel$ $(x_2, 1)$

sta ekvivalentni ker sta na višini 1

$\bar{H}([x, u, 0]) = [x, u]$

$q(H(x, u, 0)) = q(x, u)$

$q(H)(x, u, 1) = \ldots$

X je kontraktibilen

$\Longleftrightarrow$

$\exists$ zvezna izbira poti od poljubne točke X do izbrane točke

$\exists a \in X.\ \exists_{zv} f: X \longrightarrow \mathcal{C}(I, X).\ f(x)(0) = x,\ f(x)(1) = a$

$\Longleftrightarrow$

$X \neq \emptyset \wedge \exists$ zvezna izbira poti med točkami v X

$\exists f_{zv}: X \times X \longrightarrow \mathcal{C}(I, X).\ f(a,b)(0) = a,\ f(a,b)(1) = b$

4.8 (4.7 na vajah)

i) $\Rightarrow$ ii) Vemo iz 4.7 (4.6)

ii) $\Rightarrow$ iii) Naj bo $X$ retrakt kontraktibilnega prostora $Y$

$\exists r: Y \to X.\ \exists i: X \to Y.\ r \circ i = id$

(ima zvezni desni inverz (ekvivalentna definicija retrakcije))

$\exists y_0 \in Y.\ \exists H: Y \times I \to Y$

$(y, 0) \mapsto y$

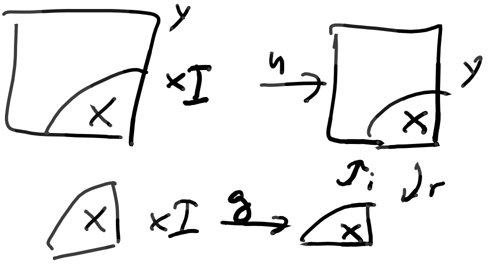
$(y, 1) \mapsto y_0$

iščemo

$g: X \times I \to X$ $\qquad x_0 = r(y_0)$

$g(x,0) = g$

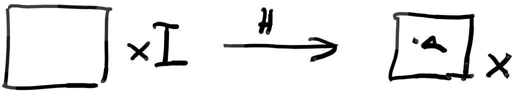
$g(x,1) = x_0$




$g(x,t) = r(h(i(x),t))$

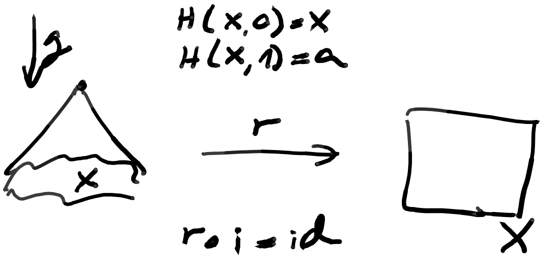
$g = r \circ h \circ (i \times id_I)$

iii) $\Rightarrow$ i)




$H(x,0) = x$

$H(x,1) = a$




$r \circ i = id$

$r([x,0]) = x$

$CX = X \times I / \{(x,1)\}$

$[x,t] \mapsto pr_1(H(x,t))$

4.9

$$I\times I/_{\text{vemo}} = M \qquad \text{vemo} \Leftrightarrow \sim$$

CM

$$I\times I/_{\text{vemo}} \times I/_{\{I\times I,\,1\}} = CM$$

Recimo da je kontraktibilen

$$\exists H: I\times I/_{\text{vemo}} \times I \longrightarrow I\times I/_{\text{vemo}}$$

$$H(x,0)=x$$
$$H(x,1)=1$$

vzamimo podprostor A =  [illegible]

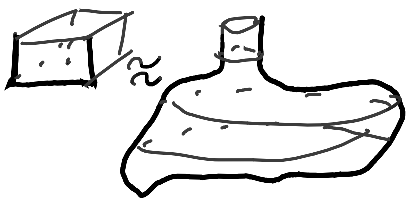
ki je krožnica topološka

torej je tudi A kontraktibilna

u. 10

Neprazen konveksen prostor je kontraktibilen

Splošno: zvezdast prostor je kontraktibilen



$\mathcal{N}$ .. Razred normalnih topoloških prostorov

$Y \in AE(\mathcal{N})$ je absolutni ekstenzor za razred normalnih prostorov

kadar za $\forall X \in \mathcal{N}$ in $\forall$ zaprto podmnožico $A \subseteq X$ velja, da lahko zvezno preslikavo $A \to Y$ razširimo do zvezne preslikave $X \to Y$

$$\begin{tikzcd} X \arrow[drr, dotted] & & \\ A^{zap} \arrow[u, hook] \arrow[rr, "f"'] & & Y \end{tikzcd}$$

Tietze: $I \in AE(\mathcal{N})$

---

- Produkt AE je AE

$$\begin{tikzcd} X & \\ A \arrow[u, hook] \arrow[r] & Y' \times Y'' \end{tikzcd}$$

razširimo vsako komponento

---

- Retrakt AE je AE

---

- $X \in AE(\mathcal{N}) \Longrightarrow X \neq \emptyset \wedge X$ povezan s potmi

---

- $AE(\mathcal{N}) \cap \mathcal{N} \subseteq AR(\mathcal{N})$

absolutni retrakt

Na tabli: 4. ~~[illegible]~~ 11

4.12 $A, B^{\text{cl}} \subseteq \mathbb{R}^n$

$$
\begin{tikzcd}
X \arrow[drr, dotted, no head] & & \\
C \arrow[u, hook] \arrow[rr] & & A \cup B
\end{tikzcd}
$$

$\mathbb{R}^n \in AE$

Dokažimo $A \cup B$ je retrakt $\mathbb{R}^n$