

7.1.

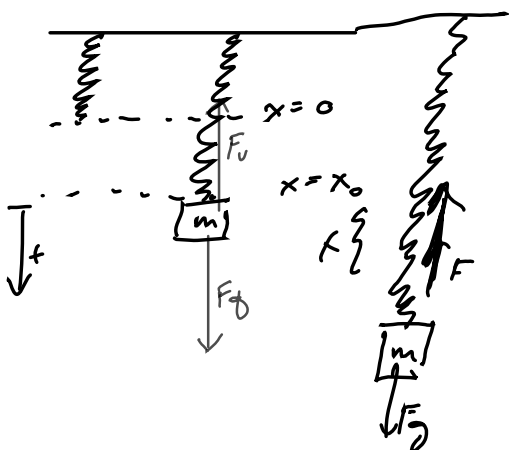
$$l = 1 \text{ m}$$

$$k = 500 \text{ N/m}$$

$$m = 8 \text{ kg}$$

$$a_0 = 0,1 \text{ m}$$

$$x_0 = ?$$



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

$$F_g + F_{v2} = m \cdot \ddot{a} = \ddot{x}(t)$$

$$+mg - k(x+x_0) = m\ddot{a}(t)$$

$$\underbrace{mg - kx_0 - kx}_{=0} = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\ddot{x} = \frac{-kx}{m} \quad \frac{-k}{m} = \omega_0^2$$

$$x(t) = \sin(\omega t) \cdot A + B \cos(\omega t)$$

anfangsbedingungen:

$$x(0) = a_0$$

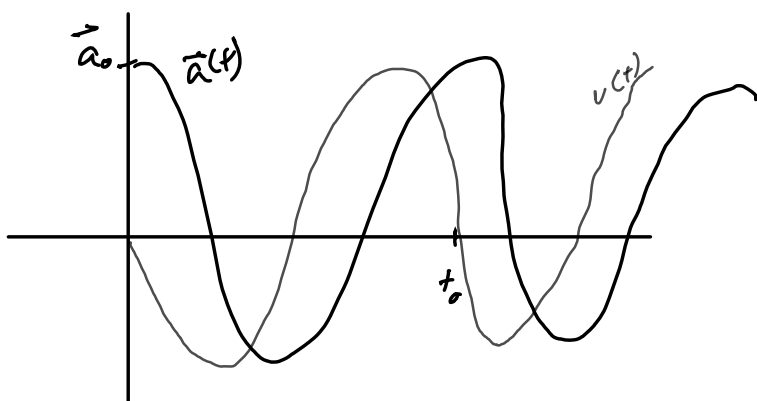
$$a_0 = x(0) = B$$

$$v(t) = \dot{x}(t)$$

$$v(0) = \dot{x}(0) = A\omega \cos(\omega t) - B\omega \sin(\omega t)$$

$$0 = A\omega \Rightarrow A = 0$$

$$x(t) = a_0 \cos(\omega t)$$



$$\cos(\omega t_0) = 1 \quad \omega t_0 = 2\pi \quad t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \sqrt{m}}{\sqrt{k}}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \dots \quad \text{Eigenfrequenz}$$

$$\omega_0 = 7,9 \text{ /s}$$

Energije nihanja

$$W = \frac{mv^2}{2} + mg(-x) + \frac{k(x+x_0)^2}{2} - \frac{k}{2}x_0^2$$

$$= \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 = 2,5 \text{ J}$$

$$l = 3\text{m}$$

$$\varphi_0 = 3,5^\circ$$

$$t = 15\text{s}$$

$$N = ?$$

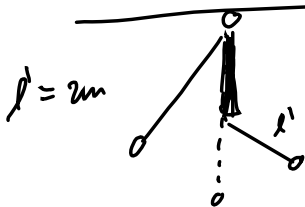
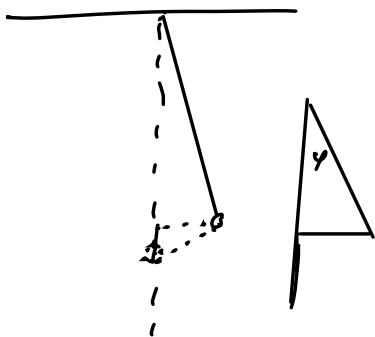
$$V_{\max} = ?$$

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$N = \frac{t}{T_0} = 4,32$$

$$v = \sqrt{2gh}$$

$$h = l - \cos\varphi \cdot l$$



$$T_0 = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l'}{g}}$$

$$= \frac{\pi}{\sqrt{g}} (\sqrt{l} - \sqrt{l'}) = 3,15\text{s}$$

$$N = \frac{t}{T_0} =$$

$$\varphi(t) = \varphi_0 \cos(\omega t)$$

\nwarrow ω - angular frequency

$$V_{\max} = l \cdot \omega$$

\nwarrow ω - angular frequency

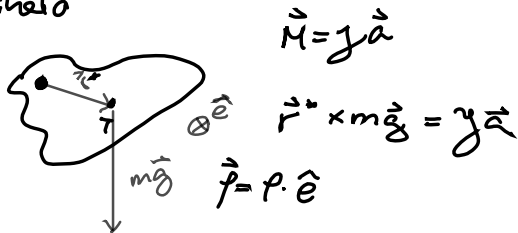
$$\omega(t) = \dot{\varphi}(t) = -\varphi_0 \omega \sin(\omega t)$$

ω - angular frequency

$$V_{\max} = l \cdot \varphi_0 \omega$$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

Tedno nihalo



$$r^* m g \sin \varphi (-\hat{e}) = y \cdot \ddot{\varphi} \hat{e}$$

$$\ddot{\varphi} = -\frac{m g r^*}{y} \sin \varphi$$

$$\varphi \ll 1$$

$$\ddot{\varphi} \approx -\underbrace{\frac{m g r^*}{y}}_{\omega^2} \varphi$$

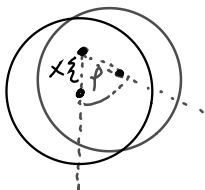
$$T_0 = 2\pi \sqrt{\frac{y}{m g r^*}}$$

Matematično nihalo:

$$y = m l^2$$

$$r^* = l \quad T_0 = 2\pi \sqrt{\frac{l}{g}}$$

7.7.)



$$R = 0,5 \text{ m}$$

t_0 min

debi osnove ozi
translacija

$$y = \frac{1}{2} m R^2 + m x^2$$

$$t_0(x) = ?$$

$$t_0 = 2\pi \sqrt{\frac{y}{mgx}} =$$

$$t_a = 2\pi \sqrt{\frac{\frac{1}{2} R^2 + x^2}{gx}}$$

da bismo nađemo:

$$\frac{2x(gx) - g(\frac{1}{2}R^2 + x^2)}{gx^2} = 0$$

$$2gx^2 - gx^2 - \frac{1}{2}R^2g$$

$$gx^2 = \frac{1}{2}R^2g$$

$$x = \sqrt{\frac{1}{2}} \frac{R}{\sqrt{2}}$$

(7.8)

$$l = 1 \text{ m}$$

$$m = 0,5 \text{ kg}$$

$$k = 5 \text{ N/m}$$

$$V = ?$$



$$\phi \ll 1$$

$$\Sigma \vec{M} = J \cdot \ddot{\alpha}$$

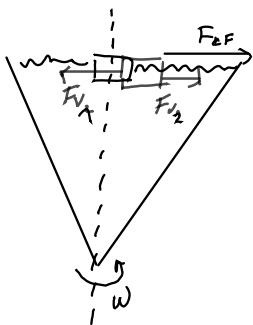
$$\underbrace{\ddot{\alpha}(t)}_{\ddot{\phi}(t)} = \underbrace{-F_g \cdot \frac{l}{2} \sin \phi}_{\phi(t)} - \underbrace{kx \cdot l \cdot \cos \phi}_{\substack{\text{reaction} \\ \ddot{x}(t) \\ l \cdot \phi(t)}}$$

$$\ddot{\phi}(t) = - \frac{m g \cdot l}{2 J} \phi(t) - \frac{k l^2}{J} \phi(t) =$$

$$= - \frac{3g}{2l} \phi(t) - \frac{3k \phi(t)}{m} = - \left(\frac{3g}{2l} + \frac{3k}{m} \right) \phi(t) \quad \omega_0$$

$$\omega_0 = \sqrt{\frac{3g}{2l} + \frac{3k}{m}} = 6,7 / \text{s}$$

$$2\pi V = \omega_0 \quad V = \frac{\omega_0}{2\pi} = 1,06 \text{ Hz}$$



$$m = 20g$$

$$k_1 = 1 \text{ N/m}$$

$$k_2 = 3 \text{ N/m}$$

$$\omega = 10/s$$

$$V = ?$$

$$\sum F = ma$$

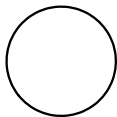
$$ma = -k_1 x - k_2 x + m \ddot{a}_r$$

$$\ddot{x}(t) = - \left(\frac{k_1 + k_2 - m\omega^2}{m} \right) x$$

$$\omega_0 = \sqrt{\frac{k_1 + k_2 - m\omega^2}{m}} = 10/s$$

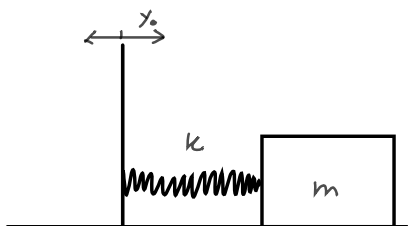
$$V = 1, \text{ Hz}$$

Faucaultova nihele





7.25



$$y_0 = 5 \text{ cm}$$

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

$$k = \frac{N}{m}$$

$$\beta = 0.2 / s$$

$$\nu = 12 \text{ Hz}$$

$$x_0 = ?$$

$$\delta = ?$$

$$x_{0 \max} = ?$$

$$v_{\max} = ?$$

$$m \ddot{x} = -k(x-y) - 2\beta m \dot{x}$$

$$\ddot{x} + 2\beta \dot{x} + \frac{k}{m} x = \frac{k}{m} y \rightarrow y_0 \cos(\omega t)$$

$$\omega^2 = \frac{k}{m}$$

Nehomogena diferencialna enačba

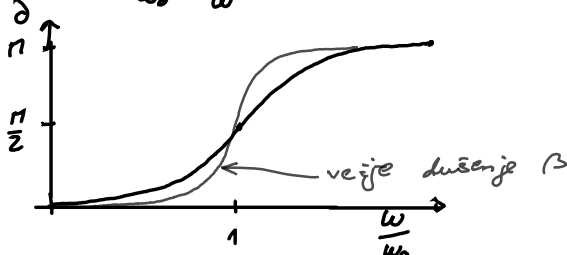
$$\text{rešitev: } x(t) = x_0 \cdot \cos(\omega t - \delta) + x_0' e^{-\beta t} \cos(\omega t - \delta')$$

$$\sqrt{\omega^2 - \beta^2}$$

se zadržuje, ne
pa zanima, ko se
ustati

Rezultati:

$$\tan \delta = \frac{2\beta \omega}{\omega_0^2 - \omega^2}$$



$$\frac{x_0}{y_0} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta \omega)^2}}$$



vrh resonance
krivulje

$$\omega_{\max} = \sqrt{\omega_0^2 - 2\beta^2}$$

Rezultati:

$$\omega_0 = \sqrt{\frac{k}{m}} = 10 / s$$

$$\delta = 179.7^\circ$$

$$\Rightarrow \nu = 1.6 \text{ Hz}$$

$$x_0 = 0.89 \text{ mm}$$

$$v_{\max} \approx 1.6 \text{ Hz}$$

$$x_{0 \max} = 1.25 \text{ m}$$

mec pri vsiljenem nihanju

$$P = \frac{dA}{dt} = \frac{F \dot{x}}{dt} = Fv = 2\beta m v \dot{x} =$$

$$v = \dot{x} = x_0 (-\sin(\omega t - \delta)) \cdot \omega$$

$$\bar{P} = 2\beta m x_0^2 \omega^2 \overline{\sin^2(\omega t - \delta)} = \beta m x_0^2 \omega^2$$

||
1
2

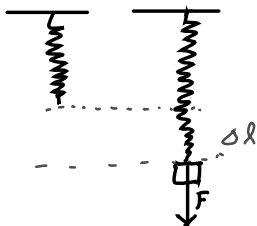
$$\bar{P} \propto \omega^2 \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}$$

$$\frac{dP}{d\omega} = \frac{2\omega[(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2] - [4\omega(\omega_0^2 - \omega^2) + 8\beta^2\omega]}{((\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2)^2} = 0$$

splošno nihanje

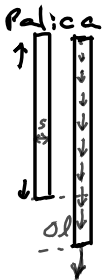
χ 14.3

Elastomehanika: Hookov zakon



$$F = k \cdot \Delta l$$

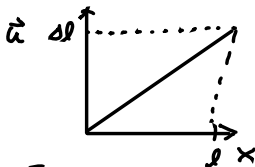
Vpeljava prožnostnega modula



→ majhne raztezke

$$\frac{F}{S} = \frac{\Delta l}{l} \cdot E \quad [E] = \frac{N}{m^2} = Pa$$

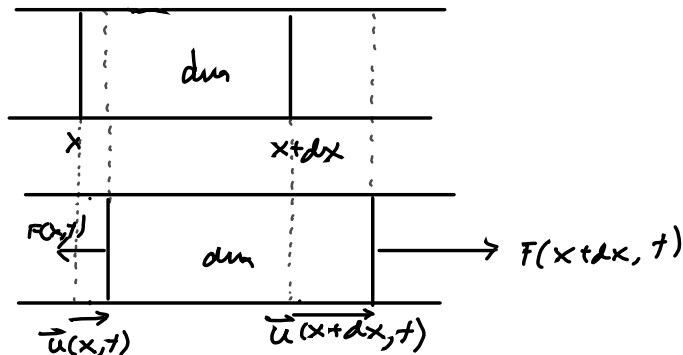
← vektorsko polje



$$\frac{du}{dx} = \frac{dl}{l}$$

$$\frac{F}{S} = E \frac{du}{dx}$$

Valovanje v elastični palici



$$F(x+dx, t) - F(x, t) = dm \cdot a(x + \frac{dx}{2}, t)$$

$$\mathcal{E} \frac{du}{dx} (x+dx, t) - \mathcal{E} \frac{du}{dx} (x, t) = dm \cdot \ddot{u}(x + \frac{dx}{2}, t)$$

$$dx \rightarrow 0$$

$$\mathcal{E} \frac{d^2 u}{dx^2} (x + \frac{dx}{2}, t) \cdot dx = \rho \cancel{dx} \ddot{u}(x + \frac{dx}{2}, t)$$

$$\frac{\partial^2 u}{\partial x^2} = \left(\frac{\rho}{\mathcal{E}} \right) \frac{\partial^2 u}{\partial t^2}$$

↓

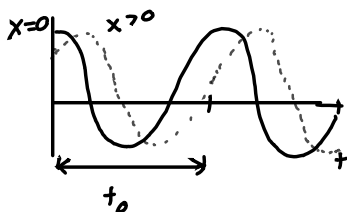
$$\frac{1}{c^2}$$

$$u(x, t) = \phi(x+ct) + \psi(x-ct)$$

$$c = \sqrt{\frac{\mathcal{E}}{\rho}}$$

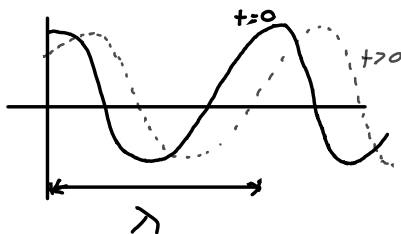
Periodična motnja

$$u(x,t) = u_0 \cdot \cos(\omega t - kx)$$



Na danem mestu

$$\omega T_0 = 2\pi$$
$$\omega = \frac{2\pi}{T_0} = 2\pi \nu$$



ob danem času

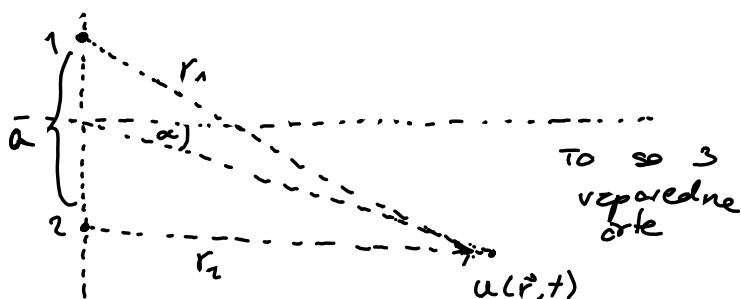
$$c = \frac{\delta x}{\delta t} = \frac{\lambda}{T_0} \Rightarrow c = \lambda \nu$$

$$a = \frac{3}{10} \lambda$$

1. izvor zakasnjjen za $\frac{1}{4}$ nihanja $\Rightarrow \delta = \frac{\pi}{2}$

smeri opazitev = ?

smo deleč od anten



$$r_1, r_2 \gg a$$

$$u = \underbrace{u_0 \sin(\omega t - k r_1 - \delta)}_{\text{prvi izvor}} + \underbrace{u_0 \sin(\omega t - k r_2)}_{\text{2. izvor}}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$u = u_0 \sin \left(\frac{2\omega t - k(r_1 + r_2) - \delta}{2} \right) \cos \left(\frac{k(r_2 - r_1) - \delta}{2} \right)$$

amplitude

točke za katere je $r_1 - r_2$ konstanta

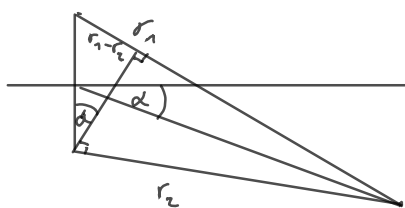
$$\frac{k}{2}(r_2 - r_1) - \frac{\delta}{2} = n\pi \quad n \in \mathbb{Z}$$

točje
priče hiperbole

$j \propto u^2$ mora biti maksimalen
(ker iščemo optike)
↖ sorazmerno

odločimo se za neke

n in potem gledamo da
je tudi na levi strani
konstantno



$$r_1 - r_2 = a \sin \alpha$$

$$-\frac{k}{2}(a \sin \alpha) - \frac{\delta}{2} = n\pi$$

$$\sin \alpha = \frac{n\pi + \frac{\delta}{2}}{-\frac{ak}{2}} = \frac{2\pi n + \delta}{-ak}$$

k valovni vektor

$$k = \frac{2\pi}{\lambda} \quad a = \frac{3}{10} \lambda$$

$$\sin \alpha = \frac{\lambda(2\pi n + \delta)}{-2\pi a} = \frac{5(2\pi n + \delta)}{-3\pi}$$

$$\sin \alpha = -\frac{10}{3} \left(n + \frac{1}{6} \right)$$

rečemo $n=0$

$$\sin \alpha_0 = -\frac{10}{3} \cdot \frac{1}{6} = -\frac{5}{6}$$

$$\Rightarrow \alpha_0 = -56,4$$

$$n=1 \Rightarrow \sin \alpha_1 = -\frac{10}{3} \cdot \frac{5}{6} = -\frac{25}{6}$$

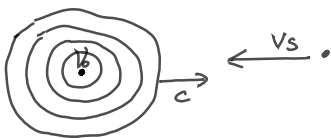
$$\alpha_1 = \text{err}$$

$$n=-1 \Rightarrow \sin \alpha_{-1} = -\frac{10}{3} \cdot \left(-\frac{3}{6} \right) = \frac{5}{3}$$

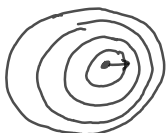
$$\nu = 1485 \text{ kHz}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \frac{\text{m}}{\text{s}}}{1485 \cdot 10^3 / \text{s}} = 202 \text{ m}$$

Dopplerjev pojav



$$N = \frac{ct}{\lambda} + \frac{v_s t}{\lambda} + \frac{v t}{\lambda}$$



$$\lambda = \frac{c}{v_0} + \frac{v}{v_0} + \frac{v_v}{v_0}$$

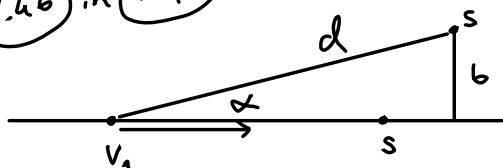
$$V_s = \frac{N}{t} = \frac{c + v + v_s}{c + v + v_v} \cdot v_0$$

$$= \frac{c + (v + v_s)}{v - (v_0 - v)}$$

h.rost
sprejemnika
glede na veter

h.rost
oddajnika
glede na veter

(7.46) in (7.47)

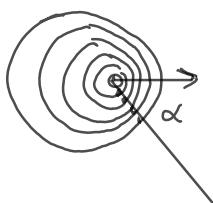


$$v_1 = 100 \frac{\text{km}}{\text{h}} = 27,8 \frac{\text{m}}{\text{s}}$$

$$V_0 = 1000 \text{ Hz}$$

a) $V_s = ?$ $V_s = \frac{c + V_s}{c - v_1} V_0$ $V_s = 0$ $c = 340 \frac{\text{m}}{\text{s}}$
 $V_1 = 27,8 \frac{\text{m}}{\text{s}}$
 $V_s = 189 \text{ Hz}$

b) $d = 100 \text{ m}$
 $b = 500 \text{ m}$

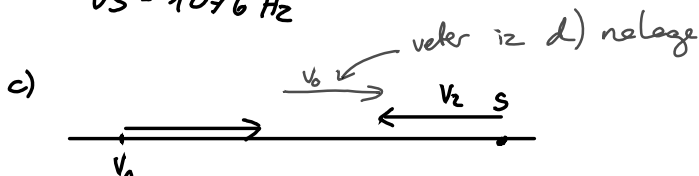


$$V_s = \frac{c}{c - v_1 \cos \alpha} V_0$$

$$\sin \alpha = \frac{b}{d} = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$

$$\cos \alpha = \frac{\sqrt{3}}{2}$$

$$V_s = 1076 \text{ Hz}$$

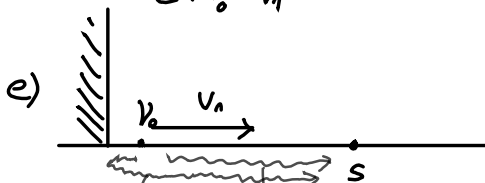


$$v_2 = 80 \frac{\text{km}}{\text{h}} = 22 \frac{\text{m}}{\text{s}}$$

$$V_s = \frac{c + v_2}{c - v_1} V_0 = 1160 \text{ Hz}$$

d) $v_0 = 30 \frac{\text{km}}{\text{h}} = 8,3 \frac{\text{m}}{\text{s}}$
 \uparrow
 v_{veler}

$$V_s = \frac{c + v_0 + v_2}{c + v_0 - v_1} = 1156 \text{ Hz}$$



$$V_{s_1} = \frac{c}{c + v_1} V_0$$

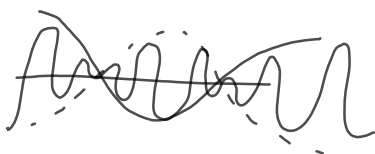
$$V_{s_2} = \frac{c}{c - v_1} V_0$$

$$V_s = V_{s_1} + V_{s_2} = c V_0 \left(\frac{1}{c + v_1} + \frac{1}{c - v_1} \right) =$$

$$= c V_0 \left(\frac{c - v_1 + c + v_1}{c^2 - v_1^2} \right) = c V_0 \left(\frac{2c}{c^2 - v_1^2} \right)$$

$$\cos(\omega t) + \cos(\omega'' t) =$$

$$2 \cos\left(\frac{\omega + \omega''}{2} t\right) \cos\left(\frac{\omega - \omega''}{2} t\right)$$



slušmo utripanje