

ket: elemen<sup>h</sup> ke<sup>h</sup> b<sup>h</sup>  $\mathbb{Z}[i]$

6.10

$\Rightarrow$  asosiasi:  $m+ni$

$$x = u(m+ni) = um + un i$$

ket: so konjugi:

$$(x+yi)(a+bi) = 1$$

$$xa - yb + i(ay + xb) = 1$$

$$ay + xb = 0 \quad xa - yb = 1$$

$$x = \frac{-ay}{b} \quad -\frac{a^2 y}{b} - yb = 1$$

$$y = \frac{-b}{a^2 + b^2} \quad -\frac{y}{b}(a^2 + b^2) = 1$$

$$x = \frac{a}{a^2 + b^2}$$

$$|a| \leq |a^2| \Rightarrow$$

$$a^2 + b^2 \in \{0, 1\} \Rightarrow a^2 + b^2 = 1$$

$$\Rightarrow a \in \{0, 1\}$$

$$b \in \{0, 1\}$$

$$\text{konjugi} \quad \{1, -1, i, -i\}$$

$$\text{asoc: } \{m+ni, -m-ni, n-mi, -n+mi\}$$

$$d \in \mathbb{Z} \\ \mathbb{Z}[\sqrt{d}] = \{m+n\sqrt{d}; m, n \in \mathbb{Z}\}$$

1) Pokaži  $\mathbb{Z}[\sqrt{d}]$  je podkolebar  $\mathbb{C}$

2) Množica  $\mathbb{Q}[\sqrt{d}] = \{g+r\sqrt{d}; g, r \in \mathbb{Q}\}$  je podpodje  
 $\mathbb{C}$  generirano z  $\mathbb{Z}[\sqrt{d}]$

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1)  $\mathbb{Z}[\sqrt{d}] \subseteq \mathbb{C}$

zapišot z seštevanje, množenje, evle  
 z množenje:

$$(m+n\sqrt{d})(x+y\sqrt{d}) = mx + nyd + \sqrt{d}(nx + ym)$$

$$1 \cdot 1 = 1 = 1 + 0 \cdot \sqrt{d}$$

$$\text{seštevanje: } (m+n\sqrt{d}) - (x+y\sqrt{d}) = (m-x) + (n-y)\sqrt{d}$$

2) invertiranje

$$\frac{m+n\sqrt{d}}{x+y\sqrt{d}} = \frac{(m+n\sqrt{d})(x-y\sqrt{d})}{x^2-yd} = \frac{c+e\sqrt{d}}{x^2-yd} \checkmark$$

↙ faktor

Automorfizmi  $\mathbb{Z}[\sqrt{d}] = \{id; \sqrt{d} \mapsto \pm\sqrt{d}\}$

Norma  $N(x) = x\sigma(x)$

$$N(g+r\sqrt{d}) = (g+r\sqrt{d})(g-r\sqrt{d}) = g^2 - r^2d$$

$$3) \quad \forall x, y \in \mathbb{Z}[\sqrt{a}] : N(xy) = N(x)N(y)$$

$$\begin{aligned} N(xy) &= xy\sigma(xy) = xy\sigma(x)\sigma(y) = x\sigma(x)y\sigma(y) \\ &= N(x) \cdot N(y) \end{aligned}$$

$$4) \quad x \in \mathbb{Z}[\sqrt{a}] \text{ ist invertierbar} \Leftrightarrow N(x) = \pm 1$$

$$xy = 1 \Rightarrow N(xy) = N(1)$$

$$N(x)N(y) = 1$$

$$N(x) = \frac{1}{N(y)}$$

$$N(y) \in \mathbb{Z} \wedge N(x) \in \mathbb{Z}$$

$$\Rightarrow N(y) \in \{\pm 1\}$$

$$\Rightarrow N(x) \in \{\pm 1\}$$

6)

$$p \in \mathbb{P}; \quad N(x) = \pm p \Rightarrow x \text{ nicht invertierbar}$$

in  $N(x) = \pm p$

rechen da  $x$  invertierbar aber nicht invertierbar

$$x = ab$$

$$N(x) = N(a)N(b) \neq \pm p \Rightarrow N(a) = 1 \vee N(b) =$$

$$4) \quad \Leftarrow N(x) = 1, \quad x\sigma(x) = \pm 1 \Rightarrow x^{-1} = \sigma(x)$$

5) zu prüfen

hier da  $N(x) = \pm 1 \Rightarrow x$  invertierbar.

$d \mid -1 \Rightarrow 1, -1$  sta edine obrnljive v  $\mathbb{Z}[\sqrt{d}]$

$$x \text{ obrnljiv} \Rightarrow N(x) N(y) = \pm 1$$

$$N(a+b\sqrt{d}) = (a+b\sqrt{d})(a-b\sqrt{d}) = a^2 - b^2d = a^2 + b^2|d|$$

$\begin{matrix} \vee \\ 1 \end{matrix}$

$$\text{če je } b^2 > 1 \Rightarrow N > 1$$

$$\Rightarrow b = 0$$

$a^2$  mora biti 1  $\Rightarrow a = \pm 1$  ote edini;

morebiti  
mimo de sta deliliva to ne je rezultat

Pokaži da so  $1+i$ ,  $7+8i$ , 3 nerazcepni v  $\mathbb{Z}[i]$

Recimo da so razcepni:

$$1+i = xy \quad N(xy) = x\sigma(x)y\sigma(y) = 2$$

prebrala

$p \rightarrow$  nerazcepno

$$N(7+8i) = 49+64 = \text{velika} = 113 \text{ prebrala}$$

~~Prebrala~~ Nema je 9

$$3 = xy \quad N(xy) = x\sigma(x)y\sigma(y) = 9$$

vsej 2 morebiti 1

$$\cancel{x\sigma(x)} N(x) = N(\sigma(x)) = \cancel{x\sigma(x)\sigma\sigma(x)} = \cancel{x\sigma(x)}$$

$$\underline{N(x)} = \underline{n} \Rightarrow \underline{n} \text{ razcepna}$$

$$N(x) = n = x\sigma(x)$$

6) Pāiņā visdeliktējie elementi  $2 \in \mathbb{Z}[i]$

$$x|2 \Leftrightarrow 2 = xy \text{ un } nd \nmid y$$

$$N(x, y) = 4 \quad N(x)N(y) = 4$$

$$x\sigma(x)y\sigma(y) = 4 = 2 \cdot 2 = xy\sigma(xy) = 2\sigma(xy)$$

$$\Rightarrow \sigma(xy) = 2 = \sigma(x)\sigma(y) \Rightarrow$$

$$N(x) = 2 \quad x = \pm 1 \pm i$$

$$x = \pm 2$$

$$x = \pm 2i$$

go use normu:

1)

$$a = 3 + 4i \quad \mathcal{J} = m^2 + n^2$$

$$b = 1 - 3i$$

$$|a| = 3 + 16 = 19$$

$$|b| = 1 + 9 = 10$$

Auguri del 100!

$$a = kb + r$$

$$\frac{a}{b} = k + \frac{r}{b}$$

$$\in \mathbb{Z}[i] \in \mathbb{Q}[i]$$

$$a = 3 - 4i$$

$$b = 1 - 3i$$

$$\frac{a}{b} = \frac{(3-4i)(1+3i)}{10} = \frac{3+12+i(3-4)}{10} = 1 + \frac{1}{2} + \frac{1}{2}i$$

$$\frac{1}{2}(1+i)(1-3i) = \frac{1}{2}(1+3+i(1-3)) = 2-i$$

$$a = b + 2-i \quad /: (2-i)$$

$$\frac{a}{2-i} = \frac{b}{2-i} + 1 \quad \Rightarrow$$

$$\frac{a}{2-i} - \frac{b}{2-i} = 1 \quad \Rightarrow \quad a - b = 2-i$$

2)

$$a=6$$

$$b=2+2\sqrt{5}$$

$$\mathbb{Z}[\sqrt{5}]$$

$$c=2 \Rightarrow \frac{N(a)}{2} = \frac{36}{2} = 3 \quad \frac{N(b)}{2} = \frac{4+4\cdot 5}{2} = \frac{24}{2} = 12$$

$$\text{E.g. } N(a) \Rightarrow \text{remainders } d|a, d|b \Rightarrow \begin{matrix} 6 \cdot 2 \cdot 3 \\ \parallel \quad \parallel \\ 6^2 \quad 6 \cdot 4 \end{matrix}$$

$$J(c) = 1+5=6$$

$$N(\gcd(a,b)) = \gcd(N(a), N(b)) = \gcd(36, 24) = 12 = x^2 \cdot 5y^2$$

$$y \in \{1, 2\}$$

$$\downarrow$$

$$x^2 = 3 \quad x^2 = 5$$

Take note

3) UFD  $\Rightarrow$  ged darlegen

$$a = \prod_{i \in I} p_i^{k_i}$$

$$b = \prod_{i \in I} p_i^{l_i}$$

$$d = \gcd(a, b) = \prod_{i \in I} p_i^{\min\{k_i, l_i\}}$$

$$d|a \wedge d|b$$

Neg  $b \mid a \iff c = \prod_{i \in I} p_i^{m_i} \quad c|a \wedge c|b$

$$a = c\alpha \quad \alpha = \prod_{i \in I} p_i^{n_i}$$

$$\text{Q: } a = \prod p_i^{n_i + m_i} \Rightarrow \prod p_i^{k_i} \Rightarrow m_i \leq k_i$$

$\forall i \text{ so } m_i < l_i \Rightarrow f_i$   
 $c|d$



$$\mathbb{Z}[\sqrt{-2}] \text{ additiv}$$

a)  $\forall a, b \in \mathbb{Z}[\sqrt{-2}]$  rek.  $a = gb + r$   $\wedge r = 0 \vee \sigma(r) < \sigma(b)$

b)  $\sigma(a) \leq \sigma(g, b) \forall a, b \in \mathbb{Z}[\sqrt{-2}]$

$$\sigma: m + \sqrt{-2}n = m^2 + 2n^2$$

Weg der b weg

$a, b \in \mathbb{Z}[\sqrt{-2}]$  nehmen  $a = x + \sqrt{-2}y$   $b = m + \sqrt{-2}n$

$$\frac{a}{b} = g_1 + g_2 \sqrt{-2}$$

$$a = ([g_1] + [g_2]\sqrt{-2})b + ([g_1 - [g_1]] + ([g_2 - [g_2]]\sqrt{-2}))b$$