

## Formule

$$\# \text{permutacij} = |\{\sigma : A \xrightarrow{\sim} A\}| = n!$$

$$B \subseteq A \Rightarrow |A - B| = |A| - |B|$$

$$|A \times B| = |A| |B|$$

Št neurejenih izborov podmnocice k od n =  $\binom{n}{k}$

multinomski:  $n_1$  tipa 1  $n_2$  tipa 2, ...  $n_k$  tipa k

$$\# \text{permutacij neurejena} = \frac{n!}{n_1! \dots n_k!}$$

①  $J = \sum$    
 uha, salety, riba, zelenjava, sladoledi,  
 predjed mess tatica  
 gurajel gledic

$2 \cdot 3 \cdot 2$  je st možnih kada  
 ||

12

② Kup kart 52 kart. Karte razdelimo na 4 igralce  
 vsake 13: B

1. Koliko načinov lahko karte teko razdelimo
  2. kolikor je tam imenovanih nepravilnih karte sem enega tipa (pla, sace, križ karo)
- 

$$u! \cdot \binom{52}{13} \binom{39}{13} \binom{26}{13} \binom{13}{13} =$$

$$= \frac{52!}{13!(52-13)! 13! \dots 13!} = \frac{52!}{(13!)^4} u!$$

$$2. 1, \sqrt{u!}$$

(3) konstrukcija na kli:

izvještaj 702 putovanja

Deklaracija: Najmanj dva prebivalca imaju  
enaku zaštitnicu (25 ženk)

$$\begin{array}{r} 2 \\ \frac{25 \cdot 25}{50} = 625 \\ 125 \\ \hline 625 \end{array} \quad 702 - 625 > 0$$

(4) izvještaj 10 kavet

2 rdeči, 3 zelene, 5 modre

- 1). # možnosti da je prvo redko izvoljen  
red modra

— — — — — — — —

1.

$$2. \frac{8!}{1! 2! 5!}$$

$$3. \frac{7!}{2! 5!} + 2 \cdot \frac{7!}{4! 1! 2!}$$

4.

R - Z - Z - Z -



$$\frac{\binom{10}{4} \cdot 6}{2} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{2 \cdot 3 \cdot 4 \cdot 2} \cdot 6 = \frac{1260}{2} = 630$$

X

modre

$$\binom{10}{5} 4 = \frac{10!}{5! 5!} 4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2} \cdot 4 =$$

=

ii) matematică de la prima vedere este  
rezolvarea problemelor

$$\binom{10}{5} \cdot \binom{4}{2} = \frac{6}{2} - \cdot - - - \uparrow$$

~~$\binom{10}{5}$~~  6 +  ~~$\binom{10}{5}$~~  3

$$\frac{\frac{10!}{5!5!2!}}{\frac{10!}{5!}} - \binom{10}{5} = \frac{\frac{10!}{5!} \left( \frac{1}{3!2!} - \frac{1}{5!} \right)}{\frac{10!}{5!}} =$$

$$= \frac{10!}{5!} \left( \frac{5! - 12}{5!2!} \right) =$$

$$= \frac{10!}{5!} \frac{4 \cdot 5 - 2}{5!2!} = \frac{10!}{5!5!} \frac{18}{2}$$

$$\binom{10}{5} 9$$

1) 3 rdeček

1 zelené

5 belih

zelené 2

$$\text{i)} \quad \Omega = ? \quad \Omega = \binom{V}{2} \quad V = \{r, r, r, z, b, b, b\}$$

$$\Omega = \{ (r, r), (r, b), (r, z), (z, b), (z, r), \\ (b, b), (b, r), (b, z) \}$$

$$\text{ii)} \quad \text{najprej rdeča: } \frac{3}{9} = \frac{1}{3}$$

$$\text{iii)} \quad R_1 = \frac{1}{3} \quad \text{drugi zeleni: } \frac{8}{9} \cdot \frac{1}{8} = \frac{1}{9}$$

$$P(R_1 \cup Z_2) = P(R_1) + P(Z_2) - P(R_1 \cap Z_2)$$

$$P(R_1 \cap Z_2) : \frac{3}{9} \cdot \frac{1}{8}$$

$$P(\dots) = \frac{4}{9} - \frac{3}{8 \cdot 8} = \frac{29}{8 \cdot 9}$$

2) 2 or n both

i)  $\bar{C}_{ijk} = \{\text{neijten in k-tam kerken zulicene kopter}\}$

$P(\bar{C}_{ijk}) = ?$  fukre j,k,n

$$\begin{aligned}\bar{C}_{ijk} &= \left(\frac{n}{n+2}\right)^{j-1} \frac{2}{n+2} \left(\frac{n+1}{n+2}\right)^{k-j-1} \left(\frac{1}{n+2}\right) = \\ &= \frac{2 \cdot n^{j-1}}{(n+2)^{j+k-j-1+1}} (n+1)^{k-j-1} = 2 n^{j-1} \frac{(n+1)^{k-j-1}}{(n+2)^k}\end{aligned}$$

$X=k$

$$P(X=k) = P\left(\bigcup_{i=1}^{k-1} \bar{C}_{i,k}\right) = \sum_{i=1}^{k-1} P(\bar{C}_{i,k}) = \frac{2}{(n+2)^k} \sum_{i=1}^k n^{j-1} (n+1)^{k-j-1}$$

$$\begin{aligned}n^{j-1} (n+1)^{k-j-1} &= \frac{n^j}{n(n+1)} \cdot (n+1)^{k-1} = \left(\frac{n}{n+1}\right)^j \left(\frac{n+1}{n}\right)^{k-1} \\ &= \frac{2(n+1)^{k-1}}{n(n+2)^k} \cdot \frac{1 - \left(\frac{n}{n+1}\right)^k}{1 - \frac{n}{n+1}} = \frac{2((n+1)^k - n^k)}{n(n+2)^k}\end{aligned}$$

ii)  $P(\bar{C}_1 | X=k)$

$$\begin{aligned}P\left(\bigcup_{i=2}^k \bar{C}_{i,k}\right) &= P(X=k) - P(\bar{C}_{1,k}) = \frac{2((n+1)^{k-1} - n^{k-1})}{(n+2)^k} - \\ &= \frac{2}{n(n+2)^k} ((n+1)^k (n+1) - n^k) = \frac{1}{n(n+2)^k} ((n+1)^{k+1} - n^k)\end{aligned}$$

$$\text{cc: } \frac{2}{(n+2)^k} ((n+1)^{k-1} ((n+2)^k - n^{k-1})) =$$

$$2(n+1)^{k-1} - \frac{n^{k-1}}{(n+2)^k}$$

3)

$$K \dots \text{konkav} \Rightarrow P(A|K) = 1$$

$$L \dots \text{linear} \Rightarrow P(A|L) = r$$

$$r=0 \Rightarrow P(A)=1$$

$$P(A) = P(A|K)P(K) + P(A|L)P(L) = \\ 1 \cdot p + (1-p)(1-r)$$

$$P(V_3|V_2) = \frac{P(V_3 \cap V_2)}{P(V_2)} = \frac{P(V_3)}{P(V_2)}$$

$$P(V_2) = P(V_2|C)P(C) + P(V_2|L)P(L) = \\ = 1 \cdot p + (1-p)(1-r)$$

$$P(V_3) = P(V_3|C)p + P(V_3|L)(1-p) =$$

$$1 \cdot p + (1-r)^2(1-p)$$

$$\Rightarrow P(V_3|V_2) = \frac{p + (1-r)^2(1-p)}{p + (1-r)(1-p)}$$

fff

1) Mecēmo keranec, dokle se pede nejmenj eno sterilo in en grob

$$P\{zgrb\} = p \quad \text{or } p < 1$$

Naj bo  $\{X=k\} = \{\text{prvič pede nejmenj } 1 \text{ sterila in zgrb}\}$   
ne keranec k

Meli so redaj neodvisno

$$\text{Računite } P(X \geq k)$$

$$\begin{aligned} P(X \geq k) &= \sum_{n=k}^{\infty} P(X=n) = \sum_{n=k}^{\infty} (p^{n-1}(1-p) + (1-p)p^{n-1}) = \\ &= \frac{(1-p)}{p^k} \sum_{n=0}^{\infty} p^n + \frac{p}{(1-p)^{k-1}} \sum_{n=0}^{\infty} (1-p)^n = \\ &\quad \cancel{\frac{(1-p)}{p^{k-1}}} \frac{1}{1-p} + \cancel{\frac{p}{(1-p)^{k-1}}} \frac{1}{1-(1-p)} = \\ &= \frac{1}{p^{k-1}} + \frac{1}{(1-p)^{k-1}} \end{aligned}$$

v prvič  $n-1$  nedn  $\rightarrow$  je samo zdrav

$$\text{pletet } p^{k-1} + (1-p)^{k-1}$$

$$p=0 \vee p=1 \Rightarrow P(X \geq k) = 0$$

$$= \frac{1 - (1-p)^{k-1}p^{k-1}}{p^{k-1}(1-p^{k-1})} = \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1} - p^{2k-1}} \quad ???$$

2)

Mecmə kevənec deklər ne deklino d.  
 m qəbar ali mə stəvil

max nəticə:  $m+n$  min nəticə:  $\min\{m, n\}$

$$P(\xi_{\text{əqr}}(k)) = p$$

$$P(X=k) = \sum_{i=0}^k p^i (1-p)^{k-i} \quad \begin{array}{l} \text{6 deklino ali mə qəbar} \\ \text{vali nəticəli} \end{array}$$

$$P(X \geq k+1; \text{dəklino mə qəbar}) = p^m (p-1)^{k-m}$$

$$+ p^{k-m} (p-1)^m$$

$k$  metar  
 m jılıq nəticə rəsədə  $k < 2m$

$$p^m (p-1)^{k-m} + p^{k-m} (p-1)^m$$

$$P(X=\underline{k}) =$$

$$P(X=\underline{k-1} \text{ təm. nəticə } m-1 \text{ qəbar})$$

$\{G=k\} = \{$  pade  $m-1$  qbov  $\vee k-1$  kerelih,  
poten  $p$  spek valde qrb  $\}$

$\{S=k\} = \{$  Na  $k-1$  kerelih val  $m-k$  stevi; poten  
 $p$  spek stevi  $\}$

$$\binom{k-1}{m-1} (p^m (1-p)^{k-m}) + p^{k-m} (1-p)^m)$$

3)

$$X \sim \text{Bin}(n, \frac{1}{2})$$

$$P(X=k) = \binom{n}{k} \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{n-k} = \binom{n}{k} \frac{1}{2^n}$$

Definiramo slučajni spremeničjuke.

$y \geq k=0, \dots, n$  velja  $P(Z) \sim P(X, y)$

$$P(X=k, y=k+1) = P(X=k) \frac{n-k}{n} \quad (a)$$

$$\text{in } P(X=k, y=k-1) = P(X=k) \frac{k}{n} \quad (b)$$

$$\text{in } P(X=k, y=l) = 0 \quad \text{če } |k-l| > 1 \text{ ali } l=k$$

Razenj te parabolita slučajne spremeničjuke  $y$

Resitev:

Popolna vrednost

$$P(Y=\emptyset) = \sum_{k=0}^n P(X=k, Y=\emptyset) =$$

$$= P(X=\emptyset-1, Y=\emptyset) + P(X+\emptyset, Y=\emptyset)$$

$$P(Y=\emptyset) = \binom{n}{\emptyset-1} \frac{1}{2^n} \frac{n-\emptyset+1}{n} + \binom{n}{\emptyset+1} \frac{1}{2^n} \frac{\emptyset+1}{n} =$$

$$= \frac{1}{2^n} \left( \frac{n!}{(\emptyset-1)! (n-(\emptyset-1))!} \cdot \frac{n(\emptyset-1)}{n} + \right.$$

$$\left. \frac{n!}{(\emptyset+1)! (n-(\emptyset+1))!} \cdot \frac{\emptyset+1}{n} \right)$$

$$= \frac{1}{2^n} \left( \binom{n-1}{\emptyset-1} + \binom{n-1}{\emptyset} \right) = \frac{1}{2^n} \binom{n}{\emptyset}$$

4)

Naj bodo  $X \sim B\left(\frac{1}{2}\right) \Rightarrow X=0$  je verjetnostjo  $\frac{1}{2}$  in  
 $X=1$  je verjetnostjo  $\frac{1}{2}$

$$Y \sim B\left(\frac{1}{2}\right)$$

$$Z = Z(X, Y)$$

$$P(Z) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \quad " \in \mathbb{R}^4 "$$

$\uparrow$   
 $P(X=1, Y=0)$

Veličja: \*  $0 \leq p_{ij} \leq 1$

$$\sum_{0 \leq j \leq 1} p_{ij} = 1$$

$$\text{Redno de } P(Z) = \begin{pmatrix} \frac{1}{n} + \delta & \frac{1}{n} - \delta \\ \frac{1}{n} - \delta & \frac{1}{n} + \delta \end{pmatrix}$$

kdej sta  $X$  in  $Y$  neodvisno od  $\delta$

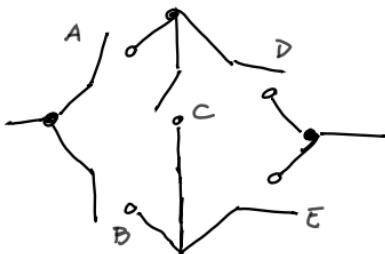
$$-\frac{1}{n} \leq \delta \leq \frac{1}{n}$$

Dejstvo:  $X \perp Y \stackrel{\text{readj.}}{\iff} \det(P_{ij}) = 0$

$$\det(P_{ij}) = \left( \frac{1}{n} - \delta \right)^2 - \left( \frac{1}{n} + \delta \right)^2 = \delta$$

$$\Rightarrow X \perp Y \Leftrightarrow \delta = 0$$

5) V verzje



Hoder slikale prøvse  
el. tak z vjerojatnoščjo  $\frac{1}{3}$

in posamezne  
stikele so  
med seboj  
neodvisne

$A = \{\text{edgedek da tak gre preko } A\}$

Kolikor je vjerojatnost da verzje gre preko?

$$\frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) \quad ??$$

$A \qquad D \qquad C \qquad E \qquad B \qquad E \qquad C \qquad D$

$$\frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} - \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} \cdot \frac{1}{3} - \underbrace{\frac{1}{3} \cdot \frac{1}{3}}_{\text{odstran}} = \frac{1}{9} \quad ??$$

$A \qquad D$

$$\frac{2}{3} \left( \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} (\dots)$$

↑  
zgolj

stetigk. v. PZ

$$P(x=k) = P(H_c) \cdot P(x=k|H_c) + P(x=k|H_{\bar{c}}) P(H_{\bar{c}}) + P(H_{\bar{c}}) P(x=k|H_{\bar{c}})$$

$$P(x=0|H_{\bar{c}}) = 1$$

$$P(x=k|H_{\bar{c}}) = 0$$

$$P(x=0|H_c) = 0$$

$$P(x=k|H_c) = P(x=k-1)$$

$$P(x=0|H_c)$$

$$k=0 \Rightarrow$$

$$P(x=0) = (1-p) P(x=0) + 1 \cdot p^2 + p^{(1-p)} \cdot 0$$

$$P(x=0)(1 - 1 + p) = p^2$$

$$P(x=0) = p$$

$$k \neq 0 \Rightarrow$$

$$P(x=k) = P(x=k)(1-p) + P(x=k-1)p^2 + p^{(p-1)} P(x=k-1)$$

f España

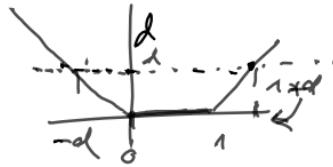
Naj bo  $X \in U[3, 3]$  in definiramo

$$\{D = d\} = \{ \max(X, [0, 1]) = d \}$$

Izraunajte komutativne parodelidene funkcije na  $F_d(D)$  in napisite njen graf

$$F_D(d) = P(D \leq d) \quad D = g(X) \text{ za } g(x) = d(X, [0, 1])$$

$$g(x) = \begin{cases} -x & ; x \leq 0 \\ 0 & ; 0 \leq x \leq 1 \\ x-1 & ; x > 1 \end{cases}$$



gostoto:  
 $f_X(x) = \begin{cases} \frac{1}{3-(-3)} = \frac{1}{6} & ; x \in [3, 3] \\ 0 & ; \text{sicer} \end{cases}$

$$F_X(x) = \int_{-\infty}^x f_X(x) dx = \int_{-3}^x f_X(x) dx = \begin{cases} \frac{x+3}{6} & ; -3 \leq x \leq 3 \\ 0 & ; x < -3 \\ 1 & ; x > 3 \end{cases}$$

Kaj so dogodki  $\{D \leq d\}$ ?

$$d < 0 \Rightarrow \{D \leq d\} = \emptyset$$

$$d = 0 \Rightarrow \{D \leq d\} = [0, 1] \quad F_D(0) = P(X \in [0, 1]) = F_X(1) - F_X(0)$$

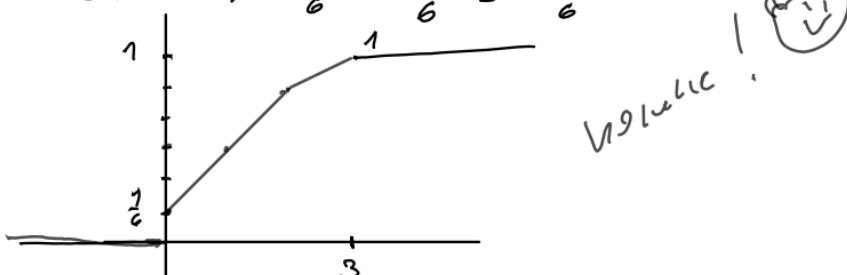
$$d \in [0, 2] \Rightarrow \{D \leq d\} = [-d, 1+d] \quad P_D(d) = F_X(1+d) - F_X(-d)$$

$$d \in [2, 3] \Rightarrow \{D \leq d\} = [-d, 3] \quad P_D(d) = F_X(3) - F_X(-d)$$

$$F_D(0) = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$F_D(d \in [0, 2]) = \frac{d+4}{6} - \frac{-d+3}{6} = \frac{2d+1}{6}$$

$$F_D(d \in [2, 3]) = \frac{5}{6} - \frac{-d+3}{6} = \frac{3+d}{6}$$



Viglj učilc!

$$f_D = F'_D(d) = \begin{cases} 0 & ; d < 0 \\ \frac{1}{3} & ; d \in [0, 2] \\ \frac{1}{6} & ; d \in [2, 3] \\ 0 & ; d > 3 \end{cases}$$

Definirajmo  $X \sim \text{Cauchy}$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

a) Izračunajte  $\gamma = \frac{1}{1+x^2}$

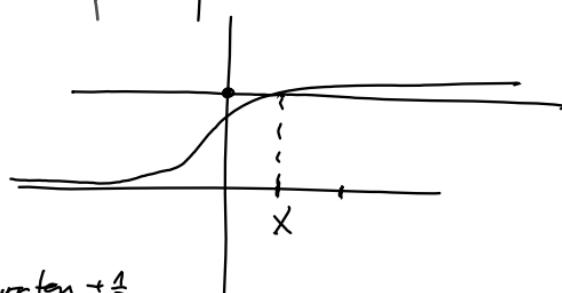
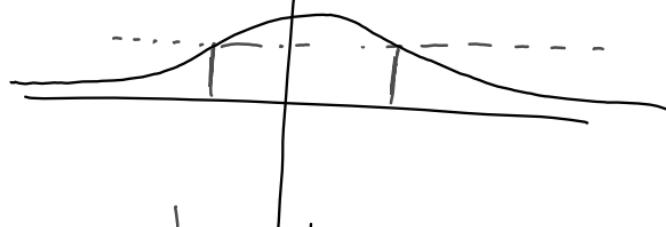
Namig:  $F_Y(y) = P(Y \leq y) = P\left(\frac{1}{1+x^2} \leq y\right) =$

$$= P\left(\frac{1}{y} \leq 1+x^2\right) = P\left(\sqrt{\frac{1}{y}-1} \leq x\right) = \int_{-\infty}^x f_X(t) dt$$

$$P\left(\sqrt{\frac{1}{y}-1} \leq x\right) = \frac{1}{\pi} \arctan(x) \Big|_{-\infty}^x = \frac{1}{\pi} \arctan x + \frac{1}{2}$$

$$P\left(\sqrt{\frac{1}{y}-1} \leq x\right) - P\left(-\sqrt{\frac{1}{y}-1} \leq x\right) =$$

$$\frac{1}{\pi} \arctan x + \frac{1}{2}$$



$$y = \frac{1}{\pi} \arctan x + \frac{1}{2}$$

$$\tan(\pi x - \frac{\pi}{2})$$

$$P(Y \leq y) = P_X(X^2 \geq \frac{1}{y} - 1) = \quad Y = \frac{1}{1+X^2}$$

$$= P_{x \geq 0}(X \geq \sqrt{\frac{1-y}{y}}) + P_{x < 0}(X \leq -\sqrt{\frac{1-y}{y}}) =$$

$$= 2 P(X \geq \sqrt{\frac{1-y}{y}}) = 2(1 - F_X(\sqrt{\frac{1-y}{y}})) = \overset{\text{det}}{F_Y(y)}$$

simetrija

$$f_X(y) = \frac{d}{dx} F_X(x)$$

izemo  $f_Y(y)$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

$$\therefore f_Y(y) = F'_Y(y)$$

$$\underline{\underline{f_Y(y)}} = \left( 2 - 2F_X\left(\sqrt{\frac{1-y}{y}}\right) \right)' = \sqrt{\frac{1}{y}-1}$$

$$= -2 f_X\left(\sqrt{\frac{1-y}{y}}\right) \frac{-\frac{1}{y^2}}{2\sqrt{\frac{1}{y}-1}} =$$

$$= -2 \frac{1}{\pi(1+\frac{1-y}{y})} \cdot \frac{-1}{2y^2\sqrt{\frac{1-y}{y}}} = \frac{1}{\pi} \frac{\sqrt{y}}{y^2\sqrt{1-y}}$$

$$Y = \frac{1}{1+x^2}$$

$$F_Y(y) = P_Y(Y \leq y) = P_X\left(\frac{1}{1+x^2} \leq y\right) =$$

$$\stackrel{?}{=} P_X(x^2 \geq \frac{1}{y} - 1) = \text{Doluje, ker je } y > 0$$

$$\begin{aligned} \ker Y: \Omega \rightarrow (0, 1] \\ \text{in } \ker Y: 1+x^2 > 0 \end{aligned}$$

$$= P(X \geq \sqrt{\frac{1-y}{y}}) + P(X \leq -\sqrt{\frac{1-y}{y}}) = 2P(X \geq \sqrt{\frac{1-y}{y}}) =$$

↑  
simetrija

$$= 2(1 - F_X(\sqrt{\frac{1-y}{y}})) \quad f_X(x) = \frac{1}{\pi(1+x^2)}$$

$$F'_X(y) = f_X(y), \text{če obstaja}$$

$$\Rightarrow f'_Y(y) = F'_X(y) = -2F_X\left(\sqrt{\frac{1-y}{y}}\right)$$

$$= -2f_X\left(\sqrt{\frac{1-y}{y}}\right) \frac{1}{2}\sqrt{\frac{y}{1-y}} \cdot \frac{-y-1+y}{y^2} =$$

$$= f_X\left(\sqrt{\frac{1-y}{y}}\right) \sqrt{\frac{y}{1-y}} \cdot \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \frac{1}{1+\frac{1-y}{y}} \sqrt{\frac{y}{1-y}} \cdot \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \frac{y}{1} \frac{1}{y^2} \sqrt{\frac{y}{1-y}} = \frac{1}{\pi y} \sqrt{\frac{y}{1-y}} = \underline{\underline{\frac{1}{\pi \sqrt{y(1-y)}}}}$$

		Defn		
		No	yes	
White	No	141	19	160
	yes	149	17	
		290	36	326

Definiramo  $R: \{\text{white, black}\} \rightarrow \mathbb{R}$   
 $R(\text{white}) = 0$   
 $R(\text{black}) = 1$   
 $D: \{\overset{\text{No}}{\text{defn}}, \overset{\text{yes}}{\text{defn}}\}$

$$\underline{X} = (R, D)$$

$$\underline{U} = \{\text{sterilo } X=(i,j)\}$$

Vprašanje?

$$1. P(D=1 | R=i) ; i \in \{0, 1\}$$

$$2. D \perp R ? \quad (\text{Ali sta neodvisni spremenljivke?})$$

$$\text{Lahko ocenimo } \hat{P}_{ij} = \frac{\#(X; X=(i,j))}{N=326}$$

$$\hat{P}_{01} = \begin{pmatrix} \frac{141}{326} & \frac{19}{326} \\ \frac{149}{326} & \frac{17}{326} \end{pmatrix} \quad \hat{P}_{00} + \hat{P}_{01} = \text{vrednost dejstva} - P(R=0)$$

$$\hat{P}_{11} = \begin{pmatrix} \frac{141}{326} & \frac{19}{326} \\ \frac{149}{326} & \frac{17}{326} \end{pmatrix} = P(R=1)$$

$$P(D=0) \quad P(D=1)$$

$$1) P(D=1 | R=0) = \frac{P_{01}}{P_{00} + P_{01}} = 0,11$$

$$\frac{P_{11}}{P_{10} + P_{11}} = 0,10$$

1.

n gledalca = n min. jekrem.

1.12.

$X_{...}$  je gledalca k: deki razenj jekre

$$E(X) = ?$$

$A_k = \{k\text{-ti obiskovalec deki eva jo jekre}$

$$P(A_k) = \frac{1}{n}$$

Definicija:  $I_k = \begin{cases} 1 & ; A_k \text{ se zgodi} \\ 0 & ; \text{Ne } A_k \end{cases}$

$$X = \sum_{k=1}^n I_k$$

$$E(X) = \sum E(I_k) = \sum \frac{1}{n} = n \cdot \frac{1}{n} = 1$$

$$\sum + P(I_k = 1)$$

||  
o al: 1  
↓

2. naloage

$$\bullet \rightsquigarrow \bullet \rightsquigarrow \bullet$$

$$P(X=k) = \frac{k}{(k+n)!}$$

X ... st heraka i gre

$$P(X \geq k) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdots \frac{1}{k} \cdot \frac{1}{k+1} = \frac{1}{(k+1)!}$$

$$\sum_{k=x}^{\infty} P(X=k)$$

$$E(X) = ?$$

$$I_k \dots \begin{cases} 1 & : \text{ne } k \text{ sekunde} \\ 0 & ; \end{cases}$$

$$P(X=k)$$

$$E(X) = \sum_{k=0}^{\infty} k \frac{1}{(k+1)!} = \sum_{k=0}^{\infty} \frac{k^2}{(k+1)!}$$

$$E(X) = (\sum_{k=0}^{\infty} I_k) = \sum E(I_k) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} = e - 1$$

3. Relage

$X$  weare skewre spementjivke

$$f_x = \begin{cases} \frac{1}{\sqrt{2\pi} x^{\frac{3}{2}}} e^{-\frac{1}{2x}} & x > 0 \\ 0 & \text{else} \end{cases}$$

$Z \sim N(0, 1)$  pakezite

$$\begin{aligned} F_x(x) &= 2P(Z > \frac{1}{\sqrt{x}}) = 2(1 - F_z(z \leq \frac{1}{\sqrt{x}})) \\ &= 2(1 - \Phi(\frac{1}{\sqrt{x}})) \end{aligned}$$

$$\underline{F_x(x)} = \int_0^x \frac{1}{\sqrt{2\pi} x^{\frac{3}{2}}} e^{-\frac{1}{2x}} dx = 2 \int_{\frac{1}{\sqrt{x}}}^{\frac{1}{\sqrt{2}}} \cancel{\frac{1}{\sqrt{2\pi}}} \cancel{x} e^{-\frac{1}{2}\cancel{x^2}} =$$

$$\cancel{\Phi}(+)\cancel{=} 1 \int_{-\infty}^{\frac{1}{\sqrt{x}}} \dots 2 \int_{\frac{1}{\sqrt{x}}}^{\infty} \cancel{\frac{1}{\sqrt{2\pi}}} \cancel{x} e^{-\frac{1}{2}\cancel{x^2}} =$$

$$u^2 = \frac{1}{x} \quad u^4 =$$

$$2u du = -\frac{1}{x^2} dx \quad = dx - \frac{2u}{u^4}$$

$$dx = -2 \frac{1}{u^3} du$$

$$X \sim U(a, b) \quad 0 < a < b < 1$$

$$Y = -\frac{1}{\lambda} \ln X \quad \lambda > 0$$

$$f_Y(y) = \begin{cases} \frac{1}{b-a} & y \in (a, b) \\ 0 & ; \end{cases}$$

$$P(Y \leq y) = P(-\frac{1}{\lambda} \ln X \leq y) = P(\lambda \ln X \leq -y) =$$

$$P(e^{\lambda x} \leq Y) = F_Y(e^{\lambda x}) = 1 - \int_{-\infty}^{e^{\lambda x}} \frac{1}{b-a} dx =$$

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injektivität  
||  
a

$$= \begin{cases} 0 & e^{\lambda x} < a \\ 1 - \frac{e^{\lambda x} - a}{b-a} & e^{\lambda x} \in (a, b) \\ 1 & e^{\lambda x} \geq b \end{cases}$$

$$\left\{ 1 - \frac{e^{\lambda x} - a}{b-a} \quad e^{\lambda x} \in (a, b) \right.$$

$$e^{\lambda x} \in (a, b)$$

$$\rightarrow x \in (\ln a, \ln b)$$

$$\lambda x \in (-\ln b, \ln a)$$

$$x \in \left( -\frac{\ln b}{\lambda}, -\frac{\ln a}{\lambda} \right)$$

$$P(x=k, y=l) = \begin{cases} \frac{1}{2 \ln 2} \frac{1}{l 2^k} & k \in \mathbb{N}, l \in \{1, \dots, k\} \\ 0 & \text{otherwise} \end{cases}$$

$$P(x=k) = ?$$

$$P(y=l) = ?$$

$$P(x=k) = \sum_l P(x=k, y=l) = \sum_{l=1}^k \frac{1}{2 \ln 2} \frac{1}{l 2^k} = \frac{1}{2^{k+1} \ln 2} \sum_{l=1}^k \frac{1}{l}$$

$$P(y=l) = \sum_k P(x=k, y=l) = \frac{1}{2 \ln 2} \cdot \sum_{k=l}^{\infty} \frac{1}{2^k} =$$

$$\frac{1}{2 \ln 2} \cdot \frac{1}{2^l} \frac{1}{(1 - \frac{1}{2}) \cdot 2} = \frac{1}{\ln 2 2^l}$$

$$P(x=k)P(y=l) = \frac{1}{2 \cdot (\ln 2)^2} \frac{1}{2^l 2^k} \cdot \overbrace{\sum_{i=1}^k \frac{1}{i}}^{\log^2 n} \quad \text{n: next to zero}$$

Vaga

• b príčkov

•  $r < b$  označení

My záležitosti lze rozdělit  
príčkov

$X = \text{stejné záležitosti označené  
z výsledkem}$   
 $y = \text{břez výsledkem}$

$$\textcircled{1} \quad \text{Var}(X) = ?$$

$$\textcircled{2} \quad \text{Var}(X) = \frac{b-n}{b-1} \text{Var}(x)$$

Nasvet: Upozorbi:  $I_k =$  degadlo de je k-té príčov označené

\textcircled{3} kej k pomoći: ze metodologije získání parametrů

Resíku:  $X \sim \text{Bin}(n, \frac{r}{b})$

$$Y \sim \text{HipGeo}(n, r, b)$$

$$\text{Iz predavanj: } E(X) = n \frac{r}{b} \quad E(Y) = n \frac{r}{b}$$

$$E(X^2) = \frac{n^2 r^2}{b^2} + n \frac{(b-r)r}{b^2}$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = n \frac{(b-r)r}{b^2}$$

$$E(Y^2) = \sum k P(Y^2=k) \approx \sum k^2 P(Y=k)$$

$$E(Y) = \sum k P(Y=k) = \sum P(I_k=1) =$$

~~$$P(I_k \text{ je r. n. záležitost}) = \frac{1}{b}$$~~

$$E(Y) = \sum_{k=1}^n P(I_k) = \frac{n r}{b}$$

$$E(Y^2) = \sum k^2 P(Y=k) = \sum k (k P(Y=k))$$

Věmo:  $\text{Var}(y) = \sum_{\substack{i \leq j \leq n \\ i \neq j}} \text{cov}(I_i, I_j)$

$$\text{cov}(x, y) = E(xy) - E(x)E(y)$$

$$\text{Var}(y) = \sum ($$

Resitav prejšnje naloge:

$$y = \sum_{k=1}^n I_k \quad \text{var}(y) = \sum_{k=1}^n \text{var} I_k + \sum_{\substack{1 \leq i, j \leq n \\ i \neq j}} \text{cov}(I_i, I_j) =$$
$$= \sum \text{var} I_k + 2 \sum_{1 \leq i < j \leq n} \text{cov}(I_i, I_j) =$$

$$\ker I_k \sim B\left(\frac{r}{b}\right)$$

$$\hookrightarrow = n \text{var} I_1 + n(n-1) \text{cov}(I_1, I_j)$$

$$\text{Od predavanj vemo } \underbrace{E(I_k) = E(I_k^2)}_{\text{vedno velja}} = \frac{r}{b}$$

$$\text{var}(I_k) = \frac{r}{b} - \left(\frac{r}{b}\right)^2 = \frac{r}{b} \left(1 - \frac{r}{b}\right)$$

$$\text{cov}(I_i, I_j) = E(I_i I_j) - E(I_i) E(I_j) =$$

$$= P(I_i=1, I_j=1) - \left(\frac{r}{b}\right)^2 =$$

$$P(I_i=1, I_j=1) = P(I_i=1 | I_j=1) P(I_j) = \frac{r-1}{b-1} \frac{r}{b}$$

$$\text{cov}(I_i, I_j) = \frac{r}{b} \left( \frac{r-1}{b-1} - \frac{r}{b} \right)$$

$$\text{var}(y) = \frac{n r}{b} \left(1 - \frac{r}{b}\right) + n(n-1) \frac{r}{b} \left( \frac{r-1}{b-1} - \frac{r}{b} \right)$$

Vaga:

$$\text{Naj bodo } M = IA + (1-I)B$$

$A, B \dots$  spremenljive skozi

$I \sim \text{Ber}(p)$  indikator

Predpostavite da je  $I$  neodvisno od  $A$  in od  $B$

$$\text{Dokaz: } E(M) = pE(A) + (1-p)E(B)$$

Razloži:

$$\begin{aligned} E(M) &= E(IA + (1-I)B) = E(IA) + E((1-I)B) = \\ &= E(I)E(A) + E(1-I)E(B) = P(I=1)E(A) + P(I=0)E(B) = \\ &= pE(A) + (1-p)E(B) \end{aligned}$$

n dni:

$$t=1, \dots, n$$

$X_t, \dots, \tilde{X}_t$  litvak/minute

$$X_t = I_t Y_t + (1-I_t) Z_t$$

•  $T = \{1, \dots, n\}$  je den spramenke iz pre v drug raz

$$I_t = \begin{cases} 1 & t < T \\ 0 & t \geq T \end{cases} \quad Y_t = \text{Pos}(x_1) \quad Z_t = \text{Pos}(x_2)$$

$I_t$  je neodvisna od  $Y_t$  in od  $Z_t$ ,  $\forall t$

①  $I_t$  je P?  $I_t = \text{Ber}(\rho)$

② Najdite izraz za  $E(X_t)$

③ Kaj je  $E(X_t^2)$   $E(Y_t^2) = \lambda_1^2 + \lambda_1$

Rješenje:

①  $P(I_t = 1) = P(t < T) = P(T > t) = \frac{n-t}{n}$

②  $E(X_t) = E(I_t Y_t + (1-I_t) Z_t) =$

$$E(I_t) E(Y_t) + E(1-I_t) E(Z_t)$$

$$\frac{n-t}{n} \lambda_1 + \frac{t}{n} \lambda_2$$

$$\text{var}(X_t) = \lambda_1 = \text{var}(Y_t)$$

③  $E(X_t^2) = E(I_t^2 Y_t^2 + 2I_t(1-I_t)Y_t Z_t + (1-I_t)^2 Z_t^2) =$

$$= \frac{n-t}{n} E(Y_t^2) + \frac{t}{n} E(Z_t^2) =$$

$$= (\lambda_1^2 + \lambda_1) + \frac{1}{n} (\lambda_2^2 + \lambda_2 - \lambda_1^2 - \lambda_1) =$$

$$= \lambda_1^2 + \lambda_1 + \frac{1}{n} (\lambda_2 - \lambda_1)(\lambda_2 + \lambda_1 + 1)$$