Nihanje

$$\nu = \frac{1}{t_0} \qquad \omega = 2\pi\nu \qquad t_0 = \frac{2\pi}{\omega}$$

$$\sum \vec{F} = m\vec{a} \qquad \sum M = J\alpha$$

Enačba: $\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = 0$

β ... koeficient dušenja

ω_0 ..., krožna frekvenca

$$D = 4\beta^2 - 4\omega_0^2$$

$D < 0$ ...., podkritično dušenje

$$y = Be^{-\beta t}\sin(\omega t + \delta)$$

$$\omega^2 = \omega_0^2 - \beta^2$$

$D = 0$ .... kritično dušenje

$$y = (B_1 + B_2 t)e^{-\beta t}$$

$D < 0$, ... nadkritično dušenje

$$y = B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t}$$

kjer sta λ_1, λ_2 rešitvi $\lambda^2 + 2\beta\lambda + \omega_0^2 = 0$

$$\lambda_{12} = -\beta \pm |\omega| \quad ; \ \omega^2 = \omega_0^2 - \beta^2$$

Vijačna vzmet (linearni zakon upora)

$$F_u = C \cdot \dot{y} \quad \ldots \text{sila upora}$$

$$\beta = \frac{C}{2m} \qquad \omega_0^2 = \frac{k}{m}$$

Matematično nihalo (ni dušenja)

$$\ddot{\varphi} + \frac{g}{l}\dot{\varphi} = 0$$

$$\varphi << 1 \Rightarrow \omega_0^2 = \frac{g}{l}$$

Fizično nihalo (ni dušenja)

$$\omega_0^2 = \frac{m g l^*}{J_z}$$

J_z ... vztrajnostni moment okoli osi vrtenja

Greenove funkcije

Nihalo vzbujamo s silo $F(t)$

$$x(t) = \int_0^t \frac{F(J)}{m\omega} e^{-\beta(t-J)} \sin(\omega t - \omega J)\, dJ$$

Vsiljeno nihanje (sinusno)

$$\ddot{y} + 2\beta \dot{y} + \omega_0^2 y = A_0 \sin(\omega_v t)$$

amplituda

frekvenca vsiljevanja

Rešitev: $y = y_h + y_p$

homogeni del

partikularni del

$$y = B e^{-\beta t} \sin(\omega t + \delta) \quad (\omega^2 = \omega_0^2 - \beta^2)$$

nastavek $B_p \sin(\omega_v t - \delta_p)$

$$\tan\delta_p = \frac{2\omega_v \beta}{\omega_0^2 - \omega_v^2} \qquad B_p = \frac{A_0}{\omega_v \sqrt{\frac{(\omega_0^2 - \omega_v^2)^2}{\omega_v^2} + 4\beta^2}}$$

B_p je funkcija ω_v. Kdaj je B_p maksimalen

$$\omega_m = \omega_0 \sqrt{1 - \frac{2\beta^2}{\omega_0^2}}$$

Sklopljeno nihanje

Zapišemo Newtonove zakone za vsako nihalo posebej zapišemo v obliko

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2 (x_1 - x_2) = 0$$

$$\ddot{x}_2 + \omega_1^2 x_2 + \omega_2 (x_1 - x_2) = 0$$

$x_a = x_1 + x_2$ $\quad x_b = x_1 - x_2$ potem prevedemo

$$x_1 = C_1 \sin(\omega_a t + \delta_a) + C_2 \sin(\omega_b t + \delta_b)$$

$$x_2 = C_1 \sin(\omega_a t + \delta_a) - C_2 \sin(\omega_b t + \delta_b)$$

$$\omega_a^2 = \omega_1^2 \qquad \omega_b^2 = \omega_1^2 + 2\omega_2^2$$

EKVIVALENTNO

$$x_1 = A\sin(\omega_a t) + B\cos(\omega_a t) - C\sin(\omega_b t) - D\cos(\omega_b t)$$

$$x_2 = A\sin(\omega_a t) + B\cos(\omega_a t) + C\sin(\omega_b t) + D\cos(\omega_b t)$$

Taylorjevi približki

$(1+x)^{\alpha} = 1+\alpha x$

$\sin x = x$

$\tan x = x$

$\cos x = 1 - \frac{1}{2}x^2$

Valovanje

$$u(x,t) = f(x-ct) + g(x+ct) \qquad f,g \in C^2(\mathbb{R})$$

iz začetnih pogojev dobimo
$u(0,\dots)$ in $u(\dots,0)$

$u(x,0) = A(x) \quad u_t(x,0) = B(x) \Rightarrow$

$$u(x,t) = \frac{1}{2}\left(A(x-ct) + A(x+ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} B(x)\,dx$$

Robni pogoji

x_0 krajišče palice

- v x_0 palica vpeta v steno $\Rightarrow u(x_0,t) = 0$
- v x_0 prost konec palice $\Rightarrow u_x(x_0,t) = 0$

Hitrost valovanja

palica: $c = \sqrt{\frac{E}{\rho}}$ $\qquad E\dots$ prožnostni modul

struna: $c = \sqrt{\frac{F}{\rho S}}$

vijačna vzmet: $c = l\sqrt{\frac{k}{m}}$

kapljevina: $c = \sqrt{\frac{1}{\chi S \rho}}$ $\qquad \chi\dots$ adiabatska stisljivost

plin: $c = \sqrt{\frac{\kappa R T}{M}}$

Sinusno valovanje

če so robni pogoji

$$u(x=0,t) = u_0 \sin(-\omega t + \delta) \Longrightarrow$$

$$u(x,t) = u_0 \sin(kx - \omega t + \delta)$$

$$k = \frac{\omega}{c} \quad \text{.... valovno število}$$

$$\omega = 2\pi\nu \qquad \lambda = \frac{\nu}{c} \qquad k = \frac{2\pi}{\lambda}$$

Stojno sinusno valovanje

$$u(x,t) = u_0 \sin(kx + \omega t + \delta_1) + u_0 \sin(kx - \omega t + \delta_2)$$

prosta na obeh koncih $\Longrightarrow$

$$k = k_n = \frac{n\pi}{l} \qquad \lambda_n = \frac{2l}{n} \qquad \nu_n = \frac{nc}{2l}$$

Energija sinusnega valovanja

$$w = w_0 \cos^2(kx - \omega t + \delta) \qquad w_0 = u^2 \omega^2 \rho$$

$$\bar{w} = \frac{1}{2} u_0^2 \omega^2 \rho$$

$P = cS\bar{w}$ .....energijski tok valovanja [W]

$j = c\bar{w}$ .....gostota energijskega toka

ZVOK

j.....jakost slišimo: $[10^{-12}\frac{W}{m^2}, 1\frac{W}{m^2}]$

glasnost = $10 \cdot \log_{10}\frac{j}{j_0}$ [db] $j_0 = 10^{-12}\frac{W}{m^2}$

$$\Delta p = -\frac{\omega u_0}{\chi c}\cos(kx - \omega t + \delta)$$

χ.... stisljivost plina

$$\chi = -\frac{1}{V}\frac{\partial V}{\partial p} > 0$$

izotermno stiskanje: $\chi = \frac{1}{p}$

adiabatno stiskanje: $\chi = \frac{1}{p\kappa}$

Dopplerjev pojav

$$\nu_s = \frac{c + v + v_s}{c + v - v_0} \nu_0$$

v … hitrost vetra

oddajnik in sprejemnik se gibljeta proti drug drugemu, veter gre v desno

Prvi letnik

$$x(t) = x_0 + v_0 t + \frac{at^2}{2} \longrightarrow t = \sqrt{\frac{2h}{g}}$$

$$v = v_0 + at$$

$$v = \frac{ds}{dt}$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$a = \frac{dv}{dt}$$

$$x = \frac{1}{2} \Delta v \cdot t$$

$$\omega = \frac{d\varphi}{dt}$$

$$\varphi = \omega_0 t + \frac{\alpha t^2}{2}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega^2 = \omega_0^2 + 2\alpha\varphi$$

$$\omega = \omega_0 + \alpha t$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\varphi = \frac{1}{2} \Delta\omega t$$

$$v = \omega r$$

$$\omega = \frac{2\pi}{t_0} = 2\pi\nu$$

$$a_t = r\alpha$$

$$W_k = \frac{mv^2}{2}$$

$$G = mv$$

$$Ft = \Delta G$$

$$W_p = mgh$$

$$W_e = \frac{kx^2}{2}$$

$$A = Fx$$

$$F = xk$$

$\Gamma = J\omega \qquad \Gamma = G r$

$M = Fr = \frac{d\Gamma}{dt} = J\alpha = J\frac{a_T}{r}$

$F_g = G\frac{m_1 m_2}{r^2} \qquad W_p = -G\frac{m_1 m_2}{r}$

$n = \frac{pV}{RT} \qquad W_n = mc\Delta T \qquad W_k = \frac{2}{3}k_B T$

$c_p \geq c_v$

enoatomen plin

$\Delta V = \beta V \Delta T \qquad PV = N k_B T \qquad \frac{R}{N_A} = k_B$

$A = -\int p\,dV \qquad c_p = c_v + \frac{R}{M}$

idealni plini

$1: \frac{3}{2}k_B T \rightarrow c_v = \frac{3}{2}\frac{R}{M} \quad c_p = \frac{5}{2}\frac{R}{M}$

$2: \frac{5}{2}k_B T \rightarrow c_v = \frac{5}{2}\frac{R}{M} \quad c_p = \frac{7}{2}\frac{R}{M}$

$n: \frac{6}{2}k_B T \rightarrow c_v = 3\frac{R}{M} \quad c_p = 4\frac{R}{M}$

$\kappa = \frac{c_p}{c_v} \qquad 1: \frac{5}{3} \quad 2: \frac{7}{5} \quad n: \frac{4}{3}$

adiabatno: $\Delta W_n = A \qquad Q = 0$

$TV^{\kappa-1} = \text{konst} \rightarrow pV^{\kappa} = \text{konst}$