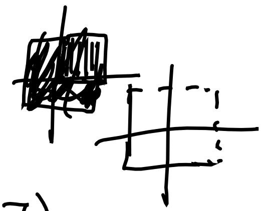
1.1.

$A = [-1,1] \times [-1,1]$

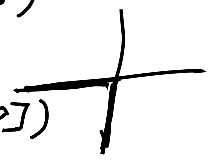
$g : \mathbb{R}^2 \to \mathbb{R}^2/A$ hoc



a) $g_*((-\infty, 0] \times (-\infty, 0])$

~~zapte~~ ni odp ni zap ~~ni. otw~~ zap

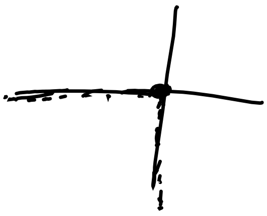
$g^*(g_*((-\infty,] \times (-\infty, 0]))$



b) $g_*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

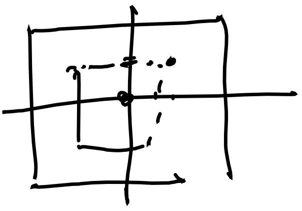
ni odpte

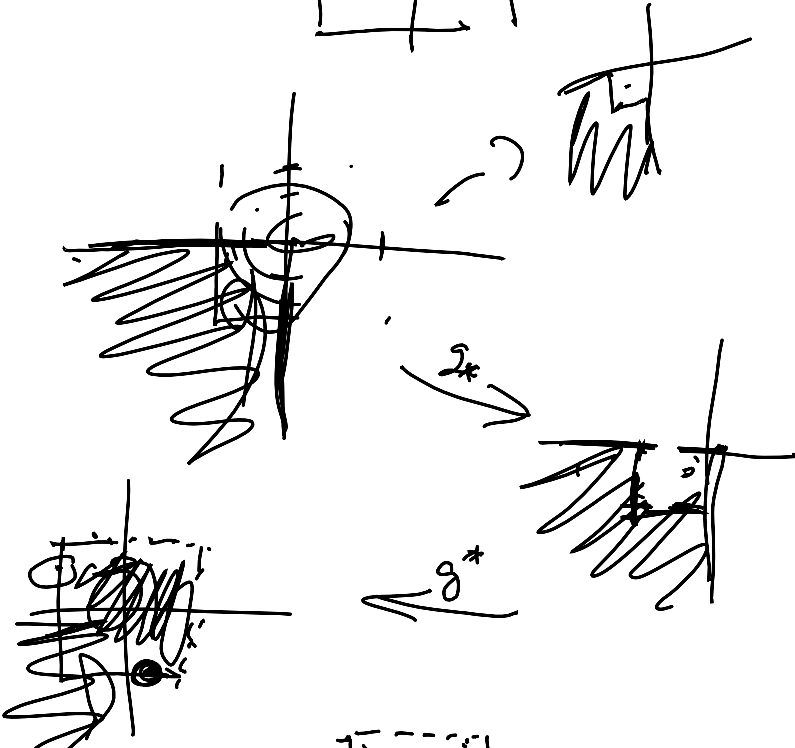
ni zapte

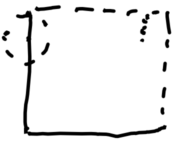


c) $g_*([-2,2] \times [-2,2])$

odpte

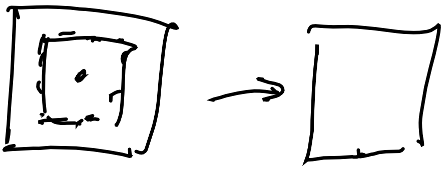






5)

$g^*(g([-2,2] \times [-2,2])$



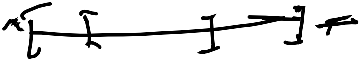
zapte

ni odp

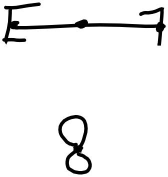
c) nizdy jezy

7^{4}

a) $[-2,2] / [-1,1] \cong [-1,1]$



b) $[-1,1] / \{-1,0,1\}$



$\cong$ [illegible]

$\mathbb{R}^2$

c)

$$\mathbb{R}^n / K(0,1) \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[rd, "f"] \arrow[d] & \\
\mathbb{R}^n / K(0,1) & \mathbb{R}^n
\end{tikzcd}$$

$$f(\vec{a}) := \begin{cases} 0 \,; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} \,; & a \in \mathbb{R}^n - \{\overline{K}(0,1)\} \end{cases}$$

d) $\mathbb{R}^n/_\sim$ $x \sim y \Leftrightarrow \|x\| = \|y\|$

$\mathbb{R}^n/_\sim \overset{\sim}{\longrightarrow} [0,\infty)$

$f: \mathbb{R}^n \longrightarrow [0,\infty)$

$f(\vec{x}) = \|\vec{x}\|$ je sur

$a \in [0,\infty)$ $(a,0,0\ldots) \mapsto a$

$[x]=[y] \Leftrightarrow \|x\|=\|y\| \Leftrightarrow f(x)=f(y)$

je zvezna

iščemo preslikavo s, da velja $f \circ s = id_{[0,\infty)}$

$s: a \longmapsto (a,0\ldots\ldots)$

Dokažimo da če $r \circ s = id_Y$

$\Rightarrow$ r kvocientna, s vložitev

$S \subseteq Y$ taka da $r^*(S)$ odp v X

$S = s^*(r^*(S)) =$

....

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S$ ■

$\Longrightarrow$ S je odprta

f)

$$S^n \times [-1, 1]) \big/ _{\{S^n \times \{-1\}, S^n \times \{1\}\}} \cong S^{n+1}$$

$$f: X \to S^{n+1}$$

$$(x, t) \longmapsto (xb, t) = (\sqrt{1-t^2}\, x, t$$

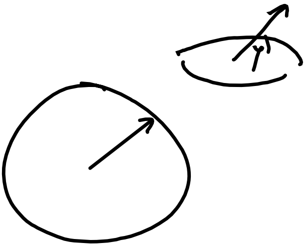
$\nearrow$

anstatt voller

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1 - \|t\|^2} = \sqrt{1-t^2}$$



$$G/_H = \{ g \cdot H \, ; g \in G\} =$$

$$= \{\{g \cdot h \, ; h \in H\} \, ; g \in G\}$$

G top. grupa

$a \in G$

$$L_a : G \longrightarrow G \qquad \text{leva translacija}$$
$$x \longmapsto a \cdot x$$

$a, b \in G$

$$h : G \longrightarrow G$$
$$h(a) = b \qquad h = ?$$

možnosti

$ba^{-1}x$

$xa^{-1}b$

bxa^{-1}

$a^{-1}xb$

$$L_{ba^{-1}} : x \longmapsto ba^{-1}x$$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\iff$ $ba^{-1}A$ okolica $b \in G$

$\exists U^{odp} \subseteq A$. $a \in U$

$b \in ba^{-1}A$

$a \in U \Rightarrow \underset{\overset{\|}{b}}{ba^{-1}a} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \overset{\subseteq A}{\text{odprta}}$ v G

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ H okolica $1 \Rightarrow H$ odp in zap v G

$a \in aH \subseteq H$

$\Rightarrow H$ je okolica vsake svoje točke

$G-H$ je odp.

$aH \cap H = \emptyset \Rightarrow aH = H$

$a \in G-H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$

vsak element ima okolico, ki ne seka $H \Rightarrow H$ je zaprta

c) C komponenta $\quad 1 \in C$

$\Rightarrow$ C zaprta edinka v G

$C \leq G$

$L_a : x \mapsto ax$ je homomorfizem za $\forall a \in C$

$\forall a \in C .\ L_a \subseteq C$

L_a ohranja povezanost

Prav tako $\quad L_a \cdot 1 = a \Rightarrow$

$$L_{a*}C \cap C \Rightarrow L_{a*}C \subseteq C$$

invertiranje: invertiranje je tudi homeo $\quad i : x \mapsto x^{-1}$

$\Rightarrow$ po istih argumentih

Ali je edinka?

$\forall a \in G .\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \mapsto axa^{-1}$ je homeomorfizem

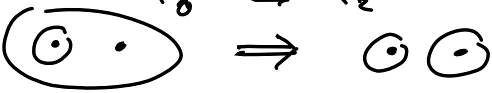
$\Rightarrow$ je povezano in $a\, id\, a^{-1} = id \in C$

$\Rightarrow$ $aCa^{-1} \subseteq C$

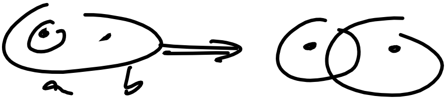
ker je kompozitum dveh translacij (leva in desna)

d) za G je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$



$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U,\ b \notin U$ BŠZS

$$U^{-1} = \{a^{-1} : a \in U\}$$

$aU^{-1}b \ni b \qquad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U,\ a = ac^{-1}b$

$$\Rightarrow b = c \Rightarrow b \in U \quad \times$$

$T_1 \Rightarrow T_2$

$\exists U, V \overset{odp}{\in} G.\quad a \in U,\ b \in V.\quad a \notin V \quad b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$$\Delta_G = f^{*}(\{1\})$$

$$f : (x, y) \longmapsto xy^{-1}$$

~~za~~ zvezna praslika ~~za~~ tk je zaprta

② T_{cc} je T_1 in ni T_2

$\Rightarrow$ $(\mathbb{R}, +)$ ni top grupa za T_{cc}

(ker 1.d)

(2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

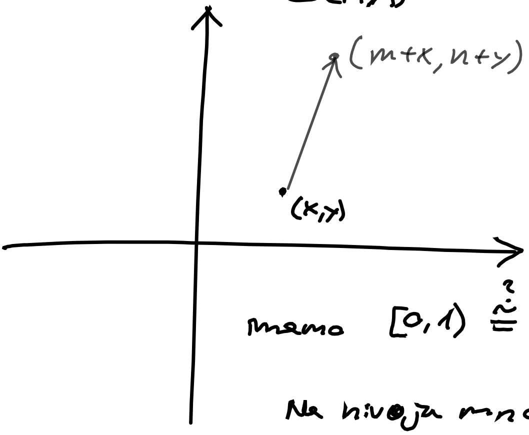
a) $(m,n)(x,y) := (m+x, n+y)$

$\Phi : g \longmapsto \underbrace{(a \longmapsto g\cdot a)}$

bijekcija (izomorfizem)

$\Phi : G \longrightarrow \mathrm{Bij}(A)$ ← grupa za komponiranje

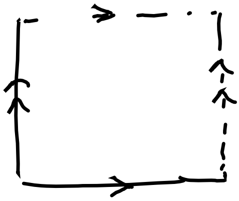
V bistvu: $\Phi : G \longrightarrow (\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ)$



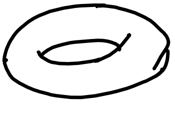
mamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ja

za nivoju topologije ? NE



Dobimo torus



$\cong S^1\times S^1$

$f : \mathbb{R}^2 \longrightarrow S^1\times S^1$

$(x,y) \longmapsto (e^{2\pi i x}, e^{2\pi i y})$

Tuk nism poslušala več ☹