7.1.

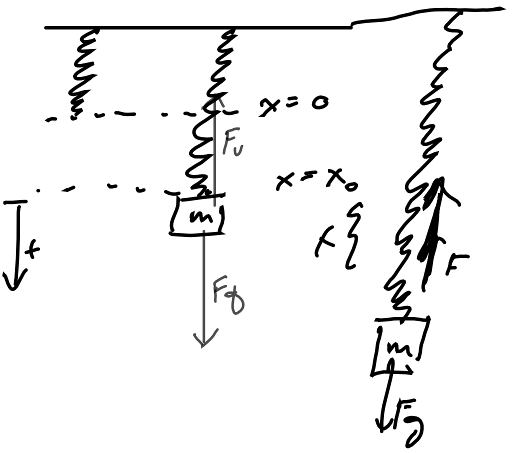
$l = 1\,m$

$k = 500\,N/m$

$m = 8\,kg$

$a_0 = 0{,}1\,m$

$x_0 = ?$



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

$$F_g + F_{vz} = m\cdot\vec{a} \qquad = \ddot{x}(t)$$

$$+mg - k(x_{(t)} + x_0) = m\vec{a}(t)$$

$$\underbrace{mg - kx_0}_{=0} - kx = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\ddot{x} = \frac{-kx}{m} \qquad -\frac{k}{m} = \omega_0^2$$

$$x(t) = \sin(\omega t)\cdot A + B\cos(\omega t)$$

začetni pogoji:

$$x(0) = \vec{a}_0$$

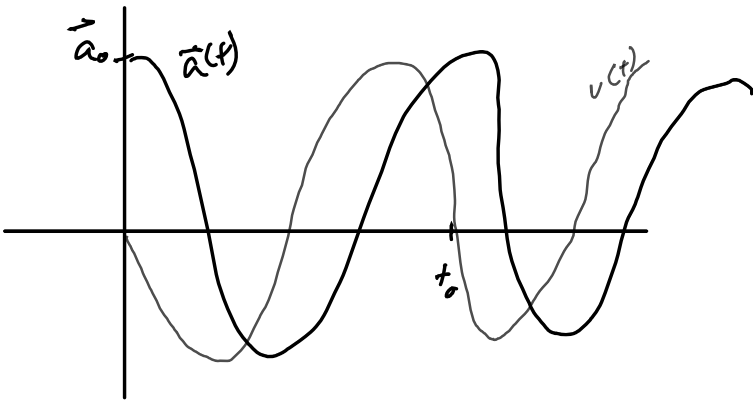
$$\vec{a}_0 = x(0) = B$$

$$v(t) = \dot{x}(t)$$

$$v(0) = \dot{x}(0) = A\omega\cos(\omega t) - B\omega\sin(\omega t)$$

$$0 = A\omega \Rightarrow A = 0$$

$$x(t) = a_0\cos(\omega t)$$



$$\cos(\omega t) = 1$$

$$\omega t_0 = 2\pi \qquad t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{m}}{\sqrt{k}}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$ ... krožna frekvenca

$$\omega_0 = 7{,}9\ \frac{1}{s}$$

Energija nihanja

$$W = \frac{mv^2}{2} + mg(-x) + \frac{k(x+x_0)^2}{2} - \frac{k}{2}x_0^2$$

$$= \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}ka_0^2 = 2{,}5\ \mathrm{J}$$

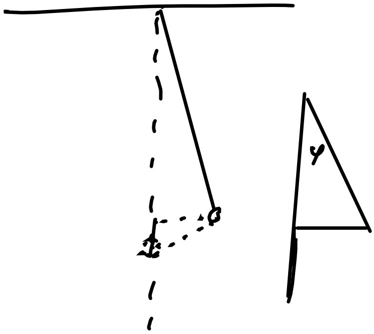
$l = 3m$

$\varphi_0 = 3{,}5^\circ$

$t = 15\,s$

$N = ?$

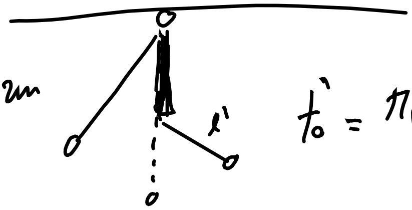
$V_{max} = ?$



$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$N = \frac{t}{t_0} = 4{,}32$$

$$v = \sqrt{\cancel{k}\, g h \cdot 2} \qquad h = l - \cos\varphi \cdot l$$



$l' = 2m$

$$t_0' = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l'}{g}} =$$

$$= \frac{\pi}{\sqrt{g}}\left(\sqrt{l} - \sqrt{l'}\right) = 3{,}15\,s$$

$$N = \frac{t}{t_0} =$$

$\varphi(t) = \varphi_0 \cos(\omega t)$ ← lastna frekvenca

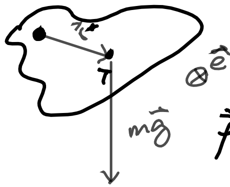
$V_{max} = l \cdot \Omega_{max}$ ← kotna hitrost

$$\Omega(t) = \dot{\varphi}(t) = -\underbrace{\varphi_0 \omega}_{\Omega_{max}} \sin(\omega t)$$

$$V_{max} = l \cdot \varphi_0 \omega$$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

Težno nihalo



$$\vec{M} = J\vec{\alpha}$$

$$\vec{r}^* \times m\vec{g} = J\vec{\alpha}$$

$$\vec{\varphi} = \varphi \cdot \hat{e}$$

$$r^* m g \sin\varphi \, (-\hat{e}) = J \cdot \ddot{\varphi} \, \hat{e}$$

$$\ddot{\varphi} = -\frac{mgr^*}{J}\sin\varphi$$

$$\varphi \ll 1$$

$$\ddot{\varphi} \approx \underbrace{-\frac{mgr^*}{J}}_{\omega^2}\varphi$$

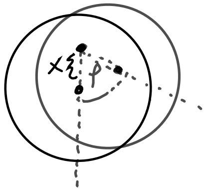
$$t_0 = 2\pi\sqrt{\frac{J}{mgr^*}}$$

Matematično nihalo:

$$J = ml^2$$

$$r^* = l \qquad t_0 = 2\pi\sqrt{\frac{l}{g}}$$

7.7.



$R = 0,5\,m$

t_0 min

oko osnovne osi   translacija

$J = \frac{1}{2} m R^2 + m x^2$

$t_0(x) = ?$

$$t_0 = 2\pi \sqrt{\frac{J}{mgx}} =$$

$$t_a = 2\pi \sqrt{\frac{\frac{1}{2}R^2 + x^2}{gx}}$$

tražimo max:

$$\frac{2x(gx) - g\left(\frac{1}{2}R^2 + x^2\right)}{gx^2} = 0$$

$$2gx^2 - gx^2 - \frac{1}{2}R^2 g$$

$$\cancel{g}x^2 = \frac{1}{2}R^2 \cancel{g}$$

$$x = \cancel{\sqrt{\tfrac{1}{2}}}\ \frac{R}{\sqrt{2}}$$

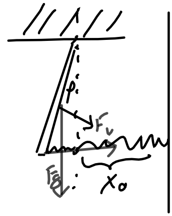
7.8

$l = 1\,m$

$m = 0{,}5\,kg$

$k = 5\,N/m$

$\nu = ?$



$\varphi << 1$

$\sum \vec{M} = J \cdot \vec{\alpha}$

$$J\alpha = -F_g \cdot \frac{l}{2}\underbrace{\sin\varphi}_{\approx \varphi(t)} - \overbrace{\underbrace{kx}_{x(t) = l\cdot\varphi(t)}}^{F_v} \cdot l \cdot \underbrace{\cos\varphi}_{\approx 1}$$

racica

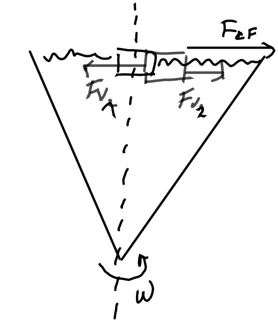
$\alpha(t) = \ddot{\varphi}(t)$

$$\ddot{\varphi}(t) = -\frac{mg \cdot l}{2J}\varphi(t) - \frac{kl^2}{J}\varphi(t) =$$

$$= -\frac{3g}{2l}\varphi(t) - \frac{3k\varphi(t)}{m} = -\Big(\underbrace{\frac{3g}{2l} + \frac{3k}{m}}_{\omega_0}\Big)\varphi(t)$$

$$\omega_0 = \sqrt{\frac{3g}{2l} + \frac{3k}{m}} = 6{,}7/s$$

$2\pi\nu = \omega_0 \qquad \nu = \frac{\omega_0}{2\pi} = 1{,}06\ Hz$



$m = 20g$

$k_1 = 1 \, N/m$

$k_2 = 3 \, N/m$

$\omega = 10/s$

$\nu = ?$

$$\sum F = ma$$

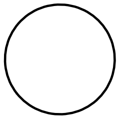
$$ma = -k_1 x - k_2 x + m\overset{\omega^2 x}{\overset{\parallel}{a_r}}$$

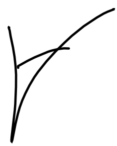
$$\ddot{x}(t) = -\left(\frac{k_1 + k_2 - m\omega^2}{m}\right) x$$

$$\omega_0 = \sqrt{\frac{k_1 + k_2 + m\omega^2}{m}} = 10/s$$

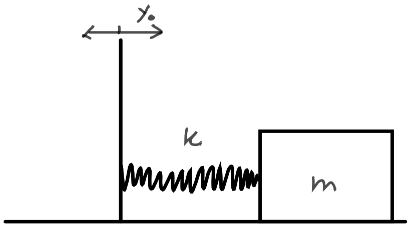
$$\nu = 1{,}6 \, Hz$$

Foucaultova nihalo





(7.25)



$y_0 = 5\,cm$

$m = 100\,g = 0{,}1\,kg$

$k = \frac{N}{m}$

$\beta = 0{,}2/s$

$\nu = 12\,Hz$

$x_0 = ?$

$\delta = ?$

$x_{0max} = ?$

$\nu_{max} = ?$

$$m\ddot{x} = -k(x-y) - 2\beta m\dot{x}$$

$$\ddot{x} + 2\beta\dot{x} + \frac{k}{m}x = \frac{k}{m}y \rightarrow y_0\cos(\omega t)$$

$$\omega^2 = \frac{k}{m}$$

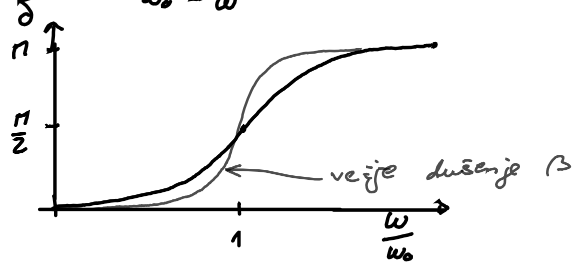
Nehomogena diferencialna enačba

nastavek: $x(t) = X_0 \cdot \cos(\omega t - \delta) + \cancel{+ x_0' e^{-\beta t} \cos(\omega' t - \delta')}$, $\omega' = \sqrt{\omega^2 - \beta^2}$

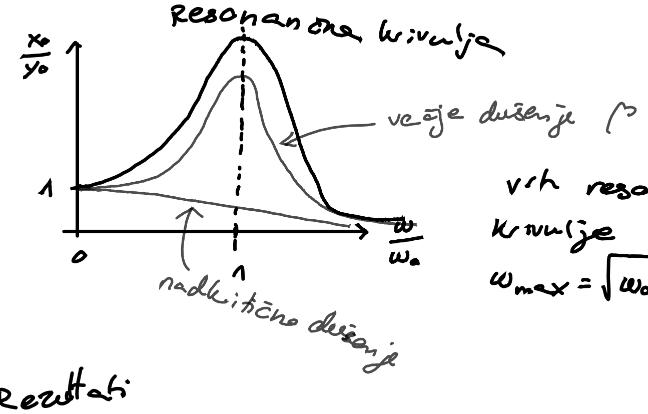
se zaduši, nas pa zanima kaj se ustali

Rezultati:

$$\tan\delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$$



$$\frac{x_0}{y_0} = \frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2 + (2\beta\omega)^2}}$$



vrh resonančne krivulje

$\omega_{max} = \sqrt{\omega_0^2 - 2\beta^2}$

Rezultati

$\omega_0 = \sqrt{\frac{k}{m}} = 10/s \Rightarrow \nu = 1{,}6\,Hz$

$\delta = 179{,}7^\circ$

$X_0 = 0{,}89\,mm$

$\nu_{max} \approx 1{,}6\,Hz$

$x_{0max} = 1{,}25\,m$

moč pri vsiljenem nihanju

$$P = \frac{dA}{dt} = \frac{F\,dx}{dt} = Fv = 2\beta m v^2_{(t)} =$$

$$v = \dot{x} = x_0 (-\sin(\omega t) - \delta) \cdot \omega$$

$$\bar{P} = 2\beta m x_0^2 \omega^2 \underbrace{\overline{\sin^2(\omega t - \delta)}}_{= \frac{1}{2}} = \beta m x_0^2 \omega^2$$

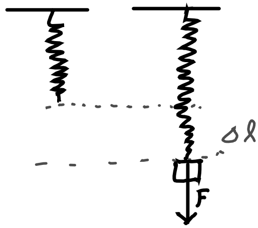
$$\bar{P} \propto \omega^2 \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}$$

$$\frac{dP}{d\omega} = \frac{2\omega\left[(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2\right] - \left[4\omega(\omega_0^2 - \omega^2) + 8\beta^2\omega\right]}{\left((\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2\right)^2} = 0$$

splosno nihanje

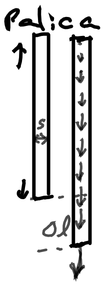
14.3

Elastomehanika: Hookov zakon



$$F = k \cdot \Delta l$$

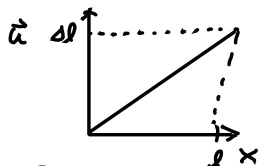
Vpeljava prožnostnega modula



za majhne raztezke

$$\frac{F}{S} = \frac{\Delta l}{l} \cdot E \qquad [E] = \frac{N}{m^2} = Pa$$

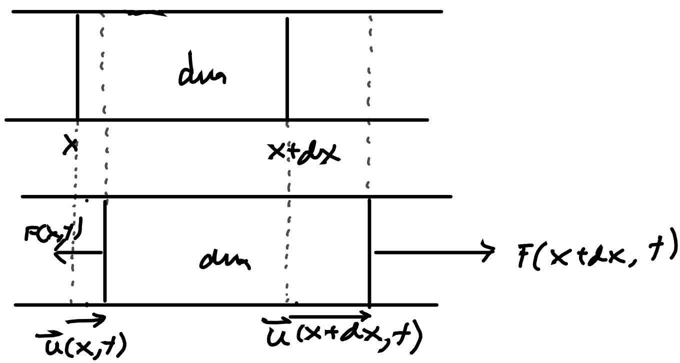
vektorska polje



$$\frac{du}{dx} = \frac{dl}{l}$$

$$\frac{F}{S} = E \frac{du}{dx}$$

Valovanje v elastični palici



$$F(x+dx,t) - F(x,t) = dm \cdot a\left(x+\frac{dx}{2},t\right)$$

$$SE\frac{du}{dx}(x+dx,t) - SE\frac{du}{dx}(x,t) = dm \cdot \ddot{u}\left(x+\frac{dx}{2},t\right)$$

$$dx \longrightarrow 0$$

$$\cancel{S}E\frac{d^2u}{dx^2}\left(x+\frac{dx}{2},t\right)\cdot\cancel{dx} = \rho\cancel{S}\,\cancel{dx}\,\ddot{u}\left(x+\frac{dx}{2},t\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E}\frac{\partial u^2}{\partial t^2}$$

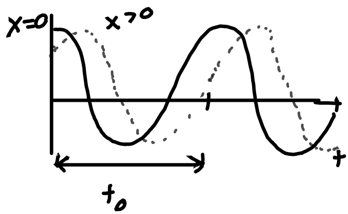
$$\frac{\rho}{E} \rightarrow \frac{1}{c^2}$$

$$u(x,t) = \overleftarrow{\phi(x+ct)} + \overrightarrow{\psi(x-ct)}$$

$$c = \sqrt{\frac{E}{\rho}}$$

Periodična motnja

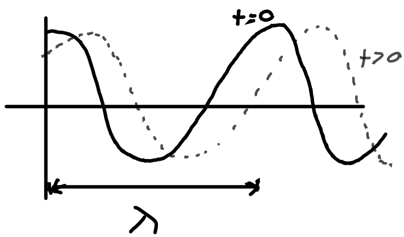
$$u(x,t)=u_0\cdot\cos(\omega t-kx)$$



Na danem mestu

$$\omega t_0=2\pi$$

$$\omega=\frac{2\pi}{t_0}=2\pi\nu$$



ob danem času

$$c=\frac{\Delta x}{\Delta t}=\frac{\lambda}{t_0}\Rightarrow c=\lambda\nu$$