

7.1.

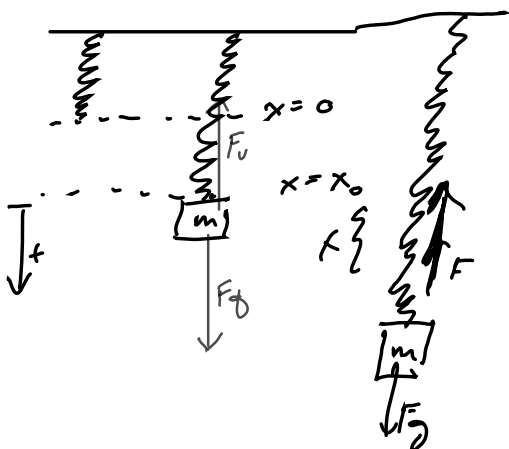
$$l = 1 \text{ m}$$

$$k = 500 \text{ N/m}$$

$$m = 8 \text{ kg}$$

$$a_0 = 0,1 \text{ m}$$

$$x_0 = ?$$



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

$$F_g + F_{v2} = m \cdot \ddot{a} = \ddot{x}(t)$$

$$+ mg - k(x_0 + x) = m \ddot{a}(t)$$

$$\underbrace{mg - kx_0}_{=0} - kx = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\ddot{x} = \frac{-kx}{m} \quad \frac{-k}{m} = \omega_0^2$$

$$x(t) = \sin(\omega t) \cdot A + B \cos(\omega t)$$

anfangsbedingungen:

$$x(0) = a_0$$

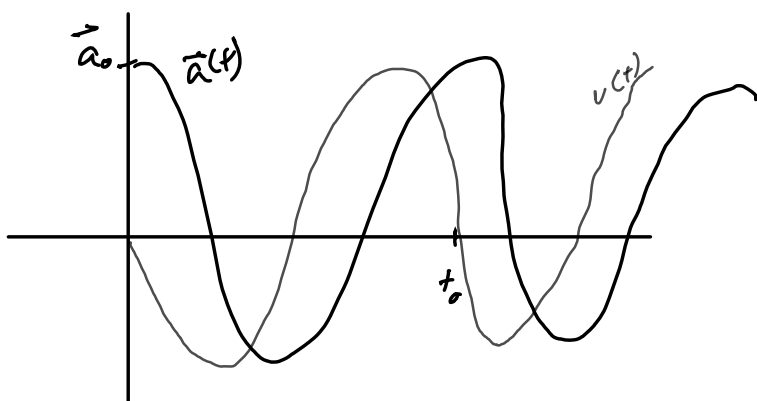
$$a_0 = x(0) = B$$

$$v(t) = \dot{x}(t)$$

$$v(0) = \dot{x}(0) = A\omega \cos(\omega t) - B\omega \sin(\omega t)$$

$$0 = A\omega \Rightarrow A = 0$$

$$x(t) = a_0 \cos(\omega t)$$



$$\cos(\omega t_0) = 1 \quad \omega t_0 = 2\pi \quad t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \sqrt{m}}{\sqrt{k}}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \dots \quad \text{natürliche Frequenz}$$

$$\omega_0 = 7,9 \text{ /s}$$

Energije nihanja

$$W = \frac{mv^2}{2} + mg(-x) + \frac{k(x+x_0)^2}{2} - \frac{k}{2}x_0^2$$

$$= \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 = 2,5 \text{ J}$$

$$l = 3\text{m}$$

$$\varphi_0 = 3,5^\circ$$

$$t = 15\text{s}$$

$$N = ?$$

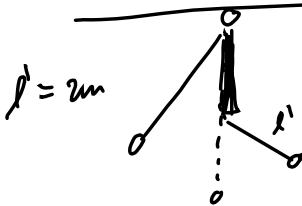
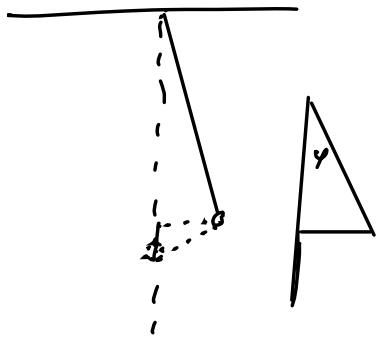
$$V_{\max} = ?$$

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$N = \frac{t}{T_0} = 4,32$$

$$v = \sqrt{2gh}$$

$$h = l - \cos\varphi \cdot l$$



$$T_0 = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l'}{g}}$$

$$= \frac{\pi}{\sqrt{g}} (\sqrt{l} - \sqrt{l'}) = 3,15\text{s}$$

$$N = \frac{t}{T_0} =$$

$$\varphi(t) = \varphi_0 \cos(\omega t)$$

$\nwarrow$   $\omega$  - angular frequency

$$V_{\max} = l \cdot \omega$$

$\nwarrow$   $\omega$  - angular frequency

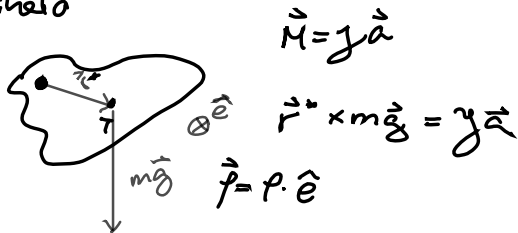
$$\omega(t) = \dot{\varphi}(t) = -\varphi_0 \omega \sin(\omega t)$$

$\omega$  - angular frequency

$$V_{\max} = l \cdot \varphi_0 \omega$$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

Tedno nihalo



$$r^* m g \sin \varphi (-\hat{e}) = J \cdot \ddot{\varphi} \hat{e}$$

$$\ddot{\varphi} = -\frac{m g r^*}{J} \sin \varphi$$

$$\varphi \ll 1$$

$$\ddot{\varphi} \approx -\underbrace{\frac{m g r^*}{J}}_{\omega^2} \varphi$$

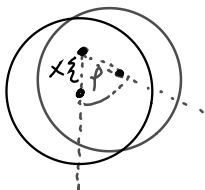
$$T_0 = 2\pi \sqrt{\frac{J}{m g r^*}}$$

Matematično nihalo:

$$J = m l^2$$

$$r^* = l \quad T_0 = 2\pi \sqrt{\frac{l}{g}}$$

7.7.)



$$R = 0,5 \text{ m}$$

$t_0$  min

debi osnove ozi  
translacija

$$y = \frac{1}{2} m R^2 + m x^2$$

$$t_0(x) = ?$$

$$t_0 = 2\pi \sqrt{\frac{y}{mgx}} =$$

$$t_a = 2\pi \sqrt{\frac{\frac{1}{2} R^2 + x^2}{gx}}$$

da bismo našli:

$$\frac{2x(gx) - g(\frac{1}{2}R^2 + x^2)}{gx^2} = 0$$

$$2gx^2 - gx^2 - \frac{1}{2}R^2g$$

$$gx^2 = \frac{1}{2}R^2g$$

$$x = \sqrt{\frac{1}{2}} \frac{R}{\sqrt{2}}$$

(7.8)

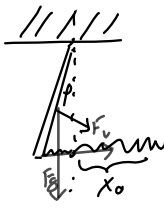
$$l = 1 \text{ m}$$

$$m = 0,5 \text{ kg}$$

$$k = 5 \text{ N/m}$$

---


$$V = ?$$



$$\phi \ll 1$$

$$\Sigma \vec{M} = J \cdot \ddot{\alpha}$$

$$\underbrace{\ddot{\alpha}(t)}_{\parallel \ddot{\phi}(t)} = - \underbrace{F_g}_{\parallel} \cdot \underbrace{\frac{l}{2}}_{\parallel} \sin \underbrace{\phi(t)}_{\parallel} - \underbrace{F_v}_{\parallel} \cdot \underbrace{k x(t)}_{\parallel} \cdot \underbrace{l \cdot \cos \phi(t)}_{\parallel}$$

reaction

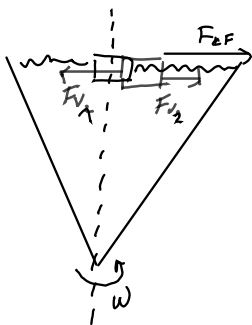
$$\ddot{\phi}(t) = - \frac{m g \cdot l}{2 J} \phi(t) - \frac{k l^2}{J} \phi(t) =$$

$$= - \frac{3g}{2l} \phi(t) - \frac{3k \phi(t)}{m} = - \left( \frac{3g}{2l} + \frac{3k}{m} \right) \phi(t)$$

$\omega_0$

$$\omega_0 = \sqrt{\frac{3g}{2l} + \frac{3k}{m}} = 6,7 / \text{s}$$

$$2\pi V = \omega_0 \quad V = \frac{\omega_0}{2\pi} = 1,06 \text{ Hz}$$



$$m = 20g$$

$$k_1 = 1 \text{ N/m}$$

$$k_2 = 3 \text{ N/m}$$

$$\omega = 10/s$$

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$$V = ?$$

$$\sum F = ma$$

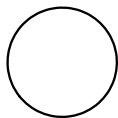
$$ma = -k_1 x - k_2 x + m \ddot{a}_r$$

$$\ddot{x}(t) = - \left( \frac{k_1 + k_2 - m\omega^2}{m} \right) x$$

$$\omega_0 = \sqrt{\frac{k_1 + k_2 - m\omega^2}{m}} = 10/s$$

$$V = 1, \text{ Hz}$$

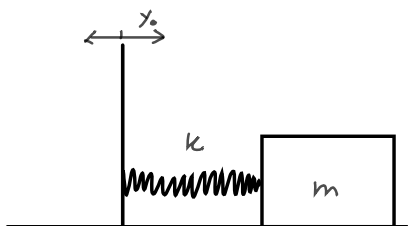
Faucaultova nihele







(7.25)



$$y_0 = 5 \text{ cm}$$

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

$$k = \frac{N}{m}$$

$$\beta = 0.2 / s$$

$$\nu = 12 \text{ Hz}$$

$$x_0 = ?$$

$$\delta = ?$$

$$x_{0 \max} = ?$$

$$v_{\max} = ?$$

$$m \ddot{x} = -k(x-y) - 2\beta m \dot{x}$$

$$\ddot{x} + 2\beta \dot{x} + \frac{k}{m} x = \frac{k}{m} y \rightarrow y_0 \cos(\omega t)$$

$$\omega^2 = \frac{k}{m}$$

Nehomogena diferencialna enačba

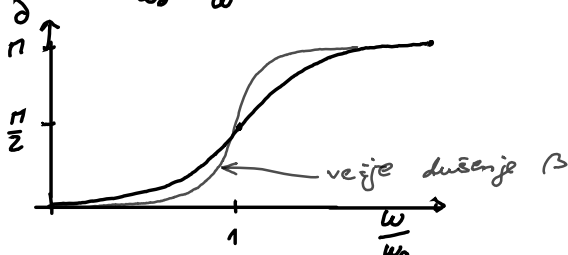
$$\text{rešitev: } x(t) = x_0 \cdot \cos(\omega t - \delta) + x_0' e^{-\beta t} \cos(\omega t - \delta')$$

$$\sqrt{\omega^2 - \beta^2}$$

se zadržuje, ne  
pa zanima, ko se  
ustati

Rezultati:

$$\tan \delta = \frac{2\beta \omega}{\omega_0^2 - \omega^2}$$



$$\frac{x_0}{y_0} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta \omega)^2}}$$



vrh resonance  
krivulje

$$\omega_{\max} = \sqrt{\omega_0^2 - 2\beta^2}$$

Rezultati:

$$\omega_0 = \sqrt{\frac{k}{m}} = 10 / s$$

$$\delta = 179.7^\circ$$

$$\Rightarrow \nu = 1.6 \text{ Hz}$$

$$x_0 = 0.89 \text{ mm}$$

$$v_{\max} \approx 1.6 \text{ Hz}$$

$$x_{0 \max} = 1.25 \text{ m}$$

mec pri vsiljenem nihanju

$$P = \frac{dA}{dt} = \frac{F \dot{x}}{dt} = Fv = 2\beta m v \dot{x} =$$

$$v = \dot{x} = x_0 (-\sin(\omega t - \delta)) \cdot \omega$$

$$\bar{P} = 2\beta m x_0^2 \omega^2 \overline{\sin^2(\omega t - \delta)} = \beta m x_0^2 \omega^2$$

||  
1  
2

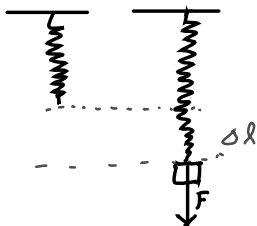
$$\bar{P} \propto \omega^2 \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}$$

$$\frac{dP}{d\omega} = \frac{2\omega[(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2] - [4\omega(\omega_0^2 - \omega^2) + 8\beta^2\omega]}{((\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2)^2} = 0$$

splošno nihanje

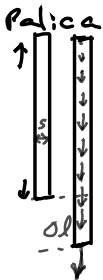
$\chi$  14.3

# Elastomehanika: Hookov zakon



$$F = k \cdot \Delta l$$

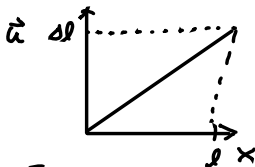
Vpeljava prožnostnega modula



→ majhne raztezke

$$\frac{F}{S} = \frac{\Delta l}{l} \cdot E \quad [E] = \frac{N}{m^2} = Pa$$

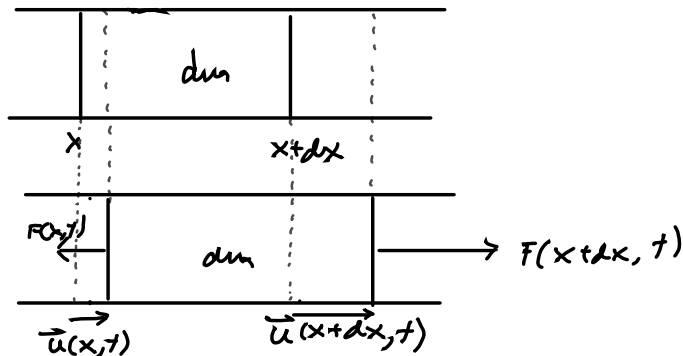
← vektorsko polje



$$\frac{du}{dx} = \frac{dl}{l}$$

$$\frac{F}{S} = E \frac{du}{dx}$$

# Valovanje v elastični palici



$$F(x+dx, t) - F(x, t) = dm \cdot a(x + \frac{dx}{2}, t)$$

$$\mathcal{E} \frac{du}{dx} (x+dx, t) - \mathcal{E} \frac{du}{dx} (x, t) = dm \cdot \ddot{u}(x + \frac{dx}{2}, t)$$

$$dx \rightarrow 0$$

$$\mathcal{E} \frac{d^2 u}{dx^2} (x + \frac{dx}{2}, t) \cdot dx = \rho \cancel{dx} \ddot{u}(x + \frac{dx}{2}, t)$$

$$\frac{\partial^2 u}{\partial x^2} = \left( \frac{\rho}{\mathcal{E}} \right) \frac{\partial^2 u}{\partial t^2}$$

$$\downarrow$$

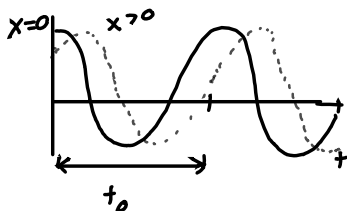
$$\frac{1}{c^2}$$

$$u(x, t) = \phi(x+ct) + \psi(x-ct)$$

$$c = \sqrt{\frac{\mathcal{E}}{\rho}}$$

## Periodična motnja

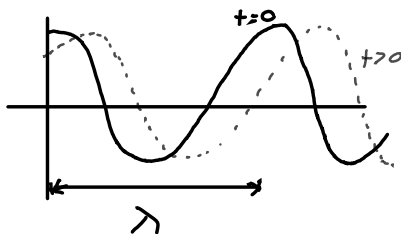
$$u(x,t) = u_0 \cdot \cos(\omega t - kx)$$



Na danem mestu

$$\omega T_0 = 2\pi$$

$$\omega = \frac{2\pi}{T_0} = 2\pi \nu$$



ob danem času

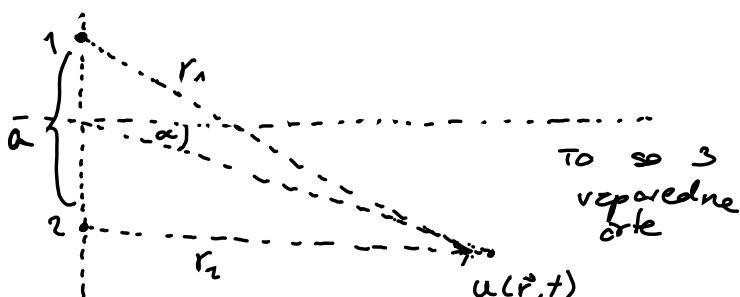
$$c = \frac{\delta x}{\delta t} = \frac{\lambda}{T_0} \Rightarrow c = \lambda \nu$$

$$a = \frac{3}{10} \lambda$$

1. izvor zakasnjjen za  $\frac{1}{4}$  nihanja  $\Rightarrow \delta = \frac{\pi}{2}$

smeri opazitev = ?

smerno deleč od anten



$$r_1, r_2 \gg a$$

$$u = \underbrace{u_0 \sin(\omega t - k r_1 - \delta)}_{\text{prvi izvor}} + \underbrace{u_0 \sin(\omega t - k r_2)}_{\text{2. izvor}}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$u = u_0 \sin \left( \frac{2\omega t - k(r_1 + r_2) - \delta}{2} \right) \cos \left( \frac{k(r_2 - r_1) - \delta}{2} \right)$$

amplitude

točke za katere je  $r_1 - r_2$  konstanta

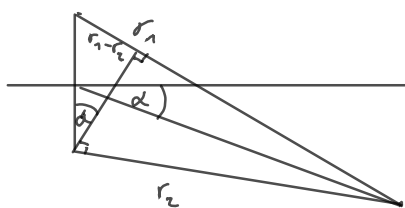
$$\frac{k}{2}(r_2 - r_1) - \frac{\delta}{2} = n\pi \quad n \in \mathbb{Z}$$

točje  
priče hiperbole

$j \propto u^2$  mora biti maksimalen  
(ker iščemo optike)  
↖ sorazmerno

odločimo se za neke

$n$  in potem gledamo da  
je tudi na levi strani  
konstantno



$$r_1 - r_2 = a \sin \alpha$$

$$-\frac{k}{2}(a \sin \alpha) - \frac{\delta}{2} = n\pi$$

$$\sin \alpha = \frac{n\pi + \frac{\delta}{2}}{-\frac{ak}{2}} = \frac{2\pi n + \delta}{-ak}$$

$k$  .... valovni vektor

$$k = \frac{2\pi}{\lambda} \quad a = \frac{3}{10} \lambda$$

$$\sin \alpha = \frac{\lambda(2\pi n + \delta)}{-2\pi a} = \frac{5(2\pi n + \delta)}{-3\pi}$$

$$\sin \alpha = -\frac{10}{3} \left( n + \frac{1}{4} \right)$$

rečemo  $n=0$

$$\sin \alpha_0 = -\frac{10}{3} \cdot \frac{1}{4} = -\frac{5}{6}$$

$$\Rightarrow \alpha_0 = -56,4$$

$$n=1 \Rightarrow \sin \alpha_1 = -\frac{10}{3} \cdot \frac{5}{4} = -\frac{25}{6}$$

$$\alpha_1 = \text{err}$$

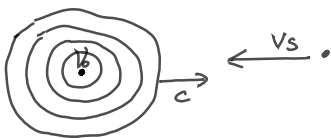
$$n=-1 \Rightarrow \sin \alpha_{-1} = -\frac{10}{3} \cdot \left( -\frac{3}{4} \right) = \frac{5}{2}$$



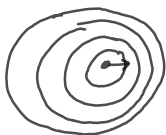
$$\nu = 1485 \text{ kHz}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \frac{\text{m}}{\text{s}}}{1485 \cdot 10^3 / \text{s}} = 202 \text{ m}$$

# Dopplerjev pojav



$$N = \frac{ct}{\lambda} + \frac{v_s t}{\lambda} + \frac{v t}{\lambda}$$



$$\lambda = \frac{c}{v_0} + \frac{v}{v_0} + \frac{v_v}{v_0}$$

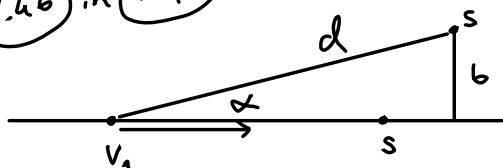
$$V_s = \frac{N}{t} = \frac{c + v + v_s}{c + v + v_v} \cdot v_0$$

$$= \frac{c + (v + v_s)}{v - (v_0 - v)}$$

h.rost  
sprejemnika  
glede na veter

h.rost  
oddajnika  
glede na veter

(7.46) in (7.47)

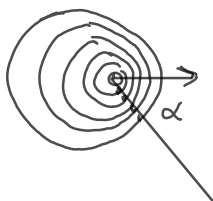


$$v_1 = 100 \frac{\text{km}}{\text{h}} = 27,8 \frac{\text{m}}{\text{s}}$$

$$V_0 = 1000 \text{ Hz}$$

a)  $V_s = ?$   $V_s = \frac{c + V_s}{c - V_1} V_0$   $V_s = 0$   $c = 340 \frac{\text{m}}{\text{s}}$   
 $V_1 = 27,8 \frac{\text{m}}{\text{s}}$   
 $V_s = 189 \text{ Hz}$

b)  $d = 100 \text{ m}$   
 $b = 500 \text{ m}$



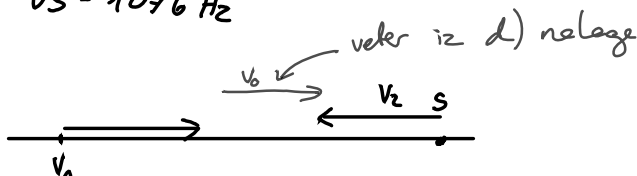
$$V_s = \frac{c}{c - V_1 \cos \alpha} V_0$$

$$\sin \alpha = \frac{b}{d} = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$

$$\cos \alpha = \frac{\sqrt{3}}{2}$$

$$V_s = 1076 \text{ Hz}$$

c)



$$v_2 = 80 \frac{\text{km}}{\text{h}} = 22 \frac{\text{m}}{\text{s}}$$

$$V_s = \frac{c + v_2}{c - v_1} V_0 = 1160 \text{ Hz}$$

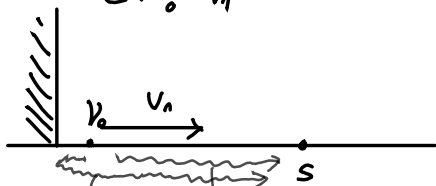
d)

$$v_0 = 30 \frac{\text{km}}{\text{h}} = 8,3 \frac{\text{m}}{\text{s}}$$

↑  
veter

$$V_s = \frac{c + v_0 + v_2}{c + v_0 - v_1} = 1156 \text{ Hz}$$

e)



$$V_{s_1} = \frac{c}{c + v_1} V_0$$

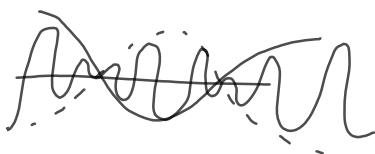
$$V_{s_2} = \frac{c}{c - v_1} V_0$$

$$V_s = V_{s_1} + V_{s_2} = c V_0 \left( \frac{1}{c + v_1} + \frac{1}{c - v_1} \right) =$$

$$= c V_0 \left( \frac{c - v_1 + c + v_1}{c^2 - v_1^2} \right) = c V_0 \left( \frac{2c}{c^2 - v_1^2} \right)$$

$$\cos(\omega t) + \cos(\omega'' t) =$$

$$2 \cos\left(\frac{\omega + \omega''}{2} t\right) \cos\left(\frac{\omega - \omega''}{2} t\right)$$



slušmo utripanje

15.4.

✓ zamudile 20 minut

(naloge 10.13)

$$\int_V d\vec{v} \vec{D} \vec{v} \int_{\partial V} \vec{D} \vec{n} dS$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\int_{\partial V} \vec{D} \cdot d\vec{S} = \epsilon_0 \oint \vec{E} \cdot d\vec{S}$$

$$Q = \epsilon_0 \oint \vec{E} \cdot d\vec{S}$$

← Nabla

$$\vec{E}_\perp = \frac{\lambda}{2\pi \epsilon_0 \rho}$$

$$Q = \epsilon_0 \oint \vec{E} \cdot d\vec{S}$$

place vch in dne velj

$$\lambda \cdot l = \epsilon_0 \oint E_\perp dS + 0 + 0$$

$$\lambda \cdot l = \epsilon E_\perp \oint dS = Q = \epsilon E_\perp 2\pi \rho \cdot l$$

$$E_\perp = \frac{\lambda}{\epsilon_0 2\pi \rho}$$

$$W_{\text{el}} + W_{\text{me}} = \text{konst}$$

$$\frac{mv_0^2}{2} + W_{\text{pe}}(a) = \frac{mv_a^2}{2} + W_{\text{pe}}(b)$$

$$\frac{m}{2} (v_a^2 - v_0^2) = W_{\text{pe}}(b) - W_{\text{pe}}(a)$$

$$\int_a^b F d\rho = - \int_a^b \frac{\lambda}{2\pi \epsilon_0 \rho} d\rho = - \frac{\lambda e}{2\pi \epsilon_0} \ln \frac{b}{a} = \frac{\lambda e}{2\pi \epsilon_0} \ln \frac{a}{b}$$

$$\vec{v}_a = \vec{v}_b$$

$$= \frac{m}{2} \sin^2 \gamma \cdot v_a^2$$

kot pri poslovnem metu

$$\ln \frac{a}{b} = \frac{\pi \epsilon_0 m \sin^2 \gamma}{\lambda e}$$

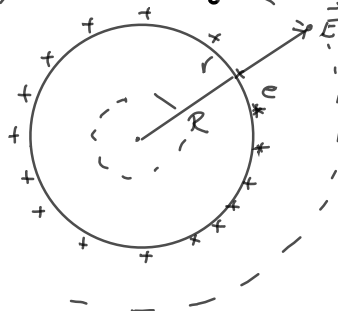
$$b = 16.8 \mu\text{m}$$

$$W_{ef}(\vec{r}_2) - W_p(\vec{r}_1) = -e \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} d\vec{r} \quad /: e$$

$$\Phi(\vec{r}_2) - \Phi(\vec{r}_1) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

10. 18

10. 20

Kovinska kugla, ko nosi naboj  $e$ 

$$\vec{E} = ? \quad r > R$$

 $\leftarrow S \text{ površina}$ 

$$e = \epsilon_0 \int_S \vec{E} \cdot d\vec{S}$$

 $\vec{E}$  je konstanta

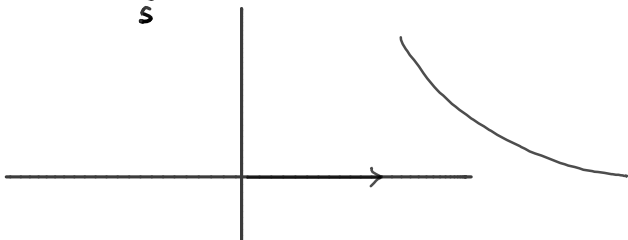
$$e = \epsilon_0 E 4\pi r^2$$

$$\Rightarrow E = \frac{e}{\epsilon_0 4\pi r^2}$$

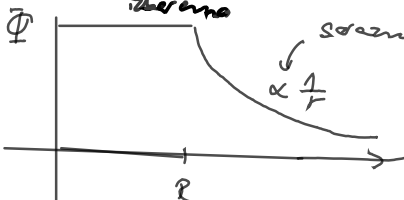
 $r < R$ 

u kernima znatnoji nič naboj

$$e = \epsilon_0 \int_S E dS \Rightarrow E = 0$$



$$\Phi(r) = \underbrace{\Phi(\infty)}_{\text{izborena}} - \int_{\infty}^r \vec{E} \cdot d\vec{r} = - \int_{\infty}^r \frac{e}{\epsilon_0 4\pi r^2} dr = \frac{e}{\epsilon_0 4\pi} \left( \frac{1}{r} - 0 \right) = \frac{e}{\epsilon_0 4\pi r}$$



$$\Phi(r < R) = \Phi(R) = \frac{e}{\epsilon_0 4\pi R}$$

Enekomerno nebite krogle

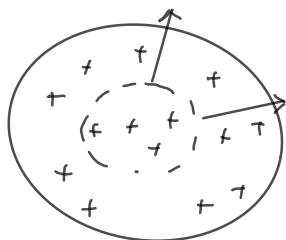
$r > R \Rightarrow$  isto kot prej

$$E = \frac{e}{\epsilon_0 4\pi r^2}$$

$$\phi(r) = \frac{e}{\epsilon_0 4\pi r}$$

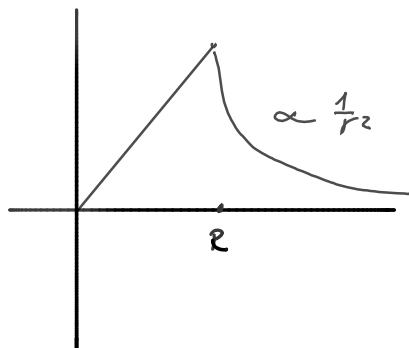
$r < R \Rightarrow$

$$e = \epsilon_0 \int_S E dS$$



$$e' = \epsilon_0 E 4\pi r^2 \quad e' = \frac{3e}{4\pi R^3} \cdot 4\pi r^2 = \frac{e r^3}{R^3}$$

$$E = \frac{e r^3}{R^3 \epsilon_0 4\pi r^2} = \frac{e r}{R^3 \epsilon_0 4\pi}$$



$$\Phi(r) = \Phi(R) + \int_R^r \frac{e r}{\epsilon_0 4\pi R^3} dr =$$

$$\Phi(R) + \frac{e r^2}{\epsilon_0 8\pi R^3} \Big|_R^r = \Phi(R) + \frac{e r^2}{\epsilon_0 8\pi R^3} + \frac{e}{\epsilon_0 8\pi R} =$$

$$= \frac{e}{4\pi \epsilon_0} \left( \frac{1}{R} + \frac{r^2}{2 \cdot R^3} + \frac{R^2}{2R^3} \right) = \frac{e}{4\pi \epsilon_0 R \epsilon_0} \left( \frac{3}{2} - \frac{r^2}{2R^2} \right)$$

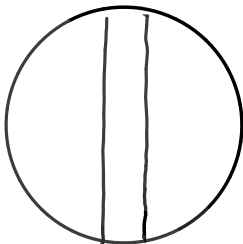
$$\vec{E} = \frac{e}{\epsilon_0 4\pi r^3} \vec{r}$$



V z-mude

$$e_1 \cdot E(\vec{r}) = -e_1 \frac{e_2 \vec{r}}{4\pi\epsilon_0 R^3} =$$

b)



$$M = 6 \cdot 10^{24} \text{ kg}$$

$$R = 6400 \text{ km}$$

$$F_e = \frac{e_1 e_2}{4\pi\epsilon_0 r^2}$$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$e_i \leftrightarrow m_i$$

$$\frac{1}{4\pi\epsilon_0} \rightarrow G$$

$$m a = F_g = -m G \frac{M}{R^2}$$

lehte zamenjamo kv  
rednosti

$$\ddot{r} + G \frac{M}{R^3} r = 0$$

$$V = \frac{\sqrt{G \frac{M}{R^3}}}{2\pi} \Rightarrow T_0 = \frac{1}{V} = \frac{2\pi}{\sqrt{G \frac{M}{R^3}}} = 84,7 \text{ min}$$

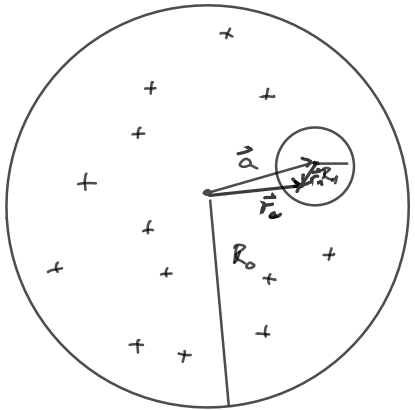
10.27

$$r_0 = 10 \text{ m}$$

$$\rho_e = 2 \mu\text{As/m}^3$$

$$R_1 = 2m$$

$$a = 7m$$



$E = ?$  v voltlin.



$$P_A = -P_C$$

$$\vec{E}_3 = \vec{E}_0 + \vec{E}_1 \quad \text{mejh na kagle}$$

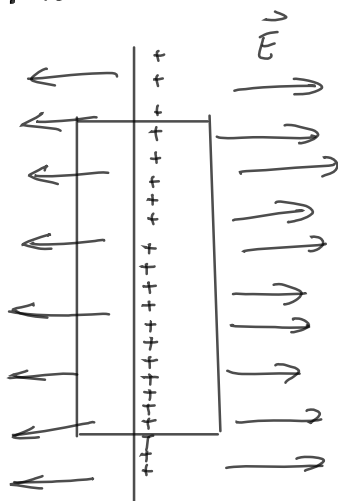
$$\frac{e \vec{r}_0}{4\pi\epsilon_0 R_0^3} + \frac{e \vec{r}_1}{4\pi\epsilon_0 R_1^3} =$$

$$= \frac{\rho_e \frac{4\pi R_e^3}{3} \vec{r}_e}{4\pi \epsilon_0 R_e^3} - \frac{\rho_e \cdot \frac{4\pi R_i^3}{3} \vec{r}_i}{4\pi \epsilon_0 R_i^3} =$$

$$= \frac{\rho_e (\vec{r}_0 - \vec{r}_1)}{3 \epsilon_0} = \frac{\rho_e}{3 \epsilon_0} \vec{a}$$

$$|\vec{E}_s| = \frac{2 \cdot 10^{-6} \text{ As} \cdot 7 \text{ m}}{\text{m}^2 \cdot 3 \cdot 8.85 \cdot 10^{-12} \text{ As}}$$

10.18  $\rightarrow$  10.30



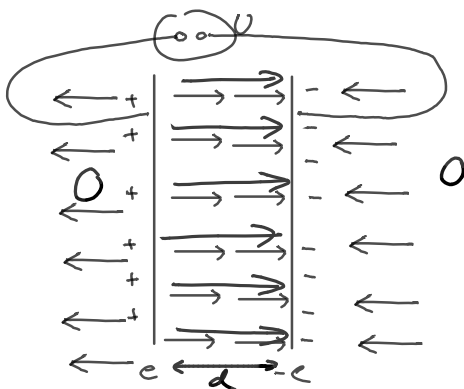
enakomerno  
nahite  
ploščo

$$\oint_S \vec{E} \cdot \vec{n} \cdot d\vec{s} = \oint_S \vec{E} \cdot \vec{n} \cdot d\vec{s} = \epsilon_0 \oint_S \vec{E} \cdot \vec{n} \cdot d\vec{s} =$$

(OK)  $\nwarrow$  vzporedno  $\nearrow$  konstantno

$$= 2\epsilon_0 E \mathcal{P}(\text{ploščo}) + 0$$

$$E = \frac{e}{2\epsilon_0 S} = \frac{\sigma}{2\epsilon_0}$$



$$E = \frac{\sigma}{\epsilon_0}$$

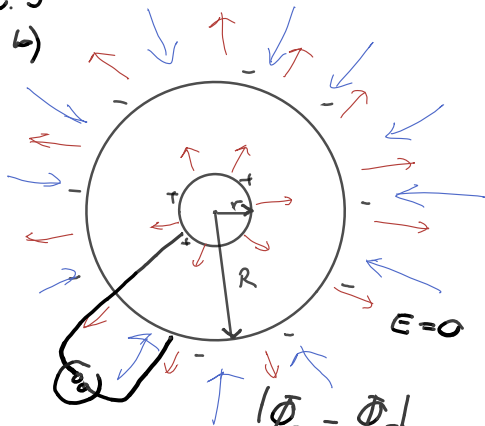
$$\Delta\Phi = - \int \vec{E} \cdot d\vec{r} = U = E \cdot d = \frac{\sigma}{\epsilon_0} d = \frac{ed}{\epsilon_0}$$

$$e = \frac{U \epsilon_0}{d} = U \cdot C$$

$$C = \frac{e}{U} \quad [F = \frac{As}{V}]$$

10.30

b)



$$|\Phi_2 - \Phi_1|$$

$$|\Delta\Phi| = U = \left| \frac{e}{4\pi\epsilon_0 R} - \frac{e}{4\pi\epsilon_0 r} \right| = \left| \frac{e(r-R)}{4\pi\epsilon_0 Rr} \right| =$$

$$= \frac{e(R-r)}{4\pi\epsilon_0 Rr}$$

$$C = \frac{e}{U} = \frac{4\pi\epsilon_0 Rr}{(R-r)}$$

(10.4.)

Ne tabl:



$$I = \frac{U [V]}{R [\Omega]} [A]$$

6.5

11.1

$$R = \left\{ \frac{l}{S} \right\}$$

$$A = W_{pe} = e'' U$$

$$P = I U$$

$$S_1 = 0,1 \text{ mm}^2$$

$$l_1 = 100 \text{ m}$$

$$\xi_1 = 0,1 \frac{\Omega \text{ mm}^2}{\text{m}}$$

$$S_2 = 0,2 \text{ mm}^2$$

$$l_2 = 28 \text{ m}$$

$$\xi_2 = 0,5 \frac{\Omega \text{ mm}^2}{\text{m}}$$

$$U = 220 \text{ V}$$

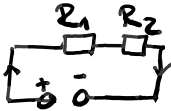
a) zaporedna vezava

$$\frac{P_a}{P_b} = ?$$

b) vzporedna vezava

$$P = \frac{A e}{t} = U I = U \left( \frac{U}{R} \right) = \frac{U^2}{R} = R I^2$$

a)



$$R_1 = 100 \Omega$$

$$R_2 = 10 \Omega$$

$$R = R_1 + R_2 \quad \text{ker } U - I R = 0$$

$$U - I_a R_1 - I_a R_2 = 0$$

$$R = R_1 + R_2$$

$$I_a = \frac{U}{R_1 + R_2} = 1,234 \text{ A}$$

$$P_a = U \cdot I_a = \underline{\underline{284,7 \text{ W}}}$$

b)



$$U = I_1 R_1 = I_2 R_2$$

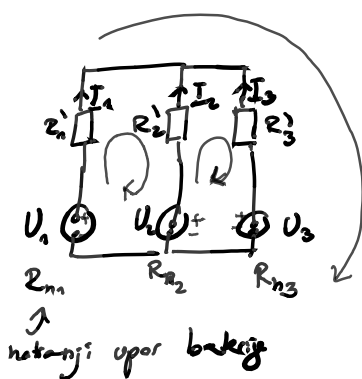
$$I = I_1 + I_2 = \frac{U}{R_1} + \frac{U}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} =$$

$$I = 5,34 \text{ A}$$

$$P_b = U I = \underline{\underline{1175,4 \text{ W}}}$$

11,11



$$U_1 = 1V$$

$$U_2 = 2V$$

$$U_3 = 3V$$

$$R_{n1} = 15\Omega$$

$$R_{n2} = 20\Omega$$

$$R_{n3} = 30\Omega$$

$$R_1' = 85\Omega$$

$$R_2' = 180\Omega$$

$$R_3' = 270\Omega$$

$$R_1 = R_{n1} + R_1'$$

$$R_1 = 100\Omega$$

$$R_2 = 200\Omega$$

$$R_3 = 300\Omega$$

$$U_1 - I_1 R_1 + I_2 R_2 - U_2 = 0 \quad (\text{lev: krog})$$

$$U_2 - I_2 R_2 + I_3 R_3 - U_3 = 0 \quad (\text{desno: krog})$$

$$U_1 - I_1 R_1 + I_3 R_3 - U_3 = 0 \quad (\text{cel krog})$$

Linearna odvisnost (dve enačbi, tri neznanke)

$$I_1 + I_2 + I_3 = 0 \quad (\text{Kirchoffov zakon})$$

$$I_1 = -I_2 - I_3$$

$$U_1 + I_2 R_1 + I_3 R_1 + I_2 R_2 - U_2 = 0$$

$$U_2 - I_2 R_2 + I_3 R_3 - U_3 = 0$$

$$I_2 = \frac{U_2 - U_3 + I_3 R_3}{R_2}$$

$$U_1 + \frac{R_1}{R_2} (U_2 - U_3 + I_3 R_3) + (U_2 - U_3 + I_3 R_3) - U_2 = 0$$

$$\frac{R_1 R_3}{R_2} I_3 + R_3 I_3 = U_2 - U_1 - \frac{R_1}{R_2} (U_2 - U_3) - U_2 + U_3$$

$$I_3 = \frac{R_2 (U_3 - U_1) - R_1 (U_2 - U_3)}{R_1 R_3}$$

$$I_3 = \frac{200\Omega (2V) - 100\Omega (1V)}{100\Omega \cdot 300\Omega} =$$

$$= \frac{500\Omega V}{100 \cdot 300\Omega} = \frac{5}{300} V = 0,016 A$$

To je narobe :-

$$I_3 = 4,5k mA$$

$$I_2 = 1,81 mA$$

$$I_1 = -6,35 mA$$

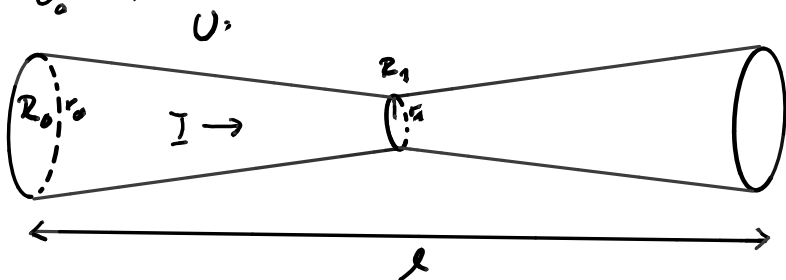
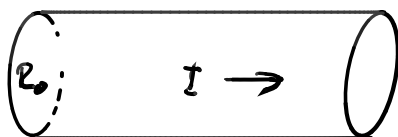
11.3

$$S_0 = 3 \text{ mm}^2$$

$$S_1 = 1 \text{ mm}^2$$

$$l = 15 \text{ cm}$$

$$\frac{U}{U_0} = 2$$

 $U_0$ :

$$\frac{U}{U_0} = \frac{RI}{R_0 I} = \frac{R}{R_0}$$

$$R_0 = \frac{l}{S_0}$$

$$R = \int dR = 2 \int_0^{l/2} \frac{dx}{\pi r(x)^2}$$

$$r(x) = \frac{2(r_0 - r_1)}{l} x + r_1$$

$$u = r(x)$$

$$du = \frac{2(r_0 - r_1)}{l} dx$$

$$\begin{aligned} R &= 2 \int_{r_1}^{r_0} \frac{2(r_0 - r_1)}{\pi l u^2} du = 4 \frac{r_0 - r_1}{\pi l} \left( -\frac{1}{u} \right) \Big|_{r_1}^{r_0} = \\ &= \frac{4}{\pi l} (r_0 - r_1) \left( \frac{1}{r_1} - \frac{1}{r_0} \right) = \\ &= \frac{4}{\pi l} \frac{(r_0 - r_1)^2}{r_0 r_1} = \frac{2}{\pi} \frac{l (r_0 - r_1)^2}{r_0 r_1} \end{aligned}$$

$$\pi r_0^2 = S_0$$

$$r_0 = \sqrt{\frac{S_0}{\pi}} \quad r_1 = \sqrt{\frac{S_1}{\pi}}$$

$$R = \frac{2}{\pi} \frac{\pi \left( \frac{S_0}{\pi} + 2\sqrt{S_0 S_1} + \frac{S_1}{\pi} \right)}{\sqrt{S_0 S_1}} =$$

$$= \frac{2}{\pi} \frac{(S_0 + S_1 + 2\sqrt{S_0 S_1})}{\sqrt{S_0 S_1}}$$

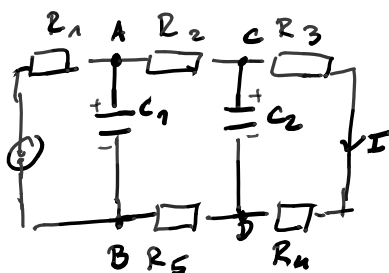
$$\frac{U}{U_0} = \frac{R}{R_0} =$$

mit sm se zuehile

$$R = \frac{l}{\pi r_0 r_1}$$

$$\frac{U}{U_0} = \frac{r_0}{r_1} = \frac{\sqrt{S_0}}{\sqrt{S_1}} = \sqrt{3}$$





$$U_0 = 6V$$

$$R_1 = 4\Omega$$

$$R_2 = 1\Omega$$

$$R_3 = 5\Omega$$

$$R_4 = 5\Omega$$

$$R_5 = 1\Omega$$

$$C_1 = 5\mu F$$

$$C_2 = 0,1\mu F$$

Pe de lașim căreie cele 2 condensatoare ne fac să știm

$$\Rightarrow R = R_1 + R_2 + R_3 + R_4 + R_5$$

$$[F = \frac{As}{V}]$$

$$C = \frac{q}{U}$$

$U_{AB}$

$$q_1 = ?$$

$$q_2 = ?$$

$$q_1 = C_1 U_{AB}$$

$$q_2 = C_2 U_{CB}$$

$$U_0 - IR_1 - U_{AB} = 0$$

$$U_0 - IR_1 - IR_2 - U_{CB} - IR_5 = 0$$

$$U_0 - I(R_1 + R_2 + R_3 + R_4 + R_5) = 0$$

$$I = \frac{U_0}{R_n} = 0,5A$$

$$U_{AB} = U_0 - IR_1 = 6V$$

$$U_{CB} = U_0 - IR_1 - IR_2 - IR_5 = 5V$$

$$q_1 = C_1 \cdot U_{AB} = 30\mu As$$

$$q_2 = 0,5\mu As$$

$V_{\text{comadib}}$  per ore

11.13 Primenje kondenzatorje

$$R = 1 \text{ M}\Omega$$

$$C = 300 \text{ nF}$$

$$t_{1,2} = ?$$

$$C = \frac{q}{U}$$



$$(U_0 +) U_C + U_R = 0$$

$$(U_0) \quad -\frac{q}{C} - IR = 0$$

advisnood case

$$-\frac{e(t)}{C} = R \frac{de}{dt}$$

$$de = -\frac{e(t)}{RC} dt$$

$$\frac{de}{e} = -\frac{dt}{RC}$$

$$\ln e \Big|_0^t = \left(-\frac{t}{RC}\right) \Big|_0^t$$

$$\ln \frac{e}{e_0} = -\frac{t}{RC}$$

$$e = e_0 e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = \frac{1}{2}$$

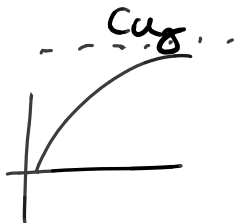
$$-\frac{t}{RC} = \ln \frac{1}{2}$$

$$t = \ln 2 \cdot RC$$

$$1 \cdot 10^6 \Omega \cdot 300 \cdot 10^{-9} \text{ F} \cdot 0,69 = 0,21 \text{ s}$$

Pdnenje kondenzatorja

$$e(t) = CU_S (1 - e^{-\frac{t}{RC}})$$



# Magnetna polje

12.1

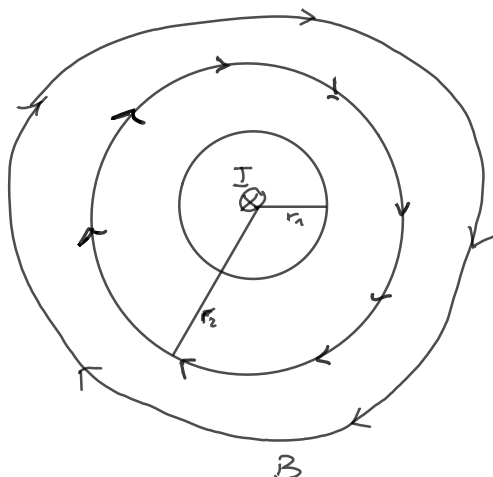
$$R = 1 \text{ cm}$$

$$I = 1000 \text{ A}$$

$$r_1 = 0,5 \text{ cm}$$

$$r_2 = 2,0 \text{ cm}$$

$$B(r) = ?$$



$$\oint \vec{B} d\vec{s} = 0$$

Ampere:

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = \frac{1}{\mu_0} \int_S \underbrace{\text{rot } \vec{B}}_{\substack{\text{je ... gostota} \\ \text{električnega} \\ \text{toka}}} d\vec{s} \quad \frac{1}{\mu_0} \oint \vec{B} d\vec{s} = I_{\text{objekt}}$$

$$\vec{B} = \mu_0 \vec{H}$$

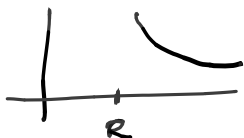
↑ gostota      ↓ jakost

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$$

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = I$$

$$\frac{B}{\mu_0} \int ds = \frac{B}{\mu_0} 2\pi r$$

$$B(r) = \frac{I \mu_0}{2\pi r} = 0,01 \frac{\text{Vs}}{\text{m}^2}$$



⇒

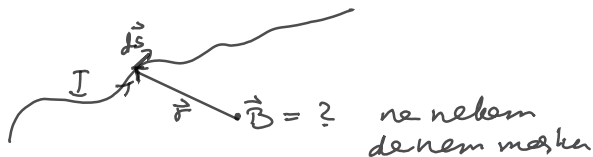
$$\frac{S}{I} = \frac{S'}{I'} = \frac{1}{j} \Rightarrow I = \frac{S'}{S} I' = \frac{4\pi r_1^2}{4\pi R^2} I' =$$

$$= \frac{B'}{\mu_0} 2\pi r_1 \Rightarrow$$

$$\frac{\mu_0 r_1}{2\pi R^2} I' = B'$$

$$\Rightarrow B(r_1) = \frac{\mu_0 r_1}{2\pi R^2} I = 0,01 \text{ T}$$

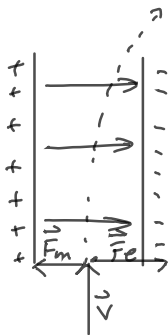
Biot-Savartova rovnice



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{r} \times d\vec{s}}{r^3}$$

12.4.

Lorentzova sila  $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$



$B = ?$  da se  
pot elek na ukl.

$$U = 50V$$

$$d = 1cm$$

$$\vec{F}_e = -\vec{F}_m$$

$$E = v \cdot B$$

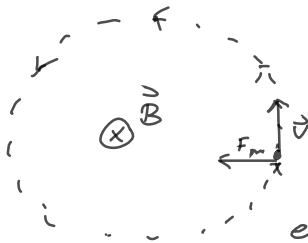
$$\frac{U}{d}$$

(B za najmanjšo  
pot 5 cm)

Prej: A je prav?

Rešja na tbi: Mel smo dva podatka, da  
smo porabi

$$B = \frac{U}{dv} = \frac{50V}{0,01m \cdot 10^6 \frac{m}{s}} = \left[ \frac{Vs}{m^2} \right] 4,8$$



$$F_m = F_{cp}$$

$$e v B = m \omega^2 r$$

$$e \omega r B = m \omega^2 r$$

$$\omega = \frac{eB}{m} = \omega_0$$

ciklotronska  
frekvence

$$e v B = m \frac{v^2}{r}$$

$$r = \frac{m v}{e B}$$

$$dA = \vec{F}_m d\vec{x} = e(\vec{v} \times \vec{B}) \vec{v} dt = 0$$

$\nearrow$   
 pravokotna