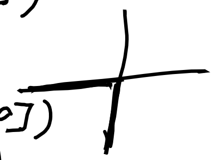
1.1.

$A = [-1,1] \times [-1,1]$

$g : \mathbb{R}^2 \to \mathbb{R}/A$ hoc

a) $g_*((-\infty, 0] \times (-\infty, 0])$

~~zapte~~ niodp nitj zap

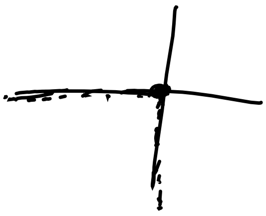


$g^*(g_*((-\infty,0] \times (-\infty,0]))$

b) $g_*(\mathbb{R}^2 - (-\infty,0] \times (-\infty,0])$

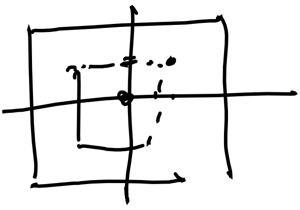
ni oapta

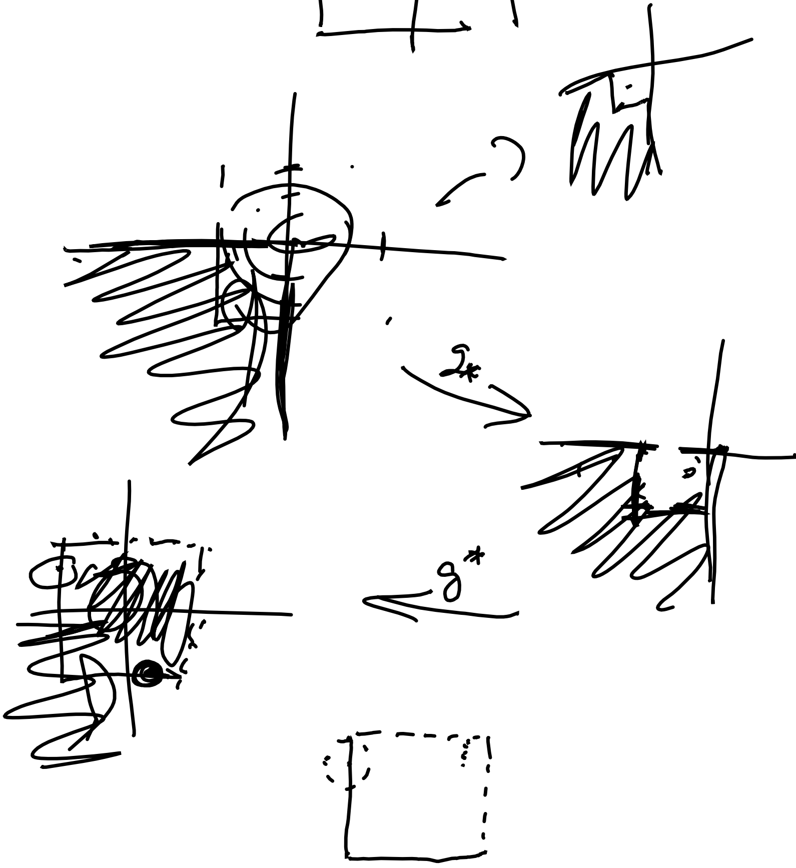
ni zapta



c) $g_*([-2,2] \times [-2,2])$

odpta





5)

$g^*(g([-2,2] \times [-2,2])$

zapta

ni odp

c) niedy jezy

7.4

a) $[-2,2] / [-1,1]$ 

$\cong [-1,1]$

b) $[-1,1] / \{-1,0,1\}$ 

$\cong$ 

$\mathbb{R}^2$

c)

$$\mathbb{R}^n / K(0,1) \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[r, "f"] \arrow[d] & \mathbb{R}^n \\
\mathbb{R}^n / K(0,1) &
\end{tikzcd}$$

$$f(\vec{a}) := \begin{cases} 0 \,; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} \,; & a \in \mathbb{R}^n - \overline{K}(0,1) \end{cases}$$

d)

$\mathbb{R}^n/_\sim \qquad x \sim y \Leftrightarrow \|x\| = \|y\|$

$\mathbb{R}^n/_\sim \cong [0,\infty)$

$f: \mathbb{R}^n \longrightarrow [0,\infty)$

$f(\vec{x}) = \|\vec{x}\|$ je surj

$a \in [0,\infty) \qquad (a,0,0\ldots) \mapsto a$

$[x]=[y] \Leftrightarrow \|x\|=\|y\| \Leftrightarrow f(x)=f(y)$

je zvezna

iščemo preslikavo s, da velja $f \circ s = id_{[0,\infty)}$

$s: a \longmapsto (a,0\ldots\ldots)$

Dokažimo da če $r \circ s = id_X$

$\Rightarrow r$ kvocientna, sub?

$S \subseteq Y$ taka da $r^*(S)$ odp v X

$S = s^*(r^*(S)) =$

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S \quad \blacksquare$

$\Longrightarrow S$ je odprta

f)

$$\left(S^n \times [-1,1]\right) / \{S^n \times \{-1\}, S^n \times \{1\}\} \approx S^{n+1}$$

$$f: X \to S^{n+1}$$

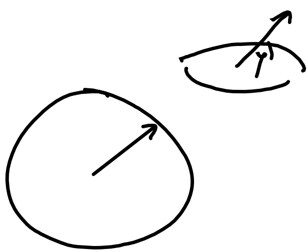
$$(x,t) \longmapsto (xb, t) = (\sqrt{1-t^2}\, x, t$$

anstatt voller

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{ g \cdot H \, ; g \in G \} =$$

$$= \{ \{ g \cdot h \, ; h \in H \} \, ; g \in G \}$$

G top. grupa

$a \in G$

$$L_a : G \longrightarrow G$$
$$x \longmapsto a \cdot x$$

leva translacija

$a, b \in G$

$$h : G \longrightarrow G$$

$h(a) = b$ $\qquad h = ?$

$$L_{ba^{-1}} : x \longmapsto ba^{-1}x$$

možnosti

$ba^{-1}x$

$xa^{-1}b$

bxa^{-1}

$a^{-1}xb$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\iff$ $ba^{-1}A$ okolica $b \in G$

$\exists U^{odp} \subseteq A \,.\, a \in U$

$b \in ba^{-1}A$

$a \in U \Rightarrow \underbrace{ba^{-1}a}_{b} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \overset{\subseteq A}{\text{odprta}}$ v G

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ H okolica 1 $\Rightarrow$ H odp in zap v G

$a \in aH \subseteq H$

$\Rightarrow$ H je okolica vsake svoje točke

G - H je odp.

$aH \cap H = \emptyset \Rightarrow aH = H$

$a \in G - H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$ vsak element ima okolico, ki ne seka H $\Rightarrow$ H je zaprta

c) C komponenta $1 \in C$

$\Rightarrow C$ zaprta edinka v G

$C \leq G$

$L_a : x \mapsto ax$ je homeomorfizem za $\forall a \in C$

$\forall a \in C.\ L_a \subseteq C$

L_a ohranja povezanost

Prav tako $L_a \cdot 1 = a \Rightarrow$

$L_{a*}C \cap C \Rightarrow L_{a*}C \subseteq C$

invertiranje: invertiranje je tudi homeo $i : x \mapsto x^{-1}$

$\Rightarrow$ po istih argumentih ~~...~~

Ali je edinka?

$\forall a \in G.\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \mapsto axa^{-1}$ je homeomorfizem

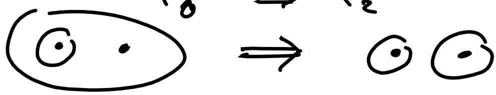
$\Rightarrow$ jo povezano in $a\, id\, a^{-1} = id \in C$

$\Rightarrow aCa^{-1} \subseteq C$

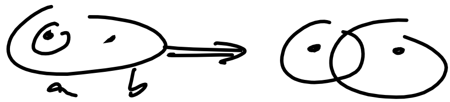
ker je kompozitum dveh translacij (leva in desna)

d) za G je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$



$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G$. $a \in U$. $b \notin U$ BŠZS

$U^{-1} = \{a^{-1} : a \in U\}$

$aU^{-1}b \ni b$ $\quad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U$. $a = ac^{-1}b$

$\Rightarrow b = c \Rightarrow b \in U$ ⚡

$T_1 \Rightarrow T_2$

$\exists U, V \underset{\text{odp}}{\in} G$. $a \in U$. $b \in V$. $a \notin V$ $b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$\Delta_G = f^{*}(\{1\})$

$f : (x, y) \mapsto xy^{-1}$

zvezna preslikava za k je zaprta

(2) T_{cc} je T_1 in ni T_2

$\Rightarrow$ $(\mathbb{R}, +)$ ni top grupa za T_{cc}

(ker 1.d)

(2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

d $(m,n)(x,y) := (m+x,\ n+y)$

$$\Phi : g \longmapsto \underbrace{(a \longmapsto g\cdot a)}_{\text{bijekcija (izomorfizem)}}$$

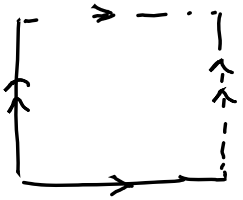
$\Phi : G \longrightarrow \mathrm{Bij}(A) \longleftarrow$ grupa za komponiranje

V širšem: $\Phi : G \longrightarrow \left(\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ\right)$

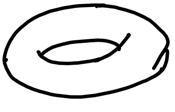
mamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ja

za nivoju topologije ? NE



Dobimo torus



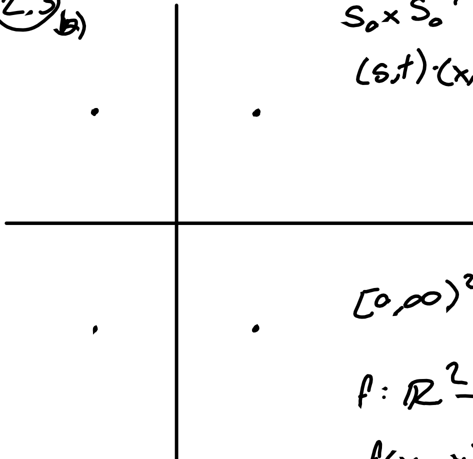
$\cong S^1\times S^1$

$f : \mathbb{R}^2 \longrightarrow S^1\times S^1$

$(x,y) \longmapsto (e^{2\pi i x},\ e^{2\pi i y})$

Tuk nism poslušala več :)

(2.3) b)



$S_0 \times S_0 \curvearrowright \mathbb{R}^2$

$(s,t)\cdot(x,y) = (sx, ty)$

$[0,\infty)^2$

$f : \mathbb{R}^2 \to [0,\infty)^2$

$f(x,y) = (|x|, |y|)$

$s : [0,\infty)^2 \to \mathbb{R}^2$

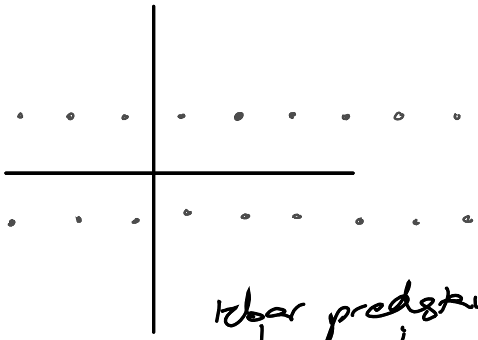
$(x,y) \mapsto (x,y)$

f, s zvezni $\Rightarrow f$ kvociente

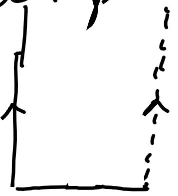
c)

$\mathbb{Z} \times S^0 \curvearrowright \mathbb{R}^2$

$(m,t)\cdot(x,y) = (m+x,\ ty)$



Izbor predstavnikov



$S_1 \times [0,\infty)$

$f: \mathbb{R}^2 \longrightarrow \overset{\subseteq \mathbb{C}}{S^1} \times [0,\infty)$

$(x,y) \longmapsto (e^{i2\pi x}, |y|)$

zvezna ✓

surjektivna ✓

Po standardnem postopku tudi identifikacije med delovanje

$((n-\frac{1}{n}, 0))_n$

slika zaporedja zap v $\mathbb{R}^2$

f-slike pa ni zaprta v $S^1 \times [0,\infty)$

$(1,0)$ je v zaprtju ni pa v f-sliki

Produkt dveh odprtih preslikav je odprta

$h: \mathbb{R} \longrightarrow [0,\infty)$

$x \longmapsto |x|$

Dovolj preveriti na bazi

$0 \notin (a,b)$: $h(a,b) = (\min\{|a|,|b|\}, \max\{\dots\})$

Če $0 \in (a,b)$: $h(a,b) = [0, \max\{|a|,|b|\})$

$g: \mathbb{R} \to S^1 \subseteq \mathbb{C}$

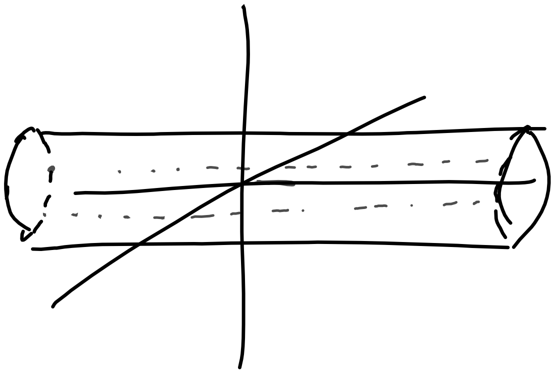
$x \mapsto e^{i2\pi x}$

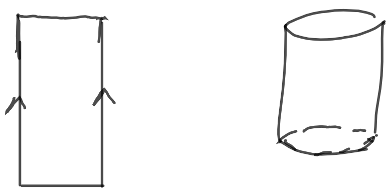
Baza $\mathbb{R}$ a/l/i: intervali dolžine <1 $\Rightarrow$ slike odprte

d)

$\mathbb{Z} \times S_0 \curvearrowright \mathbb{R} \times S^1 \subseteq \mathbb{R}^3$

$(m,t).(x,y,z) := (m+x, y, tz)$

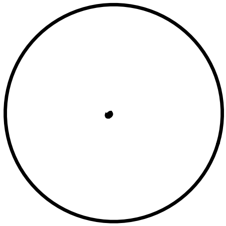




$\mathbb{R} \times S_1 \longrightarrow S_1 \times [-1,1]$

$(t, y,z) \longmapsto (e^{2i\pi t}, y)$

d)

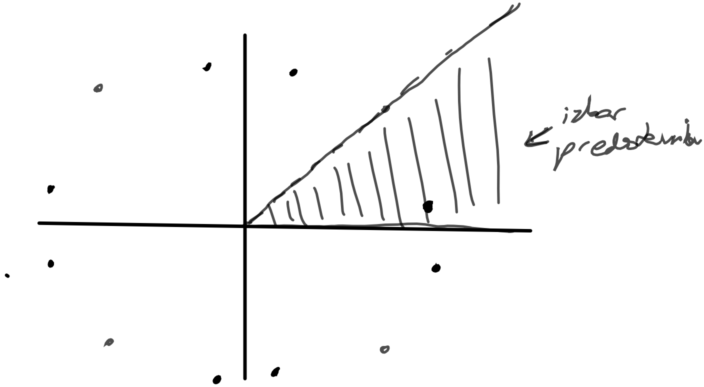


$$\vec{x} \sim \vec{y} \Longleftrightarrow \|x\| = \|y\|$$

$$f : \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(a)



$$\begin{bmatrix}1&0\\0&-1\end{bmatrix} \cong -1 \in \mathbb{C} \quad \begin{bmatrix}0&-1\\1&0\end{bmatrix} \cong i$$

Topologija ker ostane

$$Y = \{(x,y) \in \mathbb{R}^2;\ y < x;\ x > 0$$

$$f: \mathbb{R}^2 \longrightarrow Y$$

$$(x,y) \longmapsto (\max(|x|,|y|);\ \min(|x|,|y|)$$

A retrakcija $\Rightarrow$ kvocientna v ožjem smislu

2 4

$2x = g + x$

$\mathbb{R}/\mathbb{Q}$ ni math. vložiti v noben

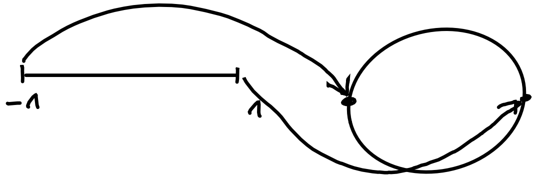
← po delovanju grupe

$U^{odp} \subseteq \mathbb{R}/\mathbb{Q}$

Ubit vse

(3.1) 8.4

a)
$X = [-1, 1]$ $A = \{-1, 1\}$
$Y = S^1$ $f(x) := (x, 0)$



$Z = [-1, 1] \times \{0\} \cup S^1$

$g : X + Y \longrightarrow Z$
$in_1(x) \longmapsto (x, 0)$
$in_2(y) \longmapsto y$

ekvivalenčni razredi: $(in_1(1,0); in_2(1,0)$ in
$in_1(-1,0)$ in $in_2(-1,0))$ neenostavno

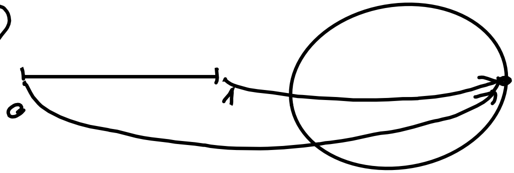
$g(in_1(1,0)) = 1,0$
$g(in_2(1,0)) = 1,0$ podobno z drug

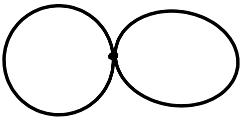
zveznost je, ker sta posamezni zvezni
in ker se ujemata na preseku

Potrebno še pokazati še da loči ekv. razrede

slikamo iz kompakta v Hausdorffov

b)





$$Z = Y \cup S((2,0), 1)$$

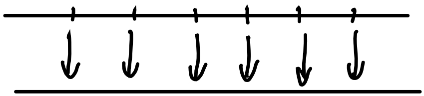
$$g: X+Y \longrightarrow Z$$

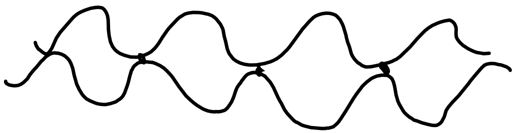
$$in_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$in_2(x,y) \longmapsto (-x+2, y)$$

Preveriti moramo

- loči ekv razr
- konst na ekv razr
- zvez, surj. kvocientna v ožjem smislu

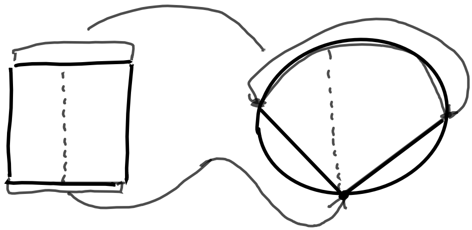




$|\sin x| \cup -|\sin x|$

$x \longmapsto$

d)



$Z = S^1 \cup \{(x,y) \in \mathbb{R}^2 : |x-1| \leq 1 \leq \sqrt{1-x^2}\}$

$g: X + Y \longrightarrow Z$

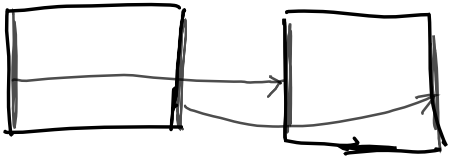
$in_2(z) \longmapsto z$

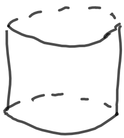
$in_1(x,y) \longmapsto (0,-1) + \frac{y+1}{2}\left(x, \sqrt{1-x^2}+1\right)$

kompaktna v hausdorffov

e)

X:





$Z = S_1 \times [-1, 1]$

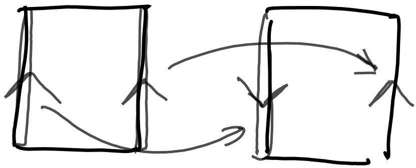
$g: X + Y \longrightarrow Z$

$in_1(x, y) \longmapsto (x, -\sqrt{1-x^2}, y)$

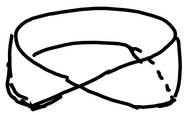
$in_2(x, y) \longmapsto (x, \sqrt{1-x^2}, y)$

kompakt $\vee$ hausdorff ✓

[illegible] ~~ste~~ identifizieren [illegible] f)



Möbiusov trak

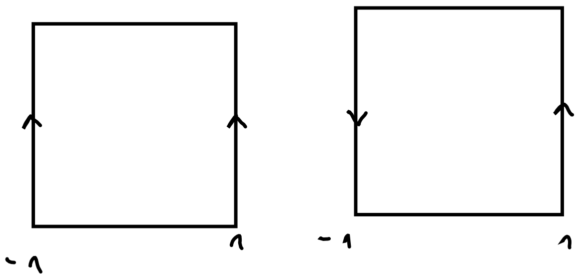


Parametrizacija möbiusovega traku

$x(u,v) = (1 + \frac{v}{2}\ csc\ \frac{u}{2}\ os\ u$

$y(u,v) =$

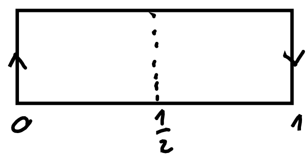
$z(u,v) =$



15.4

$X + Y \longrightarrow M$ ... Möbisov trak

$M = [0,1]\times[0,1]/\sim$



\begin{tikzcd}
X+Y \arrow[r, "g'"] \arrow[d, "q"'] \arrow[dr, "g"] & {[0,1]^2} \arrow[d, "q_m"] \\
X \cup_f Y \arrow[r, dashed] & M
\end{tikzcd}

$$in_1(u,v) \mapsto \left(\frac{u+1}{4}, \frac{v+1}{2}\right)$$

$$in_2(u,v) \longmapsto \left(\frac{3-u}{4}, \frac{v+1}{2}\right)$$

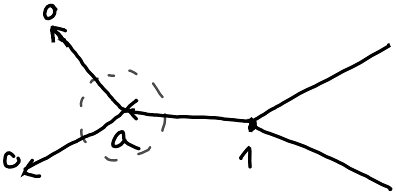
$$x \sim y \Leftrightarrow g(x) = g(x)$$

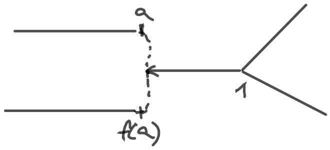
$$x \sim y \Leftrightarrow g'(x) \sim g'(y)$$

$(a,1] \longrightarrow (0,\infty)$

$f(x)=x$

zlepek $(0,\infty) \cup_f (a,\infty)$ je zlepek za katere hausdorfove

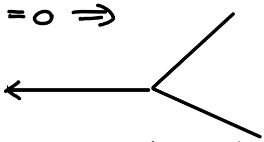




Poglejmo točko a

Vsaka okolica od a in od f(a) seka te del $(0,1]$ za $\forall a \in (0,1)$

za $a=0 \Rightarrow$



vložimo v evklidski prostor

$(0,\infty) \cup_f (0,\infty) \longrightarrow \mathbb{R}^2$

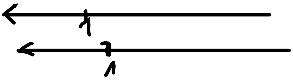
$x \in in_1(0,1] \longmapsto (0,x)$

$x \in in_1(1,\infty) \longmapsto (x, x-1)$

$x \in in_2(1,\infty) \longmapsto (x, -x+1$

je hausdorfov?

lokalna kompaktnost



kompakti: zaprti intervali v $(0,\infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh premicah

kompakti grejo v kompakte tarij

(3.3)

$X/A \approx X \cup_f \mathbb{1}$

$$\begin{tikzcd}
X \arrow[d, "f"'] & X+\mathbb{1} \arrow[l, "g'"'] \arrow[dl, "g"'] \arrow[d, "g"] \\
X/A \arrow[r, "\approx"] & X \cup_f \mathbb{1}
\end{tikzcd}$$

$1 \in \mathbb{1}$

$h:$
$[x] \longmapsto x \quad ; x \notin A$
$[a] \longmapsto 1 \quad : a \in A$

je homeomorfizem

$g' : X+\mathbb{1} \longrightarrow X$
$in_1 x \longmapsto x$
$in_2 \smile \longmapsto a \quad a \in A$

$g = f \circ g'$

$u \sim v \Leftrightarrow g'(u) \sim_A g'(v)$

$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$

$f(g'(u)) = f(g'(v)) \Leftrightarrow$

$[g'(u)] \sim [g'(v)]$

Ekv razredi v $X \cup_f \mathbb{1}$

$in_1 x = \{x\} \quad x \notin A$

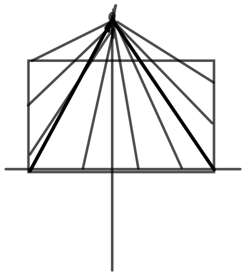
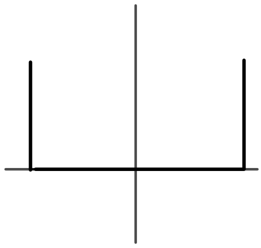
$in_1 a = A \cup \smile \quad a \in A$

$in_2 \smile = A \cup \smile$

$[in_1 x] = \{ in_1 x \}$

$(in_1)_*(A) \cup \{ in_2 \smile \}$

Retrakti, homotopije, ekstenzije, ... [illegible]

Projeciramo iz $(0,2)$ na rob

$y > 2x+2$: $\left(-1,\ 2-\frac{y-2}{x}\right)$

$2x+2 > y < -2x+2$: $\left(\frac{2x}{2-y},\ 0\right)$

$y > -2x+2$ $\left(1,\ \frac{y-2}{x}+2\right)$

$(a,b) \rightsquigarrow p_1 : y = \frac{b-2}{a}x+2$

pri $x=-1$

$y = -\frac{b-2}{a}+2$

$p_2 : y = \frac{b-2}{a}x+2 \qquad a \neq 0$

$y=0 \to x = \frac{-2a}{b-2} = \frac{2a}{2-b}$

$a=a$

$x=1$:

$y = \frac{b-2}{a}+2$

Ali je to id na X?

$x=-1$: $\left(-1,\ 2-\frac{y-2}{-1}\right) = (-1,\ 2+y-2) = (-1,y)$

$y=0$: $\left(\frac{2x}{2},0\right) = (x,0)$

$x=1$: $\left(1,\ \frac{y-2}{1}+2\right) = (1,y)$ ✓

Ali homotopna id na X

$H(x,y,t) = t(x,y) + (1-t)r(x,y)$

zvezna, ker so vsi kosi zvezni

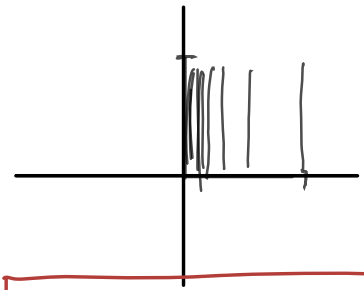
f homotopna g

ko $\exists\, H: X \times [0,1] \longrightarrow Y$

$$(x,0) \longmapsto f(x)$$

$$(x,1) \longmapsto g(x)$$

zvezna



$\boxed{\text{Retrakt } T_2 \text{ prostora je zaprt}}$

Recimo da $\exists r$ retrakt $: X \to Y$

$r|_Y = id$

daljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

mora pokriti točke $(\frac{1}{n}, 0)$

(ker gre vse pt iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1]$ čez točke

Torej

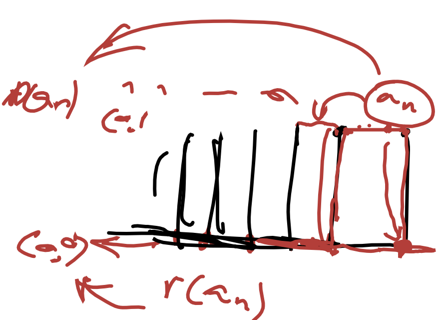
Naj bo zaporedje a_n; $r(a_n) = \frac{1}{n}$

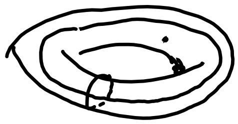
$a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim a_n = (0,1)$

$\lim r(a_n) = (0,0)$

r ni zvezna





$G \curvearrowright X$

$(X, G) \longrightarrow X$

$(x, g) \longmapsto x \cdot g$

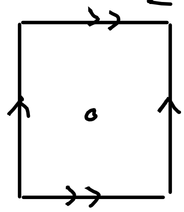
Deformacijska retrakcija "ima homotopijo do id"
je zvezna preslikava $H : X \times [0,1] \to X$.

$H(x,0) = x$ za $\forall x \in X$, $H(a,1) = a$ za $\forall a \in A$ $\wedge$ $H(x,1) \in A$ za $\forall x \in X$

$X \times [0,1] \longrightarrow$

drugo krožnica

ena krožnica



$$\begin{tikzcd}
([-1,1]^2 - \{(0,0)\}) \times [0,1] \arrow[r, "h"] \arrow[d, "\xi \times id_{[0,1]}"'] \arrow[dr, "g \circ h"] & {[-1,1]^2 - \{(0,0)\}} \arrow[d, "g"] \\
X \times [0,1] \arrow[r, dashed, "\text{zvezna preslikava}"'] & X
\end{tikzcd}$$

$$h : (\vec{x}, t) \longmapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x}(1-t)$$

$g \circ h$ mora biti konstantna na ekv. razredih

$$x \sim y \Rightarrow g \circ h(x) = g \circ h(y)$$
$$\Longleftrightarrow$$
$$[h(x)] = [h(y)]$$
$$\Longleftrightarrow$$
$$h(x) \sim h(y)$$

h ekvivalentne netrivialne razrede ohrani na mestu

$\Rightarrow$

$$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1,1]^2 - \{0,0\} \times [0,1]$$

$$\Longleftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$$

$$\Rightarrow \frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1)(1 - t_1)$$

$$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|} t_2 + (x_1, y_1)(1 - t_2)$$