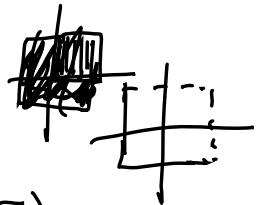


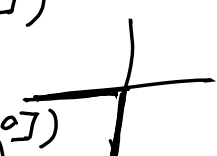
1.1.

$$A = [-1, 1] \times [-1, 1]$$

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}/A \quad \text{hoc}$$


a) $g^*((-\infty, 0] \times (-\infty, 0])$

ni: opty
ni: opty
 $g^*((-\infty, 0] \times (-\infty, 0])$



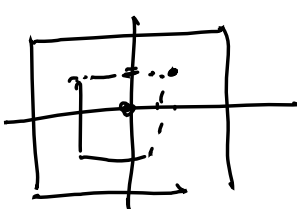
b) $g^*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

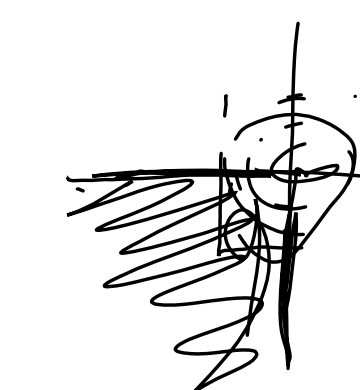
ni: opty
ni: opty



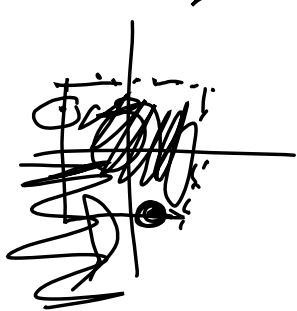
c) $g^*([-2, 2] \times [-2, 2])$

opty






g^*

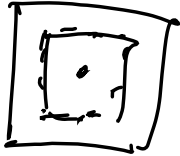


g^*

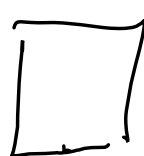


5)

$$g^*([2, 2] \times [2, 2])$$



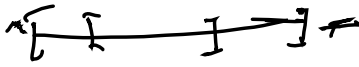
\rightarrow



opty
ni: opty

d) ni: opty

14

$$a) [-2, 2] / [-1, 1] \cong [-1, 1]$$


$$b) [-1, 1] / \mathbb{Z} \cdot 1, 0, 1, 2$$

$$[-1, 1]$$

$$\cong \text{circle}$$

$$c) \mathbb{R} / \mathbb{Z}$$

8

$$\mathbb{R}^n /_{K(0,1)} \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{array}{ccc} \mathbb{R}^n & \xrightarrow{f} & \mathbb{R}^n \\ \downarrow & & \\ \mathbb{R}^n /_{K(0,1)} & & \end{array}$$

$$f(\vec{a}) := \begin{cases} 0; & \text{ce } |\vec{a}| \leq 1 \\ \vec{a} - \frac{\vec{a}}{\|\vec{a}\|}; & \text{ce } \vec{a} \in \mathbb{R}^n - \overline{K(0,1)} \end{cases}$$

$$d) \quad \mathbb{R}^n / \sim \quad x \sim y \Leftrightarrow \|x\| = \|y\|$$

$$\mathbb{R}^n / \sim \xrightarrow{\sim} [0, \infty)$$

$$f: \mathbb{R}^n \longrightarrow [0, \infty)$$

$$f(\vec{x}) = \|\vec{x}\| \text{ je sur}$$

$$a \in [0, \infty) \quad (a, 0, 0, \dots) \mapsto a$$

$$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$$

je zvezna

iščemo preslikavo s , da

$$\text{velja } f \circ s = \text{id}_{[0, \infty)}$$

$$s: a \mapsto (a, 0, \dots)$$

Dokazimo da je $r \circ s = \text{id}_Y$

$\Rightarrow r$ kvocientna, surjektivna

$$S \subseteq Y \text{ takoda } r^*(S) \text{ odpr. v } X$$

$$S = s^*(r^*(S)) =$$

$$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$$

$$\text{id}_Y^*(S) = S$$

$\Rightarrow S$ je odprta

f)

$$S^n \times [-1, 1] / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \rightarrow S^{n+1}$$

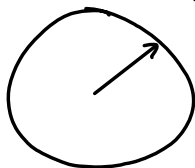
$$(x, t) \mapsto (xb, t) = (\sqrt{1-t^2}x, t)$$

\uparrow
anotado valla

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{g \cdot h; g \in G\} =$$

$$= \{ \{g \cdot h; h \in H\}; g \in G \}$$

G top. grupe

$$a \in G$$

$$L_a: G \longrightarrow G$$
$$x \longmapsto a \cdot x \quad \text{leva transkacija}$$

$$a, b \in G$$

$$h: G \longrightarrow G$$

$$h(a) = b \quad h = ?$$

$$L_{a^{-1}}: x \longmapsto ba^{-1}x$$

možnosti

$$ba^{-1}x$$
$$xa^{-1}b$$
$$bxa^{-1}$$
$$a^{-1}xb$$

topološke grupe zahtevajo

povsod isto, ker lahko vzelo

točko prestikano v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ desica $\Leftrightarrow b a^{-1} A$ desica $b \in G$

$$\exists U^{*dr} \subseteq A, a \in U$$

$$b \in b a^{-1} A$$

$$a \in U \Rightarrow b a^{-1} a \in \underbrace{b a^{-1} U}_b \subseteq b a^{-1} A$$

ker je $L_{b a^{-1}}$ homeomorfizem je

$$b a^{-1} U \stackrel{\subseteq A}{\text{odprta v } G}$$

\Leftarrow potem velja tudi obratno

b) $H \leq G$ H dedice 1 $\Rightarrow H$ odv: n zap v G

$$a \in aH \subseteq H$$

$\Rightarrow H$ je dedica
vsake svoje tocke

G/H je odv.

$$aH \cap H = \emptyset \Rightarrow aH = H$$

$$a \in G/H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$$

vsak element ima dedico ki ne
seka $H \Rightarrow H$ je zaprta

c) C komponente $1 \in C$

$\Rightarrow C$ zaprt edinka v G

$$C \subseteq G$$

$L_a: x \mapsto ax$ je homomorfizem za $\forall a \in C$

$$\forall a \in C. L_a \subseteq C$$

L_a ohranja povezanost

$$\text{Pravi tako } L_a \cdot 1 = a \Rightarrow$$

$$L_a \cdot C \cap C \Rightarrow L_a C \subseteq C$$

invertiranje: invertiranje je tudi x homeo $i: x \mapsto x^{-1}$
 \Rightarrow po istih argumentih ~~na~~

Ali je edinka?

$$\forall a \in G. aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$$

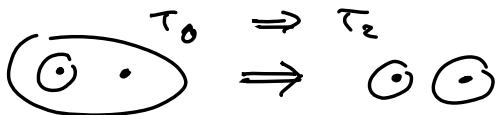
$x \mapsto axa^{-1}$ je homeomorfizem

\Rightarrow je povezano in $a \text{ id } a^{-1} = \text{id} \in C$

$$\Rightarrow aCa^{-1} \subseteq C$$

\rightarrow ker je kompozitna tvel transkicij
(leva in desna)

d) za G je



$a, b \in G$

memo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U, b \notin U \quad \text{BŠZS}$

$$U^{-1} = \{a^{-1} : a \in U\}$$

$$aU^{-1}b \ni b \quad a \mapsto b$$

Predmet da $a \in aU^{-1}b$

$$\exists c \in U, a = ac^{-1}b$$

$$\Rightarrow b = c \Rightarrow b \in U \quad \times$$

$$\tau_1 \Rightarrow \tau_2$$

$\exists U, V \subseteq G, a \in U, b \in V, a \notin V, b \notin U$

$\Delta \subseteq G \times G$ je $\Delta \subseteq G \times G$

$$\Delta_0 = f^*(\{1\})$$

$$f: (k, x) \mapsto xy^{-1}$$

za svaku grupu G je Δ_0

$$\textcircled{2} \quad T_{cc} \in T_1 \text{ in } n: T_2$$

$$\Rightarrow (\mathbb{R}, +) \text{ nichtgruppe zu } T_{cc}$$

$$(\ker 1, d)$$

$$(2.3) \quad \mathbb{Z} \times \mathbb{Z} \hookrightarrow \mathbb{R}^2$$

$$A \quad (m, n)(x, y) := (m+x, n+y)$$

$$\Phi: g \mapsto (a \mapsto g \cdot a)$$

bijekcija (izomorfizam)

$$\Phi: G \rightarrow \text{Bij}(A) \leftarrow \text{grupa}$$

za kompozicije

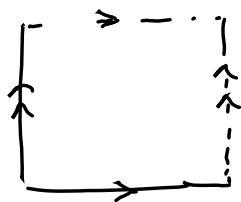
$$\text{Ubi slučaju: } \Phi: G \rightarrow (\text{Konec}(A), \circ)$$

$$\mathcal{L}(AA)$$



$$\text{mamo } [0, 1) \stackrel{?}{\cong} \mathbb{R}^2 / \Phi$$

Na nivouju množice
za nivouju topologije? NE



Dobimo torus

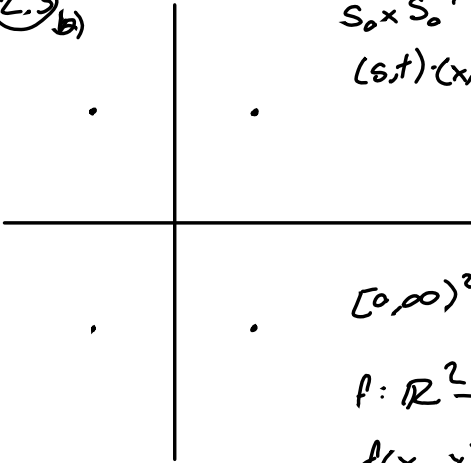
$$\bigcirc \cong S^1 \times S^1$$

$$f: \mathbb{R}^2 \rightarrow S^1 \times S^1$$

$$(x, y) \mapsto (e^{2\pi i x}, e^{2\pi i y})$$

Tak nismo pokazali već :

(2.3) b)



$$S_0 \times S_0 \hookrightarrow \mathbb{R}^2$$
$$(s, t) \cdot (x, y) = (sx, ty)$$

$$[0, \infty)^2$$

$$f: \mathbb{R}^2 \rightarrow [0, \infty)^2$$

$$f(x, y) = (|x|, |y|)$$

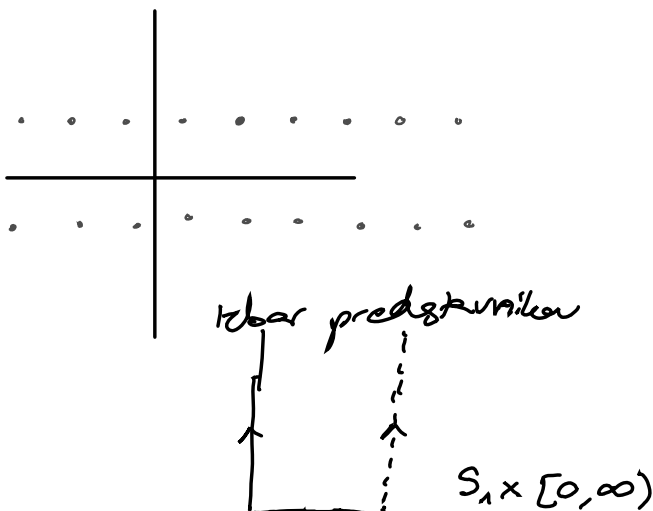
$$S: [0, \infty)^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (x, y)$$

f, S surjektiv $\Rightarrow f$ Isomorphism

$$c) \mathbb{Z} \times S^1 \hookrightarrow \mathbb{R}^2$$

$$(m, t) \cdot (x, y) = (m+x, ty)$$



$$f: \mathbb{R}^2 \rightarrow S^1 \times [0, \infty)$$

$$(x, y) \mapsto (e^{i2\pi x}, |y|)$$

warna ✓
surjektivne ✓

Po standardnem postopku radi identifikacije med delovanji

$$((n - \frac{1}{n}), 0)_n$$

slike zaporedja zap v \mathbb{R}^2
f-slike pa ni zap. v $S^1 \times [0, \infty)$

(1, 0) je v zaprtju mpa v f-sliki

Produkt dveh adgehtih preslikav je adgeht

$$h: \mathbb{R} \longrightarrow [0, \infty)$$

$$x \mapsto |x|$$

Dovolj prevesti ne bazi

$$0 \notin (a, b): h(a, b) = [\min\{|a|, |b|\}, \max\{|a|, |b|\}]$$

$$0 \in (a, b): h(a, b) = [0, \max\{|a|, |b|\}]$$

$$g: \mathbb{R} \rightarrow S^1 \subseteq \mathbb{C}$$

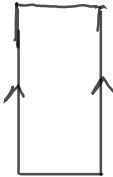
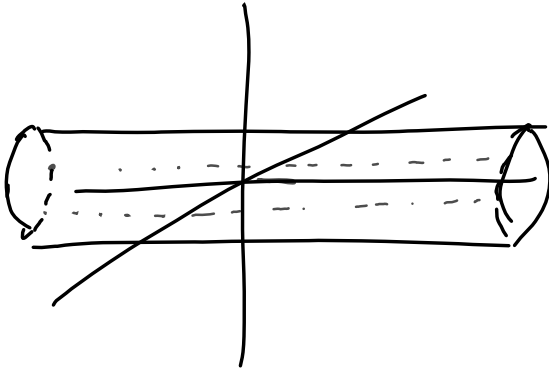
$$x \mapsto e^{i2\pi x}$$

Baza 2 a_1, \dots intervalitne $< 1 \Rightarrow$
slabe
adgeht lah

d)

$$\mathbb{Z} \times S^1 \hookrightarrow \mathbb{R} \times S^1 \subset \mathbb{R}^3$$

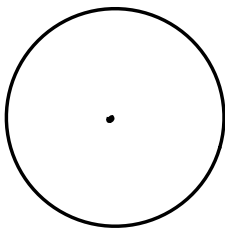
$$(m, t), (x, y, z) := (m+x, y, tz)$$



$$\mathbb{R} \times S^1 \longrightarrow S^1 \times [1, 1]$$

$$(t, y, z) \longmapsto (e^{i\eta t}, y)$$

\mathbb{R}^n

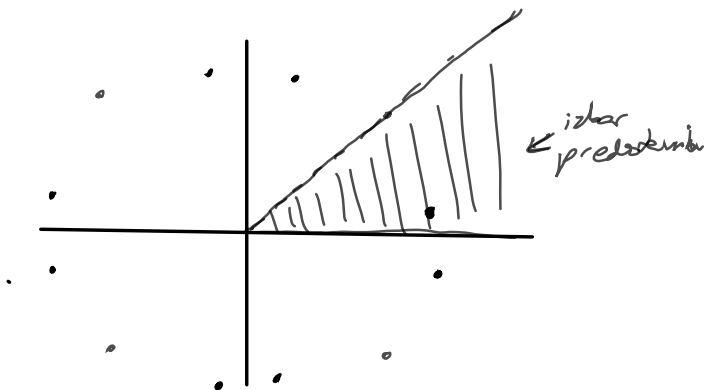


$$\vec{x} \sim \vec{y} \Leftrightarrow \|\vec{x}\| = \|\vec{y}\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(A)



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cong -1 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \dots$$

Topologija ker ...

$$Y = \{(x, y) \in \mathbb{R}^2; y < x; x > 0\}$$

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$(x, y) \longmapsto (\max(x, y), \min(x, y))$$

A retrakcija \Rightarrow koeficienta v
 ožjem or maku

2(4)

$$2x = g + x$$

\mathbb{R}/\mathbb{Q} ni matric vektorien, v. n. n. n.

$$U = \mathcal{L} \subseteq \mathbb{R}/\mathbb{Q} \quad \leftarrow \text{po delikanten grupe}$$

Ubit v. n.

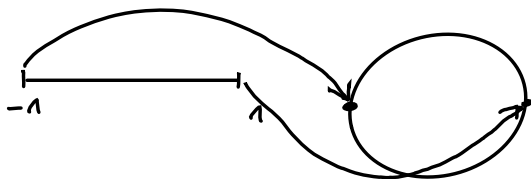
3.1

8.4

a)

$$X = [-1, 1] \quad A: \{-1, 1\}$$

$$Y = S^1 \quad f(x) := (x, 0)$$



$$Z = [-1, 1] \times \{0\} \cup S^1$$

$$g: X + Y \longrightarrow Z$$

$$in_1(x) \mapsto (x, 0)$$

$$in_2(y) \mapsto y$$

ekvivalenčni razredi: $(in_1(1, 0); in_2(1, 0))$ in $(in_1(-1, 0); in_2(-1, 0))$ neenotni

$$g(in_1(1, 0)) = 1, 0$$

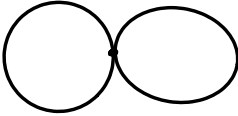
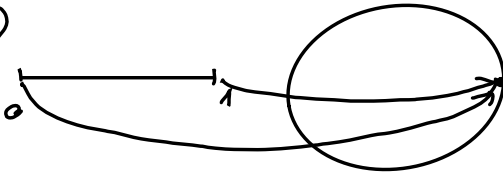
$$g(in_2(1, 0)) = 1, 0 \quad \text{padeta na 2. drug}$$

wezast pa, ker sta funkciji zvezni;
in ker ~~se~~ se ajimata na presku

Potrebo se dokazati še da loči dve razredi

Sikamo iz kompaktna v Hausdorffu

b)



$$Z = Y \cup S((2,0), 1)$$

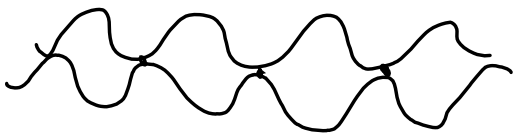
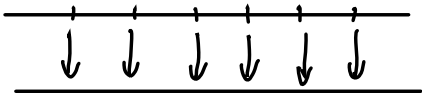
$$g: X + Y \longrightarrow Z$$

$$\text{in}_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$\text{in}_2(x, y) \longmapsto (-x + 2, y)$$

Preveriti moramo

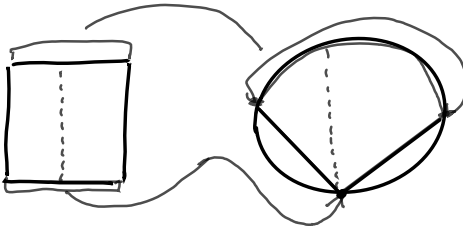
- loči dve raze
- konst na dve raze
- zvez, surj, kvocientne v ožjem smislu



$$|\sin x| \cup -|\sin x|$$

$x \mapsto$

d)



$$Z = S^1 \cup \{(x, y) \in \mathbb{R}^2; |x-1| \leq 1 \leq \sqrt{1-x^2}\}$$

$$\partial: x+y \longrightarrow Z$$

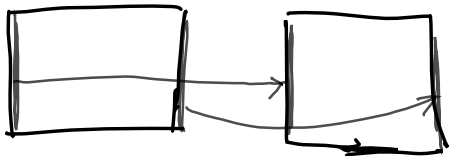
$$\text{in}_2(z) \longmapsto z$$

$$\text{in}_1(x, y) \longmapsto (0, -1) + \frac{y+1}{2} (x, \sqrt{1-x^2}+1)$$

kompatte u hausdorff

c)

$X:$



$$Z = S_1 \times [-1, 1]$$

$$g: X + Y \longrightarrow Z$$

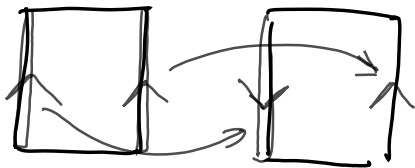
$$in_1(x, y) \longmapsto x(\sqrt{1-x^2}, y)$$

$$in_2(x, y) \longmapsto (x, \sqrt{1-x^2}, y)$$

komplett v. Hausdorff ✓

oder sie ist nicht Hausdorff

f)



Möbiussens trake



Parametrisa cje Möbiussens traku

$$x(u, v) = \left(1 + \frac{v}{2} \csc \frac{u}{2} \cos u\right)$$

$$y(u, v) =$$

$$z(u, v) =$$