

Formule

$$\# \text{ permutacij} = |\{\sigma: A \xrightarrow{\sim} A\}| = n!$$

$$B \subseteq A \Rightarrow |A-B| = |A| - |B|$$

$$|A \times B| = |A| |B|$$

Št neurejenih izborov podmnožice k od $n = \binom{n}{k}$

multinomski: n_1 tipa 1 n_2 tipa 2, ..., n_k tipa k

$$\# \text{ permutacij neurejeno} = \frac{n!}{n_1! \dots n_k!}$$

$$① \quad T = \{ \underbrace{\text{juha, solata, riba}}_{\text{predjed}}, \underbrace{\text{zelenjava, meso}}_{\text{glavni jed}}, \underbrace{\text{sladoled, tortica}}_{\text{sladic}} \}$$

2 · 3 · 2 je št možnosti

12

⑦ kup kart 52 kart. karte razdelimo na 4 igralce
vsaki 13: 13

1. koliko načinov lahko karte tako razdelimo
2. kolikšna je verjetnost, da ima vsak igralec karte same ene vrste (pik, krd, krd, krd)

$$4! \cdot \binom{52}{13} \binom{39}{13} \binom{26}{13} \binom{13}{13} =$$

$$= \frac{52!}{13! (52-13)! 13! \dots 13!} = \frac{52!}{(13!)^4} 4!$$

2. 1, $\sqrt{4!}$

③ konstantjevice na kili inna 702 p. r. m.

Dleži: Najmanj dva prebivalca inake
enaki za 25 dni 125 rok

$$\begin{array}{r} 2 \\ 25 \cdot 25 = 625 \\ \hline 50 \\ 125 \\ \hline 625 \end{array}$$

$$702 - 625 = 77$$

④ inna 10 klet

2 rdeči, 3 zelene, 5 modrih

1). # možnih klet za 10 prebivalcev inake
pod prazno

— — — — —

1. 0

$$2. \frac{8!}{1!2!5!}$$

$$3. \frac{7!}{2!5!} + 2 \cdot \frac{7!}{4!1!2!}$$

4.

2 2 2 2



$$\frac{\binom{10}{4} \cdot 6}{2} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{2 \cdot 3 \cdot 4 \cdot 2} = \frac{1260}{2} = 630$$

X modce

$$\binom{10}{5} 4 = \frac{10!}{5!5!} 4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2} \cdot 4 =$$

=

- 1) 3 rdece
1 zelena
5 belih

izbornica 2

i) $\Omega = ?$ $\Omega = \begin{pmatrix} V \\ 2 \end{pmatrix}$ $V = \{r, r, r, z, b, b, b, b\}$

$$\Omega = \{ (r, r), (r, b), (r, z), (z, b), (z, r), \\ (b, b), (b, r), (b, z) \}$$

ii) najprij rdeca: $\frac{3}{9} = \frac{1}{3}$

iii) $P_1 = \frac{1}{3}$ druga zelena: $\frac{8}{9} \cdot \frac{1}{8} = \frac{1}{9}$

$$P(R_1 \cup Z_2) = P(R_1) + P(Z_2) - P(R_1 \cap Z_2)$$

$$P(R_1 \cap Z_2): \frac{3}{9} \cdot \frac{1}{8}$$

$$P(\dots) = \frac{4}{9} - \frac{3}{8 \cdot 9} = \frac{29}{8 \cdot 9}$$

2) 2 orni: n belh

i) $\bar{C}_{j,k} = \{ \text{najtem in kateri k-tem zloženem krogu} \}$
 $\mathcal{P}(\bar{C}_{j,k}) = ?$ funkcije j, k, n

$$\bar{C}_{j,k} = \left(\frac{n}{n+2} \right)^{j-1} \frac{2}{n+2} \left(\frac{n+1}{n+2} \right)^{k-j-1} \left(\frac{1}{n+2} \right) =$$

$$= \frac{2 \cdot n^{j-1}}{(n+2)^{j+k-j-1+1} (n+1)^{k-j-1}} = 2 n^{j-1} \frac{(n+1)^{k-j-1}}{(n+2)^k}$$

$X = k$

$$\mathcal{P}(X, k) = \mathcal{P}\left(\bigcup_{i=1}^{k-1} \bar{C}_{i,k}\right) = \sum_{i=1}^{k-1} \mathcal{P}(\bar{C}_{i,k}) = \frac{2}{(n+2)^k} \sum_{j=1}^k n^{j-1} (n+1)^{k-j-1}$$

$$n^{j-1} (n+1)^{k-j-1} = \frac{n^j}{n(n+1)} (n+1)^{k-1} = \left(\frac{n}{n+1} \right)^j \frac{(n+1)^{k-1}}{n}$$

$$= \frac{2(n+1)^{k-1}}{n(n+2)^k} \frac{1 - \left(\frac{n}{n+1} \right)^k}{1 - \frac{n}{n+1}} = \frac{2((n+1)^k - n^k)}{n(n+2)^k}$$

iii) $\mathcal{P}(\bar{C}_1 | X = k)$

$$\mathcal{P}\left(\bigcup_{i=2}^k \bar{C}_{i,k}\right) = \mathcal{P}(X=k) - \mathcal{P}(\bar{C}_{1,k}) = \frac{2(n+1)^{k-1} - n^{k-1}}{(n+2)^k} =$$

$$= \frac{2}{n(n+2)^k} \left((n+1)^k (n+1) - n^k \right) = \frac{1}{n(n+2)^k} \left((n+1)^{k+1} - n^k \right)$$

$$\text{oz: } \frac{2}{(n+2)^k} \left((n+1)^{k-1} ((n+2)^k - n^{k-1}) \right) =$$

$$2(n+1)^{k-1} \frac{(n+2)^k - n^{k-1}}{(n+2)^k}$$

3)

K ... konservative $\leadsto P(A|K) = 1$

L ... leiser $\leadsto P(A|L) = r$

$$r=0 \Rightarrow P(A)=1$$

$$P(A) = P(A|K)P(K) + P(A|L)P(L) = 1 \cdot p + (1-p)(1-r)$$

$$P(V_3|V_2) = \frac{P(V_3 \cap V_2)}{P(V_2)} = \frac{P(V_3)}{P(V_2)}$$

$$P(V_2) = P(V_2|C)P(C) + P(V_2|L)P(L) = 1 \cdot p + (1-p)(1-r)$$

$$P(V_3) = P(V_3|C)p + P(V_3|L)(1-p) =$$

$$\Rightarrow P(V_3|V_2) = \frac{1 \cdot p + (1-r)^2(1-p)}{p + (1-r)(1-p)}$$

Y

1) Mečemo karanc, dokler ne pade najmanj eno steno in en grob

$$P(\{ \text{grob} \}) = p \quad 0 < p < 1$$

Naj bo $\{X=k\} = \{ \text{prvič pade najmanj } k \text{ sten in } k \text{ grob} \}$
ne karaku kj

Mehi so nekaj neodvisni

Radungite $P(X \geq k)$

$$P(X \geq k) = \sum_{n=k}^{\infty} P(X=n) = \sum_{n=k}^{\infty} (p^{n-1} (1-p) + (1-p)^{n-1} p) =$$

$$= \frac{(1-p)}{p^{k-1}} \sum_{n=0}^{\infty} p^n + \frac{p}{(1-p)^{k-1}} \sum_{n=0}^{\infty} (1-p)^n =$$

$$\frac{(1-p)}{p^{k-1}} \frac{1}{1-p} + \frac{p}{(1-p)^{k-1}} \frac{1}{1-(1-p)} =$$

$$= \frac{1}{p^{k-1}} + \frac{1}{(1-p)^{k-1}}$$

v prvih $n-1$ nehit ~~ne~~ je samo isto

$$p^{k-1} + (1-p)^{k-1}$$

$$p=0 \vee p=1 \Rightarrow p(X > k) = 0$$

$$= \frac{1}{p^{k-1} (1-p)^{k-1}} = \frac{(1-p)^{k-1} + p^{k-1}}{p^{k-1} - p^{2k-1}} \quad ???$$

- 2) Mečemo kovance deklet ne dajmo d.
m grobar ali na stavl

max netov: $m+n$ min netov: $\min\{m, n\}$

$$P(\xi_{gr}(3)) = p$$

$$P(\xi = k) =$$

$$\xi X = k = p \cdot \mathbb{I} \in \{0, 1\} \text{ doimo ali m grobo}$$

vali metvil



$$P(\{X \text{ grobar, pri } k \text{ at m grobar}\}) = p^m (p-1)^{k-m} + p^{k-m} (p-1)^m$$

~~k metar~~
~~m jik je neke vrste~~ : $k < 2m$

$$p^m (p-1)^{k-m} + p^{k-m} (p-1)^m$$

$$P(X = \xi) =$$

$$P(X = \xi_{k-1} \text{ tem mestu } p \text{ de } m-1 \text{ grobar})$$

$\{G=k\} = \{ \text{pade } m-1 \text{ qbov } \vee k-1 \text{ kaschih,} \\ \text{potem } p \text{ spet } p \text{ de qrb} \}$

$\{S=k\} = \{ \text{Na } k-1 \text{ kender } p \text{ de } m-1 \text{ stevijo, potem} \\ \text{na spet stevilo} \}$

$$\binom{k-1}{m-1} (p^m (1-p)^{k-m} + p^{k-m} (1-p)^m)$$

3)

$$X \sim \text{Bin}(n, \frac{1}{2})$$

$$P(X=k) = \binom{n}{k} \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{n-k} = \binom{n}{k} \frac{1}{2^n}$$

Definiramo slučajni spremenljivki.

Y za $k=0, \dots, n$ velja $P(Z) \sim P(X, Y)$

$$P(X=k, Y=k+1) = P(X=k) \frac{n-k}{n} \quad (a)$$

$$\text{in } P(X=k, Y=k-1) = P(X=k) \frac{k}{n} \quad (b)$$

$$\text{in } P(X=k, Y=l) = 0 \quad \text{če } |k-l| > 1 \text{ ali } l=k$$

Razenaj te parazidbita slučajne spremenljivke Y

Rezult:

Popolna verjetnost

$$P(Y=l) = \sum_{k=0}^n P(X=k, Y=l) =$$

$$= P(X=l-1, Y=l) + P(X=l+1, Y=l)$$

$$P(Y=l) = \binom{n}{l-1} \frac{1}{2^n} \frac{n-l+1}{n} + \binom{n}{l+1} \frac{1}{2^n} \frac{l+1}{n} =$$

$$= \frac{1}{2^n} \left(\frac{n!}{(l-1)!(n-(l-1))!} \cdot \frac{n-l+1}{n} + \right.$$

$$\left. \frac{n!}{(l+1)!(n-(l+1))!} \cdot \frac{l+1}{n} \right)$$

$$= \frac{1}{2^n} \left(\binom{n-1}{l-1} + \binom{n-1}{l} \right) = \frac{1}{2^n} \binom{n}{l}$$

u)

Naj bodo $X \sim B(\frac{1}{2}) \Rightarrow X=0$ z verjetnostjo $\frac{1}{2}$ in
 $X=1$ z verjetnostjo $\frac{1}{2}$

$$Y \sim B(\frac{1}{2})$$

$$Z = Z(X, Y)$$

$$P(Z) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \quad " \in \mathbb{R}^{u \times u}$$

$$\uparrow$$

$$P(X=1, Y=0)$$

Velja: $\ast \ 0 \leq p_{ij} \leq 1$

$$\ast \sum_{0 \leq i, j \leq 1} p_{ij} = 1$$

$$\text{Recimo da } P(Z) = \begin{pmatrix} \frac{1}{u} + \delta & \frac{1}{u} - \delta \\ \frac{1}{u} - \delta & \frac{1}{u} + \delta \end{pmatrix}$$

kde sta X in Y neodvisna od δ

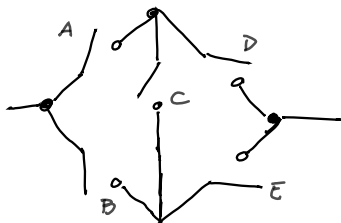
$$-\frac{1}{u} \leq \delta \leq \frac{1}{u}$$

Dajmo: $X \perp Y \Leftrightarrow \det(p_{ij}) = 0$ \leftarrow neodvisn

$$\det(p_{ij}) = \left(\frac{1}{u} + \delta\right)^2 - \left(\frac{1}{u} - \delta\right)^2 = \delta$$

$$\Rightarrow X \perp Y \text{ če je } \delta = 0$$

5) V vezju



vsake slikala prepusce
el. take z vrjetnostjo $\frac{1}{3}$

in posamezne
stihale so
med seboj
neodvisne

A \Rightarrow izdegetek da tak gre preko A

Kolikšno je vrjetnost da vezje prepusce?

$$\frac{1}{3} \left(\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} \left(\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) \quad ??$$

A D C E B E C D

$$\frac{1}{3} \left(\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} - \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} \cdot \frac{1}{3} - \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \quad ??$$

A D

odgovor

$$\frac{2}{3} \left(\frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) + \frac{1}{3} (\dots)$$

\uparrow
odgovor

stgda w p8

$$\downarrow \\ P(X=k) = P(H_c) \cdot P(X=k|H_c) + P(X=k|H_g) \cdot P(H_g) + P(H_g) \cdot P(X=k|H_g)$$

$$P(X=0|H_g) = 1$$

$$P(X=k|H_g) = 0$$

$$P(X=0|H_g) = 0$$

$$P(X=k|H_g) = P(X=k-1)$$

$$P(X=0|H_c)$$

$$k=0 \Rightarrow$$

$$P(X=0) = (1-p)P(X=0) + 1 \cdot p^2 + p(1-p) \cdot 0$$

$$P(X=0)(1 - 1+p) = p^2$$

$$P(X=0) = p$$

$$k \neq 0 \Rightarrow$$

$$P(X=k) = P(X=k)(1-p) + \cancel{P(X=k-1)p^2} + p(p-1)P(X=k-1)$$

V españa

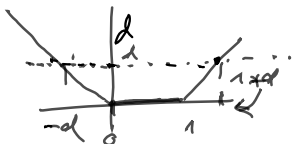
Naj bo $X \in U[-3, 3]$ in definirano

$$\{D=d\} = \{ \max(X, [0, 1]) = d \}$$

Izračunajte kumulativno porazdelitveno funkcijo na $F_D(D)$ in narišite njen graf

$$F_D(d) = P(D \leq d) \quad D = g(X) \text{ za } g(x) = d(X, [0, 1])$$

$$g(x) = \begin{cases} -x & ; x \leq 0 \\ 0 & ; 0 \leq x \leq 1 \\ x-1 & ; 1 \leq x \end{cases}$$



gostota:

$$f_X(x) = \begin{cases} \frac{1}{3-(-3)} = \frac{1}{6} & ; x \in [-3, 3] \\ 0 & ; \text{sicer} \end{cases}$$

$$F_X(x) = \int_{-\infty}^x f_X(x) dx = \int_{-3}^x f_X(x) dx = \begin{cases} \frac{x+3}{6} & ; -3 \leq x \leq 3 \\ 0 & ; x < -3 \\ 1 & ; x > 3 \end{cases}$$

Kaj so dogodki $\{D \leq d\}$?

$$d < 0 \Rightarrow \{D \leq d\} = \emptyset$$

$$d = 0 \Rightarrow \{D \leq d\} = [0, 1] \quad F_D(0) = P(X \in [0, 1]) = F_X(1) - F_X(0)$$

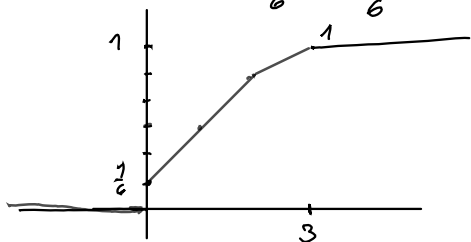
$$d \in [0, 2] \Rightarrow \{D \leq d\} = [-d, 1+d] \quad P_D(d) = F_X(1+d) - F_X(-d)$$

$$d \in [2, 3] \Rightarrow \{D \leq d\} = [-d, 3] \quad P_D(d) = F_X(3) - F_X(-d)$$

$$F_D(0) = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$F_D(d \in [0, 2]) = \frac{d+4}{6} - \frac{-d+3}{6} = \frac{2d+1}{6}$$

$$F_D(d \in [2, 3]) = \frac{5}{6} - \frac{-d+3}{6} = \frac{3+d}{6}$$



Wagellic! 

$$f_D = F'_D(d) = \begin{cases} 0 & ; d < 0 \\ \frac{1}{3} & ; d \in [0, 2] \\ \frac{1}{6} & ; d \in [2, 3] \\ 0 & ; d > 3 \end{cases}$$

Definicija: $X \sim \text{Cauchy}$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

a) Izračunaj $Y = \frac{1}{1+X^2}$

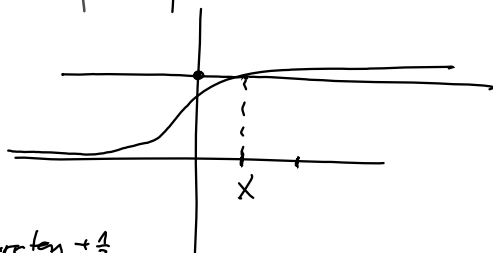
Namig: $F_Y(y) = P(Y \leq y) = P\left(\frac{1}{1+X^2} \leq y\right) =$

$$= P\left(\frac{1}{y} \leq 1+X^2\right) = P\left(\sqrt{\frac{1}{y}-1} \leq X\right) = \int_{-\infty}^X f_X(x) dx$$

$$P\left(\sqrt{\frac{1}{y}-1} \leq X\right) = \frac{1}{\pi} \arctan(x) \Big|_{-\infty}^x = \frac{1}{\pi} \arctan x + \frac{1}{2}$$

$$P\left(\sqrt{\frac{1}{y}-1} \leq X\right) - P\left(-\sqrt{\frac{1}{y}-1} \leq X\right) =$$

$$\left[\frac{1}{\pi} \arctan x + \frac{1}{2}\right]$$



$$y = \frac{1}{\pi} \arctan + \frac{1}{2}$$

$$\tan\left(\pi x - \frac{\pi}{2}\right)$$

$$P(Y \leq y) = P_X(X^2 \geq \frac{1}{y} - 1) = \quad Y = \frac{1}{1+X^2}$$

$$= P\left(X \geq \sqrt{\frac{1-y}{y}}\right) + P\left(X \leq -\sqrt{\frac{1-y}{y}}\right) =$$

$$= 2 P\left(X \geq \sqrt{\frac{1-y}{y}}\right) = 2\left(1 - F_X\left(\sqrt{\frac{1-y}{y}}\right)\right) \stackrel{\text{def}}{=} F_Y(y)$$

↑
simetrija

$$f_X(y) = \frac{d}{dx} F_X(x)$$

išeno $f_Y(y)$

$$f_X(x) = \frac{1}{\pi(1+x^2)}$$

$$f_Y(y) = F_Y'(y)$$

$$\underline{f_Y(y)} = \left(2 - 2F_X\left(\sqrt{\frac{1-y}{y}}\right)\right)' = \sqrt{\frac{1}{y}-1}$$

$$= -2 f_X\left(\sqrt{\frac{1-y}{y}}\right) \cdot \frac{-\frac{1}{y^2}}{2\sqrt{\frac{1}{y}-1}} =$$

$$= -2 \cdot \frac{1}{\pi(1+\frac{1-y}{y})} \cdot \frac{-1}{2y^2\sqrt{\frac{1-y}{y}}} = \frac{1}{\pi} \cdot \frac{\sqrt{y}}{y^2\sqrt{1-y}}$$

$$F_Y(y) = P_Y(Y \leq y) = P_X\left(\frac{1}{1+X^2} \leq y\right) =$$

$$\stackrel{2}{=} P_X(X^2 \geq \frac{1}{y} - 1) = \text{Deluje, ker je } y > 0$$

ker $Y: \Omega \rightarrow (0, 1]$
in ker je $1+X^2 > 0$

$$= P(X \geq \sqrt{\frac{1-y}{y}}) + P(X \leq -\sqrt{\frac{1-y}{y}}) = 2P(X \geq \sqrt{\frac{1-y}{y}}) =$$

↑
simetrija

$$= 2(1 - F_X(\sqrt{\frac{1-y}{y}})) \quad f_X^{\omega} = \frac{1}{\pi(1+x^2)}$$

$$F_X'(y) = f_X(y), \text{ če obstaja}$$

$$\Rightarrow f_Y(y) = F_Y'(y) = -2 F_X'(\sqrt{\frac{1-y}{y}})$$

$$= -2 f_X(\sqrt{\frac{1-y}{y}}) \frac{1}{2} \sqrt{\frac{y}{1-y}} \cdot \frac{-y-1+y}{y^2} =$$

$$= f_X(\sqrt{\frac{1-y}{y}}) \sqrt{\frac{y}{1-y}} \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \frac{1}{1+\frac{1-y}{y}} \sqrt{\frac{y}{1-y}} \frac{1}{y^2} =$$

$$= \frac{1}{\pi} \frac{y}{1} \frac{1}{y^2} \sqrt{\frac{y}{1-y}} = \frac{1}{\pi y} \sqrt{\frac{y}{1-y}} = \underline{\underline{\frac{1}{\pi \sqrt{y(1-y)}}}}$$

	Delež		
	No	yes	
White	141	19	160
Black	149	17	166
	290	36	326

Definiramo $R: \{\text{white, black}\} \rightarrow \mathbb{R}$

$$R(\text{white}) = 0$$

$$R(\text{black}) = 1$$

$$D: \{\overset{\text{No}}{\text{delež}}, \overset{\text{yes}}{\text{delež}}\}$$

$$\underline{X} = (R, D)$$

$$\underline{U} = \{\text{število } X = (i, j)\}$$

Vprašanja?

$$1. P(D=1 | R=i) ; i \in \{0, 1\}$$

2. $D \perp R$? (Ali sta neodvisni spremenljivki?)

Lahko ocenimo $\hat{P}_{ij} = \frac{\#(X; X=(i, j))}{N=326}$

$$\hat{P}_{ij} = \begin{pmatrix} \frac{141}{326} & \frac{19}{326} \\ \frac{149}{326} & \frac{17}{326} \end{pmatrix} \quad \begin{matrix} \hat{P}_{00} + \hat{P}_{01} = \text{verjetnost da je bel} = P(R=0) \\ \hat{P}_{10} + \hat{P}_{11} = P(R=1) \end{matrix}$$

$P(D=0) \quad P(D=1)$

$$1) P(D=1 | R=0) = \frac{P_{01}}{P_{00} + P_{01}} = 0,11$$

$$\frac{P_{11}}{P_{10} + P_{11}} = 0,10$$

1.

1.12.

n gledalcev z omini joku.

X ... št. gledalca ki dobi nasej joku

$$E(X) = ?$$

$A_k = \{k\text{-ti doiskavalec dobi svojo joku}\}$

$$P(A_k) = \frac{1}{n}$$

Definirajmo $I_k = \begin{cases} 1 & ; A_k \text{ se zgodi} \\ 0 & ; \neg A_k \end{cases}$

$$X = \sum_{k=1}^n I_k$$

$$E(X) = \sum E(I_k) = \sum \frac{1}{n} = n \cdot \frac{1}{n} = 1$$

$$\parallel \sum + P(I_k = 1) \quad \begin{matrix} \text{od: } 1 \\ \downarrow \end{matrix}$$

2. naloga

• → • → •

$$P(X=k) = \frac{k}{(k+1)!}$$

X... št. herakla igre

$$P(X=k) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \dots \cdot \frac{1}{k} \cdot \frac{1}{k+1} = \frac{1}{(k+1)!}$$

$$\sum_{k=0}^{\infty} P(X=k)$$

$$E(X) = ?$$

$$I_k \dots \begin{cases} 1 & : \text{na } k \text{ sekundi} \\ 0 & ; \end{cases}$$

$$E(X) = \sum k \frac{k}{(k+1)!} = \sum \frac{k^2}{(k+1)!}$$

$$E(X) = E\left(\sum_{k=0}^{\infty} I_k\right) = \sum E(I_k) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} = e - 1$$

3. Aufgabe

X weisse Glühbirne spemeltjike

$$f_X = \begin{cases} \frac{1}{\sqrt{2\pi}} x^{\frac{3}{2}} e^{-\frac{1}{2x}} & \text{za } x > 0 \\ 0 & \text{sonst} \end{cases}$$

$Z \sim N(0, 1)$ paketeite

$$F_X(x) = P_Z\left(Z > \frac{1}{\sqrt{x}}\right) = 2\left(1 - F_Z\left(Z \leq \frac{1}{\sqrt{x}}\right)\right) \\ = 2\left(1 - \Phi\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$F_X(x) = \int_0^x \frac{1}{\sqrt{2\pi}} x^{\frac{3}{2}} e^{-\frac{1}{2x}} dx = 2 \int_{\frac{1}{\sqrt{x}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2u^2}} du =$$

$$\Phi(1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^1 e^{-\frac{1}{2}u^2} du$$

$$2 \int_{\frac{1}{\sqrt{x}}}^{\infty} \dots = 2\left(1 - \Phi\left(\frac{1}{\sqrt{x}}\right)\right)$$

$$u^2 = \frac{1}{x} \quad u^4 = \frac{1}{x^2}$$

$$2u du = -\frac{1}{x^2} dx \quad = dx - \frac{2}{u}$$

$$dx = -2 \frac{1}{u^3} du$$

$$X \sim U(a, b) \quad 0 < a < b < 1$$

$$Y = -\frac{1}{\lambda} \ln X \quad \lambda > 0$$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b) \\ 0 & ; \end{cases}$$

$$P(Y \leq y) = P\left(-\frac{1}{\lambda} \ln X\right) = P(-\lambda y \leq -\ln X) =$$

$$P\left(\underbrace{e^{-\lambda y}}_{\text{injection}} \leq X\right) = F_X(e^{-\lambda y}) = \int_{-\infty}^{e^{-\lambda y}} \underbrace{\frac{1}{b-a}}_a ; =$$

$$= \begin{cases} \frac{e^{-\lambda y} - a}{b-a} & ; e^{-\lambda y} < a \text{ not} \\ 1 & ; e^{-\lambda y} \geq b \end{cases}$$

$$\begin{cases} 1 - \frac{e^{-\lambda y} - a}{b-a} & e^{-\lambda y} \in (a, b) \\ 0 & \end{cases}$$

$$e^{-\lambda x} \in (a, b)$$

$$-\lambda x \in (\ln a, \ln b)$$

$$\lambda x \in (-\ln b, \ln a)$$

$$x \in \left(-\frac{\ln b}{\lambda}, \frac{\ln a}{\lambda}\right)$$

$$P(X=k, Y=l) = \begin{cases} \frac{1}{2 \ln 2} \frac{1}{l 2^k} & k \in \mathbb{N} \quad l \in \{1, \dots, k\} \\ 0 & \text{sonst} \end{cases}$$

$$P(X=k) = ?$$

$$P(Y=l) = ?$$

$$P(X=k) = \sum_l P(X=k, Y=l) = \sum_{l=1}^k \frac{1}{2 \ln 2} \frac{1}{l 2^k} =$$

$$\frac{1}{2^{k+1} \ln 2} \sum_{l=1}^k \frac{1}{l}$$

$$P(Y=l) = \sum_k P(X=k, Y=l) = \frac{1}{2 \ln 2} \cdot \sum_{k=l}^{\infty} \frac{1}{2^k} =$$

$$\frac{1}{2 \ln 2} \cdot \frac{1}{2^l} \frac{1 \cdot 2}{(1 - \frac{1}{2}) \cdot 2} = \frac{1}{\ln 2 2^l}$$

$$P(X=k)P(Y=l) = \frac{1}{2 \cdot (\ln 2)^2 2^l 2^k} \cdot \sum_{i=1}^k \frac{1}{i} \quad \# \log^2 \quad \text{w. nachher}$$

Vaja

• b ptickov

• $r < b$ označnik

Na izočetu leta izberejo n ptickov

X = število obratnih označnik z vračanjem
 Y = brez vračanja

① $\text{var}(X) = ?$

② $\text{var}(X) = \frac{b-n}{b-1} \text{var}(Y)$

Nasvet: Uporabi: $I_k = \text{dogodi se, da je } k\text{-ta ptica označena}$

③ kaj to pomeni: za metodologijo zbiranj - porabe

rešitev: $X \sim \text{Bin}(n, \frac{r}{b})$
 $Y \sim \text{HipGeo}(n, r, b)$

Iz predevanj: $E(X) = n \frac{r}{b}$ $E(Y) = n \frac{r}{b}$
 $E(X^2) = \frac{n^2 r^2}{b^2} + n \frac{(b-r)r}{b^2}$

$\text{var}(X) = E(X^2) - E(X)^2 = \underline{\underline{n \frac{(b-r)r}{b^2}}}$

$E(Y^2) = \sum k P(Y^2=k) \leadsto \sum k^2 P(Y=k)$

$E(Y) = \sum k P(Y=k) = \sum_k P(I_k=1) =$

~~I_1, \dots, I_n ptica je izbrana~~ $\{I_k\}_{k=1}^n$

$P(I_k \text{ je r. označ.}) = \frac{r}{b}$

$E(Y) = \sum_{i=1}^n P(I_i) = \frac{n r}{b}$

$E(Y^2) = \sum k^2 P(X=k) = \sum k (k P(X=k))$

remo: $\text{var}(Y) = \sum \text{var } I_k + \sum_{\substack{1 \leq i, j \leq n \\ i \neq j}} \text{cov}(I_i, I_j)$

$\text{cov}(X, Y) = E(XY) - E(X)E(Y)$

$\text{var}(Y) = \sum ($

Rešitev prejšnje naloge:

$$y = \sum_{k=1}^n I_k \quad \text{var}(y) = \sum_{k=1}^n \text{var} I_k + \sum_{\substack{1 \leq i, j \leq n \\ i \neq j}} \text{cov}(I_i, I_j) = \\ = \sum \text{var} I_k + 2 \sum_{1 \leq i < j \leq n} \text{cov}(I_i, I_j) =$$

$$\text{ker } I_k \sim B\left(\frac{r}{b}\right)$$

$$\hookrightarrow = n \text{var} I_1 + n(n-1) \text{cov}(I_1, I_j)$$

Od predavanj vemo $\overbrace{E(I_k) = E(I_k^2)} \rightarrow \text{vedno velja}$ $= \frac{r}{b}$

$$\text{var}(I_k) = \frac{r}{b} - \left(\frac{r}{b}\right)^2 = \frac{r}{b} \left(1 - \frac{r}{b}\right)$$

$$\text{cov}(I_i, I_j) = E(I_i I_j) - E(I_i) E(I_j) =$$

$$= P(I_i=1, I_j=1) - \left(\frac{r}{b}\right)^2 =$$

$$P(I_1=1, I_j=1) = P(I_i=1 | I_j=1) P(I_j=1) = \frac{r-1}{b-1} \frac{r}{b}$$

$$\text{cov}(I_i, I_j) = \frac{r}{b} \left(\frac{r-1}{b-1} - \frac{r}{b} \right)$$

$$\text{var}(y) = \frac{nr}{b} \left(1 - \frac{r}{b}\right) + n(n-1) \frac{r}{b} \left(\frac{r-1}{b-1} - \frac{r}{b} \right)$$

Vaja:

$$\text{Maj bado } M = IA + (1-I)B$$

$A, B \dots$ spremenljive slučajne

$I \sim \text{Ber}(p)$ indikator

Predpostavimo da je I neodvisno od A in od B

$$\text{Dokazi: } E(M) = pE(A) + (1-p)E(B)$$

Rešitev:

$$E(M) = E(IA + (1-I)B) = E(IA) + E((1-I)B) =$$

$$= E(I)E(A) + E(1-I)E(B) = P(I=1)E(A) + P(I=0)E(B) =$$
$$= pE(A) + (1-p)E(B)$$

n dni:

$$t=1, \dots, n$$

$X_t, \dots, \mathcal{I}^t$ ktkov/minuta

$$X_t = I_t Y_t + (1-I_t) Z_t$$

$T = U[1, \dots, n]$ je dan spremenik iz prve v drugo fazo

$$I_t = \begin{cases} 1 & t < T \\ 0 & t \geq T \end{cases} \quad Y_t \sim \text{Pois}(\lambda_1) \quad Z_t \sim \text{Pois}(\lambda_2)$$

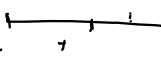
I_t je neodvisna od Y_t in od $Z_t \quad \forall t$

① kaj je P ? $I_t = \text{Ber}(p)$

② Najdite izraz za $E(X_t)$

③ kaj je $E(X_t^2)$ $E(Y_1^2) = \lambda_1^2 + \lambda_1$

Rešitev:

$$① P_{\text{pr}}(I_t=1) = P(t < T) = P(T > t) = \frac{n-t}{n}$$


$$\begin{aligned} ② E(X_t) &= E(I_t Y_t + (1-I_t) Z_t) = \\ &= E(I_t) E(Y_t) + E(1-I_t) E(Z_t) \\ &= \frac{n-t}{n} \lambda_1 + \frac{t}{n} \lambda_2 \end{aligned} \quad \text{var}(X_t) = \lambda_1 = \text{var}(Y_t)$$

$$\begin{aligned} ③ E(X_t^2) &= E(I_t^2 Y_t^2 + 2I_t(1-I_t) Y_t Z_t + (1-I_t)^2 Z_t^2) = \\ &= \frac{n-t}{n} E(Y_t^2) + \frac{t}{n} E(Z_t^2) = \\ &= (\lambda_1^2 + \lambda_1) + \frac{1}{n} (\lambda_2^2 + \lambda_2 - \lambda_1^2 - \lambda_1) = \\ &= \lambda_1^2 + \lambda_1 + \frac{1}{n} (\lambda_2 - \lambda_1) (\lambda_2 + \lambda_1 + 1) \end{aligned}$$