I Mehansko nihanje in valovanje

II električno polje

III električni tok

IV magnetno polje

V elektrodinamika

VI posebna teorija relativnosti

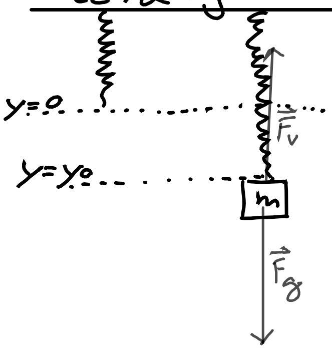
VII zaključek

I Mehansko nihanje in valovanje

Enostavna nihala

Enačba dušenega nihanja

<u>Utež na vijačni vzmeti</u>



$$F_g = \begin{bmatrix} 0 \\ -m g_0 \\ 0 \end{bmatrix}$$ smer navzdol

$g_0 \approx 10 \ m/s^2$

$$F_v = \begin{bmatrix} 0 \\ -k y_0 \\ 0 \end{bmatrix}$$ k... koeficient vzmeti N/m

$y_0 < 0$

$-k y_0 > 0 \Rightarrow$ smer je navzgor

neto sila

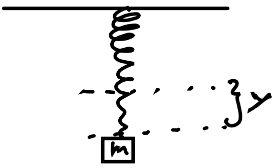
$$\vec{a} = 0 \Leftrightarrow \vec{F} = m\vec{a} = 0$$

$$\vec{F}_g + \vec{F}_v = 0$$

$$-m g_0 - k y_0 = 0 \Rightarrow$$

$$m g_0 = -k y_0$$

$$y_0 = \frac{m g_0}{k}$$



$\vec{F}_g = -m g_0 \hat{e}_y$

$\vec{F}_v = -k y \hat{e}_y$

$\vec{F}_u$ .... sila upora

$\vec{F}_u = -\vec{c}\,\vec{v}$ (linearna sila upora) $\qquad v_y = \dot{y} = \frac{dy}{dt} \neq 0$

$c > 0$ sorazmerna z viskoznostjo tekočine in površino uteži

$\vec{F}_u = -c\dot{y}\hat{e}_y$

$\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$

$\vec{F} = m \cdot \vec{a} \quad ; \ \vec{a} = \ddot{y}\hat{e}_y$

$-c\dot{y}\hat{e}_y - k y \hat{e}_y - m g_0 \hat{e}_y = m \ddot{y} \hat{e}_y$

$\Big(\underbrace{\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m} y + g_0}_{=0}\Big)\hat{e}_y = 0$

Vpeljemo $\beta := \frac{c}{m} \qquad [\beta] = s^{-1}$

$\omega_0^2 = \frac{k}{m} \qquad [\omega_0^2] = s^{-2}$

$\ddot{y} + \beta\dot{y} + \omega_0^2 y + g_0 = 0$

$y_0 = \frac{m g_0}{k}$

$g_0 = \frac{y_0 k}{m} = y_0 \omega^2$

$\ddot{y} + \beta\dot{y} + \omega_0^2 (y - y_0) = 0$

$\dot{y} + \beta y + \omega_0^2 y = \omega_0^2 y_0$

- diferencialna enačba za y 2. reda
- linearna enačba
- konstantni koeficienti
- nehomogena (pogojno, ker je lahko z lahkoto prevedemo v homogeno)

$y' := y - y_0$

$\dot{y}' = (y - y_0)^{\bullet} = \dot{y}$

$\ddot{y}' = \ddot{y}$

Dobimo: $\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0$

je homogena :)

$$\ddot{y}' + \beta\dot{y}' + \omega_0^2 y = 0$$

Nastavek: $y' = Ae^{\lambda t}$, A, λ konstanti

$[\lambda] = s^{-1}$

$[A] = m$

$\dot{y}' = \lambda y$

$\ddot{y}' = \lambda^2 y$

$$(\lambda^2 + \beta\lambda + \omega_0^2)\underbrace{Ae^{\lambda t}}_{\neq 0 \text{ ko } A\neq 0} = 0 \quad \text{za } \forall t$$

$$\lambda^2 + \beta\lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 =: -4\omega^2$$

$$(\omega^2 := \omega_0^2 - (\tfrac{\beta}{2})^2)$$

$D < 0 \Rightarrow 4\omega^2 > 0$: podkritično dušenje

$$\sqrt{D} = \sqrt{-4\omega^2} = \sqrt{-1}\, 2\omega \quad ; \omega = +\sqrt{\omega^2}$$

$$\Rightarrow \lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = \frac{-\beta}{2} \pm \omega i$$

$$y_1' = A_1 e^{\lambda_1 t} = A_1 \exp((-\tfrac{\beta}{2} + i\omega)t) =$$

$$= A_1 \exp(-\tfrac{\beta}{2}t)\exp(i\omega t)$$

$$y_2' = A_2 \exp\{-\tfrac{\beta}{2}t\}\exp(-i\omega t)$$

$$\left.\begin{aligned} \ddot{y}_1' + \beta\dot{y}_1' + \omega_0^2 y_1' &= 0 \\ \ddot{y}_2' + \beta\dot{y}_2' + \omega_0^2 y' &= 0 \end{aligned}\right\} +$$

$$(\ddot{y}_1' + \ddot{y}_2') + \beta(\dot{y}_1' + \dot{y}_2') + \omega_0(y_1' - y') = 0$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}(A_1\exp(i\omega t) + A_2\exp\{-i\omega t\})$$

Eulerjeva enačba

$$\exp\{\pm i\omega t\} = \cos\{\omega t\} \pm i\sin(\omega t)$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}((A_1 + A_2)\cos(\omega t) + i(A_1 - A_2)\sin\omega t$$

$$= e^{-\frac{\beta}{2}t}(B_1\cos(\omega t) + B_2\sin(\omega t))$$

$$= B e^{-\frac{\beta}{2}t}\sin(\omega t + \delta)$$

Fazni zamik

$\sin(\omega t + \delta) =$
$\sin(\omega t)\cos\delta + \sin\delta\cos\omega t$

$B > 0$; δ = fazni zamik

$$B e^{-\frac{\beta}{2}t}(\sin\omega t \sin\delta + \cos\omega t \sin\delta)$$

$$= e^{-\frac{\beta}{2}t}(B\sin\delta\cos(\omega t) + B\cos\omega t\sin\delta)$$

$$B_1 = B\sin\delta$$

$$B_2 = B\cos\delta$$

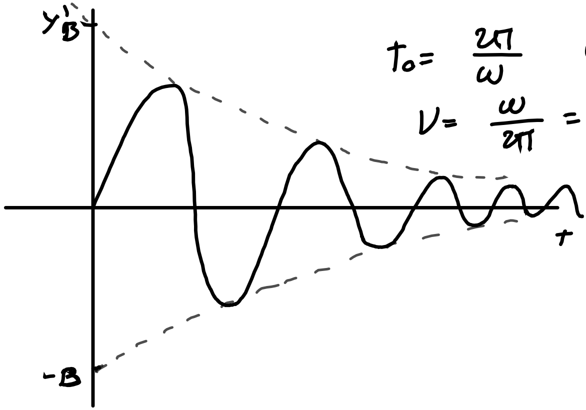
$$\tan\delta = \frac{B_1}{B_2}$$

$$B^2 = B_1^2 + B_2^2$$

$$B = \sqrt{B_1^2 + B_2^2}$$

Primer:

$\delta = 0 \Rightarrow y' = B\, e^{-\frac{\beta}{2}t} \sin(\omega t)$



$t_0 = \dfrac{2\pi}{\omega}$ $\quad \omega = 2\pi\nu$

$\nu = \dfrac{\omega}{2\pi} = \dfrac{1}{t_0}$

$\delta = \frac{\pi}{2}$

$$y'(t) = Be^{-\frac{\beta}{2}t} \sin\left(\omega t + \frac{\pi}{2}\right) = \sin \omega t \cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2}$$

$$= Be^{-\frac{\beta}{2}t} \cos(\omega t)$$

$\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0$

$y' = y - y_0$ → odmik obremenjene vzmeti v ravnovesni legi

↳ odmik od konca neobremenjene vzmeti

$\omega_0^2 = \frac{k}{m}$ (>0)

$\beta \propto C \propto \eta$

↖ sorazmerno

Nastavek $y' = Ae^{\lambda t}$

$\Longrightarrow \lambda^2 + \beta\lambda + \omega_0^2 = 0$

$D = \beta^2 - 4\omega_0^2 = -4\omega^2 \qquad \omega^2 = \omega_0^2 - \left(\frac{\beta}{2}\right)^2$

a) $D < 0 \Rightarrow (\omega^2 > 0)$

$y' = Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta)$; $\omega = \sqrt{\omega^2} =$

$= \sqrt{\omega_0^2 - \left(\frac{\beta}{2}\right)^2}$

Zelo šibko dušenje $\left(\frac{\beta}{2}\right)^2 << \omega_0 \Rightarrow$

$$t_0 = \frac{2\pi}{\sqrt{\omega_0^2}} = 2\pi\sqrt{\frac{m}{k}}$$

Primer:

$m = 500\,g = 0{,}5\,kg$

$k y_0 = -m g_0$

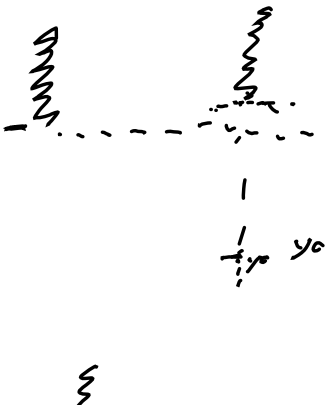
$$k = -\frac{m g_0}{y_0} \Rightarrow \frac{m}{k} = -\frac{y_0}{g_0}$$

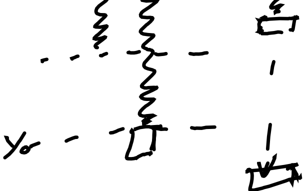
$y_0 = -0{,}4\ m$

$$\frac{m}{k} = \frac{0{,}4\,m}{10\,m/s^2} = 0{,}04\,s^2 =$$

$$\sqrt{\frac{m}{k}} = 2\cdot 10^{-1} = 0{,}2\,s$$

$$\Rightarrow t_0 = 2\pi \cdot 0{,}2s \cong 1{,}2s$$





B in δ dobimo iz začetnih pogojev

$$y' = Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta)$$

$$\dot{y}' = -\frac{\beta}{2}Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta) + \omega Be^{-\frac{\beta}{2}t}\cos(\omega t + \delta)$$

$$\dot{y}'(0) = -\frac{\beta}{2}B\sin\delta + \omega B\cos\delta =$$

$$= B\left(-\frac{\beta}{2}\sin\delta + \omega\cos\delta\right)$$

$$y'(0) = B\sin\delta$$

$$r = \frac{y'(0)}{\dot{y}'(0)} = \frac{\sin\delta}{\omega\cos\delta - \frac{\beta}{2}\sin\delta} = \frac{\tan\delta}{\omega - \frac{\beta}{2}\tan\delta}$$

$$\Rightarrow \delta = \arctan\left(\frac{r\omega}{1 + \frac{r\beta}{2}}\right)$$

$$\Rightarrow B = \frac{y'(0)}{\sin\delta}$$

$$\dot{y}'(0) = B\omega\sin\delta$$

$$\Rightarrow B = \frac{\dot{y}'(0)}{\pm\omega} = \frac{|\dot{y}'(0)|}{\omega}$$

Energija nihala

$$W_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{y}^2$$

$$W_{pr} = \frac{1}{2} k y^2 = \frac{1}{2} k (y' + y_0)^2$$

$$W_p = m g_0 (y' + y_0)$$

Skupna $\boxed{W = W_k + W_p + W_{pr}}$

a) W za zelo šibko dušenje?

$$\frac{\beta}{2} t \ll 1$$

$$\left(\frac{\beta}{2}\right)^2 \ll \omega_0^2 \Rightarrow \omega \simeq \omega_0 = \sqrt{\frac{k}{m}}$$

$$y' = B \sin(\omega_0 t + \delta)$$

$$\dot{y}' = \omega_0 B \cos(\omega_0 t + \delta)$$

$$W_k = \frac{1}{2} m \omega_0^2 B^2 \cos^2(\omega_0 t + \delta) =$$

$$= \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} B^2 \cos^2(\omega_0 t + \delta)$$

$$W_{pr} = \frac{1}{2} k B^2 \sin^2(\omega_0 t + \delta) + \frac{1}{2} k y_0^2 + k y_0 B \sin(\omega_0 t + \delta)$$

$$W_p = m g_0 B \sin(\omega_0 t + \delta) + m g_0 y_0$$

$$W = \frac{1}{2} k B^2 \underbrace{\left(\sin^2(\omega_0 t + \delta) + \cos^2(\omega_0 t + \delta)\right)}_{1}$$

$$+ (k y_0 + m g_0) B \sin(\omega t + \delta) +$$

$$+ \frac{1}{2} k y_0^2 + m g_0 y_0$$

$$= \frac{1}{2} k B^2 + \frac{1}{2} k y_0^2 + m g_0 y_0 = \text{konst.}$$

b) kritično dušenje

$D=0 \quad (\omega=0)$ (kritično dušenje

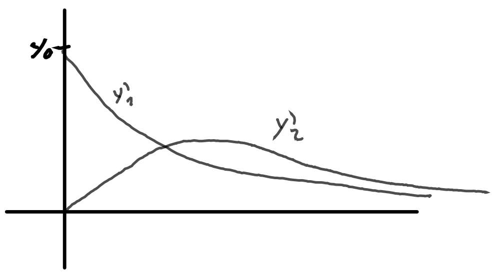
$D=\beta^2-4\omega_0^2 \Rightarrow \omega_0=\frac{\beta}{2}$

$\lambda_{1,2}=\frac{-\beta\pm\sqrt{D}}{2}=-\frac{\beta}{2}$

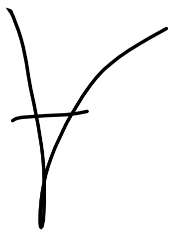
$\Rightarrow y_1'=B_1 e^{-\frac{\beta}{2}t}$

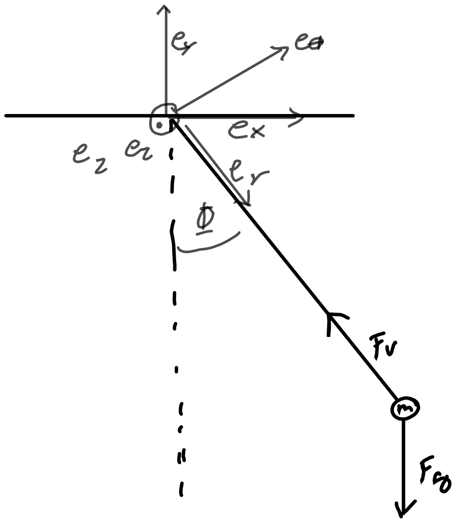
$y_2'=B_2 t e^{-\frac{\beta}{2}t}$ je tudi rešitev (DN)

$\Longrightarrow y'=y_1'+y_2'=(B_1+Bt)e^{-\frac{\beta}{2}t}$



20.2





e_ϕ, e_z, e_r ... cilindrične koordinate

$$\vec{F} = \vec{F_g} + \vec{F_v} = m\vec{a}$$

$$F_g = -mg_0 \hat{e_y}$$

$$F_v = -F_v \hat{e}_r$$

$$\vec{r} = l \cdot \vec{e_r}$$

$$\vec{r} \times \vec{F} = l\vec{e}_r \times (-mg_0 \hat{e_y} - F_v \hat{e}_r) =$$

$$= -mg_0 l \, \hat{e}_r \times \hat{e_y} - lF_v \underbrace{\hat{e_r} \times \vec{e_r}}_{=0}$$

$$= -mg_0 l \hat{e}_r \times (\hat{e_r} \cos\phi + \hat{e}_\phi \sin\phi)$$

$$= -mg_0 l \sin\Phi \, \hat{e}_r \times \hat{e}_\phi =$$

$$= -mg_0 l \sin\Phi \, \hat{e}_z$$

$$\vec{r} \times \vec{F} = m \cdot \vec{r} \times \vec{a} = m \cdot l\hat{e}_r \times (a \hat{e}_\phi) =$$

$$\vec{a} = a \cdot e_\Phi \qquad = mal \, \hat{e}_z$$

$$a = a_t = l\ddot{\Phi}$$

$$mg_0 l \sin\phi \, e_z = ml\ddot{\Phi} e_z$$

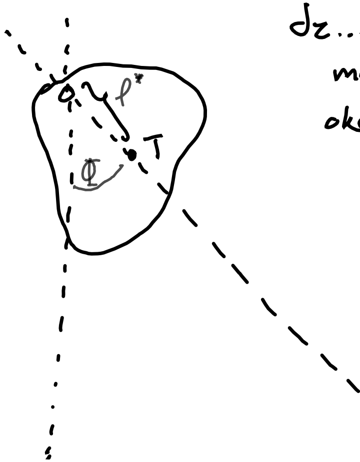
$$ml(l\ddot{\Phi} + g_0 \sin\Phi) \, e_z = 0 \quad (e_z \neq \vec{0})$$

$$-l\ddot{\Phi} = g_0 \sin\Phi$$

$$\ddot{\Phi} + \frac{g_0}{l} \sin\Phi = 0$$

$$\approx \ddot{\Phi} + \underbrace{\frac{g_0}{l}}_{\omega_0^2} \Phi = 0$$

$$\ddot{\Phi} + \omega_0^2 \Phi = 0$$



J_z .... vstrajnostni moment za vrtenje okoli fiksne osi

primer: palica

$$J_z = \frac{1}{3} m l^2 \qquad l^* = \frac{l}{2}$$

$$\sin\Phi \approx \Phi$$

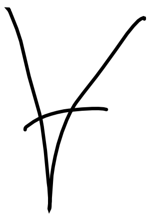
$$\Rightarrow \ddot{\Phi} + \omega_0^2 \Phi = 0$$

$$\omega_0^2 = \frac{m g l^*}{J_z} = \frac{3 m g l}{2 \cdot m l^2} = \frac{3}{2}\frac{g_0}{l}$$

Nitno nihalo:

$$J_z = m l^2 \qquad l^* = l$$

$$\omega_0^2 = \frac{m g_0 l}{m l^2} = \frac{g_0}{l}$$



21.2

Nism σe shkele

$F = F_0 \sin(\omega_v t) \qquad \omega_v = 2\pi \nu_v$

$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = \frac{F_0}{m} \sin(\omega_v t)$

$\omega_0^2 = \frac{k}{m} \qquad \beta = \frac{C}{m} \qquad y' = y - y_0$ raztezek ko je obtežena vzmet mirna

$y' = y'_h + y'_p$ ← partikularna rešitev

homogena

$y'_p = B \sin(\omega t - \delta_p)$

$B_p \{ (\omega_0^2 - \omega_v^2) [\cos\delta_p \sin(\omega_v t) - \sin\delta_p \sin(\omega_v t)]$

$+ \omega_v \beta [\cos\delta_p \cos(\omega_v t) + \sin\delta_p \sin(\omega_v t)] \}$

$= \frac{F_0}{m} \sin(\omega_v t) \qquad$ za $\forall t$

a) $t_1 = 0 \Rightarrow \sin\omega_v t = 0, \ \cos\omega_v t = 1$

$B_p \{ -(\omega_0^2 - \omega_v^2) \sin\delta_p + \omega_v \beta \cos\delta_p \} = 0$

$\Rightarrow \tan\delta_p = \frac{\omega_v \beta}{\omega_0^2 - \omega_v^2}$

$\cos\delta_p = \pm \frac{(\omega_0^2 - \omega_v^2)}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$

$\sin\delta_p = \pm \frac{\omega_v \beta}{\sqrt{\quad}}$

$\omega_v \to 0 \Rightarrow \tan\delta_p \to +0 \Rightarrow \delta_p \to 0$

$\omega_v \to \uparrow \omega_0 \Rightarrow \tan\delta_p \to +\infty \Rightarrow \delta_p \to \uparrow \frac{\pi}{2}$

$\omega_v \to \downarrow \omega_0 \Rightarrow \tan\delta_p \to -\infty \Rightarrow \delta_p \to \downarrow \frac{\pi}{2}$

$\omega_v \to \infty \Rightarrow \tan\delta_p \to -0 \Rightarrow \delta_p \to \pi$

$\omega_v \to 0 \Rightarrow \delta_p \to 0 \Rightarrow \cos\delta_p \to +1$

$\cos\delta_p \to \frac{\pm \omega_0^2}{\sqrt{\omega_0^4}} = \pm 1$ očitno mora biti +

b) $t_2 = \frac{\pi}{2\omega_v} \Rightarrow \omega_v t_2 = \frac{\pi}{2} \Rightarrow \sin(\omega_v t) = 1$

$\cos(\omega_v t) = 0$

$B_p \{ (\omega_0^2 - \omega_v^2) \cos\delta_p + \omega_v \beta \sin\delta_p \} = \frac{F_0}{m}$

$B_p \left\{ \frac{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta}} \right\} = \frac{F_0}{m}$

$\Rightarrow B_p = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$

$\omega_v \to 0 \Rightarrow B_p \to \frac{F_0}{m} \cdot \frac{1}{\omega_0^2} = \frac{F_0 \cdot m}{m \cdot k} = \frac{F_0}{k}$

$\omega_v \to \infty \Rightarrow B_p \to 0$

$B_p(\omega_v) = \max$

Kdaj doseže maksimum?
Torej kdaj je v resonanci?

Ko je imenovalec najmanjši

$(\omega_0^2 - \omega_v^2)^2 + \beta^2\omega_v^2$ je najmanjši

$$\frac{d}{d\omega_v}\left((\omega_0^2-\omega_v^2)^2 + \beta^2\omega_v^2\right) = 0$$

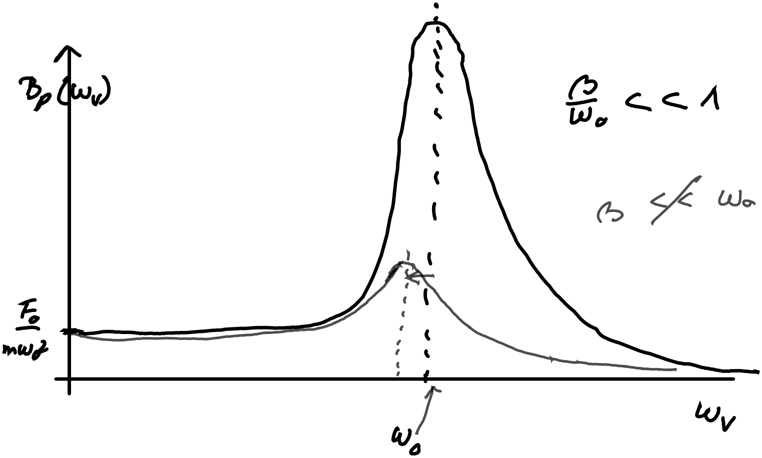
$$2(\omega_0^2 - \omega_v^2)(-2\omega_v) + 2\omega_v\beta^2 = 0$$

$$-2(\omega_0^2 - \omega_v) + \beta^2 = 0$$

$$\omega_v^2 = \omega_0^2 - \frac{\beta^2}{2}$$

$$\omega_v = +\sqrt{\omega_0^2 - \frac{\beta^2}{2}} = \omega_0\sqrt{1 - \frac{\beta^2}{2\omega_0^2}}$$

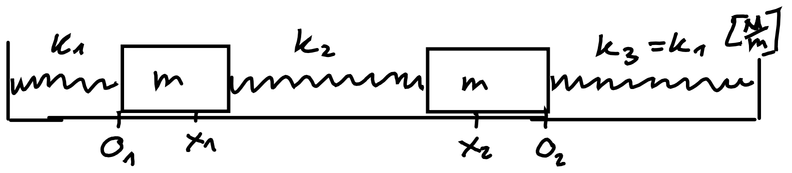
$$\beta << \omega_0 \Rightarrow \omega_v = \omega_0$$



$B_p(\omega_V)$
$\frac{\beta}{\omega_0} << 1$
$\beta \not\ll \omega_0$
$\frac{F_0}{m\omega_0^2}$
ω_0
ω_V

Sklopljeno nihanje

- $\beta \longrightarrow 0$ dušenje pošljemo proti 0
- za začetek: simetrični primer



z drugo vzmetjo, ki ji bomo rekli tretja vzmet

$\vec{F}_{1\to 1,r}$ ... sila prve vzmeti na prvi voziček ko je v ravnovesni legi

$$\left.\begin{aligned}\vec{F}_{1\to 1,r} &= -k_1 \Delta l_{1,r}\hat{e}_x \\ \vec{F}_{2\to 1,r} &= +k_2 \Delta l_{2,r}\hat{e}_x\end{aligned}\right\} + = 0$$

v ravnovesju

$$\left.\begin{aligned}F_{2\to 2,r} &= -k_2 \Delta l_{2,r}\hat{e}_x \\ F_{3\to 2,r} &= +\underset{k_3}{k_1} \Delta l_{3,r}\hat{e}_x\end{aligned}\right\} + = 0$$

$$\vec{F}_{1\to 1} = \vec{F}_{1\to 1,r} - k_1 x_1 \hat{e}_x$$

$$\vec{F}_{2\to 1} = \vec{F}_{2\to 1,r} - k_2(x_1 - x_2)\hat{e}_x$$

$$\vec{F}_{2\to 2} = \vec{F}_{2\to 2,r} + k_2(x_1 - x_2)\hat{e}_x$$

$$\vec{F}_{3\to 2} = \vec{F}_{3\to 2,r} - k_1 x_2 \hat{e}_x$$

$$\vec{F}_{1\to 1} + \vec{F}_{2\to 1} = m\ddot{x}_1\hat{e}_x$$

$$\cancel{F_{1\to 1,r}} - k_1 x_1 \hat{e}_x + \cancel{F_{2\to 1,r}} - k_2(x_1 - x_2)\hat{e}_x = $$
$$= m\ddot{x}_1\hat{e}_x$$

$$m\ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0$$

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0$$

$$\omega_1^2 = \frac{k_1}{m} \qquad \omega_2^2 = \frac{k_2}{m}$$

Zelo podobno $\ddot{x}_2 + \omega_1^2 x_2 - \omega_2^2(x_1 - x_2) =$

$$\left.\begin{aligned}x_a &= x_1 + x_2 \\ x_b &= x_1 - x_2\end{aligned}\right\} \Rightarrow x_1 = \frac{x_a + x_b}{2}$$

$$x_2 = \frac{x_a - x_b}{2}$$

$\ddot{x}_2 + \omega_1^2 x_2 - \omega_2(x_1 - x_2)$
$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2(x_1 - x_2)$
$(\ddot{x_1 + x_2}) + \omega_1^2(x_1 + x_2)$
$(\ddot{x}_2 - \ddot{x}_1) + (x_1 - x_2)($
$(\omega_1^2 + 2\omega_2)$

seštejemo enačbi:

$$\ddot{x}_1 + \ddot{x}_2 + \omega_1^2 x_1 + \omega_1^2 x_2 = 0$$

$$(\ddot{x_1 + x_2}) + \omega_1^2 (x_1 + x_2) = 0$$

$$\ddot{x}_a + \omega_a^2 x_a = 0 \qquad \omega_1 = \omega_a$$

$$\ddot{x}_b + \omega_b^2 x_b = 0 \qquad \omega_b = \sqrt{\omega_1^2 + 2\omega_2^2}$$

$$x_a = B_a \sin(\omega_a t + \delta_a)$$

$$x_b = B_B \sin(\omega_b t + \delta_b)$$

$$\Rightarrow x_1 = \underbrace{\frac{B_a}{2}}_{B_1}\sin(\omega_a t + \delta_a) + \underbrace{\frac{B_b}{2}}_{B_2}\sin(\omega_b t + \delta_b)$$

$$x_2 = B_1 \sin(\omega_a t + \delta_a) - B_2 \sin(\omega_b t + \delta_b)$$

$$B_1, B_2, \delta_a, \delta_b = ?$$

začetni pogoji: $x_1(t=0)$ $\dot{x}_1(t=0)$
$x_2(t=0)$ $\dot{x}_2(t=0)$

iz tega izvemo

Primer

$x_1(t=0)=x_0 \quad (>0) \qquad \dot{x}_1(t=0)=0$

$x_2(t=0)=0 \qquad \dot{x}_2(t=0)=0$

$$\dot{x}_1 = B_1\omega_a\cos(\omega_a t+\delta_a)+B_2\omega_b\cos(\omega_b t+\delta_b)$$

$$x_2 = B_1\omega_a\cos(\omega_a t+\delta_a)-B_2\omega_b\cos(\omega_b t+\delta_b)$$

$$\left.\begin{aligned} X_0 &= B_1\sin\delta_a + B_2\sin(\delta_b) \\ 0 &= B_1\sin\delta_a - B_2\sin\delta_b \end{aligned}\right\} +,\ -$$

$$\left.\begin{aligned} 0 &= B_1\omega_a\cos(\delta_a)+B_2\omega_b\cos(\delta_b) \\ 0 &= B_1\omega_a\cos(\delta_a)-B_2\omega_b\cos(\delta_b) \end{aligned}\right\} +,\ -$$

$$X_0 = 2B_1\sin\delta_a = 2B_2\sin\delta_B$$

$$2B_1\omega_a\cos\delta_a = 0 \Rightarrow \cos\delta_a = 0$$

$$\Rightarrow \quad B_1 = B_2 = \frac{x_0}{2} \qquad \Rightarrow \begin{array}{l} \delta_a = \pm\frac{\pi}{2} \\ \delta_b = +\frac{\pi}{2} \end{array}$$

V 7.3

mentalna
odsotna

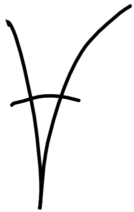
Lastna nihanja sestavljenega nihala

$$\left.\begin{aligned} x_1 &= B\sin(\omega_a t + \delta_a) + B_2 \sin(\omega_b t + \delta_b) \\ x_2 &= \ldots \end{aligned}\right\}$$

$$\begin{cases} x_1 = B_1 \sin(\omega_c t + \delta_1) \\ x_2 = B_2 \sin(\omega_c t + \delta_2) \end{cases}$$

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0$$

$$\ddot{x}_2 + \omega_2^2 x_2 - \omega_2^2 (x_1 - x_2) = 0$$



11.3

$u(x,t) = ?$

$$u(x,t) = f(x-ct) + g(x+ct)$$

$\eta := x - ct$

$x := x + ct$

ena izmed možnosti:

$$u(x,t) = f(x-ct)$$

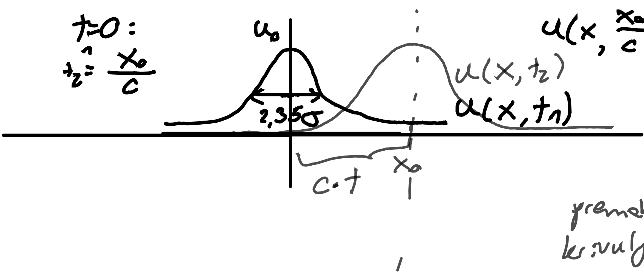
npr: $u(x,t) = u_0 e^{-\frac{(x-ct)^2}{2\sigma^2}}$

$$u(x,0) = U_0 e^{-\frac{x^2}{2\sigma^2}}$$

$$u\left(x, \frac{x_0}{c}\right) = U_0 e^{-\frac{(x-x_0)^2}{2\sigma}}$$

$t_1 = 0$:

$t_2 = \frac{x_0}{c}$



sama premaknjena krivulja

$$\Rightarrow v = \left|\frac{du}{dt}\right| = \left|\frac{\partial u}{\partial t}\right| = \left|\frac{\partial}{\partial t} f(x-ct)\right| = |f'| c$$

$v \neq c$

$$u(x,t) = f(x-ct)$$

a) $\underline{u(x,t) = u(x-ct, 0)}$

$x \longrightarrow x' = x - ct$

$t \rightarrow t' = 0$

$u(x',t') = u(x-ct, 0) = f(x', ct') = f(x-ct) =$

$= u(x,t)$

Če poznamo $u(x',t')$ ob času $t'=0$ za $\forall x'$

Potem poznam $u(x,t)$ za $\forall x$ po $\forall$ času t

b) $u(x,t) = u(0, t - \frac{x}{c})$

$x \rightarrow x' = 0$

$t \rightarrow t' = t - \frac{x}{c}$

$u(x',t') = u(0, t - \frac{x}{c}) =$

$f(0 - c(t - \frac{x}{c})) = f(x-ct) = u(x,t)$

Če poznamo $u(x'=0, t')$ za $\forall t'$

$\Rightarrow$ poznam $u(x,t)$ za $\forall x, \forall t$

Potujoče sinusno valovanje

$u(x,t) = f(x-ct)$

$u(x=0, t-\frac{x}{c}) = u_0 \sin(-\omega t + \delta)$; $\quad u_0 > 0$

$u(x,t) = ?$

$u(x,t) = u(0, t-\frac{x}{c}) = u_0 \sin(-\omega(t-\frac{x}{c}) + \delta) =$

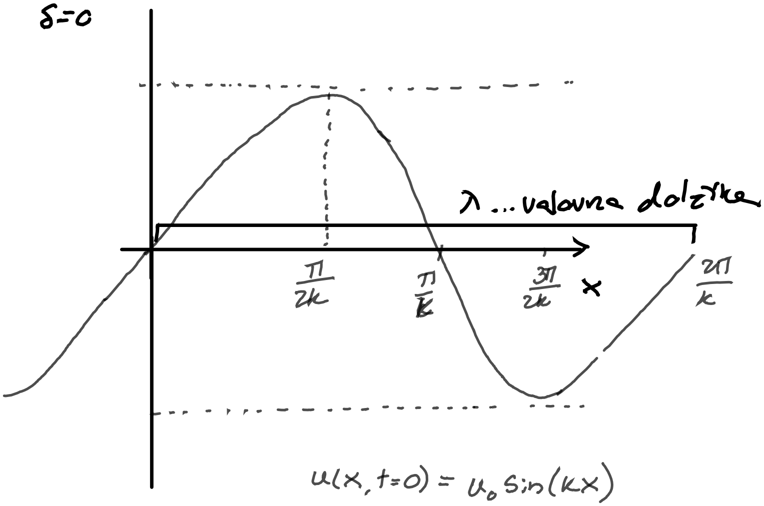
$= u_0 \sin(kx - \omega t + \delta)$; $\quad k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$

$[k] = \frac{1}{m}$

$t=0$:

$u(x, t=0)$

$\delta = 0$



$u(x, t=0) = u_0 \sin(kx)$

$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{2\pi \nu} = \frac{c}{\nu}$

$c = \lambda \nu$

$u(x,t) = u_{0,1} \sin(kx - \omega t + \delta_1) + u_{02} \sin(kx + \omega t + \delta_2)$

Valovanje v dve smeri:

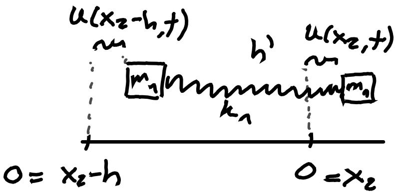
Robni pogoji in stoječe sinusno valovanje

a) togo vpeta vzmet

$$u(x_1,t) = u(x_2,t) = 0$$

b) Vzmet (palica) s prostima koncema, x_1, x_2

Model:



$$h' - h = u(x_2) - u(x_2 - h)$$

$$F_{x_2-h \to x_2} = -k_1 \Delta h$$

$$= -\frac{k\ell}{h}\left(u(x_2,t) - u(x_2-h,t)\right)$$

$$m_1 \ddot{u}(x_2,t) = -k\ell\left(\frac{u(x_2,t) - u(x_2-h,t)}{h}\right)$$

$$m_1 = \frac{m}{\ell h} \leftarrow \text{št vseh uteži}$$

$$\frac{m}{\ell h}\frac{\partial^2 u(x_2,t)}{\partial t^2} = -k\ell\left(\frac{u((x_2'+h),t) - u(x^2,t)}{h}\right)$$

$$x_2 = x_2' + h$$

$$n \to \infty \Leftrightarrow h \to 0 \quad \Rightarrow \quad h\ddot{u} \to 0$$

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x^2-h} \overset{h\to 0}{=} \left.\frac{\partial u(x,t)}{\partial x}\right|_{x^2}$$

$$\left.\frac{du(x,t)}{\partial x}\right|_{x=x_2} = \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=1} = 0$$

$$\frac{\partial u}{\partial x}\Big|_{x=0} = \frac{\partial u}{\partial x}\Big|_{x=L} = 0 \quad \text{za } \forall t$$

$\uparrow$ dolžina palice

$$u(x,t) = u_1(x,t) + u_2(x,t)$$

$$\left.\begin{aligned} u_1 &= u_0 \sin(kx - \omega t + \delta_1) \\ u_2 &= u_0 \sin(kx + \omega t + \delta_2) \end{aligned}\right\} +$$

$$= 2u_0 \sin\left(kx + \frac{\delta_1+\delta_2}{2}\right) \cos\left(-\omega t + \frac{\partial_1 + \partial_2}{2}\right)$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} = k u_0 \cos(kx - \omega t + \delta_1) + k u_0 \cos(kx + \omega t + \delta_2)$$

$$= 2k u_0 \cos\left(kx + \frac{\delta_1+\delta_2}{2}\right) \cos\left(-\omega t + \frac{\delta_1 - \delta_2}{2}\right)$$

$$\frac{\partial u}{\partial x}\Big|_{x=0} = 2k u_0 \cos\left(\frac{\delta_1+\delta_2}{2}\right) \cos\left(-\omega t + \frac{\delta_1-\delta_2}{2}\right) = 0 \quad \text{za } \forall t$$

$$\Rightarrow \frac{\delta_1+\delta_2}{2} = \frac{\pi}{2}$$

$$\cos\left(kx + \frac{\pi}{2}\right) = -\sin(kx)$$

$$\sin\left(kx + \frac{\pi}{2}\right) = \cos(kx)$$

$$u(x,t) = 2u_0 \cos(kx) \cos\left(-\omega t + \frac{\delta_1-\delta_2}{2}\right)$$

$$\frac{du}{dx} = -2k u_0 \sin(kx) \cos\left(-\omega t + \frac{\delta_1-\delta_2}{2}\right)$$

$$\frac{du}{dx}\Big|_{x=L} = 0 \Rightarrow$$

$$-2k u_0 \sin(kL) \cos\left(-\omega t + \frac{\delta_1-\delta_2}{2}\right) = 0 \quad \text{za } \forall t$$

$$kL = 0 + \pi n, \quad n \in \mathbb{Z}$$

$$k = \frac{n\pi}{L} = \frac{\omega}{c} = \frac{2\pi \nu_n}{c}$$

$$\nu_n = \frac{nc}{2L} \qquad n \in \mathbb{N}$$

$$u(x,t) = 2u_0 \cos\left(\frac{n\pi}{L} x\right) \cos\left(-2\pi \nu_n t + \frac{\delta_1-\delta_2}{2}\right)$$

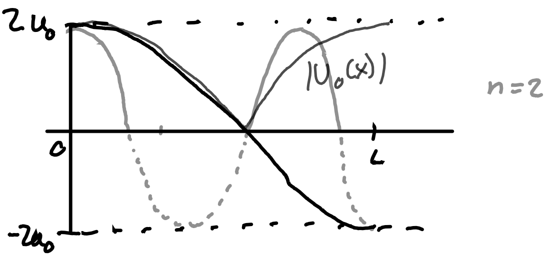
Dobimo stoječe valovanje. nima več bner:

$$u(x,t) = 2u_0 \cos\left(\frac{n\pi}{L}x\right)\cos\left(-2\pi\nu_n t + \frac{\delta_1-\delta_2}{2}\right)$$

$$n=0 \Rightarrow u(x,t) = 2u_0\cos\left(\frac{\delta_1-\delta_2}{2}\right)$$
$$\nu = 0$$

$$n=1 \rightarrow \nu_1 = \frac{c}{2L}$$

$$u(x,t) = \underbrace{2u_0\cos\left(\frac{\pi}{L}x\right)}_{|U_0(x)|}\cos\left(\frac{\pi c}{L}t + \frac{\delta_1-\delta_2}{2}\right)$$



$$\nu_1 = \frac{c}{2L} = \frac{1}{2L}\sqrt{\frac{E}{\rho}} \longrightarrow E$$

$$\rho = \frac{m}{V} = \frac{m}{LS}$$

10. Energija valovanja

- potovanje motnje $\longrightarrow$ potovanje energije
- valovanje po vijačni vzmeti

$x;\ x_{in}\ x+h$

$$W_k = \frac{1}{2} m_1 \cdot v^2 = \frac{1}{2} m_1 \left|\frac{\partial u(x,t)}{\partial t}\right|^2 = \frac{1}{2}\frac{mh}{l}\left|\frac{\partial u(x,t)}{\partial t}\right|^2$$

$$\Rightarrow \frac{W_k}{h} = \frac{1}{2}\frac{m}{l}\left|\frac{\partial u(x,t)}{\partial t}\right|^2$$

$$W_{pr} = \frac{1}{2}k_1\left[u(x+h,t) - u(x,t)\right]^2 =$$

$$= \frac{1}{2}\frac{kl}{h}\left[u(x+h,t) - u(x,t)\right]^2 =$$

$$= \frac{1}{2} kl\cancel{h}\left(\frac{u(x+h,t - u(x,t)}{h}\right)^2 =$$

$$\frac{W_{pr}}{h} = \frac{1}{2}kl\left[\frac{u(x+h,t) - u(x,t)}{h}\right]^2$$

$$\Big\downarrow h \to 0$$

$$\boxed{\lim_{h\to 0}\frac{W_{pr}}{h} = \frac{1}{2}kl\left|\frac{\partial u(x,t)}{\partial x}\right|^2} =$$

$$= \frac{1}{2}kl\cdot\frac{l}{m}\cdot\frac{m}{l}\left|\frac{\partial u(x,t)}{\partial x}\right|^2 =$$

$$\boxed{= \frac{1}{2}c^2\frac{m}{l}\left|\frac{\partial u(x,t)}{\partial x}\right|^2}$$

$$c^2 = \frac{kl^2}{m}$$

Prožna palica

$$\left.\begin{array}{l} m = \rho V = \rho S \cdot V \\ k = \frac{ES}{l} \end{array}\right\} \Rightarrow \frac{W_k}{h}(x,t) =$$

$$\frac{1}{2} \rho S \left|\frac{\partial u}{\partial t}\right|^2$$

$$\boxed{w_k = \frac{W_k}{\Delta V} = \frac{W_k}{Sl} = \frac{1}{2}\rho \left|\frac{du}{dt}\right|^2}$$

gostota kinetične energije

$$\frac{W_p}{h} = \frac{1}{2} c^2 \frac{m}{l} \left|\frac{\partial u}{\partial x}\right|^2$$

$$\bar{w} = \frac{1}{2} \rho \, \omega^2 u_0^2$$ ... povprečna gostota energije

$\omega = 2\pi\nu$

$$\bar{W} = \bar{w}\, dV \qquad dV = Sct \quad ; \quad S \perp c$$
$$= \bar{w}\, Sct$$

energija, ki v času t prepotuje skozi S

$$P := \frac{\bar{W}}{t} = \frac{\bar{w} \cdot Sct}{t} = \bar{w}\, Sc$$

$$[P] = \frac{J}{m^3} \frac{m^2 \cdot m}{s} = \frac{J}{s} = W$$ ... wat

$$j_w := \frac{P}{S}$$ ... jakost energijskega toka

$$j_w = \frac{P}{S} = \bar{w} c \qquad [j_w] = \frac{W}{m^2}$$

Primer:

zvok v zraku

$$c^2 = \frac{1}{\chi \rho}$$

gostota

stisljivost

$$pV = \frac{m}{M} RT$$

$M = 29\,kg$

$R = 8300\ J/K$

$[T] = K$

$$\chi := -\frac{1}{V}\frac{\partial V}{\partial p} \qquad \chi > 0$$

$$\frac{pV}{T} = \frac{p_0 V_0}{T_0}$$

$\Rightarrow$) $T = T_0 = konst$ (izotermna

$$pV = p_0 V_0 = K$$

$$\Rightarrow V = \frac{K}{p} \Rightarrow \frac{1}{V} = \frac{p}{K}$$

$$\boxed{\chi = \left(-\frac{1}{V}\right)\left(-\frac{K}{p^2}\right) = \frac{1}{p}}$$

$$\chi = \frac{1}{p} \longrightarrow \frac{1}{\kappa p}$$

kappa

$$c^2 = \frac{\kappa p RT}{p M} = \frac{\kappa RT}{M}$$

$\kappa = 1{,}4$ za vse dvoatomne molekule

$$\Rightarrow c = \sqrt{\frac{\kappa RT}{M}} = \sqrt{\frac{1{,}4 \cdot 8300\,\frac{J}{K} \cdot 300\,K}{29\,kg}} \approx 340\,m/s$$

$\nu_{max} \simeq 1\,kHz$ .... nese uho je najbolj občutljiva za to

$j_0 = 10^{-12}\,\frac{W}{m^2}$ ... tolikšna jakost je potrebna, da sliši ν_{max}

$u_0 = ?$

$$j_0 = \frac{1}{2}\rho \omega^2 u_0^2 \cdot c \qquad \rho \approx 1{,}2\,\frac{kg}{m^3}$$

$$\Rightarrow u_0^2 = \frac{2 j_0}{\rho (2\pi \nu_{max})^2 c}$$

Dopplerjev pojav

ν: frekvenca oddajnika ($t_0 = \frac{1}{\nu}$)

λ: valovna dolžina zvoka v snovi (npr. zraku)

c: hitrost zvoka v snovi (zraku)

ν': frekvenca, ki jo sliši sprejemnik

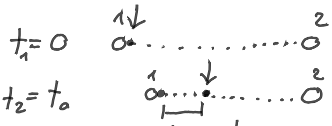
a) sprejemnik miruje glede na zrak ($v_2 = 0$), oddajnik se mu približuje s hitrostjo v_1 (glede na zrak)

$t_1 = 0$ odda prvi pisk

$t_2 = t_0$ odda drugi pisk

pri tem se premakne za $t_0 \cdot v_1$ proti sprejemniku

prvo čelo se premakne za $t_0 \cdot c$ proti sprejemniku



$\lambda = ct_0 - v_1 t_0 = t_0(c - v_1)$

$$\boxed{\nu' = \frac{c}{\lambda} = \frac{c}{(c-v_1)t_0} = \frac{1}{1-\frac{v_1}{c}}\nu}$$

− za približevanje

+ za oddaljevanje

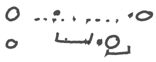
b) oddajnik miruje ($v_1 = 0$), sprejemnik se približuje z v_2

Po zraku proti sprejemniku potujejo valovi z valovno dolžino λ

$t_1 = 0$

$t_2 = t_0'$

$\frac{1}{t_0'} = \nu'$



$c_0 t_0' + v_2 t_0' = \lambda = \frac{c}{\nu}$

$t_0'(c_0 + v_2) = \frac{c}{\nu}$

$t_0' = \frac{1}{\nu'}$ $\qquad \frac{c_0 + v_2}{\nu'} = \frac{c}{\nu}$

$$\boxed{\nu' = \nu \cdot \frac{c + v_2}{c} = \nu\left(1 + \frac{v_2}{c}\right)}$$

$$\boxed{\nu' = \nu \, \frac{1 \pm \frac{v_2}{c}}{1 \mp \frac{v_1}{c}}}$$

predpostavke:

T1: vpliv okolice upade z razdaljo

$$\lim_{d_{min}\to\infty} \vec{F} = 0$$

skupna sila okolice

najmanjša razdalja okolice do predmeta

T2: definicija inercialnega op. sistema

$$d_{min} \to \infty \xRightarrow{T1} \vec{F} = 0$$ skupne sile okolice

$$S: \vec{a} = \vec{0} \Rightarrow S \text{ inercialni}$$

T3: Definicija količine neto sile $\vec{F}$

T2: S inercial

m... masa v sistemu v katerem telo miruje

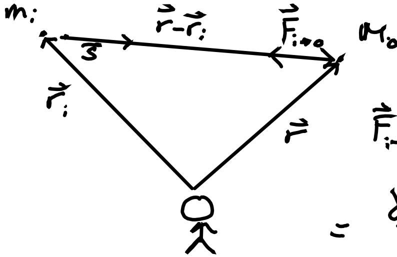
$$\vec{a} \Rightarrow \vec{F} := m\vec{a}$$

v inercialnem sistemu

T4: Princip superpozicije: $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots$

$$\Rightarrow \vec{F} = \sum_{i=0}^{n} \vec{F}_i$$

Primer: $\vec{F}_g$

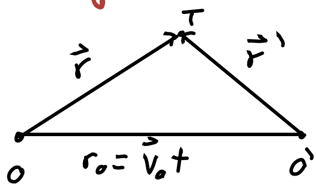


$$\vec F_{i\to 0} = -\gamma \frac{m_i M_0}{|\vec r - \vec r_i|^2}\hat e_i =$$

$$= \frac{\gamma\, m_1 M_0 (\vec r - \vec r_i)}{|\vec r - \vec r_i|^3}$$

$$\vec F_{0\to i} = \frac{\gamma M_0 m_i (\vec r_i - \vec r)}{|\vec r - \vec r_i|^3} = -\vec F_{i\to 0}$$

Galilejeve transformacije



$$\vec{r} = \vec{r}_0 + \vec{r}' \implies \vec{r}' = \vec{r} - \vec{r}_0$$

$$t' = t$$

$$S: \vec{v} = \frac{d\vec{r}}{dt}$$

$$S': \vec{v}' = \frac{d\vec{r}'}{dt'} = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(r - r_0) = \frac{dr}{dt} - \frac{d(v_0 \cdot t)}{dt} =$$

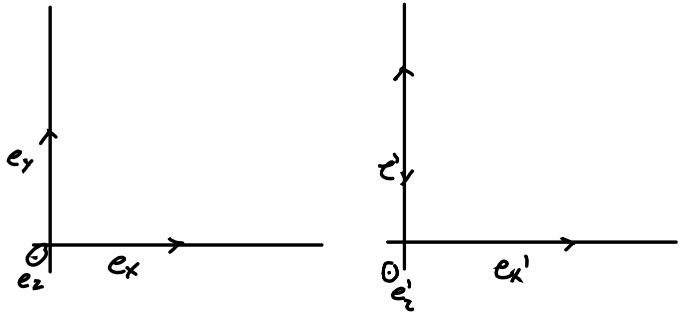
$$= v - v_0$$

$$S: \vec{a} = \frac{dv}{dt}$$

$$S': \vec{a}' = \frac{dv'}{dt'} = \frac{dv'}{dt} = \frac{d}{dt}(v - v_0) = \frac{dv}{dt} - \frac{dv_0}{dt} =$$

$$= \vec{a} - 0 = \vec{a}$$

$$\vec{F}' = m\vec{a}' = m\vec{a} = F$$



$$r_0 = v_0 t\, e_x = \begin{bmatrix} v_0 t \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{r} = x e_x + y e_y + z e_z = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{r}\,' = x' e_x' + y' e_y' + z' e_z' = x' e_x' + y e_y' + z e_z' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\vec{r}\,' = r - r_0 = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} v_0 t \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x - v_0 t \\ y \\ z \end{bmatrix}$$

$$c_0 t' = c_0 t$$

$$x' = x - \frac{v_0}{c_0} c_0 t = x - \beta_0 c_0 t$$

$$y' = y$$

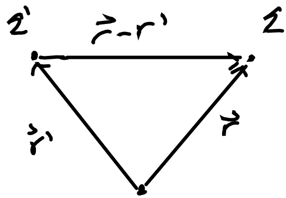
$$z' = z$$

$$^4x = \begin{bmatrix} c_0 t \\ x \\ y \\ z \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Električna sila

(Kolombov zakon)



$$F_{e'\to e} = \frac{ee'}{4\pi\varepsilon_0}\frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$F_{m'\to m} = -\gamma \frac{mm'(\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$e_0 = 1{,}6\cdot 10^{-19}\,As$$

$$e = -e_0 = -1{,}6\cdot 10^{-19}\,As$$

$$\varepsilon_0 = 8{,}9\cdot 10^{-12}\,\frac{As}{Vm}$$

$$e = N e_0$$

3. Električno polje

$\vec{F}(\vec{r})$ ... sila

$\vec{E}$ ... vektor. el. polja oz jakost. el. polja

q ... naboj

$$F_{el,q} = q \cdot \vec{E} \qquad \vec{E} = \frac{F_{el,q}}{q}$$

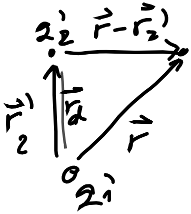
$$\vec{E}_{q'(\vec{r}')}(\vec{r}) = \frac{\vec{F}_{q' \to q}}{q} = \frac{q' (\vec{r} - \vec{r}')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}'|^3}$$

$$[F_{q' \to q}] = \frac{As^2 V m m}{As \; m^3} = \frac{VAs}{m} = N$$

$$[E] = \frac{V}{m}$$

$\forall$ 27.3 odsotna zadnjih 10 minut

Primer:



$$q_1' = q > 0$$

$$q_2' = -q \quad (<0)$$

$$r_2' = -r_d$$

$$\vec{E}(\vec{r}) = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = \frac{\overset{=q}{q_1'}\,(\vec{r} - \vec{r}_1')}{4\pi\varepsilon_0\,|\vec{r} - \vec{r}_1'|^3} = \frac{q\vec{r}}{4\pi\varepsilon_0\,|\vec{r}|^3}$$

$$E_2 = \frac{q_2'\,(\vec{r} - \vec{r}_2')}{4\pi\varepsilon_0\,|r - \vec{r}_2'|^3} = -\frac{q\,(\vec{r} - \vec{r}_2')}{4\pi\varepsilon_0\,|\vec{r} - \vec{r}_2'|^3}$$

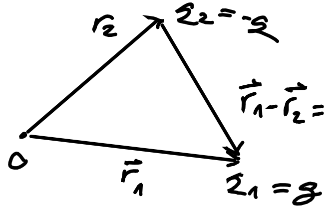
$$E \cong \frac{1}{4\pi\varepsilon_0 r^3}\left(3p_e \cdot \hat{e}_r)\hat{e}_r - \vec{p}_e\right\}$$

$$\hat{e}_r = \frac{\vec{r}}{r} \qquad \vec{p}_e := q\vec{r}_d$$

Navor na $\vec{p}_e$ v homogenem polju $\vec{E}$

$\vec{e}_1 \to e_1 : \vec{F}_{el,e_1} = e_1 \vec{E}(\vec{r}_1) = e\vec{E}$

$e_2 \to e_2 : F_{el,e_2} = e_2 \vec{E}(\vec{r}_2) = -e\vec{E}$



$\vec{r}_1 - \vec{r}_2 = r_d$ (od neg. do poz.)

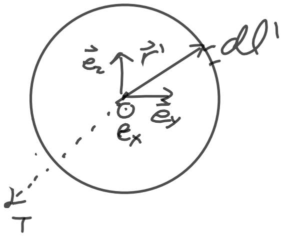
$$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1 = \vec{r}_1 \times e\vec{E} = e\,\vec{r}_1 \times \vec{E}$$

$$M_2 = \vec{r}_2 \times \vec{F}_2 = \vec{r}_2 \times (-e\vec{E}) = -e\,\vec{r}_2 \times \vec{E}$$

$$M = M_1 + M_2 = e\vec{r}_1 \times \vec{E} - e\,r_2 \times \vec{E} =$$

$$e(\vec{r}_1 - \vec{r}_2) \times \vec{E} = e\,\vec{r}_d \times \vec{E} = \vec{p}_{el} \times \vec{E}$$

E na simetrali enakomerno nabite zanke



$$\vec{r} = r\hat{e}_x$$

$$\vec{r}' = r'\cos\varphi\, e_y + r'\sin\varphi\, e_z$$

$$\vec{r} - \vec{r}' = r\hat{e}_x - r\cos\varphi\, \hat{e}_y - r'\sin\varphi\, e_z'$$

$$|\vec{r} - \vec{r}'|^3 = (r^2 + r'^2)^{\frac{3}{2}}$$

$$\rho_l = \frac{e'}{2\pi r'} = \frac{dq'}{dl'} = \frac{q'}{r'd\varphi}$$

$$\Rightarrow \quad dq' = \rho_l\, r'd\varphi$$

$$dE_p(\vec{r}) = \frac{dq'}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} =$$

$$= \frac{\rho_l r' (r\vec{e}_x - r\cos\varphi\, e_y - r\sin\varphi\, \vec{e}_z)\, d\varphi}{4\pi\varepsilon_0 (r^2 + r'^2)^{\frac{3}{2}}}$$

$$\Rightarrow \vec{E}(\vec{r}) = \int dE_p = \int \frac{\rho_l r'}{4\pi\varepsilon_0} \left[\frac{r\hat{e}_x I_1 - r' e_y I_2 - r'\hat{e}_z I_3}{(r^2 + r'^2)^{\frac{3}{2}}}\right] d\varphi$$

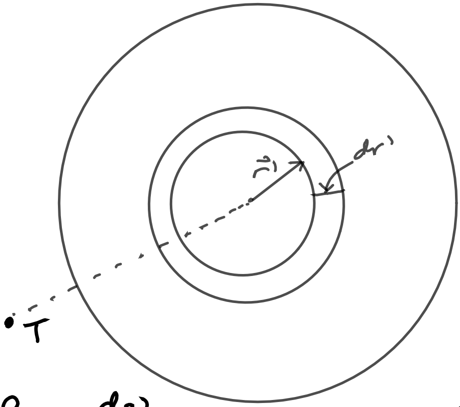
$$I_1 = \int_0^{2\pi} d\varphi = 2\pi$$

$$I_2 = \int_0^{2\pi} \cos\varphi\, d\varphi = 0$$

$$I_3 = \int_0^{2\pi} \sin\varphi\, d\varphi = 0$$

$$\vec{E}(\vec{r}) = \frac{\rho_l r' r 2\pi\, \hat{e}_x}{4\pi\varepsilon_0 (r^2 + r'^2)^{\frac{3}{2}}} = \frac{q r\ \hat{e}_x}{4\pi\varepsilon_0 (r^2 + r'^2)^{\frac{3}{2}}}$$

Ne vidim na tablo oznake sa tke
mage se pravike



$$S = \pi R^2$$

$$\rho_{s'} = \frac{q'}{s'} = \frac{q'}{\pi R^2}$$

$$dS' = 2\pi r' dr'$$

(površine zenke)

$$\rho_{s'} = \frac{dq'}{dS'} \Rightarrow dq' = \rho_s' dS' = \frac{q'}{\pi R^2} 2\pi r' dr' =$$

$$= \frac{q' 2r' dr'}{R^2}$$

$$dE_{r'}(\vec{r}) = \frac{dq' \, r}{4\pi \varepsilon_0 (r^2 + r'^2)^{\frac{3}{2}}} \hat{e}_x =$$

$$\frac{q' r r' dr}{2\pi \varepsilon_0 (r^2 + r'^2)^{\frac{3}{2}} R^2} \hat{e}_x$$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{q' r}{2\pi \varepsilon_0 R^2} \hat{e}_x I$$

$$I = \int_0^R \frac{r' dr'}{(r^2 + r'^2)^{\frac{3}{2}}} = \frac{1}{r}\left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{r^2}}}\right)$$

$$\vec{E}(\vec{r}) = \frac{q'}{\pi r^2} \frac{1}{3\varepsilon_0} \cdot \left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{r^2}}}\right)$$

$$= \frac{\rho_l}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{r^2}}}\right)$$

$$\frac{r}{R} << 1 \Rightarrow \sqrt{1 + \frac{R^2}{r^2}} = \sqrt{\frac{R^2}{r^2}} = \frac{R}{r}$$

$$\Downarrow$$

$$\frac{R}{r} >> 1$$

$$\vec{E}(r) \approx \frac{\rho_{s'}}{2\varepsilon_0}\left[1 - \frac{r}{R}\right] \approx \frac{\rho_{l'}}{2\varepsilon_0}$$