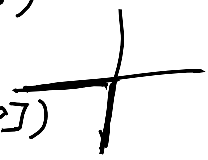
1.1.

$A = [-1,1] \times [-1,1]$

$g: \mathbb{R}^2 \to \mathbb{R}^2/A$ hoc

a) $g_*((-\infty, 0] \times (-\infty, 0])$

~~zapta~~ ~~w. otw~~ ni odp ni zap

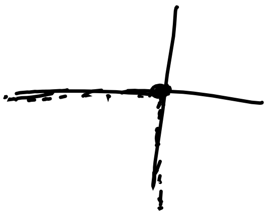


$g^*(g_*((-\infty,] \times (-\infty, 0])$

b) $g_*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

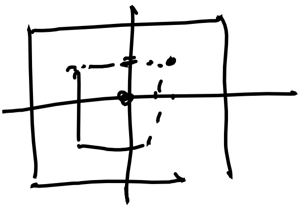
ni odpta

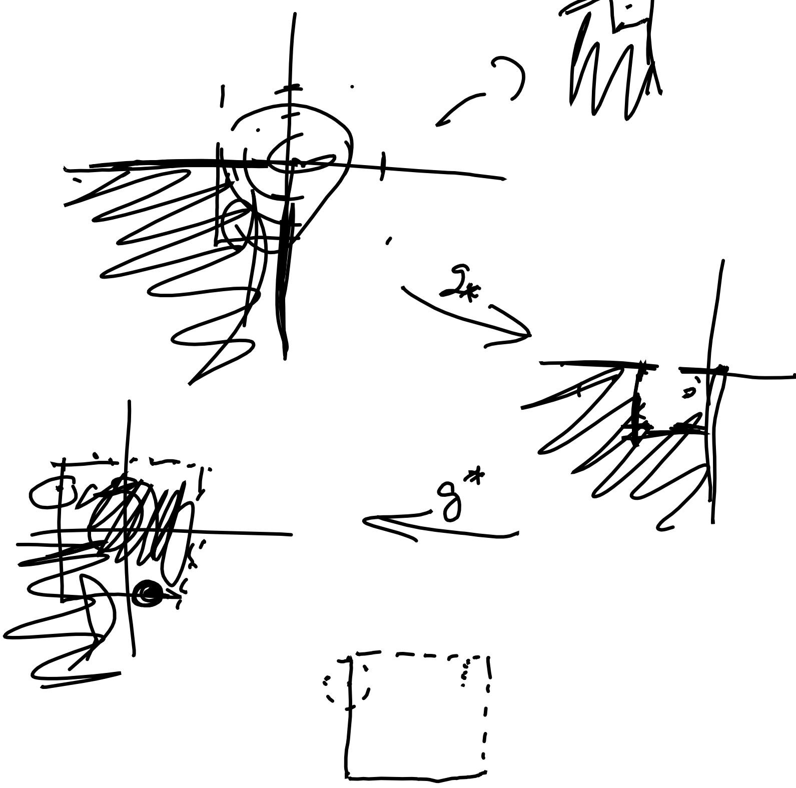
ni zapta



c) $g_*([-2,2] \times [-2,2])$

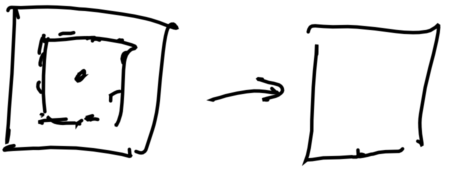
odpta





5)

$g^*(g([-2,2] \times [-2,2])$



zapta

ni odp

c) niedz jeszcz

7.4

a) $[-2,2] / [-1,1]$ 

$\simeq [-1,1]$

b) $[-1,1] / \{-1,0,1\}$ 

$\simeq$

$\mathbb{R}^2$

c)

$$\mathbb{R}^n / K(0,1) \xrightarrow{\cong} \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[d] \arrow[dr, "f"] & \\
\mathbb{R}^n / K(0,1) & \mathbb{R}^n
\end{tikzcd}$$

$$f(\vec{a}) := \begin{cases} 0 \,; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} \,; & a \in \mathbb{R}^n - \overline{K(0,1)} \end{cases}$$

d)

$\mathbb{R}^n/_{\sim} \qquad x \sim y \Leftrightarrow \|x\| = \|y\|$

$\mathbb{R}^n/_{\sim} \cong [0, \infty)$

$f: \mathbb{R}^n \longrightarrow [0, \infty)$

$f(\vec{x}) = \|\vec{x}\|$ je surj

$a \in [0, \infty) \qquad (a, 0, 0 \ldots) \mapsto a$

$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$

je zvezna

iščemo preslikavo s, da velja $\quad f \circ s = id_{[0,\infty)}$

$s: a \longmapsto (a, 0 \ldots\ldots)$

Dokažimo da če $r \circ s = id_Y$

$\Rightarrow r$ kvocientna, sulažim

$S \subseteq Y$ taka da $r^*(S)$ odp v X

$S = s^*(r^*(S)) =$

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S$ ■

$\Longrightarrow S$ je odprta

f)

$$S^n \times [-1,1] \big/ \{S^n \times \{-1\}, S^n \times \{1\}\} \approx S^{n+1}$$

$f: X \to S^{n+1}$

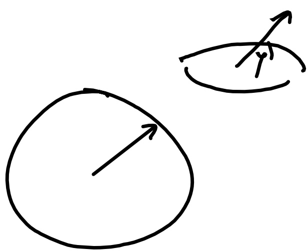
$$(x,t) \longmapsto (xb, t) = (\sqrt{1-t^2}\,x, t$$

anstatt voller

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{ g \cdot H ; g \in G \} =$$

$$= \{ \{ g \cdot h ; h \in H \} ; g \in G \}$$

G top. grupa

$a \in G$

$L_a : G \longrightarrow G$

$x \longmapsto a \cdot x$ leva translacija

$a, b \in G$

$h : G \longrightarrow G$

$h(a) = b$ $h = ?$

možnosti

$L_{ba^{-1}} : x \longmapsto ba^{-1}x$

$ba^{-1}x$

$xa^{-1}b$

bxa^{-1}

$a^{-1}xb$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\iff ba^{-1}A$ okolica $b \in G$

$\exists U^{odp} \subseteq A \,.\, a \in U$

$b \in ba^{-1}A$

$a \in U \Rightarrow \underbrace{ba^{-1}a}_{= b} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \overset{\subseteq A}{\text{odprta}}$ v G

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ H okolica $1 \Rightarrow H$ odp in zap v G

$a \in aH \subseteq H$

$\Rightarrow H$ je okolica vsake svoje točke

$G - H$ je odp.

$aH \cap H = \emptyset \Rightarrow aH = H$

$a \in G - H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$ vsak element ima okolico, ki ne seka $H \Rightarrow H$ je zaprta

c) C komponenta $1 \in C$

$\Rightarrow$ C zaprta edinka v G

$C \leq G$

$L_a: x \longmapsto ax$ je homomorfizem za $\forall a \in C$

$\forall a \in C.\ L_a \subseteq C$

L_a ohranja povezanost

Prav tako $L_a \cdot 1 = a \Rightarrow$

$L_{a*} C \cap C \Rightarrow L_{a*} C \subseteq C$

invertiranje: invertiranje je tudi homeo $i: x \longmapsto x^{-1}$

$\Rightarrow$ po istih argumentih ~~…~~

Ali je edinka?

$\forall a \in G.\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \longmapsto axa^{-1}$ je homeomorfizem

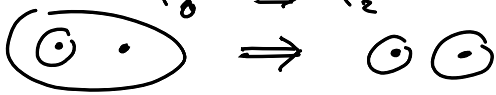
$\Rightarrow$ ji povezano in $a\, id\, a^{-1} = id \in C$

$\Rightarrow aCa^{-1} \subseteq C$

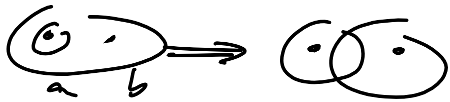
ker je kompozitum dveh translacij (leva in desna)

d) za G je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$



$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G$ $\quad a \in U,\ b \notin U$ BŠZS

$U^{-1} = \{a^{-1} : a \in U\}$

$aU^{-1}b \ni b \quad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U.\ a = ac^{-1}b$

$\Rightarrow b = c \Rightarrow b \in U$ ⨳

$T_1 \Rightarrow T_2$

$\exists U, V \in G$ odp. $\quad a \in U,\ b \in V,\ a \notin V,\ b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$\Delta_G = f^*(\{1\})$

$f : (x, y) \mapsto xy^{-1}$

~~za~~ zvezna praslika ~~zap~~ 1 je zaprta

(2) T_{ee} je T_1 in ni T_2

$\Rightarrow$ $(\mathbb{R}, +)$ ni top grupa za T_{ee}

(ker 1.d)

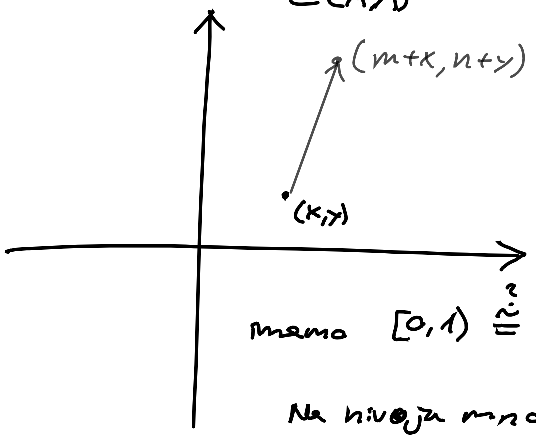
(2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

d $(m,n)(x,y) := (m+x,\, n+y)$

$$\Phi : g \longmapsto \underbrace{(a \longmapsto g\cdot a)}_{\text{bijekcija (izomorfizem)}}$$

$\Phi : G \longrightarrow \mathrm{Bij}(A)$ ← grupa za komponiranje

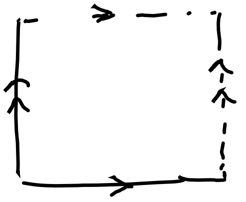
V splošnem: $\Phi : G \longrightarrow (\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ)$



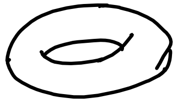
Imamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ✓

Na nivoju topologije ? NE



Dobimo torus



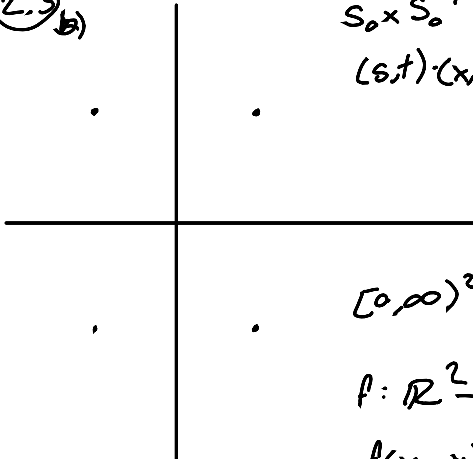
$\cong S^1 \times S^1$

$f : \mathbb{R}^2 \longrightarrow S^1 \times S^1$

$(x,y) \longmapsto (e^{2\pi i x}, e^{2\pi i y})$

Tuk nism poslušala več :'(

(2.3) (b)



$S_0 \times S_0 \curvearrowright \mathbb{R}^2$

$(s,t)\cdot(x,y) = (sx, ty)$

$[0,\infty)^2$

$f : \mathbb{R}^2 \to [0,\infty)^2$

$f(x,y) = (|x|, |y|)$

$s : [0,\infty)^2 \to \mathbb{R}^2$

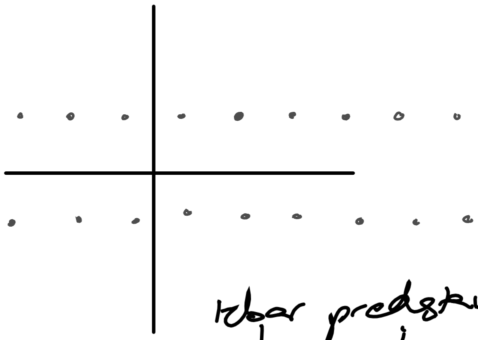
$(x,y) \longmapsto (x,y)$

f, s zvezni $\Rightarrow f$ kvociente

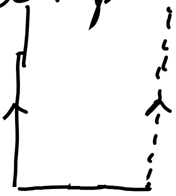
c)

$\mathbb{Z} \times S^0 \curvearrowright \mathbb{R}^2$

$(m,t)\cdot(x,y) = (m+x,\ ty)$



Izbor predstavnikov



$S_1 \times [0,\infty)$

$f: \mathbb{R}^2 \longrightarrow \underset{\subseteq \mathbb{C}}{S^1} \times [0,\infty)$

$(x,y) \longmapsto (e^{i2\pi x}, |y|)$

zvezna ✓

surjektivna ✓

Po standardnem postopku tvdi identifikacije med delovanje

$((n-\frac{1}{n}),0))_n$

slike zaporedja zap v $\mathbb{R}^2$

f-slike pa ni zaprta v $S^1 \times [0,\infty)$

$(1,0)$ je v zaprtju ampak ni v f-sliki

Produkt dveh odprtih preslikav je odprta

$h: \mathbb{R} \longrightarrow [0,\infty)$

$x \longmapsto |x|$

Dovolj preveriti na bazi

$0 \notin (a,b)$: $h(a,b) = (\min\{|a|,|b|\}, \max\{\dots\})$

Če $0 \in (a,b)$: $h(a,b) = [0, \max\{|a|,|b|\})$

$g: \mathbb{R} \to S^1 \subseteq \mathbb{C}$

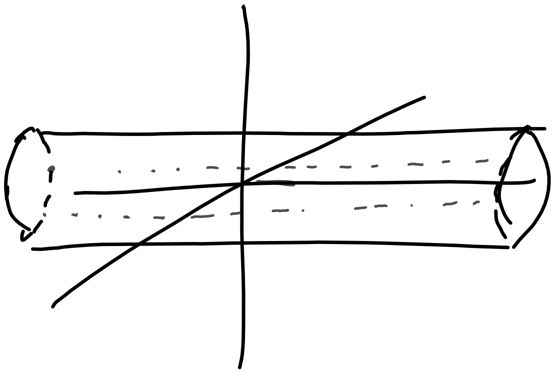
$x \mapsto e^{i2\pi x}$

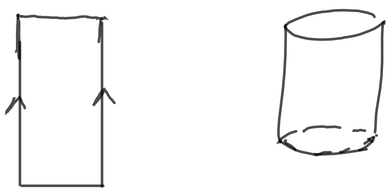
Baza $\mathbb{R}$ odprti intervali dolžine <1 $\Rightarrow$ slike odprte lahk

d)

$\mathbb{Z} \times S_0 \curvearrowright \mathbb{R} \times S^1 \subseteq \mathbb{R}^3$

$(m,t).(x,y,z) := (m+x, y, tz)$

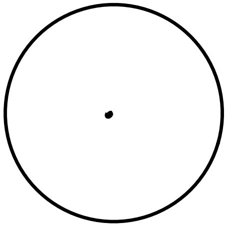




$\mathbb{R} \times S_1 \longrightarrow S_1 \times [-1,1]$

$(t, (y,z)) \longmapsto (e^{2i\pi t}, y)$

a)

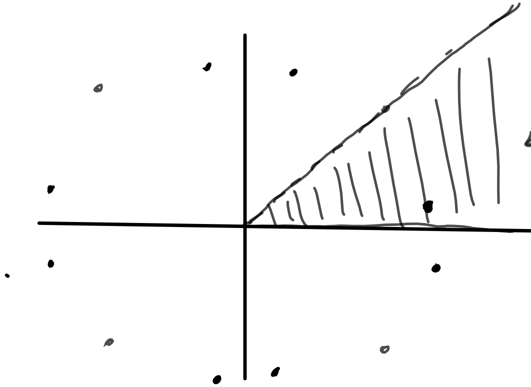


$$\vec{x} \sim \vec{y} \Leftrightarrow \|x\| = \|y\|$$

$$f : \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(a)



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cong -1 \in \mathbb{C} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx i$$

Topologija ker ...

$Y = \{(x,y) \in \mathbb{R}^2 ;\ y < x;\ x > 0$

$f: \mathbb{R}^2 \longrightarrow Y$

$(x,y) \longmapsto (\max(|x|,|y|) ; \min(|x|,|y|)$

A retrakcija $\Rightarrow$ kvocientna v ožjem smislu

2(4)

$2x = g + x$

$\mathbb{R}/\mathbb{Q}$ ni moth- vlozizi v rdan

← po delovanju grupe

U° dg $\subseteq \mathbb{R}/\mathbb{Q}$

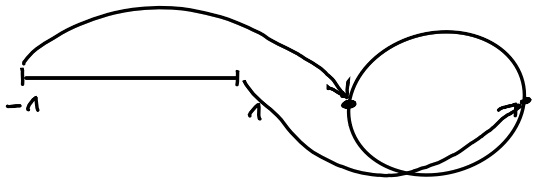
Ubit vse

3.1 8.4

a)

$X = [-1, 1]$ $A := \{-1, 1\}$

$Y = S^1$ $f(x) := (x, 0)$



$Z = [-1, 1] \times \{0\} \cup S^1$

$g : X + Y \longrightarrow Z$

$in_1(x) \longmapsto (x, 0)$

$in_2(y) \longmapsto y$

ekvivalenčni razredi: $(in_1(1,0), in_2(1,0)$ in $in_1(-1,0)$ in $in_2(-1,0))$ neenoglavo

$g(in_1(1,0)) = 1,0$

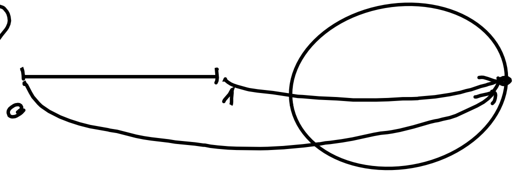
$g(in_2(1,0)) = 1,0$ podatna z drug

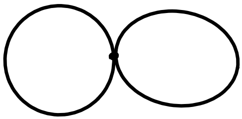
zveznost pa, ker sta pospremetni zvezni in ker ~~a~~ se ujemata na preseku

Potrebno še pokazati še da loči ekv. razrede

slikamo iz kompakta v Hausdorfov

b)





$$Z = Y \cup S((2,0),1)$$

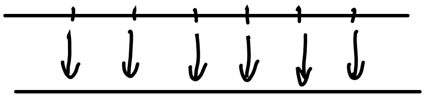
$$g: X + Y \longrightarrow Z$$

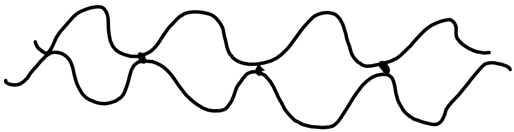
$$in_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$in_2(x,y) \longmapsto (-x+2, y)$$

Preveriti moramo

- loči ekv razr
- konst na ekv razr
- zvez, surj, kvocientna v ožjem smislu

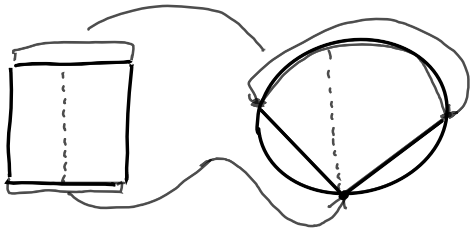




$|\sin x| \cup -|\sin x|$

$x \longmapsto$

d)



$Z = S^1 \cup \{(x,y) \in \mathbb{R}^2 ;\ |x-1| \leq 1 \leq \sqrt{1-x^2}\ \}$

$g : X + Y \longrightarrow Z$

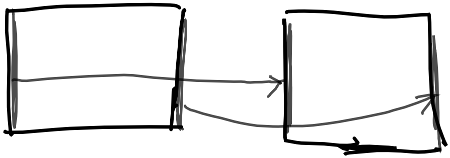
$\quad in_2(z) \longmapsto z$

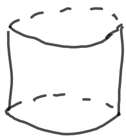
$\quad in_1(x,y) \longmapsto (0,-1) + \frac{y+1}{2}\left(x, \sqrt{1-x^2}+1\right)$

kompaktna v hausdorfov

e)

X:





$Z = S_1 \times [-1, 1]$

$g: X + Y \longrightarrow Z$

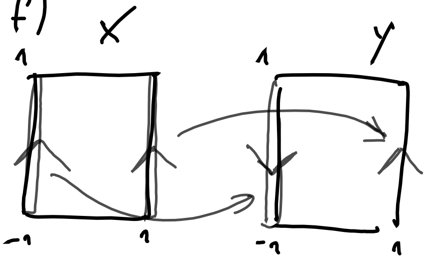
$in_1(x, y) \longmapsto (-\sqrt{1-x^2}, y)$

$in_2(x, y) \longmapsto (\sqrt{1-x^2}, y)$

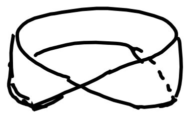
kompakt v hausdorff ✓

[illegible] [illegible] identifizierung

f)



Möbiusov trak



Parametrizacija möbiusovega traku

$$in_1(u,v) \longmapsto (x,y,z)$$

$$x(u,v) = \left(\cos\frac{v\pi}{2}\left(2+u\cos\frac{v\pi}{4}\right)\right)$$

$$y(u,v) = \left(\sin\frac{v\pi}{2}\left(2+u\cos\frac{v\pi}{4}\right)\right)$$

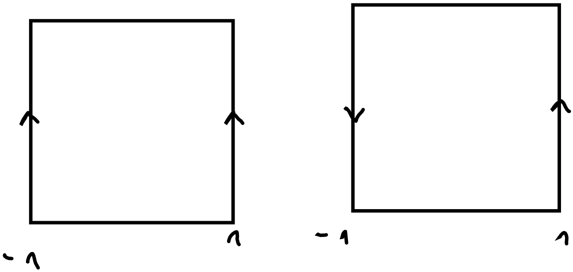
$$z(u,v) = u\sin\frac{v\pi}{2}$$

$$in_2(u,v) \longmapsto (x,y,z)$$

$$x = \cos\frac{v\pi}{2}\left(2+u\cos\frac{v\pi}{4}\right)$$

$$y = -\sin\frac{v\pi}{2}\left(2+u\cos\frac{v\pi}{4}\right.$$

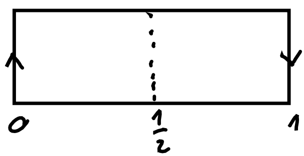
$$z = -u\sin\frac{v\pi}{2}$$



15.4

$X + Y \longrightarrow M$ ... Möbisov trak

$M = [0,1]\times[0,1]/\sim$



$$\begin{tikzcd}
X + Y \arrow[r, "g'"] \arrow[d, "q"'] \arrow[dr, "g"] & {[0,1]^2} \arrow[d, "q_m"] \\
X \cup_f Y \arrow[r, dashed] & M
\end{tikzcd}$$

$$in_1(u,v) \longmapsto \left(\frac{u+1}{4}, \frac{v+1}{2}\right)$$

$$in_2(u,v) \longmapsto \left(\frac{3-u}{4}, \frac{v+1}{2}\right)$$

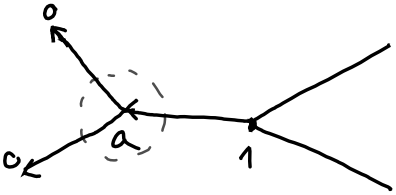
$$x \sim y \Longleftrightarrow g(x) = g(x)$$

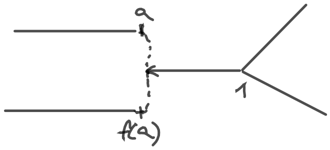
$$x \sim y \Longleftrightarrow g'(x) \sim g'(y)$$

$(a, 1] \longrightarrow (0,\infty)$

$f(x) = x$

zlepek $(0,\infty) \cup_f (a,\infty)$ je zlepek za katere hausdorffove

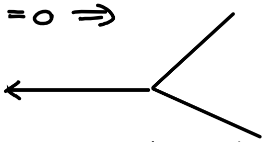




Poglejmo točko a

Vsaka okolica od a in od f(a) seka te del $(0,1]$ za $\forall a \in (0,1)$

za $a = 0 \Rightarrow$



Vložimo v evklidski prostor

$(0,\infty) \cup_f (0,\infty) \longrightarrow \mathbb{R}^2$

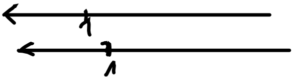
$x \in_{in_1} (0,1] \longmapsto (0,x)$

$x \in in_1 (1,\infty) \longmapsto (x, x-1)$

$x \in_{in_2} (1,\infty) \longmapsto (x, -x+1$

je hausdorffov?

lokalna kompaktnost



kompakti: zaprti intervali v $(0,\infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh premicah

kompakti grejo v kompakte tja

3.3

$X/A \approx X \cup_f \mathbb{1}$

$$\begin{tikzcd}
X \arrow[d, "f"'] & X + \mathbb{1} \arrow[l, "g'"'] \arrow[dl, "g"'] \arrow[d, "g"] \\
X/A \arrow[r, "\approx"] & X \cup_f \mathbb{1}
\end{tikzcd}$$

$1 \in \mathbb{1}$

$h:$
$[x] \longmapsto x \quad ; x \notin A$
$[a] \longmapsto 1 \quad ; a \in A$

je homeomorfizem

$g' : X + \mathbb{1} \longrightarrow X$
$in_1 x \longmapsto x$
$in_2 \ddot{\smile} \longmapsto a \quad a \in A$

$g = f \circ g'$

$u \sim v \Leftrightarrow g'(u) \sim_A g'(v)$

$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$

$f(g'(u)) = f(g'(v)) \Leftrightarrow$

$[g'(u)] \sim [g'(v)]$

Ekv razredi v $X \cup_f \mathbb{1}$

$in_1 x = \{x\} \quad x \notin A$

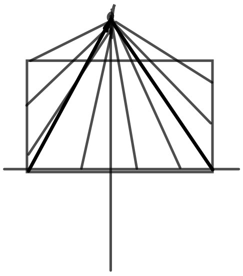
$in_1 a = A \cup \ddot{\smile} \quad a \in A$

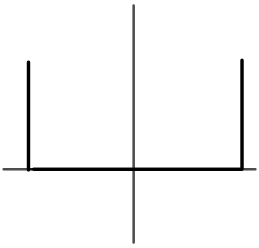
$in_2 \ddot{\smile} = A \cup \ddot{\smile}$

$[in_1 x] = \{in_1 x\}$

$(in_1)_*(A) \cup \{in_2 \ddot{\smile}\}$

Retrakti, homotopije, ekstenzije … [illegible]





Projeciramo iz $(0,2)$ na rob

$y > 2x+2$: $\left(-1,\ 2-\frac{y-2}{x}\right)$

$2x+2 > y < -2x+2$: $\left(\frac{2x}{2-y},\ 0\right)$

$y > -2x+2$: $\left(1,\ \frac{y-2}{x}+2\right)$

$(a,b) \rightsquigarrow p_1 : y = \frac{b-2}{a}x+2$

pri $x=-1$

$y = -\frac{b-2}{a}+2$

$p_2 : y = \frac{b-2}{a}x+2 \qquad a \neq 0$

$y=0 \rightarrow x = \frac{-2a}{b-2} = \frac{2a}{2-b}$

$a=a$

$x=1$:

$y = \frac{b-2}{a}+2$

Ali je to id na Y?

$x=-1$: $\left(-1,\ 2-\frac{y-2}{-1}\right) = (-1,\ 2+y-2) = (-1,y)$

$y=0$: $\left(\frac{2x}{2}, 0\right) = (x,0)$

$x=1$: $\left(1, \frac{y-2}{1}+2\right) = (1,y)$ ✓

Ali homotopne id na X

$H(x,y,t) = t(x,y) + (1-t)r(x,y)$

zvezna, ker so vsi kosi zvezni

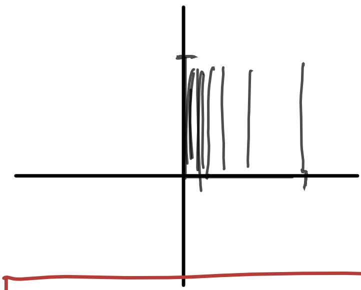
f homotopna g

ko $\exists\, H: X \times [0,1] \longrightarrow Y$

$(x,0) \longmapsto f(x)$

$(x,1) \longmapsto g(x)$

zvezna



$\boxed{\text{Retrakt } T_2 \text{ prostora je zaprt}}$

Recimo da $\exists r$ retrakt $: X \to Y$

$r|_Y = id$

daljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

mora pokriti točke $(\frac{1}{n}, 0)$

(ker gre vse pt iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1]$ čez točke

Torej

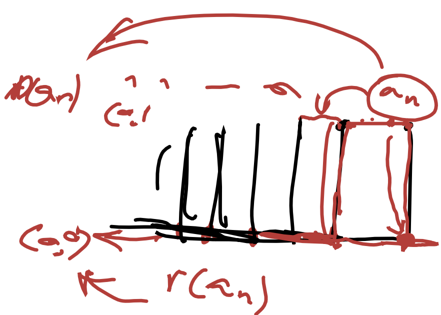
Naj bo zaporedje a_n; $r(a_n) = \frac{1}{n}$

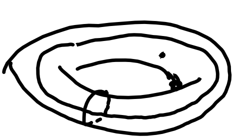
$a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim a_n = (0,1)$

$\lim r(a_n) = (0,0)$

r ni zvezna





$G \curvearrowright X$

$(X, G) \longrightarrow X$

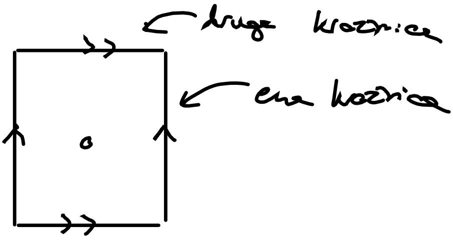
$(x, g) \longmapsto x \cdot g$

Deformacijska retrakcija "ima homotopijo do id"

je zvezna preslikava $H : X \times [0,1] \to X$.

$H(x,0) = x$ za $\forall x \in X$, $H(a,1) = a$ za $\forall a \in A$ $\wedge$ $H(x,1) \in A$ za $\forall x \in X$

$X \times [0,1] \longrightarrow$



$$\begin{tikzcd}
\left([-1,1]^2 - \{(0,0)\}\right) \times [0,1] \arrow[r, "h"] \arrow[d, "q \times id_{[0,1]}"'] \arrow[dr, "g\circ h"] & {[-1,1]^2 - \{(0,0)\}} \arrow[d, "g"] \\
X \times [0,1] \arrow[r, dashed, "\text{zvezna preslikava}"'] & X
\end{tikzcd}$$

$h : (x, t) \longmapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x}(1-t)$

$g \circ h$ mora biti konstantna na ekv. razredih

$x \sim y \Rightarrow g\circ h(x) = g \circ h(y)$

$\Longleftrightarrow$

$[h(x)] = [h(y)]$

$\Longleftrightarrow$

$h(x) \sim h(y)$

h ekvivalentne netrivialne razrede ohrani na mestu

$\Rightarrow$

$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1,1]^2 - \{(0,0)\} \times [0,1]$

$\Longleftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$

$\Rightarrow$ - - - $\frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1)(1 - t_1)$

$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|} t_2 + (x_1, y_1)(1 - t_2)$

Izrek

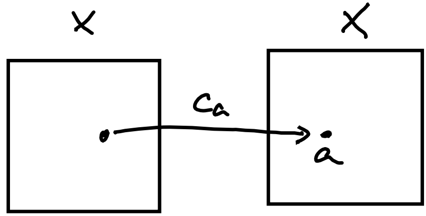
q kvocientna in X kompakten T_2 prostor

$\Longrightarrow$ $q \times id_X$ je kvocientna

Bolj splošno:

q kvocientna, X lokalna kompaktna

$\Longrightarrow$ $q \times id$ kvocientna



$\Rightarrow$ naj bo X povezan s potmi:

Naj bo γ ta pot med a in b poljubna

$\gamma(0)=a \qquad \gamma(1)=b$

$H: X\times[0,1] \longrightarrow X$

$x,t \longmapsto \gamma(t)$

$H=\gamma\circ pr_2$ takoj je zvezna

$\Leftarrow$

Recimo $\forall a,b\in X$ velja $c_a \simeq c_b$

Naj bosta a,b poljubna

$\exists H.\ X\times[0,1] \longrightarrow X$

$H_0 = c_a$

$H_1 = c_b$

$\gamma(t) = H(a,t)$

4.5

a) $f: S^n \to S^n$ n: sur

homotopna konstanti

$a \notin f_*(S^n)$ b je nasprotni tej

$$H: S^n \times [0,1] \longrightarrow S^n$$

$$(x, t) \longmapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$$

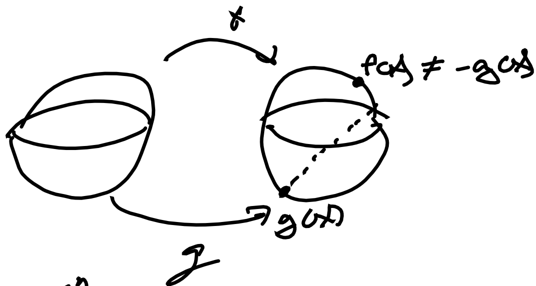
Ali je slučajno $(1-t)f(x) + t \cdot x = 0$

$\Rightarrow$ če gre ta daljica skozi 0

ampak a ni zalogi vrednosti, zato nikoli nobena daljica skozi 0,0

b) $f, g : S^n \longrightarrow S^n$

$f(x) \neq -g(x) \quad \forall\, x \in S^n \Longrightarrow f \simeq g$



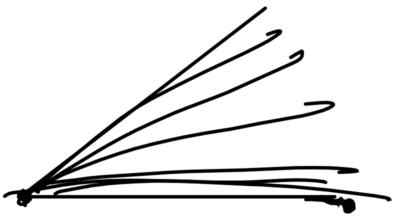
$H : \cancel{X}^{\,S^n} \times [0,1] \longrightarrow S^n$

$$(x,t) \longmapsto \frac{(1-t)(g(x)) - t f(x)}{\|(1-t)g(x) - t f(x)\|}$$

4.6 $X = \{[0,1]\times\{0\}\} \cup \{(x, \frac{x}{n}),\ x\in[0,1] \wedge n\in\mathbb{N}\}$

$H: X\times[0,1] \longrightarrow CX$

$(x,y),\ t \ = (1,t)(x,y)$



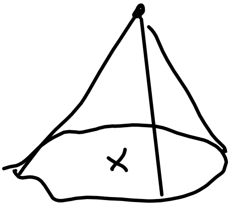
Recimo da je G podpolna krepka deformacijska retrakcija X-a na tisto točko

$\forall n\in\mathbb{N}\ \exists\, t_n \in[0,1]\quad G\left(\left(1, \frac{1}{n}\right), t_n\right) = (0,0)$

ker je $[0,1]$ kompaktna lahko izberemo konvergentno podzaporedje t_{i_n}

?

$$CX = X \times [0,1] \,/\, X \times \{1\}$$



$$CX \times [0,1] \longrightarrow CX$$

$$\begin{tikzcd}
X \times [0,1] \times [0,1] \arrow[r, "H"] \arrow[d, "q \times id"] & X \times [0,1] \arrow[d, "q"] \\
CX \times [0,1] \arrow[r, dashed] & CX
\end{tikzcd}$$

ne moremo nujno seštevat

$$H : (x, u, t) \longmapsto \cancel{(x,1)t + (x,u)(1-t)}$$

$$(x,\ u(1-t) + t)$$

Preveriti moramo še na

$$(x_1, 1, t) \quad \sim_H \quad (x_2, 1, t)$$

edini netrivialni ekvivalenčni razred

$$\overset{\Vert}{(x_1, 1)} \qquad \overset{\Vert}{(x_2, 1)}$$

sta ekvivalentni, ker sta na višini 1

$$\bar{H}([x, u, 0]) = [x, u]$$

$$q(H(x, u, 0)) = q(x, u)$$

$$q(H)(x, u, 1) = \dots$$

X je kontraktibilen

$\Longleftrightarrow$

$\exists$ zvezna izbira poti od poljubne točke X do izbrane točke

$\exists a \in X.\ \exists_{zv} f: X \longrightarrow \mathcal{C}(I, X).\ f(x)(0) = x,\ f(x)(1) = a$

$\Longleftrightarrow$

$X \neq \emptyset \wedge \exists$ zvezna izbira poti med točkami v X

$\exists f_{zv}: X \times X \longrightarrow \mathcal{C}(I, X).\ f(a,b)(0) = a,\ f(a,b)(1) = b$

4.8 (4.7 na učilnici)

i) $\Rightarrow$ ii) Vemo iz 4.7 (4.6)

ii) $\to$ iii) Naj bo X retrakt kontraktibilnega prostora Y

$\exists r: Y \to X.\ \exists i: X \to Y.\ r \circ i = id$

(ima zvezni desni inverz (ekvivalentna definicija retrakcije))

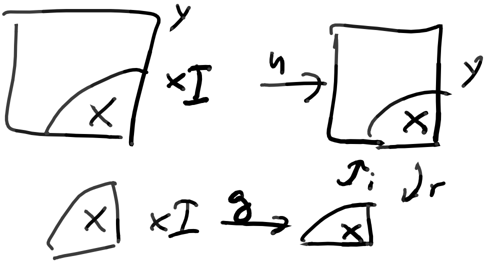
$$\exists y_0 \in Y.\ \exists H: Y \times I \to Y$$
$$(y,0) \mapsto y$$
$$(y,1) \mapsto y_0$$

iščemo

$g: X \times I \to X$ $\quad x_0 = r(y_0)$

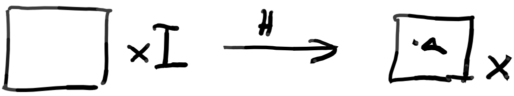
$g(x,0) = x$

$g(x,1) = x_0$



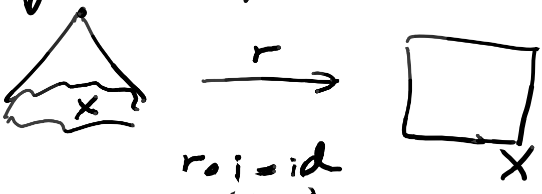
$$g(x,t) = r(h(i(x),t))$$
$$g = r \circ h \circ (i \times id_I)$$

iii) $\Rightarrow$ i)



$H(x,0) = x$

$H(x,1) = a$



$r \circ i = id$

$r([x,0]) = x$

$$CX = X \times I / \{(x,1)\}$$

$$[x,t] \longmapsto pr_1(H(x,t))$$

4.9

$$I \times I /_{\text{vemo}} = M \qquad \text{vemo} \Leftrightarrow \sim$$

CM

$$I \times I /_{\text{vemo}} \times I /_{\{I\times I, 1\}} = CM$$

Recimo da je kontraktibilen

$\exists H : I\times I/_{\text{vemo}} \times I \longrightarrow I \times I/_{\text{vemo}}$

$H(x,0) = x$

$H(x,1) = 1$

Vzamimo podprostor $A =$ [illegible] [illegible]

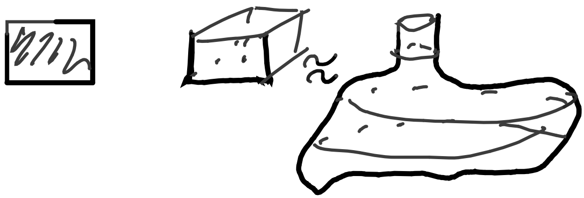
ki je krožnica $\Rightarrow$ polovka

torej je tudi A kontraktibilna

4.10

Neprazen konveksen prostor je kontraktibilen

Splošno: zvezdast prostor je kontraktibilen



$\mathcal{N}$ .. Razred normalnih topoloških prostorov

$Y \in AE(\mathcal{N})$ je absolutni ekstenzor za razred normalnih prostorov

kadar za $\forall X \in \mathcal{N}$ in $\forall$ zaprto podmnožico $A \subseteq X$ velja, da lahko zvezno preslikavo $A \to Y$ razširimo do zvezne preslikave $X \to Y$

$$\begin{tikzcd} X \arrow[drr, dotted] & & \\ A^{zap} \arrow[u, hook] \arrow[rr, "f"'] & & Y \end{tikzcd}$$

Tietze: $I \in AE(\mathcal{N})$

- Produkt AE je AE

$$\begin{tikzcd} X & \\ A \arrow[u, hook] \arrow[r] & Y' \times Y'' \end{tikzcd}$$

razširimo vsako komponento

- Retrakt AE je AE

- $X \in AE(\mathcal{N}) \Rightarrow X \neq \emptyset \wedge X$ povezan s potmi

- $AE(\mathcal{N}) \cap \mathcal{N} \subseteq AR(\mathcal{N})$

absolutni retrakt

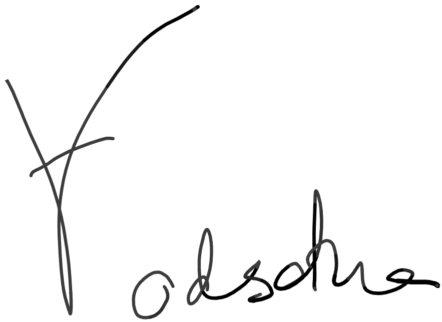
Na tabl: 4. ~~[illegible]~~ 11

4. 12 $\quad A, B^{\text{zap}} \subseteq \mathbb{R}^n$

$$\begin{tikzcd}
X \arrow[dr, dotted] & \\
C \arrow[u, hook] \arrow[r] & A \cup B
\end{tikzcd}$$

$\mathbb{R}^n \in AE$

Dokažimo $A \cup B$ je retrakt $\mathbb{R}^n$



4.19

$v \in \mathbb{R}^n \setminus \{0\}$ $\qquad Av = \lambda v$

BSZS $\|v\| = 1$

$\|Av\| = \|\lambda v\| = |\lambda| = \lambda$

$Av = \lambda v \stackrel{?}{=} \|Av\| v$

$$v = \frac{Av}{\|Av\|}$$

iščemo negibno točko preslikave

$S^{n-1} \longrightarrow S^{n-1}$

$v \longmapsto \dfrac{Av}{\|Av\|}$ ne nujno dobro definirana preslikava, ker $\ker A$ ni nujno $\{0\}$

$X = \{(x_1, \dots x_n) \in S^{n-1} \mid x_1, \dots, x_n \geq 0\} \approx B^{n-1}$

če $v \in X$ so vsi $x_i \geq 0$ in niso vsi $= 0$

$\Rightarrow AV$ ima pozitivne komponente, posebej $\neq 0$

$B^{n-1} \longrightarrow B^{n-1}$ ima negibno točko

Mnogoterost

a) $\{\frac{1}{n}, n \in \mathbb{N}\}$ števnost + diskretnost

= je mnogoterost

b) $\{\frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$

v 0 ni mnogoterost

če bi jo imela bi morala biti iz $\mathbb{R}^0 = \{*\}$

ali $\mathbb{R}^0_+$ ker ne dosega

torej točka 0 mora v vsaki okolici

vsebovati delci ~~$\{\}$~~ od a ker pomeni

da mora biti $\{0\}$ odprta, ker pa ni:

$$\boxed{\text{Števna vsota n-mnt je n-mnt}}$$

20.5

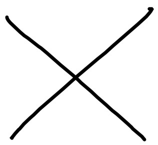
c) $\approx \overset{0\text{ dim}}{\{-1,1\}} \times \overset{1\text{ dim}}{\mathbb{R}} \approx \mathbb{R}+\mathbb{R}$

Produkt mnogoterosti je mnogoterost

$\Rightarrow$ 1 mnogoterost

Tudi: števna vsota n-mnogoterosti je n mnogoterost

d)



ni mnogoterost v $(0,0)$

$(1,1)$ ima 1-mnogoterost na delcu

Torej v $(0,0)$ mora biti 1 mnogoterost

Trdimo da ni: $\approx \mathbb{R}$ ali $\mathbb{R}_+$

naj bo U poljubna okolica točke

Ni dovolj preverjati na bazi

$U - \{(0,0)\}$ ima vsaj 4 komponente

če bi U bil homeomorfen $\mathbb{R}$ ali $\mathbb{R}_+$

bi razpadel na 1 ali 2 komponenti

$\rightarrow$ ~~ni~~ ~~[illegible]~~ U ni ~~[illegible]~~

ni mnogoterost

Naredimo vložitev 1dim mnogoterosti brez roba in ∞ preslikava ni odprta pa dom ni mnogoterost

---

$f: \mathbb{R} \longrightarrow X$

$f(x) = (x,x)$

$f_*(\mathbb{R})$ mora biti odprt

$(0,0)$ je robna točka za sliko

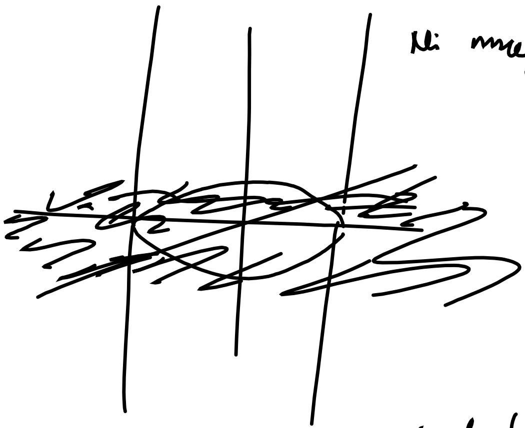
ker vsebuje $(\varepsilon,-\varepsilon)$ za dovolj majhne ε v vsaki okolici

e) $(\mathbb{R}^2 \times \{0\}) \cup (S^1 \times \mathbb{R})$

(za d nalogo)

Izrek o invarianci odprtih množic za mnogoterosti

Če sta M, N istodimenzionalni mnogoterosti M brez roba in $f: M \to N$ zvezna injekcija, potem je f odprta vložitev v $\operatorname{int} N$



Ni mnogoterost

ker bi bil vložitev $f: \mathbb{R}^2 \to X$

$x \mapsto (x, 0)$

(ampak bi bil "mnogoterost" (kot v $(0,0)$))

$0,0$ ima okolico $K_{\mathbb{R}^2}((0,0), \frac{1}{2}) \times \{0\} \approx \mathbb{R}^2$

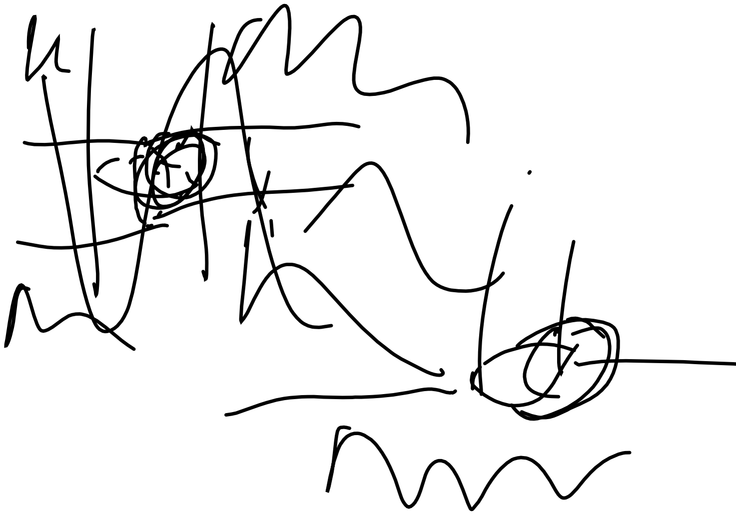
$f_*(\mathbb{R}^2)$ ni odprta ker

$(1,0,0)$ je robna za S^1 ...

$\forall U \ni (0,0,0)$

$(1,0,\varepsilon) \in U$

f)



vložitev:

$\swarrow$ z mnogoterosti

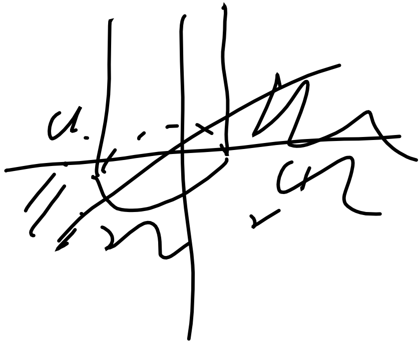
$f: S^1 \times \{R\} \to X$

$x \mapsto x$

robov $(1,0,0)$

$\partial(M \times N) = \overset{\emptyset}{\partial M} \times N \cup M \times \overset{\emptyset}{\partial N}$

$(1, 0, \overset{\varepsilon\, 0}{\cancel{0}})$ je v robu [illegible] ze [illegible] g)



je 2 mnogterost

$\approx S^1 \times \mathbb{R}$

$f: X \longrightarrow S^1 \times \mathbb{R}$

$x = re^{i\varphi}, r = 1$

$f:\quad x \longmapsto x$ ~~$\varphi \in \ldots$~~ ~~$x \ldots = 1$~~

$(re^{i\varphi}\ 0) \longmapsto (e^{i\varphi}, -r+1)$

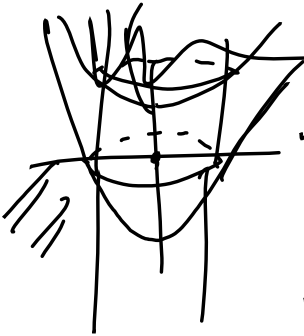
$r=1 \Rightarrow e^{i\varphi}, -1+1 =$ se ujame

elementarne

inverz

$$e^{i\varphi}, z \longmapsto \begin{cases} e^{i\varphi}, z & z \geq 0 \\ (-z+1)\, e^{i\varphi}, 0 & z \leq 0 \end{cases}$$

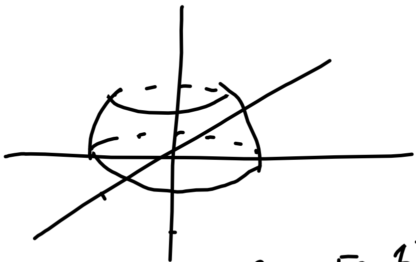
h)



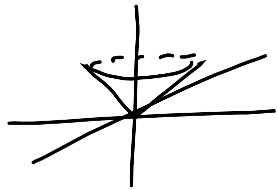
ni [illegible]

Vlasto mapen

5.2



$\approx S_1 \times [0, \frac{1}{2})$



$\approx B_n$

$\downarrow$

pr opheri-ga

$\leftarrow$

prej dokaz je

DN

nima
lastnosti
nesočne točke

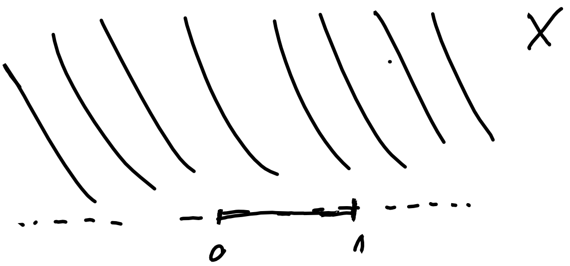
ima
lastnost
nesočne točke

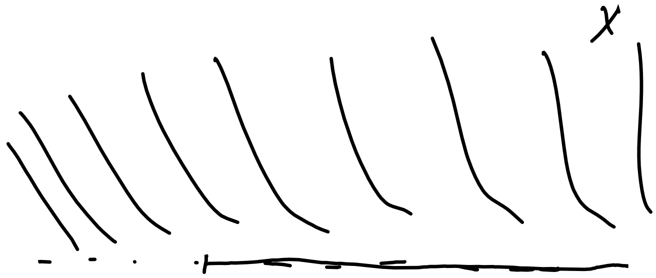
črta gre za $90°$

Rob ima
2 komponenti

Rob ima
eno komponento

5.3





X ima 2 nemnogoterostni točki

Y pa ima 1

v (0,0) ni mnogoterost ker ni lokalno kompaktna v tej točki

$Z = \mathbb{R} \times [0,\infty)$ Recimo da sta si homeomorfna

~~$Z \to X$ v zaprto~~

~~$Z \to Y$ v notranjost~~

~~Rec~~ vložimo $[0,\infty)$ tako v

$$\underbrace{Z \hookrightarrow X \xrightarrow{h} Y}_{\text{zvezna injekcija se slika v notranjost}} \hookrightarrow \mathbb{R}^2$$

$h_*(Z)$ je odprta pod $\mathbb{R}^2$

in $h_*(Z) \subseteq Y \Rightarrow$ odprta v Y

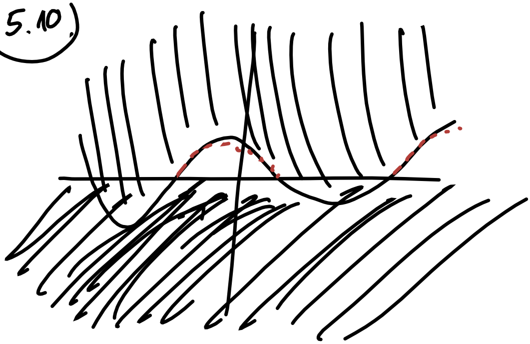
$\Rightarrow h_*(Z) \subseteq Z$

in enako pri inverzu

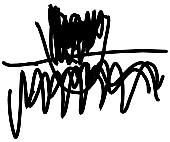
$(h^{-1})_*(Z) \subseteq Z$

$\Rightarrow h|_{[0,1]\times\{0\}} \to [0,\infty)\times\{0\}$ je dobro definiran homeomorfizem $\times$

5.10

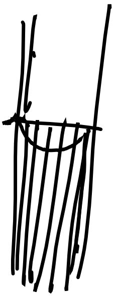


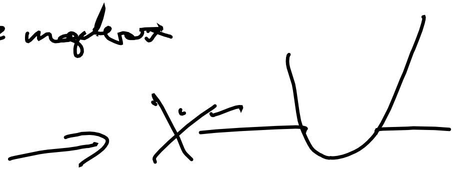
a) $p_{-1}(x) = x^2 - 1$



X_{p-1} ni lok. kompakten v $(-1, 0)$ ker se vedno dobi zaporedje $(-1, \varepsilon_n)$

Y_{p-1} je mnogoterost



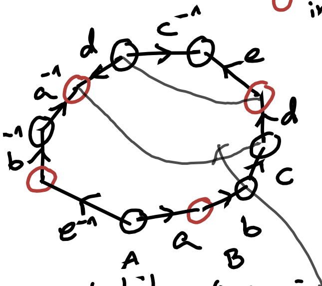


$-\pi$ $\quad$ 0

-1 $\quad$ 1

$x, y \longmapsto (y + \sqrt{1-x^2}\, e^{i\frac{(x+1)}{2}\pi}$

○ in ○ sta si enaka



ker se pojavi:
vsaka črka natanko dvakrat
je to mnogoterost

Ni orientabilna, ker je möbiusov trak

0-celice: 1
1-celice: 5
2-celice: 1

$\chi(X) = -3$

ker je neorientabilna bo to nP

$$\boxed{\chi(nP) = 2 - n}$$

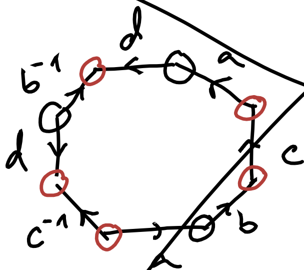
$$\boxed{\chi(nT) = 2 - 2n}$$

$$2 - n = -3$$

$$n = 5$$

$$X \approx 5P$$

je ploskev
neorientabilna
ni sklenjena

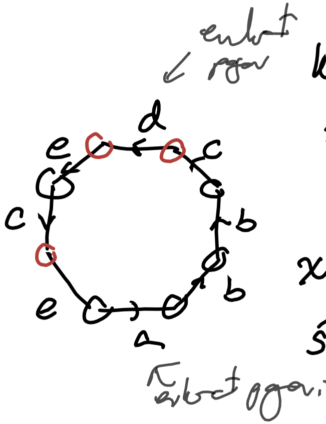


0-celice: 2

1-celice: 5

2-celice: 1

napačna
naloga :(



komp. plosku

n: sklenjene

n: orientabilna

$\chi(D) = 2 - 5 + 1$

št robnih komponent: 2

$D' = D$ kjer na vsako robno komponento prilepimo disk:

$\Longrightarrow$ dodamo 2 2-celici:

0-celice: 2
1-celice: 5
2-celice: 3

$\chi(D') = 0$

nP:

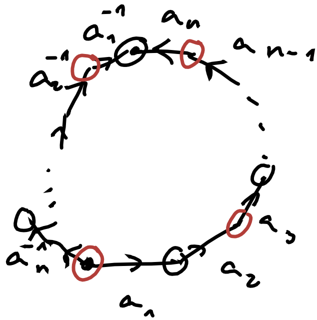
$2 - n = 0$

$n = 2$

$D' = 2P$

hehe

$\Rightarrow$ D je 2P ~~brez~~ ~~dveh~~ z dvema luknjama



kompaktna orientabilna sklenjena

n sodo: 0-celic: 1

n liho: 0-celic: 2

1-celice: n

2-celic: 1

n sodo: $\chi(G) = 1 - n + 1 = 2 - n$

n liho: $\chi(G) = 2 - n + 1 = 3 - n$

$$2 - 2m = 2 - n \qquad\qquad 2 - 2m = 3 - n$$

$$2m = n \qquad\qquad m = \frac{n-1}{2}$$

$$m = \frac{n}{2}$$

$$G \approx \frac{n}{2} T \qquad\qquad G \approx \frac{n-1}{2} T$$

5.14

a) sistem enačb:

$X \# Y \approx 2T \qquad X \# T \approx Y \# K$

$\partial(X\#Y) \approx \partial X + \partial Y$

$X, Y \overset{\Rightarrow}{} $ sklenjeni orientabilni

X neorientabilno $\Longrightarrow$ ni rešitve

b) $X \# Y \# Z \approx 2K \# T$ ← sklenjena ploskev

$X \# Y \approx Z \# 2T$

$X \# Z \approx Y \# K$

X ali Z neorientabilna

X, Y, Z sklenjene

Z neorientabilen $\Rightarrow$ X ali Y sta neorientabilna

$$\boxed{\chi(X\#Y) = \chi(X) + \chi(Y) - 2}$$

$x + y - 2 + z - 2 \approx \chi(4P) + \chi(T) - 2$

$x + y + z - \cancel{4} \approx \cancel{2-4} + \cancel{2} - \cancel{2} - \cancel{2}$

$x + y + z = 0$

$2K + T$ je neorientabilna $\Rightarrow$ $2 - n = 0$

$X \# Y \# Z \approx 2P$

$x + y - 2 = z + 2 - 2\cdot 2 - 2$

$x + y - z = 2 - 4 = -2$

$x + z - 2 = y + 2 - 2$

$x - y + z = 2$

$2y = -2 \qquad 2z = 2 \qquad x = 0$

$y = -1 \qquad z = 1$

$$\boxed{\text{Orientabilne ploskve imajo sodo } \chi}$$

Z neorientabilna $\Rightarrow$

$X \approx 3P$

$Z \approx P$

Y je lahko orientabilna ali ne

$Y \approx T$ če orientabilna

$Y \approx \underset{\substack{\| \\ K}}{2P}$ če neorientabilna

$X \# M \approx Y \# P$

$\boxed{\pi_0 \text{ je aditivno}}$

$X \# Y \approx M \# 3P$

$\partial(M \# N) = \partial M + \partial N$

$\pi_0(\partial Y) = 1 + \pi_0(\partial X)$

št komponent za povezanost s potmi = št robnih komponent

$\pi_0(\partial X) + \pi_0(\partial Y) = 1$

$\pi_0(\partial X) + \cancel{1} + \pi(\partial X) = \cancel{1}$

$\pi_0(\partial X) = 0 \Rightarrow \pi_0(\partial Y) = 1$

X sklenjena

$\boxed{\text{Möbiusov trak} = \text{projektivna ravnina brez enega diska}}$

$\chi(M) = \chi(P) - 1 = 0$

$x + y - \cancel{2} = 0 + 2 - 3 - \cancel{2}$

$x + 0 - \cancel{2} = y + 1 - \cancel{2}$

$x - y = 1$

$x + y = -1$

$x = 0$

$y = -1$

$X \approx T \lor X \approx 2P$

$X \approx T \Rightarrow Y \approx T$

5.15

a)

orientabilna

$\chi(A)$

$\chi(X \cup \text{Trak}) = \chi(X) - 1$ 

$B^n \cup_{\partial B^n} B^n \cong S^n$

- - - - - . - - - - -

$B^n + B^n \longrightarrow S^n$

$in_1 \; x_1 \ldots x_n \longmapsto (x_1 \ldots x_{n-1}, \sqrt{1 - x_1^2 - x_2^2 - \ldots - x_{n-1}^2})$

$in_2 \; x_1 \ldots x_n \longmapsto (x_1 \ldots x_{n-1}, -\sqrt{1 - x_1^2 \ldots - x_{n-1}^2})$

vzemi:

prave identifikacije:

$\|x_1 \ldots x_n\| = 1 \longmapsto x_1, \ldots x_n, 0$

$\longmapsto x_1 \ldots x_n, 0$

je kvocient na eno razredih

če $\vec{r} \sqrt{1 - |r|} = r, -\sqrt{1-|r|}$

potem $\vec{r} = \vec{r}$

$\sqrt{1-|r|} = -\sqrt{1-r}$

$\Rightarrow 1 - |r| = 0$

$|r| = 0$

surjektivnost

$f: S^{n-1} \to S^{n-1}$

$F: B^n \to B^n \quad F|_{S^{n-1}} = f$

$$F(\vec{r}) = \begin{cases} \|\vec{r}\| f\left(\frac{\vec{r}}{\|\vec{r}\|}\right) \\ 0 \; ; \; \vec{r} = 0 \end{cases}$$

$$\lim_{t \to 0} \|t \cdot e\| \, f\left(\frac{t\vec{e}}{\|t\|}\right) = \lim_{t \to 0} t \cdot f(e) = 0$$

c) $D_1 \approx B^n \approx B_2$

$X \approx S^n$

$X = D_1 \cup D_2 \longrightarrow S^n$

$h_1: D_1 \to B_n$ hom.

$h_2: D_2 \to B_n$

$$x \mapsto \begin{cases} h_1(x) & ; \sqrt{1 - h(x)^2} \\ h_2(x) & -\sqrt{1 - h(x)^2} \end{cases}$$ ni dobro definirano

$\partial D_1 \xrightarrow{h_1/\text{zožen}} S_{n-1}$

∂D_2

$||$

$S_{n-1} \xrightarrow{\approx} \partial D_1 \xrightarrow{h_1/S^{n-1}} S_{n-1}$

obstaja F ~~da~~ velja $B_n \to B_n$

$F: B_n \to B_n$

$F/_{S_n}$: ✓

F ~~(h_2^{-1})~~:

$F \circ (h_2^{-1}): D_2 \longrightarrow B_n$

$$x \begin{cases} h(x) \; , \; \sqrt{1 - h_2(x)^2} \\ F \circ (h_2^{-1}(x)) \; , \; -\sqrt{1 - |F(h_2(x))|} \end{cases}$$