$x = 0,1$

Pokaži da velja $*$

$* \quad x = \sum_{i=1}^{\infty} (2^{-4i} + 2^{-4i-1}$

b) binarni zapis za x

c) zapiši v IEEE formatu

IEEE754

enojna natančnost $P(2, 24, -125, 128)$

$$(-1)^{\sigma} (1+m)\, 2^{\tilde{e}-127}$$

m dolžine 23

$\tilde{e}$ dolžine 8

σ dolžine 1

dvojna natančnost $P(2, 53, -1021, 1024)$

$$(-1)^{\sigma} (1+m)\, 2^{\tilde{e}-1023}$$

m 52

$\tilde{e}$ 11

σ 1

a) $X = \sum_{i=1}^{\infty} 2^{-4i} + \ldots = \frac{\frac{1}{16}}{1-\frac{1}{16}} + \frac{1}{2}\,\frac{\frac{1}{16}}{1-\frac{1}{16}} = \frac{\cancel{3}}{2\cdot\cancel{16}}\,\frac{\cancel{16}}{\cancel{15}\,5} = \frac{1}{10}$

$= 0{,}1$

b) $0{,}000110011001 1 = 0{,}0\overline{0011}_{(2)}$

c)

$1{,}1\overline{0011}\cdot 2^{-4} =$

$1 + 0{,}1\overline{0011}\cdot 2^{-4}$

$0{,}100110\ldots\ldots001101$

$\tilde{e} - 127 = -4 \Rightarrow \tilde{e} = 123 = 1111011$

$123 : 2 = 61 \quad 1$

$61 : 2 = 30 \quad 1$

$30 : 2 = 15 \quad 0$

$15 : 2 = 7 \quad 1$

$7 : 2 = 3 \quad 1$

$3 : 2 = 1 \quad 1$

$1 : 2 = 0 \quad 1$

$x = 2^{-1} + 2^{-k} + 2^{-t}$

$x^2 + y^2$ izračunamo z izrazom

$y = 2^{-1} + 2^{-k}$

$x \cdot x - y \cdot y$

$k = \frac{t}{2} + 1$

$t = 2k - 2$

z obravnavo relativne napake pokažite da izračuni dosledno stabilen

$$x^2 = (2^{-1} + 2^{-k} + 2^{-t})^2 = 2^{-2} + 2^{-2k} + 2^{-2t} + 2 \cdot 2^{-1-k} + 2 \cdot 2^{-k-t} + 2 \cdot 2^{-t-1} =$$

$$\cancel{2^{-2}} + \cancel{2^{-2k}} + \underset{\substack{\| \\ 2^{-2t}}}{2^{-4k+4}} + \cancel{2^{-k}} + 2^{-k-t+1} + 2^{-t}$$

$$y^2 = (2^{-1} + 2^{-k})^2 = \cancel{2^{-2}} + \cancel{2 \cdot 2^{-k}} + \cancel{2^{-2k}}$$

$$x^2 - y^2 = 2^{-4k+4} + 2^{-k-t+1} + 2^{-t}$$

$$= 2^{-2t} + 2^{-t} + 2^{-k-t+1}$$

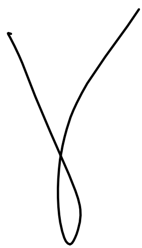
$$fl(x^2) = \overset{}{0{,}0}\overset{-2}{1}\ldots\overset{-k}{1}\ldots\overset{-t}{1}\,0\,\vdots\,\overset{-2k}{1}\ldots\overset{-t-k+1}{1}\ldots =$$

$$= 0{,}01\ldots1\ldots\overset{-t}{1}1$$

$$fl(y^2) = 0{,}01\ldots\overset{-k}{1}\ldots\overset{-t}{0}0\,\vdots\,1 = 0{,}01\ldots\overset{-k}{1}\ldots01\overset{-t}{0}$$

ni več desno zadi

zato izberemo število s sodo zadnjo števko, ker je 1 izven meje -2^{k} zadnjega mesta v ~~[illegible]~~ zapisu



22.10

$g(x) = -x^2 + 8x - 12 \qquad x_{r+1} = g(x_r)$

$\lim\limits_{r\to\infty} x_r = 4$ za $\forall x_0 \in (3,5)$

$\iff$

$\lim\limits_{r\to\infty} |x_r - 4| = 0$

$= |x_{r+1} - 4|$

$\lim |x_{r+1} - 4| = \lim\limits_{r\to\infty} |-x_r^2 + 8x_r - 16| =$

$= \lim\limits_{r\to\infty} |x_r - 4|^2 = \lim\limits_{r\to\infty} |x_{r-1} - 4|^4 = \dots = \lim\limits_{r\to\infty} |x_0 - 4|^{2^{r+1}}$

konvergira za $|x_0 - 4| < 1 \to x_0 \in (3,5)$

proti 0

Red konvergence

$\alpha \ldots$ negibna točka

$$g'(\alpha) = g''(\alpha) = \ldots = g^{(p-1)}(\alpha) = 0 \qquad g^{(p)} \neq 0$$

potem je red konvergence enak p

a) pokažite da lahko $\sqrt{\ }$ pozitivnega števila $a>$ izračunamo z iteracijo $x_{r+1} = \underbrace{x_r \cdot \frac{x_r^2+3a}{3x_r^2+a}}_{g(x_r)}$

b) določite red konvergence

c) dokažite da iteracija konvergira k $\sqrt{a}$ za $\forall$ začetni približek $x_0>0$

1. Preverimo če je $\sqrt{a}$ negibna točka iteracijske funkcije

$$\sqrt{a} \cdot \frac{\sqrt{a}^2+3a}{3\sqrt{a}^2+a} = \sqrt{a} \cdot \frac{4a}{4a} = \sqrt{a}$$

1. negibnost
2. privlačnost

2. preverjamo privlačnost $g'(\sqrt{a}) < 1$

$$g(x) = \frac{x^3+3ax}{3x^2+a}$$

$$g'(x) = \frac{(3x^2+3a)(3x^2+a) - 6x(x^3+3ax)}{(3x^2+a)^2} =$$

$$g'(\sqrt{a}) = \frac{(3a+3a)(3a+a) - 6a^2 - 18a\cdot a}{(3a+a)^2} =$$

$6a \cdot 3a + 6a^2$

$$= 0 < 1$$

b) $$g''(x) = \left(3\frac{\cancel{3}(x^2+a)(3x^2+a) - \cancel{6}\,2x^2(x^2+3a)}{(3x^2+a)^2}\right)' =$$

$$= 3\frac{\left((2x+a)(3x^2+a) + 6x(x^2+a) - 4x(x^2+3a) - 4x^3\right)(3x^2+a)^{2}}{}$$

$$- \frac{2\cancel{(3x^2+a)}\left((x^2+a)(3x^2+a) - 2x^2(x^2+3a)\right)}{(3x^2+a)^{\cancel{4}\,3}}$$

$$= -3\frac{(6x^3+3ax^2+2ax+a+6x^3+6a-4x^3+12xa-4x^3)(3x^2+a)^2}{}$$

$$- \frac{\qquad}{}$$

$$g'' = \frac{\overset{h_1}{48xa}\;\overset{h_2}{(x^2+a)}}{(3x^2+a)}$$

$$g''' = h_1'h_2 + h_1h_2' = h_1'(x)(x^2+a) + h_1(x)2x$$

$$g'''(\sqrt{a}) = h_1(\sqrt{a}) \cdot 0 + h_1(\sqrt{a})2\sqrt{a}$$

$$h_1\sqrt{a} = \frac{48a\sqrt{a}}{3a+a}$$

$$g'''\sqrt{a} = \frac{48 \cdot 2a^2}{(4a)^3} = \frac{3}{2a} \quad >0 \neq 0$$

$\Rightarrow$ red konvergence $=3$

c) $g(x) = x\frac{x^2+3a}{3x^2+a}$

Ločimo primere ko $x_0 < \sqrt{a}$

$x_1 > x_0$?

$x_1 > \sqrt{a}$ ali $x_1 < \sqrt{a}$?

$$x_1 = \cancel{x_0}\frac{x_0^2+3a}{3x_0^2+a} > \cancel{x_0}$$

$$x_0^2+3a > 3x_0^2+a$$

$$2a > 2x_0^2 \quad \checkmark$$

$$\sqrt{a} > x_0 \quad \checkmark$$

$x_1 < \sqrt{a}$

$$x_0(x_0^2+3a) < \sqrt{a}(3x_0^2+a)$$

$$\Leftarrow$$

$$\sqrt{a}(x_0^2+3a) < \sqrt{a}(3x_0^2+a)$$

$$2a < 2x_0$$

Sledi: $x_1 < \sqrt{a}$

Torej x_r naraščajo in so omejeni

konvergirajo nekam

konvergira k $\sqrt{a}$?

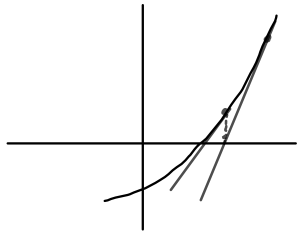
$x_0 \in (\sqrt{a}, \infty)$

pokažemo da imamo padajoče navzdol omejeno zaporedje

Tangentna metoda

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$x_{r+1} = g(x_r)$$



Babilonska metoda za računanje $\sqrt{a}$ $a>0$ temelji na iteraciji

$$x_{r+1} = \frac{1}{2}\left(x_r + \frac{a}{x_r}\right)$$

a) Preverite, da iteracija ustreza tangentni metodi za funkcijo $f(x) = x^2 - a$

b) določite red konvergence

c) Dokažite da iteracija konvergira k $\sqrt{a}$ za $\forall x_0 > 0$

a)
$$g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right) \stackrel{?}{=} x - \frac{f(x)}{f'(x)} = x - \frac{x^2-a}{2x} =$$
$$= \frac{2x^2 - x^2 + a}{2x} = \frac{1}{2}\left(x + \frac{a}{x}\right) \checkmark$$

b)
$$g'(x) = \frac{1}{2}\left(1 - \frac{a}{x^2}\right) \rightsquigarrow g'(\sqrt{a}) = 0$$
$$g''(x) = \frac{2}{2}\left(\frac{a}{x^3}\right) = \frac{a}{2x^3} \quad \frac{1}{\sqrt{a}} > 0$$

red konvergence je 2

c) $g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$

g nenaraščajoča za $x \in (\sqrt{a}, \infty)$

$\sqrt{a} < x_{r+1} < x_r$

še za

$\sqrt{a} < \frac{1}{2}\left(x_r + \frac{a}{x_r}\right) < x_r$

$2\sqrt{a}x_r < x_r^2 + a < 2x_r^2$

$a < x_r^2$ ✓

$x_r^2 - 2\sqrt{a}x_r + a > 0$

$(x_r - \sqrt{a})^2 > 0$

padajoče navzdol omejeno zaporedje

$x_0 \in (0, \sqrt{a}) \Rightarrow x_1 \in (\sqrt{a}, \infty)$

$x_1 = \frac{1}{2}\left(x_0 + \frac{a}{x_0}\right) \overset{?}{>} \sqrt{a}$

$x_0^2 + a > 2x_0\sqrt{a}$

$x_0^2 - 2x_0\sqrt{a} + a > 0$

$(x_0 - \sqrt{a})^2 > 0$ ✓

Naj bo $f \in \mathcal{C}^2$ α njena enostavna ničla

a) Dokažite, da metoda

$$x_{r+1} = x_r - \frac{2f(x_r)\cdot f'(x_r)}{2f'(x_r)^2 - f(x_r)f''(x_r)}$$

ustreza tangentni metodi za funkcijo $F(x) = \dfrac{f(x)}{\sqrt{|f'(x)|}}$

b) poenostavite metodo za $f(x) = x^2 - a$

a)
$$F'(x) = \frac{f'(x)\sqrt{|f'(x)|} - \sqrt{|f'(x)|}'}{|f'(x)|} =$$

$$\sqrt{|f'(x)|}' = \frac{f''(x)}{2\sqrt{|f'(x)|}} \, \frac{f'(x)}{|f'(x)|}$$

$$= \frac{\dfrac{f'(x)}{\sqrt{|f'(x)|}}\left(1 - \dfrac{f''(x)}{2|f'(x)|}\right)}{|f'(x)|} = \frac{\operatorname{sgn} f \left(1 - \frac{1}{2}\operatorname{sgn} f\right)}{\sqrt{|f'(x)|}} =$$

$$= \frac{\operatorname{sgn} f - \frac{1}{2}}{\sqrt{|f'(x)|}}$$

$$\frac{F(x)}{F'(x)} = \frac{f(x)\,\cancel{\sqrt{|f'(x)|}}}{\cancel{\sqrt{|f'(x)|}}\left(\operatorname{sgn} f - \frac{1}{2}\right)} = \underline{\underline{\frac{f(x)}{\operatorname{sgn} f - \frac{1}{2}}}}$$