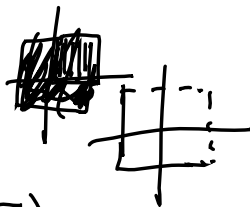


1.1.

$$A = [-1, 1] \times [-1, 1]$$

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}/A$$

hoc



a)  $g^*((-\infty, 0] \times (-\infty, 0])$

ni:  $-\infty$  do  $0$  ni:  $0$  do  $-\infty$

$$g^*((-\infty, 0] \times (-\infty, 0])$$

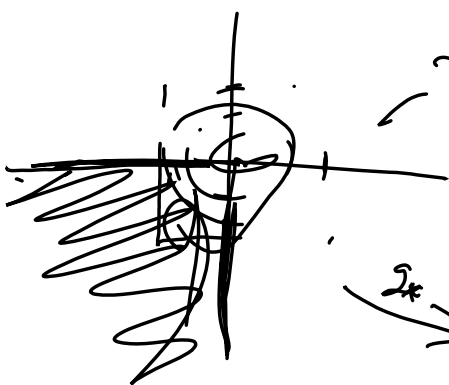
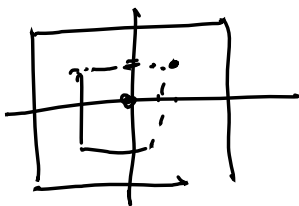
b)  $g^*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

ni:  $0$  do  $-\infty$   
ni:  $-\infty$  do  $0$



c)  $g^*([-2, 2] \times [-2, 2])$

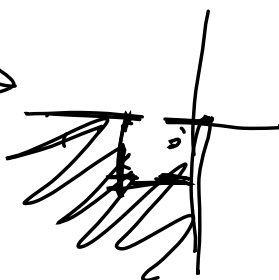
ni:  $-2$  do  $2$



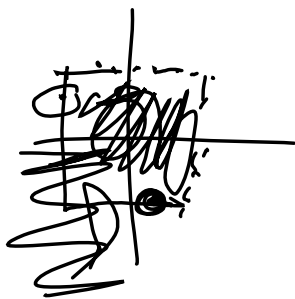
$g^*$



$g^*$

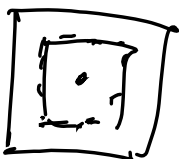


$g^*$

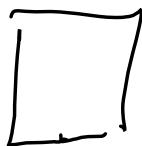


5)

$$g^*([2, 2] \times [2, 2])$$



$\rightarrow$

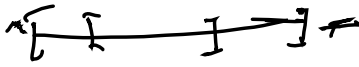


ni:  $2$  do  $2$

ni:  $2$  do  $2$

d) ni:  $2$  do  $2$

14

$$a) [-2, 2] / [-1, 1] \cong [-1, 1]$$


$$b) [-1, 1] / \{0\} \cong \mathbb{R} \cup \{\infty\}$$



$$\cong \mathbb{R} \cup \{\infty\}$$

$$c) \mathbb{R} / \mathbb{Z}$$

8

$$\mathbb{R}^n /_{K(0,1)} \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{array}{ccc} \mathbb{R}^n & \xrightarrow{f} & \mathbb{R}^n \\ \downarrow & & \\ \mathbb{R}^n /_{K(0,1)} & & \end{array}$$

$$f(\vec{a}) := \begin{cases} 0; & \text{ce } |\vec{a}| \leq 1 \\ \vec{a} - \frac{\vec{a}}{\|\vec{a}\|}; & \text{ce } \vec{a} \in \mathbb{R}^n - \overline{K(0,1)} \end{cases}$$

$$d) \quad \mathbb{R}^n / \sim \quad x \sim y \Leftrightarrow \|x\| = \|y\|$$

$$\mathbb{R}^n / \sim \xrightarrow{\sim} [0, \infty)$$

$$f: \mathbb{R}^n \longrightarrow [0, \infty)$$

$$f(\vec{x}) = \|\vec{x}\| \text{ je sur}$$

$$a \in [0, \infty) \quad (a, 0, 0, \dots) \mapsto a$$

$$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$$

je zvezna

iščemo preslikavo  $s$ , da

$$\text{velja } f \circ s = \text{id}_{[0, \infty)}$$

$$s: a \mapsto (a, 0, \dots)$$

Dokazimo da je  $r \circ s = \text{id}_Y$

$\Rightarrow r$  kvocientna, surjekcija

$$S \subseteq Y \text{ takoda } r^*(S) \text{ odpr } \vee X$$

$$S = \dots \dots \dots s^*(r^*(S)) =$$

$$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$$

$$\text{id}_Y^*(S) = S$$

$\Rightarrow S$  je odprta

f)

$$S^n \times [-1, 1] / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \rightarrow S^{n+1}$$

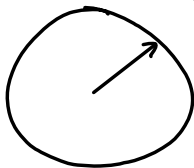
$$(x, t) \mapsto (xb, t) = (\sqrt{1-t^2}x, t)$$

$\nearrow$   
anotado valla

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{g \cdot h; g \in G\} =$$

$$= \{ \{g \cdot h; h \in H\}; g \in G \}$$

$G$  top. grupe

$$a \in G$$

$$L_a: G \longrightarrow G$$
$$x \longmapsto a \cdot x \quad \text{leva transkacija}$$

$$a, b \in G$$

$$h: G \longrightarrow G$$

$$h(a) = b \quad h = ?$$

$$L_{a^{-1}}: x \longmapsto ba^{-1}x$$

možnosti

$$ba^{-1}x$$
$$xa^{-1}b$$
$$bxa^{-1}$$
$$a^{-1}xb$$

topološke grupe zahtevajo

povsod isto, ker lahko vzelo

točko prestikano v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$  desica  $\Leftrightarrow b a^{-1} A$  desica  $b \in G$

$$\exists U^{*dr} \subseteq A, a \in U$$

$$b \in b a^{-1} A$$

$$a \in U \Rightarrow b a^{-1} a \in \underbrace{b a^{-1} U}_b \subseteq b a^{-1} A$$

ker je  $L_{b a^{-1}}$  homeomorfizem je

$$b a^{-1} U \stackrel{\subseteq A}{\text{odprta v } G}$$

$\Leftarrow$  potem velja tudi obratno

b)  $H \leq G$        $H$  dedice 1  $\Rightarrow H$  odv: n zap v  $G$

$$a \in aH \subseteq H$$

$\Rightarrow H$  je dedica  
vsake svoje tocke

$G-H$  je odv.

$$aH \cap H = \emptyset \Rightarrow aH = H$$

$$a \in G-H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$$

vsek element ima dedico ki ne  
seka  $H \Rightarrow H$  je zaprta



c) C komponente  $1 \in C$

$\Rightarrow C$  zaprt edinka v  $G$

$$C \subseteq G$$

$L_a: x \mapsto ax$  je homomorfizem za  $\forall a \in C$

$$\forall a \in C. L_a \subseteq C$$

$L_a$  ohranja povezanost

$$\text{Pravi tako } L_a \cdot 1 = a \Rightarrow$$

$$L_a \cdot C \cap C \Rightarrow L_a C \subseteq C$$

invertiranje: invertiranje je tudi  $^x$  homeo  $i: x \mapsto x^{-1}$   
 $\Rightarrow$  po istih argumentih ~~na~~

Ali je edinka?

$$\forall a \in G. aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$$

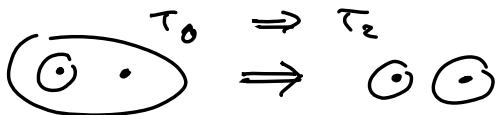
$x \mapsto axa^{-1}$  je homeomorfizem

$\Rightarrow$  je povezano in  $a \text{ id } a^{-1} = \text{id} \in C$

$$\Rightarrow aCa^{-1} \subseteq C$$

$\rightarrow$  ker je kompozitna tvel transkacij  
(leva in desna)

d) za  $G$  je



$a, b \in G$

memo  $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U, b \notin U \quad \text{BŠZS}$

$$U^{-1} = \{a^{-1} : a \in U\}$$

$$aU^{-1}b \ni b \quad a \mapsto b$$

Predmet da  $a \in aU^{-1}b$

$$\exists c \in U, a = ac^{-1}b$$

$$\Rightarrow b = c \Rightarrow b \in U \quad \times$$

$$\tau_1 \Rightarrow \tau_2$$

$$\exists U, V \subseteq G, \quad a \in U, b \in V, \quad a \notin V \quad L \notin U$$

$$\Delta \subseteq G \times G \text{ je zgrajen } \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$\Delta_0 = f^*(\{1\})$$

$$f: (k, x) \mapsto xy^{-1}$$

za vsake preslikave  $\Rightarrow k$  je zgrajen

$$\textcircled{2} \quad T_{cc} \in T_1 \text{ in } n: T_2$$

$$\Rightarrow (\mathbb{R}, +) \text{ nichtgruppe zu } T_{cc}$$

$$(\text{ker } 1, d)$$

$$(2.3) \quad \mathbb{Z} \times \mathbb{Z} \hookrightarrow \mathbb{R}^2$$

$$A \quad (m, n)(x, y) := (m+x, n+y)$$

$$\Phi: g \mapsto (a \mapsto g \cdot a)$$

bijekcija (izomorfizam)

$$\Phi: G \rightarrow \text{Bij}(A) \leftarrow \text{grupa}$$

za kompozicije

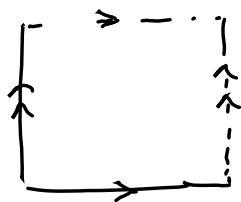
$$\text{Ubišdu: } \Phi: G \rightarrow (\text{Konec}(A), \circ)$$

$$\mathcal{L}(AA)$$



$$\text{mamo } [0, 1) \cong \mathbb{R}^2 / \Phi$$

Na nivouju množice  
za nivouju topologije? NE



Dobimo torus

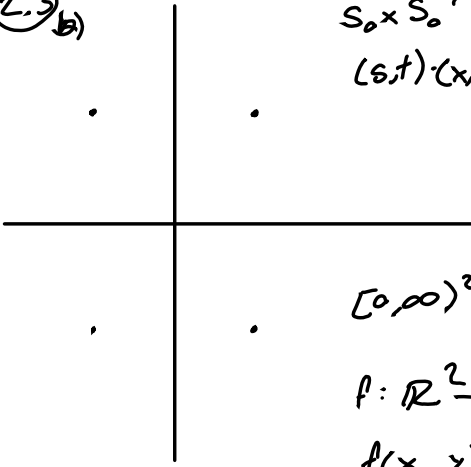
$$\bigcirc \cong S^1 \times S^1$$

$$f: \mathbb{R}^2 \rightarrow S^1 \times S^1$$

$$(x, y) \mapsto (e^{2\pi i x}, e^{2\pi i y})$$

Tuk nismo pokazali več :

(2.3) b)



$$S_0 \times S_0 \hookrightarrow \mathbb{R}^2$$
$$(s, t) \cdot (x, y) = (sx, ty)$$

$$[0, \infty)^2$$

$$f: \mathbb{R}^2 \rightarrow [0, \infty)^2$$

$$f(x, y) = (|x|, |y|)$$

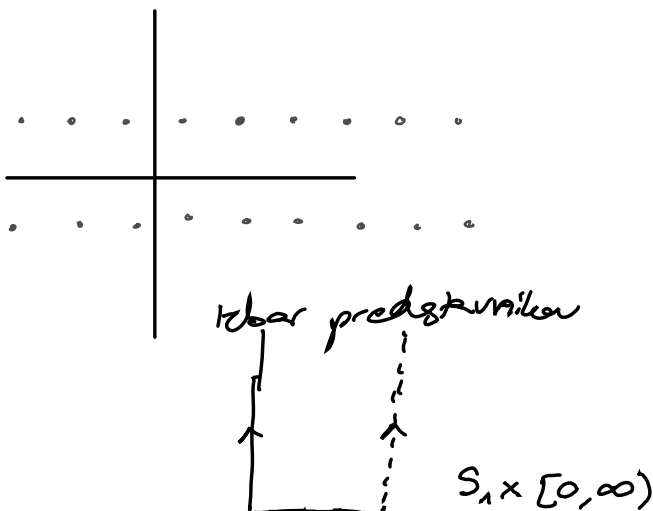
$$S: [0, \infty)^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (x, y)$$

$f, S$  surjektiv  $\Rightarrow f$  Isomorphism

$$c) \mathbb{Z} \times S^1 \hookrightarrow \mathbb{R}^2$$

$$(m, t) \cdot (x, y) = (m+x, ty)$$



$$f: \mathbb{R}^2 \rightarrow S^1 \times [0, \infty)$$

$$(x, y) \mapsto (e^{i2\pi x}, |y|)$$

warna ✓  
surjektivna ✓

Po standardnem postopku radi identifikacije med delovanji

$$((n - \frac{1}{n}), 0)_n$$

slike zaporedja zap v  $\mathbb{R}^2$   
f-slike pa ni zap. v  $S^1 \times [0, \infty)$

(1, 0) je v zaprtju mpa v f-sliki

Produkt dveh adgičnih preslikav je adgičen

$$h: \mathbb{R} \longrightarrow [0, \infty)$$

$$x \mapsto |x|$$

Dovolj preveriti ne bomo

$$0 \notin (a, b): h(a, b) = [\min\{|a|, |b|\}, \max\{|a|, |b|\}]$$

$$0 \in (a, b): h(a, b) = [0, \max\{|a|, |b|\}]$$

$$g: \mathbb{R} \rightarrow S^1 \subseteq \mathbb{C}$$

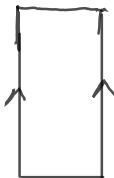
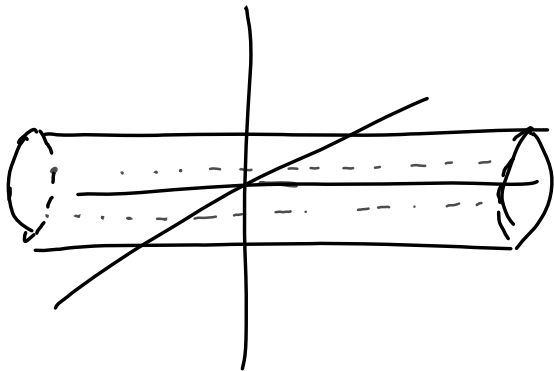
$$x \mapsto e^{i2\pi x}$$

Baza 2  $a_1, \dots$  intervalitne  $< 1 \Rightarrow$   
slabe  
adgične

d)

$$\mathbb{Z} \times S^1 \hookrightarrow \mathbb{R} \times S^1 \subset \mathbb{R}^3$$

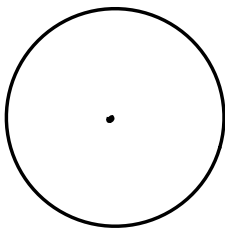
$$(m, t), (x, y, z) := (m+x, y, tz)$$



$$\mathbb{R} \times S^1 \longrightarrow S^1 \times [1, 1]$$

$$(t, y, z) \longmapsto (e^{i\eta t}, y)$$

$\mathbb{R}^n$



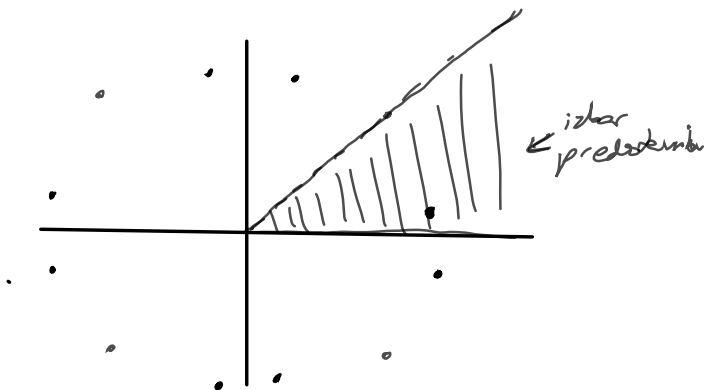
$$\vec{x} \sim \vec{y} \Leftrightarrow \|\vec{x}\| = \|\vec{y}\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???



f(A)



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cong -1 \in \mathbb{C} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong i$$

Topologija ker matrike

$$Y = \{ (x, y) \in \mathbb{R}^2; y < x; x > 0 \}$$

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$(x, y) \longmapsto (\max(x, y), \min(x, y))$$

A retrakcija  $\Rightarrow$  koeficienta v  
 ožjem smislu

2(4)

$$2x = g + x$$

$\mathbb{R}/\mathbb{Q}$  n: many values, not

$U = \mathbb{Q} \subseteq \mathbb{R}/\mathbb{Q}$  ← for different groups

Ubit vde

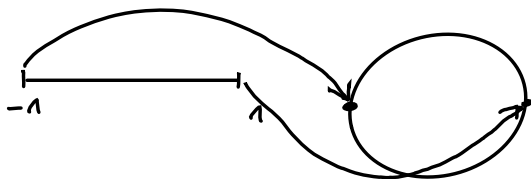
3.1

8.4

a)

$$X = [-1, 1] \quad A: \{-1, 1\}$$

$$Y = S^1 \quad f(x) := (x, 0)$$



$$Z = [-1, 1] \times \{0\} \cup S^1$$

$$g: X + Y \longrightarrow Z$$

$$in_1(x) \mapsto (x, 0)$$

$$in_2(y) \mapsto y$$

ekvivalenčni razredi:  $(in_1(1, 0); in_2(1, 0))$  in  $(in_1(-1, 0); in_2(-1, 0))$  neenotni

$$g(in_1(1, 0)) = 1, 0$$

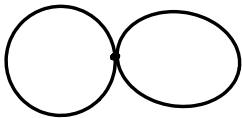
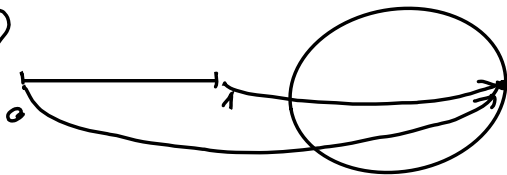
$$g(in_2(1, 0)) = 1, 0 \quad \text{padeta na 2. drug}$$

wezast pa, ker sta funkciji zvezni  
in ker ~~se~~ se ajimata na presku

potrebno seveda tudi še da loči dve razredi

Sikamo iz kompaktna v Hausdorff

b)



$$Z = Y \cup S((2,0), 1)$$

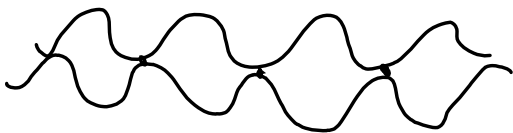
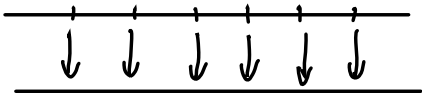
$$g: X + Y \longrightarrow Z$$

$$\text{in}_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$\text{in}_2(x, y) \longmapsto (-x + 2, y)$$

Preveriti moramo

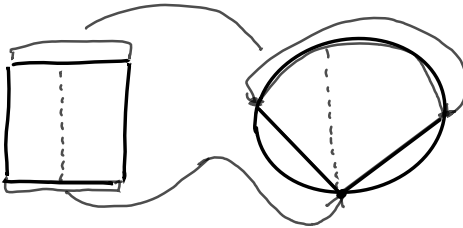
- loči dve raze
- konst na dve raze
- zvez, surj, kvocientne v ožjem smislu



$$|\sin x| \cup -|\sin x|$$

$$x \mapsto$$

d)



$$Z = S^1 \cup \{(x, y) \in \mathbb{R}^2; |x-1| \leq 1 \leq \sqrt{1-x^2}\}$$

$$\partial: x+y \longrightarrow Z$$

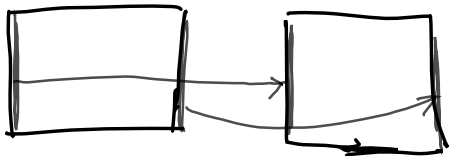
$$\text{in}_2(z) \longmapsto z$$

$$\text{in}_1(x, y) \longmapsto (0, -1) + \frac{y+1}{2} (x, \sqrt{1-x^2}+1)$$

kompatte u hausdorff

c)

$X:$



$$Z = S_1 \times [-1, 1]$$

$$g: X + Y \longrightarrow Z$$

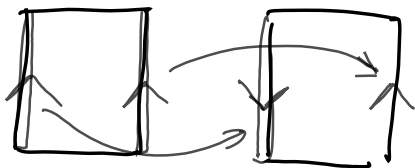
$$in_1(x, y) \longmapsto x(\sqrt{1-x^2}, y)$$

$$in_2(x, y) \longmapsto (x, \sqrt{1-x^2}, y)$$

kompakt & hausdorff ✓

also separabel &

f)



Möbiussens trake



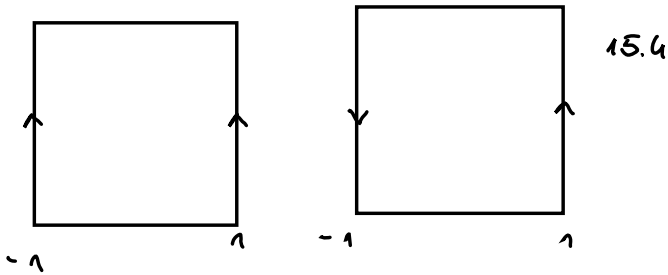
Parametrisa ena möbiussens traku

$$x(u, v) = \left(1 + \frac{v}{2} \csc \frac{u}{2} \cos u\right)$$

$$y(u, v) =$$

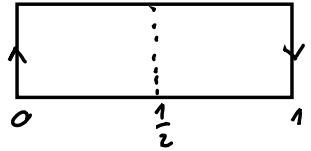
$$z(u, v) =$$





$X + Y \longrightarrow M \dots$  Möbiustrack

$$M = [0, 1] \times [0, 1] / \sim$$



$$\begin{array}{ccc}
 X + Y & \xrightarrow{\delta'} & [0, 1]^2 \\
 \downarrow 2 & \searrow g & \downarrow 2m \\
 X \cup_f Y & \dashrightarrow & M
 \end{array}$$

$$in_1(u, v) \mapsto \left( \frac{u+1}{4}, \frac{v+1}{2} \right)$$

$$in_2(u, v) \mapsto \left( \frac{3-u}{4}, \frac{v+1}{2} \right)$$

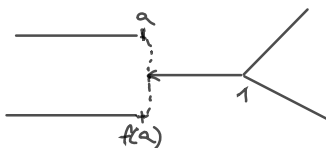
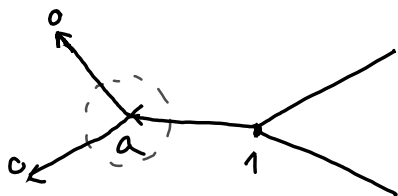
$$x \sim y \iff g(x) = g(y)$$

$$x \sim y \iff g'(x) \sim g'(y)$$

$$(a, 1] \longrightarrow (0, \infty)$$

$$f(x) = x$$

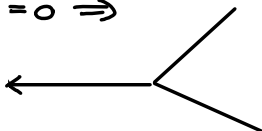
zlepak  $(0, \infty) \cup_p (0, \infty)$  je zlepak za  
katere hausdorffove



Poglejmo točko  $a$

Vseke desnice od  $a$  in od  $f(a)$  sekata  
del  $(0, 1]$  za  $\forall a \in (0, 1)$

$$\text{za } a = 0 \Rightarrow$$



Vločimo v evklidski prostor

$$(0, \infty) \cup_p (0, \infty) \longrightarrow \mathbb{R}^2$$

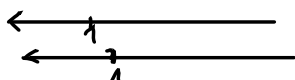
$$x \in {}_n(0, 1] \longmapsto (0, x)$$

$$x \in {}_n(1, \infty) \longmapsto (x, x-1)$$

$$x \in \underset{\text{inz}}{f}_x(1, \infty) \longmapsto (x, -x+1)$$

je hausdorffov?

lokalna kompaktnost



kompaktni: zaprti intervali v  $(0, \infty)$

v 1: zaprt interval  $[\frac{1}{2}, \frac{3}{2}]$  na obeh  
premica

konkretne grupe v konkrtni teoriji

3.3)

$$X/A \approx X \cup_f 1$$

$$\begin{array}{ccc}
 X & \xleftarrow{g'} & X + 1 \\
 f \downarrow & \nearrow g & \downarrow 2 \\
 X/A & \xrightarrow{\approx} & X \cup_f 1
 \end{array}
 \quad 1 \in 1$$

h:

$$\begin{aligned}
 [x] &\mapsto x && : x \notin A \\
 [a] &\mapsto 1 && : a \in A
 \end{aligned}$$

is homeomorphism

$$\begin{aligned}
 g' : X + 1 &\rightarrow X \\
 \text{in}_1 x &\mapsto x \\
 \text{in}_2 a &\mapsto 1 \quad a \in A
 \end{aligned}$$

$$g = f \circ g'$$

$$u \sim v \Leftrightarrow g'(u) \sim_f g'(v)$$

$$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$$

$$f(g'(u)) = f(g'(v)) \Leftrightarrow$$

$$[g'(u)] \sim [g'(v)]$$

Else we have:  $v \in X \cup_f 1$

$$\text{in}_1 x = \{x\} \quad x \notin A$$

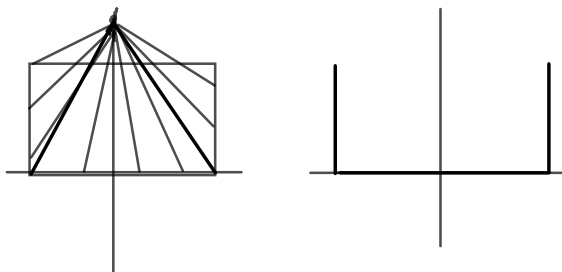
$$\text{in}_1 a = A \cup \{a\} \quad a \in A$$

$$\text{in}_2 a = A \cup \{a\}$$

$$[\text{in}_1 x] = \{[\text{in}_1 x]\}$$

$$(\text{in}_1)_*(A) \cup \{[\text{in}_2 a]\}$$

Retrakti, homotopije, ekvivalen. relacije



Projiciramo iz  $(0,2)$  na rob

$$y > 2x+2 : (-1, 2 - \frac{y-2}{a})$$

$$2x+2 > y < -2x+2 : (\frac{2x}{2-y}, 0)$$

$$y > -2x+2 \quad (1, \frac{y-2}{a} + 2)$$

$$(a,b) \leadsto p_1: y = \frac{b-2}{a}x + 2$$

$$\text{pri } x = -1 \\ y = -\frac{b-2}{a} + 2$$

$$p_2: y = \frac{b-2}{a}x + 2 \quad a \neq 0$$

$$y=0 \rightarrow x = \frac{-2a}{b-2} = \frac{2a}{2-b}$$

$$x = 1:$$

$$y = \frac{b-2}{a} + 2$$

Ali je to id na  $y$ ?

$$x = -1: (-1, 2 - \frac{y-2}{-1}) = (-1, 2+y-2) = (-1, y)$$

$$y=0: (\frac{2x}{2}, 0) = (x, 0)$$

$$x=1: (1, \frac{y-2}{a} + 2) = (1, y) \quad \checkmark$$

Ali homotopije id na  $X$

$$H(x, y, t) = t(x, y) + (1-t)r(x, y)$$

zveza, ker so vsi kosi zveza

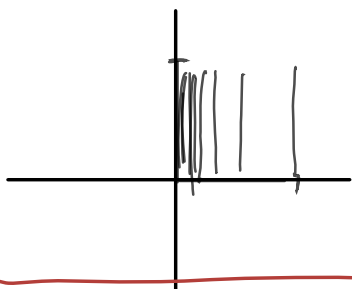
$f$  homotopy  $g$

$$h \in H: X \times [0, 1] \longrightarrow Y$$

$$(x, 0) \longmapsto f(x)$$

$$(x, 1) \longmapsto g(x)$$

where



Retrakt  $T_2$  prostora je zaprt

Recimo da  $\exists r$  retracts:  $X \rightarrow Y$

$$r|_Y = id$$

deljica  $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

morajo biti točke  $(\frac{1}{n}, 0)$

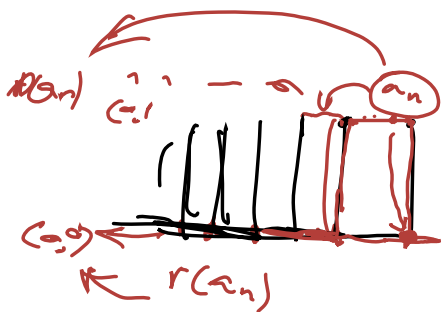
ker gre vsa  $f$  iz  $(\frac{1}{n+1}, 1)$  do  $(\frac{1}{n}, 1)$   
torej  $ce z kda$

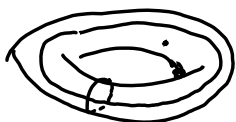
May bo zmedje  $a_n$ ;  $r(a_n) = \frac{1}{n}$

$\lim a_n = (0, 1)$   $a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim r(a_n) = (0, 0)$

$r \nmid a$ : wewa





$$G \sim X$$

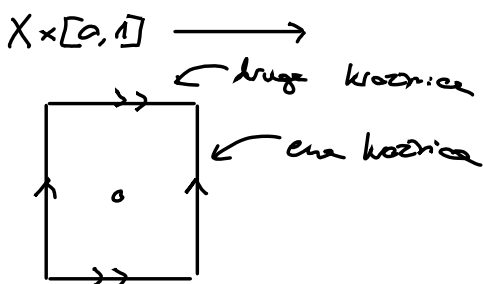
$$(x, 6) \rightarrow x$$

$$(x, g) \mapsto xg$$

Deformacijske relikve "ima homotopiju do id"  
je zvezna preslika  $H: X \times [0, 1] \rightarrow X$ .

$$H(x, 0) = x \quad , \quad H(a, 1) = a \quad \wedge \quad H(x, 1) \in A$$

$$\forall x \in X \quad \forall a \in A \quad \forall x \in X$$



$$([-1, 1]^2 - \{0, 0\}) \times [0, 1] \xrightarrow{h} [-1, 1]^2 - \{0, 0\}$$

$$\downarrow \begin{matrix} \text{ } \\ \text{ } \end{matrix} \begin{matrix} \text{ } \\ \text{ } \end{matrix} \downarrow$$

$$X \times [0, 1] \xrightarrow{\text{zvezna preslika}} X$$

$$h: (x, t) \mapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x} (1-t)$$

$g \circ h$  mora biti konstantna na dvi. razdeli

$$x \sim y \Rightarrow g \circ h(x) = g \circ h(y)$$

$$\Leftrightarrow [h(x)] = [h(y)]$$

$$\Leftrightarrow h(x) \sim h(y)$$

$h$  ekvivalentne razrede obrani na moške

$\Rightarrow$

$$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1, 1]^2 - \{0, 0\} \times [0, 1]$$

$$\Leftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$$

$\Rightarrow$

$$\frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1) (1-t_1)$$

$$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|_\infty} t_2 + (x_2, y_2) (1-t_2)$$

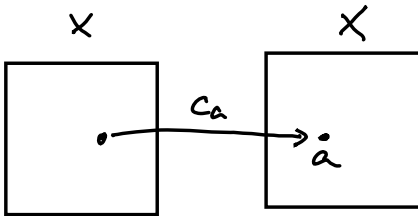
Izrek

$q$  kvocientna in  $X$  kompakten  $T_2$  prostor  
 $\Rightarrow q \times id_X$  je kvocientna

Boj splošno:

$q$  kvocientna,  $X$  lokalna kompaktna  
 $\Rightarrow q \times id$  kvocientna





$\Rightarrow$  naj bo  $X$  povezan s ptki:

Naj bo  $\gamma$  te pot med  $a$  in  $b$  po vrsti

$$\gamma(0) = a \quad \gamma(1) = b$$

$$H: X \times [0, 1] \rightarrow X$$

$$(x, t) \mapsto \gamma(t)$$

$$H = \gamma \circ \text{pr}_2 \text{ torej je zvezna}$$

$\Leftarrow$

Recimo, da  $\forall a, b \in X$  velja  $c_a \simeq c_b$   
 Naj bosta  $a, b$  poljubna

$$\exists H: X \times [0, 1] \rightarrow X$$

$$H_0 = c_a$$

$$H_1 = c_b$$

$$\gamma(t) = H(a, t)$$

4.5

a)  $f: S^n \rightarrow S^n$   $n$ : sur

homotopie kvečati  
.....

$a \notin f_*(S^n)$  b je nasprotni tej

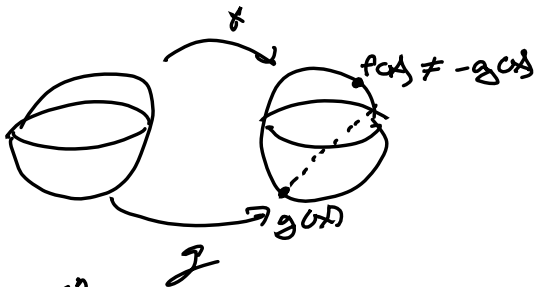
$$H: S^n \times [0, 1] \rightarrow S^n$$

$$(x, t) \mapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$$

Ali je dvčtjro  $(1-t)f(x) + t \cdot x = 0$   
 $\Rightarrow$  če greš te delice skozi 0  
 ampak  $\hat{a}$  ni zlogi vrednosti, da  
 nihle ne gre delice skozi 0,0

$$b) f, g: S^n \rightarrow S^n$$

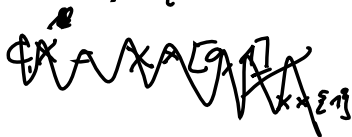
$$f(x) \neq -g(x) \quad \forall x \in S^n \rightarrow f \approx g$$



$$H: S^n \times [0, 1] \rightarrow S^n$$

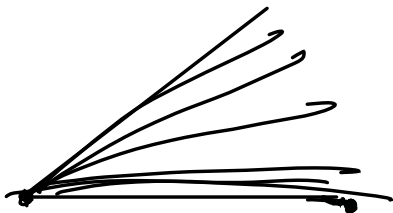
$$(x, t) \mapsto \frac{(1-t)g(x) - tf(x)}{\|(1-t)g(x) - tf(x)\|}$$

$$4.6 \quad X = \{ [0, 1] \times \{0\} \} \cup \{ x, \frac{x}{n}, x \in [0, 1] \text{ } n \in \mathbb{N} \}$$



$$H: X \times [0, 1] \longrightarrow CX$$

$$(x, y), t \longmapsto (1, t)(x, y)$$



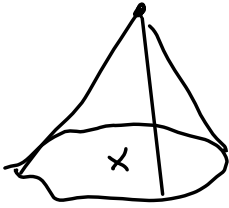
Rečimo da je  $H^G$  pogodna lokalna deformacija  
retrakcije  $x \mapsto$  na isto telo

$$\forall n \in \mathbb{N}. \exists t_n \in [0, 1] \text{ } G(1, \frac{1}{n}, t_n) = 0, 0$$

ker je  $[0, 1]$  kompaktno lahko izberemo konvergen  
podsekvencu  $t_n$

?

$$CX = X \times [0, 1] / \sim$$



$$CX \times [0, 1] \longrightarrow CX$$

$$X \times [0, 1] \times [0, 1] \xrightarrow{H} X \times [0, 1]$$

$$\begin{array}{ccc} \downarrow \text{id} & & \downarrow \text{id} \\ CX \times [0, 1] & \dashrightarrow & CX \end{array}$$

$$H: (x, u, t) \mapsto (x, 1) + \cancel{(x, u)(1-t)} + (x, u(1-t) + t)$$

ne moremă  
ceterat  
într-o

Preveriti maxima suma

$$(x_1, 1, t) \sim_H (x_2, 1, t)$$

|| echin nedrviachdu; valenchi razed

$$(x_1, 1) \quad || \quad (x_2, 1)$$

ste echivalenti la o ste ne u, din 1

$$\bar{H}([x, u, 0]) = [x, u]$$

$$\partial(H(x, u, 0)) = \partial(x, u)$$

$$\partial(H(x, u, 1)) = \dots$$

$X$  je kontraktilen

$\Leftrightarrow$

$\exists$  nema izbira poti od poljubne točke  $X$  do izbrane točke

$$\exists a \in X. \exists f: X \rightarrow \mathcal{C}(I, X), f(x)(0) = x, f(x)(1) = a$$

$\Leftrightarrow$

$X \neq \emptyset \wedge \exists$  nema izbira poti med točkami  $\forall x$

$$\exists f \omega: X \times X \rightarrow \mathcal{C}(I, X), f(a, b)(0) = a, f(a, b)(1) = b$$

u.8 (u.7 ne uči ni ci.)

i)  $\Rightarrow$  ii) Vemo iz u.7 (u.6)

ii)  $\rightarrow$  iii) Naj bo  $X$  rektak kontrakcijskega prostora  $Y$

$$\exists r: Y \rightarrow X. \exists i: X \rightarrow Y. r \circ i = \text{id}$$

(ima zvezni desni inverz (ekvivalentna definicija retrakcije))

$$\exists y_0 \in Y. \exists H: Y \times I \rightarrow Y$$

$$(y, 0) \mapsto y$$

$$(y, 1) \mapsto y_0$$

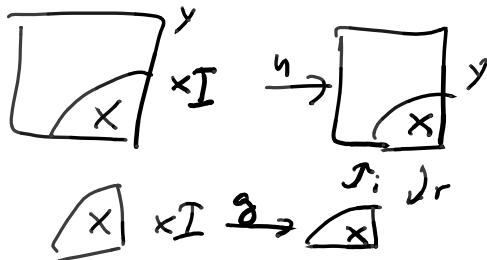
izamo

$$g: X \times I \rightarrow X$$

$$x_0 = r(y_0)$$

$$g(x, 0) = g$$

$$g(x, 1) = x_0$$



$$g(x, t) = r(h(i(x), t))$$

$$g = r \circ h \circ (i \times \text{id}_I)$$

iii)  $\Rightarrow$  i)

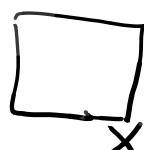
$$\square \times I \xrightarrow{H} \square \times \{a\}$$

$$H(x, 0) = x$$

$$H(x, 1) = a$$



$$\xrightarrow{r}$$



$$r \circ i = \text{id}$$

$$r([x, 0]) = x$$

$$CX = X \times I / \{x, 1\}$$

$$[x, t] \mapsto p_1(H(x, t))$$

4.9

$$I \times I / \sim_{\text{veno}} = M$$

$$\text{veno} \Leftrightarrow \sim$$

CM

$$I \times I / \sim_{\text{veno}} \times I / \sim_{\{[I, 1]\}} = \text{CM}$$

Recimo da je kontraktilen

$$\exists H: I \times I / \sim_{\text{veno}} \times I \rightarrow I \times I / \sim_{\text{veno}}$$

$$H(x, 0) = x$$

$$H(x, 1) = 1$$

Uzemimo podprostor  $A = \{x \in I \times I / \sim_{\text{veno}} \mid x \text{ je na } I \times \{0\} \text{ ali } I \times \{1\}\}$

ki je krožnica topološka

tačje je tudi  $A$  kontraktilen



u. 10

Megecazen konveksen prostor je  
kontraktilen

Splazni: zvezdast prostor je kontraktilen



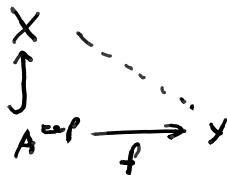
$\mathcal{N}$ . Razred normalnih topoloških prostov  
 $\mathcal{Y} \in \mathcal{AE}(\mathcal{N})$  je absolutni destenar za razred  
 normalnih prostov

kadar za  $\forall X \in \mathcal{N}$  in  $\forall$  topološko podmnožico

$A \subseteq X$  velja, da lahko vsakega preslikavo

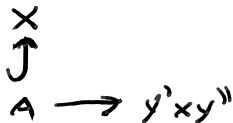
$A \rightarrow Y$  razširimo do zvezne preslikave

$X \rightarrow Y$



Torej:  $\mathcal{I} \in \mathcal{AE}(\mathcal{N})$

• Produkt  $\mathcal{AE}$  je  $\mathcal{AE}$



razširimo v vsaki komponenti

• Retrakt  $\mathcal{AE}$  je  $\mathcal{AE}$

•  $X \in \mathcal{AE}(\mathcal{N}) \Rightarrow X \neq \emptyset$  in  $X$  povezan s pdm:

•  $\mathcal{AE}(\mathcal{N}) \cap \mathcal{N} \subseteq \mathcal{AR}(\mathcal{N})$

↖ absolutni retracts

Ne tabl: 4. ~~1824~~ 11

4. 12

$$A, B \subseteq \mathbb{R}^n$$



$$\mathbb{R}^n \in \mathcal{A}\mathcal{E}$$

Dokazimo  $A \cup B$  je rektakl  $\mathbb{R}^n$

Fodsch

4.19

$$v \in \mathbb{R}^n \setminus \{0\}$$

$$Av = \lambda v$$

$$\text{vsz } \|v\| = 1$$

$$\|Av\| = \|\lambda v\| = |\lambda| \cdot \|v\| = |\lambda|$$

$$Av = \lambda v \stackrel{?}{=} \|Av\| v$$

$$v = \frac{Av}{\|Av\|}$$

iščemo neko točko preslike  
 $S^{n-1} \rightarrow S^{n-1}$

$v \mapsto \frac{Av}{\|Av\|}$  ne nujno dobro definirana  
 preslika, ker  
 $\ker A$  ni nujno  $\{0\}$

$$X = \{ (x_1, \dots, x_n) \in S^{n-1} \mid x_1, \dots, x_n \geq 0 \} \approx B^{n-1}$$

če  $v \in X$  so vsi  $x_i \geq 0$  in niso vsi  $= 0$

$\Rightarrow Av$  ima pozitivne komponente,  
 vsakej  $\neq 0$

$B^{n-1} \rightarrow B^{n-1}$  na neko točko

## Mnogoterost

a)  $\{\frac{1}{n}, n \in \mathbb{N}\}$  stanoš + distrehoš  
= je mnogoterost

b)  $\{\frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$

$\forall 0$  ni mnogoterost

če bi jo imela bi našla točki  $x \in \mathbb{R}^0 = \{0\}$   
ali  $\mathbb{R}_+^0$  kar ne obstaja

Torej točka 0 ni možna točka  
vseh delov ~~to~~ od a) kar pomeni  
da

ni o točki  $\{0\}$  - točka, kar pomeni:

Stano vsota n-mnt je n-mnt

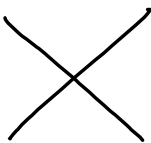
c)  $\approx \mathbb{Z}-1, 1\} \times \mathbb{R} \approx \mathbb{R} + \mathbb{R}$

Produkt množic je množica

$\Rightarrow$  1 množica

Tudi: Ste na vsaki n-množici je množica

d)



ni množica v  $(0,0)$

$(1,1)$  ima 1-množico na delu

Torej v  $(0,0)$  mora biti 1 množica

Veljavo da n:  $\approx \mathbb{R}$  ali  $\mathbb{R}_+$

naj bo  $U$  poljubna delica tebe

Ni dovolj preverjati na bazi

$U = \{(0,0)\}$  ima vsaj 4 komponente

če bi  $U$  bil homeomorfen  $\mathbb{R}$  ali  $\mathbb{R}_+$

bi razdelil na 1 ali 2 komponenti

$\Rightarrow$  ~~ni množica~~  $U$  ni množica

Navedene vložitve tadin množicah brez  
roba in se predstavlja n: od tega p den  
ni množica

$f: \mathbb{R} \rightarrow X$   
 $f(x) = (x, x)$

$f_*(\mathbb{R})$  mora biti od tega  
 $(0,0)$  je roba  
toda se isto  
ker vsebuje  $(e, -e)$  za  
dovolj majhen  $e > 0$   
vse delice



$$e) (\mathbb{R}^2 \times \{0\}) \cup (S^1 \times \mathbb{R})$$

(2a d naloge)

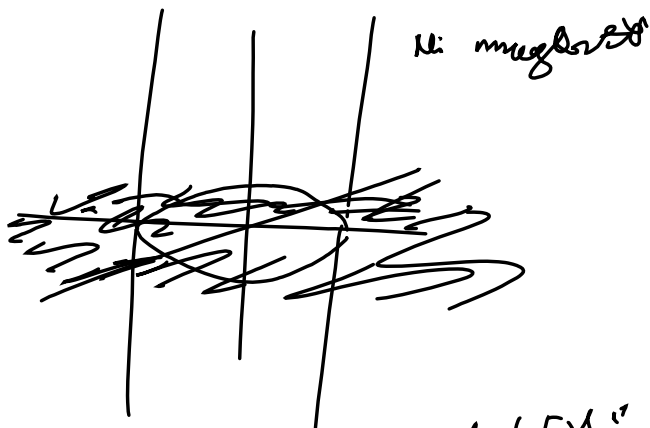
(za 1 nalogu)

Izrek o invarianci opstarih modic za mnogokrati

ce se  $M, N$  istodimenzionalni mnogobladi

$M$  brez roba in  $f: M \rightarrow N$  zveza mekaja.

Potem je f odprta vložitev v int N



ker bhl vltro

$$f: \mathbb{R}^2 \rightarrow X$$

$$x \mapsto (x, 0)$$

$$f_*(\mathbb{R}^2) \text{ is a } \mathbb{R} \text{ ker}$$

•  $(1, 0, a)$  je rana zaslava

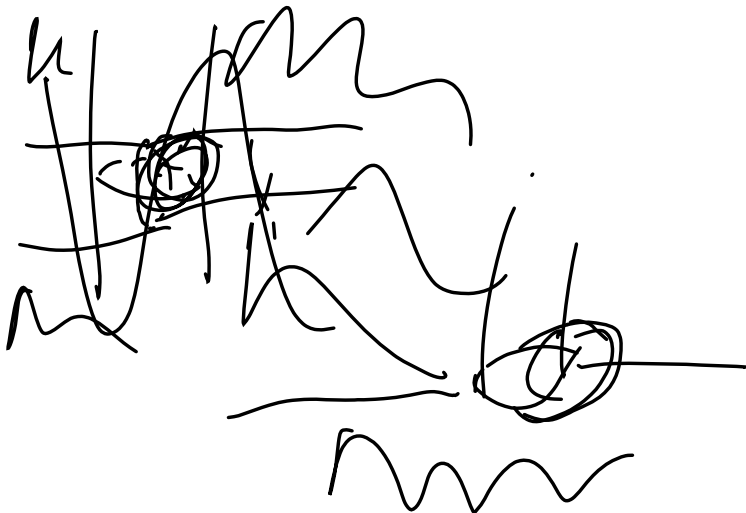
$$\forall U \supset (0,0,0)$$

$$(1, 0, \varepsilon) \in U$$

(ungeb. 6. 2. 17)  
unversch.  
(Kauf v. (0,0))

0,0 ; me dedico  
 $K(0, \frac{1}{2}) \times \mathbb{R}^2$

f)



Voorbeeld:

$$f: S^1 \times \mathbb{R}^3 \rightarrow X$$

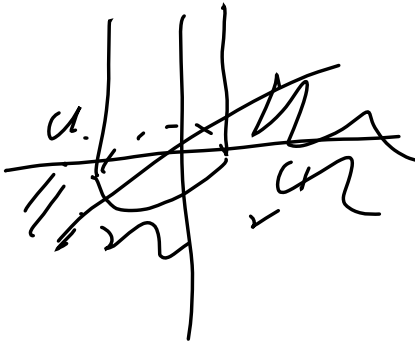
$$x \mapsto x$$

Waarvoor  $(1, 0, 0)$

$$\partial(M \times N) = \partial M \times N \cup M \times \partial N$$

$(1, 0, 0)$  is voor de te tekenen

g)



$\mathbb{R}$  &  $\mathbb{N}$  together

$$\approx S^1 \times \mathbb{R}$$

$$f: X \rightarrow S^1 \times \mathbb{R}$$

$$x = re^{it}, r = e_1$$

$$f: X \mapsto X \quad \text{for } x \geq 0 \quad x \mapsto x$$

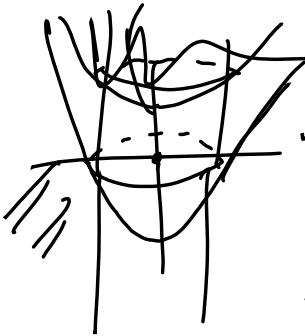
$$(re^{it}, 0) \mapsto (e^{it}, r+1)$$

$r=1 \Rightarrow e^{it}, r+1 =$  same element

inverse

$$e^{it}, z \mapsto \begin{cases} e^{it}, z & z \geq 0 \\ (z+1)e^{it}, 0 & z < 0 \end{cases}$$

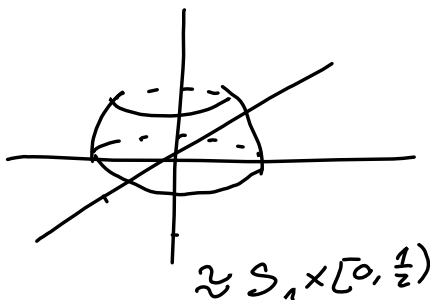
## h)



ni ~~the~~ ~~house~~  
~~the~~ ~~house~~  
more ~~the~~ ~~house~~

Visto meter

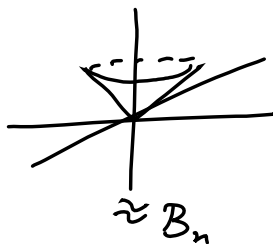
5.2



DN

ima  
lastni  
negativne točke  
crte je za 90°

Pol ima  
2 kon. pomeni



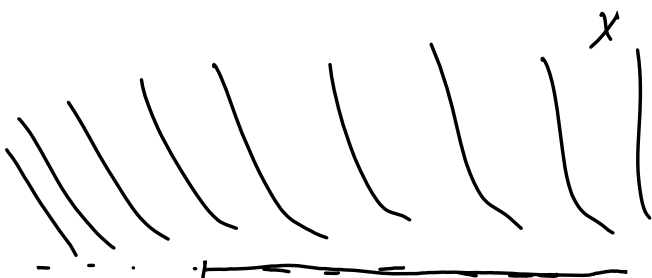
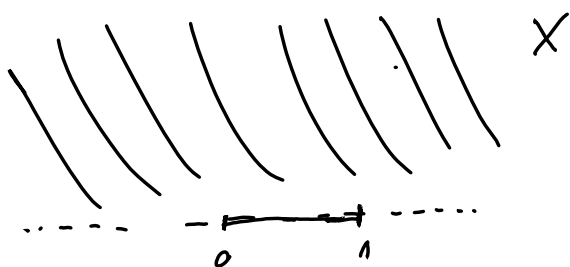
↓  
pr. ophajanja

Rej. delov

ima  
lastnost  
negativne točke

Pol ima  
ena kon. pomeni

5.3



$X$  ima 2 nenegativni točki  
 $Y$  pa ima 1

$v(0, \infty)$  ni množica ker ni  
 lokalno kompaktna v tej točki

$Z = \mathbb{R} \times (0, \infty)$  ker v določeni si konvergenci

$Z \rightarrow X$  v notranjost  
 $Z \rightarrow Y$  v notranjost

Preobrazba  $(0, \infty)$  skozi  $v$

$$Z \hookrightarrow X \xrightarrow{h} Y \hookrightarrow \mathbb{R}^2$$

zveza injektivna se skozi  $v$  notranjost

$h_*(Z)$  je odprta pod  $\mathbb{R}^2$

$$\text{in } h_*(Z) \subseteq Y \Rightarrow \mathcal{A} \cup Y$$

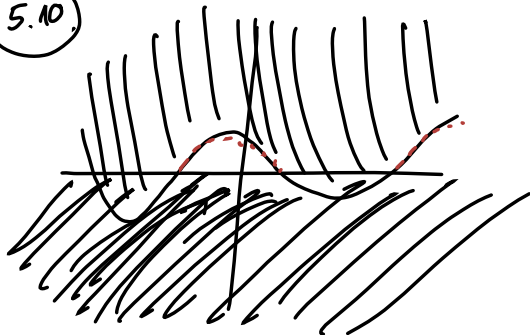
$$\Rightarrow h_*(Z) \subseteq Z$$

je eden od množic

$$(h^{-1})_*(Z) \subseteq Z$$

$\Rightarrow h|_{[0, \infty)} \rightarrow [0, \infty) \times \mathbb{R}$  je detektor  
 nenulovnih  $X$

5.10.



a)  $p_{-1}(x) = x^2 - 1$



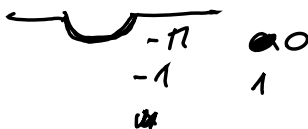
$x_{p-1}$  ni lok kengitwa

$v (-1, 0)$

kwa ku vuto dalya

$\exists$  equation  $(-1, \epsilon_n)$

$x_{p-1}$  je maglenza



$x, y \mapsto (y + \sqrt{1-x^2}) e^{i \frac{(x+1)\pi}{2}}$