1.1.

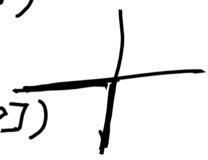
$A = [-1,1] \times [-1,1]$

$g: \mathbb{R}^2 \to \mathbb{R}^2/A$ hoc

a) $g_*((-\infty,0]\times(-\infty,0])$

~~zaprta~~ ni odp niti ~~ni zap~~ zap

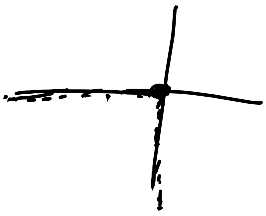
$g^*(g_*((-\infty,0]\times(-\infty,0]))$



b) $g_*(\mathbb{R}^2-(-\infty,0]\times(-\infty,0])$

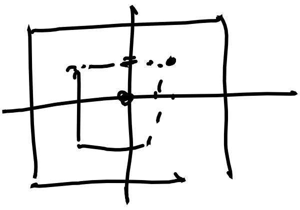
ni odprta

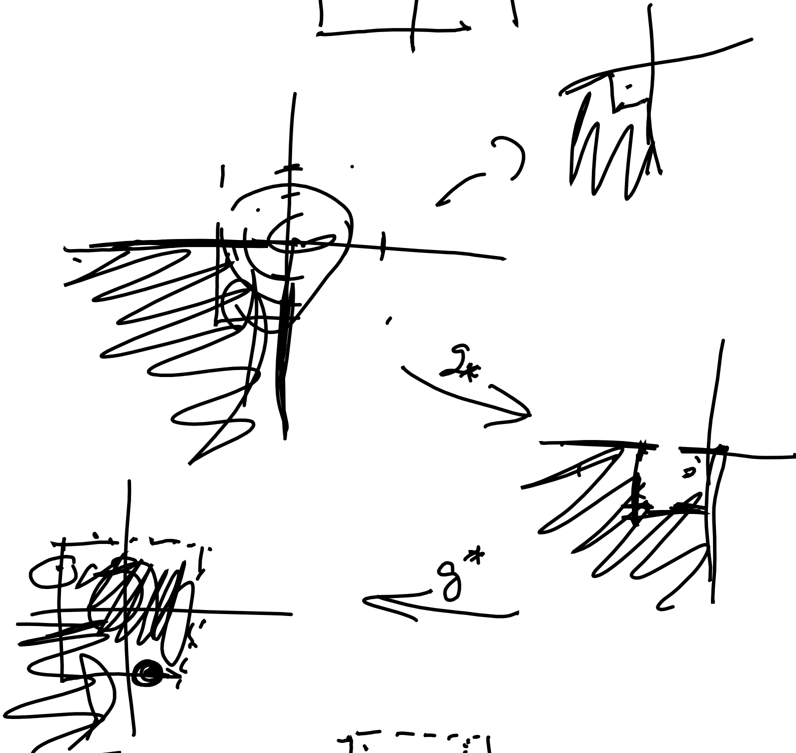
ni zaprta

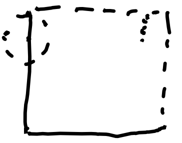


c) $g_*([-2,2]\times[-2,2])$

odprta

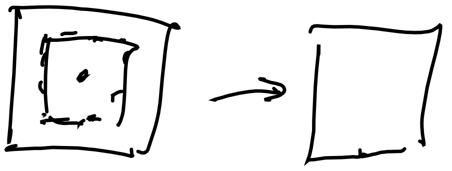






5)

$g^*(g([-2,2]\times[-2,2])$



zaprta

ni odp

c) ni odp je zap

7.4

a) $[-2,2]/[-1,1]$ 

$\cong [-1,1]$

b) $[-1,1]/\{-1,0,1\}$ 

$\cong$ 

$\mathbb{R}^2$

c)]

$$\mathbb{R}^n / K(0,1) \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{tikzcd}
\mathbb{R}^n \arrow[r, "f"] \arrow[d] & \mathbb{R}^n \\
\mathbb{R}^n / K(0,1) &
\end{tikzcd}$$

$$f(\vec{x}) := \begin{cases} 0 \ ; & \text{če je } |\vec{a}| \leq 1 \\ \vec{a} - \dfrac{\vec{a}}{\|\vec{a}\|} \ ; & a \in \mathbb{R}^n - \overline{K}(0,1) \end{cases}$$

d)

$\mathbb{R}^n/_\sim \qquad x \sim y \Leftrightarrow \|x\| = \|y\|$

$\mathbb{R}^n/_\sim \cong [0,\infty)$

$f: \mathbb{R}^n \longrightarrow [0,\infty)$

$f(\vec{x}) = \|\vec{x}\|$ je surj

$a \in [0,\infty) \qquad (a,0,0\dots) \longmapsto a$

$[x]=[y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$

je zvezna

iščemo preslikavo s, da velja $f \circ s = id_{[0,\infty)}$

$s: a \longmapsto (a,0,\dots\dots)$

Dokažimo da če $r \circ s = id_Y$

$\Rightarrow r$ kvocientna, subjektivna

$S \subseteq Y$ taka da $r^*(S)$ odp v X

$S = s^*(r^*(S)) =$

$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$

$id_Y^*(S) = S$

$\Rightarrow S$ je odprta

f)

$$S^n \times [-1,1]) / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \longrightarrow S^{n+1}$$

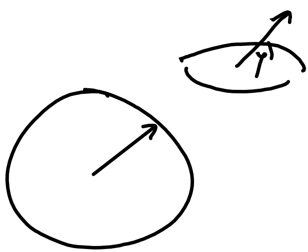
$$(x,t) \longmapsto (xb, t) = (\sqrt{1-t^2}\,x, t$$

anotato voller

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1+t^2}$$



$$G/_H = \{ g \cdot H \,;\, g \in G \} =$$

$$= \{ \{ g \cdot h \,;\, h \in H \} \,;\, g \in G \}$$

G top. grupa

$a \in G$

$L_a : G \longrightarrow G$

$x \longmapsto a \cdot x$ leva translacija

$a, b \in G$

$h : G \longrightarrow G$

$h(a) = b$ $\quad h = ?$

možnosti

$L_{ba^{-1}} : x \longmapsto ba^{-1}x$

$ba^{-1}x$

$xa^{-1}b$

bxa^{-1}

$a^{-1}xb$

Topološke grupe izgledajo povsod isto, ker lahko vsako točko preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ okolica $\iff ba^{-1}A$ okolica $b \in G$

$\exists U^{odp} \subseteq A \,.\, a \in U$

$b \in ba^{-1}A$

$a \in U \Rightarrow \underbrace{ba^{-1}a}_{= b} \in ba^{-1}U \subseteq ba^{-1}A$

Ker je $L_{ba^{-1}}$ homeomorfizem je

$ba^{-1}U \underset{\text{odprta}}{\subseteq A}$ v G

$\Leftarrow$ potem velja tudi obratno

b) $H \leq G$ $\quad$ H okolica 1 $\Rightarrow$ H odp in zap v G

$$a \in aH \subseteq H$$

$\Rightarrow$ H je okolica vsake svoje točke

G - H je odp.

$aH \cap H = \emptyset \Rightarrow aH = H$

$a \in G - H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$ vsak element ima okolico, ki ne seka H $\Rightarrow$ H je zaprta

c) C komponenta $1 \in C$

$\Rightarrow C$ zaprta edinka v G

$C \leq G$

$L_a : x \longmapsto ax$ je homeomorfizem za $\forall a \in C$

$\forall a \in C.\ L_a \subseteq C$

L_a ohranja povezanost

Prav tako $L_a \cdot 1 = a \Rightarrow$

$L_{a*} C \cap C \Rightarrow L_{a*} C \subseteq C$

invertiranje: invertiranje je tudi homeo $i : x \longmapsto x^{-1}$

$\Rightarrow$ po istih argumentih

Ali je edinka?

$\forall a \in G.\ aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$

$x \longmapsto axa^{-1}$ je homeomorfizem

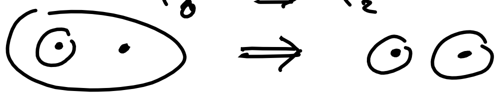
$\Rightarrow$ je povezano in $a\,id\,a^{-1} = id \in C$

$\Rightarrow aCa^{-1} \subseteq C$

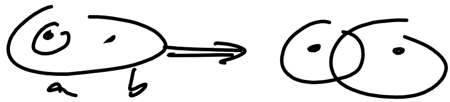
ker je kompozitum dveh translacij (leva in desna)

d) za G je

$T_0 \Rightarrow T_2$



$T_0 \Rightarrow T_1$



$a, b \in G$

imamo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U,\ b \notin U$ BŠZS

$U^{-1} = \{a^{-1} : a \in U\}$

$aU^{-1}b \ni b \quad a \mapsto b$

Recimo da $a \in aU^{-1}b$

$\exists c \in U,\ a = ac^{-1}b$

$\Rightarrow b = c \Rightarrow b \in U$ ⨳

$T_1 \Rightarrow T_2$

$\exists U, V \overset{odp}{\in} G.\quad a \in U,\ b \in V.\quad a \notin V \quad b \notin U$

$\Delta_G \in G \times G$ je zaprta v $G \times G$

$\Delta_G = f^*(\{1\})$

$f: (x, y) \mapsto xy^{-1}$

zvezna preslikava ker je zaprta

(2) T_{cc} je T_1 in ni T_2

$\Rightarrow$ $(\mathbb{R}, +)$ ni top grupa za T_{cc}

(ker 1.d)

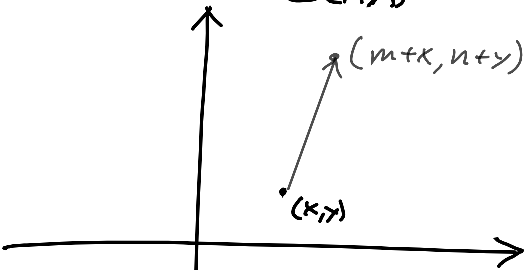
(2.3) $\mathbb{Z}\times\mathbb{Z} \curvearrowright \mathbb{R}^2$

d $(m,n)(x,y) := (m+x, n+y)$

$$\Phi : g \longmapsto \underbrace{(a \longmapsto g\cdot a)}_{\text{bijekcija (izomorfizem)}}$$

$\Phi : G \longrightarrow \mathrm{Bij}(A) \longleftarrow$ grupa za komponiranje

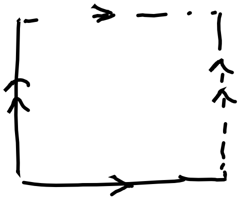
V bistvu: $\Phi : G \longrightarrow (\underset{\mathcal{C}(A,A)}{\mathrm{Homeo}(A)}, \circ)$



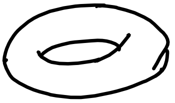
imamo $[0,1) \overset{?}{\cong} \mathbb{R}^2/\Phi$

Na nivoju množic ja

na nivoju topologije ? NE



Dobimo torus



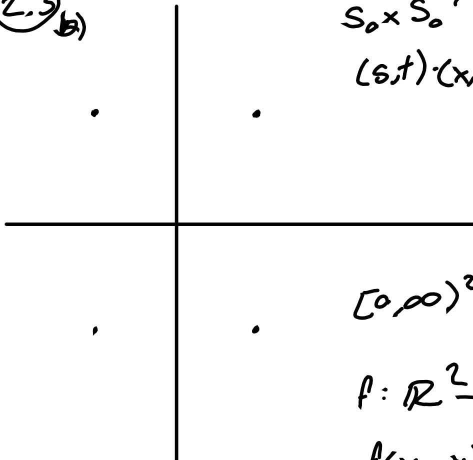
$\cong S^1\times S^1$

$f : \mathbb{R}^2 \longrightarrow S^1\times S^1$

$(x,y) \longmapsto (e^{2\pi i x}, e^{2\pi i y})$

Tuki nisem poslušala več :)

(2,3) b)



$S_0 \times S_0 \curvearrowright \mathbb{R}^2$

$(s,t)\cdot(x,y) = (sx, ty)$

$[0,\infty)^2$

$f : \mathbb{R}^2 \to [0,\infty)^2$

$f(x,y) = (|x|, |y|)$

$s : [0,\infty)^2 \to \mathbb{R}^2$

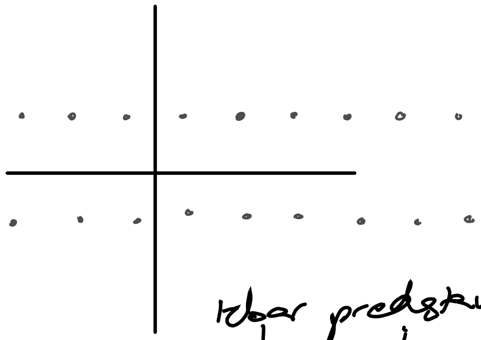
$(x,y) \mapsto (x,y)$

f, s conti $\Rightarrow f$ cuociente

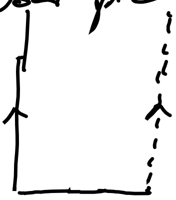
c)

$\mathbb{Z} \times S^0 \curvearrowright \mathbb{R}^2$

$(m,t)\cdot(x,y) = (m+x,\ ty)$



Izbor predstavnikov



$S_1 \times [0,\infty)$

$f : \mathbb{R}^2 \to S^1 \times [0,\infty)$, $S^1 \subseteq \mathbb{C}$

$(x,y) \mapsto (e^{i2\pi x}, |y|)$

zvezna ✓
surjektivna ✓

Po standardnem postopku tudi identifikacije med delovanje

$((n-\frac{1}{n}), 0))_n$

slika zaporedja zap v $\mathbb{R}^2$
f-slika pa ni zaprta v $S^1 \times [0,\infty)$

$(1,0)$ je v zaprtju ampak ni v f-sliki

Produkt dveh odprtih preslikav je odprta

$h : \mathbb{R} \to [0,\infty)$
$x \mapsto |x|$

Dovolj preveriti na bazi

$0 \notin (a,b)$: $h(a,b) = (\min\{|a|,|b|\}, \max\{\ldots\})$
Če $0 \in (a,b)$: $h(a,b) = [0, \max\{|a|,|b|\})$

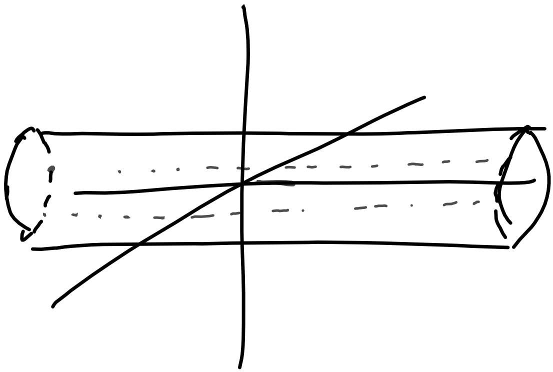
$g : \mathbb{R} \to S^1 \subseteq \mathbb{C}$
$x \mapsto e^{i2\pi x}$

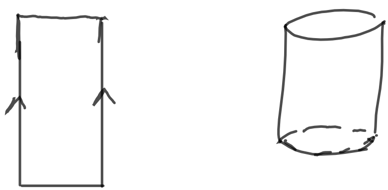
Baza $\mathbb{R}$ odprti intervali dolžine <1 $\Rightarrow$ slike odprte

d)

$\mathbb{Z} \times S_0 \curvearrowright \mathbb{R} \times S^1 \subseteq \mathbb{R}^3$

$(m,t).(x,y,z) := (m+x, y, tz)$

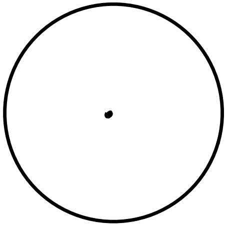




$\mathbb{R} \times S_1 \longrightarrow S_1 \times [-1,1]$

$(t, y,z) \longmapsto (e^{2i\pi t}, y)$

a)

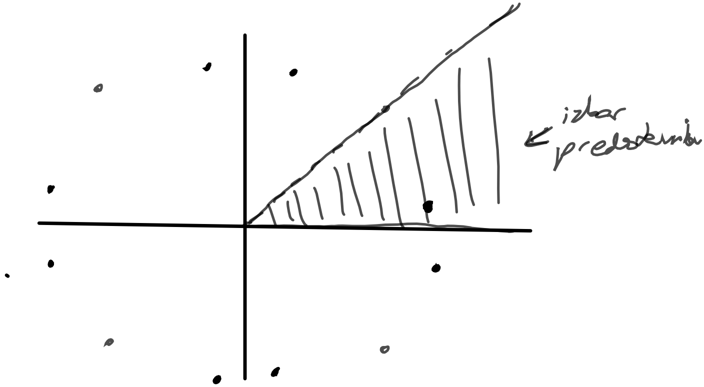


$$\vec{x} \sim \vec{y} \iff \|x\| = \|y\|$$

$$f : \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(a)



$$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \cong -1 \in \mathbb{C} \qquad \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \cong i$$

Topologija ker [illegible]

$$Y = \{(x,y) \in \mathbb{R}^2 ;\ y < x ;\ x > 0$$

$$f : \mathbb{R}^2 \longrightarrow Y$$

$$(x,y) \longmapsto (\max(|x|,|y|) ;\ \min(|x|,|y|)$$

A retrakcija $\Rightarrow$ kvocientna v ožjem smislu

2(4)

$2x = g + x$

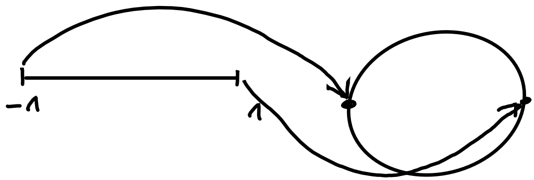
$\mathbb{R}/\mathbb{Q}$ ni mete vložen v noben

$U^{odp} \subseteq \mathbb{R}/\mathbb{Q}$ ← po delovanju grupe

Ubit vse

3.1 8.4

a)
$X = [-1, 1]$ $A: \{-1, 1\}$
$Y = S^1$ $f(x) := (x, 0)$



$Z = [-1, 1] \times \{0\} \cup S^1$

$g : X + Y \longrightarrow Z$
$in_1(x) \longmapsto (x, 0)$
$in_2(y) \longmapsto y$

ekvivalenčni razredi: $(in_1(1,0), in_2(1,0)$ in $in_1(-1,0)$ in $in_2(-1,0))$ neenostavno

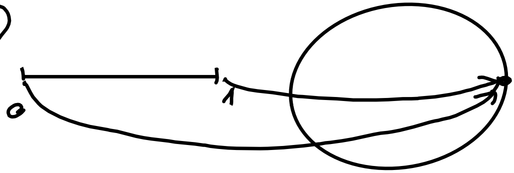
$g(in_1(1,0)) = 1,0$
$g(in_2(1,0)) = 1,0$ podatek Z drug

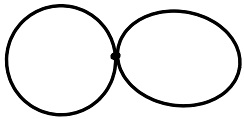
zveznost ja, ker sta posamezni zvezni in ker se ujemata na preseku

Potrebno še pokazati še da loči ekv. razrede

slikamo iz kompakta v Hausdorffov

b)





$$Z = Y \cup S((2,0), 1)$$

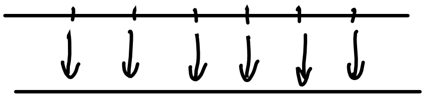
$$g : X + Y \longrightarrow Z$$

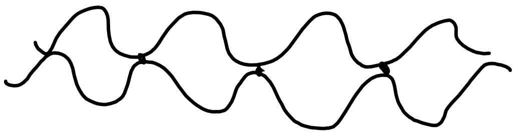
$$in_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$in_2(x,y) \longmapsto (-x+2, y)$$

Preveriti moramo

- loči ekv razr
- konst na ekv razr
- zvez, surj, kvocientna v ožjem smislu

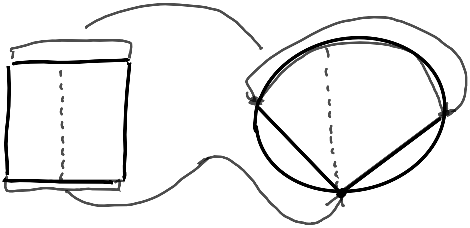




$|\sin x| \cup -|\sin x|$

$x \longmapsto$

d)



$Z = S^1 \cup \{(x,y) \in \mathbb{R}^2 ;\ |x-1| \leq 1 \leq \sqrt{1-x^2}\ \}$

$g : X + Y \longrightarrow Z$

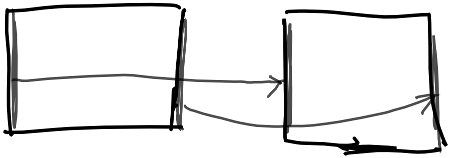
$\quad in_2(z) \longmapsto z$

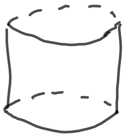
$\quad in_1(x,y) \longmapsto (0,-1) + \frac{y+1}{2}\left(x, \sqrt{1-x^2}+1\right)$

kompaktna v hausdorfov

e)

X:





$Z = S_1 \times [-1, 1]$

$g: X + Y \longrightarrow Z$

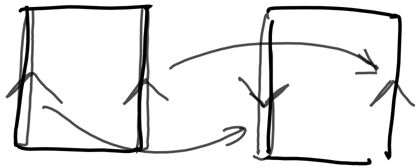
$in_1(x,y) \longmapsto (x, -\sqrt{1-x^2}, y)$

$in_2(x,y) \longmapsto (x, \sqrt{1-x^2}, y)$

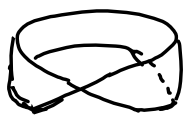
kompakt v hausdorff ✓

[illegible] ~~ste~~ identifizieren [illegible]

f)



Möbiusov trak



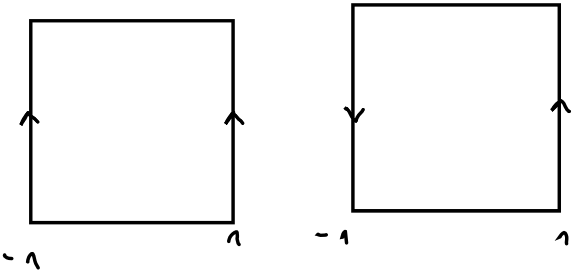
Parametrizacija möbiusovega traku

$x(u,v) = (1 + \frac{v}{2} \cos \frac{u}{2}) \cos u$

$y(u,v) =$

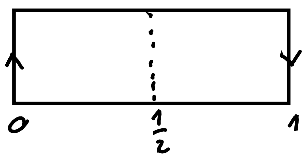
$z(u,v) =$

15.4



$X + Y \longrightarrow M$ ... Möbisov trak

$M = [0,1]\times[0,1]/\sim$



$$\begin{tikzcd}
X + Y \arrow[r, "g'"] \arrow[d, "2"'] \arrow[dr, "g"] & {[0,1]^2} \arrow[d, "2m"] \\
X \cup_f Y \arrow[r, dashed] & M
\end{tikzcd}$$

$$in_1(u,v) \longmapsto \left(\frac{u+1}{4}, \frac{v+1}{2}\right)$$

$$in_2(u,v) \longmapsto \left(\frac{3-u}{4}, \frac{v+1}{2}\right)$$

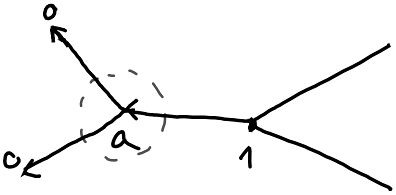
$x \sim y \Longleftrightarrow g(x) = g(x)$

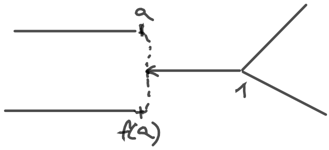
$x \sim y \Longleftrightarrow g'(x) \sim g'(y)$

$(a, 1] \longrightarrow (0,\infty)$

$f(x) = x$

zlepek $(0,\infty) \cup_f (a,\infty)$ je zlepek za katere hausdorffove

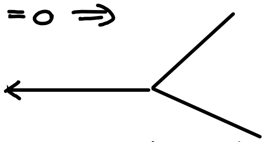




Poglejmo točko a

Vsaka okolica od a in od f(a) seka te del $(0,1]$ za $\forall a \in (0,1)$

za $a = 0 \Rightarrow$



Vložimo v evklidski prostor

$(0,\infty) \cup_f (0,\infty) \longrightarrow \mathbb{R}^2$

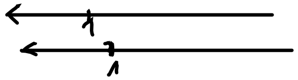
$x \in in_1(0,1] \longmapsto (0,x)$

$x \in in_1(1,\infty) \longmapsto (x, x-1)$

$x \in in_2(1,\infty) \longmapsto (x, -x+1)$

je hausdorfov?

lokalna kompaktnost



kompakti: zaprti intervali v $(0,\infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh premicah

kompakti gredo v kompakte torej

(3.3)

$$X/A \approx X \cup_f 1\!\!1$$

$$\begin{tikzcd}
X \arrow[d, "f"'] & & X+1\!\!1 \arrow[ll, "g'"'] \arrow[dll, "g"'] \arrow[d, "g"] \\
X/A \arrow[rr, "\approx"] & & X \cup_f 1\!\!1
\end{tikzcd}$$

$1 \in 1\!\!1$

$h:$

$[x] \longmapsto x \quad ; x \notin A$

$[a] \longmapsto 1 \quad : a \in A$

je homeomorfizem

$g' : X+1\!\!1 \longrightarrow X$

$in_1 x \longmapsto x$

$in_2 \smile \longmapsto a \quad a \in A$

$g = f \circ g'$

$u \sim v \Leftrightarrow g'(u) \sim_A g'(v)$

$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$

$f(g'(u)) = f(g'(v)) \Leftrightarrow$

$[g'(u)] \sim [g'(v)]$

Ekv razredi v $X \cup_f 1\!\!1$

$in_1 x = \{x\} \quad x \notin A$

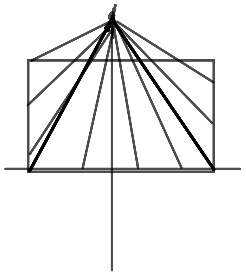
$in_1 a = A \cup \smile \quad a \in A$

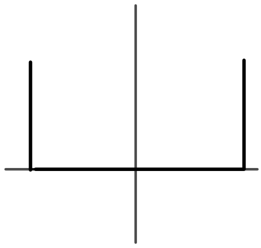
$in_2 \smile = A \cup \smile$

$[in_1 x] = \{in_1 x\}$

$(in_1)_*(A) \cup \{in_2 \smile\}$

Retrakti, homotopije, ekstenzije. ...regije kot





Projeciramo iz $(0,2)$ na rob

$y > 2x+2$: $\left(-1,\ 2-\frac{y-2}{x}\right)$

$2x+2 > y < -2x+2$: $\left(\frac{2x}{2-y},\ 0\right)$

$y > -2x+2$ $\left(1,\ \frac{y-2}{x}+2\right)$

$(a,b) \rightsquigarrow p_1$: $y = \frac{b-2}{a}x+2$

pri $x=-1$
$y = -\frac{b-2}{a}+2$

p_2: $y = \frac{b-2}{a}x+2 \quad a \neq 0$

$y=0 \to x = \frac{-2a}{b-2} = \frac{2a}{2-b}$

$a = a$

$x = 1$:
$y = \frac{b-2}{a}+2$

Ali je to id na Y?

$x=-1$: $\left(-1,\ 2-\frac{y-2}{-1}\right) = (-1,\ 2+y-2) = (-1,y)$

$y=0$: $\left(\frac{2x}{2},0\right) = (x,0)$

$x=1$: $\left(1,\ \frac{y-2}{1}+2\right) = (1,y)$ ✓

Ali homotopna id na X

$H(x,y,t) = t(x,y) + (1-t)r(x,y)$

zvezna, ker so vsi kosi zvezni

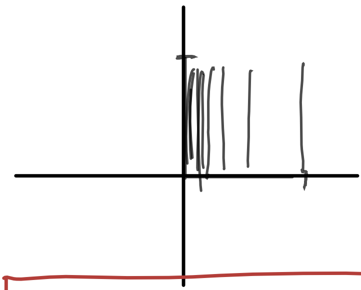
f homotopna g

ko $\exists\ H: X \times [0,1] \longrightarrow Y$

$$(x,0) \longmapsto f(x)$$

$$(x,1) \longmapsto g(x)$$

zvezna



$\boxed{\text{Retrakt } T_2 \text{ prostora je zaprt}}$

Recimo da $\exists r$ retrakt $: X \to Y$

$r|_Y = id$

daljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

mora pokriti točke $(\frac{1}{n}, 0)$

(ker gre vse pt iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1]$ čez točko

Torej

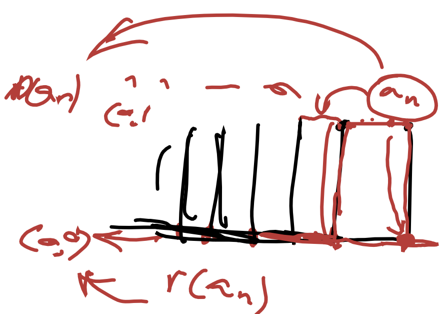
Naj bo zaporedje a_n; $r(a_n) = \frac{1}{n}$

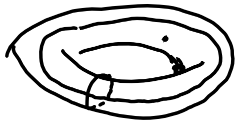
$a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim a_n = (0,1)$

$\lim r(a_n) = (0,0)$

r ni zvezna





$G \curvearrowright X$

$(X, G) \longrightarrow X$

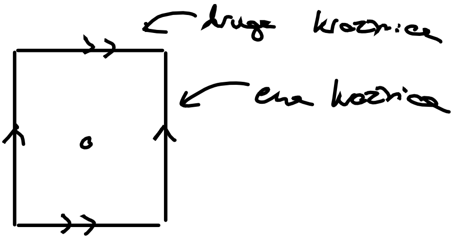
$(x, g) \longmapsto x \cdot g$

Deformacijska retrakcija "ima homotopijo do id"

je zvezna preslikava $H: X \times [0,1] \to X$.

$H(x,0) = x$ za $\forall x \in X$, $\quad H(a,1) = a$ za $\forall a \in A$ $\quad \wedge \quad H(x,1) \in A$ za $\forall x \in X$

$X \times [0,1] \longrightarrow$



$$\begin{tikzcd}
\left([-1,1]^2 - \{(0,0)\}\right) \times [0,1] \arrow[rr, "h"] \arrow[d, "q \times id_{[0,1]}"] \arrow[drr, "g \circ h"] & & {[-1,1]^2 - \{(0,0)\}} \arrow[d, "g"] \\
X \times [0,1] \arrow[rr, dashed, "\text{zvezna preslikava}"'] & & X
\end{tikzcd}$$

$h: (x,t) \longmapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x}(1-t)$

$g \circ h$ mora biti konstantna na ekv. razredih

$x \sim y \Rightarrow g \circ h(x) = g \circ h(y)$

$\Longleftrightarrow$

$[h(x)] = [h(y)]$

$\Longleftrightarrow$

$h(x) \sim h(y)$

h ekvivalentne netrivialne razrede ohrani na meje

$\Longrightarrow$

$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1,1]^2 - \{(0,0)\} \times [0,1]$

$\Longleftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$

$\Longrightarrow$ --- $\frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1)(1 - t_1)$

$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|} t_2 + (x_1, y_1)(1 - t_2)$

Izrek

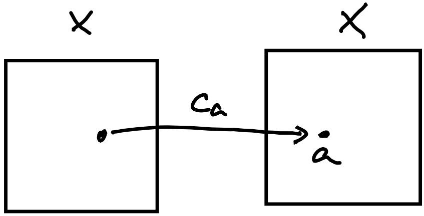
q kvocientna in X kompakten T_2 prostor

$\Longrightarrow q \times id_X$ je kvocientna

Bolj splošno:

q kvocientna, X lokalna kompaktna

$\Longrightarrow q \times id$ kvocientna



$\Rightarrow$ naj bo X povezan s potmi:

Naj bo γ ta pot med a in b poljubna

$\gamma(0)=a \qquad \gamma(1)=b$

$H : X\times[0,1] \longrightarrow X$

$x,t \longmapsto \gamma(t)$

$H=\gamma\circ pr_2$ torej je zvezna

$\Leftarrow$

Recimo da $\forall a,b\in X$ velja $c_a \simeq c_b$

Naj bosta a,b poljubna

$\exists H.\ X\times[0,1] \longrightarrow X$

$H_0 = c_a$

$H_1 = c_b$

$\gamma(t) = H(a,t)$

4.5

a) $f\ S^n \to S^n$ n: sur

homotopne konstanti

$a \not\in f_*(S^n)$ b je nasprotni točj

$H\ S^n \times [0,1] \longrightarrow S^n$

$(x, t) \longmapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$

Ali je slučajno $(1-t)f(x) + t \cdot x = 0$

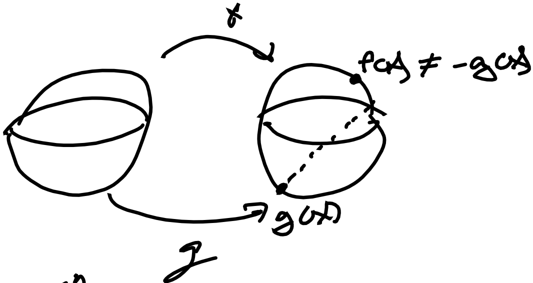
$\Rightarrow$ če gre ta daljica skozi 0

ampak a ni zalogi vrednosti, tako

nikoli nobena daljica skozi 0,0

b) $f, g : S^n \longrightarrow S^n$

$f(x) \neq -g(x) \quad \forall\, x \in S^n \Longrightarrow f \simeq g$



$H : S^n \times [0, 1] \longrightarrow S^n$

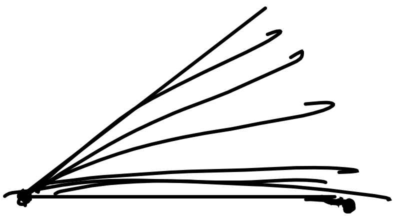
$(x, t) \longmapsto \dfrac{(1-t)(g(x)) - t f(x)}{\|(1-t)g(x) - t f(x)\|}$

4.6 $X = \{[0,1]\times\{0\}\} \cup \{x, \frac{x}{n}, x\in[0,1] \land n\in\mathbb{N}\}$

$\cancel{CX = X\times[0,1] / X\times\{1\}}$

$H: X\times[0,1] \longrightarrow CX$

$(x,y), t = ((1-t)(x,y))$



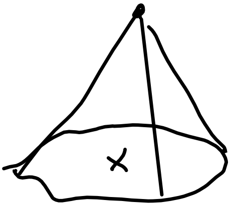
Recimo da je ~~H~~ G podpolna krepka deformacijska retrakcija X-a na tisto točko

$\forall n\in\mathbb{N} . \exists t_n \in[0,1]$ $G((1, \frac{1}{n}), t_n) = (0,0)$

ker je $[0,1]$ kompaktno lahko izberemo konvergentno podzaporedje t_{i_n}

?

$$CX = X \times [0,1] \big/ X \times \{1\}$$



$$CX \times [0,1] \longrightarrow CX$$

$$\begin{tikzcd}
X \times [0,1] \times [0,1] \arrow[r, "H"] \arrow[d, "q \times id"] & X \times [0,1] \arrow[d, "q"] \\
CX \times [0,1] \arrow[r, dashed] & CX
\end{tikzcd}$$

nemaremo nujno seštevat

$$H : (x, u, t) \longmapsto \cancel{(x,1)t + (x,u)(1-t)}$$

$$(x, u(1-t)+t)$$

Preveriti moramo samo

$$(x_1, 1, t) \sim_H (x_2, 1, t)$$

edini netrivialni ekvivalenčni razred ,

$$\begin{matrix} (x_1,1,t) & & (x_2,1,t) \\ \| & & \| \\ (x_1, 1) & & (x_2, 1) \end{matrix}$$

sta ekvivalentni ker sta na višini 1

$$\bar{H}([x,u,0]) = [x,u]$$

$$q(H(x,u,0)) = q(x,u)$$

$$q(H)(x,u,1) = \ldots$$

X je kontraktibilen

$\Longleftrightarrow$

$\exists$ zvezna izbira poti od poljubne točke X do izbrane točke

$\exists a \in X.\ \exists_{zv} f: X \longrightarrow \mathcal{C}(I, X).\ f(x)(0) = x,\ f(x)(1) = a$

$\Longleftrightarrow$

$X \neq \emptyset \wedge \exists$ zvezna izbira poti med točkami v X

$\exists f_{zv}: X \times X \longrightarrow \mathcal{C}(I, X).\ f(a,b)(0) = a,\ f(a,b)(1) = b$

4.8 (4.7 na učilnici)

i) $\Rightarrow$ ii) Vemo iz 4.7 (4.6)

ii) $\Rightarrow$ iii) Naj bo X retrakt kontraktibilnega prostora Y

$\exists r: Y \to X.\ \exists i: X \to Y.\ r \circ i = id$
(ima zvezni desni inverz (ekvivalentna definicija retrakcije))

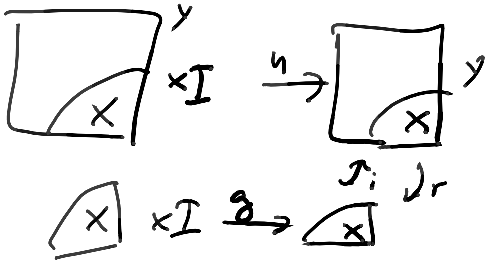
$$\exists y_0 \in Y.\ \exists H: Y \times I \to Y$$
$$(y,0) \mapsto y$$
$$(y,1) \mapsto y_0$$

iščemo

$g: X \times I \to X$ $\qquad x_0 = r(y_0)$

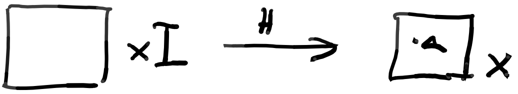
$g(x,0) = g$

$g(x,1) = x_0$

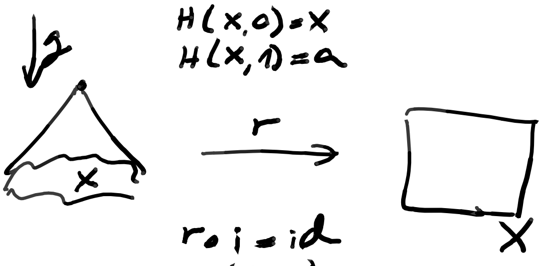


$$g(x,t) = r(h(i(x),t))$$
$$g = r \circ h \circ (i \times id_I)$$

iii) $\Rightarrow$ i)



$H(x,0) = x$
$H(x,1) = a$



$r \circ i = id$
$r([x,0]) = x$

$$CX = X \times I / \{(x,1)\}$$

$$[x,t] \mapsto pr_1(H(x,t))$$

4.9

$$I \times I /_{\text{vemo}} = M \qquad \text{vemo} \Longleftrightarrow \sim$$

CM

$$I \times I /_{\text{vemo}} \times I /_{\{I \times I, 1\}} = CM$$

Recimo da je kontraktibilen

$$\exists H : I \times I /_{\text{vemo}} \times I \longrightarrow I \times I /_{\text{vemo}}$$

$$H(x,0) = x$$
$$H(x,1) = 1$$

Vzamimo podprostor A = [illegible] [illegible]

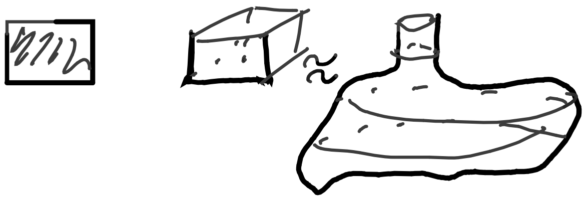
ki je krožnica topološka

torej je tudi A kontraktibilna

4.10

Neprazen konveksen prostor je kontraktibilen

Splošno: zvezdast prostor je kontraktibilen



$\mathcal{N}$ .. Razred normalnih topoloških prostorov

$Y \in AE(\mathcal{N})$ je absolutni ekstenzor za razred normalnih prostorov
kadar za $\forall X \in \mathcal{N}$ in $\forall$ zaprto podmnožico $A \subseteq X$ velja, da lahko zvezno preslikavo $A \to Y$ razširimo do zvezne preslikave $X \to Y$

$$\begin{tikzcd} X \arrow[dr, dotted] & \\ A^{zap} \arrow[u, hook] \arrow[r, "f"'] & Y \end{tikzcd}$$

Tietze: $I \in AE(\mathcal{N})$

- Produkt AE je AE

$$\begin{tikzcd} X & \\ A \arrow[u, hook] \arrow[r] & Y' \times Y'' \end{tikzcd}$$

razširimo vsako komponento

- Retrakt AE je AE

- $X \in AE(\mathcal{N}) \implies X \neq \emptyset \land X$ povezan s potmi

- $AE(\mathcal{N}) \cap \mathcal{N} \subseteq AR(\mathcal{N})$

absolutni retrakt

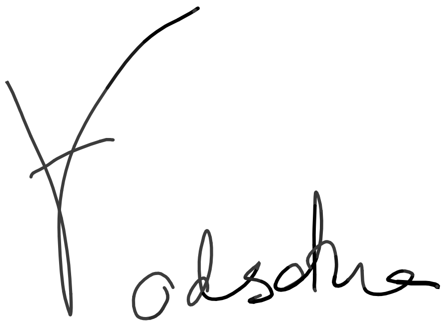
Na tabli: 4. 11

4.12 $\quad A, B^{\text{zapl}} \subseteq \mathbb{R}^n$

$$\begin{tikzcd}
X \arrow[dr, dotted, dash] & \\
C \arrow[u, hook] \arrow[r, Rightarrow] & A \cup B
\end{tikzcd}$$

$\mathbb{R}^n \in AE$

Dokažimo $A \cup B$ je retrakt $\mathbb{R}^n$



4.19

$v \in \mathbb{R}^n \setminus \{0\}$ $\qquad Av = \lambda v$

BSZS $\|v\| = 1$

$\|Av\| = \|\lambda v\| = |\lambda| = \lambda$

$Av = \lambda v \overset{?}{=} \|Av\| v$

$$v = \frac{Av}{\|Av\|}$$

iščemo negibno točko preslikave

$S^{n-1} \longrightarrow S^{n-1}$

$v \longmapsto \dfrac{Av}{\|Av\|}$ ne nujno dobro definirana preslikava, ker $\ker A$ ni nujno $\{0\}$

$X = \{(x_1, \ldots x_n) \in S^{n-1} \mid x_1, \ldots, x_n \geq 0\} \approx B^{n-1}$

če $v \in X$ so vsi $x_i \geq 0$ in niso vsi $= 0$

$\Rightarrow Av$ ima pozitivne komponente, posebej $\neq 0$

$B^{n-1} \longrightarrow B^{n-1}$ ima negibno točko

Mnogoterost

a) $\{\frac{1}{n}, n \in \mathbb{N}\}$ števnost + diskretnost

= je mnogoterost

b) $\{\frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$

v 0 ni mnogoterost

če bi jo imela bi morala biti iz $\mathbb{R}^0 = \{1\}$

ali $\mathbb{R}^0_+$ ker ne dovolj

torej točka 0 mora biti okolica

vsake točke ~~...~~ od a ko fermi: ...

mora biti $\{0\}$ odprta, ker pa ni:

$$\boxed{\text{Števna vsota n-mnt je n-mnt}}$$

20.5

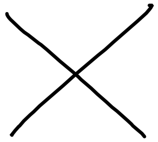
c) $\approx \underbrace{\{-1,1\}}_{0\,\text{dim}}\times \underbrace{\mathbb{R}}_{1\,\text{dim}} \approx \mathbb{R}+\mathbb{R}$

Produkt mnogoterosti je mnogoterost

$\Rightarrow$ 1 mnogoterost

Tudi: števna vsota n-mnogoterosti je n-mnogoterost

d)



ni mnogoterost v $(0,0)$

$(1,1)$ ima 1-mnogoterost na delcu

Torej v $(0,0)$ mora biti 1 mnogoterost

Trdimo da ni: $\approx \mathbb{R}$ ali $\mathbb{R}_+$

naj bo U poljubna okolica točke

Ni dovolj preverjati na bazi

$U-\{(0,0)\}$ ima vsaj 4 komponente

če bi U bil homeomorfen $\mathbb{R}$ ali $\mathbb{R}_+$

bi razpadel na 1 ali 2 komponenti

$\rightarrow$ ~~ni ...~~ U ni ~~...~~

ni mnogoterost

Naredimo vložitev 1dim mnogoterosti brez roba in $\approx$ preslikava ni odprta pa dem ni mnogoterost

$f:\mathbb{R}\to X$

$f(x)=(x,x)$

$f_*(\mathbb{R})$ mora biti odprt

$(0,0)$ je robna točka za sliko

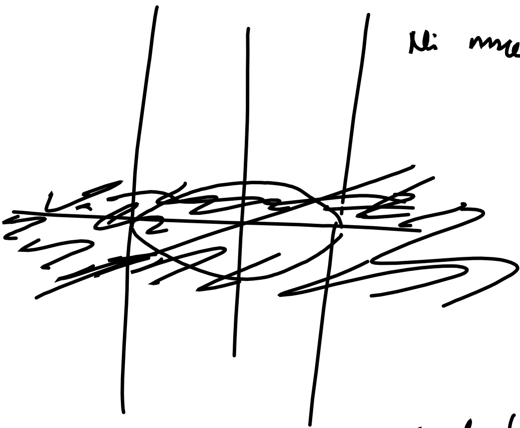
ker vsebuje $(\varepsilon,-\varepsilon)$ za dovolj majhne $\varepsilon \neq 0$ v večji okolici

e) $(\mathbb{R}^2\times\{0\})\cup(S^1\times\mathbb{R})$

(za d nalogo)

Izrek o invarianci odprtih množic za mnogoterosti

Če sta M,N istodimenzionalni mnogoterosti M brez roba in $f: M\to N$ zvezna injekcija, potem je f odprta vložitev v $\operatorname{int} N$



Ni mnogoterost

ker bi lahko vložili

(mogla bi biti 2 mnogoterost (lokalno v $(0,0)$))

$f:\mathbb{R}^2\to X$

$x\mapsto(x,0)$

$0,0$ ima okolico $K_{\mathbb{R}^2}((0,0),\frac{1}{2})\times\{0\}\approx\mathbb{R}^2$

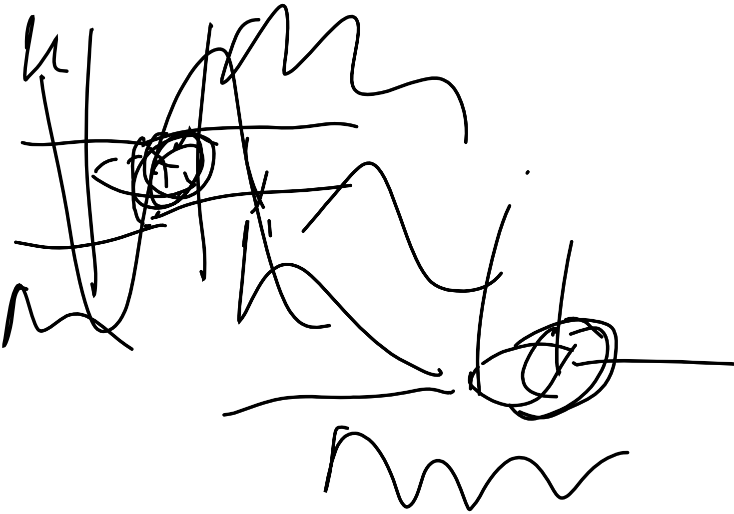
$f_*(\mathbb{R}^2)$ ni odprta ker

$(1,0,0)$ je robna za $S^1\times$

$\forall\, U\ni(0,0,0)$

$(1,0,\varepsilon)\in U$

f)



vložíte:

z meetest

$f: S^1 \times \{\mathbb{R}\} \to X$

$x \mapsto x$

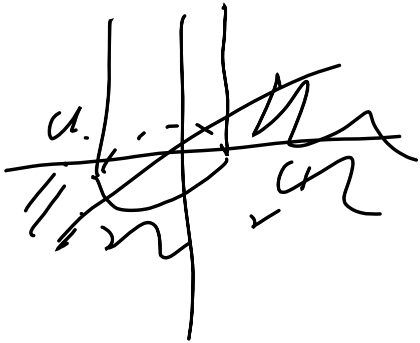
nafukov $(1,0,0)$

$$\partial(M\times N) = \underset{\emptyset}{\partial M}\times N \cup M\times \underset{\emptyset}{\partial N}$$

$(1,0,0) \in$ je v okolí ... že

[illegible]

g)



je ~~mnogterost~~

$\approx S^1 \times \mathbb{R}$

$f: X \longrightarrow S^1 \times \mathbb{R}$

$x = re^{i\varphi}, r = 1$

$f:\ x \longmapsto x$ ~~$\varphi \ge x \ge 0$~~ ~~$x \ge 1$~~

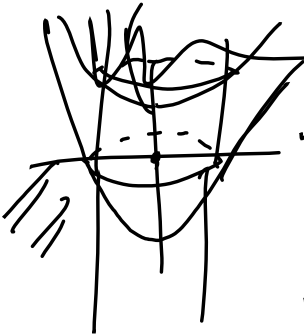
$(re^{i\varphi}, 0) \longmapsto (e^{i\varphi}, -r+1)$

$r = 1 \Rightarrow e^{i\varphi}, -1+1 =$ se ujeme elementarne

inverz

$$e^{i\varphi}, z \longmapsto \begin{cases} e^{i\varphi}, z & z \ge 0 \\ (-z+1)\, e^{i\varphi}, 0 & z \le 0 \end{cases}$$

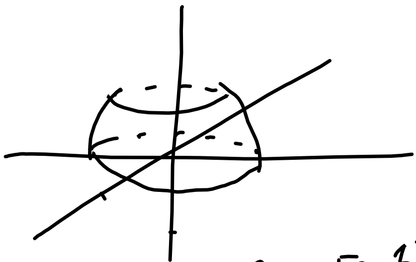
h)



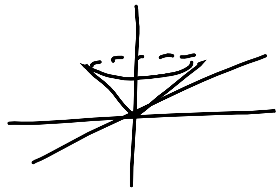
ni

Vlastno mnozen

5.2



$\approx S_1 \times [0, \frac{1}{2})$



$\approx B_n$

$\downarrow$

pr ogleda - rga

$\leftarrow$

prejodkaj

DN

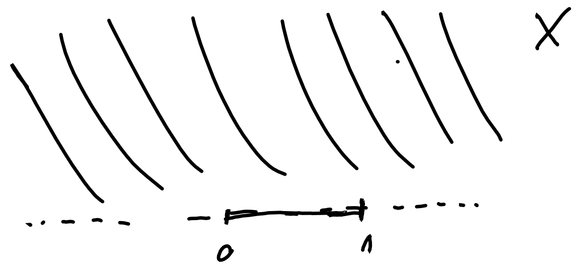
nima
lastnosti
negibne točke

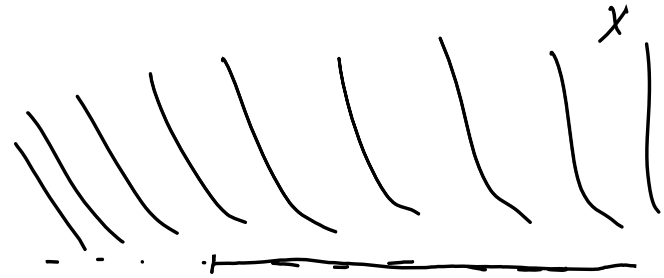
ima
lastnost
negibne točke

Rob ima
2 komponenti

Rob ima
eno komponento

5.3





X ima 2 nemnogoterostni točki

Y pa ima 1

v $(0,0)$ ni mnogoterost ker ni lokalno kompaktna v tej točki

$Z = \mathbb{R} \times [0,\infty)$ Recimo da sta si homeomorfna

~~$Z \to X$ v notranjost~~

~~$Z \to Y$ v notranjost~~

~~Recimo~~ dobimo $[0,\infty)$ [illegible] v

$$\underbrace{Z \hookrightarrow X}_{\text{zvezna injekcija se slika v notranjost}} \xrightarrow{h} Y \hookrightarrow \mathbb{R}^2$$

$h_*(Z)$ je odprta pod $\mathbb{R}^2$

in $h_*(Z) \subseteq Y \Rightarrow$ odprt v Y

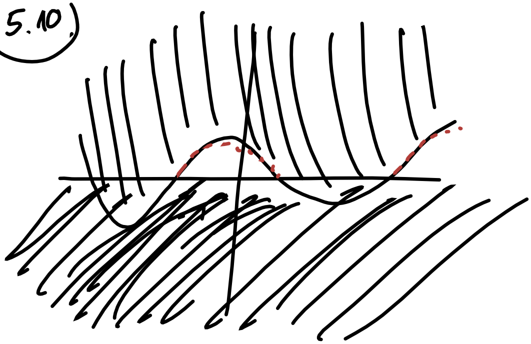
$\Rightarrow h_*(Z) \subseteq Z$

po enakem premisleku

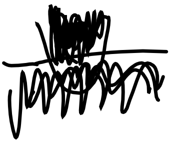
$(h^{-1})_*(Z) \subseteq Z$

$\Rightarrow h|_{[0,1]\times\{0\}} \to [0,\infty)\times\{0\}$ je [illegible] [illegible] $\times$

5.10.

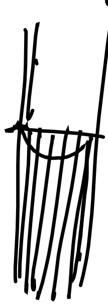


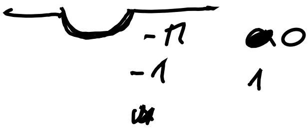
a) $p_{-1}(x) = x^2 - 1$



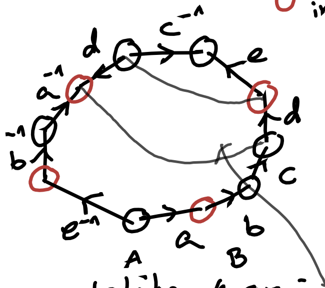
X_{p-1} ni lok. kompakten v $(-1, 0)$ ker se vedno dobi ∃ zaporedje $(-1, \varepsilon_n)$

Y_{p-1} je mnogoterost





$x, y \longmapsto (y + \sqrt{1-x^2}\, e^{i\frac{(x+1)}{2}\pi}$



○ in ○ sta si ena

ker se ponovi:
vsaka točka nekako dvakrat
je to mnogoterost

Ni orientabilna, ker je möbiusov trak

0-celice: 1
1-celice: 5
2-celice: 1

$\chi(X) = -3$

ker je neorientabilna bo to nP

$$\boxed{\chi(nP) = 2 - n}$$

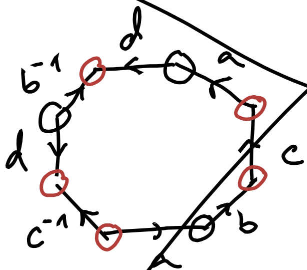
$$\boxed{\chi(nT) = 2 - 2n}$$

$2 - n = -3$

$n = 5$

$X \approx 5P$

je ploskev
neorientabilna
ni sklenjena

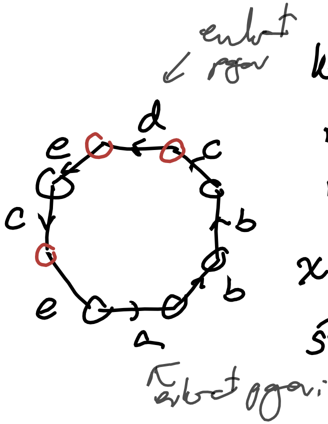


0-celice : 2

1-celice : 5

2-celice : 1

napaka
naloge :(



komp. ploskev

n: sklenjena

n: orientabilna

$\chi(D) = 2-5+1$

št robnih komponent: 2

D' = D kjer na vsako robno komponento prilepimo disk:

$\Longrightarrow$ dodamo 2 2-celici:

0-celice: 2

1-celice: 5

2-celice: 3

$\chi(D') = 0$

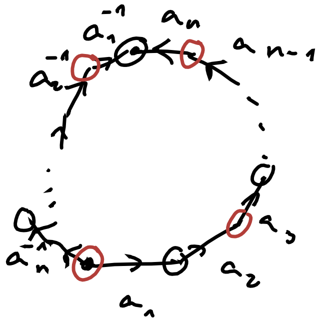
nP:

$2 - n = 0$

$n = 2$

$D' = 2P$

$\Rightarrow$ D je 2P ~~[illegible]~~ z dvema luknjama



kompaktna orientabilna sklenjena

n sodo: 0-celic: 1

n liho: 0-celic: 2

1-celice: n

2-celic: 1

n sodo: $\chi(G) = 1 - n + 1 = 2 - n$

n liho: $\chi(G) = 2 - n + 1 = 3 - n$

$$2 - 2m = 2 - n \qquad\qquad 2 - 2m = 3 - n$$

$$2m = n \qquad\qquad m = \frac{n-1}{2}$$

$$m = \frac{n}{2}$$

$$G \approx \frac{n}{2} T \qquad\qquad G \approx \frac{n-1}{2} T$$

5.14

a) sistem enačb:

$X \# Y \approx 2T \qquad X \# T \approx Y \# K$

$\partial(X\#Y) \approx \partial X + \partial Y$

$X, Y \stackrel{\Rightarrow}{\text{sklenjeni}}$ orientabilni

X neorientabilen $\Longrightarrow$ ni rešitve

b) $X \# Y \# Z \approx 2K \# T$ ← sklenjena ploskev

$X \# Y \approx Z \# 2T$

$X \# Z \approx Y \# K$

X ali Z neorientabilna

x, y, z sklenjene

Z neorientabilen $\Rightarrow$ X ali Y sta neorientabilna

$$\boxed{\chi(X\#Y) = \chi(X) + \chi(Y) - 2}$$

$x + y - 2 + z - 2 \approx \chi(4P) + \chi(T) - 2$

$x + y + z - \cancel{4} \approx \cancel{2-4} + \cancel{2} - \cancel{2} - \cancel{2}$

$x + y + z = 0$

$2k + T$ je neorientabilna $\Rightarrow 2 - n = 0$

$X \# Y \# Z \approx 2P$

$x + y - 2 = z + 2 - 2 \cdot 2 - 2$

$x + y - z = 2 - 4 = -2$

$$\boxed{\text{Orientabilne ploskve imajo sodo } \chi}$$

$x + z - 2 = y + 2 - 2$

$x - y + z = 2$

$2y = -2 \qquad 2z = 2 \qquad x = 0$

$y = -1 \qquad z = 1$

Z neorientabilna $\Rightarrow$

$X \approx 3P$

$Z \approx P$

Y je lahko orientabilen ali ne

$Y \approx T$ če orientabilen

$Y \approx \underset{\substack{\parallel \\ K}}{2P}$ če neorientabilen

$X \# M \approx Y \# P$

$\boxed{\pi_0 \text{ je aditivno}}$

$X \# Y \approx M \# 3P$

$\partial(M \# N) = \partial M + \partial N$

$\pi_0(\partial Y) = 1 + \pi_0(\partial X)$

št komponent za povezanost s potmi = št robnih komponent

$\pi_0(\partial X) + \pi_0(\partial Y) = 1$

$\pi_0(\partial X) + \cancel{1} + \pi(\partial X) = \cancel{1}$

$\pi_0(\partial X) = 0 \implies \pi_0(\partial Y) = 1$

X sklenjena

$\boxed{\text{Möbiousov trak = projektivna ravnina brez enega diska}}$

$\chi(M) = \chi(P) - 1 = 0$

$x + y - \cancel{2} = 0 + 2 - 3 - \cancel{2}$

$x + 0 - \cancel{2} = y + 1 - \cancel{2}$

$x - y = 1$

$x + y = -1$

$x = 0$

$y = -1$

$X \approx T \lor X \approx 2P$

$X \approx T \implies Y \approx T$

5.15

a)

orientabilna

$\chi(A)$

$\chi(X \cup \text{Trak}) = \chi(X) - 1$