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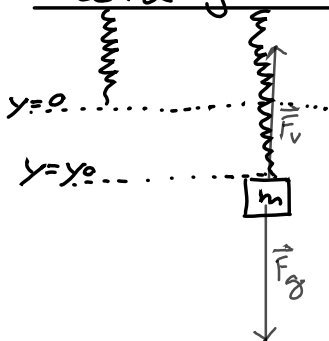
VII zaključek

I Mehansko nihanje in valovanje

Enostavna nihala

Enačba dušenega nihanja

Utěz na vijachi: vzmeti



$$\vec{F}_g = \begin{bmatrix} 0 \\ -mg_0 \\ 0 \end{bmatrix}$$

smer navzdol

$$g_0 \approx 10 \text{ m/s}^2$$

$$\vec{F}_v = \begin{bmatrix} 0 \\ -ky_0 \\ 0 \end{bmatrix}$$

k... koeficient vzmeti N/m
 $y_0 < 0$

$$-ky_0 > 0 \Rightarrow \text{smer je navzgor}$$

neto sila

$$\vec{a} = 0 \Leftrightarrow \vec{F} = m\vec{a} = 0$$
$$\vec{F}_g + \vec{F}_v = 0$$

$$-mg_0 - ky_0 = 0 \Rightarrow$$

$$mg_0 = -ky_0$$

$$y_0 = \frac{mg_0}{k}$$

$$\vec{F}_g = -mg_0 \hat{e}_y$$

$$\vec{F}_v = -k y \hat{e}_y$$

$$\vec{F}_u \dots \text{sila upora}$$

$$\vec{F}_u = -c \vec{v} \quad (\text{linearne sila upora}) \quad v_y = \dot{y} = \frac{dy}{dt} \neq 0$$

$c > 0$ sorazmerna z viskoznostjo tekočina in površino uteži

$$\vec{F}_u = -c \dot{y} \hat{e}_y$$

$$\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$$

$$\vec{F} = m \cdot \vec{a} \quad ; \quad \vec{a} = \ddot{y} \hat{e}_y$$

$$-c \dot{y} \hat{e}_y - k y \hat{e}_y - m g_0 \hat{e}_y = m \ddot{y} \hat{e}_y$$

$$\underbrace{\left(\ddot{y} + \frac{c}{m} \dot{y} + \frac{k}{m} y + g_0 \right)}_{=0} \hat{e}_y = 0$$

$$\text{Vpeljemo } \beta := \frac{c}{m} \quad [\beta] = s^{-1}$$

$$\omega_0^2 = \frac{k}{m} \quad [\omega_0^2] = s^{-2}$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y + g_0 = 0$$

$$y_0 = \frac{m g_0}{k}$$

$$g_0 = \frac{y_0 k}{m} = y_0 \omega_0^2$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 (y - y_0) = 0$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y = \omega_0^2 y_0$$

- diferencialna enačba za y 2. reda
- linearne enačbe
- konstantni koeficienti
- nehomogena (pogojno, ker je lahko z lahkoto prevedemo v homogeno)

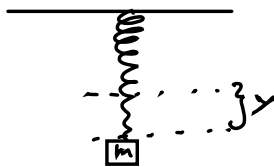
$$y' := y - y_0$$

$$\dot{y}' = (\dot{y} - \dot{y}_0) = \dot{y}$$

$$\ddot{y}' = \ddot{y}$$

$$\text{Dobimo: } \ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = 0$$

je homogeno :



$$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y = 0$$

$$\text{Nastavek: } y' = A e^{\lambda t}, \quad A, \lambda \text{ konstante}$$

$$[\lambda] = s^{-1}$$

$$\dot{y}' = \lambda y$$

$$[\lambda] = m$$

$$\ddot{y}' = \lambda^2 y$$

$$(\lambda^2 + \beta \lambda + \omega_0^2) A e^{\lambda t} = 0 \quad \text{za } \forall t \quad \sim \neq 0 \text{ to } A \neq 0$$

$$\lambda^2 + \beta \lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 = -4\omega^2$$

$$(\omega^2 = \omega_0^2 - (\frac{\beta}{2})^2)$$

$$D < 0 \Rightarrow 4\omega^2 > 0 : \text{podkritično dušenje}$$

$$\sqrt{D} = \sqrt{-4\omega^2} = \sqrt{-1} 2\omega \quad ; \quad \omega = \sqrt{\omega^2}$$

$$\Rightarrow \lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = \frac{-\beta}{2} \pm i\omega$$

$$y_1' = A_1 e^{\lambda_1 t} = A_1 \exp\left(-\frac{\beta}{2}t + i\omega t\right) =$$

$$= A_1 \exp\left(-\frac{\beta}{2}t\right) \exp(i\omega t)$$

$$y_2' = A_2 \exp\left(-\frac{\beta}{2}t\right) \exp(-i\omega t)$$

$$\begin{cases} \ddot{y}_1' + \beta \dot{y}_1' + \omega_0^2 y_1' = 0 \\ \ddot{y}_2' + \beta \dot{y}_2' + \omega_0^2 y_2' = 0 \end{cases} \quad \int t$$

$$(\ddot{y}_1' + \ddot{y}_2') + \beta(\dot{y}_1' + \dot{y}_2') + \omega_0(y_1' - y_2') = 0$$

$$\Rightarrow y' = \exp\left(-\frac{\beta}{2}t\right) (A_1 \exp(i\omega t) + A_2 \exp(-i\omega t))$$

Eulerjeva enačba

$$\exp(\pm i\omega t) = \cos(\omega t) \pm i \sin(\omega t)$$

$$\Rightarrow y' = \exp\left(-\frac{\beta}{2}t\right) ((A_1 + A_2) \cos(\omega t) + i(A_1 - A_2) \sin \omega t)$$

$$= e^{-\frac{\beta}{2}t} (B_1 \cos(\omega t) + B_2 \sin(\omega t))$$

$$= B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) \quad \leftarrow \text{Fazni zamik}$$

$$B > 0; \quad \delta = \text{fazni zamik}$$

$$B e^{-\frac{\beta}{2}t} (\sin \omega t \sin \delta + \cos \omega t \cos \delta)$$

$$= e^{-\frac{\beta}{2}t} (B \sin \delta \cos(\omega t) + B \cos \omega t \sin \delta)$$

$$B_1 = B \sin \delta$$

$$B_2 = B \cos \delta$$

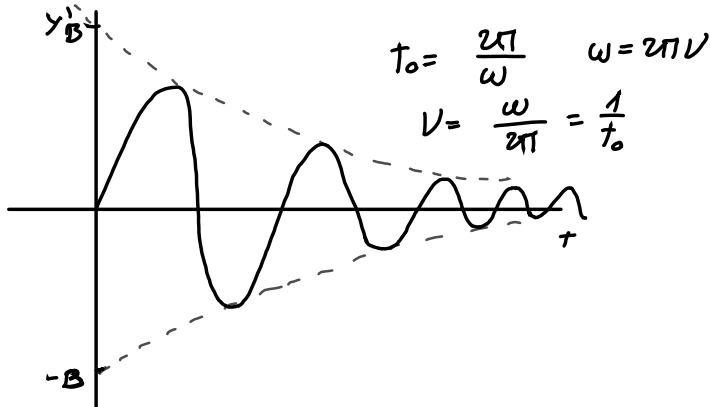
$$\tan \delta = \frac{B_1}{B_2}$$

$$B^2 = B_1^2 + B_2^2$$

$$B = \sqrt{B_1^2 + B_2^2}$$

Primer :

$$\delta = 0 \Rightarrow y = B e^{-\frac{\beta}{2}t} \sin(\omega t)$$



$$\delta = \frac{\pi}{2}$$

$$\begin{aligned} y'(t) &= B e^{-\frac{\beta}{2}t} \sin\left(\omega t + \frac{\pi}{2}\right) = \sin \omega t \cos \frac{\pi}{2} + \cos \omega t \sin \frac{\pi}{2} \\ &= B e^{-\frac{\beta}{2}t} \cos(\omega t) \end{aligned}$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y = 0$$

$y' = y - y_0 \rightarrow$ odmik do ravnovesja vzeti v ravnovesni točki
 \hookrightarrow odmik od konca neobremenjene vzeti

$$\omega_0^2 = \frac{k}{m} (> 0)$$

$$\beta \propto C \propto M$$

\nwarrow sorazmerne

Nastavek $y' = A e^{\lambda t}$

$$\Rightarrow \lambda^2 + \beta \lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 = -4\omega^2 \quad \omega^2 = \omega_0^2 - \left(\frac{\beta}{2}\right)^2$$

a) $D < 0 \Rightarrow (\omega^2 > 0)$

$$y' = B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) ; \omega = \sqrt{\omega^2} = \sqrt{\omega_0^2 - \left(\frac{\beta}{2}\right)^2}$$

Zelo gibko dušenje $(\frac{3}{2})^2 \ll \omega_0 \Rightarrow$

$$T_0 = \frac{2\pi}{\sqrt{\omega_0^2}} = 2\pi \sqrt{\frac{m}{k}}$$

Primer:

$$m = 500 \text{ g} = 0,5 \text{ kg}$$

$$k y_0 = -m g_0$$

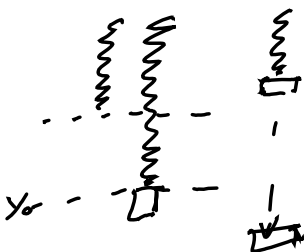
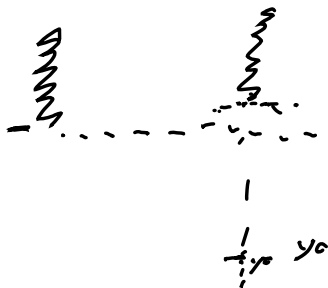
$$k = -\frac{m g_0}{y_0} \Rightarrow \frac{m}{k} = -\frac{y_0}{g_0}$$

$$y_0 = -0,4 \text{ m}$$

$$\frac{m}{k} = \frac{0,4 \text{ m}}{10 \text{ m/s}^2} = 0,04 \text{ s}^2 =$$

$$\sqrt{\frac{m}{k}} = 2 \cdot 10^{-1} = 0,2 \text{ s}$$

$$\Rightarrow T_0 = 2\pi \cdot 0,2 \text{ s} \approx 1,2 \text{ s}$$



B in δ dobimo iz začetnih pogojev

$$y' = B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta)$$

$$\dot{y}' = -\frac{\beta}{2} B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) + \omega B e^{-\frac{\beta}{2}t} \cos(\omega t + \delta)$$

$$\dot{y}'(0) = -\frac{\beta}{2} B \sin \delta + \omega B \cos \delta =$$

$$y'(0) = B \sin \delta \quad = B \left(-\frac{\beta}{2} \sin \delta + \omega \cos \delta \right)$$

$$r = \frac{y'(0)}{\dot{y}'(0)} = \frac{\sin \delta}{\omega \cos \delta - \frac{\beta}{2} \sin \delta} = \frac{\tan \delta}{\omega - \frac{\beta}{2} \tan \delta}$$

$$\Rightarrow \delta = \arctan \left(\frac{r \omega}{1 + \frac{\beta r}{2}} \right)$$

$$\Rightarrow B = \frac{y'(0)}{\sin \delta}$$

$$\dot{y}'(0) = B \omega \sin \delta$$

$$\Rightarrow B = \frac{\dot{y}'(0)}{\pm \omega} = \frac{|\dot{y}'(0)|}{\omega}$$

Energija nihala

$$W_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{y}^2$$

$$W_{pr} = \frac{1}{2} k y^2 = \frac{1}{2} k (y) + y_0)^2$$

$$W_p = m g_0 (y + y_0)$$

$$\text{Skupna } \boxed{W = W_k + W_p + W_{pr}}$$

a) W za zelo slobko dušenje ?

$$\frac{B}{z} + \ll 1$$

$$\left(\frac{B}{z}\right)^2 \ll \omega_0^2 \Rightarrow \omega \approx \omega_0 = \sqrt{\frac{k}{m}}$$

$$y' = B \sin(\omega_0 t + \delta)$$

$$\dot{y}' = \omega_0 B \cos(\omega_0 t + \delta)$$

$$\begin{aligned} W_k &= \frac{1}{2} m \omega_0^2 B^2 \cos^2(\omega_0 t + \delta) = \\ &= \frac{1}{2} m \frac{k}{m} B^2 \cos^2(\omega_0 t + \delta) \end{aligned}$$

$$W_{pr} = \frac{1}{2} k B^2 \sin^2(\omega_0 t + \delta) + \frac{1}{2} k y_0^2 + k y_0 B \sin(\omega_0 t + \delta)$$

$$W_p = m g_0 B \sin(\omega_0 t + \delta) + m g_0 y_0$$

$$\begin{aligned} W &= \frac{1}{2} k B^2 \underbrace{(\sin^2(\omega_0 t + \delta) + \cos^2(\omega_0 t + \delta))}_{1} \\ &\quad + (k y_0 + m g_0) B \sin(\omega_0 t + \delta) + \\ &\quad + \frac{1}{2} k y_0^2 + m g_0 y_0 \\ &= \frac{1}{2} k B^2 + \frac{1}{2} k y_0^2 + m g_0 y_0 = \text{konst.} \end{aligned}$$

b) kritično dušenje

$D=0$ ($\omega=0$) (kritično dušenje)

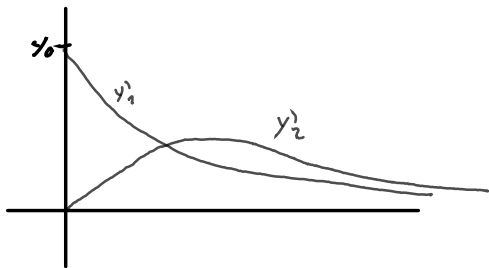
$$D = \beta^2 - 4\omega^2 \Rightarrow \omega_0 = \frac{\beta}{2}$$

$$\lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = -\frac{\beta}{2}$$

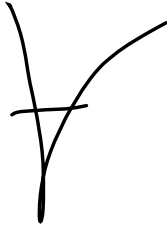
$$\Rightarrow y_1' = B_1 e^{-\frac{\beta}{2}t}$$

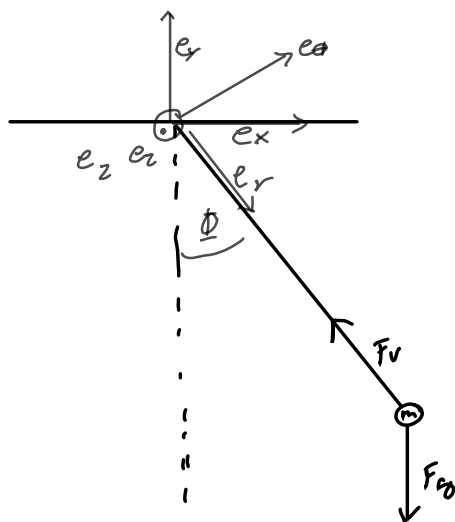
$$y_2' = B_2 t e^{-\frac{\beta}{2}t} \text{ je tudi rešitev (DN)}$$

$$\Rightarrow y' = y_1' + y_2' = (B_1 + B_2 t) e^{-\frac{\beta}{2}t}$$



20.2





$e_\rho, e_z, e_\phi \dots$ cylindrische
Koordinate

$$\vec{F} = \vec{F}_g + \vec{F}_v = m\vec{a}$$

$$F_g = -mg_0 \hat{e}_z$$

$$F_v = -F_v \hat{e}_r$$

$$\vec{r} = l \cdot \vec{e}_r$$

$$\vec{r} \times \vec{F} = l \vec{e}_r \times (-mg_0 \hat{e}_z - F_v \hat{e}_r) =$$

$$= -mg_0 l \hat{e}_r \times \hat{e}_z - \underbrace{l F_v \hat{e}_r \times \hat{e}_r}_{=0}$$

$$= -mg_0 l \hat{e}_r \times (\hat{e}_z \cos\phi + \hat{e}_\phi \sin\phi)$$

$$= -mg_0 l \sin\phi \hat{e}_r \times \hat{e}_\phi =$$

$$= -mg_0 l \sin\phi \hat{e}_z$$

$$\vec{r} \times \vec{F} = m \cdot \vec{r} \times \vec{a} = m \cdot l \hat{e}_r \times (a \hat{e}_\phi) =$$

$$\vec{a} = a \cdot \hat{e}_\phi = m a l \hat{e}_z$$

$$a = a_\phi = l \ddot{\Phi}$$

$$mg_0 l \sin\phi \hat{e}_z = m l \ddot{\Phi} \hat{e}_z$$

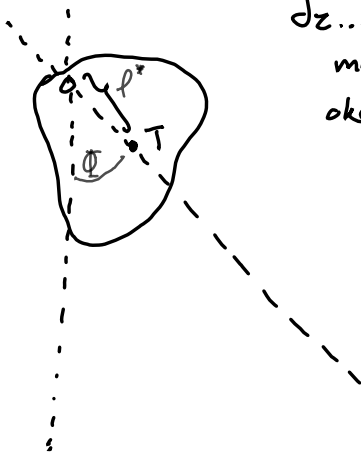
$$m l (l \ddot{\Phi} + g_0 \sin\phi) \hat{e}_z = 0$$

$$-l \ddot{\Phi} = g_0 \sin\phi$$

$$\ddot{\Phi} + \frac{g_0}{l} \sin\phi = 0$$

$$\approx \underbrace{\ddot{\Phi}}_{\omega_0^2} + \frac{g_0}{l} \Phi = 0$$

$$\ddot{\Phi} + \omega_0^2 \Phi = 0$$



J_z vs trajnostni
moment za vrtenje
okrog fiksne osi

primer: palica

$$J_z = \frac{1}{3} m l^2 \quad l^* = \frac{l}{2}$$

$$\sin \Phi \approx \Phi$$

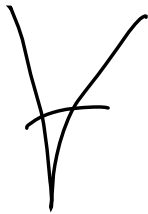
$$\Rightarrow \ddot{\Phi} + \omega_0^2 \Phi = 0$$

$$\omega_0^2 = \frac{m g l^*}{J_z} = \frac{\cancel{2} m g \cancel{2} l}{2 \cdot \cancel{2} m l^2} = \frac{3}{2} \frac{g}{l}$$

N; horizontalo:

$$J_z = m l^2 \quad l^* = l$$

$$\omega_0^2 = \frac{m g_0 l}{m l^2} = \frac{g_0}{l}$$



21.2

Nism & shkala