

I Mehansko nihanje in valovanje

II električno polje

III električni tok

IV magnetno polje

V elektrodinamika

VI posebna teorija relativnosti

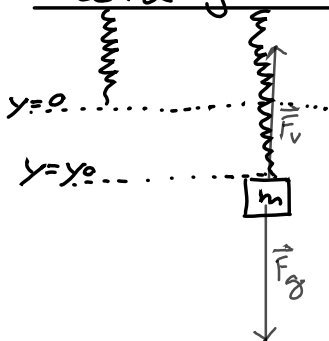
VII zaključek

I Mehansko nihanje in valovanje

Enostavna nihala

Enačba dušenega nihanja

Utěz na vijachi: vzmeti



$$\vec{F}_g = \begin{bmatrix} 0 \\ -mg_0 \\ 0 \end{bmatrix}$$

smer navzdol

$$g_0 \approx 10 \text{ m/s}^2$$

$$\vec{F}_v = \begin{bmatrix} 0 \\ -ky_0 \\ 0 \end{bmatrix}$$

k... koeficient vzmeti N/m
 $y_0 < 0$

$$-ky_0 > 0 \Rightarrow \text{smer je navzgor}$$

neto sila

$$\vec{a} = 0 \Leftrightarrow \vec{F} = m\vec{a} = 0$$
$$\vec{F}_g + \vec{F}_v = 0$$

$$-mg_0 - ky_0 = 0 \Rightarrow$$

$$mg_0 = -ky_0$$

$$y_0 = \frac{mg_0}{k}$$

$$\vec{F}_g = -mg_0 \hat{e}_y$$

$$\vec{F}_v = -k y \hat{e}_y$$

$$\vec{F}_u \dots \text{sila upora}$$

$$\vec{F}_u = -c \vec{v} \quad (\text{linearne sila upora}) \quad v_y = \dot{y} = \frac{dy}{dt} \neq 0$$

$c > 0$ sorazmerna z viskoznostjo tekočina in površino uteži

$$\vec{F}_u = -c \dot{y} \hat{e}_y$$

$$\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$$

$$\vec{F} = m \cdot \vec{a} \quad ; \quad \vec{a} = \ddot{y} \hat{e}_y$$

$$-c \dot{y} \hat{e}_y - k y \hat{e}_y - m g_0 \hat{e}_y = m \ddot{y} \hat{e}_y$$

$$\underbrace{\left(\ddot{y} + \frac{c}{m} \dot{y} + \frac{k}{m} y + g_0 \right)}_{=0} \hat{e}_y = 0$$

$$\text{Vpeljemo } \beta := \frac{c}{m} \quad [\beta] = s^{-1}$$

$$\omega_0^2 = \frac{k}{m} \quad [\omega_0^2] = s^{-2}$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y + g_0 = 0$$

$$y_0 = \frac{m g_0}{k}$$

$$g_0 = \frac{y_0 k}{m} = y_0 \omega_0^2$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 (y - y_0) = 0$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y = \omega_0^2 y_0$$

- diferencialna enačba za y 2. reda
- linearne enačbe
- konstantni koeficienti
- nehomogena (pogojno, ker je lahko z lahkoto prevedemo v homogeno)

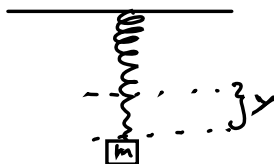
$$y' := y - y_0$$

$$\dot{y}' = (\dot{y} - \dot{y}_0) = \dot{y}$$

$$\ddot{y}' = \ddot{y}$$

$$\text{Dobimo: } \ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = 0$$

je homogena :



$$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y = 0$$

Nastavek: $y' = A e^{\lambda t}$, A, λ konstante

$$[\lambda] = s^{-1}$$

$$\dot{y}' = \lambda y$$

$$[\lambda] = m$$

$$\ddot{y}' = \lambda^2 y$$

$$(\lambda^2 + \beta \lambda + \omega_0^2) A e^{\lambda t} = 0 \quad \lambda \neq 0 \text{ to } A \neq 0$$

$$\lambda^2 + \beta \lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 = -4\omega^2$$

$$(\omega^2 = \omega_0^2 - (\frac{\beta}{2})^2)$$

$$D < 0 \Rightarrow 4\omega^2 > 0 : \text{podkritično dušenje}$$

$$\sqrt{D} = \sqrt{-4\omega^2} = \sqrt{-1} 2\omega ; \omega = \sqrt{\omega^2}$$

$$\Rightarrow \lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = \frac{-\beta}{2} \pm i\omega$$

$$y_1' = A_1 e^{\lambda_1 t} = A_1 \exp\left(-\frac{\beta}{2}t + i\omega t\right) =$$

$$= A_1 \exp\left(-\frac{\beta}{2}t\right) \exp(i\omega t)$$

$$y_2' = A_2 \exp\left(-\frac{\beta}{2}t\right) \exp(-i\omega t)$$

$$\begin{cases} \ddot{y}_1' + \beta \dot{y}_1' + \omega_0^2 y_1' = 0 \\ \ddot{y}_2' + \beta \dot{y}_2' + \omega_0^2 y_2' = 0 \end{cases} \quad \int t$$

$$(\ddot{y}_1' + \ddot{y}_2') + \beta(\dot{y}_1' + \dot{y}_2') + \omega_0(y_1' - y_2') = 0$$

$$\Rightarrow y' = \exp\left(-\frac{\beta}{2}t\right) (A_1 \exp(i\omega t) + A_2 \exp(-i\omega t))$$

Eulerjeva enačba

$$\exp(\pm i\omega t) = \cos(\omega t) \pm i \sin(\omega t)$$

$$\Rightarrow y' = \exp\left(-\frac{\beta}{2}t\right) ((A_1 + A_2) \cos(\omega t) + i(A_1 - A_2) \sin \omega t)$$

$$= e^{-\frac{\beta}{2}t} (B_1 \cos(\omega t) + B_2 \sin(\omega t))$$

$$= B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) \quad \leftarrow \text{Fazni zamik}$$

$$B > 0 ; \delta = \text{fazni zamik}$$

$$B e^{-\frac{\beta}{2}t} (\sin \omega t \sin \delta + \cos \omega t \cos \delta)$$

$$= e^{-\frac{\beta}{2}t} (B \sin \delta \cos(\omega t) + B \cos \omega t \sin \delta)$$

$$B_1 = B \sin \delta$$

$$B_2 = B \cos \delta$$

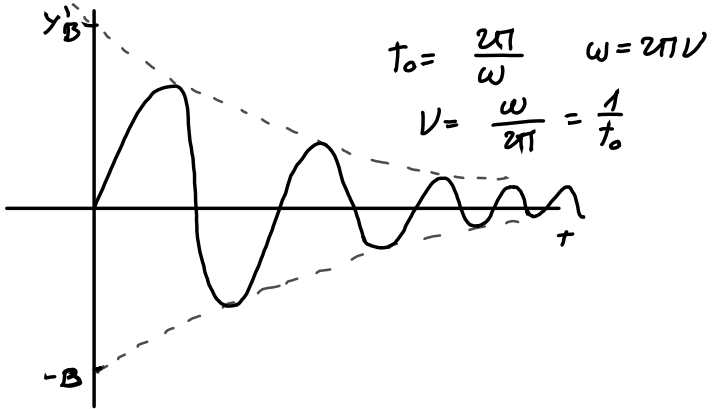
$$\tan \delta = \frac{B_1}{B_2}$$

$$B^2 = B_1^2 + B_2^2$$

$$B = \sqrt{B_1^2 + B_2^2}$$

Primer :

$$\delta = 0 \Rightarrow y = B e^{-\frac{\beta}{2}t} \sin(\omega t)$$



$$\delta = \frac{\pi}{2}$$

$$\begin{aligned} y'(t) &= B e^{-\frac{\beta}{2}t} \sin\left(\omega t + \frac{\pi}{2}\right) = \sin \omega t \cos \frac{\pi}{2} + \cos \omega t \sin \frac{\pi}{2} \\ &= B e^{-\frac{\beta}{2}t} \cos(\omega t) \end{aligned}$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y = 0$$

$y' = y - y_0 \rightarrow$ odmik do ravnovesja vzeti v ravnovesni točki
 \hookrightarrow odmik od konca neobremenjene vzeti

$$\omega_0^2 = \frac{k}{m} (> 0)$$

$$\beta \propto c \propto \eta$$

\nwarrow sorazmerne

Nastavek $y' = A e^{\lambda t}$

$$\Rightarrow \lambda^2 + \beta \lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 = -4\omega^2 \quad \omega^2 = \omega_0^2 - \left(\frac{\beta}{2}\right)^2$$

a) $D < 0 \Rightarrow (\omega^2 > 0)$

$$y' = B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) ; \omega = \sqrt{\omega^2} = \sqrt{\omega_0^2 - \left(\frac{\beta}{2}\right)^2}$$

Zelo gibko dušenje $(\frac{3}{2})^2 \ll \omega_0 \Rightarrow$

$$T_0 = \frac{2\pi}{\sqrt{\omega_0^2}} = 2\pi \sqrt{\frac{m}{k}}$$

Primer:

$$m = 500 \text{ g} = 0,5 \text{ kg}$$

$$k y_0 = -mg_0$$

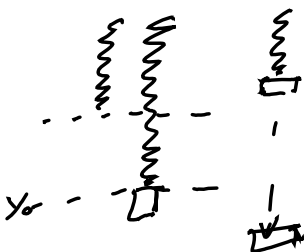
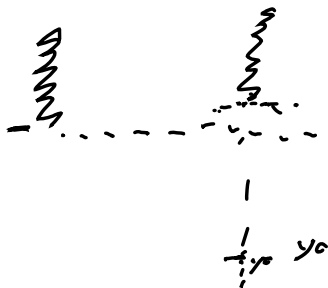
$$k = -\frac{mg_0}{y_0} \Rightarrow \frac{m}{k} = -\frac{y_0}{g_0}$$

$$y_0 = -0,4 \text{ m}$$

$$\frac{m}{k} = \frac{0,4 \text{ m}}{10 \text{ m/s}^2} = 0,04 \text{ s}^2 =$$

$$\sqrt{\frac{m}{k}} = 2 \cdot 10^{-1} = 0,2 \text{ s}$$

$$\Rightarrow T_0 = 2\pi \cdot 0,2 \text{ s} \approx 1,2 \text{ s}$$



B in δ dobimo iz začetnih pogojev

$$y' = B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta)$$

$$\dot{y}' = -\frac{\beta}{2} B e^{-\frac{\beta}{2}t} \sin(\omega t + \delta) + \omega B e^{-\frac{\beta}{2}t} \cos(\omega t + \delta)$$

$$\dot{y}'(0) = -\frac{\beta}{2} B \sin \delta + \omega B \cos \delta =$$

$$y'(0) = B \sin \delta \qquad \qquad \qquad = B \left(-\frac{\beta}{2} \sin \delta + \omega \cos \delta \right)$$

$$r = \frac{y'(0)}{\dot{y}'(0)} = \frac{\sin \delta}{\omega \cos \delta - \frac{\beta}{2} \sin \delta} = \frac{\tan \delta}{\omega - \frac{\beta}{2} \tan \delta}$$

$$\Rightarrow \delta = \arctan \left(\frac{r \omega}{1 + \frac{\beta r}{2}} \right)$$

$$\Rightarrow B = \frac{y'(0)}{\sin \delta}$$

$$\dot{y}'(0) = B \omega \sin \delta$$

$$\Rightarrow B = \frac{\dot{y}'(0)}{\pm \omega} = \frac{|\dot{y}'(0)|}{\omega}$$

Energija nihala

$$W_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{y}^2$$

$$W_{pr} = \frac{1}{2} k y^2 = \frac{1}{2} k (y + y_0)^2$$

$$W_p = m g_0 (y + y_0)$$

$$\text{Skupna } \boxed{W = W_k + W_p + W_{pr}}$$

a) W za zelo slobko dušenje ?

$$\frac{B}{z} + \ll 1$$

$$\left(\frac{B}{z}\right)^2 \ll \omega_0^2 \Rightarrow \omega \approx \omega_0 = \sqrt{\frac{k}{m}}$$

$$y' = B \sin(\omega_0 t + \delta)$$

$$\dot{y}' = \omega_0 B \cos(\omega_0 t + \delta)$$

$$\begin{aligned} W_k &= \frac{1}{2} m \omega_0^2 B^2 \cos^2(\omega_0 t + \delta) = \\ &= \frac{1}{2} m \frac{k}{m} B^2 \cos^2(\omega_0 t + \delta) \end{aligned}$$

$$W_{pr} = \frac{1}{2} k B^2 \sin^2(\omega_0 t + \delta) + \frac{1}{2} k y_0^2 + k y_0 B \sin(\omega_0 t + \delta)$$

$$W_p = m g_0 B \sin(\omega_0 t + \delta) + m g_0 y_0$$

$$\begin{aligned} W &= \frac{1}{2} k B^2 \underbrace{(\sin^2(\omega_0 t + \delta) + \cos^2(\omega_0 t + \delta))}_{1} \\ &\quad + (k y_0 + m g_0) B \sin(\omega_0 t + \delta) + \\ &\quad + \frac{1}{2} k y_0^2 + m g_0 y_0 \\ &= \frac{1}{2} k B^2 + \frac{1}{2} k y_0^2 + m g_0 y_0 = \text{konst.} \end{aligned}$$

b) kritično dušenje

$D=0$ ($\omega=0$) (kritično dušenje)

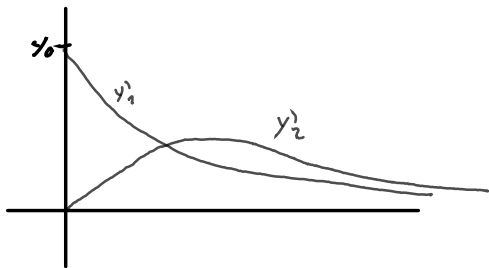
$$D = \beta^2 - 4\omega^2 \Rightarrow \omega_0 = \frac{\beta}{2}$$

$$\lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = -\frac{\beta}{2}$$

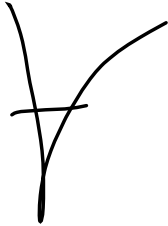
$$\Rightarrow y_1' = B_1 e^{-\frac{\beta}{2}t}$$

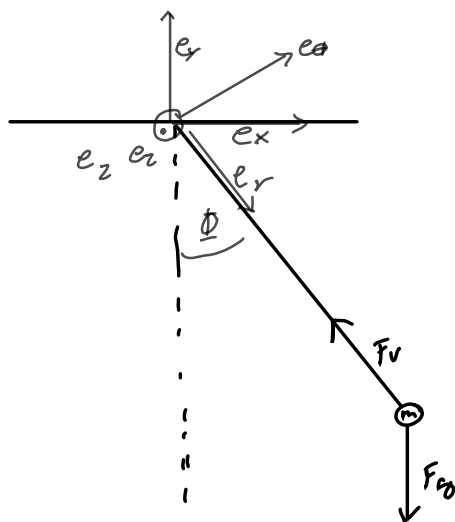
$$y_2' = B_2 t e^{-\frac{\beta}{2}t} \text{ je tudi rešitev (DN)}$$

$$\Rightarrow y' = y_1' + y_2' = (B_1 + B_2 t) e^{-\frac{\beta}{2}t}$$



20.2





$e_\phi, e_z, e_r \dots$ cylindrische
Koordinate

$$\vec{F} = \vec{F}_g + \vec{F}_v = m\vec{a}$$

$$F_g = -mg_0 \hat{e}_z$$

$$F_v = -F_v \hat{e}_r$$

$$\vec{r} = l \cdot \vec{e}_r$$

$$\vec{r} \times \vec{F} = l \vec{e}_r \times (-mg_0 \hat{e}_z - F_v \hat{e}_r) =$$

$$= -mg_0 l \hat{e}_r \times \hat{e}_z - \underbrace{l F_v \hat{e}_r \times \vec{e}_r}_{=0}$$

$$= -mg_0 l \hat{e}_r \times (\hat{e}_z \cos \phi + \hat{e}_\phi \sin \phi)$$

$$= -mg_0 l \sin \phi \hat{e}_r \times \hat{e}_\phi =$$

$$= -mg_0 l \sin \phi \hat{e}_z$$

$$\vec{r} \times \vec{F} = m \cdot \vec{r} \times \vec{a} = m \cdot l \hat{e}_r \times (a \hat{e}_\phi) =$$

$$\vec{a} = a \cdot \hat{e}_\phi = m a l \hat{e}_z$$

$$a = a_\phi = l \ddot{\phi}$$

$$mg_0 l \sin \phi \hat{e}_z = m l \ddot{\phi} \hat{e}_z$$

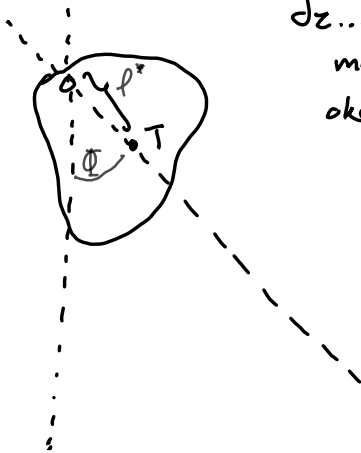
$$m l (l \ddot{\phi} + g_0 \sin \phi) \hat{e}_z = 0$$

$$-l \ddot{\phi} = g_0 \sin \phi$$

$$\ddot{\phi} + \frac{g_0}{l} \sin \phi = 0$$

$$\approx \underbrace{\ddot{\phi}}_{\omega_0^2} + \frac{g_0}{l} \phi = 0$$

$$\ddot{\phi} + \omega_0^2 \phi = 0$$



J_z vsrājnosni
moment za vrtenje
okrog fiksne osi

primer: palica

$$J_z = \frac{1}{3} m l^2 \quad l^* = \frac{l}{2}$$

$$\sin \Phi \approx \Phi$$

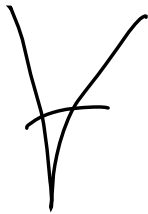
$$\Rightarrow \ddot{\Phi} + \omega_0^2 \Phi = 0$$

$$\omega_0^2 = \frac{m g l^*}{J_z} = \frac{\cancel{2} m g \cancel{2} l}{2 \cdot \cancel{2} m l^2} = \frac{3}{2} \frac{g}{l}$$

N;ho n;helo:

$$J_z = m l^2 \quad l^* = l$$

$$\omega_0^2 = \frac{m g_0 l}{m l^2} = \frac{g_0}{l}$$



21.2

Nism & shkala

$$F = F_0 \sin(\omega_v t) \quad \omega_v = 2\pi \nu$$

$$\ddot{y} + \beta \dot{y} + \omega_0^2 y = \frac{F_0}{m} \sin(\omega_v t)$$

$$\omega_0^2 = \frac{k}{m} \quad \beta = \frac{c}{m}$$

$$y' = y - y_0 \quad \text{rozliczek ko obtezan, vzmet mirna}$$

$$y' = y_h' + y_p' \quad \leftarrow \begin{array}{l} \text{homogena} \\ \text{partikularna} \\ \text{resitev} \end{array}$$

$$y_p' = B \sin(\omega_v t - \delta_p)$$

$$B_p \left\{ (\omega_0^2 - \omega_v^2) [\cos \delta_p \sin(\omega_v t) - \sin \delta_p \cos(\omega_v t)] + \omega_v \beta [\cos \delta_p \cos(\omega_v t) + \sin \delta_p \sin(\omega_v t)] \right\} = \frac{F_0}{m} \sin(\omega_v t) \quad \forall t$$

$$a) t_1 = 0 \Rightarrow \sin \omega_v t = 0, \cos \omega_v t = 1$$

$$B_p \{ -(\omega_0^2 - \omega_v^2) \sin \delta_p + \omega_v \beta \cos \delta_p \} = 0$$

$$\Rightarrow \tan \delta_p = \frac{\omega_v \beta}{\omega_0^2 - \omega_v^2}$$

$$\cos \delta_p = \pm \frac{(\omega_0^2 - \omega_v^2)}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$$

$$\sin \delta_p = \pm \frac{\omega_v \beta}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$$

$$\omega_v \rightarrow 0 \Rightarrow \tan \delta_p \rightarrow +0 \Rightarrow \delta_p \rightarrow 0$$

$$\omega_v \rightarrow \uparrow \omega_0 \Rightarrow \tan \delta_p \rightarrow +\infty \Rightarrow \delta_p \rightarrow \uparrow \frac{\pi}{2}$$

$$\omega_v \rightarrow \downarrow \omega_0 \Rightarrow \tan \delta_p \rightarrow -\infty \Rightarrow \delta_p \rightarrow \downarrow \frac{\pi}{2}$$

$$\omega_v \rightarrow \infty \Rightarrow \tan \delta_p \rightarrow -0 \Rightarrow \delta_p \rightarrow \pi$$

$$\omega_v \rightarrow 0 \Rightarrow \delta_p \rightarrow 0 \Rightarrow \cos \delta_p \rightarrow +1$$

$$\cos \delta_p \rightarrow \frac{\pm \omega_0^2}{\sqrt{\omega_0^4}} = \pm 1 \quad \text{ocisto mora biti +}$$

$$b) t_2 = \frac{\pi}{2\omega_v} \Rightarrow \omega_v t_2 = \frac{\pi}{2} \Rightarrow \sin(\omega_v t) = 1, \cos(\omega_v t) = 0$$

$$B_p \left\{ (\omega_0^2 - \omega_v^2) \cos \delta_p + \omega_v \beta \sin \delta_p \right\} = \frac{F_0}{m}$$

$$B_p \left\{ \frac{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}} \right\} = \frac{F_0}{m}$$

$$\Rightarrow B_p = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2 \beta^2}}$$

$$\omega_v \rightarrow 0 \Rightarrow B_p \rightarrow \frac{F_0}{m} \cdot \frac{1}{\omega_0^2} = \frac{F_0 \cdot m}{m \cdot k} = \frac{F_0}{k}$$

$$\omega_v \rightarrow \infty \Rightarrow B_p \rightarrow 0$$

$$B_p(\omega_v) = \max$$

Kdaj dosežemo maksimum?

Taj, kda je v resonanci?

ko je menovalec najmanjši

$$(\omega_0^2 - \omega_v^2)^2 + \beta^2 \omega_v^2 \text{ je najmanjši}$$

$$\frac{d}{d\omega_v} ((\omega_0^2 - \omega_v^2)^2 + \beta^2 \omega_v^2) = 0$$

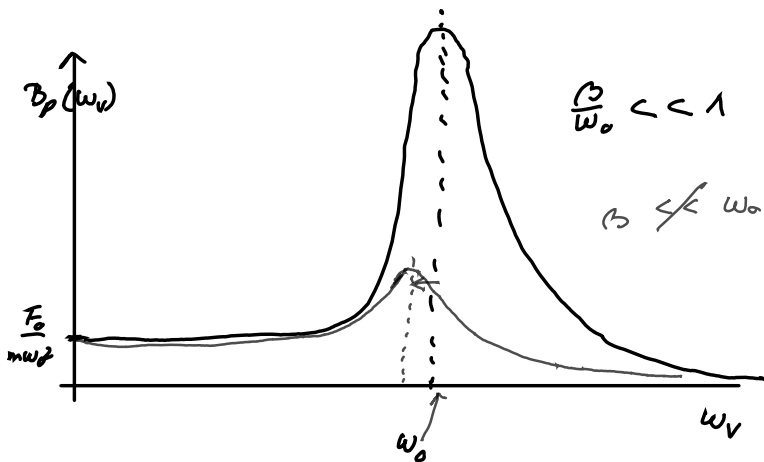
$$2(\omega_0^2 - \omega_v^2)(-2\omega_v) + 2\omega_v\beta^2 = 0$$

$$-2(\omega_0^2 - \omega_v^2) + \beta^2 = 0$$

$$\omega_v^2 = \omega_0^2 - \frac{\beta^2}{2}$$

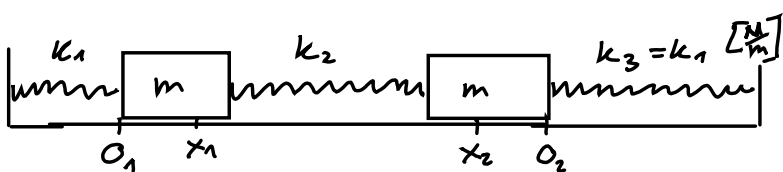
$$\omega_v = +\sqrt{\omega_0^2 - \frac{\beta^2}{2}} = \omega_0 \sqrt{1 - \frac{\beta^2}{2\omega_0^2}}$$

$$\beta \ll \omega_0 \Rightarrow \omega_v = \omega_0$$



Sklopljeno nihanje

- $\rho \rightarrow 0$ dušenje posredno proti 0
- za začetek: simetrični primer



"z druge vzmetje, k-j; bomo rekli: tretja vzmet"

$\vec{F}_{1 \rightarrow 1, r}$... sile, ue vzmeti na prvi; vozicek ko je v ravnovesni legi

$$\left. \begin{aligned} \vec{F}_{1 \rightarrow 1, r} &= -k_1 \Delta l_{1, r} \hat{e}_x \\ \vec{F}_{2 \rightarrow 1, r} &= +k_2 \Delta l_{2, r} \hat{e}_x \end{aligned} \right\} + = 0$$

v ravnovezju

$$\left. \begin{aligned} \vec{F}_{2 \rightarrow 2, r} &= -k_2 \Delta l_{2, r} \hat{e}_x \\ \vec{F}_{3 \rightarrow 2, r} &= +k_3 \Delta l_{3, r} \hat{e}_x \end{aligned} \right\} + = 0$$

$$\vec{F}_{1 \rightarrow 1} = \vec{F}_{1 \rightarrow 1, r} - k_1 x_1 \hat{e}_x$$

$$\vec{F}_{2 \rightarrow 1} = \vec{F}_{2 \rightarrow 1, r} - k_2 (x_1 - x_2) \hat{e}_x$$

$$\vec{F}_{2 \rightarrow 2} = \vec{F}_{2 \rightarrow 2, r} + k_2 (x_1 - x_2) \hat{e}_x$$

$$\vec{F}_{3 \rightarrow 2} = \vec{F}_{3 \rightarrow 2, r} - k_1 x_2 \hat{e}_x$$

$$\vec{F}_{1 \rightarrow 1} + \vec{F}_{2 \rightarrow 1} = m \ddot{x}_1 \hat{e}_x$$

$$\vec{F}_{1 \rightarrow 1, r} - k_1 x_1 \hat{e}_x + \vec{F}_{2 \rightarrow 1, r} - k_2 (x_1 - x_2) \hat{e}_x = m \ddot{x}_1 \hat{e}_x$$

$$m \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = 0$$

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0$$

$$\omega_1^2 = \frac{k_1}{m} \quad \omega_2^2 = \frac{k_2}{m}$$

Zelo podobno $\ddot{x}_2 + \omega_1^2 x_2 - \omega_2^2 (x_1 - x_2) = 0$

$$x_a = x_1 + x_2$$

$$x_b = x_1 - x_2$$

$$\Rightarrow \begin{aligned} x_1 &= \frac{x_a + x_b}{2} \\ x_2 &= \frac{x_a - x_b}{2} \end{aligned}$$

seštejemo enačbi:

$$\ddot{x}_1 + \ddot{x}_2 + \omega_1^2 x_1 + \omega_1^2 x_2 = 0$$

$$(\ddot{x}_1 + \ddot{x}_2) + \omega_1^2 (x_1 + x_2) = 0$$

$$\ddot{x}_a + \omega_a^2 x_a = 0$$

$$\ddot{x}_b + \omega_b^2 x_b = 0$$

$$\begin{aligned} \omega_a &= \omega_1 \\ \omega_b &= \sqrt{\omega_1^2 + 2\omega_2^2} \end{aligned}$$

$$x_a = B_a \sin(\omega_a t + \delta_a)$$

$$x_b = B_b \sin(\omega_b t + \delta_b)$$

$$\Rightarrow x_1 = \underbrace{\frac{B_a}{2}}_{B_1} \sin(\omega_a t + \delta_a) + \underbrace{\frac{B_b}{2}}_{B_2} \sin(\omega_b t + \delta_b)$$

$$x_2 = B_1 \sin(\omega_a t + \delta_a) - B_2 \sin(\omega_b t + \delta_b)$$

$$B_1, B_2, \delta_a, \delta_b = ?$$

začetni pogoji: $x_1(t=0) \quad \dot{x}_1(t=0)$
 $x_2(t=0) \quad \dot{x}_2(t=0)$

iz tega izvedemo

Primer

$$x_1(t=0) = x_0 (>0) \quad \dot{x}_1(t=0) = 0$$

$$x_2(t=0) = 0 \quad \dot{x}_2(t=0) = 0$$

$$\dot{x}_1 = B_1 \omega_a \cos(\omega_a t + \delta_a) + B_2 \omega_b \cos(\omega_b t + \delta_b)$$

$$x_2 = B_1 \omega_a \cos(\omega_a t + \delta_a) - B_2 \omega_b \cos(\omega_b t + \delta_b)$$

$$x_0 = B_1 \sin \delta_a + B_2 \sin(\delta_b) \quad \left. \begin{array}{l} + \\ - \end{array} \right\}$$

$$0 = B_1 \sin \delta_a - B_2 \sin \delta_b$$

$$0 = B_1 \omega_a \cos(\delta_a) + B_2 \omega_b \cos(\delta_b) \quad \left. \begin{array}{l} + \\ - \end{array} \right\}$$

$$0 = B_1 \omega_a \cos(\delta_a) - B_2 \omega_b \cos(\delta_b)$$

$$\rightarrow x_0 = 2B_1 \sin \delta_a = 2B_2 \sin \delta_b$$

$$2B_1 \omega_a \cos \delta_a = 0 \Rightarrow \cos \delta_a = 0$$

\Rightarrow

$$B_1 = B_2 = \frac{x_0}{2}$$

$$\delta_a = \pm \frac{\pi}{2}$$

$$\Rightarrow \therefore \delta_b = + \frac{\pi}{2}$$

V

7.3

mentales
adsotha

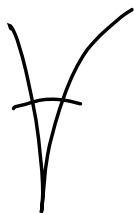
lastna nihanja sestavljenega
sistema

$$\left. \begin{aligned} x_1 &= B \sin(\omega t + \delta_a) + B_2 \sin(\omega_2 t + \delta_b) \\ x_2 &= \dots \end{aligned} \right\}$$

$$\rightarrow \begin{cases} x_1 = B_1 \sin(\omega_1 t + \delta_1) \\ x_2 = B_2 \sin(\omega_2 t + \delta_2) \end{cases}$$

$$\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0$$

$$\ddot{x}_2 + \omega_2^2 x_2 - \omega_2^2 (x_1 - x_2) = 0$$



11.3

$$u(x, t) = ?$$

$$\eta := x - ct$$

$$\chi := x + ct$$

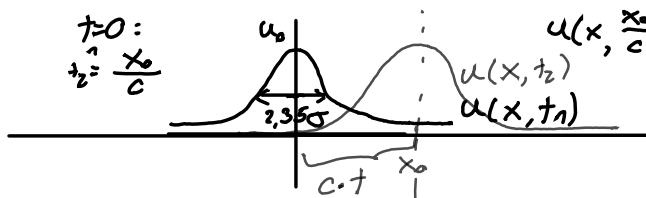
$$u(x, t) = f(x - ct) + g(x + ct)$$

ena rešena možnost:

$$u(x, t) = f(x - ct)$$

$$\text{npr: } u(x, t) = u_0 e^{-\frac{(x-ct)^2}{2\sigma^2}}$$

$$t=0: \\ t_2 = \frac{x_0}{c}$$



$$u(x, 0) = u_0 e^{-\frac{x^2}{2\sigma^2}}$$

$$u(x, \frac{x_0}{c}) = u_0 e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

↑
samo
premakljena
krivulja

$$\Rightarrow v = \left| \frac{du}{dt} \right| = \left| \frac{\partial u}{\partial t} \right| = \left| \frac{\partial}{\partial t} f(x - ct) \right| = |f'|c$$

$$v \neq c$$

$$u(x,t) = f(x-ct)$$

$$a) \quad u(x,t) = u(x-ct, 0)$$

$$x \rightarrow x' = x - ct$$

$$t \rightarrow t' = 0$$

$$u(x', t') = u(x-ct, 0) = f(x', ct') = f(x-ct) = u(x, t)$$

Če poznamo $u(x', t')$ ob času $t'=0$ za $\forall x'$
Potem poznam $u(x, t)$ za $\forall x$ po \forall času t

$$b) \quad u(x, t) = u(0, t - \frac{x}{c})$$

$$x \rightarrow x' = 0$$

$$t \rightarrow t' = t - \frac{x}{c}$$

$$u(x', t') = u(0, t - \frac{x}{c}) =$$

$$f(0 - c(t - \frac{x}{c})) = f(x - ct) = u(x, t)$$

Če poznamo $u(x'=0, t')$ za $\forall t'$
 \Rightarrow poznam $u(x, t)$ za $\forall x, \forall t$

Putujoče sinusno valovanje

$$u(x, t) = f(x - ct)$$

$$u(x=0, t = \frac{x}{c}) = u_0 \sin(-\omega t + \delta); \quad u_0 > 0$$

$$u(x, t) = ?$$

$$u(x, t) = u(0, t - \frac{x}{c}) = u_0 \sin(-\omega(t - \frac{x}{c}) + \delta) =$$

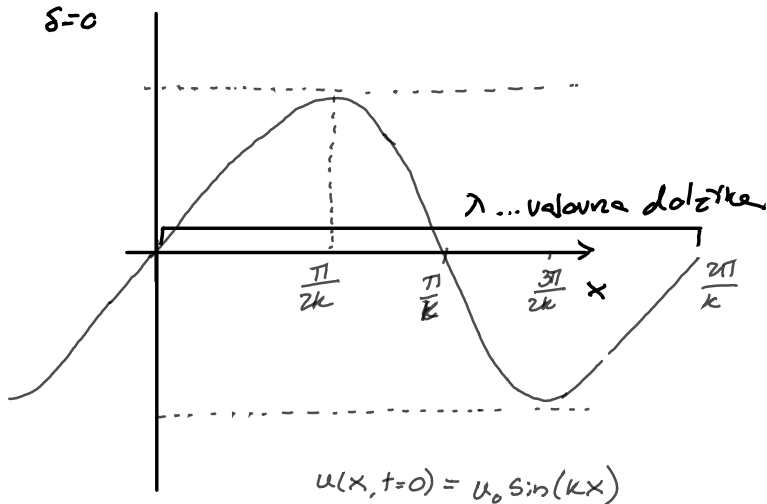
$$= u_0 \sin(kx - \omega t + \delta); \quad k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$$

$$[k] = \frac{1}{m}$$

$$t=0:$$

$$u(x, t=0)$$

$$\delta=0$$



$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{2\pi\nu} = \frac{c}{\nu}$$

$$c = \lambda\nu$$

$$u(x, t) = u_{0,1} \sin(kx - \omega t + \delta_1) + u_{0,2} \sin(kx + \omega t + \delta_2)$$

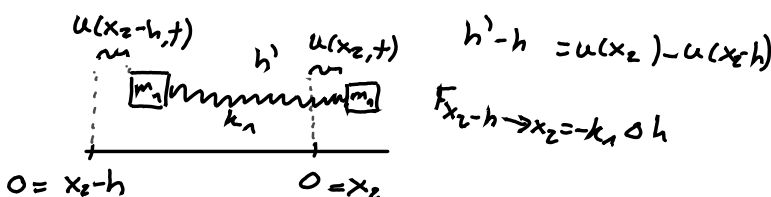
Valovanje v obe smeri

Robni pogoji in stojče sinusno valovanje

a) tog vpeta vzmet
 $u(x_1, t) = u(x_2, t) = 0$

b) Vzmet (palica) s prostima koncema, x_1, x_2

Model:



$$m_1 \ddot{u}(x_2, t) = -k l \left(\frac{u(x_2, t) - u(x_2 - h, t)}{h} \right) = -\frac{k l}{h} (u(x_2, t) - u(x_2 - h, t))$$

$$m_1 = \frac{m}{l h} \leftarrow \text{sta vseh uteži}$$

$$\frac{m}{l h} \frac{\partial^2 u(x_2, t)}{\partial t^2} = -k l \left(\frac{u(x_2' + h, t) - u(x_2', t)}{h} \right)$$

$$x_2 = x_2' + h$$

$$h \rightarrow \infty \Leftrightarrow h \rightarrow 0 \Rightarrow h \ddot{u} \rightarrow 0$$

$$\left. \frac{\partial u(x, t)}{\partial x} \right|_{x_2 - h} \stackrel{h \rightarrow 0}{=} \left. \frac{\partial u(x, t)}{\partial x} \right|_{x_2}$$

$$\left. \frac{\partial u(x, t)}{\partial x} \right|_{x=x_2} = \left. \frac{\partial u(x, t)}{\partial x} \right|_{x=1} = 0$$

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = \left. \frac{\partial u}{\partial x} \right|_{x=L} = 0 \quad \forall t$$

↑
dolžna pogo

$$u(x, t) = u_1(x, t) + u_2(x, t)$$

$$\left. \begin{aligned} u_1 &= u_0 \sin(kx - \omega t + \delta_1) \\ u_2 &= u_0 \sin(kx + \omega t + \delta_2) \end{aligned} \right\} +$$

$$= 2u_0 \sin\left(kx + \frac{\delta_1 + \delta_2}{2}\right) \cos\left(-\omega t + \frac{\delta_1 + \delta_2}{2}\right)$$

$$\begin{aligned} \Rightarrow \frac{\partial u}{\partial x} &= \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} = k u_0 \cos(kx - \omega t + \delta_1) + \\ &\quad + k u_0 \cos(kx + \omega t + \delta_2) \\ &= 2k u_0 \cos\left(kx + \frac{\delta_1 + \delta_2}{2}\right) \cos\left(-\omega t + \frac{\delta_1 + \delta_2}{2}\right) \end{aligned}$$

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = 2k u_0 \cos\left(\frac{\delta_1 + \delta_2}{2}\right) \cos\left(-\omega t + \frac{\delta_1 + \delta_2}{2}\right) \stackrel{!}{=} 0 \quad \forall t$$

$$\Rightarrow \frac{\delta_1 + \delta_2}{2} = \frac{\pi}{2}$$

$$\cos\left(kx + \frac{\pi}{2}\right) = -\sin(kx)$$

$$\sin\left(kx + \frac{\pi}{2}\right) = \cos(kx)$$

$$u(x, t) = 2u_0 \cos(kx) \cos\left(-\omega t + \frac{\delta_1 - \delta_2}{2}\right)$$

$$\frac{du}{dx} = -2k u_0 \sin(kx) \cos\left(-\omega t + \frac{\delta_1 - \delta_2}{2}\right)$$

$$\left. \frac{du}{dx} \right|_{x=L} = 0 \Rightarrow$$

$$-2k u_0 \sin(kL) \cos\left(-\omega t + \frac{\delta_1 - \delta_2}{2}\right) = 0 \quad \forall t$$

$$kL = 0 + \pi n, \quad n \in \mathbb{Z}$$

$$k = \frac{n\pi}{L} = \frac{\omega}{c} = \frac{2\pi\nu_n}{c}$$

$$\nu_n = \frac{nc}{2L} \quad n \in \mathbb{N}$$

$$u(x, t) = 2u_0 \cos\left(\frac{n\pi}{L} x\right) \cos\left(-2\pi\nu_n t + \frac{\delta_1 - \delta_2}{2}\right)$$

Dobimo stojee valovanje. nima več bier:

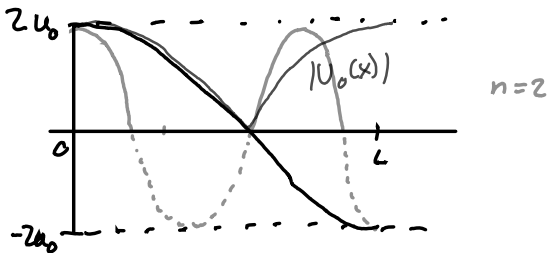
$$u(x,t) = 2u_0 \cos\left(\frac{n\pi}{L}x\right) \cos\left(-2\pi\nu_n t + \frac{\delta_1 - \delta_2}{2}\right)$$

$$n=0 \Rightarrow u(x,t) = 2u_0 \cos\left(\frac{\delta_1 - \delta_2}{2}\right)$$

$$\nu = 0$$

$$n=1 \rightarrow \nu_1 = \frac{c}{2L}$$

$$u(x,t) = \underbrace{2u_0 \cos\left(\frac{\pi}{L}x\right)}_{|u_0(x)|} \cos\left(\frac{\pi c}{L}t + \frac{\delta_1 - \delta_2}{2}\right)$$



$$\nu_1 = \frac{c}{2L} = \frac{1}{2L} \sqrt{\frac{E}{\rho}} \rightarrow E$$

$$\rho = \frac{m}{V} = \frac{m}{LS}$$

10. Energija valovanja

- potovanje matrike \rightarrow potovanje energije
- valovanje po vijačni vzmeti

x ; x in $x+h$

$$W_k = \frac{1}{2} m_1 \cdot v^2 = \frac{1}{2} m_1 \left| \frac{\partial u(x,t)}{\partial t} \right|^2 = \frac{1}{2} \frac{m_1 h}{l} \left| \frac{\partial u(x,t)}{\partial t} \right|^2$$

$$\Rightarrow \frac{W_k}{h} = \frac{1}{2} \frac{m}{l} \left| \frac{\partial u(x,t)}{\partial t} \right|^2$$

$$\begin{aligned} W_{pr} &= \frac{1}{2} k_1 [u(x+h,t) - u(x,t)]^2 = \\ &= \frac{1}{2} \frac{k}{h} l [u(x+h,t) - u(x,t)]^2 = \\ &= \frac{1}{2} k l h \left(\frac{u(x+h,t) - u(x,t)}{h} \right)^2 = \end{aligned}$$

$$\frac{W_{pr}}{h} = \frac{1}{2} k l \left[\frac{u(x+h,t) - u(x,t)}{h} \right]^2$$

$\downarrow h \rightarrow 0$

$$\lim_{h \rightarrow 0} \frac{W_{pr}}{h} = \frac{1}{2} k l \left| \frac{\partial u(x,t)}{\partial x} \right|^2 =$$

$$= \frac{1}{2} k l \cdot \frac{l}{m} \cdot \frac{m}{l} \left| \frac{\partial u(x,t)}{\partial x} \right|^2 =$$

$$= \frac{1}{2} c^2 \frac{m}{l} \left| \frac{\partial u(x,t)}{\partial x} \right|^2$$

$$c^2 = \frac{k l^2}{m}$$

Pročna polica

$$\left. \begin{aligned} m &= \rho V = \rho S \cdot l \\ k &= \frac{ES}{l} \end{aligned} \right\} \Rightarrow \frac{W_k}{h}(x,t) = \frac{1}{2} \rho S \left| \frac{\partial u}{\partial t} \right|^2$$

$$\boxed{W_k = \frac{W_k}{\Delta V} = \frac{W_k}{Sl} = \frac{1}{2} \rho \left| \frac{\partial u}{\partial t} \right|^2}$$

↑
gusto kinetične energije

$$\frac{W_p}{h} = \frac{1}{2} c^2 \frac{m}{l} \left| \frac{\partial u}{\partial x} \right|^2$$

$$\bar{w} = \frac{1}{2} \rho \omega^2 u_0^2 \dots \text{povprečna gostota energije}$$

$$\omega = 2\pi\nu$$

$$\bar{W} = \bar{w} \, dV \quad dV = S \, dt \quad ; \quad S \perp c$$

$$= \bar{w} S \, dt$$

energija, ki v času t prepotuje skozi S

$$P := \frac{\bar{W}}{t} = \frac{\bar{w} \cdot S \, dt}{t} = \bar{w} S$$

$$[P] = \frac{\frac{J}{m^3} \cdot \frac{m^2 \cdot m}{s}}{s} = \frac{J}{s} = W \dots \text{vat}$$

$$j_w := \frac{P}{S} \dots \text{jakost energijskega toka}$$

$$j_w = \frac{P}{S} = \bar{w} c \quad [j_w] = \frac{W}{m^2}$$

Primer:

zrak v zraku

$$c^2 = \frac{1}{\chi \rho}$$

\uparrow stisljivost \nwarrow gostota

$$pV = \frac{m}{M} RT$$

$$M = 29 \text{ kg}$$

$$R = 8300 \text{ J/K}$$

$$[T] = K$$

$$\chi = -\frac{1}{V} \frac{\partial V}{\partial p} \quad \chi > 0$$

$$\frac{pV}{T} = \frac{p_0 V_0}{T_0}$$

$$\Rightarrow T = T_0 = \text{konst} \quad (\text{izoterma})$$

$$pV = p_0 V_0 = K$$

$$\Rightarrow V = \frac{K}{p} \Rightarrow \frac{1}{V} = \frac{p}{K}$$

$$\boxed{\chi = \left(-\frac{1}{V}\right) \left(-\frac{K}{p^2}\right) = \frac{1}{p}}$$

$$\chi = \frac{1}{p} \rightarrow \frac{1}{K_p}$$

\nwarrow K_p kap

$$c^2 = \frac{K_p R T}{\rho M} = \frac{K R T}{M}$$

$$K = 1,4 \quad \text{za vse dvoatomne molekule}$$

$$\Rightarrow c = \sqrt{\frac{K R T}{M}} = \sqrt{\frac{1,4 \cdot 8300 \text{ J} \cdot 300 \text{ K}}{29 \text{ kg}}} \approx 340 \frac{\text{m}}{\text{s}}$$

$V_{\text{max}} \approx 1 \text{ kHz}$... nase uho je najbolj občutljiva za to

$$j_0 \approx 10^{-12} \frac{\text{W}}{\text{m}^2} \quad \dots \text{toliko sta jakost j \& troke, da slišimo } V_{\text{max}}$$

$$U_0 = ?$$

$$j_0 = \frac{1}{2} \rho \omega^2 U_0^2 \cdot c \quad \rho \approx 1,2 \frac{\text{kg}}{\text{m}^3}$$

$$\Rightarrow U_0^2 = \frac{2 j_0}{\rho (2\pi V_{\text{max}})^2 c}$$

Doplerjev pojav

ν : frekvenca oddajnika ($t_0 = \frac{1}{\nu}$)

λ : valovna dolžina zvoka v snovi (zraku)

c : hitrost zvoka v snovi (zraku)

ν' : frekvenca, ki jo sliši sprejemnik

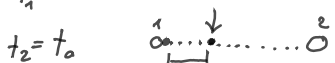
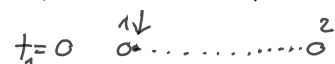
a) sprejemnik miruje glede na zrak ($v_2 = 0$), oddajnik se mu približuje s hitrostjo v_1 (glede na zrak)



$t_1 = 0$ odda prvi pisk

$t_2 = t_0$ odda drugi pisk

Pri tem se premakne za $t_0 \cdot v_1$ proti sprejemniku
prvo čelo se premakne za $t_0 \cdot c$ proti sprejemniku



$$\lambda = ct_0 - v_1 t_0 = t_0 (c - v_1)$$

$$\left[\nu' = \frac{c}{\lambda} = \frac{c}{(c - v_1)t_0} = \frac{1}{1 - \frac{v_1}{c}} \nu \right]$$

↑
- za približevanje
+ za oddaljevanje

b) oddajnik miruje ($v_1 = 0$), sprejemnik se približuje z v_2

Po zraku proti sprejemniku potujejo valovi z valovno dolžino λ

$$t_1 = 0 \quad \frac{1}{t_0} = \nu'$$

$$t_2 = t_0'$$

$$c t_0' + v_2 t_0' = \lambda = \frac{c}{\nu}$$

$$t_0' (c + v_2) = \frac{c}{\nu}$$

$$t_0' = \frac{1}{\nu'}$$

$$\frac{c + v_2}{\nu'} = \frac{c}{\nu}$$

$$\left[\nu' = \nu \cdot \frac{c + v_2}{c} = \nu \left(1 + \frac{v_2}{c} \right) \right]$$

$$\left[\nu' = \nu \frac{1 \pm \frac{v_2}{c}}{1 \mp \frac{v_1}{c}} \right]$$

predpostavke:

T1: vpliv okolice upade z razdaljo

$\lim_{d \rightarrow \infty} \vec{F} = 0$
↖ skupna sila okolice
↖ najmanjša razdalja okolice do predmeta