I Mehansko nihanje in valovanje

II elektricno polje

III elektricni tok

IV magnetno polje

V elektrodinamika

VI posebna teorija relativnosti

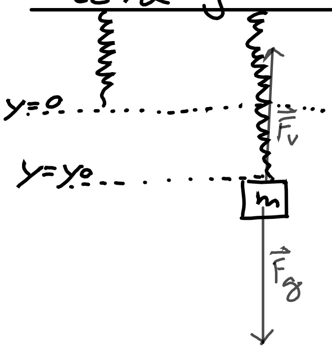
VII zaključek

I Mehansko nihanje in valovanje

Enostavna nihala

Enačba dušenega nihanja

<u>Utež na vijačni vzmeti</u>



$$F_g = \begin{bmatrix} 0 \\ -mg_0 \\ 0 \end{bmatrix}$$ smer navzdol

$g_0 \approx 10\ m/s^2$

$$F_v = \begin{bmatrix} 0 \\ -ky_0 \\ 0 \end{bmatrix}$$ k… koeficient vzmeti N/m

$y_0 < 0$

$-ky_0 > 0 \Rightarrow$ smer je navzgor

neto sila

$$\vec{a} = 0 \Leftrightarrow \vec{F} = m\vec{a} = 0$$

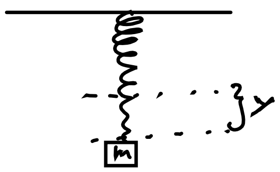
$$\vec{F}_g + \vec{F}_v = 0$$

$$-mg_0 - ky_0 = 0 \Rightarrow$$

$$mg_0 = -ky_0$$

$$y_0 = \frac{mg_0}{k}$$

$\vec{F}_g = -m g_0 \hat{e}_y$



$\vec{F}_v = -k y \hat{e}_y$

$\vec{F}_u$ .... sila upora

$\vec{F}_u = -c\vec{v}$ (linearna sila upora) $\quad v_y = \dot{y} = \frac{dy}{dt} \neq 0$

$c>0$ sorazmerna z viskoznostjo tekočine in površino uteži

$\vec{F}_u = -c\dot{y}\hat{e}_y$

$\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$

$\vec{F} = m\cdot\vec{a}$; $\vec{a} = \ddot{y}\hat{e}_y$

$-c\dot{y}\hat{e}_y - ky\hat{e}_y - mg_0\hat{e}_y = m\ddot{y}\hat{e}_y$

$(\underbrace{\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y + g_0}_{=0})\hat{e}_y = 0$

Vpeljemo $\beta := \frac{c}{m}$ $\quad [\beta] = s^{-1}$

$\omega_0^2 = \frac{k}{m}$ $\quad [\omega_0^2] = s^{-2}$

$\ddot{y} + \beta\dot{y} + \omega_0^2 y + g_0 = 0$

$y_0 = \frac{mg_0}{k}$

$g_0 = \frac{y_0 k}{m} = y_0\omega^2$

$\ddot{y} + \beta\dot{y} + \omega_0^2(y - y_0) = 0$

$\dot{y} + \beta y + \omega_0^2 y = \omega_0^2 y_0$

- diferencialna enačba za y 2. reda
- linearna enačba
- konstantni koeficienti
- nehomogena (pogojno, ker je lahko z lahkoto prevedemo v homogeno)

$y' := y - y_0$

$\dot{y}' = (\dot{y - y_0}) = \dot{y}$

$\ddot{y}' = \ddot{y}$

Dobimo: $\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0$

je homogena :)

$$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y = 0$$

Nastavek: $y' = Ae^{\lambda t}$, A, λ konstanti

$[\lambda] = s^{-1}$

$[A] = m$

$\dot{y}' = \lambda y$

$\ddot{y}' = \lambda^2 y$

$$(\lambda^2 + \beta\lambda + \omega_0^2)\underbrace{Ae^{\lambda t}}_{\neq 0 \text{ ko } A\neq 0} = 0 \quad \text{za } \forall t$$

$$\lambda^2 + \beta\lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 =: -4\omega^2$$

$$(\omega^2 := \omega_0^2 - (\tfrac{\beta}{2})^2)$$

$D < 0 \Rightarrow 4\omega^2 > 0$: podkritično dušenje

$\sqrt{D} = \sqrt{-4\omega^2} = \sqrt{-1}\, 2\omega$; $\omega = +\sqrt{\omega^2}$

$$\Rightarrow \lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = \frac{-\beta}{2} \pm \omega i$$

$$y_1' = A_1 e^{\lambda_1 t} = A_1 \exp(-\tfrac{\beta}{2} + i\omega)t) =$$

$$= A_1 \exp(-\tfrac{\beta}{2}t)\exp(i\omega t)$$

$$y_2' = A_2 \exp\{-\tfrac{\beta}{2}t)\exp(-i\omega t)$$

$$\left.\begin{aligned} &\ddot{y}_1' + \beta\dot{y}_1' + \omega_0^2 y_1' = 0 \\ &\ddot{y}_2' + \beta\dot{y}_2' + \omega_0^2 y' = 0 \end{aligned}\right\} +$$

$$(\ddot{y}_1' + \ddot{y}_2') + \beta(\dot{y}_1' + \dot{y}_2') + \omega_0(y_1' - y') = 0$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}\left(A_1\exp(i\omega t) + A_2\exp\{-i\omega t\}\right)$$

Eulerjeva enačba

$$\exp\{\pm i\omega t\} = \cos\{\omega t\} \pm i\sin(\omega t)$$

$$\Rightarrow y' = \exp\{-\tfrac{\beta}{2}t\}\{(A_1 + A_2)\cos(\omega t) + i(A_1 - A_2)\sin\omega t$$

$$= e^{-\frac{\beta}{2}t}(B_1\cos(\omega t) + B_2\sin(\omega t))$$

$$= B e^{-\frac{\beta}{2}t}\sin(\omega t + \delta)$$ ← Fazni zamik

$B > 0$; δ = fazni zamik

$$B e^{-\frac{\beta}{2}t}(\sin\omega t\sin\delta + \cos\omega t\sin\delta)$$

$$= e^{-\frac{\beta}{2}t}(B\sin\delta\cos(\omega t) + B\cos\omega t\sin\delta)$$

$B_1 = B\sin\delta$

$B_2 = B\cos\delta$

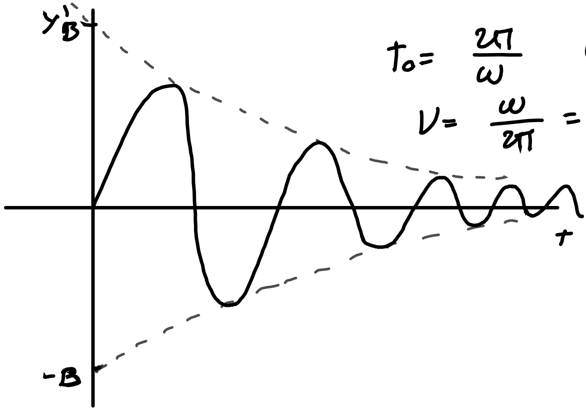
$$\tan\delta = \frac{B_1}{B_2}$$

$B^2 = B_1^2 + B_2^2$

$$B = \sqrt{B_1^2 + B_2^2}$$

Primer:

$$\delta = 0 \Rightarrow y' = B\, e^{-\frac{\beta}{2}t} \sin(\omega t)$$



$$t_0 = \frac{2\pi}{\omega} \qquad \omega = 2\pi\nu$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{t_0}$$

$\delta = \frac{\pi}{2}$

$$y'(t) = B e^{-\frac{\beta}{2}t} \sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t \cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2}$$

$$= B e^{-\frac{\beta}{2}t} \cos(\omega t)$$

$$\ddot{y}' + \beta \dot{y}' + \omega_0^2 y' = 0$$

$y' = y - y_0$ → odmik doremenjene vzmeti v ravnovesni legi
↳ odmik od konca neobremenjene vzmeti

$$\omega_0^2 = \frac{k}{m} \ (>0)$$

$\beta \propto c \propto \eta$
↖ sorazmerno

Nastavek $y' = Ae^{\lambda t}$

$$\Rightarrow \lambda^2 + \beta\lambda + \omega_0^2 = 0$$

$$D = \beta^2 - 4\omega_0^2 = -4\omega^2 \qquad \omega^2 = \omega_0^2 - \left(\frac{\beta}{2}\right)^2$$

a) $D < 0 \Rightarrow (\omega^2 > 0)$

$$y' = Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta) \ ; \ \omega = \sqrt{\omega^2} =$$
$$= \sqrt{\omega_0^2 - \left(\frac{\beta}{2}\right)^2}$$

Zelo šibko dušenje $\left(\frac{\beta}{2}\right)^2 << \omega_0 \Rightarrow$

$$t_0 = \frac{2\pi}{\sqrt{\omega_0^2}} = 2\pi\sqrt{\frac{m}{k}}$$

Primer:

$m = 500\,g = 0{,}5\,kg$

$k y_0 = -m g_0$

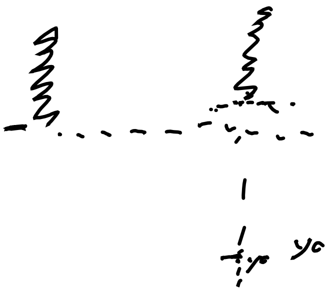
$$k = -\frac{m g_0}{y_0} \Rightarrow \frac{m}{k} = -\frac{y_0}{g_0}$$

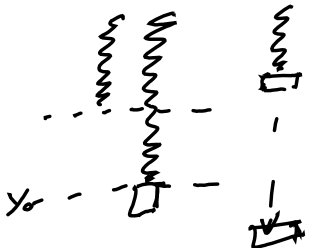
$y_0 = -0{,}4\,m$

$$\frac{m}{k} = \frac{0{,}4\,m}{10\,m/s^2} = 0{,}04\,s^2 =$$

$$\sqrt{\frac{m}{k}} = 2\cdot 10^{-1} = 0{,}2\,s$$

$$\Rightarrow t_0 = 2\pi\cdot 0{,}2\,s \cong 1{,}2\,s$$





B in δ dobimo iz začetnih pogojev

$$y' = Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta)$$

$$\dot{y}' = -\frac{\beta}{2}Be^{-\frac{\beta}{2}t}\sin(\omega t + \delta) + \omega Be^{-\frac{\beta}{2}t}\cos(\omega t + \delta)$$

$$\dot{y}'(0) = -\frac{\beta}{2}B\sin\delta + \omega B\cos\delta =$$

$$= B\left(-\frac{\beta}{2}\sin\delta + \omega\cos\delta\right)$$

$$y'(0) = B\sin\delta$$

$$r = \frac{y'(0)}{\dot{y}'(0)} = \frac{\sin\delta}{\omega\cos\delta - \frac{\beta}{2}\sin\delta} = \frac{\tan\delta}{\omega - \frac{\beta}{2}\tan\delta}$$

$$\Rightarrow \delta = \arctan\left(\frac{r\omega}{1 + \frac{r\beta}{2}}\right)$$

$$\Rightarrow B = \frac{y'(0)}{\sin\delta}$$

$$\dot{y}'(0) = B\omega\sin\delta$$

$$\Rightarrow B = \frac{\dot{y}'(0)}{\pm\omega} = \frac{|\dot{y}'(0)|}{\omega}$$

Energija nihala

$$W_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{y}^2$$

$$W_{pr} = \frac{1}{2} k y^2 = \frac{1}{2} k (y' + y_0)^2$$

$$W_p = m g_0 (y' + y_0)$$

Skupna $\boxed{W = W_k + W_p + W_{pr}}$

a) W za zelo šibko dušenje ?

$$\frac{\beta}{2} t << 1$$

$$\left(\frac{\beta}{2}\right)^2 << \omega_0^2 \Rightarrow \omega \simeq \omega_0 = \sqrt{\frac{k}{m}}$$

$$y' = B \sin(\omega_0 t + \delta)$$

$$\dot{y}' = \omega_0 B \cos(\omega_0 t + \delta)$$

$$W_k = \frac{1}{2} m \omega_0^2 B^2 \cos^2(\omega_0 t + \delta) =$$

$$= \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} B^2 \cos^2(\omega_0 t + \delta)$$

$$W_{pr} = \frac{1}{2} k B^2 \sin^2(\omega_0 t + \delta) + \frac{1}{2} k y_0^2 + k y_0 B \sin(\omega_0 t + \delta)$$

$$W_p = m g_0 B \sin(\omega_0 t + \delta) + m g_0 y_0$$

$$W = \frac{1}{2} k B^2 \underbrace{\left(\sin^2(\omega_0 t + \delta) + \cos^2(\omega_0 t + \delta)\right)}_{1}$$

$$+ (k y_0 + m g_0) B \sin(\omega t + \delta) +$$

$$+ \frac{1}{2} k y_0^2 + m g_0 y_0$$

$$= \frac{1}{2} k B^2 + \frac{1}{2} k y_0^2 + m g_0 y_0 = \text{konst.}$$

b) kritično dušenje

$D=0 \quad (\omega=0)$ (kritično dušenje

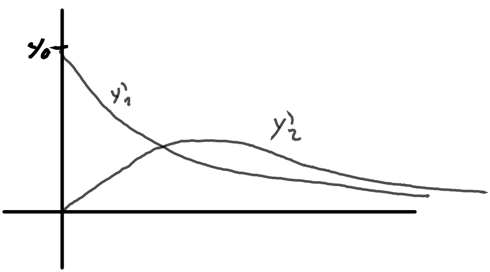
$D = \beta^2 - 4\omega_0^2 \Rightarrow \omega_0 = \frac{\beta}{2}$

$\lambda_{1,2} = \frac{-\beta \pm \sqrt{D}}{2} = -\frac{\beta}{2}$

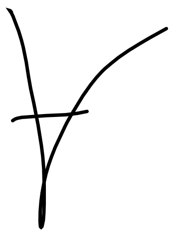
$\Rightarrow y_1' = B_1 e^{-\frac{\beta}{2}t}$

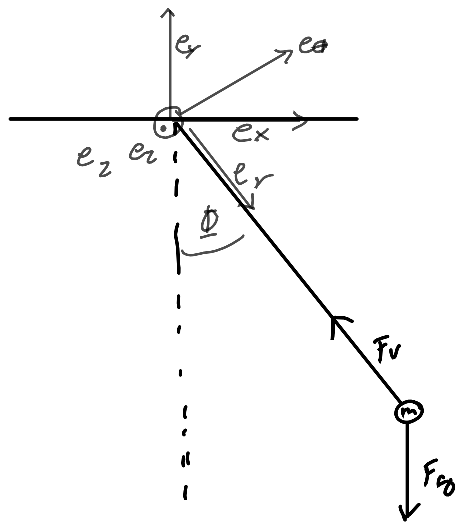
$y_2' = B_2 t e^{-\frac{\beta}{2}t}$ je tudi rešitev (DN)

$\Rightarrow y' = y_1' + y_2' = (B_1 + Bt) e^{-\frac{\beta}{2}t}$



20.2





e_ϕ, e_z, e_r ... cilindrične koordinate

$$\vec{F} = \vec{F_g} + \vec{F_v} = m\vec{a}$$

$$F_g = -mg_0\,\hat{e_y}$$

$$F_v = -F_v\,\hat{e}_r$$

$$\vec{r} = l\cdot\vec{e_r}$$

$$\vec{r}\times\vec{F} = l\vec{e}_r \times (-mg_0\hat{e_y} - F_v\hat{e}_r) =$$

$$= -mg_0 l\,\hat{e}_r\times\hat{e_y} - \underbrace{lF_v\,\hat{e_r}\times\vec{e_r}}_{=0}$$

$$= -mg_0\,l\hat{e_r}\times(\hat{e_r}\cos\phi + \hat{e}_\phi \sin\phi)$$

$$= -mg_0 l\sin\Phi\,\hat{e_r}\times\hat{e}_\phi =$$

$$= -mg_0 l\sin\Phi\,\hat{e_z}$$

$$\vec{r}\times\vec{F} = m\cdot\vec{r}\times\vec{a} = m\cdot l\hat{e}_r\times(a\,\hat{e}_\phi) =$$

$$\vec{a} = a\cdot e_\Phi \qquad = mal\,\hat{e}_z$$

$$a = a_t = l\ddot{\Phi}$$

$$mg_0 l\sin\phi\, e_z = ml\ddot{\Phi}e_z$$

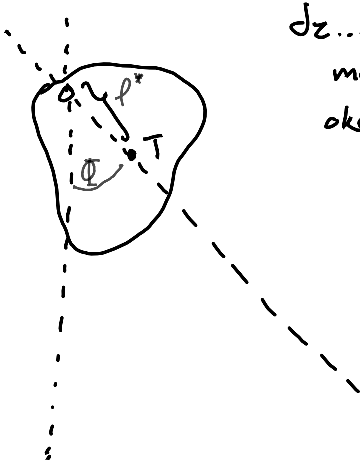
$$ml(l\ddot{\Phi} + g_0\sin\Phi)\,e_z \neq \vec{0} = 0$$

$$-l\ddot{\Phi} = g_0\sin\Phi$$

$$\ddot{\Phi} + \frac{g_0}{l}\sin\Phi = 0$$

$$\approx \ddot{\Phi} + \underbrace{\frac{g_0}{l}}_{\omega_0^2}\Phi = 0$$

$$\ddot{\Phi} + \omega_0^2\Phi = 0$$



J_z.... vstrajnostni moment za vrtenje okrog fiksne osi

primer: palica

$$J_z = \frac{1}{3} m l^2 \qquad l^* = \frac{l}{2}$$

$$\sin\Phi \approx \Phi$$

$$\Rightarrow \ddot{\Phi} + \omega_0^2 \Phi = 0$$

$$\omega_0^2 = \frac{m g l^*}{J_z} = \frac{3 m g l}{2 m l^2} = \frac{3}{2}\frac{g_0}{l}$$

Nitno nihalo:

$$J_z = m l^2 \qquad l^* = l$$

$$\omega_0^2 = \frac{m g_0 l}{m l^2} = \frac{g_0}{l}$$