

$$x = 0, 1$$

Pokaz de refe *

$$x = \sum_{i=1}^{\infty} (2^{-i}; + 2^{-4}; - 1)$$

b) binarni zapis za x

c) zapis v IEEE forme

IEEE754

dvojna notranost $P(2, 24, -125, 128)$

$$(-1)^0 (1+m) 2^{\tilde{e}-127}$$

m dolzina 23

e dolzina 8

0 dolzina 1

dvojna notranost $P(2, 53, -1021, 1024)$

$$(-1)^0 (1+m) 2^{\tilde{e}-1023}$$

m 52

e 11

0 1

$$a) X = \sum_{i=1}^{\infty} 2^{-4i} = \frac{\frac{1}{16}}{1 - \frac{1}{16}} + \frac{1}{2} \cdot \frac{\frac{1}{16}}{1 - \frac{1}{16}} = \frac{2}{2 \cdot 15} \cdot \frac{16}{15} = \frac{1}{10} = 0,1$$

$$b) 0,0001100110011 = 0,0\overline{0011}_{(2)}$$

$$c) 1,1\overline{0011} \cdot 2^{-4} = \\ 1 + 0,1\overline{0011} \cdot 2^{-4}$$

$$0,100110\dots 001101$$

$$\tilde{e}^{-123}_{-4} \Rightarrow e = 123 = 1111011$$

$$\begin{array}{rcl} 123 : 2 = 61 & 1 \\ 61 : 2 = 30 & 1 \\ 30 : 2 = 15 & 0 \\ 15 : 2 = 7 & 1 \\ 7 : 2 = 3 & 1 \\ 3 : 2 = 1 & 1 \\ 1 : 2 = 0 & 1 \end{array}$$

$$x = 2^{-1} + 2^{-k} + 2^{-t}$$

$$y = 2^{-1} + 2^{-k}$$

$$k = \frac{t}{2} + 1$$
$$t = 2k - 2$$

$x^2 + y^2$ - izračunamo z

izrazom

$$x \cdot x - y \cdot y$$

z obravnava relativne napake podelje de
izračuni direktno stabilen

$$x^2 = (2^{-1} + 2^{-k} + 2^{-t})^2 = 2^{-2} + 2^{-2k} + 2^{-2t} + 2 \cdot 2^{-1-k} + 2 \cdot 2^{-k-t} + 2 \cdot 2^{-t-1} =$$

$$\cancel{2^{-2}} + \cancel{2^{-2k}} + 2^{\cancel{-2} - 2t} + \cancel{2^{-k}} + \cancel{2^{-k-t+1}} + \cancel{2^{-t}}$$

$$y^2 = (2^{-1} + 2^{-k})^2 = 2^{-2} + \cancel{2 \cdot 2^{-k}} + \cancel{2^{-2k}}$$

$$x^2 - y^2 = 2^{-4k+4} + \cancel{2^{-k-t+1}} + \cancel{2^{-t}}$$

$$= 2^{-2t} + \cancel{2^{-t}} + \cancel{2^{-k-t+1}}$$

$$f(x) = 0,01 \dots 1 \dots 101 \dots 1 \dots =$$

$$= 0,01 \dots 1 \dots \overset{-t}{11}$$

$$f(y) = 0,01 \dots 1 \dots \overset{-k}{001} = 0,01 \dots 1 \dots \overset{-k}{010}$$

niveč denar zdi:

zato izberemo stevilo
soda zdi jo stevko, ker je
1 ne more -2^k zadnjice
 $v \cancel{zgled} \underline{zgled}$



22.10

$$g(x) = -x^2 + 8x \leq 12 \quad x_{r+1} = g(x_r)$$

$$\lim_{r \rightarrow \infty} x_r = 4 \quad \forall x_0 \in (3, 5)$$



$$\lim_{r \rightarrow \infty} |x_r - 4| = 0$$

$$= \lim_{r \rightarrow \infty} |x_r - 4|$$

$$\lim_{r \rightarrow \infty} |x_r - 4| = \lim_{r \rightarrow \infty} |-x_r^2 + 8x_r - 16| =$$

$$= \lim_{r \rightarrow \infty} |x_r - 4|^2 = \lim_{r \rightarrow \infty} |x_{r-1} - 4|^4 = \dots = \lim_{r \rightarrow \infty} |x_0 - 4|^{2^{r+1}}$$

$$\text{Konvergenzraum } |x_0 - 4| < 1 \Rightarrow x_0 \in (3, 5)$$

praktisch 0

Red konvergencija

α ... negibna točka

$$g'(\alpha) = g''(\alpha) = \dots = g^{(p-1)}(\alpha) = 0 \quad g^{(p)} \neq 0$$

potom je red konvergencija jednak p

- pokazite da tačka \sqrt{a} je pozitivne točke tevile \Rightarrow
izracunamo $x_{r+1} = x_r \cdot \frac{x_r^2 + 3a}{3x_r^2 + a}$
- dokazite red konvergencije
- dokazite da iteracija konvergira k \sqrt{a} za \forall
začetna priblizka $x_0 > 0$

1. Prouzimo da je \sqrt{a} negibna točka iteracijske funkcije

$$\sqrt{a} \cdot \frac{\sqrt{a}^2 + 3a}{3\sqrt{a}^2 + a} = \sqrt{a} \cdot \frac{4a}{4a} = \sqrt{a}$$

1. negibnost
2. privlačnost

2. preverjamo privlačnost $|g'(\sqrt{a})| < 1$

$$g(x) = \frac{x^3 + 3ax}{3x^2 + a}$$

$$g'(x) = \frac{(3x^2 + 3a)(3x^2 + a) - 6x(x^3 + 3ax)}{(3x^2 + a)^2} =$$

$$g'(\sqrt{a}) = \frac{(3a + 3a)(3a + a) - 6a^2 - 18a^2}{(3a + a)^2} =$$

$$= 0 < 1$$

$$b) g'''(x) = \left. \frac{3((x^2 + a)(3x^2 + a) - 6x^2(x^2 + 3a))}{(3x^2 + a)^2} \right|'$$

$$= \frac{3((2x + a)(3x^2 + a) + 6x(x^2 + a) - 4x(x^2 + 3a) - 4x^3)(3x^2 + a)^2}{(3x^2 + a)^3}$$

$$- \frac{2(3x^2 + a)((x^2 + a)(3x^2 + a) - 2x^2(x^2 + 3a))}{(3x^2 + a)^4}$$

$$= -3 \frac{(6x^3 + 3ax^2 + 2ax + a + 6x^3 + 6a - 4x^3 + 12xa - 4x^3)(3x^2 + a)^2}{(3x^2 + a)^5}$$

$$g''' = \frac{h_1 h_2 + h_1 h_2}{(3x^2 + a)}$$

$$g''' = h_1' h_2 + h_1 h_2' = h_1'(x^2 + a) + h_1(x) 2x$$

$$g'''(\sqrt{a}) = h_1(\sqrt{a}) \cdot 0 + h_1(\sqrt{a}) 2\sqrt{a}$$

$$h_1(\sqrt{a}) = \frac{48a\sqrt{a}}{3a + a}$$

$$g'''(\sqrt{a}) = \frac{48 \cdot 2a^2}{(4a)^3} = \frac{3}{2a} > 0 \neq 0$$

\Rightarrow red konvergencije = 3

$$c) g(x) = x \frac{x^2 + 3a}{3x^2 + a}$$

Icoutna pravice k $x_0 < \sqrt{a}$

$$x_1 > x_0 ?$$

$$x_1 > \sqrt{a} \text{ aho } x_1 < \sqrt{a} ?$$

$$x_1 = \cancel{x_0} \frac{\cancel{x_0^2 + 3a}}{\cancel{3x_0^2 + a}} \cancel{x_0}$$

$$x_0^2 + 3a > 3x_0^2 + a$$

$$2a > 2x_0^2 \quad \checkmark$$

$$\sqrt{a} > x_0 \quad \checkmark$$

$$x_1 < \sqrt{a}$$

$$x_0(x_0^2 + 3a) < \sqrt{a}(3x_0^2 + a)$$



$$\sqrt{a}(x_0^2 + 3a) < \sqrt{a}(3x_0^2 + a)$$

$$2a < 2x_0$$

Sled: $x_1 < \sqrt{a}$

Torej x_1 nekonvergira in so s tem
konvergirajo nemen

konvergirajo k \sqrt{a} ?

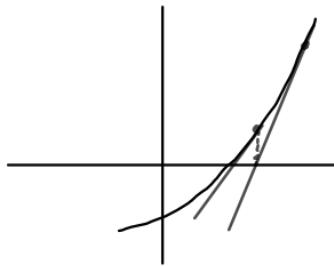
$$x_0 \in (\sqrt{a}, \infty)$$

pokazemo da imamo podujace
naredbo s mejno razvelje

Tangentna metoda

$$g(x) = x - \frac{f(x)}{f'(x)}$$

$$x_{r+1} = g(x_r)$$



Babilonska metoda za računanje \sqrt{a} ažo temelji na iteraciji:

$$x_{r+1} = \frac{1}{2}(x_r + \frac{a}{x_r})$$

a) Preverite, da iteracija ustreza tangentni metodi za funkcijo $f(x) = x^2 - a$

b) dokažite red konvergencije

c) Dokažite da iteracija konvergira k \sqrt{a} za $\forall x > 0$

$$\begin{aligned} a) \quad g(x) &= \frac{1}{2}(x + \frac{a}{x}) \stackrel{?}{=} x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - a}{2x} = \\ &= \frac{2x^2 - x^2 + a}{2x} = \frac{1}{2}\left(x + \frac{a}{x}\right) \checkmark \end{aligned}$$

$$b) \quad g'(x) = \frac{1}{2}\left(1 - \frac{a}{x^2}\right) \Rightarrow g'(\sqrt{a}) = 0$$

$$g''(x) = \frac{1}{2}\left(\frac{a}{x^3}\right) \stackrel{?}{=} \frac{a}{\sqrt{a}^3} \cdot \frac{1}{\sqrt{a}} > 0$$

red konvergencije je 2

$$c) g(x) = \frac{1}{2} \left(x + \frac{a}{x} \right)$$

g nár-eccjočca $\Rightarrow x \in (\sqrt{a}, \infty)$

$$\sqrt{a} < x_{r+1} < x_r$$

$\exists c \in$

$$\sqrt{a} < \frac{1}{2} \left(x_r + \frac{a}{x_r} \right) < x_r$$

$$2\sqrt{a}x_r < x_r^2 + a < 2x_r^2$$

$$\underbrace{\quad}_{a < x_r^2} \checkmark$$

$$x_r^2 - 2\sqrt{a}x_r + a > 0$$

$$(x_r - \sqrt{a})^2 > 0$$

pedojočce náročné a megene reprez.

$$x_0 \in (0, \sqrt{a}) \Rightarrow x_1 \in (\sqrt{a}, \infty)$$

$$x_1 = \frac{1}{2} \left(x_0 + \frac{a}{x_0} \right) > \sqrt{a}$$

$$x_0^2 + a > 2x_0\sqrt{a}$$

$$x_0^2 - 2x_0\sqrt{a} + a > 0$$

$$(x_0 - \sqrt{a})^2 > 0 \quad \checkmark$$

Naj bo $f \in C^2$ & njena enostavna nica

a) Določite, da metode

$$x_{r+\bar{r}} = x_r - \frac{2f(x_r) \cdot f'(x_r)}{2f'(x_r)^2 - f(x_r) \cdot f''(x_r)} \quad \text{ustreza tangenti:}$$

metodi za funkcijo $F(x) = \frac{f(x)}{\sqrt{|f'(x)|}}$

b) poenostavide metoda za $f(x) = x^2 - a$

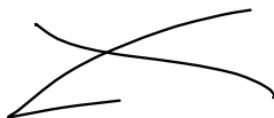
$$a) F'(x) = \frac{f'(x)\sqrt{|f'(x)|} - \sqrt{|f'(x)|}}{|f'(x)|} =$$

$$\sqrt{|f'(x)|} = \frac{f''(x)}{2\sqrt{|f'(x)|}} \cdot \frac{f'(x)}{|f'(x)|}$$

$$= \frac{\frac{f'(x)}{\sqrt{|f'(x)|}} \left(1 - \frac{f''(x)}{2|f'(x)|} \right)}{|f'(x)|} = \frac{\operatorname{sgn} f \left(1 - \frac{1}{2} \operatorname{sgn} f \right)}{\sqrt{|f'(x)|}} =$$

$$= \frac{\operatorname{sgn} f - \frac{1}{2}}{\sqrt{|f'(x)|}}$$

$$\frac{F(x)}{F'(x)} = \frac{f(x)}{\sqrt{|f'(x)|}} \cdot \frac{\sqrt{|f'(x)|}}{\operatorname{sgn} f - \frac{1}{2}} = \frac{f(x)}{\operatorname{sgn} f - \frac{1}{2}}$$



1.1

$$P(2, 3, -1, 3) \quad \pm m.b^e \quad L \leq e \leq v$$

$$P(b, +, L, U)$$

↑
bezüg

0	e: -1	1	2	3
0,100	0,01	1	10	100
0,101	0,0101	1,01	10,1	101
0,110	0,011	1,1	11	110
0,111	0,0111	1,11	11,1	111

1.2

$$P(2, 0, -10, -10)$$

$$x = 47,712$$

$$\begin{array}{r} 47:2=23 \text{ in } 1 \\ 07 \\ \hline 100 \\ 100 \end{array} \quad 1+2+2^2+2^3+2^5=$$

$$\begin{array}{r} 23:2=11 \text{ in } 1 \\ 11:2=5 \text{ in } 1 \\ 5:2=2 \text{ in } 1 \\ 2:2=1 \text{ in } 0 \\ 1:2=1 \text{ in } 1 \\ \hline 32 \\ 8 \\ 4 \\ 3 \\ \hline 47 \end{array} \quad 10111$$

$$\frac{0,712 \cdot 2}{1,424} \quad 1 \quad 0,101111010 \cdot 10^6$$

$$\frac{0,424 \cdot 2}{0,848} \quad 0 \quad ,$$

$$\frac{0,848 \cdot 2}{1,696} = 1$$

$$\frac{0,696 \cdot 2}{1,392} \quad 1$$

$$\frac{0,392 \cdot 2}{0,784} \quad 0$$

$$0,1 = \sum_{i=1}^{\infty} (z^{-u_i} + z^{-u_{i+1}})$$

$$\sum_{i=1}^{\infty} (z^{-4})^i = \frac{1}{1-z^{-4}} = \frac{z^4}{z^4-1}$$

$$\frac{1}{2} \sum_{i=1}^{\infty} (z^{-4})^i = \frac{z^3}{z^4-1}$$

$$+ = \frac{z^4 + z^3}{15} = \frac{24}{15} = \frac{8}{5} = \frac{16}{10} = 1,6$$

$$\frac{3}{2} \cdot \frac{16}{15} = \frac{8}{5} =$$

1) 2.3

$$f(x) = x^5 - 10x + 1$$

$$f(0) = 1$$

$$f(0,2) = 0,2^5 - 2 + 1 < 0$$

the vsejeno nico

če je vec, ne stevimo te de

$$f(x) = 5x^4 - 10 = 0$$

$$x^4 = 2$$

$$(2) \quad x = \sqrt[4]{2} \text{ n: ned } 0 \text{ in } 0,2$$

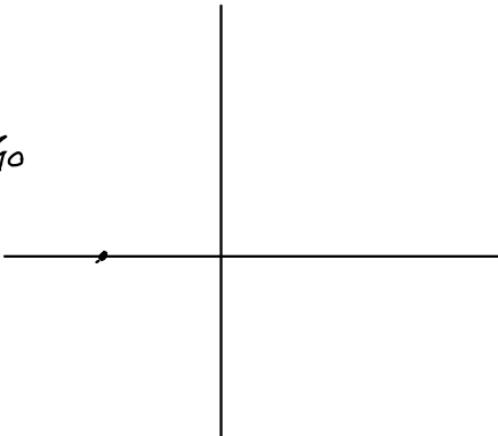
$$x_0 = 0$$

$$x_{r+1} = g(x_r)$$

$$g(x) = (x^5 + 1)/10$$

$$x_{r+1} = \frac{(x_r^5 + 1)}{10}$$

$$\lim_{r \rightarrow \infty} f(x_r)$$



$$g'(x) = \frac{5x^4}{10} \quad g'(0) = 0 <$$

$$g(g(0)) = g\left(\frac{1}{10}\right) = \frac{\frac{1}{10^5} + 1}{10} = \frac{10^5 + 1}{10^6}$$

$$g(x) = -x^2 + 8x - 12$$

$$g(x) = x$$

$$-x^2 + 8x - 12 - x = 0$$

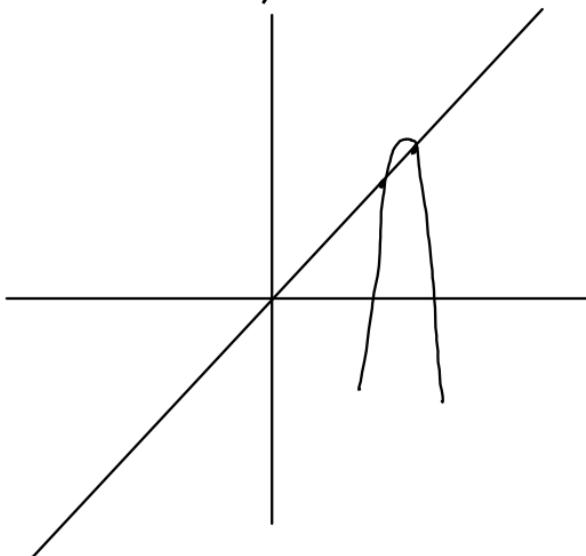
$$-x^2 + 7x - 12 = 0$$

$$-(x-3)(x-4) = 0$$

$$g'(x) = -2x + 8 \quad |-2x + 8| < 1$$

$$g'(3) = 2 \quad \text{add} \quad e\left(\frac{7}{2}, \frac{9}{2}\right)$$

$$g'(4) = 0 \quad \text{remove}$$



$$b_n = \frac{10}{3}b_{n-1} - b_{n-2}$$

$$b_{n-2} = \frac{10}{3}b_{n-1} - b_n$$

2,7 nologe

$$f(x) = (x^2 - a)x = x^3 - ax$$

$$g(x) = x \frac{x^2 + 3x}{3x^2 + a} \quad \text{je}$$

neigbore take x : \sqrt{a} , $-\sqrt{a}$, 0

ze x_0 > 0 konvergiert k Ja

$$x_r - \sqrt{a} = g(x_{r-1}) - \sqrt{a} = \frac{(x_{r-1} - \sqrt{a})^3}{3x_{r-1}^2 + a} < 0$$

$\overbrace{\hspace{10em}}^{>0}$

$$\Rightarrow x_{r-1} - \sqrt{a} < 0$$

die je streeg verantwoorde? $x_r < \sqrt{a}$

2.19 nelage

$$f(x) = x^2 - a \quad a > 0$$

Hallyjeva metoda

$$g(x) = x - \frac{2f(x)f'(x)}{2f'(x)^2 - f(x)f''(x)}$$

$$f'(x) = 2x \quad f''(x) = 2$$

$$g(x) = x - \frac{2(x^2 - a)2x}{2(2x)^2 - (x^2 - a)2} =$$

$$= x - \frac{4x^3 - 4ax}{8x^2 - 2x^2 + 2a} = \frac{2x^3 + 6ax}{6x^2 + 2a} =$$

$$= \frac{x(x^2 + 3a)}{3x^2 + a}$$

2.15 metode

$$x_{r+1} = \frac{x_{r-1}x_r + a}{x_{r-1} + x_r}$$

$$x_0, x_1 > \sqrt{a}$$

sekantna metoda

$$x_{r+1} = x_r - \frac{f(x_r)(x_r - x_{r-1})}{f(x_r) - f(x_{r-1})}$$

$$f(x) = x^2 - a$$

$$\begin{aligned} x_{r+1} &= x_r - \frac{(x_r^2 - a)(x_r - x_{r-1})}{x_r^2 - a - x_{r-1}^2 + a} = \\ &= \frac{x_r^3 - x_{r-1}x_r - x_r^3 + ax_r + x_{r-1}x_r^2 - ax_{r-1}}{x_r^2 - x_{r-1}^2} = \\ &= \frac{x_{r-1}x_r(x_r - x_{r-1}) + a(x_r - x_{r-1})}{x_r^2 - x_{r-1}^2} = \frac{x_{r-1}x_r + a}{x_r + x_{r-1}} \end{aligned}$$

$$\underline{x_{r+1} < x_r}$$

$$x_{r+1} < \frac{x_{r-1}x_r + a}{x_r + x_{r-1}} \Rightarrow x_{r+1}x_r + x_{r+1}x_{r-1} < x_{r-1}x_r + a$$

$$2x_{r+1}\cancel{x_{r-1}} < \cancel{x_{r-1}x_r} + a$$

$$x_{r+1} < \frac{x_r + a}{2} < x_r$$

...

3.3 norme

$$N_\infty(A) = \max |a_{ij}|$$

1) N_∞ ist metrische norm

$$N(A) N(B) = N(AB)$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \quad \times$$

2) $\|A\| = n N_\infty(A)$ für matrizen norm
positive definit

- $\|A\| \geq 0$: gleich ist das, in $n > 0$

$$\|A\| = 0 \Leftrightarrow n = 0 \vee a_{ij} = 0 \quad \forall a_{ij} \Rightarrow A = 0$$

• homogenität

$$\|\alpha A\| = n N(\alpha A) = n |\alpha| N(A) = |\alpha| \|A\| \quad \checkmark$$

• strk. verallgemeinert

$$\|A+B\| = n N_\infty(A+B) = n \max_j |a_{ij} + b_{ij}| \leq n \max_j |a_{ij}| + n \max_j |b_{ij}|$$

• metrische abstand

$$\leq \|A\| + \|B\|$$

$$\|A \cdot B\| \leq \|A\| \|B\|$$

$$\|A \cdot B\| = \left\| \sum_{c=1}^n a_{ic} b_{cj} \right\| \leq n \sum_{c=1}^n |b_{cj}|$$

$$n \max_j |a_{ij}| \left| \sum_{c=1}^n b_{cj} \right| \leq \max_j |a_{ij}| \max_j |b_{cj}| \leq \|A\| \|B\|$$

3.8 norme

$$\|A\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |k_{ij}|^2}$$

$$\|A\|_2 = \max \underbrace{\lambda(A^H A)}_{\text{lastne vrednost od } A^H A}$$

$$1) \frac{1}{\sqrt{n}} \|A_F\| \leq \|A\|_2 \leq \|A\|_F$$

$$\det(A^H A \rightarrow I) = 0$$

3.10

$$\|A\|_2 \leq \|A\|_F$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^H = A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$A^H A = \begin{bmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{bmatrix}$$

expand &

LU razceg

• brez pivotiranja (zadnjic =)

• delno pivotiranje $PA = LU$

$$Ax = b$$

$$L U X = Pb$$

$\underbrace{}_y$

$$\begin{aligned} 1) Ly &= Pb \\ 2) Ux &= y \end{aligned}$$

• kompletno pivotiranje $Ly = Pb$ 2) $Ux = y$ 3) $Q^{-1}x = z$

$$A = \begin{bmatrix} 2 & 1 & -2 & 1 \\ 2 & 1 & -4 & 2 \\ 3 & -2 & 3 & -1 \\ -1 & 3 & -1 & 1 \end{bmatrix}$$

izrazenje LU razceg in telo
 $\det(A)$

$$\xrightarrow{\left[\begin{array}{cccc} 3 & -2 & 3 & -1 \\ 2 & 1 & -4 & 2 \\ 2 & 1 & -2 & 1 \\ -1 & 3 & -1 & 1 \end{array} \right] \rightsquigarrow \left[\begin{array}{cccc} 3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ \frac{2}{3} & \frac{1}{3} & -4 & \frac{5}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 & \frac{2}{3} \end{array} \right]}$$

1 - (2)($\frac{2}{3}$)

$$\xrightarrow{P \rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right]} \quad 1 - (-2)(\frac{2}{3})$$

$$\left[\begin{array}{cccc} 3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ \frac{2}{3} & 1 & 2 & -1 \\ -\frac{1}{3} & 1 & 6 & -2 \end{array} \right] \rightsquigarrow \left[\begin{array}{cccc} 3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ -\frac{1}{3} & 1 & 6 & -2 \\ \frac{2}{3} & 1 & \frac{1}{3} & -\frac{1}{3} \end{array} \right]$$

$$\xrightarrow{\left[\begin{array}{cccc} -3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ -\frac{1}{3} & 1 & 6 & -2 \\ \frac{2}{3} & 1 & 2 & -1 \end{array} \right] \rightsquigarrow \left[\begin{array}{cccc} -3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ -\frac{1}{3} & 1 & 6 & -2 \\ \frac{2}{3} & 1 & \frac{1}{3} & -\frac{1}{3} \end{array} \right]}$$

-1 - (- $\frac{2}{3}$)

$$U = \left[\begin{array}{cccc} -3 & -2 & 3 & -1 \\ \frac{2}{3} & \frac{1}{3} & -6 & \frac{8}{3} \\ -\frac{1}{3} & 1 & 6 & -2 \\ \frac{2}{3} & 1 & \frac{1}{3} & -\frac{1}{3} \end{array} \right] \quad L = \left[\begin{array}{cccc} 1 & & & \\ \frac{2}{3} & 1 & & \\ -\frac{1}{3} & 1 & 1 & \\ \frac{2}{3} & 1 & \frac{1}{3} & 1 \end{array} \right]$$

$$P = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right]$$

$$\det(A) = \frac{\det(U) \det(L)}{\det(P)} = \det(U) = (-7)(-2) = -14$$

$$\begin{aligned} "1" &= (-1)^{\text{st zem. vrstic}} \\ "2" & \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 2 & -6 \\ -3 & 3 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ 14 \\ 0 \end{bmatrix}$$

LU s komplexem Produktionsplan + system $Ax=b$

$$A = \begin{bmatrix} 1 & 2 & -6 \\ 4 & 2 & -6 \\ -3 & 3 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -6 & 2 & 4 \\ -3 & 2 & 1 \\ 3 & 3 & -3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -6 & 2 & 4 \\ \frac{1}{2} & 1 & -1 \\ \frac{1}{2} & 4 & -1 \end{bmatrix} \rightsquigarrow$$

$$\begin{bmatrix} -6 & 2 & 4 \\ \frac{1}{2} & 4 & -1 \\ \frac{1}{2} & 1 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -6 & 2 & 4 \\ \frac{1}{2} & 4 & -1 \\ \frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & \\ \frac{1}{2} & 1 & \\ \frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix} \quad U = \begin{bmatrix} -6 & 2 & 4 \\ 0 & 4 & -1 \\ 0 & 0 & -\frac{3}{4} \end{bmatrix} \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} & 1 & 1 \\ 1 & & \end{bmatrix}$$

$$Ly = Pb$$

$$\begin{bmatrix} 1 & & \\ \frac{1}{2} & 1 & \\ \frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 0 \\ 8 \end{bmatrix}$$

$$y_1 = 14$$

$$y_2 = \frac{14}{2} + y_2 = 0 \Rightarrow y_2 = -7$$

$$y_3 = -\frac{3}{4}$$

$$Uz = y$$

$$\textcircled{1} \quad x = Qz$$

a) $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 4 & 5 & 2 & 0 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ Izračunajte LU razceg brez pivotiranja
Kaj opazite

b) Nekaj bo $\begin{bmatrix} a_{11} b_1 \\ a_{12} \\ \vdots \\ a_{nn} b_n \end{bmatrix}$ splošna 3-diagonarna matrika

Zapisite algoritam za razceg te tridiagonale matrike in preglejte st. operacij
koliko operacij potrebujemo za reševanje sistema

$$Ax = z$$

a) $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 4 & -7 & 2 & 0 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 3 & 0 & 0 \\ 4 & -7 & 2 & 0 \\ 0 & -\frac{2}{3} & \frac{11}{3} & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 3 & 0 & 0 \\ 4 & -7 & 2 & 0 \\ 0 & -\frac{2}{3} & \frac{11}{3} & 6 \\ 0 & 0 & \frac{2}{3} & -\frac{20}{11} \end{bmatrix}$ $6 \cdot 7 = 42$
 $L = I \quad U = A \rightsquigarrow \quad 2 - \frac{42}{11} = -\frac{20}{11}$

b) for i in range ($1, n$):

$$= L[i][i-1] = A[i][i-1] / A[i-1][i-1]$$

$$U[i][i] = A[i][i] - L[i][i-1] \cdot A[i-1][i]$$

$$u_1 = a_1$$

for $i = (1:n)$

$$l_{ii} = \frac{c_i}{u_i}$$

$$u_{i+1} = a_{i+1} - l_{ii} \cdot b_i$$

end

$n-1$ deljenja s L

$n-1$ množenj

$n-1$ odštevanj

$3 \cdot (n-1)$ operacij

$$Ax = z$$

$$Ly = z: \quad y_1 = z_1 \quad y_2 = l_{12}y_1 + z_2 \quad \dots \quad y_i = z_i - l_{i-1}y_{i-1}$$

$2 \cdot (n-1)$ operacij

$$Ux = z: \quad x_n = u_n^{-1} z_n$$

~~$b_{n-1} x_n + u_{n-1} x_{n-1} = z_n$~~

$$x_{n-1} = \frac{z_n - b_{n-1} x_n}{u_{n-1}}$$

$$= 3 \cdot (n-1) + 1 = 3n-2 \text{ operacij}$$

skupno $5n-4$ operacij;

Opisite postopek za reševanje sistema linearnih enačb

$$\begin{bmatrix} U & -I \\ B & L \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}, \text{ kjer je } B = LU \text{ nesgubačna}$$

Presteje števile operacij

$$\begin{bmatrix} Ux - y \\ Bx + Ly \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$Bx + Ly = L(Ux + Ly) = L(Ux + Iy)$$

$$y = Ux - a \quad Ux = y + a$$

$$Bx + Ly = L(Ux + Ux - a) = L(2Ux - a) =$$

$$= L(\underbrace{2y + a}_z) = b$$

$$1. \quad Lz = b \quad \text{n}^2 \text{ operacij}$$

$$2. \quad y = \frac{z - a}{2} \quad 2n \text{ operacij}$$

$$3. \quad Ux - a + y \quad n + \underbrace{n^2 + n}_{\substack{\text{obrat.} \\ \text{svo}}} =$$

stopej: $2n + 4n$

15.11

✓ menyatakan
bahwa

Razceq Choleskega

$A \in \mathbb{R}^{n \times n}$ simetrična, pozitivno definitska
 $(\forall x \in \mathbb{R}^n : x^T A x > 0)$

$$A = V \cdot V^T \quad V = \begin{pmatrix} & & \\ & \ddots & \\ & & 1 \end{pmatrix} \quad \text{nesingularna s pozitivnim diag. elementi}$$

for $i = 1:n$

$$V_{ii} := \sqrt{a_{ii} - \sum_{k=1}^{i-1} v_{ik}^2}$$

for $j = i+1:n$

$$v_{ij} := \frac{1}{v_{ii}} \left(a_{jj} - \sum_{k=1}^{j-1} v_{jk} v_{ik} \right)$$

a) Dados ferter Choleskega (v)

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 8 & -2 & 8 \\ -2 & -2 & 14 & -11 \\ 3 & 8 & -11 & 15 \end{bmatrix}$$

b) Nøj bo

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 8 & 0 & 3 \\ 0 & 0 & 3 & -4 \\ -4 & 3 & -4 & \alpha \end{bmatrix}$$

i) zulæsse \propto jf to poz. bet.

ii) ~~$\alpha = 23$~~ 23. Række sætten

$$Ax = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}$$

$$v_{ij} := \frac{1}{v_{ii}} (a_{ij} - \sum_{k=1}^{j-1} v_{ik} v_{kj})$$

$$v_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} v_{ik}^2}$$

$$v_{21} = \frac{1}{1}(2-0)$$

$$v_{22} = \sqrt{18 - 2^2} = 4\sqrt{2}$$

$$v_{32} = \frac{1}{2}(-2 - (-4)) = 1$$

$$v_{42} = \frac{1}{2}(8 - (3 \cdot 2)) = 1$$

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 8 & 0 & 3 \\ 0 & 0 & 3 & -4 \\ -4 & 3 & -4 & \alpha \end{bmatrix}$$

$$V = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\alpha > 14$$

Dan⁺ s^e matriki A in B $\in \mathbb{R}^{n \times n}$, B je pozitivno definitna. Sestavite učinkovit postopek za izračun sledi od $A^T B^{-1} A$ in prestejet c st. operacij.

Ne jekabl:

reševanje sistemov nehomogenih enačb

$$f_1(x_1 \dots x_n) = 0$$

$$f_2(x_1 \dots x_n) = 0$$

$$\vdots$$
$$f_n(x_1 \dots x_n) = 0$$

$$\begin{aligned} F(x_1 \dots x_n) &= \begin{bmatrix} f_1(x_1 \dots x_n) \\ \vdots \\ f_n(x_1 \dots x_n) \end{bmatrix} \\ F(\underline{x}) &= 0 \end{aligned}$$

Jakobijska iteracija

$$G(x) = \begin{bmatrix} g_1(\underline{x}) \\ \vdots \\ g_n(\underline{x}) \end{bmatrix} \quad x^{(r+1)} = G(x^{(r)})$$

Seidlova iteracija

$$x_i^{(r+1)} = g_i(x_1^{(r+1)}, \dots, x_{i-1}^{(r+1)}, x_i^{(r)}, \dots, x_n^{(r)})$$

- \underline{x} je privlačna nežilna točka, če obstaja tečna metrična norma, da velja $\|J_G(\underline{x})\| < 1$
(G mora biti odvedljiva v \underline{x})

Potem \exists dolga $\mathcal{R} \ni a$. $\forall \underline{x}^{(0)} \in \mathcal{R}$ reševanje konvergira k \underline{x}

$$\text{Dan je sistem enačb} \quad x = \sin\left(\frac{2x-y}{u}\right)$$

$$y = \cos\left(\frac{x+2y}{u}\right)$$

a) pri začetnem pribl.žku $x^0 = \begin{bmatrix} 2\pi \\ 0 \end{bmatrix}$ nevedite

dva koraka Jakobijske in se idemo iteracije

b) Pokažite, da iteraciji konvergirajo za A začetni pribl.žek

$$G(x,y) = \begin{bmatrix} \sin\left(\frac{2x-y}{u}\right) \\ \cos\left(\frac{x+2y}{u}\right) \end{bmatrix}$$

$$x^1 = G\left(\begin{bmatrix} 2\pi \\ 0 \end{bmatrix}\right) = \begin{bmatrix} \sin\left(\frac{4\pi}{u}\right) \\ \cos\left(\frac{2\pi}{u}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x^2 = G\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Sledi:

$$x^1 = G\left(\begin{bmatrix} 2\pi \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x^2 = G\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} \sin\frac{1}{u} \\ \cos\left(-\sin\frac{1}{u} + 2\right) \end{bmatrix}$$

$$G(x, y) = \begin{bmatrix} \sin\left(\frac{2x-y}{u}\right) \\ \cos\left(\frac{x+2y}{u}\right) \end{bmatrix}$$

$$\int_G(x, y) = \begin{bmatrix} \cos\left(\frac{2x-y}{u}\right) \cdot \frac{1}{2} - \cos\left(\frac{2x-y}{u}\right) \frac{1}{u} \\ -\underbrace{\sin\left(\frac{x+2y}{u}\right) \cdot \frac{1}{u}}_{|| < 1} - \sin\left(\frac{x+2y}{u}\right) \cdot \frac{1}{2} \end{bmatrix}$$

Vektorraum \mathbb{R}^n normen

Verteilung auf Abs. und Orient. von \mathbb{R}^n :

$$\left| \int_G(x, y) \right|_p = \max \left\{ \left| \cos\left(\frac{2x-y}{u}\right) \frac{1}{2} + \frac{\cos\left(\frac{2x-y}{u}\right)}{u} \right|, \dots, y \leq \right. \\ \left. \leq \max \left\{ \frac{1}{2} + \frac{1}{u}, \frac{1}{2} + \frac{1}{u} \right\} < 1 \right.$$

\rightarrow für festes x, y kein Abstand problem
Koeffizienten kann es nur problem

zu seidde:

$$G(x, y) = \begin{bmatrix} \sin\left(\frac{2x-y}{u}\right) \\ \cos\left(\frac{\sin\left(\frac{2x-y}{u}\right) + 2y}{u}\right) \end{bmatrix}$$

$$\int_G = \begin{bmatrix} \frac{1}{2} \cos\left(\frac{2x-y}{u}\right) - \frac{1}{u} \cos\left(\frac{2x-y}{u}\right) \\ -\frac{1}{2} \cos\left(\frac{2x-y}{u}\right) \sin\left(\frac{\sin\left(\frac{2x-y}{u}\right) + 2y}{u}\right), \dots \end{bmatrix}$$

Spur der Matrix $\| \mathbb{R}^n \text{ norme in } \mathbb{R}^n \text{ mit } u = 1 \text{ ist }$

Newthover metode

$$x^{(r+1)} = x^{(r)} - J_F(x^{(r)})^{-1} F(x^{(r)})$$

\curvearrowleft

$$J_F(x^{(r)}) (x^{(r+1)} - x^{(r)}) = -F(x^{(r)})$$

1) $J_F(x^{(r)}) \Delta x^{(r)} = -F(x^{(r)})$

2) $x^{(r+1)} = \Delta x^{(r)} + x^{(r)}$

$$\text{Dan je sistem } \begin{aligned} x^2 + y^2 &= 4 \\ x^2 - y^2 &= 1 \end{aligned}$$

a) Naredite 2. korak newtonove metode

$$\text{pri } x^{(0)} = (2, 1)$$

$$x = \begin{bmatrix} x \\ y \end{bmatrix} \quad F = \begin{bmatrix} x^2 + y^2 - 4 \\ x^2 - y^2 - 1 \end{bmatrix}$$

$$\nabla F = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$$

$$\nabla_F(x^0) = \begin{bmatrix} u & v \\ u & -v \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u\bar{x} + v\bar{y} \\ u\bar{x} - v\bar{y} \end{bmatrix} = - \begin{bmatrix} \frac{1}{2} \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} u & v \\ u & -v \end{bmatrix} \sim \begin{bmatrix} u & v \\ 0 & -u \end{bmatrix} \sim y = \frac{1}{u}$$

$$u\bar{x} + v\bar{y} = -1$$

$$u\bar{x} = -\frac{3}{2}$$

$$x = -\frac{3}{8}$$

3. 12

✓ 1. nelage

Predolacení systému:

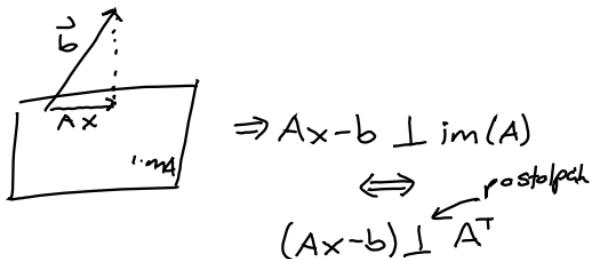
$$Ax = b \quad A \in \mathbb{R}^{m \times n}, m \geq n \quad b \in \mathbb{R}^m$$

Isčeme x , k i minimizovat $\|Ax - b\|_2$

A polnější rang \Rightarrow jednoznačné řešení

Normalní systém

$$Ax \in \text{Im}(A)$$



$A^T A$ je simetrické, pozitivně definitní
 \Rightarrow lze upravit pomocí rozložení Choleskova

$$\begin{aligned} &\Leftrightarrow \\ &A^T(Ax - b) = 0 \\ &\Leftrightarrow \\ &A^T A x = A^T b \end{aligned}$$

$$1) A^T A = VV^T$$

$$2) A^T A x = A^T b$$

$$\underbrace{VV^T x}_y = A^T b$$

$$V_y = A^T b$$

f podana v tabuli

$$f(-1) = \frac{11}{4} \quad \text{Poisci parabolo, ki se}$$

$$f(0) = \frac{7}{4} \quad \text{po metodi najmanjih}$$

$$f(1) = \frac{1}{4} \quad \text{kvadratov najbolje prilega}$$

$$f(2) = \frac{13}{4} \quad \text{funkciji } f. \text{ Izpeljite normale in ga resite z racijem Cholesega}$$

$$p(x) = ax^2 + bx + c$$

$$p(-1) = a - b + c = \frac{11}{4}$$

$$A \begin{bmatrix} c \\ b \\ a \end{bmatrix} = b$$

$$p(0) = c = \frac{7}{4}$$

$$p(1) = a + b + c = \frac{1}{4}$$

$$p(2) = 4a + 2b + c = \frac{13}{4}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix}$$

$$\text{Cholesky} \quad V = \begin{bmatrix} 2 \\ 1 & \sqrt{5} \\ 3 & \frac{8-3}{\sqrt{5}} & \sqrt{18-9-5} \end{bmatrix} =$$

$$= \begin{bmatrix} 2 \\ 1 & \sqrt{5} \\ 3 & \sqrt{5} & 2 \end{bmatrix} \quad V y = A^T b$$

$$A^T b = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 11 \\ 7 \\ 1 \\ 13 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 & \sqrt{5} \\ 3 & \sqrt{5} & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 16 \end{bmatrix} \Rightarrow \begin{aligned} y_1 &= 4 \\ 4 + \sqrt{5}y_2 &= 4 \Rightarrow y_2 = 0 \\ 12 + 2y_3 &= 16 \Rightarrow y_3 = 2 \end{aligned}$$

$$V^T x = y$$

$$\begin{bmatrix} 2 & 1 & 3 \\ \sqrt{5} & \sqrt{5} & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \begin{aligned} x_3 &= 1 \\ \sqrt{5}(x_2^2 + 1) &= 0 \Rightarrow x_2 = -1 \\ 2x_1 - 1 + 3 &= 4 \end{aligned}$$

$$x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} a &= 1 \\ b &= -1 \\ c &= 1 \end{aligned}$$

$$x_1 = 1$$

$$\underline{\underline{p(x) = x^2 - x + 1}}$$

QR razcep

$$A = QR \quad A \in \mathbb{R}^{m \times n} \quad m > n$$

$Q \in \mathbb{R}^{m \times n}$
ortonormirani
stolpcí

$R \in \mathbb{R}^{n \times n}$
zaginje
tričotna

$$Q^T Q = I$$

$$Q Q^T \neq I \quad \text{ker vrtice } n:so$$

ortonormirane

• Modificiran Gram-Schmidov postopek

$$k=1, \dots, n$$

$$g_k = a_k$$

$$i=1, \dots, k-1$$

$$r_{ik} = g_i^T g_k$$

$$g_k = g_k - r_{ik} g_i$$

$$r_{kk} = \|g_k\|_2$$

$$g_k = \frac{g_k}{r_{kk}}$$

S pomočjo MGS (modificiran gram Schmidt) izračunajte QR razceg za matriko

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{2} & 0 & -2\sqrt{2} \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2\sqrt{3} \end{bmatrix} \quad Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$$

$$g_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \|g_1\| = \sqrt{2}$$

$$g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \rightsquigarrow r_{12} = 0 \quad r_{22} = \sqrt{2}$$

$$g_2 = g_2 - 0 \quad s_2 = \frac{g_2}{r_{22}}$$

$$g_3 = \begin{bmatrix} -1 \\ -3 \\ 1 \\ 1 \end{bmatrix} \rightsquigarrow r_{13} = -\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} = -2\sqrt{2}$$

$$g_3 = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \rightsquigarrow$$

$$r_{23} = 0 \quad r_{33} = \sqrt{12} = 2\sqrt{3},$$

$$g_3 = g_3 \rightsquigarrow$$

$$g_3 = \frac{1}{2\sqrt{3}} g_3$$

Modificiran Gramm-Schmidt

$k = 1 : n$

$$g_k = a_k$$

$j = 1, \dots, k-1$

$$r_{jk} = g_j^\top g_k$$

$$g_k = g_k - r_{jk} g_j$$

$$r_{kk} = \|g_k\|_2$$

$$g_k = \frac{1}{r_{kk}} g_k$$

Pri reševanju sistema z MGS naredimo QR razceg na razstavljeni matotku

$$\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} Q & g_{n+1} \end{bmatrix} \begin{bmatrix} R & Z \\ 0 & g \end{bmatrix} \quad \left(\begin{array}{l} Ax = b \\ \min_x \|Ax - b\|_2 \end{array} \right)$$

Rešljeno $Rx = z$ $\min_x \|Ax - b\|_2 = g$

Od zadnjic

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -3 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{1}{2} \end{bmatrix} R = \begin{bmatrix} \sqrt{2} & 0 & -2\sqrt{2} \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

b) Preko QR razcepa rešite predstavljen sistem

$$Ax = \begin{bmatrix} 4 \\ 6 \\ -1 \\ 2 \end{bmatrix} \quad \text{Po metodi nejmajžih kvadrata.}$$

Kriterij je $\min \|Ax - b\|_2$

$$\begin{aligned} [A; b] &= \begin{bmatrix} 1 & 0 & -1 & 4 \\ 1 & 0 & -3 & 6 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 2 \end{bmatrix} & g_u &= \begin{bmatrix} 6 \\ 6 \\ -1 \\ 2 \end{bmatrix} \\ j=1 & r_{1u} = \frac{4}{\sqrt{2}} + \frac{6}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \end{aligned}$$

$j=2:$

$$r_{2u} = -\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{2}} = -\frac{3}{\sqrt{2}} = \frac{\sqrt{2} \cdot 3}{2} \quad g_u = g_u - 5\sqrt{2} e_1$$

$$g_u = g_u + \frac{-3}{\sqrt{2}} e_2 = \begin{bmatrix} -1 \\ 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \quad g_u = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 2 \end{bmatrix}$$

$j=3$

$$r_{3u} = \frac{1}{2}$$

$$g_u = g_u + \frac{1}{2} g_3 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$r_{4u} = \sqrt{\frac{9}{16} + 2} = \sqrt{\frac{9}{8}} = \frac{3}{2\sqrt{2}} =$$

$$g_u = \frac{1}{r_{4u}} g_u = \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Rx = z$$

$$R = \begin{bmatrix} \sqrt{2} & 0 & -2\sqrt{2} \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5\sqrt{2} \\ -\frac{3}{2}\sqrt{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$x_3 = -\frac{1}{4}$$

$$x_2 = -\frac{3}{2}$$

$$\sqrt{2} x_1 - 2\sqrt{2} = 5\sqrt{2}$$

$$x_1 = \frac{1}{2} + 5 = \frac{3}{2}$$

$$x = \begin{bmatrix} \frac{5}{2} \\ -\frac{3}{2} \\ -\frac{1}{4} \end{bmatrix}$$

$$\min_x \|Ax - b\|_2 = 2$$

✓ 10.12
1.w -

Razširjeni QR razcep

$$A = \tilde{Q} \tilde{R}$$

\tilde{Q} : mxm ortogonalna $\tilde{Q}^T \tilde{Q} = \tilde{Q} \tilde{Q}^T = I$

\tilde{R} : mxn zgora trapecna

$$A = \tilde{Q} \tilde{R} = \begin{bmatrix} Q & Q_1 \end{bmatrix} \begin{bmatrix} R \\ 0 \end{bmatrix}$$

Reševanje sistema: $\sqrt{\text{množenje } \tilde{Q} \text{ ne spremem}}$
 z - rene

$$\|Ax - b\|_2 = \|\tilde{Q} \tilde{R} x - b\|_2 = \|\tilde{Q}^T (\tilde{Q} \tilde{R} x - b)\|_2 =$$

$$= \|\tilde{R} x - Q^T b\| = \left\| \begin{bmatrix} R \\ 0 \end{bmatrix} x - \begin{bmatrix} Q^T \\ Q_1^T \end{bmatrix} b \right\|$$

x, b : to minizira dobimo iz $Rx = Q^T b$

$$\text{minimum} = \|Q_1^T b\|_2$$

Givensove rotacije

Vhodni podatki: A, b

Izhod: $\tilde{R} Q^T b$

for $i=1:n$

for $k=i+1:m$

if $a_{ki} \neq 0$:

$$R_{ik}^T = I$$

$$r = \sqrt{a_{ii}^2 + a_{ki}^2}$$

$$c = \frac{a_{ii}}{r}$$

$$s = \frac{a_{ki}}{r}$$

$$(R_{ik}^T)_{ii} = c$$

$$(R_{ik}^T)_{ik} = s$$

$$R_{ik}^T ([i, k], [i, k]) = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

$$A = R_{ik}^T A$$

$$b = R_{ik}^T b$$

Izhod: $A (= \tilde{R})$, $b (= \tilde{Q}^T b)$

Naloga: s pomočjo grebenovih robcej rešite preddelen sistem. Koliko je min. $\|Ax - b\|$?

$$\begin{bmatrix} 3 \\ -4 \\ 12 \\ -84 \end{bmatrix} \times = \begin{bmatrix} 7 \\ -1 \\ 5 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ \frac{-75}{25} \\ \frac{5}{25} \\ \frac{84}{25} \end{bmatrix}$$

$i=1 \quad k=2$

$$r = 5 \\ c = \frac{3}{5} \\ s = -\frac{4}{5}$$

$$R = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \\ 1 & 1 \end{bmatrix}$$

RA ↑
RB ↑

$$\frac{144}{25} \\ \frac{25}{169}$$

$i=1 \quad k=3$

$$r = \sqrt{5^2 + 12^2} = 13 \\ c = \frac{5}{13} \quad s = \frac{12}{13}$$

$$R = \begin{bmatrix} \frac{5}{13} & 0 & \frac{12}{13} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{12}{13} & 0 & \frac{5}{13} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

RA ↗
RB ↗

$i=1 \quad k=4$

$$r = \sqrt{13^2 + 84^2} = 85 \\ c = \frac{13}{85} \quad s = -\frac{84}{85}$$

$$R = \begin{bmatrix} \frac{13}{85} & -\frac{84}{85} \\ \frac{84}{85} & \frac{13}{85} \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 85 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ \frac{5}{5} \\ -\frac{35}{13} \\ \frac{84}{13} \end{bmatrix} \quad Q = 1 \\ \rightsquigarrow R = 85$$

$$Q_1^\top b = \begin{bmatrix} 5 \\ -\frac{35}{13} \\ \frac{84}{13} \end{bmatrix}$$

$$\text{minimum: } \sqrt{85 + \frac{35^2}{13^2} + \frac{84^2}{13^2}}$$

$$=\sqrt{85}$$

X merkantil 15 krt

Singapore
in
26.12

Interpolacija

Izšemo polinom, ki bo potekal skozi dene tocke. Za $n+1$ takih obstaja enoten polinom stopnje n , ki jih interpolira.

Lagrangeva oblika interpolacijskega polinoma

Lagrangev: bazični polinomi:

$$l_{i,n}(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{(x - x_j)}{(x_i - x_j)}$$

interpolacijski polinom:

$$P(x) = \sum_{i=0}^n f(x_i) l_{i,n}(x)$$

Dodatek L.I.P.

$$f(x) = \frac{40}{x+1} \quad v \text{ dodatek } \left\{ 0, \frac{1}{3}, \frac{2}{3}, 1 \right\}$$

$n=3$

$$l_0 = \frac{x - \frac{1}{3}}{-\frac{1}{3}} \cdot \frac{x - \frac{2}{3}}{-\frac{2}{3}} \cdot \frac{x-1}{-1} = (3x-1)(\frac{3}{2}x-1)(x-1)$$

$$l_1 = \frac{x}{\frac{1}{3}} \cdot \frac{x - \frac{2}{3}}{-\frac{1}{3}} \cdot \frac{x-1}{-\frac{2}{3}} + 40 = f(x_1)$$

$$l_2 = \frac{x}{\frac{2}{3}} \cdot \frac{x - \frac{1}{3}}{\frac{1}{3}} \cdot \frac{x-1}{-\frac{1}{3}} + 30 = f(x_2)$$

$$l_3 = \frac{x}{1} \cdot \frac{x - \frac{1}{3}}{\frac{2}{3}} \cdot \frac{x-2}{\frac{1}{3}} + 20 = f(x_3)$$

$$P(x) = \frac{3}{2}(x - \frac{2}{3})(x-1)(3x-1)(40) + 30x + \frac{60}{36}$$

$$+ \frac{3}{2}x(x - \frac{1}{3})(24(x-1) + 60(x - \frac{2}{3})) = \frac{40}{16}$$

$$= \frac{6}{2} (3x-2)(x-1)(-30x+40) + \frac{-24}{16}$$

$$\frac{6}{2} (3x^2-1)(36x-16) =$$

$$= \frac{6}{2} (9x-4)(-10(3x^2-5x+2) + 12x^2-4) =$$

$$= \frac{6}{2} (9x-4)(-18x^2-50x-24) =$$

$$= 6(9x-4)(-9x^2-25x-12) =$$

$$= 6(x^3(-81) + x^2(36) + x(-3 \cdot 12 + 4 \cdot 28)) + 12 \cdot 4$$

Newtonova oblike interpolacijskega polinoma

Baza iz premaknjeneih potenc

$$1, \quad x - x_0, \quad (x - x_0)(x - x_1), \quad \dots, \quad (x - x_0) \dots (x - x_{n-1})$$

$$p(x) = \sum_{i=0}^n \underbrace{[x_0, x_1, \dots, x_i]}_{\text{deljene difference}} \cdot f(x - x_0) \dots (x - x_{i-1})$$

$\overset{n; \text{ nule}}{\underset{j}{\dots}}$

$$[x_0, \dots, x_k] f = \frac{[x_1, \dots, x_k] f - [x_0, \dots, x_{k-1}] f}{x_k - x_0}$$

x_i	$[x_i]$	$[,]f$	L, If
0	40	$\frac{30-40}{\frac{1}{3}} = -10$	40
$\frac{1}{3}$	30	$\frac{24-30}{\frac{2}{3}} = -18$	$\frac{-18+30}{\frac{2}{3}} = 18$
$\frac{2}{3}$	24	$\frac{20-24}{\frac{1}{3}} = -12$	$\frac{-12+18}{\frac{2}{3}} = 9$
1	20		$\frac{9-12}{1} = -3$

$$p(x) = 40 - 30(x-0) + 18x(x-\frac{1}{3}) - 9x(x-\frac{1}{3})(x-\frac{2}{3})$$

$$40 - 30x + 18x^2 - 6x - 9x^3 + 9x^2 - 2 =$$

$$= 40 - 38x + 27x - 9x^3$$

Vemo \rightarrow preferency

$$p(x) - p(x) = w(x) \frac{f^{(n+1)}(\xi)}{(n+1)!} \quad w(x) = \prod_{i=1}^n (x-x_i)$$

$$w(x) = x(x-\frac{1}{3})(x-\frac{2}{3})(x-1)$$