Furierove vrste

$$FV(f) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx$$

$$\boxed{\begin{array}{l} f \text{ soda} \Rightarrow b_n = 0 \\ f \text{ liha} \Rightarrow a_n = 0 \end{array}}$$

Parsevalove enakost

$$\frac{1}{\pi} \int_{-\pi}^{\pi} |f(x)|^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$\int x \cos(nx)\,dx = \frac{x \sin(nx)}{n} + \frac{\cos(nx)}{n^2} + C$$

$$\int x \sin(nx)\,dx = -\frac{x \cos(nx)}{n} + \frac{\sin(nx)}{n^2} + C$$

$$\int x^2 \cos(nx)\,dx = \frac{x^2}{n} \sin(nx) + \frac{2x}{n^2} \cos(nx) - \frac{2}{n^3} \sin(nx)$$

$$\int x^2 \sin(nx)\,dx = -\frac{x^2}{n} \cos x + \frac{2x}{n^2} \sin(nx) + \frac{2}{n^3} \cos(nx)$$

$$\int \cos(nx)\,dx + i\int \sin(nx)\,dx = \int e^{inx}\,dx$$

Kotne zadeve

$$\sin x \sin y = -\frac{1}{2}(\cos(x+y) - \cos(x-y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

$$\sin x \cos y = \frac{1}{2}(\cos(x+y) + \sin(x-y))$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

Integral:

$$X_T = \frac{\int_k x \, dm}{m(K)} = \frac{\int_k x \rho \, ds}{\int_k \rho \, ds}$$

$$ds = |\dot{\vec{r}}(t)| dt = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \, dt$$

$$\int_K u \, ds = \int_a^b u(\vec{r}(t)) |\dot{\vec{r}}(t)| dt$$

$$l(K) = \int_K ds = \int_a^b |\dot{\vec{r}}(t)| dt$$

$$\int_K \vec{R} \, d\vec{r} = \int_K \vec{R} \cdot \vec{T} \, ds = \int_a^b \vec{R}(\vec{r}(t)) \cdot \dot{\vec{r}}(t) \, dt$$

$$\iint_\Sigma u \, dS = \iint_D u(\vec{r}(s,t)) \sqrt{EG - F^2} \, ds \, dt =$$

$$\iint_D u(\vec{r}(s,t)) |\vec{r}_s \times \vec{r}_t| \, ds \, dt$$

$$\iint_{\vec{\Sigma}} \vec{R} \, d\vec{S} = \iint_D \vec{R} \cdot \vec{N} \, dS = \iint_D \vec{R}(\vec{r}) \cdot (\vec{r}_s \times \vec{r}_t) \, ds \, dt$$

$$\int_K X\,dx + Y\,dy + Z\,dz = \int_K (X, Y, Z)\,d\vec{r}$$

$$\iint_\Sigma X\,dz\,dy + Y\,dx\,dz + Z\,dx\,dy = \iint_\Sigma (X, Y, Z)\,d\vec{S}$$

$$\iint_D |\vec{r}_u \times \vec{r}_v|\,du\,dv = \iint_D \sqrt{EG - F^2}\,du\,dv$$

$$E = |\vec{r}_u|^2 \quad F = \vec{r}_u \cdot \vec{r}_v \qquad G = |\vec{r}_v|^2$$

Površina grafa $f \in C^1(D)$

$$\iint_D \sqrt{1 + f_x^2 + f_y^2}$$

Površina torusa $0 < a < R$: $P = 2\pi a\, 2\pi R$

Gradient $\vec{\nabla} u = (u_x, u_y, u_z) = \operatorname{grad} u$

divergenca $\vec{\nabla} \cdot \vec{R} = X_x + Y_y + Z_z = \operatorname{div} \vec{R}$

rotor $\vec{\nabla} \times \vec{R} = (Z_y - Y_z, X_z - Z_x, Y_x - X_y) = \operatorname{rot} \vec{R}$

$$S \xrightarrow{\text{grad}} V \xrightarrow{\text{rot}} V \xrightarrow{\text{div}} S$$

če naredimo dva zaporedna koraka pride 0

$\operatorname{div}(\operatorname{rot}(\vec{R})) = 0$ $\qquad$ $\operatorname{rot}(\operatorname{grad}(u)) = 0$

$$\operatorname{div} \circ \operatorname{grad} = \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$$

Polje u je harmonično če $\Delta u = 0$

Polje $\vec{R}$ je potencialno če $\exists u.\ \vec{R} = \operatorname{grad} u$

Polje $\vec{R}$ ima vektorski potencial če je $\vec{R} = \operatorname{rot} \vec{f}$ za nek $\vec{f}$

Polje $\vec{R}$ je irotacionalno če $\operatorname{rot} \vec{R} = 0$

Polje $\vec{R}$ je solenoidalno če $\operatorname{div} \vec{R} = 0$

$$\vec{f} = \operatorname{grad} u \Longrightarrow \int_K \vec{f}\, d\vec{r} = u(b) - u(a)$$

Gauss

D omejena odprta

- rob iz končnega števila odsekoma gladkih ploskev

$\vec{R} \in C^1(\bar{D})$

$$\oiint_{\partial D} \vec{R}\, d\vec{S} = \iiint_D \operatorname{div} \vec{R}\, dV$$

Green

D omejena odprta

- končno število odsekoma gladkih krivulj za rob

$X, Y \in C^1(\bar{D})$

$$\int_{\partial D} X\,dx + Y\,dy = \iint_D (Y_x - X_y)\,dx\,dy$$

Stokes

Σ omejena odsekoma gladka,

- rob iz končnega števila odsekoma gladkih krivulj

$\vec{R} \in C^1(\bar{\Sigma})$

$$\int_{\partial \Sigma} \vec{R}\, d\vec{r} = \iint_{\Sigma} \operatorname{rot} \vec{R}\, d\vec{S}$$

Uporabno

$$\operatorname{grad} \frac{1}{|\vec r - \vec a|} = -\frac{\vec r - \vec a}{|\vec r - \vec a|^3}$$ je solenoidalno

$$\operatorname{rot}(\vec r \times \vec a) = -2\vec a$$

$$\operatorname{grad} \frac{\vec a \vec r}{\vec b \vec r} = \frac{\vec r \times (\vec a \times \vec b)}{(\vec b \vec r)^2}$$

$$\operatorname{grad}(\operatorname{div} \vec f) = \operatorname{rot}(\operatorname{rot} \vec f) + \Delta \vec f$$

$$\operatorname{div}(\vec f \times \vec g) = \vec g(\operatorname{rot} \vec f) - \vec f(\operatorname{rot} \vec g)$$

$$\operatorname{rot}(\vec f \times \vec g) = (\operatorname{div} \vec g)\vec f - (\operatorname{div} \vec f)\vec g - (\vec f \operatorname{grad})\vec g + (\vec g \operatorname{grad})\vec f$$

$$(\vec f \operatorname{grad})\vec g = (\vec f(\operatorname{grad} g_1), \vec f(\operatorname{grad} \vec g_2), \dots)$$

$$\operatorname{div}(u\vec F) = u \operatorname{div} \vec F + \vec\nabla u \cdot \vec F$$

$$\operatorname{div}(\vec A \times \vec B) = \vec B \operatorname{rot} \vec A - \vec A \operatorname{rot} \vec B$$

$$\iint_{\partial D} u \frac{dv}{d\vec n}\, dS = \iiint_D (\operatorname{grad} u \cdot \operatorname{grad} v + u \Delta v)\, dV$$

$$\iint_{\partial D} \left(v \frac{du}{\partial \vec n} - u \frac{\partial v}{\partial \vec n}\right) dS = \iiint_D (u \Delta v - v \Delta u)\, dV$$

$D \subseteq \mathbb{R}^3$ je zvezdasto $\Rightarrow$

- $\vec{R}$ je potencialna $\Leftrightarrow$ $\operatorname{rot}\vec{R} = \vec{0}$

  (velja tudi za $\mathbb{R}^2$)

- $\vec{R}$ ima vektorski potencial ($\exists\, \vec{G} \in C^2(\overline{D})$. $\vec{R} = \operatorname{rot}\vec{G}$) $\Leftrightarrow$ $\operatorname{div}\vec{R} = 0$

- $\vec{G}$ je vektorski potencial $\Rightarrow$ $\vec{G}_0 + \operatorname{grad} u$ je tudi vektorski potencial

- $\operatorname{div}\vec{G} = \operatorname{div}(\operatorname{grad} u) \Rightarrow \vec{G} = \operatorname{rot}\vec{F} + \operatorname{grad} u$