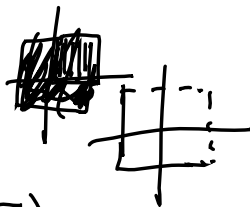


1.1.

$$A = [-1, 1] \times [-1, 1]$$

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}/A$$

hoc



a) $g^*((-\infty, 0] \times (-\infty, 0])$

ni: opty
ni: opty

$$g^*((-\infty, 0] \times (-\infty, 0])$$

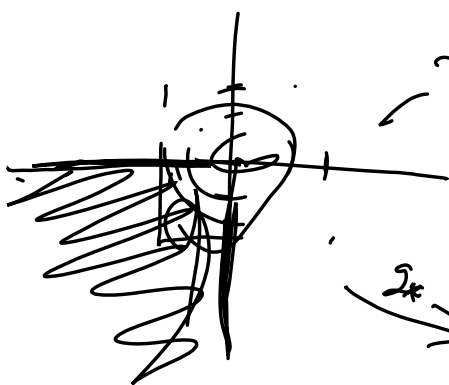
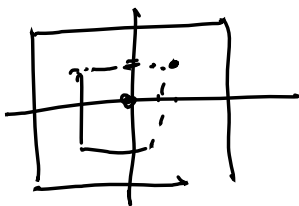
b) $g^*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

ni: opty
ni: opty

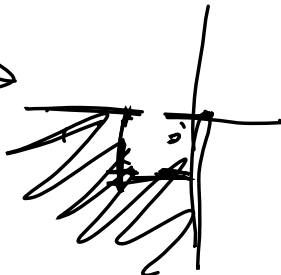


c) $g^*([-2, 2] \times [-2, 2])$

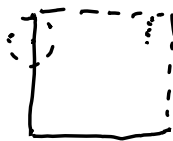
opty



g^*

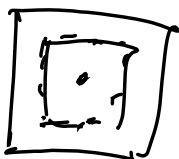


g^*

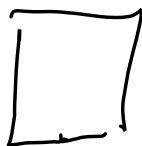


5)

$$g^*([2, 2] \times [2, 2])$$



\rightarrow

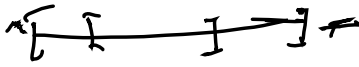


opty

ni: opty

d) ni: opty

14

$$a) [-2, 2] / [-1, 1] \cong [-1, 1]$$


$$b) [-1, 1] / \mathbb{Z} \cdot 1, 0, 1, 2$$

$$[-1, 1]$$

$$\cong \text{circle}$$

$$c) \mathbb{R} / \mathbb{Z}$$

8

$$\mathbb{R}^n /_{K(0,1)} \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{array}{ccc} \mathbb{R}^n & \xrightarrow{f} & \mathbb{R}^n \\ \downarrow & & \\ \mathbb{R}^n /_{K(0,1)} & & \end{array}$$

$$f(\vec{a}) := \begin{cases} 0; & \text{ce } |\vec{a}| \leq 1 \\ \vec{a} - \frac{\vec{a}}{\|\vec{a}\|}; & \text{ce } \vec{a} \in \mathbb{R}^n - \overline{K(0,1)} \end{cases}$$

$$d) \quad \mathbb{R}^n / \sim \quad x \sim y \Leftrightarrow \|x\| = \|y\|$$

$$\mathbb{R}^n / \sim \xrightarrow{\sim} [0, \infty)$$

$$f: \mathbb{R}^n \longrightarrow [0, \infty)$$

$$f(\vec{x}) = \|\vec{x}\| \quad \text{je sur}$$

$$a \in [0, \infty) \quad (a, 0, 0, \dots) \mapsto a$$

$$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$$

je zvezna

iščemo preslikavo s , da

$$\text{velja} \quad f \circ s = \text{id}_{[0, \infty)}$$

$$s: a \longmapsto (a, 0, \dots)$$

Dokazimo da je $r \circ s = \text{id}_Y$

$\Rightarrow r$ kvocientna, surjekcija

$$S \subseteq Y \text{ takoda } r^*(S) \text{ odpr. v } X$$

$$S = \underline{\dots\dots\dots} s^*(r^*(S)) =$$

$$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$$

$$\text{id}_Y^*(S) = S$$



$\Rightarrow S$ je odprta

f)

$$S^n \times [-1, 1] / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \rightarrow S^{n+1}$$

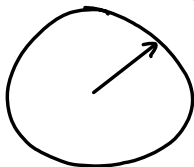
$$(x, t) \mapsto (xb, t) = (\sqrt{1-t^2}x, t)$$

\uparrow
anotado valla

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1-\|t\|^2} = \sqrt{1-t^2}$$



$$G/H = \{g \cdot h ; g \in G\} =$$

$$= \{ \{g \cdot h ; h \in H\} ; g \in G \}$$

G top. grupe

$$a \in G$$

$$L_a: G \longrightarrow G$$
$$x \longmapsto a \cdot x \quad \text{leva transkacija}$$

$$a, b \in G$$

$$h: G \longrightarrow G$$

$$h(a) = b \quad h = ?$$

$$L_{a^{-1}}: x \longmapsto ba^{-1}x$$

možnosti

$$ba^{-1}x$$
$$xa^{-1}b$$
$$bxa^{-1}$$
$$a^{-1}xb$$

topološke grupe zahtevajo

povsod isto, ker lahko vzelo

točko prestikano v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ desica $\Leftrightarrow b a^{-1} A$ desica $b \in G$

$$\exists U^{*dr} \subseteq A, a \in U$$

$$b \in b a^{-1} A$$

$$a \in U \Rightarrow \underbrace{b a^{-1} a}_{b} \in b a^{-1} U \subseteq b a^{-1} A$$

ker je $L_{b a^{-1}}$ homeomorfizem je

$$b a^{-1} U \stackrel{\subseteq A}{\text{odprta v } G}$$

\Leftarrow potem velja tudi obratno

b) $H \leq G$ H dedice 1 $\Rightarrow H$ odg: n zap v G

$$a \in aH \subseteq H$$

$\Rightarrow H$ je dedica
vsake svoje tocke

$G-H$ je adf.

$$aH \cap H = \emptyset \Rightarrow aH = H$$

$$a \in G-H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$$

vsek element ima dedico ki ne
seka $H \Rightarrow H$ je zaprta

c) C komponente $1 \in C$

$\Rightarrow C$ zaprt edinka v G

$$C \subseteq G$$

$L_a: x \mapsto ax$ je homomorfizem za $\forall a \in C$

$$\forall a \in C. L_a \subseteq C$$

L_a ohranja povezanost

$$\text{Pravi tako } L_a \cdot 1 = a \Rightarrow$$

$$L_a \cdot C \cap C \Rightarrow L_a C \subseteq C$$

invertiranje: invertiranje je tudi x homeo $i: x \mapsto x^{-1}$
 \Rightarrow po istih argumentih ~~na~~

Ali je edinka?

$$\forall a \in G. aC = Ca \Leftrightarrow aCa^{-1} \subseteq C$$

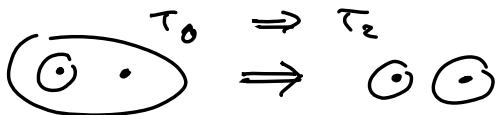
$x \mapsto axa^{-1}$ je homeomorfizem

\Rightarrow je povezano in $a \text{ id } a^{-1} = \text{id} \in C$

$$\Rightarrow aCa^{-1} \subseteq C$$

\rightarrow ker je kompozitna tvel transkacij
(leva in desna)

d) za G je



memo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U, b \notin U \quad \text{BŠZS}$

$$U^{-1} = \{a^{-1} : a \in U\}$$

$$aU^{-1}b \ni b \quad a \mapsto b$$

Predmet da $a \in aU^{-1}b$

$$\exists c \in U, a = ac^{-1}b$$

$$\Rightarrow b = c \Rightarrow b \in U \quad \times$$

$$\tau_1 \Rightarrow \tau_2$$

$\exists U, V \subseteq G, a \in U, b \in V, a \notin V, b \notin U$

$\Delta \subseteq G \times G$ je $\Delta \subseteq G \times G$

$$\Delta_0 = f^*(\{1\})$$

$$f: (k, x) \mapsto xy^{-1}$$

za neka pravi $\Rightarrow k$ je pravi

$$\textcircled{2} \quad T_{cc} \in T_1 \text{ in } n: T_2$$

$$\Rightarrow (\mathbb{R}, +) \text{ nichtgruppe zu } T_{cc}$$

$$(\ker 1, d)$$

$$(2.3) \quad \mathbb{Z} \times \mathbb{Z} \hookrightarrow \mathbb{R}^2$$

$$A \quad (m, n)(x, y) := (m+x, n+y)$$

$$\Phi: g \mapsto (a \mapsto g \cdot a)$$

bijekcija (izomorfizam)

$$\Phi: G \rightarrow \text{Bij}(A) \leftarrow \text{grupa}$$

za kompozicije

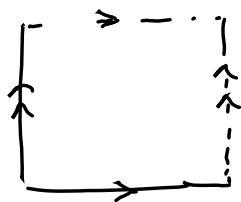
$$\text{Ubi slučaju: } \Phi: G \rightarrow (\text{Konec}(A), \circ)$$

$$\mathcal{L}(AA)$$



$$\text{mamo } [0, 1) \cong \mathbb{R}^2 / \Phi$$

Na nivouju množice
za nivouju topologije? NE



Dobimo torus

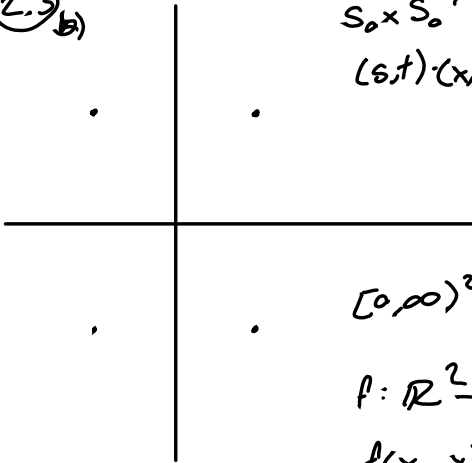
$$\bigcirc \cong S^1 \times S^1$$

$$f: \mathbb{R}^2 \rightarrow S^1 \times S^1$$

$$(x, y) \mapsto (e^{2\pi i x}, e^{2\pi i y})$$

Tuk nismo pokazali već :

(2.3) b)



$$S_0 \times S_0 \hookrightarrow \mathbb{R}^2$$
$$(s, t) \cdot (x, y) = (sx, ty)$$

$$[0, \infty)^2$$

$$f: \mathbb{R}^2 \rightarrow [0, \infty)^2$$

$$f(x, y) = (|x|, |y|)$$

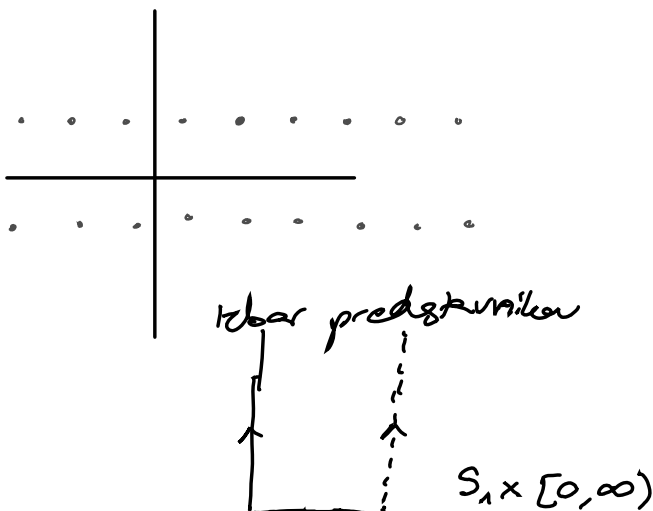
$$S: [0, \infty)^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (x, y)$$

f, S surjektiv $\Rightarrow f$ Isomorphism

$$c) \mathbb{Z} \times S^1 \hookrightarrow \mathbb{R}^2$$

$$(m, t) \cdot (x, y) = (m+x, ty)$$



$$f: \mathbb{R}^2 \rightarrow S^1 \times [0, \infty)$$

$$(x, y) \mapsto (e^{i2\pi x}, |y|)$$

zema \checkmark
surjektiven \checkmark

Po standardnem postopku radi identifikacije med delovnicami

$$((n - \frac{1}{n}), 0)_n$$

slike zaporedja zap v \mathbb{R}^2
f-slike pa ni zap, v $S^1 \times [0, \infty)$

(1,0) je v zaprtju npr v f-sliki

Produkt dveh aditivnih preslik je aditiv

$$h: \mathbb{R} \longrightarrow [0, \infty)$$

$$x \mapsto |x|$$

Dovolj preveriti ne bomo

$$0 \notin (a, b): h(a, b) = [\min\{|a|, |b|\}, \max\{|a|, |b|\}]$$

$$0 \in (a, b): h(a, b) = [0, \max\{|a|, |b|\}]$$

$$g: \mathbb{R} \rightarrow S^1 \subseteq \mathbb{C}$$

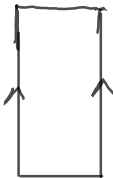
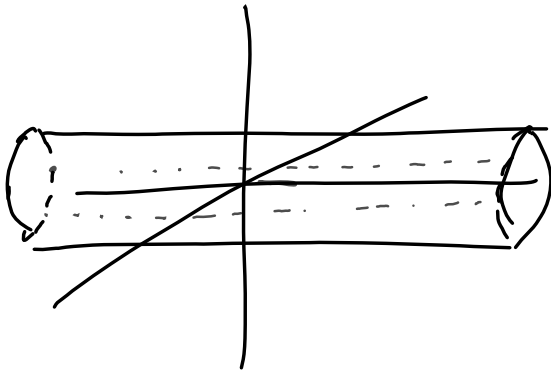
$$x \mapsto e^{i2\pi x}$$

Prva 2 a₁, ..., a_n imajo dolžino < 1 \Rightarrow slika aditivna

d)

$$\mathbb{Z} \times S^1 \hookrightarrow \mathbb{R} \times S^1 \subset \mathbb{R}^3$$

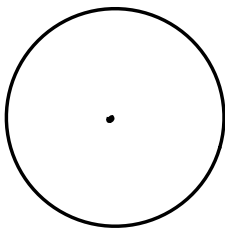
$$(m, t), (x, y, z) := (m+x, y, tz)$$



$$\mathbb{R} \times S^1 \longrightarrow S^1 \times [1, 1]$$

$$(t, y, z) \longmapsto (e^{i\eta t}, y)$$

\mathbb{R}^n

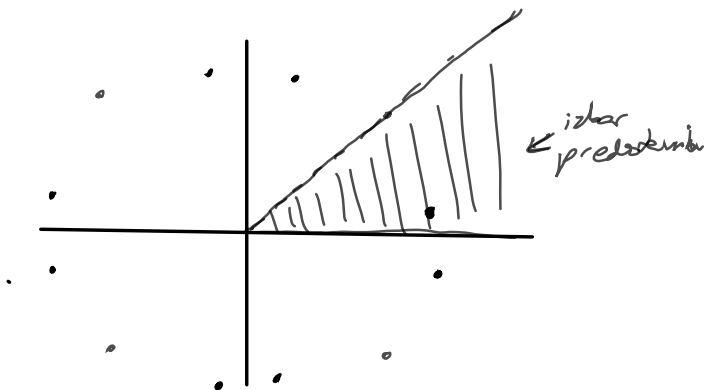


$$\vec{x} \sim \vec{y} \Leftrightarrow \|\vec{x}\| = \|\vec{y}\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???

f(A)



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cong -1 \in \mathbb{C} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong i$$

Topologija ker matrike

$$Y = \{(x, y) \in \mathbb{R}^2; y < x; x > 0\}$$

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$(x, y) \longmapsto (\max(x, y), \min(x, y))$$

A retrakcija \Rightarrow kvadranta v
 ožjem or maku

2(4)

$$2x = g + x$$

\mathbb{R}/\mathbb{Q} ni matric vektorien, v. n. n. n.

$U = \mathcal{L} \subseteq \mathbb{R}/\mathbb{Q}$ ← per delikate grupe

Ubit v. n.

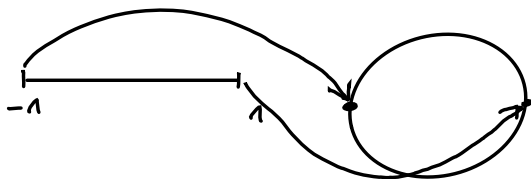
3.1

8.4

a)

$$X = [-1, 1] \quad A: \{-1, 1\}$$

$$Y = S^1 \quad f(x) := (x, 0)$$



$$Z = [-1, 1] \times \{0\} \cup S^1$$

$$g: X + Y \longrightarrow Z$$

$$in_1(x) \longmapsto (x, 0)$$

$$in_2(y) \longmapsto y$$

ekvivalenčni razredi: $(in_1(1, 0); in_2(1, 0))$ in $(in_1(-1, 0); in_2(-1, 0))$ neenakovredni

$$g(in_1(1, 0)) = 1, 0$$

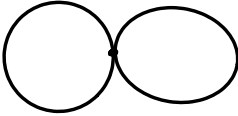
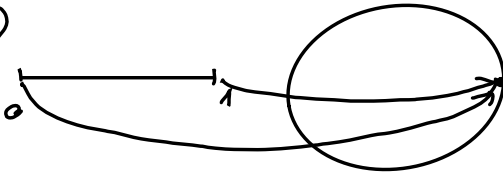
$$g(in_2(1, 0)) = 1, 0 \quad \text{padeta na 2. drug}$$

vernost pa, ker sta funkciji zvezni; in ker ~~se~~ se ujmeta na preseku

potrebno seveda tudi še da loči dvi. razrede

Sikamo iz kompaktna v Hausdorffa

b)



$$Z = Y \cup S((2,0), 1)$$

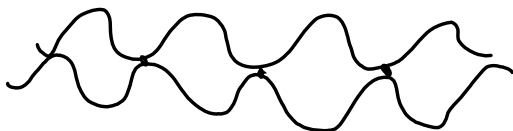
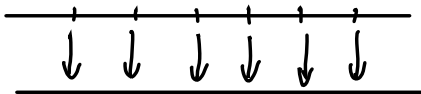
$$g: X + Y \longrightarrow Z$$

$$\text{in}_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$\text{in}_2(x, y) \longmapsto (-x + 2, y)$$

Preveriti moramo

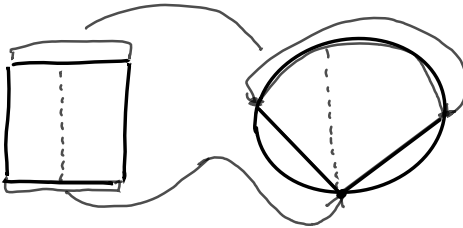
- loči dve raze
- konst na dve raze
- zvez, surj, kvocientne v ožjem smislu



$$|\sin x| \cup -|\sin x|$$

$x \mapsto$

d)



$$Z = S^1 \cup \{(x, y) \in \mathbb{R}^2; |x-1| \leq 1 \leq \sqrt{1-x^2}\}$$

$$\partial: x+y \longrightarrow Z$$

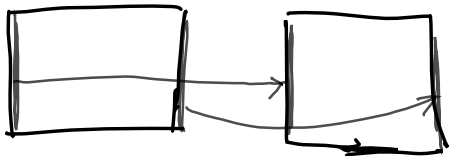
$$\text{in}_2(z) \longmapsto z$$

$$\text{in}_1(x, y) \longmapsto (0, -1) + \frac{y+1}{2} (x, \sqrt{1-x^2}+1)$$

kompatte u hausdorff

c)

$X:$



$$Z = S^1 \times [-1, 1]$$

$$g: X + Y \longrightarrow Z$$

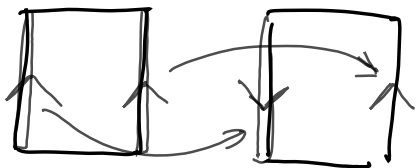
$$in_1(x, y) \longmapsto (x, \sqrt{1-x^2}, y)$$

$$in_2(x, y) \longmapsto (x, -\sqrt{1-x^2}, y)$$

hemispheren v. halbkugeln ✓

oder sie identifizieren

f)



Möbiussens trake

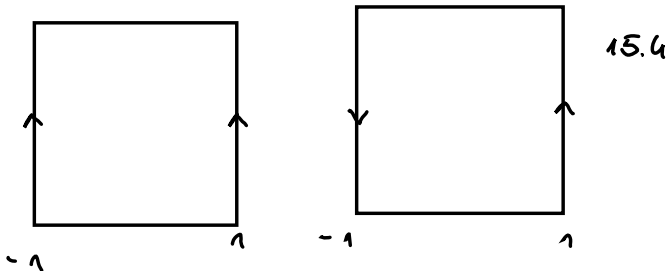


Parametrisa ena möbiussens traku

$$x(u, v) = \left(1 + \frac{v}{2} \csc \frac{u}{2} \cos u\right)$$

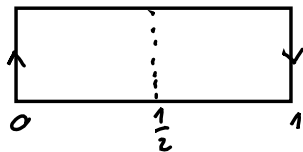
$$y(u, v) =$$

$$z(u, v) =$$



$X + Y \longrightarrow M \dots$ Möbiustrack

$$M = [0, 1] \times [0, 1] / \sim$$



$$\begin{array}{ccc}
 X + Y & \xrightarrow{\delta'} & [0, 1]^2 \\
 \downarrow 2 & \searrow g & \downarrow 2m \\
 X \cup_f Y & \dashrightarrow & M
 \end{array}$$

$$in_1(u, v) \mapsto \left(\frac{u+1}{4}, \frac{v+1}{2} \right)$$

$$in_2(u, v) \mapsto \left(\frac{3-u}{4}, \frac{v+1}{2} \right)$$

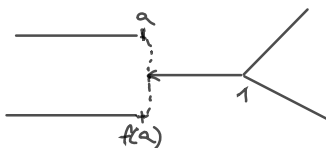
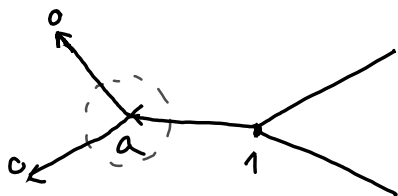
$$x \sim y \Leftrightarrow g(x) = g(y)$$

$$x \sim y \Leftrightarrow g'(x) \sim g'(y)$$

$$(a, 1] \longrightarrow (0, \infty)$$

$$f(x) = x$$

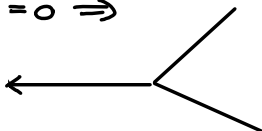
zlepak $(0, \infty) \cup_p (0, \infty)$ je zlepak za
katere hausdorffove



Poglejmo točko a

Vseke dedice od a in od $f(a)$ sekata
del $(0, 1]$ za $\forall a \in (0, 1)$

$$\text{za } a = 0 \Rightarrow$$



Vločimo v evklidski prostor

$$(0, \infty) \cup_p (0, \infty) \longrightarrow \mathbb{R}^2$$

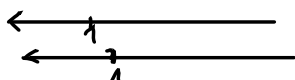
$$x \in (0, 1] \longmapsto (0, x)$$

$$x \in (1, \infty) \longmapsto (x, x-1)$$

$$x \in \bigcup_{n \in \mathbb{Z}} (1, \infty) \longmapsto (x, -x+n)$$

je hausdorffov?

lokalna kompaktnost



kompleti: zaprti intervali v $(0, \infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh
premica

kompleti: zaprti intervali v $(0, \infty)$

3.3)

$$X/A \approx X \cup_f 1$$

$$\begin{array}{ccc}
 X & \xleftarrow{g'} & X + 1 \\
 f \downarrow & \nearrow g & \downarrow 2 \\
 X/A & \xrightarrow{\approx} & X \cup_f 1
 \end{array}
 \quad 1 \in 1$$

h:

$$\begin{aligned}
 [x] &\mapsto x & : x \notin A \\
 [a] &\mapsto 1 & : a \in A
 \end{aligned}$$

is homeomorphism

$$\begin{aligned}
 g' : X + 1 &\rightarrow X \\
 \text{in}_1 x &\mapsto x \\
 \text{in}_2 a &\mapsto 1 \quad a \in A
 \end{aligned}$$

$$g = f \circ g'$$

$$u \sim v \Leftrightarrow g'(u) \sim_f g'(v)$$

$$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$$

$$f(g'(u)) = f(g'(v)) \Leftrightarrow$$

$$[g'(u)] \sim [g'(v)]$$

Else we have: $v \in X \cup_f 1$

$$\text{in}_1 x = \{x\} \quad x \notin A$$

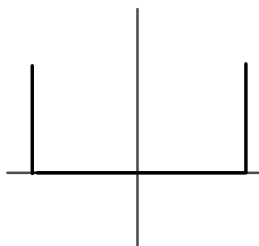
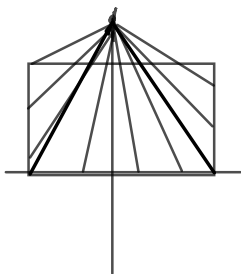
$$\text{in}_1 a = A \cup \{a\} \quad a \in A$$

$$\text{in}_2 a = A \cup \{a\}$$

$$[\text{in}_1 x] = \{[\text{in}_1 x]\}$$

$$(\text{in}_1)_*(A) \cup \{[\text{in}_2 a]\}$$

Retrakti, homotopije, ekvivalen. relacije



Projiciramo iz $(0,2)$ na rob

$$y > 2x+2 : (-1, 2 - \frac{y-2}{a})$$

$$2x+2 > y < -2x+2 : (\frac{2x}{2-y}, 0)$$

$$y > -2x+2 \quad (1, \frac{y-2}{a} + 2)$$

$$(a,b) \leadsto p_1: y = \frac{b-2}{a}x + 2$$

$$\text{pri } x = -1 \\ y = -\frac{b-2}{a} + 2$$

$$p_2: y = \frac{b-2}{a}x + 2 \quad a \neq 0$$

$$y=0 \rightarrow x = \frac{-2a}{b-2} = \frac{2a}{2-b}$$

$$x=1:$$

$$y = \frac{b-2}{a} + 2$$

Ali je to id na y ?

$$x = -1: (-1, 2 - \frac{y-2}{-1}) = (-1, 2+y-2) = (-1, y)$$

$$y=0: (\frac{2x}{2}, 0) = (x, 0)$$

$$x=1: (1, \frac{y-2}{a} + 2) = (1, y) \quad \checkmark$$

Ali homotopije id na X

$$H(x, y, t) = t(x, y) + (1-t)r(x, y)$$

zven, ker so vsi kosi zven

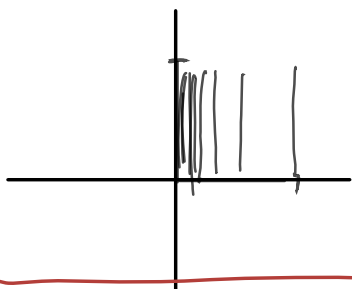
f homotopy g

$$\text{let } H: X \times [0, 1] \longrightarrow Y$$

$$(x, 0) \longmapsto f(x)$$

$$(x, 1) \longmapsto g(x)$$

where



Retrakt T_2 prostora je zaprt

Recimo da $\exists r$ retracts: $X \rightarrow Y$

$$r|_Y = id$$

deljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

morajo biti točke $(\frac{1}{n}, 0)$

ker gre vsa f iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1)$
če z ~~ne~~ ~~ne~~ ~~ne~~

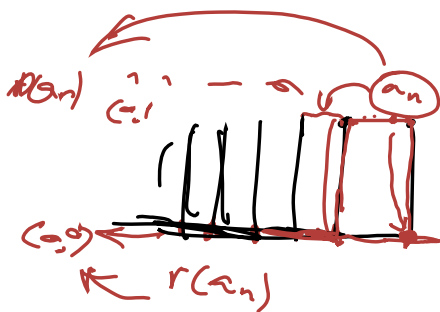
Torej

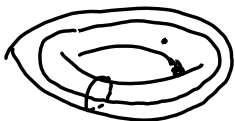
May bo opredelje a_n ; $r(a_n) = \frac{1}{n}$

$\lim a_n = (0, 1)$ $a_n \in [\frac{1}{n}, 1] \times \{1\}$

$\lim r(a_n) = (0, 0)$

$r \nmid a$: ~~ne~~ ~~ne~~ ~~ne~~





$$G \sim X$$

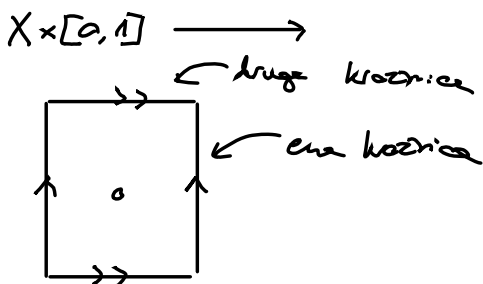
$$(x, 6) \rightarrow X$$

$$(x, 8) \mapsto x \cdot g$$

Deformacijske relikve "ima homotopiju do id"
je zvezna preslika $H: X \times [0, 1] \rightarrow X$.

$$H(x, 0) = x \quad , \quad H(a, 1) = a \quad \wedge \quad H(x, 1) \in A$$

$$\forall x \in X \quad \forall a \in A \quad \forall x \in X$$



$$([-1, 1]^2 - \{0, 0\}) \times [0, 1] \xrightarrow{h} [-1, 1]^2 - \{0, 0\}$$

$$\downarrow \begin{matrix} \text{ } \\ \text{ } \end{matrix} \begin{matrix} \text{ } \\ \text{ } \end{matrix} \downarrow$$

$$X \times [0, 1] \xrightarrow{\text{zvezna preslika}} X$$

$$h: (x, t) \mapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x} (1-t)$$

$g \circ h$ mora biti konstantna na ekv. razdeli

$$x \sim y \Rightarrow g \circ h(x) = g \circ h(y)$$

$$\Leftrightarrow [h(x)] = [h(y)]$$

$$\Leftrightarrow h(x) \sim h(y)$$

h ekvivalentne razrede obravnava na maske

\Rightarrow

$$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1, 1]^2 - \{0, 0\} \times [0, 1]$$

$$\Leftrightarrow t_1 = t_2 \wedge [x_1, y_1] = [x_2, y_2]$$

$$\Rightarrow \frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1) (1-t_1)$$

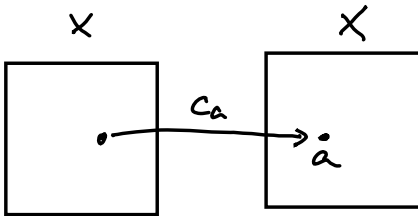
$$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|_\infty} t_2 + (x_2, y_2) (1-t_2)$$

Izrek

q kvocientna in X kompakten T_2 prostor
 $\Rightarrow q \times id_X$ je kvocientna

Boj splošno:

q kvocientna, X lokalna kompaktna
 $\Rightarrow q \times id$ kvocientna



\Rightarrow naj bo X povezan s ptn:

Naj bo γ te pot med a in b po glasu

$$\gamma(0) = a \quad \gamma(1) = b$$

$$H: X \times [0, 1] \rightarrow X$$

$$(x, t) \mapsto \gamma(t)$$

$$H = \gamma \circ pr_2 \text{ torej je zvezna}$$

\Leftarrow

Recimo, da $\forall a, b \in X$ velja $c_a \simeq c_b$
 Naj bosta a, b poljubna

$$\exists H: X \times [0, 1] \rightarrow X$$

$$H_0 = c_a$$

$$H_1 = c_b$$

$$\gamma(t) = H(a, t)$$

4.5

a) $f: S^n \rightarrow S^n$ n : sur

homotopie kadeh

$a \notin f_*(S^n)$ b je nasprotni tej

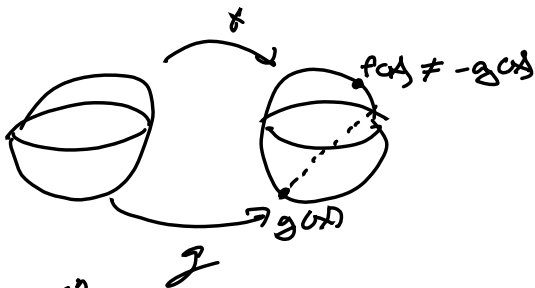
$$H: S^n \times [0, 1] \rightarrow S^n$$

$$(x, t) \mapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$$

Ali je dvčtjro $(1-t)f(x) + t \cdot x = 0$
 \Rightarrow če graf te dalice skozi 0
 ampak \hat{a} ni zlogi vrednosti, da
 niha ne gre dalje skozi 0,0

$$b) f, g: S^n \rightarrow S^n$$

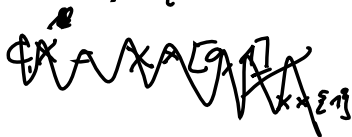
$$f(x) \neq -g(x) \quad \forall x \in S^n \rightarrow f \approx g$$



$$H: S^n \times [0, 1] \rightarrow S^n$$

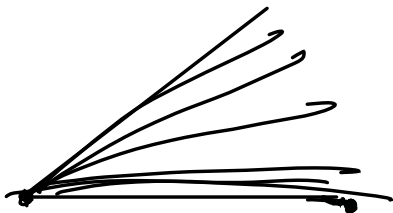
$$(x, t) \mapsto \frac{(1-t)g(x) - tf(x)}{\|(1-t)g(x) - tf(x)\|}$$

$$4.6 \quad X = \{ [0, 1] \times \{0\} \} \cup \{ x, \frac{x}{n}, x \in [0, 1] \text{ } n \in \mathbb{N} \}$$



$$H: X \times [0, 1] \longrightarrow CX$$

$$(x, y), t \longmapsto (1, t)(x, y)$$



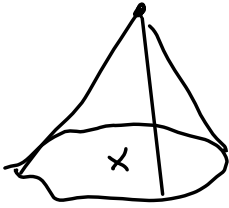
Reálna le je H^G poljuben krepka deformacija
retrakcije x na isto telo

$$\forall n \in \mathbb{N}. \exists t_n \in [0, 1] \quad G\left(1, \frac{1}{n}, t_n\right) = 0, 0$$

ker je $[0, 1]$ kompaktno lahko izberemo konvergen
podsekvenco t_n

?

$$CX = X \times [0, 1] / \sim$$



$$CX \times [0, 1] \longrightarrow CX$$

$$X \times [0, 1] \times [0, 1] \xrightarrow{H} X \times [0, 1]$$

$$\begin{array}{ccc} \downarrow \text{id} & & \downarrow \text{id} \\ CX \times [0, 1] & \dashrightarrow & CX \end{array}$$

$$H: (x, u, t) \mapsto (x, 1) + \cancel{(x, u)(1-t)} + (x, u(1-t) + t)$$

ne moreno
ceterat
nife

Preveriti maximo samo

$$(x_1, 1, t) \sim_H (x_2, 1, t)$$

|| ekvivalentni razredi u t=1

$$(x_1, 1) \quad || \quad (x_2, 1)$$

ste ekvivalentni kod t=1

$$\bar{H}([x, u, 0]) = [x, u]$$

$$\partial(H(x, u, 0)) = \partial(x, u)$$

$$\partial(H(x, u, 1)) = \dots$$

X je kontraktilen

\Leftrightarrow

\exists nema izbira poti od poljubne točke X do izbrane točke

$$\exists a \in X. \exists f: X \rightarrow \mathcal{C}(I, X), f(x)(0) = x, f(x)(1) = a$$

\Leftrightarrow

$X \neq \emptyset \wedge \exists$ nema izbira poti med točkami $\forall x$

$$\exists f \omega: X \times X \rightarrow \mathcal{C}(I, X), f(a, b)(0) = a, f(a, b)(1) = b$$

u.8 (u.7 ne uči niči)

i) \Rightarrow ii) Vemo iz u.7 (u.6)

ii) \rightarrow iii) Naj bo X rektet kontraktibilnega prostora Y

$$\exists r: Y \rightarrow X. \exists i: X \rightarrow Y. r \circ i = \text{id}$$

(ima zvezni desni inverz (ekvivalentna definicija retrakcije))

$$\exists y_0 \in Y. \exists H: Y \times I \rightarrow Y$$

$$(y, 0) \mapsto y$$

$$(y, 1) \mapsto y_0$$

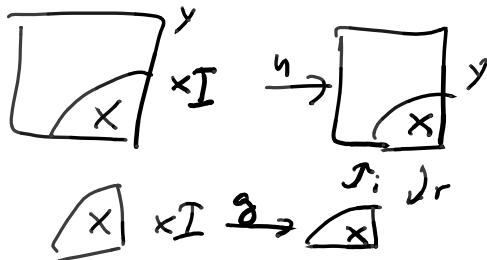
izamo

$$g: X \times I \rightarrow X$$

$$x_0 = r(y_0)$$

$$g(x, 0) = g$$

$$g(x, 1) = x_0$$



$$g(x, t) = r(h(i(x), t))$$

$$g = r \circ h \circ (i \times \text{id}_I)$$

iii) \Rightarrow i)

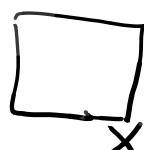
$$\square \times I \xrightarrow{H} \square \times \{a\}$$

$$H(x, 0) = x$$

$$H(x, 1) = a$$



$$\xrightarrow{r}$$



$$r \circ i = \text{id}$$

$$r([x, 0]) = x$$

$$CX = X \times I / \{x, 1\}$$

$$[x, t] \mapsto p_1(H(x, t))$$

4.9

$$I \times I / \sim_{\text{veno}} = M$$

$$\text{veno} \Leftrightarrow \sim$$

CM

$$I \times I / \sim_{\text{veno}} \times I / \sim_{\{[I, 1]\}} = \text{CM}$$

Recimo da je kontraktilen

$$\exists H: I \times I / \sim_{\text{veno}} \times I \rightarrow I \times I / \sim_{\text{veno}}$$

$$H(x, 0) = x$$

$$H(x, 1) = 1$$

Uzemimo podprostor $A = \{x \in I \times I / \sim_{\text{veno}} \mid x \text{ je na } I \times \{0\} \text{ ali } I \times \{1\}\}$

ki je krožnica topološka

tačje je tudi A kontraktilen

u. 10

Megecazen konveksen prostor je
kontraktilen

Splazni: zvezdast prostor je kontraktilen



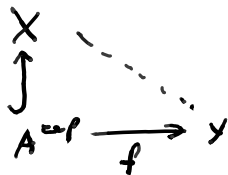
\mathcal{N} . Razred normalnih topoloških prostov
 $\mathcal{Y} \in \mathcal{AE}(\mathcal{N})$ je absolutni destenar za razred
 normalnih prostov

kadar za $\forall X \in \mathcal{N}$ in \forall topološko podmnožico

$A \subseteq X$ velja, da lahko vsakega preslikavo

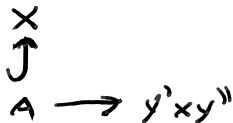
$A \rightarrow Y$ razširimo do zvezne preslikave

$X \rightarrow Y$



Torej: $\mathcal{I} \in \mathcal{AE}(\mathcal{N})$

• Produkt \mathcal{AE} je \mathcal{AE}



razširimo v vsaki komponenti

• Retrakt \mathcal{AE} je \mathcal{AE}

• $X \in \mathcal{AE}(\mathcal{N}) \Rightarrow X \neq \emptyset$ in X povezan s pdm:

• $\mathcal{AE}(\mathcal{N}) \cap \mathcal{N} \subseteq \mathcal{AR}(\mathcal{N})$

↖ absolutni retracts

Ne tabl: 4. ~~1824~~ 11

4. 12

$$A, B \subseteq \mathbb{R}^n$$



$$\mathbb{R}^n \in \mathcal{A}\mathcal{E}$$

Dokazujeme $A \cup B$ je rektifikabilno \mathbb{R}^n

Fodsch

4.19

$$v \in \mathbb{R}^n \setminus \{0\}$$

$$Av = \lambda v$$

$$\text{vsz } \|v\| = 1$$

$$\|Av\| = \|\lambda v\| = |\lambda| \cdot \|v\| = |\lambda|$$

$$Av = \lambda v \stackrel{?}{=} \|Av\| v$$

$$v = \frac{Av}{\|Av\|}$$

iščemo neko točko preslike
 $S^{n-1} \rightarrow S^{n-1}$

$v \mapsto \frac{Av}{\|Av\|}$ ne nujno dobro definirana
 preslika, ker
 $\ker A$ ni nujno $\{0\}$

$$X = \{ (x_1, \dots, x_n) \in S^{n-1} \mid x_1, \dots, x_n \geq 0 \} \approx B^{n-1}$$

če $v \in X$ so vsi $x_i \geq 0$ in niso vsi $= 0$

$\Rightarrow Av$ ima pozitivne komponente,
 vsakej $\neq 0$

$B^{n-1} \rightarrow B^{n-1}$ na neko točko

Mnogoterost

a) $\{\frac{1}{n}, n \in \mathbb{N}\}$ stanoš + distrehoš
= je mnogoterost

b) $\{\frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$

$\forall 0$ ni mnogoterost

če bi jo imela bi našla točki iz $\mathbb{R}^0 = \{0\}$
ali \mathbb{R}_+^0 kar ne obstaja

Torej točka 0 ni možna točka
vseh delov ~~to~~ od a b) fenoš

moč biti $\{0\}$ - točka, kar pa ni:

Stane vsota n-mnt je n-mnt