

7.1.

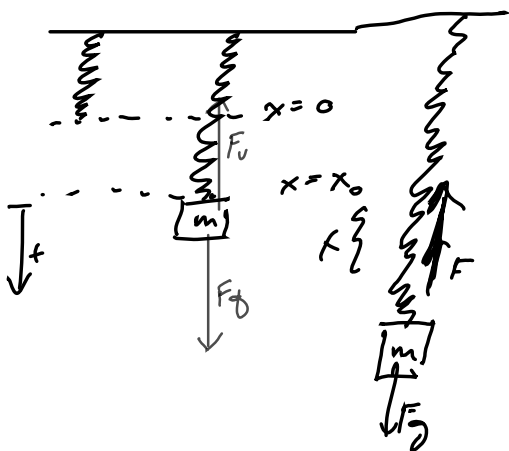
$$l = 1 \text{ m}$$

$$k = 500 \text{ N/m}$$

$$m = 8 \text{ kg}$$

$$a_0 = 0,1 \text{ m}$$

$$x_0 = ?$$



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

$$F_g + F_{v2} = m \cdot \ddot{a} = \ddot{x}(t)$$

$$+ mg - k(x_0 + x) = m \ddot{a}(t)$$

$$\underbrace{mg - kx_0}_{=0} - kx = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\ddot{x} = \frac{-kx}{m} \quad \frac{-k}{m} = \omega_0^2$$

$$x(t) = \sin(\omega t) \cdot A + B \cos(\omega t)$$

anfangsbedingungen:

$$x(0) = a_0$$

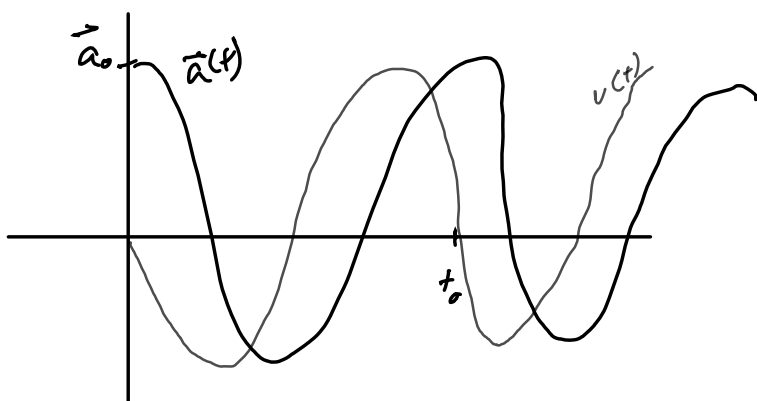
$$a_0 = x(0) = B$$

$$v(t) = \dot{x}(t)$$

$$v(0) = \dot{x}(0) = A\omega \cos(\omega t) - B\omega \sin(\omega t)$$

$$0 = A\omega \Rightarrow A = 0$$

$$x(t) = a_0 \cos(\omega t)$$



$$\cos(\omega t_0) = 1 \quad \omega t_0 = 2\pi \quad t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \sqrt{m}}{\sqrt{k}}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \dots \quad \text{natürliche Frequenz}$$

$$\omega_0 = 7,9 \text{ /s}$$

Energije nihanja

$$W = \frac{mv^2}{2} + mg(-x) + \frac{k(x+x_0)^2}{2} - \frac{k}{2}x_0^2$$

$$= \frac{mv^2}{2} + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 = 2,5 \text{ J}$$

$$l = 3\text{m}$$

$$\varphi_0 = 3,5^\circ$$

$$t = 15\text{s}$$

$$N = ?$$

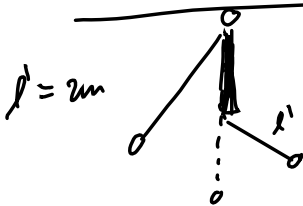
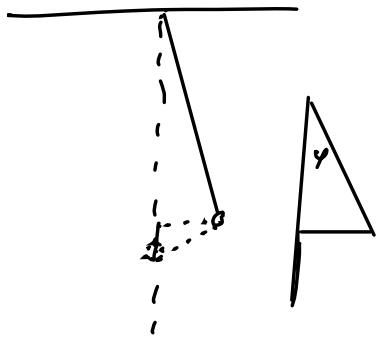
$$V_{\max} = ?$$

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$N = \frac{t}{T_0} = 4,32$$

$$v = \sqrt{2gh}$$

$$h = l - \cos\varphi \cdot l$$



$$T_0 = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l'}{g}}$$

$$= \frac{\pi}{\sqrt{g}} (\sqrt{l} + \sqrt{l'}) = 3,15\text{s}$$

$$N = \frac{t}{T_0} =$$

$$\varphi(t) = \varphi_0 \cos(\omega t)$$

\nwarrow ω - angular frequency

$$V_{\max} = l \cdot \omega$$

\nwarrow ω - angular frequency

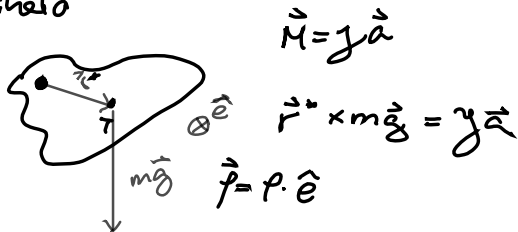
$$\omega(t) = \dot{\varphi}(t) = -\varphi_0 \omega \sin(\omega t)$$

ω - angular frequency

$$V_{\max} = l \cdot \varphi_0 \omega$$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

Tedno nihalo



$$r^* m g \sin \varphi (-\hat{e}) = J \cdot \ddot{\varphi} \hat{e}$$

$$\ddot{\varphi} = -\frac{m g r^*}{J} \sin \varphi$$

$$\varphi \ll 1$$

$$\ddot{\varphi} \approx -\underbrace{\frac{m g r^*}{J}}_{\omega^2} \varphi$$

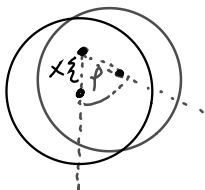
$$T_0 = 2\pi \sqrt{\frac{J}{m g r^*}}$$

Matematično nihalo:

$$J = m l^2$$

$$r^* = l \quad T_0 = 2\pi \sqrt{\frac{l}{g}}$$

7.7.)



$$R = 0,5 \text{ m}$$

t_0 min

debi osnove translacija

$$y = \frac{1}{2} m R^2 + m x^2$$

$$t_0(x) = ?$$

$$t_0 = 2\pi \sqrt{\frac{y}{mgx}} =$$

$$t_a = 2\pi \sqrt{\frac{\frac{1}{2} R^2 + x^2}{gx}}$$

da bismo našli:

$$\frac{2x(gx) - g(\frac{1}{2}R^2 + x^2)}{gx^2} = 0$$

$$2gx^2 - gx^2 - \frac{1}{2}R^2g$$

$$gx^2 = \frac{1}{2}R^2g$$

$$x = \sqrt{\frac{1}{2}} \frac{R}{\sqrt{2}}$$

(7.8)

$$l = 1 \text{ m}$$

$$m = 0,5 \text{ kg}$$

$$k = 5 \text{ N/m}$$

$$V = ?$$



$$\phi \ll 1$$

$$\Sigma \vec{M} = J \cdot \ddot{\alpha}$$

$$\underbrace{\ddot{\alpha}(t)}_{\ddot{\phi}(t)} = - \underbrace{F_g}_{\frac{mg}{2}} \cdot \underbrace{\frac{l}{2}}_{\phi(t)} \sin \phi - \underbrace{F_v}_{\frac{kx}{l}} \cdot \underbrace{l}_{\phi(t)} \cos \phi$$

reaction

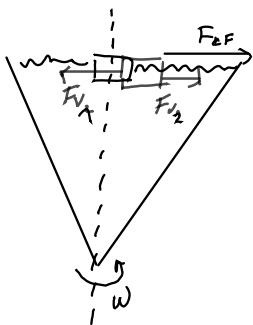
$$\ddot{\phi}(t) = - \frac{m g \cdot l}{2 J} \phi(t) - \frac{k l^2}{J} \phi(t) =$$

$$= - \frac{3g}{2l} \phi(t) - \frac{3k \phi(t)}{m} = - \left(\frac{3g}{2l} + \frac{3k}{m} \right) \phi(t)$$

ω_0

$$\omega_0 = \sqrt{\frac{3g}{2l} + \frac{3k}{m}} = 6,7 / \text{s}$$

$$2\pi V = \omega_0 \quad V = \frac{\omega_0}{2\pi} = 1,06 \text{ Hz}$$



$$m = 20g$$

$$k_1 = 1 \text{ N/m}$$

$$k_2 = 3 \text{ N/m}$$

$$\omega = 10/s$$

$$\nu = ?$$

$$\sum F = ma$$

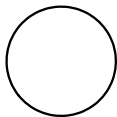
$$ma = -k_1x - k_2x + m\ddot{a}_r$$

$$\ddot{x}(t) = -\left(\frac{k_1 + k_2 - m\omega^2}{m}\right)x$$

$$\omega_0 = \sqrt{\frac{k_1 + k_2 - m\omega^2}{m}} = 10/s$$

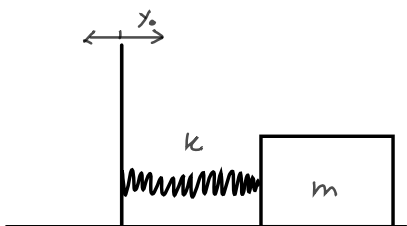
$$\nu = 1, \text{ Hz}$$

Faucaultova nihele





7.25



$$y_0 = 5 \text{ cm}$$

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

$$k = \frac{N}{m}$$

$$\beta = 0.2 / s$$

$$\nu = 12 \text{ Hz}$$

$$x_0 = ?$$

$$\delta = ?$$

$$x_{0 \max} = ?$$

$$v_{\max} = ?$$

$$m \ddot{x} = -k(x-y) - 2\beta m \dot{x}$$

$$\ddot{x} + 2\beta \dot{x} + \frac{k}{m} x = \frac{k}{m} y \rightarrow y_0 \cos(\omega t)$$

$$\omega^2 = \frac{k}{m}$$

Nehomogena diferencialna enačba

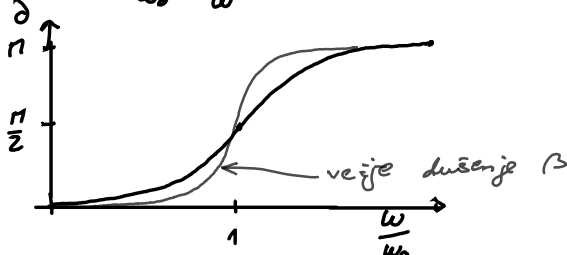
$$\text{rešitev: } x(t) = x_0 \cdot \cos(\omega t - \delta) + x_0' e^{-\beta t} \cos(\omega t - \delta')$$

$$\sqrt{\omega^2 - \beta^2}$$

se zadržuje, ne
pa zanima, ko se
ustati

Rezultati:

$$\tan \delta = \frac{2\beta \omega}{\omega_0^2 - \omega^2}$$



$$\frac{x_0}{y_0} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta \omega)^2}}$$



vrh resonance
krivulje

$$\omega_{\max} = \sqrt{\omega_0^2 - 2\beta^2}$$

Rezultati:

$$\omega_0 = \sqrt{\frac{k}{m}} = 10 / s$$

$$\delta = 179.7^\circ$$

$$\Rightarrow \nu = 1.6 \text{ Hz}$$

$$x_0 = 0.89 \text{ mm}$$

$$v_{\max} \approx 1.6 \text{ Hz}$$

$$x_{0 \max} = 1.25 \text{ m}$$

mec pri vsiljenem nihanju

$$P = \frac{dA}{dt} = \frac{F \dot{x}}{dt} = Fv = 2\beta m v \dot{x} =$$

$$v = \dot{x} = x_0 (-\sin(\omega t - \delta)) \cdot \omega$$

$$\bar{P} = 2\beta m x_0^2 \omega^2 \overline{\sin^2(\omega t - \delta)} = \beta m x_0^2 \omega^2$$

||
1
2

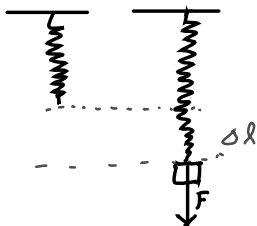
$$\bar{P} \propto \omega^2 \frac{1}{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}$$

$$\frac{dP}{d\omega} = \frac{2\omega[(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2] - [4\omega(\omega_0^2 - \omega^2) + 8\beta^2\omega]}{((\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2)^2} = 0$$

splošno nihanje

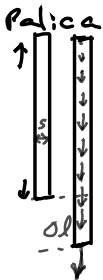
f 14.3

Elastomehanika: Hookov zakon



$$F = k \cdot \Delta l$$

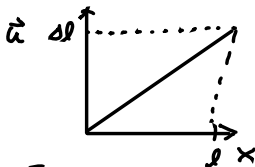
Vpeljava prožnostnega modula



→ majhne raztezke

$$\frac{F}{S} = \frac{\Delta l}{l} \cdot E \quad [E] = \frac{N}{m^2} = Pa$$

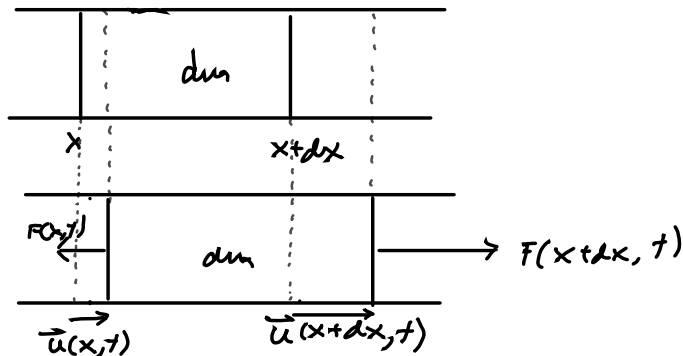
← vektorsko polje



$$\frac{du}{dx} = \frac{dl}{l}$$

$$\frac{F}{S} = E \frac{du}{dx}$$

Valovanje v elastični palici



$$F(x+dx, t) - F(x, t) = dm \cdot a(x + \frac{dx}{2}, t)$$

$$\mathcal{E} \frac{du}{dx} (x+dx, t) - \mathcal{E} \frac{du}{dx} (x, t) = dm \cdot \ddot{u}(x + \frac{dx}{2}, t)$$

$$dx \rightarrow 0$$

$$\mathcal{E} \frac{d^2u}{dx^2} (x + \frac{dx}{2}, t) \cdot dx = \rho \cancel{dx} \ddot{u}(x + \frac{dx}{2}, t)$$

$$\frac{\partial^2 u}{\partial x^2} = \left(\frac{\rho}{\mathcal{E}} \right) \frac{\partial^2 u}{\partial t^2}$$

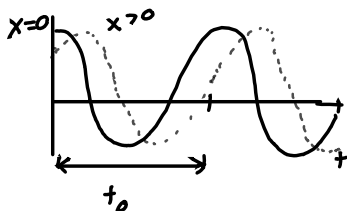
\downarrow
 $\frac{1}{c^2}$

$$u(x, t) = \phi(x+ct) + \psi(x-ct)$$

$$c = \sqrt{\frac{\mathcal{E}}{\rho}}$$

Periodična motnja

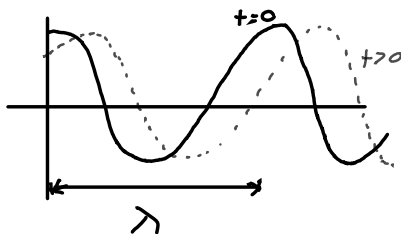
$$u(x,t) = u_0 \cdot \cos(\omega t - kx)$$



Na danem mestu

$$\omega T_0 = 2\pi$$

$$\omega = \frac{2\pi}{T_0} = 2\pi \nu$$



ob danem času

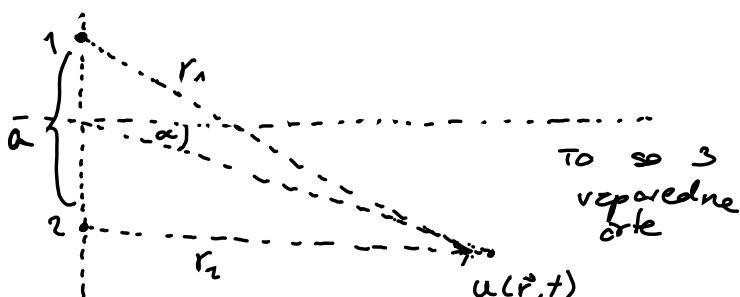
$$c = \frac{\delta x}{\delta t} = \frac{\lambda}{T_0} \Rightarrow c = \lambda \nu$$

$$a = \frac{3}{10} \lambda$$

1. izvor zakasnjjen za $\frac{1}{4}$ nihaja $\Rightarrow \delta = \frac{\pi}{2}$

smeri opazitev = ?

smo deleč od anten



$$r_1, r_2 \gg a$$

$$u = \underbrace{u_0 \sin(\omega t - kr_1 - \delta)}_{\text{prvi izvor}} + \underbrace{u_0 \sin(\omega t - kr_2)}_{\text{2. izvor}}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$u = u_0 \sin \left(\frac{2\omega t - k(r_1 + r_2) - \delta}{2} \right) \cos \left(\frac{k(r_2 - r_1) - \delta}{2} \right)$$

amplitude

točke za katere je $r_1 - r_2$ konstanta

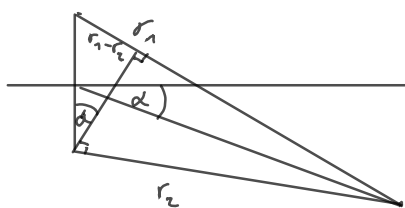
$$\frac{k}{2}(r_2 - r_1) - \frac{\delta}{2} = n\pi \quad n \in \mathbb{Z}$$

Torej
priče hiperbole

$j \propto u^2$ mora biti maksimalen
(ker iščemo optike)
Sorazmerno

odločimo se za neke

n in potem gledamo da
je tudi na levi strani
konstantno



$$r_1 - r_2 = a \sin \alpha$$

$$-\frac{k}{2}(a \sin \alpha) - \frac{\delta}{2} = n\pi$$

$$\sin \alpha = \frac{n\pi + \frac{\delta}{2}}{-\frac{ak}{2}} = \frac{2\pi n + \delta}{-ak}$$

k valovni vektor

$$k = \frac{2\pi}{\lambda} \quad a = \frac{3}{10} \lambda$$

$$\sin \alpha = \frac{\lambda(2\pi n + \delta)}{-2\pi a} = \frac{5(2\pi n + \delta)}{-3\pi}$$

$$\sin \alpha = -\frac{10}{3} \left(n + \frac{1}{6} \right)$$

rečemo $n=0$

$$\sin \alpha_0 = -\frac{10}{3} \cdot \frac{1}{6} = -\frac{5}{6}$$

$$\Rightarrow \alpha_0 = -56,4$$

$$n=1 \Rightarrow \sin \alpha_1 = -\frac{10}{3} \cdot \frac{5}{6} = -\frac{25}{6}$$

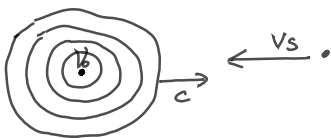
$$\alpha_1 = \text{err}$$

$$n=-1 \Rightarrow \sin \alpha_{-1} = -\frac{10}{3} \cdot \left(-\frac{3}{6} \right) = \frac{5}{2}$$

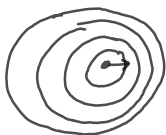
$$\nu = 1485 \text{ kHz}$$

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \frac{\text{m}}{\text{s}}}{1485 \cdot 10^3 / \text{s}} = 202 \text{ m}$$

Dopplerjev pojav



$$N = \frac{ct}{\lambda} + \frac{v_s t}{\lambda} + \frac{v t}{\lambda}$$



$$\lambda = \frac{c}{v_0} + \frac{v}{v_0} + \frac{v_v}{v_0}$$

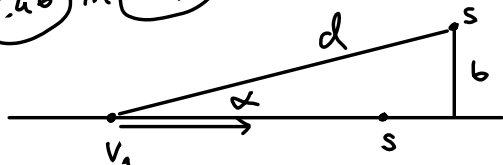
$$V_s = \frac{N}{t} = \frac{c + v + v_s}{c + v + v_v} \cdot v_0$$

$$= \frac{c + (v + v_s)}{v - (v_0 - v)}$$

h.rost
spejalka
glede na veter

h.rost
oddajnika
glede na veter

(7.46) in (7.47)

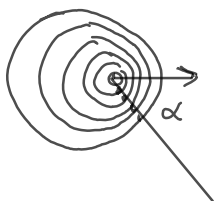


$$v_1 = 100 \frac{\text{km}}{\text{h}} = 27,8 \frac{\text{m}}{\text{s}}$$

$$V_0 = 1000 \text{ Hz}$$

a) $V_s = ?$ $V_s = \frac{c + V_s}{c - V_1} V_0$ $V_s = 0$ $c = 340 \frac{\text{m}}{\text{s}}$
 $V_1 = 27,8 \frac{\text{m}}{\text{s}}$
 $V_s = 189 \text{ Hz}$

b) $d = 100 \text{ m}$
 $b = 500 \text{ m}$



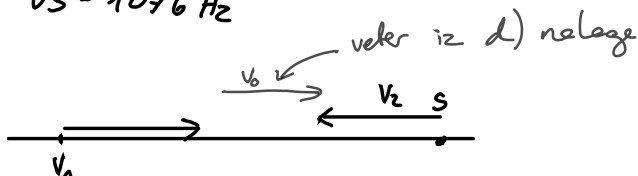
$$V_s = \frac{c}{c - V_1 \cos \alpha} V_0$$

$$\sin \alpha = \frac{b}{d} = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$

$$\cos \alpha = \frac{\sqrt{3}}{2}$$

$$V_s = 1076 \text{ Hz}$$

c)



$$v_2 = 80 \frac{\text{km}}{\text{h}} = 22 \frac{\text{m}}{\text{s}}$$

$$V_s = \frac{c + v_2}{c - v_1} V_0 = 1160 \text{ Hz}$$

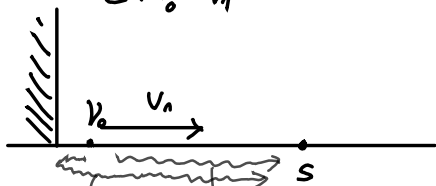
d)

$$v_0 = 30 \frac{\text{km}}{\text{h}} = 8,3 \frac{\text{m}}{\text{s}}$$

↑
veter

$$V_s = \frac{c + v_0 + v_2}{c + v_0 - v_1} = 1156 \text{ Hz}$$

e)



$$V_{s_1} = \frac{c}{c + v_1} V_0$$

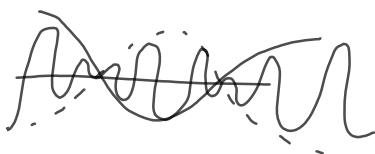
$$V_{s_2} = \frac{c}{c - v_1} V_0$$

$$V_s = V_{s_1} + V_{s_2} = c V_0 \left(\frac{1}{c + v_1} + \frac{1}{c - v_1} \right) =$$

$$= c V_0 \left(\frac{c - v_1 + c + v_1}{c^2 - v_1^2} \right) = c V_0 \left(\frac{2c}{c^2 - v_1^2} \right)$$

$$\cos(\omega t) + \cos(\omega'' t) =$$

$$2 \cos\left(\frac{\omega + \omega''}{2} t\right) \cos\left(\frac{\omega - \omega''}{2} t\right)$$



slušmo utripanje

15.4.

✓ zamudile 20 minut

(naloge 10.13)

$$\int_V d\vec{v} \vec{D} \vec{v} \int_{\partial V} \vec{D} \vec{n} dS$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\int_{\rho_e W} \epsilon_0 \oint \vec{E} \cdot d\vec{S}$$

$$e = \epsilon_0 \oint \vec{E} d\vec{S}$$

← Nabla

$$\vec{E}_\perp = \frac{\lambda}{2\pi \epsilon_0 \rho}$$

$$e = \epsilon_0 \oint \vec{E} \vec{n} dS$$

place vch in dno velj

$$\lambda \cdot l = \epsilon_0 \oint E_\perp dS + 0 + 0$$

$$\lambda \cdot l = \epsilon E_\perp \oint dS = \epsilon = \epsilon E_\perp 2\pi \rho \cdot l$$

$$E_\perp = \frac{\lambda}{\epsilon_0 2\pi \rho}$$

$$W_{ke} + W_{pe} = \text{konst}$$

$$\frac{mv_0^2}{2} + W_{pe}(a) = \frac{mv_a^2}{2} + W_{pe}(b)$$

$$\frac{m}{2} (v_a^2 - v_0^2) = W_{pe}(b) - W_{pe}(a)$$

$$\int_a^b F d\rho = - \int_a^b e \frac{\lambda}{2\pi \epsilon_0 \rho} d\rho = - \frac{\lambda e}{2\pi \epsilon_0} \ln \frac{b}{a} = \frac{\lambda e}{2\pi \epsilon_0} \ln \frac{a}{b}$$

$$\vec{v}_{a''} = \vec{v}_b$$

$$= \frac{m}{2} \sin^2 \gamma \cdot v_a^2$$

kot pri poslovnem metu

$$\ln \frac{a}{b} = \frac{\pi \epsilon_0 m \sin^2 \gamma}{\lambda e}$$

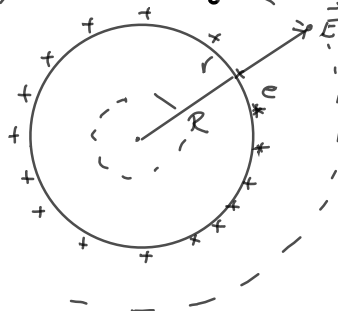
$$b = 16.8 \mu m$$

$$W_{ef}(\vec{r}_2) - W_p(\vec{r}_1) = -e \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} d\vec{r} \quad /: e$$

$$\Phi(\vec{r}_2) - \Phi(\vec{r}_1) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

10. 18

10. 20

Kovinska kugla, ko nosi naboj e 

$$\vec{E} = ? \quad r > R$$

← S površina

$$e = \epsilon_0 \int_S \vec{E} \cdot d\vec{s}$$

 \vec{E} je konstanta

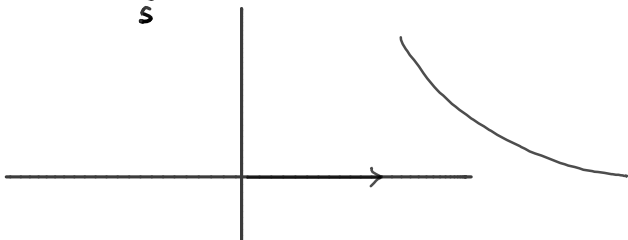
$$e = \epsilon_0 E 4\pi r^2$$

$$\Rightarrow E = \frac{e}{\epsilon_0 4\pi r^2}$$

 $r < R$

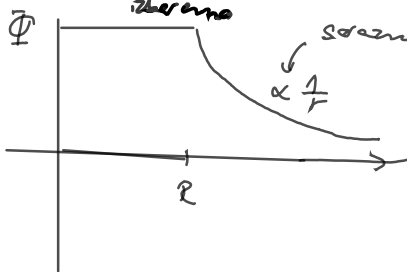
u kernima znatnoji nič naboj

$$e = \epsilon_0 \int_S E \cdot d\vec{s} \Rightarrow E = 0$$



$$\Phi(r) = \underbrace{\Phi(\infty)}_{\text{izborena}} - \int_{\infty}^r \vec{E} \cdot d\vec{r} = - \int_{\infty}^r \frac{e}{\epsilon_0 4\pi r^2} dr = \frac{e}{\epsilon_0 4\pi} \left(\frac{1}{r} - 0 \right) = \frac{e}{\epsilon_0 4\pi r}$$

srazmerno $\propto \frac{1}{r}$



$$\Phi(r < R) = \Phi(R) = \frac{e}{\epsilon_0 4\pi R}$$

Enekomerno nebite kroga

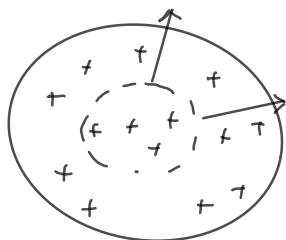
$r > R \Rightarrow$ isto kot prej

$$E = \frac{e}{\epsilon_0 4\pi r^2}$$

$$\phi(r) = \frac{e}{\epsilon_0 4\pi r}$$

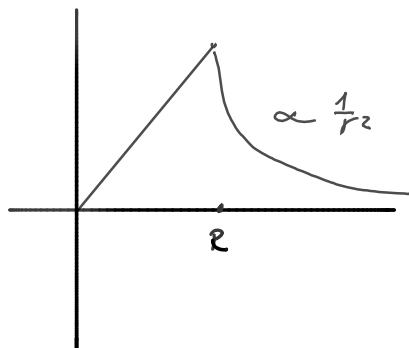
$r < R \Rightarrow$

$$e = \epsilon_0 \int_S E dS$$



$$e' = \epsilon_0 E 4\pi r^2 \quad e' = \frac{3e}{4\pi R^3} \cdot 4\pi r^2 = \frac{e r^3}{R^3}$$

$$E = \frac{e r^3}{R^3 \epsilon_0 4\pi r^2} = \frac{e r}{R^3 \epsilon_0 4\pi}$$



$$\Phi(r) = \Phi(R) + \int_R^r \frac{e r}{\epsilon_0 4\pi R^3} dr =$$

$$\Phi(R) + \frac{e r^2}{\epsilon_0 8\pi R^3} \Big|_R^r = \Phi(R) + \frac{e r^2}{\epsilon_0 8\pi R^3} + \frac{e}{\epsilon_0 8\pi R} =$$

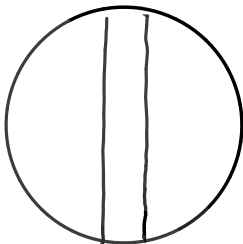
$$= \frac{e}{4\pi \epsilon_0} \left(\frac{1}{R} + \frac{r^2}{2 \cdot R^3} + \frac{R^2}{2R^3} \right) = \frac{e}{4\pi \epsilon_0 R \epsilon_0} \left(\frac{3}{2} - \frac{r^2}{2R^2} \right)$$

$$\vec{E} = \frac{e}{\epsilon_0 4\pi r^3} \vec{r}$$

V z-mude

$$e_1 \cdot E(\vec{r}) = -e_1 \frac{e_2 \vec{r}}{4\pi\epsilon_0 R^3} =$$

b)



$$M = 6 \cdot 10^{24} \text{ kg}$$

$$R = 6400 \text{ km}$$

$$F_e = \frac{e_1 e_2}{4\pi\epsilon_0 r^2}$$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$e_i \leftrightarrow m_i$$

$$\frac{1}{4\pi\epsilon_0} \rightarrow G$$

$$m a = F_g = -m G \frac{M}{R^2}$$

lehte zamenjamo kv
rednosti

$$\ddot{r} + G \frac{M}{R^3} r = 0$$

$$V = \frac{\sqrt{G \frac{M}{R^3}}}{2\pi} \Rightarrow T_0 = \frac{1}{V} = \frac{2\pi}{\sqrt{G \frac{M}{R^3}}} = 84,7 \text{ min}$$

10.27

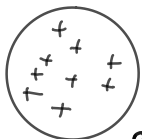
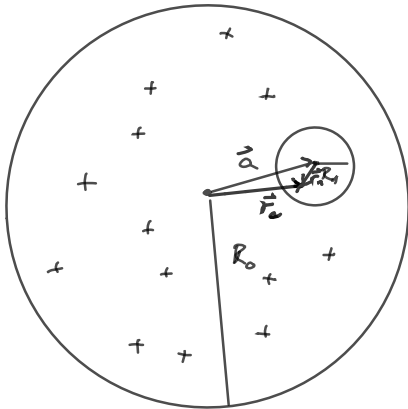
$$R_0 = 10 \text{ m}$$

$$\rho_c = 2 \mu\text{As}/\text{m}^3$$

$$R_1 = 2 \text{ m}$$

$$a = 7 \text{ m}$$

$$E = ? \text{ v } \text{veľk. in'}$$



$e \rho_c$



ρ_1

$$\rho_1 = -\rho_c$$

$$\vec{E}_s = \vec{E}_0 + \vec{E}_1$$

celková veľkosť vektoru \vec{E}_s je rovná veľkosti \vec{E}_0 a \vec{E}_1 (miejh na krogla)

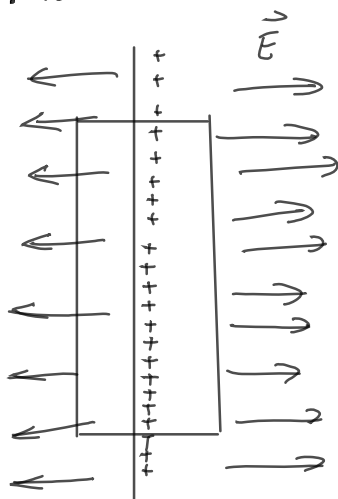
$$\frac{\rho_c \vec{r}_0}{4\pi\epsilon_0 R_0^3} + \frac{\rho_1 \vec{r}_1}{4\pi\epsilon_0 R_1^3} =$$

$$= \frac{\rho_c \frac{4\pi R_0^3}{3} \vec{r}_0}{4\pi\epsilon_0 R_0^3} - \frac{\rho_c \cdot \frac{4\pi R_1^3}{3} \vec{r}_1}{4\pi\epsilon_0 R_1^3} =$$

$$= \frac{\rho_c (\vec{r}_0 - \vec{r}_1)}{3\epsilon_0} = \frac{\rho_c}{3\epsilon_0} \vec{a}$$

$$|\vec{E}_s| = \frac{2 \cdot 10^{-6} \text{ As} \cdot 7 \text{ m}}{\text{m}^3 \cdot 3 \cdot 8.85 \cdot 10^{-12} \text{ As}}$$

10.18 \rightarrow 10.30



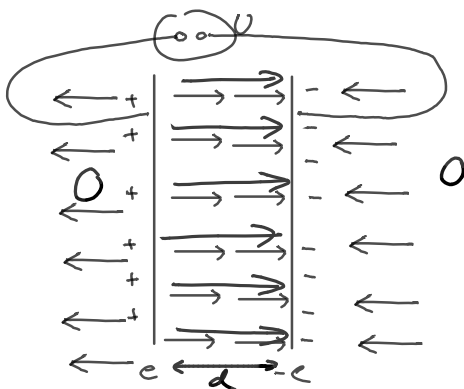
enakomerno
nahite
ploščo

$$\oint_S \vec{E} \cdot \vec{n} \, dS = \oint_S \vec{E} \cdot \vec{n} \, dS = \epsilon_0 \oint_S \vec{E} \cdot \vec{n} \, dS =$$

(OK) \nwarrow vzporedno \nearrow konstantno

$$= 2\epsilon_0 E \mathcal{P}(\text{ploščo}) + 0$$

$$E = \frac{e}{2\epsilon_0 S} = \frac{\sigma}{2\epsilon_0}$$



$$E = \frac{\sigma}{\epsilon_0}$$

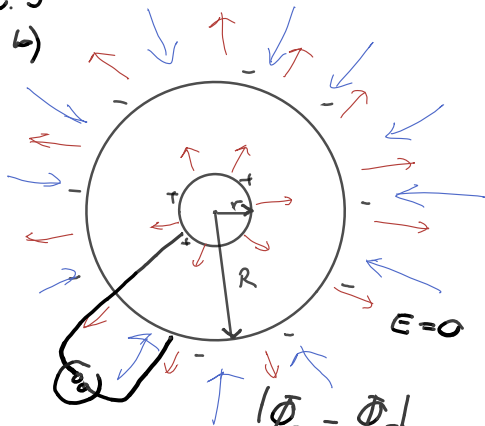
$$\Delta\Phi = - \int \vec{E} \cdot d\vec{r} = -U = E \cdot d = \frac{\sigma}{\epsilon_0} d = \frac{ed}{\epsilon_0}$$

$$e = \frac{U \epsilon_0}{d} = U \cdot C$$

$$C = \frac{e}{U} \quad [F = \frac{As}{V}]$$

10.30

b)



$$|\Phi_2 - \Phi_1|$$

$$|\Delta\Phi| = U = \left| \frac{e}{4\pi\epsilon_0 R} - \frac{e}{4\pi\epsilon_0 r} \right| = \left| \frac{e(r-R)}{4\pi\epsilon_0 Rr} \right| =$$

$$= \frac{e(R-r)}{4\pi\epsilon_0 Rr}$$

$$C = \frac{e}{U} = \frac{4\pi\epsilon_0 Rr}{(R-r)}$$

(10.4.)

Ne tabl:



$$I = \frac{U [V]}{R [\Omega]} [A]$$

6.5

11.1

$$R = \left\{ \frac{l}{S} \right\}$$

$$A = W_{pe} = e'' U$$

$$P = I U$$

$$S_1 = 0,1 \text{ mm}^2$$

$$l_1 = 100 \text{ m}$$

$$\xi_1 = 0,1 \frac{\Omega \text{ mm}^2}{\text{m}}$$

$$S_2 = 0,2 \text{ mm}^2$$

$$l_2 = 28 \text{ m}$$

$$\xi_2 = 0,5 \frac{\Omega \text{ mm}^2}{\text{m}}$$

$$U = 220 \text{ V}$$

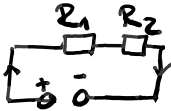
a) zaporedna vezava

$$\frac{P_a}{P_b} = ?$$

b) vzporedna vezava

$$P = \frac{A e}{t} = U I = U \left(\frac{U}{R} \right) = \frac{U^2}{R} = R I^2$$

a)



$$R_1 = 100 \Omega$$

$$R_2 = 10 \Omega$$

$$R = R_1 + R_2 \quad \text{ker } U - I R = 0$$

$$U - I_a R_1 - I_a R_2 = 0$$

$$R = R_1 + R_2$$

$$I_a = \frac{U}{R_1 + R_2} = 1,234 \text{ A}$$

$$P_a = U \cdot I_a = \underline{\underline{284,7 \text{ W}}}$$

b)



$$U = I_1 R_1 = I_2 R_2$$

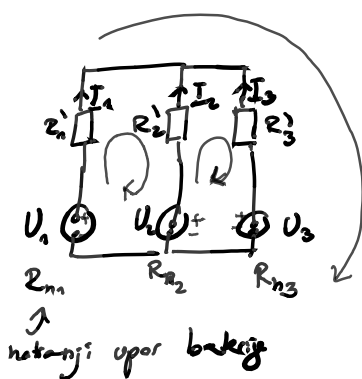
$$I = I_1 + I_2 = \frac{U}{R_1} + \frac{U}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} =$$

$$I = 5,34 \text{ A}$$

$$P_b = U I = \underline{\underline{1175,4 \text{ W}}}$$

11,11



$$U_1 = 1V$$

$$U_2 = 2V$$

$$U_3 = 3V$$

$$R_{n1} = 15\Omega$$

$$R_{n2} = 20\Omega$$

$$R_{n3} = 30\Omega$$

$$R_1' = 85\Omega$$

$$R_2' = 180\Omega$$

$$R_3' = 270\Omega$$

$$R_1 = R_{n1} + R_1'$$

$$R_1 = 100\Omega$$

$$R_2 = 200\Omega$$

$$R_3 = 300\Omega$$

$$U_1 - I_1 R_1 + I_2 R_2 - U_2 = 0 \quad (\text{lev: krog})$$

$$U_2 - I_2 R_2 + I_3 R_3 - U_3 = 0 \quad (\text{desno: krog})$$

$$U_1 - I_1 R_1 + I_3 R_3 - U_3 = 0 \quad (\text{cel krog})$$

Linearna odvisnost (dve enačbi, tri neznanke)

$$I_1 + I_2 + I_3 = 0 \quad (\text{Kirchoffov zakon})$$

$$I_1 = -I_2 - I_3$$

$$U_1 + I_2 R_1 + I_3 R_1 + I_2 R_2 - U_2 = 0$$

$$U_2 - I_2 R_2 + I_3 R_3 - U_3 = 0$$

$$I_2 = \frac{U_2 - U_3 + I_3 R_3}{R_2}$$

$$U_1 + \frac{R_1}{R_2} (U_2 - U_3 + I_3 R_3) + (U_2 - U_3 + I_3 R_3) - U_2 = 0$$

$$\frac{R_1 R_3}{R_2} I_3 + R_3 I_3 = U_2 - U_1 - \frac{R_1}{R_2} (U_2 - U_3) - U_2 + U_3$$

$$I_3 = \frac{R_2 (U_3 - U_1) - R_1 (U_2 - U_3)}{R_1 R_3}$$

$$I_3 = \frac{200\Omega (2V) - 100\Omega (1V)}{100\Omega \cdot 300\Omega} =$$

$$= \frac{500\Omega V}{100 \cdot 300\Omega} = \frac{5}{300} V = 0,016 A$$

To je narobe :-

$$I_3 = 4,5k mA$$

$$I_2 = 1,81 mA$$

$$I_1 = -6,35 mA$$

11.3

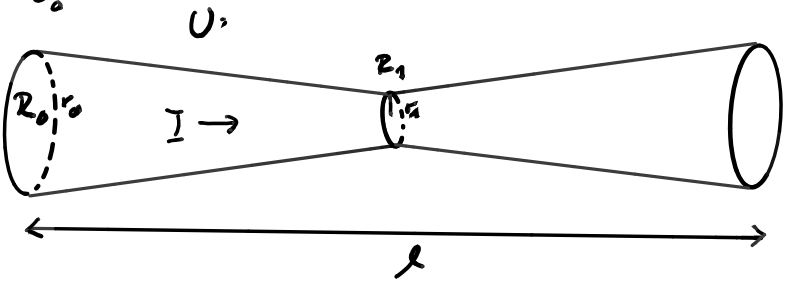
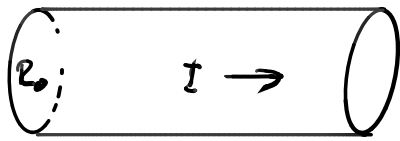
$$S_0 = 3 \text{ mm}^2$$

$$S_1 = 1 \text{ mm}^2$$

$$l = 15 \text{ cm}$$

$$\frac{U}{U_0} = 2$$

U_0 :



$$\frac{U}{U_0} = \frac{RI}{R_0 I} = \frac{R}{R_0}$$

$$R_0 = \frac{l}{S_0}$$

$$R = \int dR = 2 \int_0^{l/2} \frac{dx}{\pi r(x)^2}$$

$$r(x) = \frac{2(r_0 - r_1)}{l} x + r_1$$

$$u = r(x)$$

$$du = \frac{2(r_0 - r_1)}{l} dx$$

$$\begin{aligned} R &= 2 \int_{r_1}^{r_0} \frac{2(r_0 - r_1)}{\pi l u^2} du = 4 \frac{r_0 - r_1}{\pi l} \left(-\frac{1}{u} \right) \Big|_{r_1}^{r_0} = \\ &= \frac{4}{\pi l} (r_0 - r_1) \left(\frac{1}{r_1} - \frac{1}{r_0} \right) = \\ &= \frac{4}{\pi l} \frac{(r_0 - r_1)^2}{r_0 r_1} = \frac{2}{\pi} \frac{l (r_0 - r_1)^2}{r_0 r_1} \end{aligned}$$

$$\pi r_0^2 = S_0$$

$$r_0 = \sqrt{\frac{S_0}{\pi}} \quad r_1 = \sqrt{\frac{S_1}{\pi}}$$

$$R = \frac{2}{\pi} \frac{\pi \left(\frac{S_0}{\pi} + 2\sqrt{S_0 S_1} + \frac{S_1}{\pi} \right)}{\sqrt{S_0 S_1}} =$$

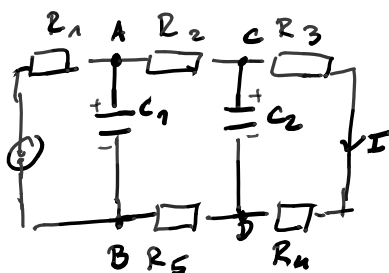
$$= \frac{2}{\pi} \frac{(S_0 + S_1 + 2\sqrt{S_0 S_1})}{\sqrt{S_0 S_1}}$$

$$\frac{U}{U_0} = \frac{R}{R_0} =$$

mit sm se zuehile

$$R = \frac{l}{\pi r_0 r_1}$$

$$\frac{U}{U_0} = \frac{r_0}{r_1} = \frac{\sqrt{S_0}}{\sqrt{S_1}} = \sqrt{3}$$



$$U_0 = 6V$$

$$R_1 = 4\Omega$$

$$R_2 = 1\Omega$$

$$R_3 = 5\Omega$$

$$R_4 = 5\Omega$$

$$R_5 = 1\Omega$$

$$C_1 = 5\mu F$$

$$C_2 = 0,1\mu F$$

Pe de lașim căreie cele
condensator ne ține tot

$$\Rightarrow R = R_1 + R_2 + R_3 + R_4 + R_5$$

$$[F = \frac{As}{V}]$$

$$C = \frac{q}{U}$$

U_{AB}

$$q_1 = ?$$

$$q_2 = ?$$

$$q_1 = C_1 U_{AB}$$

$$q_2 = C_2 U_{CB}$$

$$U_0 - IR_1 - U_{AB} = 0$$

$$U_0 - IR_1 - IR_2 - U_{CB} - IR_5 = 0$$

$$U_0 - I(R_1 + R_2 + R_3 + R_4 + R_5) = 0$$

$$I = \frac{U_0}{R_n} = 0,5A$$

$$U_{AB} = U_0 - IR_1 = 6V$$

$$U_{CB} = U_0 - IR_1 - IR_2 - IR_5 = 5V$$

$$q_1 = C_1 \cdot U_{AB} = 30\mu As$$

$$q_2 = 0,5\mu As$$

V_{comadib} per ore

11.13 Primenje kondenzatorje

$$R = 1 \text{ M}\Omega$$

$$C = 300 \text{ nF}$$

$$t_{1,2} = ?$$

$$C = \frac{q}{U}$$



$$(U_0 +) U_C + U_R = 0$$

$$(U_0) - \frac{q}{C} - IR = 0$$

advisnood case

$$-\frac{e(t)}{C} = R \frac{de}{dt}$$

$$de = -\frac{e(t)}{RC} dt$$

$$\frac{de}{e} = -\frac{dt}{RC}$$

$$\ln e \Big|_0^t = \left(-\frac{t}{RC}\right) \Big|_0^t$$

$$\ln \frac{e}{e_0} = -\frac{t}{RC}$$

$$e = e_0 e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = \frac{1}{2}$$

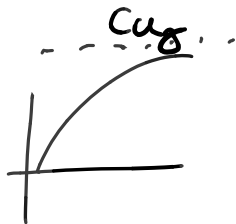
$$-\frac{t}{RC} = \ln \frac{1}{2}$$

$$t = \ln 2 \cdot RC$$

$$1 \cdot 10^6 \Omega \cdot 300 \cdot 10^{-9} \text{ F} \cdot 0,69 = 0,21 \text{ s}$$

Pdnenje kondenzatorja

$$e(t) = CU_S (1 - e^{-\frac{t}{RC}})$$



Magnetna polje

12.1

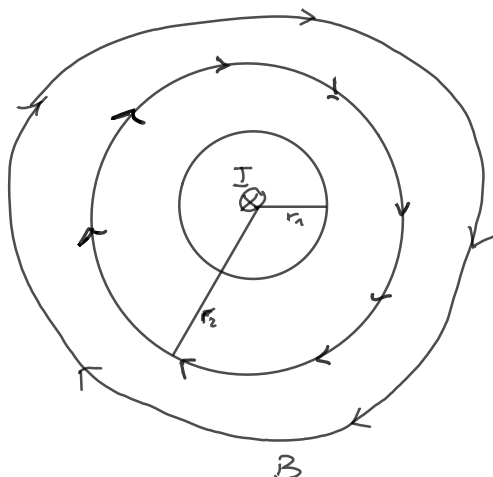
$$R = 1 \text{ cm}$$

$$I = 1000 \text{ A}$$

$$r_1 = 0,5 \text{ cm}$$

$$r_2 = 2,0 \text{ cm}$$

$$B(r) = ?$$



$$\oint \vec{B} d\vec{s} = 0$$

Ampere:

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = \frac{1}{\mu_0} \int_S \underbrace{\text{rot } \vec{B}}_{\substack{\text{je ... gostota} \\ \text{električnega} \\ \text{toka}}} d\vec{s} \quad \frac{1}{\mu_0} \oint \vec{B} d\vec{s} = I_{\text{objekt}}$$

$$\vec{B} = \mu_0 \vec{H}$$

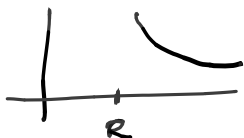
↑ gostota ↓ jakost

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$$

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = I$$

$$\frac{B}{\mu_0} \int ds = \frac{B}{\mu_0} 2\pi r$$

$$B(r) = \frac{I \mu_0}{2\pi r} = 0,01 \frac{\text{Vs}}{\text{m}^2}$$



⇒

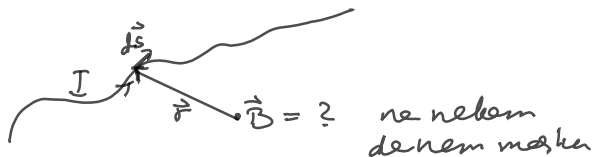
$$\frac{S}{I} = \frac{S'}{I'} = \frac{1}{j} \Rightarrow I = \frac{S'}{S} I' = \frac{4\pi r_1^2}{4\pi R^2} I' =$$

$$= \frac{B'}{\mu_0} 2\pi r_1 \Rightarrow$$

$$\frac{\mu_0 r_1}{2\pi R^2} I' = B'$$

$$\Rightarrow B(r_1) = \frac{\mu_0 r_1}{2\pi R^2} I = 0,01 \text{ T}$$

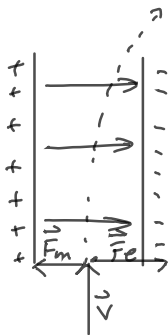
Biot-Savartova rovnice



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{r} \times d\vec{s}}{r^3}$$

12.4.

Lorentzova sila $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$



$B = ?$ da se
pot elek na ukl.

$$U = 50V$$

$$d = 1cm$$

$$\vec{F}_e = -\vec{F}_m$$

$$E = v \cdot B$$

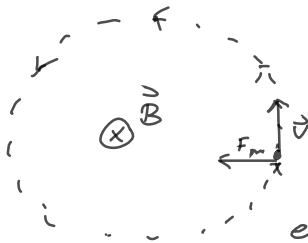
$$\frac{U}{d}$$

(B za najmanjšo
pot 5 cm)

Prej: A je prav?

Rešja na tbi: Mel smo dva podatka, da
smo porabi

$$B = \frac{U}{dv} = \frac{50V}{0,01m \cdot 10^6 \frac{m}{s}} = \left[\frac{Vs}{m^2} \right] 4,8$$



$$F_m = F_{cp}$$

$$e v B = m \omega^2 r$$

$$e \omega r B = m \omega^2 r$$

$$\omega = \frac{eB}{m} = \omega_0$$

ciklotronska
frekvence

$$e v B = m \frac{v^2}{r}$$

$$r = \frac{m v}{e B}$$

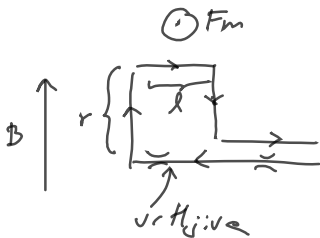
$$dA = \vec{F}_m d\vec{x} = e(\vec{v} \times \vec{B}) \vec{v} dt = 0$$

\nearrow
 pravokotna

12.10

20.5

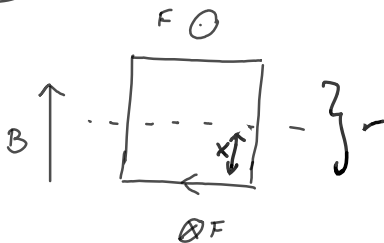
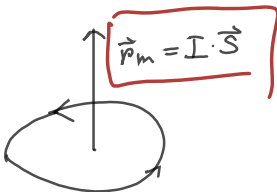
V



$$M = r \cdot F_m = r \cdot I l B = \underbrace{I S \cdot B}_{p_m} = p_m B$$

magnetni
moment

Zanka s tokom

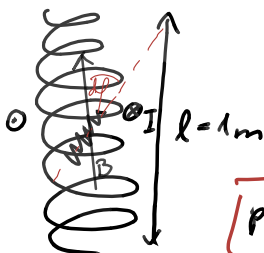


$$M = x F_m + (r - x) F_m = r F_m$$

$$M = p_m \vec{B}$$

$$M = \vec{r} \times \vec{F}_m$$

$$N_1 = 100 \quad l = 1 \text{ m} \quad I_1 = 25 \text{ A}$$



$$N_2 = 500$$

$$S_2 = 20 \text{ cm}^2$$

$$I_2 = 0,1 \text{ A}$$

$$p_m = N I \vec{S} \quad A = ?$$

$$dA = \vec{r} \cdot d\vec{p} = M dp$$

$$M = \vec{p}_m \times \vec{B} = p_m B \sin \varphi = N I_2 \cdot S B \sin \varphi$$

$B = ?$

$$\frac{1}{\mu_0} \oint \vec{B} d\vec{s} = \sum I$$

$$\frac{1}{\mu_0} (B l + 0) = N_1 I_1$$

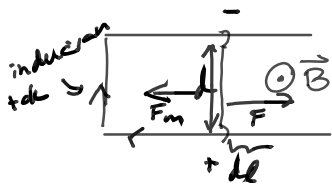
ker je tok
tam tako mogoče

$$B = \frac{N_1 I_1 \mu_0}{l}$$

$$A = \int_0^\pi N_2 S_2 I_2 \frac{\mu_0 N_1 I_1}{l} \sin \varphi d\varphi =$$

$$+ 2 N_2 S_2 I_2 \frac{\mu_0 N_1 I_1}{l} = 6,82 \cdot 10^{-4} \text{ J}$$

Elektromagnetna indukcija



$$F_m = I \vec{d} \times \vec{B}$$

$$F dl = dA e = U_i I_i dt$$

$$I_i \cdot \underbrace{d \cdot B dl}_{v \cdot dt} = U_i I_i dt$$

magneti:
polovi

$$\Phi_m = \vec{B} \cdot \vec{S}$$

$$U_i = B \cdot d \cdot v = B \cdot \left(\frac{d \cdot dl}{dt} \right) = \left(B \frac{dS}{dt} \right) = \frac{d\Phi_m}{dt}$$

$$U_i = (\vec{d} \times \vec{B}) \cdot \vec{v}$$

Φ_m hoće ostati isti

$\Phi_m = \vec{B} \cdot \vec{S}$ ako se \vec{S} povećava
mora B smanjivati

12. 18

$$\xi = \text{konst}$$

$$d = 5 \text{ cm}$$

$$B = 1 \text{ T}$$



$$\vec{a} = \text{konst}$$

$$l(I = \text{max}) = ?$$

$$l = \frac{1}{2} a t^2$$

$$v = a t$$

$$l = \frac{v^2}{2a}$$

$$v = \sqrt{2al}$$

$$I_i = \frac{U}{R} = \frac{B \cdot d \cdot v(l)}{R(l)}$$

$$= \frac{B \cdot d \cdot \sqrt{2al}}{\frac{\xi \cdot (2d + 2l)}{S}} =$$

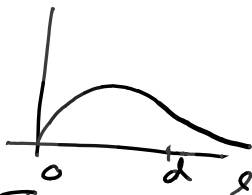
$$R = \frac{\xi \cdot l'}{S}$$

$$= \frac{S B d}{\xi} \cdot \frac{\sqrt{2al}}{(2d + 2l)}$$

$$\text{is also same as } \propto \frac{\sqrt{l}}{d + l}$$

$$d \ll l \Rightarrow I_i \propto \frac{1}{\sqrt{l}}$$

$$l \ll d \Rightarrow I_i \propto \sqrt{l}$$



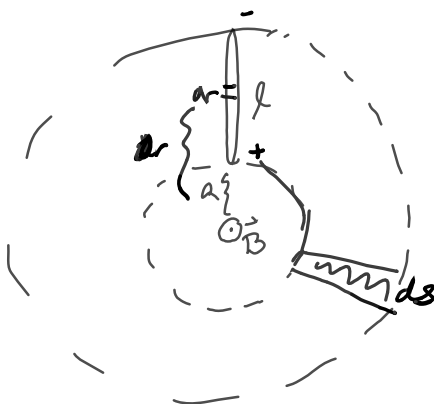
$$\left(\frac{\sqrt{l}}{d + l} \right)' = \frac{\frac{1}{2\sqrt{l}} (d + l) - \sqrt{l}}{(d + l)^2} =$$

$$= \frac{\frac{d}{2\sqrt{l}} + \frac{1}{2} \sqrt{l} - \sqrt{l}}{(d + l)^2} = \frac{\frac{d}{2\sqrt{l}} - \frac{1}{2} \sqrt{l}}{(d + l)^2} = 0$$

$$\frac{d}{2\sqrt{l}} - \frac{1}{2} \sqrt{l} = 0 \quad \cdot \sqrt{l}$$

$$d\sqrt{l} - l = 0$$

$$d = \sqrt{l}$$



$$a = 25 \text{ cm}$$

$$l = 0,5 \text{ m}$$

$$V = 2 \text{ Hz}$$

$$B = 0,5 \text{ T}$$

$$U_i = ?$$

$$U_i = B \cdot d \cdot v$$

Lorenz force

$$\boxed{F_m = e v \times B}$$

$$dU_i = B \cdot dr \cdot v$$

$$U_i = \int_a^{a+l} B \omega r dr = \frac{B\omega}{2} ((a+l)^2 - a^2) =$$

$$= \frac{B\omega}{2} (2al + l^2) = B\omega al + \frac{B\omega}{2} l^2$$

$$\stackrel{\substack{\downarrow \\ \text{Vs}_{\text{m}^2}}}{=} \frac{B\omega}{2} (l + 2a) = B\pi V (l + 2a)$$

$$= 0,5 \text{ T} \cdot 2\pi \cdot 2/s \cdot (0,5 + 0,5) \text{ m} = 1,57 \text{ V}$$

2. način

$$U_i = \frac{d\Phi_m}{dt} = \frac{B ds}{dt} =$$

$$ds = \pi((a+l)^2 - a^2) \cdot \frac{dl}{2\pi} = \frac{dl}{2} l(l+2a)$$

$$= \frac{B dl}{dt \cdot 2} l(l+2a) = \frac{B\omega l}{2} (l+2a)$$

12.20

$$\xi = 0,05 \frac{\Omega \text{ mm}^2}{\text{m}}$$

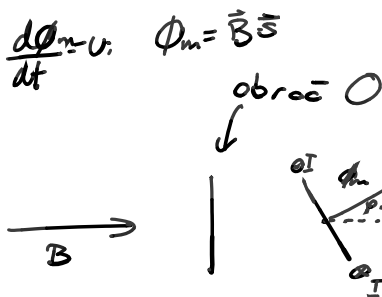
$$r = 5 \text{ cm}$$

$$S_0 = 1 \text{ mm}^2$$

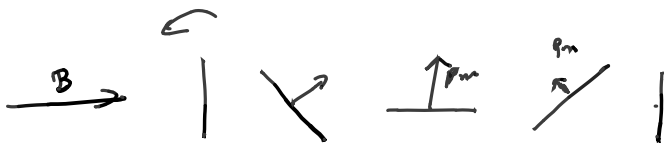
$$B = 0,7 \text{ T}$$

$$\omega = 10 \text{ Hz}$$

$$\bar{M} = ?$$



$$\phi_m = B \cdot S \cdot \cos \varphi$$



$$\vec{M} = \vec{p}_m \times \vec{B} = p_m \cdot B \cdot \sin \varphi = I_i B S \sin \varphi =$$

$$p_m = I_i \cdot S \quad I_i = \frac{U_i}{R} = \frac{U_i}{R} = \frac{d\phi_m}{dt R} =$$

$$= \frac{B \cdot S \cdot \cos \varphi \frac{d\varphi}{dt}}{R} =$$

$$= -\frac{B \cdot S \omega}{R} \sin \omega t$$

$$= -\frac{B^2 S^2 \omega}{R} \sin^2 \omega t$$

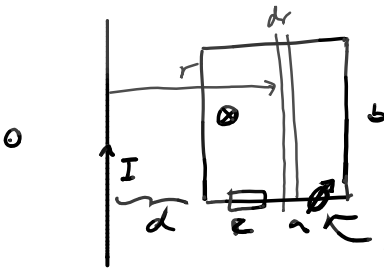
$$|M| = \frac{B^2 S^2 \omega}{R} \sin^2 \omega t \quad \overline{\sin^2} = \frac{1}{2}$$

$$\bar{M} = \frac{B^2 S^2 \omega}{2 R} = \frac{B^2 S^2 S_0 \omega}{4 \pi r \xi} = 0,06 \text{ Nm}$$

$$R = \xi \frac{2 \pi r}{S_0}$$

12.22

$$I = 300 \text{ A} \quad a = 5 \text{ cm} \quad d = 3 \text{ cm} \\ b = 10 \text{ cm} \quad R = 10 \Omega$$



balistion:
galvanometer
(voltmeter ali
amper meter)

$$F \rightarrow \int F dx$$

$$M \rightarrow \int M dt$$

$$I \rightarrow \int I dt = e \quad \text{tokam; sunek = nabaj}$$

$$\Phi_m = \int_S \vec{B} \cdot d\vec{s} = \int_a^{da} B b dr = \int_a^{da} \frac{\mu_0 I}{2\pi r} b dr =$$

$$ds = b dr$$

$$= \frac{\mu_0 I}{2\pi} b \ln \frac{da}{a} \quad \mu_0 \text{ je } I \text{ se tokel}$$

$$\frac{d\Phi_m}{dt} = U_i = \frac{I_i}{R} \quad I_i = \frac{d\Phi_m}{dt} R$$

$$\int_{\text{izloz}} I_i dt = \int_{\text{izloz}} \frac{1}{R} d\Phi_m = \frac{1}{R} \int_{\text{izloz}} d\Phi_m$$

$$\int I_i dt = \frac{1}{R} (\Phi_{m1} - \Phi_{m2}) = \frac{\mu_0 I}{2\pi R} \ln \frac{da}{a}$$

$$= 5,88 \cdot 10^{-7} \text{ As}$$