DAT026

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We hereby declare that we have both actively participated in solving every exercise. All solutions are entirely our own work, without having taken part of other solutions.

1 Problems

1.1 Chocolate Factory Problem

List of variables:

* Price P
* Quantity Q
* Production cost C
* Does buy new machine M
* Net Income I

We can calculate their other fixed yearly expenses (we presume this includes things such as property costs, salaries etc) by taking amount of total income they received, minus the known costs (production cost) i.e. $\textit{Other expenses} = \textit{(Revenue + Loss} - \textit{Quantity} \cdot (\textit{Price} - \textit{Production Cost}) = 1,300,000 + 40,000 - 80,000 \cdot (18-5) = 1,340,000 - 80,000 \cdot (13) = 1,300,000 - 1,040,000 = 260,000$

From this we gather that they have some sort of fixed expenses of about $260,000kr$ each year beyond the production cost this will be assigned to the constant F.

Our goal is to maximize profits, this is done by maximizing the net income for the company. The net income can be calculated simply by taking all the income minus all the expenses, we would also like to be able to do this for multiple years as things such as machine purchases might not pay off immediately.

$$\begin{gathered} \textit{Net Income over the next N years} = \\ N \cdot ((\textit{Price} - \textit{Production Cost}) \cdot \textit{Quantity} - \textit{Fixed Expenses}) \\ - (\textit{Does Buy New Machine} \cdot 450,000) \end{gathered}$$

We also know that the production is currently $5kr$ (a constant) but if we buy the new machine this will be decreased to $3kr$ i.e. a decrease of $2kr$ giving us the following function:

$$\textit{Production Cost} = 5 - \textit{Does Buy New Machine} \cdot 2$$

We can now quickly merge this functions into one by replacing the production cost with the function to calculate it. Here we will also exchange the variables to their variable names found above.

$$I(N) = N \cdot ((P - (5 - 2 \cdot M)) \cdot Q - F) - (M \cdot 450,000)$$

We can make a rough estimate on how the quantity changes depending on the data from the home city. From this we get that the current $80,000$ bars a year is

due to the cost of $18kr$ should the price be lowered to $17kr$ we would expect the quantity to go up by (8100/7400) (so about a 9.5% increase) to a total of $87,567$. Should we instead decrease the price be even lower to $15kr$ we would expect the quantity to go up by a total of 39% to a total of $80,000 * 1.39 = 111,351$ each year.
Now that we know all of our variables and have our objective function we only need our constraints. Since we have three different possible prices these will be three different variables that can be either 0 or 1

- $P1, P2, P3 = (0 \text{ or } 1)$
- $P1 + P2 + P3 = 1$
- $Q = P1 \cdot 111,351 + P2 \cdot 87,567 + P3 \cdot 80,000$
- Production cost, see function above.
- $M = 1 \ or \ 0$

This will also change our objective function slightly by exchanging the price P to the three new variables $P1, P2, P3$ as well as exchanging the Quantity to its constraint. This gives us the final objective function of:

$$I(N) = N \cdot (((P1 \cdot 15 + P2 \cdot 17 + P3 \cdot 18) - (5 - 2 \cdot M))$$
$$\cdot (P1 \cdot 111,351 + P2 \cdot 87,567 + P3 \cdot 80,000) - F) - (M \cdot 450,000)$$

We run this through mathematica for one year forwards ($N = 1$) and get the following result. We should set the price to $15kr$ and not buy the machine, this would lead to a income of $1,113,510kr$ and since we know we have other fixed expenses of around $260,000kr$ this would lead to a net profit of about $850,000kr$. As mentioned this was only for the first year, if we instead calculate the profits over a longer period of time say 10 years we instead get the answer to still have the price at $15kr$ but this time buy the machine. This would lead to a income of about $12,912,000kr$ after production costs and machine purchase, however we still need to count those other costs $260,000 * 10kr = 2,600,000kr$ this still leads to a $10,312,000kr$ of net income over the next 10 years.

1.2 Emergency Care Problem

(a) The variables are the different sites, $S1, S2, ..., S6$ all of them can be either 0 or 1. The constraints are then:

- $S2 + S4 \geq 1$
- $S5 + S6 \geq 1$
- $S4 + S5 \geq 1$
- $S1 + S4 \geq 1$
- $S1 + S2 + S3 + S5 \geq 1$

- $S1 + S3 + S6 \geq 1$
- $S3 + S4 \geq 1$
- $S1 = 0$ or $S1 = 1$
- $S2 = 0$ or $S2 = 1$
- $S3 = 0$ or $S3 = 1$
- $S4 = 0$ or $S4 = 1$
- $S5 = 0$ or $S5 = 1$
- $S6 = 0$ or $S6 = 1$

Where the first set of constraints symbolises that at least one of the stations which are in range of a specific region has been built and the second set just makes sure that the variables can only even be 0 or 1.

Our objective function is then:

$$S1*710000+S2*610000+S3*650000+S4*910000, S5*720000+S6*578000$$

(b) The function is already linear.

(c) Calculating this in mathematica without having the integer requirement we get that we should set the variables as follows:

- $S1 = 0.2$
- $S2 = 0.2$
- $S3 = 0.2$
- $S4 = 0.8$
- $S5 = 0.4$
- $S6 = 0.6$

For a total cost of $\approx 1,76$ million.

From this we suppose that we could conclude that it would be more efficient to build several small stations instead of fewer bigger ones, however this assumes that having half a station 10 miles away and half a station that is 6 miles is the same as having a full station 8 miles away. It also assumes that the cost of building a smaller station will decrease linearly.

(d) When we maximized the objective function with the integer requirements we get that we should build on sites $2, 4, 6$ for a total cost of about 2.1 million.

(e) We could instead think of how many people will be covered by each station, say instead of saying there should be a station within X minutes of each region. We could instead say that one station should cover X people within Y minutes range.

1.3 communications Network Problem

(a) The variables in this problem we set to be the different connections between the nodes.

Variables:

- AB
- AC
- BD
- BE
- CD
- CE
- DF
- EF

For the constraints, we first took our variables and put limits on them so for example a link cannot a negative capacity but it can also not go above its specific limit. We also had to make sure that a node could never have more capacity than what they have incoming.

Constraints:

- $DF \leq BD + CD$
- $EF \leq CE + BE$
- $BD + BE \leq AB$
- $CD + CE \leq AC$
- $0 \leq AB \leq 16$
- $0 \leq AC \leq 48$
- $0 \leq BD \leq 32$
- $0 \leq BE \leq 8$
- $0 \leq CD \leq 8$
- $0 \leq CE \leq 32$
- $0 \leq DF \leq 64$
- $0 \leq EF \leq 16$

Since the input into F is entirely based on the links DF and EF our objective functions become the sum of the usage of these nodes: $F = DF + EF$

When we ran this through mathematica it gave us the answer that the capacity would be $40Mbit/s$ by transfering the following amounts through the different links.

Node	Transfer rate used
AB	16
AC	48
BD	16
BE	0
CD	8
CE	32
DF	24
EF	16

As can be seen the link BE is not used at all, this is due to the fact that all of the capacity going into the node B needs to go through the link BD to maximize the capacity.

(b) For this part we need only to change the constraint on the link BD to have a maximum of 0. Now mathematica gave us a maximum capacity of 24 by using the different links according to the table below.

Node	Transfer rate used
AB	16
AC	48
BD	0
BE	8
CD	8
CE	32
DF	8
EF	16

(c) To conclude which links could be removed, we added 8 new variables representing the state of each link (active/inactive) and could therefor only be 0 or 1. We then multiplied the maximum capacity for each link with it's active state, modelling it being on or off. We also added the constraint that $DF + EF \geq 35$ (i.e. F should at least take $35 Mbits/s$). This gave us the following variables / constraints lists.

Variables:

- AB
- AC
- BD
- BE

- CD
- CE
- DF
- EF
- BAB
- BAC
- BBD
- BBE
- BCD
- BCE
- BDF
- BEF

Constraints:

- $DF + EF \geq 35$
- $DF \leq BD + CD$
- $EF \leq CE + BE$
- $BD + BE \leq AB$
- $CD + CE \leq AC$
- $0 \leq AB \leq 16$
- $0 \leq AC \leq 48$
- $0 \leq BD \leq 32$
- $0 \leq BE \leq 8$
- $0 \leq CD \leq 8$
- $0 \leq CE \leq 32$
- $0 \leq DF \leq 64$
- $0 \leq EF \leq 16$
- $BAB = 0$ or 1
- $BAC = 0$ or 1
- $BBD = 0$ or 1
- $BBE = 0$ or 1
- $BCD = 0$ or 1
- $BCE = 0$ or 1
- $BDF = 0$ or 1
- $BEF = 0$ or 1

Our goal now is to minimize the amount of nodes active, which is simply done by minimizing $BAB + BAC + BBD + BBE + BCD + BCE + BDF + BEF$. After running this through mathematica we got the result that only the link BE could be removed.

1.4 Shortest path as LP Problem

1.5 Bridge Problem

(a) The travel time between the cities should equal out at about 50 minutes during rush hour. This due to the fact that in the beginning everyone might take one preferred way but then some will realize that the other way will be quicker, this will hopefully at some point even out so that about 50% of the traffic will take the A,C route and the other 50% will take the other route (D,A). This would mean each route would have a constant traffic of about 10 cars per minute in each direction giving the mountain roads a time of $(10 + 10)$ 20 minutes, this plus the constant 30 minutes travel time of the bigger roads gives us 50 minutes.

(b) We believe that building the bridge would increase the travel time to 60 minutes each way. This because if we had the state we had before with 50% of the traffic in each direction taking the A/C route and D/B route (which took 50 minutes) some would soon start taking the A/D route (plus bridge) taking them about 42 minutes (20 on the first mountain road, 1 minute for the bridge and then the $20 + 1$ minutes for the other mountain road. This will in time increase the travel time through the A/D route to 61 when someone will realize that taking the B/D or A/C route would only take 60 minutes since they avoid the bridge. This will also lower the travel time for those taking the A/D route to 60 minutes $(30 + 1 + 29)$ giving us a state equilibrium.

2 Reflection

I (a) Yes, we had multiple meetings with supervisors, during this module we mostly asked questions regarding problems we encountered with Mathematica.

(b) Yes, we both attended the lecture.

(c) We were not asked to do this.

II

- **Chocolate Factory Problem:** This problem went smoothly, the math took some time but we made constant progress. We could have asked a few questions to supervisors regarding what assumptions we could make about the prices and such, as can be seen we only used the prices shown in the data from the home town and scaled them up, this is something we can think about for future modules.
- **Emergency Care Problem:** This problem was also solved sort of mechanically, we had no issues with this problem.
- **Communications Network Problem:** Same as emergency care.
- **Shortest Path as LP Problem:** We did not solve this problem.

- **Bridge Problem:** This problem took some time to just think about, especially the (b) part but no real issues arose here either.

Overall this module went quite smoothly, probably in part due to both members of the group thinking these problems were quite fun to think about and solve.

III Overall we believe that we solved the problems in this module very well, however the Chocolate Factory Problem we could have tried a bit more outside of the box, allowing for more variable prices etc.