Probabilistic Game Theory: Optimizing Strategies with Monte Carlo Simulations

Abstract

This project explores the intersection of game theory and probabilistic methods by analyzing strategic decision-making in a competitive, probabilistic game scenario. Using Monte Carlo simulations, we assess adaptive strategies for two players — Alice and Bob, who can choose between aggressive, balanced, or defensive playstyles. The simulation predicts optimal strategies based on dynamic game states, exploring both greedy and non-greedy approaches to maximize overall performance.

Introduction

This project employs game theory and dynamic programming to model player behavior, simulating thousands of iterations to analyze the impact of various strategies. The game setting involves two players, Alice and Bob, each choosing different playstyles with distinct probabilities of winning, drawing, or losing. Our goals are to optimize Alice's strategy, in the current round as well as maximizing her total score over several rounds.

Problem Setup

The game is modeled with the following features:

- **Players**: Alice and Bob
- **Actions**: Attack, Balanced, Defence
- **Outcomes**: Win (+1 point), Draw (+0.5 points), Loss (0 points)

Each round, players independently choose one of three strategies, with outcome probabilities determined by their past performance and the chosen strategies. The probabilities for each scenario are laid out in a matrix form, and the task is to evaluate and optimize Alice's strategy using probabilistic methods.

Probabilistic Modeling and Simulations

Monte Carlo simulations were employed to model the game's outcomes over multiple rounds. The simulation computes the expected results for a variety of scenarios:

- **Case 1**: Both players always attack.

- **Case 2**: Bob's strategy depends on his previous round's outcome.
- **Case 3**: Bob selects his strategy uniformly at random.

For each scenario, the following calculations were made:

1. The expected value and variance of Alice's score after a given number of rounds.
2. The optimal strategy for Alice to maximize her points against different playstyles of Bob.
3. Exploration of greedy vs. non-greedy strategies using Monte Carlo simulation to validate the findings.

Key Findings

Optimal Strategy Identification: Simulations showed that Alice's optimal strategy shifts based on Bob's behavior, particularly when he adapts his style based on past outcomes. Monte Carlo simulations validated that in some cases, a non-greedy strategy (deviating from the immediate best option) yielded better long-term performance.

Impact of Stochastic Variables: The randomness introduced by Bob's uniformly distributed choices led to unexpected advantages for Alice when optimizing her playstyle dynamically.

Greedy vs. Non-Greedy Approach: Non-greedy strategies occasionally outperformed greedy ones, particularly when Alice needed to account for future rounds, leading to higher overall scores when validated by the simulations.

Conclusion

This project demonstrates the power of game theory combined with probabilistic methods to model and optimize strategies in competitive scenarios. By simulating thousands of game rounds using Monte Carlo methods, we were able to identify optimal strategies under different game conditions. The study highlights the importance of dynamic decision-making and the potential advantages of non-greedy strategies in long-term play.