Recurrence Relation

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Linear Non-Homogeneous Recurrence Relation

- A recurrence relation is called non-homogeneous if it is in the form:

$$\boldsymbol{F_n = AF_{n-1} + BF_{n-2} + f(n) \quad ; \text{where } f(n) \neq 0}$$

- Example:

$$F_n = 3F_{n-1} + 10F_{n-2} + \boxed{3^n}$$

with $F_0 = 2$ and $F_1 = 3$.

How to solve Linear non-homogeneous Recurrence Relation

- $\Rightarrow$ $F_n = AF_{n-1} + BF_{n-2} + f(n)$; where $f(n) \neq 0$

Its associated homogeneous recurrence relation is:

$$F_n = AF_{n-1} + BF_{n-2}$$

Now the solution a_n is:

$$a_n = a_h + a_t$$

$a_h \rightarrow$ solution of the associated homogeneous recurrence relation

$a_t \rightarrow$ particular solution

How to solve Linear non-homogeneous Recurrence Relation (Cont..)

- Let $f(n) = Cx^n$;

- Let $x^2 = Ax + B$ be the characteristic equation of the associated homogeneous recurrence relation and let x_1 and x_2 be the roots, then:

I. **If $x \neq x_1$ and $x \neq x_2$, then $a_t = Dx^n$**

II. **If $x = x_1$ and $x \neq x_2$, then $a_t = Dnx^n$**

III. **If $x = x_1 = x_2$, then $a_t = Dn^2x^n$**

Q:- Solve the recurrence relation:

$$\boxed{f_n = 3f_{n-1} + 10 f_{n-2} + 7(5)^n}$$

$f(n) = C n^n$

where $f_0 = 4$ and $f_1 = 3$.

Ans Associated homogeneous rec. relⁿ:

$$F_n = 3F_{n-1} + 10 f_{n-2} \quad — (1)$$

Characteristic eqn. of (1), $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x_1 = 5, x_2 = -2$

$\therefore$ Solⁿ is: $a_n = a(5)^n + b(-2)^n \quad — (2)$

$f(n) = 7(5)^n$ (form $c\,r^n$)

$r = 5,\ r_1 = 5,\ r_2 = -2$

$\therefore\ a_t = D n\, r^n = \underline{Dn(5)^n}$ — (4)

(3) $\underbrace{Dn(5)^n} = 3\,\underbrace{D(n-1)(5)^{n-1}} + 10\,D(n-2)(5)^{n-2} + 7(5)^n$

On dividing both sides of (3) by 5^{n-2}, we get

$Dn(5)^2 = 3D(n-1)(5) + 10D(n-2)\,5^0 + 7(5)^2$

$25Dn = 15Dn - 15D + 10Dn - 20D + 175$

$35D = 175$

$D = 5$ (substitute D in (4))

So, $a_t = 5n(5)^n$

The sol^n of recurrence relation is:

$$F_n = a_h + a_t$$

$$F_n = a5^n + b(-2)^n + 5n(5)^n$$

General soln $-$ $F_n = a5^n + b(-2)^n + n(5)^{n+1}$

$F_0 = 4,\ F_1 = 3$

Put $n=0$, & $n=1$, we get $a=-2$, $b=6$.

$\therefore$ Soln is : $\underbrace{F_n = (-2)5^n + 6(-2)^n + n5^{n+1}}_{\text{Ans}}$

form $f(n) = \boxed{Cx^n}$ Rosen

$\boxed{\text{form of } f(n)}$

$a_t = Dnx^n$

$\boxed{a_t} =$

$\equiv x = x_1 \;\&\; x \neq x_2$

Example


Thank you!!! ...