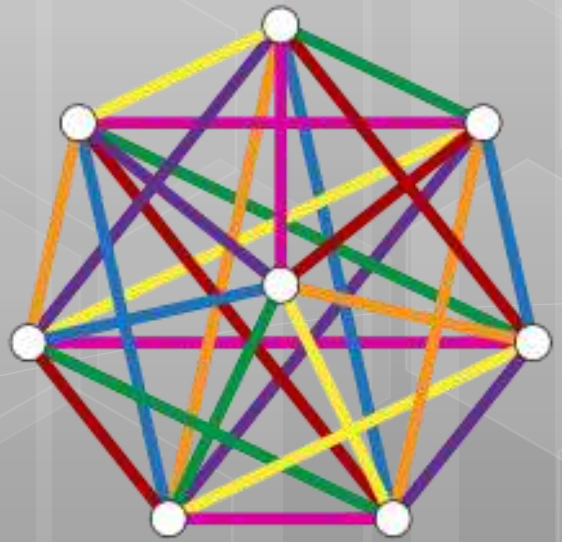

Graph Theory

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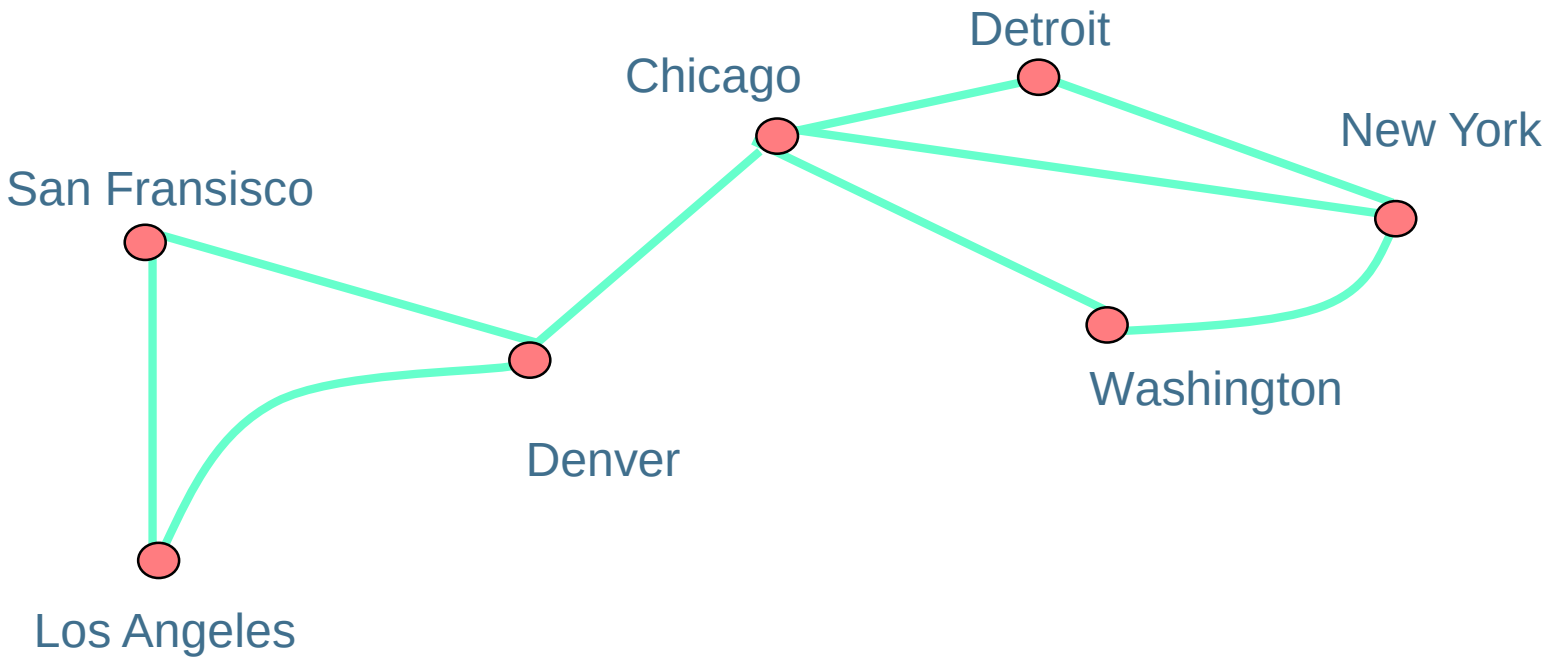
Definition

A **graph** G = (V,E) consists of V, a nonempty set of **vertices**, and E, a set of **edges**. Each edge has either one or two vertices associated with it, called its **endpoints**. An edge is said to connect its endpoints.

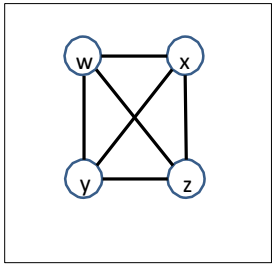


A Computer network

1. Simple Graph

A graph in which each edge connects two different vertices, and no two edges connect the same pair of vertices, then the graph is a **simple graph**.

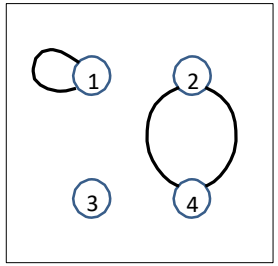
Example:

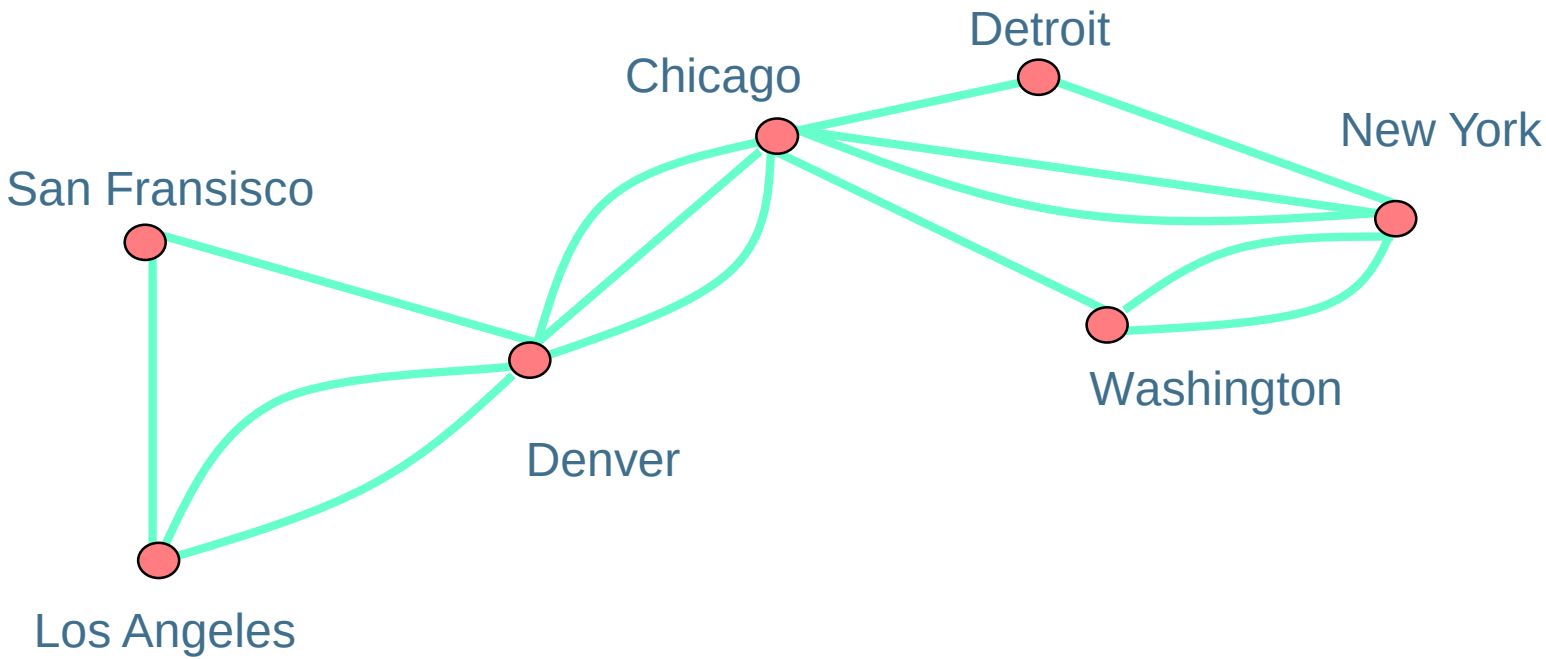


2. Multigraph

A graph in which more than one edge connecting the same pair of vertices. Graphs that may have multiple edges connecting the same vertices are called **Multigraphs**.

Example:

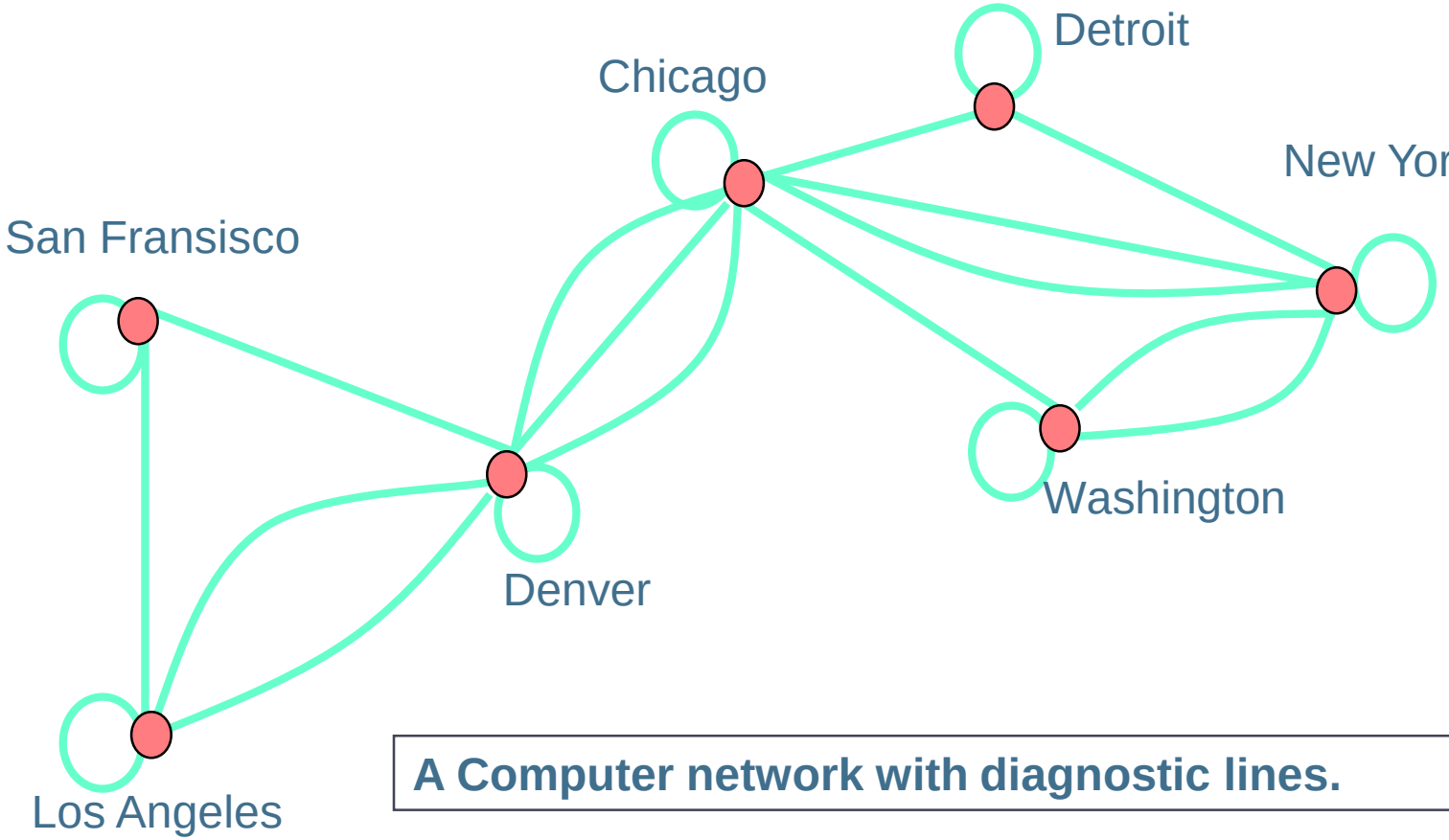




A Computer network with multiple lines.

3. Pseudo Graph

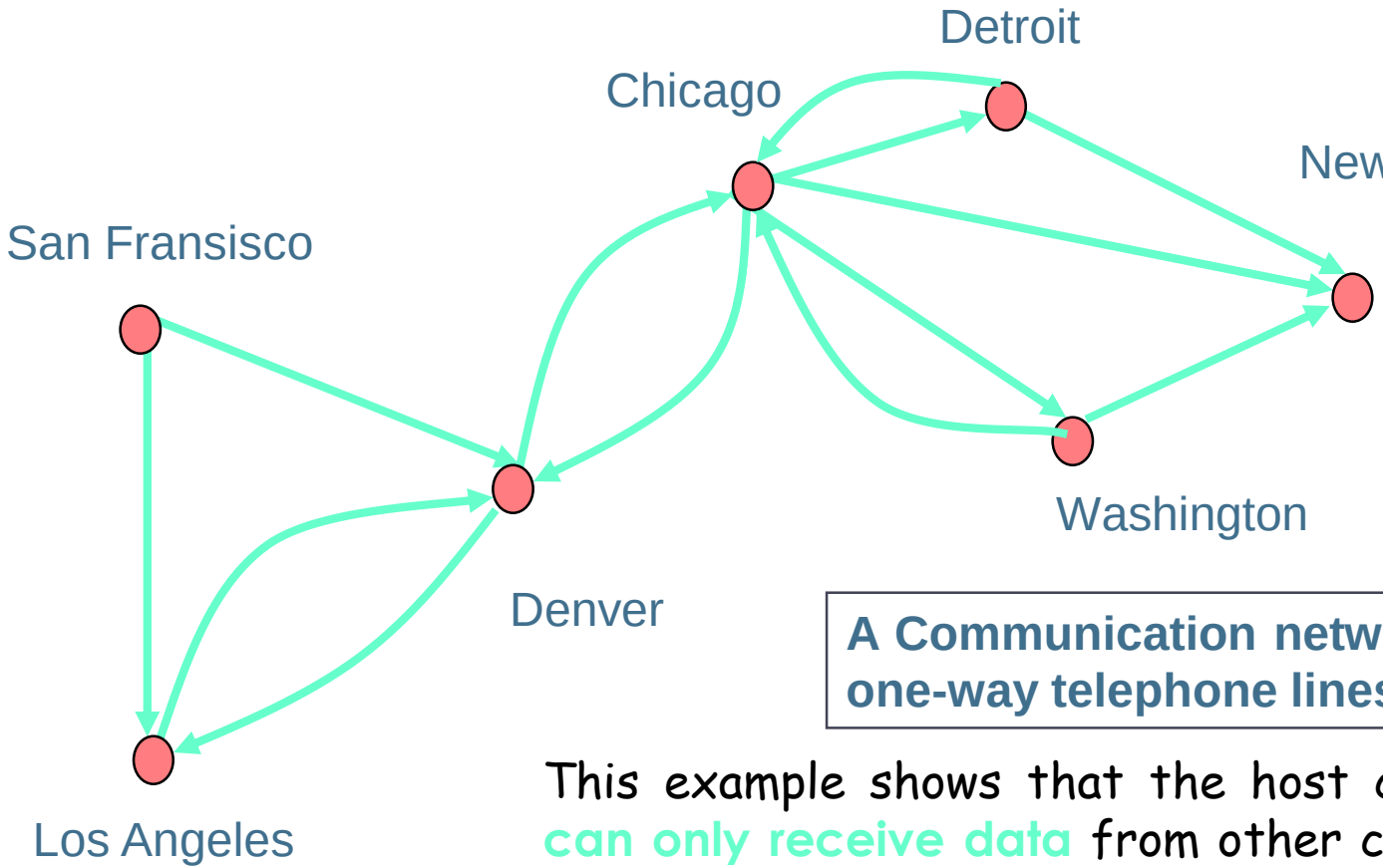
A **pseudo graph** G = (V,E) consists of a set V of vertices, a set E of edges, and a function f from E to {{u,v} | u, v ∈ V}. An edge is a loop if f(e) = {u,u} = {u} for some u ∈ V.



A Computer network with diagnostic lines.

4. Directed Graph

A **directed graph** G consists of a nonempty set V of vertices and a set E of directed edges, where each edge is associated with an ordered pair of vertices. We write G = (V, E) to denote the graph.



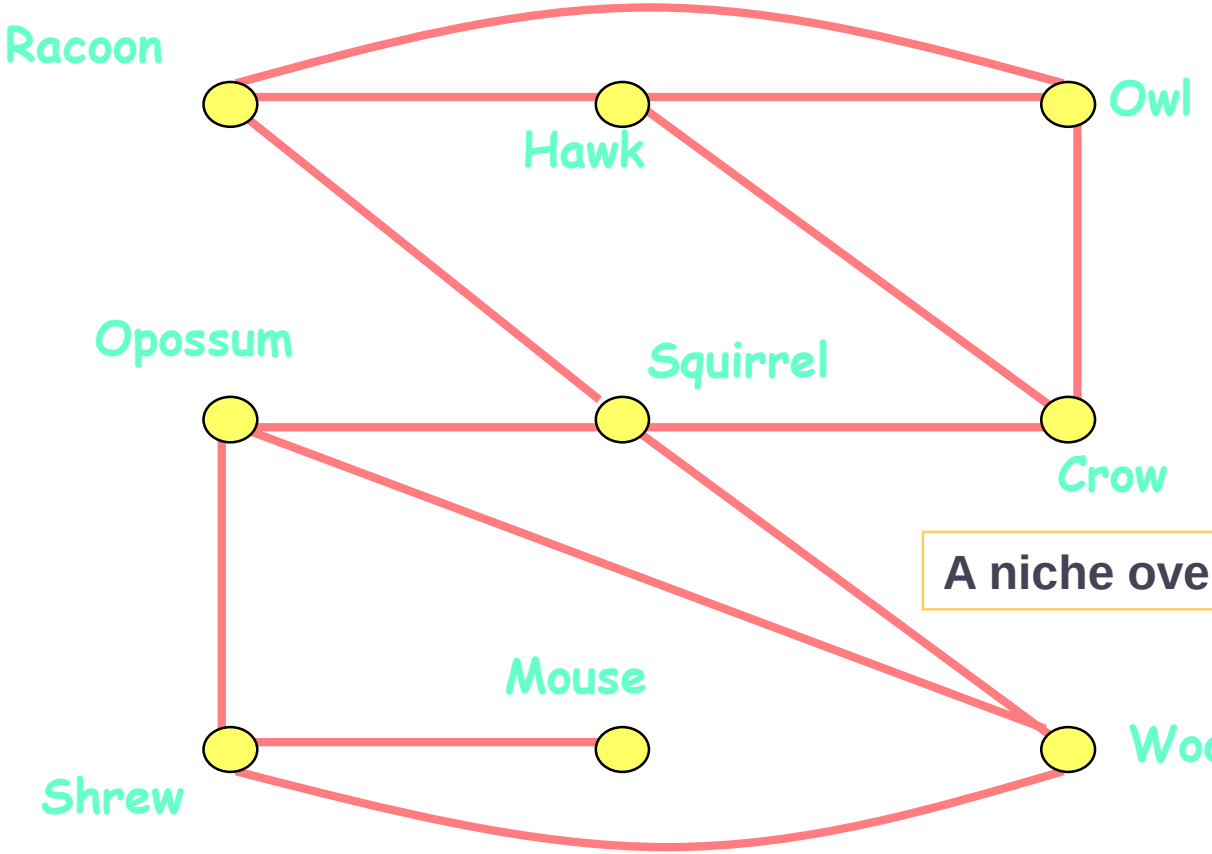
A Communication network with one-way telephone lines.

This example shows that the host computer can only receive data from other computer, it cannot emit.

5. Undirected Graph

An **undirected edge** connect two vertices if the two species represented by these vertices compete for food.

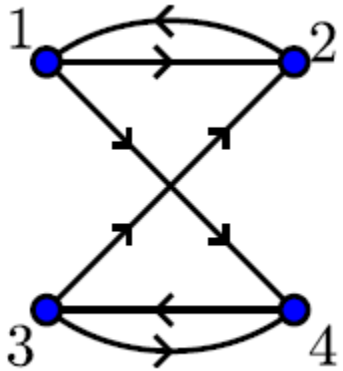
Example: Competition between species in an ecological system can be modeled using a **niche overlap graph**.



A niche overlap graph

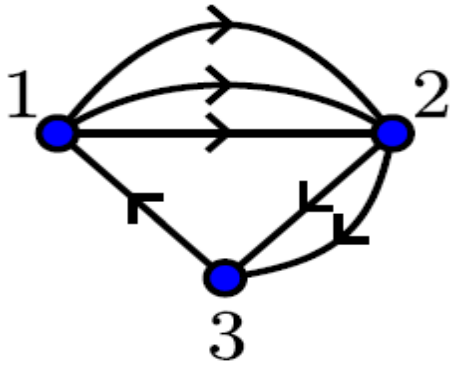
6. Simple Directed Graph

When a directed graph has no loops and has no multiple directed edges, it is called a simple directed graph.



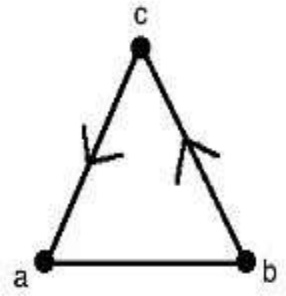
7. Directed Multigraphs

In some networks, multiple communication links between two data centers may be present. Directed graphs that may have multiple directed edges from a vertex to a second vertex are used to model such networks. We called such graphs are **directed multigraphs.**



8. Mixed Graph

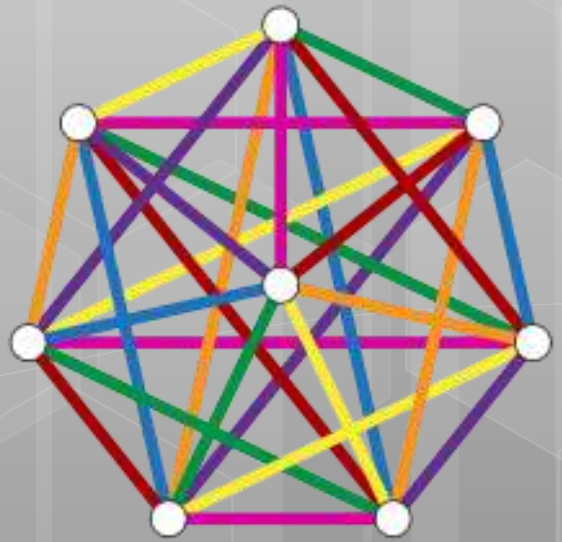
A graph with both **directed** and **undirected** edges is called a **Mixed graph**.



Graph Terminology

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed Graph	Directed and Undirected	Yes	Yes


Thank you!



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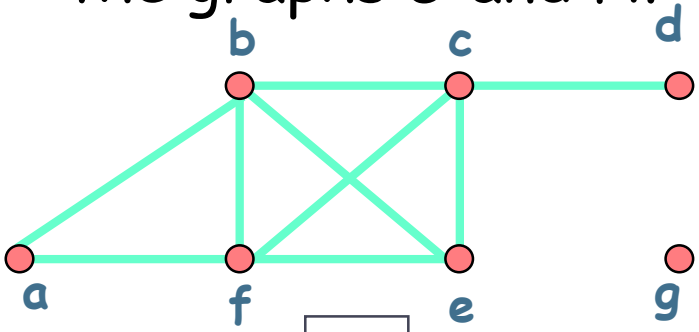
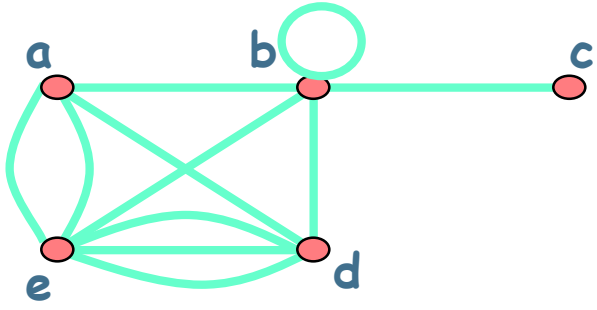
Degree of a vertex

The **degree of a vertex in an undirected graph** is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by **deg(v)**.

Example: What are the degrees of the vertices in the graphs G and H?



G



H

Solution:

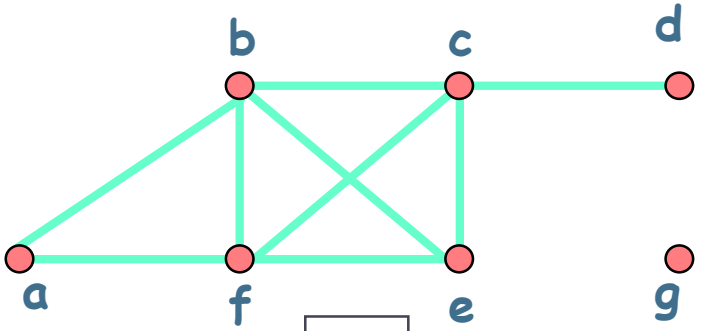
$$\text{In } G \begin{cases} deg(a) = 2 \\ deg(b) = deg(c) = deg(f) = 4 \\ deg(d) = 1 \\ deg(e) = 3 \\ deg(g) = 0 \end{cases}$$

$$\text{In } H \begin{cases} deg(a) = 4 \\ deg(b) = deg(e) = 6 \\ deg(c) = 1 \\ deg(d) = 5 \end{cases}$$

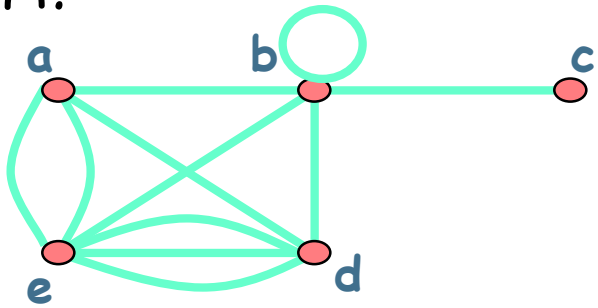
Neighborhood of a vertex

The neighborhood (neighbor set) of a vertex v in a undirected graph, denoted N(v) is the set of vertices adjacent to v.

Example: What are the neighborhood of the vertices in the graphs G and H?



G



H

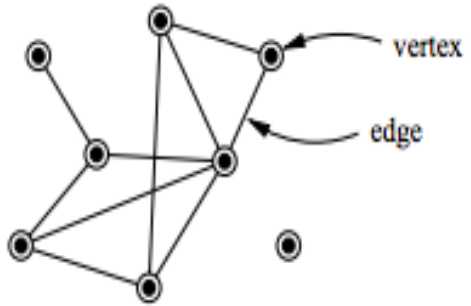
Solution:

$$\text{In G}\begin{cases} \text{N(a)} = \{\text{b, f}\} \\ \text{N(b)} = \{a, c, f, e\} \\ \text{N(c)} = \{d, e, f, b\} \\ N(\text{d}) = \{c\} \\ \text{N(e)} = \{\text{b, c, f}\} \\ \text{N(f)} = \{\text{a, b, c, e}\} \\ \text{N(g)} = \{\} \end{cases}$$

$$In\ \text{H}\begin{cases} \text{N(a)} = \{\text{b, e, d}\} \\ \text{N(b)} = \{c, e, d, b, a\} \\ N(\text{c}) = \{\text{b}\} \\ \text{N(d)} = \{\text{e, a, b}\} \\ \text{N(e)} = \{\text{a, d, b}\} \end{cases}$$

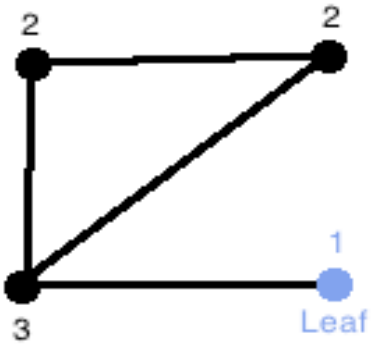
Isolated vertex

An **isolated vertex** is a vertex with degree **zero**; that is, a vertex that is not an endpoint of any edge (the example image illustrates one isolated vertex).



Pendant vertex

A **leaf vertex** (also **pendant vertex**) is a vertex with degree one.



Handshaking Theorem

Let G = (V,E) be an undirected graph with e edges. Then

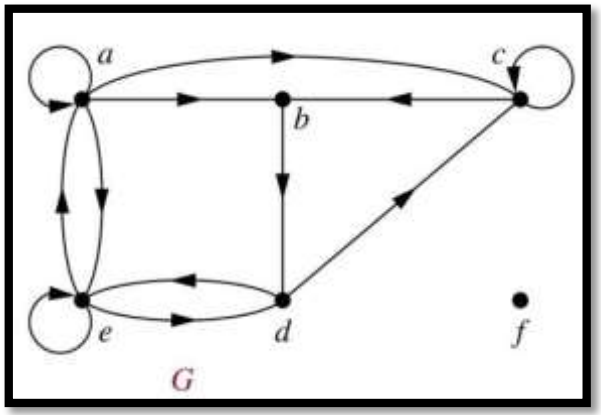
$$2e = \sum_{v \in V} \deg(v)$$

(Note that this applies even if multiple edges & loops are present.)

Example: How many edges are there in a graph with ten vertices each of degree 6 ?

Solution: Since the sum of the degrees of the vertices is 6*10 = 60 $\Rightarrow$ 2e = 60. Therefore, e = 30

Example: Find the in-degree and the out-degree of each vertex in the graph G.

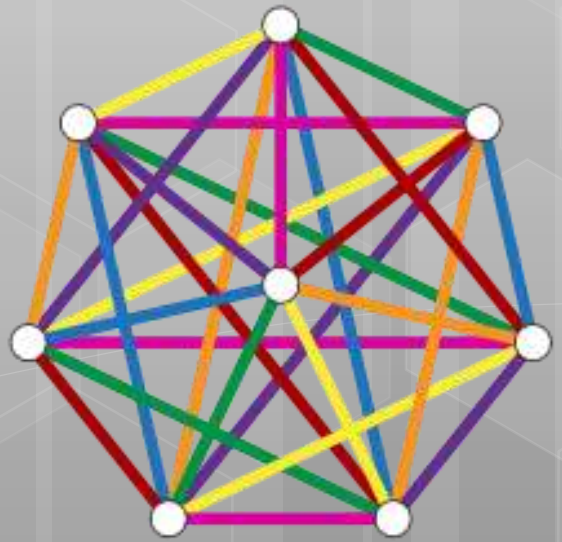


Solution: The in-degree of G are: $\deg^-(a) = 2$, $\deg^-(b) = 2$, $\deg^-(c) = 3$ $\deg^-(d) = 2$, $\deg^-(e) = 3$, and $\deg^-(f) = 0$.
The out-degree of G are: $\deg^+(a) = 4$, $\deg^+(b) = 1$, $\deg^+(c) = 2$, $\deg^+(d) = 2$, $\deg^+(e) = 3$, and $\deg^+(f) = 0$

Let G = (V,E) be a graph with directed edges. Then

$$\sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}(v) = |E|.$$


Thank you!



Graph Theory

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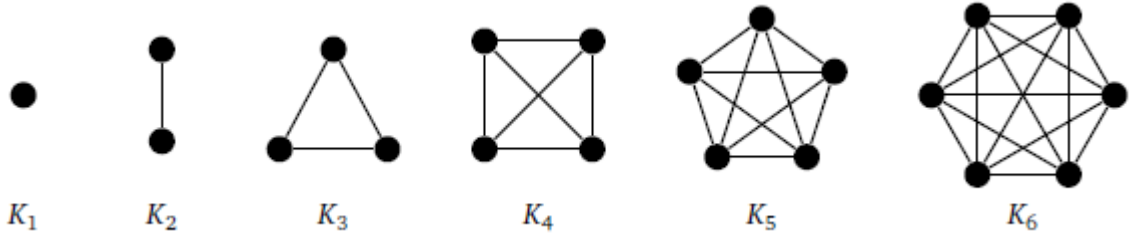
Complete Graph

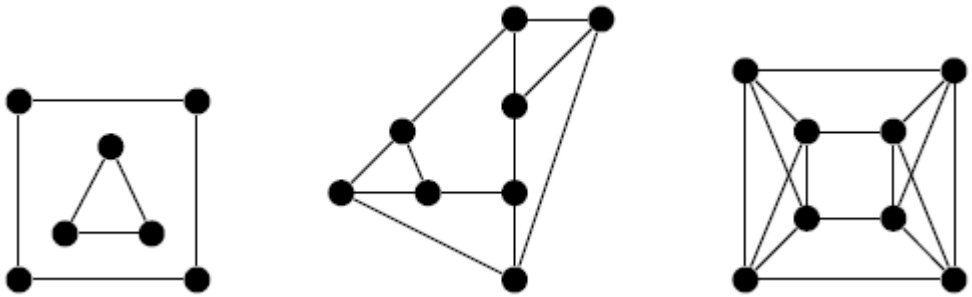
A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices. The graphs K_n for n=1, 2, 3, 4, 5, 6.

Examples :



Regular Graph

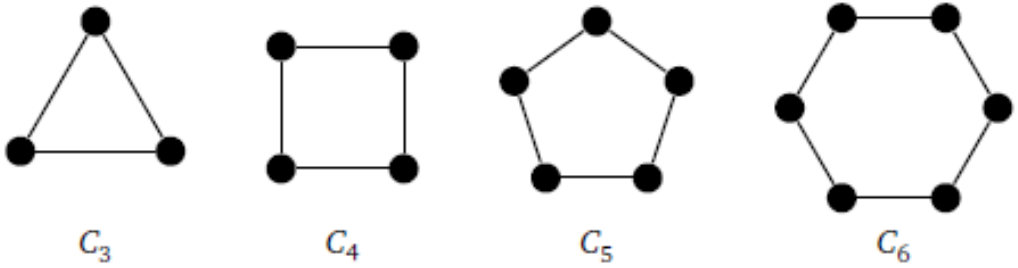
A regular graph is one in which every vertex has the same degree. The term k-regular is used to denote a graph in which every vertex has degree k. There are many types of regular graphs. Shown below are a 2-regular, a 3-regular, and a 4-regular graph.



Cycle

A cycle C_n, $n \geq 3$, is a graph that consists of n vertices $v_1, v_2, \ldots, v_n$ and n edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$

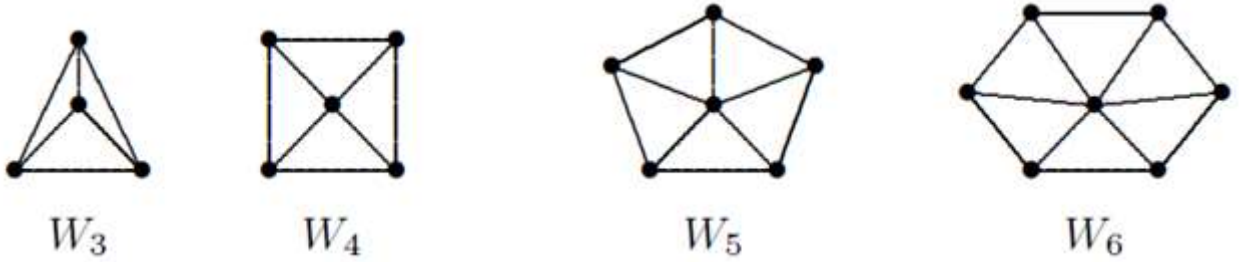
Examples :



Wheel

A wheel W_n, $n \geq 3$, is a graph that consists of a cycle C_n with an extra vertex that connects to each vertex in C_n.

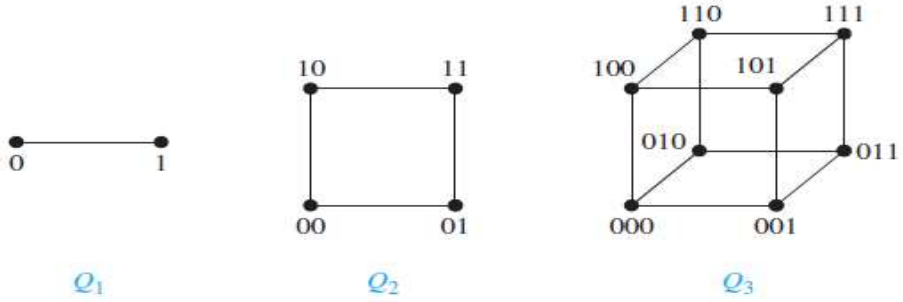
Examples :



N-dimensional Hypercube (n cube)

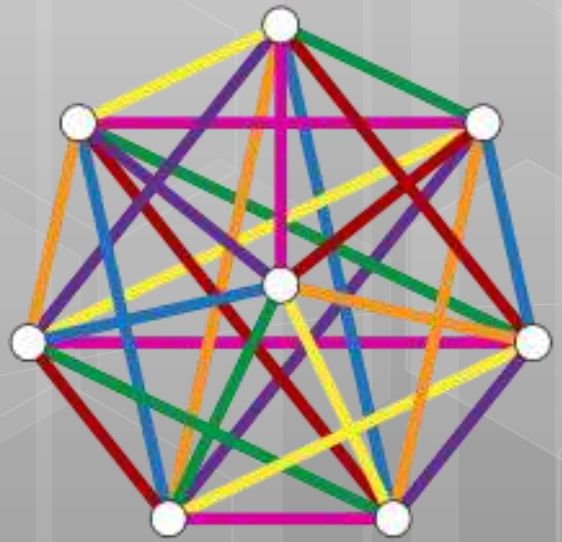
Hypercube graph Q_n is the graph formed from the vertices and edges of an n-dimensional hypercube. For instance, the cubical graph Q_3 is the graph formed by the 8 vertices and 12 edges of a three-dimensional cube. Q_n has 2^n vertices, $2^{n-1}n$ edges, and is a regular graph with n edges touching each vertex.

Examples :



The n-cube Q_n, $n = 1, 2, 3$.


Thank you!



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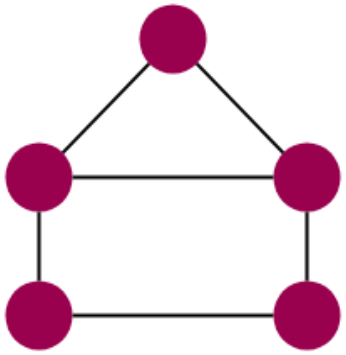
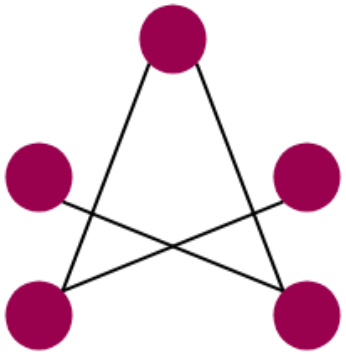
Complementary Graph

The complement of a graph G is a graph $\overline{G}$ with the same vertex set as G, but whose edges are exactly the opposite of the edges in G. That is, uv is an edge of $\overline{G}$ if and only if uv is not an edge of G. Shown below are a graph and its complement.

Example :



Graph G



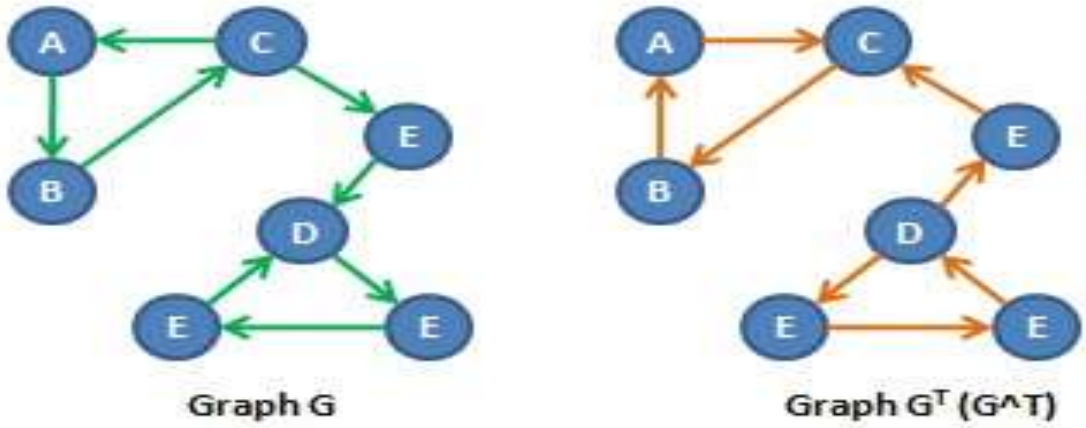
Complement Graph $\overline{G}$

One way to think of the complement is that the original plus the complement equals a complete graph. So we can get the complement by taking a complete graph and removing all the edges associated with the original graph.

Converse of a Graph

Converse, transpose or **reverse** of a directed **graph** G is another directed **graph** on the same set of vertices with all of the edges reversed compared to the orientation of the corresponding edges in G.

Example :

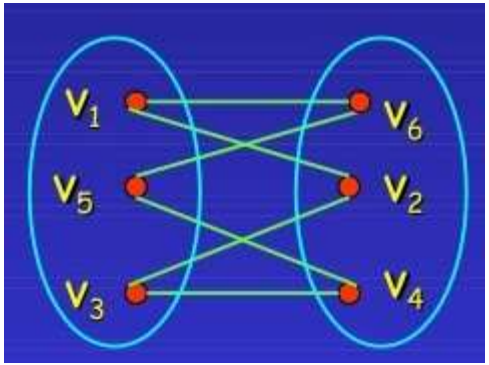


Graph G and its transpose G^T

Bipartite Graph

A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2. In other words, there are no edge in G connects either two vertices in V_1 or in V_2. When this condition holds, we call the pair (V_1, V_2) a bipartition of the vertex set V of G.

Example :

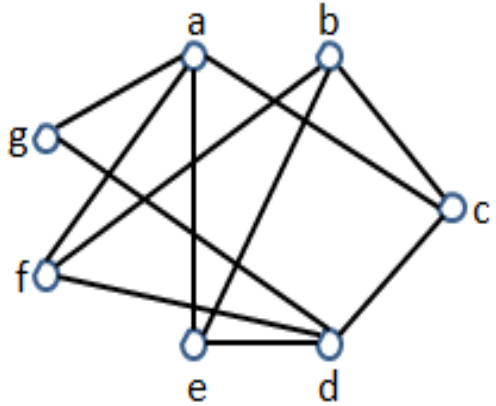


Its vertex set can be partitioned it into 2 sets:

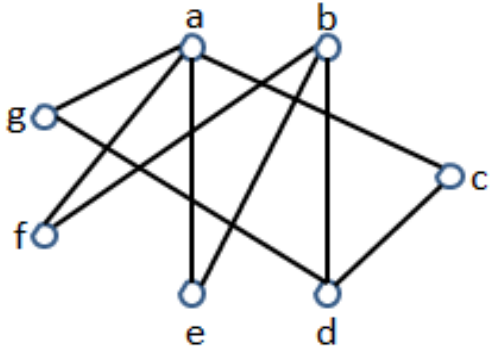
$V_1=\{v_1,v_3,v_5\}$ $V_2=\{v_2,v_4,v_6\}$

Examples

Which of the following is a bipartite graph?



G



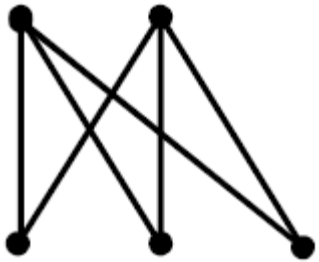
H

Theorem: A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex, so that no two adjacent vertices are assigned the same color.

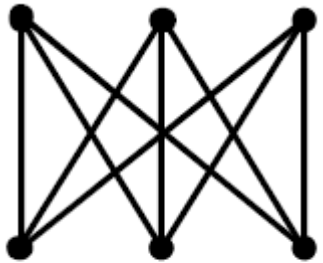
Complete Bipartite Graph

A complete bipartite graph Km,n is a graph that has its vertex set partitioned into two subsets of m and n vertices respectively with an edge between two vertices if and only if one vertex is in the first subset and other vertex is in the second subset.

Examples :

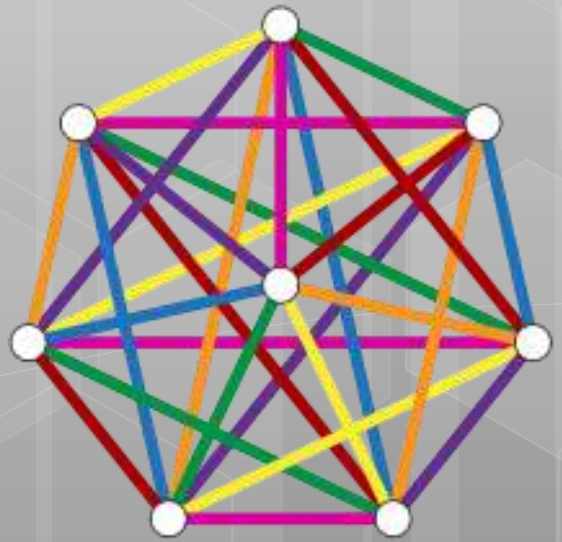


$K_{2,3}$



$K_{3,3}$


Thank you!



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Representing Graphs: Adjacency Lists

Definition: An *adjacency list* represents a graph (with no multiple edges) by specifying the vertices that are adjacent to each vertex.

Examples:

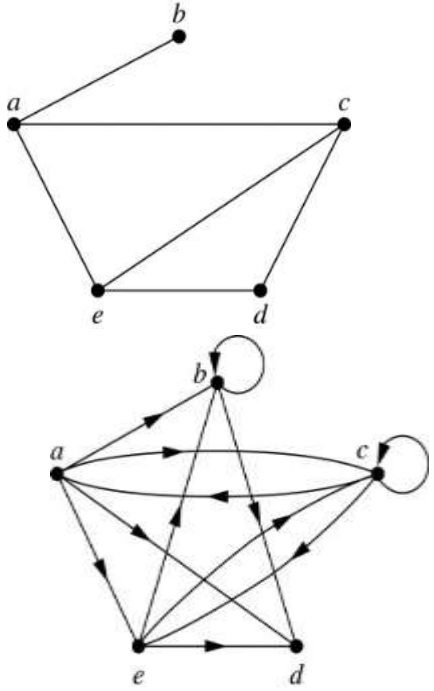


TABLE 1 An Adjacency List for a Simple Graph.

Vertex	Adjacent Vertices
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

TABLE 2 An Adjacency List for a Directed Graph.

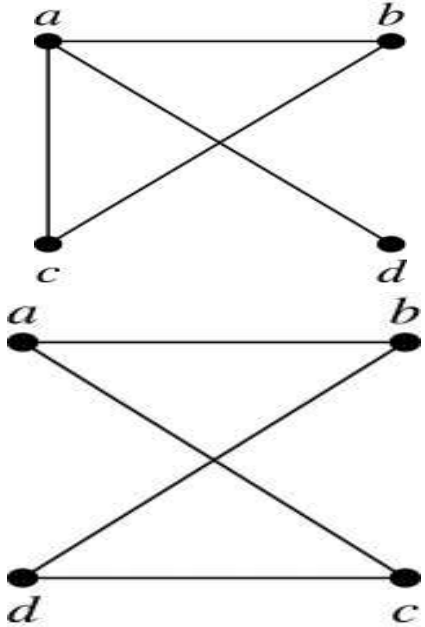
Initial Vertex	Terminal Vertices
a	b, c, d, e
b	b, d
c	a, c, e
d	
e	b, c, d

Representing Graphs: Adjacency Matrix

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge of } G, \\ 0 & \text{otherwise.} \end{cases}$$

Examples:

The vertex ordering is a, b, c, d.



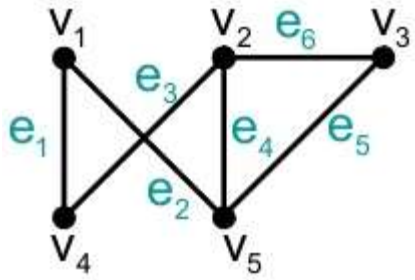
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Representing Graphs: Incidence Matrices

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i, \\ 0 & \text{otherwise.} \end{cases}$$

Example:

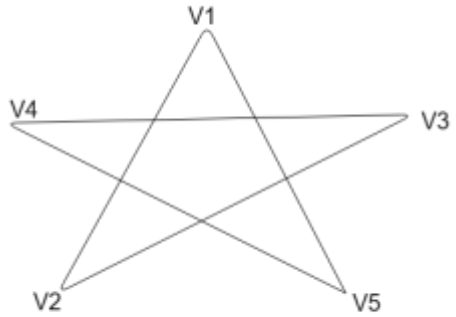


$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

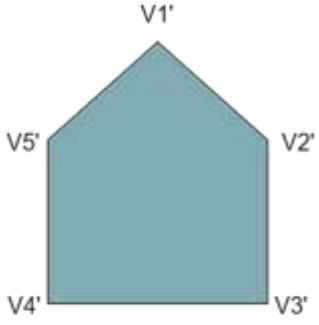
Isomorphism of Graphs

"The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic** if there exists a (one to one and onto) i.e. bijective function f from $V1$ to $V2$.

Example:



G



G'

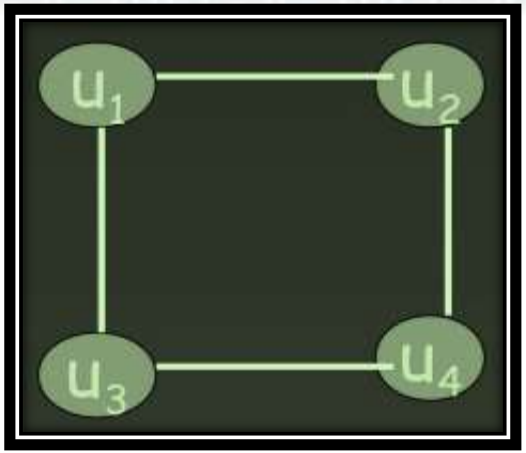
Let f be a bijective function from V to V'.

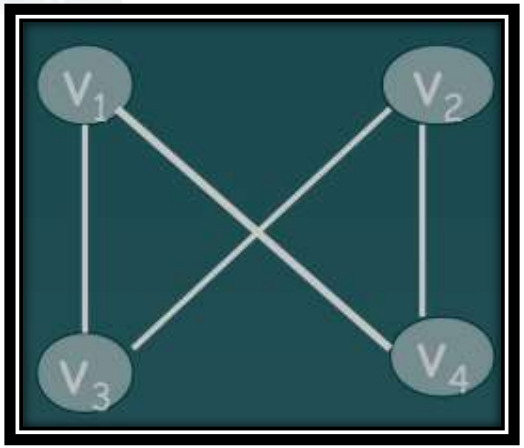
Let the correspondence between the graphs be-

$$v1' = f(v1)$$
$$v2' = f(v5)$$
$$v3' = f(v3)$$
$$v4' = f(v4)$$
$$v5' = f(v2)$$

Example:

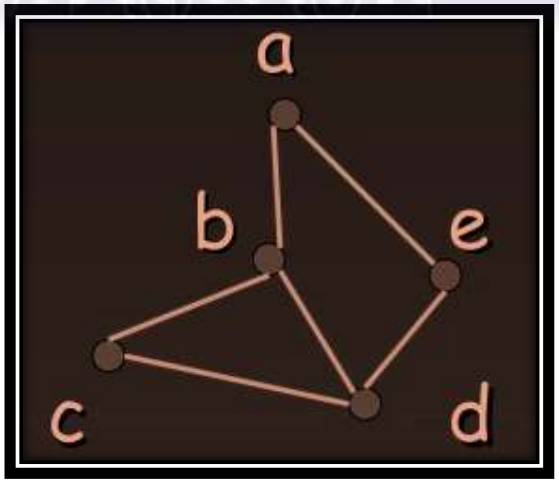
Representation example: G1 = (V1, E1) ,
G2 = (V2, E2)
$f(u_1) = v_1$, $f(u_2) = v_4$, $f(u_3) = v_3$, $f(u_4) = v_2$,

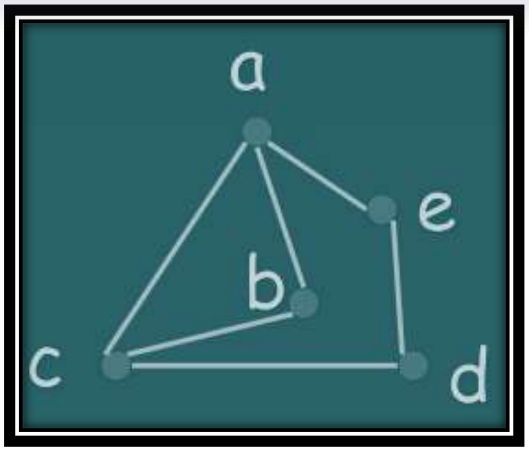




Example:

Solution: Yes, they are isomorphic, because they can be arranged to look identical. You can see this if in the right graph you move vertex b to the left of the edge {a, c}. Then the isomorphism f from the left to the right graph is: f(a) = e, f(b) = a,
f(c) = b, f(d) = c, f(e) = d.

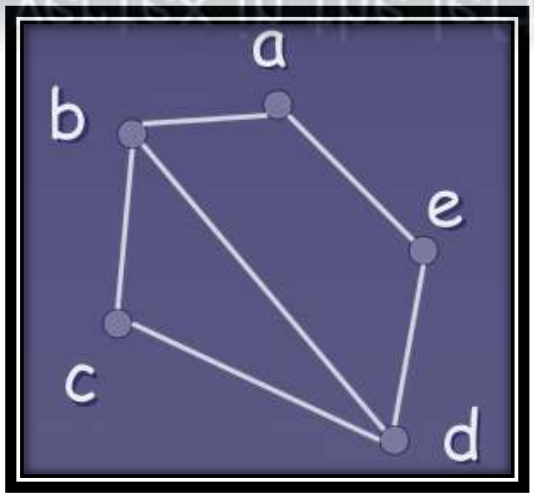


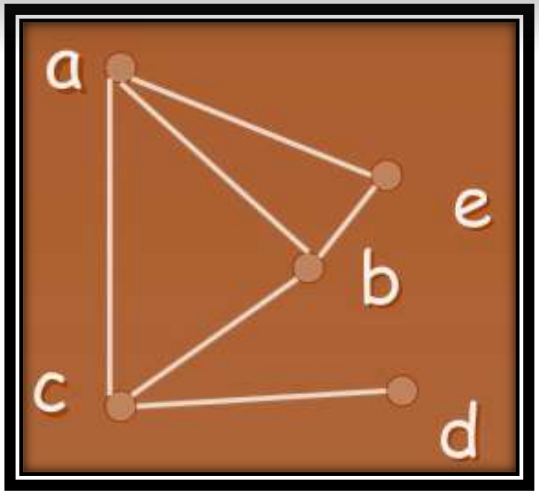


Example:

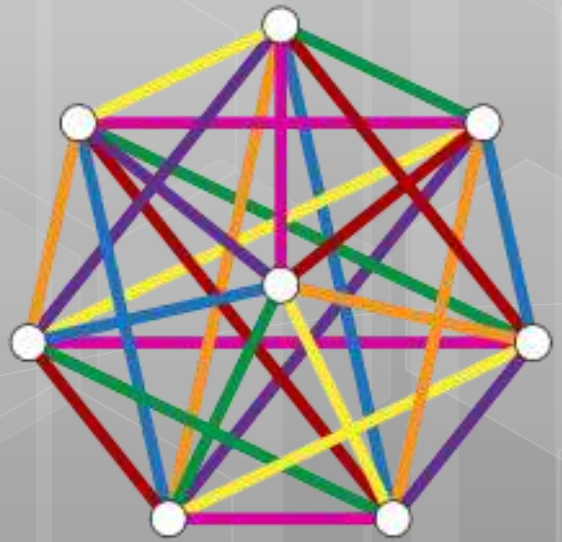
Solution: No, they are not isomorphic, because they differ in the degrees of their vertices.

Vertex d in right graph is of degree one, but there is no such vertex in the left graph.






Thank you!



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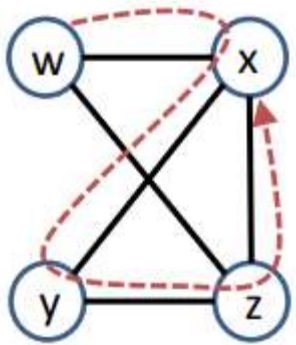
Path

A path is a sequence of edges that begins at a vertex, and travels from vertex to vertex along edges of the graph. The number of edges on the path is called the **length** of the path.

Example:

Consider the graph on the right. w→x→y→z→x corresponds to a path of length 4.

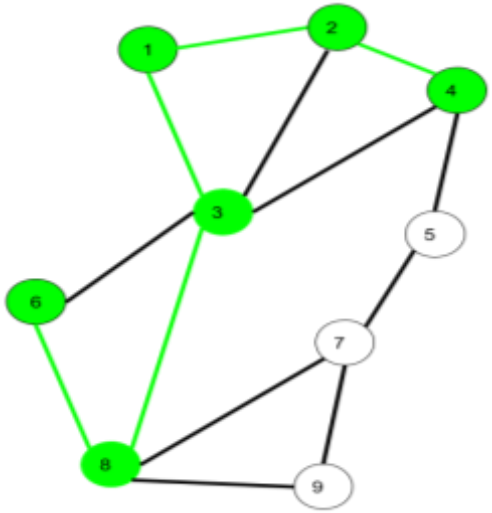


Path

It is a trail in which neither vertices nor edges are repeated i.e. if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge. As path is also a trail, thus it is also an open walk.

Vertex not repeated.
Edge not repeated.

Here 6->8->3->1->2->4 is a Path.

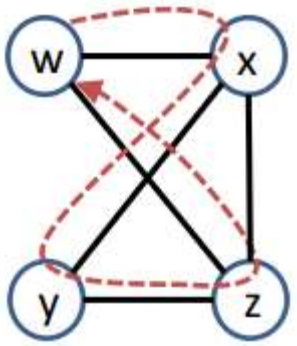


Circuit

If a path begins and ends at the same vertex, the path is also called a circuit.

Example:

Consider the graph on the right. w→x→y→z→w corresponds to a circuit of length 4.

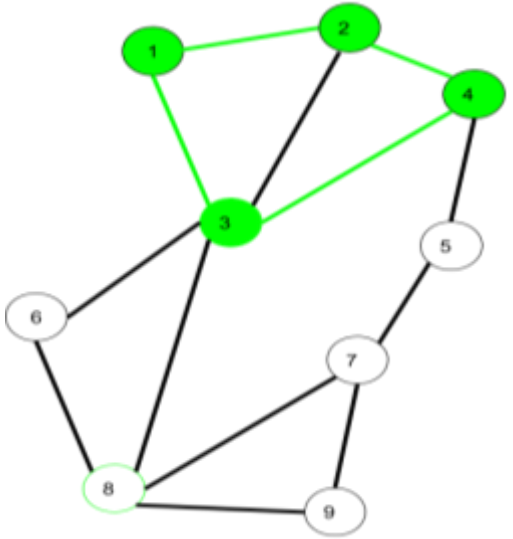


Circuit/Cycle

Traversing a graph such that we do not repeat a vertex nor we repeat a edge but the starting and ending vertex must be same i.e. we can repeat starting and ending vertex only then we get a cycle.

Vertex not repeated.
Edge not repeated.

Here 1->2->4->3->1 is a cycle.



Undirected Graph

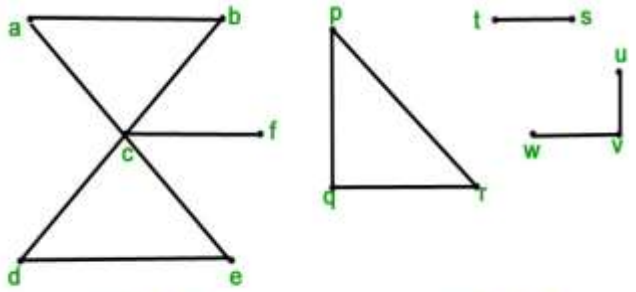
A undirected graph is said to be connected if there is a path between every pair of distinct vertices of the graph. An undirected graph that is not connected is called disconnected.

Connected Component

A connected component of a graph G is a connected subgraph of G that is not a proper subgraph of another connected subgraph of G.

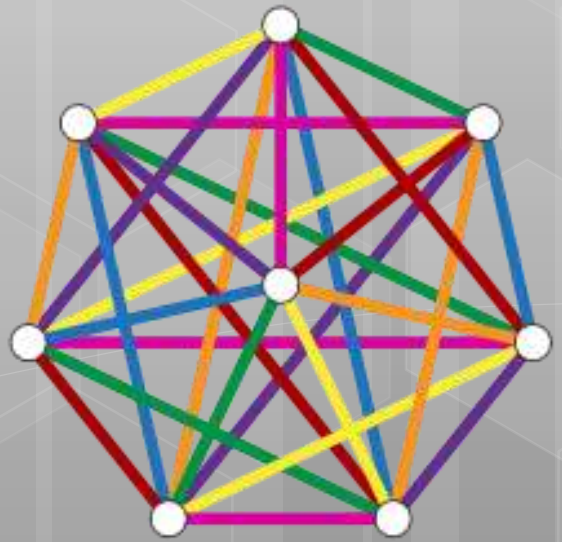
For example, in the following diagram, graph G_1 is connected and graph G_2 is disconnected. Since G_1 is connected there is only one connected component. But in case of G_2 there are three connected components.

.



Graph G1 Graph G2


Thank you!



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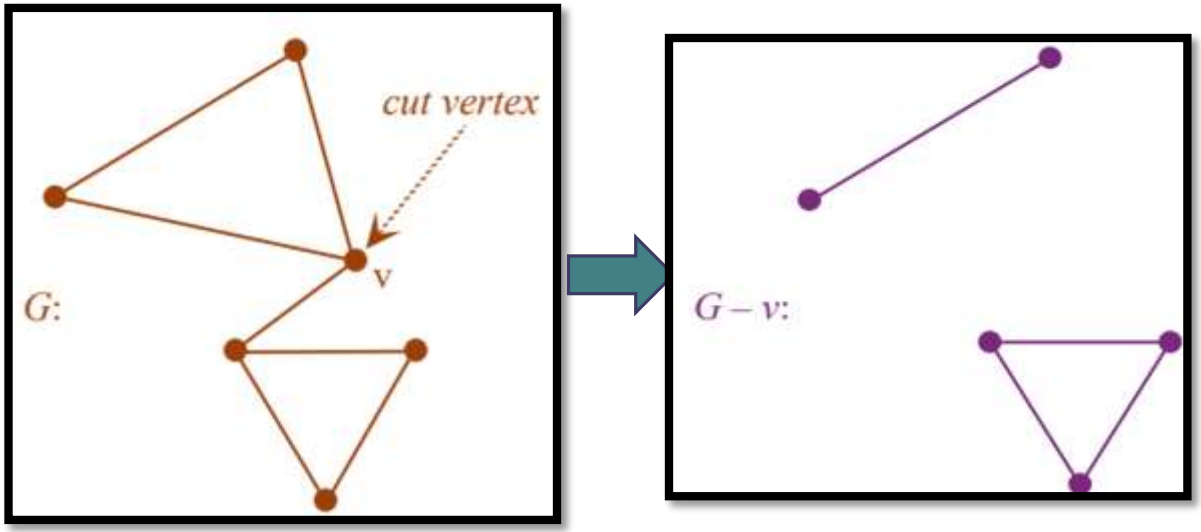
Cut Vertices

A vertex v of a graph G is a cut vertex or an articulation vertex of G if the graph G−v consists of a greater number of components than G.

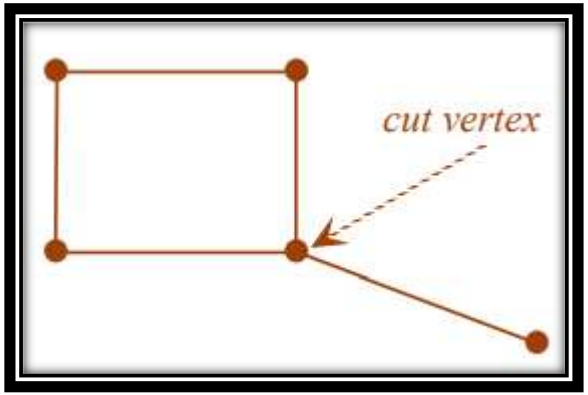
Example. v is a cut vertex of the graph below:

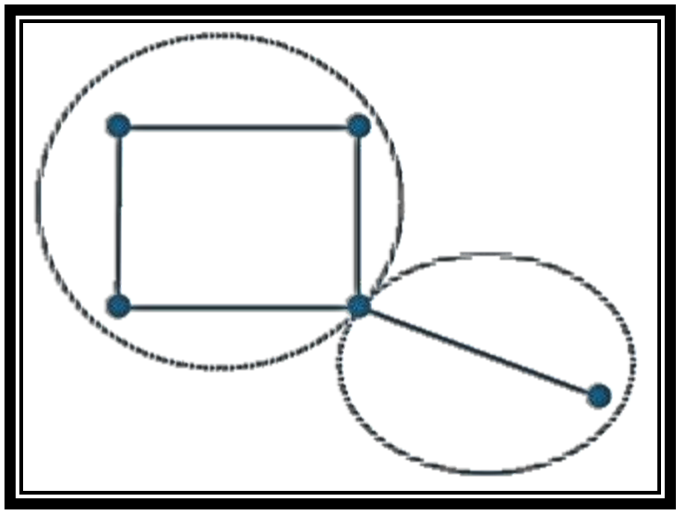


A graph is separable if it is not connected or if there exists at least one cut vertex in the graph. Otherwise, the graph is nonseparable.

Example. The graph below is separable:

.

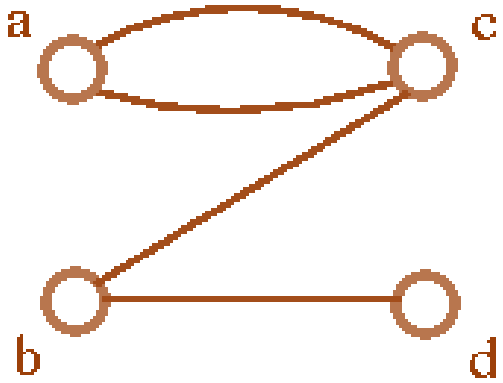




Cut Edges/Bridge

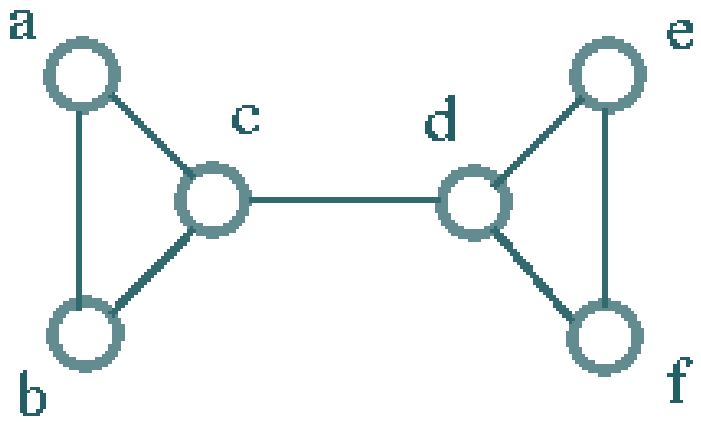
An edge whose removal produces a graph with more connected components than in the original graph is called cut edge.

Example



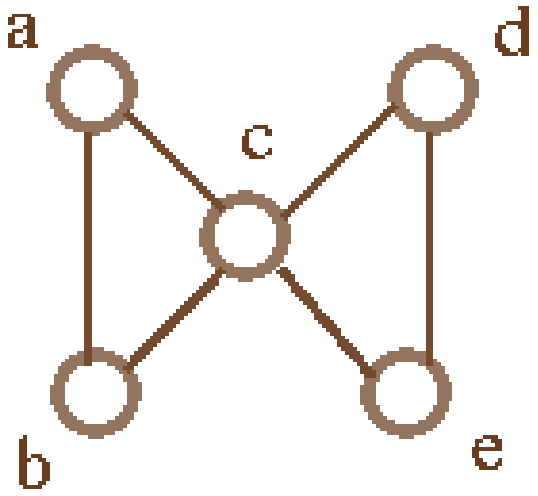
The graph G1 can be split up into two components by removing one of the edges {b, c} or {b, d}. Therefore, edge {b, c} or {b, d} is a bridge.

Example



The graph G2 can be disconnected by removing a single edge, {c, d}. Therefore, edge {c, d} is a bridge.

Example

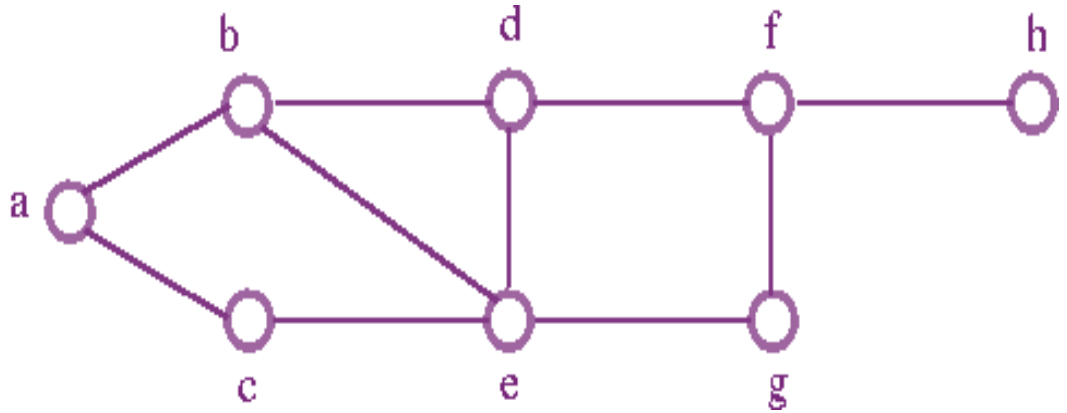


The graph G3 cannot be disconnected by removing a single edge, but the removal of two edges (such as {a, c} and {b, c}) disconnects it.

Vertex Cut

A cut-vertex is a single vertex whose removal disconnects a graph. It is important to note that the above definition breaks down if G is a complete graph, since we cannot then disconnects G by removing vertices.

Example:

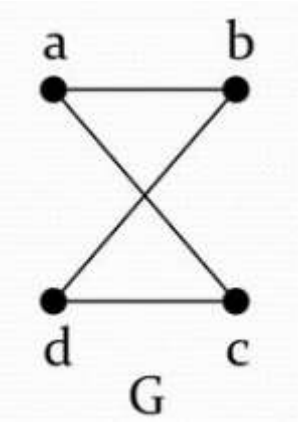


Vertex cutset of G is {b, e}.

Counting path between vertices

- How many path of length four are there from a to d in the simple graph G?

Example:

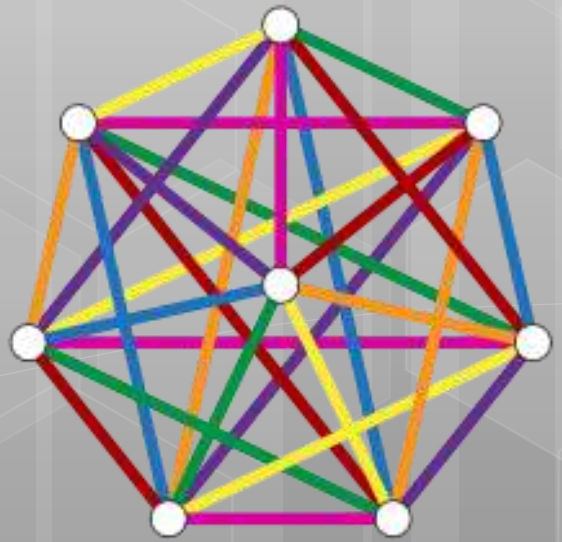


$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \qquad A^4 = \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 0 \\ 8 & 0 & 0 & 8 \end{bmatrix}$$

a→b→a→b→d
a→b→a→c→d
a→b→d→b→d
a→b→d→c→d
a→c→a→b→d
a→c→a→c→d
a→c→d→b→d
a→c→d→c→d

There are eight paths from a to d.


Thank you!



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Difference between simple path and Euler path

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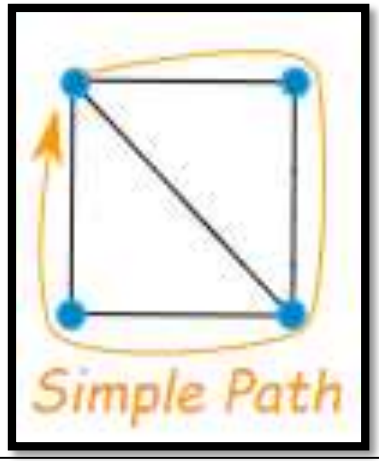
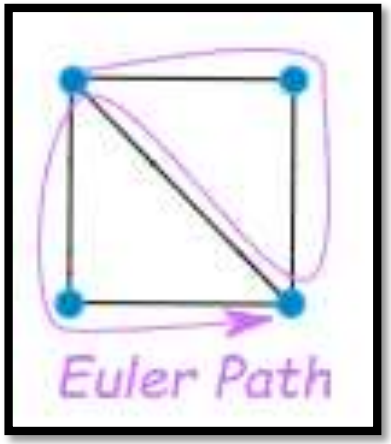


A route around a graph that visits **every vertex** once is called a **simple path**.

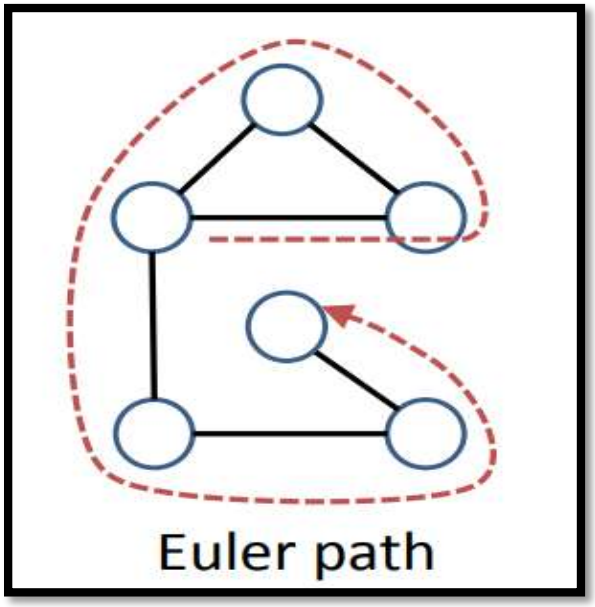


A route around a graph that visits **every edge** once is called an **Euler path**.

Euler Path

An Euler path in a multigraph G is a simple path that contains every edge of G. (So, every edge occurs exactly once in the path.)

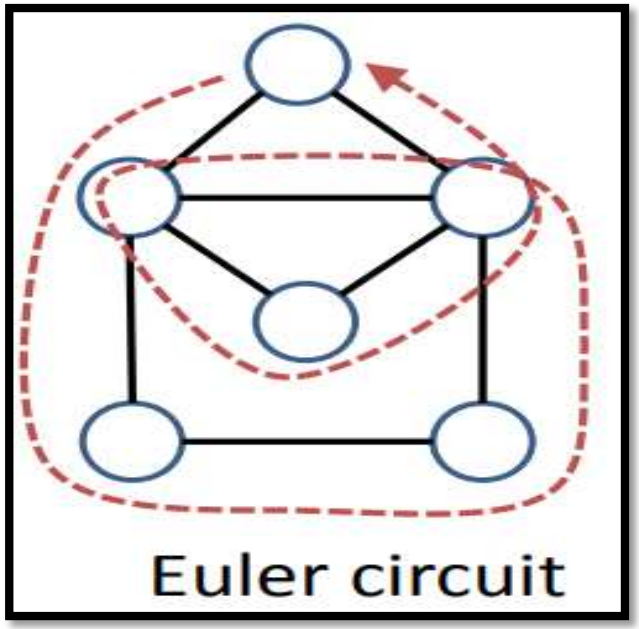
Example:



Euler Circuit

An Euler circuit in an multigraph G is a simple circuit that contains every edge of G. (So, every edge occurs exactly once in the circuit.)

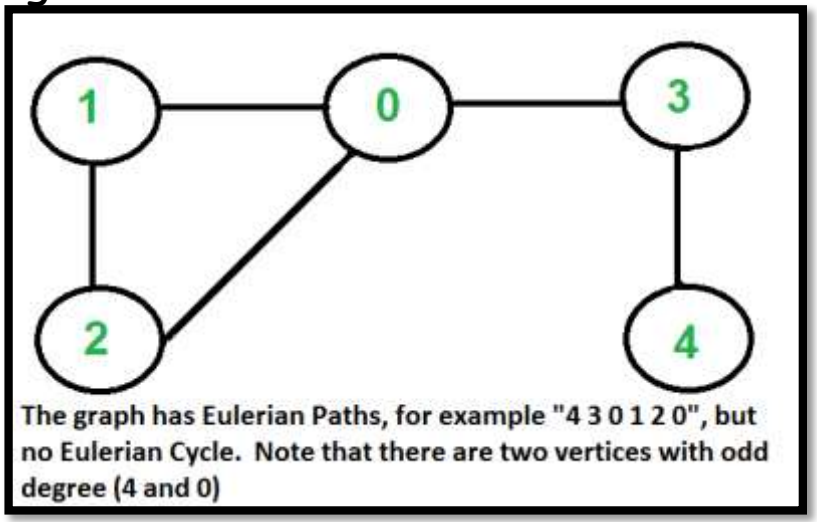
Example:



Important points related to Euler Path and Circuit

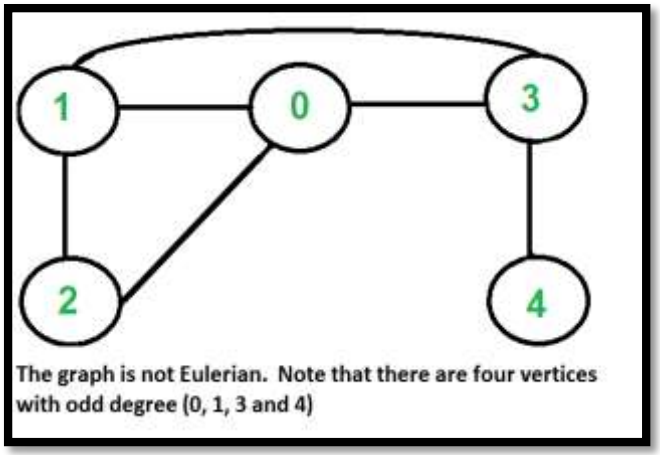
- A graph has an Euler circuit if and only if the degree of every vertex is even.
- A graph has an Euler path if and only if there are at most two vertices with odd degree.

Example:

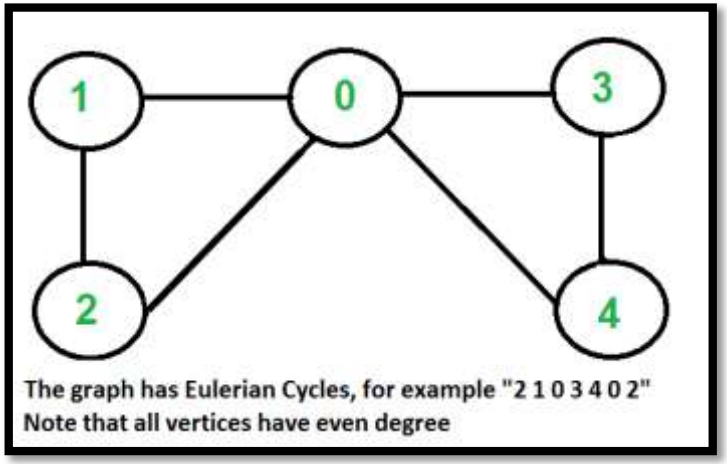


The graph has Eulerian Paths, for example "4 3 0 1 2 0", but no Eulerian Cycle. Note that there are two vertices with odd degree (4 and 0)

Examples:



The graph is not Eulerian. Note that there are four vertices with odd degree (0, 1, 3 and 4)

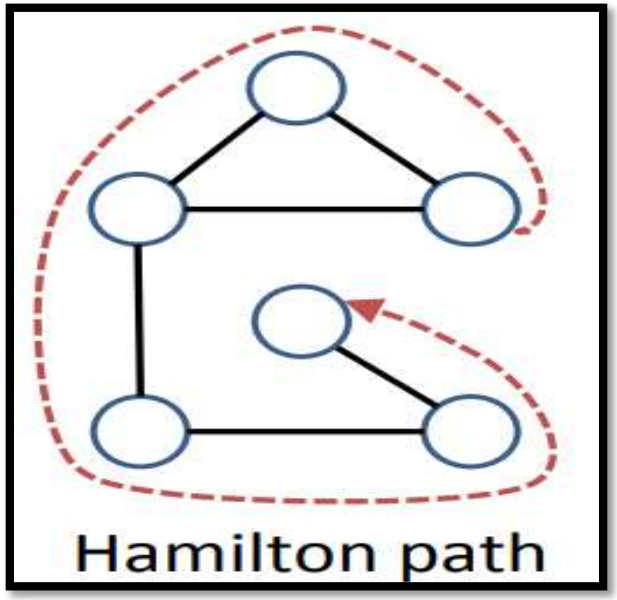


The graph has Eulerian Cycles, for example "2 1 0 3 4 0 2"
Note that all vertices have even degree

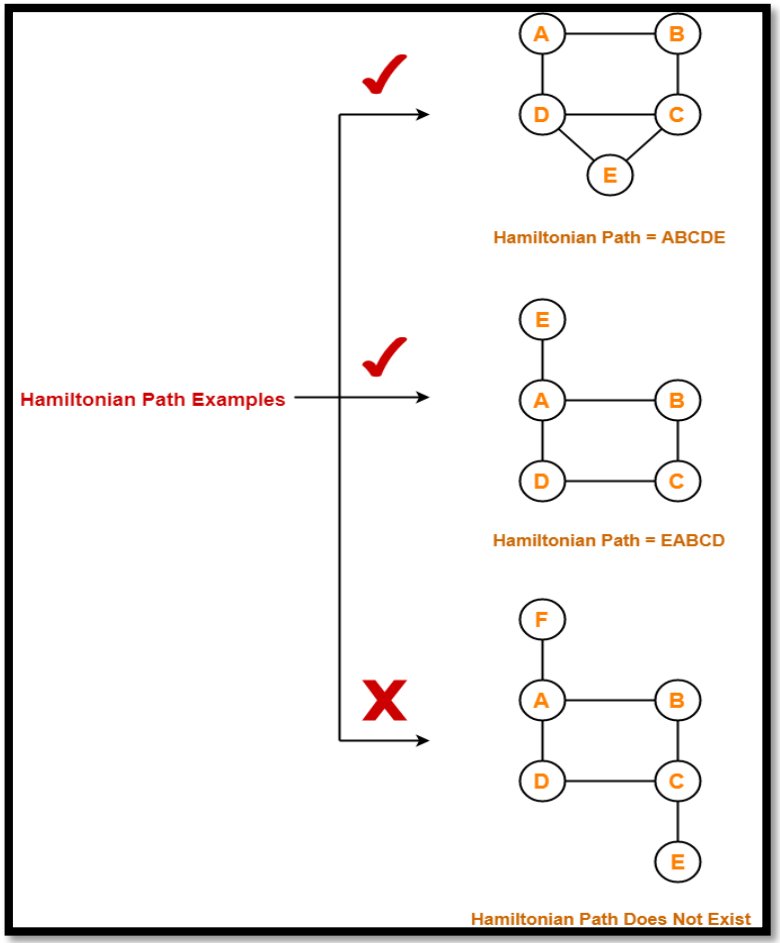
Hamiltonian Path

- A Hamiltonian path in a (undirected) graph G is a simple path that visits every vertex exactly once. (In other words, it is a tidy path that visits every vertex.)

Example:



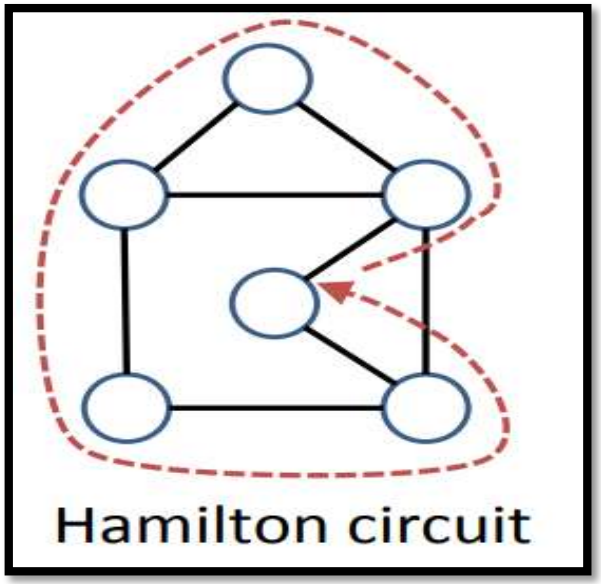
Examples:


Hamiltonian Path Examples
A
B
D
C
E
Hamiltonian Path = ABCDE
E
A
B
D
C
Hamiltonian Path = EABCD
F
A
B
D
C
E
Hamiltonian Path Does Not Exist

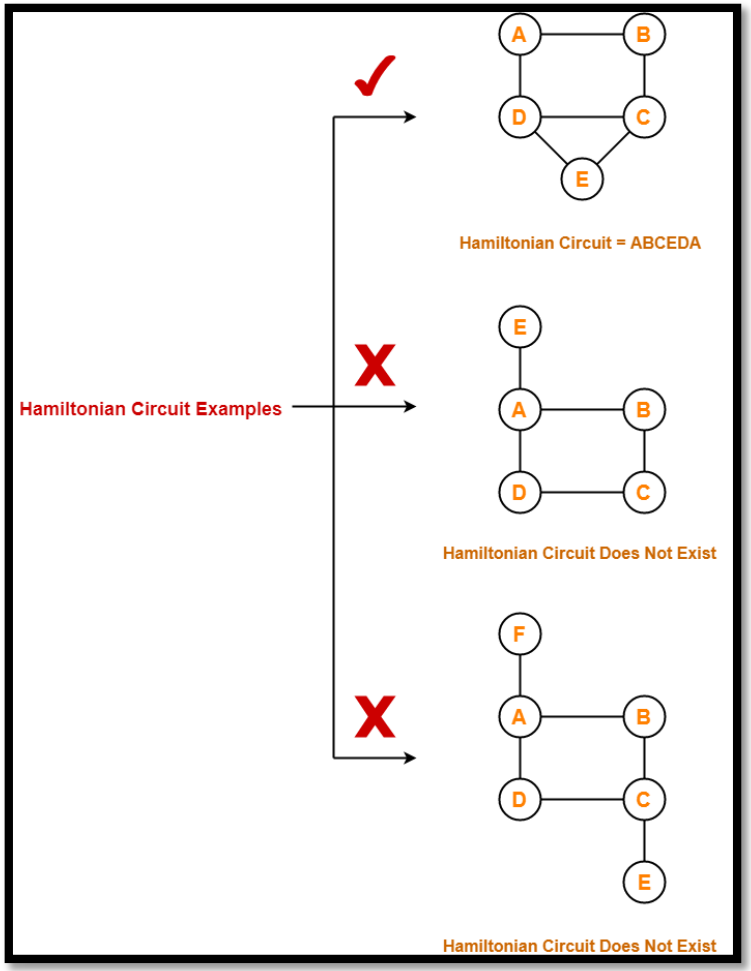
Hamiltonian Circuit

A Hamiltonian circuit in a (undirected) graph G is a simple circuit that passes through every vertex exactly once (except for the common start and end vertex, which is seen exactly twice).

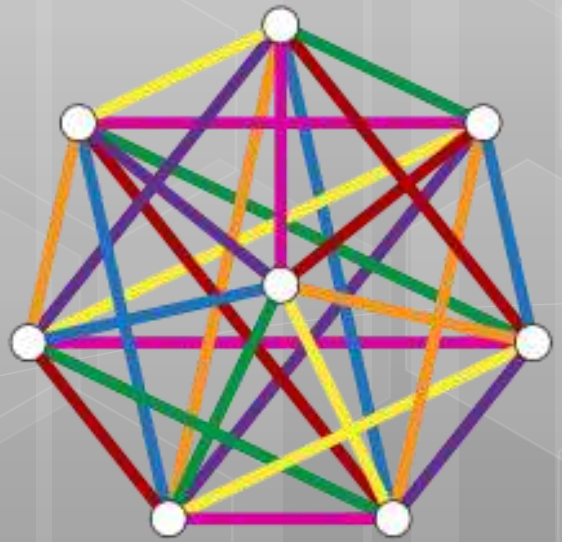
Example:



Examples:


Hamiltonian Circuit Examples
A
B
C
D
E
Hamiltonian Circuit = ABCEDA
E
A
B
D
C
Hamiltonian Circuit Does Not Exist
F
A
B
D
C
E
Hamiltonian Circuit Does Not Exist


Thank you!



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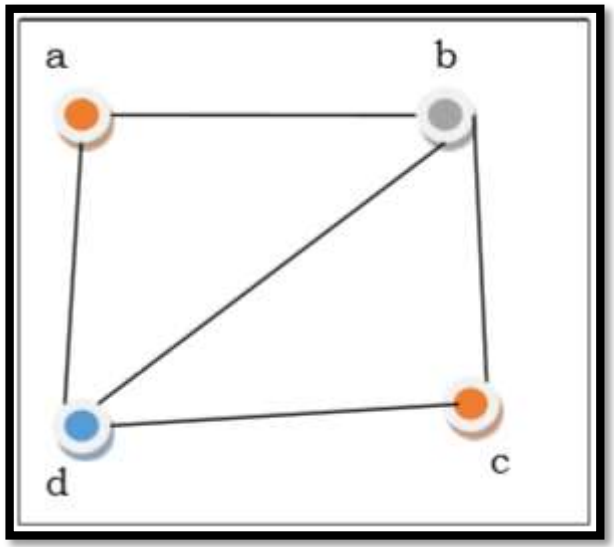
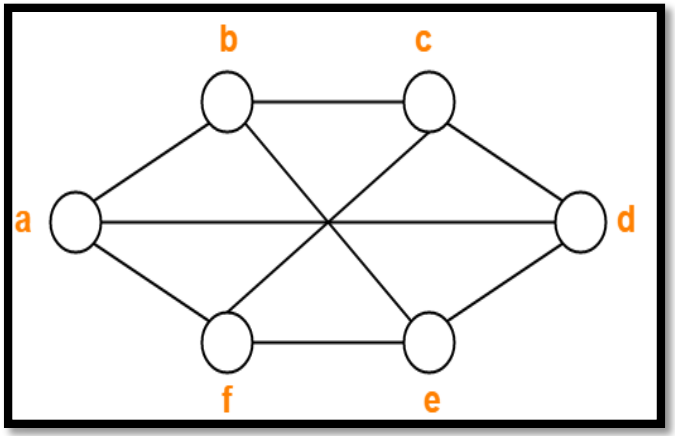
Graph Coloring

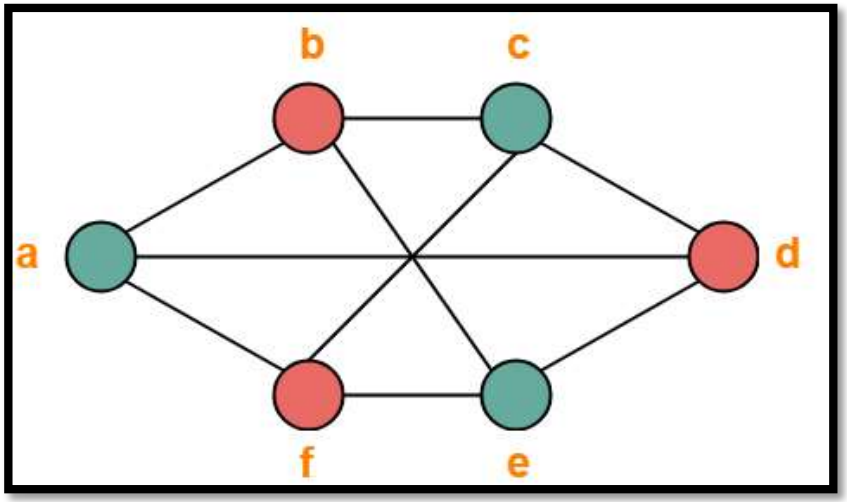
Graph coloring is the procedure of assignment of colors to each **vertex** of a **graph** G such that no adjacent vertices get same **color**. The smallest number of colors required to **color** a **graph** G is called its **chromatic** number of that **graph**. It is denoted by $\chi(G)$.

Example:

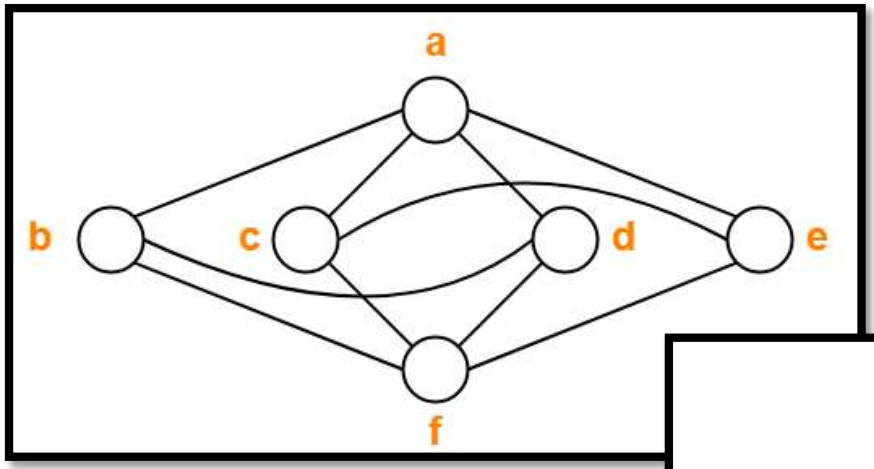


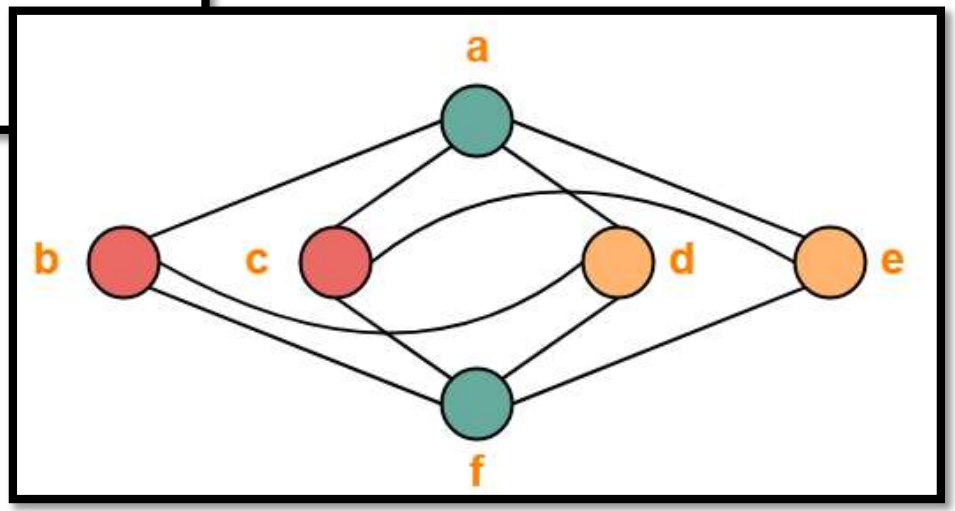
What is the chromatic number for the following graph?



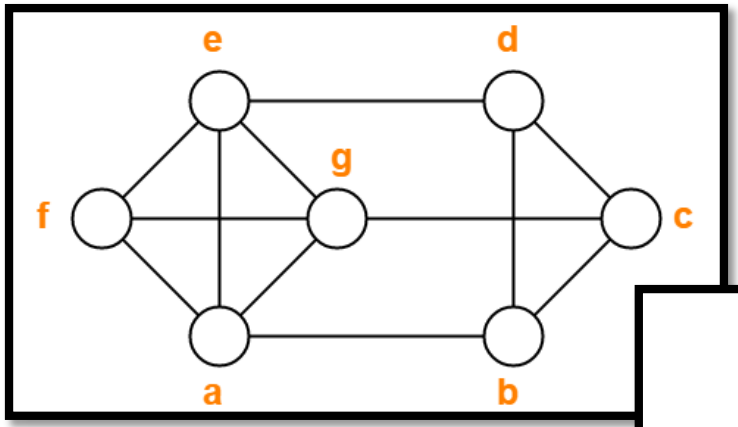


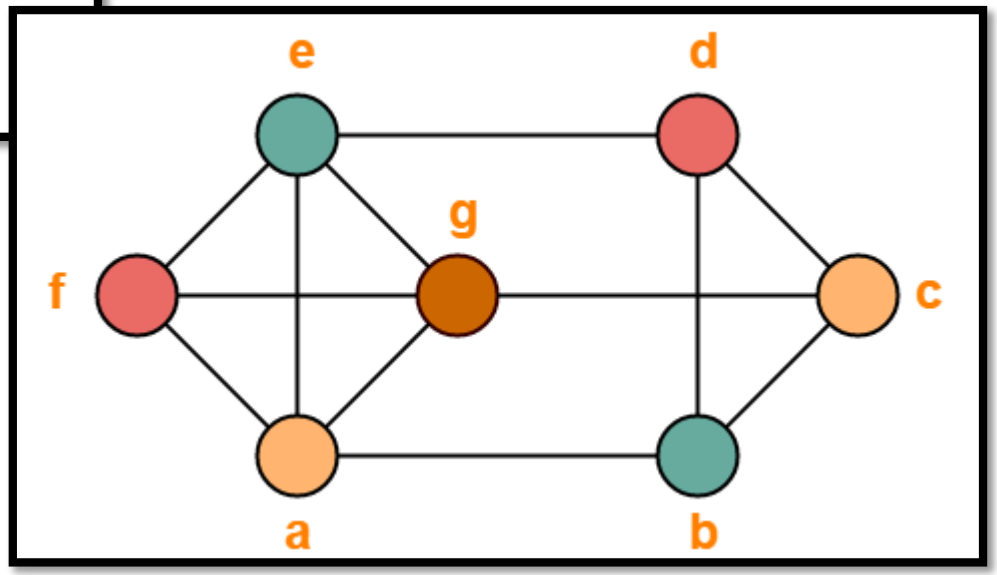
What is the chromatic number for the following graph?



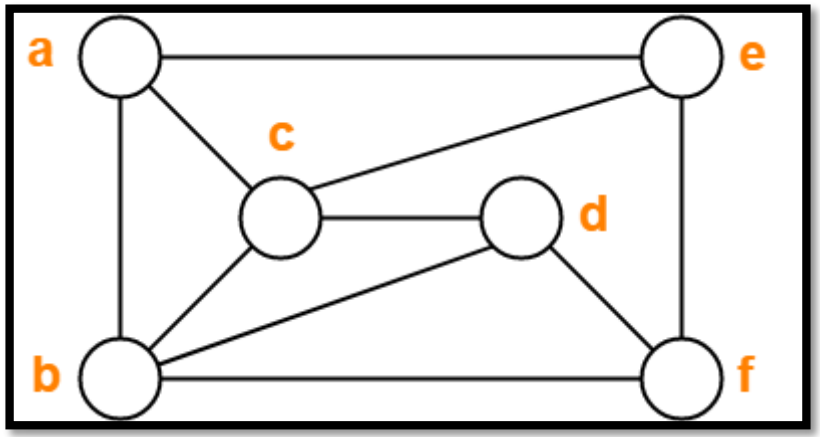


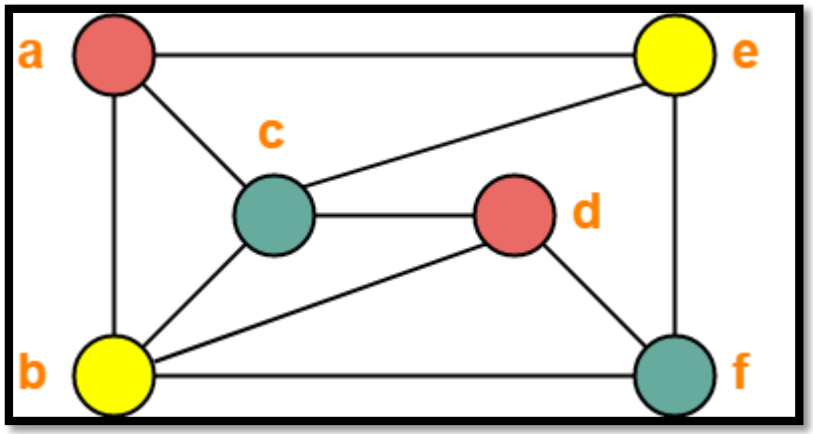
What is the chromatic number for the following graph?

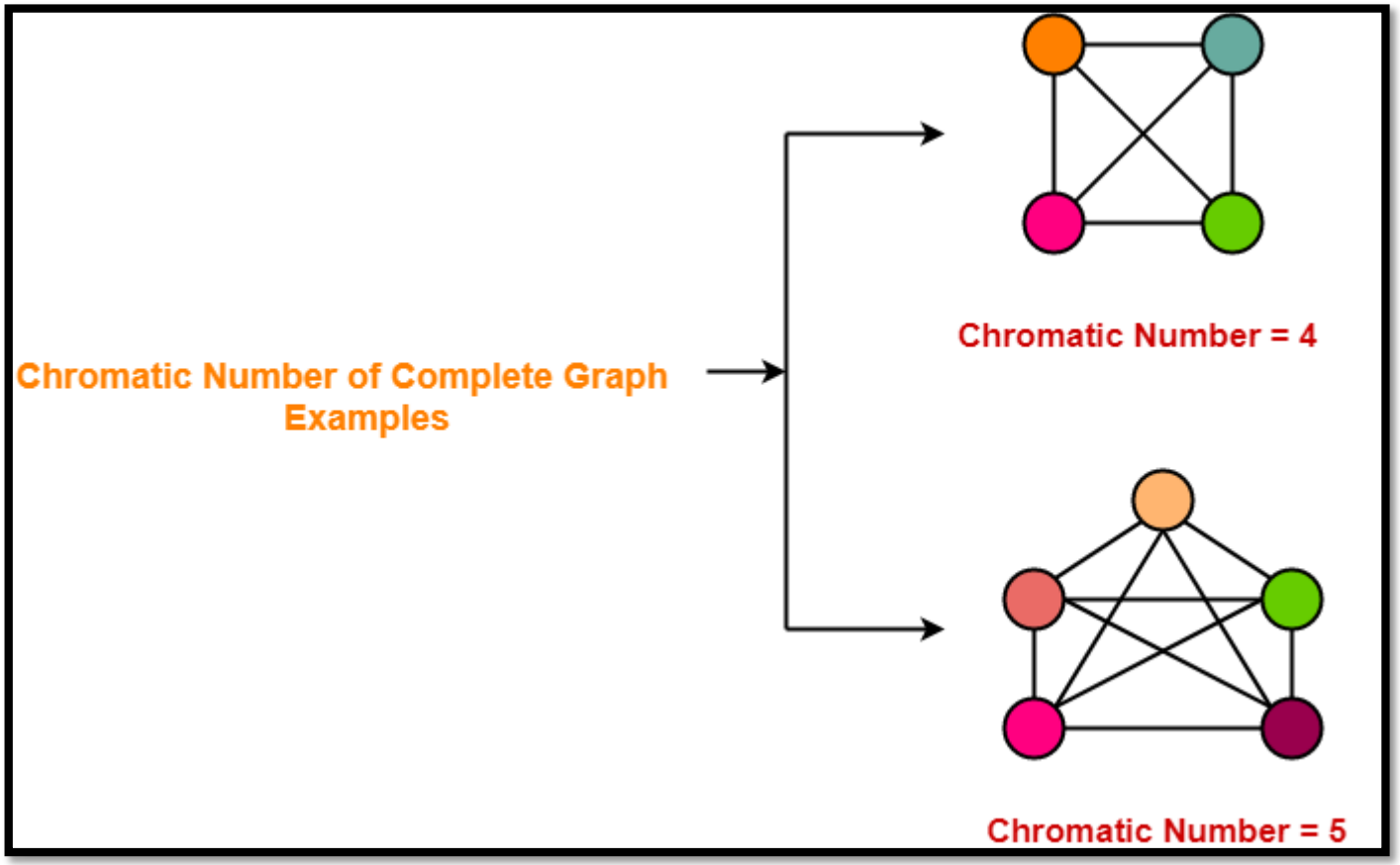


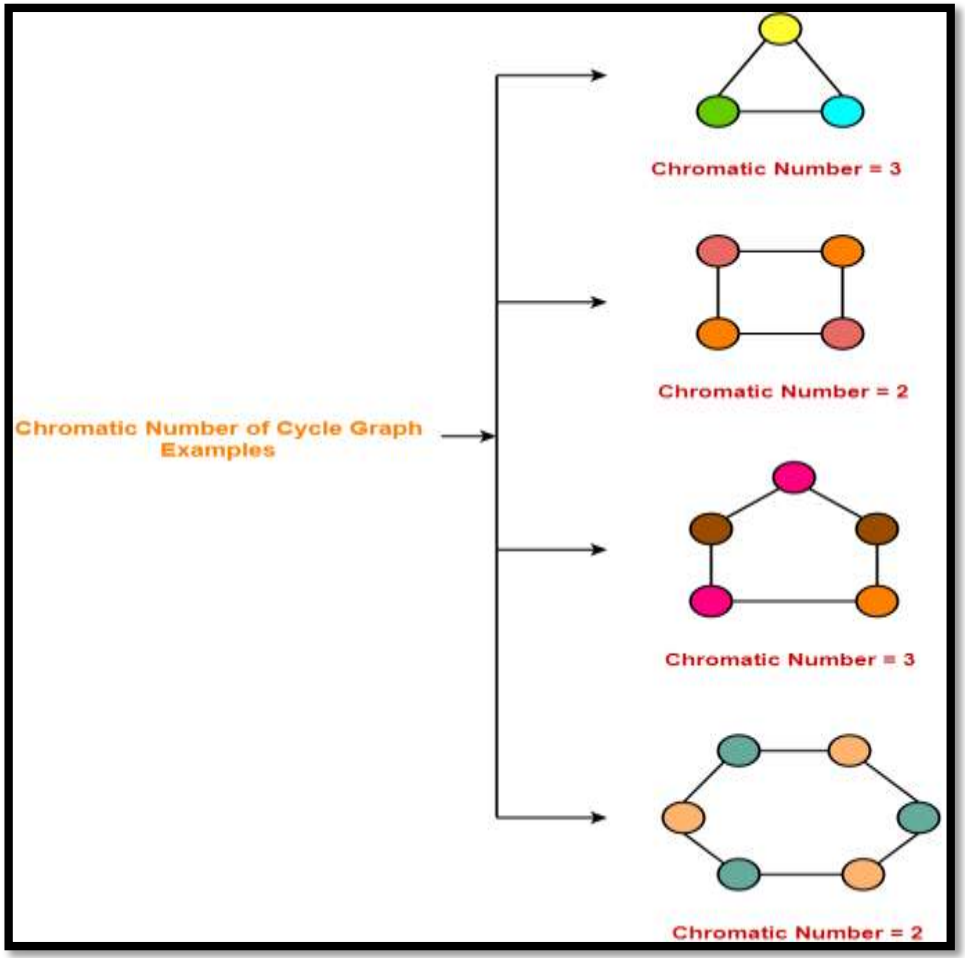


What is the chromatic number for the following graph?

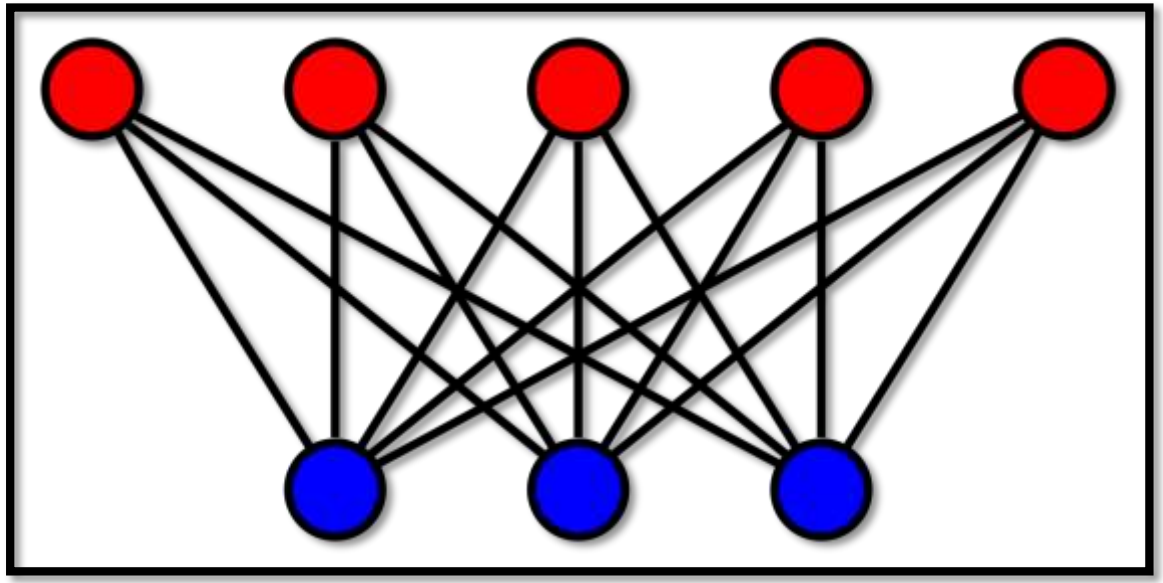





Chromatic Number of Complete Graph Examples
Chromatic Number = 4
Chromatic Number = 5


Chromatic Number of Cycle Graph Examples
Chromatic Number = 3
Chromatic Number = 2
Chromatic Number = 3
Chromatic Number = 2

What is the chromatic number of the complete bipartite graph Km,n where m and n are positive integers?

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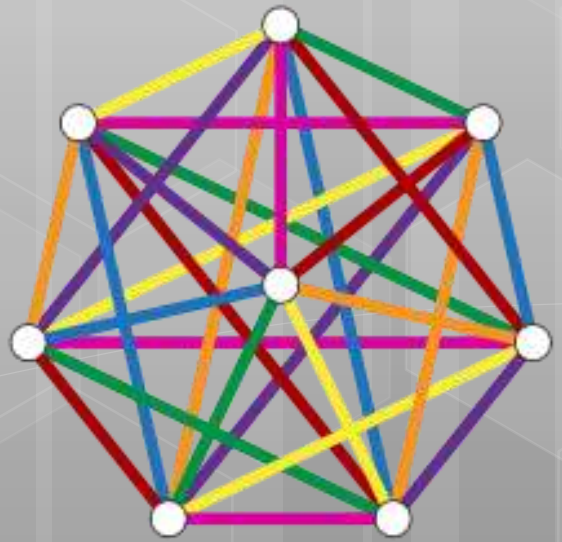

Graph Theory

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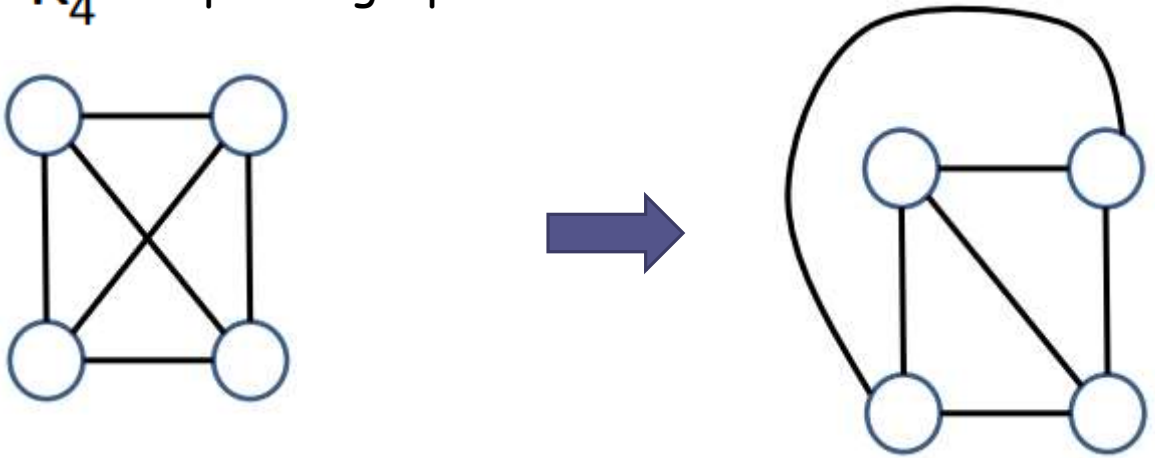
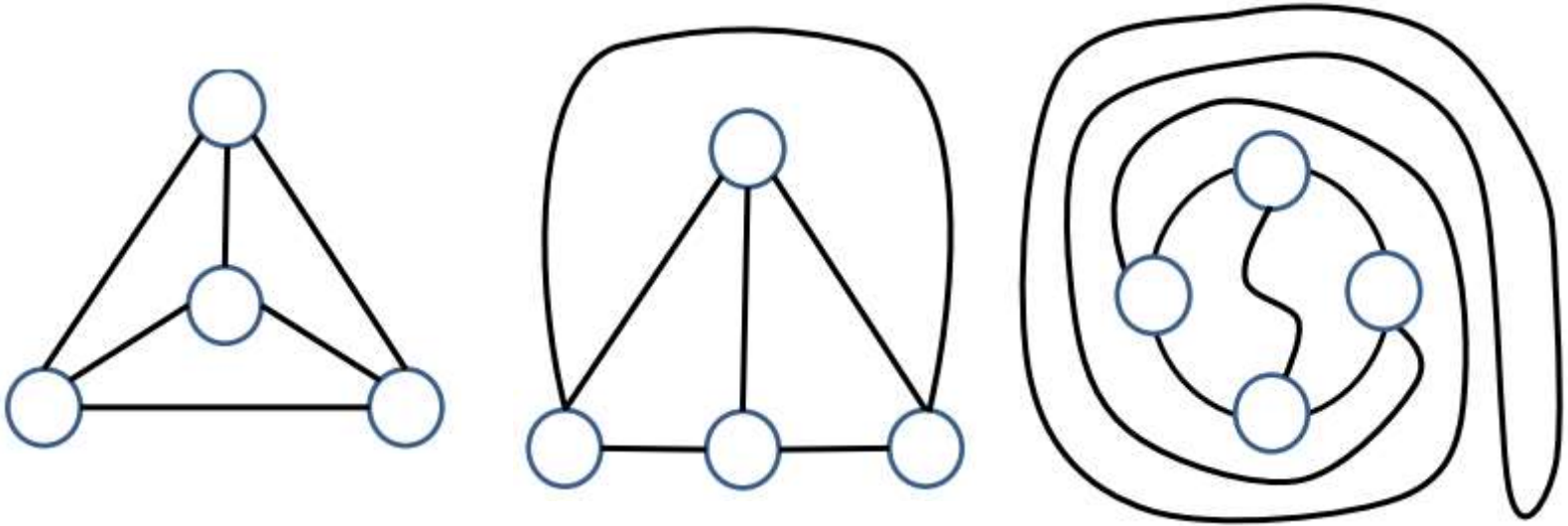
Planar Graph

A graph is called planar if it can be drawn in a plane without any edges crossing. Such a drawing is called a planar representation of the graph.

Example: K_4 is a planar graph

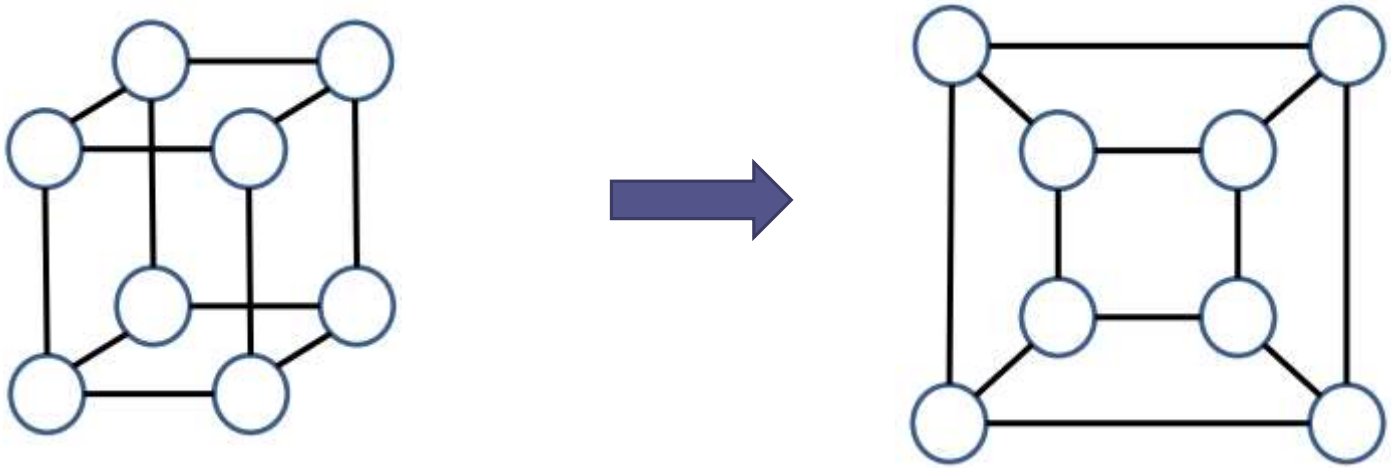


Other planar representations of K_4



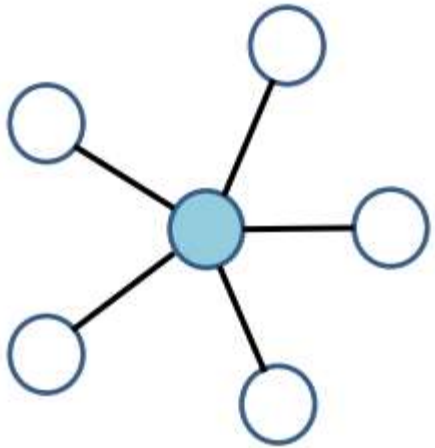
Examples of Planar Graph

Q_3 is a planar graph.

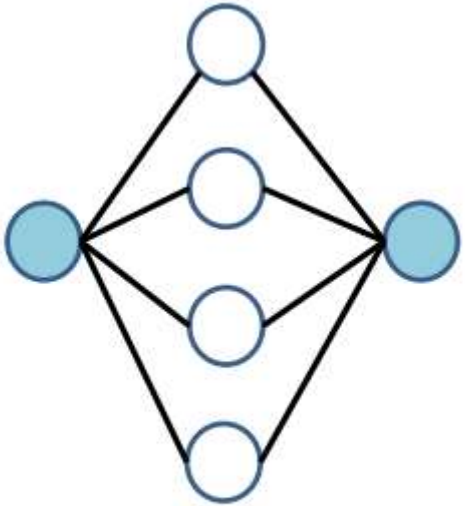


Examples of Planar Graph

$K_{1,n}$ and $K_{2,n}$ are planar graphs for all n.



$K_{1,5}$



$K_{2,4}$

Euler's Formula

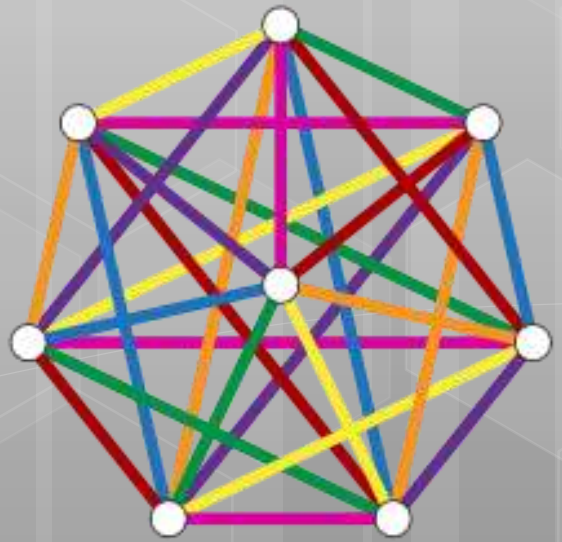
Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then

$$r = e-v+2$$

Example: Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

Solution: There is a formula for calculating the number of regions
number of regions = e-v+2
Sum of degrees of edges = 20*3 = 60 = 2*number of edges
2*e = 60
e = 30
Regions = 30-20+2 = 12


Thank you!



Graph Theory

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Shortest path algorithm (Dijkstra's Algorithm)

- An algorithm for finding the shortest path between two vertices in a graph.

- It Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

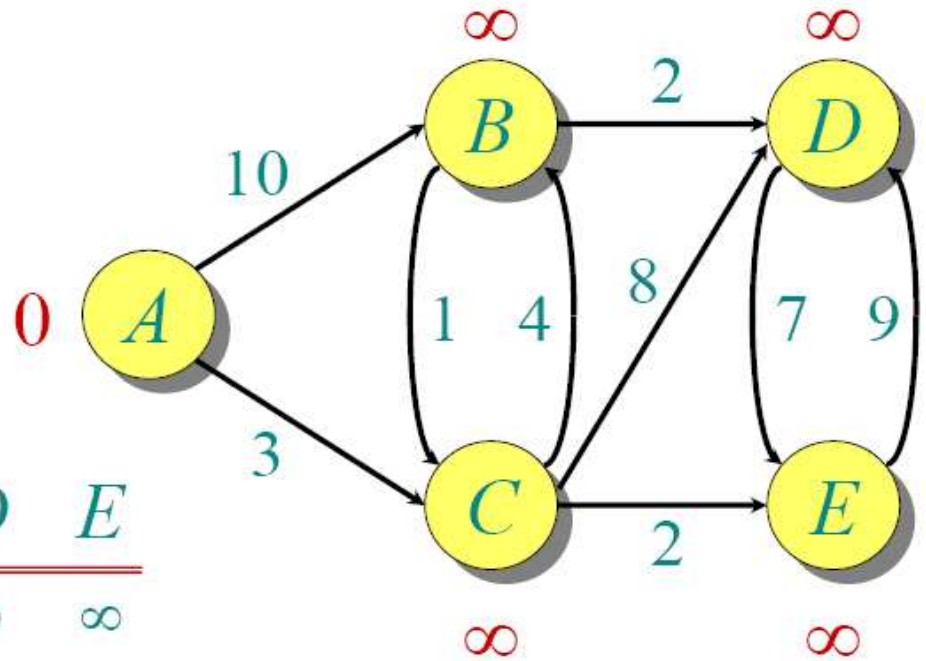
- A weighted graph associates a label (weight) with every edge in the graph. Weights are usually real numbers.

Shortest path algorithm (Dijkstra's Algorithm)

- Google Maps are the examples of real life networks.
- In any of the map each town is a vertex (node) and each road is an edge (arc).
- In real life we often want to know what is the shortest path between two places.
- The Computer cannot decide which route is the best, so it uses an algorithm to do so.
- One such algorithm is Dijkstra's.

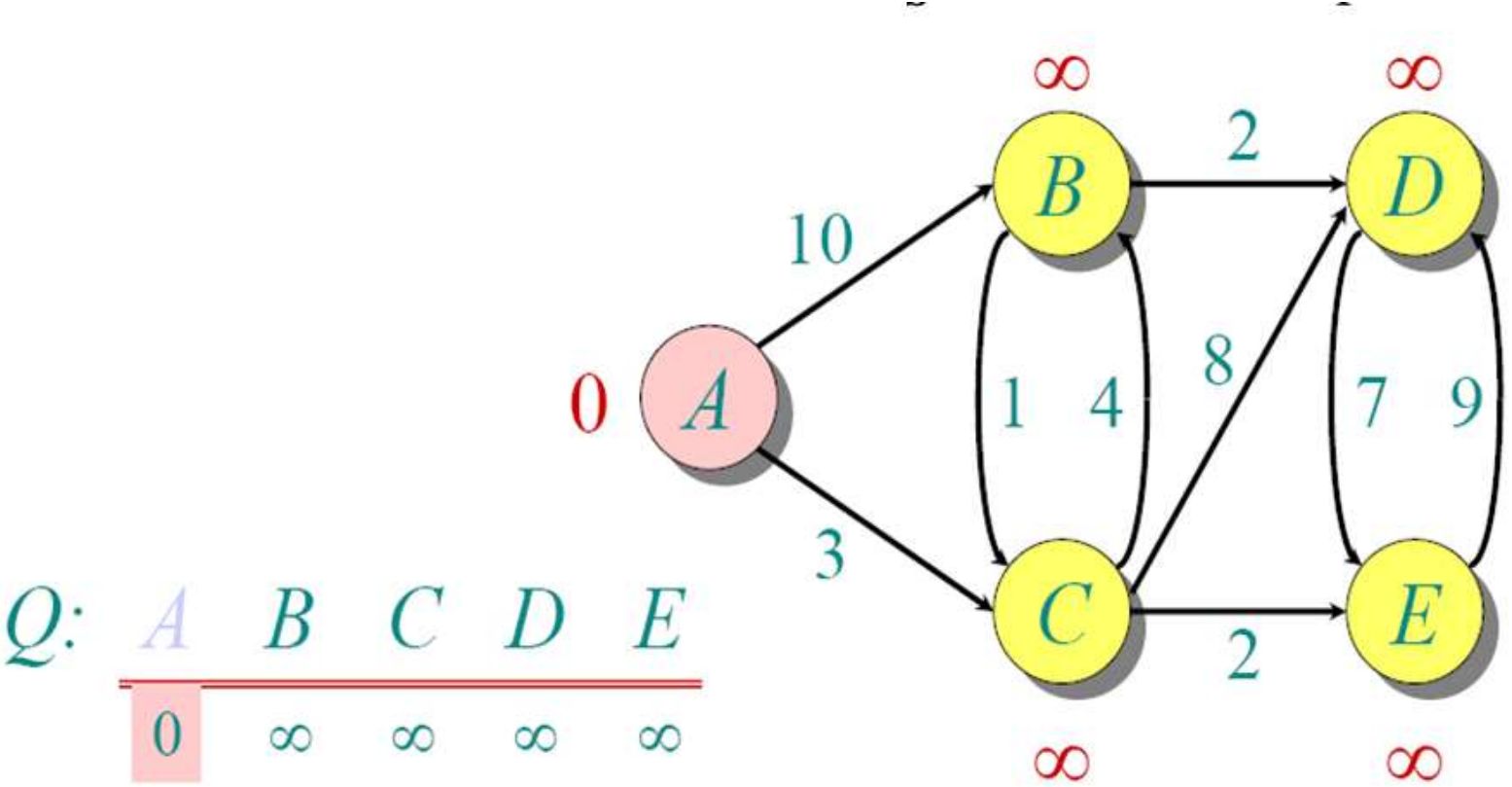
Dijkstra's Algorithm

Initialize:

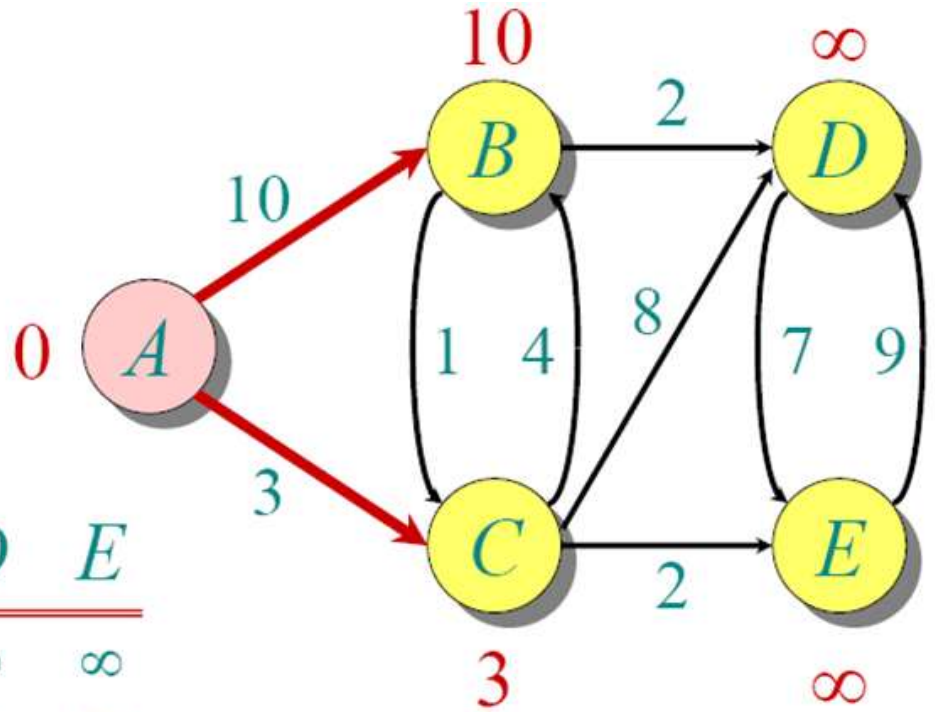


Q:	A	B	C	D	E
	0	∞	∞	∞	∞

S: {}

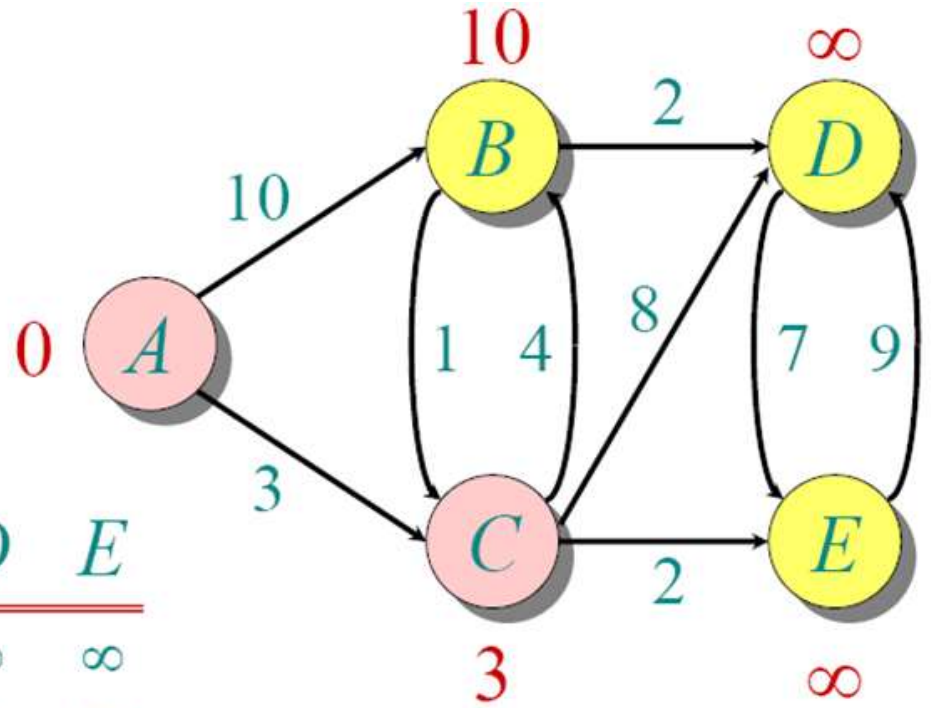


Q:	A	B	C	D	E
	0	∞	∞	∞	∞



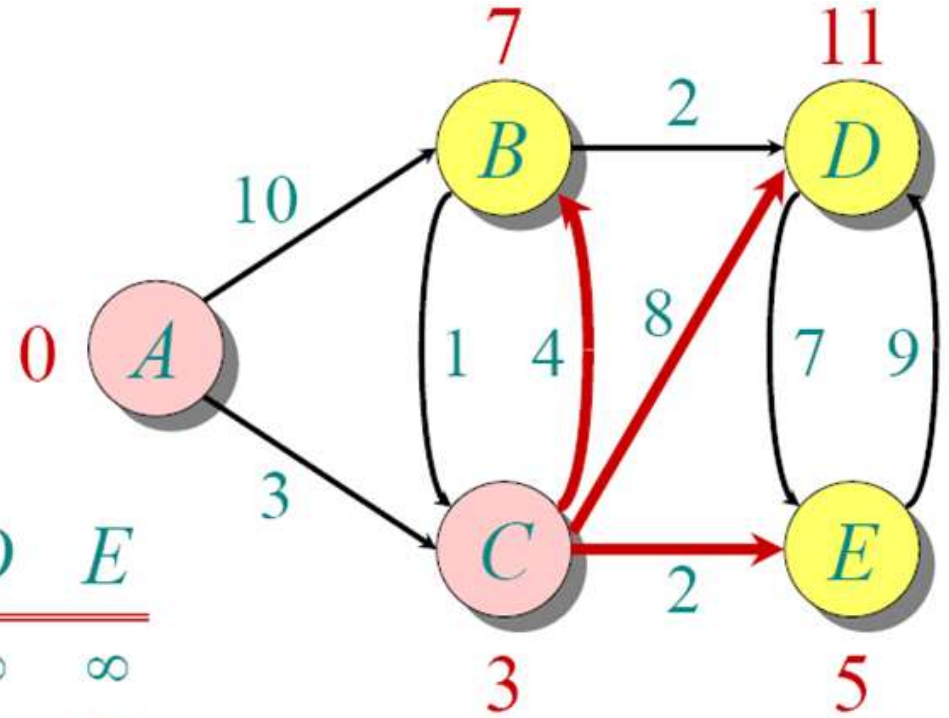
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

$S: \{ A \}$



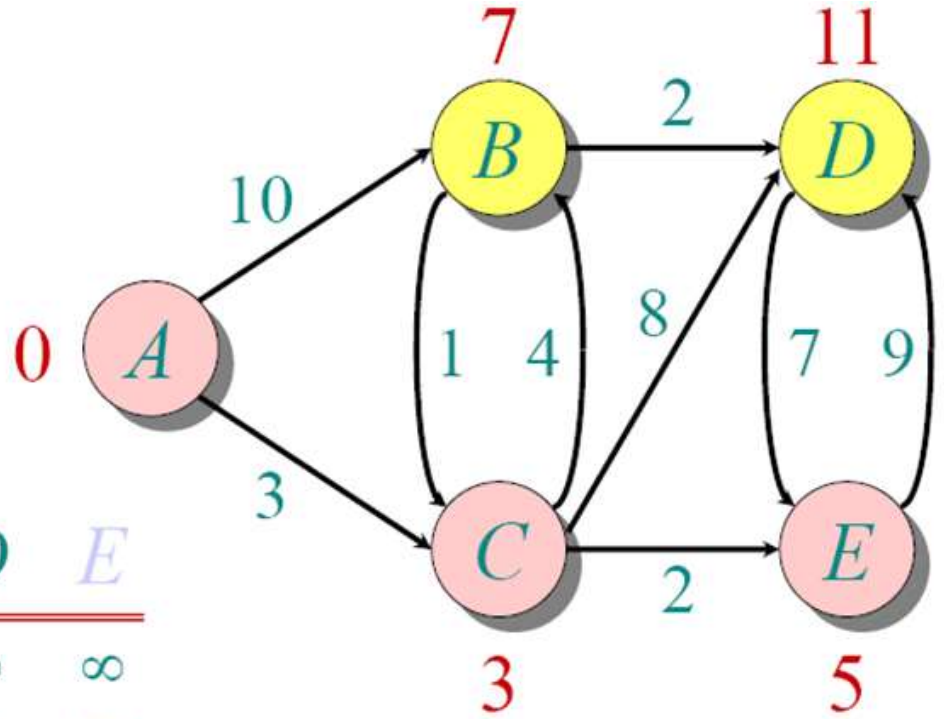
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

$S: \{ A, C \}$



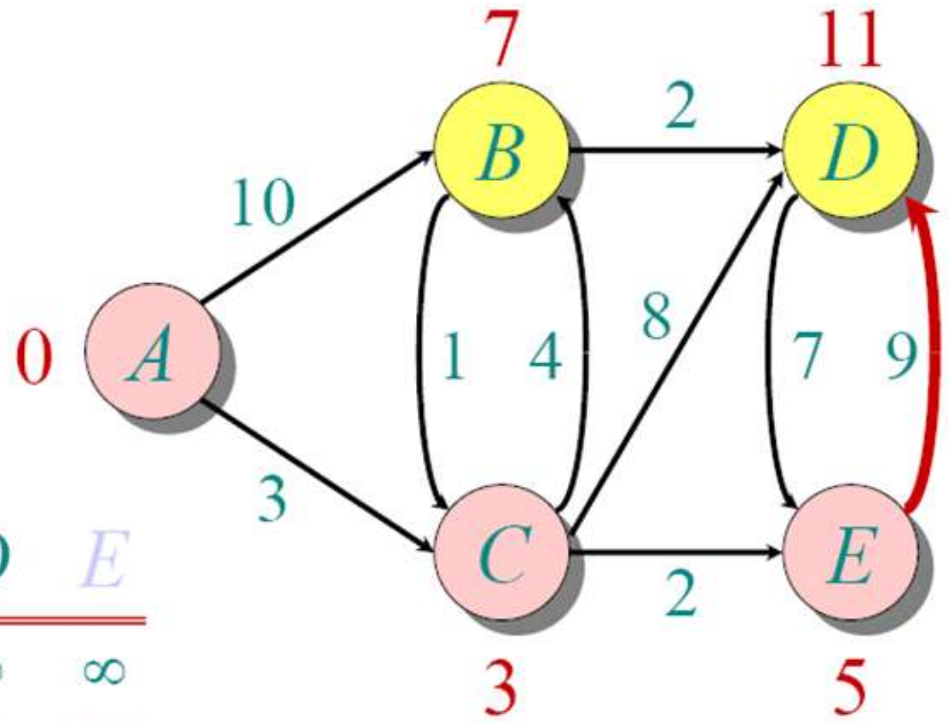
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

$S: \{ A, C \}$



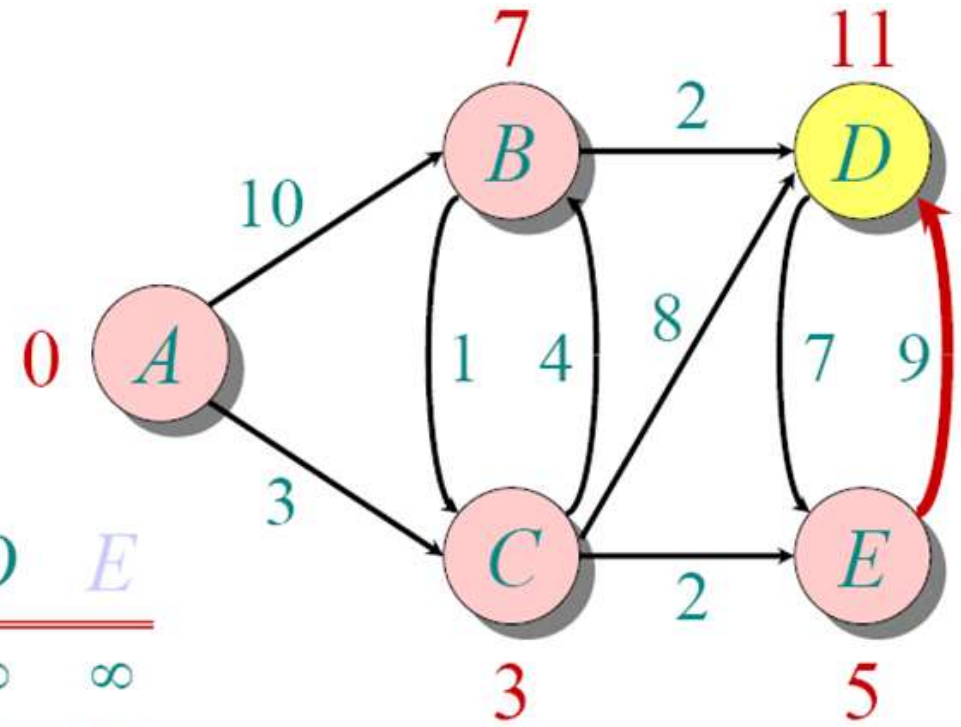
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

$S: \{ A, C, E \}$



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

$S: \{ A, C, E \}$



Q:	A	B	C	D	E
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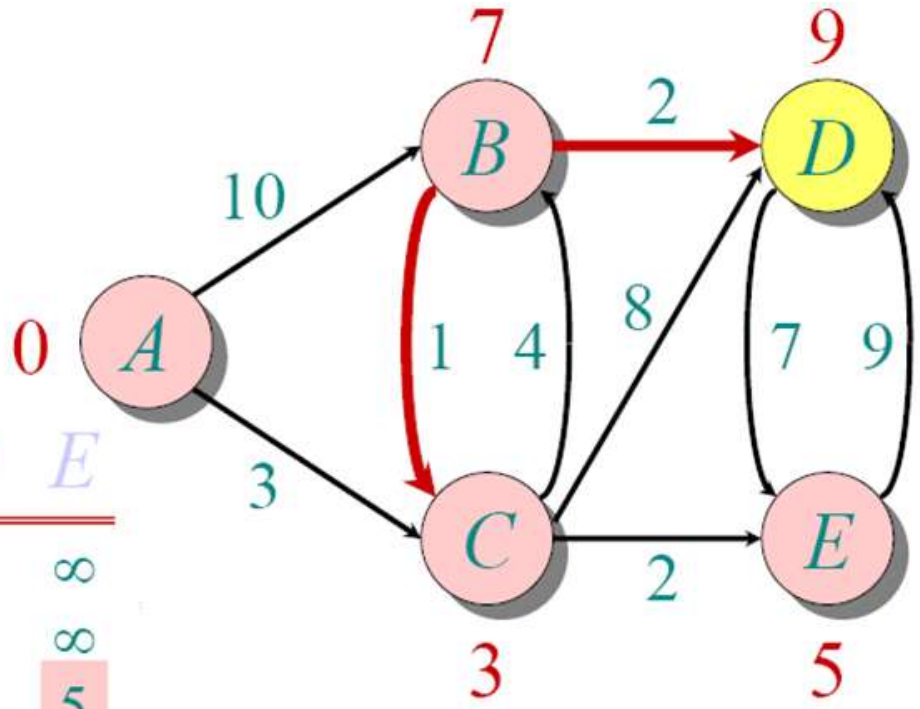
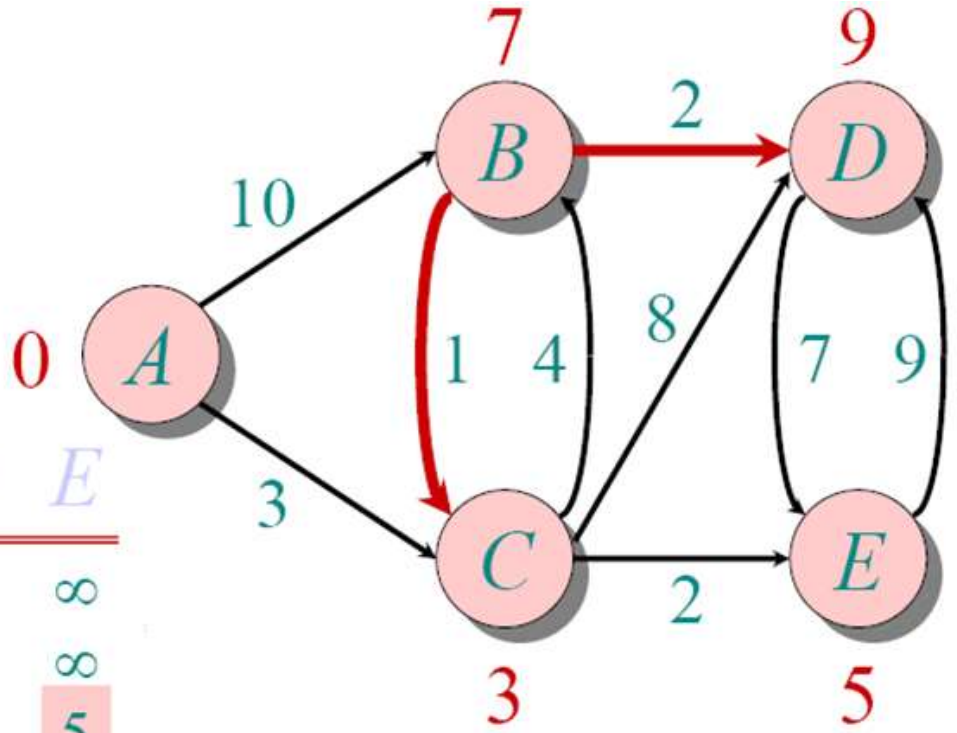
		10	3	∞	∞
		7		11	5
		7		11	

$S: \{ A, C, E, B \}$



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

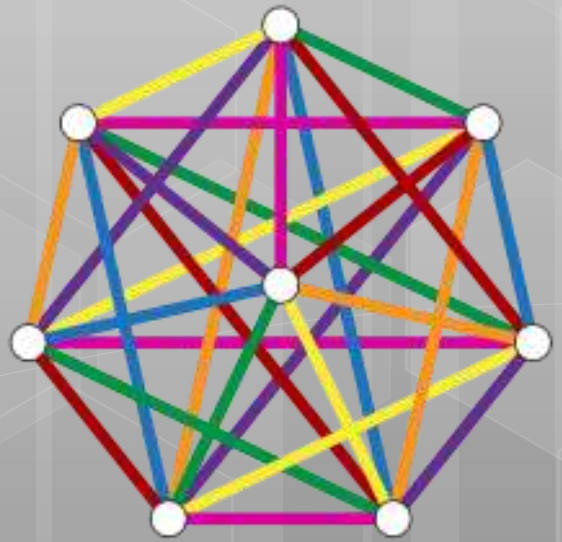
$S: \{ A, C, E, B \}$



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

$S: \{ A, C, E, B, D \}$


Thank you!



Graph Theory

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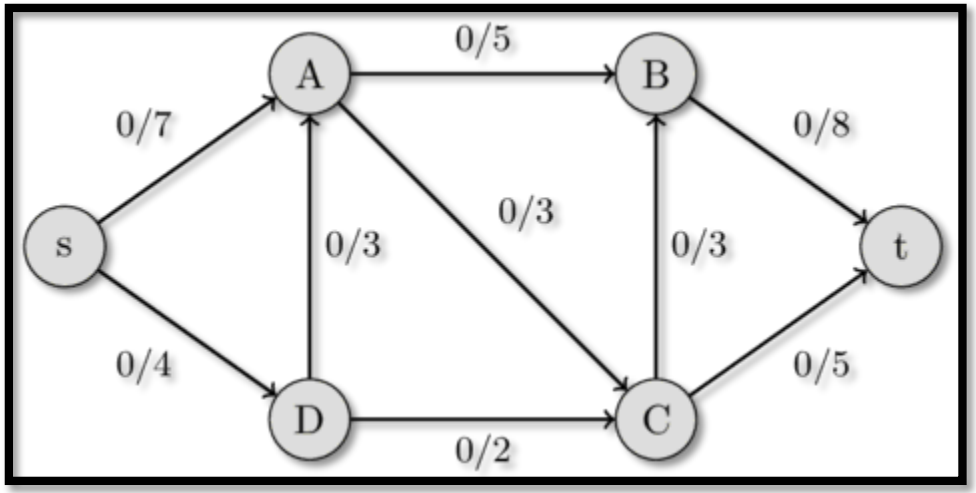
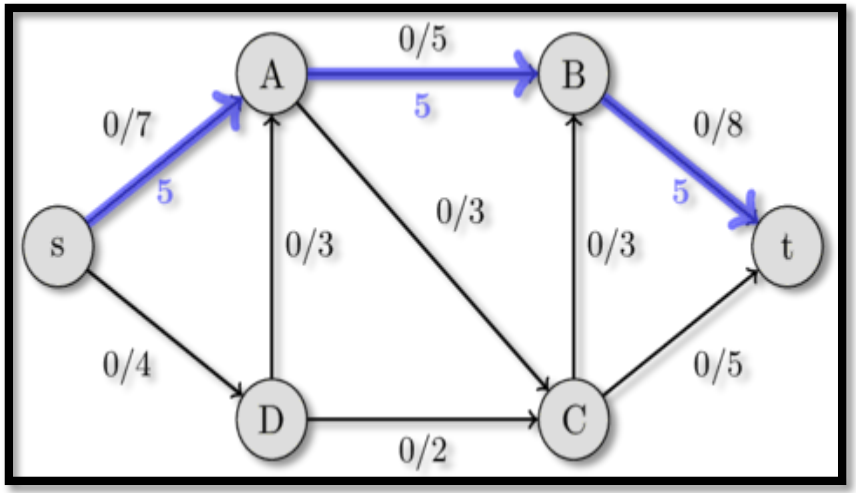
Max Flow: Ford Fulkerson Algorithm

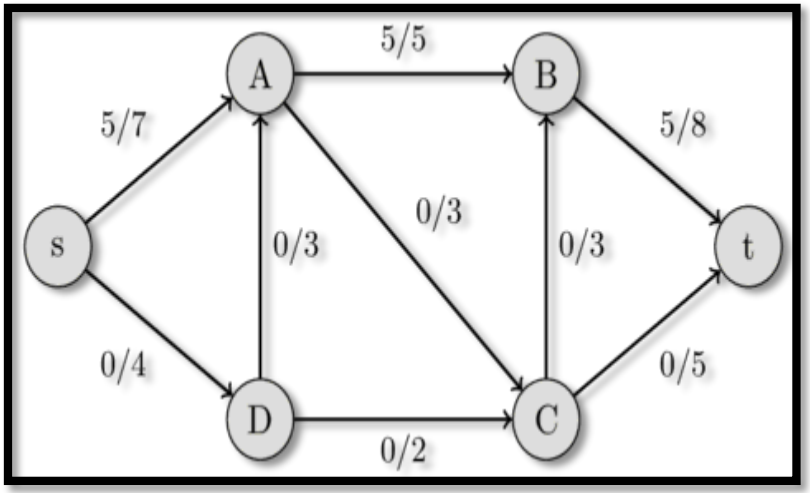


Find the path from s to t.

We can find the path s–A–B–t with the residual capacities 7, 5 and 8. Their minimum is 5, therefore we can increase the flow along this path by 5. This gives a flow of 5 for the network.

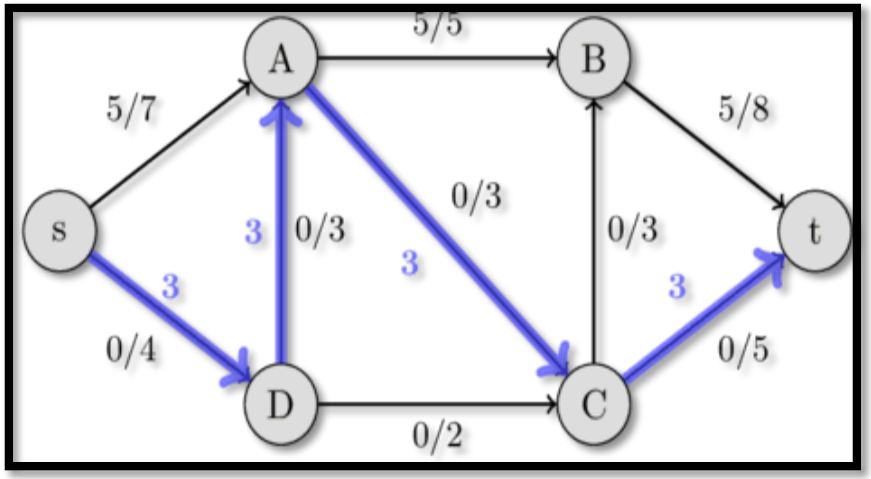
Max Flow: Ford Fulkerson Algorithm

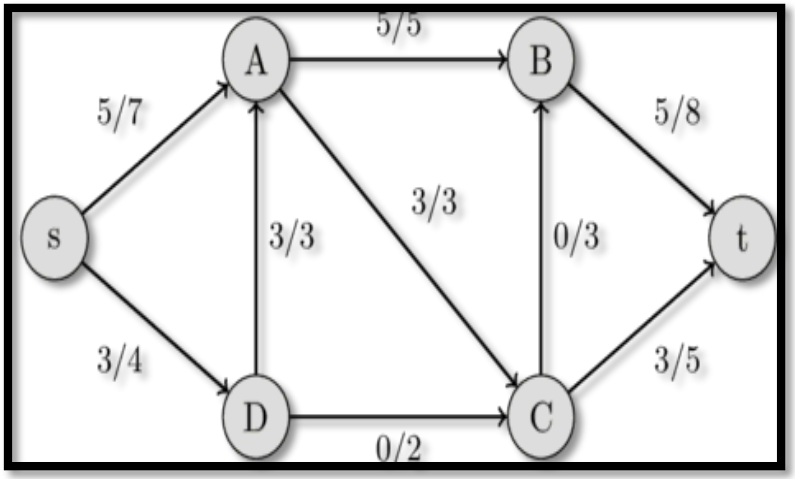




Again we look for an augmenting path, this time we find s–D–A–C–t with the residual capacities 4, 3, 3 and 5. Therefore we can increase the flow by 3 and we get a flow of 8 for the network.

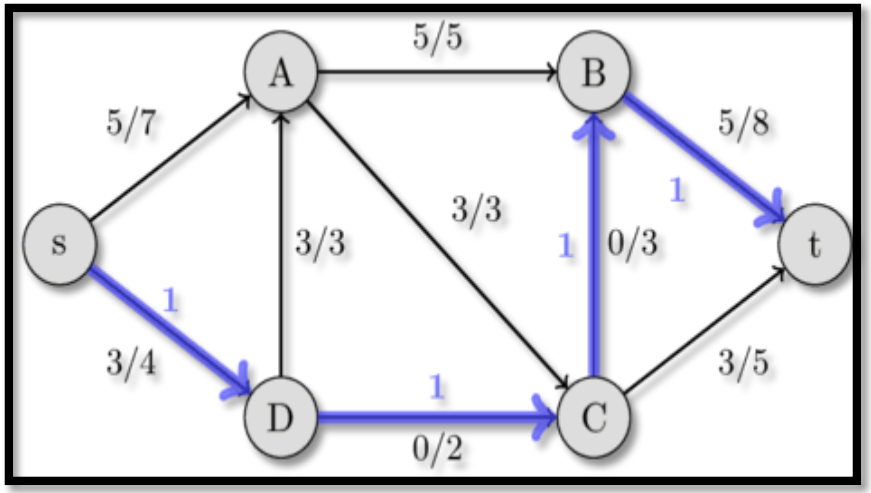
Max Flow: Ford Fulkerson Algorithm

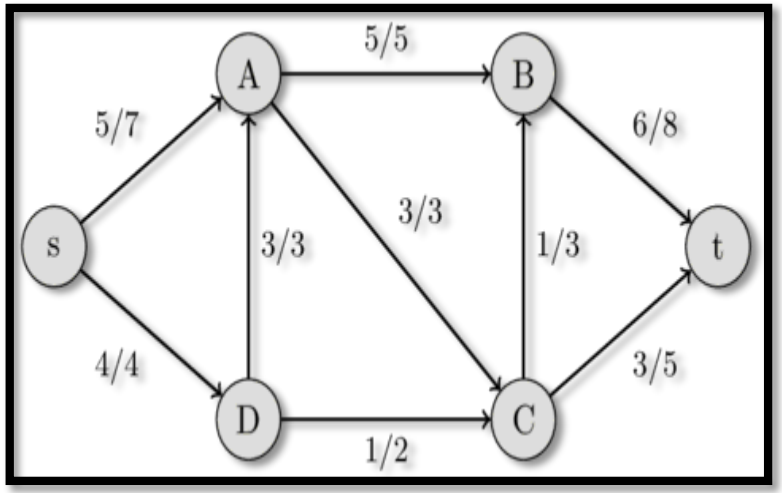




This time we find the path s–D–C–B–t with the residual capacities 1, 2, 3 and 3 and we increase by 1.

Max Flow: Ford Fulkerson Algorithm

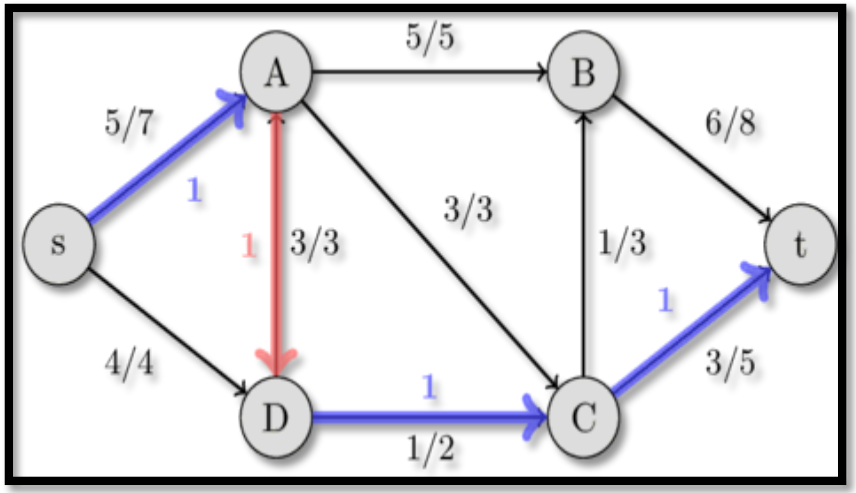


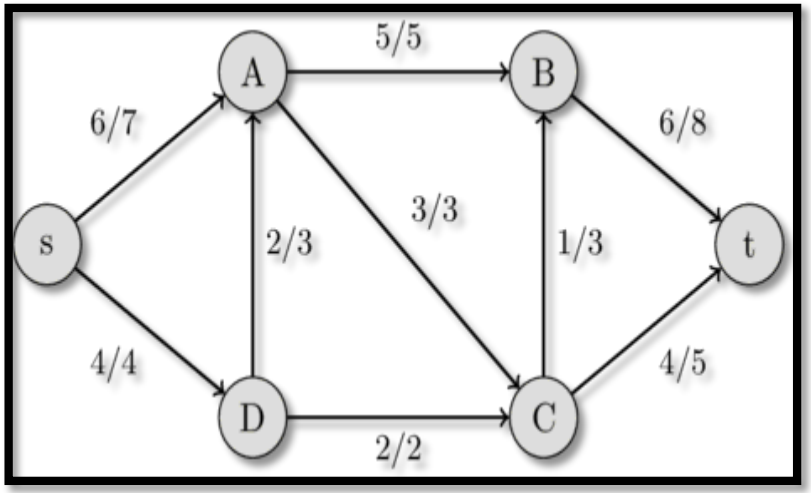


Max Flow: Ford Fulkerson Algorithm

This time we find the augmenting path s–A–D–C–t with the residual capacities 2, 3, 1 and 2. We can increase by 1. But this path is very interesting. It includes the reversed edge (A,D). In the original flow network we are not allowed to send any flow from A to D. But because we already have a flow of 3 from D to A this is possible. The intuition of it is the following: Instead of sending a flow of 3 from D to A, we only send 2 and compensate this by sending an additional flow of 1 from s to A, which allows us to send an additional flow of 1 along the path D–C–t.

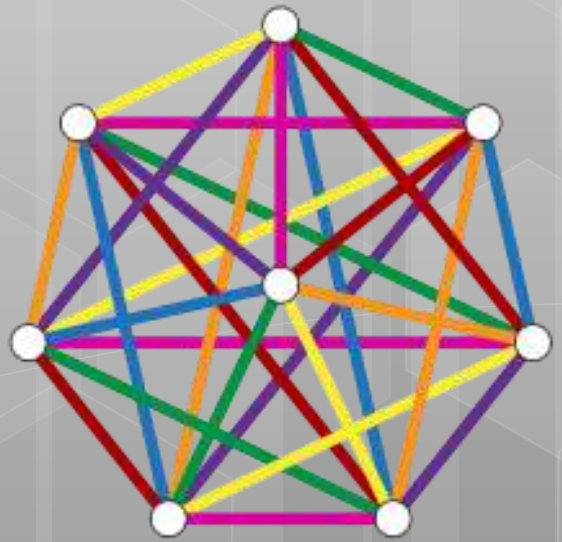
Max Flow: Ford Fulkerson Algorithm





Now it is impossible to find an augmenting path between s and t, therefore this flow of 10 is the maximal possible. We have found the maximal flow.


Thank you!



Graph Theory

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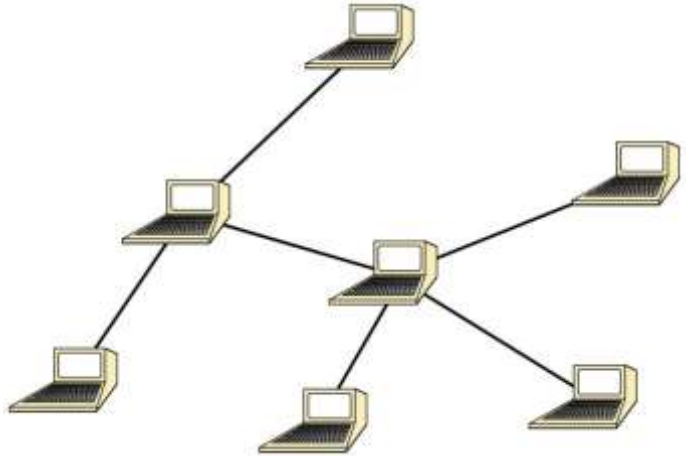
Minimum Spanning Tree

- A tree is an acyclic, undirected, connected graph.
- A spanning tree of a graph is a tree containing all vertices from the graph.
- A minimum spanning tree is a spanning tree, where the sum of the weights on the tree's edges are minimal.

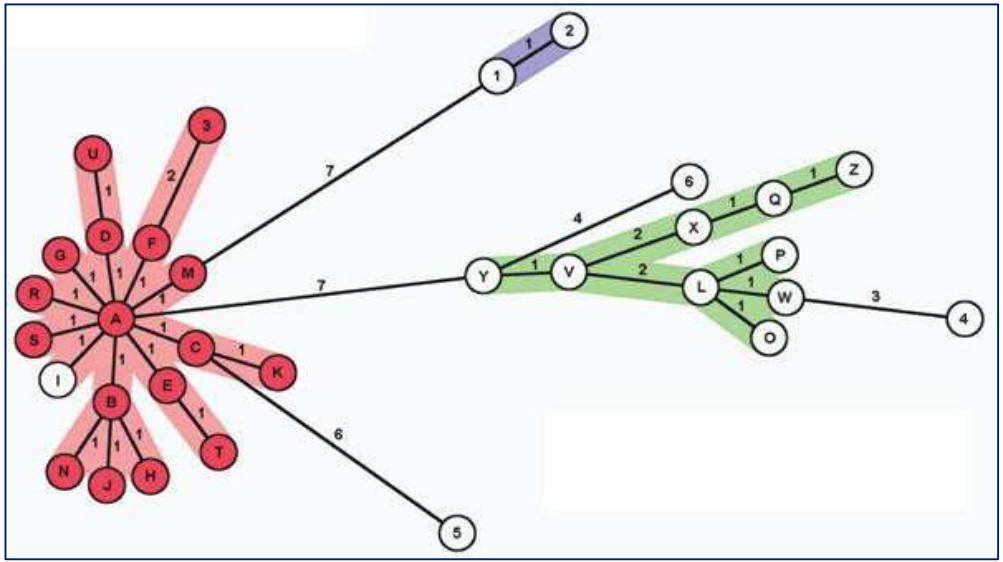
Minimum Spanning Tree

- Find a minimum-cost set of edges that connect all vertices of a graph
- Applications
  - Connect "nodes" with a minimum of "wire"
    - Networking
    - Circuit design



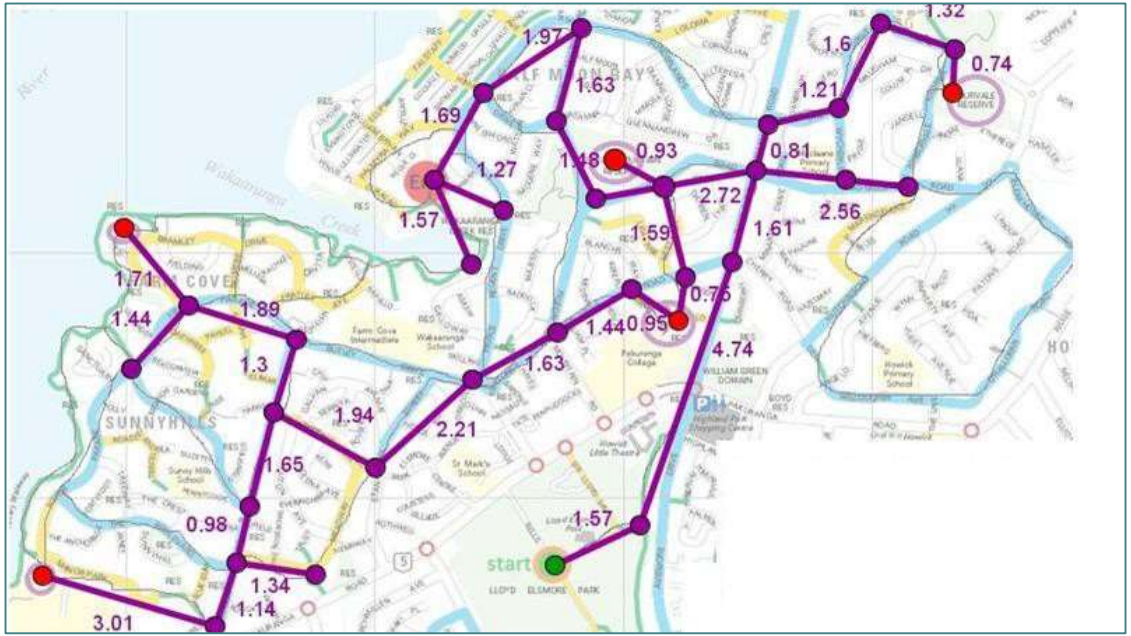
Minimum Spanning Tree

- Applications
  - Collect nearby nodes
    - Clustering, taxonomy construction



Minimum Spanning Tree

- Applications
  - Approximating graphs



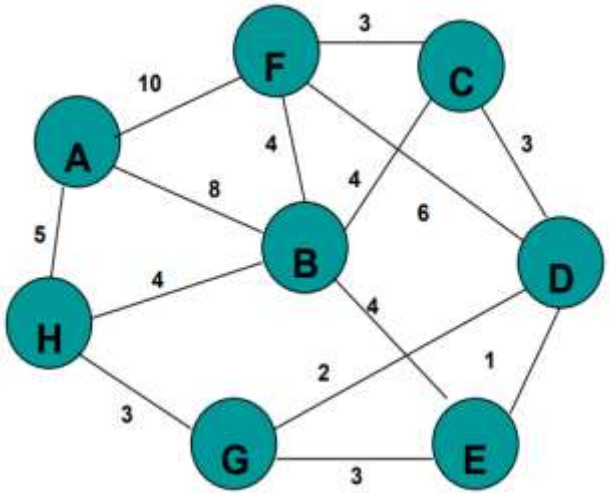
Minimum Spanning Tree

- Both Kruskal's and Prim's Algorithms work with undirected graphs
- Both work with weighted and unweighted graphs but are more interesting when edges are weighted.
- Both are greedy algorithms that produce optimal solutions.

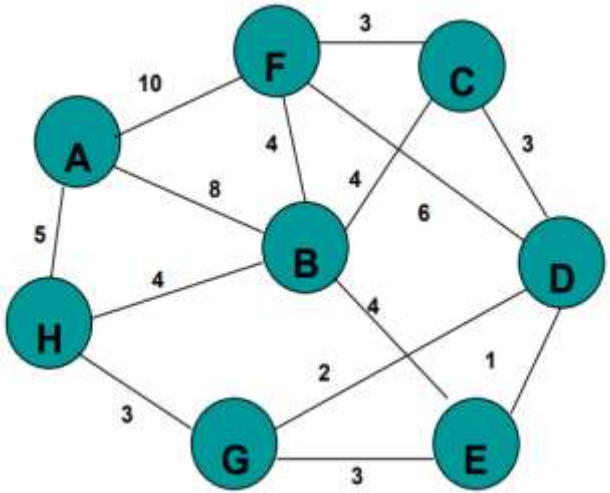
Minimum Spanning Tree

Kruskal's algorithm:

- Work with edges
- Two steps:
  - Sort edges by increasing edge weight
  - Select the first $|V| - 1$ edges that do not generate a cycle
- Walk through:



Kruskal's algorithm: Example

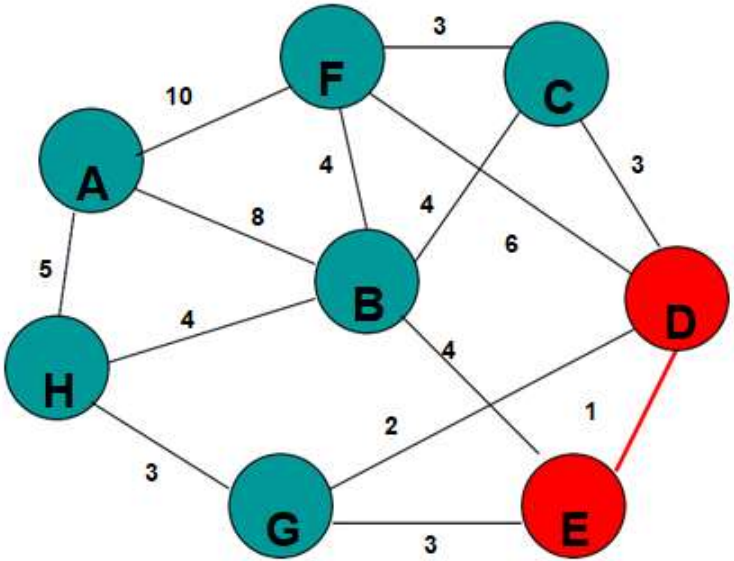


Sort the edges by increasing edge weight.

edge	d_v	
(D,E)	1	
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

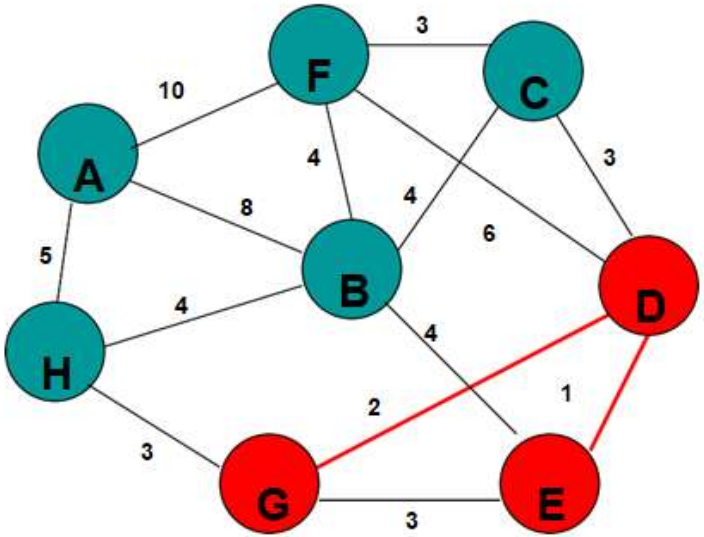


Select first |V|–1 edges which do not generate a cycle.

edge	d_v	
(D,E)	1	√
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

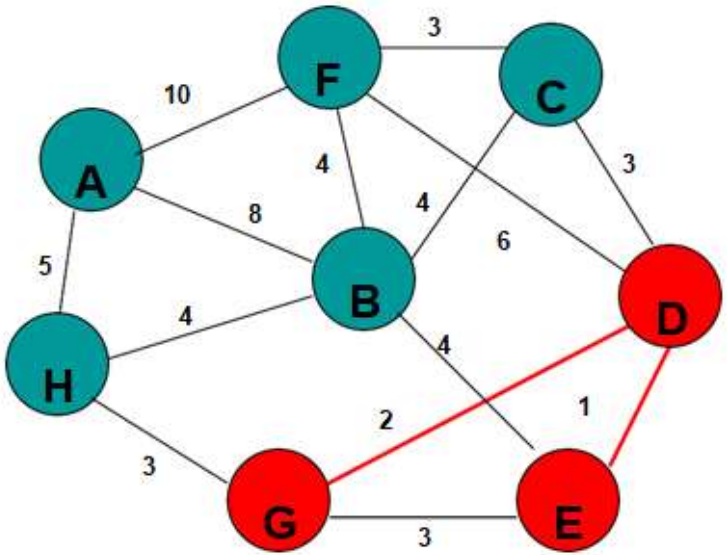


Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example



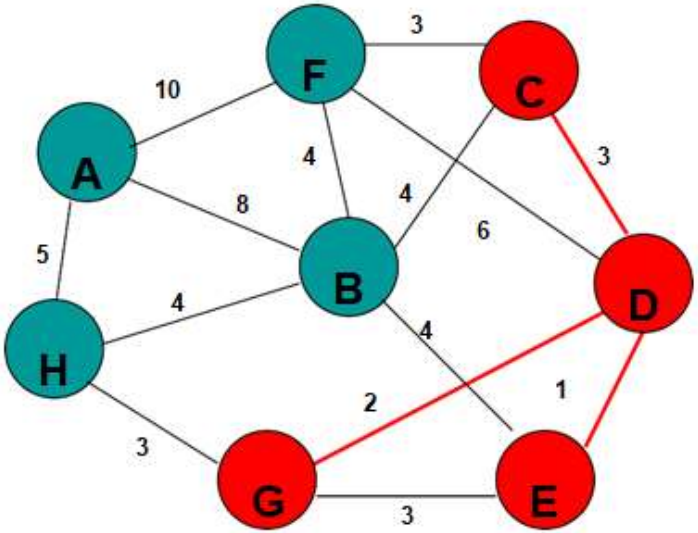
Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (E,G) would create a cycle.

Kruskal's algorithm: Example

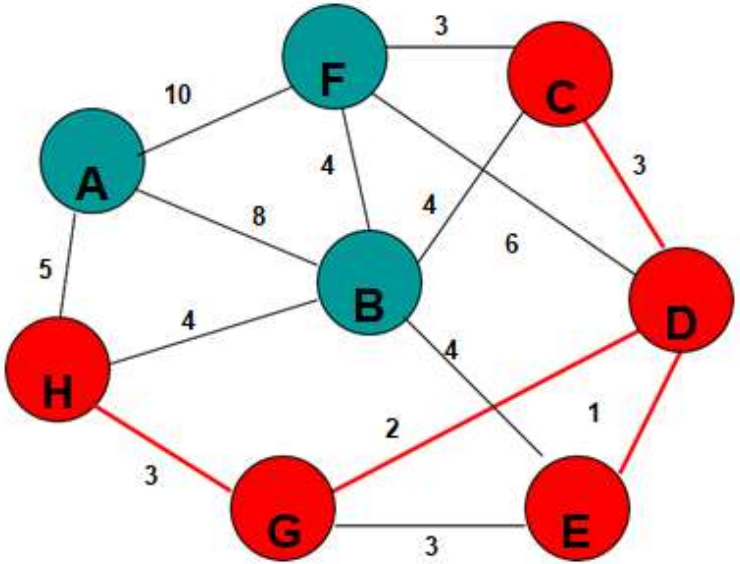


Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

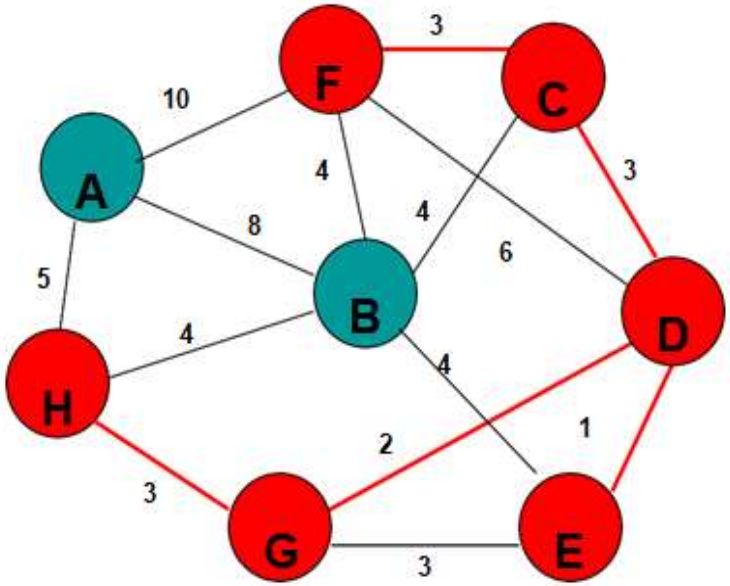


Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

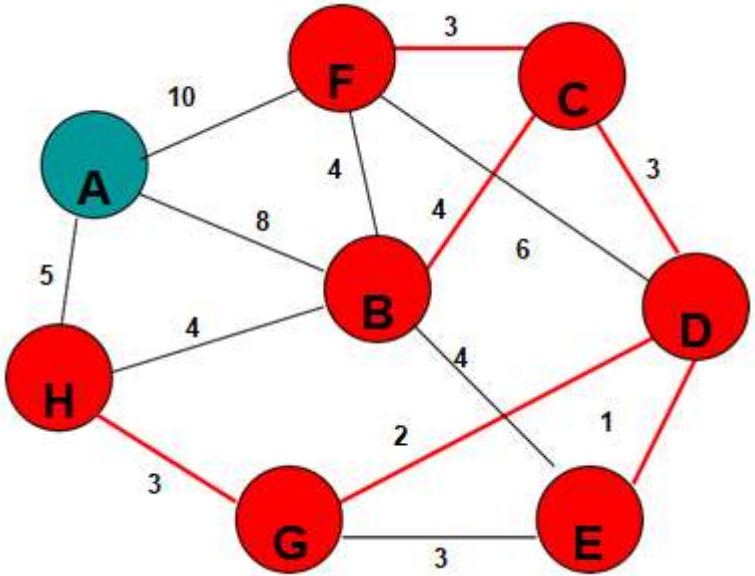


Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

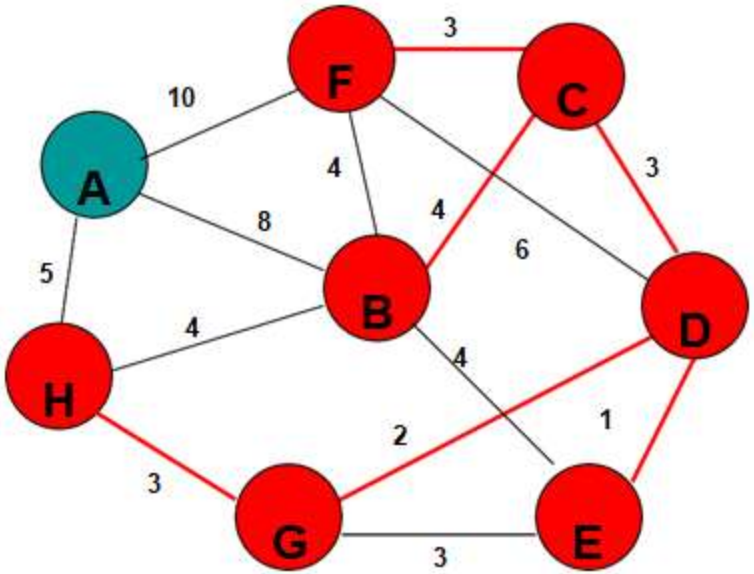


Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example



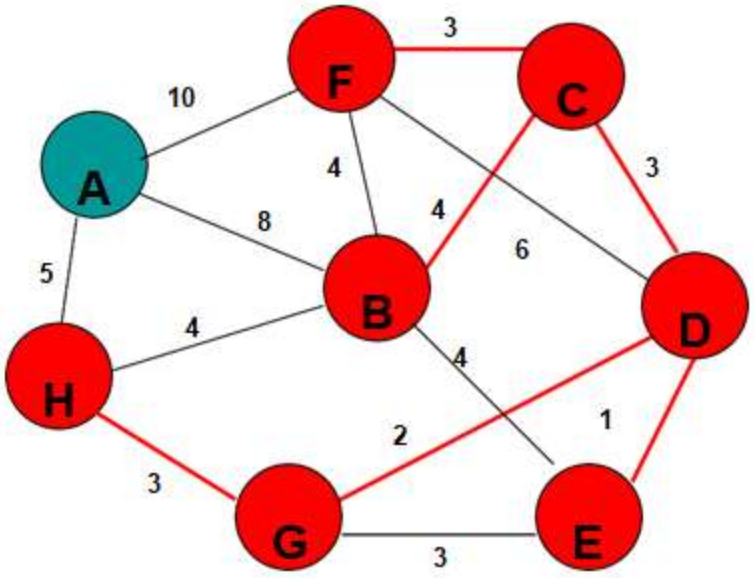
Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (B,E) would create a cycle.

Kruskal's algorithm: Example



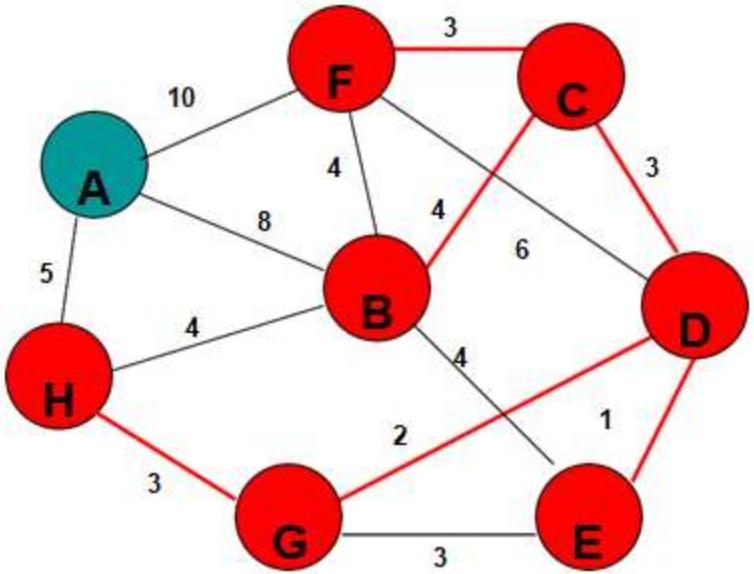
Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (B,F) would create a cycle.

Kruskal's algorithm: Example



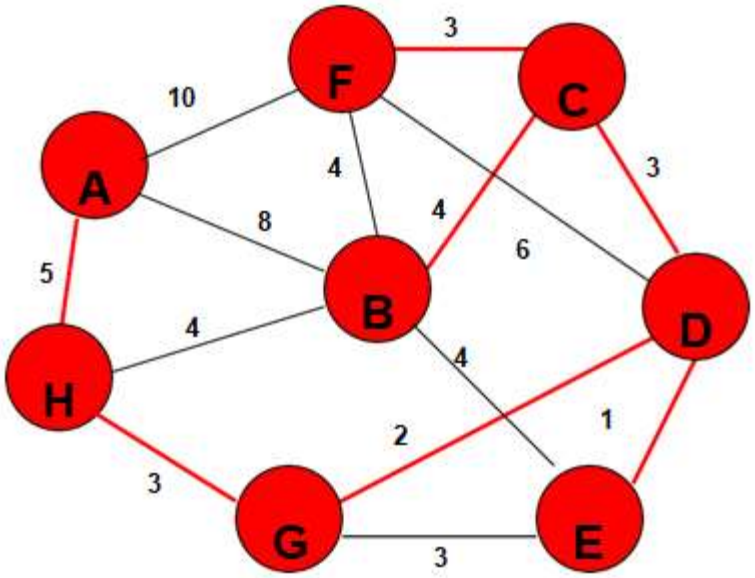
Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (B,H) would create a cycle.

Kruskal's algorithm: Example



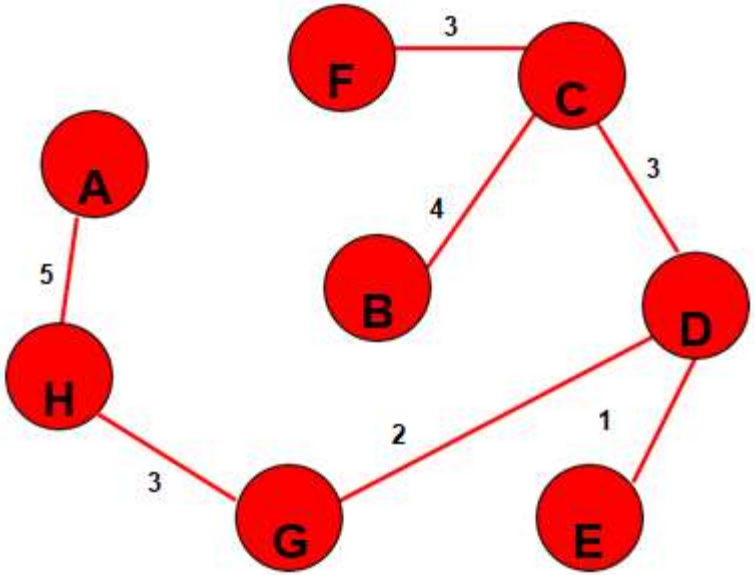
Select next |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's algorithm: Example

Select next |V|–1 edges which do not generate a cycle



edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

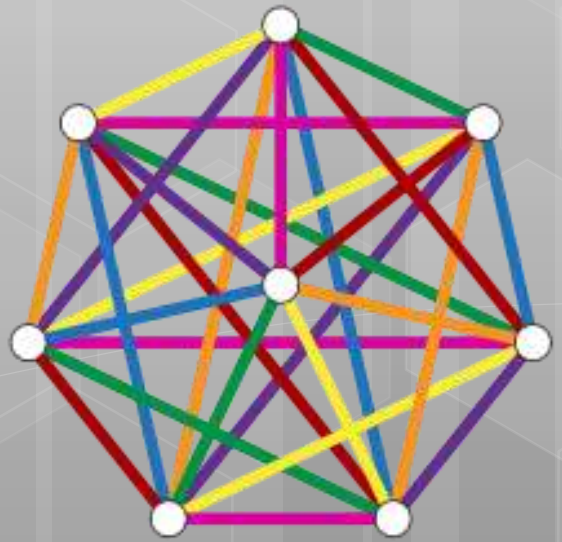
edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

not considered

Done

Total Cost = *21*


Thank you!



Graph Theory

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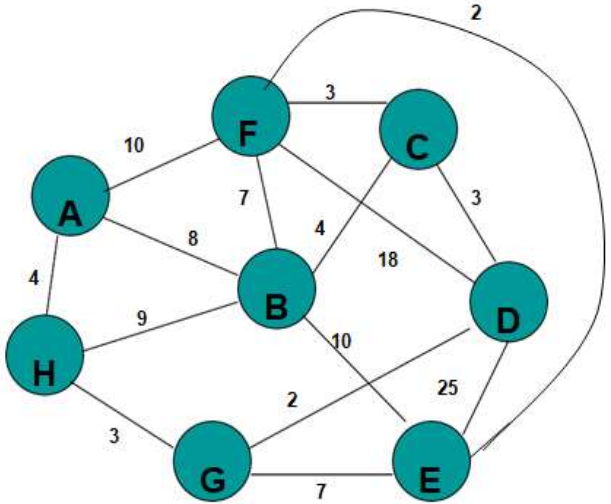
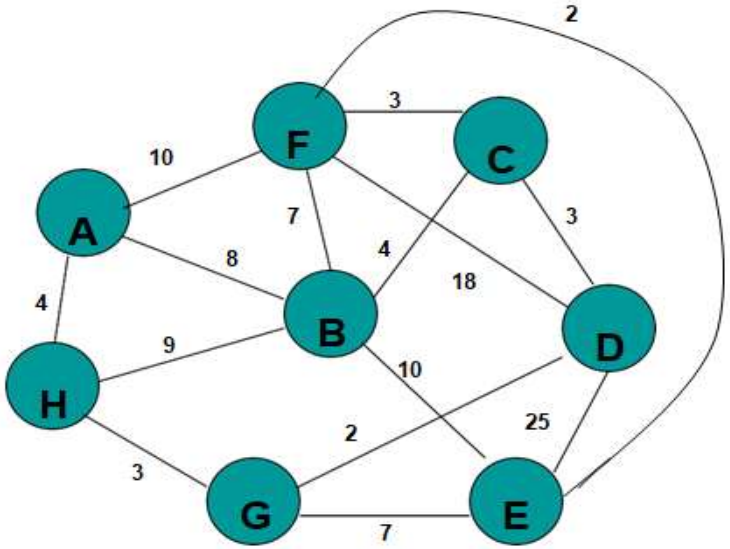
Minimum Spanning Tree

Prim's algorithm:

- Work with nodes (instead of edges)
- Two steps
  - Select node with minimum distance
  - Update distances of adjacent, unselected nodes
- Walk through:



Prim's algorithm: Example



Initialize array

	K	d_v	p_v
A	F	∞	–
B	F	∞	–
C	F	∞	–
D	F	∞	–
E	F	∞	–
F	F	∞	–
G	F	∞	–
H	F	∞	–

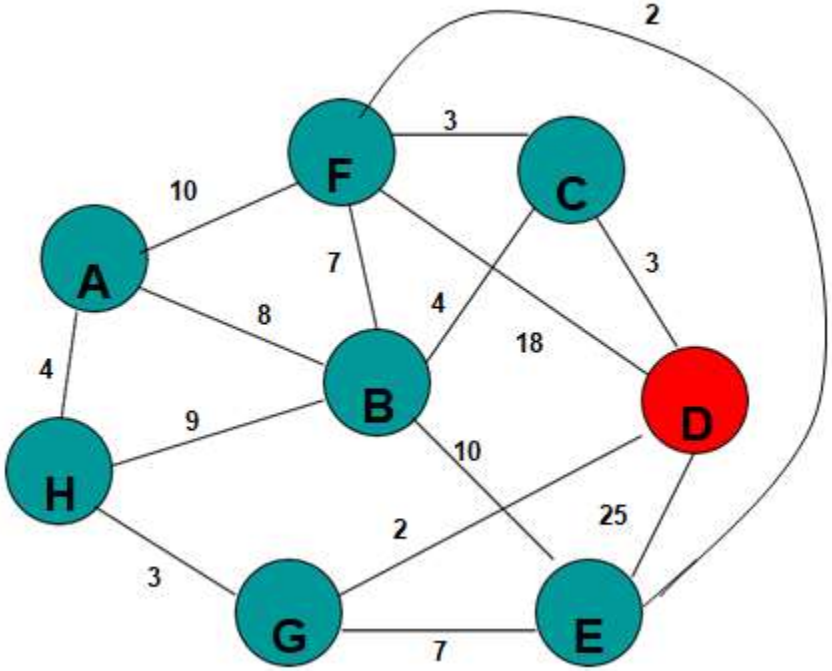
K : whether in the tree

d_v : distance to the tree

p_v : closest node that is in the tree

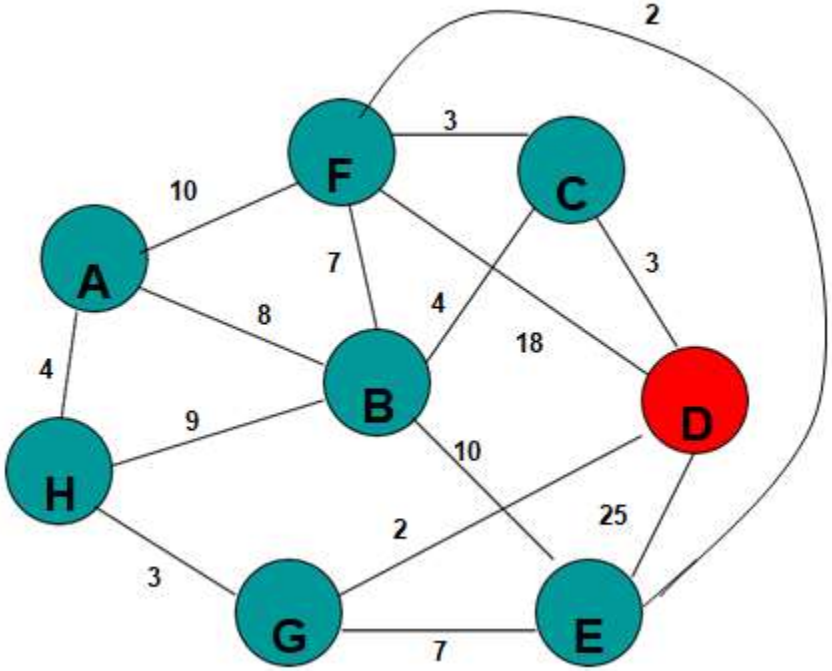
Prim's algorithm: Example

Start with any node, say D



	K	d_v	p_v
A			
B			
C			
D	T	0	–
E			
F			
G			
H			

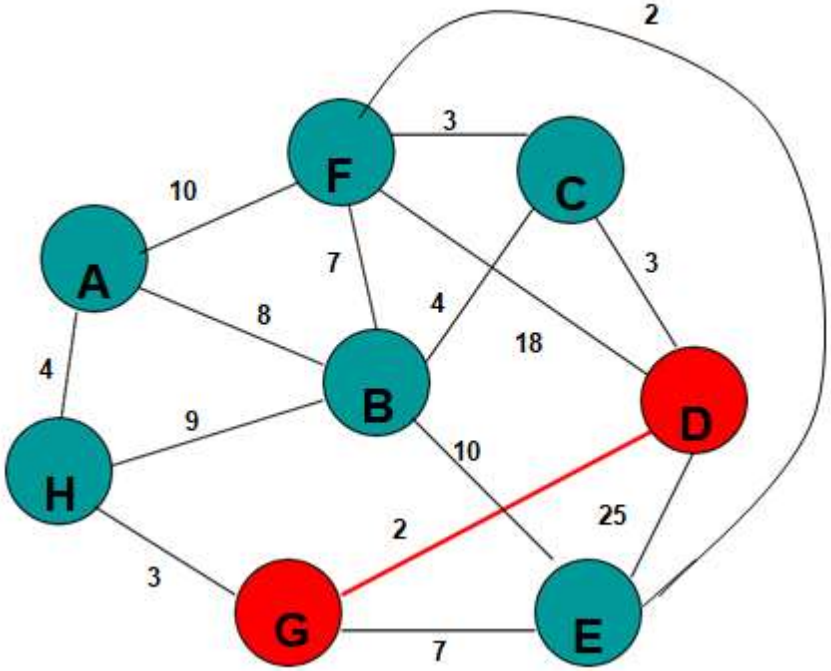
Prim's algorithm: Example



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G		2	D
H			

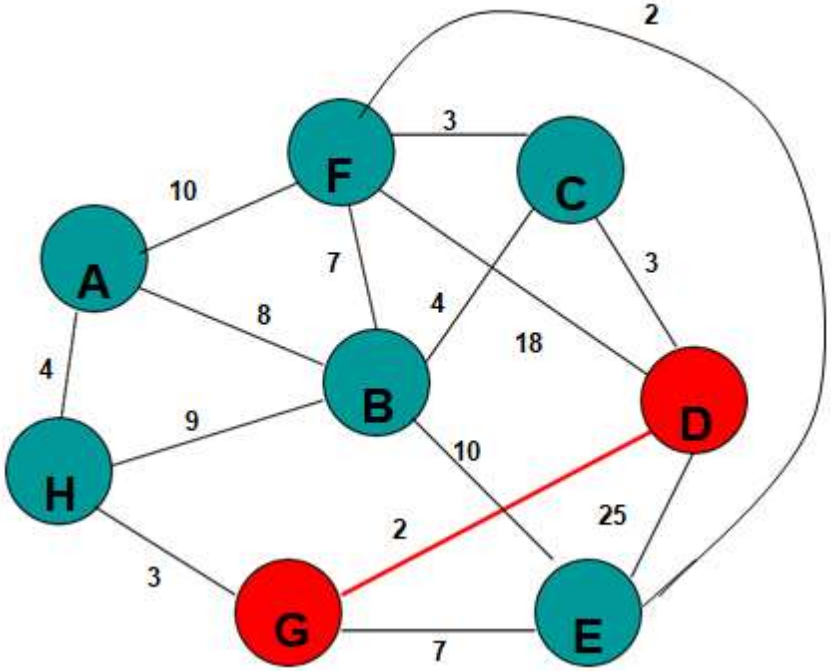
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G	T	2	D
H			

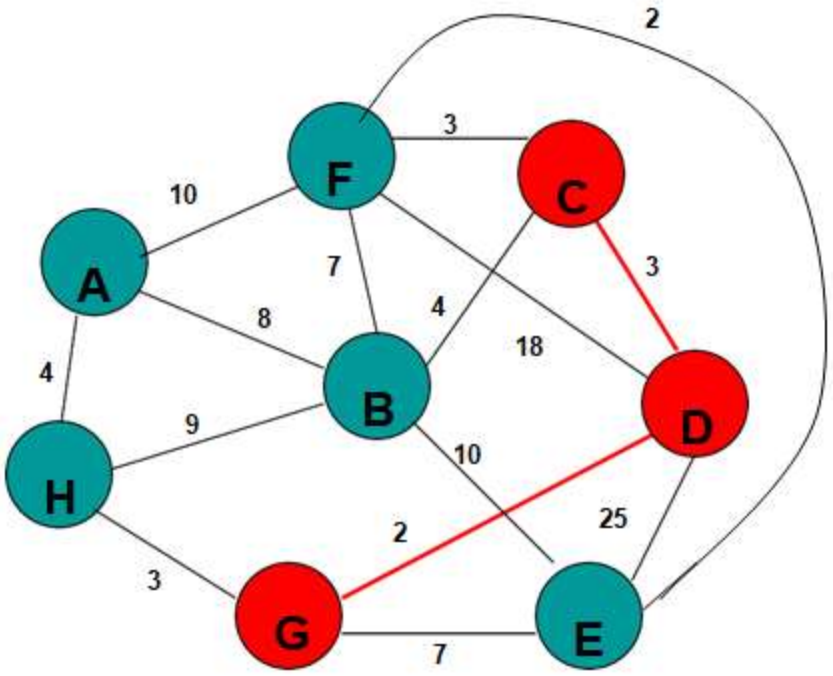
Prim's algorithm: Example



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

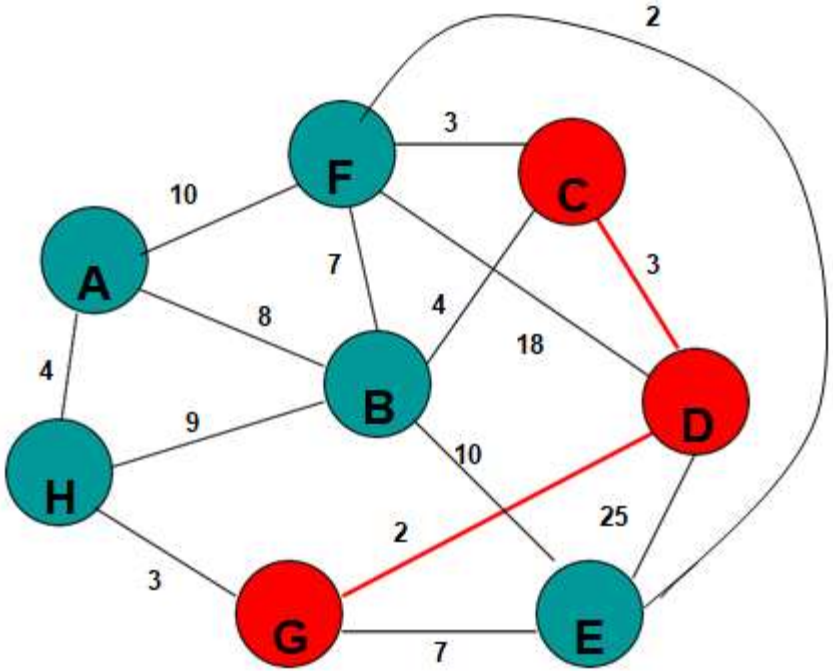
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A			
B			
C	T	3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

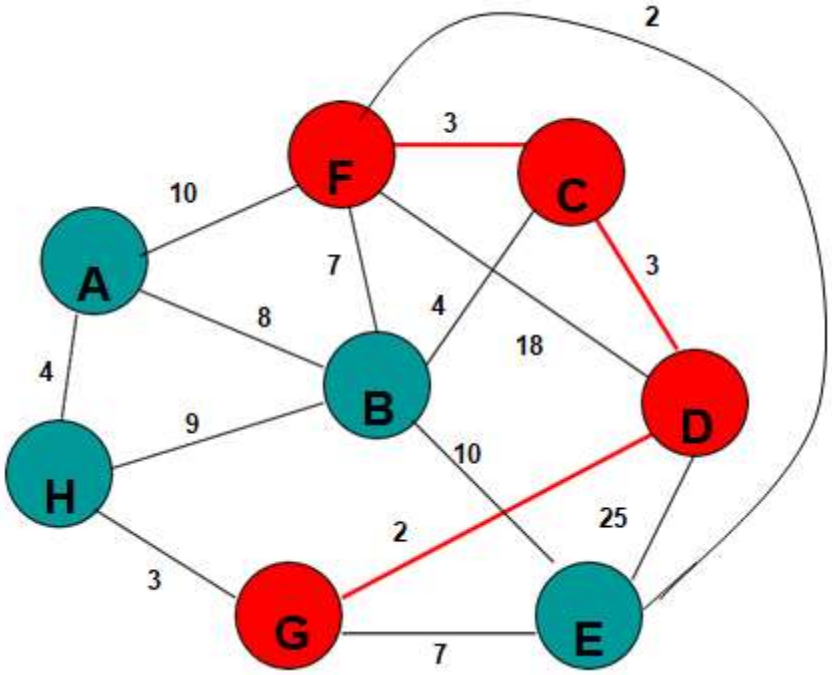
Prim's algorithm: Example



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F		3	C
G	T	2	D
H		3	G

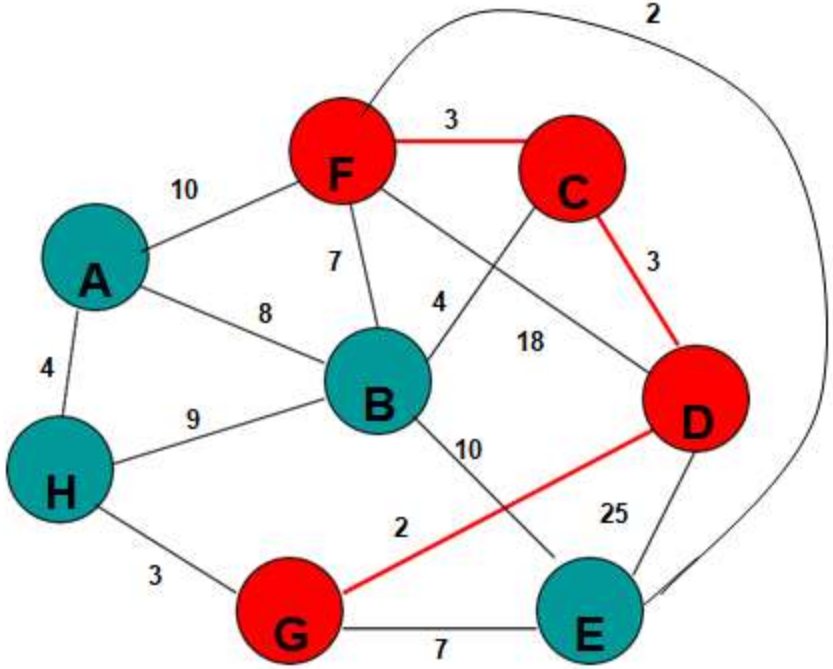
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F	T	3	C
G	T	2	D
H		3	G

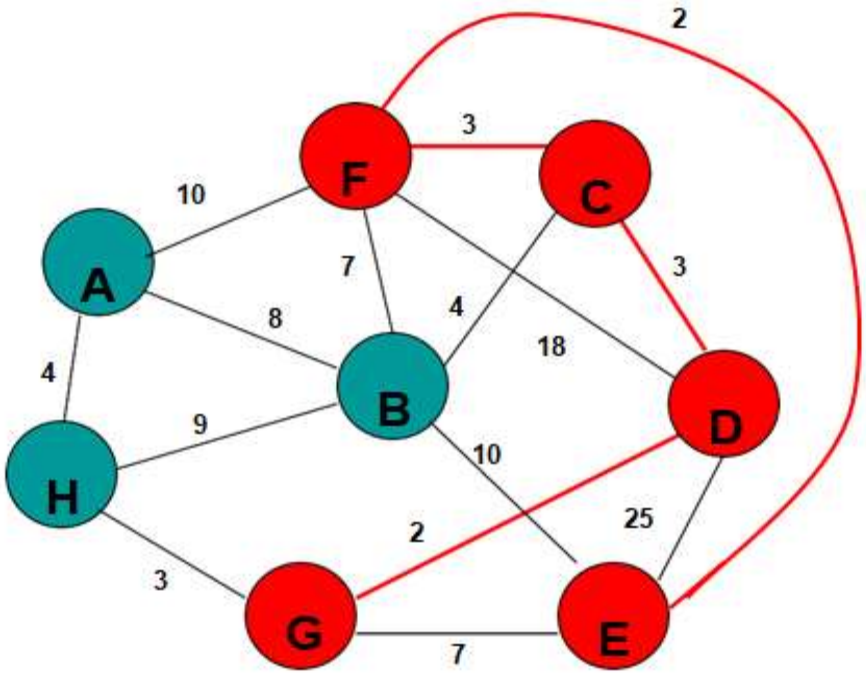
Prim's algorithm: Example



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E		2	F
F	T	3	C
G	T	2	D
H		3	G

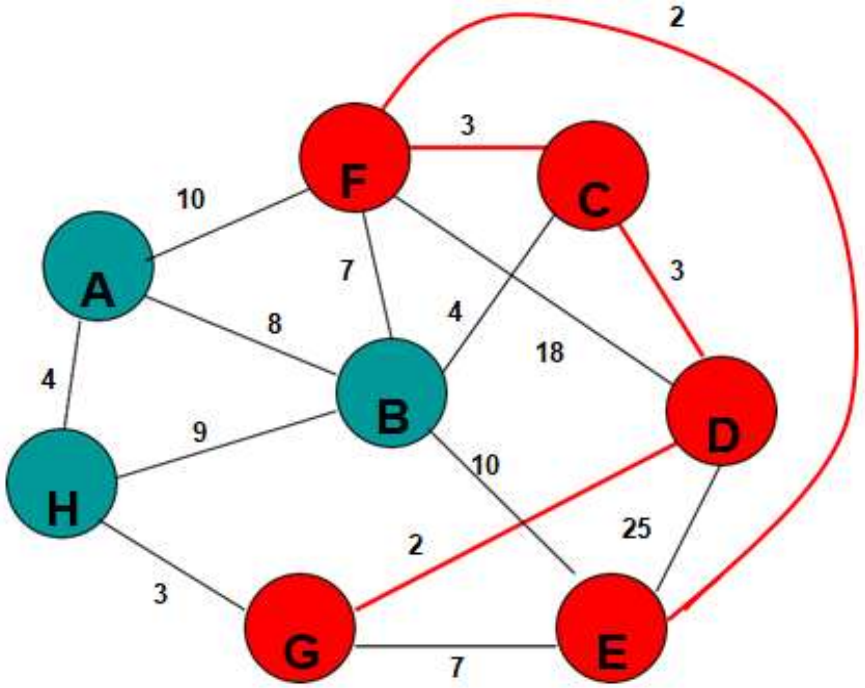
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Prim's algorithm: Example

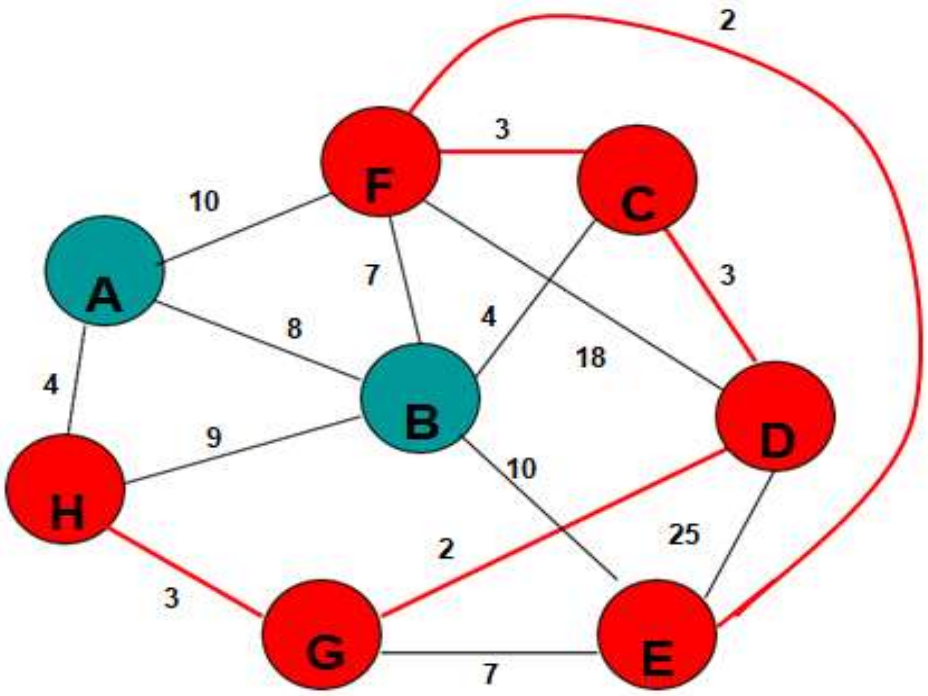


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Table entries unchanged

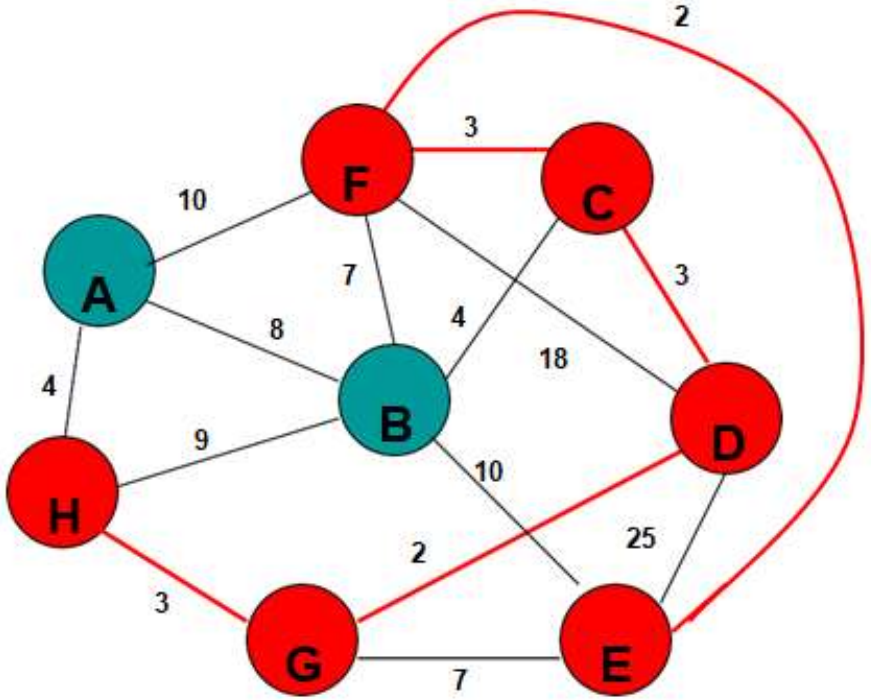
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

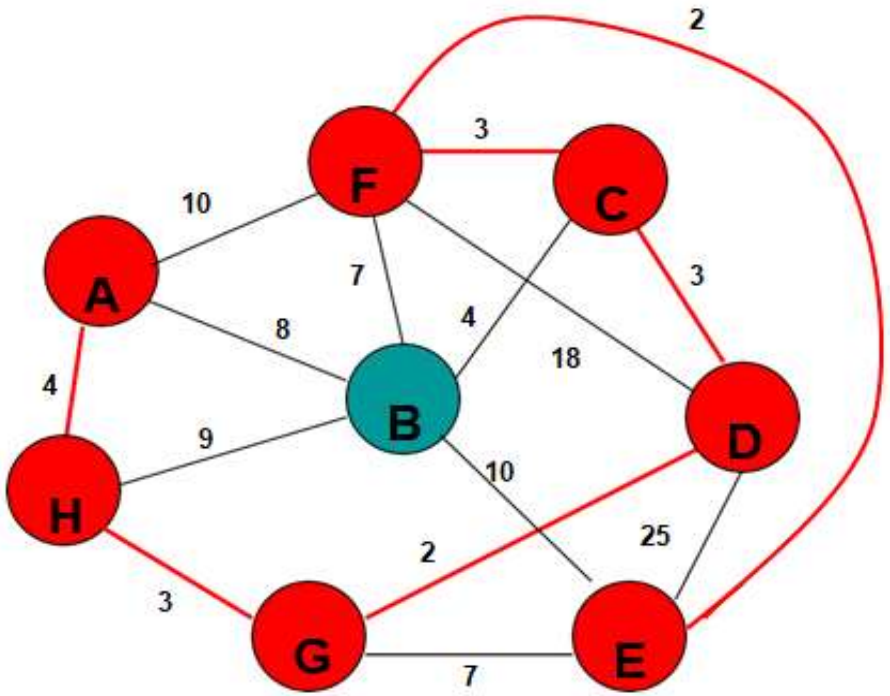
Prim's algorithm: Example



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

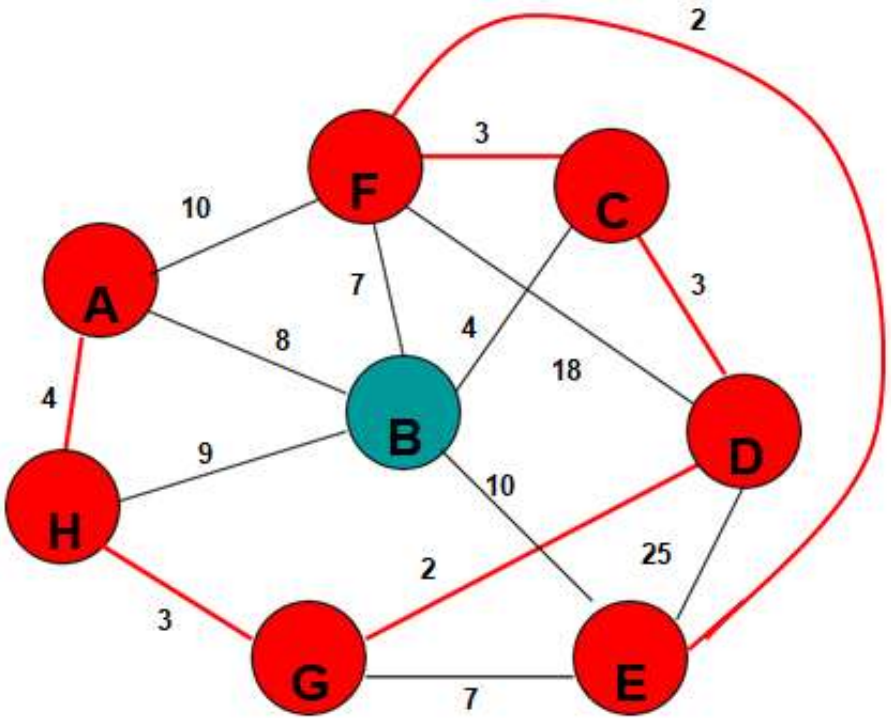
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim's algorithm: Example

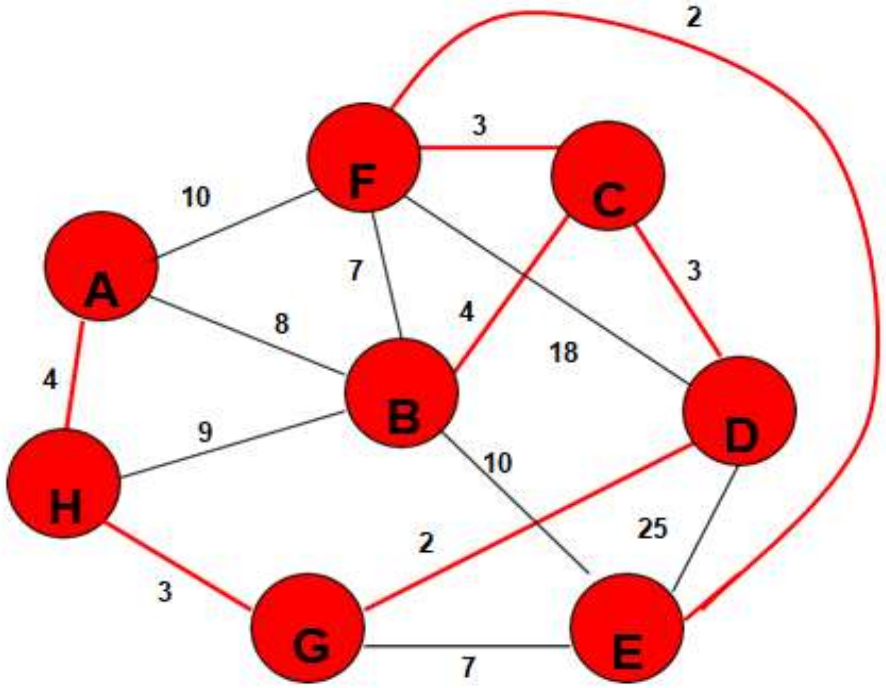


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Table entries unchanged

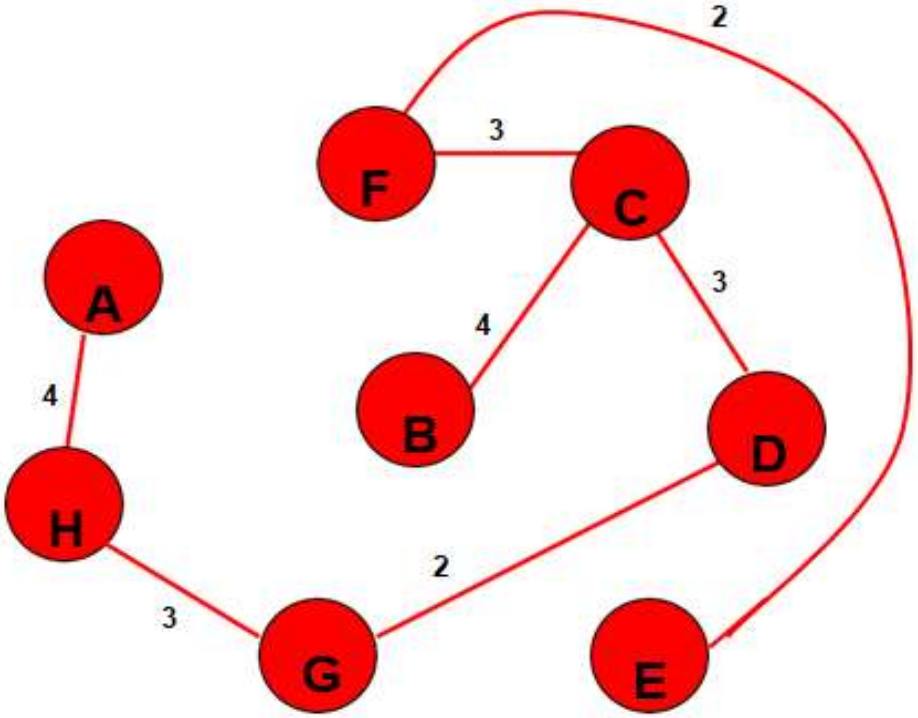
Prim's algorithm: Example



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim's algorithm: Example

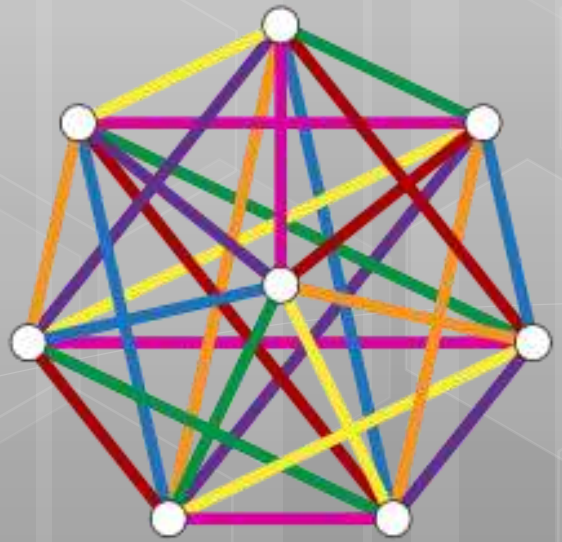


Cost of Minimum Spanning Tree = **21**

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Done


Thank you!



Graph Theory

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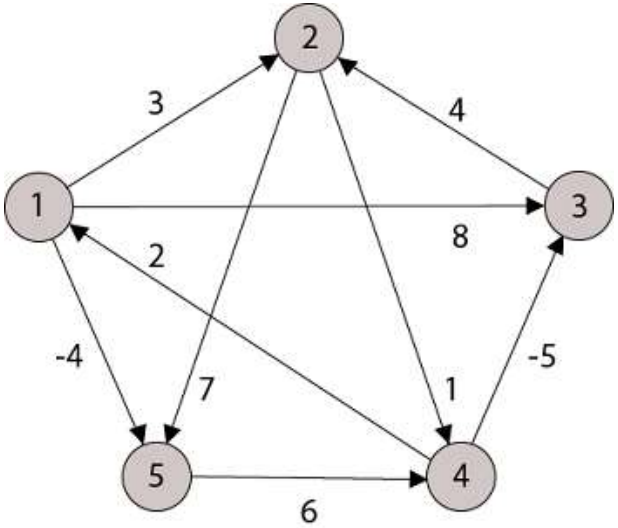
Table of Contents

Floyd Warshall's Algorithm

- Floyd Warshall Algorithm is a famous algorithm.
- It is used to solve All Pairs Shortest Path Problem.
- It computes the **shortest path between every pair of vertices** of the given graph.
- Floyd Warshall Algorithm is an example of dynamic programming approach.
- It can also handle the negative weight.

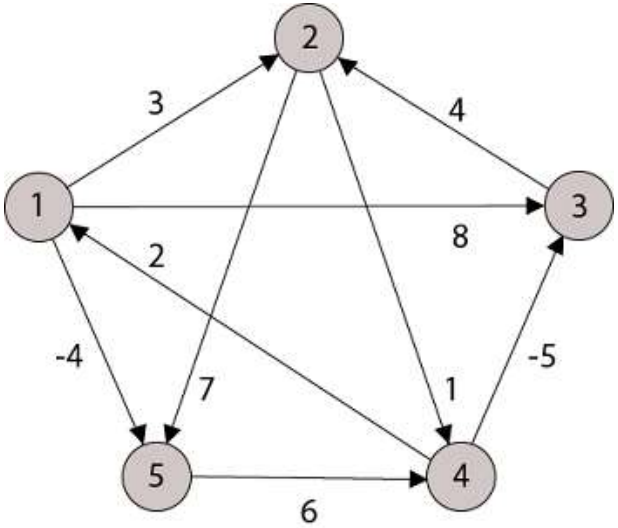
Floyd Warshall's Algorithm: Example



D(0)=

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

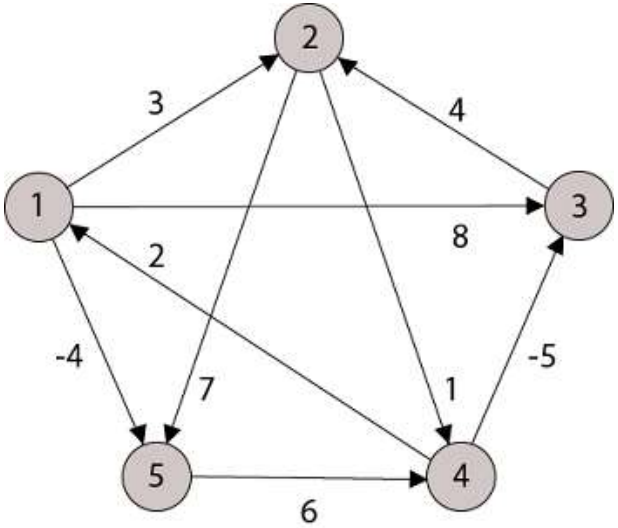
Floyd Warshall's Algorithm: Example



D(1)=

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

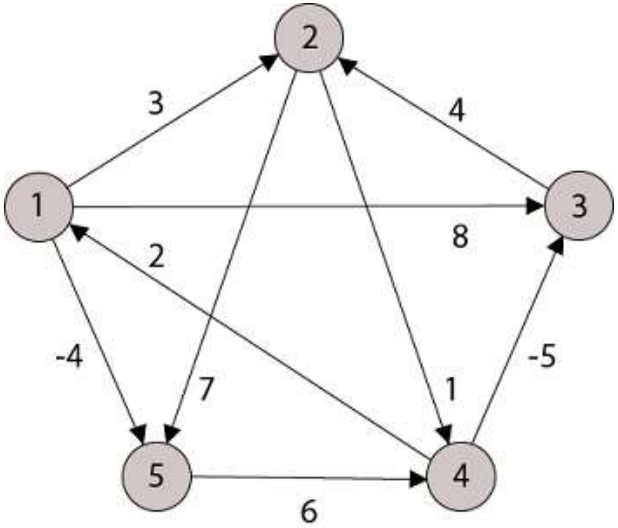
Floyd Warshall's Algorithm: Example



D(2)=

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

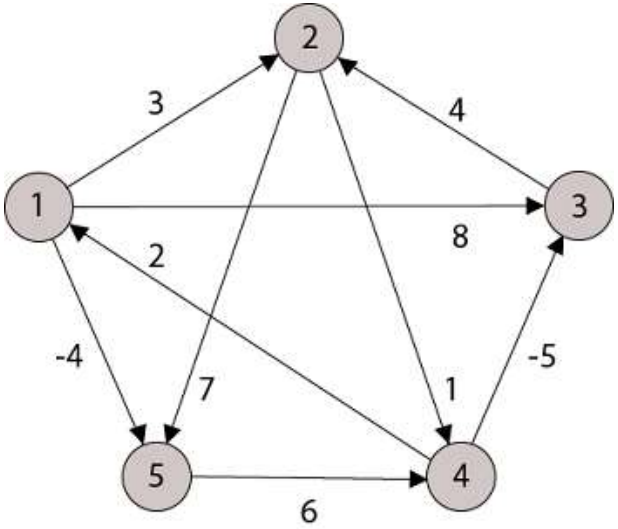
Floyd Warshall's Algorithm: Example



D(3)=

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	∞	∞	6	0

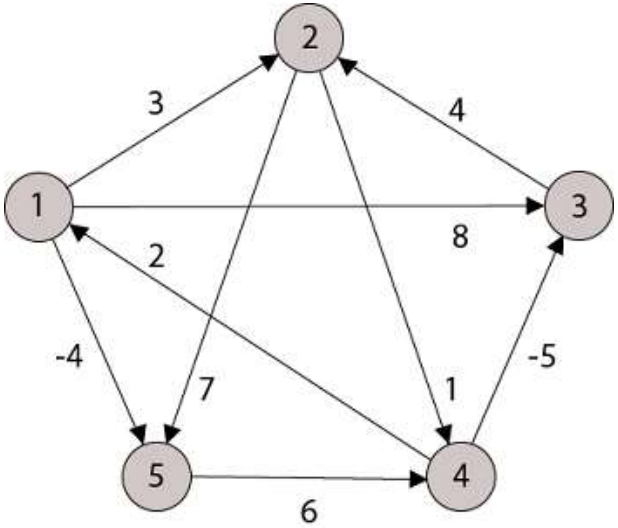
Floyd Warshall's Algorithm: Example



D(4)=

	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0

Floyd Warshall's Algorithm: Example



D(5)=

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	1	-1
3	7	4	0	5	3
4	2	-1	-5	0	-2
5	8	5	1	6	0


Thank you!