Roll Number: _____

School of Mathematics, Thapar Institute of Engineering and Technology
Mid Semester Examination (Date: 14/03/2018)

B.E.(Sem IV) Course Code: UMA031 Course Name: Optimization Techniques
Time: 2 Hours M. Marks: 30 Faculty: AK, ID, JPR, MKS, MG, SJK, MKR

1. (a) Upon completing the construction of his house, Mr. Sharma discovers that 100 square feet of plywood scrap and 120 square feet of white pine scrap are in usable form for the construction of book cases and tables. It takes 20 square feet of plywood and 20 square feet of white pine to construct a book case. It takes 10 square feet of plywood and 15 square feet of white pine to construct a table. By selling the finished products to a local furniture store, Mr. Sharma can realise a profit of Rs. 20 on each book case and Rs. 25 on each table. The linear programming formulation for the given problem is
Maximize $z = 20x_1 + 25x_2$ subject to $20x_1 + 10x_2 \leq 100$, $20x_1 + 15x_2 \leq 120$, $x_1, x_2 \geq 0$

   (i) Determine all the basic solutions of the given LPP by converting the problem to standard form and hence, find the optimal solution.

   (ii) Plot the feasible region and show the correspondence of basic feasible solutions obtained in part (i) with the corner points of the feasible region. **[4 marks]**

   (b) Solve the following linear programming problem using Two-Phase method. **[6 marks]**
Maximize $z = 2x_1 + x_2$ subject to $2x_1 + 3x_2 \geq 6$, $x_1 - x_2 \geq 1$, $x_1, x_2 \geq 0$

2. (a) Without performing Simplex iterations, formulate the linear programming problem from the following optimal table. Here, s_1 and s_2 are the slack variables. **[6 marks]**

	Basic	x_1	x_2	s_1	s_2	Solution
c_B	z	1	0	0	2	10
0	s_1	1	0	1	-1	3
2	x_2	1	1	0	1	5

   (b) Determine graphically whether the following sets are convex or not. **[2+2 marks]**

   (i) $S = \{(x, 0), (0, y); x \in \mathbb{R}, y \in \mathbb{R}\}$

   (ii) $S = \{(x, y) \in \mathbb{R}^2; xy \geq 4, x \leq 0, y \leq 0\}$

3. (a) Write the dual of the following LPP **[3 marks]**
Minimize $z = 2x_1 + 5x_2 + x_3$
subject to $x_1 - 2x_2 + x_3 \leq 6$, $3x_1 - 2x_3 = 4$, $x_1 \geq 0$, $x_2 \leq 0$ and x_3 is unrestricted in sign.

   (b) Consider the following LPP **[5+2 marks]**
Minimize $z = 5x_1 + 6x_2$ subject to $x_1 + 2x_2 \geq 2, 4x_1 + x_2 \geq 4$ and $x_1, x_2 \geq 0$

   (i) Solve the given LPP using dual simplex method.

   (ii) Find the optimal solution of its dual (without solving the dual) from the optimal table obtained in part (i).