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School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Term Examination, March 2019

B.E. IV Semester — UMA007 : Numerical Analysis

Time Limit: 02 Hours — Maximum Marks: 25

Instructor(s): Amrik Sen, A. K. Lal, Kavita Goyal, Meenu Rani, Munish Kansal, Paramjeet Singh, Sapna Sharma.

Instructions: Attempt all the questions. Organize your work, in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit.

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1. (a) Explain condition number. Check the stability of $f(x) = (\cos(x) - 1)/x^2$, when x is near 0. In case of instability, rewrite the expression to make it stable. [3 marks]

(b) How many multiplications and additions are required to determine the sum of the form $\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j$? Modify the expression to an equivalent form to reduce the number of total computations. [3 marks]

2. (a) Find the intersection point of the curves $y = \frac{1}{x}$ and $y = e^x$ in the interval $[0, 1]$ using bisection method with an accuracy 10^{-1}. [3 marks]

(b) Find the multiplicity of the root $\alpha = 0$ for the equation $e^x - x - 1 = 0$. Use modified Newton's method to find an approximation of the root by performing four iterations starting with an initial guess $x_0 = 1$. Also, compute the order of convergence. [3 marks]

3. (a) Given $g(x)$ is a continuous function on the interval $[a, b]$ such that $g(x) \in [a, b]$ for all $x \in [a, b]$ and $|g'(x)| \leq k < 1$ for all $x \in (a, b)$. Prove that for any $p_0 \in [a, b]$, the sequence generated by $p_n = g(p_{n-1}), n \geq 1$ converges to the unique fixed point p of $g(x)$ in $[a, b]$. Also, prove that $|p_n - p| < \frac{k^n}{1-k}|p_1 - p_0|$. [5 marks]

(b) Can you use Newton's method to find a root $\alpha = 5$ for the equation $(x - 5)^{\frac{1}{2}} = 0$? Justify your answer. [2 marks]

4. (a) Determine the spacing h in a table of equally spaced values of the function $f(x) = 1/x$ between 1 and 2, so that the interpolation with a quadratic interpolating polynomial yields an accuracy of 5×10^{-8}. [3 marks]

(b) Suppose $x_j = j$ for $j = 0, 1, 2, 3$ and it is known that $P_{0,1}(x) = 2x + 1, P_{1,2}(x) = x + 1$ and $P_{1,2,3}(2.5) = 3$. Find $P_{0,1,2,3}(2.5)$ using Neville's method. [3 marks]
