

# Types of Function

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# Types of Function

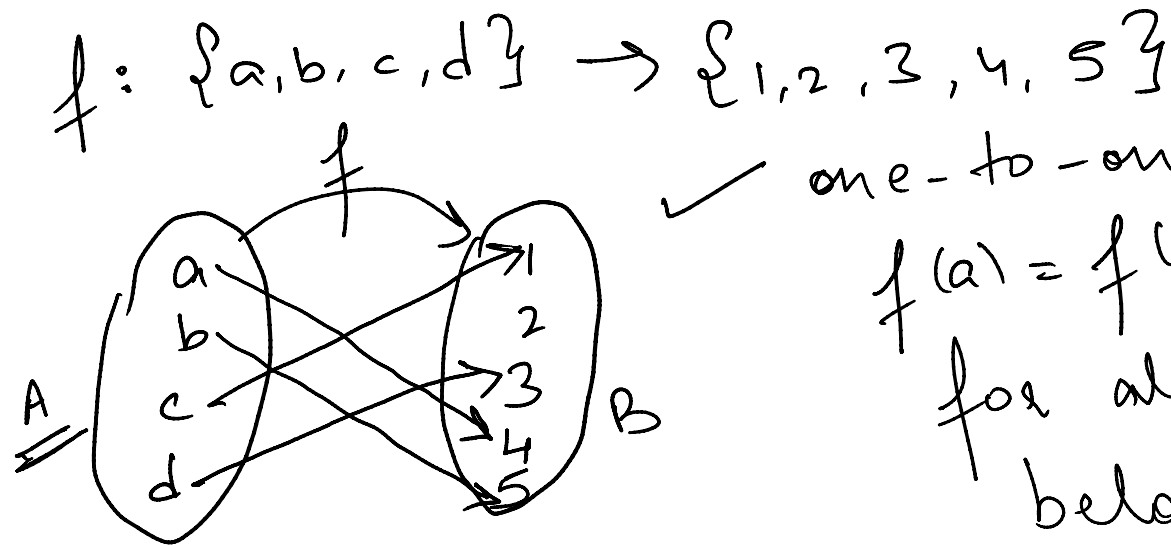
- ▶ A function can be of three types:
  - One-to-One function (Injective function)
  - Onto function (Surjective function)
  - One-to-One Correspondence (Bijective function)

# One-to-One function

- ▶ A function  $f$  is said to be *one-to-one*, or an *injection*, if and only if  $f(a) = f(b)$  implies that  $a = b$  for all  $a$  and  $b$  in the domain of  $f$ .
- ▶ A function is said to be *injective* if it is one-to-one.

# Example 1

- Determine whether the function  $f$  from  $\{a, b, c, d\}$  to  $\{1, 2, 3, 4, 5\}$  with  $\underline{f(a) = 4}$ ,  $\underline{f(b) = 5}$ ,  $\underline{f(c) = 1}$ , and  $\underline{f(d) = 3}$  is one-to-one.



✓ one-to-one.

$$f(a) = f(b) \Rightarrow a = b \quad \checkmark$$

for all elements  $a$  and  $b$   
belonging to domain of  $f$ .

## Example 2

- Determine whether the function  $f(x) = x^2$  from the set of integers to the set of integers is one-to-one.

$$\begin{aligned} f: \mathbb{Z} &\rightarrow \mathbb{Z} \\ f(x) &= x^2 \\ f(-1) &= (-1)^2 = 1 \\ f(1) &= 1^2 = 1 \end{aligned}$$

$$\underline{f(-1) = f(1)} \text{ but } \underline{-1 \neq 1}$$

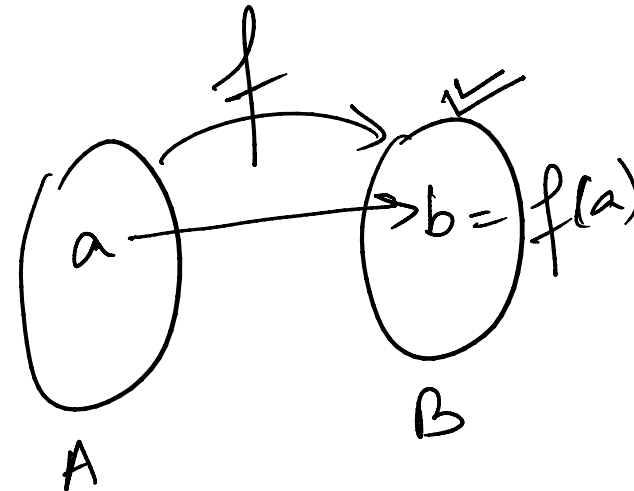
$$(f(a) = f(b) \Rightarrow a = b)$$

∴ not one-to-one

# Onto function

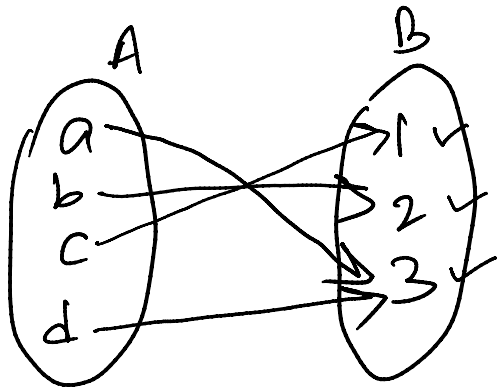
- ▶ A function  $f$  from  $A$  to  $B$  is called *onto*, or a surjection, if and only if for every element  $b \in B$  there is an element  $a \in A$  with  $f(a) = b$ .
- ▶ A function  $f$  is called surjective if it is onto.

$$f: A \rightarrow B$$



# Example 1

- Let  $f$  be the function from  $\{a, b, c, d\}$  to  $\{1, 2, 3\}$  defined by  $\underline{f(a) = 3}$ ,  $\underline{f(b) = 2}$ ,  $\underline{f(c) = 1}$ , and  $\underline{f(d) = 3}$ . Is  $\underline{f}$  an onto function?



Onto ✓  
every element  $b \in B$  has an element  
 $\downarrow$   
 $a \in A$  such that  $f(a) = b$ .

## Example 2

- Is the function  $f(x) = x^2$  from the set of integers to the set of integers onto?

$$f: \mathbb{Z} \rightarrow \mathbb{Z}$$
$$f(x) = x^2$$

3 is not the square of any integer.

Not onto.

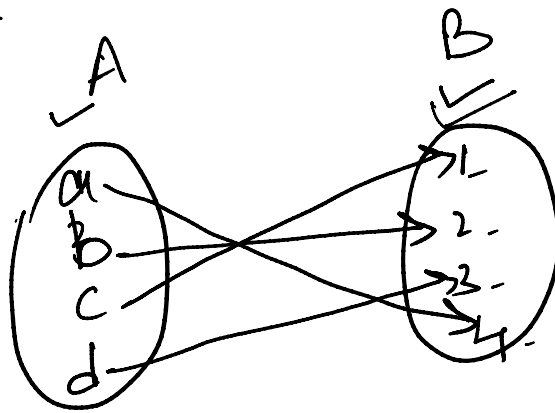


# One-to-One correspondence

- ▶ The function  $f$  is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.
- ▶ A function  $f$  is called *bijective* if it is one to one and onto.

# Example 1

- Let  $f$  be the function from  $\{a, b, c, d\}$  to  $\{1, 2, 3, 4\}$  with  $f(a) = 4, f(b) = 2, f(c) = 1$ , and  $f(d) = 3$ . Is  $f$  a bijection?



One-to-one:

$$a \neq b,$$

Onto: For  
such that  
onto.

$f(a) = f(b)$  implies that  
 $\therefore$  this fn. is one-to-one.

For every  $b \in B$ ,  $\exists a \in A$   
 $f(a) = b$ .  $\therefore$  this fn. is

$\therefore f$  is a bijection. ✓

## Example 2

- Is the function  $f(x) = x^2$  from the set of integers to the set of integers bijective?

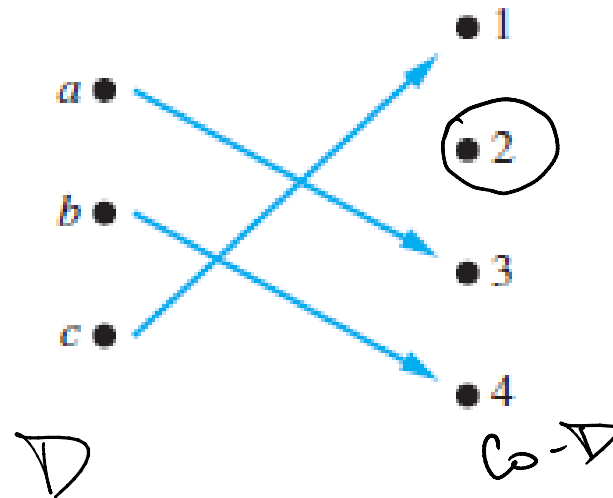
## Example 2

- Is the function  $f(x) = x^2$  from the set of integers to the set of integers bijective?

$f(n) = n^2$  is not one-to-one &  
 $f(n) = n^2$  is onto.

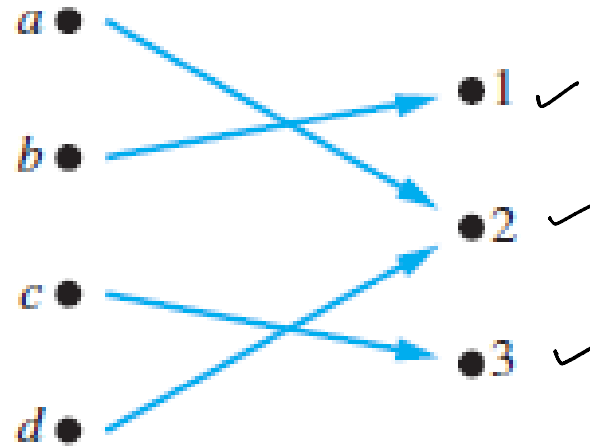
∴  $f(n) = n^2$  from  $\mathbb{Z}$  to  $\mathbb{Z}$  is not bijective.

# Examples of Different Types of Correspondences



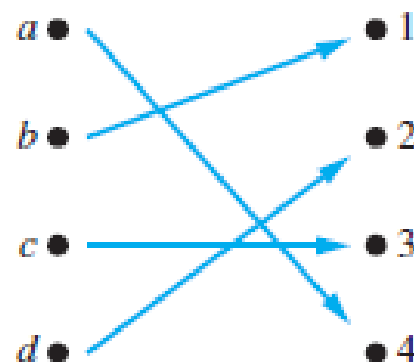
One-to-one ✓  
not onto

# Examples of Different Types of Correspondences



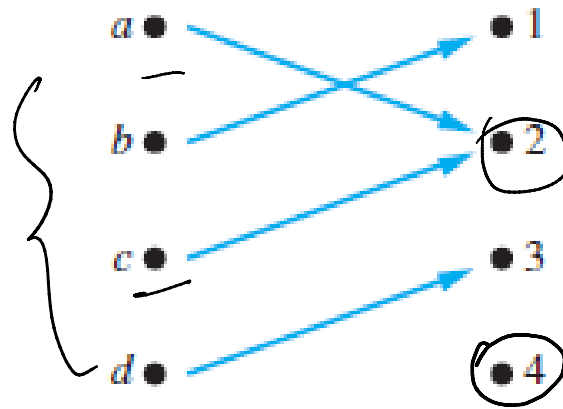
One-to-one?  
 $f(a) = f(d) = 2$   
but  $a \neq d$   
Not one-to-one  
Onto? ✓

# Examples of Different Types of Correspondences



One-to-one ✓  
Onto ✓

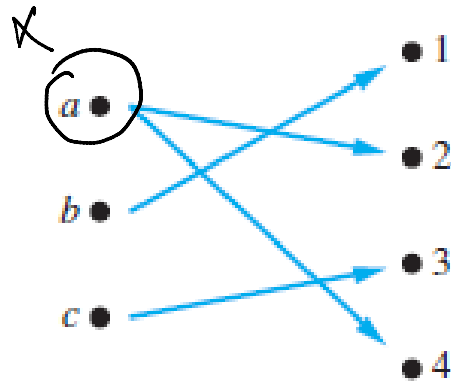
# Examples of Different Types of Correspondences



Not one-to-one  
Not onto.



# Examples of Different Types of Correspondences



Not a function.



Thanks