Discrete Mathematical Structures (UCS405)

Tutorial Sheet-6

1. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

   a) $\neg q$ **b)** $p \wedge q$ **c)** $\neg p \vee q$ **d)** $p \rightarrow \neg q$ **e)** $\neg q \rightarrow p$

   f) $\neg p \rightarrow \neg q$ **g)** $p \leftrightarrow \neg q$ **h)** $\neg p \wedge (p \vee \neg q)$

2. Let p, q, and r be the propositions

   p : Grizzly bears have been seen in the area.
   q : Hiking is safe on the trail.
   r : Berries are ripe along the trail.

   Write these propositions using p, q, and r and logical connectives (including negations).
   a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
   b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
   c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
   d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
   e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
   f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

3. State the converse, contrapositive, and inverse of each of these conditional statements.
   a) If it snows today, I will ski tomorrow.
   b) I come to class whenever there is going to be a quiz.
   c) A positive integer is a prime only if it has no divisors other than 1 and itself.

4. Use De Morgan’s laws to find the negation of each of the following statements.
   a) Jan is rich and happy.
   b) Carlos will bicycle or run tomorrow.
   c) Mei walks or takes the bus to class.
   d) Ibrahim is smart and hard working.

5. Show that each of these conditional statements is a tautology by using truth tables.

   a) $(p \wedge q) \rightarrow p$ **b)** $p \rightarrow (p \vee q)$

   c) $\neg p \rightarrow (p \rightarrow q)$ **d)** $(p \wedge q) \rightarrow (p \rightarrow q)$

6. Show that each conditional statement in question 5 is a tautology without using truth tables.

7. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

8. Find the dual of each of these compound propositions.

   a) $p \wedge \neg q \wedge \neg r$ **b)** $(p \wedge q \wedge r) \vee s$ **c)** $(p \vee \mathbf{F}) \wedge (q \vee \mathbf{T})$

9. i) Convert $p \vee q \rightarrow \neg r$ into DNF.

   ii) Convert $\neg(p \rightarrow q) \vee (r \rightarrow p)$ into CNF.