UEC-404 Signals & Systems

Tutorial #3

<u>NOTES for Signal Transformations in Time</u>

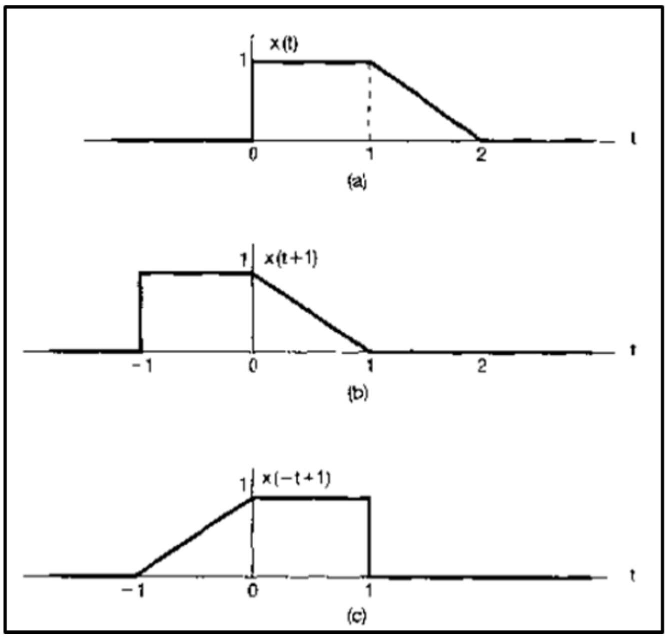
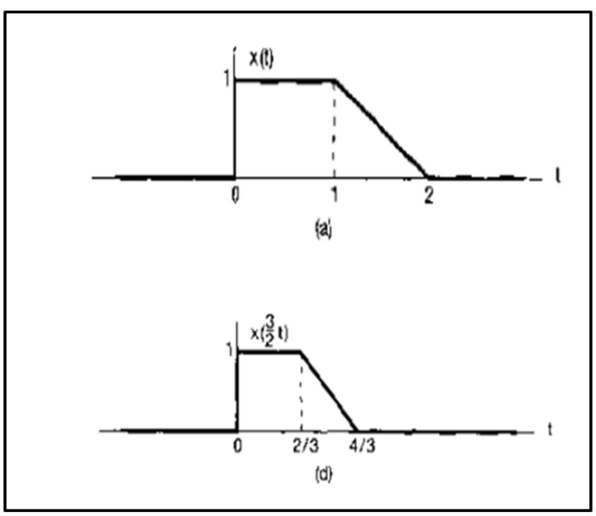


Figure 1.13 (a) The continuous-time signal $x(t)$ used in Examples 1.1–1.3 to illustrate transformations of the independent variable, (b) the time-shifted signal $x(t+1)$; (c) the signal $x(-t+1)$ obtained by a time shift and a time reversal; (d) the time-scaled signal $x(\frac{3}{2}t)$; and (e) the signal $x(\frac{3}{2}t+1)$ obtained by time-shifting and scaling.

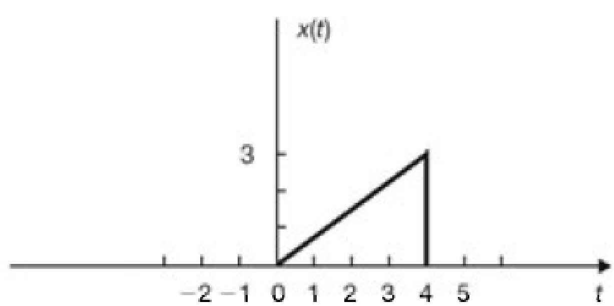


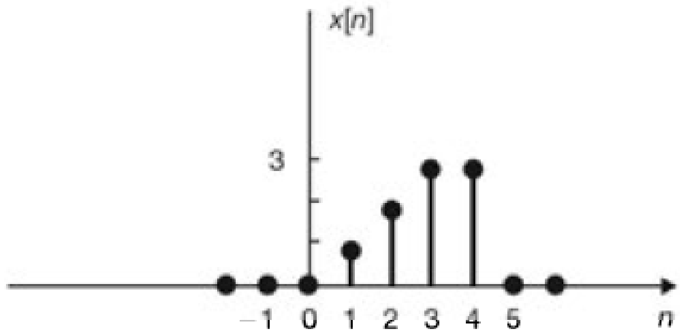
Given the signal $x(t)$, shown in Figure 1.13(a), the signal $x(\frac{3}{2}t)$ corresponds to a linear compression of $x(t)$ by a factor of $\frac{2}{3}$ as illustrated in Figure 1.13(d). Specifically we note that the value of $x(t)$ at $t = t_0$ occurs in $x(\frac{3}{2}t)$ at $t = \frac{2}{3}t_0$. For example, the value of $x(t)$ at $t = 1$ is found in $x(\frac{3}{2}t)$ at $t = \frac{2}{3}(1) = \frac{2}{3}$. Also, since $x(t)$ is zero for $t < 0$, we have $x(\frac{3}{2}t)$ zero for $t < 0$. Similarly, since $x(t)$ is zero for $t > 2$, $x(\frac{3}{2}t)$ is zero for $t > \frac{4}{3}$.

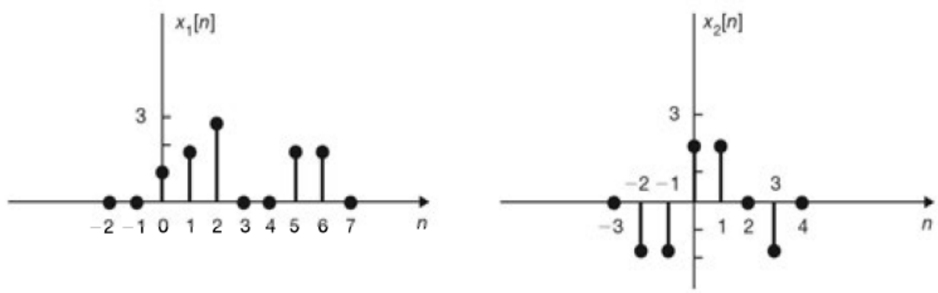
Suppose that we would like to determine the effect of transforming the independent variable of a given signal, $x(t)$, to obtain a signal of the form $x(\alpha t + \beta)$, where α and β are given numbers. A systematic approach to doing this is to first delay or advance $x(t)$ in accordance with the value of β, and then to perform time scaling and/or time reversal on the resulting signal in accordance with the value of α. The delayed or advanced signal is linearly stretched if $|\alpha| < 1$, linearly compressed if $|\alpha| > 1$, and reversed in time if $\alpha < 0$.

To illustrate this approach, let us show how $x(\frac{3}{2}t + 1)$ may be determined for the signal $x(t)$ shown in Figure 1.13(a). Since $\beta = 1$, we first advance (shift to the left) $x(t)$ by 1 as shown in Figure 1.13(b). Since $|\alpha| = \frac{3}{2}$, we may linearly compress the shifted signal of Figure 1.13(b) by a factor of $\frac{2}{3}$ to obtain the signal shown in Figure 1.13(e).

[1]	Use the sifting property of the dirac delta (impulse) function: $$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$ What are the values of the following integrals involving impulses: **[a]** $\int_{t=-\infty}^{\infty}(\cos t)\,\delta(t)dt$ Ans. 1 **[b]** $\int_{t=0}^{\pi/2}(\sin t)\,\delta(t-\pi/2)dt$ Ans. 1 **[c]** $\int_{t=\pi}^{\infty}(\sin t)\,\delta(t-\pi/2)dt$ Ans. 0 **[d]** $\int_{t=0}^{\infty}\delta(t+\pi/2)\,[\sin(t-\pi)]dt$ Ans. 0 **[e]** $\int_{t=-\infty}^{\infty}\delta(t)\,(t^3-2t^2+10t+1)dt$ Ans. 1 **[f]** $\int_{t=0}^{0}\delta(t)\,e^{2t}dt$ Ans. 1 **[g]** $\int_{t=0}^{0}10^{10}\,e^{2t}dt$ Ans. 0 **[h]** $\int_{-4}^{7}\sin(\omega t)\,(t-3)^2\delta(3t+4)dt$ Ans. $6.26\ \sin(-4/3\omega)$
[2]	**[a]** Compute the polar form of the complex numbers $e^{j(1+j)}$ and $(1+j)e^{-j\pi/2}$ **[b]** Compute the rectangular form of the complex numbers $2e^{\frac{j5\pi}{4}}$ and $e^{-j\pi}+e^{j6\pi}$
[3]	Compute the following integrals **a)** $\int_{-\infty}^{\infty}e^{-t}\,\delta(t-1)dt$ Ans. e^{-1} **b)** $\int_{0}^{\infty}e^{-t}\,\delta(t-1)dt$ Ans. e^{-1}

	c) $\int_{0}^{\infty} e^{-t}\,\delta(t+1)dt$ Ans. 0 since the interval of integration does not include the point t= -1, where the impulse is centred. **d)** $\int_{-\infty}^{\infty}(t^3+t^2+t+1)\,\delta(t)dt$ Ans. 1 **e)** $\int_{-\infty}^{\infty}\cos^2(2\pi t+0.1\pi)\,\delta(t+1)dt$ Ans. $\cos^2(0.1\pi)$ **f)** $\int_{-\infty}^{\infty} e^{-t}\,\delta(-t-1)dt$ Ans. e **g)** $\int_{-\infty}^{\infty} t^2\,\delta\left(-\frac{1}{2}t+\frac{1}{2}\right)dt$ Ans. 2 **h)** $\int_{-\infty}^{\infty} e^{t}\,\delta(3t-1)dt$ Ans. $\frac{1}{3}e^{1/3}$
[4]	Write the following sinusoids in terms of complex exponentials **a)** $x(t)=3\cos(100\pi t+15^\circ)$ **b)** $x(t)=2\cos(10\pi t-0.1\pi)$ **c)** $x(t)=5\sin(20\pi t-0.2\pi)$ **d)** $x(t)=10\sin(1000\pi t)$ **e)** $x(t)=3\sin(200\pi t-0.2\pi)$
[5]	Take the complex exponential signal $$x(t)=5e^{j(20\pi t+0.2\pi)}$$ For each of the expression below, determine the corresponding constant H: **a)** $\dot{x}(t)=H\,x(t)$ Ans. $H=j20\pi$ **b)** $\int x(t)\,dt=H\,x(t)$ Ans. $H=1/j20\pi$ **c)** $x(t-0.1)=H\,x(t)$ Ans. $H=e^{-j2\pi}=1$ Could you do the same (i.e., find a constant H in each of these expression) if $x(t)$ were not an exponential?
[6]	A continuous-time signal $x(t)$ is shown below. Sketch and label each of the following signals:  **[a]** $x(t-2)$ **[b]** $x(2t)$ **[c]** $x(t/2)$ **[d]** $x(-t)$ ***Hint*:** refer to above NOTES.

<table>
<tr><td>[7]</td><td>A discrete-time signal $x[n]$ is shown below. Sketch and label each of the following signals:


[a] $x[n-2]$
[b] $x[2n]$
[c] $x[-n]$
[d] $x[-n+2]$</td></tr>
<tr><td>[8]</td><td>Using the discrete-time signals $x_1[n]$ and $x_2[n]$ shown below, represent each of the following signals by a graph and by a sequence of numbers.


[a] $y_1[n] = x_1[n] + x_2[n]$
[b] $y_2[n] = 2x_1[n]$
[c] $y_3[n] = x_1[n]x_2[n]$</td></tr>
<tr><td>[9]</td><td>Find the even and odd components of $x(t) = e^{jt}$</td></tr>
<tr><td>[10]</td><td>Find the even and odd components of each of the following signals:
[a] $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$
[b] $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$</td></tr>
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