

Numbers in RED are index positions in array

\* The corresponding array is shown below each max-heap visualized as a nearly complete binary tree.

Q. Illustrate stepwise execution of the Heapsort algorithm on an array  $A = [44, 30, 50, 22, 60, 55, 77, 55]$  to arrange its elements in ascending order.

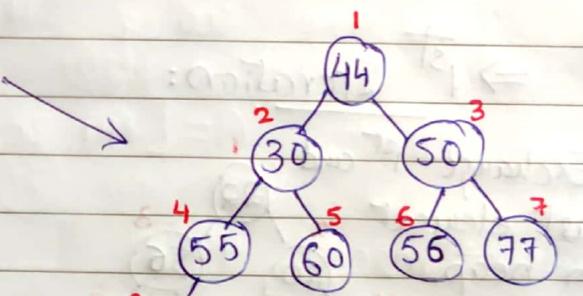
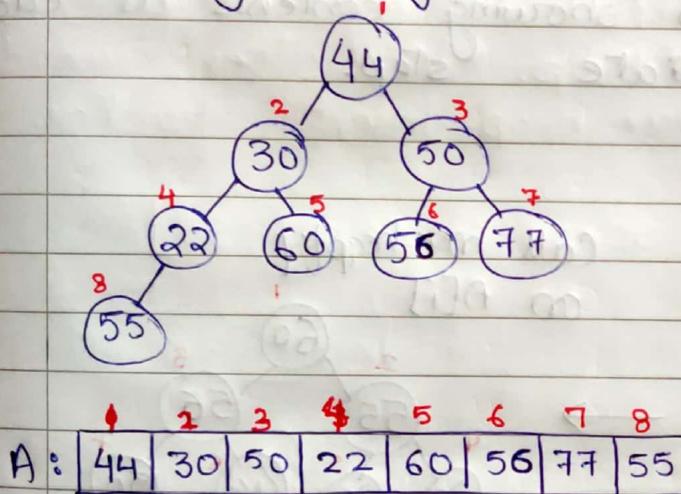
STEP I: Create max-heap.

It can be done in two ways

- ① Taking all the elements at once
- ② Taking one element, insert in the existing max-heap.

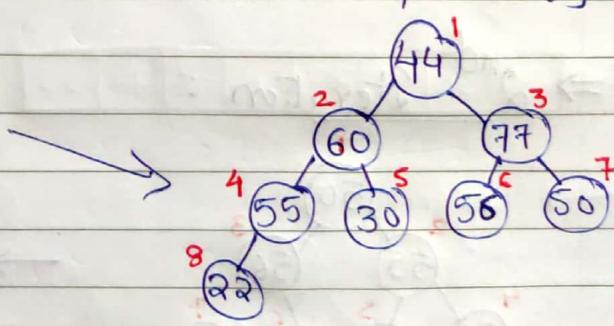
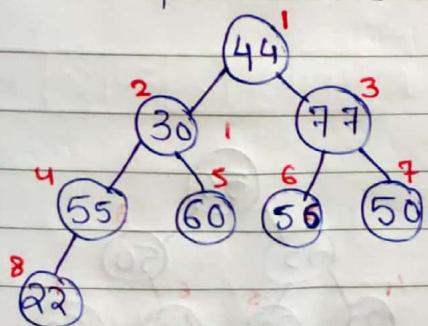
Using way ①

→ Execute max-heapify at index 4, i.e.  $A[4] = 22$ .



⇒ Execute max-heapify at index 3, i.e.  $A[3] = 50$ .

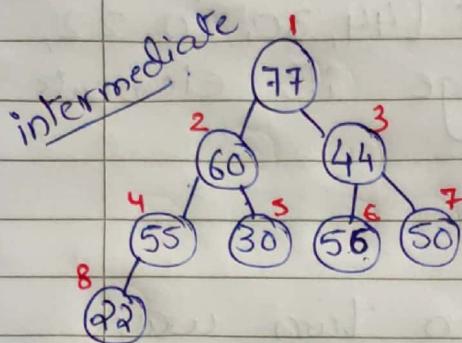
⇒ Execute max-heapify at index 2, i.e.  $A[2] = 30$



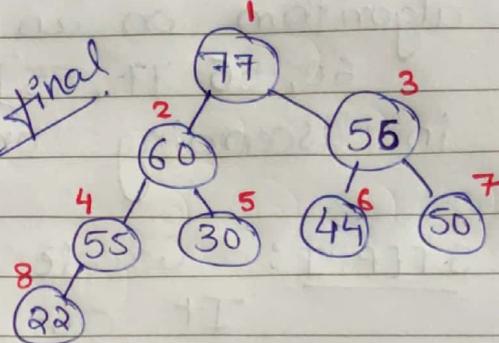
A: [44, 30, 77, 55, 60, 56, 50, 22]

A: [44, 60, 77, 55, 30, 56, 50, 22]

$\Rightarrow$  Execute max-heapify at index 1, i.e.  $A[1] = 44$ .



A :	77	60	44	55	30	56	50	22
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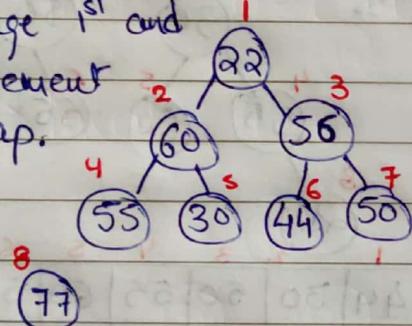


A :	77	60	56	55	30	44	50	22
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STEP II : Execute rest of the steps of heapsort algorithm showing contents at each intermediate step.

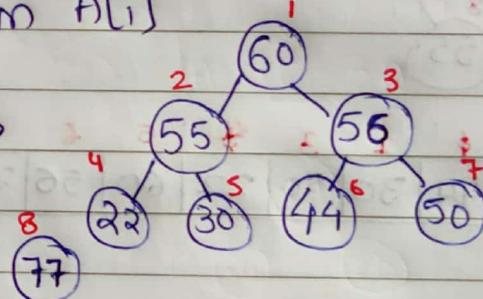
$\Rightarrow$  1<sup>st</sup> Iteration:

Exchange 1<sup>st</sup> and last element in heap.



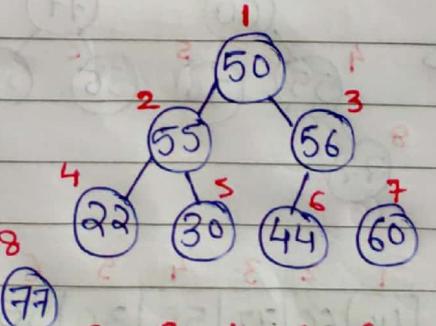
A :	22	60	56	55	30	44	50	77
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Call max-heapify on  $A[1]$

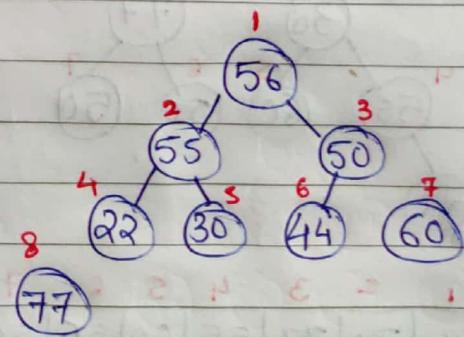


A :	60	55	56	22	30	44	50	77
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$\Rightarrow$  2<sup>nd</sup> Iteration :

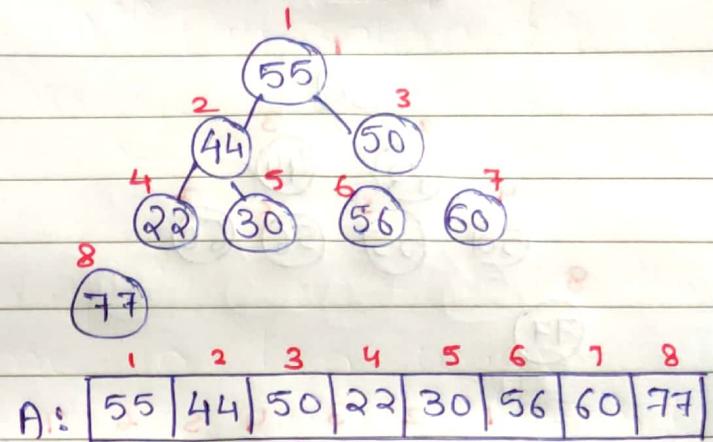
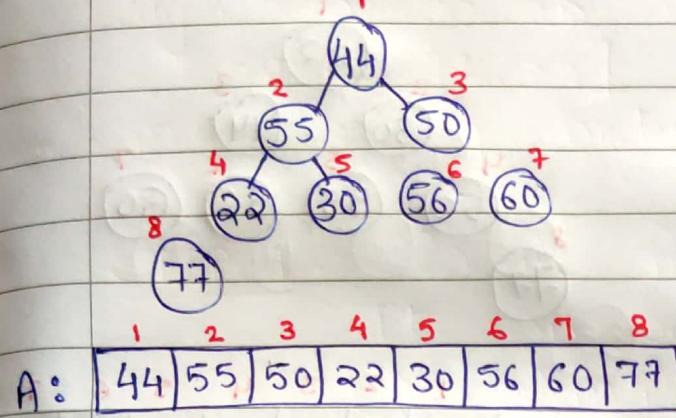


A :	50	55	56	22	30	44	60	77
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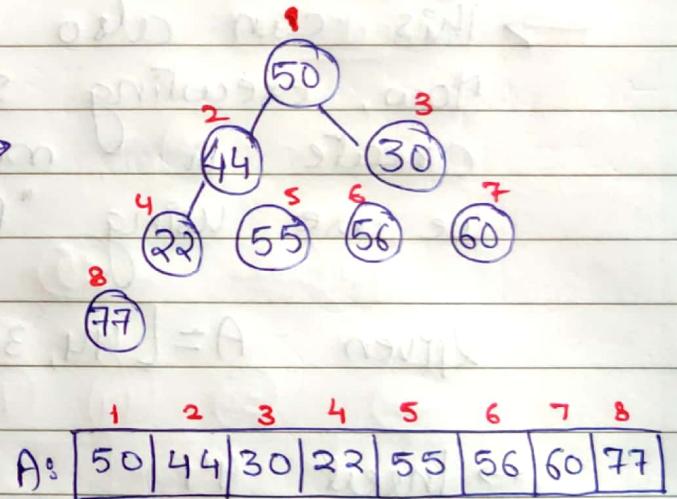
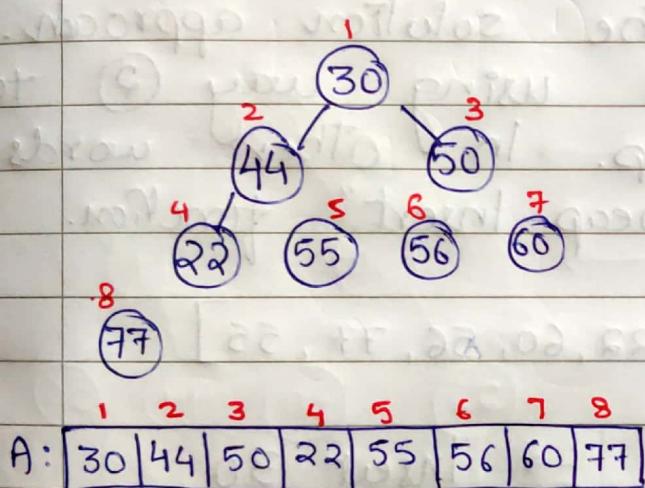


A :	56	55	50	22	30	44	60	77
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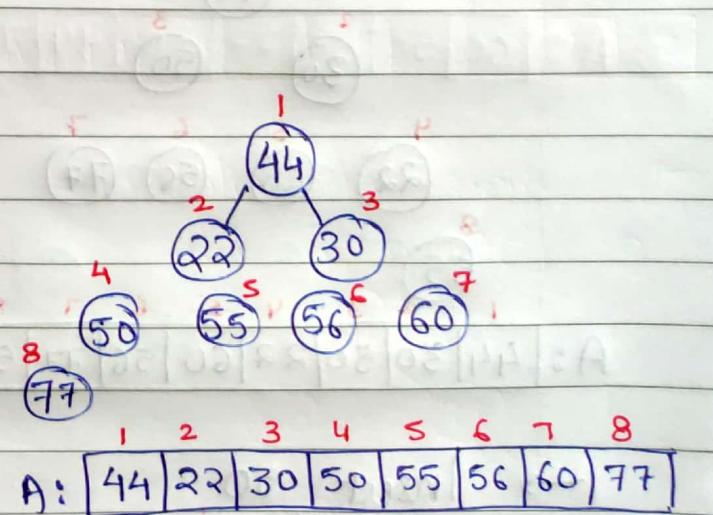
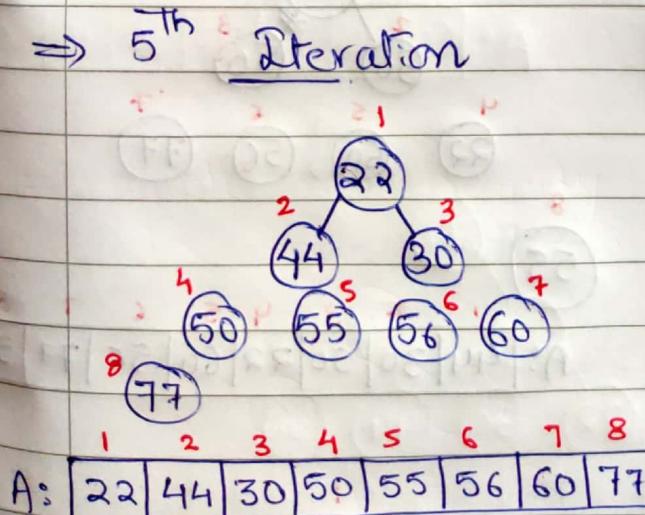
⇒ 3<sup>rd</sup> Iteration



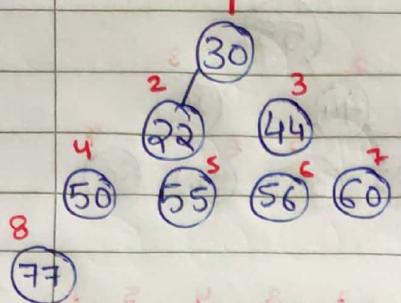
⇒ 4<sup>th</sup> Iteration



⇒ 5<sup>th</sup> Iteration

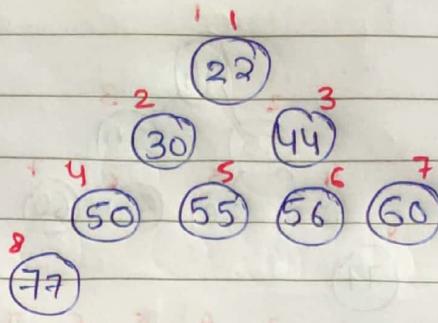


→ 6<sup>th</sup> Iteration



1	2	3	4	5	6	7	8
A: 30	22	44	50	55	56	60	77

→ 7<sup>th</sup> Iteration



1	2	3	4	5	6	7	8
A: 22	30	44	50	55	56	60	77

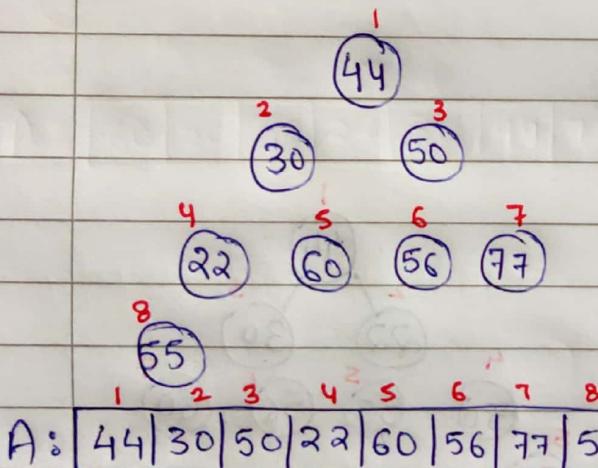
"SORTED"

→ This can also be the solution approach.

Now, executing STEP 1 using way ② to create a max-heap. In other words we are using "Max-heap-Insert" function.

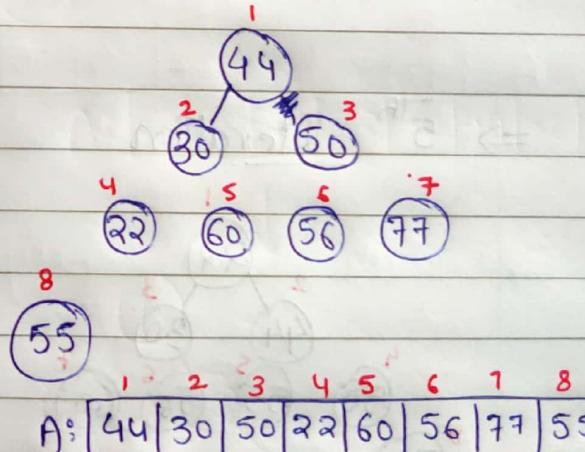
Given A = [44, 30, 50, 22, 60, 56, 77, 55]

→ Insert 44



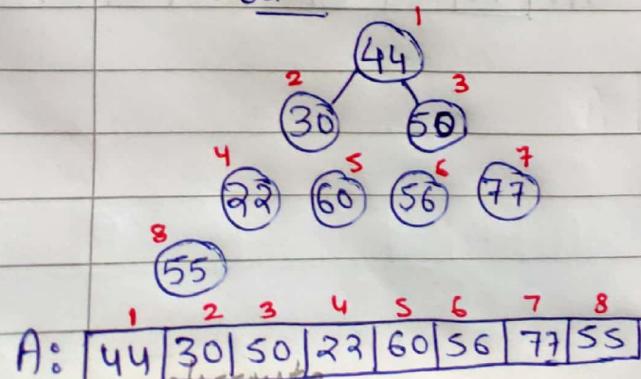
1	2	3	4	5	6	7	8
A: 44	30	50	22	60	56	77	55

→ Insert 30



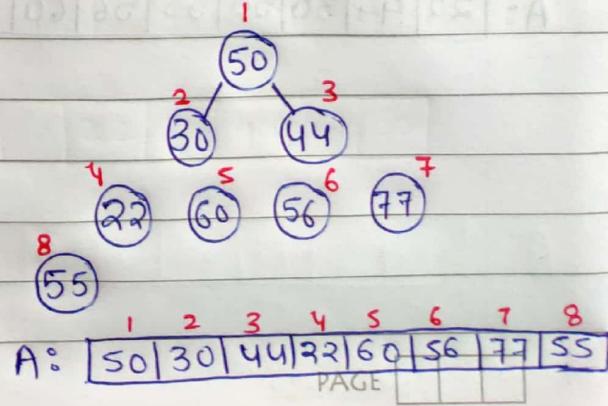
1	2	3	4	5	6	7	8
A: 44	30	50	22	60	56	77	55

→ Insert 50



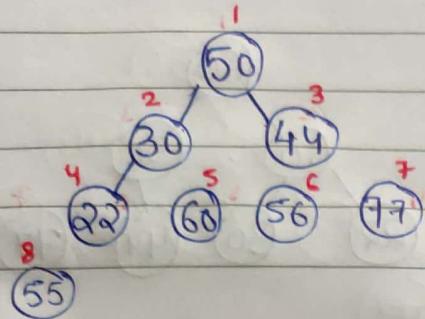
1	2	3	4	5	6	7	8
A: 44	30	50	22	60	56	77	55

→



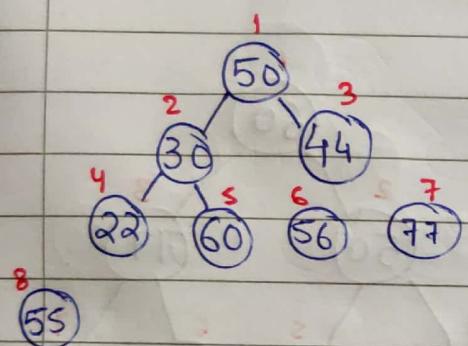
1	2	3	4	5	6	7	8
A: 50	30	44	22	60	56	77	55

→ Insert 22

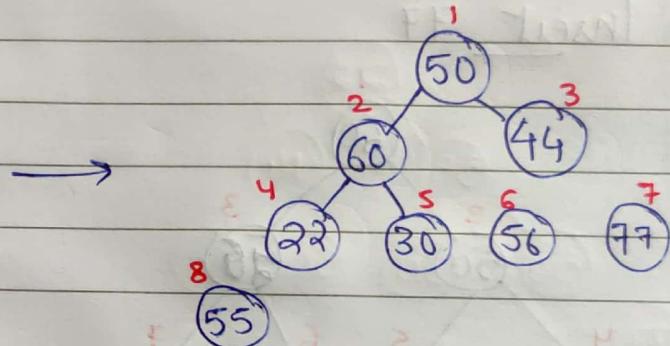


A[8]	1	2	3	4	5	6	7	8
50   30   44   22   60   56   77   55								

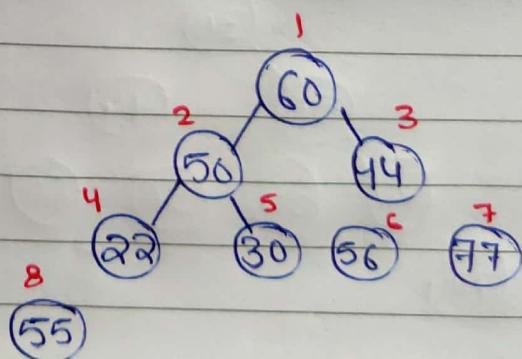
→ Insert 60



A[8]	1	2	3	4	5	6	7	8
50   30   44   22   60   56   77   55								

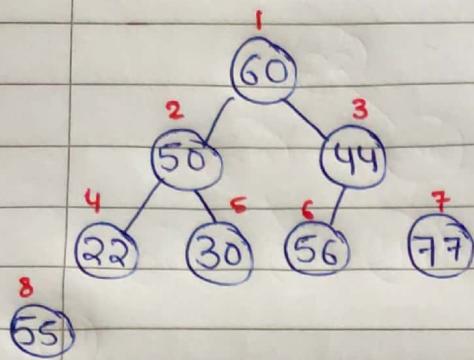


A[8]	1	2	3	4	5	6	7	8
50   60   44   22   30   56   77   55								

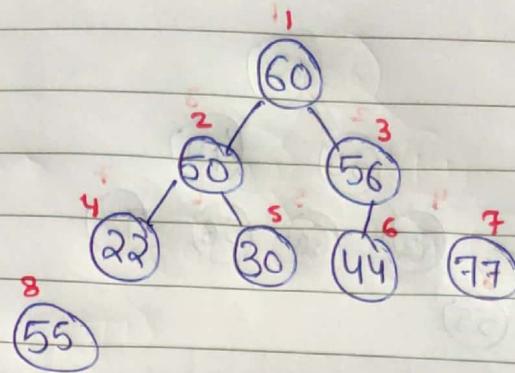


A[8]	1	2	3	4	5	6	7	8
60   50   44   22   30   56   77   55								

→ Insert 56



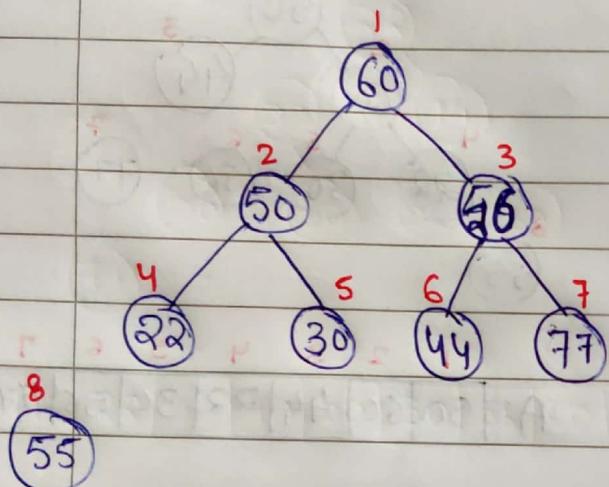
→



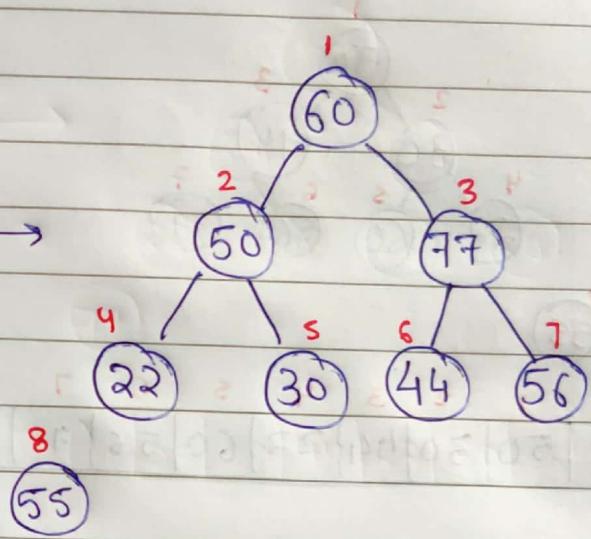
A: [60|50|44|22|30|56|77|55]

A: [60|50|56|22|30|44|77|55]

→ Insert 77

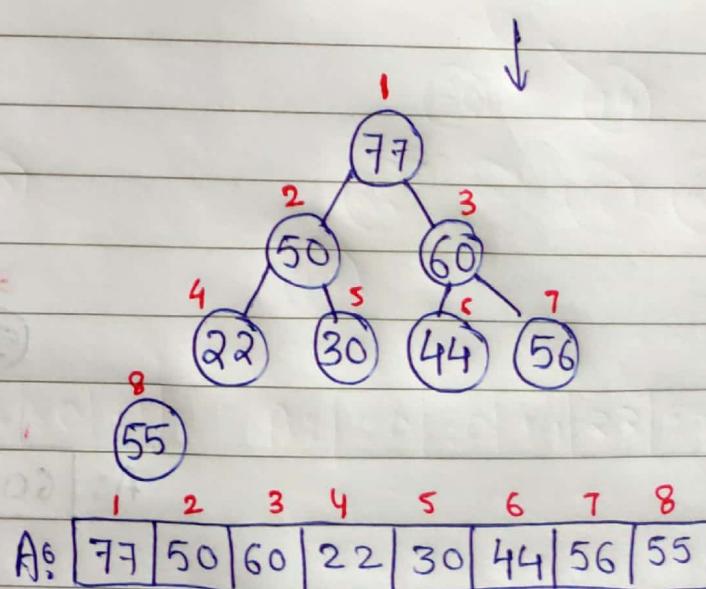


→



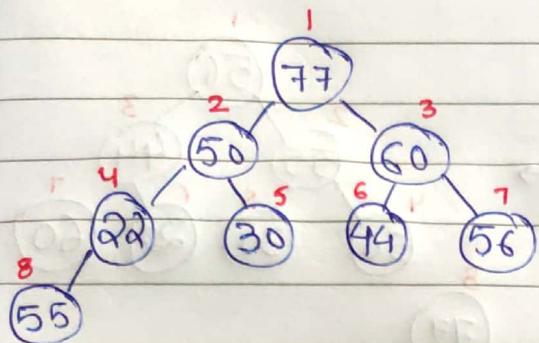
A: [60|50|56|22|30|44|77|55]

A: [60|50|77|22|30|44|56|55]

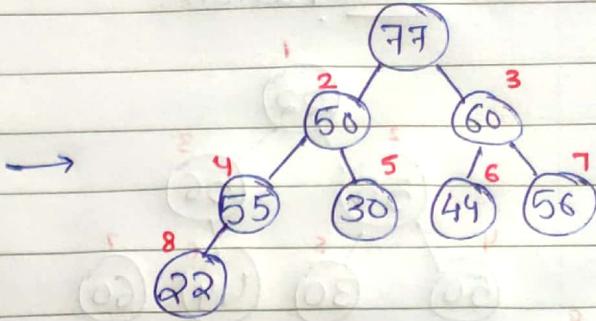


A: [77|50|60|22|30|44|56|55]

→ Insert 55

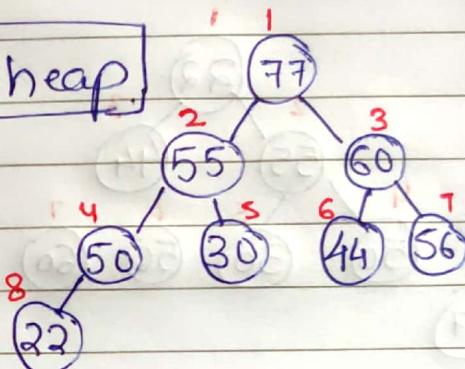


A:	77   50   60   22   30   44   56   55
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A:	77   50   60   55   30   44   56   22
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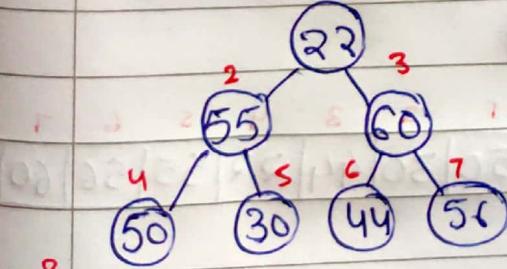
Max-heap



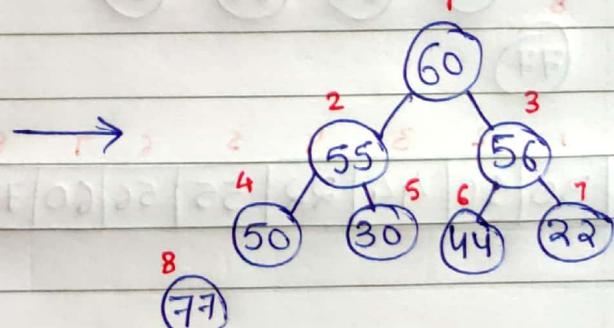
A:	77   55   60   50   30   44   56   22
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STEP II: Execute rest of the steps of heapsort algorithm showing contents at each intermediate step.

→ 1<sup>st</sup> Iteration

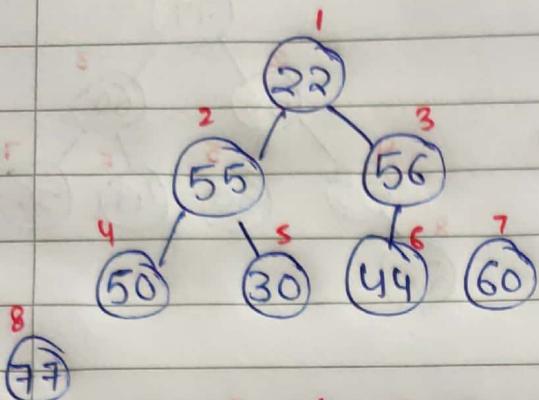


A:	22   55   60   50   30   44   56   77
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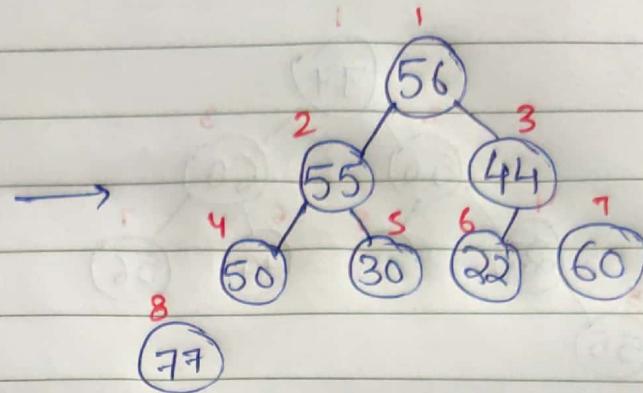


A:	60   55   56   50   30   44   22   77
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$\Rightarrow$  2<sup>nd</sup> Iteration

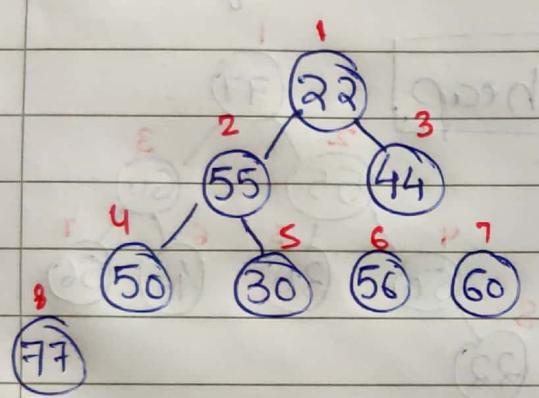


A:	22	55	56	50	30	44	60	77
----	----	----	----	----	----	----	----	----

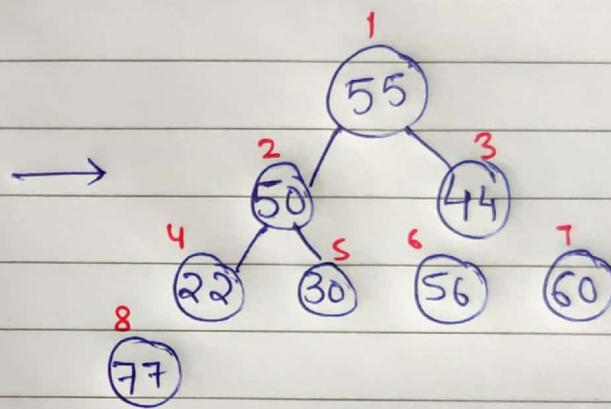


A:	56	55	44	50	30	22	60	77
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$\Rightarrow$  3<sup>rd</sup> Iteration

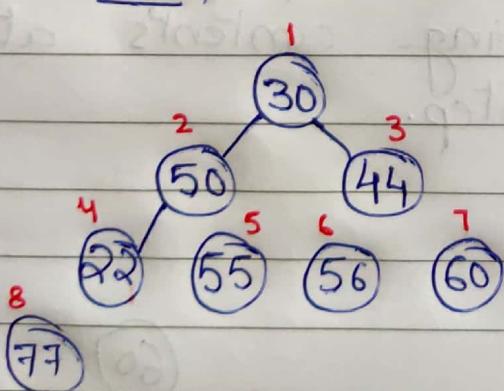


A:	22	55	44	50	30	56	60	77
----	----	----	----	----	----	----	----	----

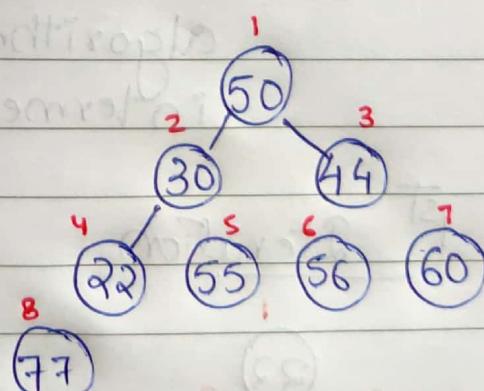


A:	55	50	44	22	30	56	60	77
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$\Rightarrow$  4<sup>th</sup> Iteration

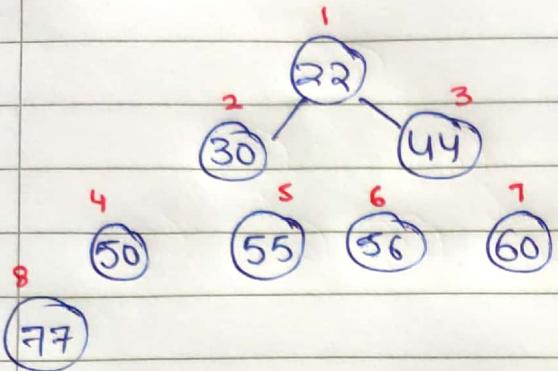


A:	30	50	44	22	55	56	60	77
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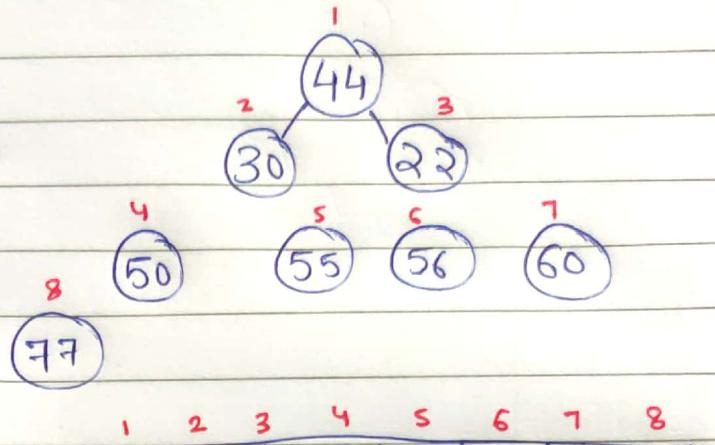
A:	50	30	44	22	55	56	60	77
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⇒ 5<sup>th</sup> Iteration



A: 

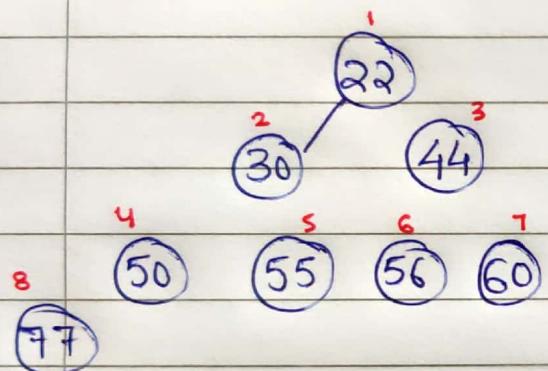
22	30	44	50	55	56	60	77
----	----	----	----	----	----	----	----



A: 

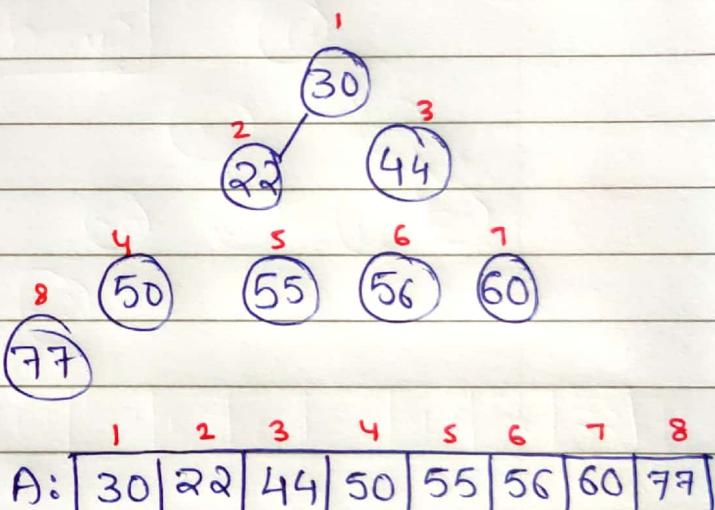
44	30	22	50	55	56	60	77
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⇒ 6<sup>th</sup> Iteration



A: 

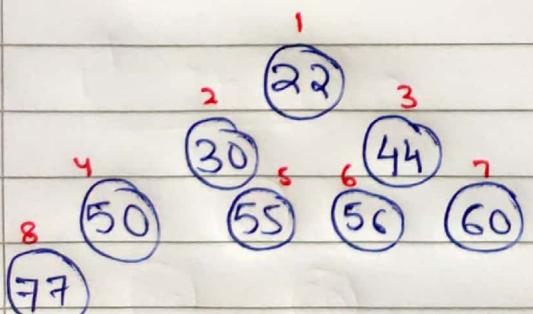
22	30	44	50	55	56	60	77
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A: 

30	22	44	50	55	56	60	77
----	----	----	----	----	----	----	----

⇒ 7<sup>th</sup> Iteration



A: 

22	30	44	50	55	56	60	77
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"SORTED"