Course: UMA 035 (Optimization Techniques)

Instructor: Dr. Amit Kumar,

Associate Professor,

School of Mathematics,

TIET, Patiala

Email: amitkumar@thapar.edu

Mob: 9888500451

Example:

Example

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

What will happen if fourth job cannot be assigned to the third person.

Solution:

Since, in the obtained optimal solution, the fourth job has not been assigned to the third person. So, solution will remain same.

Example:

What will happen if fouth job cannot be assigned to the fourth person.

Solution:

Since, in the obtained optimal solution, the fourth job has been assigned to the fourth person. So, solution will change.

To find the optimal solution, take '-' at this position after transforming the problem into minimization and balanced.

Since, maximization problem. So transform into minimization by subtracting all the elements of the table from the largest element

		Jobs			
		I	II	III	IV
Persons	A	$42-42=0$	$42-35=7$	$42-28=14$	$42-21=21$
	B	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	C	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	D	$42-24=18$	$42-20=22$	$42-16=26$	$42-12=30$

		Jobs			
		I	II	III	IV
Persons	A	0	7	14	21
	B	12	17	22	27
	C	12	17	22	27
	D	18	22	26	–

Balanced Problem (Square matrix)

Subtract minimum of each row from the elements of the corresponding row.

		Jobs			
		I	II	III	IV
Persons	A	$0-0=0$	$7-0=7$	$14-0=14$	$21-0=21$
	B	$12-12=0$	$17-12=5$	$22-12=10$	$27-12=15$
	C	$12-12=0$	$17-12=5$	$22-12=10$	$27-12=15$
	D	$18-18=0$	$22-18=4$	$26-18=8$	-

		Jobs			
		I	II	III	IV
Persons	A	0	7	14	21
	B	0	5	10	15
	C	0	5	10	15
	D	0	4	8	-

Subtract minimum of each column from the elements of the corresponding column.

		Jobs			
		I	II	III	IV
Persons	A	0−0 = 0	7−4 = 3	14−8 = 6	21−15 = 6
	B	0−0 = 0	5−4 = 1	10−8 = 2	15−15 = 0
	C	0−0 = 0	5−4 = 1	10−8 = 2	15−15 = 0
	D	0−0 = 0	4−4 = 0	8−8 = 0	-

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	6
	B	0	1	2	0
	C	0	1	2	0
	D	0	0	0	–

One 0 in first row. Select it and cut the column.

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	6
	B	0	1	2	0
	C	0	1	2	0
	D	0	0	0	–

One 0 in second row. Select it and cut the column.

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	6
	B	0	1	2	0
	C	0	1	2	0
	D	0	0	0	–

One 0 in second column. Select it and cut the row.

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	6
	B	0	1	2	0
	C	0	1	2	0
	D	0	0	0	–

No 0 left but no 0 has been assigned in third row. Not optimal soltion.

Minimum {3,6,1,2,1,2}=1

		Jobs			
		I	II	III	IV
Persons	A	0	3 – 1	6 – 1	6
	B	0	1 – 1	2 – 1	0
	C	0	1 – 1	2 – 1	0
	D	0+1	0	0	–

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	6
	B	0	0	1	0
	C	0	0	1	0
	D	1	0	0	–

One 0 in first row. Select it cut the column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	6
	B	0	0	1	0
	C	0	0	1	0
	D	1	0	0	–

One 0 in third column. Select it and cut the row.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	6
	B	0	0	1	0
	C	0	0	1	0
	D	1	0	0	–

Two 0 in second row. Choose any one, cross the other and cut the column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	6
	B	0	0	1	0
	C	0	0	1	0
	D	1	0	0	–

One 0 in third row. Select it and cut column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	6
	B	0	0	1	0
	C	0	0	1	0
	D	1	0	0	–

No 0 left and one 0 has been selected in each row. Solution is optimal.

Optimal solution:

Job I to person A

Job II to person B

Job III to person D

Job IV to person C

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Optimal profit:

42+25+15+16=98

Example:

What will happen if fourth job is always assigned to the fourth person.

Solution:

Since, maximization problem. So transform into minimization by subtracting all the elements of the table from the largest element

		Jobs			
		I	II	III	IV
Persons	A	$42-42=0$	$42-35=7$	$42-28=14$	$42-21=21$
	B	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	C	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	D	$42-24=18$	$42-20=22$	$42-16=26$	$42-12=30$

		Jobs			
		I	II	III	IV
Persons	A	0	7	14	21
	B	12	17	22	27
	C	12	17	22	27
	D	18	22	26	30

Balanced Problem (Square matrix)

Delete the fourth job and the fourth person

		I	II	III
Persons	A	0	7	14
	B	12	17	22
	C	12	17	22

Subtract minimum of each row from the elements of the corresponding row.

		Jobs		
		I	II	III
Persons	A	$0-0=0$	$7-0=7$	$14-0=14$
	B	$12-12=0$	$17-12=5$	$22-12=10$
	C	$12-12=0$	$17-12=5$	$22-12=10$

		Jobs		
		I	II	III
Persons	A	0	7	14
	B	0	5	10
	C	0	5	10

Subtract minimum of each column from the elements of the corresponding column.

		Jobs		
		I	II	III
Persons	A	$0-0=0$	$7-5=2$	$14-10=4$
	B	$0-0=0$	$5-5=0$	$10-10=0$
	C	$0-0=0$	$5-5=0$	$10-10=0$

		Jobs		
		I	II	III
Persons	A	0	2	4
	B	0	0	0
	C	0	0	0

One 0 in first row. Select it and cut the column

		Jobs		
		I	II	III
Persons	A	0	2	4
	B	0	0	0
	C	0	0	0

Two 0 in second row. Choose any one, cross the other and cut the column.

		Jobs		
		I	II	III
Persons	A	0	2	4
	B	0	0	0
	C	0	0	0

One 0 in third row. Select it and cut the column

		Jobs		
		I	II	III
Persons	A	0	2	4
	B	0	0	0
	C	0	0	0

No 0 left and one 0 has been selected in each row. Solution is optimal.

Optimal solution:

Job I to person A

Job II to person B

Job III to person C

Job IV to person D

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Optimal profit:

42+25+20+12=99

Example

		Jobs			
		I	II	III	IV
Persons	A	50	50	-	20
	B	70	40	20	30
	C	90	30	50	-
	D	70	20	60	70

Suppose an additional person becomes available for performing all four jobs at the respective costs 60, 45, 30 and 80. Is it economical to replace one of the current four persons with the new one?

Solution:

		Jobs			
		I	II	III	IV
Persons	A	50	50	-	20
	B	70	40	20	30
	C	90	30	50	-
	D	70	20	60	70
	E	60	45	30	80

Minimization Problem

Unbalanced problem (Not a square matrix)

Need to transform into balanced one

		Jobs				
		I	II	III	IV	V
Persons	A	50	50	-	20	0
	B	70	40	20	30	0
	C	90	30	50	-	0
	D	70	20	60	70	0
	E	60	45	30	80	0

Subtracting minimum (0) of each row from the elements of the corresponding row

		Jobs				
		I	II	III	IV	V
Persons	A	50	50	-	20	0
	B	70	40	20	30	0
	C	90	30	50	-	0
	D	70	20	60	70	0
	E	60	45	30	80	0

Subtracting minimum of each column from the elements of the corresponding column

		Jobs				
		I	II	III	IV	V
Persons	A	50−50	50−20	-	20−20	0
	B	70−50	40−20	20−20	30−20	0
	C	90−50	30−20	50−20	-	0
	D	70−50	20−20	60−20	70−20	0
	E	60−50	45−20	30−20	80−20	0

		Jobs				
		I	II	III	IV	V
Persons	A	0	30	-	0	0
	B	20	20	0	10	0
	C	40	10	30	-	0
	D	20	0	40	50	0
	E	10	25	10	60	0

One 0 in third row

		Jobs				
		I	II	III	IV	V
Persons	A	0	30	-	0	0
	B	20	20	0	10	0
	C	40	10	30	-	0
	D	20	0	40	50	0
	E	10	25	10	60	0

One 0 in second row

		Jobs				
		I	II	III	IV	V
Persons	A	0	30	-	0	0
	B	20	20	0	10	0
	C	40	10	30	-	0
	D	20	0	40	50	0
	E	10	25	10	60	0

One 0 in third row

		Jobs				
		I	II	III	IV	V
Persons	A	0	30	-	0	0
	B	20	20	0	10	0
	C	40	10	30	-	0
	D	20	0	40	50	0
	E	10	25	10	60	0

One 0 in first column

		Jobs				
		I	II	III	IV	V
Persons	A	0	30	-	0	0
	B	20	20	0	10	0
	C	40	10	30	-	0
	D	20	0	40	50	0
	E	10	25	10	60	0

No 0 left but no 0 has been assigned in fifth row. Not optimal.

Minimum{20, 40, 20,10,50,60,10}=10

		Jobs				
		I	II	III	IV	V
Persons	A	0	30+10	-	0	0+10
	B	20 – 10	20	0	10–10	0
	C	40–10	10	30	-	0
	D	20–10	0	40	50–10	0
	E	10–10	25	10	60–10	0

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

One 0 in third row

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

One 0 in fourth row.

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

One 0 in fifth row

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

One 0 in first row

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

One 0 in second row

		Jobs				
		I	II	III	IV	V
Persons	A	0	40	-	0	10
	B	10	20	0	0	0
	C	30	10	30	-	0
	D	10	0	40	40	0
	E	0	25	10	50	0

Optimal solution:

Job I to person E

Job II to person D

Job III to person B

Job IV to person A

Job V to person C

In actual case, there is no Job V so in actual case no Job has been assigned to person C.

Hence, person C can be replaced with the new person E.

		Jobs				
		I	II	III	IV	V
Persons	A	50	50	-	20	0
	B	70	40	20	30	0
	C	90	30	50	-	0
	D	70	20	60	70	0
	E	60	45	30	80	0

Minimum cost = 20+20+0+20+60 = 120

Example:

Write LPP for the following problem (maximization).

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Maximize($42x_{11}+35x_{12}+28x_{13}+21x_{14}+30x_{21}+25x_{22}+20x_{23}+15x_{24}+30x_{31}+25x_{32}+20x_{33}+15x_{34}+24x_{41}+20x_{42}+16x_{43}+12x_{44}$)

Subject to

$x_{11}+ x_{12}+ x_{13}+ x_{14}=1$

$x_{21}+ x_{22}+ x_{23}+ x_{24}=1$

$x_{31}+ x_{32}+ x_{33}+ x_{34}=1$

$x_{41}+ x_{42}+ x_{43}+ x_{44}=1$

$x_{11}+ x_{21}+ x_{31}+ x_{41}=1$

$x_{12}+ x_{22}+ x_{32}+ x_{42}=1$

$x_{13}+ x_{23}+ x_{33}+ x_{43}=1$

$x_{14}+ x_{24}+ x_{34}+ x_{44}=1$

x_{ij} are 0 or 1.