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Mass Transfer-I

Mass Transfer Coefficients

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Mass Transfer Coefficient

The Mass Transfer Coefficient

We know that

(Mass per unit time per unit area) α Driving Force {Fick's 1st Law}

(Rate per unit area) α Driving Force

(Rate / Area) α Driving Force

The mass transfer coefficient is defined on the following 'phenomenological basis'.

Rate of mass transfer ∝ Concentration driving force (i.e. the difference in concentration)
Rate of mass transfer ∝ Area of contact between the phases

In case of Mass transfer: Driving force is the concentration difference

For binary solutions

The mass transfer flux in case of steady state molecular diffusion in fluids under stagnant and laminar flow conditions is defined as follows: (for binary solutions):

$$N_A = \frac{N_A}{N_A + N_B}\frac{D_{AB}c}{Z} ln\frac{N_A/(N_A + N_B) - c_{A2}/c}{N_A/(N_A + N_B) - c_{A1}/c}$$

Movement of the bulk fluid particles in the turbulent condition is not yet thoroughly understood. Specifically for gases, it is fairly well known as a molecular diffusion since it is described in terms of kinetic theory.

The rate of mass transfer from the interface to the turbulent zone in the same manner can be useful for molecular diffusion. Thus the term $\frac{D_{AB}c}{Z}$ of above equation is replaced by ***F***, which is a characteristic of molecular diffusion

$$N_A = \frac{N_A}{N_A + N_B} F\, ln\frac{N_A/(N_A + N_B) - c_{A2}/c}{N_A/(N_A + N_B) - c_{A1}/c}$$

Source: *Mass Transfer Operations, 3rd ed. Robert E. Treybal (1981)*

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- $\boldsymbol{F}$ is called mass transfer coefficient and the value of $\boldsymbol{F}$ is depends on the fluid kinematics (local nature of fluid motion).
- It is a local mass transfer coefficient defined for a particular location at the interphase (phase boundary surface)

We know that

Flux α Driving Force

Flux = Coefficient × Driving Force

In case of Mass transfer: Driving force is the concentration difference

The mass transfer coefficient is defined on the following "phenomenological basis"

- **Rate of mass transfer α Concentration driving force (i.e. difference in concentration)**
- **Rate of mass transfer α Contact area between phases**

Cont…

Imagine we are interested in the transfer of mass from some interface into a well-mixed solution. We expect that the amount transferred is proportional to the concentration difference and the interfacial area:

$$N_A = k\,(C_{Ai} - C_A)$$

$$k = \frac{N_A}{(C_{Ai} - C_A)} = \frac{N_A}{\Delta C_A}$$

$$k_c = \frac{N_A}{\Delta C_A} = \frac{\text{molar flux}}{\text{concentration driving force}}$$

Remark: The flux includes both convection and diffusion

Where
N_A = *Flux at interface*
k, kc = *Mass transfer coefficient*
C_{Ai} = *Concentration at interface*
C_A = *Concentration at bulk of the solution*

Concentration (C) is defined in number of ways and standards have not been established we have variety of Mass transfer coefficients for each situation:

Source: *Mass Transfer Operations, 3rd ed. Robert E. Treybal (1981)*

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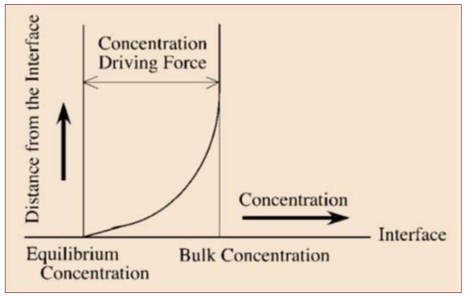

Where
N_A = *Flux at interface*
$k_G\ k_L\ k_C\ k_x\ k_y$ = *Mass transfer coefficients*
C_{Ai} = *Concentration mass (mole)*
x_{Ai}, y_{Ai} = *Concentration mass (mole) friction*
p_{Ai} = *Concentration partial Pressure*
i = *interface level i.e., 1, 2, ….*

Mass transfer in liquid phase

$$N_A = k_x(x_{A1} - x_{A2}) = k_L(C_{A1} - C_{A2})$$

Mass transfer in gas phase

$$N_A = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2}) = k_c(C_{A1} - C_{A2})$$

Source: *Mass Transfer, From Fundamentals to Modern Industrial Applications.* Koichi Asano (2006)

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Relationships between various definitions of concentration in a multicomponent system.

	mass fraction ω_A [–]	**mole fraction** x_A [–]	**partial density** ρ_A [kg m^{-3}]	**molar density** c_A [kmol m^{-3}]	**partial pressure** p_A [kPa]
mass fraction ω_A [–]	ω	$\frac{x_A M_A}{\sum_i x_i M_i}$	$\frac{\rho_A}{\sum_i \rho_i}$	$\frac{c_A M_A}{\sum_i c_i M_i}$	$\frac{p_A M_A}{\sum_i p_i M_i}$
mole fraction x_A [–]	$\frac{(\omega_A/M_A)}{\sum_i(\omega_i/M_i)}$	x_A	$\frac{\rho_A/M_A}{\sum_i(\rho_i/M_i)}$	$\frac{c_A}{\sum_i c_i}$	$\frac{p_A}{\sum_i p_i}$
partial density ρ_A [kg m^{-3}]	$\rho\omega_A$	$\frac{\rho x_A M_A}{\sum_i x_i M_i}$	ρ_A	$c_A M_A$	$\frac{M_A p_A}{RT}$
molar density c_A [kmol m^{-3}]	$\frac{\rho\omega_A}{M_A}$	$c x_A$	$\frac{\rho_A}{M_A}$	c_A	$\frac{p_A}{RT}$
partial pressure p_A [kPA]	$\frac{(\omega_A/M_A)P}{\sum_i(\omega_i/M_i)}$	$P x_A$	$\frac{RT\rho_A}{M_A}$	$c_A RT$	p_A

Mixture: $\sum_i x_i = 1,\ \sum_i \omega_i = 1,\ \rho = \sum_i \rho_i,\ c = \sum_i c_i,\ P = \sum_i p_i$

$$M = \sum_i M_i x_i = \left(\sum_i \frac{\omega_i}{M_i}\right)^{-1}$$

Source: Mass Transfer. From Fundamentals to Modern Industrial Applications. Koichi Asano (2006)

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Mass transfer coefficient	Unit	Definition	Driving force	Phase
k_y	[kmol m^{-2} s^{-1}]	$N_A^* = k_y (y_s - y_\infty)$	Δy	Gas phase
k_G	[kmol m^{-2} s^{-1} kPa^{-1}]	$N_A^* = k_G (p_s - p_\infty)$	Δp	
k_Y	[kmol m^{-2} s^{-1}]	$N_A^* = k_Y (Y_s - Y_\infty)$	ΔY	
k	[m s^{-1}]	$N_A = \rho_G k (\omega_{Gs} - \omega_{G\infty})$	$\Delta \omega_G$	
k_H	[kg m^{-2} s^{-1}]	$N_A = k_H (H_s - H_\infty)$	ΔH	
k_L	[m s^{-1}]	$N_A^* = k_L (c_s - c_\infty)$	Δc	Liquid phase
k_x	[kmol m^{-2} s^{-1}]	$N_A^* = k_x (x_s - x_\infty)$	Δx	
k_X	[kmol m^{-2} s^{-1}]	$N_A^* = k_X (X_s - X_\infty)$	ΔX	
k	[m s^{-1}]	$N_A = \rho_L k (\omega_{Ls} - \omega_{L\infty})$	$\Delta \omega_L$	

c = molar density [mol m^{-3}], H = absolute humidity [–], M_A = molecular weight [kg kmol^{-1}], N_A = mass flux [kg m^{-2} s^{-1}], $N_A^* = N_A/M_A$ = molar flux [kmol m^{-2} s^{-1}], p = partial pressure [kPa], x, y = mole fraction [–], $X = x/(1-x)$ [–], $Y = y/(1-x)$ [–], ω = mass fraction [–].

Source: Mass Transfer. From Fundamentals to Modern Industrial Applications. Koichi Asano (2006)

Relations between mass transfer coefficients

For Gas Phase

$$\because c_A = y_A \rho_M$$

$$\therefore k_y = \frac{J_A}{y_{Ai} - y_A} = \frac{J_A}{{c_{Ai}}/{\rho_M} - {c_A}/{\rho_M}} = \rho_M \frac{J_A}{c_{Ai} - c_A}$$

$$\therefore k_y = \rho_M \cdot k_c = \frac{k_c P}{RT}$$

For Liquid Phase

$$\because c_A = x_A \rho_M$$

$$\therefore k_x = \frac{J_A}{x_{Ai} - x_A} = \frac{J_A}{{c_{Ai}}/{\rho_M} - {c_A}/{\rho_M}} = \rho_M \frac{J_A}{c_{Ai} - c_A}$$

$$\therefore k_x = \rho_M \cdot k_c = \frac{k_c \rho_x}{\bar{M}}$$

Different types of Mass transfer coefficients

Diffusion of A through non-diffusing B: Flux, N_A	Diffusion of A through non-diffusing B: Mass transfer coefficient	Equimolar counterdiffusion of A and B: Flux, N_A	Equimolar counterdiffusion of A and B: Mass transfer coefficient	Unit of the mass transfer coefficient
Gas-phase mass transfer				
$k_G(p_{A1} - p_{A2})$	$k_G = \frac{D_{AB}P}{RT\delta p_{BM}}$	$k'_G(p_{A1} - p_{A2})$	$k'_G = \frac{D_{AB}}{\delta RT}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta p_A)}$
$k_y(y_{A1} - y_{A2})$	$k_y = \frac{D_{AB}P^2}{RT\delta p_{BM}}$	$k'_y(y_{A1} - y_{A2})$	$k'_y = \frac{D_{AB}P}{\delta RT}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta y_A)}$
$k_c(C_{A1} - C_{A2})$	$k_c = \frac{D_{AB}P}{\delta p_{BM}}$	$k'_c(C_{A1} - C_{A2})$	$k'_c = \frac{D_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta C_A)}$
Liquid-phase mass transfer.				
$k_L(C_{A1} - C_{A2})$	$k_L = \frac{D_{AB}}{\delta x_{BM}}$	$k'_L(C_{A1} - C_{A2})$	$k'_L = \frac{D_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta C_A)}$
$k_x(x_{A1} - x_{A2})$	$k_x = \frac{CD_{AB}}{\delta x_{BM}}$	$k'_x(x_{A1} - x_{A2})$	$k'_x = \frac{CD_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta x_A)}$
Conversion				
	$k_G RT = \frac{RT}{P} k_y = k_c$; $k_L = \frac{k_x}{C_{av}}$		$k'_c = k'_G RT = \frac{RT}{P} k'_y$; $k'_L = \frac{k'_x}{C_{av}}$	

Source: Principles of Mass Transfer. And Separation Processes: Binay K. Dutta (2007)

Example

Show the mutual relationship between the mass transfer coefficients k, k_H, k_c, k_y, and k_G.

Solution

From the definitions of mass transfer coefficients

$$N_A = \rho k (\omega_s - \omega_\infty) = k_H (H_s - H_\infty) \tag{A}$$

$$N_A/M_A = N_A^* = k_c (c_s - c_\infty) = k_y (y_s - y_\infty) = k_G (p_s - p_\infty) \tag{B}$$

$$H = \frac{\omega}{1-\omega}, \quad c = \frac{p}{RT}, \quad y = \frac{p}{P} = \frac{(M_B/M_A)\,\omega_A}{1 + \{M_B/M_A - 1\}\,\omega_A} \tag{C}$$

c = molar density [mol m^{-3}], H = absolute humidity [–], M_A = molecular weight [kg kmol^{-1}], N_A = mass flux [kg m^{-2} s^{-1}], $N_A^* = N_A/M_A$ = molar flux [kmol m^{-2} s^{-1}], p = partial pressure [kPa], x, y = mole fraction [–], $X = x/(1-x)$ [–], $Y = y/(1-x)$ [–], ω = mass fraction [–].

Source: Mass Transfer. From Fundamentals to Modern Industrial Applications. Koichi Asano (2006)

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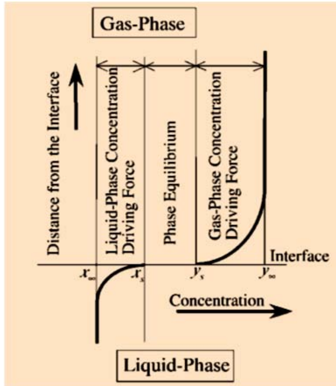
Substituting these equations into Eq. (A) or (B), we obtain the following equations:

$$k = \frac{k_H}{\rho(1-\omega_s)(1-\omega_\infty)} \qquad \text{(D)}$$

$$\begin{aligned} k_y &= k_G P = k_c(P/RT) \\ &= \left(\frac{\rho k}{M_A}\right)\left\{1+\left(\frac{M_B}{M_A}-1\right)\omega_s\right\}\left\{1+\left(\frac{M_B}{M_A}-1\right)\omega_\infty\right\}\left(\frac{M_A}{M_B}\right) \\ &\approx ck\left\{1+\left(\frac{M_B}{M_A}\right)\omega_s\right\} \end{aligned} \qquad \text{(E)}$$

Source: Mass Transfer. From Fundamentals to Modern Industrial Applications. Koichi Asano (2006)

Overall Mass Transfer Coefficients



Concentration profiles near the gas-liquid interface and overall mass transfer coefficient

$$N_A^* = k_y(y_\infty - y_s) = k_x(x_s - x_\infty) = K_y(y_\infty - y^*) = K_x(x^* - x_\infty)$$

Source: Mass Transfer. From Fundamentals to Modern Industrial Applications. Koichi Asano (2006)

Typical "Order of magnitude" of Mass Transfer coefficients and film thickness

The "order of magnitude" of the mass transfer coefficient and film thickness for typical ***gas-liquid separation equipment*** are

Gas-phase mass transfer coefficient, $k_c \sim 10^{-2}$ m/s; film thickness, $\delta \sim 1$ mm
Liquid-phase mass transfer coefficient, $k_L \sim 10^{-5}$ m/s; film thickness, $\delta \sim 0.1$ mm

Once these typical values are noted, it is rather easy to determine the orders of magnitude of other types of mass transfer coefficients. For example, at $T = 298$ K and at a 'low concentration',

$$k_G = \frac{k_c}{RT} \sim \frac{10^{-2}\,\text{m/s}}{0.08317\,[(\text{m}^3)(\text{bar})/(\text{kmol})(\text{K})]\,(298\,\text{K})} \Rightarrow k_G \sim 4\times10^{-4}\ \text{kmol/(m}^2\text{)(s)}(\Delta p,\ \text{bar})$$

Source: Principles of Mass Transfer. And Separation Processes: Binay K. Dutta (2007)

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For liquid-phase mass transfer in a 'dilute' aqueous solution,

$$k_x = k_L(\rho/M)_{av} \sim (10^{-5}\ \text{m/s})(1000/18\ \text{kmol/m}^3) \Rightarrow k_x \sim 5\times10^{-4}\ \text{kmol/(m}^2\text{)(s)}(\Delta x)$$

Since the liquid-phase diffusivity of common solutes, $D \sim 10^{-9}$ m^2/s,

$$\text{liquid film thickness, } \delta \sim D/k_L \Rightarrow \delta \sim 10^{-4}\ \text{m or } 0.1\ \text{mm}$$

Source: Principles of Mass Transfer. And Separation Processes: Binay K. Dutta (2007)

Important Dimensionless groups in Mass transfer

Dimensionless groups and their physical significance	*Analogous groups in heat transfer*
Reynolds number, $\mathrm{Re} = \frac{l v \rho}{\mu} = \frac{l v}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}}$	The same
Schmidt number, $\mathrm{Sc} = \frac{\mu}{\rho D} = \frac{\nu}{D} = \frac{\text{momemntum diffusivity}}{\text{molecular diffusivity}}$	$\mathrm{Pr} = \frac{c_p \mu}{k} = \frac{\mu/\rho}{k/\rho c_p} = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$
Sherwood number*, $\mathrm{Sh} = \frac{k_L l}{D} = \frac{k_L \Delta C}{(D/l)\Delta C} = \frac{\text{convective mass flux}}{\text{diffusive flux across a layer of thickness } l}$	$\mathrm{Nu} = \frac{hl}{k} = \frac{\text{convective heat flux}}{\text{conduction heat flux across a layer of thickness } l}$
Stanton number, $\mathrm{St}_M = \frac{\mathrm{Sh}}{(\mathrm{Re})(\mathrm{Sc})} = \frac{k_L}{v} = \frac{k_L \Delta C}{v \Delta C} = \frac{\text{convective mass flux}}{\text{flux due to bulk flow of the medium}}$	$\mathrm{St}_H = \frac{\mathrm{Nu}}{(\mathrm{Re})(\mathrm{Pr})} = \frac{h \Delta T}{v \Delta T} = \frac{\text{convective heat flux}}{\text{heat flux due to bulk flow}}$
Peclet number, $\mathrm{Pe}_M = (\mathrm{Re})(\mathrm{Sc}) = \frac{l v}{D} = \frac{v \Delta C}{(l/D)\Delta C} = \frac{\text{flux due to bulk flow of the medium}}{\text{diffusive flux across a layer of thickness } l}$	$\mathrm{Pe}_H = (\mathrm{Re})(\mathrm{Pr}) = \frac{(v \rho c_p)\Delta T}{(k/l)\Delta T} = \frac{\text{heat flux due to bulk flow}}{\text{conduction flux across a thickness } l}$
Colburn factor, $j_D = \mathrm{St}_M (\mathrm{Sc})^{2/3} = \frac{\mathrm{Sh}}{(\mathrm{Re})(\mathrm{Sc})^{1/3}}$	$j_H = \mathrm{St}_H (\mathrm{Pr})^{2/3} = \frac{\mathrm{Nu}}{(\mathrm{Re})(\mathrm{Pr})^{1/3}}$
Grashof number, $\mathrm{Gr} = \frac{l^3 \Delta \rho g}{\mu \nu}$	The same
Lewis number, Le = Sc/Pr	See Chapter 10

* The Sherwood number for gas-phase mass transfer is defined in Eq. (3.16). The suffix *M* or *D* refers to mass transfer, *H* to heat transfer. The Grashof number is the analogue of Reynolds number in free convection mass transfer. Pr = Prandtl number; Nu = Nusselt number; l = characteristic length; $\nu = \mu/\rho$ = momentum diffusivity; $\alpha = k/\rho c_p$ = thermal diffusivity; $\Delta\rho$ = difference in density because of a difference in concentration or temperature.

Source: Principles of Mass Transfer. And Separation Processes: Binay K. Dutta (2007)

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Mass transfer resistance

The inverse to mass transfer coefficient $\left(\frac{1}{K}\right)$ is termed as a ***resistance to mass transfer*** or ***mass transfer resistance***

Molecular mass transfer resistance

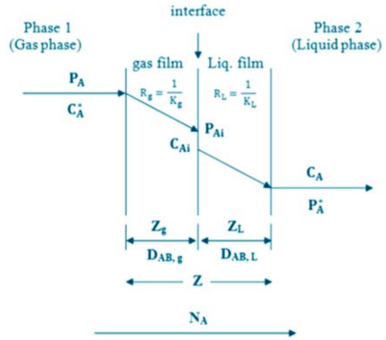
$$R_D = \frac{L}{D_{AB}}$$

Convective mass transfer resistance in gas phase

$$R_g = \frac{1}{K_g}$$

Convective mass transfer resistance in liquid phase

$$R_L = \frac{1}{K_L}$$



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References

ETH

Mass Transfer

Theories for Mass Transfer Coefficients

Lecture 9, 15.11.2017, Dr. K. Wegner

- Lecture notes/ppt of Dr. Yahya Banat **(ybanat@qu.edu.qa)**