THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY, PATIALA
Department of Electronics and Communication Engineering

UEC310 – Information and Communication Theory

TUTORIAL - 12

Q1	The number of customers arriving at a grocery store can be modeled by a Poisson process with intensity λ=20 customers per hour. a) Find the probability that there are 5 customers between 10:00 and 10:20. b) Find the probability that there are 5 customers between 10:00 and 10:20 and 7 customers between 10:20 and 11:00.
Q2	Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ. Find the probability that there are two arrivals in (0,2] and three arrivals in (1,4].

Solution of tutorial-8

A 1	Here λ=20 customers per hour so that the interval between 10:00 and 10:20 is $\tau = {}^{1}/_{3}\ hours.$ Thus if $X\ \rightarrow Number\ of\ arrivals\ in\ that\ duration$ $$X \sim Poisson\left({}^{20}/_{3}\right)$$ $$P(X=5) = \frac{\left({}^{20}/_{3}\right)^5}{5!} e^{-\left({}^{20}/_{3}\right)} =$$ Here, non-overlapping interval $I_1 = [10{:}00\ AM\ - 10{:}20\ AM]$ and $I_2 = [10{:}20\ AM\ - 11{:}00\ AM]$ $$P(5\ arrival\ in\ I_1\ and\ 7\ arrival\ in\ I_2) = P(5\ arrival\ in\ I_1\ )P(7\ arrival\ in\ I_2)$$ Length of interval $\tau_1 = {}^{1}/_{3}$ and $\tau_2 = {}^{2}/_{3}$ and $\lambda\tau_1 = {}^{20}/_{3}$ and $\lambda\tau_2 = {}^{40}/_{3}$ $$P(5\ arrival\ in\ I_1\ and\ 7\ arrival\ in\ I_2) = \frac{\left({}^{20}/_{3}\right)^5}{5!} e^{-\left({}^{20}/_{3}\right)} \frac{\left({}^{40}/_{3}\right)^7}{7!} e^{-\left({}^{40}/_{3}\right)} =$$
A 2	

Note that the two intervals $(0,2]$ and $(1,4]$ are not disjoint. Thus, we cannot multiply probabilities for each interval to obtain the desired probability. In particular,

$$(0,2] \cap (1,4] = (1,2].$$

Let X, Y, and Z be the numbers of arrivals in $(0,1]$, $(1,2]$, and $(2,4]$ respectively. Then X, and Z are independent, and

$$\begin{aligned} X &\sim Poisson(\lambda \cdot 1), \\ Y &\sim Poisson(\lambda \cdot 1), \\ Z &\sim Poisson(\lambda \cdot 2). \end{aligned}$$

Let A be the event that there are two arrivals in $(0,2]$ and three arrivals in $(1,4]$. We can use law of total probability to obtain $P(A)$. In particular,

$$\begin{aligned} P(A) &= P(X+Y=2 \text{ and } Y+Z=3) \\ &= \sum_{k=0}^{\infty} P\big(X+Y=2 \text{ and } Y+Z=3|Y=k\big)P(Y=k) \\ &= P\big(X=2, Z=3|Y=0\big)P(Y=0) + P(X=1, Z=2|Y=1)P(Y=1)+ \\ &\qquad + P(X=0, Z=1|Y=2)P(Y=2) \\ &= P\big(X=2, Z=3\big)P(Y=0) + P(X=1, Z=2)P(Y=1)+ \\ &\qquad P(X=0, Z=1)P(Y=2) \\ &= P(X=2)P(Z=3)P(Y=0) + P(X=1)P(Z=2)P(Y=1)+ \\ &\qquad P(X=0)P(Z=1)P(Y=2) \end{aligned}$$

$$\begin{aligned} &= \left(\frac{e^{-\lambda}\lambda^2}{2}\right) \cdot \left(\frac{e^{-2\lambda}(2\lambda)^3}{6}\right) \cdot \left(e^{-\lambda}\right) + \left(\lambda e^{-\lambda}\right) \cdot \left(\frac{e^{-2\lambda}(2\lambda)^2}{2}\right) \cdot \left(\lambda e^{-\lambda}\right) + \\ &\qquad \left(e^{-\lambda}\right) \cdot \left(e^{-2\lambda}(2\lambda)\right) \cdot \left(\frac{e^{-\lambda}\lambda^2}{2}\right). \end{aligned}$$