Course: UMA 035 (Optimization Techniques)

Instructor: Dr. Amit Kumar,

Associate Professor,

School of Mathematics,

TIET, Patiala

Email: amitkumar@thapar.edu

Mob: 9888500451

Convex set

Theoretically

A set S is said to a convex set if the convex linear combination of every two arbitrary points of the set S also belong to the same set S.

Mathematically

If $a_1X_1 + a_2X_2 \in S$ for all $X_1, X_2 \in S$

where,

- $a_1 >= 0$
- $a_2 >= 0$
- $a_1 + a_2 = 1$

Then, the set S will be convex.

OR

If $a_1X_1 + (1-a_1)X_2 \in S$ for all $X_1, X_2 \in S$

where,

- $0 <= a_1 <= 1$

Then, the set S will be convex.

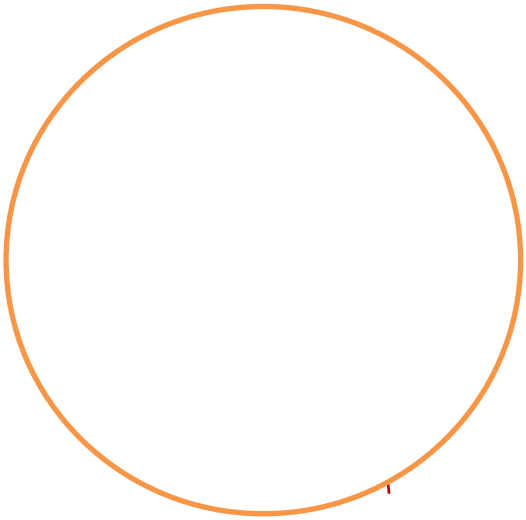
Graphically

Draw the region for the set S.

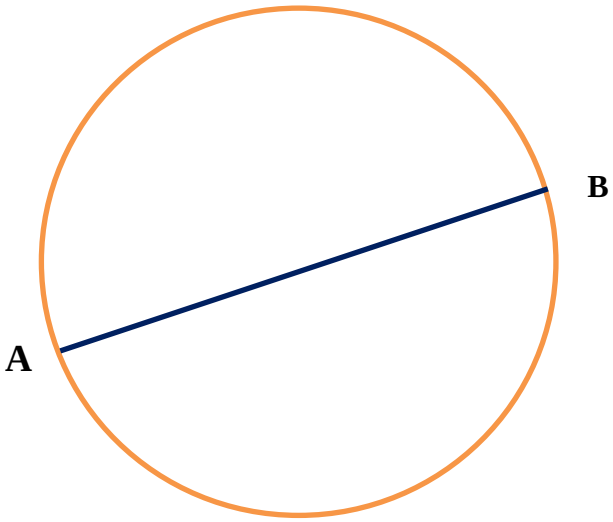
If it is possible to find any two distinct points inside the region of the set S such that some of the portion of the line segment joining the points lies

outside the region of the set S. Then the set S will not be a convex set otherwise the set S will be a convex set.

Example: Check graphically that the set $S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 = r^2\}$ is convex or not.



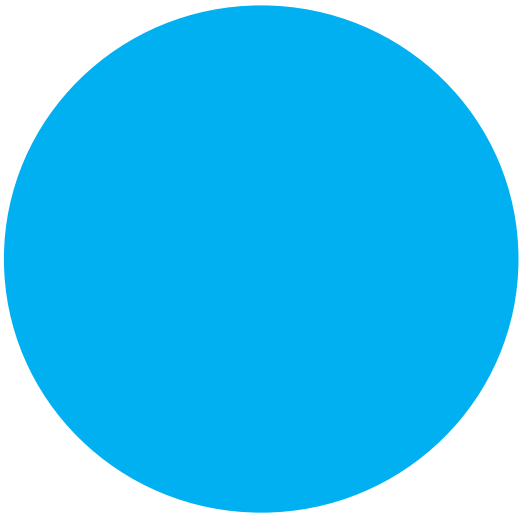
The set S is representing the boundary of the circle. So, the set S will not be convex if it is possible to find two distinct points on the boundary such that some portion of the line segment joining the considered points does not lie on the boundary of the circle.



Since, it is possible to find two points A and B on the boundary of the circle such that line segment AB does not lie on the boundary of the circle. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 < r^2\}$ is convex or not.

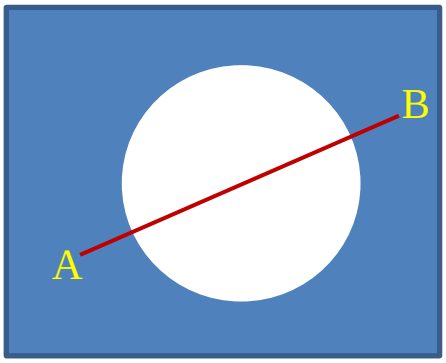
Since the set S is representing the inside region of the circle. So, the set S will not be convex if it is possible to find two distinct points lying inside the the circle such that some portion of the line segment joining the considered points does not lie inside the circle.



Since, it is not possible to find two points A and B inside the circle such that some portion of the line segment AB does not lie inside of the circle. So, the set is convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 > r^2\}$is convex or not.

Since the set S is representing the outside region of the circle. So, the set S will not be convex if it is possible to find two distinct points lying outside the circle such that some portion of the line segment joining the considered points does not lie outside the circle.

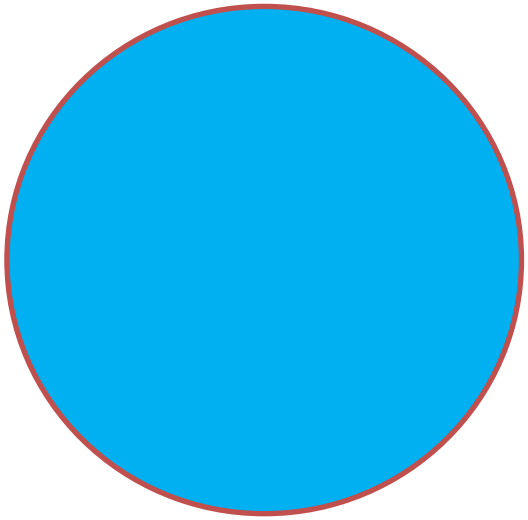


Since, it is possible to find two points A and B lying outside the circle such that some portion of the line segment AB does not lie outside the circle. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 <= r^2\}$is convex or not.

Since the set S is representing the inside region of the circle including boundary of the circle. So, the set S will not be convex if it is possible to find two distinct points inside the circle or on boundary of the circle such

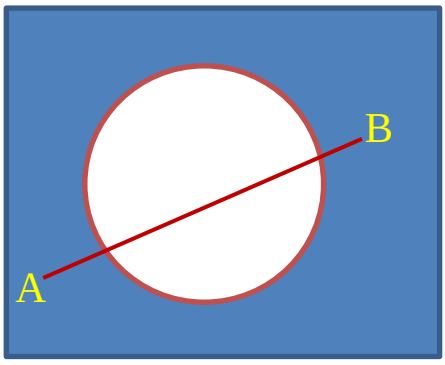
that some portion of the line segment joining the considered points either does not lies on the boundary or inside the circle.



Since, it is not possible to find two points A and B inside the circle such that some portion of the line segment AB does not lie inside of the circle or the boundary of the circle. So, the set is convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 >= r^2\}$ is convex or not.

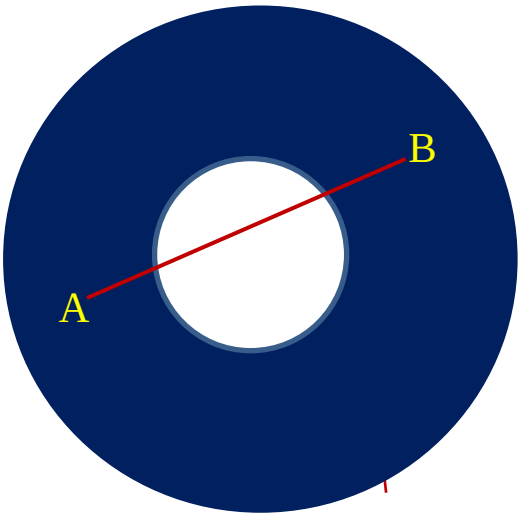
Since the set S is representing the outside region of the circle including the boundary of the circle. So, the set S will not be convex if it is possible to find two distinct points outside the region of the circle such that some portion of the line segment joining the considered points does not lie outside the region of the circle.



Since, it is possible to find two points A and B outside the circle such that some portion of the line segment AB does not lie outside of the circle. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : r_1^2 < (x_1-a)^2 + (x_2-b)^2 < r_2^2\}$ is convex or not.

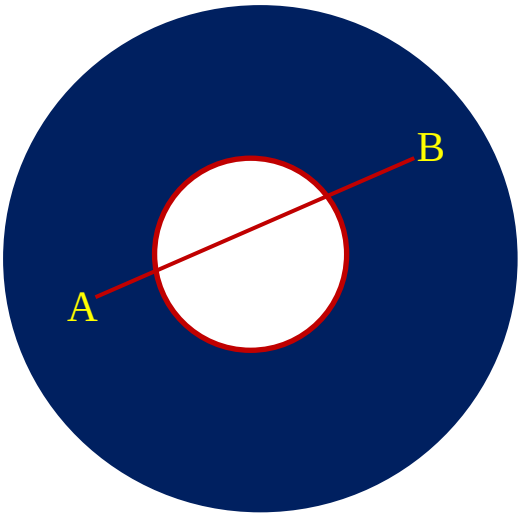
Since the set S is representing the region between two circles. So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is possible to find two points A and B in the region such that some portion of the line segment AB does not lie in the region. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : r_1^2 <=(x_1-a)^2 + (x_2-b)^2 <r_2^2\}$ is convex or not.

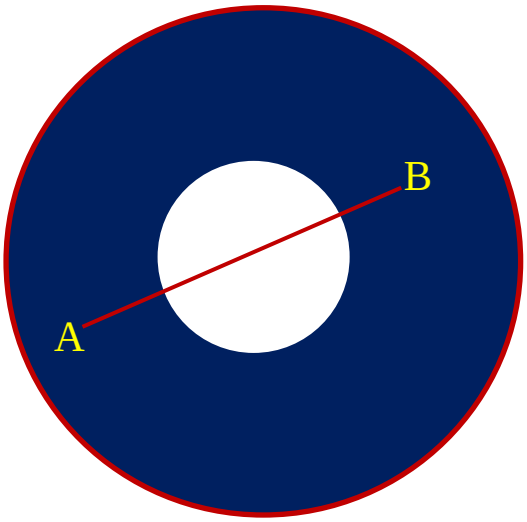
Since the set S is representing the region between two circles including the boundary of smaller circle. So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is possible to find two points A and B in the region such that some portion of the line segment AB does not lie inside the region. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : r_1^2 <(x_1-a)^2 + (x_2-b)^2 <=r_2^2\}$ is convex or not.

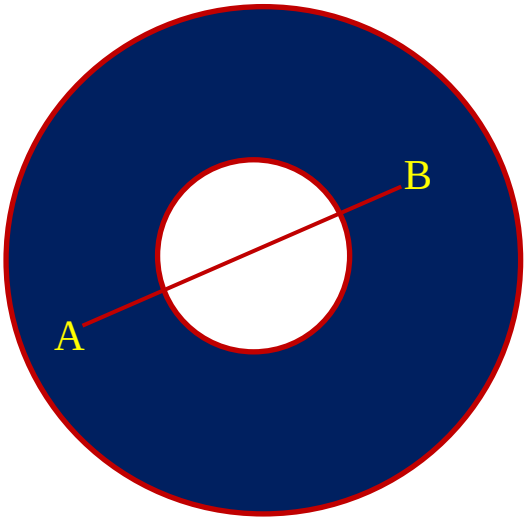
Since the set S is representing the region between two circles including the boundary of bigger circle. So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is possible to find two points A and B lying in the region such that some portion of the line segment AB does not lie in the region. So, the set is not convex.

Example: Check graphically that the set $S=\{(x_1,x_2) : r_1^2 <= (x_1-a)^2 + (x_2-b)^2 <= r_2^2\}$ is convex or not.

Since the set S is representing the region between two circles including the boundary of bigger and smaller circles. So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is possible to find two points A and B lying in the region such that some portion of the line segment AB does not lie in the region. So, the set is not convex.

Example: Check graphically that the set $S= S=\{(x_1,x_2) : 16x_2 > x_1^2\}$ or $S=\{(x,y) : 16y > x^2$ is convex or not

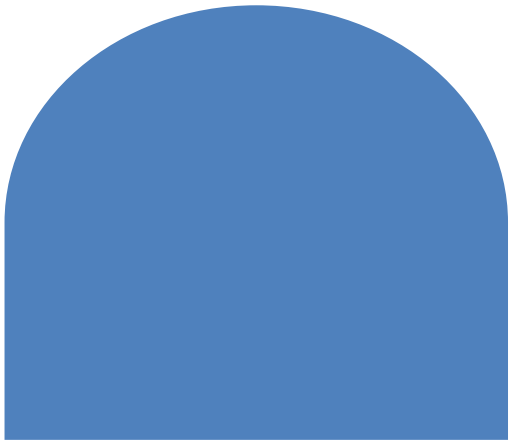
Since the set S is representing the region inside the parabola (Please consider any point of inside parabola, it will satisfy the parabola). So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.

Since, it is not possible to find two points A and B in the region such that some portion of the line segment AB does not lie in the region. So the set is convex.

Example: Check graphically that the set S= S={(x_1,x_2) : $-16x_2 > x_1^2$ } or S={(x,y) : $-16y > x^2$ is convex or not

Since the set S is representing the region inside the parabola (Please consider any point of inside parabola, it will satisfy the parabola). So, the set S will not be convex if it is possible to find two distinct points lying in

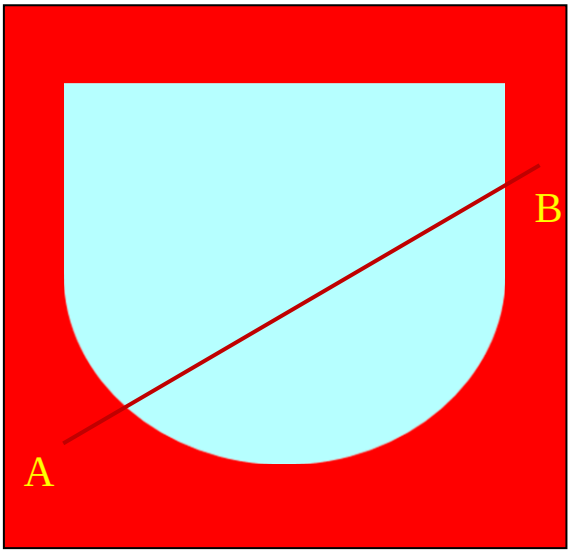
the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is not possible to find two points A and B in the region such that some portion of the line segment AB does not lie in the region. So the set is convex.

Example: Check graphically that the set S= $S=\{(x_1,x_2) : 16x_2 < x_1^2\}$ or $S=\{(x,y) : 16y < x^2$ is convex or not

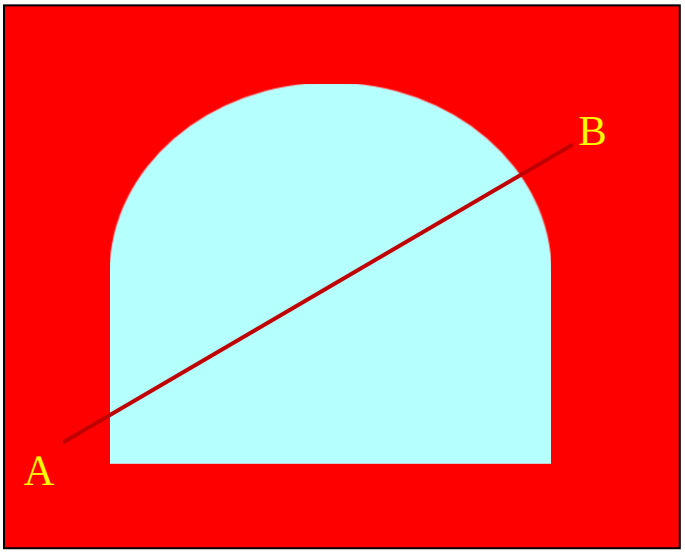
Since the set S is representing the region outside the parabola (Please consider any point of inside parabola, it will not satisfy the parabola). So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



Since, it is possible to find two points A and B in the region such that some portion of the line segment AB does not lie in the region. So the set is not convex.

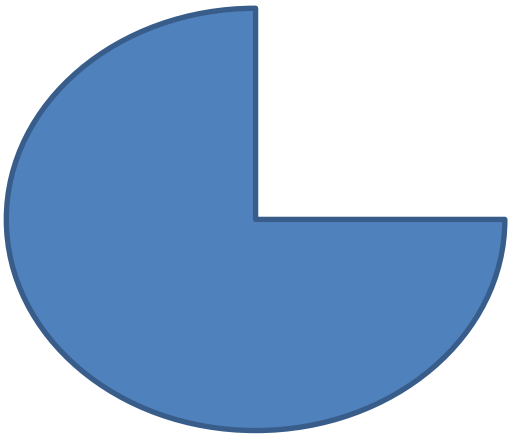
Example: Check graphically that the set S= S={(x_1,x_2) : $-16x_2 < x_1^2$ } or S={(x,y) : $-16y < x^2$ is convex or not

Since the set S is representing the region outside the parabola (Please consider any point of inside parabola, it will not satisfy the parabola). So, the set S will not be convex if it is possible to find two distinct points lying in the region such that some portion of the line segment joining the considered points does not lie in the region.



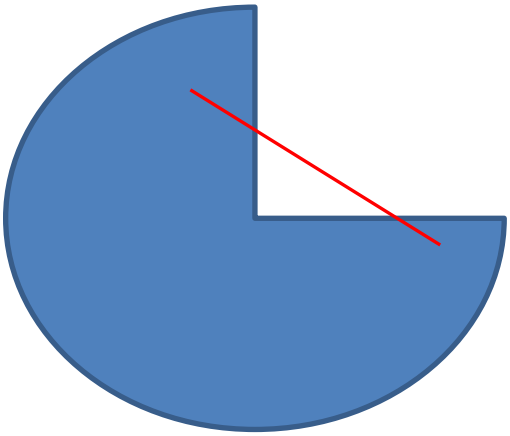
Since, it is possible to find two points A and B in the region such that some portion of the line segment AB does not lie in the region. So the set is not convex.

Example:



Set is convex or not?

No.

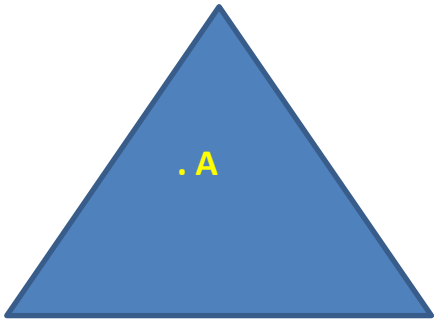


Extreme point or corner point or vertex

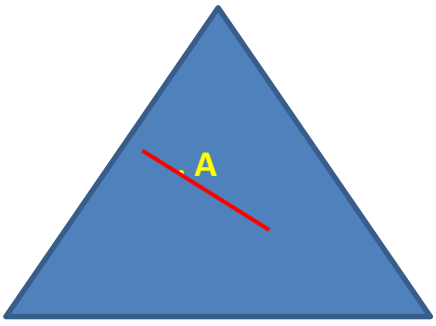
Let S be a convex set. Then, a point P belongs the set S is said to be an extreme point if it cannot be written as a convex linear combination of two other distinct points of the set S.

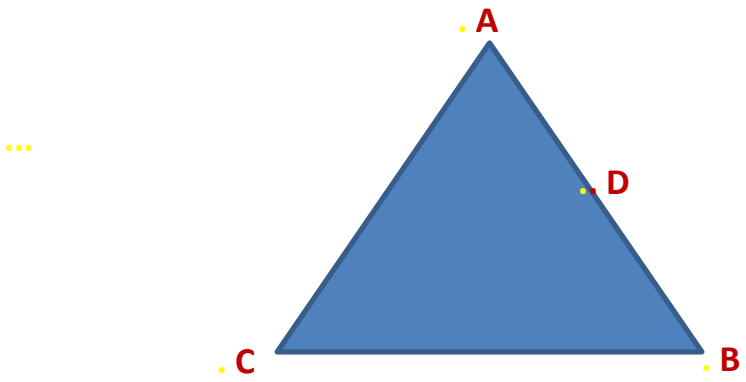
OR

Let S be a convex set. Then, a point P belongs to the set S is said to be an extreme point if it is not possible to draw a line segment in the region of S which passes the point P.



A is not an extreme point as it is possible to draw line segment in the region which passes through A.





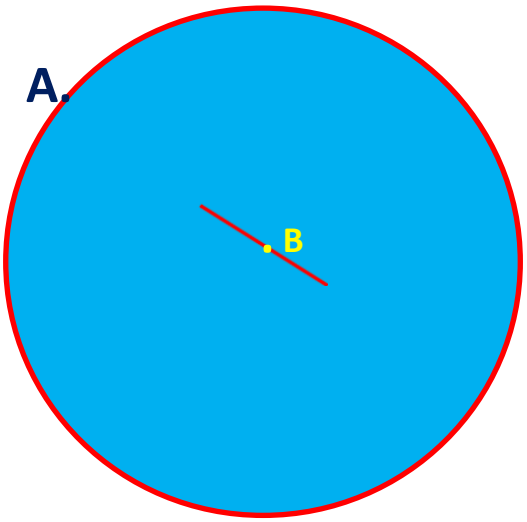
A is an extreme point as it is not possible to draw line segment in the region which passes through A.

B is an extreme point as it is not possible to draw line segment in the region which passes through A.

C is an extreme point as it is not possible to draw line segment in the region which passes through A.

D is not an extreme point as it is possible to draw line segment (AB) in the region which passes through D.

$S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 <= r^2\}$

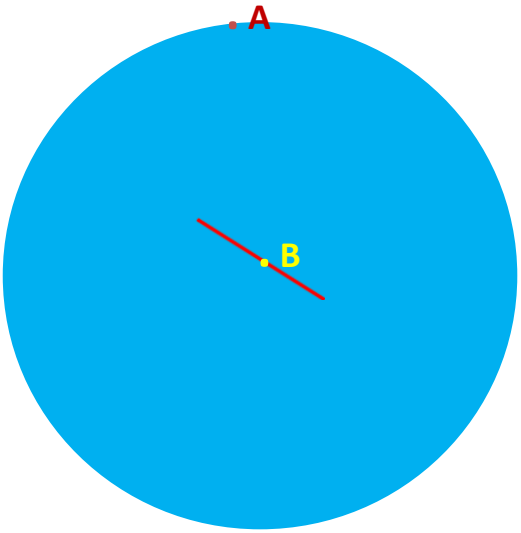


A is an extreme point as it is not possible to draw line segment in the region which passes through A.

B is not an extreme point as it is possible to draw line segment in the region which passes through B.

Remark: All the points lying on the boundaries of circle, ellipse, parabola etc. are extreme points. Hence, infinite number of extreme points in these cases.

$S=\{(x_1,x_2) : (x_1-a)^2 + (x_2-b)^2 < r^2\}$



Although, it is not possible to draw line segment in the region which passes through A. But as A does not belongs to the set. So, A is not an extreme point.

B is not an extreme point as it is possible to draw line segment in the region which passes through B.

Remark: If boundaries of circle, ellipse, parabola etc. are not included. Then, no extreme points in these cases.

Example:

How many extreme points in the set $S=\{x: 0<=x<1\}$

Ans: Only one extreme point x=0

Example:

How many extreme points in the set S={x: 0<x<1}

Ans: No extreme point

Example:

How many extreme points in the set S={x: 0<=x<=1}

Ans: Two extreme points x=0 and x=1.