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School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Term Examination, March 2018

B.E. IV Semester — UMA007 : Numerical Analysis

Time Limit: 02 Hours — Maximum Marks: 25

Instructor(s): Arvind K. Lal, Kavita Goyal, Meenu Rani, Paramjeet Singh, Raj Nandkeolyar, Vivek Sangwan.

Instructions: You are expected to answer all the questions. Organize your work, in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode is permitted.

1. (a) Find the largest interval in which $fl(x)$ must lie to approximate $\sqrt{3}$ with relative error at most 10^{-4}. [3 marks]

(b) Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulas are available to find the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad \text{and} \quad x = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0}.$$

(i) Show that both formulas are algebraically correct.

(ii) Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and three-digit rounding arithmetic to compute the x-intercept both ways. Which method is better and why? [4 marks]

2. (a) Use the bisection method to find the first five approximations c_n to the root α of the equation $x - 2\sin x = 0$ in $[1.5, 2]$. Also find the error estimate $|\alpha - c_5|$. [3 marks]

(b) What are the solutions α, if any, of the equation $x = \sqrt{1+x}$? Does the iteration $x_{n+1} = \sqrt{1+x_n}$, converge to any of these solutions assuming x_0 is chosen sufficiently close to α? [3 marks]

3. (a) Consider the system $Ax = b$, where matrix A is of order n. Show that a total of $\frac{n(n-1)(2n+5)}{6}$ additions/subtractions and $\frac{n(n^2+3n-1)}{3}$ multiplications/divisions are required to solve the system using Gauss elimination. [3 marks]

(b) Solve the following linear system

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 3 \\ 2x_1 + 7x_2 + x_3 &= 19 \\ x_1 - 3x_2 + 12x_3 &= 31 \end{aligned}$$

with three iterations of Gauss-Seidel method. Use three digit chopping arithmetic and take initial approximation $\mathbf{x}^{(0)} = \mathbf{0}$.

[3 marks]

4. (a) Let $f \in C^2[a, b]$. If α is a simple root of $f(x) = 0$ and $f'(\alpha) \neq 0$, then prove that the Newton's method generates a sequence $\{x_n\}$ converging at least quadratically to root α for any initial approximation x_0 near to α. [4 marks]

(b) The iterative scheme $x_{n+1} = \frac{x_n}{2} + \frac{5}{2x_n}$, $n \geq 0$ converges to $\sqrt{5}$. Find the order of convergence of the iterative scheme. [2 marks]