

Que:9

Compute the planar density for the BCC (100), (111) and (110) planes in terms of atomic radius r .



THE PLANAR DENSITY $\rho_P = \frac{N_e}{A}$, WHERE N_e IS THE NO. OF EFFECTIVE ATOMS IN THE PLANE AND A IS THE AREA OF THE PLANE.

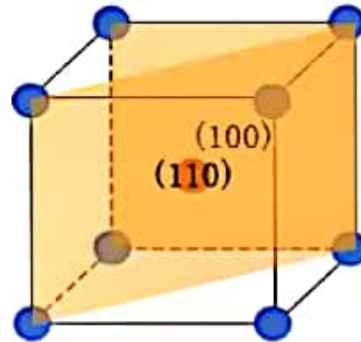
(100)

$$N_e = \frac{1}{4} \times 4 = 1$$

$$\text{Area of the plane} = a^2$$

$$\rho_P = \frac{1}{a^2} = \frac{1}{\left(\frac{4r}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{16r^2}$$



(110)

$$N_e = \frac{1}{4} \times 4 + 1 = 2$$

$$\text{Area of the plane} = \sqrt{2}a^2$$

$$\rho_P = \frac{2}{\sqrt{2}a^2} = \frac{2}{\sqrt{2}\left(\frac{4r}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{8\sqrt{2}r^2}$$

(111) plane

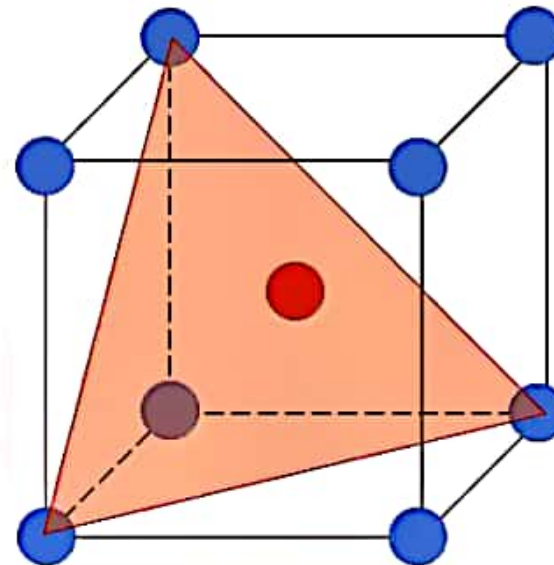
$$N_e = \frac{1}{6} \times 3 = \frac{1}{2}$$

$$\text{Area of the plane} = \frac{\sqrt{3}(\sqrt{2}a)^2}{4}$$

$$= \frac{\sqrt{3}}{2} a^2$$

$$\rho_P = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} a^2} = \frac{1}{\sqrt{3} \left(\frac{4r}{\sqrt{3}} \right)^2}$$

$$= \frac{\sqrt{3}}{16r^2}$$



Que:10

Calculate the planar density for (110) plane of BCC iron lattice in atoms per square millimeter. The lattice constant of iron is 0.287 nm.

Que:11

From an X - Ray powder diffraction of a pure element, peaks at the following 2θ values in degrees were obtained 38.7, 45.4, 65.7, 78.8, 83.0, 99.6, 112.5, 117.0, 138.1, and 164.2. Copper $K\alpha$ radiation was used. Find the lattice parameter and the crystal structure.



2θ	θ	$\sin \theta$	$\sin^2 \theta$	$A = \sin^2 \theta / \sin^2 \theta_1$	$A*2$	$A*3$
38.7	19.35	0.331	0.110	0.996	2	3
45.4	22.7	0.386	0.149	1.355	2.709	4.064 \approx 4
65.7	32.85	0.542	0.294	2.671	5.341	8.012 \approx 8
78.8	39.4	0.635	0.403	3.666	7.331	10.997 \approx 11
83	41.5	0.663	0.440	3.996	7.992	11.988 \approx 12
99.6	49.8	0.764	0.584	5.306	10.613	15.919 \approx 16
112.5	56.25	0.832	0.692	6.293	12.586	18.879 \approx 19
117	58.5	0.853	0.728	6.615	13.229	19.844 \approx 20
138.1	69.05	0.934	0.872	7.931	15.861	23.792 \approx 24
164.2	82.1	0.991	0.982	8.928	17.856	26.784 \approx 27

From the above calculations we found that the crystal structure is FCC.

- For $\theta = 19.35$,

$$(hkl) = (111)$$

We know

$$2d \sin \theta = n\lambda$$

Assuming $n=1$ and $\lambda=1.54 \text{ \AA}$ (for Cu K_α radiation)

$$2 \times d \sin 19.35 = 1.54$$

$$d = 2.32 \text{ \AA}$$

The lattice parameter 'a' can be calculated using the equation

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$2.32 = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$a = 4.026 \text{ \AA}$$



Que:12

A BCC crystal is used to measure the wavelength of some X - rays. The Bragg angle for reflection from (110) plane is 20.2° . What is the wavelength? The lattice parameter of the crystal is 3.15 \AA .

✓ BRAGG'S LAW IS GIVEN AS:

$$2d \sin \theta = n\lambda$$

✓ INTERPLANAR DISTANCE "d" IS

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

✓ $a = 3.15 \text{ \AA}$, $h=1$, $k=1$, $l=0$ FOR (110)

$$\begin{aligned} d &= \frac{3.15}{\sqrt{1^2 + 1^2 + 0^2}} \\ &= 2.227 \text{ \AA} \end{aligned}$$

✓ TAKING $\theta = 20.2^\circ$, $n=1$, FROM BRAGG'S LAW

$$\begin{aligned} 2 \times 2.227 \times \sin 20.2 &= \lambda \\ \lambda &= 1.5236 \text{ \AA} \end{aligned}$$

Que:13

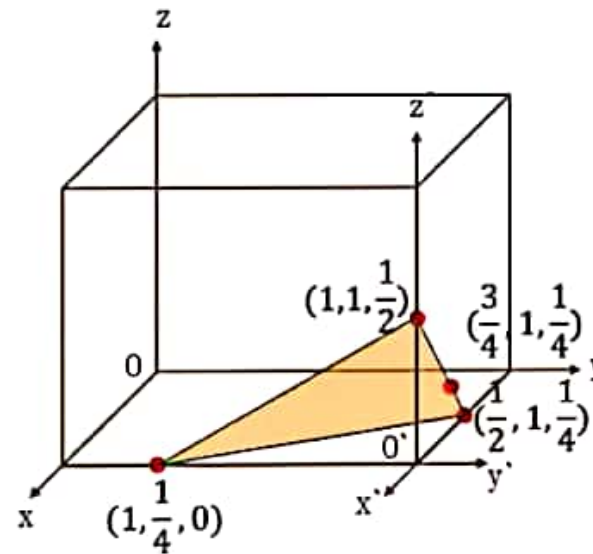
Determine the Miller indices of cubic crystal plane that intersects the position coordinates $(1, 1/4, 0)$, $(1, 1, 1/2)$, and $(3/4, 1, 1/4)$.

The given position coordinate are $(1, 1/4, 0)$, $(1, 1, 1/2)$, and $(3/4, 1, 1/4)$.

THE INTERCEPTS ON THE X, Y AND Z
AXIS ARE RESPECTIVELY $\frac{-1}{2}, \frac{-3}{4}, \frac{1}{2}$

TAKING THE RECIPROCALLS $-2, \frac{-4}{3}, 2$

THE MILLER INDICES WILL BE
 $(\bar{6}, \bar{4}, 6)$



Que:14

NaCl has the FCC lattice with
 $a = 5.63 \text{ \AA}$. What is the
spacing of $\{100\}$ plane?



✓ THE FORMULA FOR INTERPLANAR SPACING IS

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

✓ $a = 5.63 \text{ \AA}$, $h=1$, $k=0$, $l=0$.

✓ SUBSTITUTING VALUES IN ABOVE FORMULA

$$d = \frac{5.63}{\sqrt{1^2 + 0^2 + 0^2}}$$
$$= 5.63 \text{ \AA}$$



Que:15

Gold has atomic weight 197 and the density 19.3 gm/cc. What is the spacing between atoms in solid gold?



- Gold has FCC crystal structure.
- Density,

$$\rho = \frac{M}{V} = \frac{m \times N_e}{a^3}$$

M is mass of an atom, N_e is no. of effective atoms in fcc and a^3 is volume of fcc.

$$m = \frac{\text{atomic weight } (m_A)}{\text{Avagadro's number } (N_A)}$$

$$\rho = \frac{m_A \times N_e}{a^3 \times N_A}$$

$$19.3 = \frac{197 \times 4}{a^3 \times 6.022 \times 10^{23}}$$

$$a = (6.78 \times 10^{-23})^{\frac{1}{3}}$$

$$a = 4.077 \text{ \AA}$$

Que:16

Compare packing fraction for SC and FCC lattice.

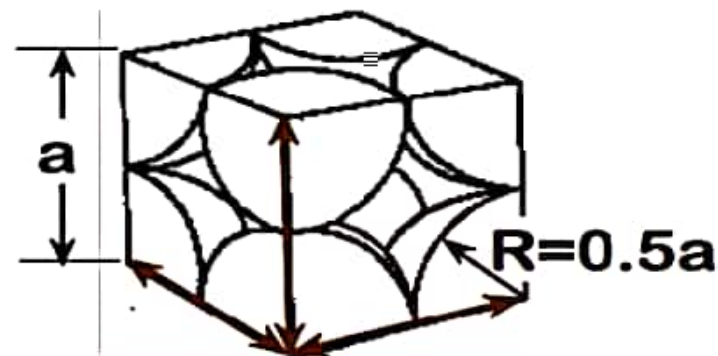


Atomic Packing Factor (APF)

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}^*}{\text{Volume of unit cell}}$$

*assume hard spheres

- APF for a simple cubic structure = 0.52



close-packed directions
contains $8 \times 1/8 =$
1 atom/unit cell

$$\text{APF} = \frac{\begin{array}{c} \text{atoms} \\ \text{unit cell} \end{array} \cdot \begin{array}{c} \text{volume} \\ \text{atom} \end{array}}{\begin{array}{c} \text{volume} \\ \text{unit cell} \end{array}}$$

The diagram shows the calculation of APF for a simple cubic structure. The numerator consists of two parts: 'atoms unit cell' (represented by a green box with the number 1) and 'volume atom' (represented by an orange box with the formula $\frac{4}{3} \pi (0.5a)^3$). The denominator is 'volume unit cell' (represented by a blue box with the formula a^3). Arrows point from the labels to their respective parts in the equation.

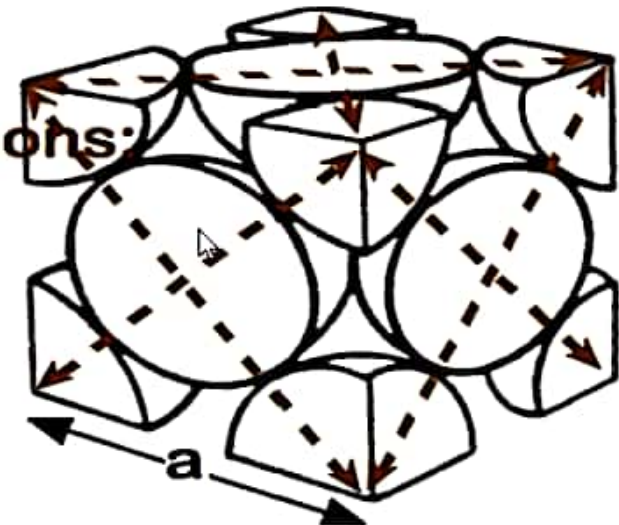
Face Centered Cubic (FCC)

- Close packed directions are face diagonals.
- Note: All atoms are identical; the face-centered atoms are shaded differently only for ease of viewing.



- Coordination # = 12
- Close-packed directions:
length = $4R$
 $= \sqrt{2} a$

Unit cell contains:
 $6 \times 1/2 + 8 \times 1/8$
 $= 4 \text{ atoms/unit cell}$



$$\text{APF} = \frac{\text{atoms/unit cell} \times \text{volume/atom}}{\text{volume/unit cell}}$$

$$\text{APF} = \frac{4 \times \frac{4}{3} \pi (\sqrt{2}a/4)^3}{a^3}$$

- APF for a FCC = 0.74

Que:17

In powder diffraction pattern for lead with radiation of $\lambda = 1.54 \text{ \AA}$ the (220) Bragg reflection angle is $\theta = 32^\circ$. What is the radius of atom?



- Lead has FCC structure. We know

$$2d \sin \theta = n\lambda$$
$$2 \times \frac{a}{\sqrt{2^2 + 2^2 + 0^2}} \sin 32^\circ = 1.54$$
$$a = 4.11 \text{ \AA}$$

Now, for FCC,

$$\sqrt{2}a = 4r$$
$$r = 1.45 \text{ \AA}$$