School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 6
Lagrange Interpolation

1. Find the unique polynomial $P(x)$ of degree 2 or less such that

$$P(1) = 1,\ P(3) = 27,\ P(4) = 64$$

using Lagrange interpolation. Evaluate $P(1.05)$.

2. For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct Lagrange interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error.

(a) $f(x) = \sin \pi x$.
(b) $f(x) = \sqrt[3]{x-1}$.
(c) $f(x) = \log_{10}(3x-1)$.

3. Let $P_3(x)$ be the Lagrange interpolating polynomial for the data $(0,0)$, $(0.5,y)$, $(1,3)$ and $(2,2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

4. Let $f(x) = \sqrt{x-x^2}$ and $P_2(x)$ be the interpolation polynomial on $x_0 = 0$, x_1 and $x_2 = 1$. Find the largest value of x_1 in $(0,1)$ for which $f(0.5) - P_2(0.5) = -0.25$.

5. Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

(a) $f(x) = \sin x,\ x_0 = 2.0,\ x_1 = 2.4,\ x_2 = 2.6,\ n = 2.$
(b) $f(x) = e^{2x}\cos 3x,\ x_0 = 0,\ x_1 = 0.3,\ x_2 = 0.6,\ n = 2.$

6. Use the Lagrange interpolating polynomial of degree two or less and four-digit chopping arithmetic to approximate $\cos 0.750$ using the following values. Find an error bound for the approximation.

$$\cos 0.698 = 0.7661,\ \cos 0.733 = 0.7432,\ \cos 0.768 = 0.7193.$$

The actual value of $\cos 0.750$ is 0.7317 (to four decimal places). Explain the discrepancy between the actual error and the error bound.

7. Determine the spacing h in a table of equally spaced values of the function $f(x) = \sqrt{x}$ between 1 and 2, so that interpolation with a quadratic polynomial will yield an accuracy of 5×10^{-4}

8. Use Neville's method to obtain the approximations for Lagrange interpolating polynomial of degrees one, two, and three to approximate the following:

$$f(8.4) \text{ if } f(8.1) = 16.94410,\ f(8.3) = 17.56492,\ f(8.6) = 18.50515,\ f(8.7) = 18.82091.$$

9. Use Neville's method to approximate $\sqrt{3}$ with the following functions and values.

(a) $f(x) = 3^x$ and the values $x_0 = -2,\ x_1 = -1,\ x_2 = 0,\ x_3 = 1$, and $x_4 = 2$.
(b) $f(x) = \sqrt{x}$ and the values $x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 4$, and $x_4 = 5$.
(c) Compare the accuracy of the approximation in parts (a) and (b).

10. Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5,y)$, $(1,3)$, and $(2,2)$. Use Neville's method to find y if $P_3(1.5) = 0$.

11. Neville's method is used to approximate $f(0.4)$, giving the following table.

CONTINUED

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{0,1} = 2.6$		
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

Determine $P_2 = f(0.5)$.

12. Suppose $x_j = j$, for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = x + 1,\ P_{1,2}(x) = 3x - 1,\ P_{1,2,3}(1.5) = 4.$$

Find $P_{0,1,2,3}(1.5)$.

13. Neville's Algorithm is used to approximate $f(0)$ using $f(-2)$, $f(-1)$, $f(1)$, and $f(2)$. Suppose $f(-1)$ was understated by 2 and $f(1)$ was overstated by 3. Determine the error in the original calculation of the value of the interpolating polynomial to approximate $f(0)$.

14. From census data, the approximate population of the United States was 150.7 million in 1950, 179.3 million in 1960, 203.3 million in 1970, 226.5 million in 1980, and 249.6 million in 1990. Using Lagrange interpolation polynomial for these data, find an approximate value for the population in 2000. Then use the polynomial to estimate the population in 1920 based on these data. What conclusion should be drawn?