

LECTURE 10

UEI407

Stable and Unstable Systems

The term bounded input refers to a finite value of input signal $x(t)$ for any value of t . Hence if input $x(t)$ is bounded then exists a constant M_x such that

$$|x(t)| \leq M_x \quad \text{and} \quad M_x < \infty \quad \text{for all value of } t.$$

Examples bounded input signals are decaying signal, impulse signal, step signal.

Examples of unbounded input signals are ramp signal and increasing exponential signal.

For a bounded input $x(t)$ to a system, if the output $y(t)$ of the system is bounded, the system is called bounded-input /bounded-output (BIBO) stable.

The term bounded refers to finite and predictable output for any value of t . Hence if output $y(t)$ is bounded then exists a constant M_y such that

$$|y(t)| \leq M_y \text{ and } M_y < \infty \text{ for all values of } t.$$

A linear time invariant system is BIBO stable if and only if its impulse response is absolutely integrable i.e., $I = \int_{-\infty}^{\infty} |h(t)| dt < \infty$

If a system has unbounded output for a bounded input, the system becomes unstable.

Test the stability of the LTI systems having impulse responses

- (a) $h(t) = e^{-5|t|}$ (b) $h(t) = e^{4t} u(t)$ (c) $h(t) = t e^{-3t} u(t)$

(a) For stability, $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

$$\begin{aligned}\int_{-\infty}^{\infty} |h(t)| dt &= \int_{-\infty}^{\infty} |e^{-5|t|}| dt = \int_{-\infty}^{\infty} e^{-5|t|} dt = \int_{-\infty}^0 e^{5t} dt + \int_0^{\infty} e^{-5t} dt \\ &= \left[\frac{e^{5t}}{5} \right]_{-\infty}^0 + \left[\frac{e^{-5t}}{-5} \right]_0^{\infty} = \frac{e^0}{5} - \frac{e^{-\infty}}{5} + \frac{e^{-\infty}}{-5} - \frac{e^0}{-5} = \frac{1}{5} - 0 - 0 + \frac{1}{5} = \frac{2}{5} < \infty\end{aligned}$$

The system is stable.

(b)

For stability, $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

$$\begin{aligned}\int_{-\infty}^{\infty} |h(t)| dt &= \int_{-\infty}^{\infty} |e^{4t} u(t)| dt = \int_0^{\infty} e^{4t} dt = \left[\frac{e^{4t}}{4} \right]_0^{\infty} \\ &= \frac{e^{\infty}}{4} - \frac{e^0}{4} \\ &= \infty - \frac{1}{4} = \infty\end{aligned}$$

The system is unstable.

(c)

For stability, $\int_{-\infty}^{\infty} |h(t)| dt < \infty$

$$\begin{aligned}\int_{-\infty}^{\infty} |h(t)| dt &= \int_{-\infty}^{\infty} |te^{-3t} u(t)| dt = \int_0^{\infty} te^{-3t} dt \\&= \left[t \times \frac{e^{-3t}}{-3} - \int 1 \times \frac{e^{-3t}}{-3} dt \right]_0^{\infty} = \left[-\frac{te^{-3t}}{3} - \frac{e^{-3t}}{9} \right]_0^{\infty} \\&= -\lim_{t \rightarrow \infty} \frac{te^{-3t}}{3} + 0 - 0 + \frac{1}{9} = \frac{1}{9}\end{aligned}$$

The system is stable.

Invertible and Non-invertible Systems

If a system has a unique relationship between its input $x(t)$ and output $y(t)$, the system is known as invertible. Therefore, for an invertible system if $y(t)$ is known , $x(t)$ can be found out ambiguously and uniquely. On the other hand, if the system does not have a unique relationship between its input and output, the system is said to be non-invertible.

For $y(t) = 5x(t)$, the system is said to be invertible whereas for $y(t) = 4x^2(t)$, the system is said to be non-invertible.

Feedback System

The system having feedback is a special class of system. In this system, the feedback is taken from output and added to input of the system shown in Figure 1.

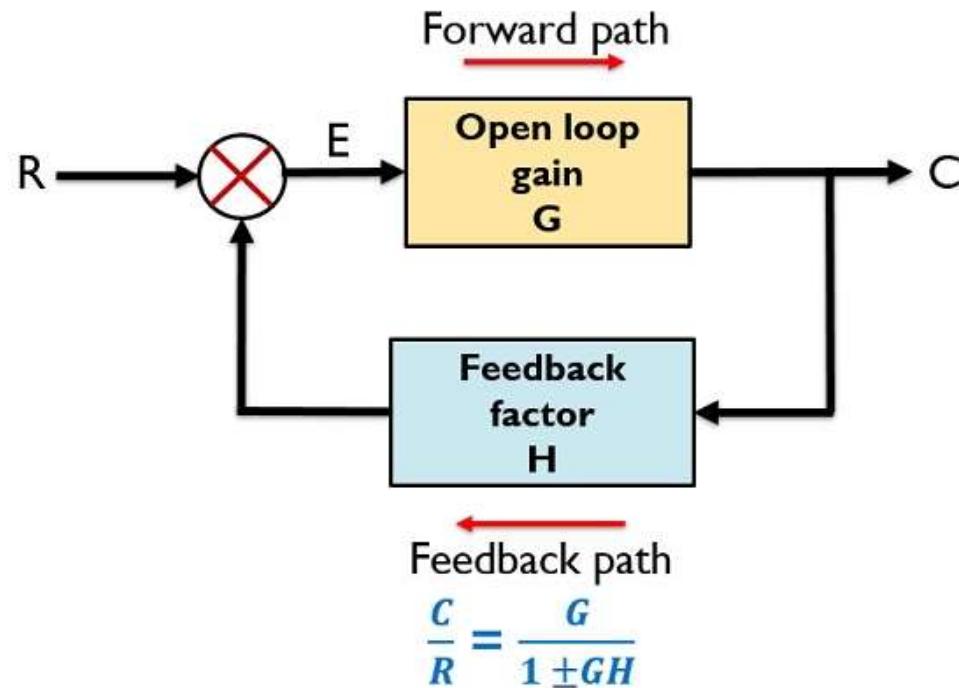


Figure 1: Feedback System

Discrete Time Signals

The discrete time signals are obtained at sampling instants because they are obtained by time sampling of continuous time signals. The exponential signal e^{-t} for $-\infty$ to ∞ is considered. Let us sample this exponential signal at time instants separated by period T. Usually, the sampling instants are represented by $t_n = nT$ and hence the sampled or discrete time signal is represented by

$$x(t_n) = x(nT)$$

$$\therefore x(nT) = \begin{cases} e^{-nT} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

If sample duration is $T = 1$ sec., we can write

$$x(nT) = \begin{cases} e^{-n} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

Therefore, $x(n)$ for $n = 0, 1, 2, 3, \dots$ becomes

$$\begin{aligned} x(n) &= \{ e^0, e^{-1}, e^{-2}, e^{-3}, e^{-4}, e^{-5}, e^{-6}, \dots \} \\ &= \{ 1, 0.368, 0.135, 0.049, 0.018, 0.0067, \dots \} \end{aligned}$$

Figure 2 shows the graphical representation of $x(n)$.

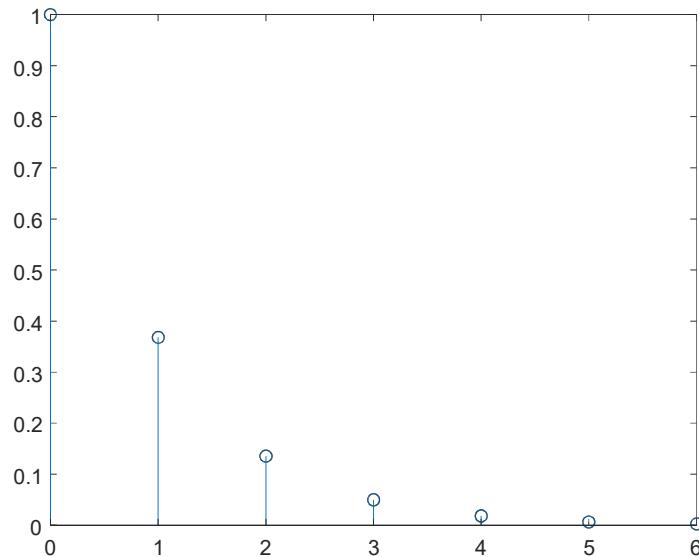


Figure 2 : Graphical representation of $x(n)$

Figure 3 shows the graphical representation of a discrete time signal.

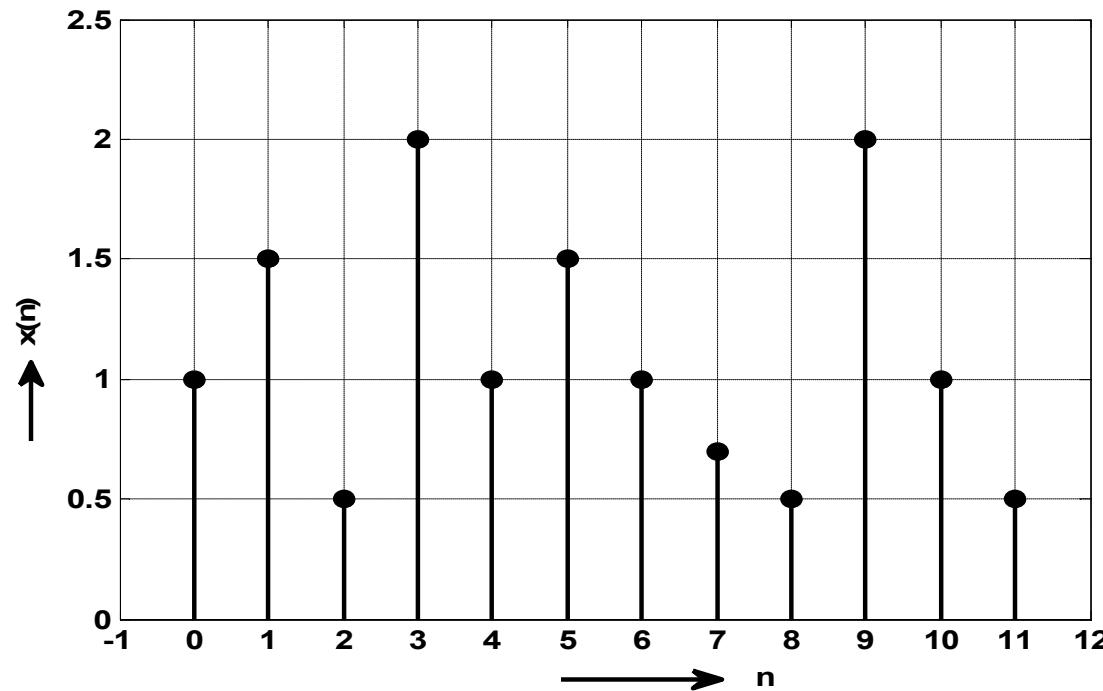


Figure 3 : Discrete time signal

The discrete time signal shown in Figure 3 can be represented mathematically as

The discrete time signal shown in Figure 3 can be represented mathematically as

$$x(n) = \begin{cases} 1 & \text{for } n = 0,4,6,10 \\ 1.5 & \text{for } n = 1,5 \\ 2 & \text{for } n = 3,9 \\ 0.5 & \text{for } n = 2,8,11 \\ 0.7 & \text{for } n = 7 \end{cases}$$

The above functional representation of discrete-time systems. The discrete-time signal can also be represented in tabular form as shown below.

n -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
x(n) 0 0 1 1.5 0.5 2 1 1.5 1 0.7 0.5 2 1 0.5 0

An infinite duration of sequence can be represented as

$$x(n) = \{\dots, 3, 2, 3, \underset{\uparrow}{1}, 5, 2, 1.5, 2.7, \dots\}$$

where the symbol \uparrow indicates the value of $x(n)$ corresponding to $n = 0$.

Another example of an infinite duration of sequence is shown below.

$$x(n) = \{1, 2, 3, 1, 6, \dots\}$$

Since ' \uparrow ' is not used in the above sequence, the first value $x(0) = 1$.

An infinite duration of sequence can be represented as

$$x(n) = \{\dots, 3, 2, 3, \underset{\uparrow}{1}, 5, 2, 1.5, 2.7, \dots\}$$

where the symbol \uparrow indicates the value of $x(n)$ corresponding to $n = 0$.

Another example of an infinite duration of sequence is shown below.

$$x(n) = \{1, 2, 3, 1, 6, \dots\}$$

Since ‘↑’ is not used in the above sequence, the first value $x(0) = 1$.

A finite duration satisfying the condition $x(n) = 0$ for $n < 0$ can be represented as

$$x(n) = \{1, 3, 2, 4\}$$

where $x(0) = 1$, $x(1) = 3$, $x(2) = 2$ and $x(3) = 4$.

For example, let us take the following signal, which is discrete time cosine wave expressed by $x(n) = \cos(0.4\pi n)$

The signal shown in Figure 4 $x(n) = \cos(0.4\pi n)$ is an array of samples for $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \dots$ and so on.

Figure 4 shows the plot of sequence of an array samples. Here $x(n)$ is defined for integer values of ‘n’ because the sample numbers are integer only.

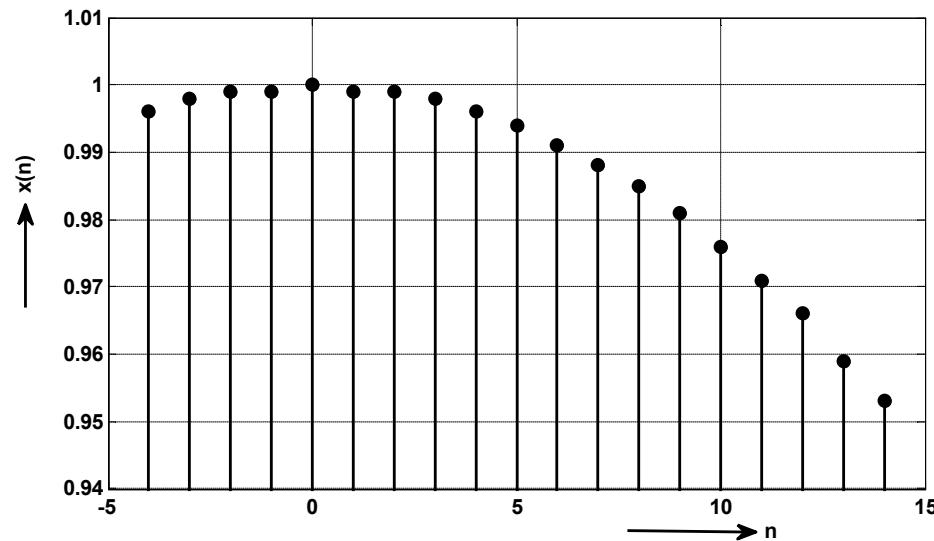


Figure 4