Tutorial Sheet-4

1. List the triples in the relation $\{(a, b, c) \mid a, b, \text{ and } c \text{ are integers with } 0 < a < b < c < 5\}$.

2. Represent each of these relations on $\{1, 2, 3\}$ with a matrix and graph (with the elements of this set listed in increasing order).
    a) $\{(1, 1), (1, 2), (1, 3)\}$
    b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
    c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
    d) $\{(1, 3), (3, 1)\}$

3. Let R be the relation represented by the matrix

    $$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

    Find the matrix representing
    a) R^{-1}.   b) comp(R)   c) R^2.

4. Let R be the relation on the set $\{0, 1, 2, 3\}$ containing the ordered pairs $(0, 1), (1, 1), (1, 2), (2, 0), (2, 2),$ and $(3, 0)$. Find the
    a) reflexive closure of R.   b) symmetric closure of R.

5. Find the transitive closures of these relations on $\{1, 2, 3, 4\}$.
    a) $\{(1, 2), (2,1), (2,3), (3,4), (4,1)\}$
    b) $\{(2, 1), (2,3), (3,1), (3,4), (4,1), (4, 3)\}$
    c) $\{(1, 2), (1,3), (1,4), (2,3), (2,4), (3, 4)\}$
    d) $\{(1, 1), (1,4), (2,1), (2,3), (3,1), (3, 2), (3,4), (4, 2)\}$

6. Use Warshall's algorithm to find the transitive closures of above relations.

7. Find the smallest relation containing the relation $\{(1, 2), (1, 4), (3, 3), (4, 1)\}$ that is
    a) reflexive and transitive.
    b) symmetric and transitive.
    c) reflexive, symmetric, and transitive.