Roll Number: _____

B.E.	Course Code: UMA007	Course Name: Numerical Analysis
Time: 3 Hours, M. Marks: 100	Name of Faculty: Dr. Isha Dhiman, Dr. Jolly Puri, Dr. Kavita.	

Note: (1) **Attempt all the questions.**
(2) **Calculator without graphing mode and alphanumeric memory is permitted.**

1. (a) Find the root of $f(x) = x^3 + 4x^2 - 10 = 0$ in the interval $[1,2]$ using bisection method by performing five iterations.

   [10 marks]

   (b) Use Newton's method to approximate the first positive real root of $\cos x - x^3 = 0$ correct to four decimal places by taking initial guess $x_0 = 0.5$.

   [8 marks]

2. (a) Perform two iterations of Gauss Seidel to solve the following linear systems, using $\mathbf{x}^{(0)} = \mathbf{0}$,

$$4x_1 + x_2 - x_3 = 5,$$
$$-x_1 + 3x_2 + x_3 = -4,$$
$$2x_1 + 2x_2 + 5x_3 = 1.$$

   [10 marks]

   (b) Use power method to find the highest in magnitude eigenvalue and the corresponding eigen vector of the matrix $\mathbf{A}$ by performing four iterations, using $\mathbf{x}^{(0)} = [1,1]^T$,

$$\mathbf{A} = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}.$$

   [10 marks]

3. (a) Suppose $x_0, x_1, \ldots, x_n$ are distinct numbers in the interval $[a,b]$ and $f \in C^{n+1}[a,b]$. Prove that for each x in $[a,b]$, there exists a number $\xi(x)$ in (a,b) such that

$$f(x) = P(x) + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x - x_0)(x - x_1)\ldots(x - x_n),$$

   where $P(x)$ is the Lagrange's interpolating polynomial of degree n.

   [10 marks]

(b) Use Newton forward difference formula to construct a polynomial of degree 3 for the following data and find $f(0.25)$

$$f(0.1) = -0.6205, f(0.2) = -0.2840, f(0.3) = 0.0066, f(0.40) = 0.2484$$

[8 marks]

4. (a) By using the method of least squares, fit a curve of the form $y = ax^b$ to the following data

x	2	3	4	5
y	27.8	62.1	110	161

[10 marks]

(b) Use Runge Kutta method of order 4 with $h = 0.2$ to find $y(0.4)$ for the following initial value problem

$$y' = y - t^2 + 1, \quad y(0) = 0.5.$$

[10 marks]

5. (a)   i. Determine the values of n and h required to approximate

$$\int_0^2 \frac{1}{x+4} dx,$$

to within 10^{-5} and hence compute the approximation using Composite Simpson's one-third rule.

[10 marks]

ii. Determine constants $a, b, c,$ and d that will produce a quadrature formula

$$\int_{-1}^1 f(x)\, dx = af(-1) + bf(1) + cf'(-1) + df'(1),$$

that has degree of precision 3.

[8 marks]

(b) Show that the function $g(x) = (x^2 - 1)/3$ has a unique fixed point in the interval $[-1, 1]$.

[6 marks]