Course: UMA 035 (Optimization Techniques)

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What is the aim of each and every person ?

To maximize …..and/or To minimize….

For example,

- **Aim of a student is to maximize his/her CGPA and/or to maximize the enjoy and/ or to minimize the study time etc.**

- **Aim of a businessman is to maximize his/her profit and/or to minimize the loss etc.**

To maximize/minimize….We may need to remember some conditions

For example

If time for end semester examination is 3 hr and you may not leave before 1 hr.

Then,

Time for appearing in the examination <=3hr.

Time for appearing in the examination >=1hr.

If there is a competitive examination of 3 hr with the condition that one cannot leave before 3hr. Then

Time for appearing in the examination =3hr.

A business man has Rs 50,000 for investing in business. Then,

Invested amount <=50,000

There may be three types of conditions

- **Time>=2 may be transformed into Time -2 >=0 (Greater than equal to)**

- **Time=3 may be transformed into Time -3 =0 (equal to)**

- **Time<=3 may be transformed into Time -3 <=0 (Less than equal to)**

We can transform a condition in such a manner that right hand side is zero.

- **Time>=2 may be transformed into Time -2 =0 (Greater than equal to)**

- **Time=3 may be transformed into Time -3 =0 (equal to)**

- **Time<=3 may be transformed into Time -3 =0 (Less than equal to)**

Finally, the aim is

Maximize/Minimize (f(X))

Subject to

g1(x) <= or = or >= 0 (First condition)

g2(x) <= or = or >= 0 (Second condition)

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gm(x) <= or = or >= 0 (m^{th} condition)

OR

Optimize (f(X))

Subject to

g1(x) <= or = or >= 0 (First condition)

g2(x) <= or = or >= 0 (Second condition)

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gm(x) <= or = or >= 0 (m^{th} condition)

- **It is called Mathematical Programming Problem (MPP)**

- **f(X) is called objective function**

- **g1(x) <= or = or >= 0 is called first constraint**

- **g2(x) <= or = or >= 0 is called second constraint**

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- **gm(x) <= or = or >= 0 is called m^{th} constraint**

Mathematical programming problem (MPP) may be classified into two major categories:

- **Linear Programming Problem (LPP)**

- **Non-linear Programming Problem (NLPP)**

Linear Programming Problem (LPP)

- **If all f(X), g1(x), g2(x),..., gm(x) are linear then MPP is called LPP.**

- **If at least one of f(X), g1(x), g2(x),..., gm(x) is non-linear then MPP is called NLPP.**

Linear function of n variables x1, x2,…, xn

a0+a1*x1+ a2*x2+…+ an*xn, where, a0, a1,…, an are real numbers.

If the given function can be compared by this function then the given function is linear otherwise non-linear.

Example

2 * x1+4*$(x2)^2$

It is function of two variables

General linear function of two variables is a0+a1*x1+ a2*x2

Comparing

- **a0=0**
- **a1=2**
- **a2*x2 cannot be compared with 4*$(x2)^2$**

2 * x1+4*$(x2)^2$ is a non-linear function

Example

2 * x1+4*x2

It is function of two variables

General linear function of two variables is a0+a1*x1+ a2*x2

Comparing

- **a0=0**
- **a1=2**
- **a2=4**

  2 * x1+4*x2 is a linear function

Example

2 * log(x1)+4*x2

It is function of two variables

General linear function of two variables is a0+a1*x1+ a2*x2

Comparing

- **a0=0**

- **a1*x1 cannot be compared with 2 * log(x1)**

2 * log(x1)+4*x2 is a non-linear function

Example

2 * |x1|+4*x2

It is function of two variables

General linear function of two variables is a0+a1*x1+ a2*x2

Comparing

- **a0=0**

- **a1*x1 cannot be compared with 2 * |x1|**

2 * |x1|+4*x2 is a non-linear function

Maximize (sin(x1)+2*x2)

Subject to

x1+x2<=3

What is objective function i.e., f(x)?

Ans: sin(x1)+2*x2

How many constraints?

Ans: One

What is constraint i.e., g1 (x)?

Ans: x1+x2-3

f(x) is linear or non-linear?

Ans: Non-linear

g1 (x) is linear or non-linear?

Ans: Linear

LPP or NLPP?

Ans: NLPP

Maximize (x1+2*x2)

Subject to

x1+x2<=3

What is objective function i.e., f(x)?

Ans: x1+2*x2

How many constraints?

Ans: One

What is constraint i.e., g1 (x)?

Ans: x1+x2-3

f(x) is linear or non-linear?

Ans: Linear

g1 (x) is linear or non-linear?

Ans: Linear

LPP or NLPP?

Ans: LPP

Maximize (x1+2*x2)

Subject to

$(x1)^2+x2<=3$

What is objective function i.e., f(x)?

Ans: x1+2*x2

How many constraints?

Ans: One

What is constraint i.e., g1 (x)?

Ans: $(x1)^2+x2-3$

f(x) is linear or non-linear?

Ans: Linear

g1 (x) is linear or non-linear?

Ans: Non-linear

LPP or NLPP?

Ans: NLPP

Maximize (x1+2*x2)

Subject to

x1+|x2| <=3

What is objective function i.e., f(x)?

Ans: x1+2*x2

How many constraints?

Ans: One

What is constraint i.e., g1 (x)?

Ans: x1+|x2| -3

f(x) is linear or non-linear?

Ans: Linear

g1 (x) is linear or non-linear?

Ans: Non-linear

LPP or NLPP?

Ans: NLPP

Mathematical Programming Problem

Maximize/Minimize (f(X))

Subject to

g1(x) <= or = or >= 0

g2(x) <= or = or >= 0

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gm(x) <= or = or >= 0

If all f(X), g1(x), g2(x),..., gm(x) are linear then MPP is called LPP.

Let

f(X)=c1*x1+ c2*x2+… + cn*xn

g1(x)= a11*x1+ a12*x2+… + a1n*xn-b1

g2(x)= a21*x1+ a22*x2+… + a2n*xn-b2

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gm(x)= am1*x1+ am2*x2+… + amn*xn-bm

Then

Maximize/Minimize (c1*x1+ c2*x2+… + cn*xn)

Subject to

a11*x1+ a12*x2+… + a1n*xn –b1<= or = or >= 0

a21*x1+ a22*x2+… + a2n*xn –b2<= or = or >= 0

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am1*x1+ am2*x2+… + amn*xn-bm <= or = or >= 0

Maximize/Minimize (c1*x1+ c2*x2+… + cn*xn)

Subject to

a11*x1+ a12*x2+… + a1n*xn <= or = or >= b1

a21*x1+ a22*x2+… + a2n*xn <= or = or >= b2

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am1*x1+ am2*x2+… + amn*xn <= or = or >= bm

It can also be rewritten as

Maximize/Minimize ($\sum_{j=1}^{n} c_j x_j$)

Subject to

$\sum_{j=1}^{n} a_{1j} x_j$ **<= or = or >=**b_1

$\sum_{j=1}^{n} a_{2j} x_j$ **<= or = or >=** b_2

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$\sum_{j=1}^{n} a_{mj} x_j$ **<= or = or >=** b_m

It can also be rewritten as

Maximize/Minimize ($\sum_{j=1}^{n} c_j x_j$)

Subject to

$\sum_{j=1}^{n} a_{ij} x_j$ **<= or = or >=** $b_i; \qquad i = 1, 2, \ldots, m$

It is called General form of a LPP

Maximize/Minimize (c1*x1+ c2*x2+… + cn*xn)

Subject to

a11*x1+ a12*x2+… + a1n*xn <= or = or >= b_1

a21*x1+ a22*x2+… + a2n*xn <= or = or >= b_2

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am1*x1+ am2*x2+… + amn*xn <= or = or >= b_m

It can also be rewritten as

Maximize/Minimize $\left(\begin{bmatrix} c_1 & \cdots & c_n \end{bmatrix}_{1\times n} \times \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \right)$

Subject to

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m\times n} \times \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \leq or = or \geq \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$$

Assuming $\begin{bmatrix} c_{11} & \cdots & c_{1n} \end{bmatrix}_{1\times n} = C,$, $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = X,$

$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m\times n} = A,$ $\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1} = b,$

Following Matrix form of LPP is obtained

Maximize/Minimize (CX)

Subject to

AX $\leq or = or \geq b$

Conclusions

General form of a LPP

Maximize/Minimize $(\sum_{j=1}^{n} c_j x_j)$

Subject to

$\sum_{j=1}^{n} a_{ij} x_j$ <= or = or >= b_i; $i = 1, 2, \dots, m$

Matrix form of a LPP

Maximize/Minimize (CX)

Subject to

$AX \le or = or \ge b$

where,

$C = \begin{bmatrix} c_{11} & \dots & c_{1n} \end{bmatrix}_{1\times n}$,

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1},$$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m\times n},$$

$$b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1},$$