School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 8
Numerical Integration

1. Approximate the following integrals using the trapezoidal and Simpson's rules.

(a) $I = \int\limits_{-0.25}^{0.25} (\cos x)^2 \, dx$.

(b) $\int\limits_{e}^{e+1} \frac{1}{x \ln x} \, dx$.

(c) Find a bound for the error using the error formula, and compare this to the actual error.

2. The quadrature formula $\int\limits_0^2 f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

3. Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int\limits_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

4. The length of the curve represented by a function $y = f(x)$ on an interval $[a, b]$ is given by the integral

$$I = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx.$$

Use the trapezoidal rule and Simpson's rule with 4 and 6 subintervals compute the length of the graph of the ellipse given with equation $4x^2 + 9y^2 = 36$.

5. Determine the values of n and h required to approximate

$$\int\limits_0^2 \frac{1}{x+4} \, dx$$

to within 10^{-4}. Use composite Trapezoidal and composite Simpson's rule.

6. The area A inside a closed curve $y^2 + x^2 = \cos x$ is given by

$$A = 4 \int_0^\alpha \left(\cos x - x^2\right)^{1/2} dx$$

where α is the positive root of the equation $\cos x = x^2$.

(a) Compute α with three correct decimals by Newton's method.

(b) Use composite trapezoidal rule with 6 subintervals to compute the area A.

7. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?

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8. Evaluate the integral

$$\int_{-1}^{1} e^{-x^2} \cos x \, dx$$

by using the Gauss-Legendre two and three point formulas.

9. Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

10. Evaluate

$$I = \int_0^1 \frac{\sin x \, dx}{2+x}$$

by subdividing the interval $[0, 1]$ into two equal parts and then by using Gauss-Legendre two point formula.

11. A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation

$$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(u)} \, du$$

Suppose that $R(v) = -v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in meters/second. If $m = 10$ kg and $v(0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s.

12. In statistics it is shown that

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \, e^{-\dfrac{x^2}{2\sigma^2}} \, dx = 1,$$

for any positive σ. The function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \, e^{-\dfrac{x^2}{2\sigma^2}}$$

is the normal density function with mean $\mu = 0$ and standard deviation σ. The probability that a randomly chosen value described by this distribution lies in $[a, b]$ is given by $\int_a^b f(x)dx$. Approximate to within 10^{-3} the probability that a randomly chosen value described by this distribution will lie in

(a) $[-\sigma, \sigma]$

(b) $[-2\sigma, 2\sigma]$

(c) $[-3\sigma, 3\sigma]$.