Functions

Ms. Rajanpreet Kaur Chahal

Assistant Professor, CSE

Thapar Institute of Engineering & Technology, Patiala

Email: rajanpreet@thapar.edu


Activity One

Function definition

- Let A and B be nonempty sets.
- A *function* f from A to B is an assignment of exactly one element of B to each element of A.
- We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.
- If f is a function from A to B, we write
  - $f : A \to B$

Domain and Range

- If f is a function from A to B, we say that A is the *domain* of f and B is the *codomain* of f.
- If $f(a) = b$, we say that b is the *image* of a and a is a *preimage* of b.
- The *range*, or *image*, of f is the set of all images of elements of A.
- Also, if f is a function from A to B, we say that f *maps* A to B.

Equal functions

- Two functions are **equal** when:
  - They have the same domain,
  - They have the same codomain,
  - They map each element of their common domain to the same element in their common codomain.

Real valued/Integer valued functions

- A function is called **real-valued** if its codomain is the set of real numbers.
- A function is called **integer-valued** if its codomain is the set of integers.
- Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.

Function addition/multiplication

- Let f1 and f2 be functions from A to **R**. Then f1 + f2 and f1f2 are also functions from A to **R** defined for all $x \in A$ by
  - $(f1 + f2)(x) = f1(x) + f2(x)$,
  - $(f1f2)(x) = f1(x)f2(x)$.

Example

Question: Let f1 and f2 be functions from **R** to **R** such that

$f1(x) = x^2$ and $f2(x) = x - x^2$.

What are the functions f1 + f2 and f1f2?

Answer:

Image of a subset

- Let f be a function from A to B and let S be a subset of A.
- The *image* of S under the function f is the subset of B that consists of the images of the elements of S.
- The image of S is denoted by $f(S)$ where $f(S) = \{t \mid \exists s \in S\ (t = f(s))\}$.

Example

- Question: Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. Consider subset $S = \{b, c, d\}$.

  What is image of S?

  Answer:

Summary

- Concept of Functions
- Domain, Codomain and Range of functions
- Equal Functions
- Real-valued and integer-valued functions
- Function addition/multiplication
- Image of a subset

Types of Function

Ms. Rajanpreet Kaur Chahal
Assistant Professor, CSE
Thapar Institute of Engineering & Technology, Patiala
Email: rajanpreet@thapar.edu

Types of Function

- A function can be of three types:
  - One-to-One function (Injective function)
  - Onto function (Surjective function)
  - One-to-One Correspondence (Bijective function)

One-to-One function

- A function f is said to be *one-to-one*, or an *injunction*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.
- A function is said to be *injective* if it is one-to-one.

Example 1

- Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.

Example 2

- Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

Onto function

- A function f from A to B is called *onto*, or a *surjection*, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.
- A function f is called *surjective* if it is onto.

Example 1

- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an onto function?

Example 2

- Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

One-to-One correspondence

- The function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.
- A function f is called *bijective* if it is one to one and onto.

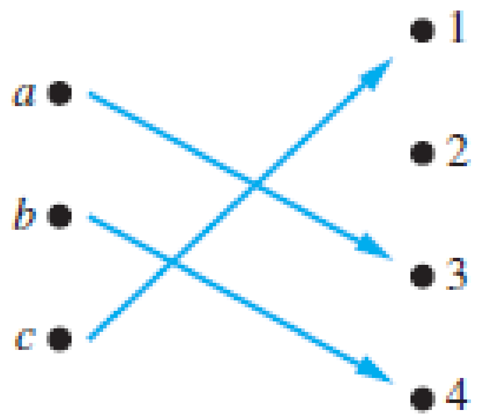
Example 1

- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f a bijection?

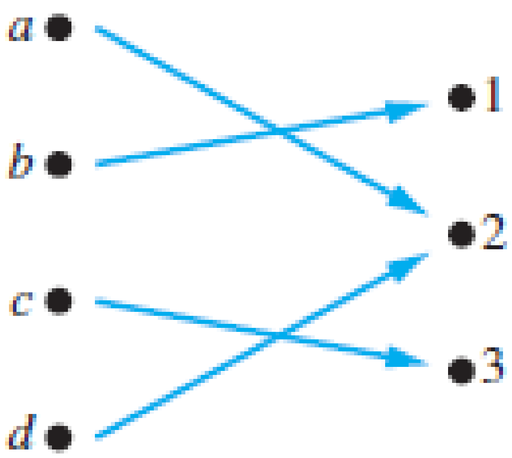
Example 2

- Is the function $f(x) = x^2$ from the set of integers to the set of integers bijective?

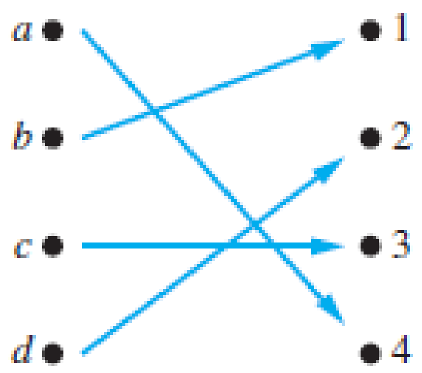
Examples of Different Types of Correspondences



Examples of Different Types of Correspondences



Examples of Different Types of Correspondences



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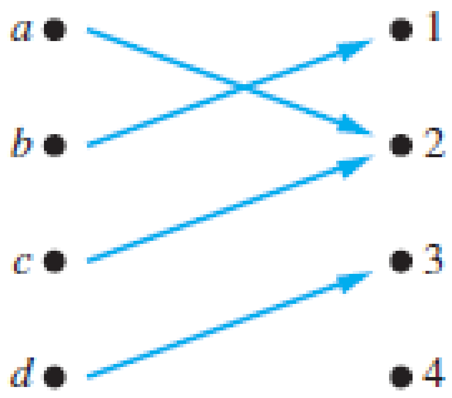
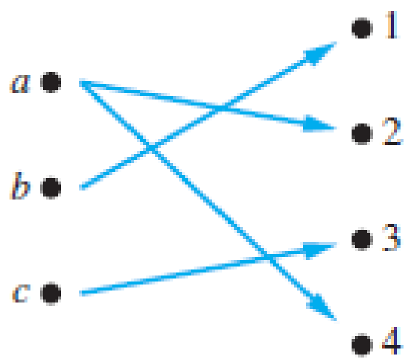


Examples of Different Types of Correspondences



Inverse Function and Compositions of Functions

Ms. Rajanpreet Kaur Chahal
Assistant Professor, CSE
Thapar Institute of Engineering & Technology, Patiala
Email: rajanpreet@thapar.edu

Inverse function

- Let f be a one-to-one correspondence from the set A to the set B.
- The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$.
- The inverse function of f is denoted by f^{-1}.
- $f^{-1}(b) = a$ when $f(a) = b$.
- A one-to-one correspondence is called **invertible** as we can define an inverse of it.
- A function is **not invertible** if it is not a one-to-one correspondence.

Function f^{-1} Is the Inverse of Function f

Example 1

- Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?

Example 2

- Let f be the function from **R** to **R** with $f(x) = x^2$. Is f invertible?

Example 3

- Let f be the function from the set of non-negative real numbers to the set of non-negative real numbers with $f(x) = x^2$. Is f invertible?

Composition of functions

- Let g be a function from the set A to the set B and let f be a function from the set B to the set C.
- The *composition* of the functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by
  - $(f \circ g)(a) = f(g(a))$.

Composition of the Functions *f* and *g*.

Example 1

- Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$. What is the composition of f and g, and what is the composition of g and f?

Example 1

- Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f?

Answer:

Note: $f \circ g \neq g \circ f$

Composition of a function and its inverse

- When the composition of a function and its inverse is formed, in either order, an identity function is obtained.
- Suppose that f is a one-to-one correspondence from the set A to the set B. Then the inverse function f^{-1} exists and is a one-to-one correspondence from B to A, i.e.,
  - $f^{-1}(b) = a$ when $f(a) = b$, and
  - $f(a) = b$ when $f^{-1}(b) = a$.
- Hence, $(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$, and $(f \circ f^{-1})(b) = f\left(f^{-1}(b)\right) = f(a) = b$.
- Also, $(f^{-1})^{-1} = f$.

Floor and Ceil Function

Ms. Rajanpreet Kaur Chahal
Assistant Professor, CSE
Thapar Institute of Engineering & Technology, Patiala
Email: rajanpreet@thapar.edu

Graphs of Functions

- Let f be a function from the set A to the set B.
- The *graph* of the function f is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } f(a) = b\}$.

Example

- Display the graph of the function $f(x) = x^2$ from the set of integers to the set of integers.

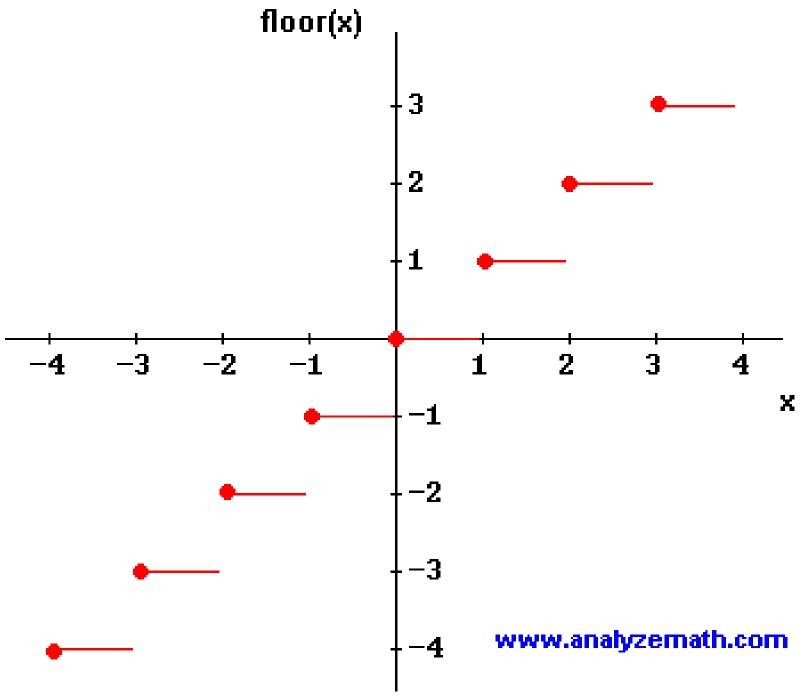
The graph of f is the set of ordered pairs of the form $(x, f(x)) = (x, x^2)$, where x is an integer

The Graph of $f(x) = x^2$ from Z to Z.

Floor function

- The ***floor function*** assigns to the real number *x* the largest integer that is less than or equal to *x*.
- The value of the floor function at *x* is denoted by $\lfloor x \rfloor$.

Graph of floor function



Ceil function

- The ***ceiling function*** assigns to the real number *x* the smallest integer that is greater than or equal to *x*.
- The value of the ceiling function at *x* is denoted by [x].

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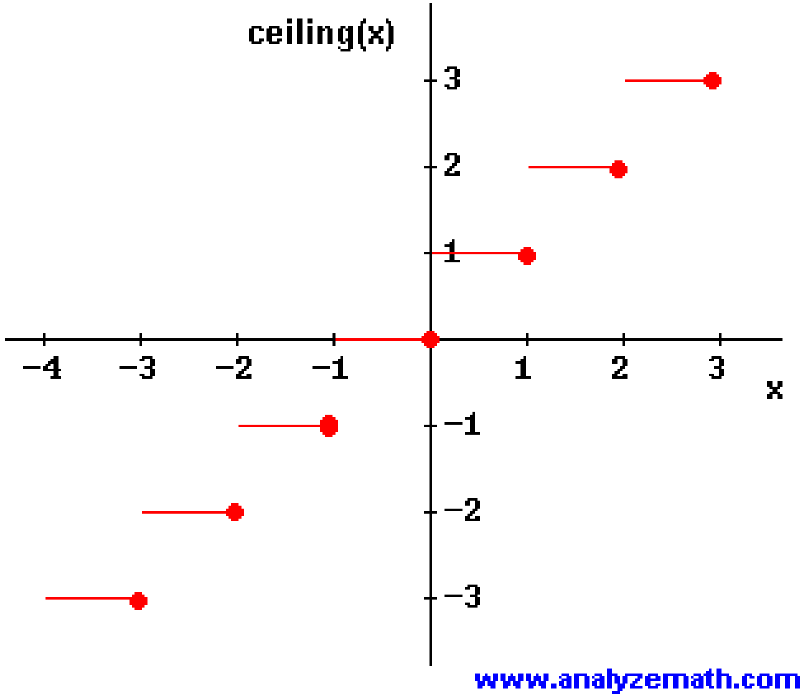
Graph of ceil function



Example 1

- Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

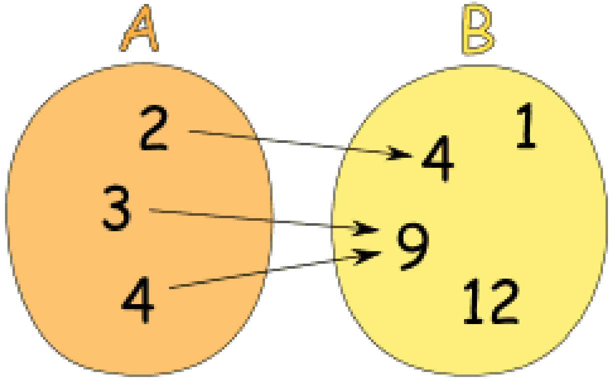
Example 2

- In asynchronous transfer mode, data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second?

Practice on Basics of Functions

DR. ASHISH GIRDHAR
DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
THAPAR INSTITUTE OF ENGINEERING & TECHNOLOGY, PATIALA

Question 1

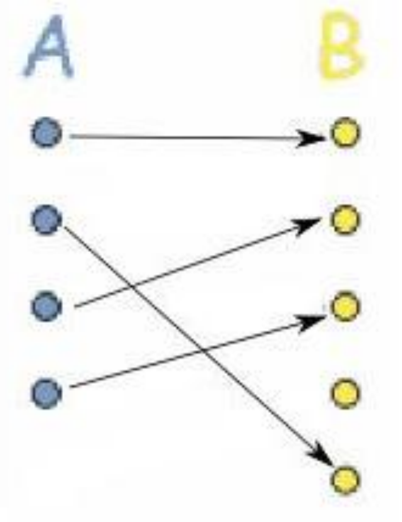


For the function illustrated above, what is the range?

Question 2

What is the domain for the function $f(x) = \dfrac{(x-2)(x-4)}{(x-1)(x-3)}$?

Question 3



The function from set A to set B is

Question 4

Which of the following functions is NOT injective (one-to-one)?

A $f(x) = x^3 + 4$ from R to R

B $f(x) = x^3 + 4$ from N to N

C $f(x) = x^2 + 4$ from R to R

D $f(x) = x^2 + 4$ from N to N

Question 5

If X = Floor(X) = Ceil(X) then :

a) X is a fractional number

b) X is a Integer

c) X is less than 1

d) none of the mentioned

Question 6

Suppose that $f: R \longrightarrow R$ and $f(x) = 3x - 8$

a) Is f^{-1} a function?

b) Find the inverse function of f.

c) Compute $f(f^{-1}(7))$ and $f^{-1}(f(7))$

Thanks