Course: UMA 035 (Optimization Techniques)

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Multi-objective linear programming problem

Maximize/Minimize $(c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n)$

Maximize/Minimize $(c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n}x_n)$

Maximize/Minimize $(c_{31}x_1 + c_{32}x_2 + \ldots + c_{3n}x_n)$

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Maximize/Minimize $(c_{k1}x_1 + c_{k2}x_2 + \ldots + c_{kn}x_n)$

Subject to

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n$ <=or =or>= b_1,

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n$ <=or =or>= b_2,

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$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n$ <=or =or>= b_m.

Example:

Maximize (Marks in Physics)

Maximize (Marks in Mathematics)

Maximize (Marks in Chemistry)

Subject to

Marks secured Student 1 in Physics =50

Marks secured Student 1 in Chemistry =60

Marks secured Student 1 in Mathematics =65

Marks secured Student 2 in Chemistry =50

Marks secured Student 2 in Mathematics =60

Marks secured Student 2 in Physics =65

	Physics	**Chemistry**	**Mathematics**
Student 1	**50**	**60**	**65**
Student 2	**65**	**50**	**60**
Who is better?	**Student 2**	**Student 1**	**Student 1**
Who is best?		**Not possible to conclude**	

The above problem is solved by transforming the multi-objective LPP into single-objective LPP as follows:

Maximize

$$\left(\frac{Marks\ in\ Phy \times Credit\ of\ Phy + Marks\ in\ Chem \times Credit\ of\ Chem + Marks\ in\ Math \times Credit\ of\ Math}{Credit\ of\ Phy + Credit\ of\ Chem + Credit\ of\ Math}\right)$$

Subject to

Marks secured student 1 in Physics =50

Marks secured student 1 in Chemistry =60

Marks secured student 1 in Mathematics =65

Marks secured student 2 in Chemistry =50

Marks secured student 2 in Mathematics =60

Marks secured student 2 in Physics =65

	Physics (Credit 5)	Chemistry (Credit 3)	Mathematics (Credit 4)
Student 1	50	60	65
Student 2	65	50	60
Average marks of Student 1	$\frac{50\times5+60\times3+65\times4}{5+3+4}$ =57.5		
Average marks of Student 2	$\frac{65\times5+50\times3+60\times4}{5+3+4} = 59.58$		
Who is best?	Student 2		

Example:

Maximize ($2x_1+3x_2$)

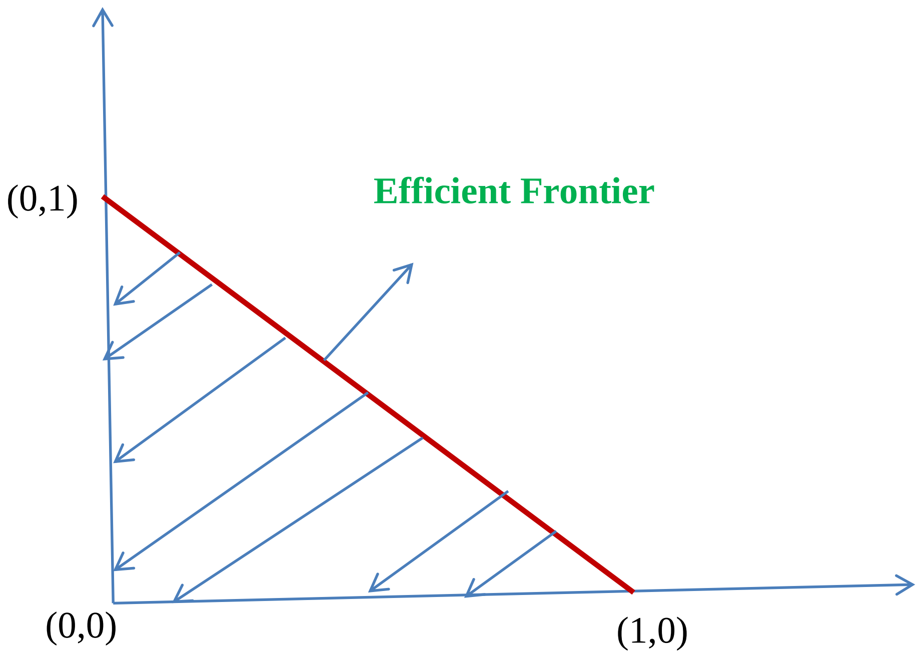
Minimize (x_1-x_2)

Maximize ($3x_1+2x_2$)

Subject to

$x_1+x_2<=1$, $x_1>=0$, $x_2>=0$.

Solution:



	$x_1=0$ and $x_2=0$	$x_1=1$ and $x_2=0$	$x_1=0$ and $x_2=1$
$2x_1+3x_2$	0	2	3 (Maximum)
x_1-x_2	0	1	-1 (Minimum)
$3x_1+2x_2$	0	3 (Maximum)	2

According to the first and second objective, the optimal solution is $x_1=0$ and $x_2=1$.

While, according to the third objective, the optimal solution is $x_1=1$ and $x_2=0$.

Efficient solutions or non-dominated solutions:

(0,1) and (1,0)

as well as

$a(0,1)+(1-a)(1,0)$

where, $0<=a<=1$.

Curve passing through all the efficient solutions is called efficient frontier.

In this example, the line segment joining (1, 0) and (0, 1) is the efficient frontier.