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Roll Number:	Name:
Thapar University, Patiala School of Mathematics	
B.E.(2nd yr, SEM-II)	Course Code: UMA031 Course Name: Optimization Techniques
May 19, 2016	Thursday, 9.00 am – 12.00 noon
Time: 3 Hours, M. Marks: 100	Name of Faculty: MKS, MKR, VKS, NK

Note: Attempt all questions in the given sequence. Assume missing data, if any, suitably.

Q.1(a) Solve the following problem by the simplex method starting with the corner point (0,10)
Min $z = 12x_1 + 10x_2$ subject to $5x_1 + x_2 \geq 10,\ 6x_1 + 5x_2 \geq 30,\ x_1 + 4x_2 \geq 8$
$x_1, x_2 \geq 0$. Also find optimal non basic feasible solution and show it on the graph. **(12)**

Q.1(b) Use the complementary slackness theorem to verify that $(n, 0, \dots, 0)$ is an optimal solution of the linear programming problem
Min $\sum_{j=1}^{n} jx_j$ subject to $\sum_{j=1}^{i} x_j \geq i\ (i = 1,2,\dots,n)$, $x_j \geq 0\ (j = 1,2,\dots,n)$. **(8)**

Q.2(a) State and prove weak duality theorem for linear programming. **(6)**

Q.2(b) Solve the following linear parametric programming problem for the entire range of t
Max $z = (3+3t)x_1 + 2x_2 + (5-6t)x_3$ subject to
$x_1 + 2x_2 + x_3 \leq 40,\ \ 3x_1 + 2x_3 \leq 60,\ x_1 + 4x_2 \leq 30,\ \ x_1, x_2, x_3 \geq 0$. The optimal table for t=0 is given below; **(10)**

B.V.	x_1	x_2	x_3	s_1	s_2	s_3	X_B(Sol.)
Z	4	0	0	1	2	0	160
x_2	-1/4	1	0	½	-1/4	0	5
x_3	3/2	0	1	0	1/2	0	30
s_3	2	0	0	-2	1	1	10

where s_1, s_2, s_3 are slack variables.

Q.2(c) Using branch and bound algorithm, solve the following integer programming problem
Max $z = 3x_1 + 4x_2$ subject to $2x_1 + x_2 \leq 6,\ 2x_1 + 3x_2 \leq 9,\ x_1, x_2 \geq 0$ and are integers. **(10)**

Q.3(a) Consider the following transportation problem (TP) **(10)**

	D_1	D_2	D_3	Availability
S_1	7	5	6	90
S_2	8	2	3	10
S_3	7	6	6	20
Demand	60	50	50	

(i) Formulate the above (TP) as a linear programming problem.

(ii) Find the optimal solution of above (TP) with restriction that it is required that the demand at the destination D_1 must be satisfied exactly. (use Vogel's approximation method for initial basic feasible solution).

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Q.3(b) A company needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of skills of the Workers. The following table summarizes the cost (in Rs.) of the assignments. Determine the optimal assignment using Hungarian method. **(12)**

		Jobs			
		1	2	3	4
Workers	1	50	30	20	80
	2	70	90	20	60
	3	60	40	50	70
	4	50	70	70	80

If an additional (fifth) worker becomes available for performing the four jobs at the respective cost of Rs.40, Rs.50, Rs.80 and Rs.50. Is it economical to replace this additional worker among the current four workers.

Q.3(c) The following table gives the normal duration, normal cost, crash duration and crash cost for different activities of a project. **(16)**

Activity	Normal duration(days)	Normal cost (In Rs.)	Crash duration(days)	Crash Cost (In Rs.)
1-2	13	1000	13	1500
2-3	10	700	4	2500
2-4	12	1500	8	3500
3-5	5	400	1	1000
4-6	12	1200	12	1600
5-6	8	1000	6	1400

(i) Determine the critical path, normal duration and normal cost of the project.

(ii) Determine the optimal cost for completing the project in 33 days.

Q.4(a) Find the all stationary points of the function **(6)**

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

and decide the nature of these points.

Q.4(b) Solve the following non linear programming problem using KKT conditions: **(10)**

Opt. $z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$ subject to $x_1 + x_2 \le 1$, $2x_1 + 3x_2 \le 6$, $x_1, x_2 \ge 0$.

Note:. Evaluated answer sheet may be seen on 24-05-16 as per given schedule in Room no. T106.

Branch	Time	Branch	Time
CIE 1-3	10:30 am	MEE 5-7	12:30 pm
CIE 4-6	11:00 am	EIC 1-3	12:45 pm
MPE 1-2	11:30 am	EIC 4-6	1:00pm
MEE 1-4	12:00noon		