TOTAL PRESSURE ON SUBMERGED SURFACES

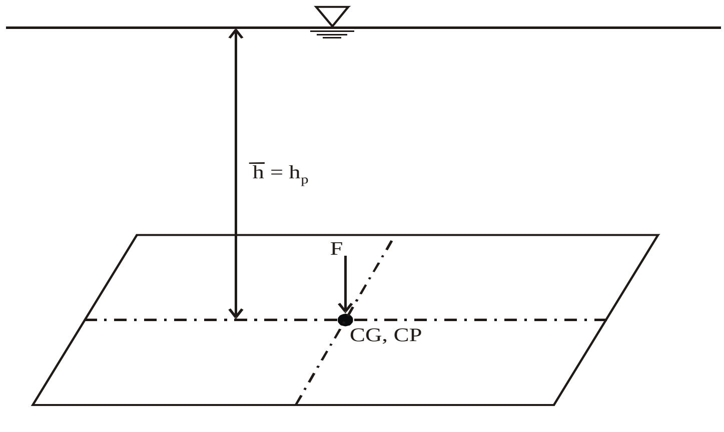
- A static mass of fluid exerts a force on the surface.
- This force is known as total pressure or thrust or hydrostatic force (if liquid is water).
- Total pressure always acts normal to the surface.
- The point where it acts on the surface is known as center of pressure (**CP**).
- ❖ Determination of total pressure and its location is required in the design of storage tanks, ships, dams etc.

Following cases of submerged surfaces will be discussed:

- ➢ Horizontal plane surface
- ➢ Vertical plane surface
- ➢ Inclined plane surface
- ➢ Curved surface

Horizontal Surface

- Consider a surface is submerged and held in a horizontal position at a depth $\bar{h}$ (also represents distance of **CG** of surface) below the free surface.



Observation: In all the cases, it is assumed that liquid is only on one side of the surface.

Example: Bottom of a storage tank is a case of horizontal surface.

- Every point on the surface is at the same depth below the free surface.

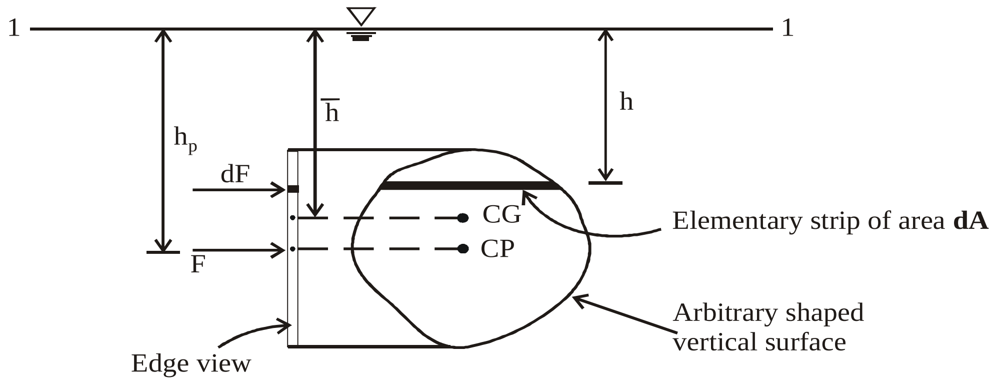
$\therefore$ Pressure intensity, **p** is constant over the whole surface and is equal to $\gamma\bar{h}$

$\therefore$ Total pressure, **F** = **pA** $\Rightarrow \mathbf{F = \gamma\bar{h}A}$

- As intensity of pressure is uniform over the surface and therefore, centroid of surface *i.e.*CG and centre of pressure *i.e.* **CP** will coincide each other.
- ❖ **F** is acting normal to the surface in the vertical downward direction at the centre of pressure.

Vertical Plane Surface

- Consider a plane surface of arbitrary shape is immersed vertically in a static mass of liquid.



- Depth of liquid varies from point to pt. on the surface.
- $\therefore$ Intensity of pressure is not constant over the surface.

✓ Analysis is carried out by dividing the surface into a number of small parallel strips.

- Pressure force on a small strip is first calculated.
- Total pressure force on the surface is obtained by integrating the pressure force on the small strip.
- Consider elementary strip of area ***dA*** at depth, ***h*** from **1-1** (free surface) and parallel to it.

❖ Pressure intensity on the elementary strip, $\mathbf{p = \gamma h}$

$\therefore$ Pressure force on the strip, $\mathbf{dF = pdA = \gamma\, h\, dA}$

$\therefore$ Total pressure on the whole surface, $\mathbf{F} = \int \gamma hdA = \gamma \int hdA$

❖ $\int hdA$ represents sum of the first moments of areas of strips about 1-1, which can also be written as $A\bar{h}$

$$\therefore F = \gamma A\bar{h}$$

Location of center of pressure (CP)

- **CP** does not coincide **CG**

❖ **CP** lies below **CG** (intensity of pressure increases with increase in the depth).

- Location is determined by applying principle of moments.

❖ Moment of the resultant force about an axis is equal to the sum of the moments of the component forces about the same axis.

- Moment of pressure force on the elementary strip about **1-1** = $\mathbf{dF \times h}$

$\therefore$ Total moment about **1-1** = $\int (dF \times h) = \int \gamma hdA \times h = \gamma \int (hdA)h$

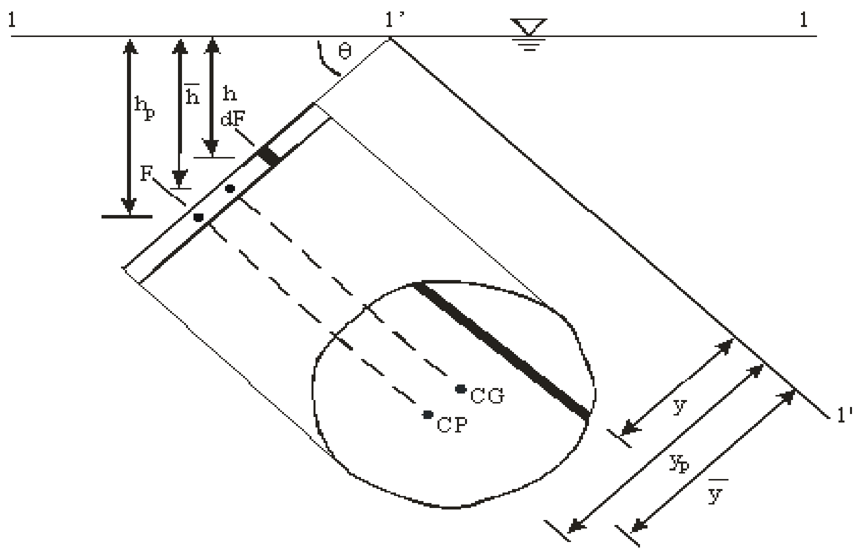
❖ **(hdA)h** represents second moment of elementary strip about **1-1.**

❖ $\therefore \int$ **(hdA)h** represents the sum of second moments of areas of all such elementary strips about 1-1 = **MI** of the surface about free surface 1-1, say $\mathbf{I_1}$

$\therefore$ Total moment of pressure forces about 1-1 = $\gamma \mathbf{I_1}$

- Also, moment of ***F*** about 1-1 = $\mathbf{F \times h_p = \gamma A\bar{h} \times h_p}$
- Applying the principle of moments, to get $\mathbf{h_p = \dfrac{I_1}{A\bar{h}}}$
- Using the theorem of parallel axis, $\mathbf{I_1 = I_G + A(\bar{h})^2}$
- $\mathbf{I_G}$ is the moment of inertia of surface about an axis passing through the centroid and parallel to free surface **1-1**, which is the intersection of the plane of vertical surface (produced) and the free surface.
- Simplify, to get $\mathbf{h_p = \bar{h} + \dfrac{I_G}{A\bar{h}}}$

Inclined Plane Surface



- Intersection of plane of inclined surface (produced) and the free surface is denoted by **1′-1′**

θ is the angle made by the edge view of surface with **1-1**

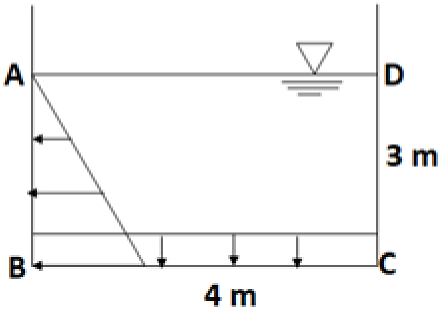
$$\mathbf{F = \gamma A\bar{h}}$$

h is the vertical distance of centre of gravity of inclined surface from free surface 1-1

$$\mathbf{h_p = \bar{h} + \frac{I_G}{A\bar{h}} \sin^2 \theta}$$

❖ I_G is the moment of inertia of the surface about an axis passing through the centroid and parallel to axis **1′-1′**.

Problem 1: A square tank of side 4 m contain water up to depth of 3 m. Determine the hydrostatic force and its location at the bottom and a side of the tank.



Solution:

$$F_{BC} = \gamma A\bar{h} = 9.810 \times 4 \times 4 \times 3 = 470.88 \text{ kN},$$

➢ acting at 3 m from free water surface.

$$F_{AB} = \gamma A\bar{h} = 9.810 \times 3 \times 4 \times (3/2) = 176.58 \text{ kN},$$

➢ acting at

$$h_p = \bar{h} + \frac{I_G}{A\bar{h}} = 1.5 + \frac{(4 \times 3^3)/12}{3 \times 4 \times 1.5} = 2 \text{ m},$$ from free water surface.

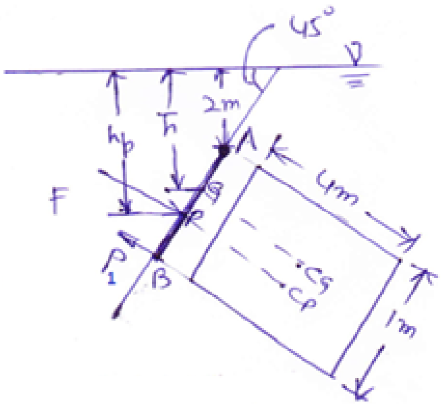
Alternatively

- F_{BC} = Weight of water above surface **BC** = Specific weight of water × Volume of water above **BC**
  = 9.810 × 4 × 4 × 3 = 470.88 kN
- F_{AB} = Area of pressure diagram × Width of tank

$$= \frac{1}{2} \times \gamma h \times h \times b = \frac{1}{2}\gamma h^2 \times b = \frac{1}{2} \times 9.810 \times 3^2 \times 4 = 176.58 \text{ kN},$$

➢ acting at **(2/3)h** i.e. **2 m** from free water surface.

Problem 2: An inclined rectangular sluice gate **AB**, 4 m wide and 1 m deep is installed to control the flow of water. The upper end **A** of the gate is hinged and lies at a vertical distance of 2 m from the free surface of water. The inclination of gate with horizontal is 45°. Find the total pressure on the gate and its location. Also, determine the normal force to be applied at lower end **B** to open it.



Solution:

$$F_{AB} = \gamma A\bar{h};$$

$$\bar{h} = (2 + 0.5 \sin 45) = 2.353 \text{ m}, \quad A = 4 \times 1 = 4 \text{ m}^2$$

$$\therefore F_{AB} = 9.810 \times 4 \times 2.353 = 92.35 \text{ kN},$$ acting at

$$h_p = \bar{h} + \frac{I_G}{A\bar{h}} \sin^2\theta, \qquad I_G = \frac{4 \times 1^3}{12}, \theta = 45°$$

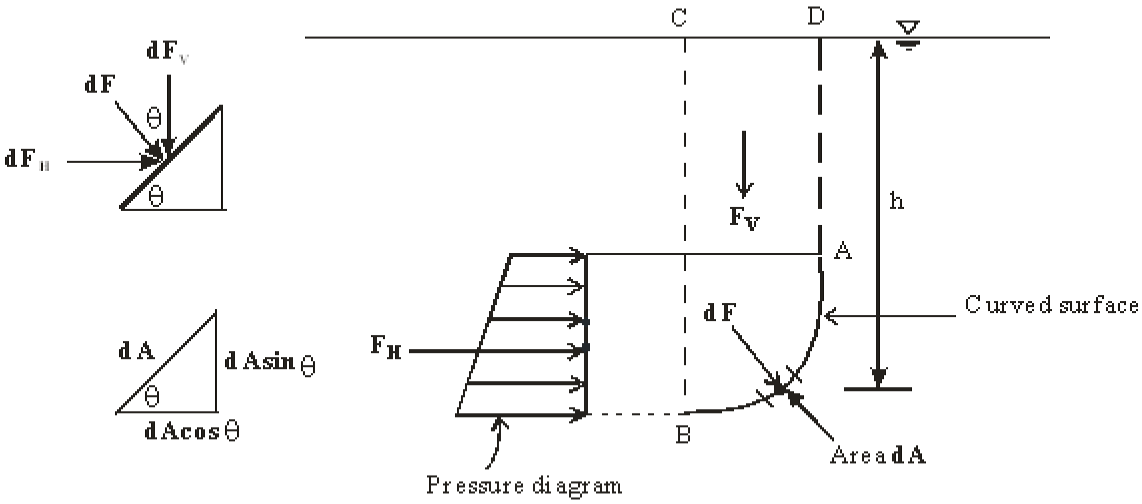
$$\therefore h_p = 2.37 \text{ m}$$

- Taking moments about **A**, to write
- **P_1× AB = F × AP;** $\qquad AP = \frac{(h_p - 2)}{\sin 45} = 0.523 \text{ m}, AB = 1 \text{m}$

$$\therefore P_1 \times 1 = 92.35 \times 0.523 \qquad \Rightarrow P_1 = 48.4 \text{ kN}$$

Curved Surface

- A curved surface ***AB*** is submerged in a static mass of fluid of specific weight γ



- At any point on the curved surface, pressure force acts normal to the surface
- ***dA*** is elementary area of curved surface lying at a depth ***h*** below the surface.

$\therefore$ Pressure force acting on **dA**, **dF = γhdA**

❖ Direction of pressure force on the elementary areas varies from point to point Also,. Thus, it is difficult to describe area **dA** mathematically.

$\therefore$ Total pressure on the curved surfaces is determined by resolving the total pressure into horizontal and vertical components.

- Horizontal component of **dF,** $dF_H = dF\sin\theta = \gamma h dA \sin\theta$
- Vertical component of **dF,** $dF_V = dF\cos\theta = \gamma h dA \cos\theta$

➢ **θ** is the angle made by ***dA*** with horizontal

$\therefore$ Total horizontal pressure, $F_H = \int dF_H = \gamma \int h dA \sin\theta$

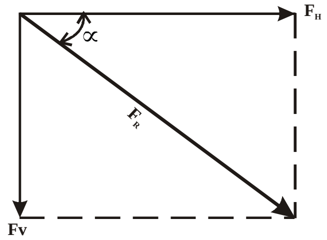
- Total vertical pressure, $F_V = \int dF_V = \gamma \int h dA \cos$
- Terms **dAsinθ** and **dAcosθ** represent the vertical and horizontal projections of **dA**, respectively.
- $\left(\gamma \int h dA \sin\theta\right)$ represents the total horizontal pressure on the projected area of curved surface on a vertical plane and is <u>acting at the center of pressure of projected area.</u>

$\therefore$ **F_H** = Projection of curved surface on a vertical plane and is acting at the **CP** of projected area.

- $\left(\gamma\int hdA\cos\theta\right)$ represents the total vertical pressure on projected area of curved surface on a horizontal plane= Total weight of liquid supported by the curved surface up to the free surface of liquid *i.e.* Weight of liquid in **ABCDA** and is acting at the centroid of area **ABCDA.**

$\therefore$ **F_V =** Weight of liquid supported by the curved surface up to the free surface of liquid and is acting at the centroid of liquid above the curved surface.

- Resultant pressure, $F_R = \sqrt{F_H^2 + F_V^2}$
- Acting at $\tan\alpha = (F_V / F_H)$ with the horizontal



Observation: In the above case, liquid is above the curved surface. In some cases, liquid may be below the curved surface i.e. underside of the curved surface is subjected to pressure force. In these cases,

F_V = Weight of **imaginary** liquid above the curved surface upto the free surface and is acting in the upward direction.

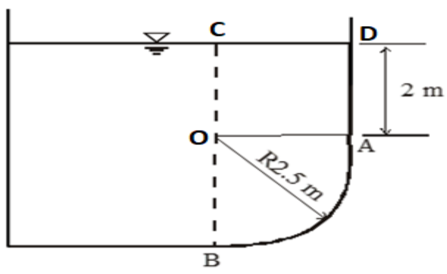
F_H remains the same.

Problem: The bottom corner of a tank containing water is a quadrant of radius 2.5 m as shown in **Figure.** The width of tank is 3 m. Determine on the curved surface **AB**,

(i) Horizontal hydrostatic force and its location

(ii) Vertical hydrostatic force and its location

(iii) Resultant hydrostatic force and its direction.



Solution:

- Horizontal hydrostatic force on the curved surface AB

= Projection of the curved surface on a vertical plane and is given by

$F_H = \gamma A\bar{h} \Rightarrow F_H = 9.810 \times 2.5 \times 3 \times \left(2 + \frac{2.5}{2}\right) = 239$ kN, acting at

$h_p = \bar{h} + \frac{I_G}{A\bar{h}}$; from free water surface. $I_G = \frac{3 \times 2.5^3}{12}$

$\therefore h_p = 3.41$ **m free water surafce**

- Vertical component of the pressure force on the curved surface AB

= Weight of water supported by the curved surface up to the free surface and is given by

- $F_V = \gamma$ (Volume of water in the rectangular portion **OADC** + Volume of water in quadrant **OAB**)

$$\therefore F_v = 9.810\left(2.5 \times 2 \times 3 + \frac{\pi}{4}(2.5)^2 \times 3\right) = 147.15 + 144.50 = 291.65 \text{ kN},$$

- acting at at the centroid of area **ABOCDA.**

- The line of action of F_V may be obtained by taking moments of its two components about line **BC** *i.e.*

$$291.65 \times x = 147.15 \times \frac{2.5}{2} + 144.50 \times \frac{4r}{3\pi}$$

- x is the distance of line of action of F_V from vertical line **BC** and the term $\frac{4r}{3\pi}$ represents distance of centroid of quadrant from straight line **OB**.

- Solving, to get
- **x = 1.156 m**

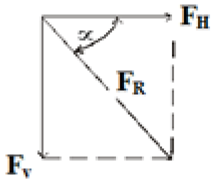
$\therefore F_V$ acts at a distance of 1.156 m from **BC.**

- Resultant hydrostatic force is given by,

$$F_R = \sqrt{F_H^2 + F_V^2} = 377.15 \text{ kN}$$

- acting at an angle α with the horizontal as shown in **Figure**.

$$\tan\alpha = \frac{F_V}{F_H} = 1.22 \Rightarrow \alpha = 50.66°$$



BUOYANCY AND FLOATATION

- When a body is immersed in a static fluid either fully or partially, it is subjected to an upward force which tends to lift or buoy it up.
- This tendency of an immersed body to be lifted up due to an upward force against the gravity force is known as buoyancy and the force is known as buoyant force or upthrust.
- The buoyant force acts through the centroid of volume of fluid displaced and its point of application is known as center of buoyancy.
- The magnitude of buoyant force can be determined by using Archimedes principle and is given by:
- Buoyant force, F_B = Weight of fluid displaced by the body

  = Specific weight of fluid × Volume of fluid displaced by the body

Observations:

(i) For fully immersed/submerged bodies, volume of fluid displaced is equal to the volume of body. Also, in this case, centre of buoyancy, **B** and centre of gravity, **G** of the body coincide each other

(ii) For floating/partially submerged bodies, top portion of the body is in contact with air whereas its lower portion is submerged in the fluid.

- ❖ Since, density of air (1.24 kg/m^3) is very small as compared to density of water (1000 kg/m^3) and therefore weight of air displaced by top portion of the body may be neglected.
- Thus, buoyant force acting on a floating body is also equal to the weight of liquid displaced by the body and is acting at the centroid of the fluid displaced by the body *i.e.* in this case, **G** and **B** do not coincide each other.

(iii) For an immersed body (fully or partially) to be in equilibrium, buoyant force acting on the body must be equal to the weight of body *i.e.* $\mathbf{F_B = W}$.

- Further, lines of action of **G** and **B** must lie along the same vertical line, so that their moment about any axis is zero.

(iv) If $\mathbf{W > F_B}$ (the body moves downwards) and if $\mathbf{W < F_B}$ (the body is lifted upward and rises until $\mathbf{W = F_B}$).

STABILITY OF SUBMERGED AND FLOATING BODIES

- Stability here means the tendency of a body to return to its original position, after it has been displaced slightly from its original position by an external force.

- When a submerged or a floating body is given a small angular displacement, the following three conditions of equilibrium may be developed:

(i) Stable equilibrium (ii) Unstable equilibrium and (iii) Neutral equilibrium.

(i) **Stable Equilibrium**

- In this case, the small angular displacement (from the equilibrium position) sets up a couple which oppose the angular displacement of the body, thereby bringing the body back to its original position.
- In other words, in this case, the overturning couple and restoring couple act in the opposite directions.

(ii) Unstable Equilibrium

- The small displacement produces a couple, which further increases the angular displacement of the body, thereby not allowing the body to return to its original position.
- In other words, in this case, the overturning couple and restoring couple act in the same direction.

(iii) Neutral Equilibrium

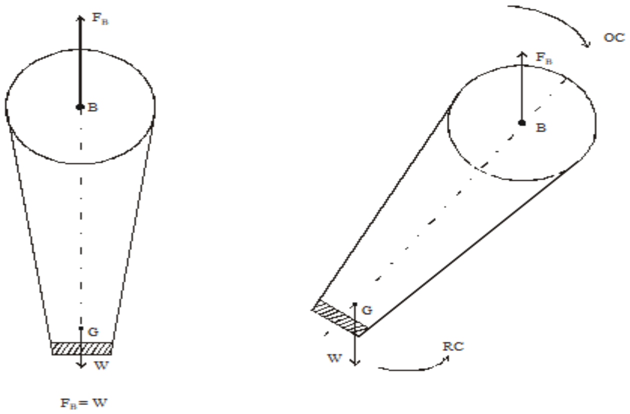
- The small displacement does not set up any couple and therefore the body adopts the new position without either returning to its original position or increasing the angular displacement.

Stability of submerged bodies:

- Stability of submerged bodies is governed by the relative position of **B** with respect to **G** (For each of the three conditions of equilibrium, the location of **B** with respect to **G** is different)

Stable Equilibrium

- Consider an aerostatic balloon and gondola system floating in air (this is a case of a submerged body).
- The buoyant force $\mathbf{F_B}$ is located at the center of balloon and acts vertically upward through **B**.
- The weight **W** carried in the gondola is acing vertically downward through **G.**

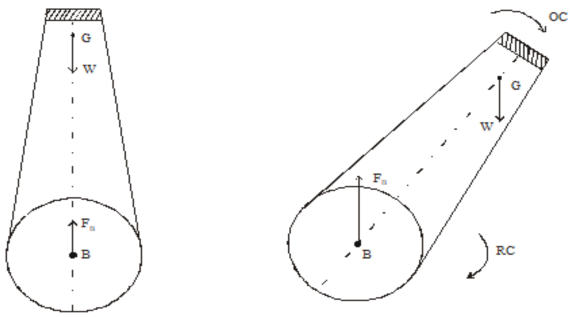


Stable equilibrium of an aerostatic balloon

- Let the system be given a small angular displacement in the clockwise direction by some external force.
- This displacement sets up a couple in the anticlockwise direction which brings the system back to its original position.
- This condition constitutes the stable equilibrium of a submerged body *i.e.* for stable equilibrium; **B** should be located above **G.**

<u>Unstable Equilibrium</u>**:**

- Consider the same system as above but in the inverted position.
- In this case, the overturning couple and the restoring couple act in the same direction *i.e*. clockwise direction, thereby not allowing the body to return to its original position.

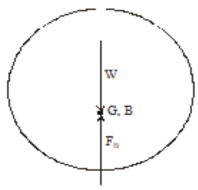


Unstable equilibrium of an aerostatic balloon

- Thus, for unstable equilibrium, **B** is located below **G.**

<u>Neutral Equilibrium:</u>

- Consider a normal balloon submerged in air.



Neutral equilibrium of a balloon

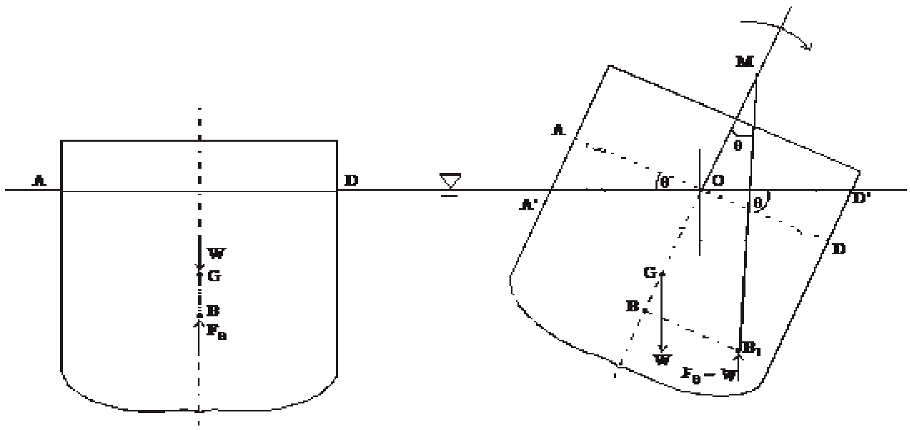
- In this case, both **B** and **G** will coincide each other. It is unaffected by the angular displacement.

Thus, in case of submerged bodies

- For stable equilibrium: **B** is above **G**
- For unstable equilibrium: **B** is below **G**
- For neutral equilibrium: **B** and **G** coincide

Stability of floating bodies:

- The stability conditions of floating bodies are different from submerged bodies.
- For the stability of floating bodies, one needs to first define metacentre and the metacentric height as explained below.
- **Figure** shows a cross-section of a floating body (a boat or a ship) in equilibrium:

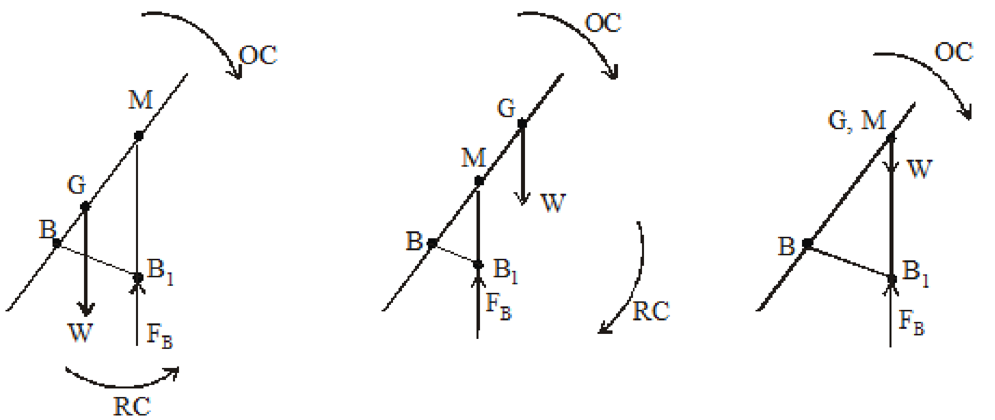


Metacentre and metacentric height

- Weight of body **W** acts vertically downward through **G** and the buoyant force $\mathbf{F_B}$ acts upward through **B**. Both these forces act along the same vertical line and **B** is located below **G.**
- Let the body is slightly tilted so that it undergoes a small angular displacement **θ** in the clockwise direction (*in the actual case, the body may get tilted due to wind and wave action*).
- Due to this displacement, a triangular wedge **AOA'** of body comes out of liquid on the left of axis and a similar triangular wedge **DOD'** on the right of axis goes inside the liquid.

- As a result, the centre of buoyancy shifts from **B to B₁**, to the right and normal to the initial vertical axis of **B** as shown in **Fig.**
- ❖ The shifting of centre of buoyancy to the right is due to the fact that the immersed portion of body on the right hand side increases while that on left hand side decreases (*i.e.* there is unequal submergence of body with respect to the axis).
- ❖ For a fixed position of loading on the body, the relative position of **G** remains unchanged.
- If a vertical line is drawn through the new position of buoyant force *i.e.* through point **B_1**, it will intersect the initial line of action of buoyant force drawn through **B**, at point **M.**
- The point **M** is known as <u>metacentre</u> and distance between **M** and **G** is called the <u>metacentric height</u>.
- ❖ The metacentric height is a measure of static stability of floating bodies (greater is the metacentric height; greater is the stability of body).

- ✓ Stability of floating bodies is governed by the relative position of **G** and **M.**
- If **M** is located above **G**, the body is in stable equilibrium.
- If **M** is located below **G**, the body is in unstable equilibrium.
- If **M** and **G** coincide, then the body is in a state of neutral equilibrium.
- The three conditions of equilibrium of are illustrated in **Figures**.



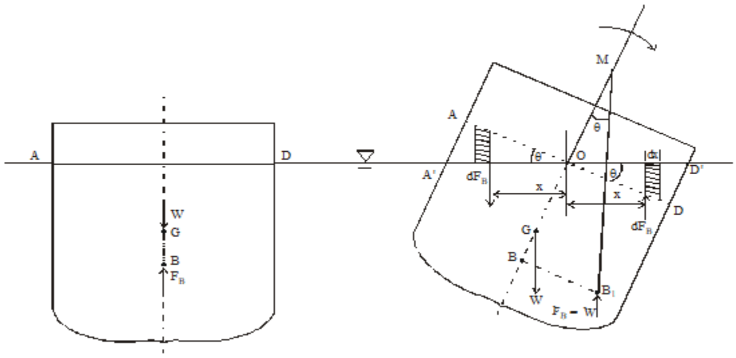
Stable equilibrium (M is above G) | Unstable equilibrium (M is below G) | Neutral equilibrium (M coincides with G)

Conditions of equilibrium of floating bodies

Determination of Metacentric Height

- Metacentric height can be determined by two methods:
  - Analytical method
  - Experimental method

(i) **Analytical method**: Consider the following figure:



- Due unsymmetrical submergence of body, the centre of buoyancy shifts from **B** to **B_1**.
- Triangular wedge **AOA'** comes out of liquid on the left of axis and a similar triangular wedge **DOD′** on the right of axis goes inside the liquid.
- The triangular wedge on the right of initial axis represents a gain in buoyant force (↑) whereas a similar triangular wedge on the left of initial axis represents an equal loss in buoyant force (↓). (This is due to the fact that volume of liquid displaced by the body remains same).

✓ Consider two small strips of wedges of thickness **dx** at a distance **x** from **O** as shown in **Figure**.

∴ Buoyant force acting on each strip = Specific weight × Volume of each strip

- Volume of each strip = $x\,\theta \times dx \times L$
  - **θ** is the angle of tilt, also known as angle of heel and **L** is the length of body measured normal to the plane of screen/paper.

∴ Buoyant force acting on each strip, $dF_B = \gamma \times x\theta \times dx \times L$

- Buoyant force on the strip of triangular wedge **AOA'** is acting in the downward direction whereas buoyant force on strip of triangular wedge **DOD′** is acting in the upward direction (loss and gain of buoyant forces) and hence these two forces form a couple.

∴ Moment of couple = $dF_B \times 2x = 2\gamma\theta x^2 L dx$

Total moment of couple on the two triangular wedges

$$= 2\gamma\theta\int x^2 L dx = \gamma\theta\left[2\int (Ldx)\times(x)\times x\right]$$

- The term $\left[2\int (Ldx)\times(x)\times x\right]$ represents the second moment of plan area of body at the water surface about the longitudinal axis (axis of tilt) *i.e.* moment of inertia **I.**
  - ❖ The body is most likely to tilt about the axis with least moment of inertia.

∴ Total moment of couple due to buoyant forces on two triangular wedges = $\gamma\,\theta\,I$ **(1)**

- Also, moment of buoyant force due to shifting of **B** to **B1** = $F_B(\overline{BB_1})$

$$= F_B(\overline{BM}\tan\theta) \qquad (2) \qquad (\overline{BB_1} = \overline{BM}\tan\theta)$$

- Equating Equations (1) and (2) , to get

$$F_B(\overline{BM}) = \gamma\, I \qquad \text{(For small angle of heel, } \tan\theta \approx \theta)$$

- As, buoyant force = Specific weight × Volume of liquid displaced by the body

$$\therefore F_B = \gamma\forall$$

$$\therefore \overline{BM} = \frac{I}{\forall}$$

∴Metacentric height, $\overline{GM} = (\overline{BM} - \overline{BG})$

$$\therefore \overline{GM} = \left(\frac{I}{\forall} - \overline{BG}\right)$$

- This is the required expression for metacentric height.
- In this equation,
  - ➢ **I** = Moment of inertia of plan of the body at the free water surface about the longitudinal axis or the axis of tilt (body is most likely to tilt about the axis with least moment of inertia. **I = Min (I_{xx}, I_{yy}).**
  - ➢ ∀ = Volume of liquid displaced by the body.
  - ➢ **BG** = Distance between the CG and the centre of buoyancy.
- If metacentric height is +ve, the body is in stable equilibrium, if GM is -ve, unstable equilibrium and if GM = 0, neutral equilibrium.

Observation:

- The total moment of couple on the two triangular wedges can also be determined as follows:
- Weight of each triangular wedge *i.e.* buoyant force acting on each triangular wedge $= \gamma\frac{1}{2}\left(\frac{b}{2}\frac{b}{2}\tan\theta\right)L$

- acting at $\frac{2}{3}\left(\frac{b}{2}\right) = \frac{b}{3}$ from O (*i.e.* centre).

- Here **L** is the length of body normal to the plane of paper and **b** is the width of body (**= AD**).

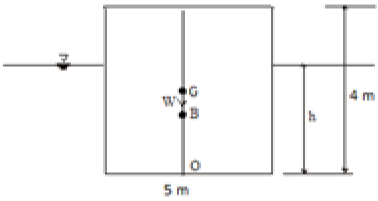
∴ Moment of couple on the two triangular wedges = $\left(\gamma \frac{b^2}{8} \tan\theta\, L\right)\left(\frac{b}{3} + \frac{b}{3}\right)$

$$= \gamma\left(\frac{Lb^3}{12}\right)\tan\theta = \gamma I \tan\theta = \gamma I \theta$$

- This expression is same as Eq. (1).

Problem: A rectangular block (**SG** = 0.6) of dimensions 10 m × 5 m × 4m is floating in water (**SG** =1.02). Find its metacentric height.

Solution:



- According to Archimedes' principle
- Weight of the body = Weight of water displaced by the body

$$\therefore 0.6 \times 9.810 \times 10 \times 5 \times 4 = 1.02 \times 9.810 \times \forall$$

$\therefore$ Volume of water displaced, $\forall$ = 117.65 m^3

$\therefore 10 \times 5 \times \mathbf{h} = 117.65$, **h** is the depth of immersion

$\therefore$ h = 2.353 m $\quad\quad \therefore \overline{OB} = h/2 = 1.1765\text{m}$

- Also, $\overline{OG} = H/2 = 4/2 = 2\text{ m}$

$$\therefore \overline{BG} = \overline{OG} - \overline{OB} = (2 - 1.1765) = 0.8235\text{ m}$$

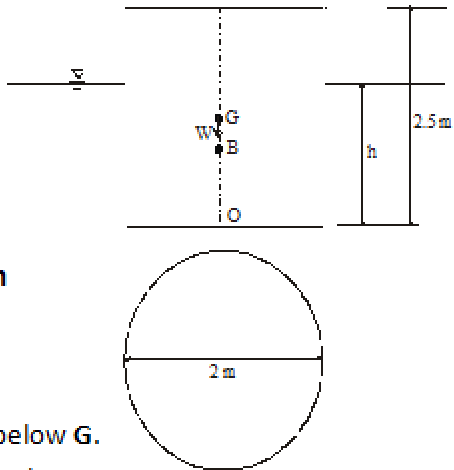
$$I_{xx} = \frac{10 \times 5^3}{12} = 104.2\,m^4, \quad I_{yy} = \frac{5 \times 10^3}{12} = 416.67\,m^4 \qquad \therefore I = \text{Min}\,(I_{xx}, I_{yy}) = 104.2\,m^4$$

$$\therefore \overline{BM} = \frac{I}{\forall} = \frac{104.2}{117.65} = 0.886\text{ m}, \qquad \overline{GM} = \left(\frac{I}{\forall} - \overline{BG}\right)$$

$$\therefore \overline{GM} = \overline{BM} - \overline{BG} = (0.886 - 0.8235) = 0.062\text{ m}$$

Problem 2: A cylindrical buoy 2 m diameter, 2.5 m high and weighing 21.5 kN, is floating in sea water (specific weight = 10.02 kN/m^3). Show that the buoy cannot float with its axis vertical.

Solution: Weight of the body = Weight of water displaced by the body



$$\Rightarrow 21.5 = 10.02 \times \forall \qquad \therefore \forall = 2.146\,m^3$$

$$\therefore \pi r^2\, x h = 2.146 \Rightarrow h = 0.684\text{ m} \qquad \therefore \overline{OB} = h/2 = 0.342\text{ m}$$

$$\overline{BM} = \frac{I}{\forall} = 0.366\text{m} \qquad \left(I = \frac{\pi}{64}(2)^4 = 0.786 m^4\right)$$

$$\overline{OG} = 1.25\text{ m} \qquad \therefore \overline{BG} = \overline{OG} - \overline{OB} = 0.908\text{ m}$$

$$\therefore \overline{GM} = (\overline{BM} - \overline{BG}) = (0.365 - 0.908) = -0.543\text{ m}$$

- As metacentric height is negative and therefore **M** is below **G**.

➢ The buoy is in unstable equilibrium and cannot float with its axis vertical.

(ii) Experimental method of finding the metacentric height

- Before making prototypes, models of boats and ships are often made and tested experimentally, for determining the metacentric height.
- This is due to the fact that it is difficult to find analytically the metacentric height of these bodies due to their irregular shapes.
- Experimental set up of determining the metacentric height consists of a ship model which is allowed to float in a tank.
- For tilting the model, a cross bar is fixed on the model. On this cross bar two movable weights are suspended. These weights can be placed at unequal distances with respect to the center of the cross bar so as to tilt the model slightly.
- By means of a pendulum, the angle of tilt is measured on a graduated arc. Pendulum and graduated arc are suitably fixed at the center of the crossbar.
- The following steps are involved in the experiment:

i. Fill the tank to about $2/3^{rd}$ full of water and determine its level.

ii. Put the ship model in the tank with the moveable weights placed across the bar and again determine the level of water in the tank.

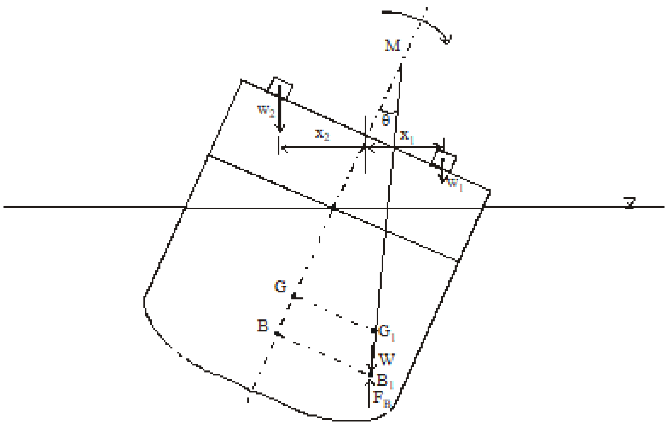
iii. Calculate the weight of ship model using Archimedes principle.

iv. Move the weights across the deck of body so as to tilt the model through a small angle.

v. Note the angle of tilt after the model comes to rest in a new position of equilibrium.

vi. In the new position of equilibrium, the body is under the action of a clockwise moment caused by the movement of weights and an anticlockwise moment due to shifting of **B** to **B_1**.

vii. Also, the movement of weights causes a parallel shift of centre of gravity of the model from **G** to **G_1**.



Experimental method of finding metacentric height

$\therefore$ Net moment due to movement of weights across the bar = $(w_1x_1 - w_2x_2)$

- Also, moment due to shifting of **G** to **G_1 =** $W(\overline{GG_1})$
- Equating, to get

$$W(\overline{GG_1}) = (w_1x_1 - w_2x_2)$$

$$\therefore W(\overline{GM}\tan\theta) = (w_1x_1 - w_2x_2) \qquad (\overline{GG_1} = \overline{GM}\tan\theta)$$

$$\therefore \overline{GM} = \frac{(w_1x_1 - w_2x_2)}{W\tan\theta}$$

- This is the required expression for finding the metacentric height experimentally. In this equation,
  - w_1 and w_2 are the moveable weights
  - x_1 and x_2 are the distances of w_1 and w_2 from the centre, respectively.
  - **W** is the total weight of the ship model including the moveable weights w_1 & w_2 .
  - θ is the angle of tilt.

Observation:

- Metacentric height is a measure of the static stability of floating bodies. Greater is the metacentric height; greater is the stability of a floating body.
- Metacentric height can be increased by placing weights in the body.
- Too much of metacentric height is also not desirable as it gives rise oscillations to the body.

TIME PERIOD OF OSCILLATION OF A FLOATING BODY

- A floating body may be set in a state of oscillations just like a simple pendulum, as if suspended at the metacentre.
- This may happen when overturning couple acting on the body is suddenly removed.
- The time period, **T** of oscillation is given by the expression:

$$T = 2\pi\sqrt{\frac{K^2}{g(\overline{GM})}}$$

  - **K** = Radius of gyration of body and can be calculated using **$I = MK^2$**.
- Here **M** is the mass of body and **I** is the moment of inertia of plan of the body at the free water surface about longitudinal axis.
- The equation shows that greater is the metacentric height, smaller is the time period of oscillation and vice versa.

- ✓ Further, a ship may have two types of oscillatory motions viz. *rolling motion and pitching motion*.
- The oscillatory motion about longitudinal axis is known as rolling motion whereas oscillatory motion about transverse axis is known as pitching motion.
- Since moment of inertia of plan of ship at water surface about transverse axis is much more than that about the longitudinal axis.
- Thus, if a ship has safe metacentric height in rolling motion then it will also be safe in pitching motion.
- The same is however not true in case of time periods of rolling and pitching.
- Eq. (1) shows that greater is the metacentric height, smaller is the time period of rolling of body.
- A smaller time period of rolling is quite uncomfortable for the passenger ships as it gives rise to more oscillations.
- Also, the ship will be subjected to undue strains which may damage its structure.
- Thus, these two requirements are contrary to each other.
- In actual practice, an optimum value of metacentric height is selected as shown below:

Type of ship	Metacentric height (m)
Cargo ships	0.3 to 1
Passenger ships	0.45 to 1.25
War ships	1 to 1.5
River crafts	Up to 3.5

- In case of passenger ships, comfort is more important than stability and hence such ships have small metacentric heights.
- In case of cargo ships, metacentric height varies with load and shifting of cargo may cause the ships to roll.
- ➢ Thus, in addition to the stability of a cargo ship, its period of roll is also required to be determined and hence cargo ships have small metacentric heights.
- In case of war ships and river crafts, stability is more important than comfort and hence such vessels have large metacentric heights.