Roll No. : ..101715113........

School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Term Examination, September 2018

B.E. III Semester | UMA007 : Numerical Analysis

Time Limit: 02 Hours | Maximum Marks: 25

Instructor(s): Kavita Goyal, Mamta Gulati, Meenu Rani, Nishu Jain, Munish Kansal, Paramjeet Singh, Parimita Roy, Sapna Sharma,, Vivek Sangwan

Instructions: You are expected to answer all the questions. Organize your work, in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode is permitted.

1. (a) Use four-digit chopping arithmetic and the formula for the roots of a quadratic equation, to find the most accurate approximations to the roots of the following quadratic equation:

$$1.002x^2 - 11.01x + 0.01265 = 0.$$

Also compute the absolute relative errors. [3 marks]

(b) Let floating point representation of a real number x is $x = (0.a_1a_2\cdots a_na_{n+1}\cdots) \times 10^e$, $a_1 \neq 0$. Let $fl(x)$ be its machine approximation with n digits by chopping, then obtain a bound for absolute relative error. [3 marks]

2. (a) Using the bisection method, determine the point of intersection of the curves given by $y = 3x$ and $y = e^x$ in the interval $[0, 1]$ with accuracy 0.1. [3 marks]

(b) Establish Newton's iterative scheme, not involving the reciprocal of x, to find $\frac{1}{x}$ and hence compute $\frac{1}{3}$ correct to 4 decimal places with $x_0 = 1$. [3 marks]

3. (a) We require to solve the following system of linear equations using LU decomposition:

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ 3x_1 + 3x_2 + 9x_3 &= 8 \\ 3x_1 + 3x_2 + 5x_3 &= 10. \end{aligned}$$

Find the matrices L and U using Gauss elimination and using these matrices, solve the system of equations. [4 marks]

(b) Use the Gauss-Seidel method to solve the linear system of equations

$$\begin{aligned} 10x_1 - x_2 &= 9 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6 \end{aligned}$$

by taking initial approximation $\mathbf{x}^{(0)} = \mathbf{0}$ with $\|x^{(k+1)} - x^{(k)}\|_\infty < 0.1$. [3 marks]

4. Let g and g' are continuous functions on $[a, b]$ and assume that g satisfy $a \leq g(x) \leq b$, $\forall x \in [a, b]$. Furthermore, assume that there is a positive constant $\lambda < 1$ with $|g'(x)| \leq \lambda$, $\forall x \in (a, b)$. Then prove that $x = g(x)$ has a unique solution α in the interval $[a, b]$ and the iterates $x_{n+1} = g(x_n)$, $n \geq 1$ converges linearly to the unique fixed point α in $[a, b]$. Also obtain the following error bound

$$|\alpha - x_n| \leq \frac{\lambda^n}{1-\lambda}|x_1 - x_0|.$$

[6 marks]

$|p_n - p|$

$|x_{(n+1)} - \alpha|$

$\frac{5/42}{3/7}$ $\frac{5}{7/2} \times \frac{2}{3} = \frac{5}{16}$ $\frac{3}{2} - \frac{3}{2}$

$3 + \frac{3}{2}$

$9 - 3/2$

$8 - 13$

$-3/2$