Course: UMA 035 (Optimization Techniques)

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Practice Sheet 1

Q2.

	T_1	T_2	T_3	Availability
B_1	8	6	5	150
B_2	6	6	6	150
B_3	10	8	4	150
B_4	8	6	4	150
Demand	200	200	200	

Solution:

The problem is of maximization (Maximize sales).

Transform into minimization by subtraction all the costs from the largest cost.

	T_1	T_2	T_3	Availability
B_1	$10-8=2$	$10-6=4$	$10-5=5$	150
B_2	$10-6=4$	$10-6=4$	$10-6=4$	150
B_3	$10-10=0$	$10-8=2$	$10-4=6$	150
B_4	$10-8=2$	$10-6=4$	$10-4=6$	150
Demand	200	200	200	

200+200+200=150+150+150+150

Balanced Transportation problem.

	T_1	T_2	T_3	Availability
B_1	2	4	5	150
B_2	4	4	4	150
B_3	0	2	6	150
B_4	2	4	6	150
Demand	200	200	200	

Do yourself

Q3.

	D_1	D_2	D_3	Availability
S_1	5	1	7	10
S_2	6	4	6	80
S_3	3	2	5	15
Demand	75	20	50	

Solution:

The problem is of Minimization.

$10+80+15 \neq 75+20+50$

Unbalanced Transportation problem.

Transform into balanced transportation problem by adding a dummy source having availability $(75+20+50)-(10+80+15)=40$

	D_1	D_2	D_3	Availability
S_1	5	1	7	10
S_2	6	4	6	80
S_3	3	2	5	15
S_4				40
Demand	75	20	50	

In the problem unknown costs are given as 5, 3, 2. If not given then assume these costs as 0.

	D_1	D_2	D_3	Availability
S_1	5	1	7	10
S_2	6	4	6	80
S_3	3	2	5	15
S_4	5	3	2	40
Demand	75	20	50	

Do yourself

Q4.

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	10	70
Demand	30	40	50	40	60	

Solution:

The problem is of minimization.

$40+60+70 \neq 30+40+50+40+60$

Unbalanced Transportation problem.

Transform into balanced transportation problem by adding a dummy source having availability $(30+40+50+40+60) - (40+60+70) = 50$

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	10	70
O_4						50
Demand	30	40	50	40	60	

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	10	70
O_4	0	0	0	0	0	50
Demand	30	40	50	40	60	

(i) Since, no transportation from O_3 to D_5. So assume the cost at this position with M (large positive real number).

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	M	70
O_4	0	0	0	0	0	50
Demand	30	40	50	40	60	

Assume M as 100 to see positive or negative like in Big-M method

Do yourself

(ii) Since, O_1 supplies exactly 20 units to destination D_5. So reduce the supply of O_1 and demand of D_5 with 20 units and assume the cost at this position with M (large positive real number).

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	M	40−20=20
O_2	15	20	13	19	16	60
O_3	18	15	18	20	10	70
O_4	0	0	0	0	0	50
Demand	30	40	50	40	60−20=40	

Assume M as 100 to see positive or negative like in Big-M method

Do yourself

(iii) Since, D_2 receives atleast 10 units from O_2. So reduce the supply of O_2 and demand of D_2 with 10 units.

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60−10=50
O_3	18	15	18	20	10	70
O_4	0	0	0	0	0	50
Demand	30	40−10=30	50	40	60−20=40	

Do yourself

Q5.

	D_1	D_2	D_3	Availability
O_1	4	5(30)	2	30
O_2	4(40)	1	3	40
O_3	3	6(20)	2	20
O_4	2	3	7(60)	60
Demand	40	50	60	

Four basic variables are given. While, the number of basic variables should be m+n−1 i.e., 4+3−1=6.

Therefore, there is a need to assume the value of two basic variables as 0 (Degenerate solution).

These variables need to be assumed in such a manner that it is not possible to construct a loop using these basic variables.

	D_1	D_2	D_3	Availability
O_1		30	0	30
O_2	40			40
O_3		20	0	20
O_4			60	60
Demand	40	50	60	

Incorrect as loop exist

	D_1	D_2	D_3	Availability
O_1		30	0	30
O_2	40		0	40
O_3		20		20
O_4			60	60
Demand	40	50	60	

No loop exists correct.

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=5$

$x_{13} \Rightarrow u_1+v_3=c_{13} \Rightarrow u_1+v_3=2$

$x_{21} \Rightarrow u_2+v_1=c_{21} \Rightarrow u_2+v_1=4$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=3$

$x_{32} \Rightarrow u_3+v_3=c_{32} \Rightarrow u_3+v_2=6$

$x_{43} \Rightarrow u_4+v_3=c_{43} \Rightarrow u_4+v_3=7$

Do yourself

6.

	Plants					
Warehouse	1	2	3	4	Sales (Price)	
Production cost	15	18	14	13		
Raw material cost	10	9	12	8		
						Availability
1	3	9	5	4	34	80
2	1	7	4	5	32	110
3	5	8	3	6	31	150
4	7	3	8	2	31	100
5	4	5	6	7	31	150
Demand	150	200	175	100		

Cost table

	Plants					
Warehouse	1	2	3	4	Sales (Price)	
						Availability
1	3+15+10=28	9+18+9=36	5+14+12=31	4+13+8=25	34	80
2	1+15+10=26	7+18+9=34	4+14+12=30	5+13+8=26	32	110
3	5+15+10=30	8+18+9=35	3+14+12=29	6+13+8=27	31	150
4	7+15+10=32	3+18+9=30	8+14+12=34	2+13+8=23	31	100
5	4+15+10=29	5+18+9=32	6+14+12=32	7+13+8=28	31	150
Demand	150	200	175	100		

Profit table (Selling price−Cost)

	Plants				
Warehouse	1	2	3	4	
					Availability
1	34−28=6	34−36=−2	34−31=3	34−25=9	80
2	32−26=6	32−34=−2	32−30=2	32−26=6	110
3	31−30=1	31−35=−4	31−29=2	31−27=4	150
4	31−32=−1	31−30=1	31−34=−3	31−23=8	100
5	31−29=2	31−32=−1	31−32=−1	31−28=3	150
Demand	150	200	175	100	

Minimization problem

	Plants				
Warehouse	1	2	3	4	
					Availability
1	9−6=3	9−(−2)=11	9−3=6	9−9=0	80
2	9−6=3	9−(−2)=11	9−2=7	9−6=3	110
3	9−1=8	9−(−4)=13	9−2=7	9−4=5	150
4	9−(−1)=10	9−1=8	9−(−3)=12	9−8=1	100
5	9−2=7	9−(−1)=10	9−(−1)=10	9−3=6	150
Demand	150	200	175	100	

Unbalanced transportation problem

$80+110+150+100+150 \neq 150+200+175+100$

$590 \neq 625$

Transform into balanced transportation problem

	Plants				
Warehouse	1	2	3	4	
					Availability
1	3	11	6	0	80
2	3	11	7	3	110
3	8	13	7	5	150
4	10	8	12	1	100
5	7	10	10	6	150
6	0	0	0	0	35
Demand	150	200	175	100	

Do yourself

Assignment Problem

LPP for a balanced transportation problem

Minimize $\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}\right)$

Subject to

$\sum_{j=1}^{n} x_{ij} = a_i \,, \quad i = 1, 2, \ldots, m \,;$

$\sum_{i=1}^{m} x_{ij} = b_j \,, \quad j = 1, 2, \ldots, n \,;$

$x_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$

LPP for a balanced assignment problem

Put

- $m = n$
- $a_i = b_j = 1$
- $x_{ij} = 0$ **or** $1.$

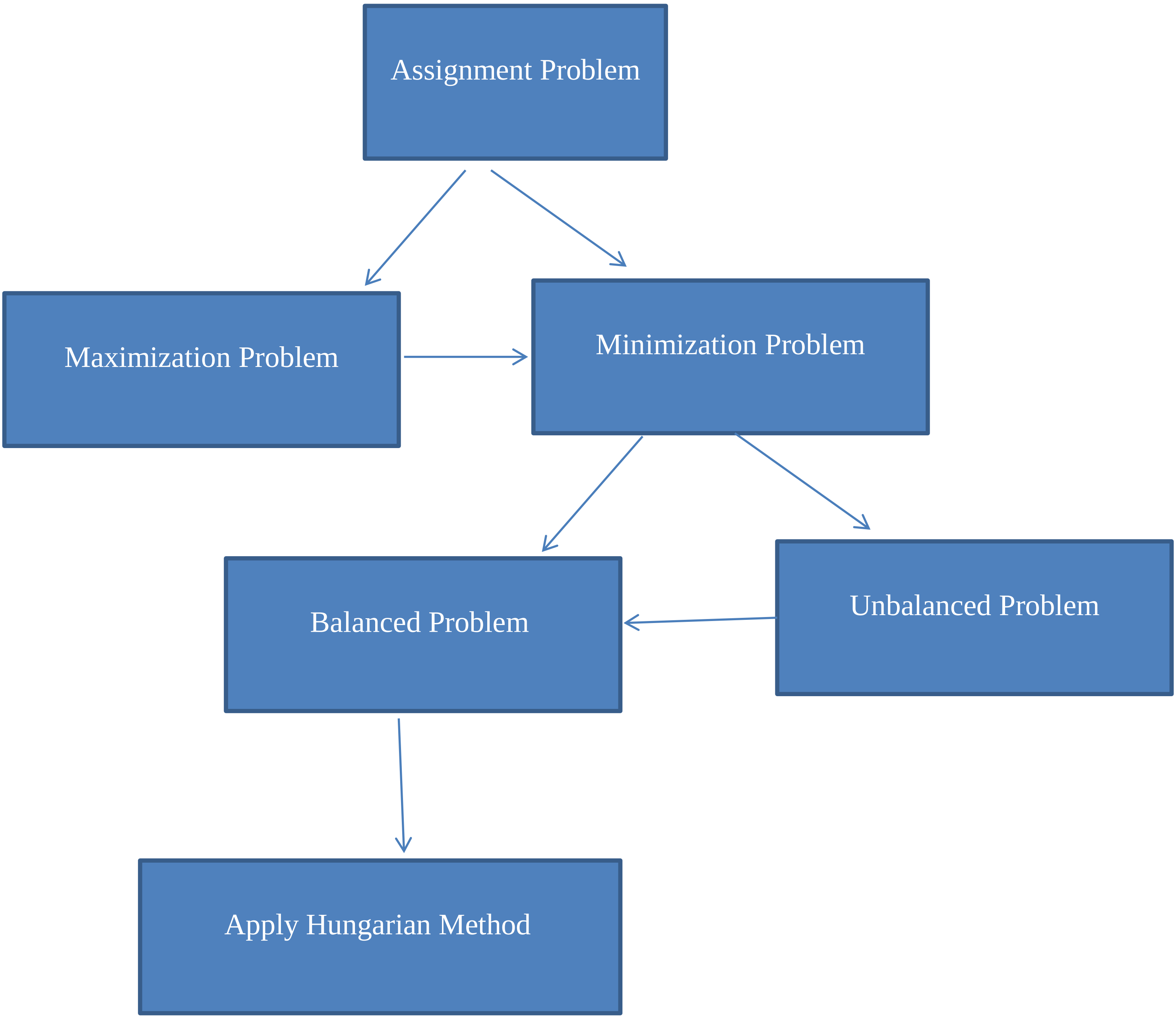
Minimize $\left(\sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}\right)$

Subject to

$\sum_{j=1}^{m} x_{ij} = 1 \,, \quad i = 1, 2, \ldots, m \,;$

$\sum_{i=1}^{m} x_{ij} = 1 \,, \quad j = 1, 2, \ldots, m \,;$

$x_{ij} = 0 \text{ or } 1, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n.$



Hungarian Method

Step 1:

Subtract the minimum element of each row from the elements of the corresponding row.

Step 2:

Subtract the minimum element of each column from the elements of the corresponding column.

Step 3:

Find that row/column in which there exists minimum number of 0. Select a 0 , cross the remaining 0 of that row and cut the corresponding column.

Step 4

Repeat Step 3 till there exist any 0 which is neither crossed nor cut by any line.

Step 5

If a 0 is selected in each row then the solution is optimal otherwise not optimal.

Step 6

If not optimal then find the minimum of those elements which does not lie on any line.

Step 7

Subtract the obtained minimum from those elements which does not lie on any line as well as add the obtained minimum to those elements which lies on the intersection of two lines.

Step 8

Repeat Steps 3 to Step 7 till the optimal solution is not obtained.

Example

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Which job should be assigned to which person so that total profit is maximum.

Solution:

Since, maximization problem. So transform into minimization by subtracting all the elements of the table from the largest element

		Jobs			
		I	II	III	IV
Persons	A	$42-42=0$	$42-35=7$	$42-28=14$	$42-21=21$
	B	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	C	$42-30=12$	$42-25=17$	$42-20=22$	$42-15=27$
	D	$42-24=18$	$42-20=22$	$42-16=26$	$42-12=30$

		Jobs			
		I	II	III	IV
Persons	A	0	7	14	21
	B	12	17	22	27
	C	12	17	22	27
	D	18	22	26	30

Balanced Problem (Square matrix)

Subtract minimum of each row from the elements of the corresponding row.

		Jobs			
		I	II	III	IV
Persons	A	$0-0=0$	$7-0=7$	$14-0=14$	$21-0=21$
	B	$12-12=0$	$17-12=5$	$22-12=10$	$27-12=15$
	C	$12-12=0$	$17-12=5$	$22-12=10$	$27-12=15$
	D	$18-18=0$	$22-18=4$	$26-18=8$	$30-18=12$

		Jobs			
		I	II	III	IV
Persons	A	0	7	14	21
	B	0	5	10	15
	C	0	5	10	15
	D	0	4	8	12

Subtract minimum of each column from the elements of the corresponding column.

		Jobs			
		I	II	III	IV
Persons	A	$0-0=0$	$7-4=3$	$14-8=6$	$21-12=9$
	B	$0-0=0$	$5-4=1$	$10-8=2$	$15-12=3$
	C	$0-0=0$	$5-4=1$	$10-8=2$	$15-12=3$
	D	$0-0=0$	$4-4=0$	$8-8=0$	$12-12=0$

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	9
	B	0	1	2	3
	C	0	1	2	3
	D	0	0	0	0

Find row/column having minimum no of 0

First Row have only one 0. Select the 0 and cut the column.

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	9
	B	0	1	2	3
	C	0	1	2	3
	D	0	0	0	0

One 0 in second column. Select 0 and cut row.

		Jobs			
		I	II	III	IV
Persons	A	0	3	6	9
	B	0	1	2	3
	C	0	1	2	3
	D	0	0	0	0

No 0 left. But no 0 has been selected in second and third row . Solution is not optimal.

Find minimum of those elements which does not lie on any line.

Minimum{3,6,9,1,2,3,1,2,3}=1

Suntract minimum from those elements which does not lie on any line and add at those elements which lies at intersection of two lines.

		Jobs			
		I	II	III	IV
Persons	A	0	3 − 1	6 − 1	9 − 1
	B	0	1 − 1	2 − 1	3 − 1
	C	0	1 − 1	2 − 1	3 − 1
	D	0+1	0	0	0

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	8
	B	0	0	1	2
	C	0	0	1	2
	D	1	0	0	0

One 0 in first row. Select 0 and cut column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	8
	B	0	0	1	2
	C	0	0	1	2
	D	1	0	0	0

One 0 in second row. Select 0 and cut column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	8
	B	0	0	1	2
	C	0	0	1	2
	D	1	0	0	0

One 0 in third column. Select 0 and cut row.

		Jobs			
		I	II	III	IV
Persons	A	0	2	5	8
	B	0	0	1	2
	C	0	0	1	2
	D	1	0	0	0

No 0 left but no 0 has been selected in third row. Solution is not optimal.

Minimum {5,8,1,2,1,2}=1

		Jobs			
		I	II	III	IV
Persons	A	0	2	$5-1$	$8-1$
	B	0	0	$1-1$	$2-1$
	C	0	0	$1-1$	$2-1$
	D	$1+1$	0+1	0	0

		Jobs			
		I	II	III	IV
Persons	A	0	2	4	7
	B	0	0	0	1
	C	0	0	0	1
	D	1 + 1	1	0	0

One 0 in first row. Select it and cut column

		Jobs			
		I	II	III	IV
Persons	A	0	2	4	7
	B	0	0	0	1
	C	0	0	0	1
	D	2	1	0	0

One 0 in fourth column. Select it and cut row

		Jobs			
		I	II	III	IV
Persons	A	0	2	4	7
	B	0	0	0	1
	C	0	0	0	1
	D	2	1	0	0

Two 0 in second row. Select any one, cross the other and cut column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	4	7
	B	0	0	0	1
	C	0	0	0	1
	D	2	1	0	0

One 0 in third row. Select it and cut column.

		Jobs			
		I	II	III	IV
Persons	A	0	2	4	7
	B	0	0	0	1
	C	0	0	0	1
	D	2	1	0	0

Optimal solution:

Job I to person A

Job II to person B

Job III to person C

Job IV to person D

		Jobs			
		I	II	III	IV
Persons	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Optimal Profit:

42+25+20+12=99