

Note: Attempt all the problems. Please attempt the parts of a problem at one place. You can assume a missing data suitably. You can use the following, wherever required:

$$\sum_{k=1}^n \frac{1}{k} = \log n + O(1); \sum_{k=1}^n k^2 = n(n+1)(2n+1)/6$$

1(a)	Assume that you are given an array $A[1 \dots n]$ of distinct numbers. This is also known that the sequence of numbers in the array is unimodal, i.e., there is an index i such that the sequence $A[1 \dots i]$ is increasing ($A[j] < A[j+1]$ for $1 \leq j < i$), and the sequence $A[i \dots n]$ is decreasing. The index i is called the mode of A . Give an $O(\log n)$ algorithm that finds the mode of A .	4			
1(b)	Assume that you are given an array $A[1 \dots n]$ of distinct numbers. Give an algorithm to reverse the elements of this array.	2			
1(c)	Prove that the average case complexity of Bubble Sort is $O(n^2)$, when we are sorting an array of size n .	4			
2(a)	Establish the correctness / incorrectness of the statements: (i) $10n^3 + 15n^4 + 100n^2 2^n = O(n^2 2^n)$, (ii) $3^n = O(2^n)$.	2			
2(b)	<table border="1"> <tr> <td>Find the Big-O complexities of the functions in Fig. 1 and Fig. 2.</td> <td> <pre>void Function (int n) { int i = 1, s = 1; while (s <= n) { i++; s = s+i; printf("!"); } //Fig. 1//</pre> </td> <td> <pre>void Function (int n) { for (int i=1; i<=n; i++) for (int j=1; j<=n; j=j+i) printf("!"); } //Fig. 2//</pre> </td> </tr> </table>	Find the Big- O complexities of the functions in Fig. 1 and Fig. 2.	<pre>void Function (int n) { int i = 1, s = 1; while (s <= n) { i++; s = s+i; printf("!"); } //Fig. 1//</pre>	<pre>void Function (int n) { for (int i=1; i<=n; i++) for (int j=1; j<=n; j=j+i) printf("!"); } //Fig. 2//</pre>	2+2
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2(c)	<p>Following functions is given. Write an equivalent mathematical function $f(n)$ for this function, and then find the Big-O complexity of this function. Use the definition of Big-O and mention the values of c and n_0 explicitly.</p> <pre>void Function (int n) { if (n <= 1) return; for (int i=1; i<=3; i++) Function(n - 1); }</pre>	4			
3(a)	What is the policy, followed in Stack implementation? Stack is a linear or non-linear data structure.	2			
3(b)	Give two situations/applications where Stack data structure will be preferred to use.	2			
3(c)	We want to implement <i>push</i> and <i>pop</i> operations on Stack with the help of a linked list. Give algorithms for these operations; taking <i>start</i> as the starting address of list, <i>start</i> → <i>info</i> as the data part of <i>node</i> and <i>start</i> → <i>next</i> as the pointer to next <i>node</i> .	4			
3(d)	What are the complexities of these operations in your algorithms?	2			