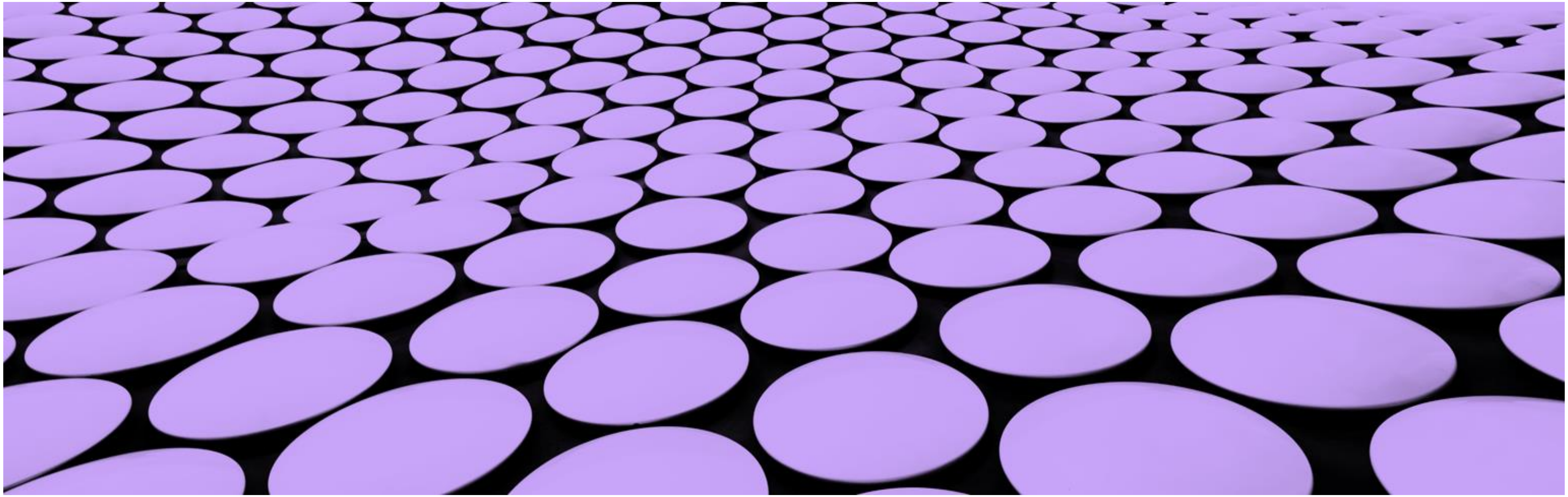
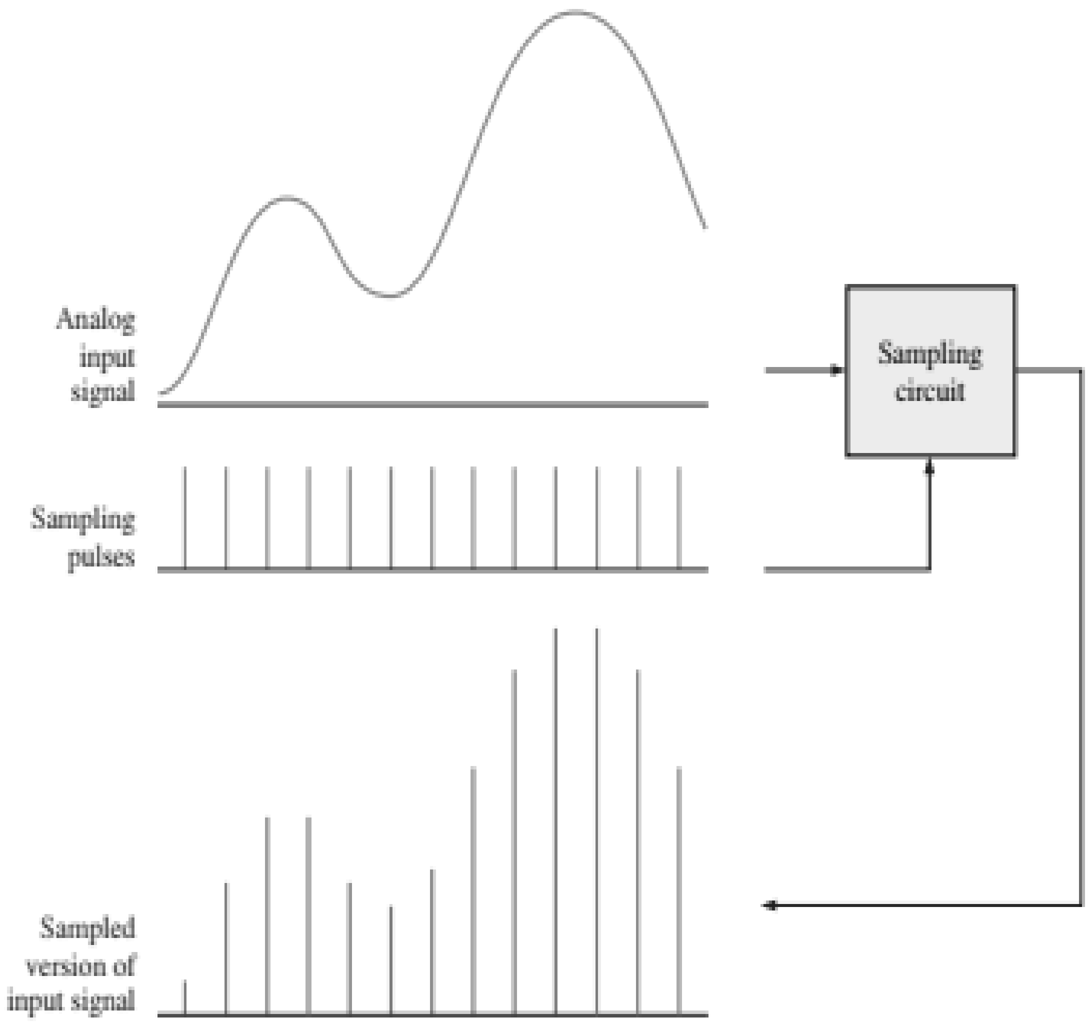
SAMPLING & QUANTIZATION



SAMPLING

- **Sampling** is the process of taking a sufficient number of discrete values at points on a waveform that will define the shape of the waveform.
- The more samples can be taken, the more accurately the signal can be defined.
- Sampling converts an analog signal into a series of impulses, each representing the amplitude of the signal at a given instant in time.



SAMPLING THEOREM

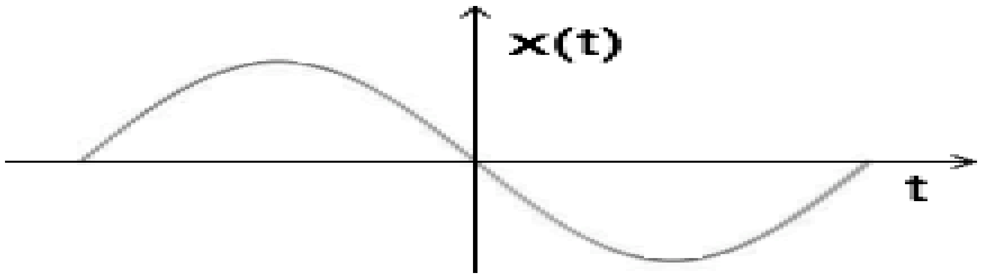
- In order to recover a continuous signal, the sampling rate must be greater than twice the highest frequency in the signal.

$$\boxed{f_{sample} \geq 2 f_m}$$

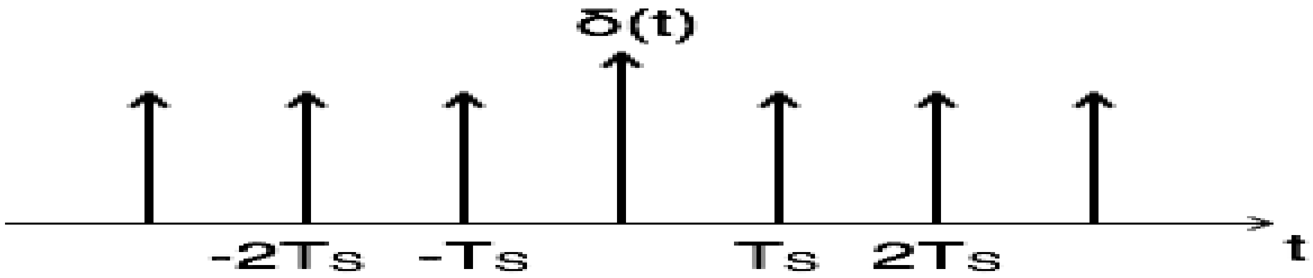
where f_{sample} = sampling frequency, and the frequency f_m is known as the Nyquist frequency .

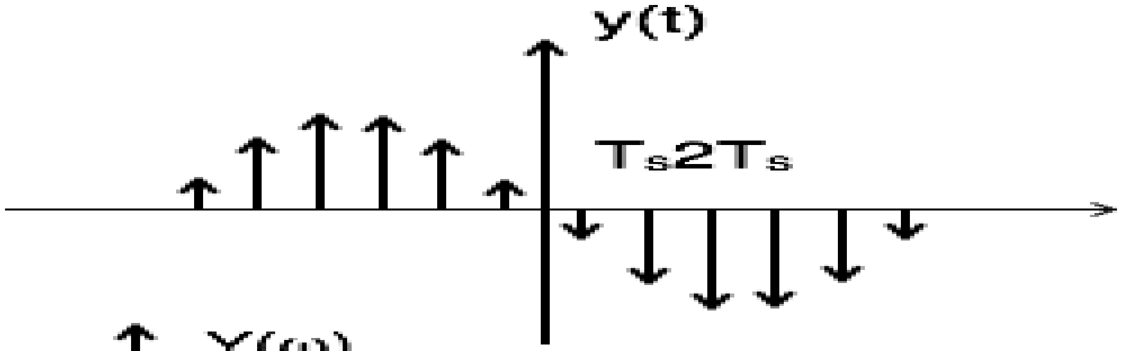
- If the signal is sampled less than this, the recovery process will produce frequencies that are entirely different than in the original signal.
- The frequency $2f_m$ is called the Nyquist rate.

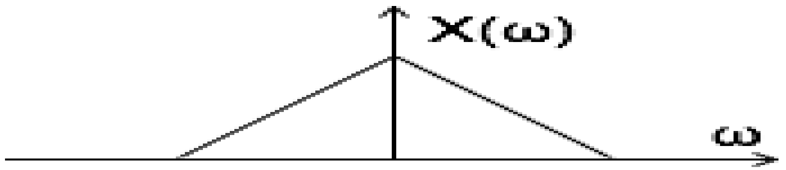
CONTD..

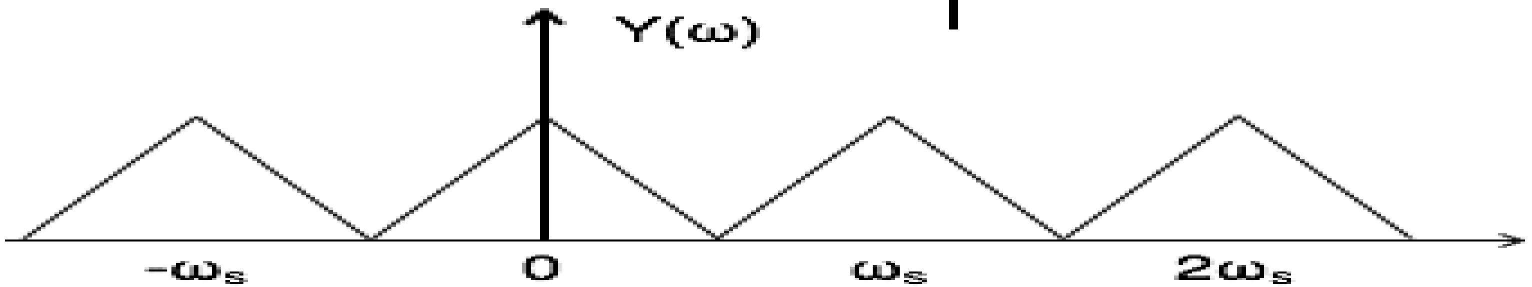

x(t)
t


x(t)
δ(t)
multiplier
y(t)


δ(t)
-2Ts
-Ts
Ts
2Ts
t


y(t)
Ts2Ts


X(ω)
ω


Y(ω)
-ωs
0
ωs
2ωs

EFFECT OF SAMPLING IN THE FREQUENCY DOMAIN

- Let $g_a(t)$ be a continuous-time signal that is sampled uniformly at $t = nT$, generating the sequence $g[n]$

$$g[n] = g_a(nT), \ -\infty < n < \infty$$

where with T being the **sampling period**

- The reciprocal of T is called the **sampling frequency** F_T

$$\boxed{F_T = 1/T}$$

- The frequency-domain representation of $g_a(t)$ is given by its continuous-time Fourier transform (CTFT)

$$G_a(j\Omega) = \int_{-\infty}^{\infty} g_a(t) e^{-j\Omega t} dt$$

- The frequency-domain representation of $g[n]$ is given by DTFT

$$G(e^{j\omega}) = \sum_{n=-\infty}^{\infty} g[n] e^{-j\omega n}$$

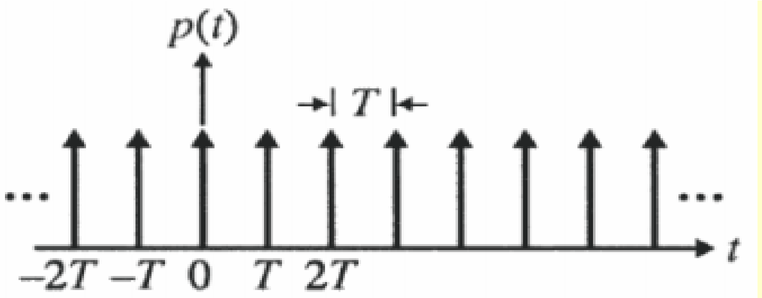
CONTD..

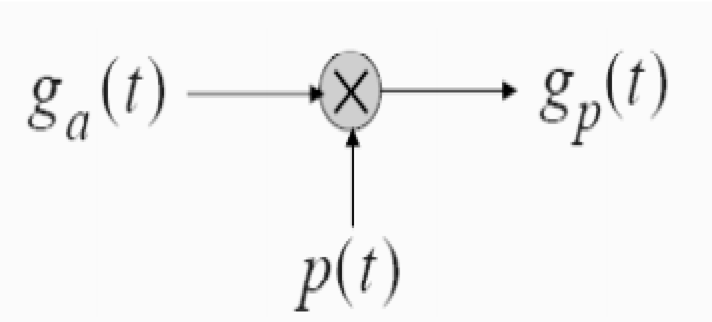
- To establish the relation between $G_a(j\Omega)$ and $G(e^{j\omega})$, the sampling operation mathematically as a multiplication of $g_a(t)$ by a **periodic impulse train** $p(t)$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- $p(t)$ is consists of a train of ideal impulses with a period T
- The multiplication operation yields an impulse train signal

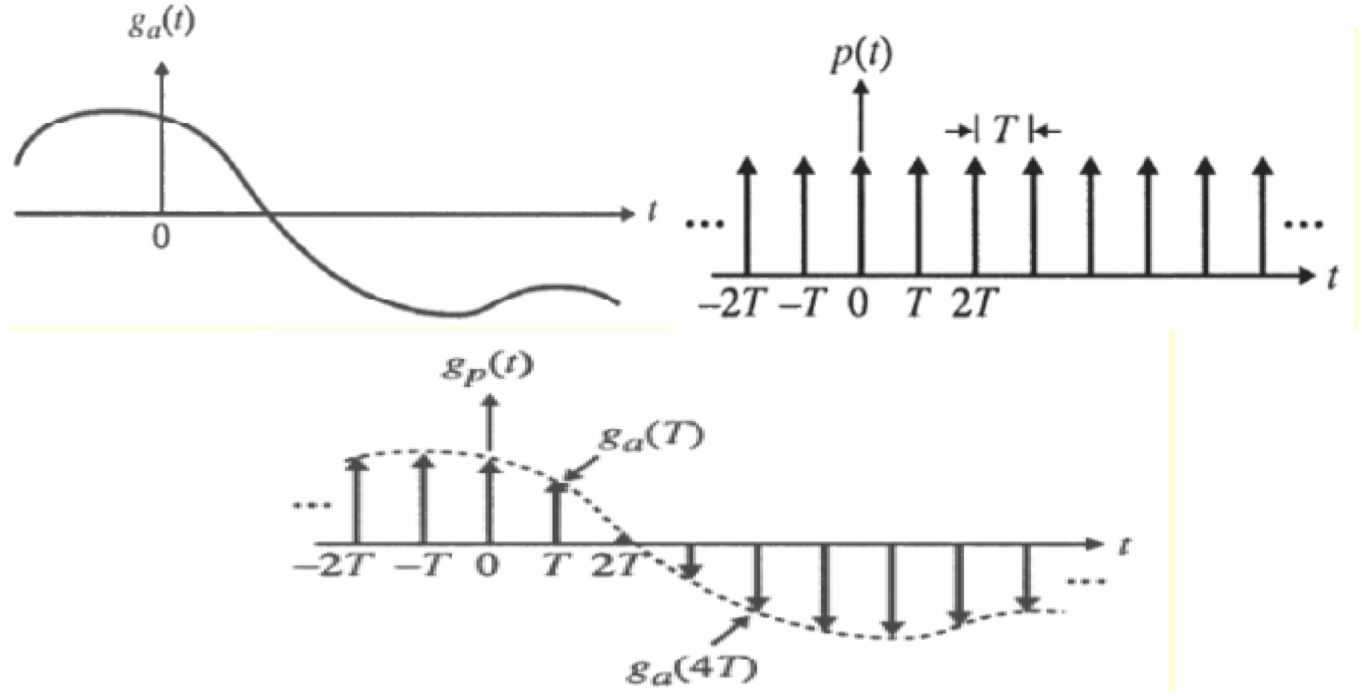
$$\boxed{g_p(t) = g_a(t)p(t) = \sum_{n=-\infty}^{\infty} g_a(nT)\delta(t - nT)}$$





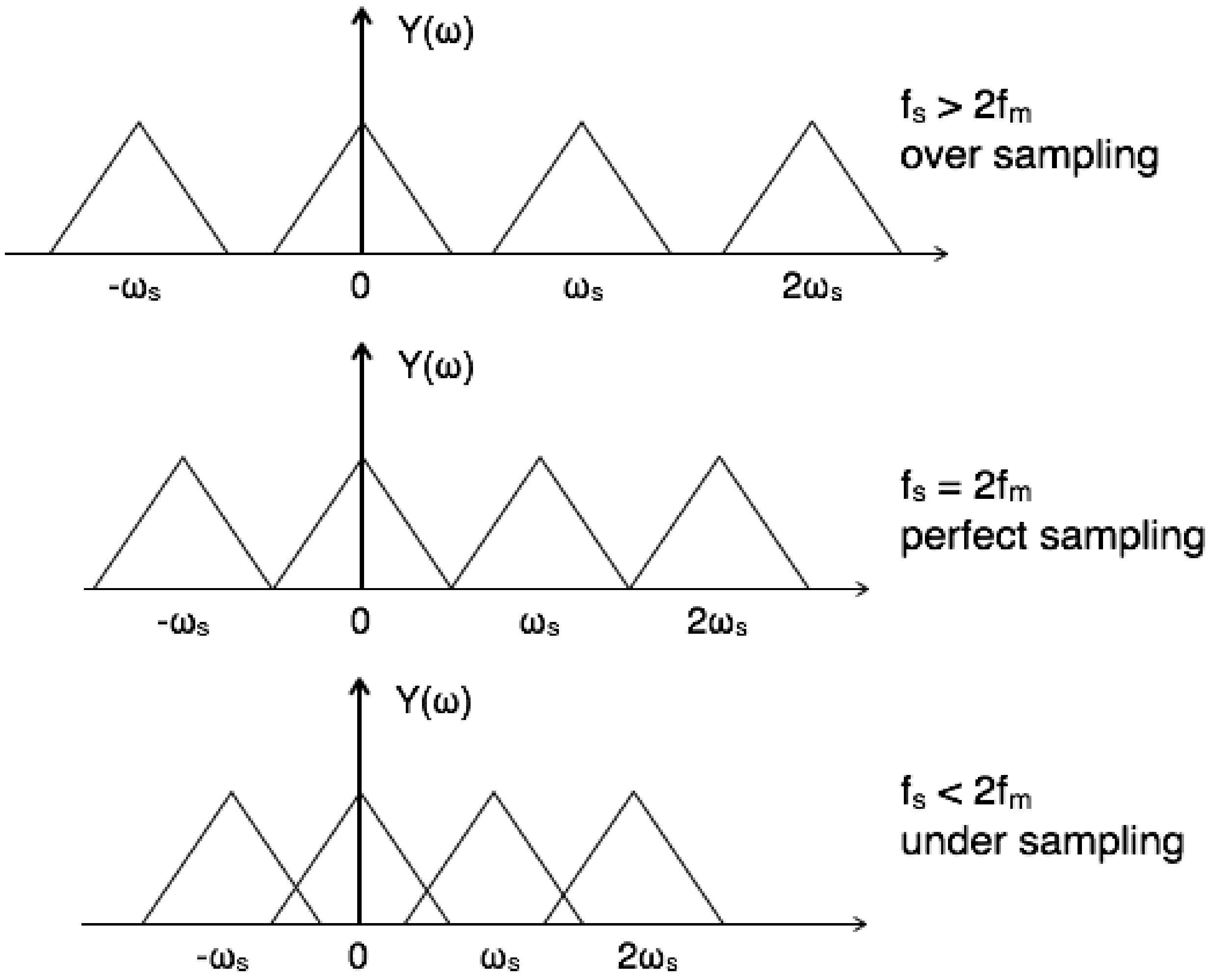
CONTDD..

- $g_p(t)$ is a continuous-time signal consisting of a train of uniformly spaced impulses with the impulse at $t = nT$ weighted by the sampled value $g_a(nT)$ of $g_a(t)$ at that instant



TYPES OF SAMPLING

- **Overs sampling** - The sampling frequency is higher than the Nyquist rate.
- **Under sampling** - The sampling frequency is lower than the Nyquist rate.
- **Critical sampling** - The sampling frequency is equal to the Nyquist rate .
- *A sampling rate of 44.1 kHz is used in mp3 decoding*



EXAMPLE

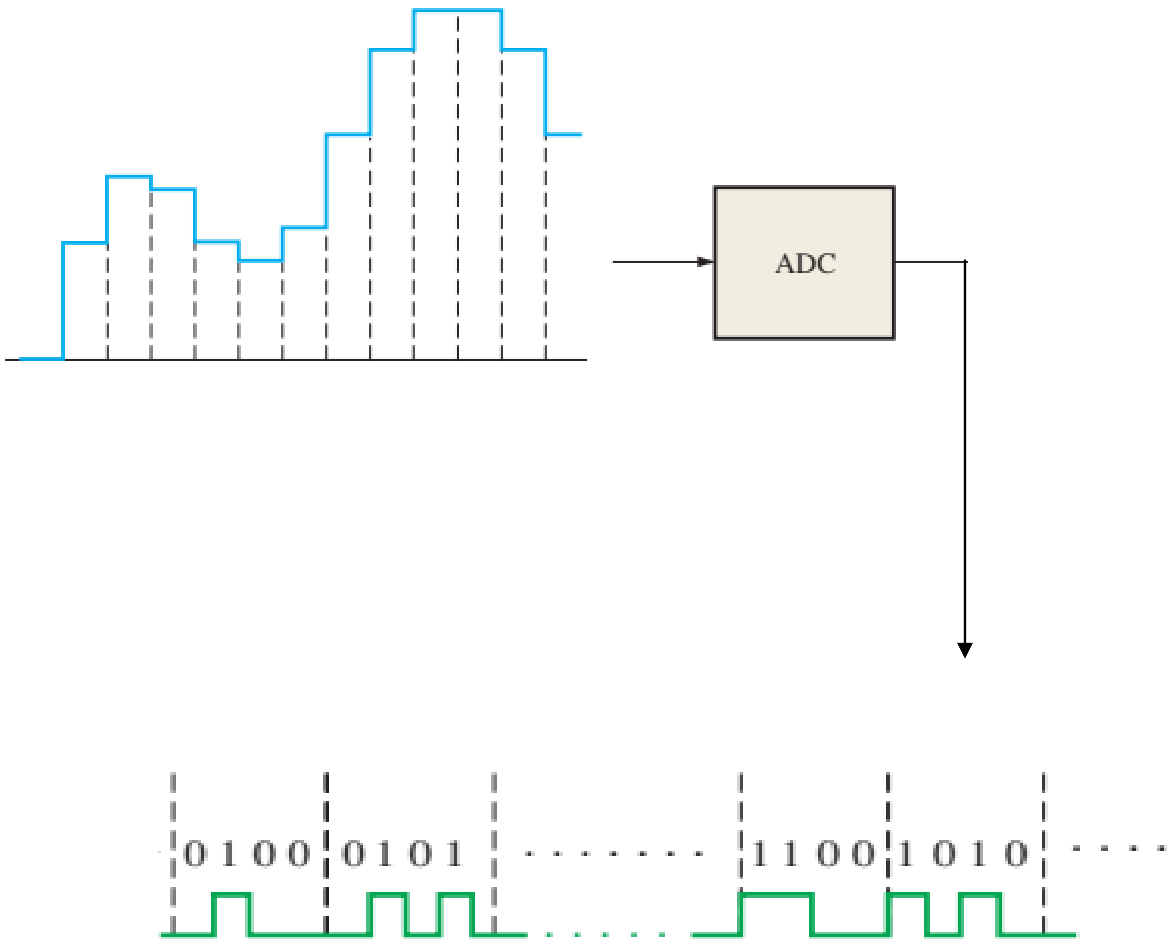
- Consider the three signals $g_1(t)$ = sin(6πt), $g_2(t)$ = sin (14πt), $g_3(t)$ = sin (26πt)
- $$\omega_1 = 6\pi,\ 2\pi f_1 = 6\pi,\ f_1 = 3$$
  $$\omega_2 = 14\pi,\ 2\pi f_2 = 14\pi,\ f_2 = 7$$
  $$\omega_3 = 26\pi,\ 2\pi f_3 = 26\pi,\ f_3 = 13$$

$$f_m = \max(f_1, f_2, f_3) = 13$$

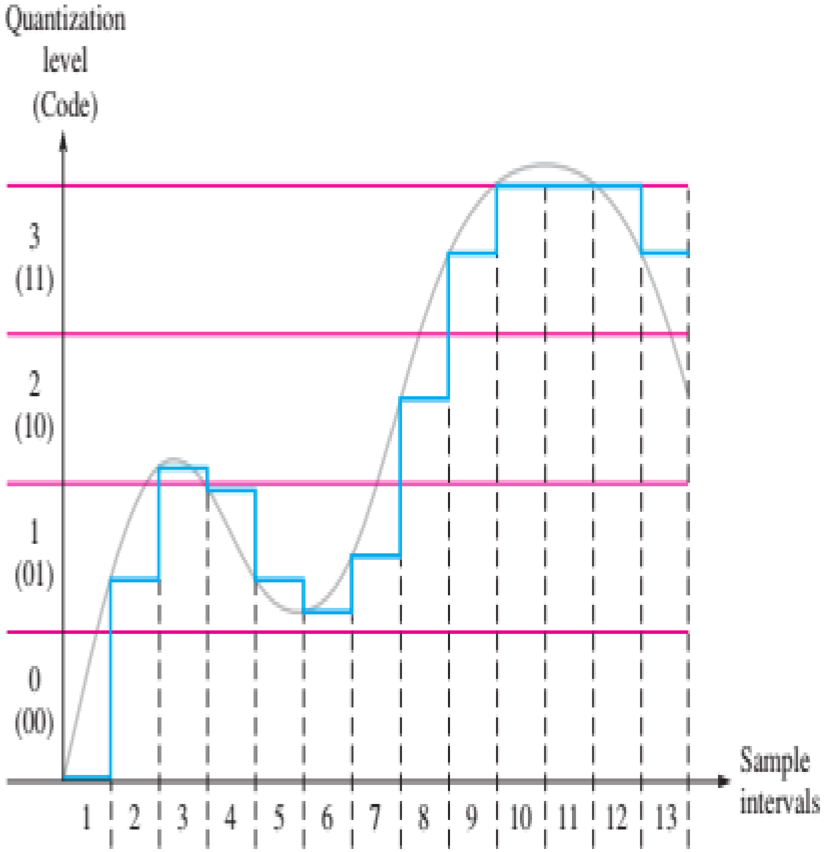
$$\boxed{f_s \geq 2 * f_m \geq 2*13 \geq 26}$$

QUANTIZATION

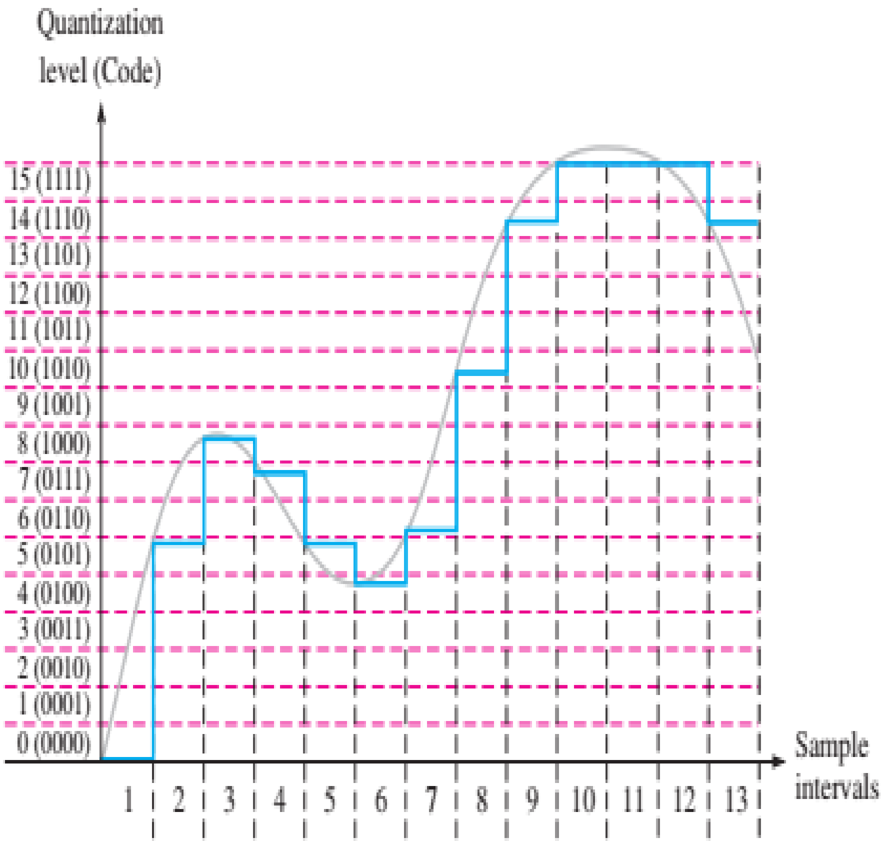
- The process of converting an analog value to a code is called **quantization**.
- During the quantization process, the ADC converts each sampled value of the analog signal to a binary code.
- The more bits that are used to represent a sampled value, the more accurate is the representation.



CONTD..



Sample Interval	Quantization Level	Code
1	0	00
2	1	01
3	2	10
4	1	01
5	1	01
6	1	01
7	1	01
8	2	10
9	3	11
10	3	11
11	3	11
12	3	11
13	3	11



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