FLUID DYNAMICS

FORCES ACTING ON A FLUID MASS IN MOTION

- Various forces acting on the fluid mass in motion are:
- Body force (Examples: Gravity force, centrifugal force, magnetic force), pressure, viscous, turbulent, surface tension and compressibility and above all inertia force.
- Body force is generally due to weight of fluid.
- Pressure force is due to the difference of pressure between two points in the direction of flow.
- Viscous force is due to the viscosity of the fluid.
- Turbulent force is due to turbulent nature of flow.
- Surface tension force is due to the cohesive property of fluid mass.
- Compressibility force is due to the elastic property of fluid.

✓ Depending upon the flow phenomenon, some of these forces may be insignificant and hence are neglected. For example,

- Surface tension force is significant only when depth of flow is very small.
- Compressibility force is significant only for compressible fluids.
- Remaining 4 forces are considered in the analysis of turbulent flow.
- For viscous fluids, turbulent force also becomes insignificant
- Remaining 3 forces are considered in laminar flow analysis
- For ideal or real fluids having small viscosity, viscous force may also be neglected

∴ For ideal fluids flow analysis, only 2 forces are considered *viz.* <u>pressure force</u> and <u>body force.</u>

✓ Mechanics of fluids *i.e.* dynamic behavior of fluid motion is also governed by Newton's second law of motion i.e.

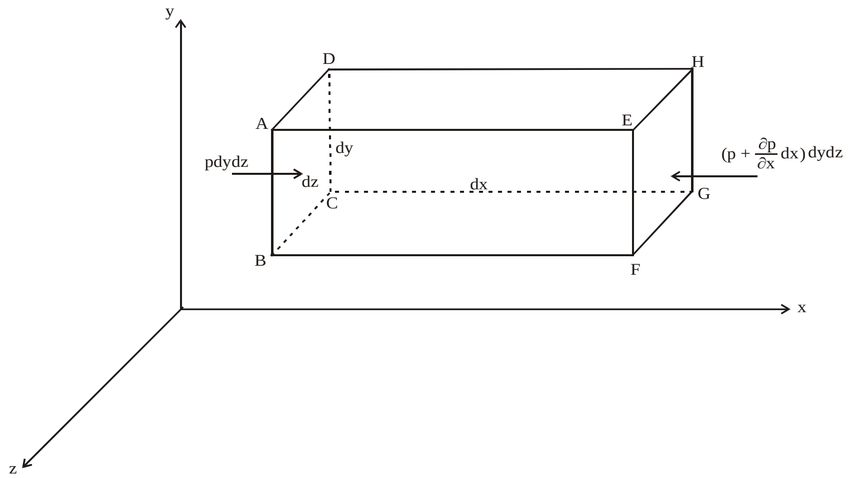
Net force in any direction = Mass × acceleration in the same direction

Bernoulli's equation

- Based on the principle of conservation of energy.
- can be derived using several approaches *viz.* applying principle of conservation of energy to a **C∀,** integrating Euler's equation of motion in Cartesian coordinates or along a stream line etc.
- Euler's equation approach is discussed hereunder only

EULER'S EQUATION OF MOTION IN CARTESIAN COORDINATES

- Consider a **C∀** in the shape of a rectangular paralleopipe as shown in **Figure.**



- Further, considering <u>ideal fluid flow</u> through the **C∀.**

  $\therefore$ Forces acting on the **C∀**are; pressure and body forces.

- Pressure force on the face **ABCD** is **pdydz** where as on the face **EFGH** is $\left(p + \frac{\partial p}{\partial x} dx\right) dydz.$

- If ρ = mass density of fluid; **X**, **Y** and **Z** are the components of body force per unit mass in the respective directions, then
- Body force along x- direction = **X (ρ dx dy dz)**
- Similarly, body forces along **y-** and **z-** directions are **Y (ρ dx dy dz)** and **Z (ρ dx dy dz)**, respectively.

❖ Here **(ρ dx dydz)** is the mass of fluid in **C∀.**

- Applying Newton's second law of motion along **x-**direction

$$X(\rho dxdydz) + pdydz - \left(p + \frac{\partial p}{\partial x} dx\right) dydz = ma_x = \rho(dxdydz)a_x$$

- Simplifying, to get

$$X - \frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) = a_x \qquad (1)$$

- Similarly, along **y-** and **z-** directions, one can write

$$Y - \frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) = a_y \quad \text{and} \quad Z - \frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) = a_z$$

➢ known as Euler's equation of motion.

- **(∂p/∂x)**, **(∂p/∂y)** and **(∂p/∂z)** are the pressure gradients along **x-**, **y-** and **z-** directions, respectively

Integration of Euler's eq. to obtain Bernoulli's equation

- Following additional assumptions are made in the analysis:

(i) As the fluid is ideal and thus flow is irrotational. $\therefore$ Potential function ϕ exists.

$$\therefore w_x = 0, w_y = 0 \text{ and } w_z = 0$$

$$w_z = 0 \Rightarrow \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \;\therefore \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Similarly, if $w_x = 0 \text{ and } w_y = 0$, one gets $\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$ and $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$

Also, $-\frac{\partial \phi}{\partial x} = u,\; -\frac{\partial \phi}{\partial y} = v \text{ and } -\frac{\partial \phi}{\partial z} = w$

(ii) There exists a force potential denoted by Ω defined as $X = -\frac{\partial \Omega}{\partial x}, Y = -\frac{\partial \Omega}{\partial y} \text{ and } Z = -\frac{\partial \Omega}{\partial z}$

(- ve sign signifies that as **x**, **y** and **z** increases, **Ω** decreases)

Equation 1 can be rewritten as: $X - \frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$$\Rightarrow -\frac{\partial \Omega}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} + \frac{\partial u}{\partial t}$$ [(Using assumptions (i) and (ii)]

$$= \frac{1}{2}\frac{\partial}{\partial x}(u^2) + \frac{1}{2}\frac{\partial}{\partial x}(v^2) + \frac{1}{2}\frac{\partial}{\partial x}(w^2) + \frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial}{\partial x}(u^2 + v^2 + w^2) + \frac{\partial u}{\partial t}$$

$$= \frac{1}{2}\frac{\partial}{\partial x}(V^2) + \frac{\partial u}{\partial t}$$

$$\Rightarrow \therefore -\frac{\partial \Omega}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}(V^2) + \frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial x}\right),$$ [(Using assumption (i)]

Simplifying, to get

$$\frac{\partial}{\partial x}\left(\frac{V^2}{2} + \Omega + \frac{p}{\rho} - \frac{\partial \phi}{\partial t}\right) = 0,$$ Integrating, to get

$$\frac{V^2}{2} + \Omega + \frac{p}{\rho} - \frac{\partial \phi}{\partial t} = F_1(y, z, t)$$

F₁ is a constant of integration, which can be a function of (**y, z and t**).

Similarly, along y- and z-directions, respectively, one can write

$$\frac{V^2}{2} + \Omega + \frac{p}{\rho} - \frac{\partial \phi}{\partial t} = F_2(x, z, t) \text{ and } \frac{V^2}{2} + \Omega + \frac{p}{\rho} - \frac{\partial \phi}{\partial t} = F_3(x, y, t)$$

Since L.H.S. of Eqs. are same.

$$\therefore F_1(y, z, t) = F_2(x, z, t) = F_3(x, y, t)$$

Since **x**, **y** and **z** are independent variables, above functions will hold good only if these variables disappear from the functional terms or in other words, functions $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ are the functions of time only or simply constants.

$$\therefore \frac{V^2}{2} + \Omega + \frac{p}{\rho} - \frac{\partial\phi}{\partial t} = \text{Constant}$$

If body force exerted on the flowing fluid is due to gravity only, then X = 0, Y = - g and Z = 0.

$$\therefore -\frac{\partial\Omega}{\partial y} = -g$$, [(Using assumption (ii)], Integrate, to get

$$\Omega = gy + C,$$ **C** is a constant of integration.

At **y = 0, Ω = 0 ∴ C = 0** $\Rightarrow \Omega = gy$

$$\therefore \frac{V^2}{2} + gy + \frac{p}{\rho} - \frac{\partial\phi}{\partial t} = \text{Constant}$$

- ❖ General form of Bernoulli's equation.

For steady flow, $\frac{\partial\phi}{\partial t} = 0, \therefore \frac{V^2}{2} + gy + \frac{p}{\rho} = C$

$$\Rightarrow \frac{V^2}{2g} + y + \frac{p}{\rho g} = C_1 \qquad \text{OR} \qquad \frac{p}{\rho g} + \frac{V^2}{2g} + y = C_1$$

- ➢ **C** and $\mathbf{C_1}$ are constants.
    - ❖ Standard form of Bernoulli's equation.

Observations:

(i) Each term represents <u>energy per unit weight</u> of flowing fluid.

- has the dimension of length and thus each term is also known as head.
    - ➢ **(p/ρg)** is known as pressure head/static head, represents ability of unit weight to do work by virtue of its pressure.
    - ➢ **(V^2/2g)** is known as velocity head/kinetic head/dynamic pressure head and it represents kinetic energy per weight.
    - ➢ ***y*** is known as potential head/elevation head/datum, represents potential energy per weight.
- Sum of pressure head, velocity head and potential head is known as <u>total head</u> or total energy per unit weight of flowing fluid
- Bernoulli's theorem may be stated as: "for an ideal (inviscid, incompressible, irrotational) and steady fluid flow, the total energy per unit weight of the flowing fluid is constant"

(ii) If Bernoulli's eq. is applied between any two points in the flowing field

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2$$

(iii) For real fluids flow, there always occurs loss of energy in the direction of flow

$\therefore$ For real fluids flow, $\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2+h_L$

- **h_L** is the loss of energy per unit weight or head loss between two points under consideration.

(iv) If between two points, some energy is supplied to the flow system or by the flow system (hydraulic machines).

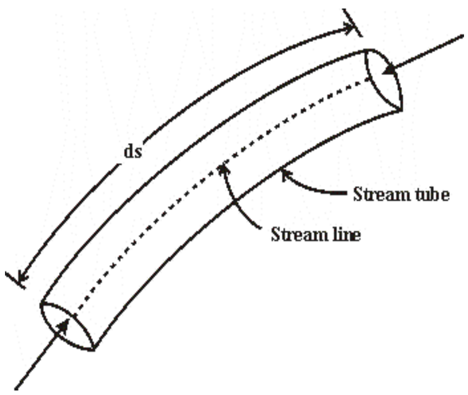
Example: water lifted by a pump or water used in the turbines for generation of hydropower

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1\pm h_m=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2+h_L$$

- **h_m** is the work done per unit weight by hydraulic machine on the fluid (**+ sign**, pump case) or by the fluid (**- sign**, turbine case).

Bernoulli's Equation along a Stream Line

- Consider **C∀** in the shape of a stream tube having cross-sectional area **A** and length **ds**
- Further, consider flow along a streamline passing through the centre of stream tube.



- Forces acting on the stream tube are: pressure forces acting at the ends and and the body force.
- If **S** is the body force per unit mass in the direction of flow, then applying Newton's 2nd law of motion along the direction of flow, to get

$$pA-(p+\frac{\partial p}{\partial s}ds)A+S(\rho A ds)=(\rho A ds)a_s$$

$$\Rightarrow\left[S-\frac{1}{\rho}\left(\frac{\partial p}{\partial s}\right)\right]=a_s \qquad \textbf{(1)}$$

- Here **p** is the pressure intensity, ρ is the mass density of fluid and $\mathbf{a_s}$ is the acceleration along the stream line.
- Equation (1) is the Euler's equation along a stream line.

Integration of Euler's equation to obtain Bernoulli's equation

- Force potential in this case is $\mathbf{S = -\frac{\partial \Omega}{\partial s}}$
- Velocity along a stream line is a function of position and time *i.e.* **V = f(s, t)**

  ∴ $\mathbf{a_s}$ along a stream line can be written as:

$$\mathbf{a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = \frac{1}{2}\frac{\partial}{\partial s}(V^2) + \frac{\partial V}{\partial t}}$$

- Substituting in Eq. (1), to write

$$\therefore -\frac{\partial \Omega}{\partial s} - \frac{1}{\rho}\left(\frac{\partial p}{\partial s}\right) = \frac{1}{2}\frac{\partial}{\partial s}(V^2) + \frac{\partial V}{\partial t}$$

- For steady flow, $\frac{\partial V}{\partial t} = 0 \Rightarrow -\frac{\partial \Omega}{\partial s} - \frac{1}{\rho}\left(\frac{\partial p}{\partial s}\right) = \frac{1}{2}\frac{\partial}{\partial s}(V^2)$

$$\Rightarrow \frac{1}{2}\frac{\partial}{\partial s}(V^2) + \frac{1}{\rho}\left(\frac{\partial p}{\partial s}\right) + \frac{\partial \Omega}{\partial s} = 0$$

- Integrating assuming constant density, to get

$$\frac{V^2}{2} + \frac{p}{\rho} + \Omega = C$$, **C** is a constant of integration.

- If body force exerted on the flowing fluid is due to gravity only, then **S = - g**

$$\therefore -\frac{\partial \Omega}{\partial s} = -g \Rightarrow -\frac{\partial \Omega}{\partial y} = -g,$$ (Take **s = y**, ∴∂**s** = ∂y)

  Integrating, $\Omega = gy + C$,**C** is a constant of integration.

- At $y = 0, \Omega = 0$ $\quad \therefore C = 0$ $\quad \Rightarrow \Omega = gy$

$$\therefore \frac{V^2}{2} + \frac{p}{\rho} + gy = C,$$ Divide by ***g***, to get

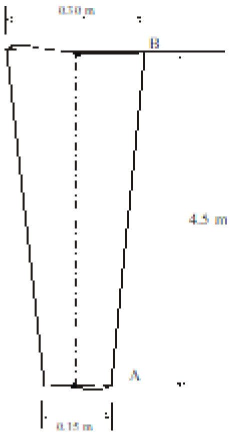
$$\frac{p}{\rho g} + \frac{V^2}{2g} + y = C_1,$$ $\mathbf{C_1}$ is a constant.

- Bernoulli's equation.

Problems:

Q1: Oil of relative density 0.80 flows through a vertical pipeline which changes in its diameter from 150 mm at section **A** to 300 mm at section **B**, section **B** being 4.5 m higher than section **A**. If pressures at **A** and **B** are 200 kN/m^2 and 140 kN/m^2, respectively, determine the direction of flow and the head loss when pipe carries a discharge of 0.11 m^3/s. Neglect losses.

Solution: Direction of flow is indicated by the difference of total head at the two sections:



- Total head at **A**, H_A $= \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + y_A$
- Total head at **B**, H_B $= \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + y_B$
- From continuity equation,

$$Q = A_A V_A = A_B V_B \qquad \Rightarrow 0.11 = \frac{\pi}{4}(0.15)^2 \times V_A = \frac{\pi}{4}(0.30)^2 \times V_B$$

$\therefore$ V_A = 6.225 m/s and V_B = 1.556 m/s

- Assuming datum is passing through **A**

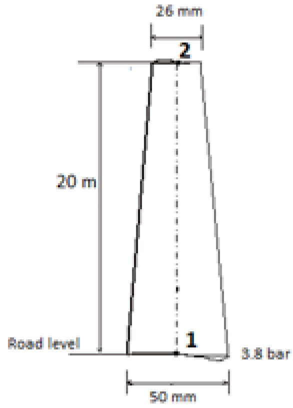
$$\therefore y_A = 0, y_B = 4.5\text{ m}$$

$\therefore$ Total head at **A**, $H_A = \frac{200 \times 10^3}{0.8 \times 9810} + \frac{(6.225)^2}{2 \times 9.81} + 0 = 27.46\text{ m}$

- Total head at **B**, $H_B = \frac{140 \times 10^3}{0.8 \times 9810} + \frac{(1.556)^2}{2 \times 9.81} + 4.5 = 22.46\text{ m}$
- Since total head at **A** is more than the total head at **B** and therefore the direction of flow is from **A** to **B** *i.e.* upward.
- Head loss is given by,

$$h_L = (H_A - H_B) = 27.46 - 22.46 = 5\text{ m}$$

Q2: Water at a pressure of 3.8 bar at road level flows into an office building with 0.6 m/s through a pipe 50 mm in diameter. The pipe tapers down to 26 mm diameter at the top floor, 20 m above. Calculate the flow velocity and the pressure in the pipe at the top floor. Neglect losses.



Solution: From continuity equation

$$A_1V_1 = A_2V_2 \qquad \Rightarrow \frac{\pi}{4}(0.05)^2 \times 0.6 = \frac{\pi}{4}(0.026)^2 \times V_2$$

$\therefore \mathbf{V_2} = 2.22$ m/s

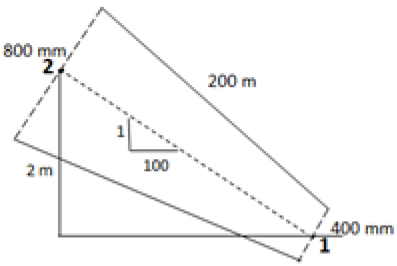
Applying Bernoulli's equation between points **1** and **2** (assuming datum is passing through **1**), to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2; \qquad (y_1 = 0, y_2 = 20 \text{ m})$$

$$\therefore \frac{3.8\times10^5}{9810} + \frac{(0.6)^2}{2\times9.81} + 0 = \frac{p_2}{9810} + \frac{(2.22)^2}{2\times9.81} + 20$$

$\therefore \mathbf{p_2} = 186480.25$ Pa $= 186.5$ kPa

Q3: A pipe 200 m long slopes down at 1 in 100 and tapers from 800 mm diameter at the higher end to 400 mm diameter at the lower end. The pipe carries oil of specific gravity 0.85 at the rate of 100 lps. If pressure gauge at the higher end reads 50 kN/m^2, determine velocities at the two ends and pressure at the lower end. Neglect losses.



Not to scale

Solution:

- **L =** 200 m, **SG** = 0.85, **Q =** 100 lps = 0.1 m^3/s

$$Q = A_1V_1 = A_2V_2 \qquad \Rightarrow 0.1 = \frac{\pi}{4}(0.4)^2 \times V_1 = \frac{\pi}{4}(0.8)^2 \times V_2$$

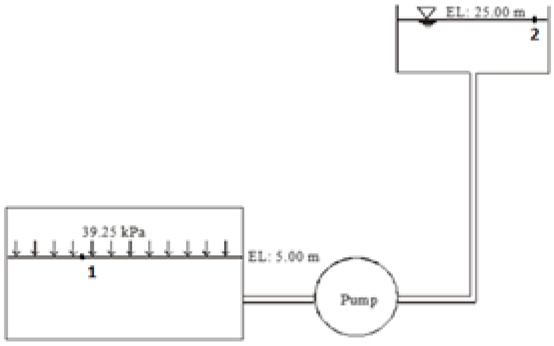
$\therefore \mathbf{V_1} = 0.796$ m/s and $\mathbf{V_2} = 0.2$ m/s

- Assuming upward flow and applying Bernoulli's equation between points **1** and **2** (assuming datum is passing through **1**), to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2; \qquad \left(y_1 = 0, y_2 = \frac{1}{100}\times200 = 2 \text{ m}\right)$$

$$\therefore \frac{p_1}{0.85\times9810} + \frac{(0.796)^2}{2\times9.81} + 0 = \frac{50\times10^3}{0.85\times9810} + \frac{(0.2)^2}{2\times9.81} + 2 \qquad \therefore \mathbf{p_1} = 66438.7 \text{ Pa} = 66.44 \textbf{ kPa}$$

Q4: A 12 HP pump with 80% efficiency is discharging oil of specific gravity 0.85 to an overhead tank as shown in **Figure**. If losses in the system are 1.2 m of oil, find the rate of flow.



Solution:

- Power of pump = 12HP = 12 × 746 = 8952 **W**

∴ Output power of pump, **P** = 0.80 × 8952 = 7161.6 **W**

- Apply Bernoulli's equation between **1** and **2**, to write

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1+h_p=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2+h_L,$$

- Here, $V_1=0, V_2=0, p_2=0$

$$\therefore \frac{39.25\times10^3}{0.85\times9810}+0+5+h_p=0+0+25+1.2 \qquad \Rightarrow h_p=16.5\text{ m}$$

- Power of pump is given by, $P=\gamma Q h_p$

$$\therefore 7161.6=0.85\times9810\times Q\times16.5$$

$$\therefore Q = 0.052\ \text{m}^3/\text{s}$$

MOMENTUM EQUATION

- Momentum is the property that a moving object has due to its mass and motion.
- Momentum is equal to the product of body's mass and its velocity – vector.

Momentum is Conserved!

- In a game of pool/billiard, if one ball <u>stops dead</u> after the collision, the other ball will move away with all the momentum. If ball is deflected, both balls will carry a portion of the momentum from collision.

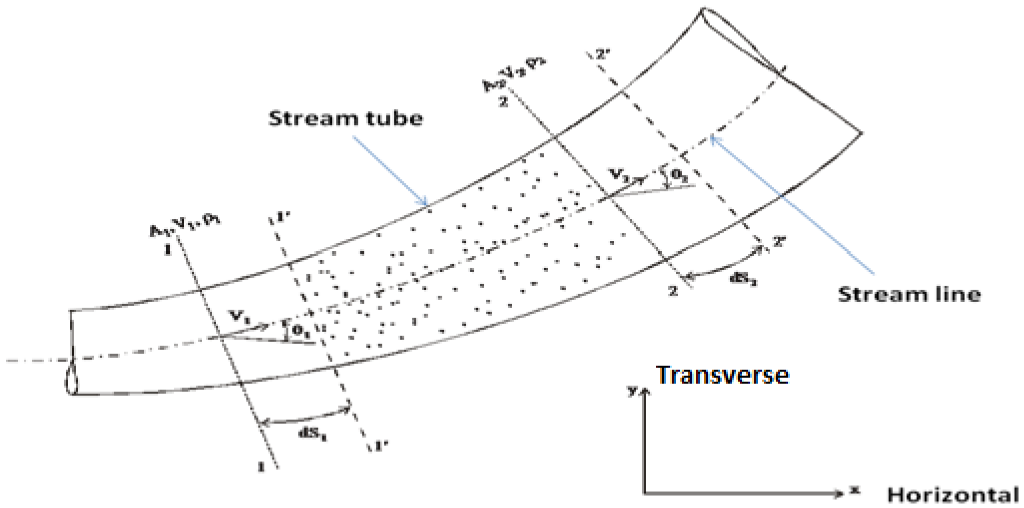
√ **ME** derived using the principle of conservation of momentum.

- Net force acting on a body in any direction is equal to the rate of change of momentum of the body in the same direction.
- Net force in any direction is (Newton's **2nd** law of motion)

$$\vec{F} = m\vec{a} = m\frac{d\vec{u}}{dt} = \frac{d}{dt}(m\vec{u})$$

❖ Momentum equation (modified form of Newton's 2nd law of motion.

- Also, $\vec{F}\,dt = d(m\vec{u})$
    - ➢ $\vec{F}\,dt$ represents impulse and $d(m\vec{u})$ represents change in momentum.
- Impulse acting on the body in any direction is equal to the change in momentum of the body in the same direction.
- This form of Eq. is applicable to finite bodies, for which action of any force is completed in a finite period of time.
- Fluid mass is continuum and any fluid motion may not be completed in a finite period of time.
- For fluids, Eq. is modified by considering the flow in a stream tube.
- Consider flow of fluid in a diverging stream tube lying on a horizontal plane (**xy**-plane)



- Consider mass of fluid between sections **1-1** and **2-2** at any time **t**
- In time **dt,** let the fluid mass moves to **1′-1′**& **2′-2′**and dS_1 & dS_2 are the displacements.

Observation: Since, region **1′-1′-2-2** is common to both **1-1-2-2**and **1′-1′-2′-2′,** thus, it will not experience any momentum change.

❖ Momentum change of fluid mass in **1-1-1′-1′** and **2-2-2′-2′,** is to be considered.

- Mass of fluid in **1-1-1′-1′** = $\rho_1 A_1 dS_1 = \rho_1 A_1 (V_1 dt)$

∴ Momentum of fluid mass in **1-1-1′-1′** = $\rho_1 A_1 V_1^2 dt$

- Similarly, momentum of fluid mass in **2-2-2′-2′** = $\rho_2 A_2 V_2^2 dt$

∴ Change in momentum of fluid mass = $(\rho_2 A_2 V_2^2 dt - \rho_1 A_1 V_1^2 dt)$

- For incompressible fluids, $\rho_1 = \rho_2 = \rho$ and discharge, $Q = A_1V_1 = A_2V_2$

∴ Change in momentum of fluid mass = $\rho Q(V_2 - V_1)dt$

- Also, change in momentum = Impulse = **F ×dt** (momentum principle)
- Equating, to get; $F = \rho Q(V_2 - V_1)$
- Here **F** is the force exerted on the fluid by the stream tube (system).
- Force exerted by the fluid on the system is (**– F**) [Newton's 3rd law].
- ❖ Equation is called linear momentum equation.
- Term (**ρQ**) is called mass rate of flow or mass flux and (**ρQV**) is called momentum rate of flow or momentum flux.**(ρQV_1)** is called momentum influx or incoming momentum rate and **(ρQV_2)** is called momentum efflux or outgoing momentum rate.

Observations:

(i) **F** is the dynamic force resulting due to change of velocity. In addition to dynamic forces, static forces are also acting at sections **1-1** and **2-2**and thus needs to be considered. Dynamic and static forces arise due to change in momentum of fluid, can be determined by using continuity, Bernoulli's and momentum equations.

(ii) If θ_1 and θ_2 are the inclinations of the stream line, at sections **1-1** and **2-2**, respectively with horizontal, then F_x and F_y can be calculated and thus the resultant force and its direction.

(iii) If stream tube is lying on the vertical plane, then F_x remains same but F_y will be modified by considering the weight of stream tube and the weight of liquid in it.

Applications of momentum equation

It is used to determine the force exerted by fluid on a flow system which involves either change in velocity or its direction and both.

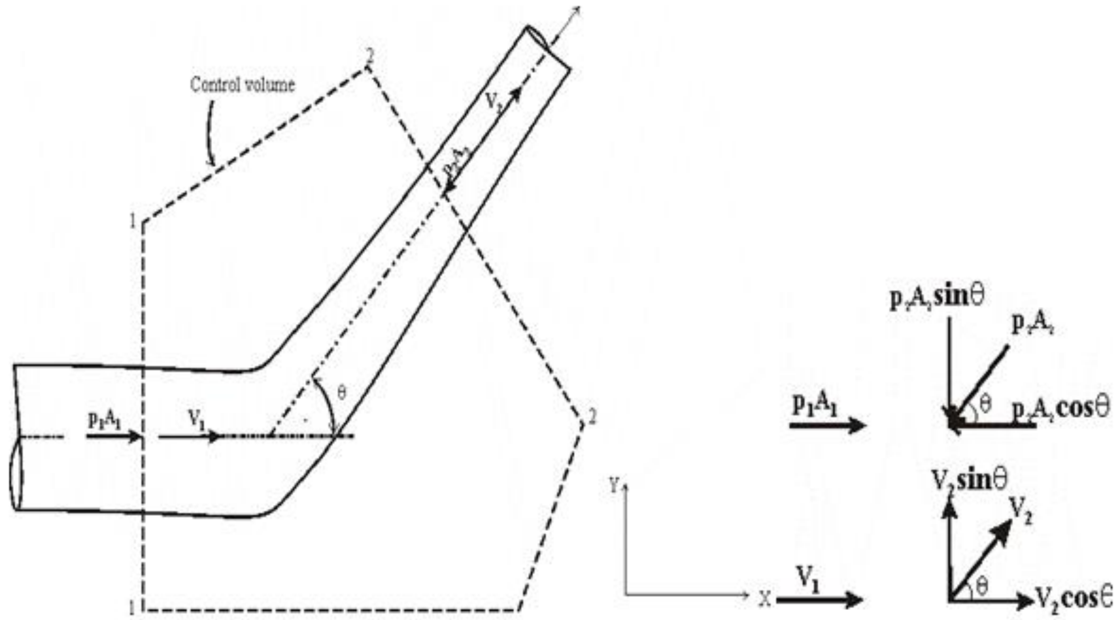
Examples:

- Force exerted on a; pipe bend, pipe nozzle, body (plates and vanes) by a fluid jet (impact of jet)
- Force caused by lift and drag of airplane wings.
- Jet propulsion (reaction of high velocity jet is utilized for the propulsion of ships, aircrafts, missiles, rockets etc.)

- Momentum equation is also used to determine the characteristics of flow when there is an abrupt change of flow section. **Examples:** Sudden enlargement and contraction of flow section, hydraulic jump in channel flow etc.

Force exerted on a <u>horizontal</u> reducing pipe bend

- Consider flow in a reducing pipe bend fitted in pipe line and lying on a <u>horizontal plane</u>.



- θ is the angle of bend and is defined as the external angle with the initial direction of flow.

✓ For solving problems, generally a control volume is selected such that incoming and outgoing flows are normal to the control surfaces.

- Two co-ordinate axes are marked.
- Static forces and velocities of flow are shown at the inlet and outlet of **C∀.**
- Momentum eq. is then applied along ***X-*** and ***Y-*** directions *i.e.*
- Net force in any direction= Rate of change of momentum in the same direction

➢ Rate of change of momentum = Mass rate of flow × Change in velocity

<u>X-direction (horizontal) [X-momentum]</u>

$$F_X + (p_1A_1 - p_2A_2\cos\theta) = \rho Q(V_2\cos\theta - V_1)$$

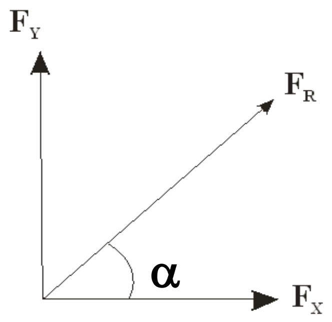
<u>Y- direction (transverse) [Y-momentum]</u>

$$F_Y + (0 - p_2A_2\sin\theta) = \rho Q(V_2\sin\theta - 0)$$

➢ **F_X** and **F_Y** are the forces acting <u>on the fluid by the bend</u> along **X-** and **Y-** directions, respectively.

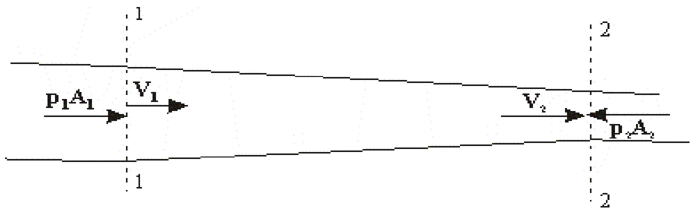
- If **F_X** and **F_Y** are +ve, then assumed direction is correct

- Resultant force, $F_R = \sqrt{F_X^2 + F_Y^2}$ and its direction with the horizontal $\tan\alpha = F_Y / F_X$.



- Resultant force on the bend is **- F_R**

Force exerted on a nozzle and reaction of jet



X-momentum

$$F_X + (p_1A_1 - p_2A_2) = \rho Q(V_2 - V_1)$$

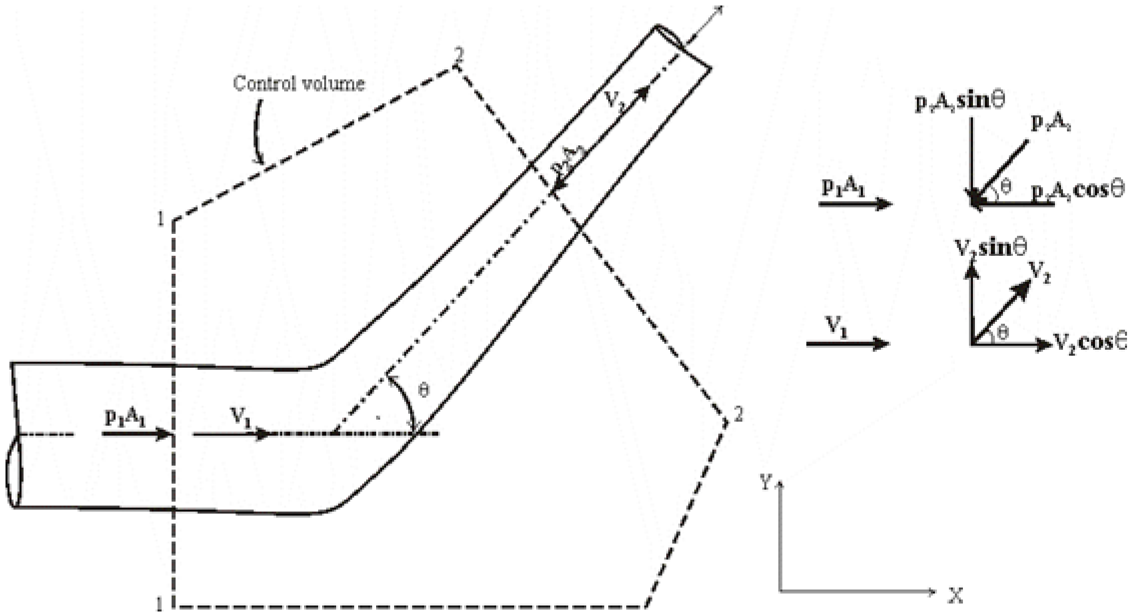
- **F_X** is the force <u>on the fluid</u> due to the action of nozzle in the **C∀.**
- Force exerted by the fluid <u>on the nozzle</u> is (**- F_X**).
- This reaction force may be used to propel the nozzle and the craft to which it is attached (aircraft, ship, rocket, submarine).

Problems:

- **Q1:** The angle of a reducing bend is 60^{o}. Its initial diameter is 300 mm and final diameter is 150 mm. The bend is fitted in a pipeline carrying a discharge of 360 lps. The pressure at the commencement of bend is 2.94 bar. The friction loss in the bend may be assumed as 10% of velocity head at exit of bend. Determine the force exerted on the bend.

Solution:

- Consider control volume as shown in **Figure.**
- Let the inlet and outlet flow sections are denoted by **1-1** and **2-2**.



- Angle of bend, θ = 60°

$$\therefore A_1 = \frac{\pi}{4}(0.3)^2 = 0.0707\ m^2 \text{ and } A_2 = \frac{\pi}{4}(0.15)^2 = 0.0177\ m^2$$

- From continuity equation,

$$Q = A_1V_1 = A_2V_2 \quad \Rightarrow 0.36 = 0.0707 \times V_1 = 0.0177 \times V_2$$

$$\Rightarrow \mathbf{V_1 = 5.1\ m/s \ \ and\ V_2 = 20.34\ m/s}$$

- Applying Bernoulli's equation between **1-1** and **2-2**, to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0.1\frac{V_2^2}{2g}$$

$$\therefore \frac{2.94 \times 10^5}{9810} + \frac{(5.1)^2}{2 \times 9.81} = \frac{p_2}{9810} + \frac{(20.34)^2}{2 \times 9.81} + 0.1\frac{(20.34)^2}{2 \times 9.81}; \qquad (p_1 = 2.94\ bar = 2.94 \times 10^5\ N/m^2)$$

- Solving for $\mathbf{p_2}$, to get

$$p_2 = 79461\ \ N/m^2$$

- Write momentum equation along **X-** and **Y-** directions, to write

X-momentum

$$F_X + (p_1A_1 - p_2A_2\cos\theta) = \rho Q(V_2\cos\theta - V_1)$$

$$\therefore F_X + (2.94\times10^5 \times 0.0707 - 79461\times0.0177\cos60) = 1000\times0.36(20.34\cos60 - 5.1)$$

- Solve for **F_X**, to get

$$F_X = -18257.4\ \text{N}\ (\rightarrow)\ \text{or}\ 18257.4\ \text{N}\ (\leftarrow)$$

Y-momentum

$$F_Y + (0 - p_2A_2\sin\theta) = \rho Q(V_2\sin\theta - 0)$$

$$\therefore F_Y + (0 - 79461\times0.0177\sin60) = 1000\times0.36(20.34\sin60)$$

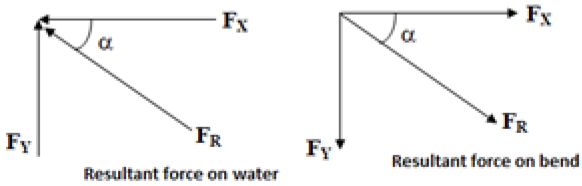
- Solve for **F_Y**, to get

$$F_Y = 7559.4\ \text{N}\ (\uparrow)$$

- Resultant force, $F_R = \sqrt{F_X^2 + F_Y^2}$

$$\Rightarrow F_R = 19769.5\ \text{N}$$

- Resultant force on bend (**- F_R**).

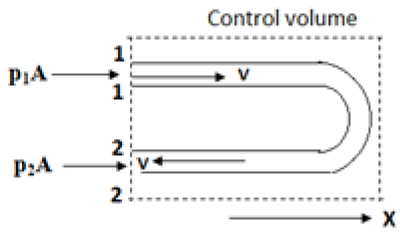


Q2: A pipe of 200 mm diameter conveying 0.18 m^3/s of water has a 180^o bend in the horizontal plane. The pressure intensities at the inlet and outlet of bend are 290 **kPa** and 280 **kPa**. Find the resultant force exerted by water on the bend.

Solution: D = 200 mm, Q = 0.18 m^3/s, **θ** = 180°, **p_1** = 290 kPa, **p_2** = 280 kPa

$$A = \frac{\pi}{4}(0.2)^2 = 0.0314\text{m}^2$$

- **$V_1 = V_2 = V = Q/A = 5.73$ m/s**



X-momentum

$$F_X + (p_1A + p_2A) = \rho Q[V - (-V)] = 2\rho QV$$

- Solve for **F_X**, to get
- **F_X = - 15835.2 N** ($\rightarrow$) or **15835.2 N** ($\leftarrow$)
- Here **F_X** is the resultant force exerted on water by the bend.

  $\therefore$ Resultant force exerted on the bend by water = **15835.2 N** ($\rightarrow$)

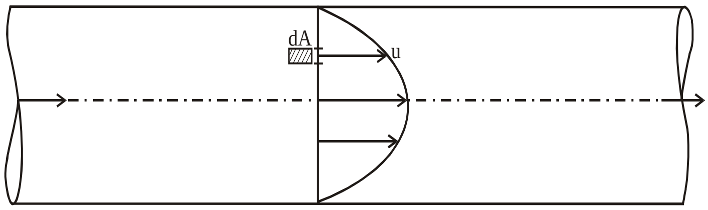
KINETIC ENERGY AND MOMENTUM CORRECTION FACTORS:

- In the derivation of Bernoulli's and momentum equations, average or uniform velocity of flow at a section (a point) is considered.
- However, in case of real fluids, velocity at any section is non-uniform.

- The effect of actual (non-uniform) velocity distribution on kinetic energy flux in Bernoulli's equation and momentum flux in momentum equation is accounted for using factors known as kinetic energy correction factor and momentum correction factor, respectively.

Analysis of kinetic energy connection factor (α)

- Consider the flow of a liquid through a pipe as shown in **Figure**.



Actual velocity distribution for flow in a pipe

- Draw the velocity distribution along a section.
- If **dA** = elementary cross sectional area of flow, **u** = local velocity of flow through **dA**, which may be assumed uniform and ρ = mass density of fluid

$\therefore$ Mass rate of flow through **dA** = $\rho u dA$

- Kinetic energy possessed by the fluid per sec or kinetic energy flux through area **dA** = $\frac{1}{2}(\rho u dA)u^2$

$\therefore$ Total kinetic energy flux through area **A** = $\frac{1}{2}\rho \int u^3 dA$ **(1)**

- If kinetic energy flux is computed using mean velocity of flow **V** then kinetic energy correction factor denoted by α has to be used.
- Kinetic energy flux through **A** using **V** = $\alpha \frac{1}{2}(\rho VA)V^2 = \alpha \frac{1}{2}\rho V^3 A$ **(2)**
- Equating Eqs. (1) and (2) and simplify, to get

$$\alpha = \frac{1}{V^3 A}\int u^3 dA \qquad \textbf{(3)}$$

- This is the required expression for kinetic energy correction factor.

Analysis of momentum correction factor (β)

- Momentum flux through area **dA**

= Mass rate of flow × local velocity through **dA** $= \rho u dA \times u = \rho u^2 dA$

$\therefore$ Total momentum flux through **A** = $\int \rho u^2 dA$

- Momentum flux calculated using mean vel. of flow, **V** = $\beta(\rho VA)V = \beta \rho V^2 A$
- Equating equations and simplify, to get

$$\beta = \frac{1}{V^2 A}\int u^2 dA \qquad \textbf{(4)}$$

- This is the required expression for momentum correction factor β.

✓ Mathematically, in Eqs. (3) and (4), numinator > denominator and therefore the value of α and β is always greater than 1.

- However, the actual value of α and β depends on the type of flow as shown in **Table**:

Type of flow	Velocity distribution equation	α	β
Laminar	$u = u_{max}(1-r^2/R^2)$	2	1.33
Turbulent	$u = u_{max}(y/R)^{1/7}$	1.03 to 1.06	1.02 to 1.05

- **Observations: (i)** The mean velocity **V** can be calculated using

$$V = \frac{1}{A}\int u dA$$

(ii) Using these correction factors, Bernoulli's and momentum equations are modified as:

$$\frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + y_2 + h_L \text{ and}$$

$$F_X + (p_1 A_1 \cos\theta_1 - p_2 A_2 \cos\theta_2) = \rho Q(\beta_2 V_2 \cos\theta_2 - \beta_1 V_1 \cos\theta_1)$$

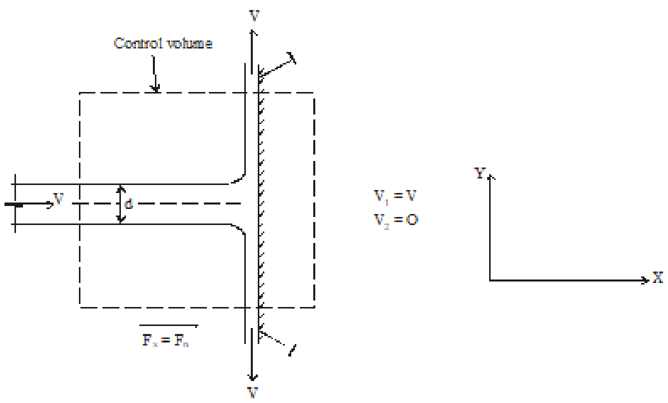
$$F_Y + (p_1 A_1 \sin\theta_1 - p_2 A_2 \sin\theta_2) = \rho Q(\beta_2 V_2 \sin\theta_1 - \beta_1 V_1 \sin\theta_1)$$

IMPACT OF JET

- When a jet of water (issuing from a nozzle) strikes an obstruction placed in its path, a force is exerted by the jet on the obstruction. This force is known as impact of jet.
- Since this force involves change of velocity and therefore momentum equation can be used to determine this force.
- Let us determine the force exerted by a fluid jet on a stationary flat plate and a curved plate.

Stationary flat plate

- Let a jet of diameter **d** and velocity **V** strikes a stationary flat plate at its centre. The plate is held normal to the jet as shown in **Figure**.



Jet striking a flat plate normal to jet

- The following assumptions are made in the analysis:

(i) Jet after striking the plate leaves it tangentially *i.e.* jet is deflected through an angle of 90°.

(ii) There is no loss of energy due to impact of jet (friction between the jet and the plate is neglected or the plate is smooth). Thus, the jet will move on and off the plate with same velocity **V.**

(iii) Pressure is atmospheric everywhere so that static forces are zero.

- Under these assumptions, apply momentum equation along **X**- and **Y**- directions, to write

X-momentum

- Force exerted on the plate, F_X = Mass rate of flow (Initial velocity – Final velocity)

$$= \rho Q(V_1 - V_2)$$

- Here, $V_1 = V$ = Initial velocity with which jet strikes the plate (striking velocity)
- $V_2 = 0$ = Final velocity along **X**-direction (plate is impervious)

➢ ρ = mass density of fluid and **Q** = discharge through the nozzle.

Y-momentum

- Force exerted on plate, F_Y = Mass rate of flow (Initial velocity – Final velocity)

$$= \rho Q[0 - (V - V)] = 0$$

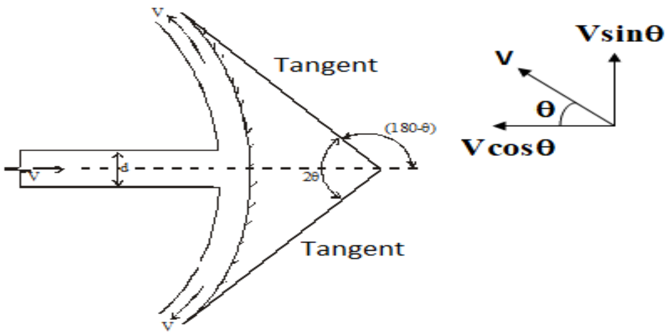
∴ The only force exerted on the plate is along **X**-direction, which is also the normal direction to the plate.

∴ Normal force exerted on the plate, $F_n = \rho QV = \rho AV^2 = \rho Q^2/A$

➢ **A** is the cross-sectional area of jet.

Curved plate

- Let a jet of water strikes a fixed symmetrical curved plate at its centre as shown in **Figure.**
- The jet after striking the vane will leave it tangentially.
- Draw two tangents at the outlet tips of vane and let **2θ** be the angle between the two tangents.



Jet striking a curved plate at its centre

- Thus, after striking the vane, the jet will be deflected on each side through (**180^o - θ**).
- Velocity at the outlet of vane can be resolved into two components i.e. **Vcosθ** and **Vsinθ**.

∴ Normal force exerted on the vane, **F_n** = $\rho Q(V_1 - V_2)$

- ➢ Here **V_1 = V and V_2 = -Vcos**θ

∴Normal force exerted on the plate, $F_n = \rho Q[V - (-V\cos\theta)]$

$$\Rightarrow F_n = \rho QV(1 + \cos\theta) \qquad \textbf{(1)}$$

Observations:

(i) When **θ = 90°**, the vane will reduce to a flat plate held normal to the jet direction and Eq. (1) reduces to:

- ➢ This is the same equation as that for a flat plate held normal to the jet direction.

(ii) When **θ = 0°**, the vane will become semi-circular (hemispherical).

∴Normal force exerted on the hemispherical vane is, **F_n** $= 2\rho QV = \dfrac{2\rho Q^2}{A}$

- ➢ This is twice of the force exerted on a flat plate held normal to the flow direction.
- ➢ **A** is the area of jet.

(iii) If some loss of energy occurs due to the impact or friction, then outgoing velocity will be less than the incoming velocity, **V**.

- If velocity at the outlet tip (outgoing velocity) is **k_LV**, then, **F_n = ρQV(1 + $K_L$cosθ**)
- ➢ **K_L** is called the loss coefficient of vane.