School of Mathematics
Thapar Institute of Engineering and Technology, Patiala
Optimization Techniques (UMA035)
Practice sheet No. 5

1. Write the duals of the following problems:
   (i) Max $z = x_1 - 2x_2 + 4x_3 - 3x_4$, s/t $x_1 + x_2 - 3x_3 + x_4 = 9, 3x_1 + 5x_2 + 2x_3 - 7x_4 \le 5$,
   $x_1 - 3x_2 + 5x_4 \ge 8, x_1, x_2, x_3, x_4 \ge 0$

   (ii) Min $z = 2x_1 + x_2 + x_3$, s/t $x_1 + x_2 - x_3 \ge 1, -2x_1 + x_3 \le 0, x_1 - x_2 + x_3 = 2,$
   $x_1 \ge 0, x_2 \le 0$

2. Show that the following problem and its dual are infeasible.
   Max $z = 8x_1 + 6x_2$, s/t $2x_1 - x_2 \ge 2, -4x_1 + 2x_2 \ge 1$, $x_1, x_2 \ge 0$

3. Write the dual of the problem: Max $z = x_1 + 2x_2 + x_3$,
   s/t $x_1 + x_2 - x_3 \le 2,\ x_1 - x_2 + x_3 = 1,\ 2x_1 + x_2 + x_3 \ge 2,\ x_1 \ge 0,\ x_2 \le 0$
   and using the duality theory show that maximum of z can not exceed one.

4. Show by inspection that the dual of the problem:
   Max $z = -2x_1 + 3x_2 + 5x_3$, s/t $x_1 - x_2 + x_3 \le 15,\ x_1, x_2, x_3 \ge 0$
   is infeasible. What can you say about the solution of the primal?

5. (i)Solve the following problem graphically. Write its dual. Then using the complementary slackness theorem obtain the solution of the dual problem.
   Maximize $z = 2x_1 + 3x_2$
   Subject to $x_1 + x_2 \le 3,\ 2x_1 + 3x_2 \ge 3,\ -x_1 + x_2 \le 0,\ x_1 \le 2,\ x_1, x_2 \ge 0$.

   (ii) Write the dual of the problem:
   Minimize $z = x_1 + 2x_2 + 3x_3 + 4x_4$,
   Subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \ge 30, 2x_2 + 3x_3 + 2x_4 \ge 40\ x_1, x_2, x_3, x_4 \ge 0$.
   Solve the dual graphically. Then using the complementary slackness theorem obtain the solution of the above problem.

6. (i)Describe the dual simplex method. Using it solve:

   $Min\ z = 2x_1 + x_2,\ s/t\ 3x_1 + x_2 \ge 3,\ 4x_1 + 3x_2 \ge 6, x_1 + 2x_2 \le 3, x_1, x_2 \ge 0$.

   (ii) Min $z = x_1 + 4x_2 + 3x_4$, s/t $x_1 + 2x_2 - x_3 + x_4 \ge 3$
   s/t $x_1 + 2x_2 - x_3 + x_4 \ge 3, -2x_1 + x_2 + 4x_3 + x_4 \ge 2,\ x_1, x_2, x_3,\ x_4 \ge 0$