Course: UMA 035 (Optimization Techniques)

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Let X_1, X_2,…, X_n , X_{n+1} be (n+1) points in R^n. Then the convex set $\{a_1X_1 + a_2X_2 +$ … $+a_nX_n + a_{n+1}X_{n+1} : a_1>=0, a_2>=0,…, a_n>=0, a_{n+1}>=0$ and $a_1 + a_2 +…+ a_n + a_{n+1} =1\}$ is called a simplex in R^n.

Simplex in R

Putting n =1,

Let X_1 and X_2 be two points in R. Then the convex set $\{a_1X_1 + a_2X_2 : a_1>=0, a_2>=0$ and $a_1 + a_2 =1\}$ is called a simplex in R.

Simplex in R^2

Putting n =2,

Let X_1, X_2 and X_3 be three points in R^2. Then the convex set $\{a_1X_1 + a_2X_2 + a_3X_3: a_1>=0, a_2>=0, a_3>=0$ and $a_1 + a_2 + a_3 =1\}$ is called a simplex in R^2.

Step 1:

Convert the considered LPP in standard form.

Step 2:

If the problem is of minimization then convert it into maximization by changing the sign of all the coefficients in the objective function.

Step 3:

Construct the following Table.

Coefficients from objective function

Coefficient of Basic Variables in Objective function (C_B)	Basic Variables	x_1 x_2 ... x_n S_1 S_2 ... S_m	Solution	Minimum Ratio
$Z_j - C_j =$				
Coefficient of first basic variable in objective function	first basic variable	Coefficients from first constraint	Right hand side of first constraint	
Coefficient of second basic variable in objective function	second basic variable	Coefficients from second constraint	Right hand side of second constraint	
⋮	⋮	⋮	⋮	
Coefficient of mth basic variable in objective function	m^{th} basic variable	Coefficients from m^{th} constraint	Right hand side of mth constraint	

First Basic variable will be that variable corresponding to which the

column $\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ will exist in the Table.

Second Basic variable will be that variable corresponding to which the

column $\begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ will exist in the Table.

Third Basic variable will be that variable corresponding to which the

column $\begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ will exist in the Table.

$\vdots$

m^{th} Basic variable will be that variable corresponding to which the column

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ will exist in the Table.

Step 4:

Calculate $Z_j - C_j$ corresponding to each variable

For x_1

[(Coefficient of first basic variable in objective function)(First element of column of x_1)+ (Coefficient of second basic variable in objective function)(Second element of column of x_1)+…+ (Coefficient of mth basic variable in objective function)(m^{th} element of column of x_1)]−[element lying outside table of column of x_1]

For x_2

[(Coefficient of first basic variable in objective function)(First element of column of x_2)+ (Coefficient of second basic variable in objective function)(Second element of column of x_2)+…+ (Coefficient of mth basic variable in objective function)(m^{th} element of column of x_2)]−[element lying outside table of column of x_2]

⋮

For x_n

[(Coefficient of first basic variable in objective function)(First element of column of x_n)+ (Coefficient of second basic variable in objective function)(Second element of column of x_n)+…+ (Coefficient of mth basic variable in objective function)(m^{th} element of column of x_n)]−[element lying outside table of column of x_n]

[(Coefficient of first basic variable in objective function)(First element of column of S_1)+ (Coefficient of second basic variable in objective function)(Second element of column of S_1)+…+ (Coefficient of mth basic variable in objective function)(m^{th} element of column of S_1)]−[element lying outside table of column of S_1]

$\vdots$

[(Coefficient of first basic variable in objective function)(First element of column of S_m)+ (Coefficient of second basic variable in objective function)(Second element of column of S_m)+…+ (Coefficient of mth basic variable in objective function)(m^{th} element of column of S_m)]−[element lying outside table of column of S_m]

Step 5:

Check that all the calculated values of $Z_j - C_j$ are >=0 or not

Case (i)

If all the calculated values of $Z_j - C_j$ are >=0 then the solution is optimal.

Case (ii)

If one or more values of $Z_j - C_j$ is negative. Then the solution is not optimal. Go to Step 6.

Step 6:

Find minimum {negative values of $Z_j - C_j$}. The variable corresponding to which the minimum exist is called entering variable.

Step 7:

Find minimum {negative values of $Z_j - C_j$}. The variable corresponding to which the minimum exist is called entering variable.

Step 8:

In the last column of the table, find minimum

$$\{\frac{number\ corresponding\ to\ the\ first\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ first\ basic\ variable\ in\ the\ column\ of\ entering\ variable^*},$$

$$\frac{number\ corresponding\ to\ the\ second\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ second\ basic\ variable\ in\ the\ column\ of\ entering\ variable^*}, \dots,$$

$$\frac{number\ corresponding\ to\ the\ m^{th}\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ m^{th}\ basic\ variable\ in\ the\ column\ of\ entering\ variable^*}\}.$$

*If the element is negative or zero then do not consider it. Only positive numbers in the denominator.

The variable corresponding to which the minimum exist is called leaving variable.

Step 8:

Construct a new table by replacing the leaving basic variable with the entering variable in the second column (Basic variables) of the table.

Step 9:

Apply the following row operations to find the elements of the table.

- $R_1 \rightarrow R_1 - (a_{1i}) * R_p / (a_{pi})$

- $R_2 \rightarrow R_2 - (a_{2i}) * R_p / (a_{pi})$

- $\vdots$

- $R_p \rightarrow R_p / (a_{pi})$

- $\vdots$

-

- $R_m \rightarrow R_m - (a_{mi}) * R_p / (a_{pi})$

These operations have been obtained as follows:

- **Write m rows:**

  R_1

  R_2

  $\vdots$

  R_m

- **Insert arrow in front of each row:**

$R_1 \rightarrow$

$R_2 \rightarrow$

$\vdots$

$R_m \rightarrow$

➢ **Insert same row after arrow**

$R_1 \rightarrow R_1$

$R_2 \rightarrow R_2$

$\vdots$

$R_m \rightarrow R_m$

➢ **Insert division sign in that row corresponding to which leaving variable exist (let R_p)**

$R_1 \rightarrow R_1$

$R_2 \rightarrow R_2$

$\vdots$

$R_p \rightarrow R_p/$

$\vdots$

$R_m \rightarrow R_m$

➢ **Insert negative sign in remaining rows**

$R_1 \rightarrow R_1-$

$R_2 \rightarrow R_2-$

$$\vdots$$

$R_p \rightarrow R_p/$

$$\vdots$$

$R_m \rightarrow R_m-$

> **Insert the elements of that column corresponding to which entering variable exist (let ith column)**

$R_1 \rightarrow R_1-(a_{1i})$

$R_2 \rightarrow R_2-(a_{2i})$

$$\vdots$$

$R_p \rightarrow R_p/(a_{pi})$

$$\vdots$$

$R_m \rightarrow R_m-(a_{mi})$

> **Multiply to all with $R_p/(a_{pi})$**

> $R_1 \rightarrow R_1-(a_{1i})*R_p/(a_{pi})$

> $R_2 \rightarrow R_2-(a_{2i})*R_p/(a_{pi})$

> $\vdots$

> $R_p \rightarrow R_p/(a_{pi})$

> $\vdots$

>

➢ $R_m \rightarrow R_m - (a_{mi}) * R_p / (a_{pi})$

Step 10:

Go to Step 5 and repeat the procedure until an optimal solution is obtained.

Example:

Solve the following LPP by the Simplex method.

Minimize $(x_1 - 3x_2 + 2x_3)$

Subject to

$3x_1 - x_2 + 3x_3 <= 7$,

$-2x_1 + 4x_2 <= 12$,

$-4x_1 + 3x_2 + 8x_3 <= 10$

$x_1 >= 0$,

$x_2 >= 0$,

$x_3 >= 0$.

Solution

Minimize $(x_1 - 3x_2 + 2x_3)$

Subject to

$3x_1 - x_2 + 3x_3 + S_1 = 7$,

$-2x_1 + 4x_2 + S_2 = 12$,

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1 >= 0$,

$x_2 >= 0,$

$x_3 >= 0,$

$S_1 >= 0,$

$S_2 >= 0,$

$S_3 >= 0.$

Maximize $(-x_1 + 3x_2 - 2x_3)$

Subject to

$3x_1 - x_2 + 3x_3 + S_1 = 7,$

$-2x_1 + 4x_2 + S_2 = 12,$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1 >= 0,$

$x_2 >= 0,$

$x_3 >= 0,$

$S_1 >= 0,$

$S_2 >= 0,$

$S_3 >= 0.$

C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									

		-1	3	-2	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									
		3	-1	3	1	0	0		
		-2	4	0	0	1	0		
		-4	3	8	0	0	1		

		−1	3	−2	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									
	S_1	3	−1	3	1	0	0		
	S_2	−2	4	0	0	1	0		
	S_3	−4	3	8	0	0	1		

		−1	3	−2	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									
	S_1	3	−1	3	1	0	0	7	
	S_2	−2	4	0	0	1	0	12	
	S_3	−4	3	8	0	0	1	10	

		−1	3	−2	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									
0	S_1	3	−1	3	1	0	0	7	
0	S_2	−2	4	0	0	1	0	12	
0	S_3	−4	3	8	0	0	1	10	

$$[(0)(0)+(0)(0)+ (0)(1)] -(0) =0$$

		−1	3	−2	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j =$									
0	S_1	3	−1	3	1	0	0	7	
0	S_2	−2	4	0	0	1	0	12	
0	S_3	−4	3	8	0	0	1	10	

$$[(0)(0)+(0)(1)+ (0)(0)] -(0) =0$$

$$[(0)(3)+(0)(-2)+ (0)(-4)] -(-1) =1$$

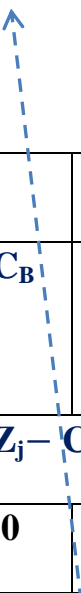$$[(0)(1)+(0)(0)+ (0)(0)] -(3) =-3$$

$$[(0)(-1)+(0)(4)+ (0)(3)] -(-2) =2$$

$$[(0)(3)+(0)(0)+ (0)(8)] -(0) =0$$

Entering variable

		3	−2	0	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j − C_j =$		1	−3	2	0	0	0		
0	S_1	3	−1	3	1	0	0	7	
0	S_2	−2	4	0	0	1	0	12	
0	S_3	−4	3	8	0	0	1	10	

		3	−2	0	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j − C_j =$		1	−3	2	0	0	0		
0	S_1	3	−1	3	1	0	0	7	7/-
0	S_2	−2	4	0	0	1	0	12	12/4=3
0	S_3	−4	3	8	0	0	1	10	10/3=3.33

Leaving Variable

C_B	Basic Variables	3	−2	0	0	0	0	Solution	Minimum Ratio
		x_1	x_2	x_3	S_1	S_2	S_3		
$Z_j - C_j =$		1	−3	2	0	0	0		
0	S_1	3	−1	3	1	0	0	7	7/-
0	S_2	−2	4	0	0	1	0	12	12/4=3
0	S_3	−4	3	8	0	0	1	10	10/3=3.33