Lecture-12

Operations on Signals

A group of basic operations is applied to an input signal to result the output signal in signal processing. Equation (1) shows that the mathematical transformation from one signal to another signal.

$$y(n) = T[x(n)] \tag{1}$$

The following are the basic set of operations:

(a) Shifting (b) Time reversal (c) Time scaling (d) Scalar multiplication

(e) Signal multiplier (f) Signal addition

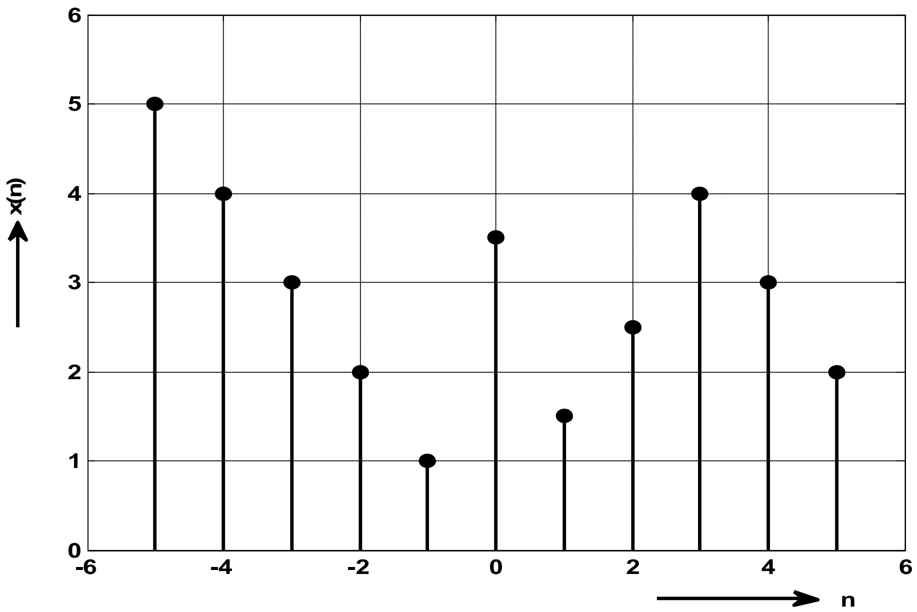
Shifting

In shift operations the input sequence is taken and the values of the sequence are shifted by an integer increment of the independent variable, which may delay or advance the sequence in time. Mathematically, the shifting operation can be represented by

$$y(n) = x(n-k)$$

where x(n) is the input and y(n) is the output.

Figure 1 shows the sequence x(n).



x(-5) = 5, x(-4) = -4, x(-3) = -3,
x(-2) = -2, x(-1) = -1,
x(0) = 3.5, x(1) = 1.5, x(2) = 2.5,
x(3) = 4, x(4) = 3, x(5) = 2

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

Figure 1 : Original Signal

For positive values of 'k', the shifting delays the sequence shown in Figure 2 for k = 3 i.e., delayed by three samples.

y(n) = x(n-3)

y(0)= x(-3) = 3

y(1) = x(-2) = 2

y(2) = x(-1) = 1

y(3) = x(0) = 3.5

y(4) = x(1) = 1.5

y(5) = x(2) = 2.5

y(6) = x(3) = 4

y(7) = x(4) = 3

y(8) = x(5) = 2

y(9) = x(6) = 0

……………

y(-1) = x(-4) = 4

y(-2) = x(-5) = 5

y(-3) = x(-6) =0

……….

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

$$y(n) = x(n-3) = \{5, 4, \underset{\uparrow}{3}, 2, 1, 3.5, 1.5, 2.5, 4, 3, 2\}$$

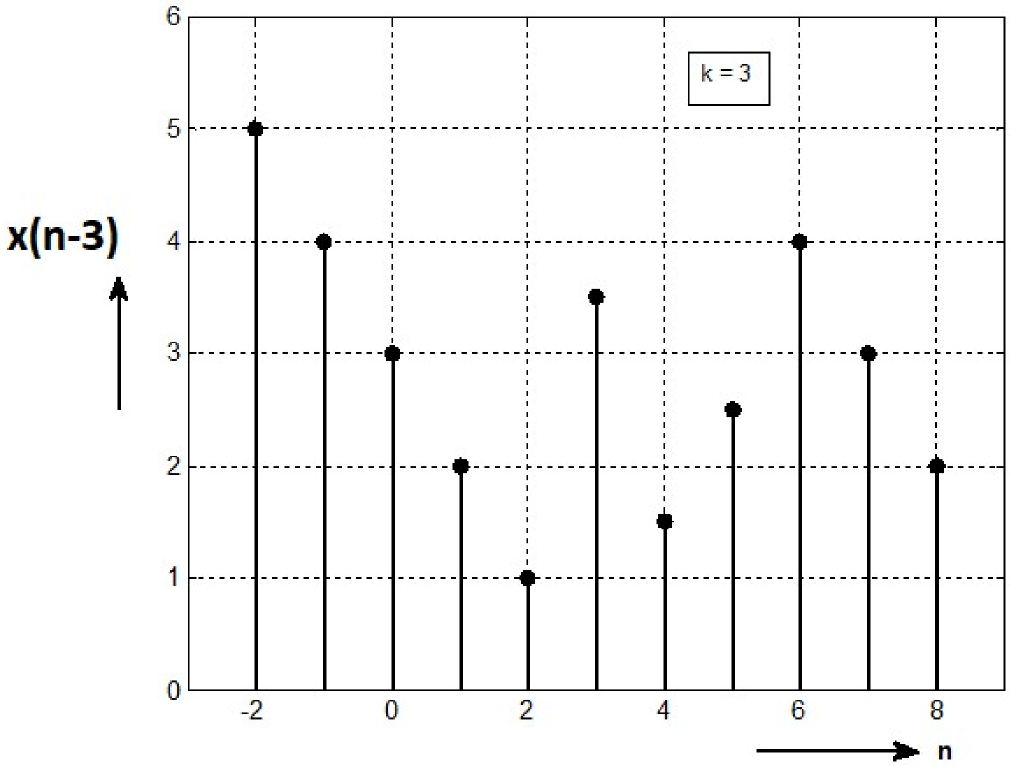


Figure 2 : Delayed by 3 samples

$$x(n-3) = \{5, 4, \underset{\uparrow}{3}, 2, 1, 3, 1.5, 2.5, 4, 3, 2\}$$

y(n) = x(n+3)

y(0)= x(3) = 4

y(1) = x(4) = 3

y(2) = x(5) = 2

y(3) = x(6) = 0

……….

y(-1) = x(2) = 2.5

y(-2) = x(1) = 1.5

y(-3) = x(0) = 3.5

y(-4) = x(-1) = 1

y(-5) = x(-2) = 2

y(-6) = x(-3) = 3

y(-7) = x(-4) = 4

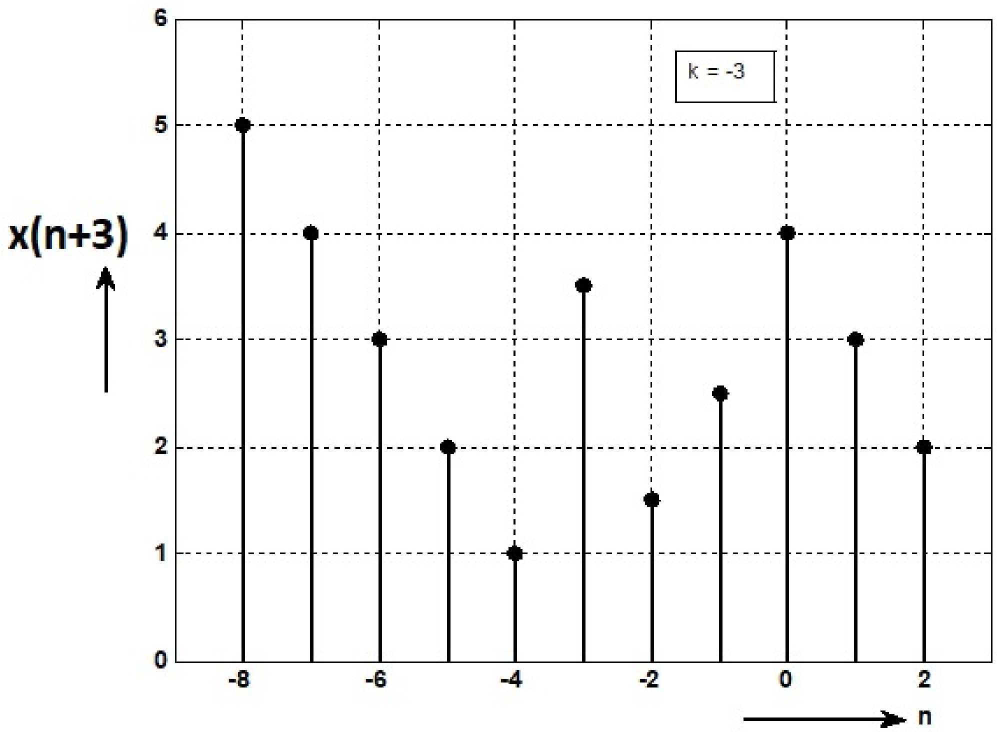
y(-8) = x(-5) = 5

y(-9) = x(-6) =0

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

$$y(n) = x(n+3) = \{5, 4, 3, 2, 1, 3.5, 1.5, 2.5, \underset{\uparrow}{4}, 3, 2\}$$

For negative values of 'k', the shifting advances the sequence shown in Figure 3 for k = –3 i.e., advanced by three samples.



$$x(n+3) = \{5, 4, 3, 2, 1, 3.5, 1.5, 2.5, \underset{\uparrow}{4}, 3, 2\}$$

Figure 3 : Advanced by three samples

Time Reversal

The folding of sequence $x(n)$ about $x = 0$ gives the time reversal of the sequence, which is denoted by $x(-n)$. Figure 4 shows the time reversal of the sequence $x(n)$.

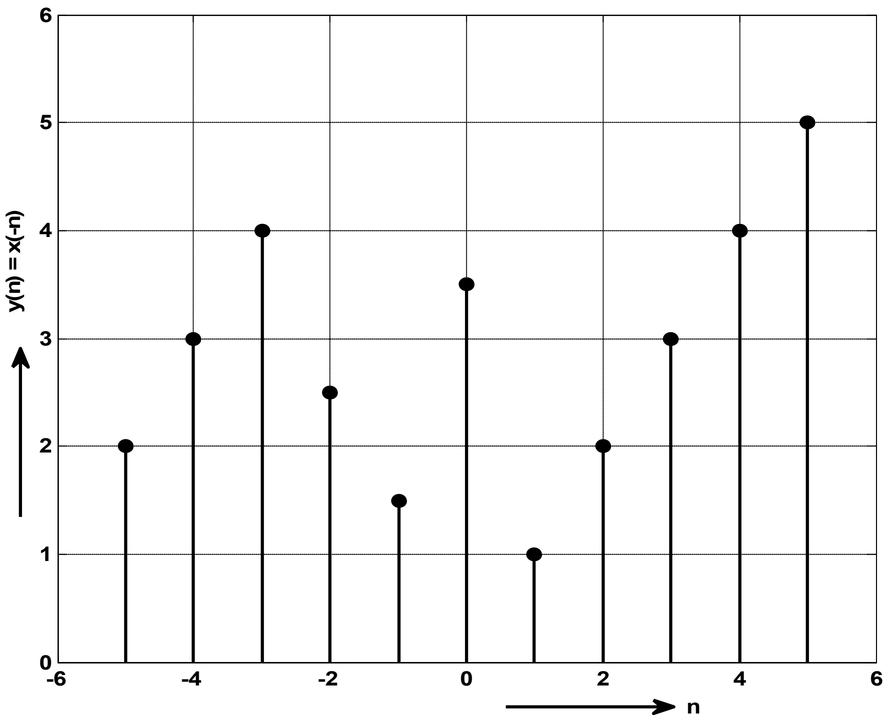
y(n) = x(-n)

y(0) = x(0), y(1) = x(-1)=1, y(2)= x(-2)=2,y(3)=x(-3)=3, y(4) = x(-4)=4, y(5) = x(-5)=5

y(-1) = x(1)=1.5, y(-2) = x(2) =2.5, y(-3) = x(3) = 4, y(-4) = x(4) = 3, y(-5) = x(5) =2

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

$$x(-n) = \{2, 3, 4, 2.5, 1.5, \underset{\uparrow}{3.5}, 1, 2, 3, 4, 5\}$$



$$x(-n) = \{2, 3, 4, 2.5, 1.5, \underset{\uparrow}{3.5}, 1, 2, 3, 4, 5\}$$

Figure 4: Time reversal of signal x(n) i.e., x(–n)

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

y(n) = x(-n+3)

y(-1) = x(4) = 3

y(-2) = x(5) = 2

y(0) = x(3) = 4

y(1) = x(2) = 2.5

y(2) = x(1) = 1.5

y(3) = x(0) = 3.5

y(4) = x(-1) = 1

y(5) = x(-2) = 2

y(6) = x(-3) = 3

y(7) = x(-4)

y(8) = x(-5)

Figure 5 shows the signal x(–n + 3), which is obtained by delaying x(–n) three units of time. Here x(–n + 3) = x(3 – n) = x[–(n–3)].

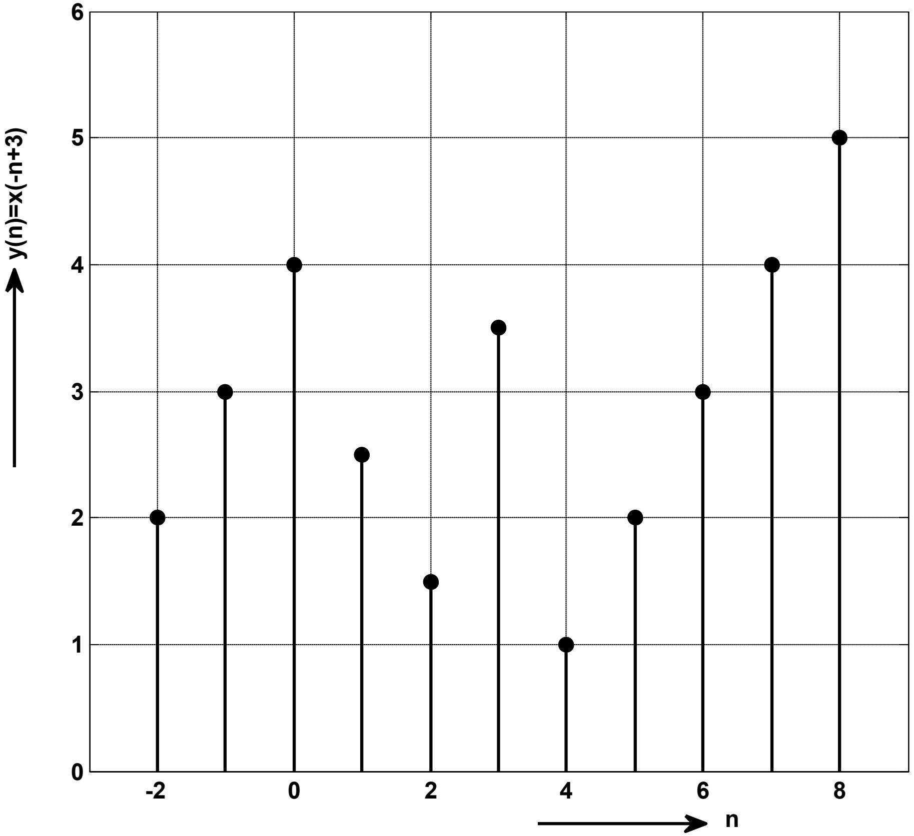


Figure 5: Plot of x(–n+3)

$$x(-n) = \{2, 3, 4, 2.5, 1.5, \underset{\uparrow}{3.5}, 1, 2, 3, 4, 5\}$$

$$x(-n+3) = \{2, 3, \underset{\uparrow}{4}, 2.5, 1.5, 3.5, 1, 2, 3, 4, 5\}$$

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

y(n) = x(-n-3)

y(1) = x(-4) = 4

y(2) = x(-5) = 5

y(0) = x(-3) = 3

y(-1) = x(-2) = 2

y(-2) = x(-1) = 1

y(-3) = x(0) = 3.5

y(-4) = x(1) = 1.5

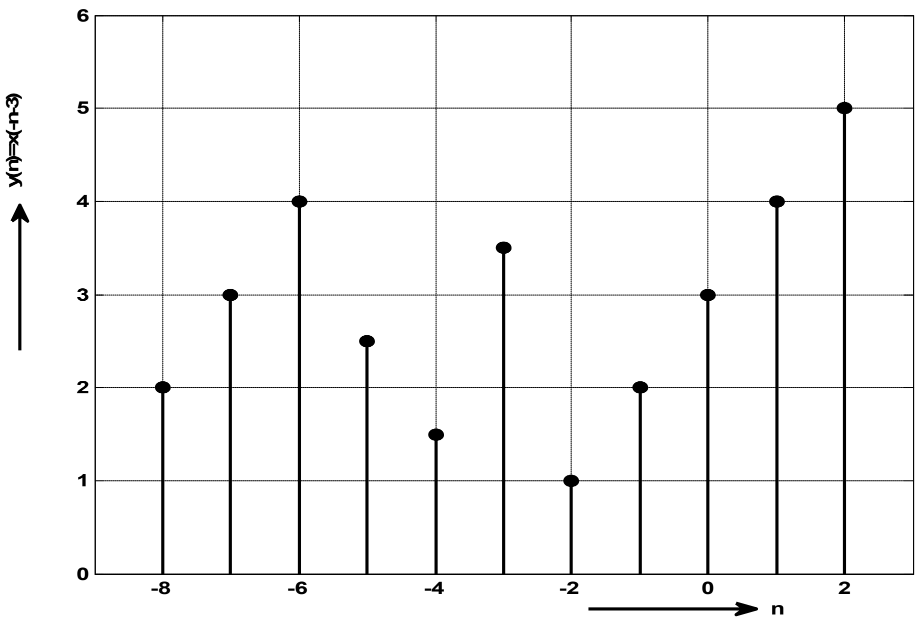
y(-5) = x(2) = 2.5

y(-6) = x(3) = 4

y(-7) = x(4) = 3

y(-8) = x(5) = 2

Figure 6 shows the signal x(–n–3), which is obtained by advancing x(–n) three units of time respectively. Here x(–n–3) = x[–(n + 3)].



$$x(-n) = \{2, 3, 4, 2.5, 1.5, 3\underset{\uparrow}{.}5, 1, 2, 3, 4, 5\}$$

$$x(-n-3) = \{2, 3, 4, 2.5, 1.5, 3.5, 1, 2, \underset{\uparrow}{3}, 4, 5\}$$

Figure 6: Plot of x(–n–3)

Time Scaling

Time scaling can be accomplished by substituting n by λn in the sequence x(n). To plot y(n) = x(2n), the following calculations are carried out.

From Figure 1, we have

$$x(n) = \{5, 4, 3, 2, 1, 3.5, 1.5, 2.5, 4, 3, 2\}$$
$$\uparrow$$

x(0) = 3.5

x(1) = 1.5 , x(–1) = 1

x(2) = 2.5, x(–2) = 2

x(3) = 4, x(–3) = 3

$x(4) = 3, \quad x(-4) = 4$

$x(5) = 2, \quad x(-5) = 5$

$x(6) = x(-6) = 0$

and so on.

Let y(n) = x(2n)

$$x(n) = \{5, 4, 3, 2, 1, \underset{\uparrow}{3.5}, 1.5, 2.5, 4, 3, 2\}$$

Now y(0) = x(0) = 3.5

y(1) = x(2) = 2.5, y(2) = x(4) = 3, y(3) = x(6) = 0, and so on.

Similarly,

y(–1) = x(–2) = 2, y(–2) = x(–4) = 4, y(–3) = x(–6) = 0 and so on.

Figure 7 shows the plot of y(n) = x(2n).

$$y(n) = x(2n) = \{4, 2, \underset{\uparrow}{3.5}, 2.5, 3\}$$

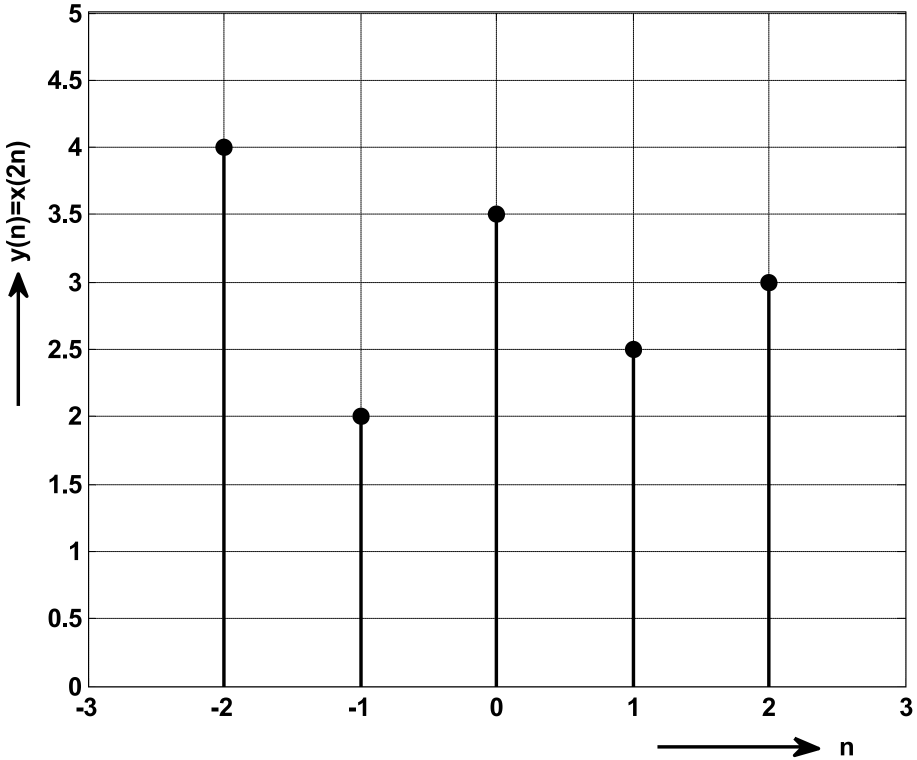


Figure 7 Plot of $y(n) = x(2n)$

Figure 7 shows that the signal x(2n) obtained by reducing the sampling on the continuous time signal by a factor 2. This process of reducing the sampling rate is known as **down sampling or decimation**.