LECTURE-14

UEI407

Discrete time Systems

The classifications of discrete time signals have been introduced. Let us introduce discrete time systems. A device or an algorithm which perform some particular operations on the discrete time signal is known as discrete time system. Figure 34 shows the input and output of a discrete time systems.

In Figure 1, y(n) is the response for the excitation x(n). The input output relationship for discrete time system is given by $y(n) = T[x(n)]$ i.e.,

$$x(n) \xrightarrow{T} y(n)$$

where T represents transformation operation which depends on the characteristic of the discrete time system.

Figure 1: Discrete time system

Classifications of Discrete Time Systems

- The discrete time systems can be classified as follows:

- Static / Dynamic
- Causal/Non–causal
- Time invariant/ Time variant
- Linear/Non–linear
- Stable/Unstable

Static and dynamic Systems

The system is said to be static if its output depends only on the present input. On the other hand, if the output of the system depends on the past input, the system is said to be dynamic.

For example, $y(n) = 5x(n)$ and $y(n) = 2x^2(n) + 3x(n)$ are the static systems whereas $y(n) = x(n) + 2\,x(n-1)$ represents the dynamic system.

Causal and Non–causal Systems

If the output of the system depends on the past and presents input only, the system is said to be causal system. On the other hand, if the output of the system depends on future inputs also, the system is also known as non–causal system.

$y(n) = x(n)+2x(n-1)$ and $y(n) = x(n)+2x(n+1)$ represent the causal and non–causal systems.

Check whether the following systems are causal or non–causal:

(i) $y(n) = x(n) + x(n-2)$

(ii) $y(n) = x(n) + x(n+2)$

(iii) $y(n) = x(3n)$

(i) The given system equation is

$$y(n) = x(n) + x(n-2)$$

The output $y(n)$ depends on the present input $x(n)$ and the past input $x(n-2)$. Therefore, the system is causal

(ii) The given system equation is

$$y(n) = x(n) + x(n+2)$$

The output $y(n)$ depends on the present input $x(n)$ and the future input $x(n+2)$. Therefore, the system is noncausal.

(iii) The given system equation is

$y(n) = x(3n)$

For $n = 1$, $y(1) = x(3)$

For $n = 2$, $y(2) = x(6)$

and so on.

The output $y(n)$ depends on the future input only. Therefore, the system is noncausal.

Time invariant and Time variant Systems

When the input output characteristic of a system does not change with shift of time origin, this system is said to be shift invariant or time invariant system. If the system has response y(n) for the excitation x(n), we have

$$x(n) \xrightarrow{T} y(n)$$

This system will be shift invariant or time invariant if,

$$x(n-k) \xrightarrow{T} y(n-k)$$

where ‘k’ is a constant.

Equation (1.46) shows that if the input is shifted by k samples, the output is also shifted by the same number of samples for any shift invariant or time invariant system.

To test this property the following points are to be checked:

- To get output y(n) of the system, it is excited by input x(n).
- The input is delayed by ‘k’ samples to get the output.

$$y(n,k) = T[x(n-k)] \quad (1)$$

where y(n,k) is the response due to delayed or shifted input.

- y(n−k) is obtained from y(n) delaying it by ‘k’ samples i.e., y(n−k) is the output delayed or shifted directly.
- The system will be shift invariant or time invariant if

  $$\bullet\, y(n,k) = y(n-k) \quad (2)$$

- The system will be shift variant or time variant when

  $$\bullet\, y(n,k) \neq y(n-k) \quad (3)$$

  for a single value of ‘k’.

Cooking rice is shift invariant operation whereas ambient temperature is shift variant parameter.

Example Determine whether the following signals are shift invariant i.e., time invariant or not.

(i) $y(n) = x(n) - x(n-2)$

(ii) $y(n) = nx(n)$

(i) Here $y(n) = x(n) - x(n-2) = T[x(n)]$

If the input is delayed by 'k' samples, the output will be

$$y(n,k) = T[x(n-k)] = x(n-k) - x(n-k-2) \tag{E1}$$

If we delay y(n) by 'k' samples, we have

$$y(n-k) = x(n-k) - x(n-k-2) \tag{E2}$$

From Eq. (E1) and Eq. (E2) we have

$$y(n,k) = y(n-k)$$

Therefore, the system is shift invariant.

(ii) Here $y(n) = nx(n) = T[x(n)]$

If the input is delayed by ‘k’ samples, the output will be

$$y(n,k) = T[x(n-k)] = nx(n-k) \tag{E3}$$

because the multiplier ‘n’ is not a part of input.

If we delay y(n) by ‘k’ samples, we have

$$y(n-k) = (n-k)\ x(n-k) \tag{E4}$$

From Eq. (E3) and Eq. (E4) we have

$$y(n,k) \neq y(n-k)$$

Therefore, the system is shift variant.

Linear and Non–linear Systems

If a system satisfies both additive and homogeneous property, the system is said to be linear. Otherwise, it will be nonlinear.

If $x_1(n)$ and $x_2(n)$ are two inputs, for additive operation

$$T[x_1(n) + x_2(n)] = T[x_1(n)] + T[x_2(n)] \qquad (4)$$

If the input is scaled by a constant and the output is also scaled by the same amount, the system is said to be homogeneous.

For any constant ‘c’ and for any input sequence x(n), if the following relation

$$T[cx(n)] = cT[x(n)] \qquad (5)$$

holds good, the system is said to be homogeneous.

Therefore, for a linear system,

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)] \tag{6}$$

where $x_1(n)$ and $x_2(n)$ are two inputs and a_1, a_2 are two constants.

Equation (6) can be further extended for ‘N’ number of input signals shown below.

$$x(n) = \sum_{k=0}^{N-1} a_k x_k(n)$$

and output of the system becomes

$$y(n) = \sum_{k=0}^{N-1} a_k y_k(n)$$

For linear system,

$$T\left[\sum_{k=0}^{N-1} a_k x_k(n)\right] = \sum_{k=0}^{N-1} a_k y_k(n)$$

Example Find the additive and homogeneous property of the following systems:

(a) $$y(n) = 5x(n) + \frac{x(n+2)x(n-1)}{x(n)}$$

(b) $$y(n) = \mathrm{Im}\{x(n)\}$$

(a) $y(n) = 5x(n) + \dfrac{x(n+2)x(n-1)}{x(n)}$

Let $y_1(n)$ and $y_2(n)$ be the responses for the inputs $x_1(n)$ and $x_2(n)$ respectively.

$$\therefore T[x_1(n) + x_2(n)] = 5[x_1(n) + x_2(n)]$$

$$+\left[\frac{\{x_1(n+2) + x_2(n+2)\}\{x_1(n-1) + x_2(n-1)\}}{x_1(n) + x_2(n)}\right]$$

$$T[x_1(n)] = 5x_1(n) + \frac{x_1(n+2)\, x_1(n-1)}{x_1(n)}$$

and $T[x_2(n)] = 5x_2(n) + \dfrac{x_2(n+2)\, x_2(n-1)}{x_2(n)}$

$$\therefore T[x_1(n)] + T[x_2(n)] \neq T[x_1(n) + x_2(n)]$$

Therefore, the system is not additive.

Again, $T[cx(n)] = 5cx(n) + \dfrac{cx(n+2)cx(n-1)}{cx(n)} = 5cx(n) + c\left[\dfrac{x(n+2)\,x(n-1)}{x(n)}\right]$

Therefore, the system is homogeneous.

(b) $\quad y(n) = \mathrm{Im}\{x(n)\}$

Let $y_1(n)$ and $y_2(n)$ be the responses for the inputs $x_1(n)$ and $x_2(n)$ respectively.

$$\therefore T[cy(n)] = Im[cx(n)] = c \ \ Im[x(n)]$$

Therefore, the system is not homogeneous in general.

$$\therefore T[x_1(n) + x_2(n)] = Im[x_1(n) + x_2(n)] \quad = Im[x_1(n)] + Im[x_2(n)]$$

$$= \ y_1(n) + y_2(n)$$

Therefore, the system is additive.

Stable and Unstable Systems

A system is said to be bounded input bounded output (BIBO) stable provided every bounded input produces a bounded output. The input is bounded only if there exists finite number of M_x such that

$$|x(n)| \leq M_x < \infty$$

On the other hand, if the output is unbounded for bounded input, the system is unstable.

Find the stable or unstable system from the following

(i) $y(n) = x^2(n)$

(ii) $y(n) = \dfrac{e^{x(n)}}{x(n-2)}$

(iii) $y(n) = \sin[x(n)]$

(iv) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

(i) Let $x(n)$ be a bounded input with $|x(n)| < M$.

Then $|y(n)| = |x(n)|^2 < M^2$

Hence $y(n)$ is stable system.

(ii) Here $y(n) = \dfrac{e^{x(n)}}{x(n-2)}$

For $x(n) = \delta(n)$, $y(n)$ is infinite for all values of n except $n = 2$.

Hence the system is not stable.

(iii) Here $y(n) = \sin[x(n)]$

$|\sin x(n)| \le 1$ for all finite values of $x(n)$.

Hence the system is stable.

(iv) Here $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

Let $x(n) = u(n)$

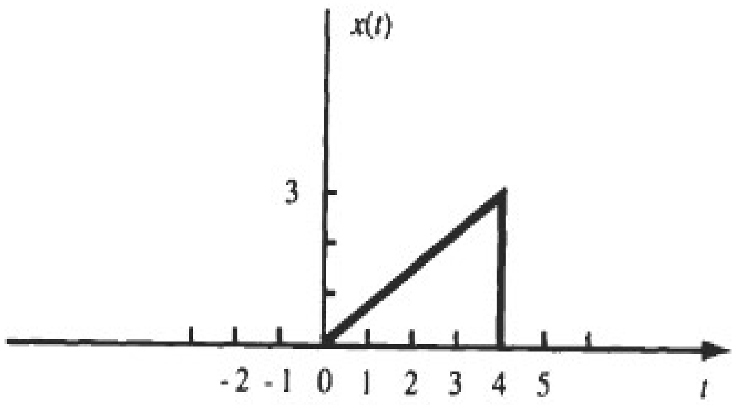
$\therefore |x(n)| \leq 1$

$\therefore y(n) = \sum_{k=-\infty}^{\infty} u(k) = \sum_{k=0}^{\infty} u(k) = \infty$

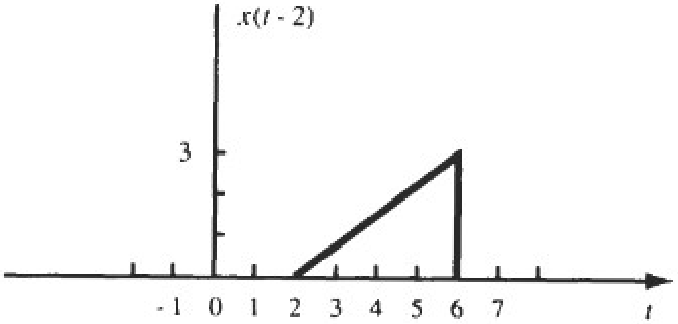
The response is unbounded and hence the system is unstable.

x(t) is shown below.

Find x(t-2), x(2t),x(t/2) and x(-t)

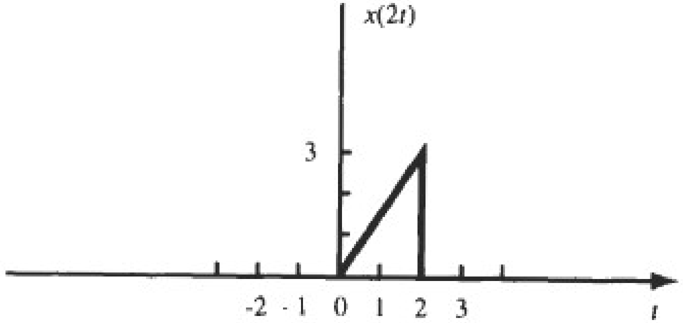


(a)



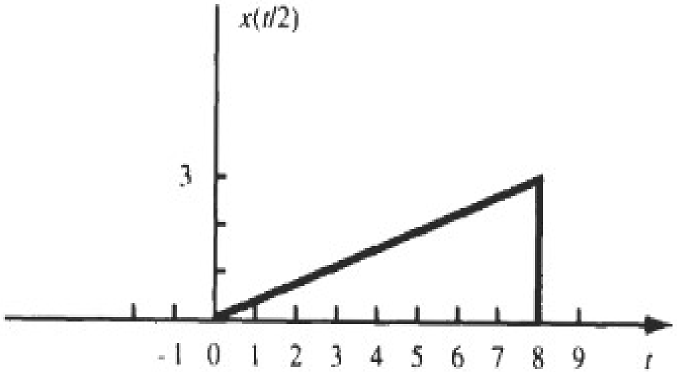
x(t - 2)
3
-1
0
1
2
3
4
5
6
7
t

(b)



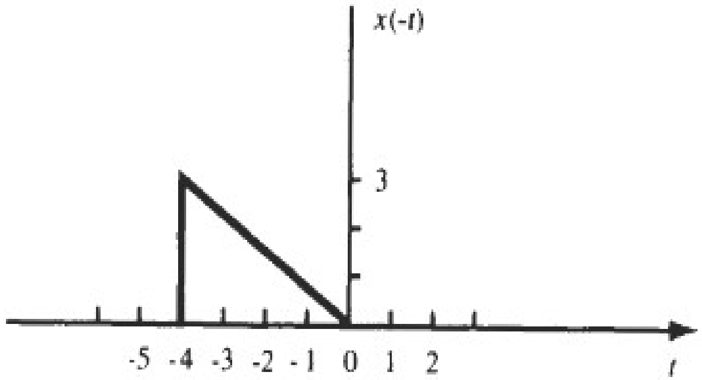
x(2t)
3
-2
-1
0
1
2
3
t

(c)



x(t/2)
3
-1
0
1
2
3
4
5
6
7
8
9
t

(d)



x(-t)
3
-5
-4
-3
-2
-1
0
1
2
t