Solids and Structures
UES017

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Consequences of Force applied on Deformable Bodies

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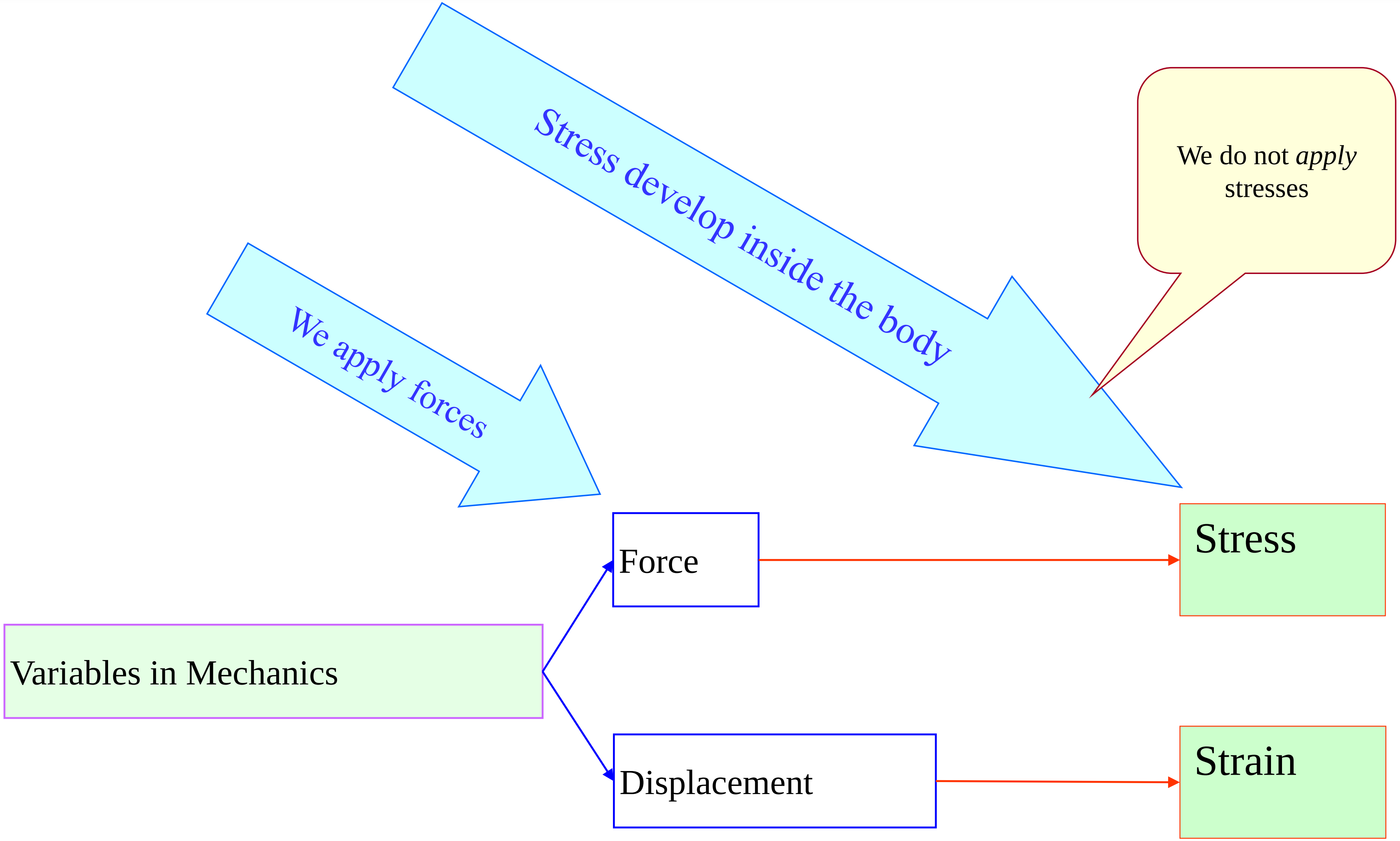
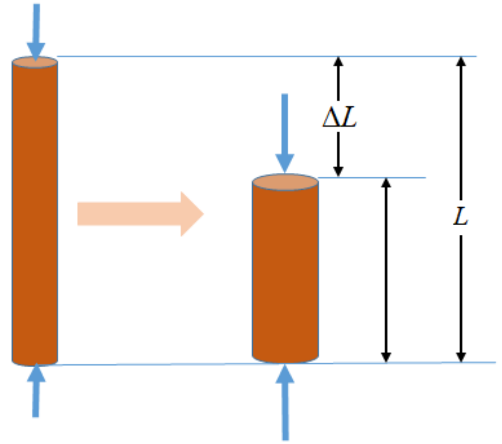
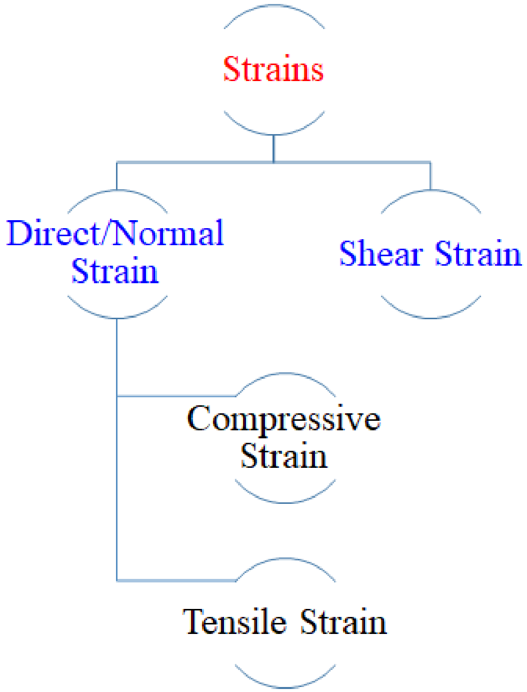


External forces and constraints give rise to a stress field within a body.

Strain: Measure of Deformation

- As the body is subjected to forces, the body deforms and distorts.
- The term *deformation* refers to geometric changes that take place in dimensions of the body (extensions or contractions), while the term *distortions* represents changes in its shape.
- The intensity of the internal force (force per unit area) is termed as stress.
- Similarly, a quantity used to provide a measure of the intensity of a deformation (deformation per unit original dimension/distortion) is called strain.
- Mathematically, strain is defined as change in dimension per unit original dimension.





Strain: Normal Strain

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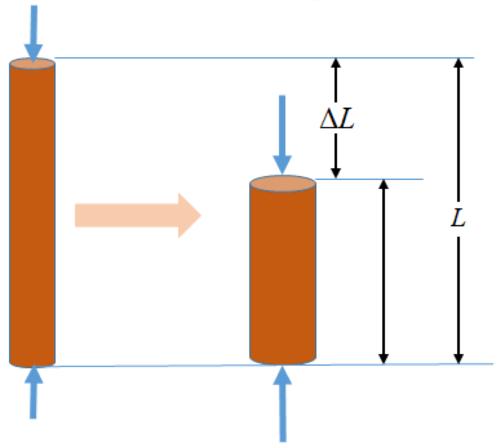
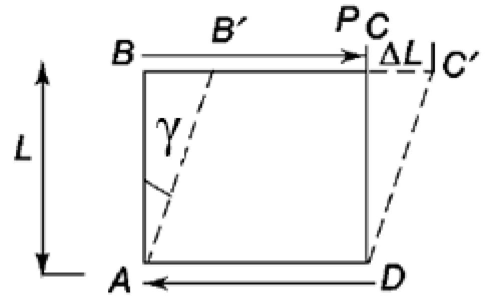
- In order to understand normal strain, consider the deformation of a simple bar under an axial load.
- Normal strain is defined as change in length of the bar per unit original length of bar.
- Normal Strain is designated by ε.
- Normal strain is a dimensionless quantity; however, these may be expressed in terms of mm/m, μm/m.
- When the elongation takes place, the normal strain is positive while it is negative when contraction takes place.
- The elongation occurs if there is axial tensile stress and hence positive normal strains are referred to as tensile strains. The contraction occurs if there is axial compressive stress and hence negative normal strains are referred to as compressive strains.



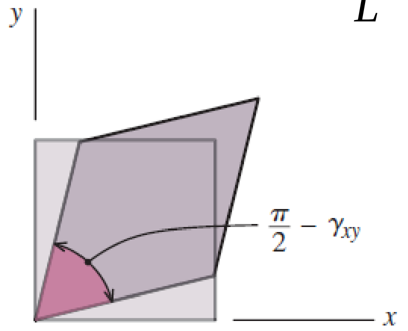
$$\varepsilon_c = -\frac{\Delta L}{L}$$

Strain: Shear Strain

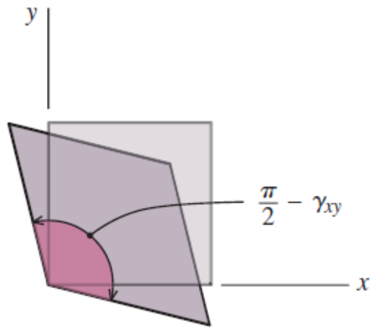
- Shear stresses acting on an object are accompanied by shear strains.
- A deformation involving a change in shape (distortion) can be used to illustrate a shear strain.
- As an aid in visualizing these strains, note that the shear stresses have no tendency to elongate or shorten the object.
- The lengths of the sides of the object do not change. Instead, the shear stresses produce a change in the shape of the element.
- The original element, which is a rectangular parallelepiped, is deformed into an oblique parallelepiped
- Because of this deformation, the angles between the side faces change.
- This angular deformation or the change in angle is termed **shear strain.**
- **The shear strain is designated by γ.**



$$\gamma \approx \tan\gamma = \frac{\Delta L}{L}$$

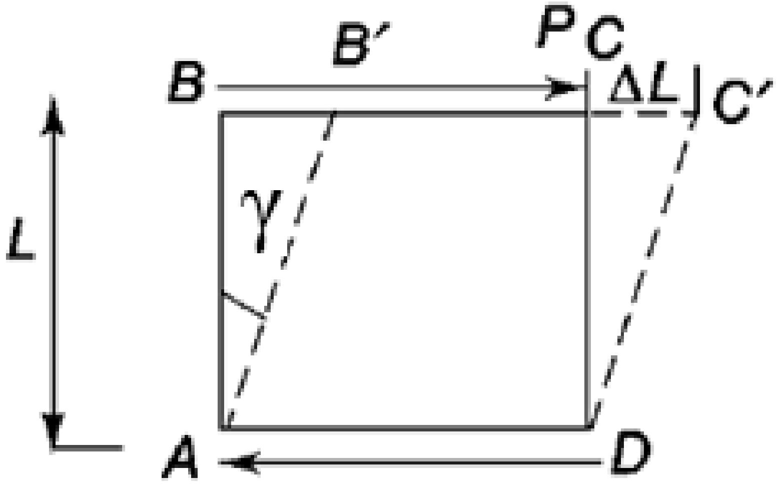


Positive Shear



Negative Shear

Positive Shear



$$\gamma \approx \tan\gamma = \frac{\Delta L}{L}$$

- Body rotates clockwise
- Reduces the angle between sides from original 90 deg to (90-ϒ)

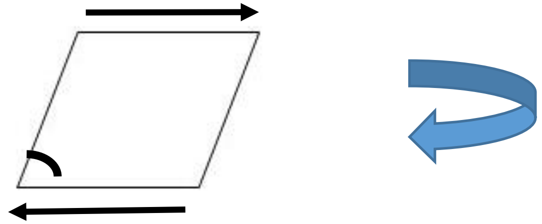
Negative Shear

- Body rotates anti-clockwise
- Increases the angle between sides from original 90 deg to (90+ϒ)

Sign Conventions for Shear Strain

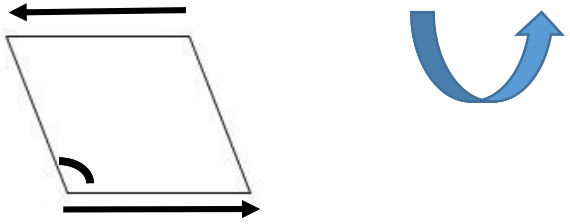
Positive shear strain:

If the angle between two positive faces (or two negative faces) are reduced.

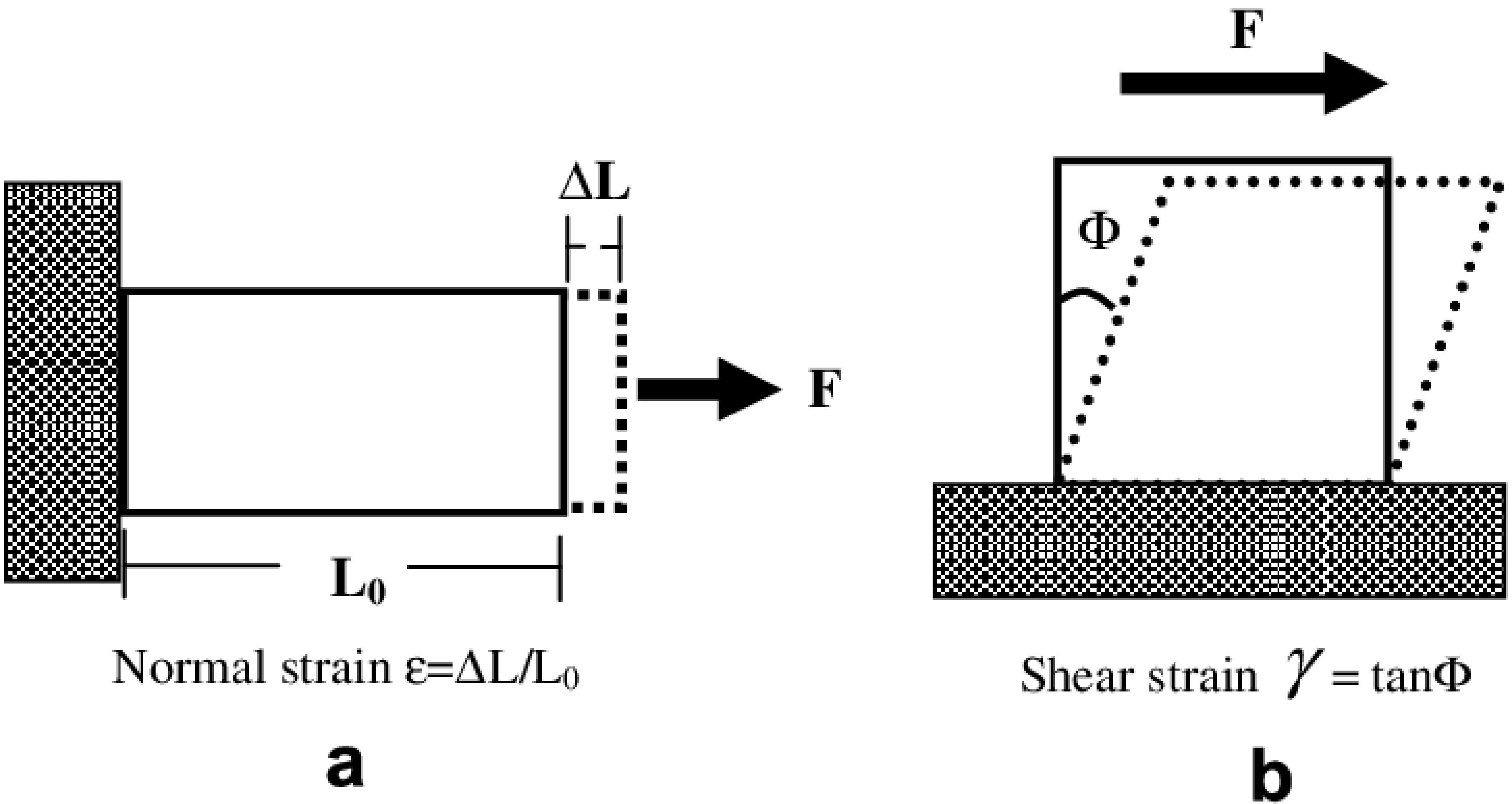


Negative shear stress:

If the angle between two positive faces (or two negative faces) are increased.

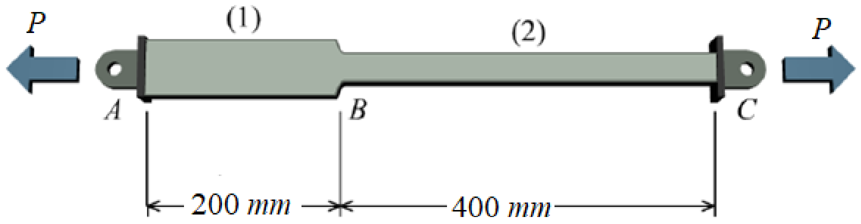


Normal Strain Vs Shear Strain



Strain: Illustrations

Illustration: When an axial load is applied to the ends of the bar shown in Fig., the total elongation of the bar between joints A and C is 2mm In segment (2), the normal strain is measured as 1300 μm/m. Determine:
(a) the elongation of segment (2).
(b) the normal strain in segment (1) of the bar.



Given Data:

To Find:

Solution:

Normal strain in (2) $\varepsilon_2 = \dfrac{\delta_2}{L_2} = 1300 \times 10^{-6} = 0.0013$

Elongation in segment (2)

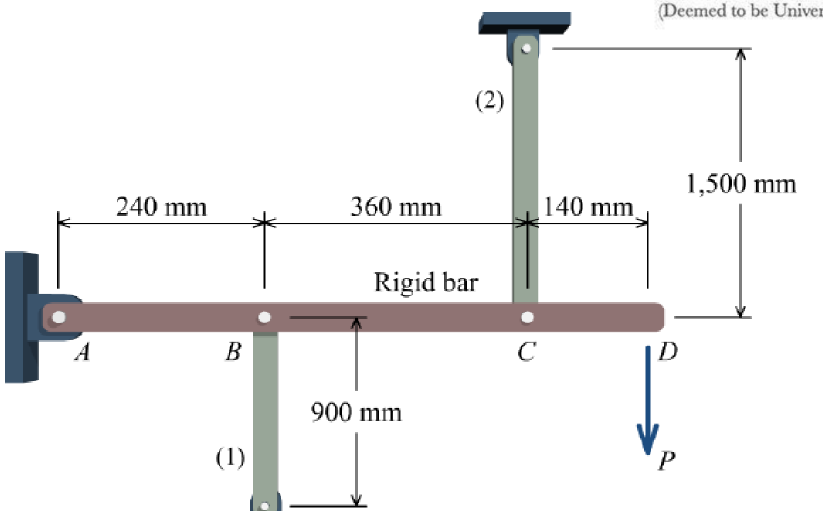
$$\delta_2 = \varepsilon_2 L_2 \quad = (0.0013)400 = 0.52mm$$

Elongation in segment (1), $\delta_1 = \delta - \delta_2 \quad = 2 - 0.52 = 1.48mm$

Normal strain in (1) $\varepsilon_1 = \dfrac{\delta_1}{L_1} = \dfrac{1.48}{200} = 0.0074$

Strain: Illustrations

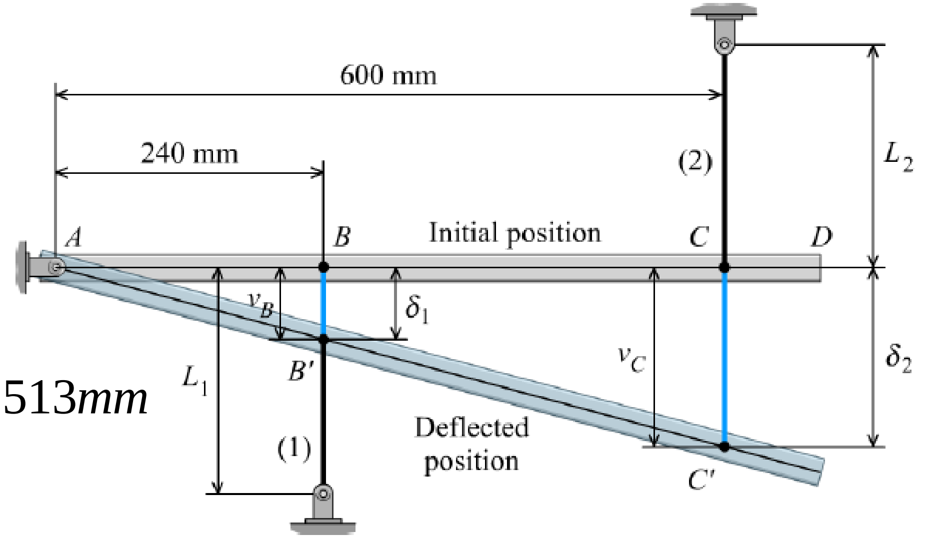
Illustration: A rigid bar ABCD is supported by two bars as shown in Fig. There is no strain in the vertical bars before load P is applied. After load P is applied, the normal strain in rod (1) is −570 μm/m. Determine the normal strain in rod (2).



Given Data:

To Find: Normal strain in rod (2)

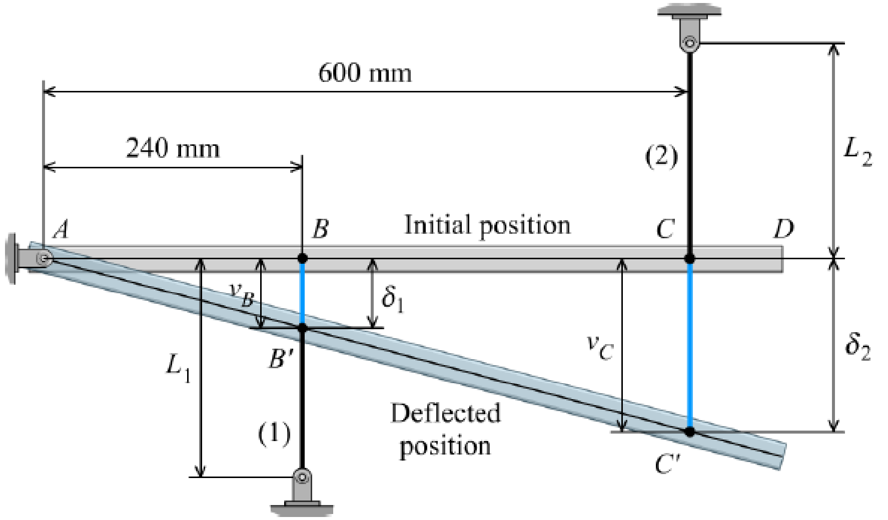
Solution: Draw the deformed shape of bar ABCD due to load P



Normal strain in rod (1) $\varepsilon_1 = \dfrac{\delta_1}{L_1} \Rightarrow \delta_1 = L_1\varepsilon_1 = (900)(-570\times10^{-6}) = -0.513mm$

$\Rightarrow \boxed{\delta_1 = -0.513mm}$

Strain: Illustrations



Deformation in rod (1)

$$\boxed{\delta_1 = -0.513mm}$$

From Similarity of Triangles in the deformed shape

$$\Delta ABB' \sim \Delta ACC' \qquad \therefore \frac{AB}{BB'} = \frac{AC}{CC'}$$

$$\Rightarrow \quad CC' = \frac{BB'}{AB} AC \quad = \frac{0.513}{240} 600 = 1.2825mm$$

Deformation in rod (2)

$$\delta_2 = CC' = 1.2825mm$$

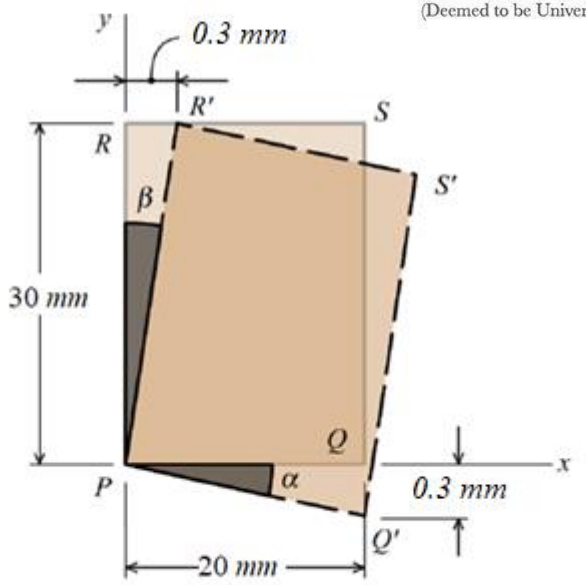
Normal Strain in rod (2)

$$\varepsilon_2 = \frac{\delta_2}{L_2} = \frac{1.2825}{1500} = 0.000855 \quad \Rightarrow \quad \boxed{\varepsilon_2 = 855\mu m / m}$$

Thus, the normal strain in rod (2) is 0.000855 or 855 $\mu m/m$

Strain: Illustrations

Illustration: A thin rectangular plate PQSR is uniformly deformed to a shape PQ’R’S’ as shown in Fig. Determine the shear strain at point P.



Solution: Angle at P before deformation = $\frac{\pi}{2}$

Angle at P after deformation = $\left(\frac{\pi}{2}+\alpha-\beta\right)$

Therefore, shear strain $\gamma=\alpha-\beta$

$$\alpha \approx \tan\alpha = \frac{0.3}{20} = 0.015$$

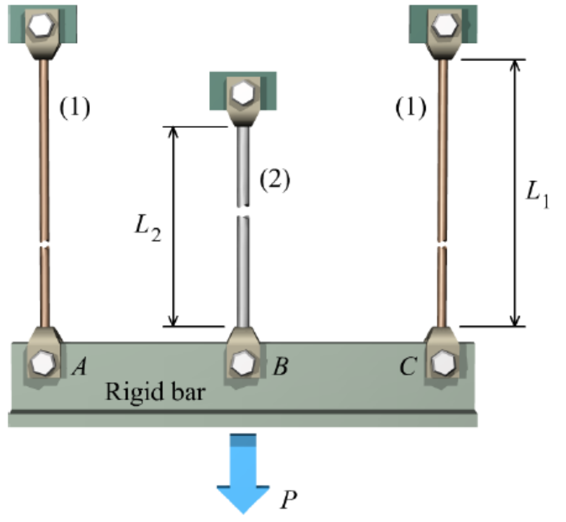
$$\beta \approx \tan\beta = \frac{0.3}{30} = 0.01$$

$$\gamma=\alpha-\beta=0.015-0.01=0.05$$

Thus, the shear strain at P is 0.05

Self Asssesment

Exercise 1: A rigid steel bar is supported by three rods, as shown in Fig. There is no strain in the rods before the load *P* is applied. After load *P* is applied, the normal strain in rods (1) is 860 μm/m. Assume initial rod lengths of L_1 = 2400 mm and L_2= 1800 mm. Determine the normal strain in rod (2).



Exercise 2: A solid circular bar is 500 mm long and is stretched to 505 mm with a force of 50 kN. The diameter of the bar is 10 mm. Calculate the associated stress and strain

Answers: (1) 0.0011467 (2) 636.6 MPa, 0.01

Stress-Strain Relationship: Tensile Test

- Different materials respond uniquely to the applied loads.
- It is important to evaluate the strength of the materials for design
- Tensile Test is one of the fundamental test to evaluate properties of materials.
- In this test, a specimen of the material, usually a round rod or a flat bar, is held between the grips and pulled with a controlled tension force.
- The tensile force is gradually increased and the corresponding elongation is recorded (using extensometer).
- The load-deflection curve may be plotted. However, it is applicable for specific specimen.
- The load and elongation data obtained in the tension test can be readily converted to stress and strain data.
- The plot so obtained between stress and strain is called the **Stress–Strain diagram**.
- The stress–strain diagram is more useful because it applies to the material in general rather than to the particular specimen used in the test.

Stress Strain Curve: Ductile Materials

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- The material that can undergo large strains before it ruptures or failures are called ductile material e.g. **mild steel**
- The **advantage of ductility** is that visible distortion may occur if the load becomes too large, thus providing an opportunity to take remedial action before a fracture occur;
- Such materials are used because they are capable of absorbing shock or energy, and if before becoming overloaded, will exhibit large deformation before failing;
- Ductility in the material is characterized by its **elongation and the reduction in area** of cross section when fracture occurs;
- Percentage reduction in area measures the amount in necking

Percent elongation

$$\%\,\text{elongation} = \frac{L_f - L_0}{L_0}$$

Percent reduction in area

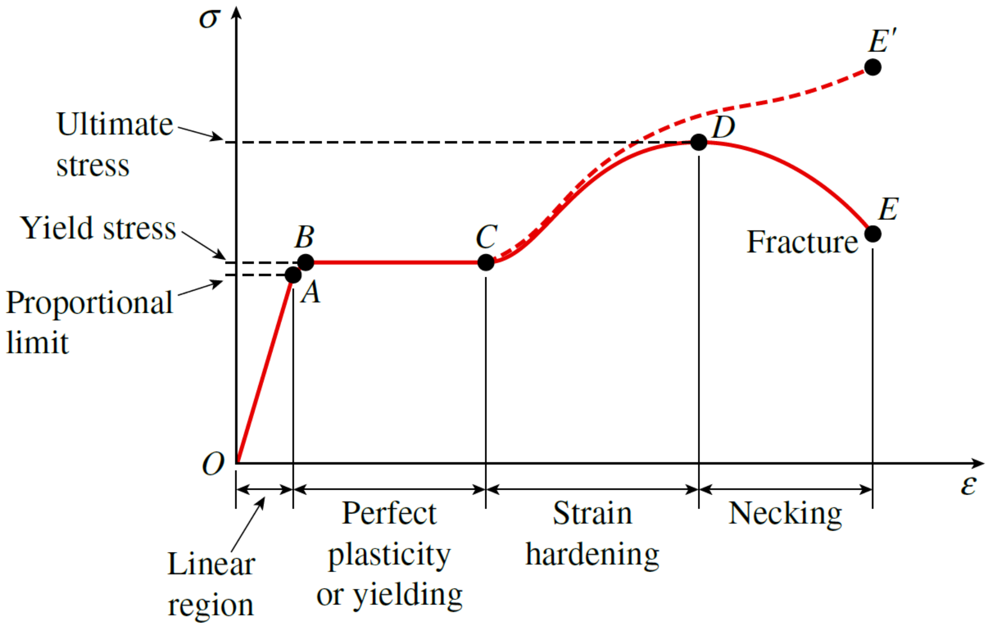
$$\%\,\text{reduction in area} = \frac{A_0 - A_f}{A_0}$$

L_0 -- original gage length;
L_f -- distance between gage marks at fracture;
A_0 -- original cross sectional area;
A_f -- final cross sectional area at fracture

Stress Strain Curve: Ductile Materials (Structural Steel)

Elastic Region OA - diagram begins with a straight line. Stress is directly proportional to strain (Hook's Law), i.e., linearly elastic **(Proportional Limit)**. If load is removed upon reaching **Elastic Limit**, specimen will return back to its original shape;

Plastic Region BC – in the region form B to C the material becomes **perfectly plastic**, which means that it can deform without an increase in the applied load. This phenomena is known as the **Yielding of the material** and stress at point B/C, is known as **Yield Stress**.



Region CD – after undergoing the large strain that occurs during yielding in the region BC, the steel begins to **Strain Harden**. During the strain hardening, ***the material undergoes change in atomic and crystalline structure*** that resulting in a ***increased resistance of the material*** to further deformation. The load reaches its maximum value and corresponding stress is called ultimate stress (point D) .

Fracture occurs at a point **E (Fracture/Rupture/Breaking Point)** on the diagram.
In the vicinity of ultimate stress, the reduction in the area of bar becomes clearly visible & necking of bar occurs.

Stress Strain Curve: Ductile Materials (Structural Steel)

Conventional Stress/Nominal stress/Engineering Stress

If the initial area of the bar is used for calculating the stress, the resulting stress is called conventional stress/ engineering stress:

$$\sigma = \frac{P}{A}$$

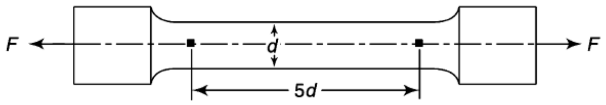
If the initial gage length is used for calculating the strain, the resulting strain is called **nominal strain**;

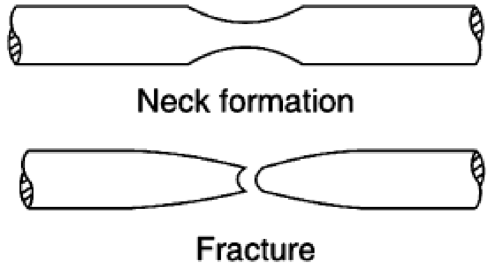
$$\varepsilon = \frac{\delta}{L_0}$$

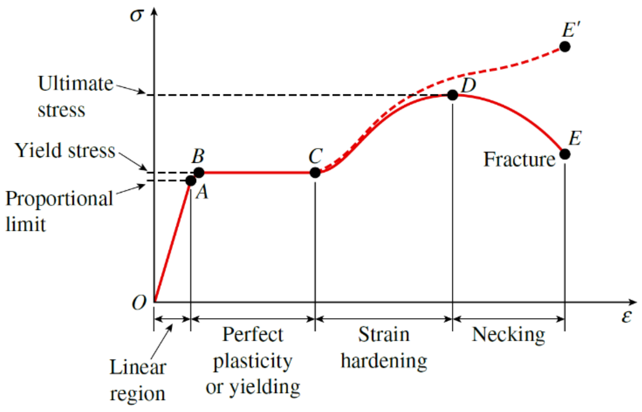
True Stress:

A more exact value of the axial stress known as true stress, can be calculated by using the actual area of the bar.

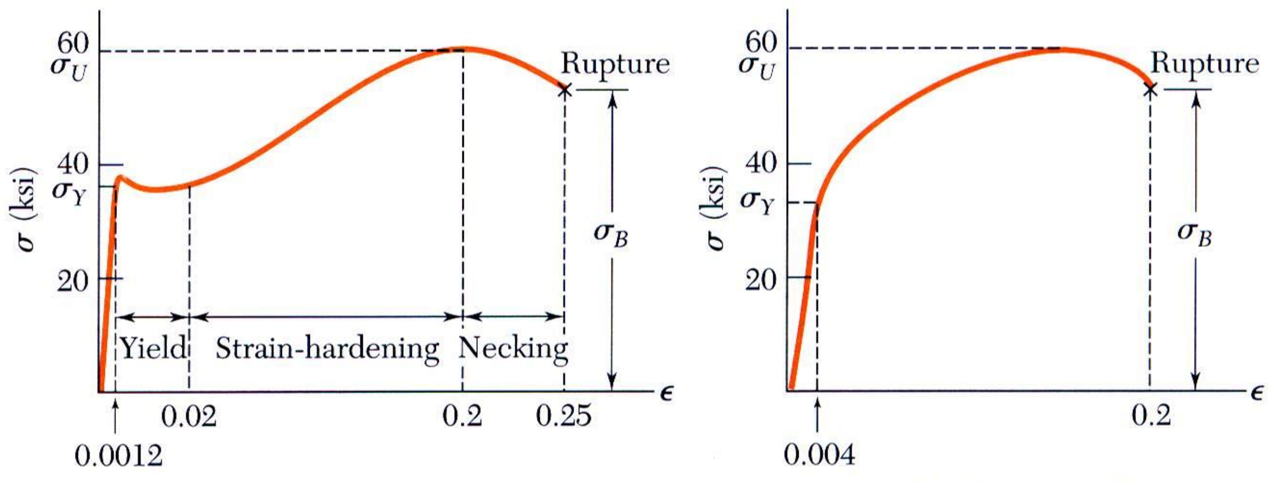
The distance between the gage marks increases as the tensile load is applied. If the actual distance is used for calculating the strain, the obtained strain is known as the natural or true strain.



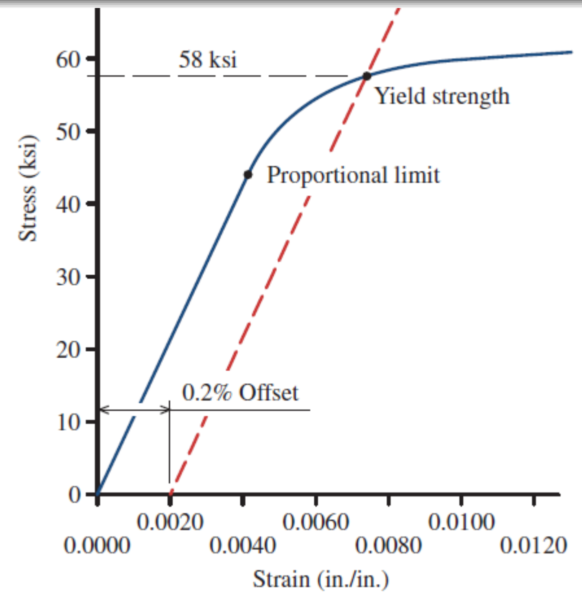




Stress Strain Curve: Ductile Materials



(a) Low-carbon steel (b) Aluminum alloy



- Most metals do not exhibit constant yielding behavior beyond the elastic range, e.g. aluminum

- It does not have well-defined yield point, thus it is standard practice to define its *yield strength* using a graphical procedure called the offset method.

Offset method (To evaluate the yield strength):

- Normally, a 0.2 % strain is chosen and from this point on the ε axis, a line parallel to initial straight-line portion of stress-strain diagram is drawn;

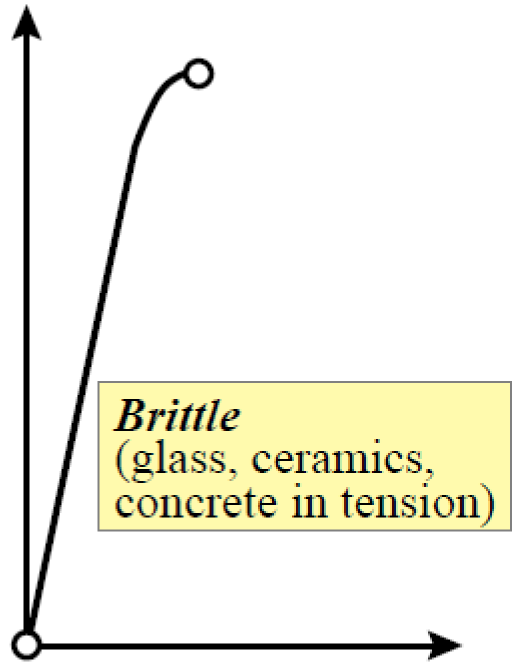
- The point where this line intersects the curve defines the yield strength.

Stress Strain Curve: Brittle Materials

- Material that exhibit little or no yielding before failure are referred to as brittle materials, e.g. cast iron, glass, stone, concrete;

- These materials fail with only little elongation after the proportional limit is exceeded and the **fracture stress is same as ultimate stress**. The high carbon steel behave in same manner.

- Ordinary glass is nearly ideal brittle material, because it exhibits no ductility;



Stress-strain diagram for Brittle material (Glass)

Stress Strain Curve: Linear Elastic Material

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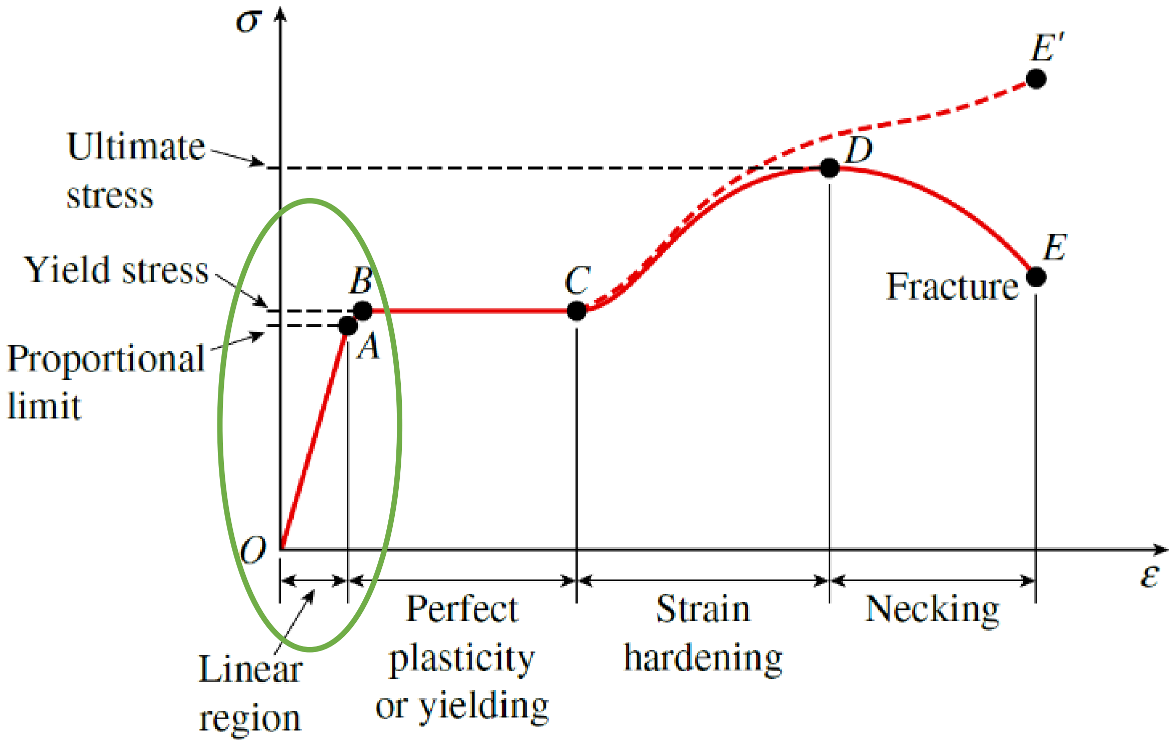
- When the load is applied to a tensile specimen so that stress-strain curve goes from O to A on the stress-strain curve.
- When the load is removed, the curve follows exactly the same curve back to the origin.
- This property of material by which it returns back to its original dimensions during unloading is **called elasticity, and the material itself is said to be elastic.**



Hooke's Law: Material Constants

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- Most engineering materials exhibit a linear relationship between stress and strain with the elastic region;

- Discovered by Robert Hooke in 1676 using springs, known as *Hooke's law;*

$$\sigma = E\varepsilon$$

- E represents the **Constant of Proportionality**, also called the Modulus of Elasticity or Young's modulus (named after English scientist Thomas Young);

- The stiffness is now in terms of stress and strain only and this constant is called the Modulus of Elasticity (E);

- $E_{\text{mild steel}}$ = 210 GPa, $E_{\text{aluminium}}$ = 70 GPa

- E has units of stress, i.e., Pascal's, MPa or GPa; 1 Pascal = 1 N/m^2

- The Hooke's Law in Shear is defined as;

$$\tau = G\gamma$$

- *G* is the **Shear modulus of Elasticity** (Modulus of Rigidity)

- It is defined as the ratio of the Shear Stress to the Shear Strain.

Hooke's Law: Material Constants

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Volumetric Strain

The ratio between the change in volume and original volume of the body is referred as volumetric strain.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

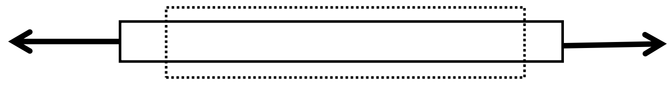
Bulk Modulus of Elasticity (K)

The ratio of normal stress to the volumetric strain is known as bulk modulus of elasticity and it is denoted by a letter K.

$$\text{Bulk modulus (K)} = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

Poisson's Ratio

- Ratio of strain in lateral direction to the strain in axial direction is known as Poisson's ratio.



$$\nu = -\frac{\textit{Lateral strain}}{\textit{axial strain}} \qquad \nu = -\frac{\Delta D/D}{\Delta L/L}$$

- Poisson's ratio is named for the famous French mathematician Simeon Denis Poisson (1781-1840).

- For structural material, the value of Poisson ratio lies between **0<ν<0.5, for metals, between 0.25 to 0.35.**

Relation between Material Constants

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The relations exist between the elastic constants for any specific material and these relations hold good for all materials within elastic range.

Relation between E and G

$$E = 2G(1+\nu)$$

Relation between E and K

$$E = 3K(1-2\nu)$$

Relation between E, G and K

$$E = \frac{9KG}{3K+G}$$

where, E, G, K and ν are defined as the Young's modulus, Shear modulus, Bulk modulus of Rigidity and Poisson's ratio.

- A material is said to be **homogenous** if it has same composition throughout the body (at all locations). Hence, **the elastic properties are same at every point in the body.**

- A material is said to be **isotropic** if it has same elastic properties in all the directions. In case of isotropic material, the number of independent elastic constants are 2 (Young's Modulus and Poisson's ratio) e.g. metal.

Allowable Stress and Allowable Loads: Factor of Safety

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- If the structural failure is to be avoided, the loads that a structure actually can support must be greater than the loads it will be required to sustain when in service.
- The actual strength of the structure must exceed the required strength.
- The ratio of actual strength to the required strength is called Factor of Safety (FOS).
- **FOS should be greater than one in order to avoid the failure.**
- If the FOS is too low, the likelihood of failure will be too high and hence, the structure will be unacceptable.
- If the FOS is high, there is wastage of material.
- FOS will depends on the type of loading, inaccuracies in construction, quality of workmanship, method of analysis.

Important Relations

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Stress: Normal and Shear Stress

$$\sigma = \frac{NormalForce}{Area} = \frac{F}{A} \qquad \tau = \frac{ShearFoce}{Area} = \frac{F}{A}$$

Strain: Normal and Shear Strain

$$\varepsilon = \frac{\Delta L}{L} \qquad \gamma \approx \tan\gamma = \frac{\Delta L}{L}$$

Stress-Strain Relations

$$\sigma = E\varepsilon \qquad \tau = G\gamma$$

Material Properties for Isotropic Materials

$$\boxed{E = \frac{\sigma}{\varepsilon}} \qquad \boxed{G = \frac{\tau}{\gamma}}$$

$$\boxed{\nu = -\frac{\Delta D/D}{\Delta L/L}} \qquad \boxed{\text{Bulk modulus (K)} = \frac{\text{Normal stress}}{\text{volumetric strain}}}$$

The relations exist between the elastic constants for any specific material and these relations hold good for all materials within elastic range.

Relation between E and G

$$E = 2G(1+\nu)$$

Relation between E and K

$$E = 3K(1-2\nu)$$

Relation between E, G and K

$$E = \frac{9KG}{3K+G}$$

where, E, G, K and ν are defined as the Young’s modulus, Shear modulus, Bulk modulus of rigidity and Poisson’s ratio.

Illustrative Examples: Material Properties

Illustration: A circular pipe of internal diameter 30 mm and thickness 4 mm is subjected to a force 30 kN and the elongation was measured as 1 mm. If the length of the pipe is 2 m, find the value of Young's Modulus of Elasticity and Stress in the pipe.

Given Data: $L = 2000mm, \quad \Delta L = 1mm, \quad P = 30kN, \quad d = 30mm, \quad t = 4mm.$

To Find: $E = ?, \sigma = ?$

Solution: D = d+2t = 30 +2(4) = 38mm

$$P = 30\text{ kN}, \quad A_{pipe} = \frac{\Pi}{4} * (D^2 - d^2) = 427.36\text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{30\ X1000}{427.36} = 70.2\text{ N/mm}^2 = 70.2\text{ MPa}$$

$$E = \frac{\sigma}{\epsilon} = \frac{70.2}{\epsilon}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{1}{2000} = 0.0005$$

$$E = \frac{\sigma}{\epsilon} = \frac{70.2}{0.0005} = 140{,}400\text{ MPa} = 140.4\text{ GPa}$$

Thus, Youngs' modulus is 140.4 GPa and stress in the pipe is 70.2 MPa

Illustrative Examples: Material Properties

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Illustration: At the proportional limit, a 30 mm wide ×10 mm thick bar elongates 3.0 mm under an axial load of 60 kN. The bar is 1.5m long. The Poisson's ratio is 0.30 for the material.

Determine: (a) Modulus of elasticity (b) Proportional limit (c) Change in each lateral dimension.

Given Data:

$$L = 1.5m = 1500mm, \quad \Delta L = 3mm, \quad P = 60kN, \quad width(B) = 30mm, \quad thickness(H) = 10mm, \quad \nu = 0.30.$$

To Find:

$$(a) E = ?, \quad (b) \sigma_{proportional} = ?, \quad (c) \Delta b = ?, \quad \Delta h = ?.$$

Solution:

Area of cross section, $A = 300mm^2$

Force at proportional limit, $P = 60kN$

Stress at proportional limit, $\sigma_{proportional} = \frac{P}{A} = \frac{60 \times 1000}{300}$

$\boxed{\sigma_{proportional} = 200N/mm^2 = 200Mpa}$ Ans. (b)

Strain in the direction of length, $\varepsilon_L = \frac{\Delta L}{L}$

$$\varepsilon_L = \frac{3}{1500} = 0.002$$

Hooke's law states that $\sigma_{proportional} = E\varepsilon_L$

$$E = \frac{\sigma_{proportional}}{\varepsilon_L} = \frac{200}{0.002} = 100000MPa \quad \boxed{\therefore E = 100GPa}$$

Illustrative Examples: Material Properties

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(C) the change in each lateral dimension.

$$\varepsilon_L = 0.002$$

Poisson ratio is defined as:, $\nu = -\frac{\varepsilon_B}{\varepsilon_L} = -\frac{\varepsilon_H}{\varepsilon_L}$

Thus, Strain in the dimension of width, $\varepsilon_B = -\nu\varepsilon_L = -0.3 \times 0.002 = -0.0006$

Hence, Change in the dimension of width, $\boxed{\Delta B = \varepsilon_B \times B = -0.0006 \times 30 = -0.018mm}$

Similarly, Strain in the dimension of thickness, $\varepsilon_H = -\nu\varepsilon_L = -0.3 \times 0.002 = -0.0006$

Hence, Change in the dimension of thickness, $\boxed{\Delta H = \varepsilon_H \times H = -0.0006 \times 10 = -0.006mm}$

Illustrative Examples: Material Properties

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Illustrations: The following data relate to a bar subjected to a tensile test: Diameter of the bar, D = 30 mm, Tensile load, P = 50 kN, Gauge length, L=300 mm, Extension of the bar is (ΔL) is 0.1mm, Change in diameter is ΔD = 0.003mm.

Calculate: (a) Poisson's ratio (b) The value of the Young's modulus and Shear Modulus

Given Data: $L = 300mm, \quad \Delta L = 0.1mm, \quad P = 50kN, \quad D = 30mm, \quad \Delta D = 0.003mm.$

To Find: $(a)\nu = ? \quad (b)E = ?, G = ?$

Solution:

$$\epsilon_L = \frac{\Delta L}{L} = \frac{0.1}{300} = 0.0003$$

$$\epsilon_D = \frac{-\Delta D}{D} = \frac{-0.003}{30} = -0.0001$$

$$\boxed{\nu} \quad \boxed{= \frac{-\Delta D/D}{\Delta L/L} = \frac{0.0001}{0.0003} = 0.33}$$

$$A_{bar} = \frac{\Pi}{4} * (D^2) = 706.85 \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{50 \, X1000}{706.85} = 70.74 \text{ N/mm}^2 = 70.74 \text{ MPa}$$

$$\boxed{E = \frac{\sigma}{\varepsilon}} = \frac{70.74}{0.0003} = 235{,}800 \text{ MPa} = 235.80 \text{ GPa}$$

$$\boxed{E = 2G(1+\nu)}$$

Substitute for E and $\boldsymbol{\nu}$, we get
G = 88.65 GPa

Stress Strain Relations: Biaxial and Triaxial Loads

Stress-Strain Relations: $\sigma = E\varepsilon$ *(Applicable only for Uniaxial Loads)*

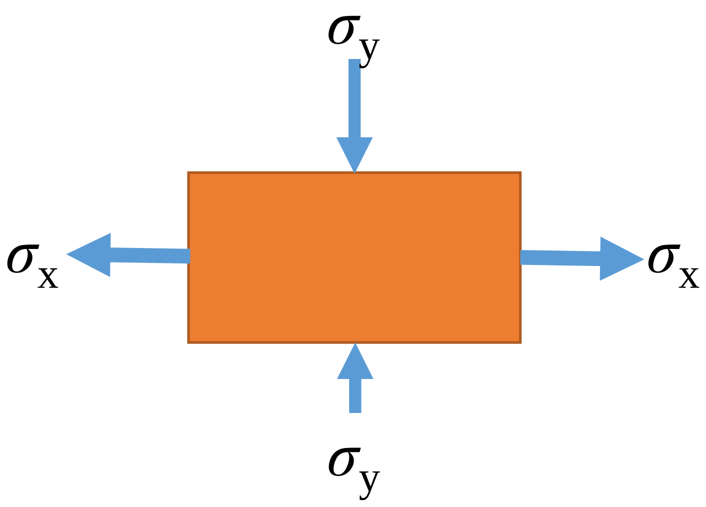
Uniaxial Loading ($\sigma_y = \sigma_z = 0$)

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{-\nu(\sigma_x)}{E}$$

$$\varepsilon_z = \frac{-\nu(\sigma_x)}{E}$$

Biaxial Loading ($\sigma_z = 0$)

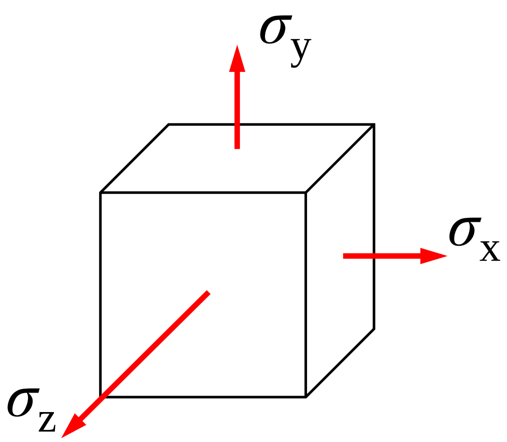


$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x)\right]$$

$$\varepsilon_z = \frac{-1}{E}\left[\nu(\sigma_x + \sigma_y)\right]$$

3D Loading Case



$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

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Numerical Illustrations

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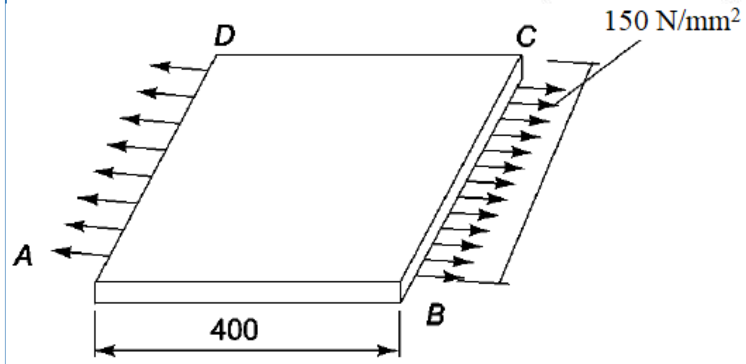
Illustration: A steel plate of dimensions 300 x 400 x 10 mm is subjected to an axial stress of 150 N/mm^2 parallel to the 400 mm side as shown. Determine the percentage change in volume of the plate. E = 200 GPa and ν = 0.3



Solution:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

Steel plate : 300mm X 400 mm X 10mm

$$\sigma_x = 150\ \text{N/mm}^2 \qquad \sigma_y = 0 = \sigma_z$$

Substituting in the expressions,

$$\epsilon x = \frac{\sigma_x}{E} = \frac{150}{200\ X 10^3} = 0.00075$$

$$\epsilon y = \frac{-\nu\sigma_x}{E} = -0.3\ X\ 0.00075 = -0.000275 = \epsilon z$$

Volumetric Strain

$$\epsilon\, v = \epsilon x + \epsilon y + \epsilon z = 0.0003$$

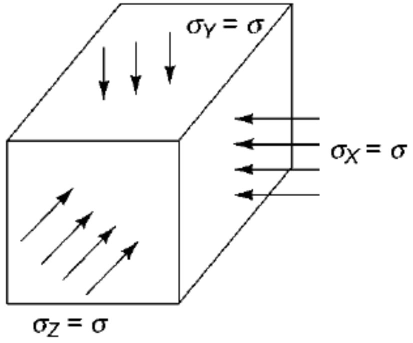
Now, $\epsilon\, v = \dfrac{dV}{V}$

% change in volume $= \dfrac{dV}{V}\ X\ 100 = 0.0003\ X\ 100 = 0.03\%$

Numerical Illustrations

Illustration: A cube of 1 m side, made of steel, change the volume by *0.075% in a sea at certain* depth. What will be the corresponding stress on each face of the cube? E = 200 GPa and v= 0.3.

Solution:



- Under water, the solid will be subjected to hydrostatic pressure (compressive) of equal magnitude on all sides
- Therefore, the strain in each direction will be equal and given by

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \frac{\sigma}{E}(1-2\nu)$$

Volumetric Strain $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3\sigma}{E}(1-2\nu)$

Also, Volumetric Strain = 0.075%

$$\therefore \frac{3\sigma}{E}(1-2\nu) = \frac{0.075}{100} \qquad \therefore \sigma = \frac{0.075\times 200\times 1000}{100\times 3\times 0.4}$$

$$\boxed{\therefore \sigma = 125 MPa}$$

Thank You

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