

## Discrete Mathematical Structures (UCS405)

### Tutorial Sheet-3

1. Let  $A = \{1, 2, 3, 4\}$ , and  $R$  is a relation defined by “ $a$  divides  $b$ ”. Write  $R$  as a set of ordered pair, draw directed graph.
2. List the ordered pairs in the relation  $R$  from  $A = \{0, 1, 2, 3, 4\}$  to  $B = \{0, 1, 2, 3\}$ , where  $(a, b) \in R$  if and only if
  - a)  $a = b$ .
  - b)  $a + b = 4$ .
  - c)  $a > b$ .
  - d)  $a | b$ .
  - e)  $\gcd(a, b) = 1$ .
  - f)  $\text{lcm}(a, b) = 2$ .
3. Let  $A = \{1, 2, 3, 4\}$ , give an example of a mapping which is (i) neither symmetric nor antisymmetric, (ii) anti-symmetric and reflexive but not transitive, (iii) transitive and reflexive but not anti-symmetric.
4. For each of these relations on the set  $\{1, 2, 3, 4\}$ , decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.
  - a)  $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
  - b)  $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
  - c)  $\{(2, 4), (4, 2)\}$
  - d)  $\{(1, 2), (2, 3), (3, 4)\}$
  - e)  $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
  - f)  $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$
5. Let  $R_1 = \{(1, 2), (2, 3), (3, 4)\}$  and  $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$  be relations from  $\{1, 2, 3\}$  to  $\{1, 2, 3, 4\}$ . Find
  - a)  $R_1 \cup R_2$ .
  - b)  $R_1 \cap R_2$ .
  - c)  $R_1 - R_2$ .
  - d)  $R_2 - R_1$ .
6. Let  $R$  be the relation  $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$ , and let  $S$  be the relation  $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$ . Find  $S \circ R$ .
7. List the 16 different relations on the set  $\{0, 1\}$ .
  - I. How many of the 16 different relations on  $\{0, 1\}$  contain the pair  $(0, 1)$ ?
  - II. Which of the 16 relations on  $\{0, 1\}$ , which are
    - a) reflexive?
    - b) irreflexive?
    - c) symmetric?
    - d) antisymmetric?
    - e) asymmetric?
    - f) transitive?
8. Let  $R$  be the relation on the set  $\{1, 2, 3, 4, 5\}$  containing the ordered pairs  $(1, 1), (1, 2), (1, 3), (2, 3), (2, 4), (3, 1), (3, 4), (3, 5), (4, 2), (4, 5), (5, 1), (5, 2)$ , and  $(5, 4)$ . Find
  - a)  $R^2$ .
  - b)  $R^3$ .
  - c)  $R^4$ .
  - d)  $R^5$ .