School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Semester Examination, December-2019

B.E. III Semester | UMA007: Numerical Analysis

Time Limit: 03 Hours | Maximum Marks: 100

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Instructions: This question paper has one printed page. You are expected to answer all the questions. Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without programming mode is permitted.

1. (a) Discuss the stability of the algorithm for calculating $f(x) = \sqrt{x+1} - \sqrt{x}$ at the point $x = 12345$. Suggest a modification if the procedure is unstable. [10 Marks]

   (b) For any $X^{(0)} \in \mathbb{R}^n$, the sequence $X^{(k)}$ defined by $X^{(k)} = TX^{(k-1)} + C$, $k \geq 1$, converges to the unique solution of $X = TX + C$ if and only if $\rho(T) < 1$. [10 Marks]

2. (a) Use secant method to find a solution of the equation $2x + 3\cos x - e^x = 0$ with initial guesses $x_0 = 0$ and $x_1 = 1$ correct to 0.05 accuracy. [8 Marks]

   (b) Solve the following system of equations

$$\begin{aligned} 3.03x - 12.1y + 14z &= -119 \\ -3.03x + 12.1y - 7z &= 120 \\ 6.11x - 14.2y + 21z &= -139 \end{aligned}$$

   using Gauss elimination method with scaled partial pivoting and three digit rounding arithmetic. [12 Marks]

3. (a) Determine the spacing h in a table of equally spaced values of the function $f(x) = \cos x$ between 0 and 1 so that the interpolation will yield an accuracy of 9×10^{-9} for quadratic interpolating polynomial. [10 Marks]

   (b) Using power method, find the largest eigen value and corresponding eigen vector of the matrix

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

   correct to 2 decimal places by taking initial guess as $[1, 1, 1]^T$. [10 Marks]

4. (a) Approximate the area beneath $f(x) = \dfrac{1}{1+x^2}$ on the interval $[-1, 1]$ using Simpson's rule with 8 subintervals. Also compare with the exact area. [8 marks]

   (b) The motion of a swinging pendulum is described by the following second-order initial value problem

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0, \theta(0) = \frac{\pi}{6}, \theta'(0) = 0,$$

   where θ is the angle with vertical axis at time t, length of the pendulum $L = 2ft$ and $g = 32.17 ft/s^2$. With $h = 0.1s$, find the angle θ at $t = 0.1$ using Runge-Kutta fourth order method. [12 Marks]

5. (a) Derive two point Gauss quadrature formula for integration. [10 Marks]

   (b) Given the initial-value problem $\dfrac{dy}{dt} = \dfrac{2}{t}y + t^2e^t, y(1) = 0, 1 \leq t \leq 1.2$. Use Euler's method with $h = 0.1$ to compute $y(1.1)$ and $y(1.2)$. Further use these values and Lagrange linear interpolation to approximate y at $t = 1.14$. [10 Marks]