

Course: UMA 035 (Optimization Techniques)

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Change in Right hand side

$$\text{Max } (3x_1 + 2x_2 + 5x_3)$$

Subject to

$$x_1 + 2x_2 + x_3 + S_1 = 430$$

$$3x_1 + 2x_3 + S_2 = 460$$

$$x_1 + 4x_2 + S_3 = 420$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0.$$

The optimal table for this LPP is as follows:

		3	2	5	0	0	0		
C _B	Basic Variables	x ₁	x ₂	x ₃	S ₁	S ₂	S ₃	Solution	Minimum Ratio
Z _j - C _j =		4	0	0	1	2	0		
2	x ₂	− $\frac{1}{4}$	1	0	$\frac{1}{2}$	− $\frac{1}{4}$	0	100	
5	x ₃	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230	
0	S ₃	2	0	0	−2	1	1	20	

(i) Find the new optimal solution if the RHS elements are 60, 64 and 59

instead of 430, 460 and 420 respectively.

- (ii) Find the new optimal solution if the RHS elements are 45, 46 and 40 instead of 430, 460 and 420 respectively.
- (iii) Within what range the RHS of first constraint can be changed without affecting the feasibility.
- (iv) Within what range the coefficient of x_2 in the objective function can be changed without affecting the optimality.

Solution:

If the given problem is solved by the simplex method then in the starting table

- S_1 will be the first basic variable
- S_2 will be the second basic variable
- S_3 will be the third basic variable

$B^{-1} = [\text{Column of } S_1 \text{ in the optimal table} \text{ Column of } S_2 \text{ in the optimal table}$

$\text{Column of } S_3 \text{ in the optimal table}]$

$$B^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

Column of solution

B⁻¹*RHS matrix

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix} = \begin{bmatrix} 100 \\ 230 \\ 20 \end{bmatrix}$$

(i) New column of solution is

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 60 \\ 64 \\ 59 \end{bmatrix} = \begin{bmatrix} 14 \\ 32 \\ 3 \end{bmatrix}$$

Since, all calculated values are ≥ 0 . So, no need to apply Dual Simplex method.

New optimal solution is

$$x_2=14$$

$$x_3=32$$

$$S_3=3$$

(ii) New column of solution is

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ 46 \\ 40 \end{bmatrix} = \begin{bmatrix} 11 \\ 23 \\ -4 \end{bmatrix}$$

Since, one value is negative so need to apply dual simplex method.

Since, all calculated values are ≥ 0 . So, no need to apply Dual Simplex method.

		3	2	5	0	0	0		
C _B	Basic Variables	x ₁	x ₂	x ₃	S ₁	S ₂	S ₃	Solution	Minimum Ratio
Z _j - C _j =		4	0	0	1	2	0		
2	x ₂	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	11	
5	x ₃	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	S ₃	2	0	0	-2	1	1	-4	

		3	2	5	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Maximum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		
2	x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	11	
5	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	S_3	2	0	0	-2	1	1	-4	

		3	2	5	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Maximum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		
2	x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	11	
5	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	S_3	2	0	0	-2	1	1	-4	

Only one negative value in fourth row

		3	2	5	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Maximum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		Only one negative value in fourth row
2	x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	11	
5	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	S_3	2	0	0	-2	1	1	-4	

		3	2	5	0	0	0		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	Solution	Maximum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		Only one negative value in fourth row
2	x_2	$\frac{1}{4}$	1	0	0	0	$\frac{1}{4}$	10	
5	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	S_1	-1	0	0	1	$-\frac{1}{2}$	$-\frac{1}{2}$	2	

New optimal solution is

$$x_2=10$$

$$x_3=23$$

$$S_1=2$$

(ii) New column of solution is

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} b \\ 46 \\ 40 \end{bmatrix} = \begin{bmatrix} \frac{b}{2} - \frac{23}{2} \\ \frac{23}{2} \\ -2b + 88 \end{bmatrix}$$

No need to apply dual simplex method, if all calculated values are ≥ 0 .

Hence,

$$\frac{b}{2} - \frac{23}{2} \geq 0 \text{ and } -2b + 88 \geq 0$$

$$b \geq 23 \text{ and } b \leq 44$$

$$23 \leq b \leq 44$$

Addition of a variable

Example:

$$\text{Min } (2x_1 + x_2 + MA_1 + MA_2)$$

Subject to

$$3x_1 + x_2 + A_1 = 3$$

$$4x_1 + 3x_2 - S_2 + A_2 = 6$$

$$x_1 + 2x_2 + S_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, S_2 \geq 0, S_3 \geq 0.$$

Optimal Table

		2	1	0	M	M	0		
C _B	Basic Variables	x ₁	x ₂	S ₂	A ₁	A ₂	S ₃	Solution	Minimum Ratio
Z _j - C _j =		0	0	- $\frac{1}{5}$	$\frac{2}{5} - M$	$\frac{1}{5} - M$	0		
2	x ₁	1	1	$\frac{1}{5}$	$\frac{3}{5}$	- $\frac{1}{5}$	0	$\frac{3}{5}$	
1	x ₂	0	0	- $\frac{3}{5}$	- $\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{6}{5}$	
0	S ₃	0	0	1	1	-1	1	0	

Find the new optimal solution if a non-negative variable x_3 having the coefficients $\frac{1}{2}$ in the objective function, 0 in the first constraint, 5 in the second constraint and 2 in the third constraint is added in the original problem.

Solution:

New LPP is

$$\text{Min } (2x_1 + x_2 + M A_1 + M A_2 + \frac{1}{2}x_3)$$

Subject to

$$3x_1 + x_2 + A_1 + 0x_3 = 3$$

$$4x_1 + 3x_2 - S_2 + A_2 + 5x_3 = 6$$

$$x_1 + 2x_2 + S_3 + 2x_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, S_2 \geq 0, S_3 \geq 0.$$

		2	1	0	M	M	0	$\frac{1}{2}$		
C_B	Basic Variables	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$-\frac{1}{5}$	$\frac{2}{5} - M$	$\frac{1}{5} - M$	0			
2	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0		$\frac{3}{5}$	
1	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0		$\frac{6}{5}$	
0	S_3	0	0	1	1	-1	1		0	

The given optimal table is for minimum as all values of $Z_j - C_j$ are ≤ 0 .

Transform it into maximization by changing the sign of $Z_j - C_j$ and sign of objective function coefficients.

		$\frac{-}{2}$	$\frac{-}{1}$	0	$-M$	$-M$	0	$-\frac{1}{2}$		
C_B	Basic Variable s	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution n	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0			
$\frac{-}{2}$	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0		$\frac{3}{5}$	
$\frac{-}{1}$	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0		$\frac{6}{5}$	
0	S_3	0	0	1	1	-1	1		0	

If the given problem is solved by the Big-M method then in the starting table

- A_1 will be the first basic variable
- A_2 will be the second basic variable
- S_3 will be the third basic variable

$B^{-1} = [\text{Column of } A_1 \text{ in the optimal table} \quad \text{Column of } A_2 \text{ in the optimal table}$

$\quad \quad \quad \text{Column of } S_3 \text{ in the optimal table}]$

$$\mathbf{B}^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Column of x_3

\mathbf{B}^{-1} (Coefficients of x_3 in constraints)

$$= \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}$$

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
C_B	Basic Variables	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0			
-2	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	-1	$\frac{3}{5}$	
-1	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	3	$\frac{6}{5}$	
0	S_3	0	0	1	1	-1	1	-3	0	

		$-\frac{1}{2}$								
C_B	Basic Variable s	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution n	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0	$-\frac{1}{2}$		
$-\frac{1}{2}$	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	-1	$\frac{3}{5}$	
$-\frac{1}{1}$	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	3	$\frac{6}{5}$	
0	S_3	0	0	1	1	-1	1	-3	0	

Solution is not optimal. The variable x_3 is entering variable.

		-2	-1	0	-M	-M	0	$\frac{1}{-2}$		
C_B	Basic Variables	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0	$\frac{1}{-2}$		
-2	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	-1	$\frac{3}{5}$	
-1	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	3	$\frac{6}{5}$	
0	S_3	0	0	1	1	-1	1	-3	0	

		-2	-1	0	-M	-M	0	$\frac{1}{-2}$		
C_B	Basic Variables	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0	$\frac{1}{-2}$		
-2	x_1	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	-1	$\frac{3}{5}$	$\frac{3}{5} / -$
-1	x_2	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	3	$\frac{6}{5}$	$\frac{6}{5} / 3$
0	S_3	0	0	1	1	-1	1	-3	0	0/-

		-2	-1	0	-M	-M	0	1 -2		
C_B	Basic Variables	x₁	x₂	S₂	A₁	A₂	S₃	x₃	Solution	Minimum Ratio
Z _j - C _j =		0	0	1 5	-2 5 + M	-1 5 + M	0	1 -2		
-2	x ₁	1	1	1 5	3 5	-1 5	0	-1	3 5	3 5 /-
-1	x ₂	0	0	-3 5	-4 5	3 5	0	3	6 5	6 5 / 3
0	S ₃	0	0	1	1	-1	1	-3	0	0 / -

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
C_B	Basic Variables	x_1	x_2	S_2	A_1	A_2	S_3	x_3	Solution	Minimum Ratio
$Z_j - C_j =$		0	$\frac{1}{6}$	$\frac{1}{5}$	$-\frac{8}{15} + M$	$-\frac{1}{10} + M$	0	0		
-2	x_1	1	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0	1	
$-\frac{1}{2}$	x_3	0	$\frac{1}{3}$	$-\frac{1}{5}$	$-\frac{4}{5}$	$\frac{1}{5}$	0	1	$\frac{2}{5}$	
0	S_3	0	1	$\frac{2}{5}$	$\frac{1}{5}$		$-\frac{2}{5}$	1	0	$\frac{6}{5}$

New optimal solution:

$$x_1=1$$

$$x_3=\frac{2}{5}$$

$$S_3=\frac{6}{5}$$

Remaining are 0 i.e., $x_2=S_2=A_1=A_2=0$

Addition of a constraint

Example:

$$\text{Min } (x_1 - 2x_2 + x_3)$$

Subject to

$$x_1 + 2x_2 - 2x_3 \leq 4$$

$$x_1 - x_3 \leq 3$$

$$2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Optimal Table

		1	-2	1	0	0	0		
C_B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	Solution	Minimum Ratio
Z _j - C _j =		-\$\frac{9}{2}\$	0	0	-\$\frac{3}{2}\$	0	-1		
-2	x ₂	3	1	0	1	0	1	6	
0	S ₂	\$\frac{7}{2}\$	0	0	\$\frac{1}{2}\$	1	1	7	
1	x ₃	\$\frac{5}{2}\$	0	1	\$\frac{1}{2}\$	0	1	4	

Find the new optimal solution if

- (i) The constraint $x_2 + x_3 \geq 9$ is added.
- (ii) The constraint $x_2 + x_3 = 10$ is added.
- (iii) The constraint $x_1 + x_2 = 4$ is added.
- (iv) The constraint $x_1 + x_2 = 7$ is added.
- (v) The constraint $x_1 + x_2 \leq 4$ is added.
- (vi) The constraint $x_1 + x_2 \geq 7$ is added.

Solution

The given optimal table is for minimization as all values of $Z_j - C_j$ are $<= 0$.

Transform it for maximization by changing the sign of values of $Z_j - C_j$ and objective function coefficients.

		-1	2	-1	0	0	0		
C_B	Basic Variables	x_1	x_2	X_3	S_1	S_2	S_3	Solution	Minimum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1		
2	x_2	3	1	0	1	0	1	6	
0	S_2	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	7	
-1	x_3	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	4	

It is obvious from the provided table that

$$x_1=0, x_2=6, x_3=4.$$

(i)

The value of $x_2 + x_3 = 6 + 4 = 10$.

Since, the obtained value is > 9 . So the constraint $x_2 + x_3 \geq 9$ is satisfying.

Hence, on adding the constraint $x_2 + x_3 \geq 9$, there will be no change in the optimal solution.

(ii)

The value of $x_2 + x_3 = 6 + 4 = 10$.

Since, the obtained value is $= 10$. So the constraint $x_2 + x_3 = 10$ is satisfying.

Hence, on adding the constraint $x_2 + x_3 = 10$, there will be no change in the optimal solution.

(iii)

The value of $x_1 + x_2 = 0 + 6 = 6$.

Since, the obtained value is 6 . So the constraint $x_1 + x_2 = 4$ is not satisfying.

Hence, on adding the constraint $x_1 + x_2 = 4$, there will be change in the optimal solution.

$x_1 + x_2 = 4$ is equivalent to $x_1 + x_2 \geq 4$ and $x_1 + x_2 \leq 4$.

Since, the constraint $x_1 + x_2 \geq 4$ is satisfying and the constraint $x_1 + x_2 \leq 4$ is not satisfying.

So to add $x_1 + x_2 = 4$ is equivalent to add the constraint $x_1 + x_2 \leq 4$.

Now to add $x_1 + x_2 \leq 4$ is equivalent to $x_1 + x_2 + S_4 = 4$

C_B	Basic Variables	x_1	x_2	X_3	S_1	S_2	S_3	S_4	Solution	Minimum Ratio
		-1	2	-1	0	0	0	0		
$\frac{9}{2}$	x_2	0	0	$\frac{3}{2}$	0	1	0	0	6	
3		1	0	1	0	1	0	0	7	
$\frac{7}{2}$	S_2	0	0	$\frac{1}{2}$	1	1	0	0	4	
$\frac{5}{2}$	x_3	0	1	$\frac{1}{2}$	0	1	0	0	4	
1	S_4	1	1	0	0	0	0	1		

In the table,

The first basic variable is x_2 . So, its column should be

1

0

0

0

The second basic variable is S_2 . So, its column should be

0

1

0

0

The third basic variable is x_3 . So, its column should be

0

0

1

0

The fourth basic variable is S_4 . So, its column should be

0

0

0

1

It is obvious that the column of the first basic variable x_2 is

1

0

0

1

instead of

1

0

0

0

Therefore, we need to apply a row operation for the fifth row such that the last element of this column is 0 and no change in columns of other basic variables.

$R_5 \rightarrow R_5 - R_2$

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	S ₄	Solution	Minimum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	x ₂	3	1	0	1	0	1	0	6	
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	S ₄	1-3 =-2 =0	1 -1 =0	0 =-1 =-1	0-1 =-1 =-1	0 -1 -1	0-1=0 -1 -1	1-0=0 -1 -1	4-6=0 -2 -2	

Since, the RHS corresponding to the variable S₄ is negative. So, there is a need to apply dual simplex method.

S₄ is leaving variable.

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	S ₄	Solution	Maximum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		Maximum
2	x ₂	3	1	0	1	0	1	0	6	$m\{$
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	$\frac{9}{2}, \frac{3}{2}$
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	$, \frac{1}{-1} \} = \frac{1}{-1}$
0	S ₄	-2	0	0	-1	0	-1	1	-2	

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	S ₄	Solution	Maximum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		Maximum
2	x ₂	3	1	0	1	0	1	0	6	$m\{$
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	$\frac{9}{2}, \frac{3}{2}$
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	$, \frac{1}{-1} \} = \frac{1}{-1}$
0	S ₄	-2	0	0	-1	0	-1	1	-2	

		-1	2	-1	0	0	0	0		
C_B	Basic Variables	x₁	x₂	x₃	S₁	S₂	S₃	S₄	Solution	Maximum Ratio
	$Z_j - C_j =$	$\frac{5}{2}$	0	0	$\frac{1}{2}$	0	0	0		
2	x₂	1	1	0	1	0	0	0	4	
0	S₂	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1	0	0	5	
-1	x₃	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0	0	0	2	
0	S₃	2	0	0	1	0	1	-1	2	

New optimal solution is

x₂=4

S₂=5

x₃=2

S₃=2

Remaining are 0 i.e., x₁=S₁=S₄=0.

(iv)

The value of $x_1 + x_2 = 0 + 6 = 6$.

Since, the obtained value is 6. So the constraint $x_1 + x_2 = 7$ is not satisfying.

Hence, on adding the constraint $x_1 + x_2 = 7$, there will be change in the optimal solution.

$x_1 + x_2 = 7$ is equivalent to $x_1 + x_2 \geq 7$ and $x_1 + x_2 \leq 7$.

Since, the constraint $x_1 + x_2 \leq 7$ is satisfying and the constraint $x_1 + x_2 \geq 7$ is not satisfying.

So to add $x_1 + x_2 = 7$ is equivalent to add the constraint $x_1 + x_2 \geq 7$.

Now to add $x_1 + x_2 \geq 7$ is equivalent to $x_1 + x_2 - S_4 = 7$ or $-x_1 - x_2 + S_4 = -7$

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	x ₃	S ₁	S ₂	S ₃	S ₄	Solution	Minimum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	x ₂	3	1	0	1	0	1	0	6	
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	S ₄	-1	-1	0	0	0	0	-1	-7	

In the table,

The first basic variable is x₂. So, its column should be

1

0

0

0

The second basic variable is S₂. So, its column should be

0

1

0

0

The third basic variable is x_3 . So, its column should be

0

0

1

0

The fourth basic variable is S_4 . So, its column should be

0

0

0

1

It is obvious that the column of the first basic variable x_2 is

1

0

0

-1

instead of

1

0

0

0

Therefore, we need to apply a row operation for the fifth row such that the last element of this column is 0 and no change in columns of other basic variables.

R₅ → R₅ + R₂

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	S ₄	Solution	Minimum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	x ₂	3	1	0	1	0	1	0	6	
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	S ₄	-1+ 3 =2	-1 +1 =0	0	0 + 1=1	0	0 + 1=1	1+0=1 -7+6=-1	-7+6=-1	

Since, the RHS corresponding to the variable S₄ is negative. So, there is a need to apply dual simplex method.

S₄ is leaving variable.

		-1	2	-1	0	0	0	0		
C _B	Basic Variables	x ₁	x ₂	X ₃	S ₁	S ₂	S ₃	S ₄	Solution	Maximum Ratio
	$Z_j - C_j =$	$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	x ₂	3	1	0	1	0	1	0	6	
0	S ₂	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x ₃	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	S ₄	2	0	0	1	0	1	1	-1	

Since, all the values in the row corresponding to leaving variable are ≥ 0 .

So, it is not possible to find entering variable.

Hence, no feasible solution exists for the transformed LPP.

(v) Same solution as in (iii)

(vi) Same solution as in (iv)