Course: UMA 035 (Optimization Techniques)

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Example: **Solve the following LPP by graphical method.**

Maximize/Minimize ($3x_1 + 2x_2$)

Subject to

$2x_1 + 4x_2 <= 8$,

$3x_1 + 5x_2 >= 15$,

$x_1 >= 0$, $x_2 >= 0$.

Solution: Since, Minimum value of x_1 and x_2 are 0. So, there is no need to transform these variables into new variables.

Draw First Cosnstraint

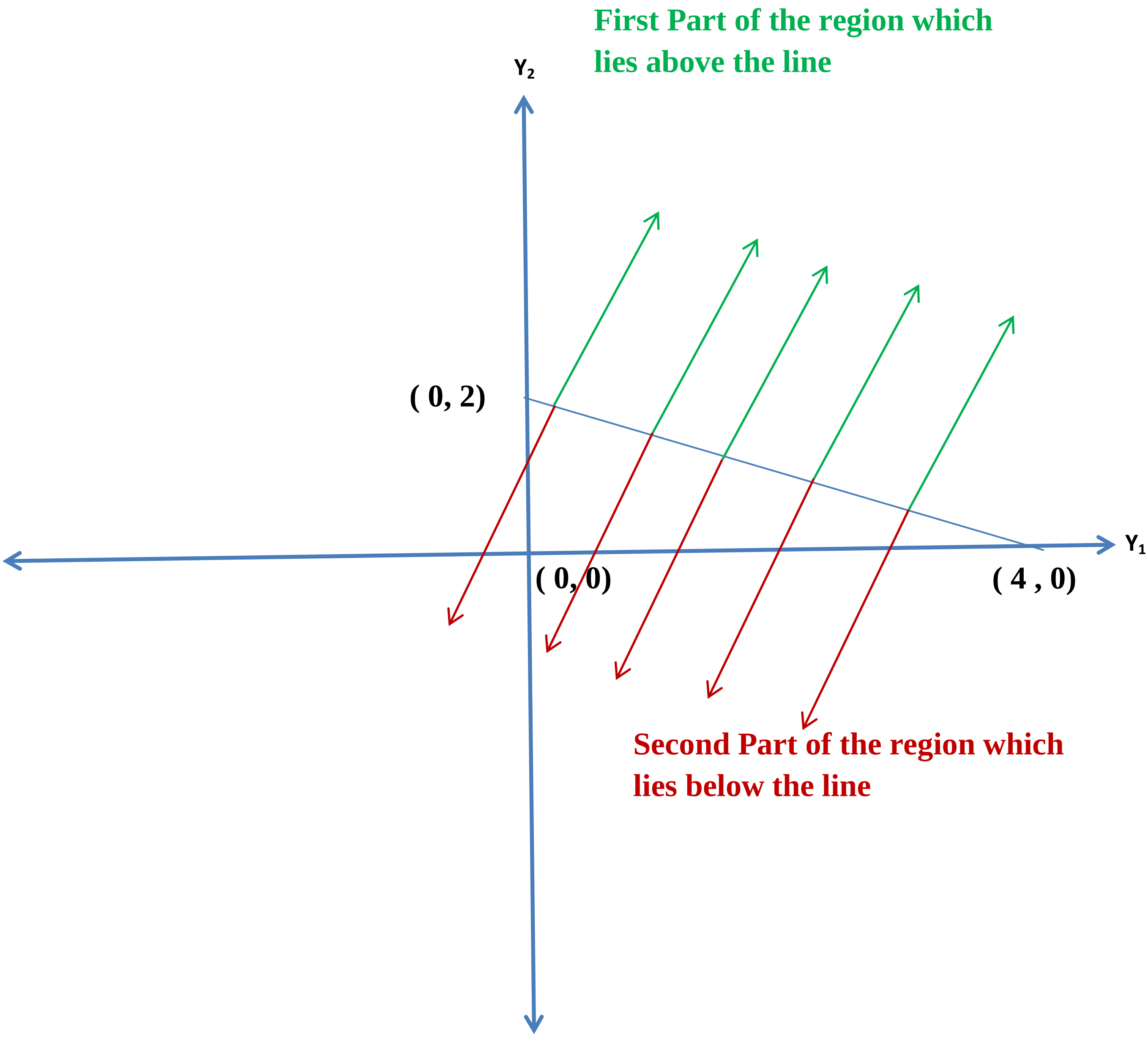
$2x_1 + 4x_2 <= 8$

Assuming $x_1 = 0$, $2x_1 + 4x_2 = 8$ implies $0 + 4x_2 = 8$ i.e., $x_2 = 2$

Therefore, first point is $(x_1, x_2) = (0, 2)$

Assuming $x_2 = 0$, $2x_1 + 4x_2 = 8$ implies $2x_1 + 0 = 8$ i.e., $x_1 = 4$

Therefore, second point is $(x_1, x_2) = (4, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

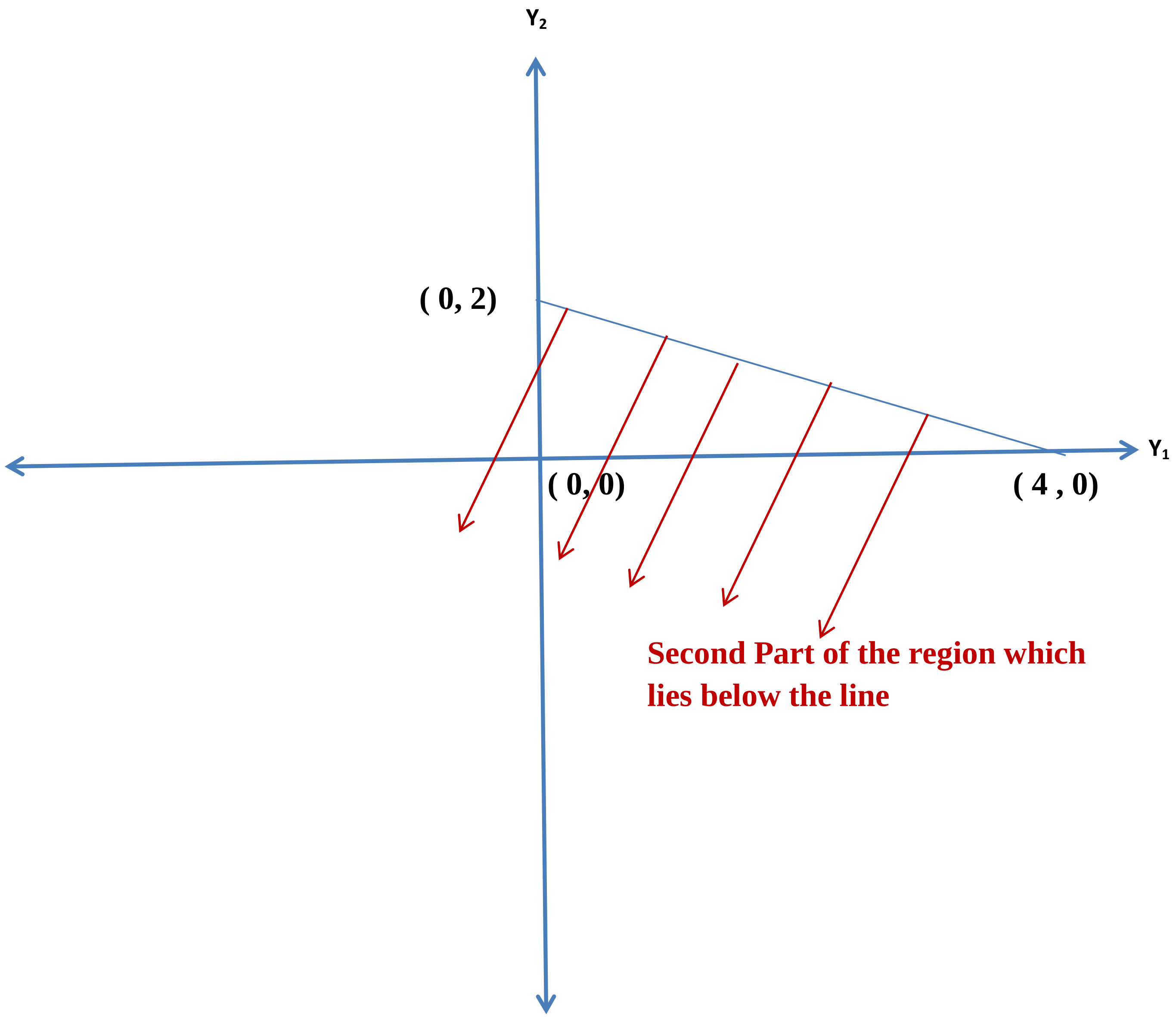
Putting (0,0) in the constraint

$2x_1+4x_2 <= 8$, we have

$0+0<=8$

$0<=8$

It is obvious that the constraint is satisfying. So, we consider the second part.



Draw Second Cosnstraint

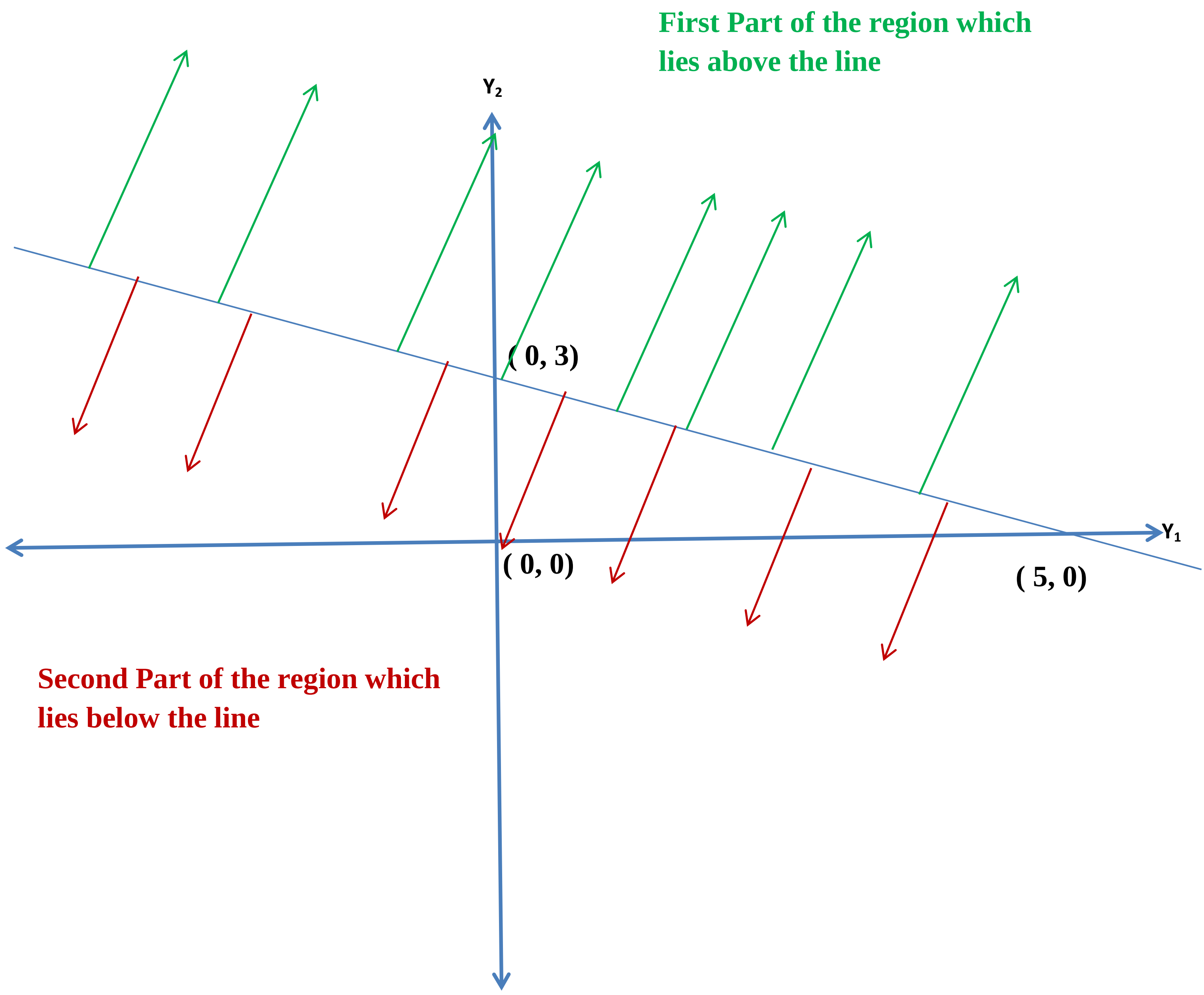
$3x_1 + 5x_2 >= 15$

Assuming $x_1 = 0$, $3x_1+5x_2 = 15$ implies $0 + 5x_2 = 15$ i.e., $x_2 = 3$

Therefore, first point is $(x_1,x_2)=(0, 3)$

Assuming $x_2 = 0$, $3x_1+5x_2 = 15$ implies $3x_1+0 = 15$ i.e., $x_1 = 5$

Therefore, second point is $(x_1,x_2)=(5, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

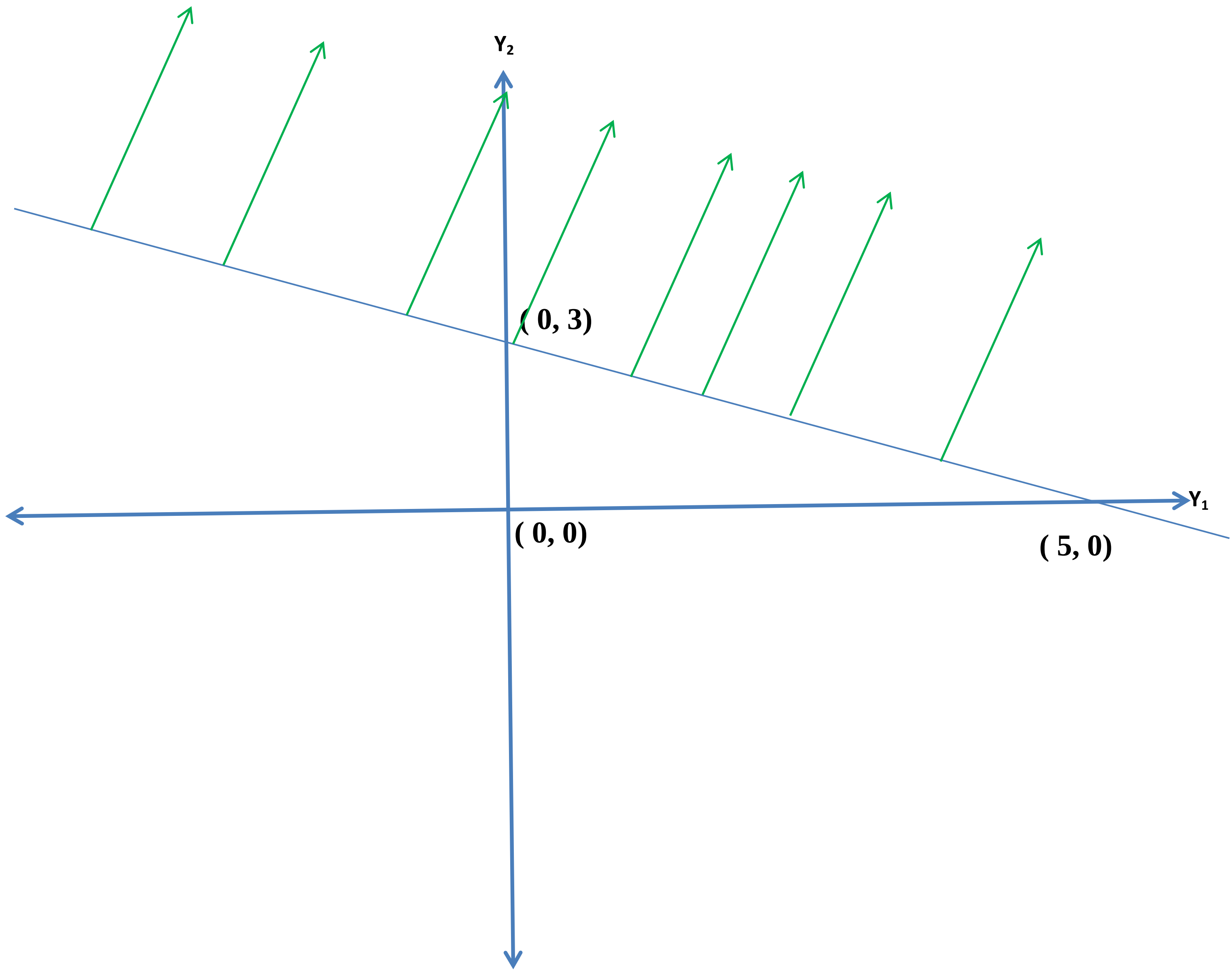
Putting (0,0) in the the constraint

$3x_1+5x_2 >= 15$, we have

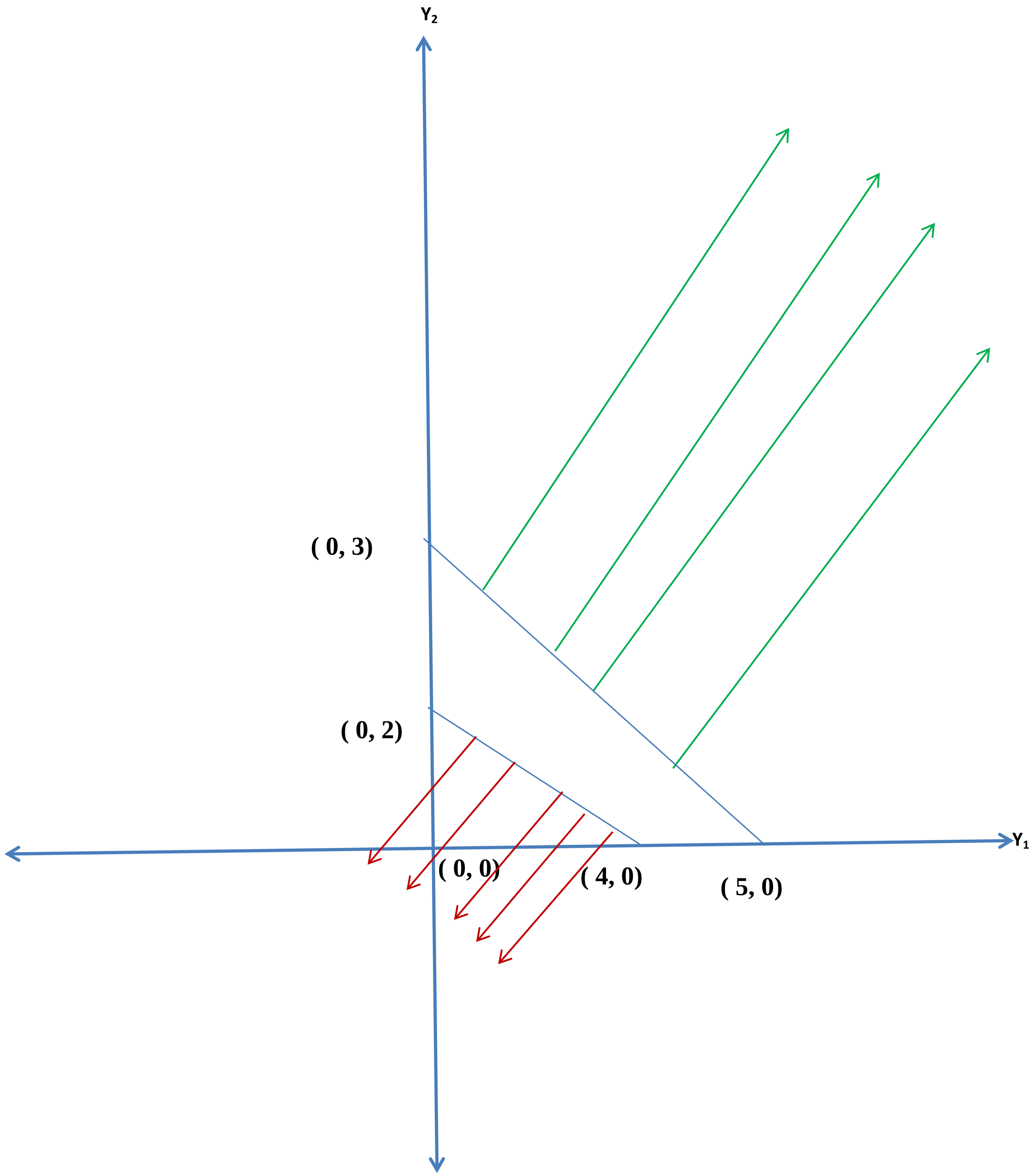
$0+0 >= 15$

$0 >= 15$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)

It is obvious that there is no common region in the first quadrant. So, no feasible region and hence, no optimal solution.

Example: Solve the following LPP by graphical method.

Maximize/Minimize ($3x_1 + 2x_2$)

Subject to

$2x_1 + 4x_2 <= 8$,

$3x_1 + 5x_2 <= 15$,

$x_1 >= 0$, $x_2 >= 0$.

Solution: Since, Minimum value of x_1 and x_2 are 0. So, there is no need to transform these variables into new variables.

Draw First Cosnstraint

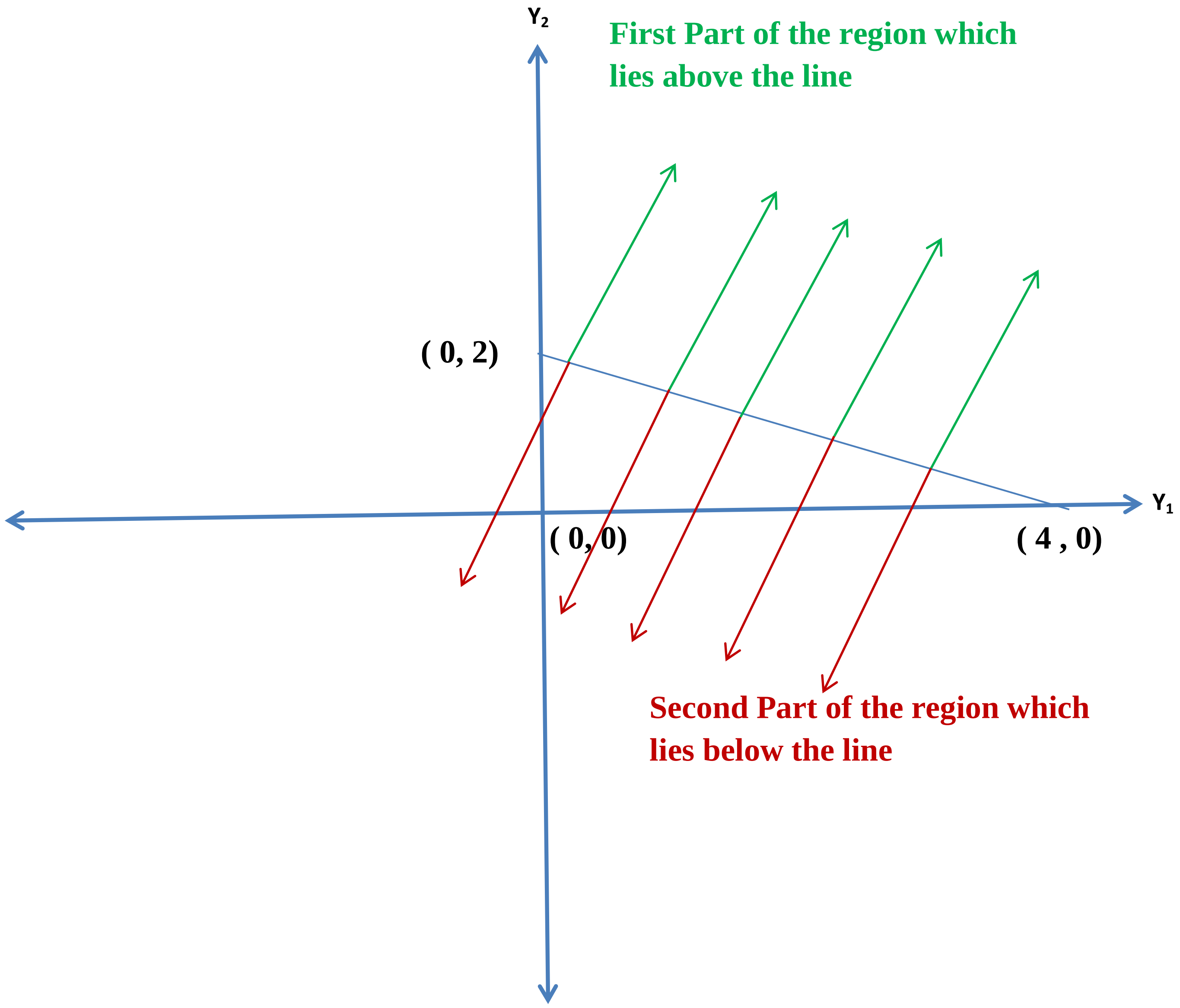
$2x_1 + 4x_2 <= 8$

Assuming $x_1 = 0$, $2x_1 + 4x_2 = 8$ implies $0 + 4x_2 = 8$ i.e., $x_2 = 2$

Therefore, first point is $(x_1, x_2) = (0, 2)$

Assuming $x_2 = 0$, $2x_1 + 4x_2 = 8$ implies $2x_1 + 0 = 8$ i.e., $x_1 = 4$

Therefore, second point is $(x_1, x_2) = (4, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

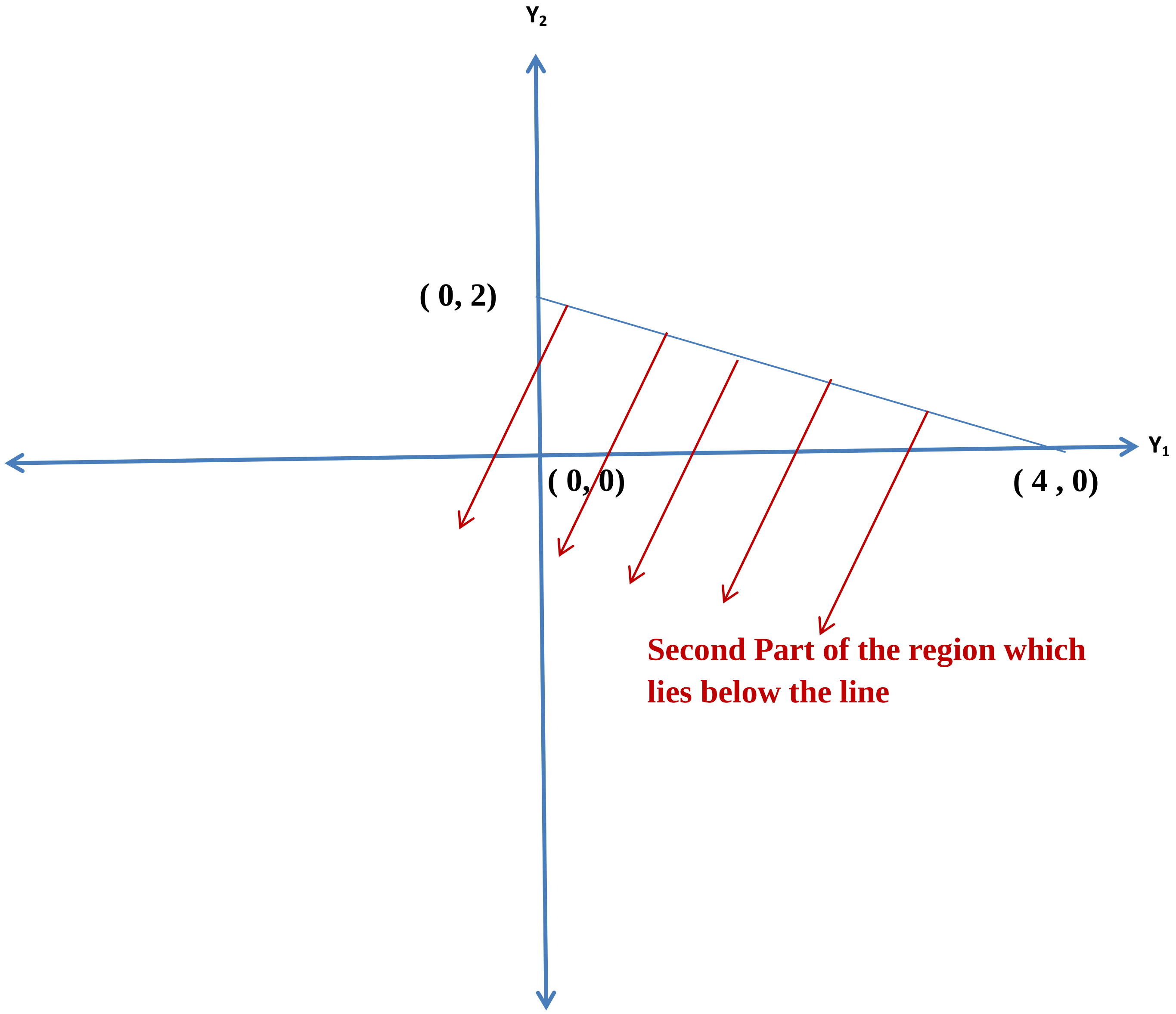
Putting (0,0) in the constraint

$2x_1+4x_2 <= 15$, we have

$0+0<=8$

$0<=8$

It is obvious that the constraint is satisfying. So, we consider the second part.



Draw Second Cosnstraint

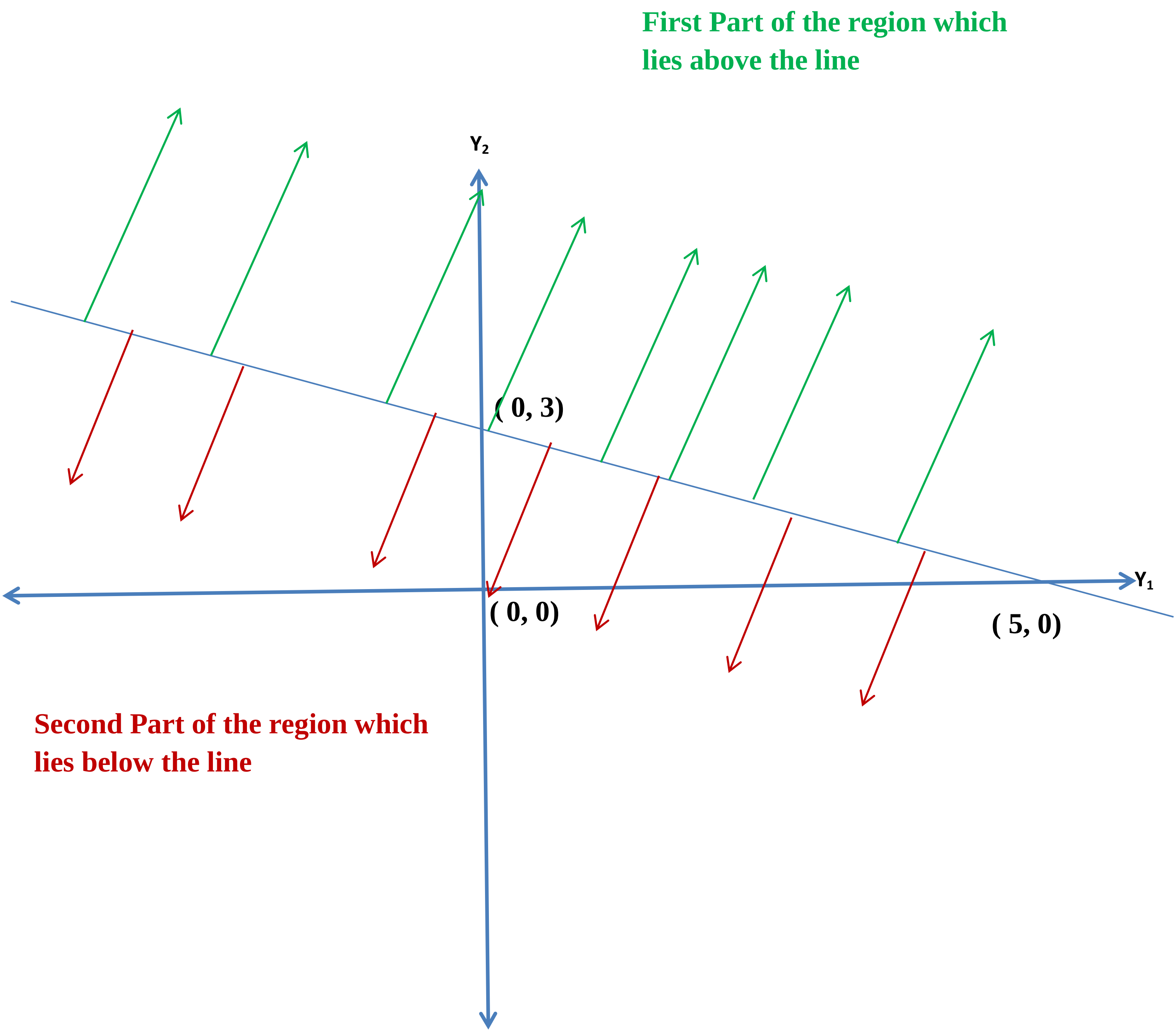
$3x_1 + 5x_2 >= 15$

Assuming $x_1 = 0$, $3x_1+5x_2 = 15$ implies $0 + 5x_2 = 15$ i.e., $x_2 = 3$

Therefore, first point is $(x_1,x_2)=(0, 3)$

Assuming $x_2 = 0$, $3x_1+5x_2 = 15$ implies $3x_1+0 = 15$ i.e., $x_1 = 5$

Therefore, second point is $(x_1,x_2)=(5, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

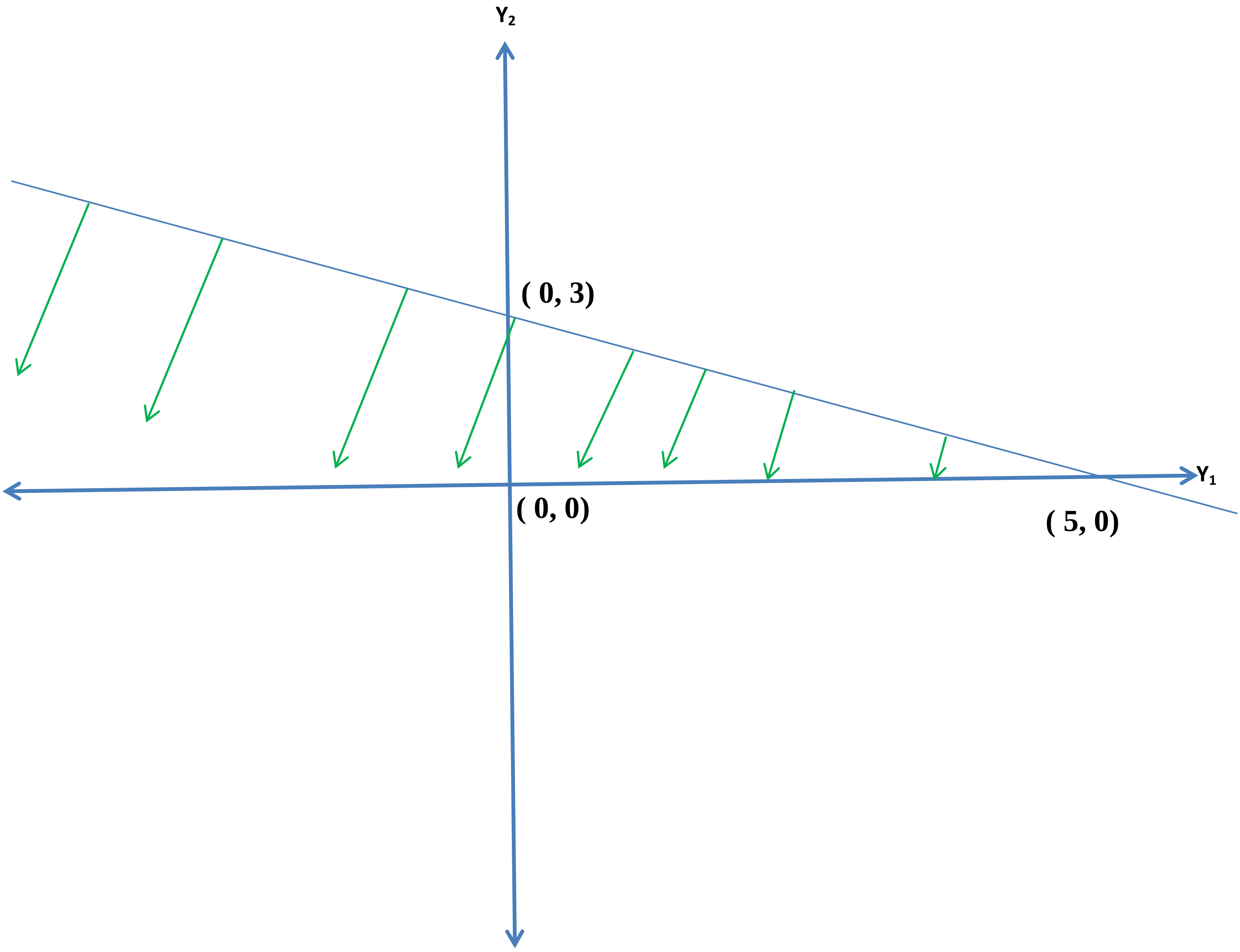
Putting (0,0) in the the constraint

$3x_1+5x_2 <= 15$, we have

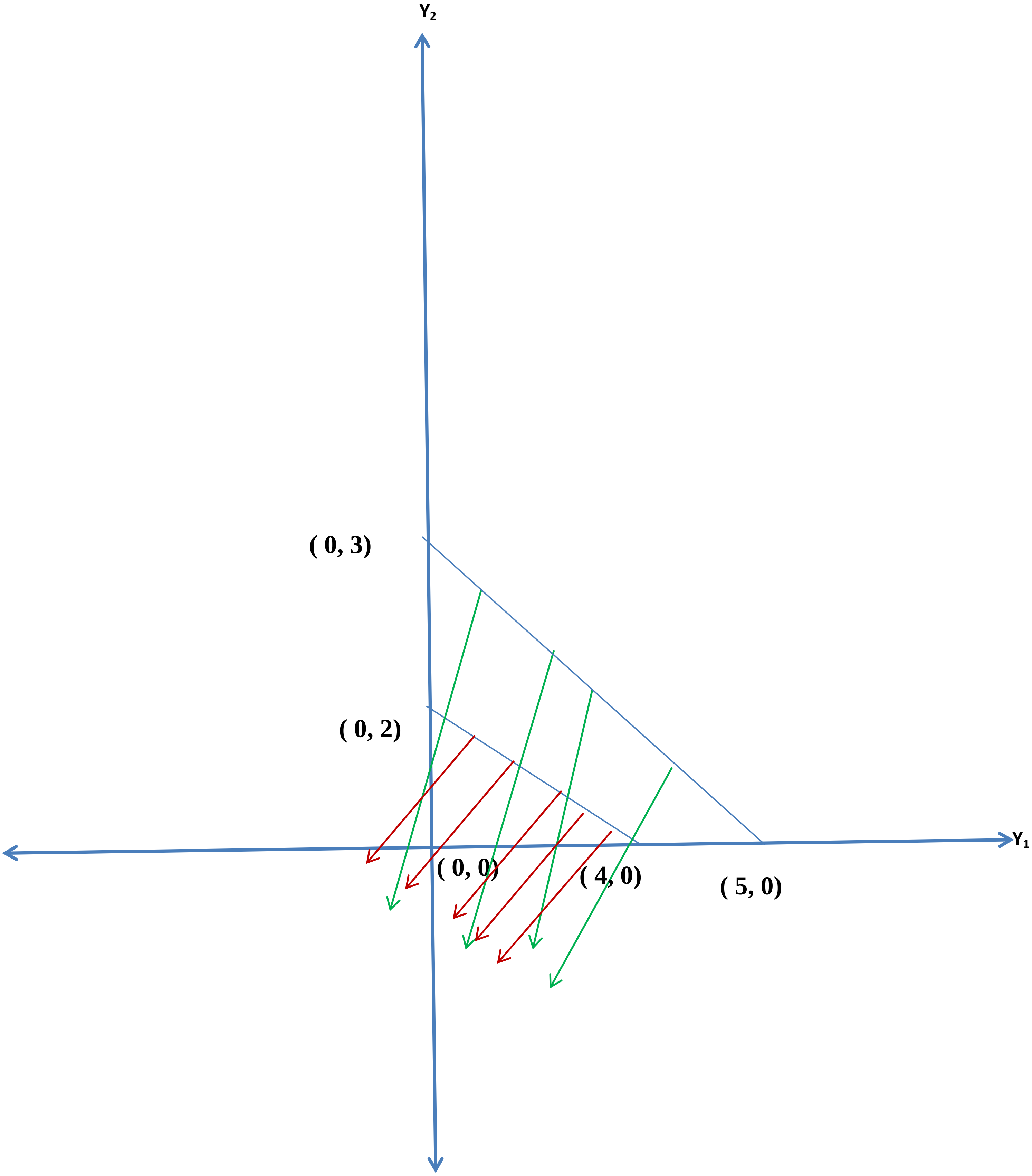
$0+0<=15$

0<= 15

It is obvious that the constraint is satisfying. So, we consider the second part.

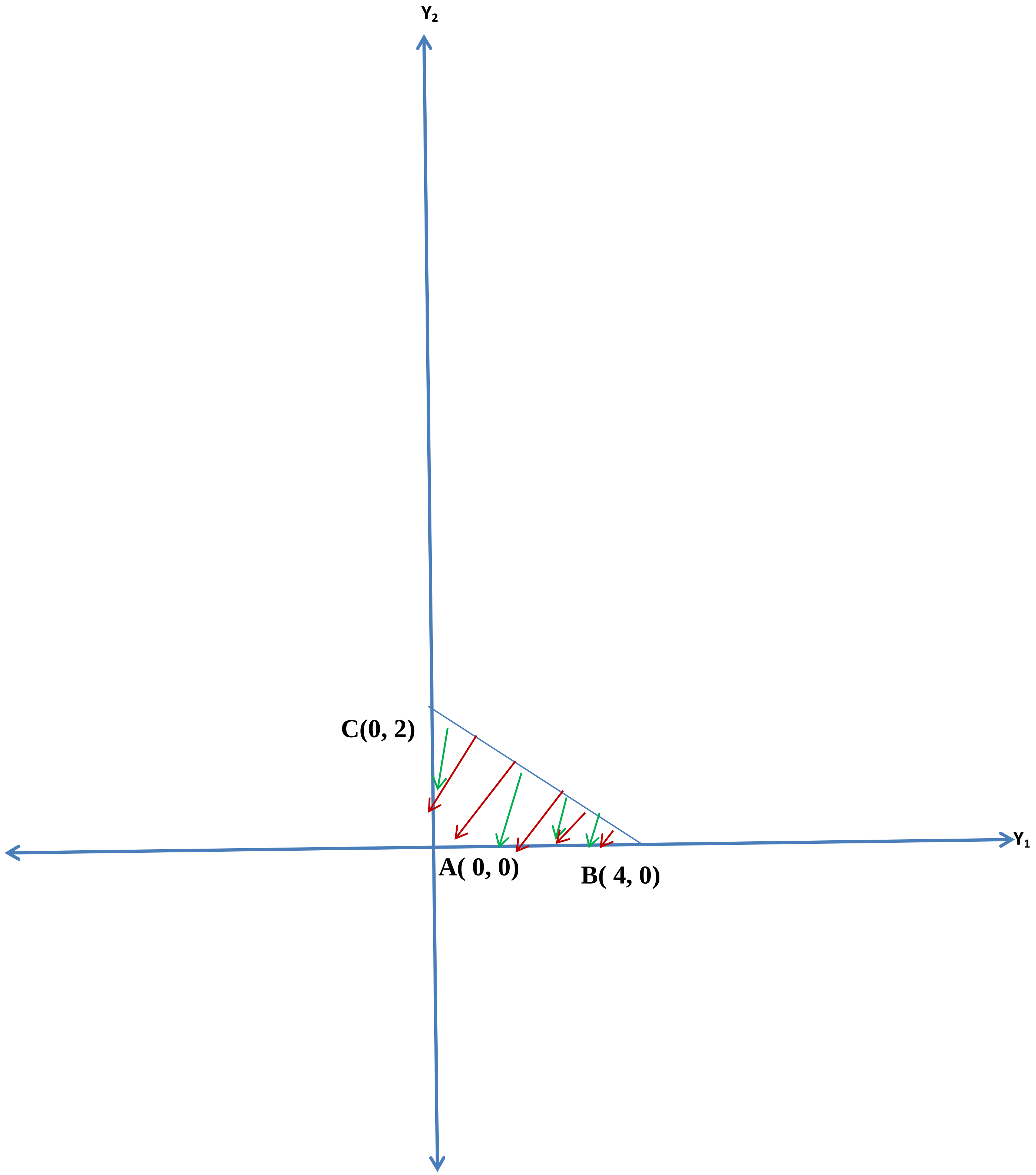


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



The feasible region is bounded so no need to add any constraint.

Extreme points or vertices or corner points

First point A (0, 0)

Second point B (4, 0)

Third point C (0, 2)

Value of the objective function $3x_1 + 2x_2$ at

- **A (0,0) is 3*0+2*0=0**
- **B (4,0) is 3*4+2*0=12**
- **C (0,2) is 3*0+2*2=4**

Since maximum {0,12,4}=12 which is corresponding to x_1=4 and x_2=0. So, if the problem is of maximization then the

- **Optimal solution is x_1=4 and x_2=0**
- **The optimal value is 12**

Since minimum {0,12,4}=0 which is corresponding to x_1=0 and x_2=0. So, if the problem is of minimization then the

- **Optimal solution is x_1=0 and x_2=0**
- **The optimal value is 0**

Example: **Solve the following LPP by graphical method.**

Maximize/Minimize $(3x_1 + 2x_2)$

Subject to

$2x_1 + 4x_2 >= 8$,

$3x_1 + 5x_2 <= 15$,

$x_1 >= 0$, $x_2 >= 0$.

Solution: Since, Minimum value of x_1 and x_2 are 0. So, there is no need to transform these variables into new variables.

Draw First Cosnstraint

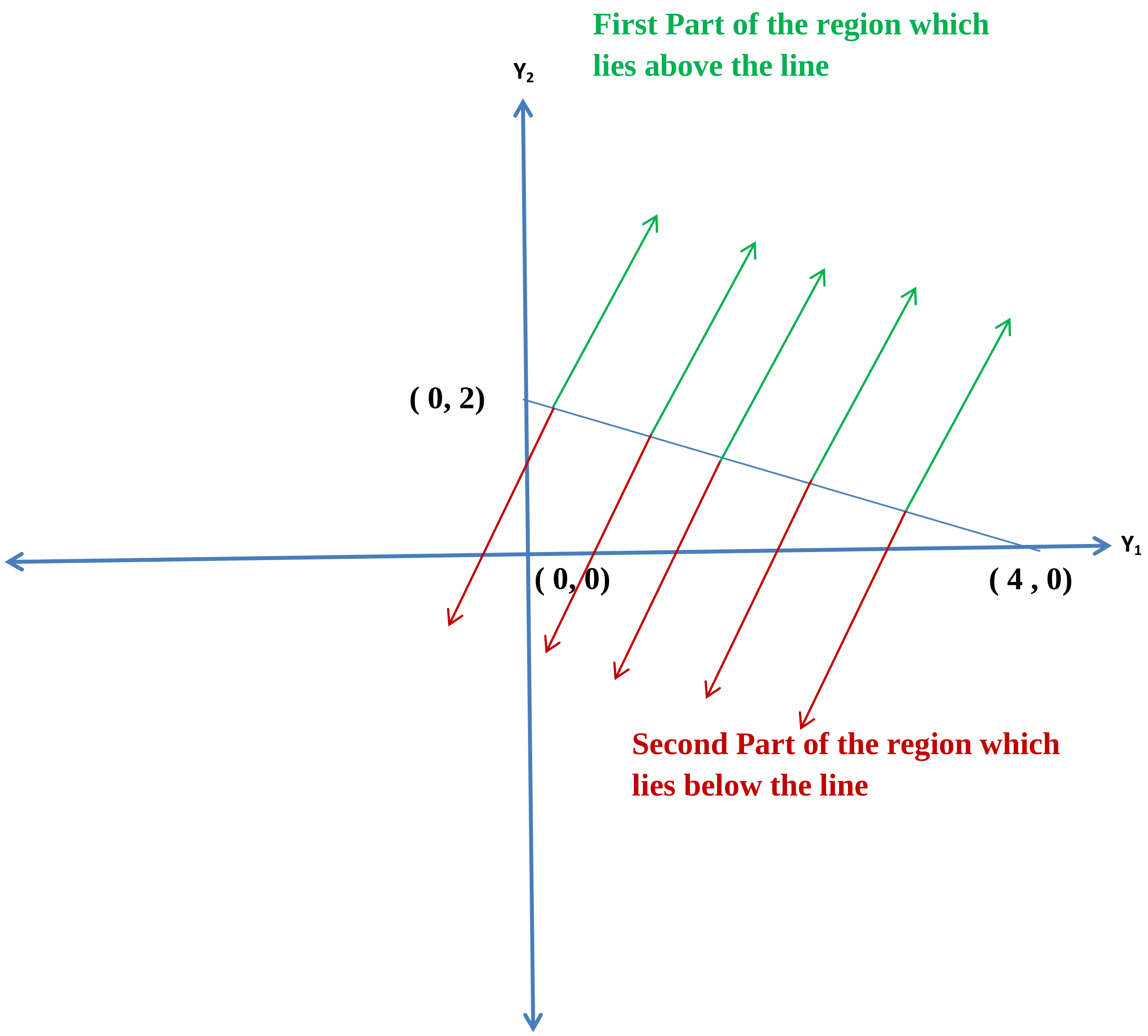
$2x_1 – 4x_2 < = 8$

Assuming $x_1 = 0$, $2x_1+4x_2 = 8$ implies $0 + 4x_2 = 8$ i.e., $x_2 = 2$

Therefore, first point is $(x_1,x_2)=(0, 2)$

Assuming $x_2 = 0$, $2x_1+4x_2 =8$ implies $2x_1 + 0 = 8$ i.e., $x_1 =4$

Therefore, second point is $(x_1,x_2)=(4, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

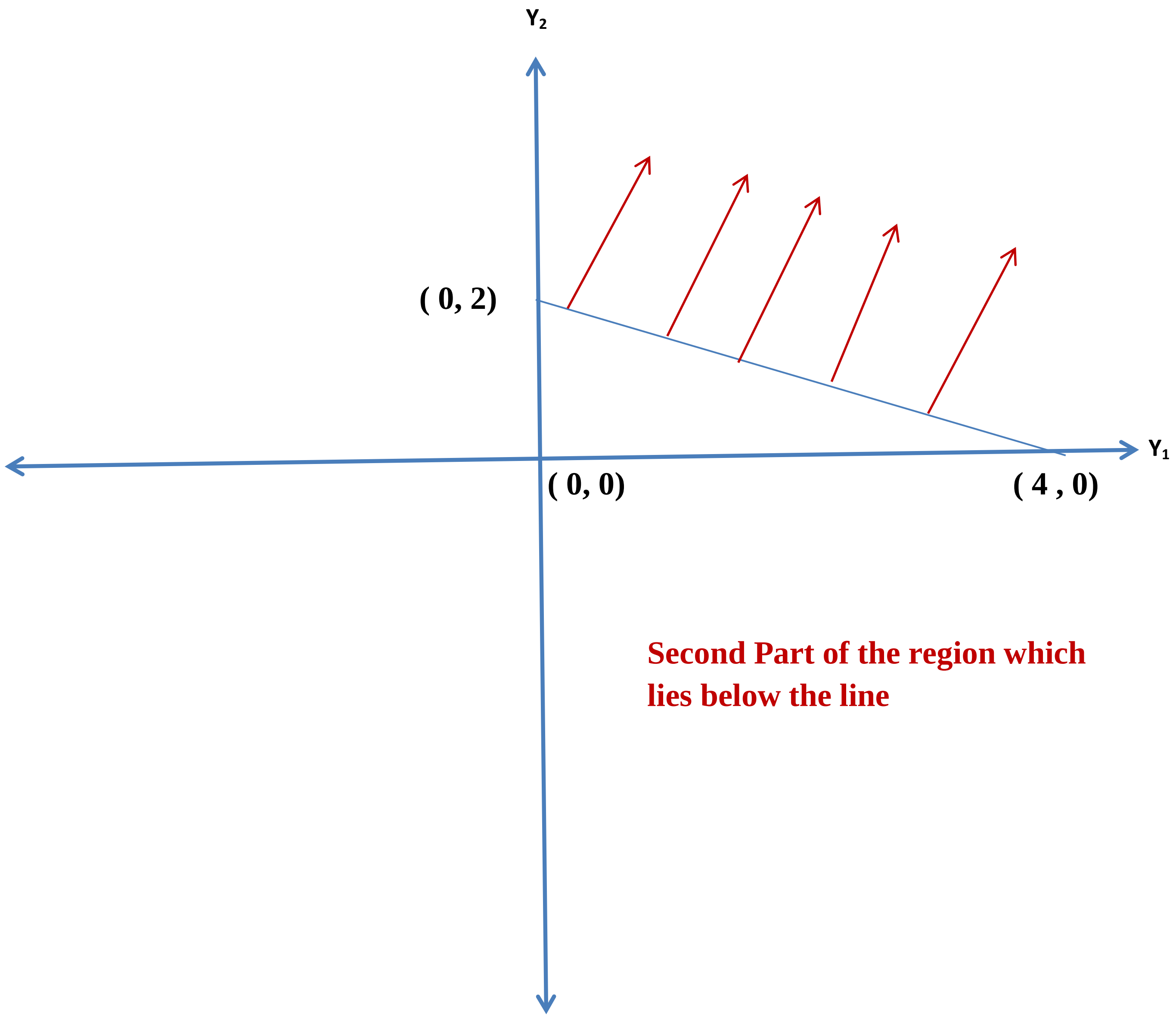
Putting (0,0) in the constraint

$2x_1+4x_2 >= 8$, we have

$0+0>=8$

$0>=8$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Draw Second Cosnstraint

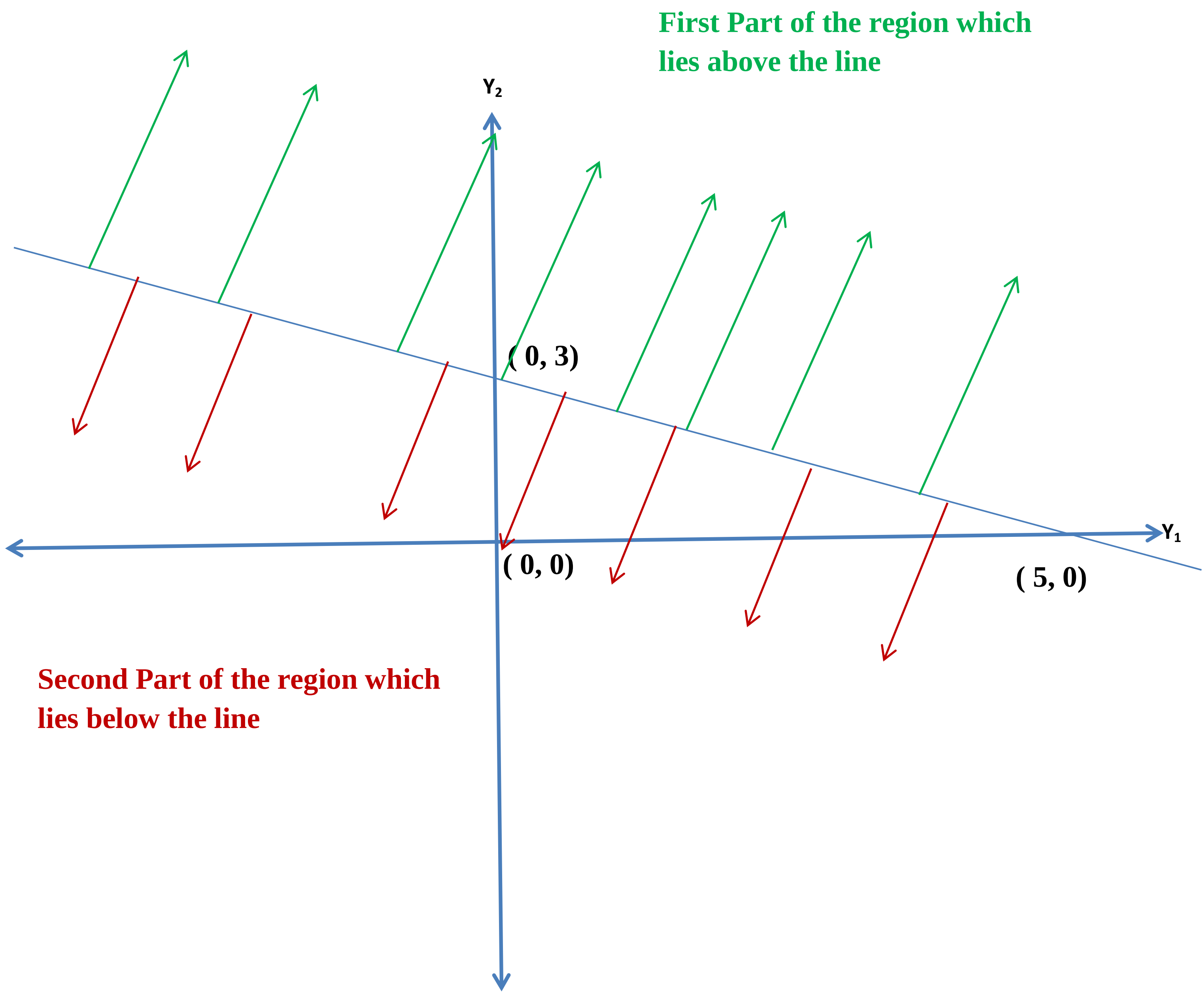
$3x_1 + 5x_2 <= 15$

Assuming $x_1 = 0$, $3x_1+5x_2 = 15$ implies $0 + 5x_2 = 15$ i.e., $x_2 = 3$

Therefore, first point is $(x_1,x_2)=(0, 3)$

Assuming $x_2 = 0$, $3x_1+5x_2 = 15$ implies $3x_1+0 = 15$ i.e., $x_1 = 5$

Therefore, second point is $(x_1,x_2)=(5, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

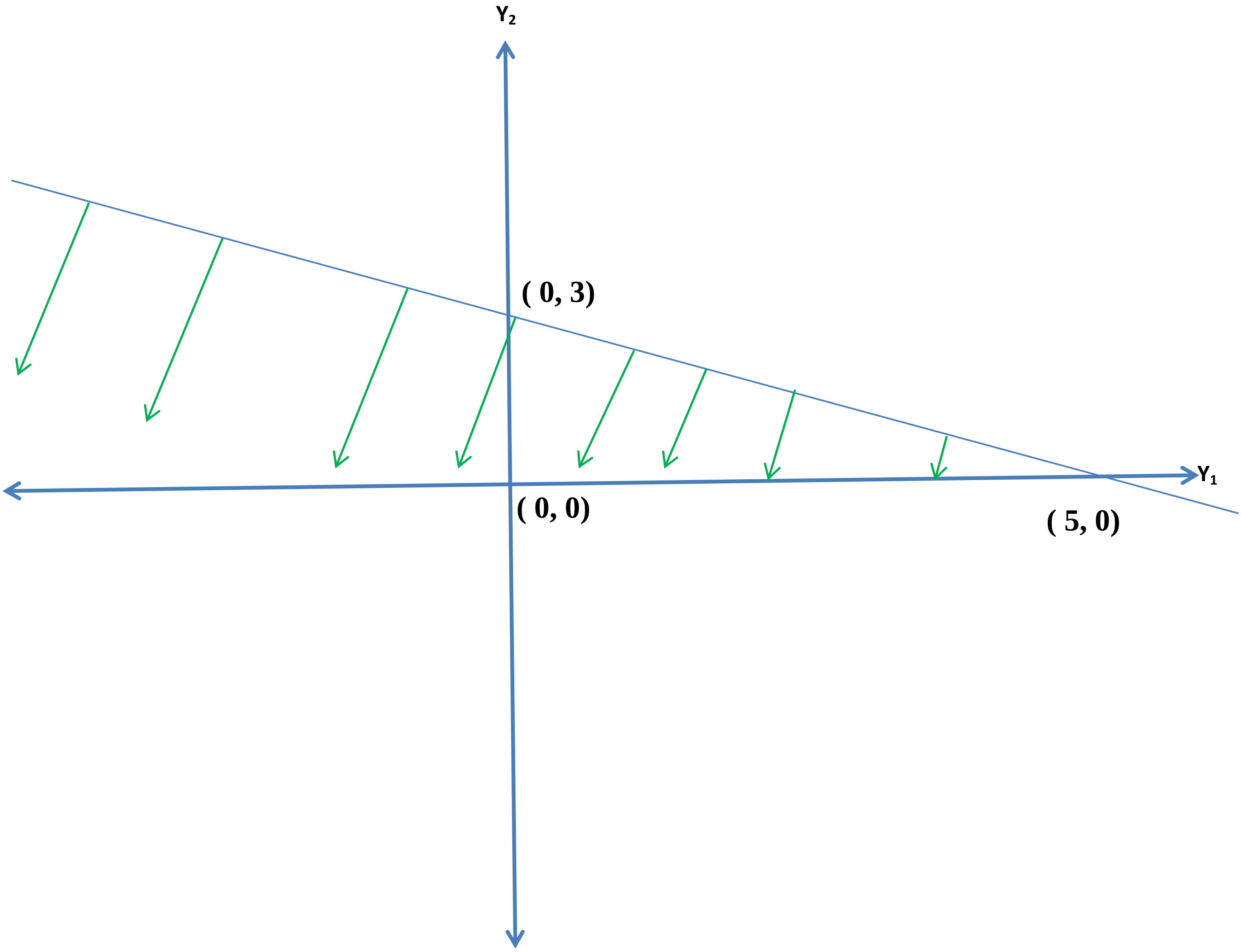
Putting (0,0) in the the constraint

$3x_1+5x_2 <= 15$, we have

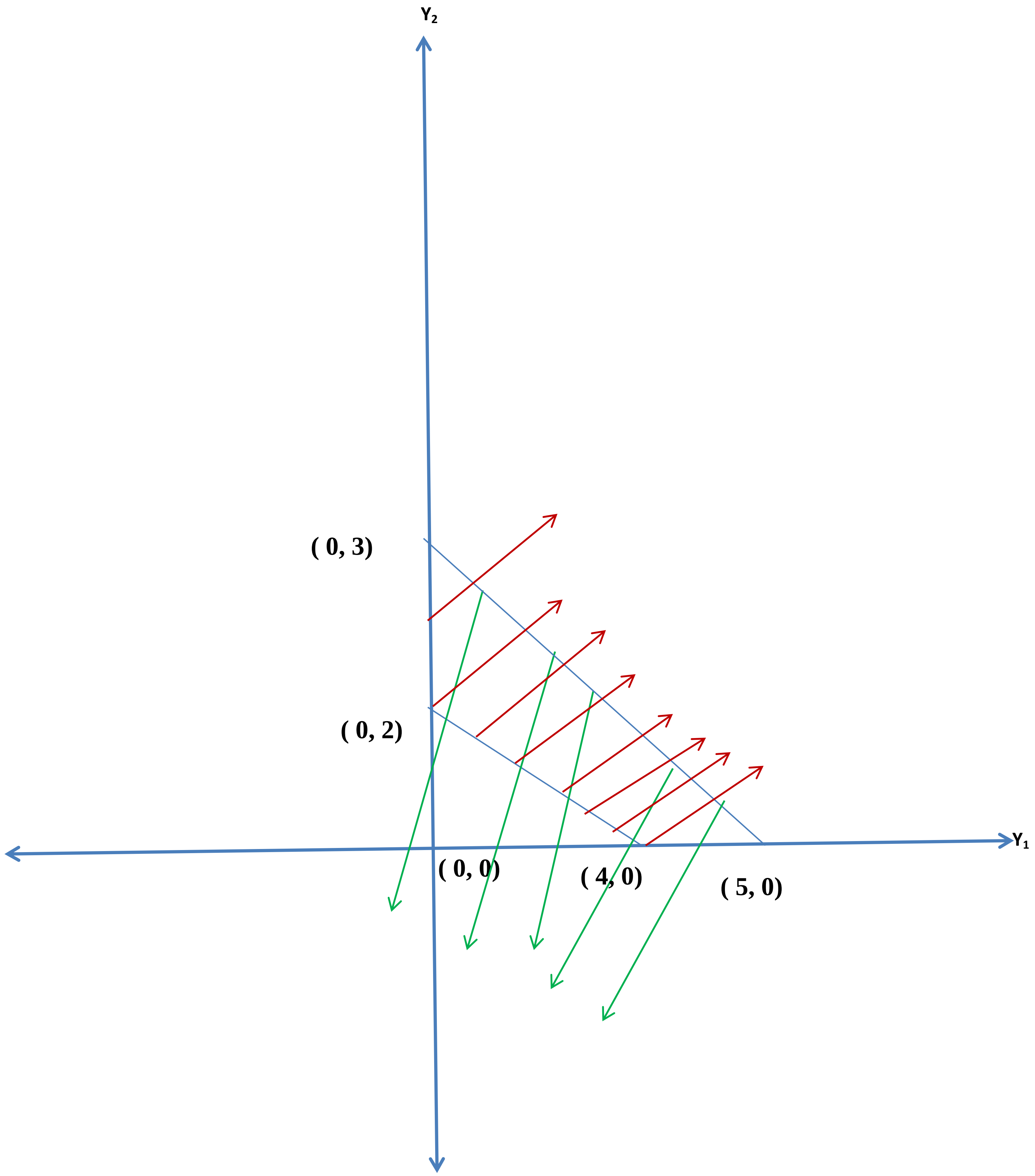
$0+0 <= 15$

$0 <= 15$

It is obvious that the constraint is satisfying. So, we consider the second part.

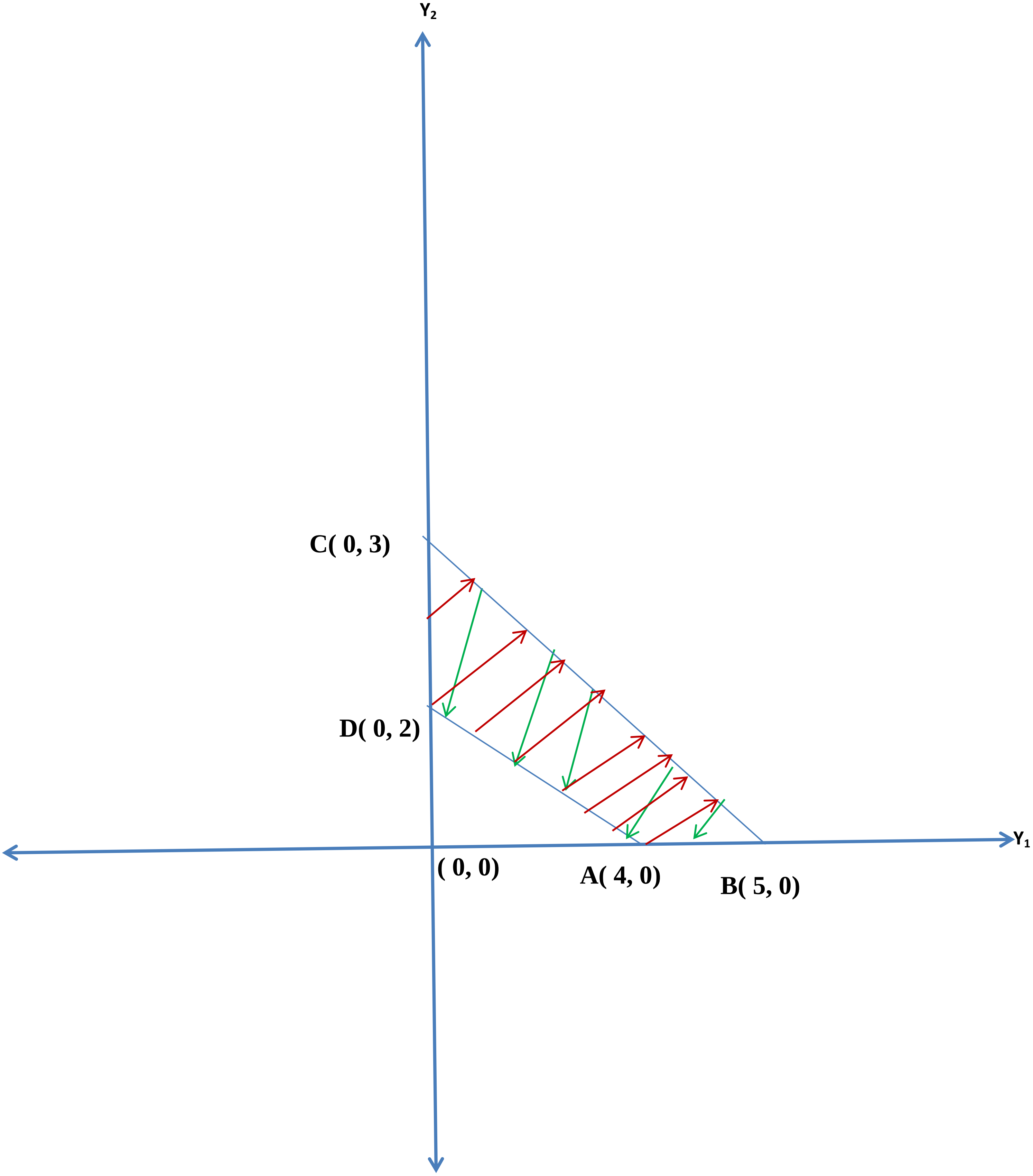


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



The feasible region is bounded so no need to add any constraint.

Extreme points or vertices or corner points

First point A (4, 0)

Second point B (5, 0)

Third point C (0, 3)

Fourth point (0, 2)

Value of the objective function $3x_1 + 2x_2$ at

- **A (4,0) is 3*4+2*0=12**
- **B (5,0) is 3*5+2*0=15**
- **C (0,3) is 3*0+2*3=6**
- **D (0, 2) is 3*0+2*2=4**

Since maximum {12,15,6,4}=15 which is corresponding to x_1=5 and x_2=0. So, if the problem is of maximization then the

- **Optimal solution is x_1=5 and x_2=0**
- **The optimal value is 15**

Since minimum {12,15,6,4}=4 which is corresponding to x_1=0 and x_2=2. So, if the problem is of minimization then the

- **Optimal solution is x_1=0 and x_2=2**

- **The optimal value is 4**

Example: Solve the following LPP by graphical method.

Maximize/Minimize ($3x_1 + 2x_2$)

Subject to

$2x_1 + 4x_2 >= 8$,

$3x_1 + 5x_2 >= 15$,

$x_1>=0, x_2>= 0$.

Solution: Since, Minimum value of x_1 and x_2 are 0. So, there is no need to transform these variables into new variables.

Draw First Cosnstraint

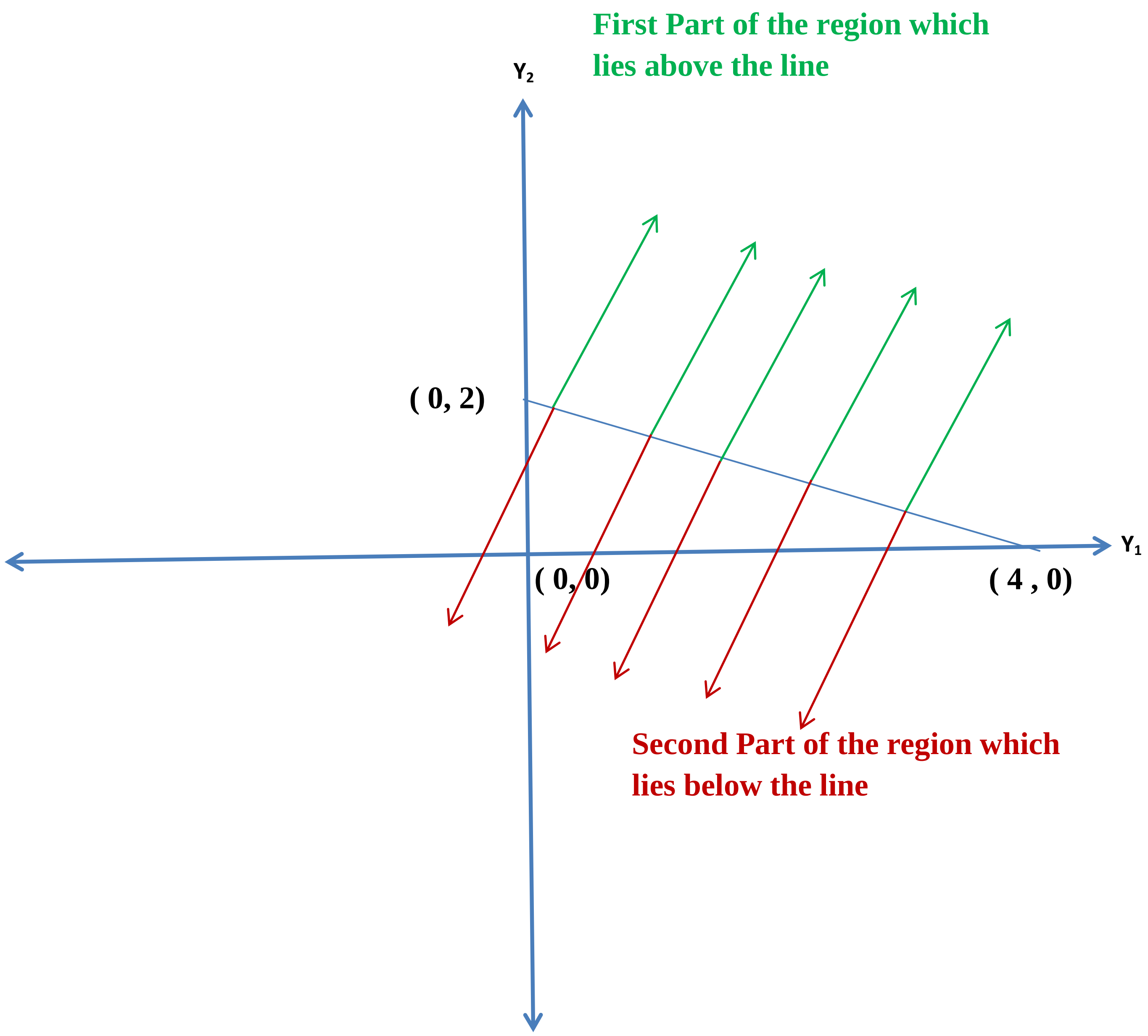
$2x_1+4x_2 >= 8$

Assuming $x_1 = 0$, $2x_1+4x_2 = 8$ implies $0 + 4x_2 = 8$ i.e., $x_2 = 2$

Therefore, first point is $(x_1,x_2)=(0, 2)$

Assuming $x_2 = 0$, $2x_1+4x_2 =8$ implies $2x_1 + 0 = 8$ i.e., $x_1 =4$

Therefore, second point is $(x_1,x_2)=(4, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

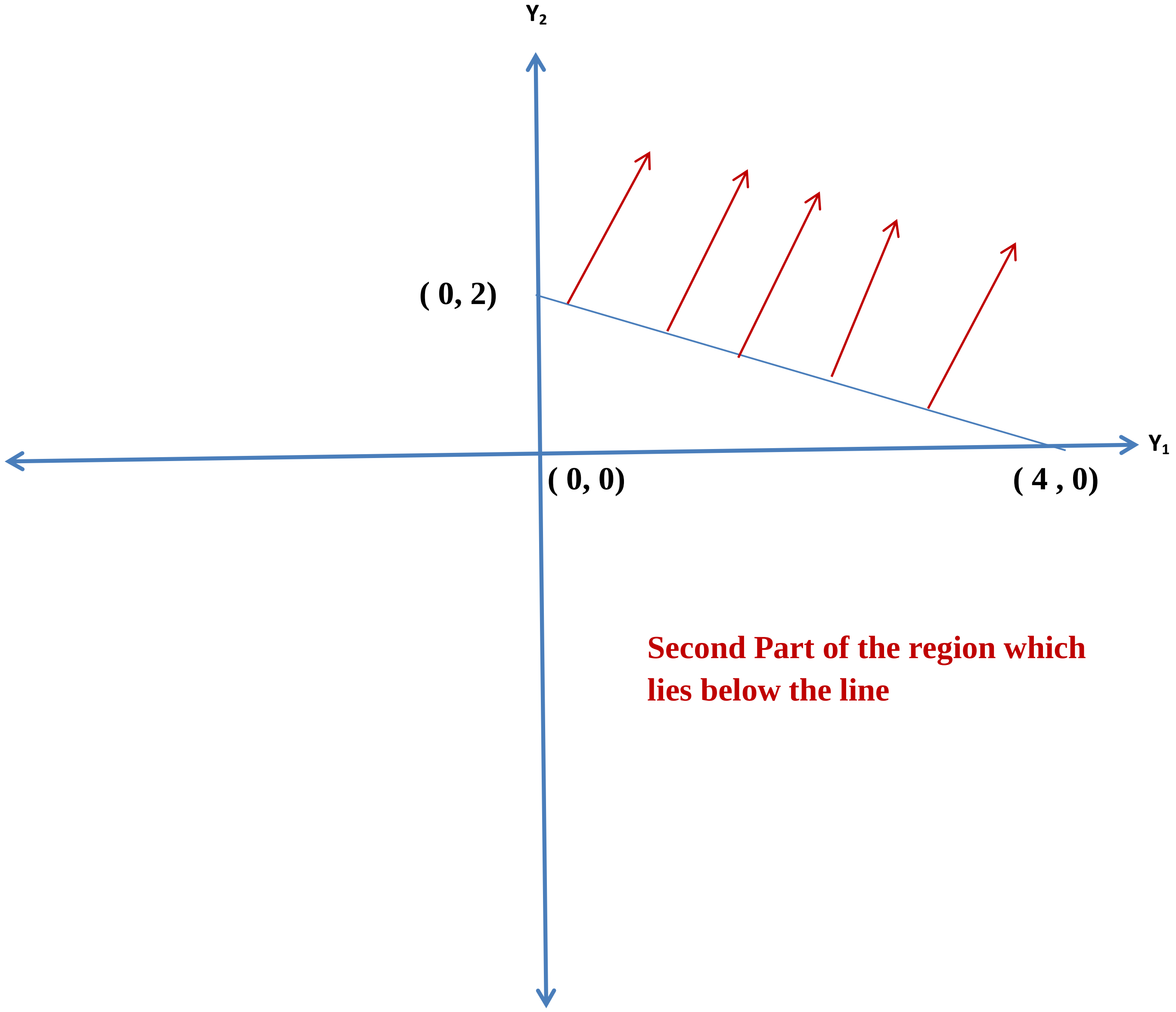
Putting (0,0) in the constraint

$2x_1+4x_2 >= 8$, **we have**

$0+0>=8$

$0>=8$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Second Part of the region which lies below the line

Draw Second Cosnstraint

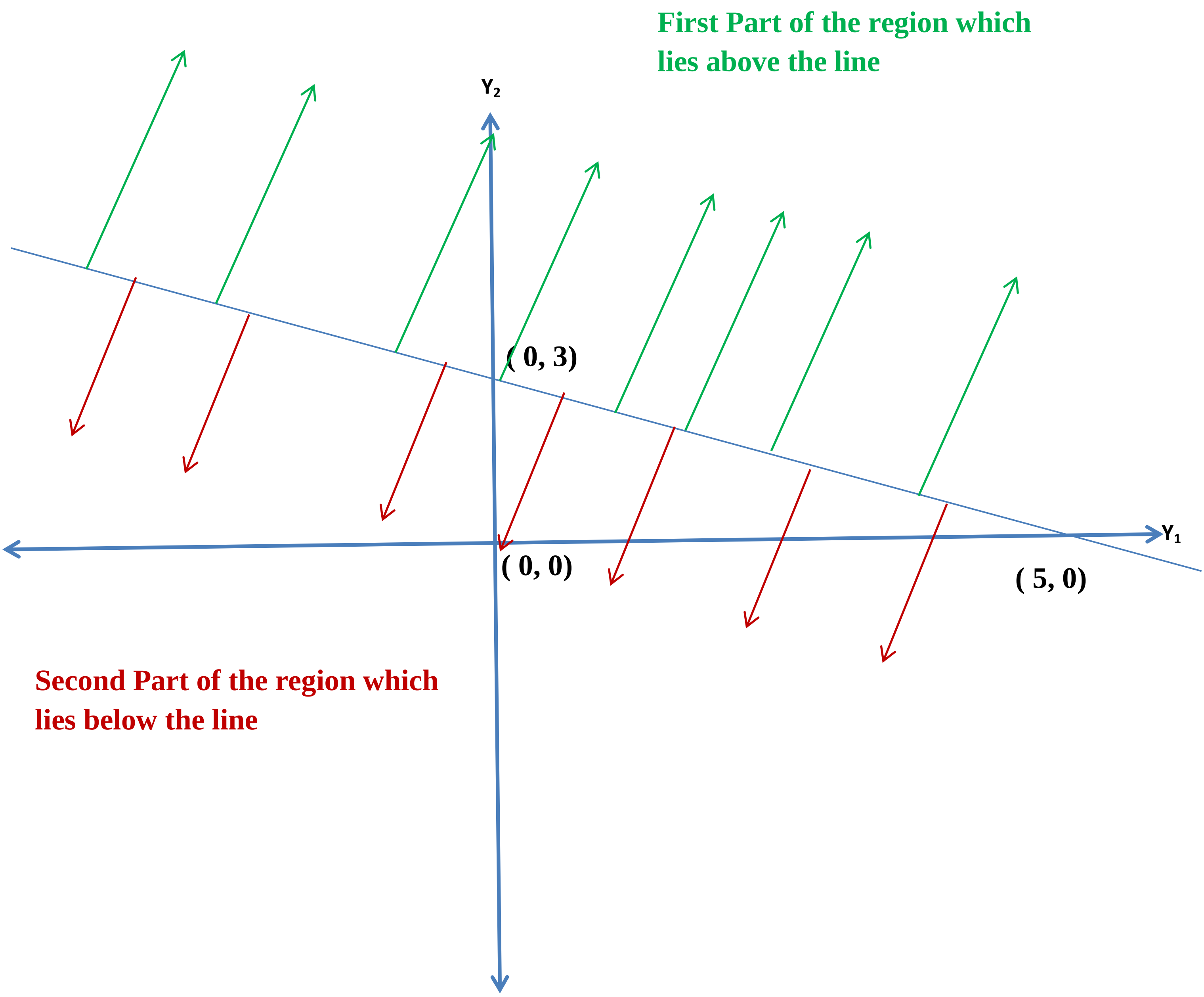
$3x_1 + 5x_2 >= 15$

Assuming $x_1 = 0$, $3x_1+5x_2 = 15$ implies $0 + 5x_2 = 15$ i.e., $x_2 = 3$

Therefore, first point is $(x_1,x_2)=(0, 3)$

Assuming $x_2 = 0$, $3x_1+5x_2 = 15$ implies $3x_1+0 = 15$ i.e., $x_1 = 5$

Therefore, second point is $(x_1,x_2)=(5, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

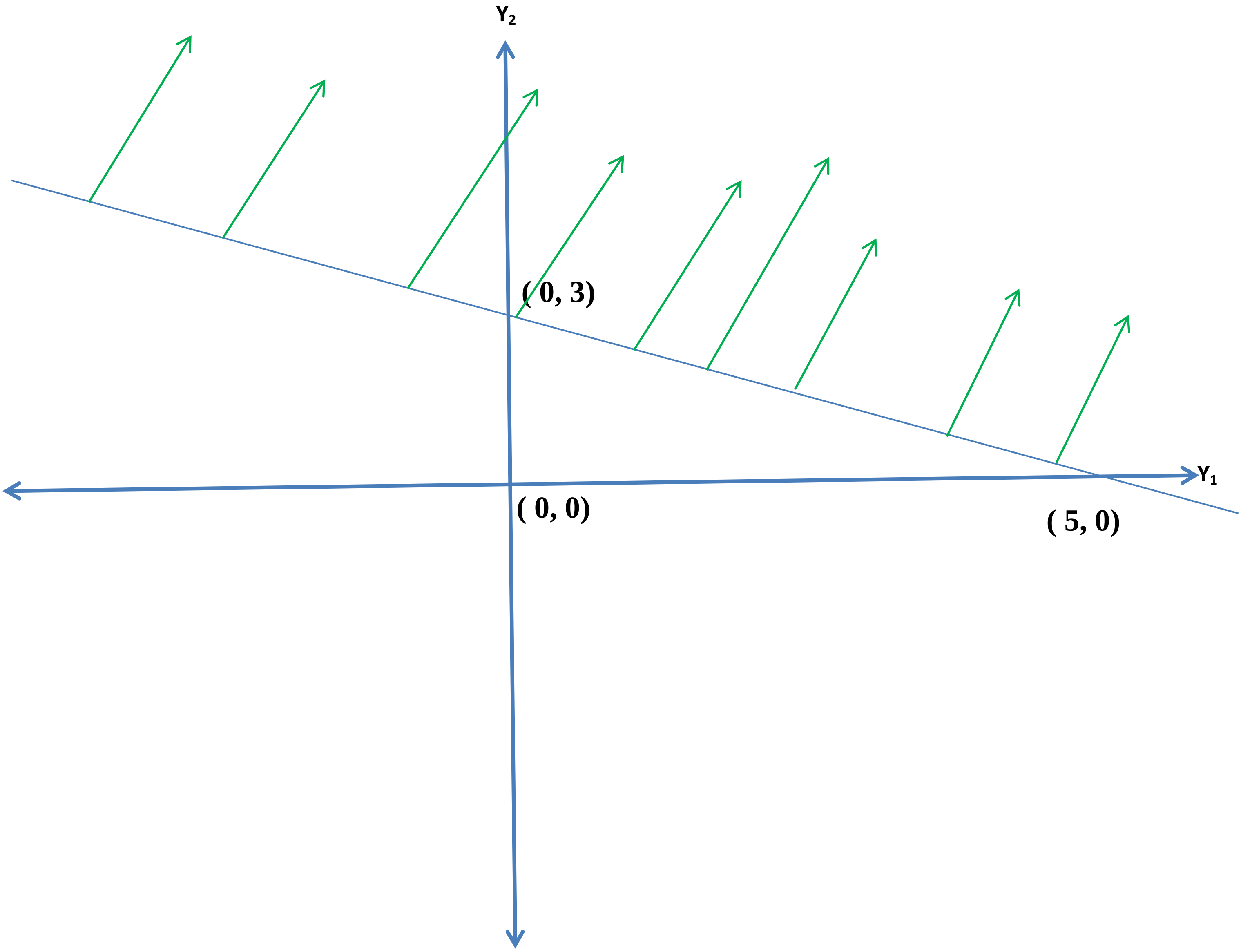
Putting (0,0) in the the constraint

$3x_1+5x_2 > = 15$, we have

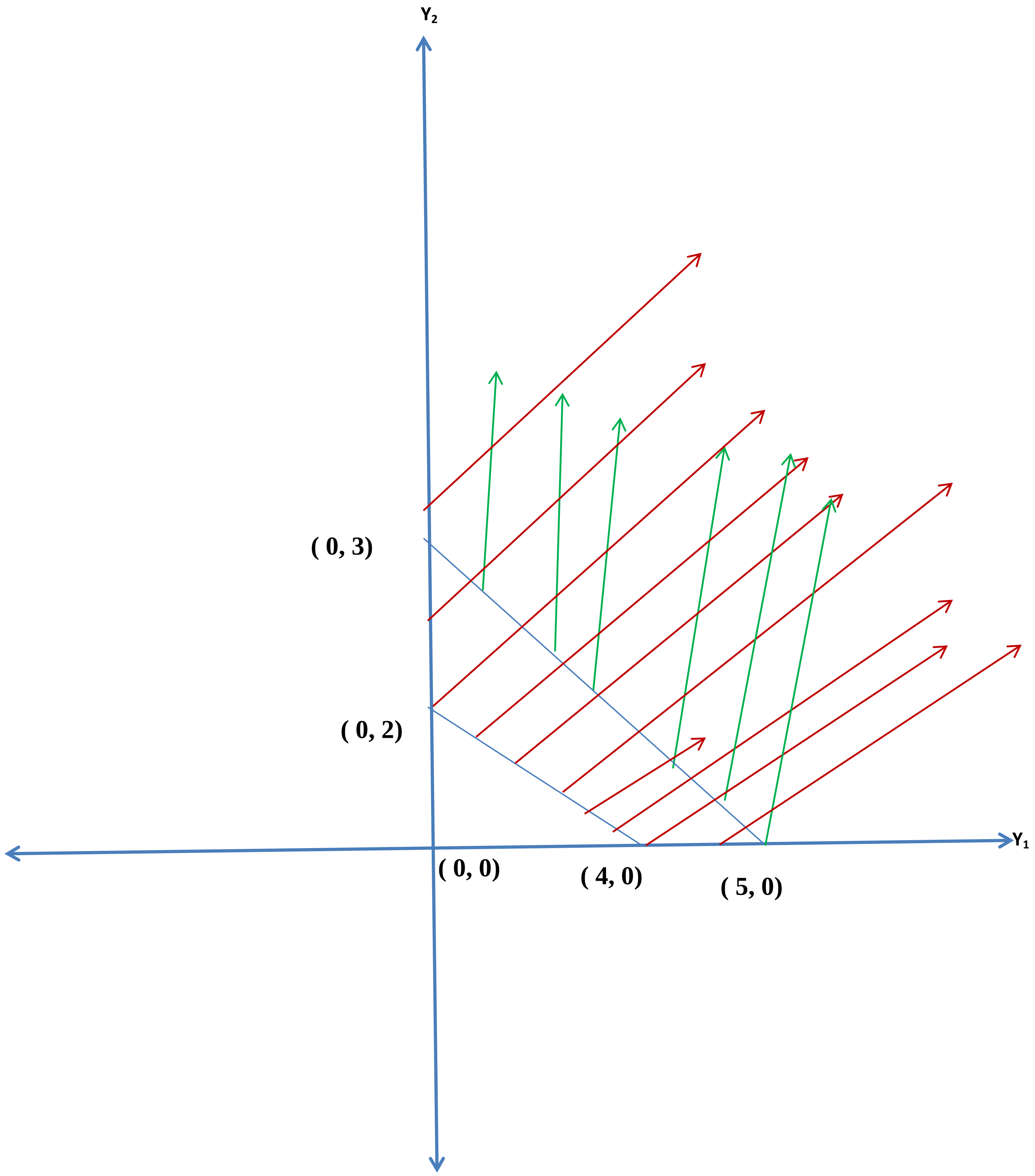
$0+0> =15$

$0>= 15$

It is obvious that the constraint is not satisfying. So, we consider the first part.

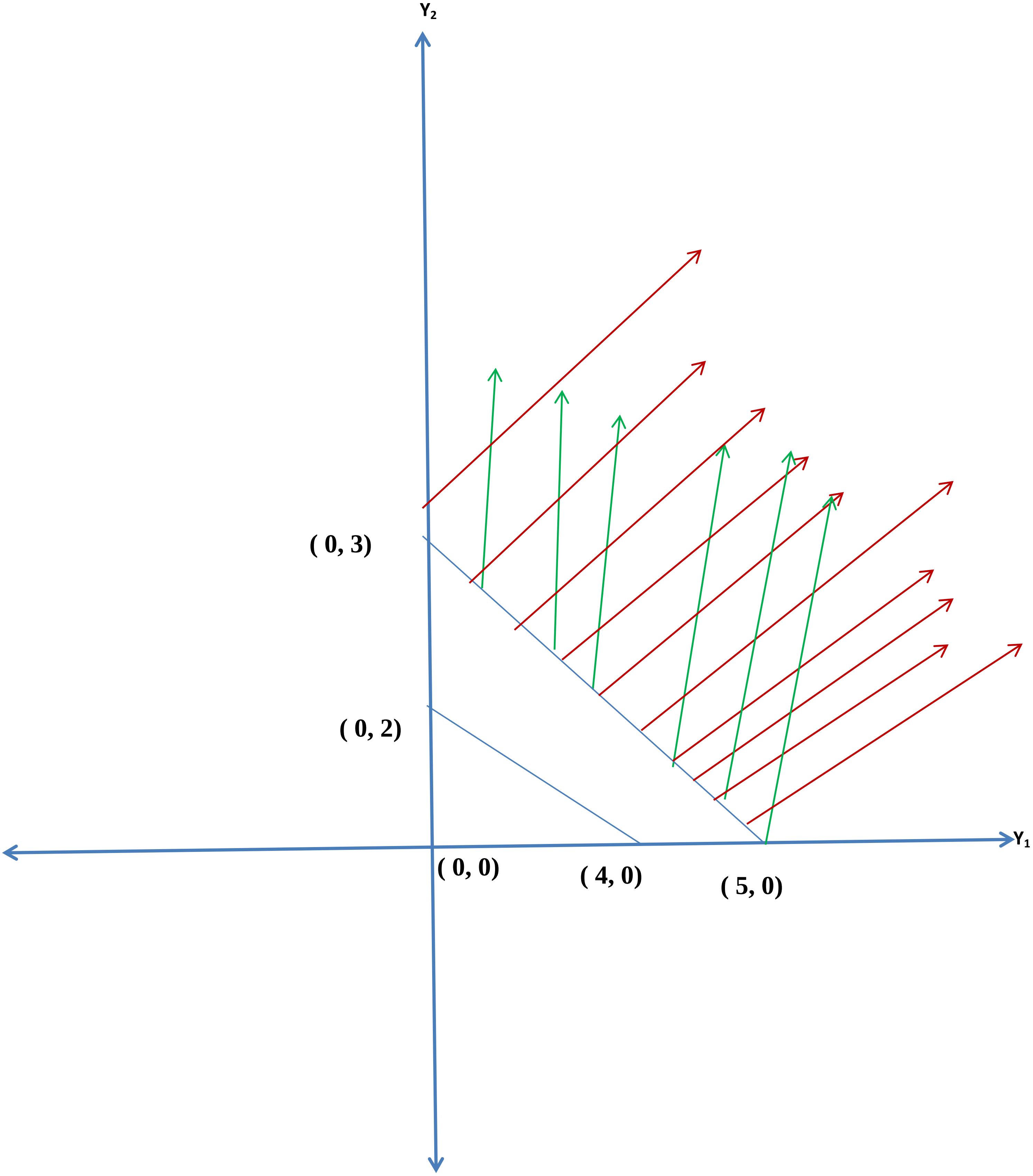


Common region of both the constraints



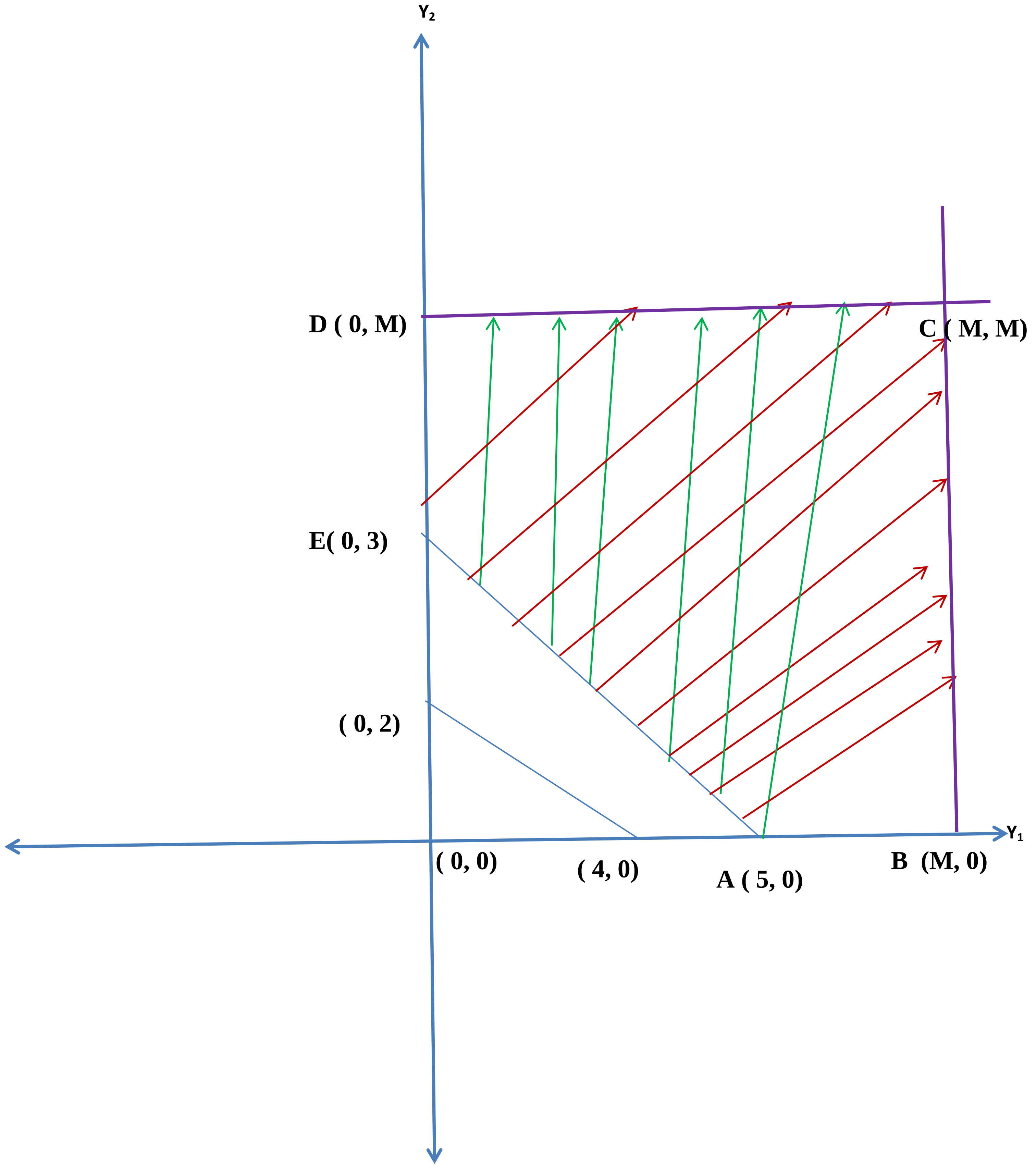
Common region of both the constraints in the first quadrant

(Feasible region)



The feasible region is unbounded and to transform it into bounded there is a need to add $x_1<=M$ and $x_2<=M$.

Including these constraints the new feasible region is as follows:



Extreme points or vertices or corner points

First point A (5, 0)

Second point B (M, 0)

Third point C (M, M)

Fourth point D (0, M)

Fifth point E (0, 3)

Value of the objective function $3x_1 + 2x_2$ at

- **A (5,0) is 3*5+2*0=15**
- **B (M,0) is 3*M+2*0=3M**
- **C (M,M) is 3*M+2*M=5M**
- **D (0, M) is 3*0+2*M=2M**
- **E (0, 3) is 3*0+2*3=6**

Since maximum {15,3M,5M,2M,6}=5M which is depending upon M. So, if the problem is of maximization then the problem has unbounded optimal solution.

Since minimum {15,3M,5M,2M,6}=6 which is corresponding to x_1=0 and x_2=3. So, if the problem is of minimization then the

- **Optimal solution is x_1=0 and x_2=3**
- **The optimal value is 6**

Example: **Solve the following LPP by graphical method.**

Maximize/Minimize ($6x_1 + 10x_2$)

Subject to

$2x_1 + 4x_2 >= 8$,

$3x_1 + 5x_2 >= 15$,

$x_1 >= 0$, $x_2 >= 0$.

Solution: Since, Minimum value of x_1 and x_2 are 0. So, there is no need to transform these variables into new variables.

Draw First Cosnstraint

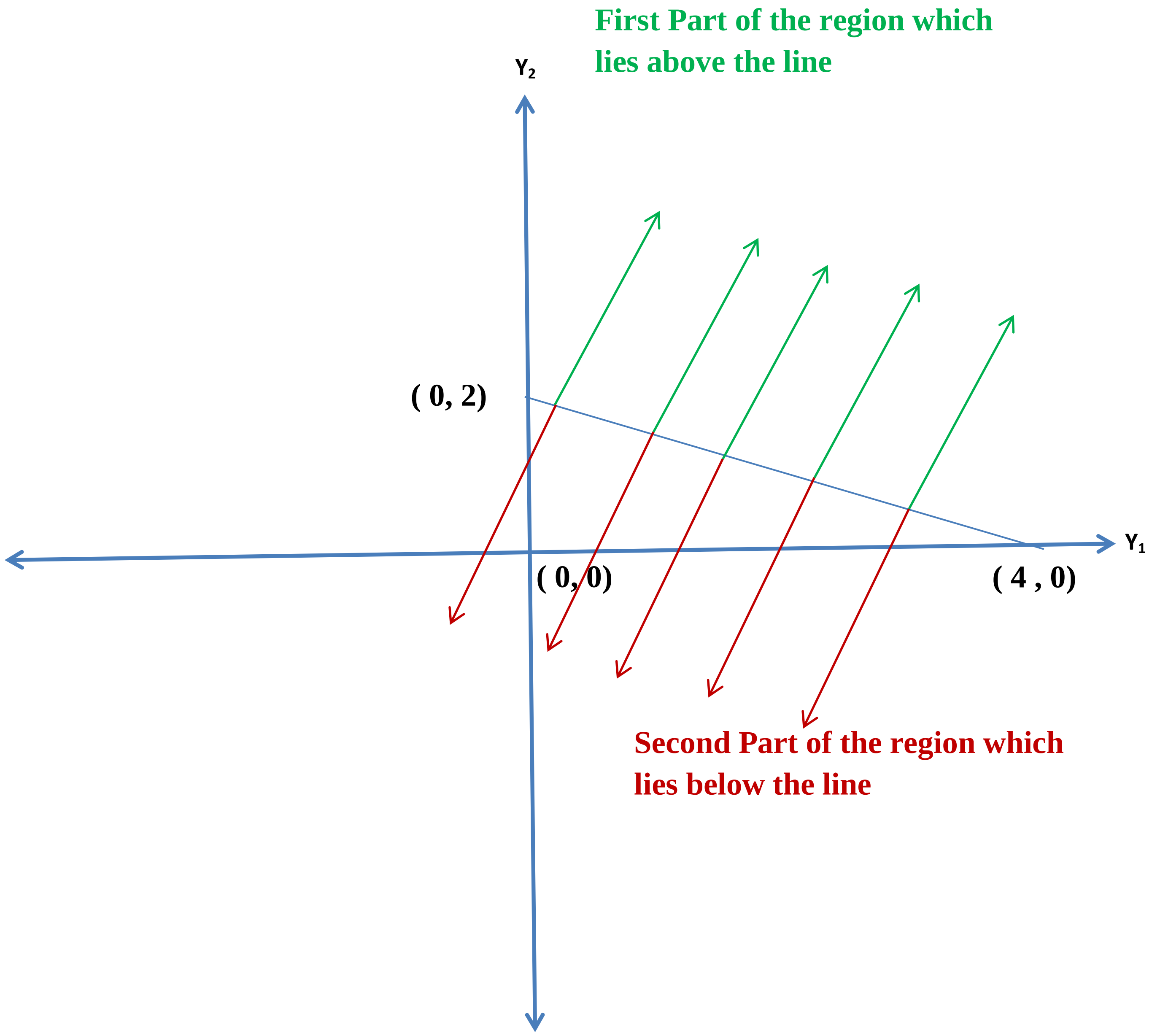
$2x_1 + 4x_2 >= 8$

Assuming $x_1 = 0$, $2x_1+4x_2 = 8$ implies $0 + 4x_2 = 8$ i.e., $x_2 = 2$

Therefore, first point is $(x_1,x_2)=(0, 2)$

Assuming $x_2 = 0$, $2x_1+4x_2 = 8$ implies $2x_1+0 = 8$ i.e., $x_1 = 4$

Therefore, second point is $(x_1,x_2)=(4, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

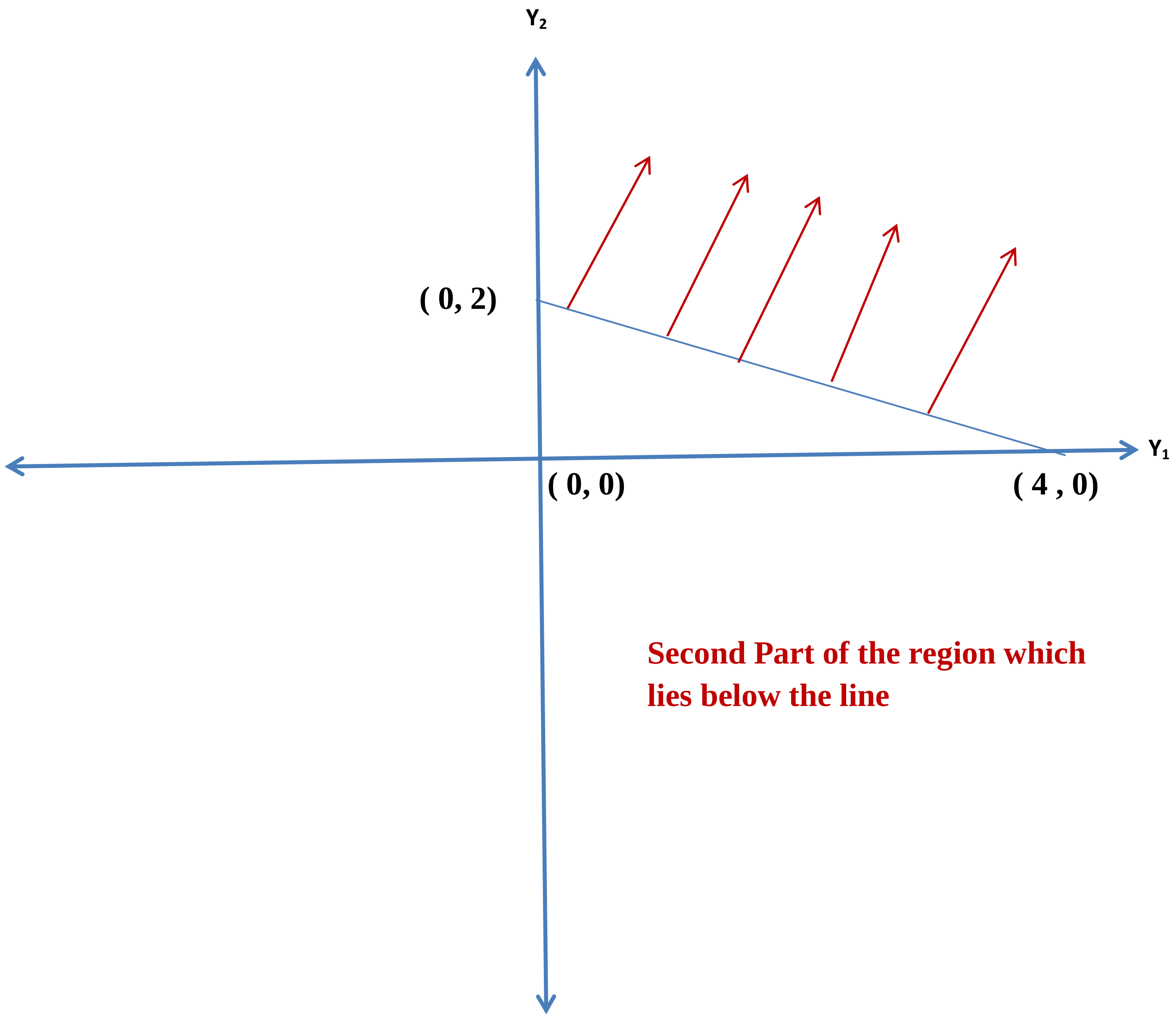
Putting (0,0) in the constraint

$2x_1+4x_2 >= 8$, we have

$0+0>=8$

$0>=8$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Draw Second Cosnstraint

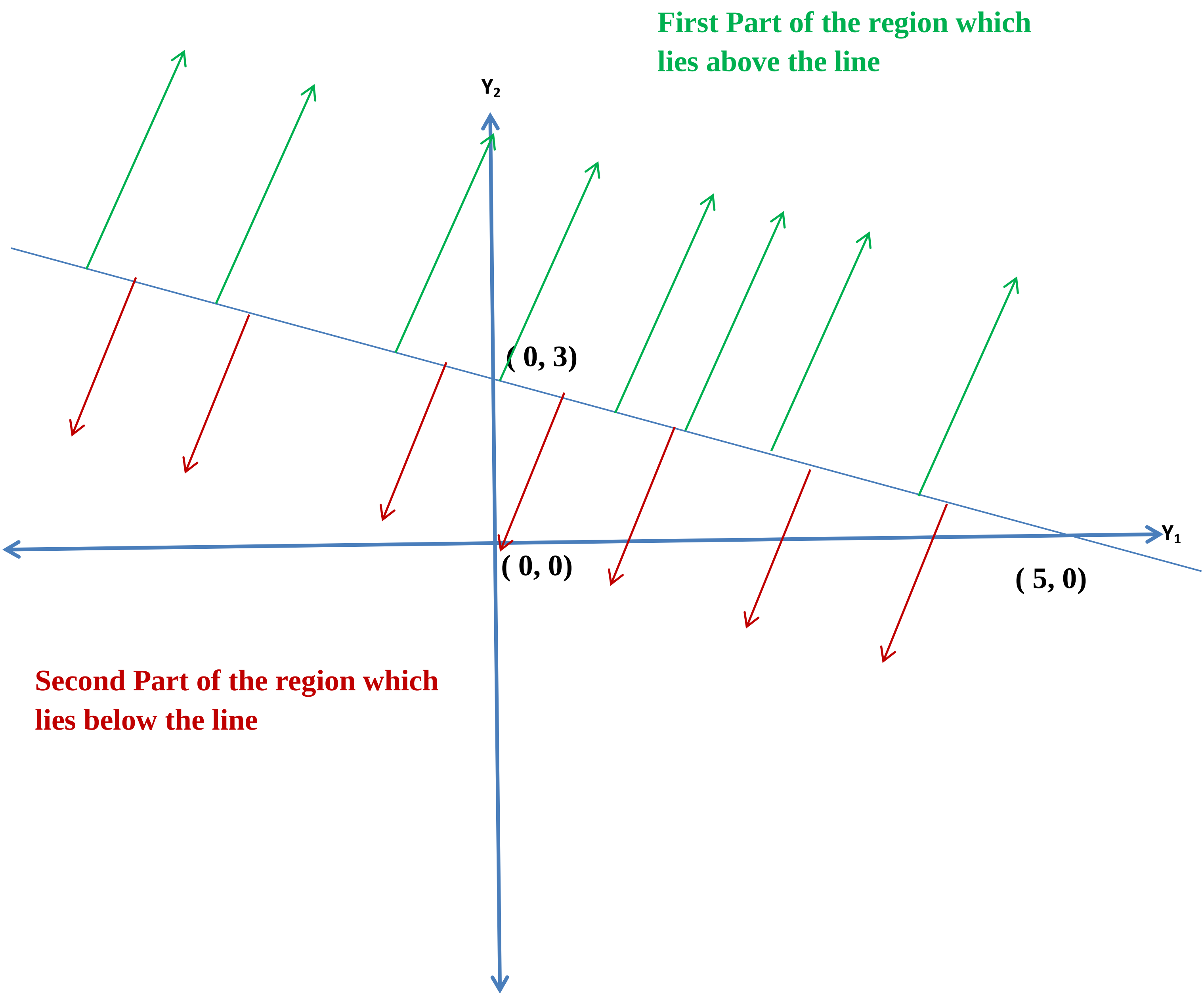
$3x_1 + 5x_2 >= 15$

Assuming $x_1 = 0$, $3x_1+5x_2 = 15$ implies $0 + 5x_2 = 15$ i.e., $x_2 = 3$

Therefore, first point is $(x_1,x_2)=(0, 3)$

Assuming $x_2 = 0$, $3x_1+5x_2 = 15$ implies $3x_1+0 = 15$ i.e., $x_1 = 5$

Therefore, second point is $(x_1,x_2)=(5, 0)$



(0, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

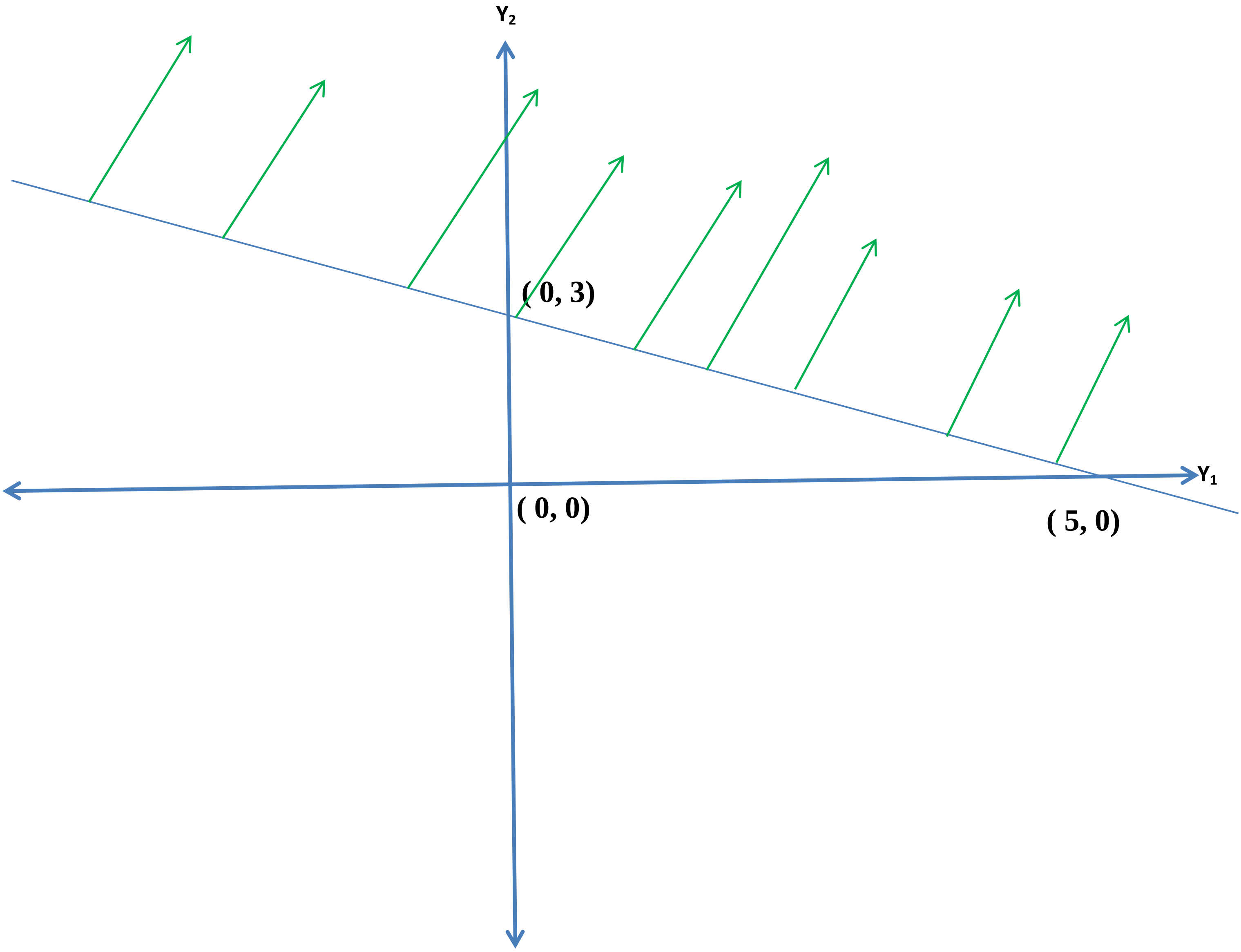
Putting (0,0) in the the constraint

$3x_1+5x_2 > = 15$, **we have**

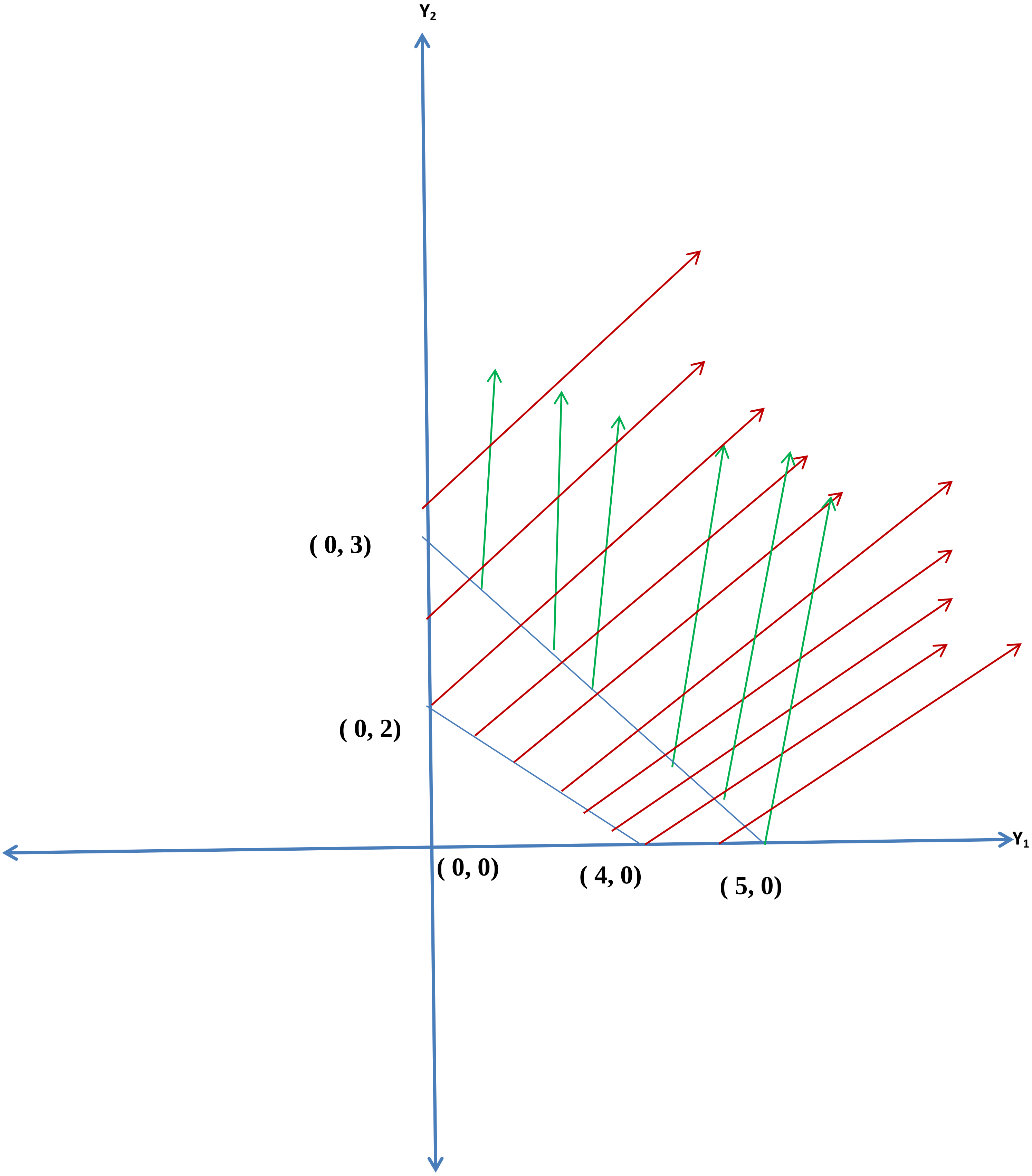
$0+0 > =15$

$0 >= 15$

It is obvious that the constraint is not satisfying. So, we consider the first part.

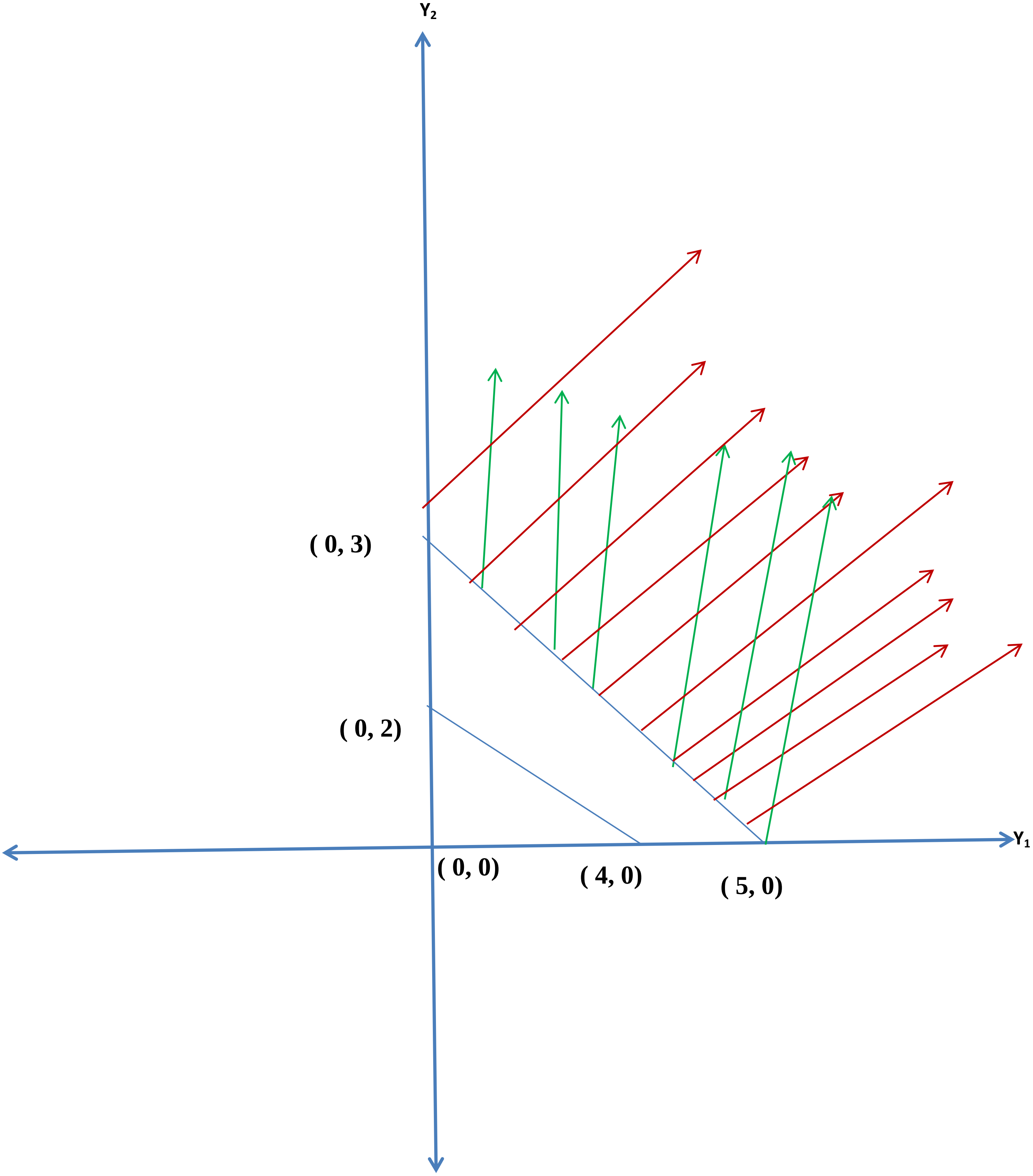


Common region of both the constraints



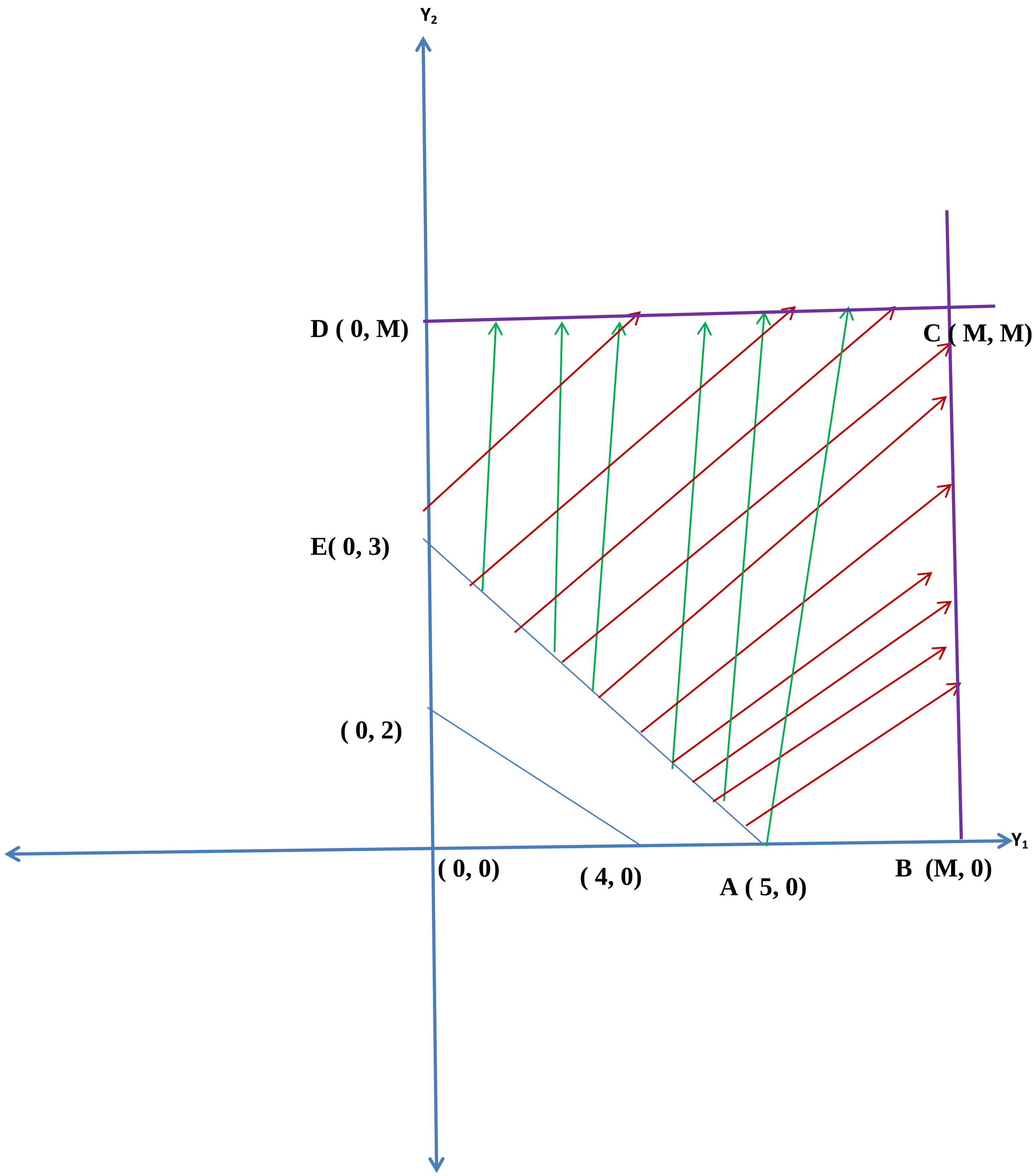
Common region of both the constraints in the first quadrant

(Feasible region)



The feasible region is unbounded and to transform it into bounded there is a need to add x_1<=M and x_2<=M.

Including these constraints the new feasible region is as follows:



Extreme points or vertices or corner points

First point A (5, 0)

Second point B (M, 0)

Third point C (M, M)

Fourth point D (0, M)

Fifth point E (0, 3)

Value of the objective function $6x_1 + 10x_2$ at

- **A (5,0) is 6*5+10*0=30**
- **B (M,0) is 6*M+10*0=6M**
- **C (M,M) is 6*M+10*M=16M**
- **D (0, M) is 6*0+10*M=10M**
- **E (0, 3) is 6*0+10*3=30**

Since maximum {30,6M,16M,10M,30}=16M which is depending upon M. So, if the problem is of maximization then the problem has unbounded optimal solution.

Since minimum {30,6M,16M,10M,30}=30 which is corresponding to

- **x_1=5 and x_2=0**

  and

- **x_1=0 and x_2=3**

So, if the problem is of minimization then the optimal solutions are

- **x_1=5 and x_2=0**
- **x_1=0 and x_2=3**

  and

  the optimal value is 30.

As discussed earlier if two optimal solutions will exist then infinite number of optimal solutions exists which can be obtained as follows:

First optimal solution	**Second optimal solution**	**Alternative optimal solutions**
x_1=5	**x_1=0**	**x_1=a*5+b*0**
x_2=0	**x_2=3**	**x_2= a*0+b*3**

where,

- **a>=0**
- **b>=0**
- **a+b=1**

For example:

If a=1/2 and b=1/2 then the alternative solution is x_1=a*5+b*0 = 5/2+0=5/2 and x_2= a*0+b*3=0+3/2=3/2.

If a=1/3 and b=2/3 then the alternative solution is x_1=a*5+b*0 = 5/3+0= 5/3 and x_2= a*0+b*3=0+(2/3)*3=2.

Remark

Alternative solution of a LPP may exist only if a constraint of the LPP will be parallel to the objective function of the LPP i.e.,

Coefficient of first variable in objective function / Coefficient of first variable in a constraint = Coefficient of second variable in objective function / Coefficient of second variable in the same constraint

In the above LPP,

Coefficient of x_1 in the objective function = 6

Coefficient of x_1 in the second constraint = 3

Coefficient of x_2 in the objective function = 10

Coefficient of x_2 in the second constraint = 5

It is obvious that

6/3 = 10/5

It is pertinent to mention that if the problem is of maximization then no alternative solution exists. But if the problem is of minimization then alternative optimal solutions exist.

Hence, if the condition will be satisfied then alternative optimal solution may or may not exist.

But, if the condition will not be satisfied then it is sure that alternative optimal solution will not exist.