School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

Mid-Semester Examination, September-2019

B.E. III Semester — UMA007 : Numerical Analysis

Time Limit: 02 Hours — Maximum Marks: 25

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Instructions: This question paper has one printed page. You are expected to answer all the questions. Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without programming mode is permitted.

1. (a) Use four-digit rounding arithmetic and the formula to find the most accurate approximations to the roots of the following quadratic equation:

$$\frac{1}{3}x^2 + \frac{123}{4}x - \frac{1}{6} = 0.$$

Also compute the absolute and relative errors. For calculation of exact roots, take nine digits rounding arithmetic. [3 marks]

(b) Let x be any given real number and $fl(x)$ be its approximation obtained by rounding upto n-digits. Find the maximum relative error bound. [4 marks]

2. (a) Determine the number of iterations needed for Bisection method to find the point of intersection of the curves $y = x^4 - 2$ and $y = x + 1$ in the interval $[1, 1.5]$ correct within an accuracy of 10^{-1}. And hence find the point of intersection of the given curves. [3 marks]

(b) Find the root of the equation $x = \tan x$ in the interval (4,5) using fixed point iteration method by taking $x_0 = 4.5$ correct to two decimal places. [3 marks]

3. (a) Show that the equation $x^3 - 7x^2 + 16x - 12 = 0$ has a double root at $x = 2$. Using modified Newton-Raphson method, starting with an initial approximation $x_0 = 1$, find the root correct to 2 decimal places. [3 marks]

(b) Given the iterative scheme $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, $n \geq 0$ with $f(\alpha) = f'(\alpha) = 0$ and $f''(\alpha) \neq 0$. Find the order of convergence of this scheme. [3 marks]

4. (a) Consider the system $Ax = b$, where matrix A is of order n. Find the total number of additions/subtractions and multiplications/divisions required in the upper triangulation process of the augmented matrix in the Gauss elimination method. [3 marks]

(b) Use Gauss-Seidel method to solve the following system of linear equations correct up to two decimal places:

$$3x_1 - x_2 + x_3 = 5$$
$$x_1 - 3x_2 + x_3 = 5$$
$$x_1 + 2x_2 + 4x_3 = 7$$

Take $[0.6667, \ -0.8148, \ 1.9907]^T$ as an initial approximation. [3 marks]