

LECTURE-11

UEI407

In science and engineering the signals used are analog in nature. These are functions of a continuous variable such as time or space. Devices like amplifiers, filters and frequency analyzers are required to process such signals because these devices will respond to the continuous variation of input signals (instantaneous amplitude). Therefore, the characteristics of these signals are changed or some desired information is extracted. Figure 1 shows the block diagram of an analog signal processing system that processes the analog signal.

Figure 1 shows the sampling operation in digital signal processing systems.

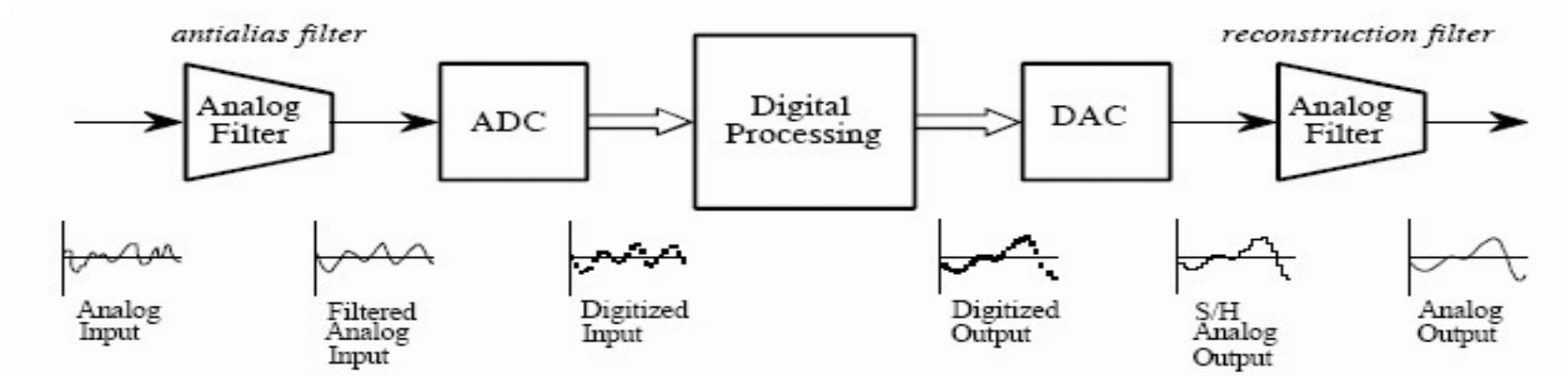


Figure 1: Sampling operation

The input in Figure 1 is from a transducer or a communication signal, which may be an EEG or ECG. This input signal is applied to an anti-aliasing filter, which is a low pass filter used to remove the high frequency noise and hence band limit the signals. The power frequency component is a large part of external noise and it can be removed easily with the help of a 50 Hz notch filter. This notch filter is used in addition to this anti-aliasing filter. An amplifier not shown in Figure 1 may be used to bring the signal up to the voltage range, which is required by the input of analog to digital converter unit.

This is required only for the signal having relatively constant magnitude during conversion from analog to digital. The digital signal processor (**DSP**) may be a large programmable digital computer or a microprocessor. This is configured to perform a specified set of operations on the input signal. The output of DSP is applied to the digital to analog converter (**DAC**). The output of DAC is continuous but it contains unwanted high frequency components. To remove high frequency components the output of DAC is applied to a reconstruction filter, which gives a smooth continuous signal.

Concept of Frequency in Discrete Time Signals

Let us find out the representation of frequency for discrete time signals and also the relationship between sampling frequency, continuous time frequency and discrete time frequency. Let us consider the following analog signal:

$$x(t) = A \cos(\omega t + \phi), \quad -\infty < t < \infty \quad (1)$$

The above represented analog signal is a continuous cosine wave having amplitude A , frequency ω and phase ϕ respectively where $\omega = 2\pi f$. The above expression can also be written as

$$\begin{aligned} x(t) &= A \cos(2\pi f t + \phi), \quad -\infty < t < \infty \\ &= \frac{A}{2} e^{j(2\pi f t + \phi)} + \frac{A}{2} e^{-j(2\pi f t + \phi)} \end{aligned} \quad (2)$$

Equation (2) shows that the cosine wave can be expressed in terms of two equal amplitude complex conjugate functions where the first and second complex exponential has the positive and negative frequency. In practice it is not possible to get the negative frequency and it is used for only mathematical expressions of signals.

Again the cosine wave is periodic with period T and it satisfies the condition

$$x(t) = x(t + T)$$

Therefore, the signal repeats after the period T. For increasing or decreasing the value of f, the cosine waves for different frequencies will be obtained, which will be distinct from each other.

It is possible to increase f upto infinity for analog cosine wave and it can be decreased to zero. Hence f satisfies $-\infty \leq f \leq \infty$. The discrete time cosine wave can be represented as

$$x(n) = A \cos(n\omega + \phi), \quad -\infty < n < \infty \quad (3)$$

where $x(n)$ is the sequence of samples of discrete time cosine wave,

ω is the frequency in radians per sample,

ϕ is the phase in radians,

n is the index of the samples and

A is the amplitude of cosine wave.

The unit of angular frequency (ω) is radians/samples and samples has no unit which is just the index of samples. Therefore, ω is expressed in radians only. The meaning of frequency in radians/sample and in radians are identical. Since $\omega = 2\pi f$, f is the frequency in cycles per sample. Again, f can also be expressed in cycles because samples has no unit. Therefore, equation (3) can also be written as

$$x(n) = A\cos(2\pi n f + \phi), \quad -\infty < n < \infty \quad (4)$$

Standard Discrete Time Signals

The standard discrete time signals are discrete time sinusoids, unit sample sequence, unit step signal, unit ramp signal and exponential signal etc. which are very useful in the analysis of discrete time systems. Let us represent the other standard signals.

Unit Sample Sequence

The unit sample sequence $\delta(n)$ which is very much similar to the unit impulse signal $\delta(t)$ can be represented mathematically as follows:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

We can write it in the sequence form as follows:

$$\delta(n) = \{\dots, 0, 0, 0, 1, 0, 0, 0, \dots\}$$

↑

‘↑’ in the above sequence represents the 0 th sample. We can write the above sequence simply as

$$\delta(n) = \{1\}$$

Figure 2 shows the graphical representation of unit sample sequence.

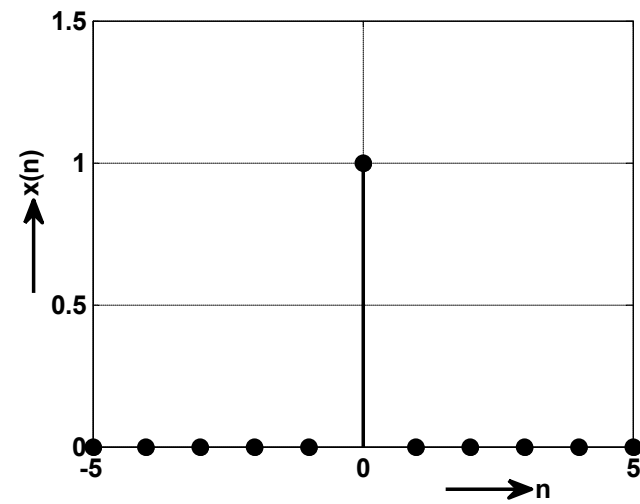


Figure 2 : Delta function

Unit Step sequence

Generally, $u(n)$ denotes the unit step sequence. All its samples have a value of '1' for $n \geq 0$. Mathematically, we can write $u(n)$ as follows:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

A unit step sequence having eight samples can be written as

$$u(n) = \{0, 0, 0, 1, 1, 1, 1, 1\}$$

↑

‘↑’ in the above sequence represents the 0 th sample. Without using ‘↑’, $u(n)$ can be written as follows:

$$u(n) = \{1, 1, 1, 1, 1\}$$

The first sample is considered as 0 th sample. Figure 3 shows the graphical representation of unit step sequence.

Since the unit step sequence is similar to standard unit step signal $u(t)$ used for continuous time systems, it is also called discrete time representation of unit step signal.

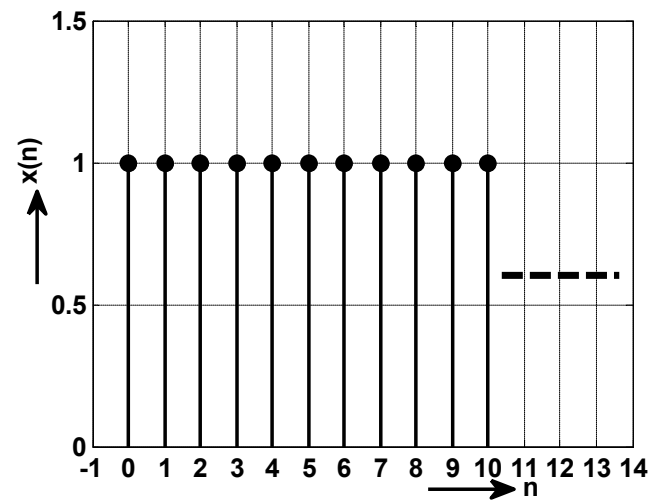


Figure 3 : Unit Step Function

Unit Ramp Sequence

The value of unit ramp sequence increases linearly with the sample number 'n' linearly. It is denoted by $u_r(n)$. Mathematically, it is defined by

$$u_r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

The sequence for unit ramp with 12 samples is given below.

$$u_r(n) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

Figure 4 shows the graphical representation of unit ramp sequence.

From Figure 4, we can observe that the amplitudes of unit ramp sequence increases linearly with the number of samples.

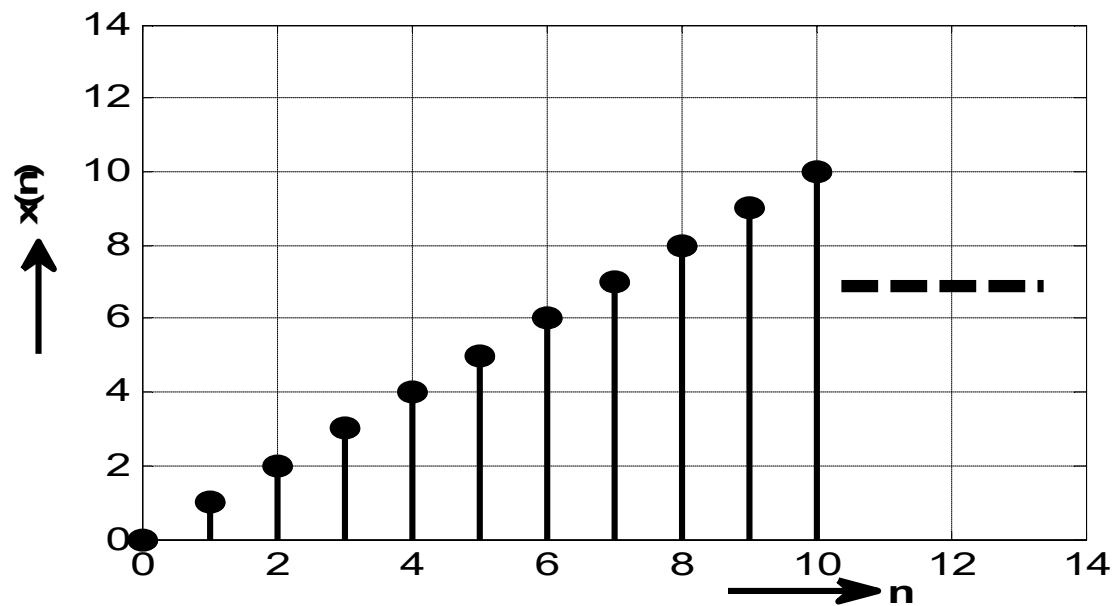


Figure 4 : Ramp Function

Exponential sequence

The exponential sequence can be represented mathematically as follows:

$$x(n) = a^n \quad (4)$$

Figure 5 and Figure 6 show the plot of a^n for $0 < a < 1$ and $a > 1$ respectively.

The exponential sequence can also be complex value when 'a' is complex valued. When 'a' is complex valued, it can be written as

$$a = re^{j\theta} \quad (5)$$

where 'r' is the magnitude of 'a' and θ is the angle of 'a'. Therefore, the sequence $x(n)$ of represented by (5) becomes

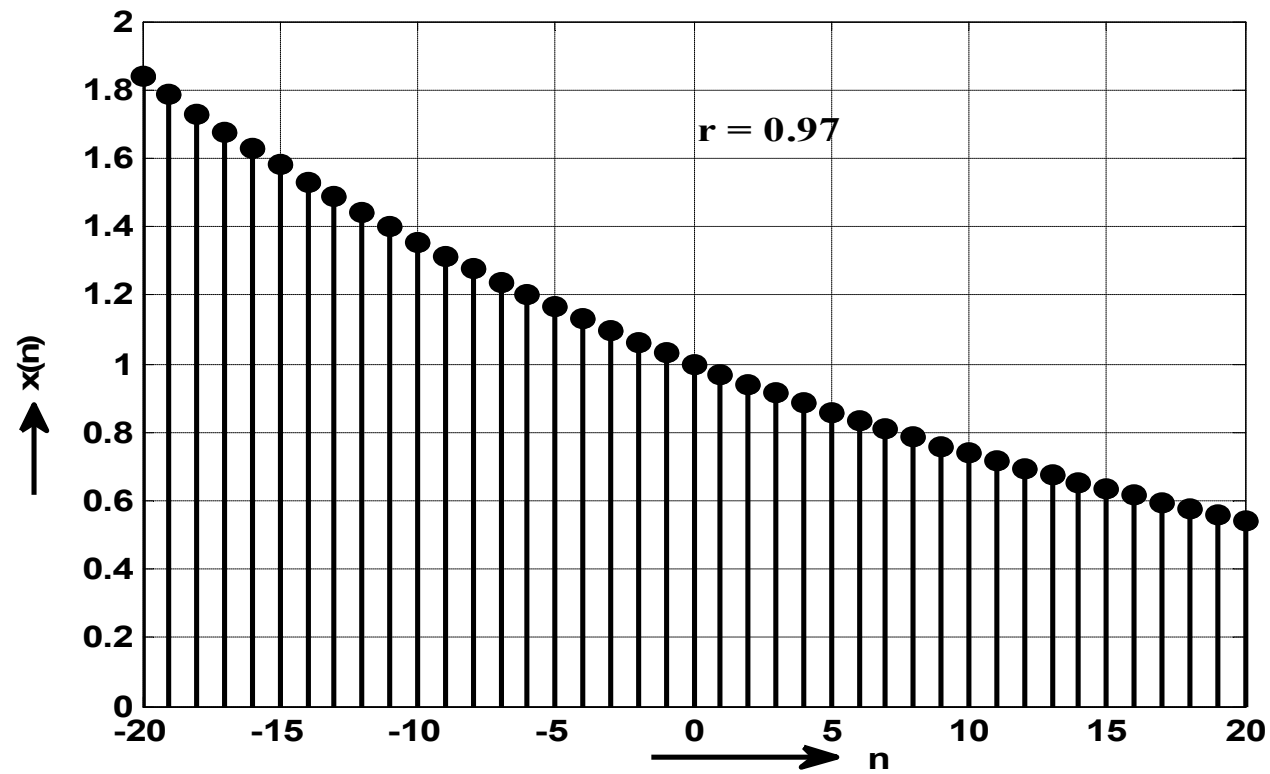


Figure 19

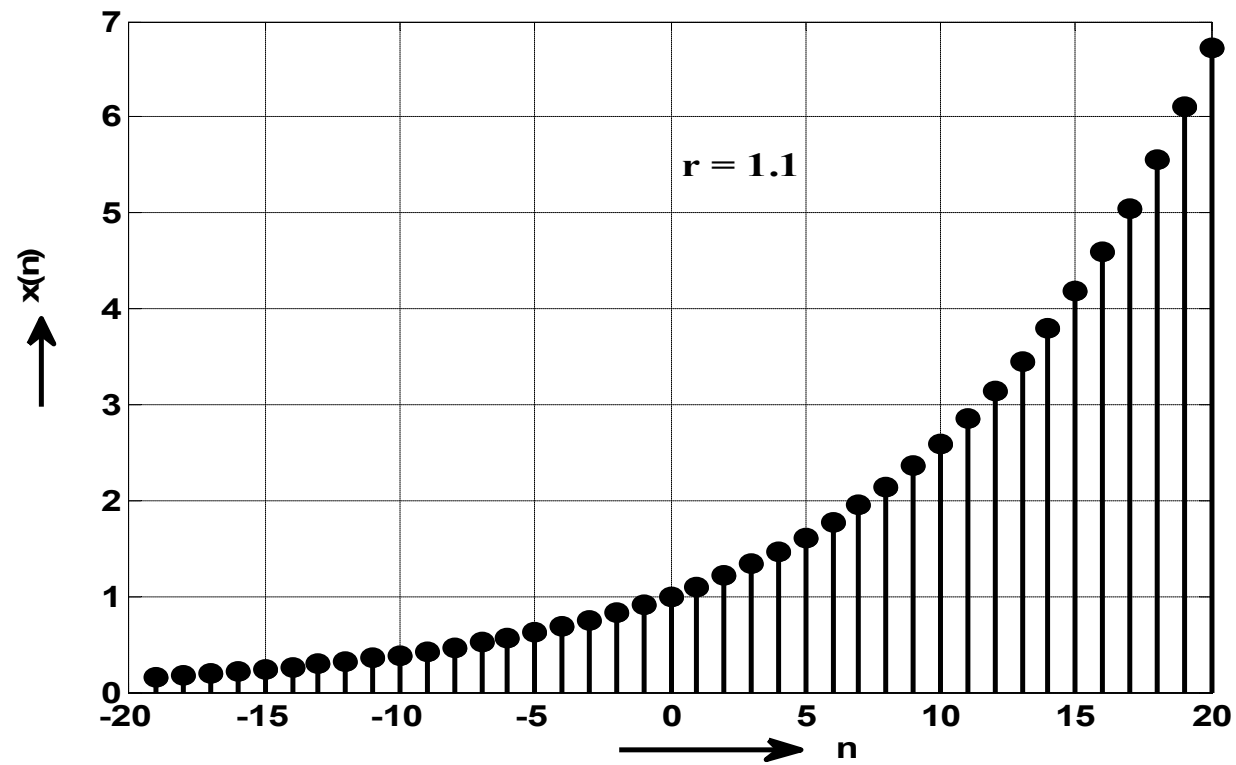


Figure 20

$$x(n) = a^n = \left[re^{j\theta} \right]^n = r^n e^{jn\theta}$$

Again, $e^{j\theta} = \cos\theta + j\sin\theta$ (6)

Using Eq.(6), Eq.(5) can be written as

$$x(n) = r^n e^{jn\theta} = r^n [\cos(n\theta) + j\sin(n\theta)] = r^n \cos(n\theta) + jr^n \sin(n\theta)$$

Therefore, each sample has real and imaginary part.

$$\text{Re of } [x(n)] = r^n \cos(n\theta) \quad \text{and} \quad \text{Im of } [x(n)] = r^n \sin(n\theta)$$

Magnitude of $x(n) = r^n$ and phase of $x(n) = n\theta$

A discrete time signal $x(n)$ is defined as

$$\begin{cases} x(n) = 1 + \frac{n}{4} & \text{for } -4 \leq n \leq -1 \\ x(n) = 1 & \text{for } 0 \leq n \leq 4 \\ x(n) = 0 & \text{elsewhere} \end{cases}$$

Find the value of $x(n)$.

$$x(n) = 1 + \frac{n}{4} \quad \text{for } -4 \leq n \leq -1$$

$$\text{For } n = -4, x(n) = 1 + \frac{-4}{4} = 0 \quad \text{For } n = -3, x(n) = 1 + \frac{-3}{4} = \frac{1}{4}$$

$$\text{For } n = -2, x(n) = 1 + \frac{-2}{4} = \frac{1}{2} \quad \text{For } n = -1, x(n) = 1 + \frac{-1}{4} = \frac{3}{4}$$

$$x(n) = 1 \quad \text{for } 0 \leq n \leq 4$$

Therefore, $x(0) = x(1) = x(2) = x(3) = x(4) = 1$ and $x(n) = 0$ elsewhere.

$$\therefore x(n) = \left\{ \dots, 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, 1, 1, 1, 1, 0, \dots \right\}$$

↑

$$x(n) = \{1,2,3,4,5,6\}$$

$$y(n) = 2x(n)$$

$$y(0)=1, x(1) = 2, x(2)=3, x(3)=4, x(4)=5, x(5)=6$$

$$y(0)=2x(0)=2, y(1)=2x(1)=4, \dots\dots\dots$$

$$y(n) = \{2,4,6,8,10,12\}$$

$$x_1(n) = \{1,2,3\}$$

$$x_2(n) = \{4,2,1\}$$

$$y(n) = x_1(n) + x_2(n)$$

$$= \{ 1+4, 2+2, 3+1 \} = \{5,4,4\}$$

$$y(n) = 0.5x(n)$$

$$y(n) = \{0.5,1,1.5,2,2.5,3\}$$