Roll Number: ________________

Thapar University, Patiala
School of Mathematics
End Semester Examination

B.E. (Second Year): Semester-I (2017 – 18) (COE/ECE/ENC)	Course Code: UMA007 Course Name: Numerical Analysis
Date: 09-12-2017	Day/Time: Saturday/9.00 AM
Time: 3 Hours, M. Marks: 100	Name of Faculty: Isha Dhiman, Jolly Puri, Kavita, Mamta Gulati, Meenu Rani, Paramjeet Singh, Raj Nandkeolyar, Sapna Sharma.

Note: (1) **Attempt all the questions.**
(2) **Calculator without graphing mode and alphanumeric memory is permitted.**

1. (a) Use four-digit rounding arithmetic to find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute and relative errors when compared with the exact roots given by $\alpha_1 = 0.0054208191$ and $\alpha_2 = -92.264646742$.

$$0.3333x^2 + 30.75x - 0.1667 = 0.$$

[10 marks]

(b) The equation $e^x - 4x^2 = 0$ has a root in $[4, 5]$. Show that we cannot find this root using fixed point iteration method with the iteration function $x = 0.5e^{x/2}$. Can you find an iteration function, which will correctly locate the root? Justify your answer.

[8 marks]

2. (a) Show that Gauss-Seidel method does not converge for the following system of equations,

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 7, \\ x_1 + x_2 + x_3 &= 2, \\ 2x_1 + 2x_2 + x_3 &= 5. \end{aligned}$$

[12 marks]

(b) Use the power method to find the smallest eigenvalue and the corresponding eigenvector of the following matrix $\mathbf{A}$ correct upto three decimal places using $\mathbf{x}^{(0)} = [1, 0]^T$:

$$\mathbf{A} = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}.$$

[8 marks]

3. (a) Let $f \in C^n[a, b]$ and $x_0, x_1, ..., x_n$ are distinct numbers in $[a, b]$. Then, show that there exists $\xi \in (a, b)$ such that

$$f[x_0, x_1, ..., x_n] = \frac{f^{(n)}(\xi)}{n!},$$

where $f[x_0, x_1, ..., x_n]$ denote the Newton's divided difference.

[8 marks]

Continue

(b) Determine the spacing h in a table of equally spaced values of the function $f(x) = \cos x$ between 1 and 2, so that interpolation will yield an accuracy of 5×10^{-8} for quadratic interpolating polynomial.

[10 marks]

4. (a) By using the method of least squares fit a curve of the form $y = c_0 x + c_1/\sqrt{x}$ to the following data

x	0.2	0.3	0.5	1	2
y	16	14	11	6	3

[10 marks]

(b) Use Runge-Kutta fourth-order method to solve the following initial value problem (IVP) for $x = 0.3$, with a step size of 0.3

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xz + 1, \quad y(0) = 0,$$
$$\frac{\mathrm{d}z}{\mathrm{d}x} = -xy, \quad z(0) = 1.$$

[10 marks]

5. (a) i. A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation

$$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(u)} \mathrm{d}u.$$

Suppose that $R(v) = -v\sqrt{v}$ for a particular fluid, and R is in Newtons and v is in meters/second. If $m = 10$ kg and $v(0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s using composite trapezoidal rule with a step size of 1.0 .

[8 marks]

ii. Find a Newton-Cotes formula

$$\int_0^1 \frac{f(x)}{\sqrt{x(1-x)}} \mathrm{d}x = af(0) + bf(1/2) + cf(1),$$

which is exact for polynomials upto highest possible degree. Then use the formula to evaluate

$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{x - x^3}}.$$

[10 marks]

(b) Show that the following initial value problem has a unique solution and hence apply Picard's method to generate $y_0(t)$. $y_1(t)$, $y_2(t)$ and $y_3(t)$,

$$y' = -y + t + 1, \quad 0 \le t \le 1, \quad y(0) = 1.$$

[6 marks]