Course: UMA 035 (Optimization Techniques)

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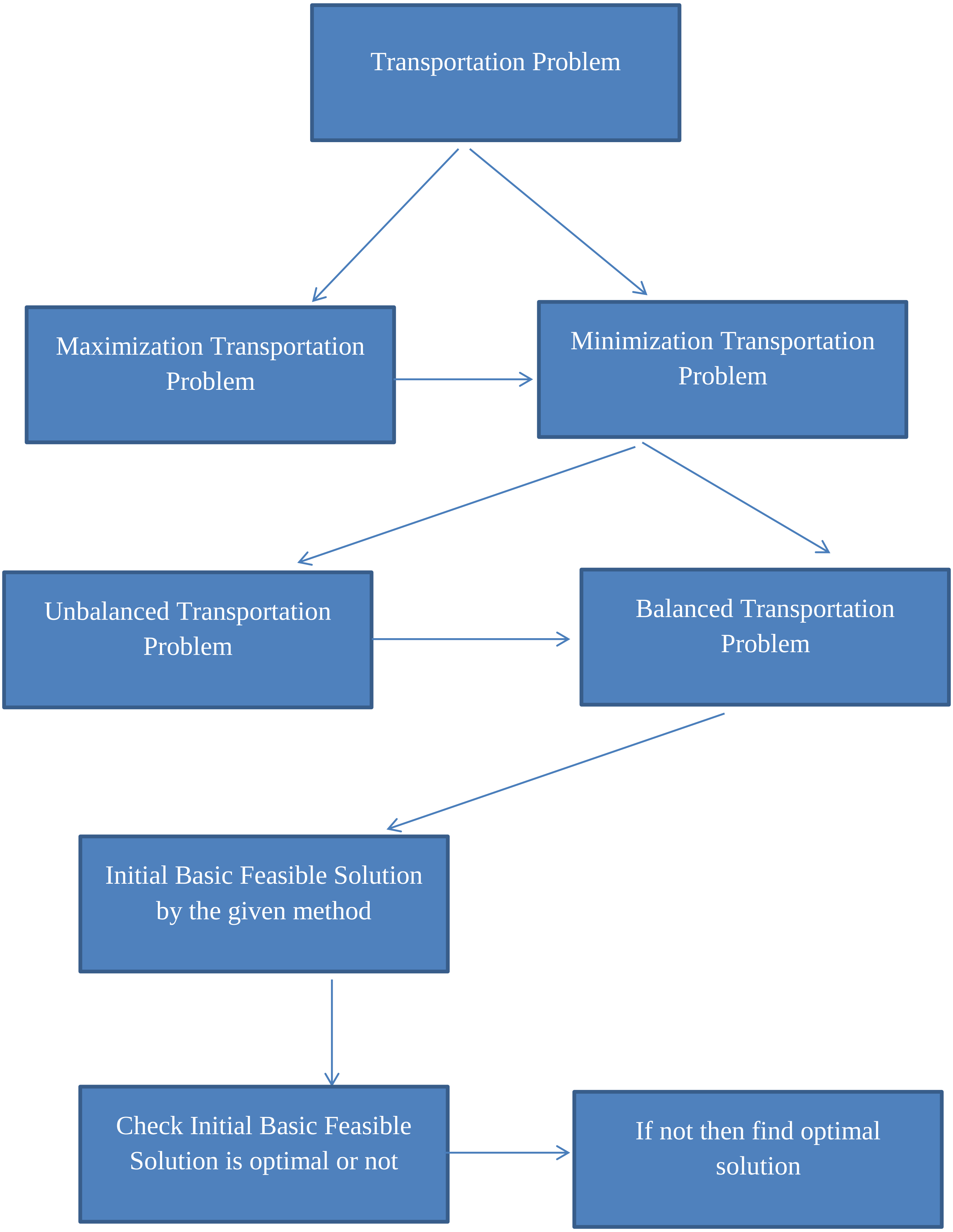
Transportation Problem
Maximization Transportation Problem
Minimization Transportation Problem
Unbalanced Transportation Problem
Balanced Transportation Problem
Initial Basic Feasible Solution by the given method
Check Initial Basic Feasible Solution is optimal or not
If not then find optimal solution

Optimal solution

$C_{ij} - (u_i + v_j) >= 0$ for all i and j.

Calculate u_i and v_j

Assume the relation $C_{ij} = u_i + v_j$ for all $m+n-1$basic variables.

Assume $u_1 = 0$ and solve the obtained equations.

Entering variable

Variable corresponding to which most negative value of $C_{ij} - (u_i + v_j)$ will occur.

Leaving variable

Write a variable θ at the position of entering variable.

Make a closed loop having starting and ending at θ by considering the movement (left, right, down, up) and turn at only basic variable.

Subtract θ from the basic variable values at first, third, fifth turning position …

Add θ in the basic variable values at second, fourth, sixth turnining positions …

Find minimum of all those values from which θ has been subtracted.

Put the obtained minimum value in place of θ to obtain a new basic feasible solution.

The variable for which value is 0 after putting θ is leaving variable.

If 0 is obtained for two or more variables then chose any one as leaving variable.

Example:

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

Write the LPP and its dual.

Solution:

30+50+35=15+30+25+45

Balanced Transportation problem.

LPP

Minimize $(x_{11}+2x_{12}+3x_{13}+4x_{14}+7x_{21}+6x_{22}+2x_{23}+5x_{24}+4x_{31}+3x_{32}+2x_{33}+7x_{34})$

Subject to

$x_{11}+x_{12}+x_{13}+x_{14}=30$

$x_{21}+x_{22}+x_{23}+x_{24}=50$

$x_{31}+x_{32}+x_{33}+x_{34}=35$

$x_{11}+x_{21}+x_{31}=15$

$x_{12}+x_{22}+x_{32}=30$

$x_{13}+x_{23}+x_{33}=25$

$x_{14}+x_{24}+x_{34}=45$

$x_{ij}>=0$, $i=1,2,3$; $j=1,2,3,4$

Dual

Maximize $(30u_1+50u_2+35u_3+15v_1+30v_2+25v_3+45v_4)$

Subject to

$u_1+v_1<=1$

$u_1+v_2<=2$

$u_1+v_3<=3$

$u_1+v_4<=4$

$u_2+v_1<=7$

$u_2+v_2<=6$

$u_2+v_3<=2$

$u_2+v_4<=5$

$u_3+v_1<=4$

$u_3+v_2<=3$

$u_3+v_3<=2$

$u_3+v_3<=7$

u_i and v_j are unrestricted.

Example:

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

Solve the problem (Apply North West Corner method to find initial basic feasible solution).

Solution:

Assume the problem is of minimization.

30+50+35=15+30+25+45

Balanced Transportation problem.

Initial Basic Feasible solution by North West Corner method

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

x_{11}=Minimum {30, 15}=15

Cut the first column and reduce the availability of the first source by 15 units.

	D_2	D_3	D_4	Availability
S_1	2	3	4	30−15=15
S_2	6	2	5	50
S_3	3	2	7	35
Demand	30	25	45	

x_{12}=Minimum {15, 30}=15

Cut the first row and reduce the demand of the second destination by 15 units.

	D_2	D_3	D_4	Availability
S_2	6	2	5	50
S_3	3	2	7	35
Demand	30−15=15	25	45	

x_{22}=Minimum {50, 15}=15

Cut the first column and reduce the availability of the second source by 15 units.

	D_3	D_4	Availability
S_2	2	5	50−15 = 35
S_3	2	7	35
Demand	25	45	

x_{23}=Minimum {35, 25}=25

Cut the first column and reduce the availability of the second source by 25 units.

	D_4	Availability
S_2	5	$35 - 25 = 10$
S_3	7	35
Demand	45	

x_{24}=Minimum {10, 45}=10

Cut the first row and reduce the demand of the fourth destination by 10 units.

	D_4	Availability
S_3	7	35
Demand	$45 - 10 = 35$	

x_{34}=Minimum {35, 35}=35

	D_1	D_2	D_3	D_4	Availa bility
S_1	1 (15)	2 (15)	3	4	30
S_2	7	6 (15)	2 (25)	5 (10)	50
S_3	4	3	2	7 (35)	35
Demand	15	30	25	45	

Initial transportation cost=1*15+2*15+6*15+2*25+5*10+7*35=480

Check solution is optimal or not

For basic variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=2$

$x_{22} \Rightarrow u_2+v_2=c_{22} \Rightarrow u_2+v_2=6$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{34} \Rightarrow u_3+v_4=c_{34} \Rightarrow u_3+v_4=7$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=2$

$u_2=4$

$v_3=-2$

$v_4=1$

$u_3=6$

For non-basic variables

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+1)=2$

$x_{14} \Rightarrow c_{14}-(u_1+v_4)= 4-(0+1)=3$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(4+1)=2$

$x_{31} \Rightarrow c_{31}-(u_3+v_1)= 4-(6+1)= -3$

$x_{32} \Rightarrow c_{32}-(u_3+v_2)= 3-(6+2)= -5$

$x_{33} \Rightarrow c_{33}-(u_3+v_3)= 2-(6-2)= -2$

Most negative value is -5.

x_{32} is entering variable.

	D_1	D_2	D_3	D_4	Availability
S_1	15	15			30
S_2		$15-\theta$	25	$10+\theta$	50
S_3		θ		$35-\theta$	35
Demand	15	30	25	45	

θ = minimum {15, 35}=15

New basic feasible solution

$x_{11}=15$

$x_{12}=15$

$x_{22}=15-\theta=15-15=0$ (Leaving Variable)

$x_{23}=25$

$x_{24}=10+\theta=10+15=25$

$x_{32}=\theta=15$

$x_{34}=35-\theta=35-15=20$

Check that the new solution is optimal or not

For Basic Variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=2$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{34} \Rightarrow u_3+v_4=c_{34} \Rightarrow u_3+v_4=7$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=2$

$u_3=1$

$v_4=6$

$u_2 = -1$

$v_2 = 3$

For Non-basic Variables

$x_{13} \Rightarrow c_{13} - (u_1 + v_3) = 3 - (0 + 3) = 0$ **Leaving variable**

$x_{14} \Rightarrow c_{14} - (u_1 + v_4) = 4 - (0 + 6) = -2$

$x_{21} \Rightarrow c_{21} - (u_2 + v_1) = 7 - (-1 + 1) = 7$

$x_{22} \Rightarrow c_{22} - (u_2 + v_2) = 6 - (-1 + 2) = 5$

$x_{31} \Rightarrow c_{31} - (u_3 + v_1) = 4 - (1 + 1) = 2$

$x_{33} \Rightarrow c_{33} - (u_3 + v_3) = 2 - (1 + 3) = -2$

The most negative value is – 2 which is corresponding to x_{14} and x_{33}. Any of these two may be considered as entering variable.

Let x_{14} as entering variable.

	D_1	D_2	D_3	D_4	Availability
S_1	15	$15-\theta$		θ	30
S_2			25	25	50
S_3		15 $+\theta$		20 $-\theta$	35
Demand	15	30	25	45	

θ=minimum {20, 15}=15

New solution is

$x_{11}=15$

$x_{12}=15-\theta=15-15=0$ leaving variable

$x_{14}=\theta=15$

$x_{23}=25$

$x_{24}=10+\theta=10+15=25$

$x_{32}=15+\theta=30$

$x_{34}=20-\theta=20-15=5$

Check that the obtained solution is optimal or not

For Basic Variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{14} \Rightarrow u_1+v_4=c_{14} \Rightarrow u_1+v_4=4$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{34} \Rightarrow u_3+v_4=c_{34} \Rightarrow u_3+v_4=7$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=4$

$u_2=1$

$v_4=4$

$u_3=3$

$v_2=0$

$v_3=1$

For Non-basic Variables

$x_{12} \Rightarrow c_{12} - (u_1+v_2) = 2-(0+0) = 2$

$x_{13} \Rightarrow c_{13} - (u_1+v_3) = 3-(0+1) = 2$

$x_{21} \Rightarrow c_{21} - (u_2+v_1) = 7-(1+1) = 5$

$x_{23} \Rightarrow c_{23} - (u_2+v_3) = 2-(1+1) = 0$

$x_{31} \Rightarrow c_{31} - (u_3+v_1) = 4-(3+1) = 0$

$x_{33} \Rightarrow c_{33} - (u_3+v_3) = 2-(1+3) = -2$

Only one negative value. So, x_{33} is entering variable.

	D_1	D_2	D_3	D_4	Availability
S_1	15			15	30
S_2			$25-\theta$	$25+\theta$	50
S_3		30	θ	$5-\theta$	35
Demand	15	30	25	45	

θ =minimum {25, 5}=5

New solution is

$x_{11}=15$

$x_{14}=15$

$x_{23}=25-\theta=20$

$x_{24}=25+\theta=30$

$x_{32}=30$

$x_{33}=\theta=5$

$x_{34}=5-\theta=0$ Leaving Variable

Check that the obtained solution is optimal or not

For Basic Variables

$x_{11}\Rightarrow u_1+v_1=c_{11}\Rightarrow u_1+v_1=1$

$x_{14}\Rightarrow u_1+v_4=c_{14}\Rightarrow u_1+v_4=4$

$x_{23}\Rightarrow u_2+v_3=c_{23}\Rightarrow u_2+v_3=2$

$x_{24}\Rightarrow u_2+v_4=c_{24}\Rightarrow u_2+v_4=5$

$x_{32}\Rightarrow u_3+v_2=c_{32}\Rightarrow u_3+v_2=3$

$x_{33}\Rightarrow u_3+v_3=c_{33}\Rightarrow u_3+v_3=2$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=5$

$u_2=-2$

$v_4=4$

$u_3=-2$

$v_2=5$

$v_3=4$

For Non-basic Variables

$x_{12} \Rightarrow c_{12}-(u_1+v_2)= 2-(0+5)=-3$

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+4)= -1$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(-2+1)= 8$

$x_{22} \Rightarrow c_{22}-(u_2+v_2)= 6-(-2+5)= 3$

$x_{31} \Rightarrow c_{31}-(u_3+v_1)= 4-(-2+1)= 5$

$x_{34} \Rightarrow c_{34}-(u_3+v_4)= 7-(-2+4)=5$

The most negative value is -3 which is corresponding to x_{12}. x_{12} is entering variable

	D_1	D_2	D_3	D_4	Availability
S_1	15	θ		$15-\theta$	30
S_2			$20-\theta$	$30+\theta$	50
S_3		$30-\theta$	$5+\theta$		35
Demand	15	30	25	45	

θ =minimum {15, 20, 30}=15

New solution is

$x_{11}=15$

$x_{12}=\theta=15$

$x_{14}=0$ Leaving Variable

$x_{23}=5$

$x_{24}=45$

$x_{32}=15$

$x_{33}=20$

Check that the obtained solution is optimal or not

For Basic Variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=2$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{33} \Rightarrow u_3+v_3=c_{33} \Rightarrow u_3+v_3=2$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$u_2=1$

$v_4=4$

$u_3=1$

$v_2=2$

$v_3=1$

For Non-basic Variables

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+1)=2$

$x_{14} \Rightarrow c_{14}-(u_1+v_4)= 4-(0+4)= 0$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(1+1)= 5$

$x_{22} \Rightarrow c_{22} - (u_2+v_2) = 6-(1+2) = 3$

$x_{31} \Rightarrow c_{31} - (u_3+v_1) = 4-(1+1) = 2$

$x_{34} \Rightarrow c_{34} - (u_3+v_4) = 7-(1+4) = 2$

All values are >=0. Solution is optimal.

	D_1	D_2	D_3	D_4	Availability
S_1	1 (15)	2 (15)	3	4	30
S_2	7	6	2 (5)	5 (45)	50
S_3	4	3 (15)	2(20)	7	35
Demand	15	30	25	45	

Transportation cost=1*15+2*15+2*5+5*45+3*15+2*20=365

Alternative solution

x_{14} is a non-basic variable and $c_{14}-(u_1+v_4)=0$. So alternative solution may exist.

Enter x_{14} to find alternative solution.

	D_1	D_2	D_3	D_4	Availability
S_1	15	$15-\theta$		θ	30
S_2			$5+\theta$	$45-\theta$	50
S_3		$15+\theta$	$20-\theta$		35
Demand	15	30	25	45	

θ=minimum {45, 20, 15}=15

New solution is

$x_{11}=15$

$x_{12}=15-\theta=0$

$x_{14}=15$

$x_{23}=20$

$x_{24} = 30$

$x_{32} = 30$

$x_{33} = 5$