Course: UMA 035 (Optimization Techniques)

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Dual of a LPP

LPP	Dual of a LPP
m constraints	m variables
n variables	n constraints
Maximize	Minimize
Minimize	Maximize

Canonical form of a maximize LPP

Maximize (CX)

Subject to

$AX <= b$

$X >= 0$

Canonical form of a minimize LPP

Minimize (CX)

Subject to

$AX >= b$

$X >= 0$

Method to write the dual of a maximize LPP

Step 1:

Convert all the variables into >=0.

Step 2:

Convert sign of all the constraints into <= as well as sign of all the variables >=.

Step 3:

Write the transpose of the coefficient matrix with new variables (called dual variables).

Step 3:

Write the transpose of the objective function matrix as RHS of constraints with >= sign.

Step 4:

Write the transpose of the RHS with new variables as minimize objective function.

Method to write the dual of a minimize LPP

Step 1:

Convert sign of all the constraints into >= as well as sign of all the variables >=.

Step 2:

Write the transpose of the coefficient matrix with new variables (called dual variables).

Step 3:

Write the transpose of the objective function matrix as RHS of constraints with <= sign.

Step 4:

Write the transpose of the RHS with new variables as maximize objective function.

Example:

Write the dual of the following LPP

Max $(4x_1 + 10x_2 - 2x_3)$

Subject to

$2x_1 + x_2 + x_3 >= 10$

$2x_1 + 5x_2 + x_3 <= 20$

$2x_1 + 3x_2 + x_3 >= 18$

$x_1 \geq 0$, $x_2 \leq 0$, x_3 unrestricted.

Sign of all the variables should be ≥ 0

$x_2 \leq 0$

$\Rightarrow 0 \leq 0 - x_2$

Assume $0 - x_2 = y_1$ i.e., $x_2 = - y_1$

x_3 is unrestricted

Assume

$x_3 = y_2 - y_3$, where, $y_2 \geq 0$ and $y_3 \geq 0$

Transformed LPP

Max $(4x_1 + 10(-y_1) - 2(y_2 - y_3)$

Subject to

$2x_1 + (-y_1) + (y_2 - y_3) \geq 10$

$2x_1 + 5(-y_1) + (y_2 - y_3) \leq 20$

$2x_1 + 3(-y_1) + (y_2 - y_3) \geq 18$

$y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$.

Max $(4x_1 - 10y_1 - 2y_2 + 2y_3)$

Subject to

$2x_1 - y_1 + y_2 - y_3 >= 10$

$2x_1 - 5y_1 + y_2 - y_3 <= 20$

$2x_1 - 3y_1 + y_2 - y_3 >= 18$

$y_1 >= 0, y_2 >= 0, y_3 >= 0.$

Sign of all the constraints should be <=0

Max $(4x_1 - 10y_1 - 2y_2 + 2y_3)$

Subject to

$-2x_1 + y_1 - y_2 + y_3 <= -10$

$2x_1 - 5y_1 + y_2 - y_3 <= 20$

$-2x_1 + 3y_1 - y_2 + y_3 <= -18$

$y_1 >= 0, y_2 >= 0, y_3 >= 0.$

Max $\left(\begin{bmatrix} 4 & -10 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}\right)$

Subject to

−2	1	−1	1	x_1	<=	−10
2	−5	1	−1	y_1	<=	20
−2	3	−1	1	y_2	<=	−18
				y_3	<=	

Min $\left(\begin{bmatrix} -10 & 20 & -18 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}\right)$

Subject to

−2	2	−2		z_1	>=	4
1	−5	3		z_2	>=	−10
−1	1	−1		z_3	>=	−2
1	−1	1			>=	2

Minimize $(-10z_1+20z_2-18z_3)$

Subject to

$-2z_1+2z_2-2z_3>=4$

$z_1-5z_2+3z_3>=-10$

$-z_1+z_2-z_3>=-2$

$z_1-z_2+z_3>=2$

$z_1>=0, z_2>=0, z_3>=0,$

Write the dual of the following LPP

Min $(4x_1+10x_2-2x_3)$

Subject to

$2x_1+x_2+x_3>=10$

$2x_1+5x_2+x_3<=20$

$2x_1+3x_2+x_3>=18$

$x_1>=0, x_2<=0, x_3$ unrestricted.

Solution:

Sign of all the variables should be $>=0$

$x_2<=0$

$\Longrightarrow 0<=0-x_2$

Assume $0-x_2=y_1$ i.e., $x_2=-y_1$

x_3 is unrestricted

Assume

$x_3=y_2-y_3$, where, $y_2>=0$ and $y_3>=0$

Min $(4x_1 + 10(-y_1) - 2(y_2 - y_3))$

Subject to

$2x_1 + (-y_1) + (y_2 - y_3) >= 10$

$2x_1 + 5(-y_1) + (y_2 - y_3) <= 20$

$2x_1 + 3(-y_1) + (y_2 - y_3) >= 18$

$y_1 >= 0, y_2 >= 0, y_3 >= 0.$

Min $(4x_1 - 10y_1 - 2y_2 + 2y_3)$

Subject to

$2x_1 - y_1 + y_2 - y_3 >= 10$

$2x_1 - 5y_1 + y_2 - y_3 <= 20$

$2x_1 - 3y_1 + y_2 - y_3 >= 18$

$y_1 >= 0, y_2 >= 0, y_3 >= 0.$

Sign of all the constraints should be >=0

Min $(4x_1 - 10y_1 - 2y_2 + 2y_3)$

Subject to

$2x_1 - y_1 + y_2 - y_3 >= 10$

$-2x_1 + 5y_1 - y_2 + y_3 >= -20$

$2x_1 - 3y_1 + y_2 - y_3 >= 18$

$y_1 >= 0, y_2 >= 0, y_3 >= 0.$

Min ($[4 \ -10 \ -2 \ 2]$ $\begin{bmatrix} x_1 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$)

Subject to

2	−1	1	−1	x_1	>=	10
−2	5	−1	1	y_1	>=	−20
2	−3	1	−1	y_2	>=	18
				y_3	>=	

Max ($[10 \ -20 \ 18]$ $\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$)

Subject to

2	−2	2	z_1	<=	4
−1	5	−3	z_2	<=	−10
1	−1	1	z_3	<=	−2
−1	1	−1		<=	2

Maximize $(10z_1 - 20z_2 + 18z_3)$

Subject to

$2z_1 - 2z_2 + 2z_3 <= 4$

$-z_1 + 5z_2 - 3z_3 <= -10$

$z_1 - z_2 + z_3 <= -2$

$-z_1 + z_2 - z_3 <= 2$

$z_1 >= 0, z_2 >= 0, z_3 >= 0,$

Maximize (CX)

Subject to

AX<=b

X>=0

Minimize ($b^T Z$)

Subject to

$A^T Z >= C^T$

Z>=0

Write the dual of the following LPP

Min $(4x_1 + 10x_2 - 2x_3)$

Subject to

$2x_1 + x_2 + x_3 <= 10$

$2x_1 + 5x_2 + x_3 <= 20$

$x_1 >= 0,\ x_2 >= 0,\ x_3 >= 0.$

Min $(4x_1 + 10x_2 - 2x_3)$

Subject to

$-2x_1 - x_2 - x_3 >= -10$

$-2x_1 - 5x_2 - x_3 >= -20$

$x_1 >= 0,\ x_2 >= 0,\ x_3 >= 0.$

Maximize $(-10z_1 - 20z_2)$

Subject to

$$-2z_1 - 2z_2 <= 4$$

$$-z_1 - 5z_2 <= 10$$

$$-z_1 - z_2 <= -2$$

$$z_1 >= 0, \ z_2 >= 0.$$

Write the dual of the following LPP

Max $(4x_1+10x_2-2x_3)$

Subject to

$2x_1+x_2+x_3<=10$

$2x_1+5x_2+x_3<=20$

$x_1>=0, x_2>=0, x_3>=0.$

Solution:

Dual

Minimize $(10z_1+20z_2)$

Subject to

$2z_1+2z_2>=4$

$z_1+5z_2>=10$

$z_1+z_2>=2$

$z_1>=0, z_2>=0.$

Minimize (CX)

Subject to

AX>=b

X>=0

Maximize ($b^T Z$)

Subject to

$A^T Z <= C^T$

Z>=0