Roll No. : ....

School of Mathematics, Thapar University, Patiala

Mid-Term Examination, March 2017

B.E. IV Semester | UMA007 : Numerical Analysis
Time Limit: 02 Hours | Maximum Marks: 25
Instructor(s): Arvind K. Lal, Jolly Puri, Paramjeet Singh, Raj Nandkeolyar, Rajvir Singh, Sapna Sharma

Instructions: You are expected to answer all the questions. Organize your work, in a reasonably neat and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode and alphanumeric memory is permitted.

1. (a) Use bisection method to find the solution of the equation $3x - e^x = 0$ in the interval $[1, 2]$ accurate within 10^{-2}. [3 marks]

(b) A calculator is defective: it can only add, subtract, and multiply. Use the Newton's Method and the defective calculator to find the reciprocal of 3.142 correct to 4 decimal places. [3 marks]

2. (a) A rectangular parallelepiped has sides of length 3 cm, 4 cm, and 5 cm, measured to the nearest centimeter. What are the best upper and lower bounds for the volume of this parallelepiped? What are the best upper and lower bounds for the surface area? [3 marks]

(b) How many multiplications and additions are required to determine a sum of the form

$$\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j \text{ ?}$$

Modify this sum to an equivalent form that reduces the number of computations. [3 marks]

3. (a) We require to solve the following system of linear equations using LU decomposition.

$$\begin{aligned} x_1 + x_2 - x_3 &= 3 \\ x_1 + 2x_2 - 2x_3 &= 2 \\ -2x_1 + x_2 + x_3 &= 1. \end{aligned}$$

Find the matrices L and U using Gauss elimination. Using those values of L and U, solve the given system of equations. [3 marks]

(b) Find the condition number $K(A)$ of the matrix

$$A = \begin{bmatrix} 1 & c \\ c & 1 \end{bmatrix}, \quad |c| \neq 1.$$

When does A become ill-conditioned? What does this say about the linear system $Ax = b$? How is $K(A)$ related to $\det(A)$? [3 marks]

4. (a) Show that the Gauss-Seidel method does not converge for the following system of equations:

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 7 \\ x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 2x_2 + x_3 &= 5. \end{aligned}$$

[3 marks]

(b) Let $g \in C[a, b]$ with $g(x) \in [a, b]$ for all $x \in [a, b]$ and $g'(x)$ is continuous function on (a, b) such that $\max\limits_{x \in [a,b]} |g'(x)| = \lambda < 1$. Show that g has a unique fixed-point $\alpha \in [a, b]$ and the fixed point iterations $x_n = g(x_{n-1})$ has the following error bound

$$|\alpha - x_n| \leq \frac{\lambda^n}{1-\lambda} |x_1 - x_0|, \; n \geq 1,$$

provided initial guess $x_0 \in [a, b]$. [4 marks]