Course: UMA 035 (Optimization Techniques)

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Limitations of algebraic method

- **Algebraic method can be used only if the feasible region (common region of all the constraints) of the considered LPP is bounded.**
- **Algebraic method cannot be used if the feasible region (common region of all the constraints) of the considered LPP is unbounded.**

Difficulties in applying algebraic method

- **To apply the algebraic method, there is a need to consider all $^{n}C_{m}$ cases.**
- **It is difficult to consider all the possible cases e.g., if n = 100 and m=2. Then number of possible cases will be $^{100}C_{2}$ = 100*99/2*1=50*99 =4950 cases**

Confusion about the existing results

In the last lecture, we have solved the following three LPPs by algebraic methods without verifying that the feasible region is bounded or unbounded. Therefore, the results of these three LPPs, obtained in the last lecture, may or may not be correct.

First LPP

Maximize $(3x_1 - 2x_2)$

Subject to

$x_1 - 4x_2 <= 5$,

$2x_1 - 8x_2 >= 10$,

$x_1 >= 2$, $x_2 >= -8$.

Second LPP

Maximize $(3x_1 - 2x_2 + x_3)$

Subject to

$x_1 - 4x_2 + x_3 < = 5$,

$x_1 >= 0$, $x_2 >= 0$, $x_3 >= 0$.

Third LPP

Maximize $(2x_1 - 8x_2 + 2x_3)$

Subject to

$x_1 - 4x_2 + x_3 < = 5$,

$x_1 >= 0$, $x_2 >= 0$, $x_3 >= 0$.

Method to draw the feasible region for a LPP having two variables

The feasible region for a LPP having two variables can be drawn as follows:

Step 1: Check that the minimum value of both the variables are 0 or not. If there exist a variable for which minimum value is not 0. Then replace such a variable with a new variable so that the minimum value the new variable is 0.

Step 2: Draw a line segment corresponding to each constraint $a_1x_1 + a_2x_2$ <=,=,>=b_1 of the LPP in the first quadrant as follows.

Case (i): If a_2 =0 then draw a line parallel to X_1 axis (X axis) passing through the point (b_1 / a_1, 0).

Case (ii): If a_1 =0 then draw a line parallel to X_2 axis (Y axis) passing through the point (0, b_1 / a_2).

Case (iii): If neither a_1 nor a_2 is 0 then

- **Put an arbitrary value of x_2 (say, 0) in $a_1x_1 + a_2x_2=b_1$ and find the value of $x_1=(b_1– a_2*0) / a_1$ and hence, the point $(x_1, x_2) = (b_1 / a_1, 0)$ on X_1 axis (or X axis).**

- **Put an arbitrary value of x_1 (say, 0) in $a_1x_1 + a_2x_2=b_1$ and find the value of $x_2=(b_1– a_1*0) / a_2$ and hence, the point $(x_1, x_2) = (0, b_1 / a_2)$ on X_2 axis (Y axis).**
- **Join the points $(b_1 / a_1, 0)$ and $(0, b_1 / a_2)$ with a line.**
- **If $b_1=0$ then join (0,0) and $((b_1– a_2*p) / a_1, p)$ or (0,0) and $(p, (b_1– a_1*p) / a_2)$, where p is any real number**

Step 3: Line will divide the whole region in to two parts. Consider any arbitrary point of one part and put in the constraint.

If the constraint is satisfied then shade the considered part otherwise shade the other part.

Step 4: Find the common shaded region of all the constraints in the first quadrant.

Step 5: The common shaded region, obtained in Step 3, is called the feasible region for the considered LPP.

Method to check that the obtained feasible region is bounded or unbounded

If the maximum values of both the decision variables x_1 and x_2 are finite in the feasible region. Then the feasible region will be bounded otherwise the feasible region will be unbounded.

Example: **Draw the feasible region for the following LPP and check that the feasible region is bounded or unbounded.**

Maximize ($3x_1 - 2x_2$)

Subject to

$x_1 - 4x_2 <= 5$,

$x_1 + 8x_2 >= 15$,

$x_1 >= 2$, $x_2 >= -8$.

Solution: Since, Minimum value of x_1 and x_2 are not 0. So, there is a need to transform these variables into new variables.

First variable

$x_1 >= 2$

may be written as

$x_1 - 2 >= 0$

Assume $x_1 - 2 = y_1$

i.e.,

$x_1 = y_1 + 2$

Replace x_1 with $y_1 + 2$ in the given LPP.

Second variable

$x_2 >= -8$

may be written as

$x_2 + 8 >= 0$

Assume $x_2 + 8 = y_2$

i.e.,

$x_2 = y_2 - 8$

Replace x_2 with $y_2 - 8$ in the given LPP.

Transformed LPP

Maximize $(3 (y_1+2) - 2(y_2-8))$

Subject to

$(y_1+2) - 4 (y_2-8) <= 5$,

$(y_1+2) + 8(y_2-8) >= 15$,

$y_1+2 >= 2$, $y_2-8 >= -8$.

Maximize $(3y_1+6 - 2y_2+16))$

Subject to

$y_1+2-4y_2+32 <= 5$,

$y_1+2 + 8y_2-64 >= 15$,

$y_1 >= 2-2$, $y_2 >= -8+8$

Maximize $(3y_1 - 2y_2 + 22)$)

Subject to

$y_1 - 4y_2 + 34 <= 5$,

$y_1 + 8y_2 - 62 >= 15$,

$y_1 >= 0$, $y_2 >= 0$

Maximize $(3y_1 - 2y_2 + 22)$)

Subject to

$y_1 - 4y_2 <= 5 - 34$,

$y_1 + 8y_2 >= 15 + 62$,

$y_1 >= 0$, $y_2 >= 0$

Maximize $(3y_1 - 2y_2 + 22)$)

Subject to

$y_1 - 4y_2 <= -29$,

$y_1 + 8y_2 >= 77$,

$y_1 >= 0$, $y_2 >= 0$

Draw First Cosnstraint

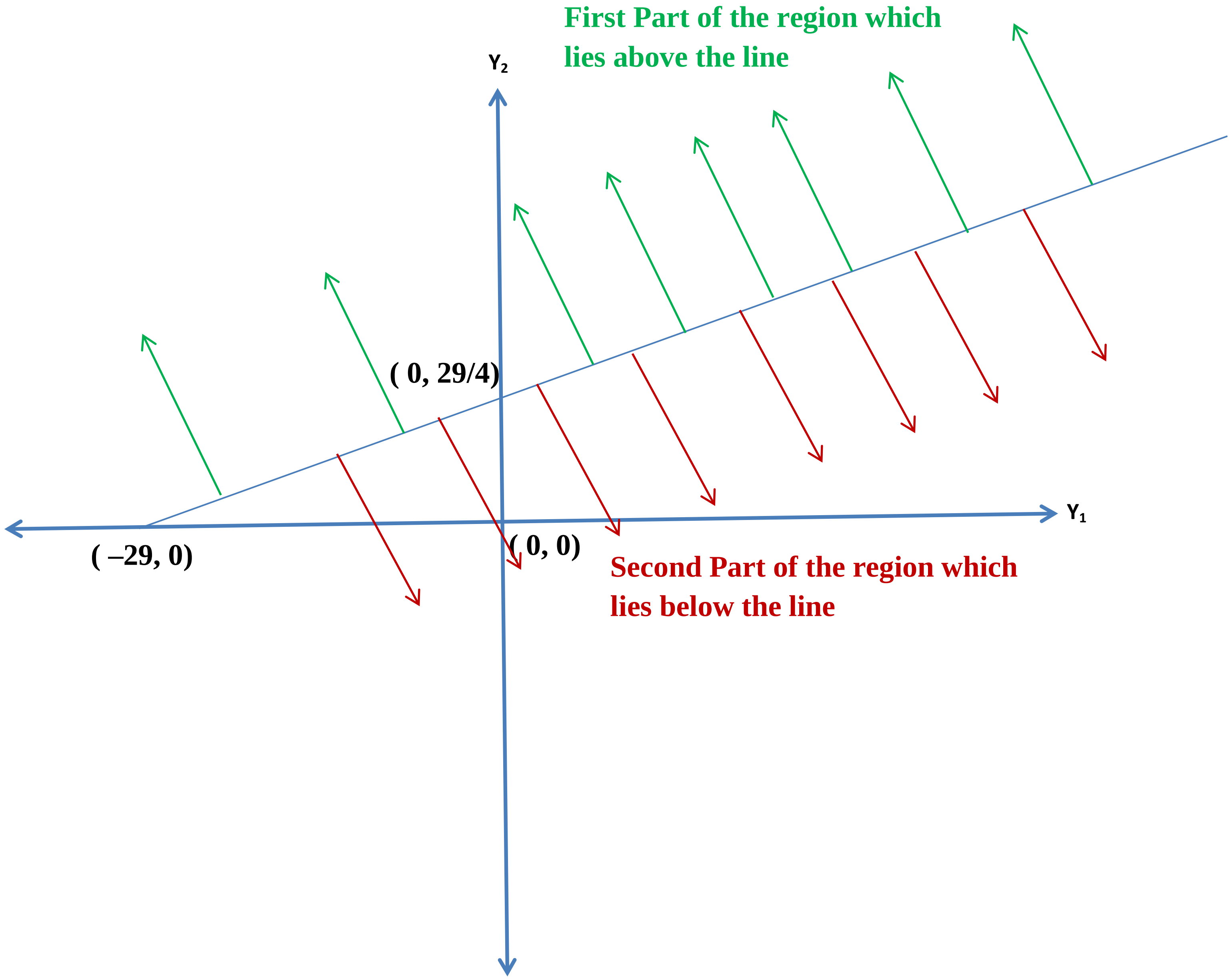
$y_1 - 4y_2 <= -29$

Assuming $y_1 = 0$, $y_1 - 4y_2 = -29$ implies $0 - 4y_2 = -29$ i.e., $y_2 = 29/4$

Therefore, first point is $(y_1, y_2) = (0, 29/4)$

Assuming $y_2 = 0$, $y_1 - 4y_2 = -29$ implies $y_1 - 0 = -29$ i.e., $y_1 = -29$

Therefore, second point is $(y_1,y_2)=(-29, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

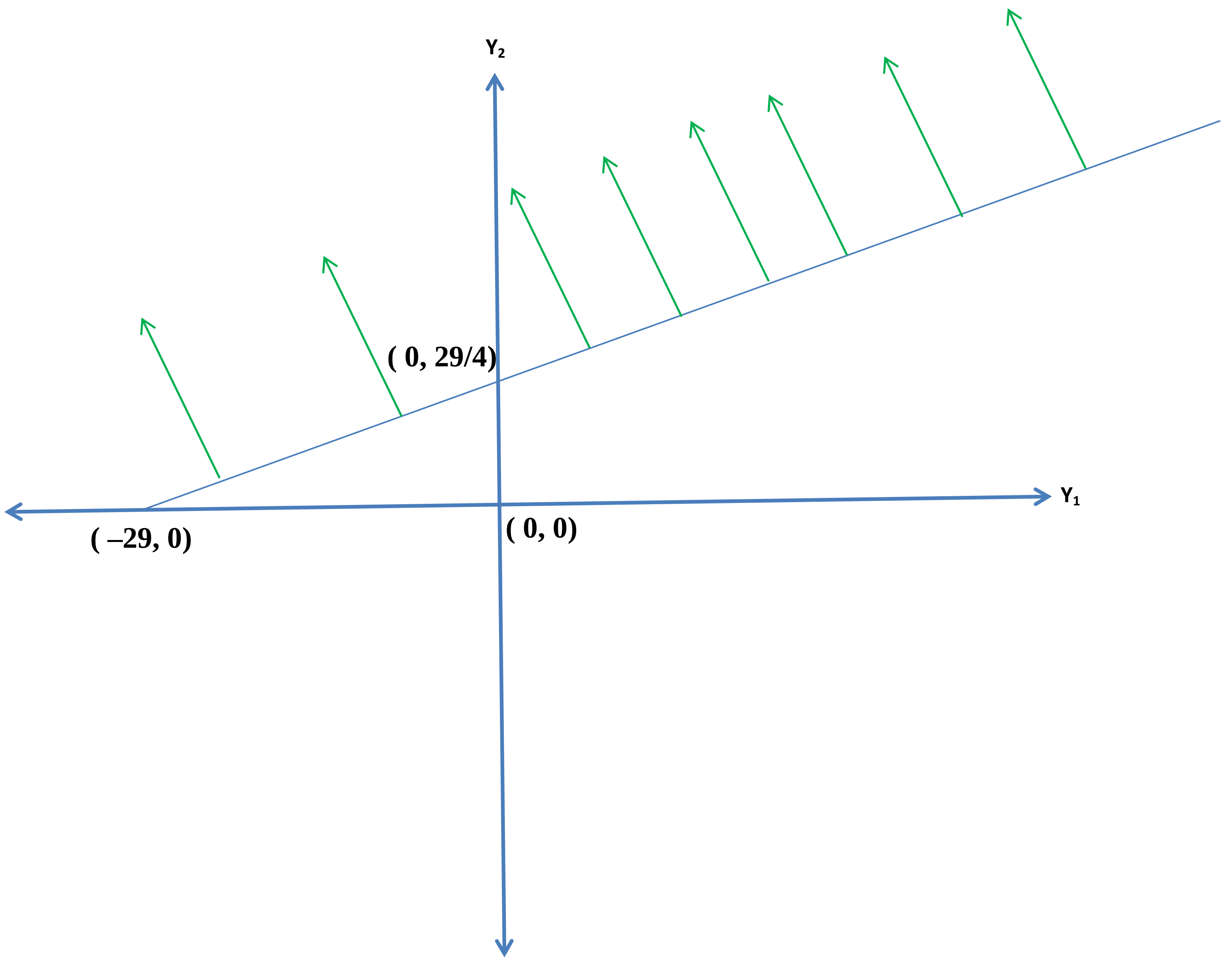
Putting (5,0) in the constraint

$y_1 - 4y_2 <= -29$, we have

$5 - 0 <= -29$

$5 <= -29$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Draw Second Cosnstraint

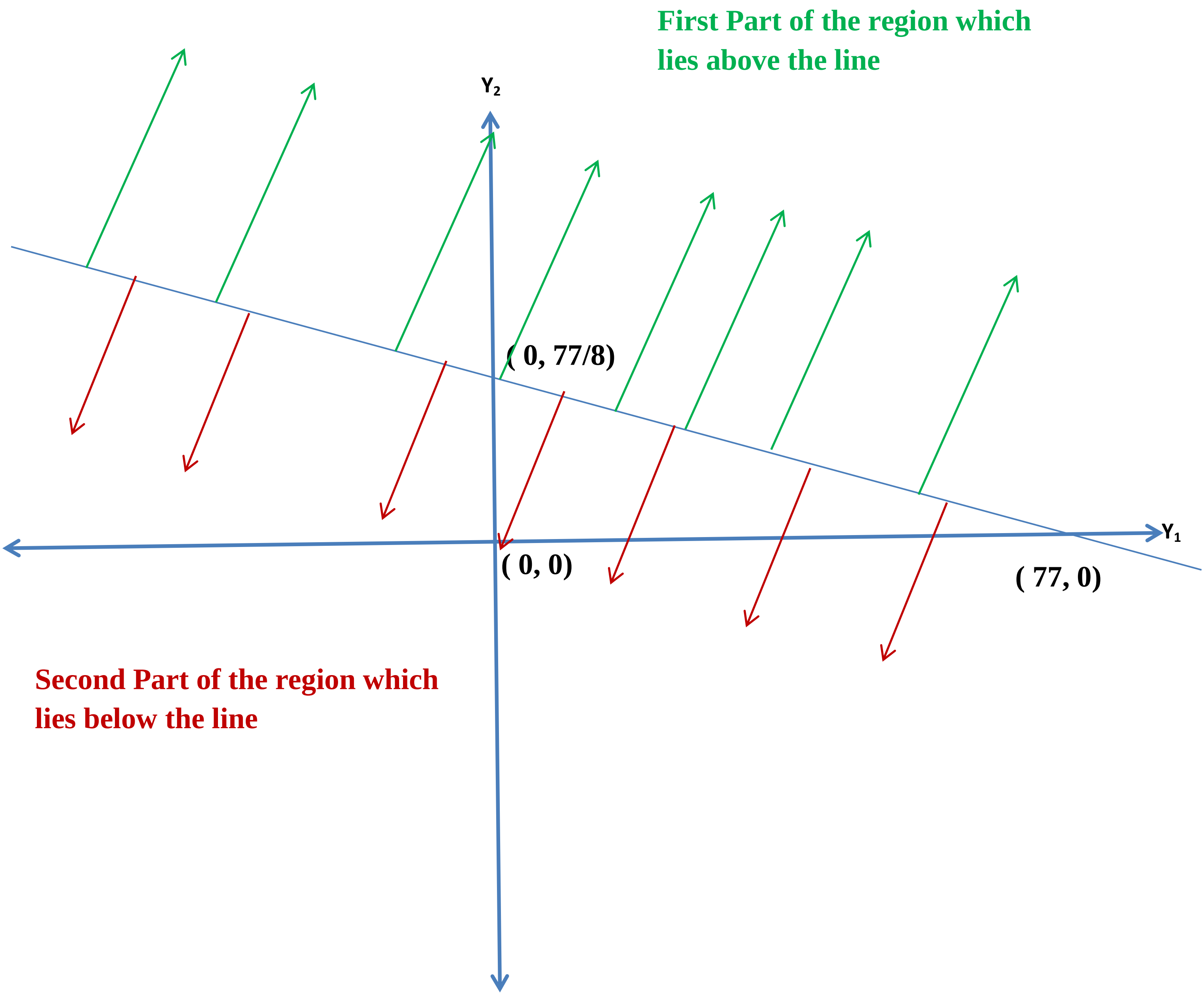
$y_1 + 8y_2 >= 77$

Assuming $y_1 = 0$, $y_1 + 8y_2 = 77$ implies $0 + 8y_2 = 77$ i.e., $y_2 = 77/8$

Therefore, first point is $(y_1, y_2) = (0, 77/8)$

Assuming $y_2 = 0$, $y_1 + 8y_2 = 77$ implies $y_1 = 77$ i.e., $y_1 = 77$

Therefore, second point is $(y_1,y_2)=(77, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint

then we will consider the second part otherwise the first part.

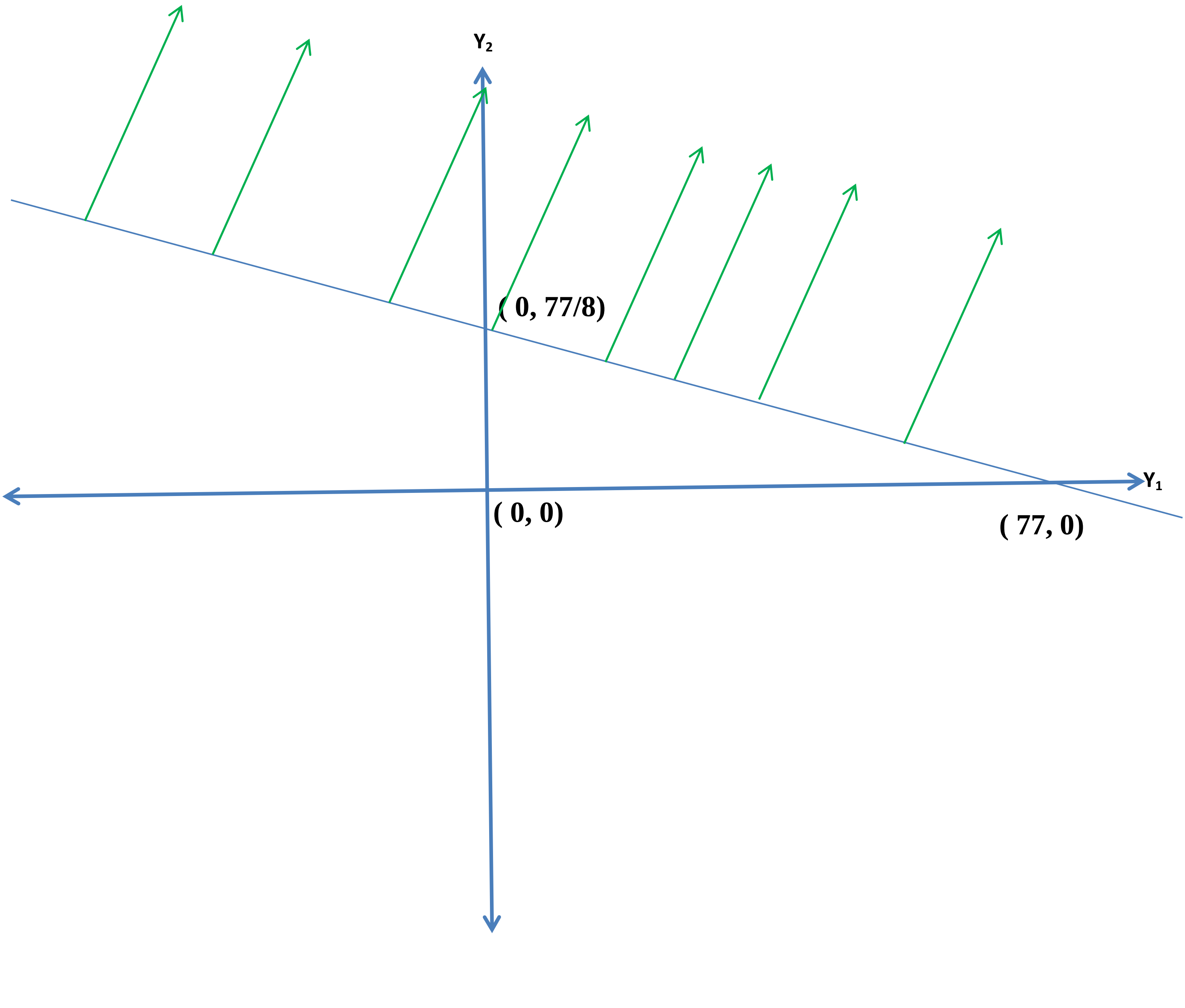
Putting (5,0) in the the constraint

$y_1+8y_2>=77$, we have

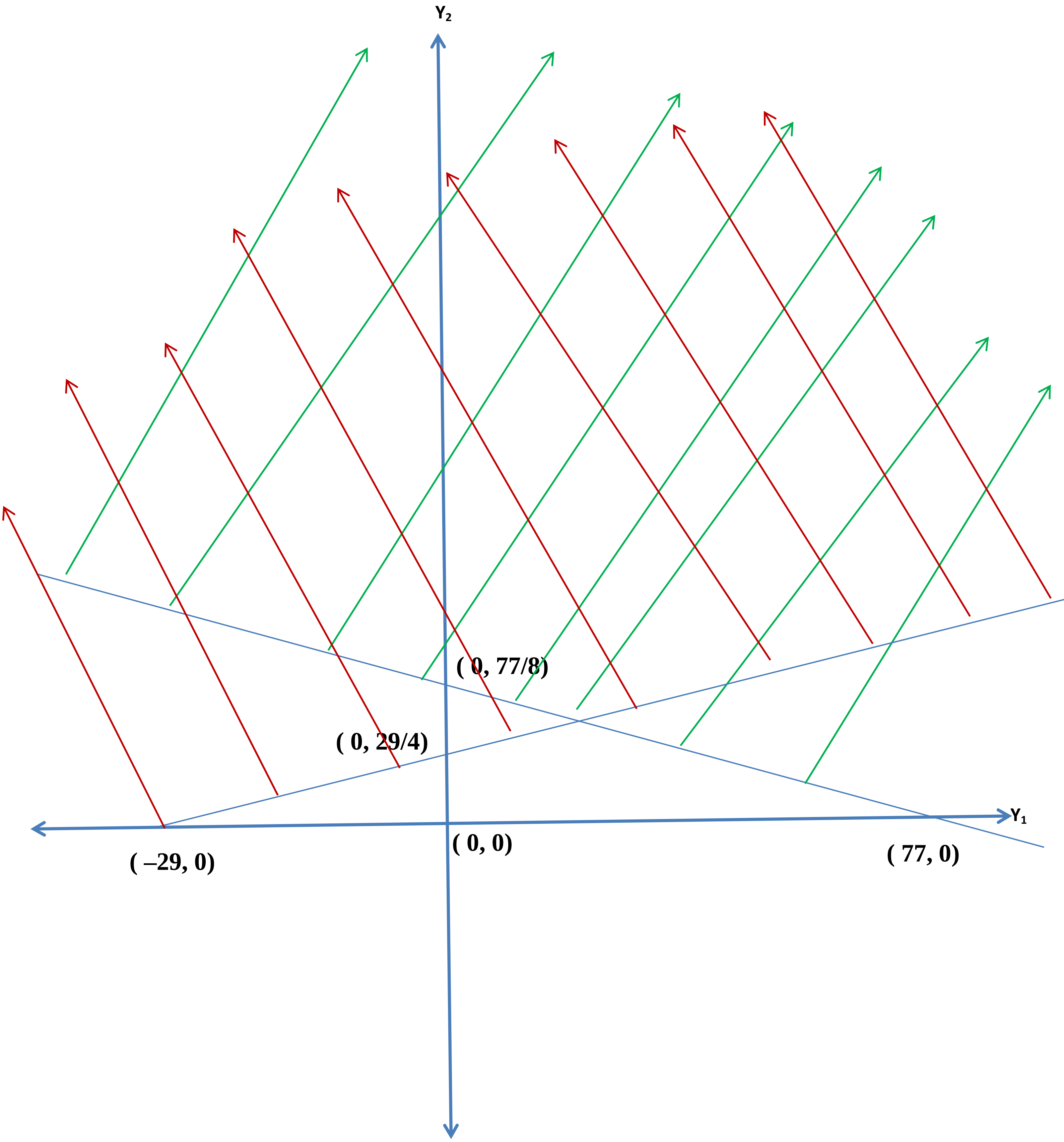
5+0> =77

5>= 77

It is obvious that the constraint is not satisfying. So, we consider the first part.

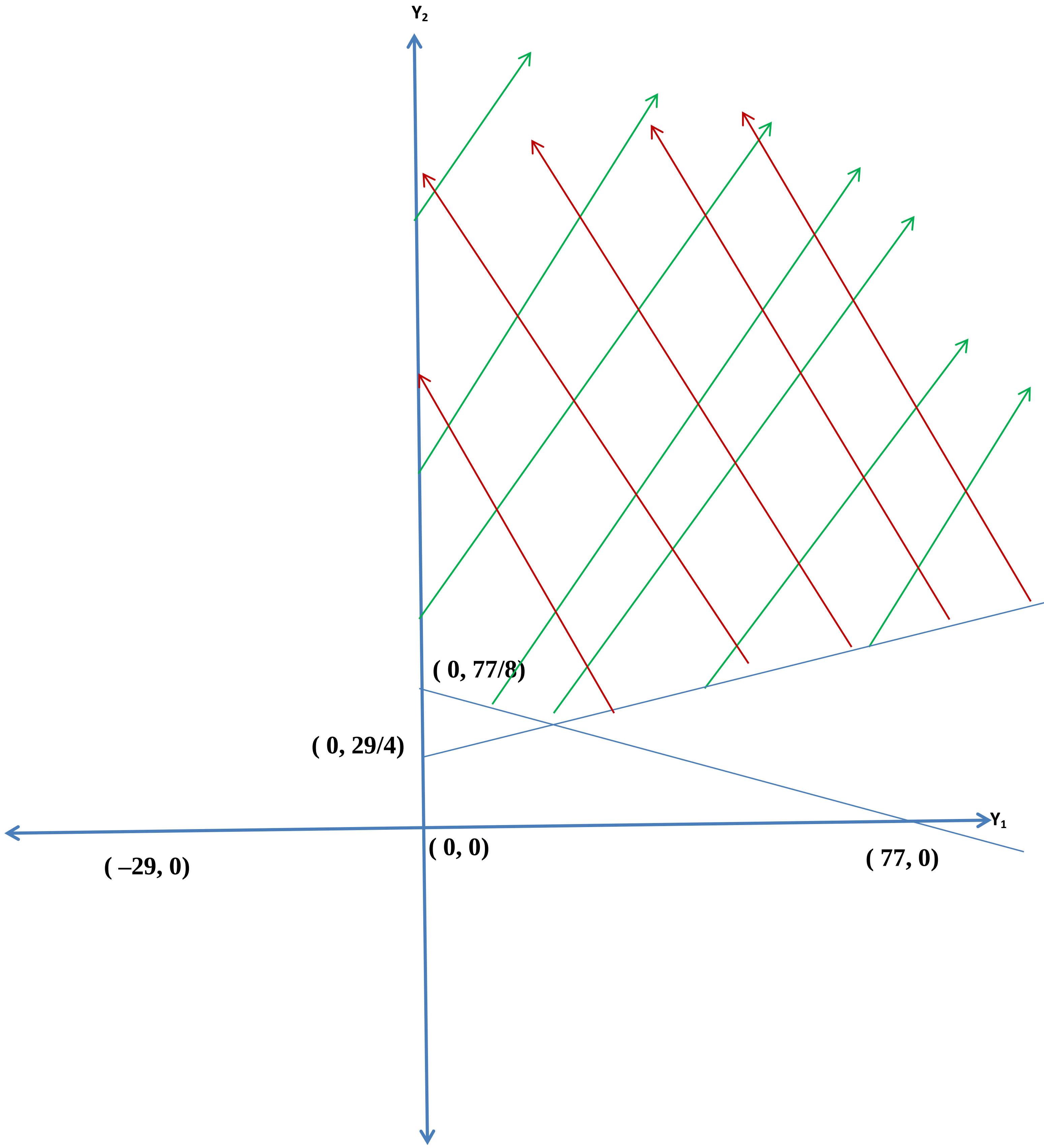


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



It is obvious that Maximum value of y_1 and the maximum value of y_2 in the feasible region is infinite. So, the feasible region is unbounded.

Example: Draw the feasible region for the following LPP and check that the feasible region is bounded or unbounded.

Maximize $(3x_1 - 2x_2)$

Subject to

$x_1 - 4x_2 <= 5$,

$x_1 + 8x_2 <= 15$,

$x_1 >= 2$, $x_2 >= -8$.

Solution: Since, Minimum value of x_1 and x_2 are not 0. So, there is a need to transform these variables into new variables.

First variable

$x_1 >= 2$

may be written as

$x_1 - 2 >= 0$

Assume $x_1 - 2 = y_1$

i.e.,

$x_1 = y_1 + 2$

Replace x_1 with y_1+2 in the given LPP.

Second variable

$x_2 >= -8$

may be written as

$x_2+8>=0$

Assume $x_2+8=y_2$

i.e.,

$x_2=y_2-8$

Replace x_2 with y_2-8 in the given LPP.

Transformed LPP

Maximize $(3\,(y_1+2) - 2(y_2-8))$

Subject to

$(y_1+2)-4\,(y_2-8) \; < = 5,$

$(y_1+2) + 8(y_2-8) <= 15,$

$y_1+2>=2,\ y_2-8>= -8.$

Maximize $(3y_1+6 - 2y_2+16))$

Subject to

$y_1+2-4y_2+32 \; < = 5,$

$y_1+2 + 8y_2–64>= 15,$

$y_1>=2–2, y_2>= –8+8$

Maximize $(3y_1– 2y_2+22))$

Subject to

$y_1–4y_2+34 < = 5,$

$y_1+ 8y_2–62<= 15,$

$y_1>=0, y_2>= 0$

Maximize $(3y_1– 2y_2+22))$

Subject to

$y_1–4y_2 < = 5–34,$

$y_1+ 8y_2<= 15+62,$

$y_1>=0, y_2>= 0$

Maximize $(3y_1– 2y_2+22))$

Subject to

$y_1–4y_2 < = –29,$

$y_1+ 8y_2<=77,$

$y_1>=0, y_2>= 0$

Draw First Cosnstraint

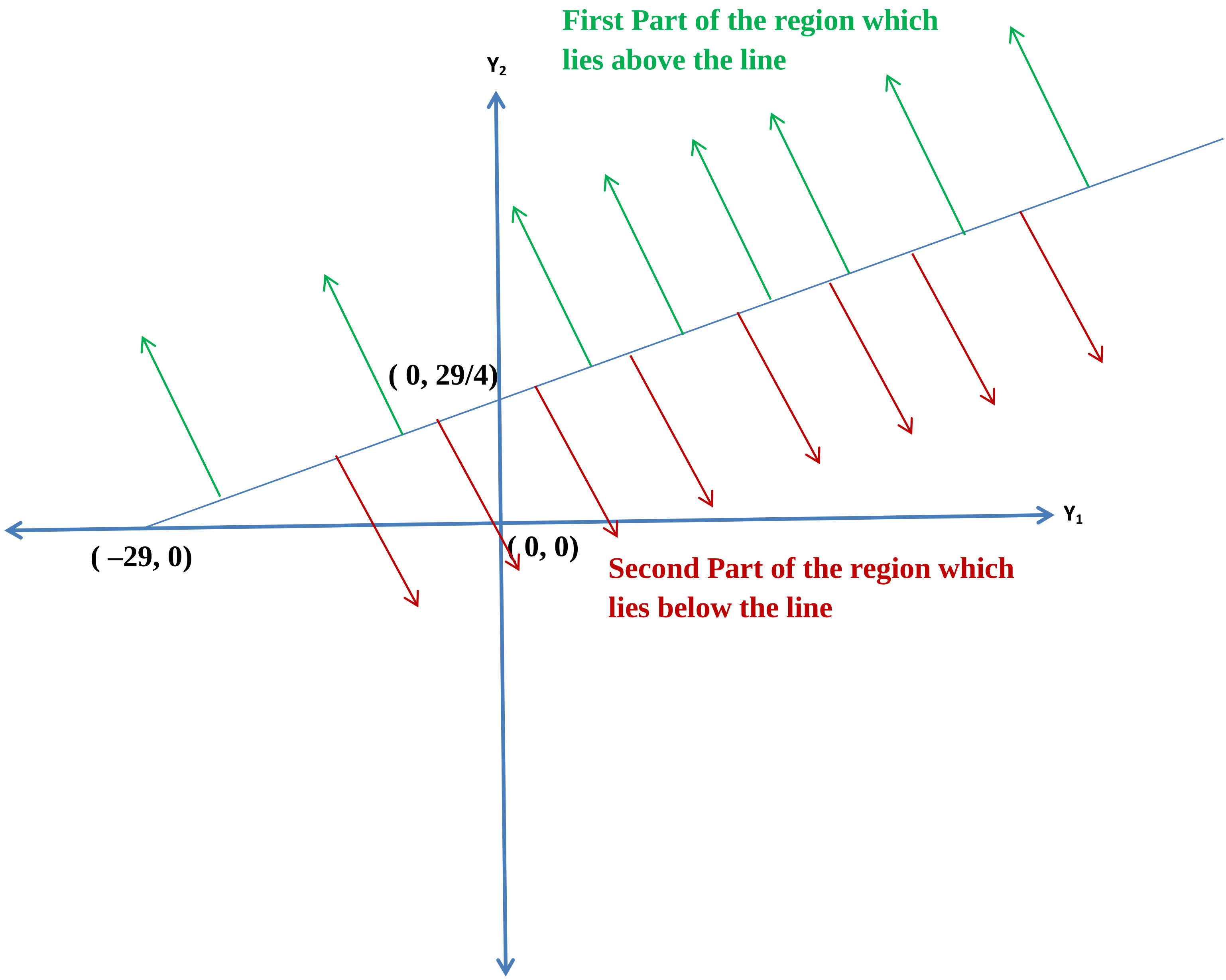
$y_1–4y_2 < = –29$

Assuming $y_1 = 0$, $y_1 – 4y_2 = –29$ implies $0 – 4y_2 = –29$ i.e., $y_2 = 29/4$

Therefore, first point is $(y_1, y_2) = (0, 29/4)$

Assuming $y_2 = 0$, $y_1 – 4y_2 = –29$ implies $y_1 – 0 = –29$ i.e., $y_1 = –29$

Therefore, second point is $(y_1, y_2) = (–29, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

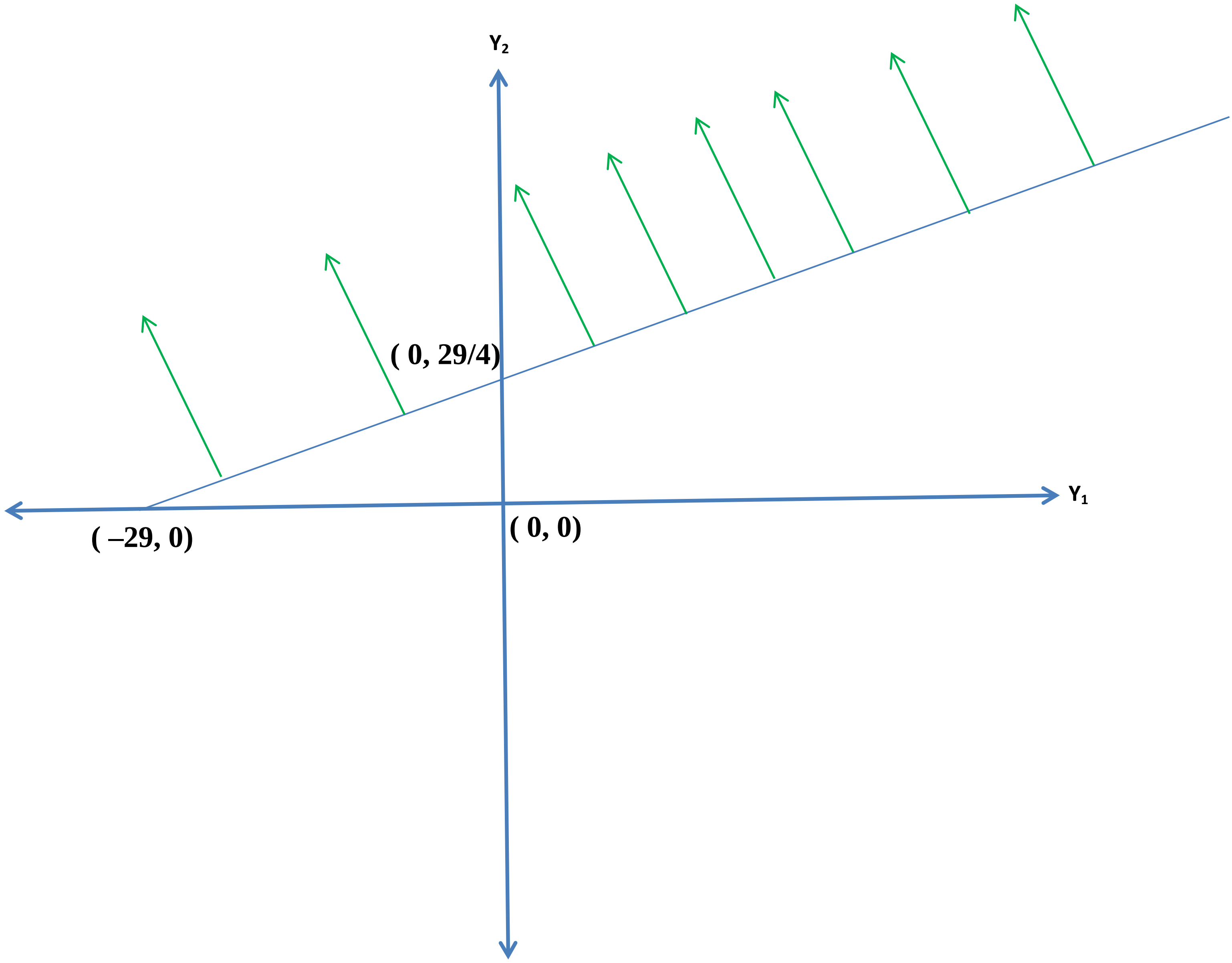
Putting (5,0) in the the constraint

$y_1 - 4y_2 <= -29$, we have

$5 - 0 <= -29$

$5 <= -29$

It is obvious that the constraint is not satisfying. So, we consider the first part.



Draw Second Cosnstraint

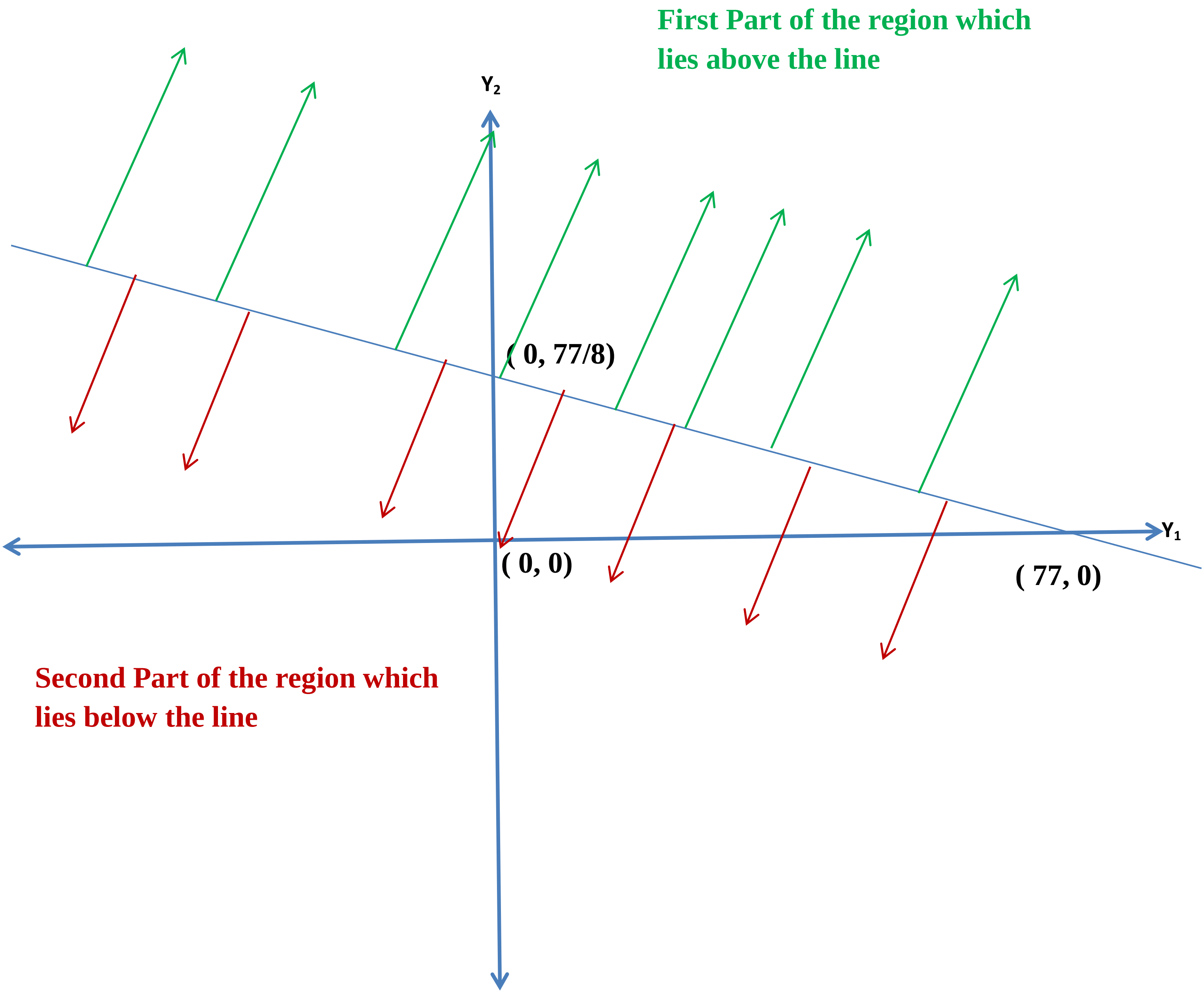
$y_1 + 8y_2 >= 77$

Assuming $y_1 = 0$, $y_1+8y_2 = 77$ implies $0 + 8y_2 = 77$ i.e., $y_2 = 77/8$

Therefore, first point is $(y_1,y_2)=(0, 77/8)$

Assuming $y_2 = 0$, $y_1+8y_2 = 77$ implies $y_1 = 77$ i.e., $y_1 = 77$

Therefore, second point is $(y_1,y_2)=(77, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

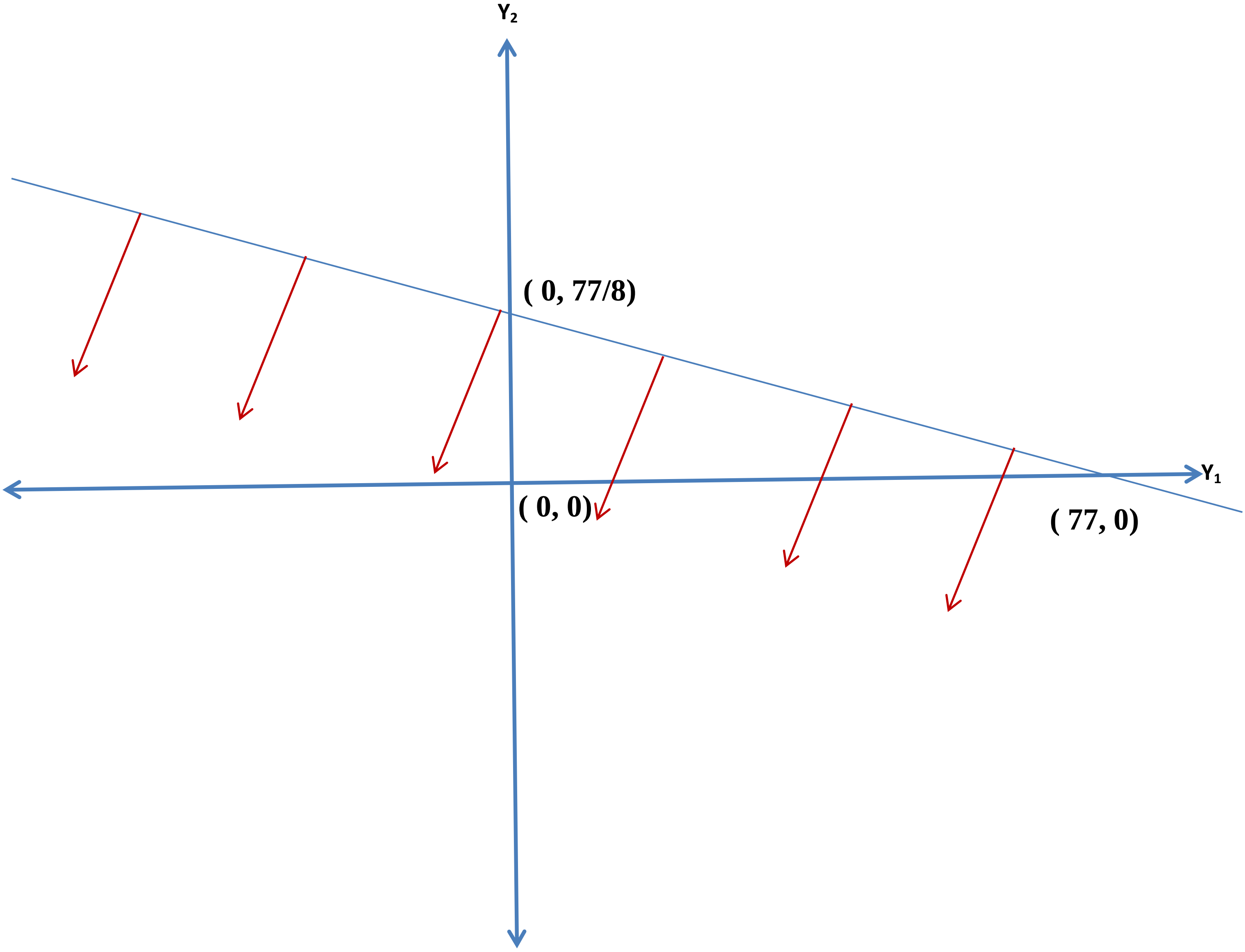
Putting (5,0) in the the constraint

$y_1+8y_2 <= 77$,   we have

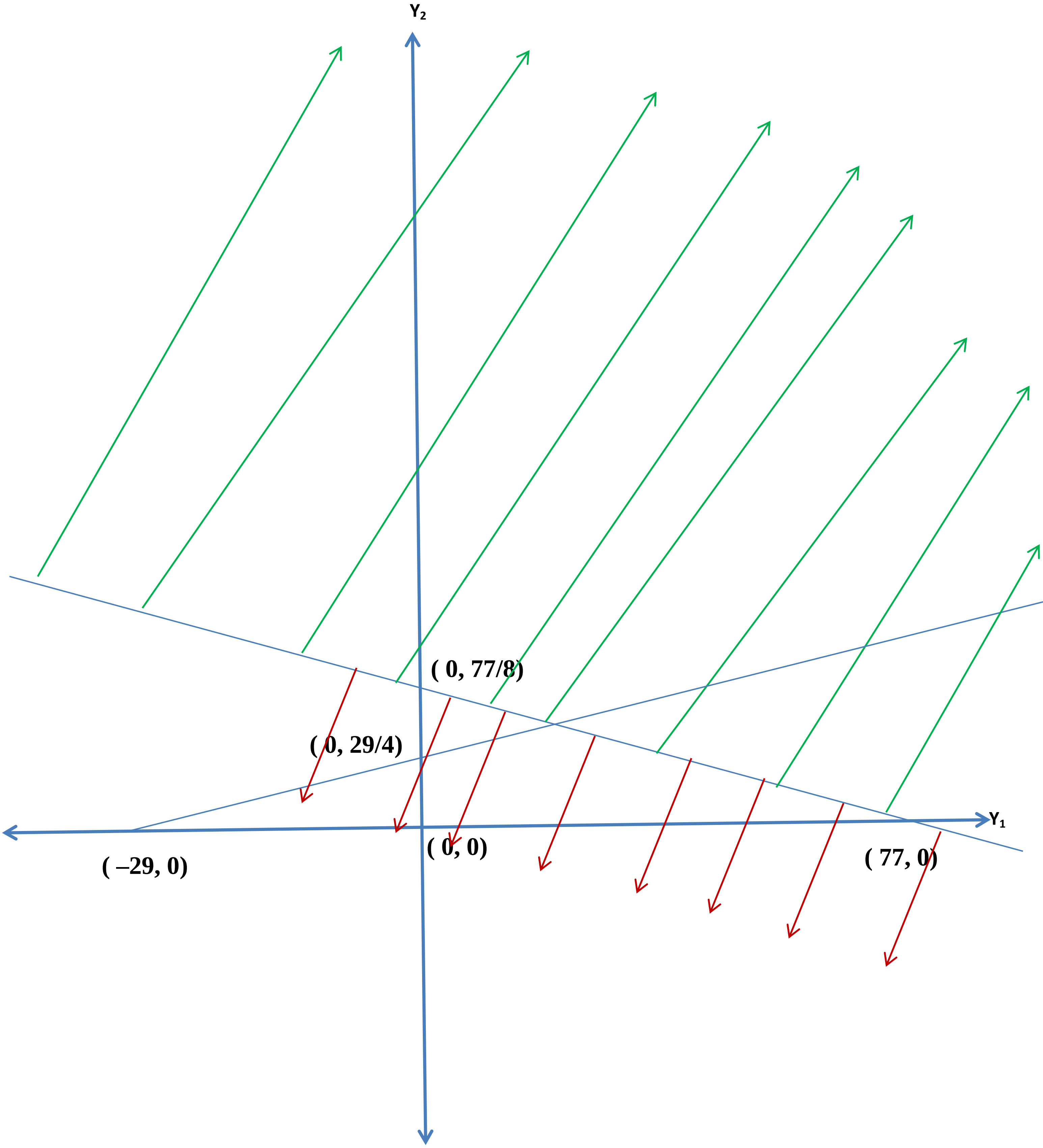
$5+0<=77$

$5<= 77$

It is obvious that the constraint is satisfying. So, we consider the second part.

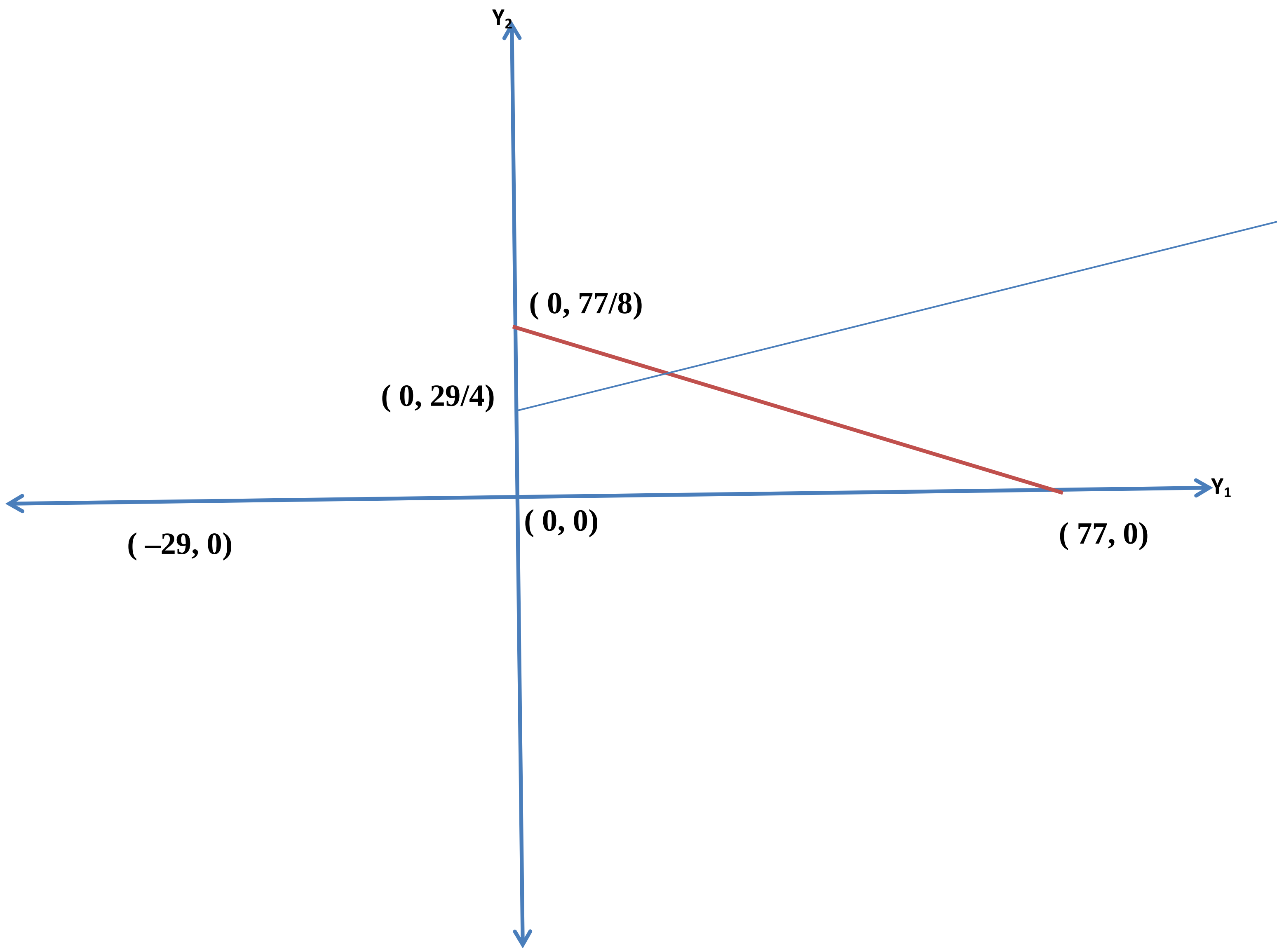


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



It is obvious that Maximum value of y_1 and the maximum value of y_2 in the feasible region is finite. So, the feasible region is bounded.

Example: **Draw the feasible region for the following LPP and check that the feasible region is bounded or unbounded.**

Maximize $(3x_1 - 2x_2)$

Subject to

$x_1 - 4x_2 >= 5$,

$x_1 + 8x_2 <= 15$,

$x_1 >= 2$, $x_2 >= -8$.

Solution: Since, Minimum value of x_1 and x_2 are not 0. So, there is a need to transform these variables into new variables.

First variable

$x_1 >= 2$

may be written as

$x_1 - 2 >= 0$

Assume $x_1 - 2 = y_1$

i.e.,

$x_1 = y_1 + 2$

Replace x_1 with $y_1 + 2$ in the given LPP.

Second variable

$x_2 >= -8$

may be written as

$x_2 + 8 >= 0$

Assume $x_2 + 8 = y_2$

i.e.,

$x_2 = y_2 - 8$

Replace x_2 with $y_2 - 8$ in the given LPP.

Transformed LPP

Maximize $(3(y_1+2) - 2(y_2-8))$

Subject to

$(y_1+2) - 4(y_2-8) > = 5,$

$(y_1+2) + 8(y_2-8) <= 15,$

$y_1+2 >= 2,\ y_2-8 >= -8.$

Maximize $(3y_1+6 - 2y_2+16))$

Subject to

$y_1+2-4y_2+32 > = 5,$

$y_1+2 + 8y_2-64 <= 15,$

$y_1 >= 2-2,\ y_2 >= -8+8$

Maximize $(3y_1 - 2y_2 + 22))$

Subject to

$y_1 - 4y_2 + 34 >= 5,$

$y_1 + 8y_2 - 62 <= 15,$

$y_1 >= 0, y_2 >= 0$

Maximize $(3y_1 - 2y_2 + 22))$

Subject to

$y_1 - 4y_2 > = 5 - 34,$

$y_1 + 8y_2 <= 15 + 62,$

$y_1 >= 0, y_2 >= 0$

Maximize $(3y_1 - 2y_2 + 22))$

Subject to

$y_1 - 4y_2 > = -29,$

$y_1 + 8y_2 <= 77,$

$y_1 >= 0, y_2 >= 0$

Draw First Cosnstraint

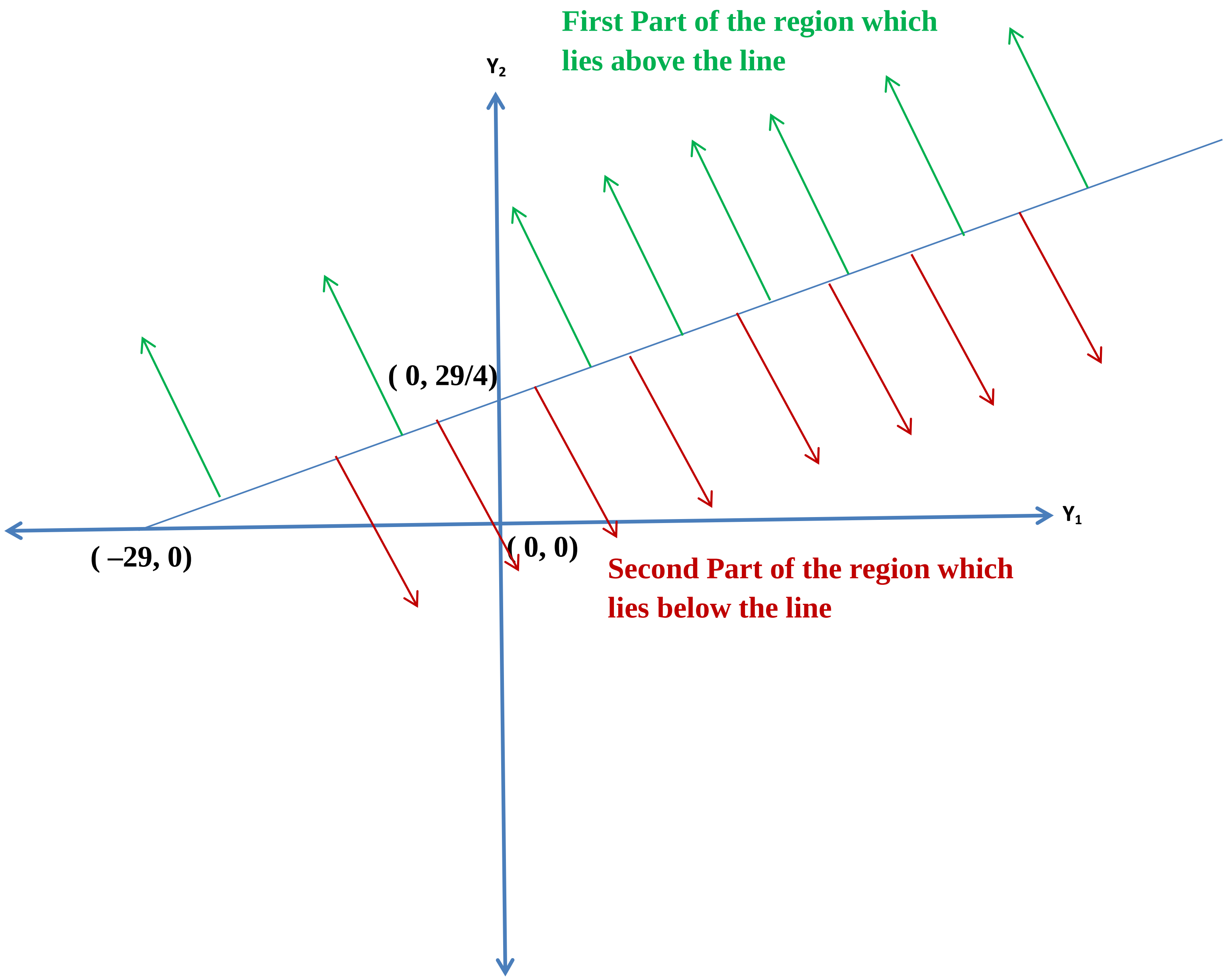
$y_1 - 4y_2 > = -29$

Assuming $y_1 = 0$, $y_1 - 4y_2 = -29$ implies $0 - 4y_2 = -29$ i.e., $y_2 = 29/4$

Therefore, first point is $(y_1, y_2) = (0, 29/4)$

Assuming $y_2 = 0$, $y_1 - 4y_2 = -29$ implies $y_1 - 0 = -29$ i.e., $y_1 = -29$

Therefore, second point is $(y_1, y_2) = (-29, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

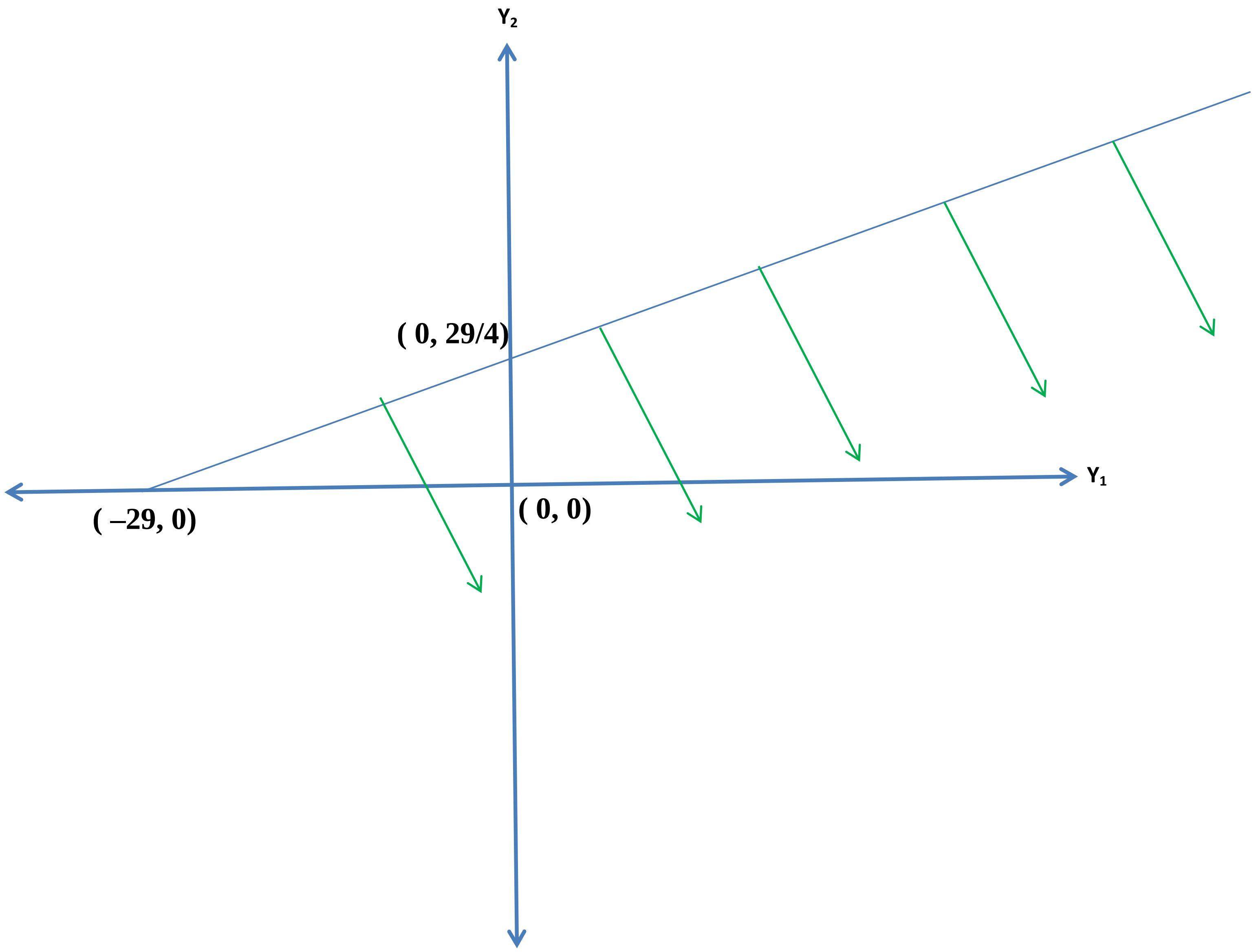
Putting (5,0) in the constraint

$y_1 - 4y_2 >= -29$, we have

$5 - 0 >= -29$

$5 >= -29$

It is obvious that the constraint is satisfying. So, we consider the second part.



Draw Second Cosnstraint

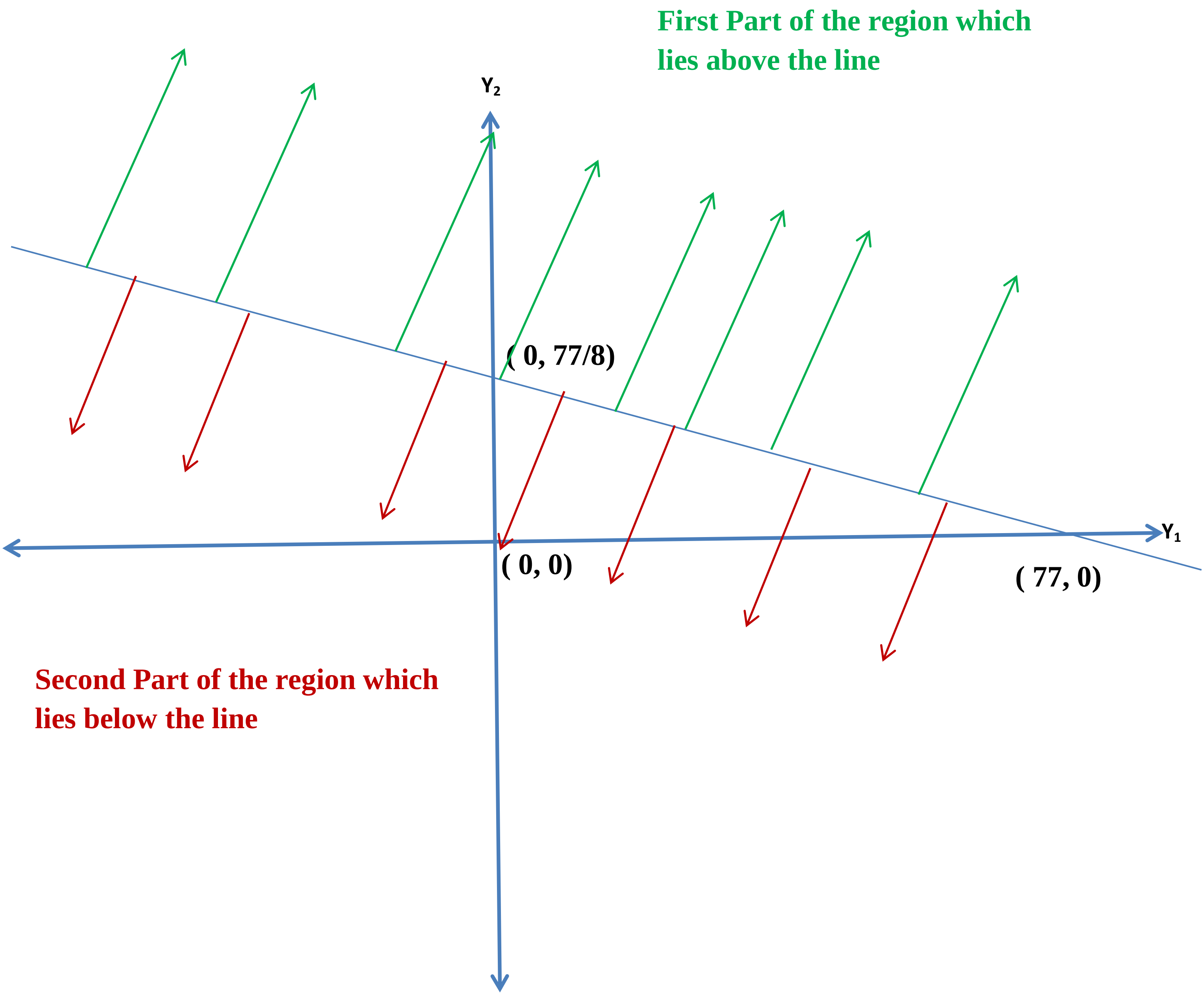
$y_1 + 8y_2 <= 77$

Assuming $y_1 = 0$, $y_1+8y_2 = 77$ implies $0 + 8y_2 = 77$ i.e., $y_2 = 77/8$

Therefore, first point is $(y_1,y_2)=(0, 77/8)$

Assuming $y_2 = 0$, $y_1+8y_2 = 77$ implies $y_1 = 77$ i.e., $y_1 = 77$

Therefore, second point is $(y_1, y_2) = (77, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

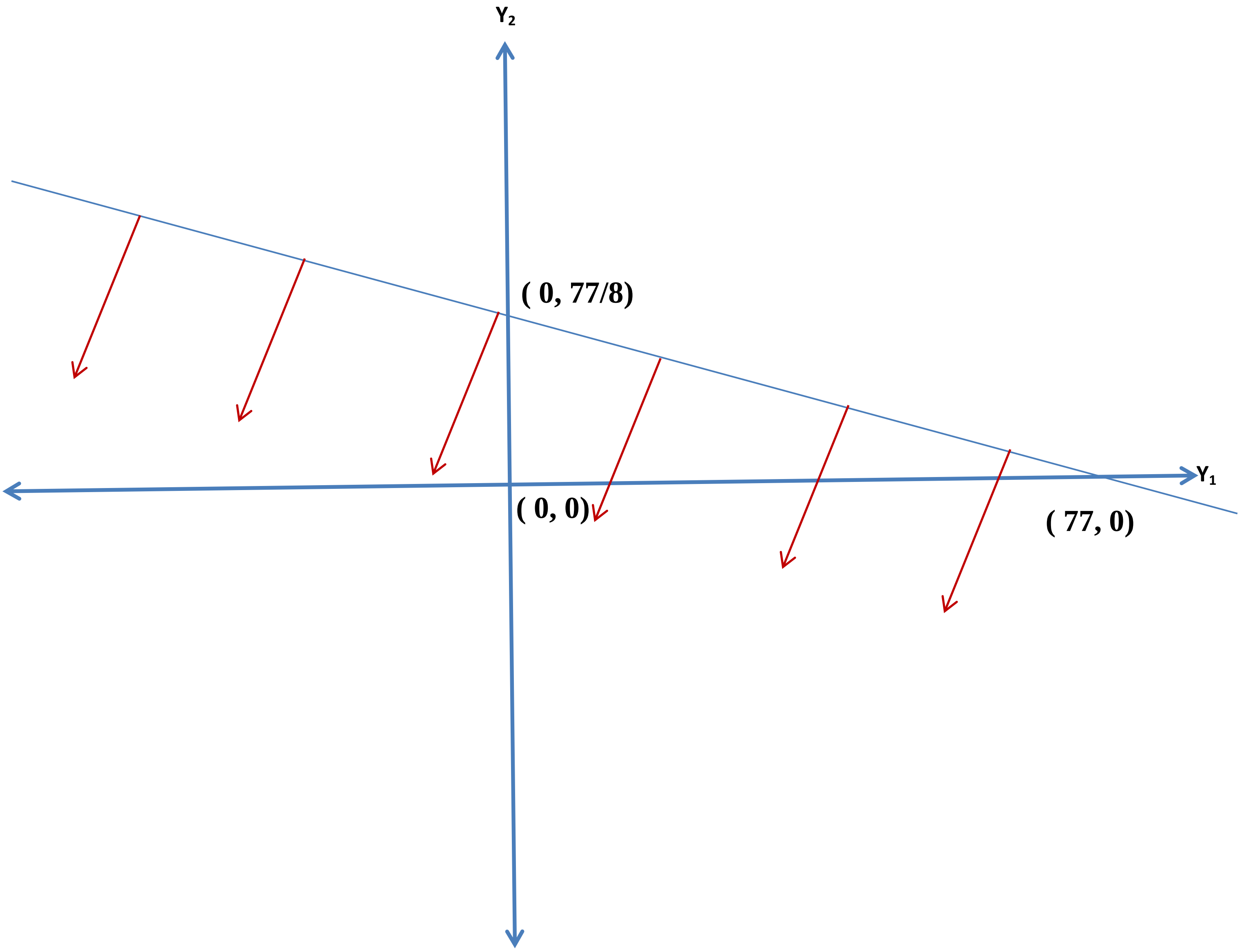
Putting (5,0) in the constraint

$y_1 + 8y_2 <= 77$, we have

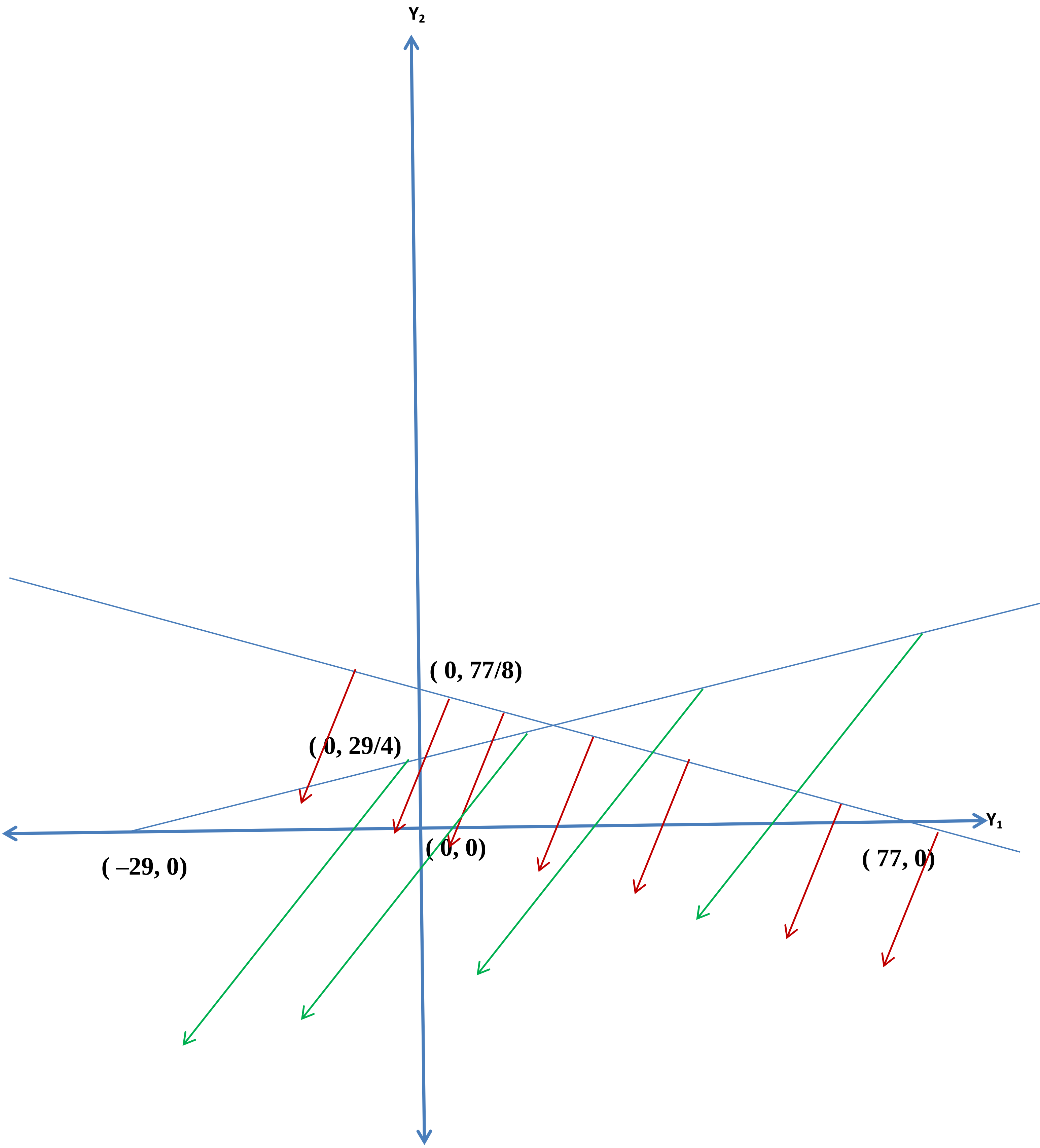
$5 + 0 <= 77$

5<= 77

It is obvious that the constraint is satisfying. So, we consider the second part.

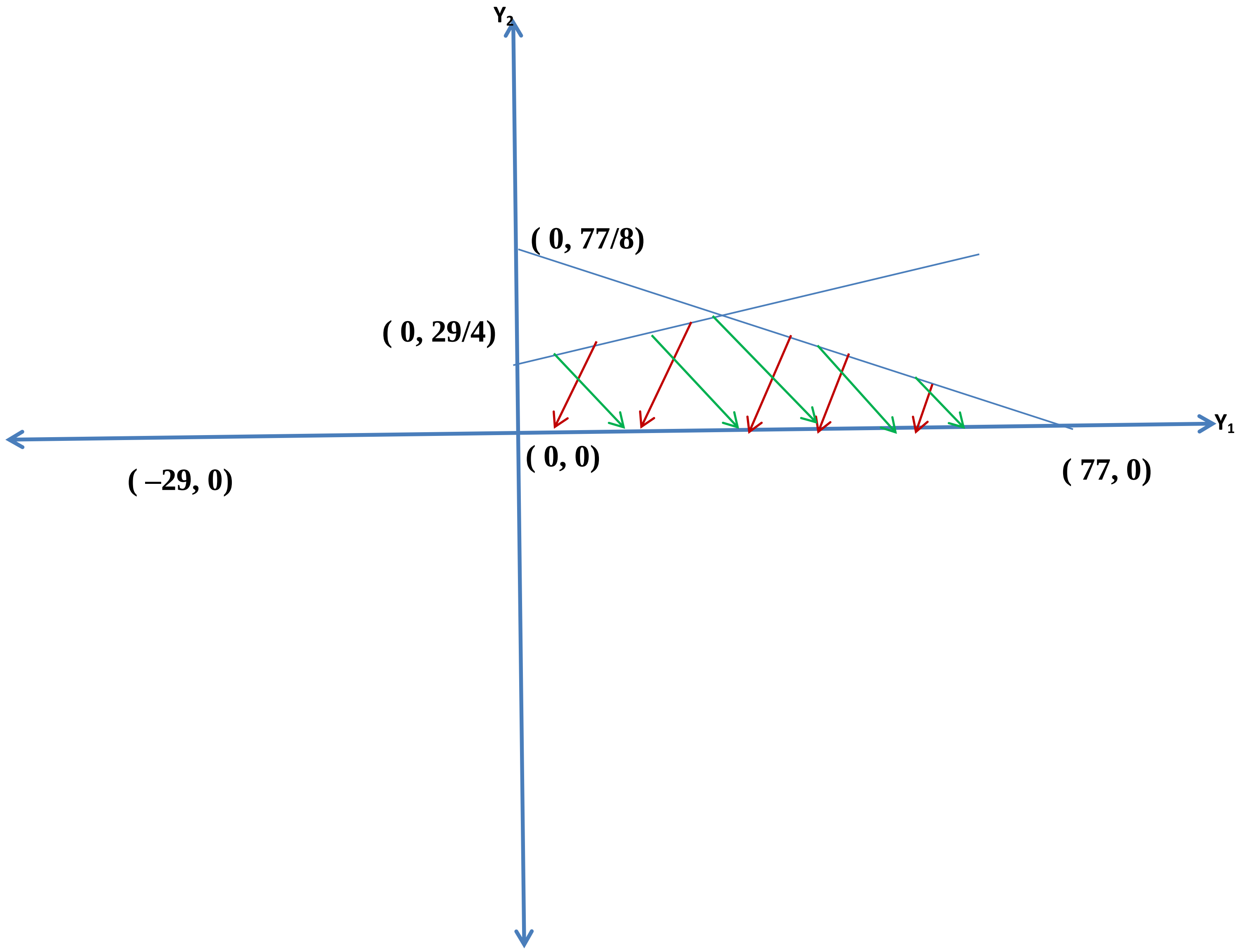


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



It is obvious that Maximum value of y_1 and the maximum value of y_2 in the feasible region are finite. So, the feasible region is bounded.

Maximize $(3y_1 - 2y_2 + 22))$

Subject to

$y_1 - 4y_2 >= -29$,

$y_1 + 8y_2 >= 77$,

$y_1 >= 0$, $y_2 >= 0$

Draw First Cosnstraint

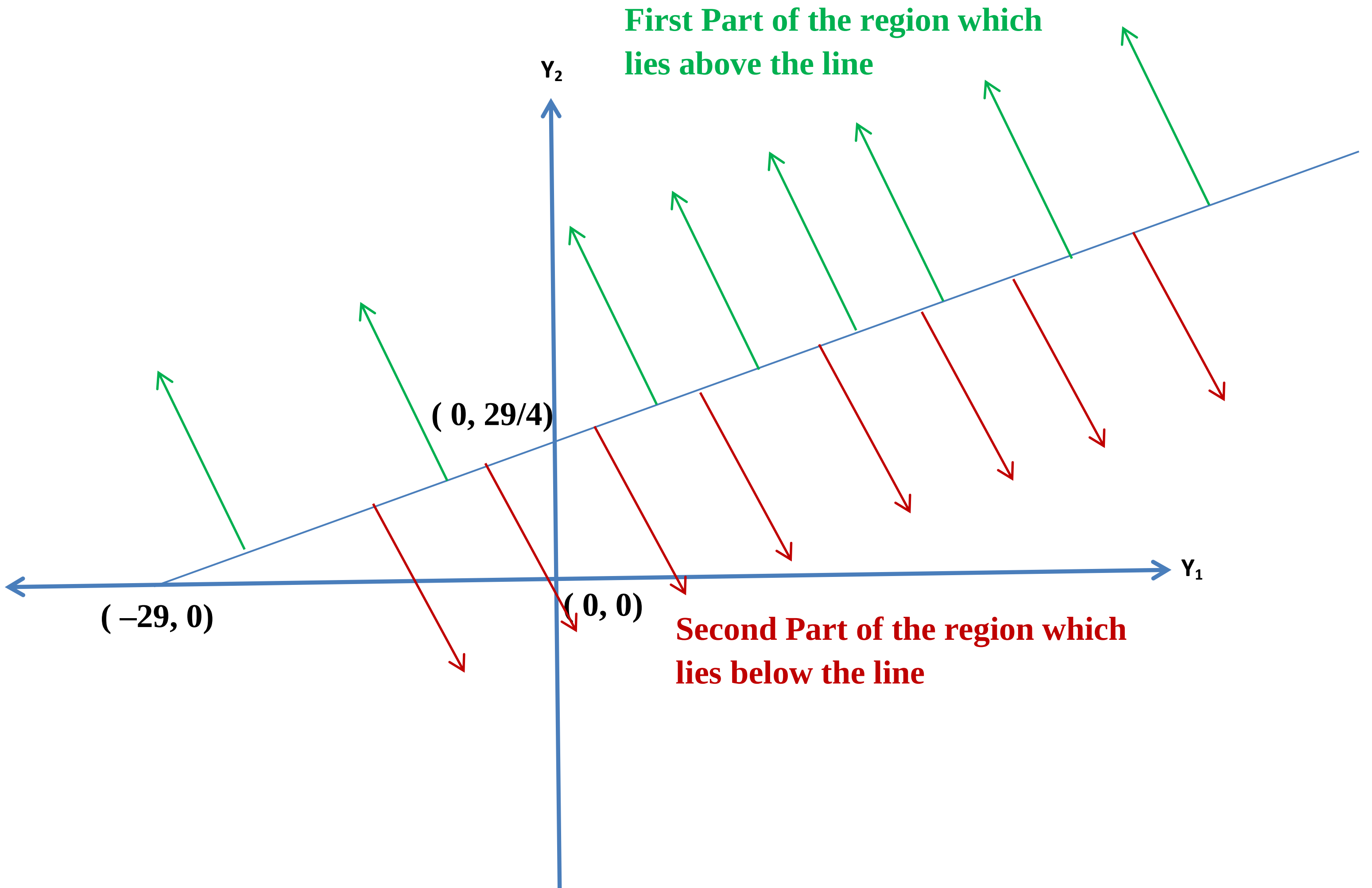
$y_1 - 4y_2 >= -29$

Assuming $y_1 = 0$, $y_1 - 4y_2 = -29$ implies $0 - 4y_2 = -29$ i.e., $y_2 = 29/4$

Therefore, first point is $(y_1, y_2) = (0, 29/4)$

Assuming $y_2 = 0$, $y_1 - 4y_2 = -29$ implies $y_1 - 0 = -29$ i.e., $y_1 = -29$

Therefore, second point is $(y_1, y_2) = (-29, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

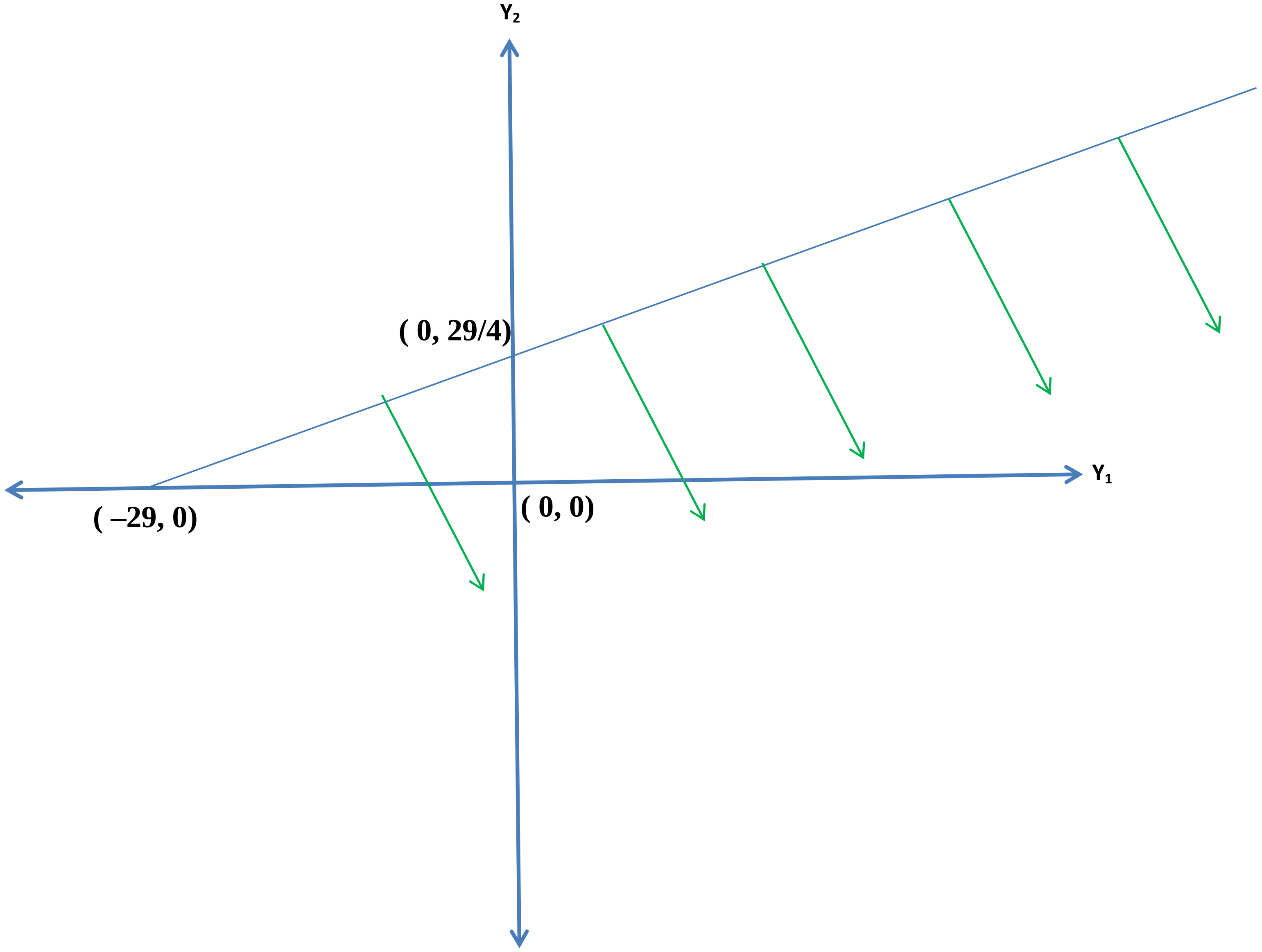
Putting (5,0) in the constraint

$y_1 - 4y_2 >= -29$, we have

$5 - 0 >= -29$

$5 >= -29$

It is obvious that the constraint is satisfying. So, we consider the second part.



Draw Second Cosnstraint

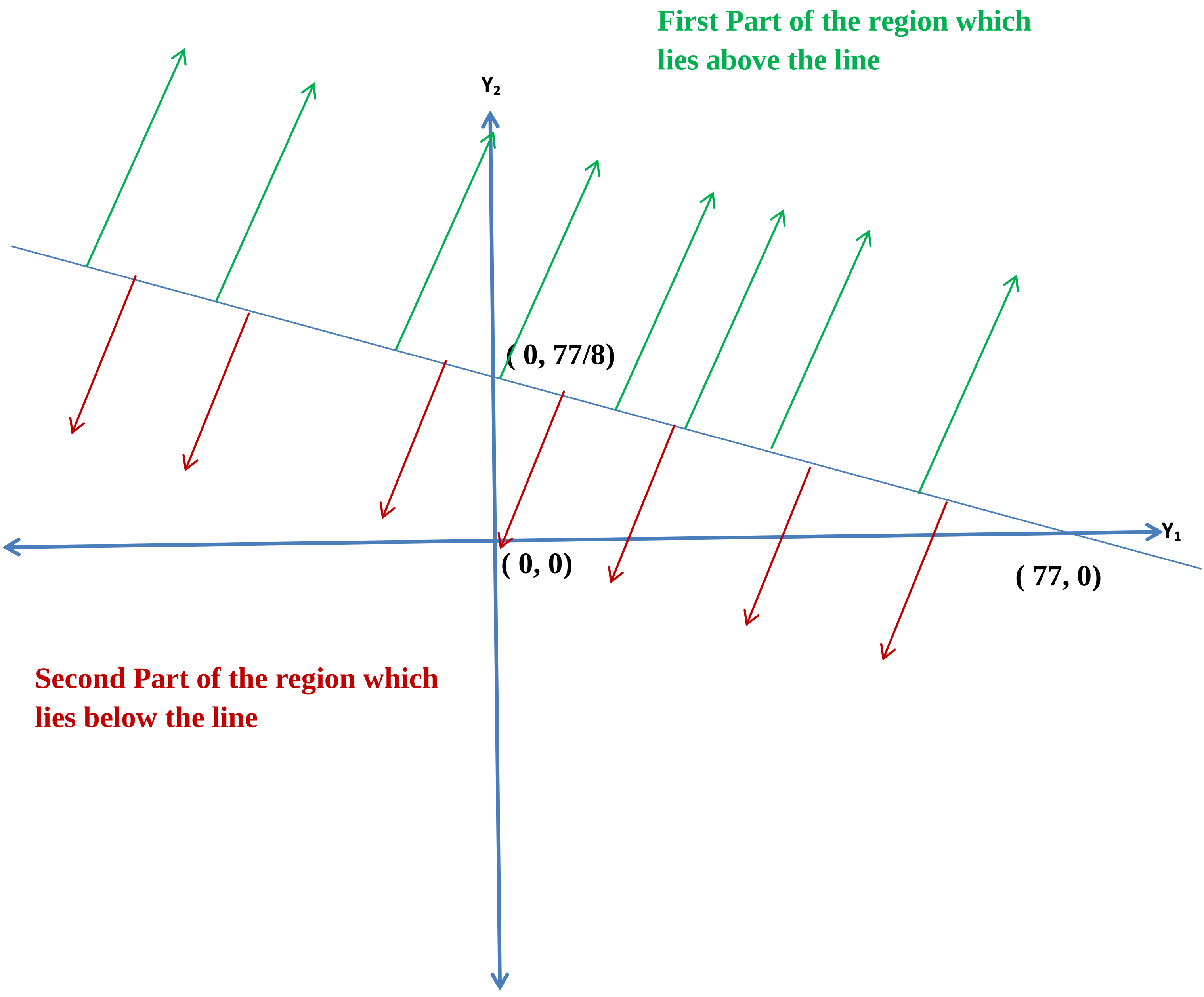
$y_1 + 8y_2 >= 77$

Assuming $y_1 = 0$, $y_1+8y_2 = 77$ implies $0 + 8y_2 = 77$ i.e., $y_2 = 77/8$

Therefore, first point is $(y_1,y_2)=(0, 77/8)$

Assuming $y_2 = 0$, $y_1+8y_2 = 77$ implies $y_1 = 77$ i.e., $y_1 = 77$

Therefore, second point is $(y_1,y_2)=(77, 0)$



(5, 0) lies in the second part of the region. If it will satisfy the constraint then we will consider the second part otherwise the first part.

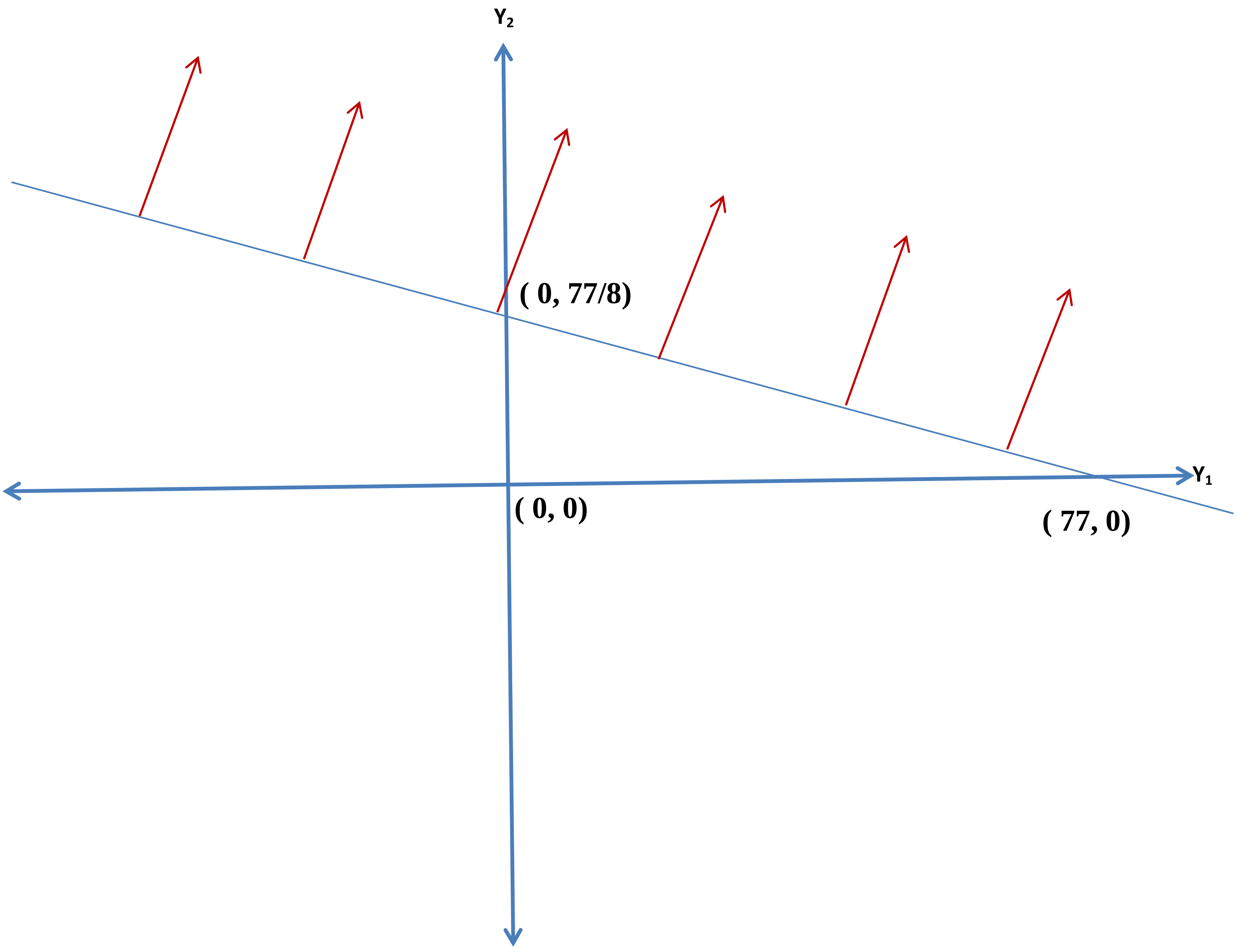
Putting (5,0) in the constraint

$y_1+8y_2 >= 77$, we have

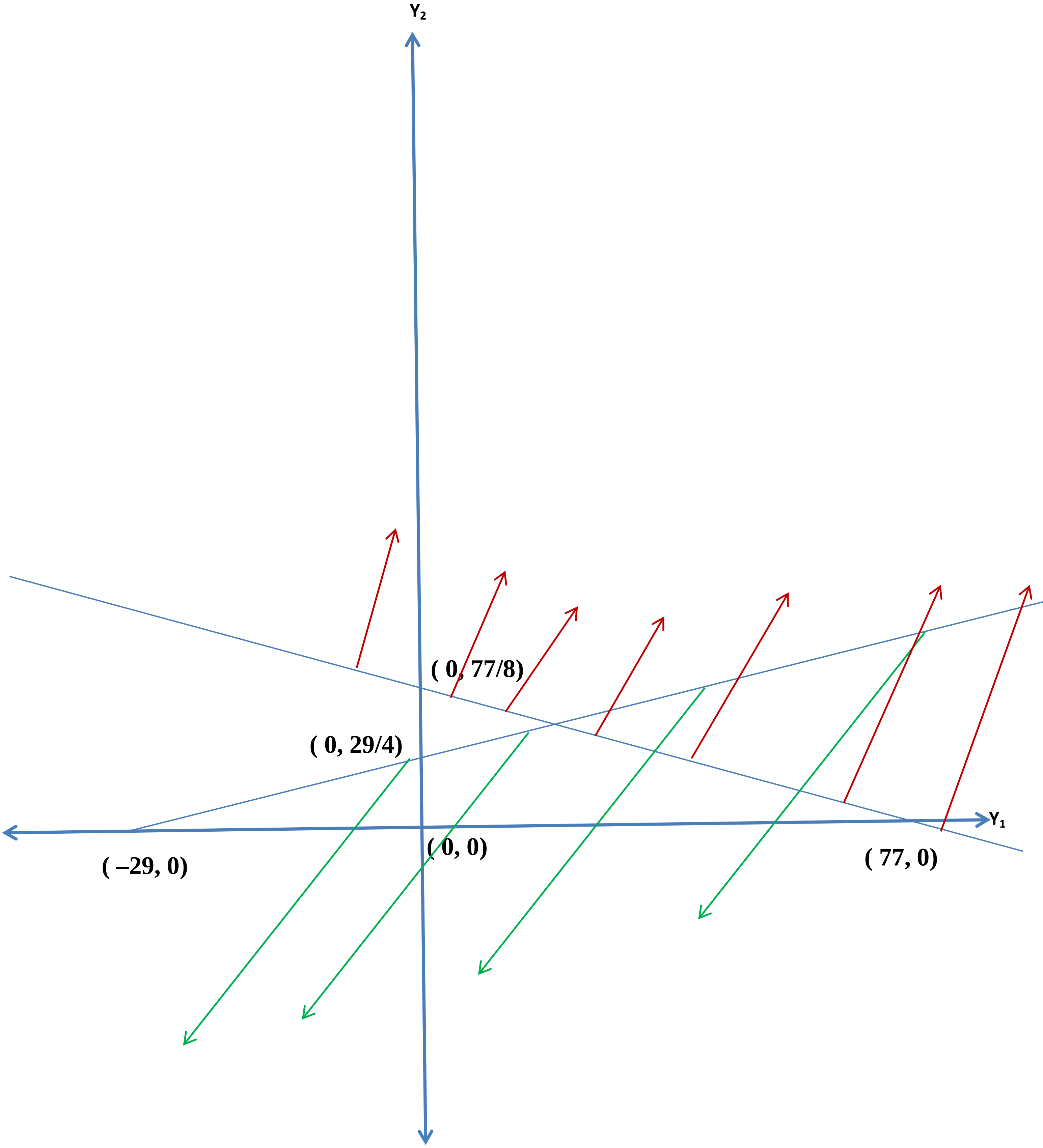
$5+0>=77$

$5>= 77$

It is obvious that the constraint is not satisfying. So, we consider the first part.

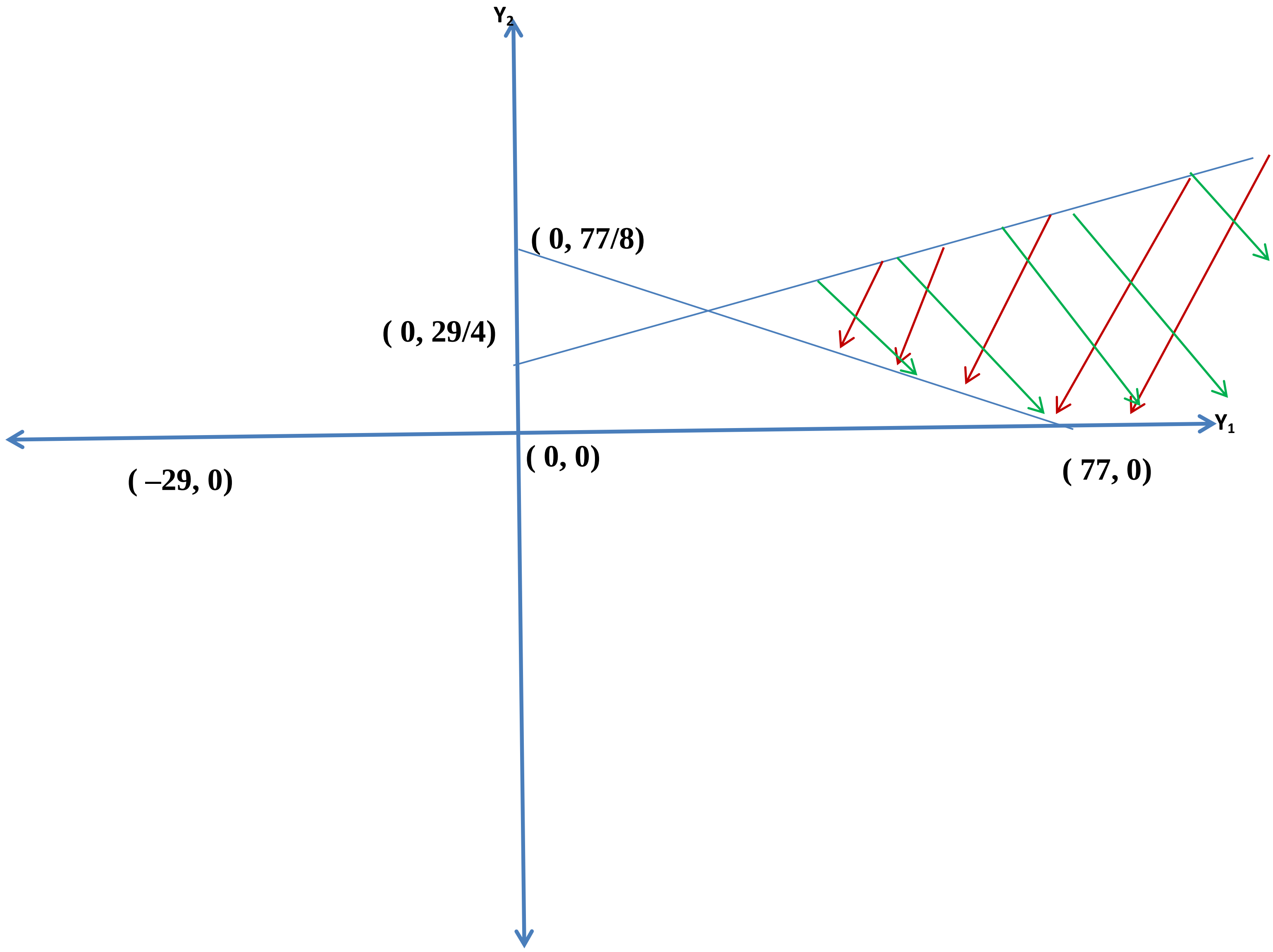


Common region of both the constraints



Common region of both the constraints in the first quadrant

(Feasible region)



It is obvious that Maximum value of y_1 in the feasible region is infinite. So, the feasible region is unbounded.