School of Mathematics, TIET
Optimization Techniques (UMA-035)
Practice Sheet # 3

1. Which of the following sets are convex

   (a) $\{(x_1, x_2) : x_1 x_2 \le 1\}$ (b) $\{(x_1, x_2) : x_1^2 + x_2^2 < 1\}$ (c) $\{(x_1, x_2) : x_1^2 + x_2^2 \ge 3\}$

   (d) $\{(x_1, x_2) : 4x_1 \ge x_2^2\}$ (e) $\{(x_1, x_2) : 0 < x_1^2 + x_2^2 \le 4\}$ (f) $\{(x_1, x_2) : x_2 - 3 \ge -x_1^2, x_1, x_2 \ge 0\}$

2. Find (i) All basic solutions (ii) All Basic feasible solutions of following equations

   (a)
   $$x_1 + x_2 + 4x_3 + 2x_4 + 3x_5 = 8$$
   $$4x_1 + 2x_2 + 2x_3 + x_4 + 6x_5 = 4$$
   $$x_1, x_2, x_3, x_4, x_5 \ge 0$$

   (b)
   $$3x_1 + 2x_2 + x_3 = 1$$
   $$5x_1 + x_2 + x_3 + x_4 = 2$$
   $$2x_1 + 5x_2 + x_3 + x_5 = 4$$
   $$x_1, x_2, x_3, x_4, x_5 \ge 0$$

3. Prove that the half space $\{\mathbf{X} \in \mathbf{R}^n : \mathbf{a}^T\mathbf{X} \ge \alpha\}$ is a convex set.

4. Let $\mathbf{S}_1$ *and* $\mathbf{S}_2$ be two disjoint nonempty set in R^n. Then show that the set $S = \{X_1 - X_2 : X_1 \in S_1, X_2 \in S_2\}$ is convex.

5. Show that a linear program with bounded feasible region is bounded and give a counter example to show that the converse need not be true.

6. Prove that the minimum of a LPP occurs on some extreme point (vertex).

7. Find all the extreme points of the set $S = \{(x_1, x_2); x_1 + 2x_2 \ge -2, -x_1 + x_2 \le 4, x_1 \le 4\}$ and represent the point (2, 3) as the convex linear combination of the extreme points of S.

8. Define the set of feasible solutions of an LPP. Prove that the set of feasible solutions of an LPP is a convex set.

9. Define the set of optimal solutions of an LPP, and then prove that the set of all optimal solution of an LPP is a convex set.

10. Does the union of two convex sets form a convex set? Justify your answer.

11. Can the set $S = \{(x_1, x_2); x_1 \ge 1 \text{ or } x_2 \ge 1\}$ be the feasible region of a LPP? Justify.

12. Prove algebraically that the set $S = \{(x_1, x_2) \mid 3(x_1)^2 + 4(x_2)^2 \le 12\}$ is a convex set.