Solutions
Tutorial sheet-7 (Basic Logic)

Ans 1: a) We have taken the conjunction of two propositions and asserted one of them. This is, according to Table 1, simplification.
b) We have taken the disjunction of two propositions and the negation of one of them, and asserted the other. This is, according to Table 1, disjunctive syllogism. See Table 1 for the other parts of this exercise as well.
c) modus ponens **d)** addition **e)** hypothetical syllogism

Ans 2: Let r be the proposition "It rains," let f be the proposition "It is foggy," let s be the proposition "The sailing race will be held," let l be the proposition "The life saving demonstration will go on," and let t be the proposition "The trophy will be awarded." We are given premises $(\neg r \vee \neg f) \to (s \wedge l)$, $s \to t$, and $\neg t$.
We want to conclude r. We set up the proof in two columns, with reasons, as in Example 6. Note that it is valid to replace subexpressions by other expressions logically equivalent to them.

Step	Reason
1. $\neg t$	Hypothesis
2. $s \to t$	Hypothesis
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \to (s \wedge l)$	Hypothesis
5. $(\neg(s \wedge l)) \to \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \to (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition, using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

Ans 3: a) If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.
b) We really can't conclude anything specific here.
c) By universal instantiation, we conclude from the first conditional statement by modus ponens that dragonflies have six legs, and we conclude by modus tollens that spiders are not insects. We could say using existential generalization that, for example, there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.
d) We can apply universal instantiation to the conditional statement and conclude that if Homer (respectively, Maggie) is a student, then he (she) has an Internet account. Now modus tollens tells us that Homer is not a student. There are no conclusions to be drawn about Maggie.
e) The first conditional statement is that if x is healthy to eat, then x does not taste good. Universal instantiation and modus ponens therefore tell us that tofu does not taste good. The third sentence says that if you eat x, then x tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.

Ans 4: a) Here x is still equal to 0, since the condition is false.
b) Here x is still equal to 1, since the condition is false.
c) This time x is equal to 1 at the end, since the condition is true, so the statement x := 1 is executed.

Ans 5: The answers given here are not unique, but care must be taken not to confuse nonequivalent sentences. Parts (c) and (f) are equivalent; and parts (d) and (e) are equivalent. But these two pairs are not equivalent to each other.
a) Some student in the school has visited North Dakota. (Alternatively, there exists a student in the school who has visited North Dakota.)
b) Every student in the school has visited North Dakota. (Alternatively, all students in the school have visited North Dakota.)
c) This is the negation of part (a): No student in the school has visited North Dakota. (Alternatively, there does not exist a student in the school who has visited North Dakota.)

d) Some student in the school has not visited North Dakota. (Alternatively, there exists a student in the school who has not visited North Dakota.)
e) This is the negation of part (b): It is not true that every student in the school has visited North Dakota. (Alternatively, not all students in the school have visited North Dakota.)
f) All students in the school have not visited North Dakota. (This is technically the correct answer, although common English usage takes this sentence to mean—incorrectly—the answer to part (e). To be perfectly clear, one could say that every student in this school has failed to visit North Dakota, or simply that no student has visited North Dakota.)

Ans 6:
a) If an animal is a rabbit, then that animal hops. (Alternatively, every rabbit hops.)
b) Every animal is a rabbit and hops.
c) There exists an animal such that if it is a rabbit, then it hops. (Note that this is trivially true, satisfied, for example, by lions, so it is not the sort of thing one would say.)
d) There exists an animal that is a rabbit and hops. (Alternatively, some rabbits hop. Alternatively, some hopping animals are rabbits.)

Ans 7:
a) We assume that this means that one student has all three animals: $\exists x(C(x) \land D(x) \land F(x))$.
b) $\forall x(C(x) \lor D(x) \lor F(x))$ c) $\exists x(C(x) \land F(x) \land \neg D(x))$
d) This is the negation of part (a): $\neg\exists x(C(x) \land D(x) \land F(x))$.
e) Here the owners of these pets can be different: $(\exists x\, C(x)) \land (\exists x\, D(x)) \land (\exists x\, F(x))$. There is no harm in using the same dummy variable, but this could also be written, for example, as $(\exists x\, C(x)) \land (\exists y\, D(y)) \land (\exists z\, F(z))$.

Ans 8:
a) Since $0 + 1 > 2 \cdot 0$, we know that $Q(0)$ is true.
b) Since $(-1) + 1 > 2 \cdot (-1)$, we know that $Q(-1)$ is true.
c) Since $1 + 1 \not> 2 \cdot 1$, we know that $Q(1)$ is false.
d) From part (a) we know that there is at least one x that makes $Q(x)$ true, so $\exists x\, Q(x)$ is true.
e) From part (c) we know that there is at least one x that makes $Q(x)$ false, so $\forall x\, Q(x)$ is false.
f) From part (c) we know that there is at least one x that makes $Q(x)$ false, so $\exists x\, \neg Q(x)$ is true.
g) From part (a) we know that there is at least one x that makes $Q(x)$ true, so $\forall x\, \neg Q(x)$ is false.

Ans 9: a) Some system is open.
b) Every system is either malfunctioning or in a diagnostic state.
c) Some system is open, or some system is in a diagnostic state.
d) Some system is unavailable.
e) No system is working. (We could also say "Every system is not working," as long as we understood that this is different from "Not every system is working.")

Ans 10:
There are many ways to write these, depending on what we use for predicates.
a) Let $F(x)$ be "There is less than x megabytes free on the hard disk," with the domain of discourse being positive numbers, and let $W(x)$ be "User x is sent a warning message." Then we have $F(30) \to \forall x\, W(x)$.
b) Let $O(x)$ be "Directory x can be opened," let $C(x)$ be "File x can be closed," and let E be the proposition "System errors have been detected." Then we have $E \to ((\forall x\, \neg O(x)) \land (\forall x\, \neg C(x)))$.
c) Let B be the proposition "The file system can be backed up," and let $L(x)$ be "User x is currently logged on." Then we have $(\exists x\, L(x)) \to \neg B$.
d) Let $D(x)$ be "Product x can be delivered," and let $M(x)$ be "There are at least x megabytes of memory available" and $S(x)$ be "The connection speed is at least x kilobits per second," where the domain of discourse for the last two propositional functions are positive numbers. Then we have $(M(8) \land S(56)) \to D(\text{video on demand})$.