Periodic and Non–periodic Signals

A continuous time signal is said to be periodic with period T_p for which the signal is advanced in time and remains unchanged. The signal x(t) must satisfy the following condition to be periodic in nature.

$$x(t) = x(t + T_P) \text{ for } -\infty \text{ to } \infty \qquad (1)$$

The examples of periodic signals are sine and cosine signals having period $2\pi/\omega_0$

If a continuous time signal x(t) does not satisfy equation (1), the signal is termed as non–periodic or aperiodic signal.

A **signal** is a **periodic signal if** it completes a pattern within a measurable time frame, called a period and repeats that pattern over identical subsequent periods. The completion of a full pattern is called a cycle. A period is defined as the amount of time (expressed in seconds) required to complete one full cycle.

Example : If $y_1(t)$ and $y_2(t)$ be the two periodic signals with two periods T_1 and T_2 respectively, find the suitable condition so that $y(t) = y_1(t) + y_2(t)$ will be periodic. Determine its fundamental period if so.

Solution:

Here the given signal $y_1(t)$ and $y_2(t)$ are periodic with periods T_1 and T_2 respectively. Therefore, we can write

$y_1(t) = y_1(t + T_1) = y_1(t + pT_1)$ where p is a positive integer

and $y_2(t) = y_2(t + T_2) = y_2(t + qT_2)$ where q is a positive integer

Since, $y(t) = y_1(t) + y_2(t)$ we can write $y(t)$ as

$$y(t) = y_1(t + pT_1) + y_2(t + qT_2) \qquad \text{(E1)}$$

Now $y(t)$ will be periodic if and only if

$$y(t + T) = y_1(t + pT_1) + y_2(t + qT_2) \qquad \text{(E2)}$$

Equation (E1) and (E2) is valid if an only if

$$pT_1 = qT_2 = T$$

i.e., $$\frac{T_1}{T_2} = \frac{q}{p}$$

= rational number

Therefore, we can conclude that the sum of two periodic signals will be periodic if the ratio of their respective period is a rational number only. The fundamental period is the least common multiple of T_1 and T_2.

Examples of **periodic signals** include the sinusoidal **signals** and periodically repeated non-sinusoidal **signals**, such as the rectangular pulse sequences used in radar. Non-**periodic signals** include speech waveforms and random **signals** arising from unpredictable disturbances of all kinds.

Amplitude, **frequency**, and **phase** are three important characteristics of a periodic signal.

Show that the complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

is periodic and that its fundamental period is $2\pi/\omega_0$.

$x(t)$ will be periodic if

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t}$$

Since

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

we must have

$$e^{j\omega_0 T} = 1$$

If $\omega_0 = 0$, then $x(t) = 1$, which is periodic for any value of T. If $\omega_0 \neq 0$, Eq. holds if

$$\omega_0 T = m 2\pi \qquad \text{or} \qquad T = m\frac{2\pi}{\omega_0} \qquad m = \text{positive integer}$$

Thus, the fundamental period T_0, the smallest positive T, of $x(t)$ is given by $2\pi/\omega_0$.

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

(a) $x(t) = \cos\left(t + \dfrac{\pi}{4}\right)$

(b) $x(t) = \sin \dfrac{2\pi}{3}t$

(c) $x(t) = \cos \dfrac{\pi}{3}t + \sin \dfrac{\pi}{4}t$

(d) $x(t) = \cos t + \sin \sqrt{2}\, t$

(e) $x(t) = \sin^2 t$

(f) $x(t) = e^{j[(\pi/2)t - 1]}$

$$(a)\quad x(t)=\cos\left(t+\frac{\pi}{4}\right)=\cos\left(\omega_0 t+\frac{\pi}{4}\right)\longrightarrow \omega_0=1$$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 2\pi$.

(b) $x(t) = \sin \frac{2\pi}{3} t \longrightarrow \omega_0 = \frac{2\pi}{3}$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 3$.

(c) $x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t = x_1(t) + x_2(t)$

where $x_1(t) = \cos(\pi/3)t = \cos \omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 6$ and $x_2(t) = \sin(\pi/4)t = \sin \omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = 8$. Since $T_1/T_2 = \frac{6}{8} = \frac{3}{4}$ is a rational number, $x(t)$ is periodic with fundamental period $T_0 = 4T_1 = 3T_2 = 24$.

(d) $x(t) = \cos t + \sin\sqrt{2}\,t = x_1(t) + x_2(t)$

where $x_1(t) = \cos t = \cos\omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 2\pi$ and $x_2(t) = \sin\sqrt{2}\,t = \sin\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \sqrt{2}\,\pi$. Since $T_1/T_2 = \sqrt{2}$ is an irrational number, $x(t)$ is nonperiodic.

(*e*) Using the trigonometric identity $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$, we can write

$$x(t) = \sin^2 t = \tfrac{1}{2} - \tfrac{1}{2}\cos 2t = x_1(t) + x_2(t)$$

where $x_1(t) = \frac{1}{2}$ is a dc signal with an arbitrary period and $x_2(t) = -\frac{1}{2}\cos 2t = -\frac{1}{2}\cos\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \pi$. Thus, $x(t)$ is periodic with fundamental period $T_0 = \pi$.

(f) $x(t) = e^{j[(\pi/2)t-1]} = e^{-j}e^{j(\pi/2)t} = e^{-j}e^{j\omega_0 t} \longrightarrow \omega_0 = \dfrac{\pi}{2}$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 4$.