

**Course: UMA 035 (Optimization Techniques)**

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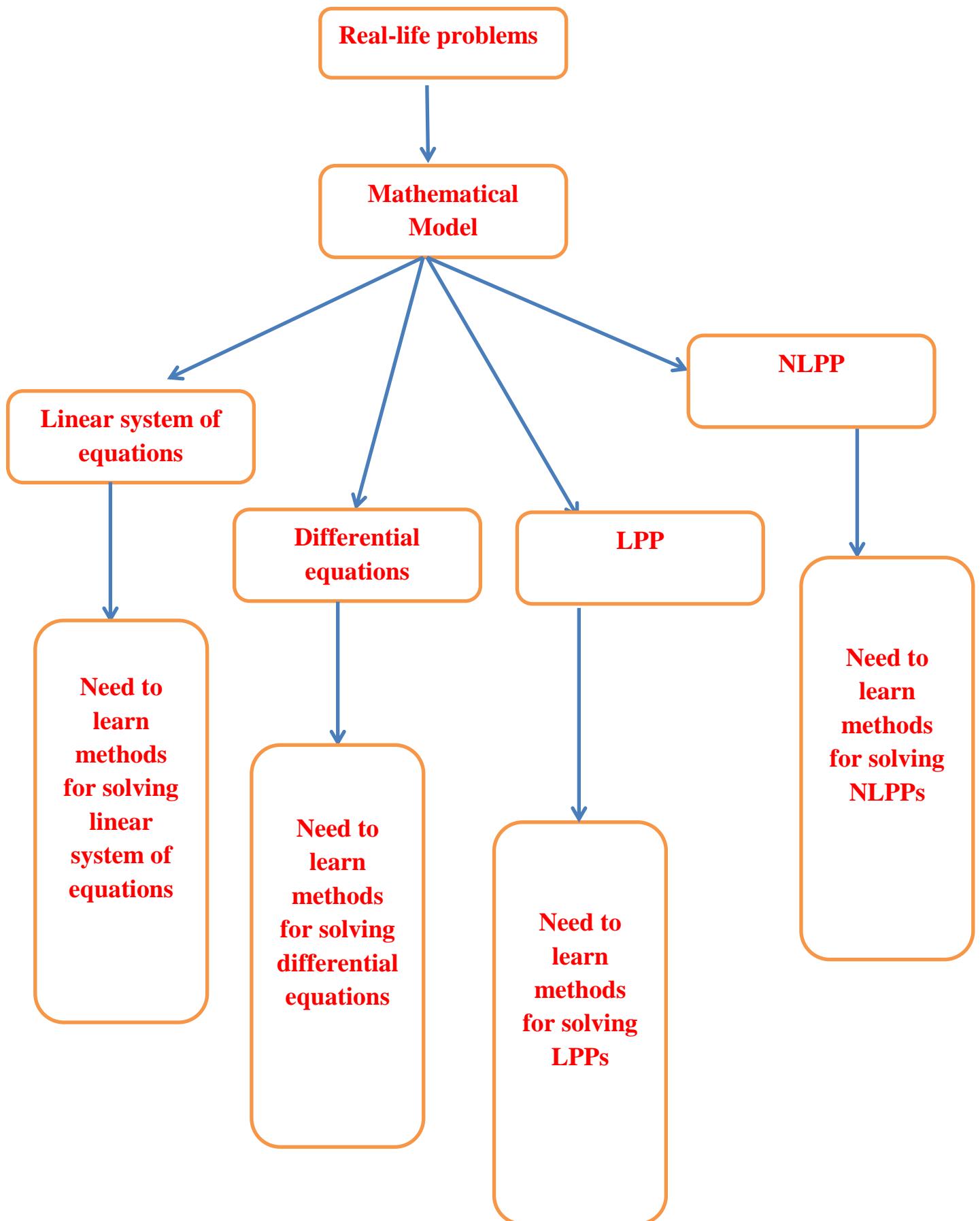
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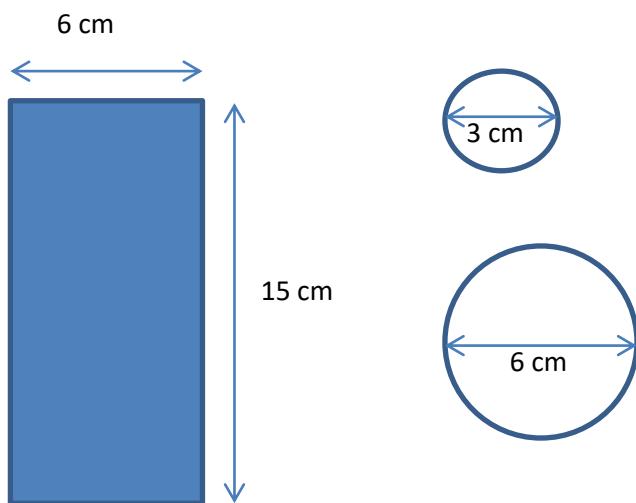
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## Need of LPP and NLPP in real-life problems



## Some real-life problems

Material Science department of TIET needs circular metallic plates of **diameter 3 cm and 6 cm** to perform experiments on heat treatment studies and requires **minimum 2500 and 1500 units** respectively. These are to be cut from parent metallic sheets of dimension **6 x 15 cm<sup>2</sup>**. Construct a mathematical model so that minimum number of parent metallic sheets are used.

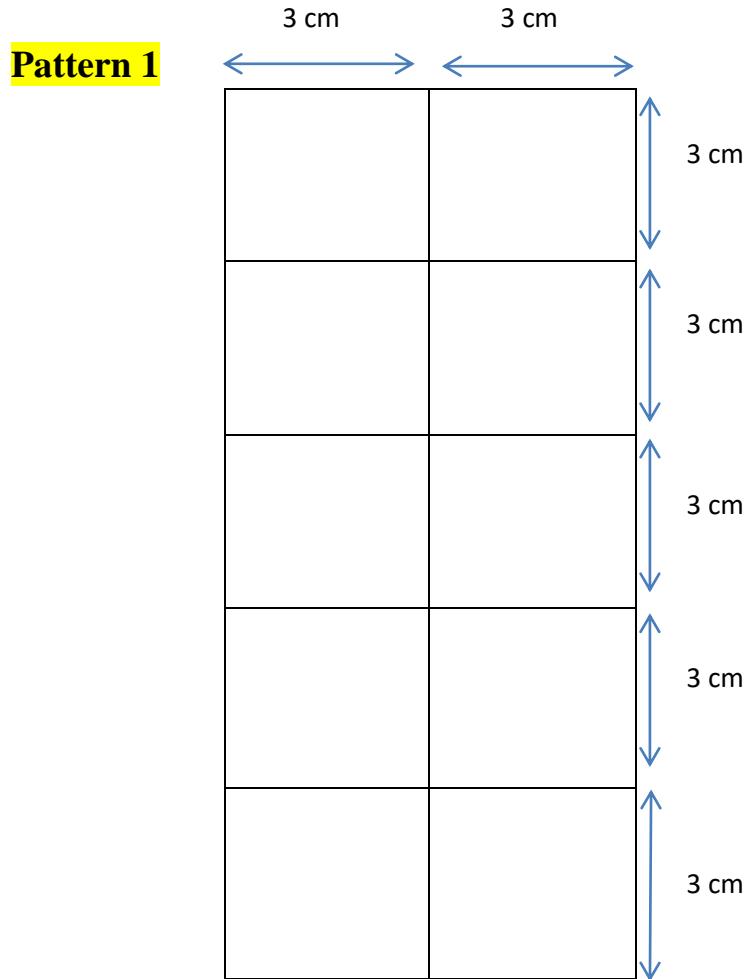


**Minimize (Total number of used rectangular metallic sheets)**

**Subject to**

**Circular plates of diameter 3 cm  $\geq 2500$**

**Circular plates of diameter 6 cm  $\geq 1500$**

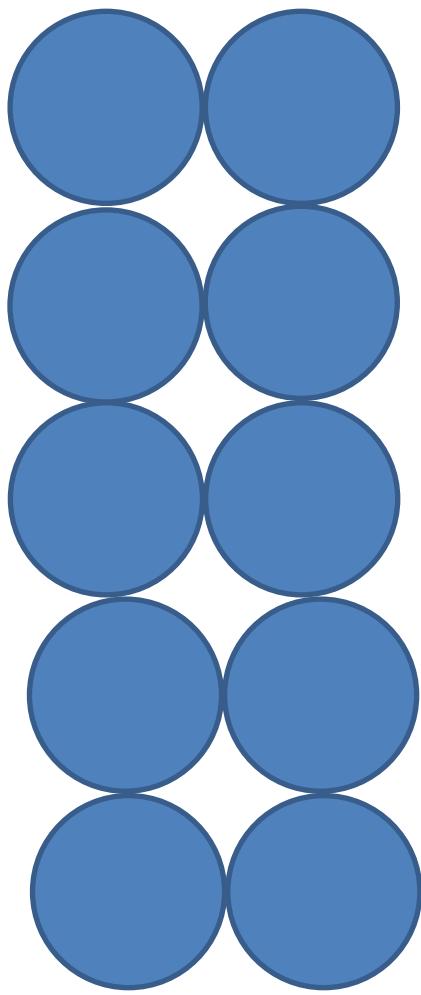


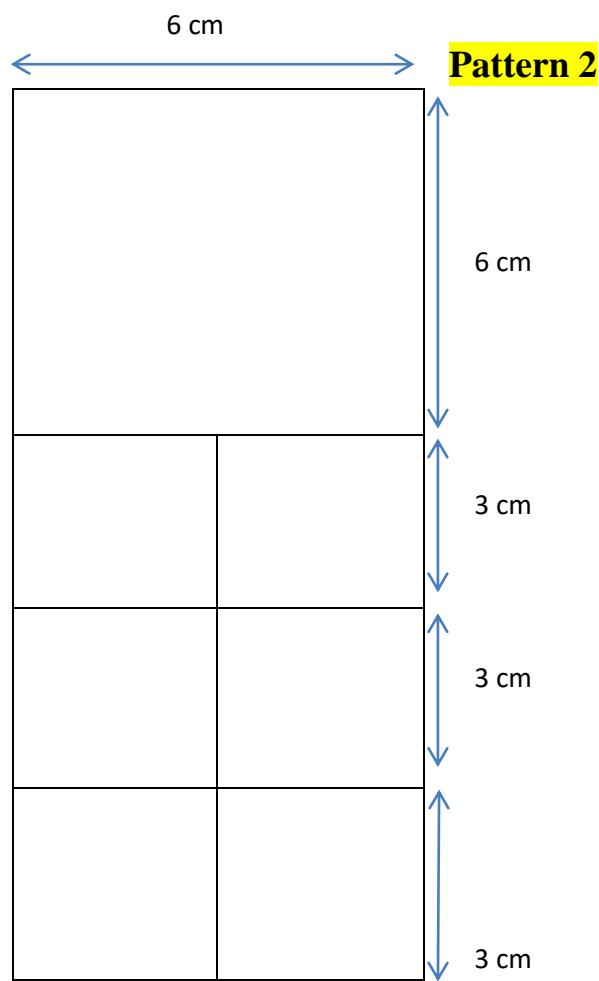
One sheet  $\longrightarrow$  6 cm  $\longrightarrow$  0 circular plates

x1 sheet  $\longrightarrow$  6 cm  $\longrightarrow$  0x1 circular plates

One sheet  $\longrightarrow$  3 cm  $\longrightarrow$  10 circular plates

x1 sheet  $\longrightarrow$  3 cm  $\longrightarrow$  10x1 circular plates



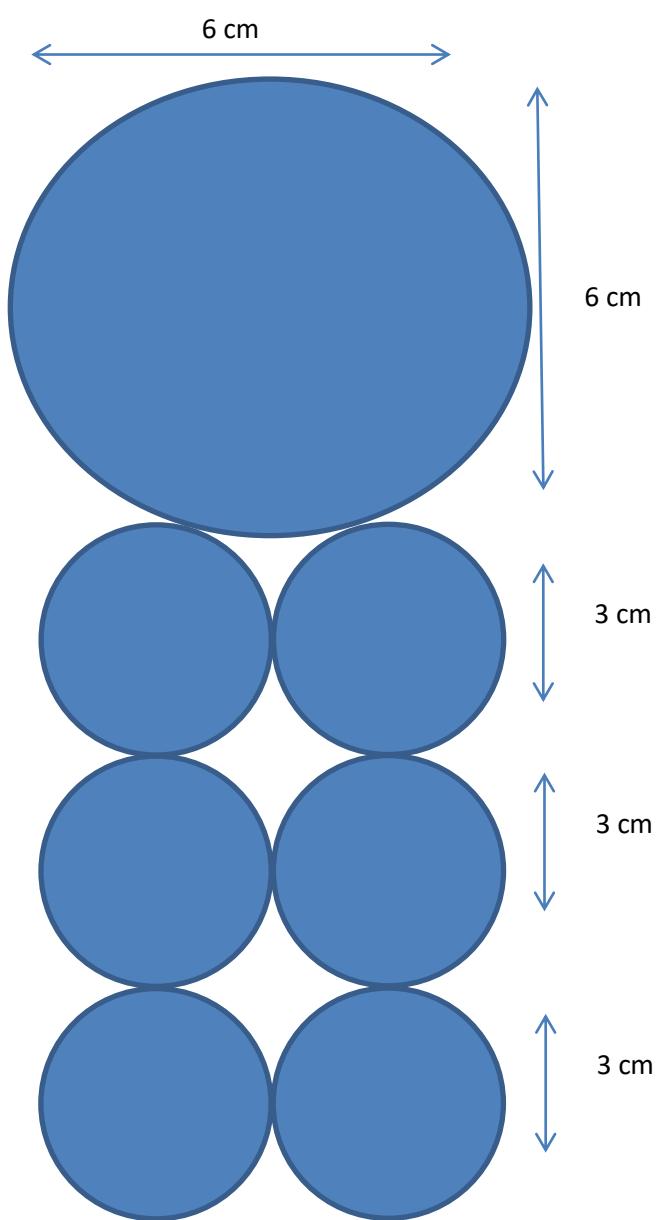


**One sheet** → 6 cm → 1 circular plates

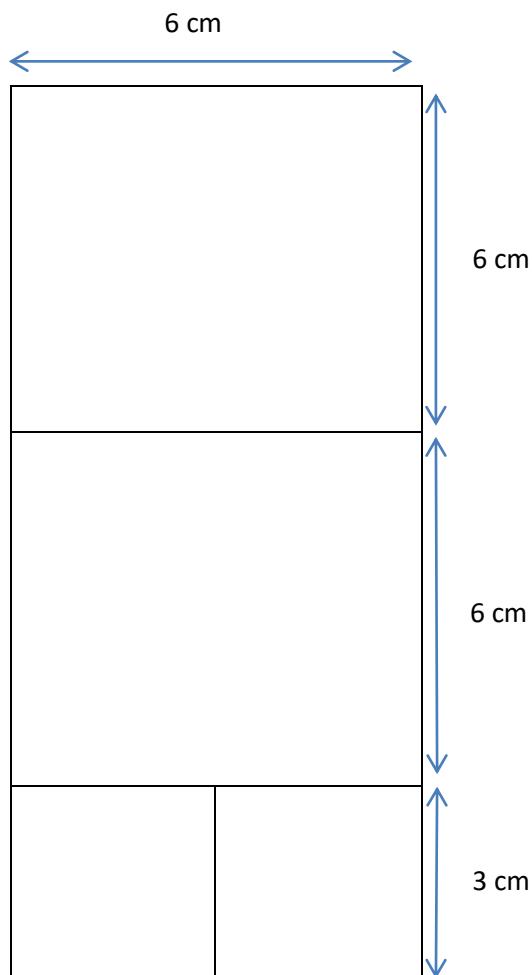
**x2 sheet** → 6 cm → 1x2 circular plates

**One sheet** → 3 cm → 6 circular plates

**x2 sheet** → 3 cm → 6x2 circular plates



### **Pattern 3**



**One sheet** → 6 cm → 2 circular plates

**x3 sheet** → 6 cm → 2x2 circular plates

**One sheet** → 3 cm → 2 circular plates

**x3 sheet** → 3 cm → 2x3 circular plates

|                  | <b>Number of circular plates having diameter 3 cm</b> | <b>Number of circular plates having diameter 6 cm</b> | <b>Number of rectangular sheets</b> |
|------------------|---|---|-------------------------------------|
| <b>Pattern 1</b> | <b>10x1</b>   | <b>0x1</b>  | <b>x1</b>                           |
| <b>Pattern 2</b> | <b>6x2</b>  | <b>1x2</b>  | <b>x2</b>                           |
| <b>Pattern 3</b> | <b>2x3</b>  | <b>2x3</b>  | <b>x3</b>                           |
| <b>Total</b>     | <b>10x1+6x2+2x3</b>                                   | <b>0x1+1x2+2x3</b>                                    | <b>x1+ x2+ x3</b>                   |

**x1, x2 and x3 physically represents the number of rectangular sheets.**

**If sheets will not be used then number of sheets will be 0.**

**If sheets will be used then number of sheets will be positive.**

**Number of sheets cannot be negative.**

**Therefore,  $x1 \geq 0, x2 \geq 0, x3 \geq 0$ .**

**Minimize (Total number of used rectangular metallic sheets)**

**Subject to**

**Circular plates of diameter 3 cm  $\geq 2500$**

**Circular plates of diameter 6 cm  $\geq 1500$**

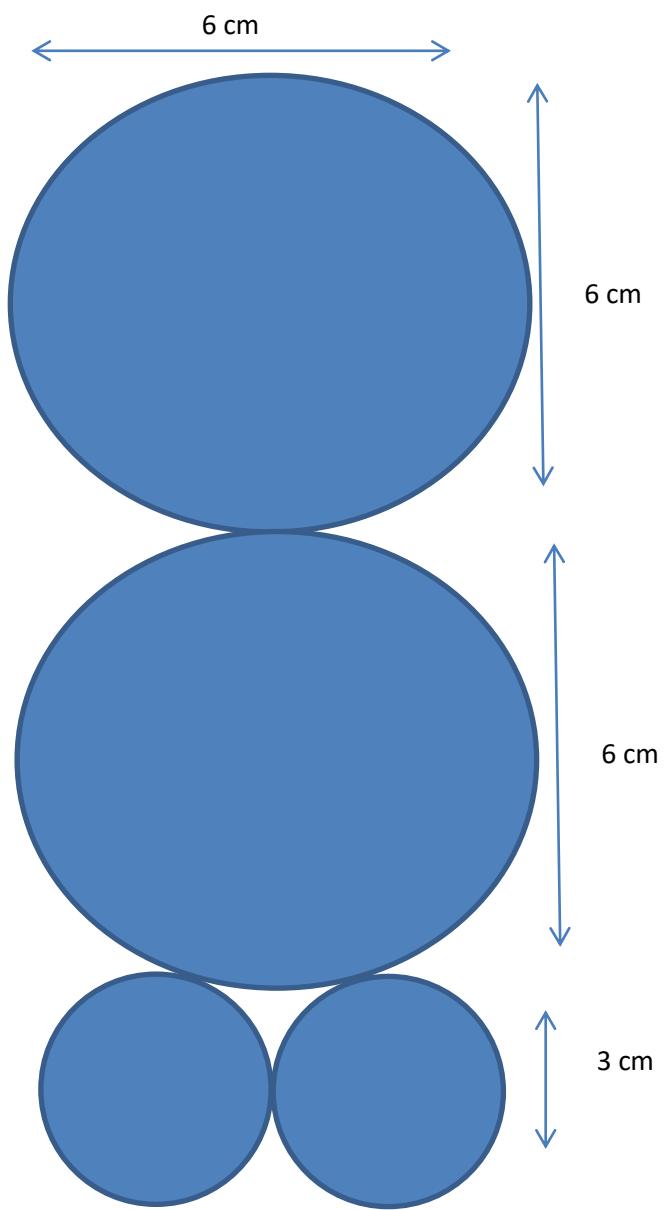
**Minimize ( $x1 + x2 + x3$ )**

**Subject to**

**$10x1+6x2+2x3 \geq 2500$**

**$0x1+1x2+2x3 \geq 1500$**

**$x1 \geq 0, x2 \geq 0, x3 \geq 0$**



A metal slitting company cuts master rolls with width 200 centimeters into subrolls of small width. Customer specifies that they need subrolls of different widths given in the following table:

| Width of subroll (in cm) | Numbers required |
|--------------------------|------------------|
| 35                       | 200              |
| 80                       | 90               |
| 90                       | 350              |
| 120                      | 850              |

The objective is to use a minimum number of master rolls to satisfy set of customers'orders. Construct the mathematical model.

**Minimize (Number of master rolls)**

**Subject to**

**Number of subrolls of width 35 = 200**

**Number of subrolls of width 80 = 90**

**Number of subrolls of width 90= 350**

**Number of subrolls of width 120 = 850**

**Pattern 1**

**All of width 35**

**$200/35 = 5.71\dots$**

**This indicates maximum 5 subrolls of width 35 can be obtained**

**Used material is  $35*5=175$**

**Wastage =  $200-175=25$**

**Pattern 2**

**If we reduce the subrolls of width 35 from 5 to 4 (Fixed) then**

**Used material is  $35*4=140$**

**Remaining =  $200-140=60$**

**Only possibility to cut from 60 is 35.**

**60 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this Pattern.**

### **Pattern 3**

**If we reduce the subrolls of width 35 from 5 to 3 then**

**Used material is  $35 \times 3 = 105$**

**Remaining =  $200 - 105 = 95$**

**95 is not a wastage as it is greater than 35.**

**The possibilities are one subroll of 80 or one subroll of 90**

#### **Pattern 3a**

**One subroll of 80**

**If we cut one subroll of 80 from 95 then remaining is  $95 - 80 = 15$ .**

**15 is wastage as it is less than 35**

#### **Pattern 3b**

**One subroll of 90**

**If we cut one subroll of 90 from 95 then wastage is  $95 - 90 = 5$ .**

**5 is wastage as it is less than 35**

### **Pattern 4**

**If we reduce the subrolls of width 35 from 5 to 2 then**

**Used material is  $35 \times 2 = 70$**

**Remaining =  $200 - 70 = 130$**

**130 is not a wastage as it is greater than 35.**

**The possibilities are one subroll of 80 or one subroll of 90 or one subroll of 120**

#### **Pattern 4a**

**One subroll of 80**

**If we cut one subroll of 80 from 130 then remaining is  $130 - 80 = 50$ .**

**50 is not wastage as we can cut subroll of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this case.**

#### **Pattern 4b**

**One subroll of 90**

**If we cut one subroll of 90 from 130 then remaining is  $130-90=40$ .**

**40 is not wastage as we can cut subroll of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this pattern.**

#### **Pattern 4c**

**One subroll of 120**

**If we cut one subroll of 120 from 130 then remaining is  $130-120=10$ .**

**10 is a wastage as it is less than 35.**

#### **Pattern 5**

**If we reduce the subrolls of width 35 from 5 to 1 then**

**Used material is  $35*1=35$**

**Remaining =  $200-35=165$**

**The possibilities are two subrolls of 80 or one subroll of 90 or one subroll of 120**

#### **Pattern 5a**

**Two subrolls of 80**

**If we cut two subrolls of 80 from 165 then remaining is  $165-80*2=5$ .**

**5 is a wastage as it is less than 35.**

#### **Pattern 5b**

**One subroll of 90**

**If we cut one subroll of 90 from 165 then remaining is  $165-90=75$ .**

**75 is not wastage as we can cut two subrolls of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this pattern.**

### **Pattern 5c**

**One subroll of 120**

**If we cut one subroll of 120 from 165 then remaining is  $165-120=45$ .**

**45 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this case.**

### **Pattern 6**

**If we reduce the subrolls of width 35 from 5 to 0 then**

**Used material is  $35*0=0$**

**Remaining =  $200-0=200$**

**The possibilities are two subrolls of 80 or two subrolls of 90 or one subroll of 120 and one subroll of 80 or one subroll of 80 and one of 90**

### **Pattern 6a**

**Two subrolls of 80**

**If we cut two subrolls of 80 from 200 then remaining is  $200-80*2=40$ .**

**40 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.**

**So we will not consider this case.**

### **Pattern 6b**

**Two subrolls of 90**

**If we cut two subrolls of 90 from 200 then remaining is  $200-90*2=20$ .**

**20 is a wastage as it is less than 35.**

### **Pattern 6c**

**One subroll of 120 and one subroll of 80**

**If we cut one subroll of 120 and one subroll of 80 from 200 then remaining is  $200-120-80=0$ .**

**0 is wastage as it is less than 35.**

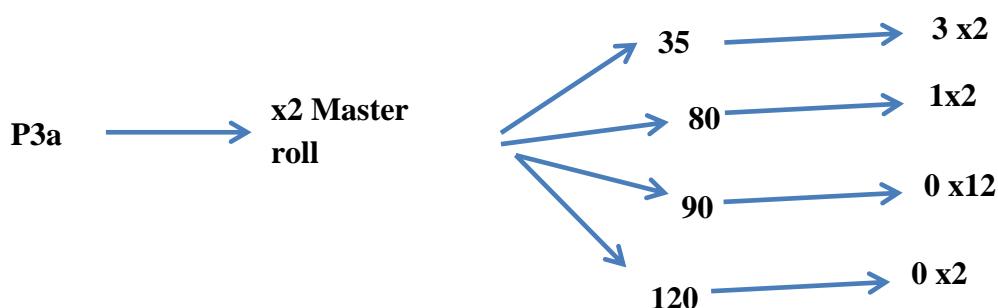
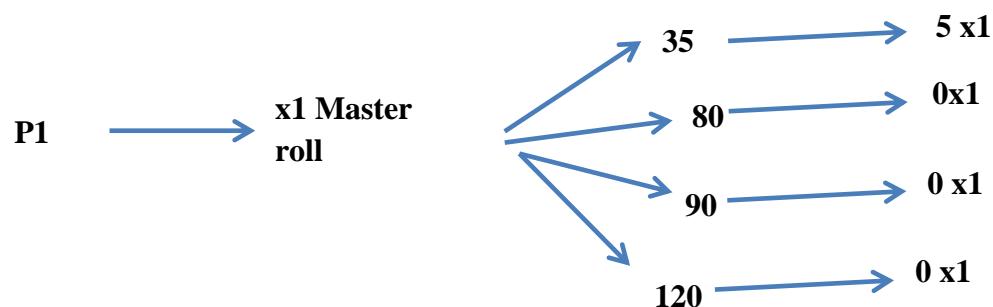
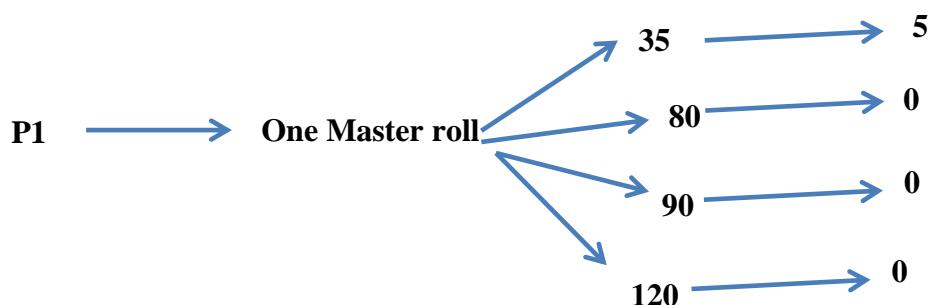
## Pattern 6d

One subroll of 80 and one subroll of 90

If we cut one subroll of 80 and one subroll of 90 from 200 then remaining is 200-90-80=30.

30 is wastage as it is less than 35.

|     | P1 | P3a | P3b | P4a | P5a | P6b | P6c | P6d |
|-----|----|-----|-----|-----|-----|-----|-----|-----|
| 35  | 5  | 3   | 3   | 2   | 1   | 0   | 0   | 0   |
| 80  | 0  | 1   | 0   | 0   | 2   | 0   | 1   | 1   |
| 90  | 0  | 0   | 1   | 0   | 0   | 2   | 0   | 1   |
| 120 | 0  | 0   | 0   | 1   | 0   | 0   | 1   | 0   |



|              | P1    | P3a   | P3b  | P4a  | P5a  | P6b  | P6c  | P6d  | Total                               |
|--------------|-------|-------|------|------|------|------|------|------|-------------------------------------|
| Master Rolls | x1    | x2    | x3   | x4   | x5   | x6   | x7   | x8   | x1+ x2+...+ x8                      |
| 35           | 5x1   | 3 x2  | 3 x3 | 2 x4 | 1 x5 | 0 x6 | 0 x7 | 0 x8 | 5x1+3 x2+3 x3+2 x4+ x5              |
| 80           | 0 x1  | 1 x2  | 0 x3 | 0 x4 | 2 x5 | 0 x6 | 1 x7 | 1 x8 | 2 x5+ x7+x8                         |
| 90           | 0 x1  | 0 x2  | 1 x3 | 0 x4 | 0 x5 | 2 x6 | 0 x7 | 1 x8 | 1 x3+2 x6+ x8                       |
| 120          | 0 x1  | 0 x2  | 0 x3 | 1 x4 | 0 x5 | 0 x6 | 1 x7 | 0 x8 | 1 x4+1 x7                           |
| Wastage      | 25 x1 | 15 x2 | 5x3  | 10x4 | 5x5  | 20x6 | 0x7  | 30x8 | 25 x1+15 x2+5x3+10x4+5x5+ 20x6+30x8 |

Minimize (Number of master rolls)

Subject to

Number of subrolls of width 35 = 200

Number of subrolls of width 80 = 90

Number of subrolls of width 90= 350

Number of subrolls of width 120 = 850

Minimize (x1+ x2+...+ x8)

Subject to

5x1+3 x2+3 x3+2 x4+ x5= 200

2 x5+ x7+x8= 90

1 x3+2 x6+ x8= 350

1 x4+1 x7= 850

x1, x2,..., x8 >=0

Minimize (Wastage)

Subject to

Number of subrolls of width 35 = 200

Number of subrolls of width 80 = 90

**Number of subrolls of width 90= 350**

**Number of subrolls of width 120 = 850**

**Minimize (25 x<sub>1</sub>+15 x<sub>2</sub>+5x<sub>3</sub>+10x<sub>4</sub>+5x<sub>5</sub>+20x<sub>6</sub>+30x<sub>8</sub>)**

**Subject to**

**5x<sub>1</sub>+3 x<sub>2</sub>+3 x<sub>3</sub>+2 x<sub>4</sub>+ x<sub>5</sub>= 200**

**2 x<sub>5</sub>+ x<sub>7</sub>+x<sub>8</sub>= 90**

**1 x<sub>3</sub>+2 x<sub>6</sub>+ x<sub>8</sub>= 350**

**1 x<sub>4</sub>+1 x<sub>7</sub>= 850**

**x<sub>1</sub>, x<sub>2</sub>,..., x<sub>8</sub> >=0**