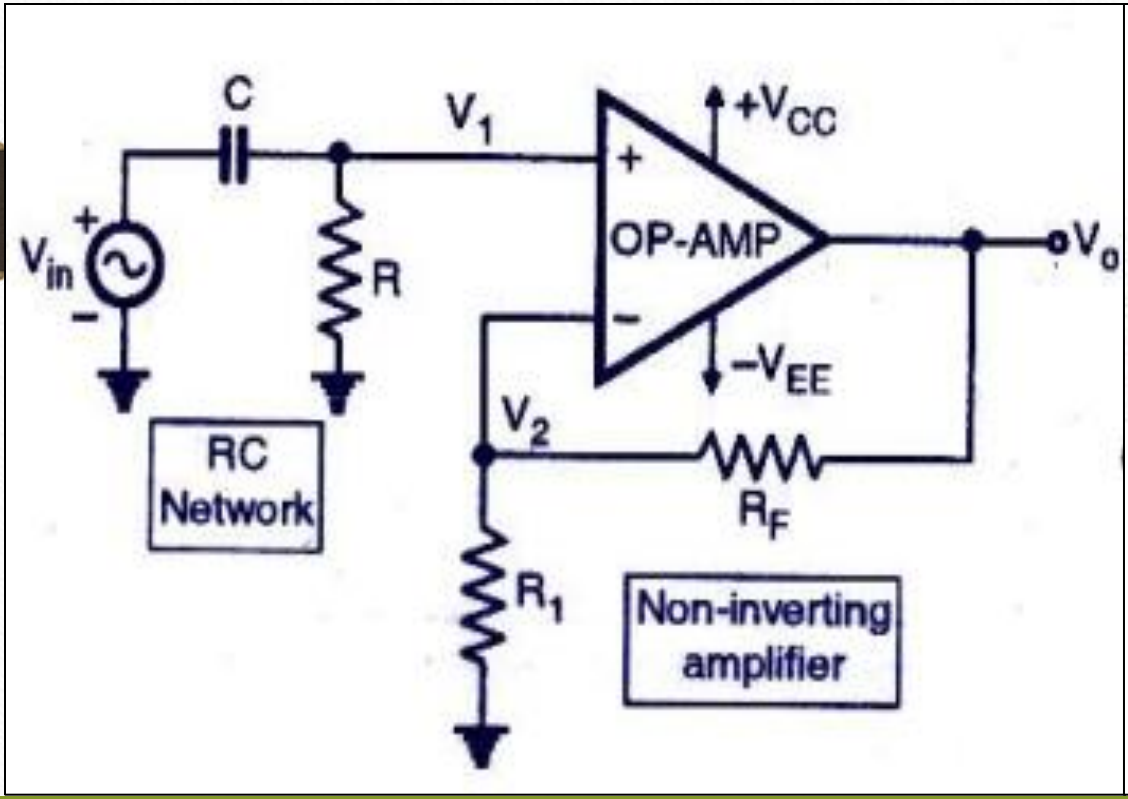
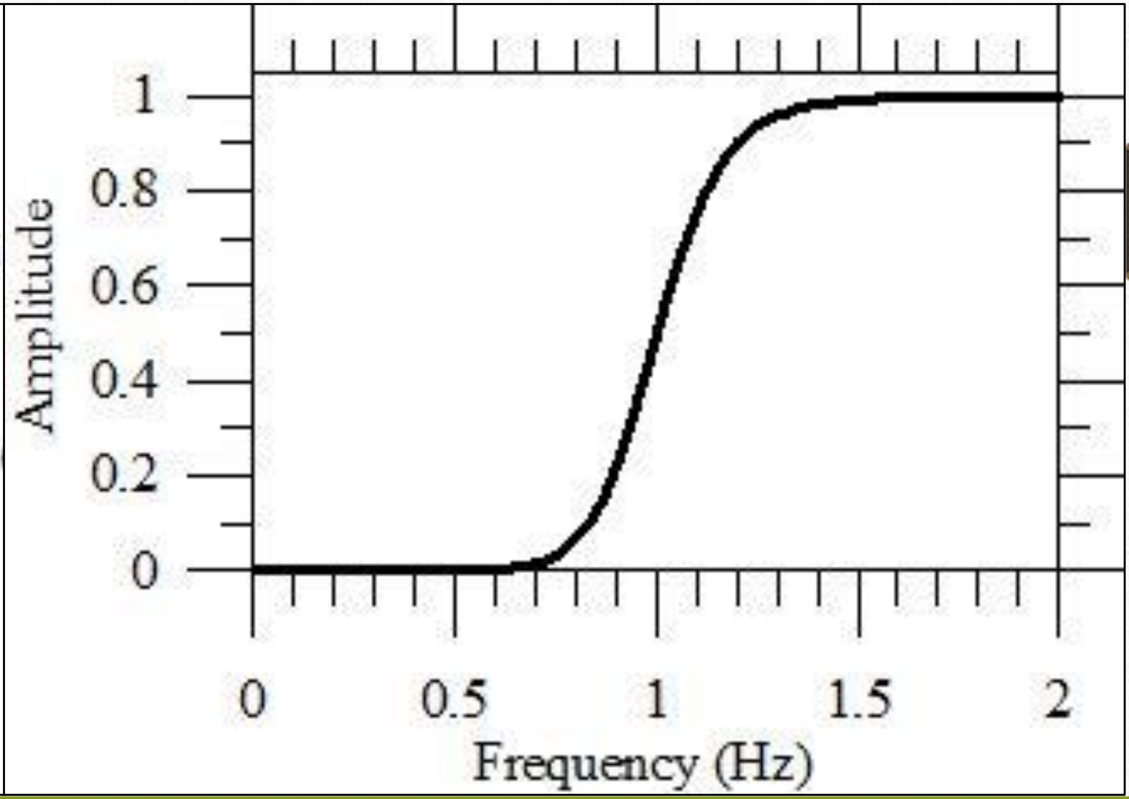
Active Filters

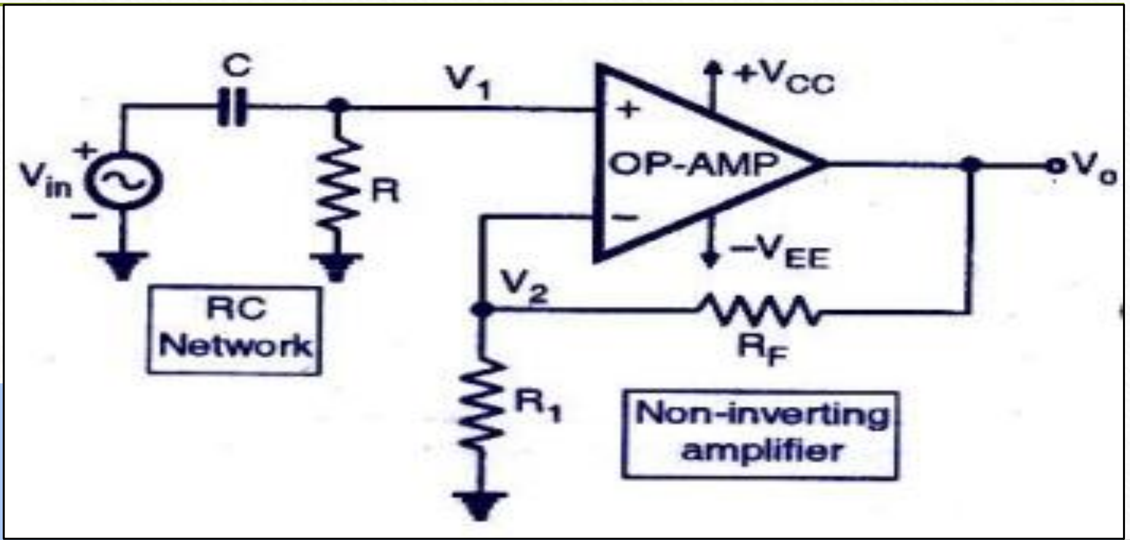
High Pass Butterworth filter & Chebyshev Filter

First order High Pass Butterworth filter





Expression of the Gain



Voltage $V_1 = \frac{R}{R - jX_C} V_{in}$

$$Where X_C = \frac{1}{2\pi fC} \qquad f_L = \frac{1}{2\pi RC}$$

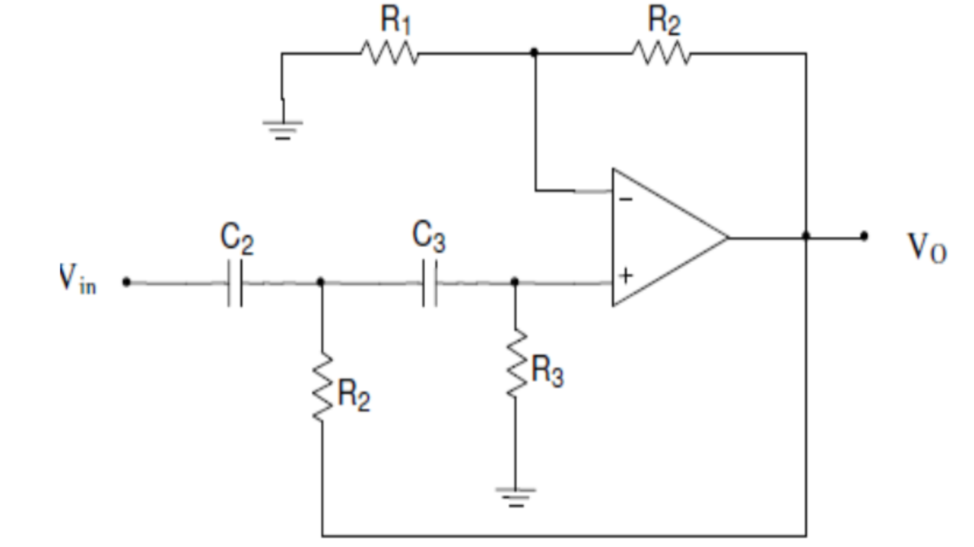
$$V_1 = \frac{R}{R - \frac{j}{2\pi fC}} V_{in} = \frac{R}{R + \frac{1}{j2\pi fC}} = \frac{(R \times j2\pi fC)}{1 + j2\pi fRC} V_{in} \qquad \frac{j\left[\frac{f}{f_L}\right]}{1 + j\left[\frac{f}{f_l}\right]} V_{in}$$

Output voltage = $V_0 = A_{VF}.V_1 = \frac{A_{VF}\left[\frac{jf}{f_L}\right]}{1 + j\frac{f}{f_L}} V_{in}$

Gain = $\frac{V_0}{V_{in}} = \frac{A_{VF}\left[\frac{jf}{f_L}\right]}{1 + j\left(\frac{f}{f_L}\right)}$

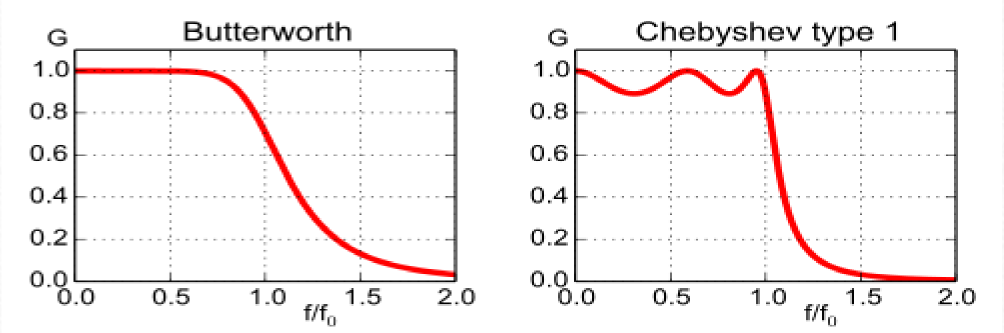
2nd order High Pass Butterworth filter

- This filter can be derived from the second order Butterworth LPF by simply interchanging the positions of R and C.
- The second order filter produces a gain roll off of +40 dB / decade in the stop band.



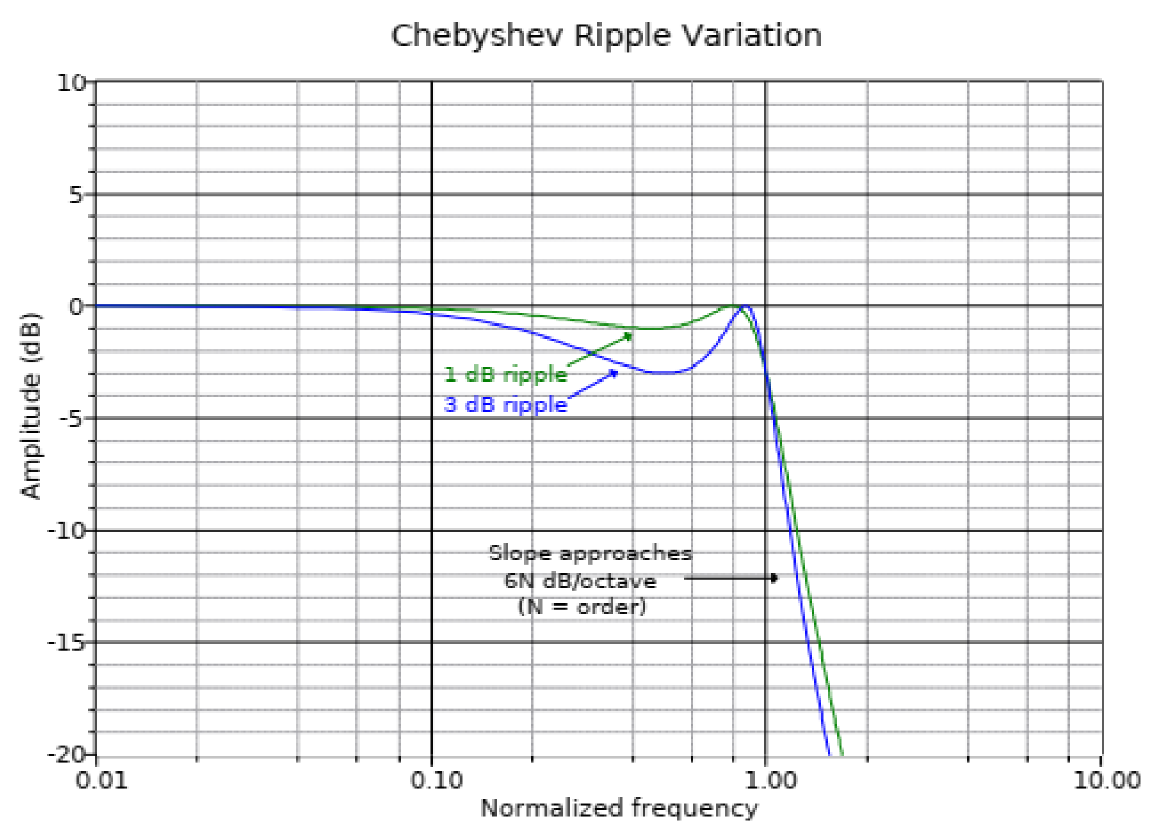
$$\left|\frac{V_O}{V_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f_L}{f}\right)^4}} \qquad f_L = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

Chebyshev filter

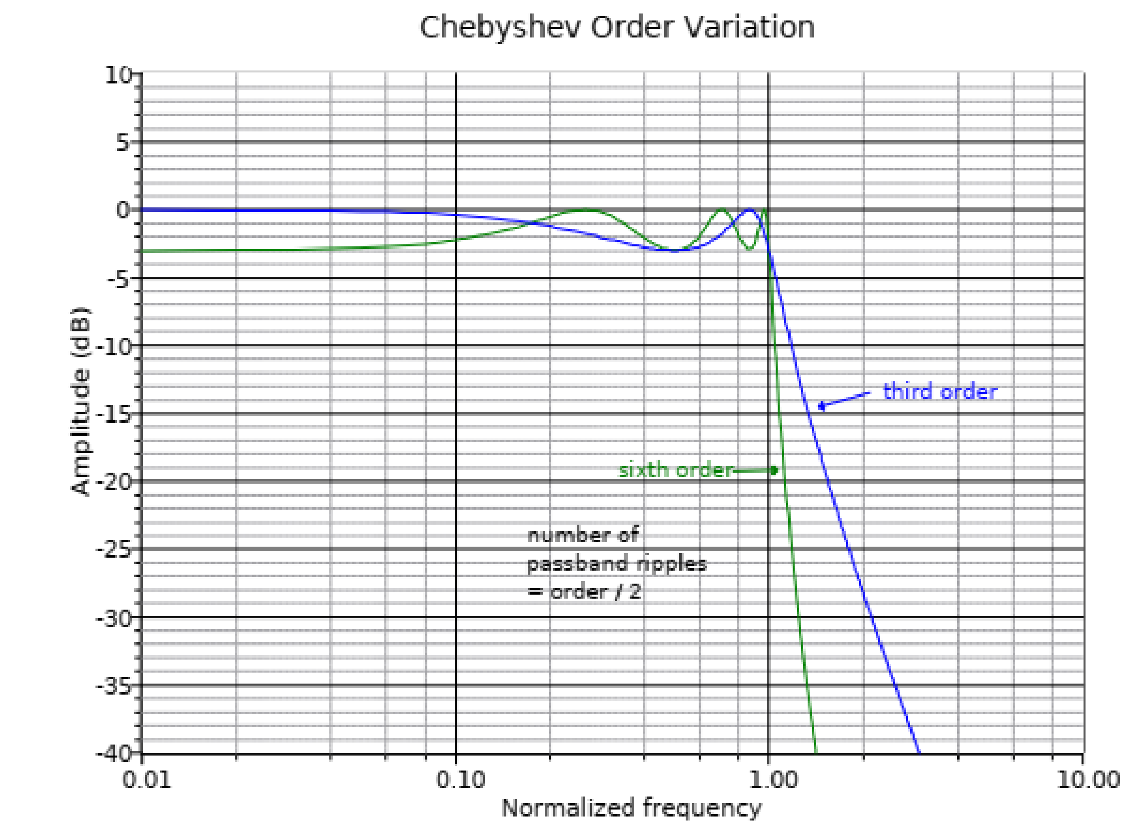


- The Chebyshev is actually a class of filters all its own.
- They are based on Chebyshev polynomials.
- The Chebyshev exhibits initial roll off rates in excess of 6 dB per octave per pole.
- This extra-fast transition is paid for in two ways: first, the phase response tends to be rather poor, resulting in a great deal of ringing when filtering pulses or other fast transients.
- The second effect is that the Chebyshev is non-monotonic.
- The passband response is not smooth; instead, ripples may be noticed.
- In fact, the height of the ripples defines a particular Chebyshev response.
- It is possible to design an infinite number of variations from less than 0.1 dB ripple to more than 3 dB ripple.
- The more ripple can be tolerate, the greater the roll off will be, and the worse the phase response will be.

Contd..



Chebyshev Ripple Variation
10
5
0
-5
-10
-15
-20
Amplitude (dB)
1 dB ripple
3 dB ripple
Slope approaches
6N dB/octave
(N = order)
0.01
0.10
1.00
10.00
Normalized frequency



Chebyshev Order Variation
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
Amplitude (dB)
third order
sixth order
number of
passband ripples
= order / 2
0.01
0.10
1.00
10.00
Normalized frequency

Chebyshev filter design

- Type-I Chebyshev Filter
- Type-II Chebyshev Filter

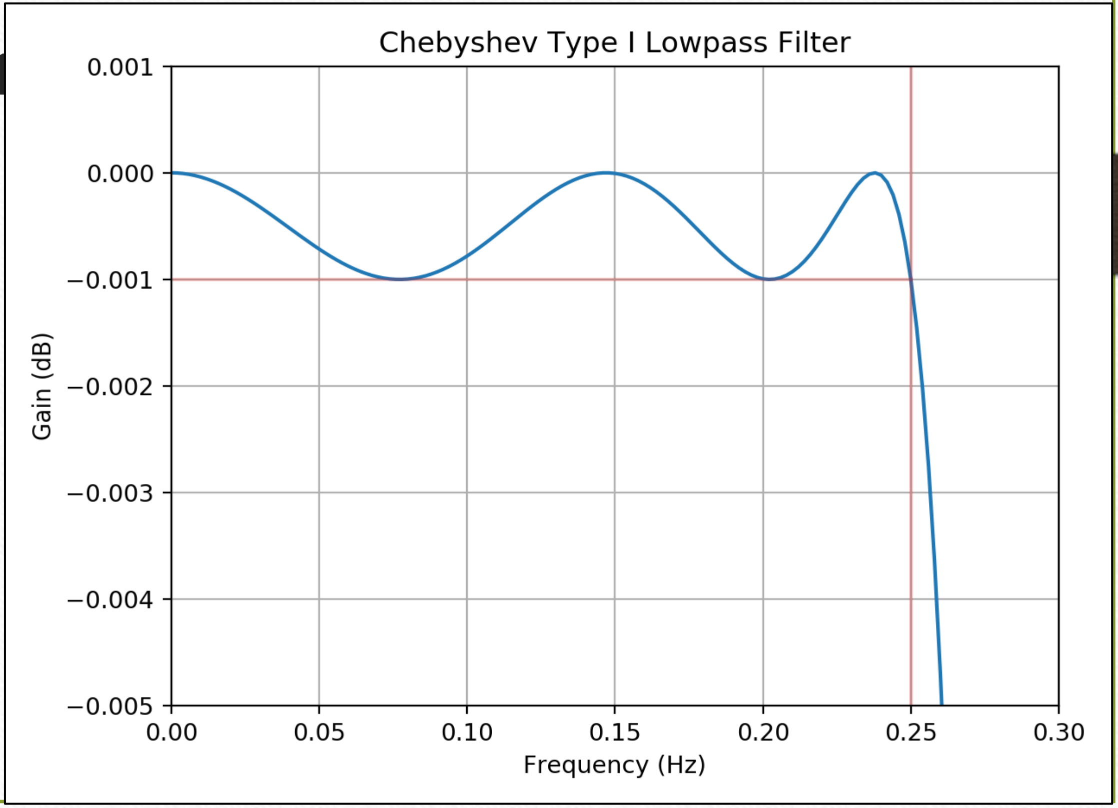
Type-I Chebyshev Filter

- These filters are all pole filters.
- In the pass band, these filters show equi-ripple behavior and they have monotonic characteristics in the stop band.
- The magnitude squared frequency response of chebyshev filter is given by:

$$|H(j\Omega)|^2 = \frac{1}{1+\in^2 C_N^2\left(\frac{\Omega}{\Omega_P}\right)}$$

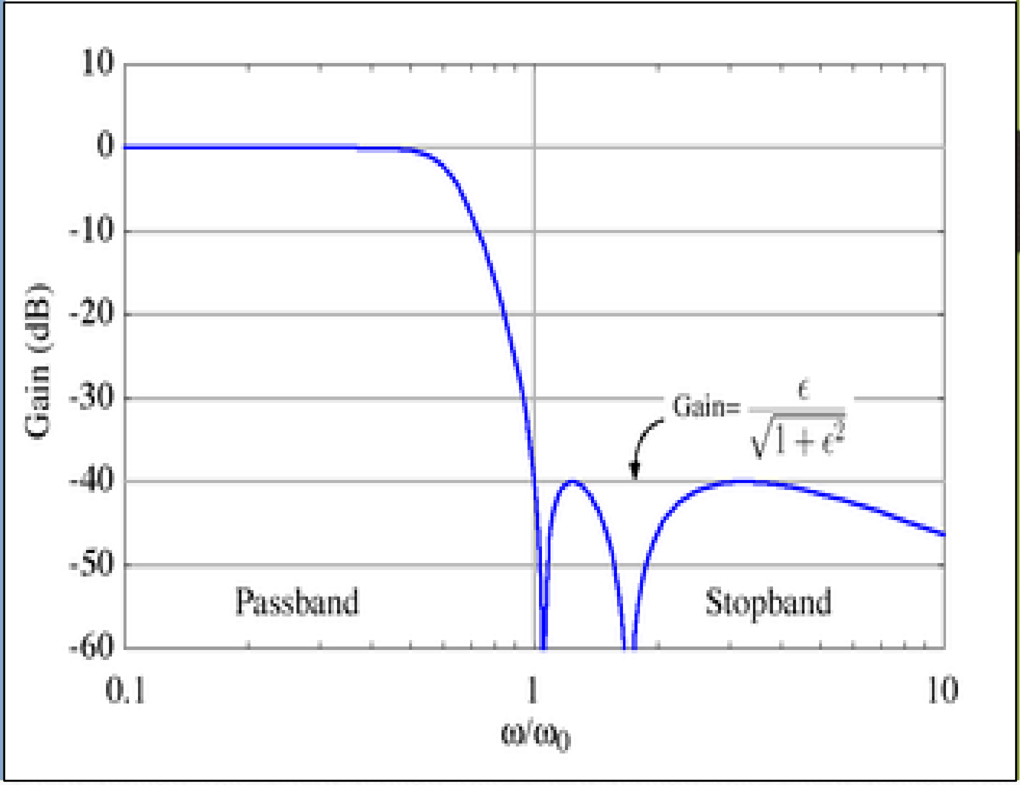
Contd..

- Where $C_N\left(\dfrac{\Omega}{\Omega_p}\right)$ is **Chebyshev polynomial**
- ε = Ripple parameter in the passband



Type-II Chebyshev Filter

- These filters contains zeros as well as poles.
- In the stop band, these filters show equiripple behaviour and they have monotonic characteristics in the pass band.
- The major difference between butterworth and chebyshev filter is that the poles of butterworth filter lie on the circle while the poles of chebyshev filter lie on ellipse



Chebyshev low pass filter design

Let A_p=Attenuation in passband
A_s=Attenuation in stop band
Ω_p=Passband edge frequency
Ω_c=Cut off frequency
Ω_s=Stopband edge frequency

Step I: Calculation of parameter ε :
It is given by :

Ripple parameter

$$\epsilon = \left[10^{0.1 A_P(dB)} - 1\right]^{1/2}$$

If A_P is not in dB then ε is calculated using the equation,

$$\epsilon = \left[\frac{1}{A_P^2} - 1\right]^{1/2}$$

Contd..

Step II: Calculation of order N of the filter:

•When stop band attenuation (As) is given in dB then 'N' is calculated using the equation:

$$-A_S(dB) = -20\log_{10}\in -6(N-1) - 20N\log_{10}(\Omega_S)$$

•When stop band attenuation (As) is not given in dB then 'N' is calculated using the equation:

$$N \geq \frac{\cosh^{-1}\left[\frac{1}{\in}\left(\frac{1}{A_S^2}-1\right)^{0.5}\right]}{\cosh^{-1}\left(\frac{\Omega_S}{\Omega_P}\right)}$$

Step III: Calculation of poles:

•The position of poles of Chebyshev filter lie on ellipse at coordinates x_k and y_k given by:

Contd..

$$X_k = r\cos\theta_k, k = 0,1,.....n-1$$

$$y_k = R\sin\theta_k, k = 0,1,.....n-1$$

$$\theta_k = \frac{\Pi}{2} + \frac{(2k+1)\Pi}{2N}$$

$$r = \Omega_p \frac{\beta^2 - 1}{2\beta}$$

Contd..

- R represents major axis of ellipse and is given by,

$$R = \Omega_p \frac{\beta^2 + 1}{2\beta}$$

- Here the parameter β is given by,

$$\beta = \left[\frac{\sqrt{1+\in^2} + 1}{\in} \right]^{\frac{1}{N}}$$

- Thus the pole positions are denoted by s_p ,

$$s_p = r \cos \theta_k + jR \sin \theta_k$$

Contd..

Step IV: Calculation of system function H(s):

- The system transfer function H(s) of analog filter is given by,

$$H(s) = \frac{k}{(s-s_0)(s-s_1)(s-s_2)\ldots}$$

- After simplification this equation can be written as,

$$H(s) = \frac{k}{s^N + b_{N-1}s^{N-1} + \ldots + b_0}$$

Here b_0 = Constant term in the denominator

- Now the value of 'k' can be calculated as follows

$$k = \begin{cases} b_0 & \text{for 'N' odd} \\ \dfrac{b_0}{\sqrt{1+\epsilon^2}} & \text{for 'N' even} \end{cases}$$

Numerical

- Design a Chebyshev analog filter with maximum passband attenuation of 2.5 dB at Ωp = 20 rad/sec and stop band
  attenuation of 30 dB at Ωs = 50 rad/sec.

Solution

Given Ap=2.5 db, Ωp = 20 rad/sec , As= 30 dB , Ωs = 50 rad/sec

Step-I: Calculation of parameter ε : Ripple parameter

$$\in=\left[10^{0.1A_P(dB)}-1\right]^{1/2}=\left[10^{0.1(2.5)}-1\right]^{1/2}=0.882$$

Step II: Calculation of order N of the filter:

$$-A_S(dB)=-20\log_{10}\in-6(N-1)-20N\log_{10}(\Omega_S)$$

$$-30=-20\log_{10}(0.882)-6(N-1)-20N\log_{10}(50)$$

After solving, N=0.95

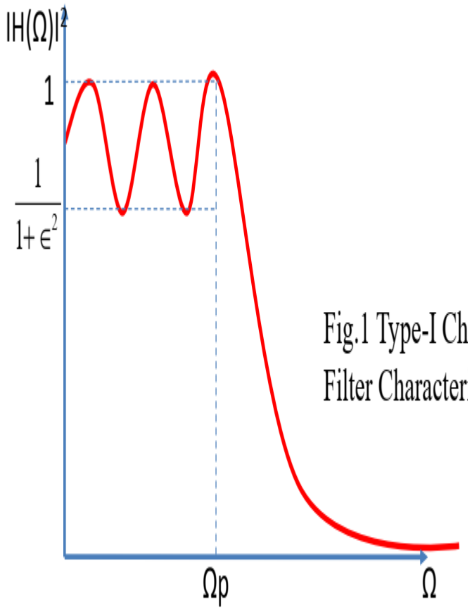


Fig.1 Type-I Chebyshev Filter Characteristics

Contd..

Step III: Calculation of poles:

- The pole positions are denoted by s_p ,

$$s_p = r\cos\theta_k + jR\sin\theta_k$$

- First we will calculate parameter 'β'

$$\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{\frac{1}{N}} = \left[\frac{\sqrt{1+(0.882)^2}+1}{0.882}\right]^{\frac{1}{1}} = 2.64$$

- Now we will calculate the values of 'r' and 'R'

$$r = \Omega_p \frac{\beta^2 - 1}{2\beta} = 20\frac{(2.64)^2 - 1}{2(2.64)} = 22.6$$

$$R = \Omega_p \frac{\beta^2 + 1}{2\beta} = 20\frac{(2.64)^2 + 1}{2(2.64)} = 30.19$$

- Now we will calculate values of θ_k

$$\theta_k = \frac{\pi}{2} + \frac{(2k+1)\pi}{2N}, \quad k=0 \text{ to } N-1$$

Contd..

$$\theta_0 = \frac{\pi}{2} + \frac{(2\times 0+1)\pi}{2\times 1} \quad = \pi$$

- Now we can write pole positions as

$$s_0 = r\cos\theta_0 + jR\sin\theta_0 = 22.6\cos\pi + j\,(30.19)\sin\pi = -22.6$$

Step IV: Calculation of system function H(s):

The system transfer function H(s) of analog filter is given by,

$$H(s) = \frac{k}{(s-s_0)} \quad = \frac{k}{(s+22.6)}$$

Here for N=1, we have

$k = b_0 = 22.6$

$$\therefore \quad H(s) = \frac{22.6}{(s-s_0)}$$

References

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- ***Terrell, D., Op Amps Design Application and Troubleshooting, Newness (1996).***