Course: UMA 035 (Optimization Techniques)

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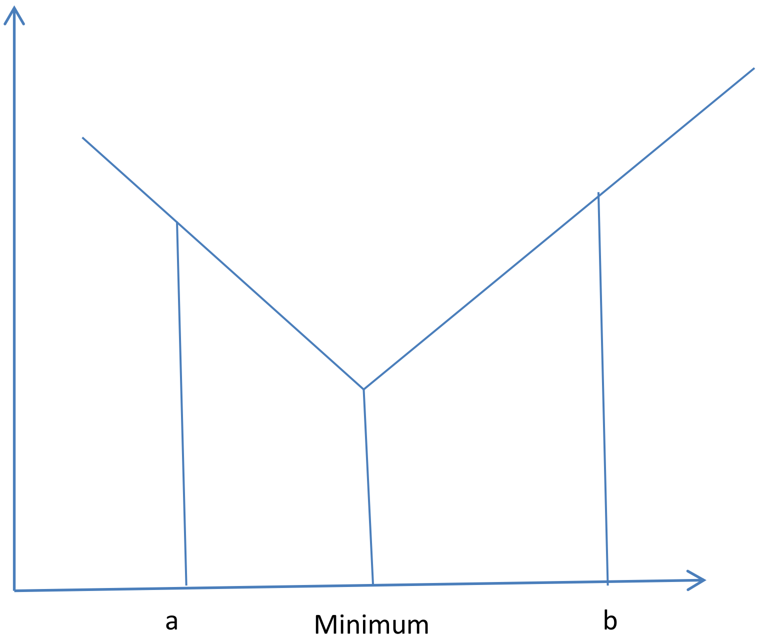
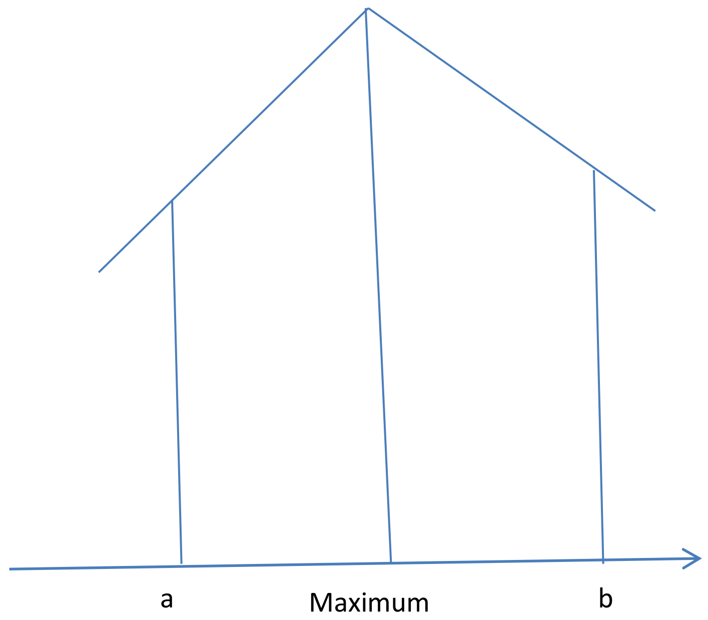
Search Techniques:

In real-life situations, it is not always possible to find an optimal solution of non-linear programming problems. In this chapter, three techniques (Dichotomous technique, Fibonacci technique and Steepest Descent technique) will be discussed to find an approximate optimal solution of non-linear programming problems without constraints.

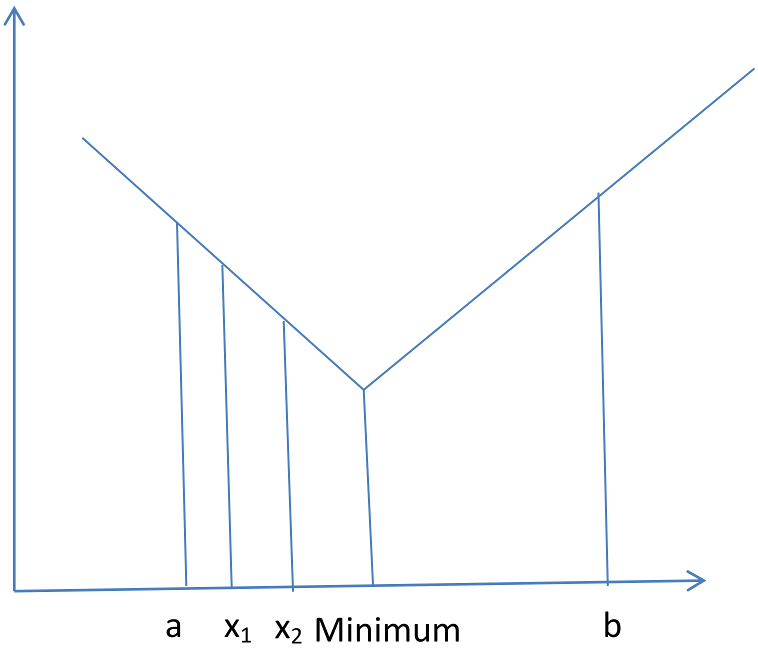
Unimodal function

A unimodal function is one that has only one peak in the given interval. Thus, a function of one variable is said to be unimodal on a given interval [a, b] if it has either unique minimum or maximum on [a, b].

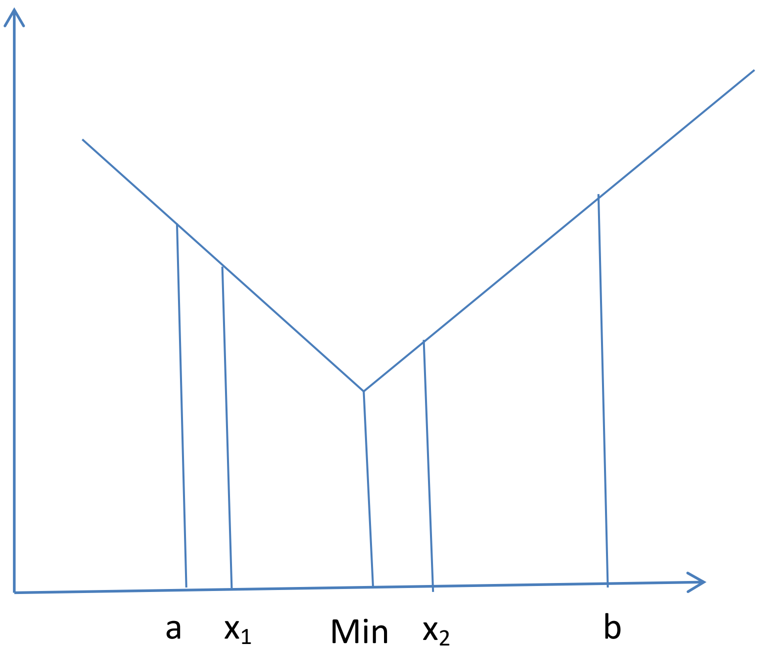



a
Maximum
b

Some results for minimum



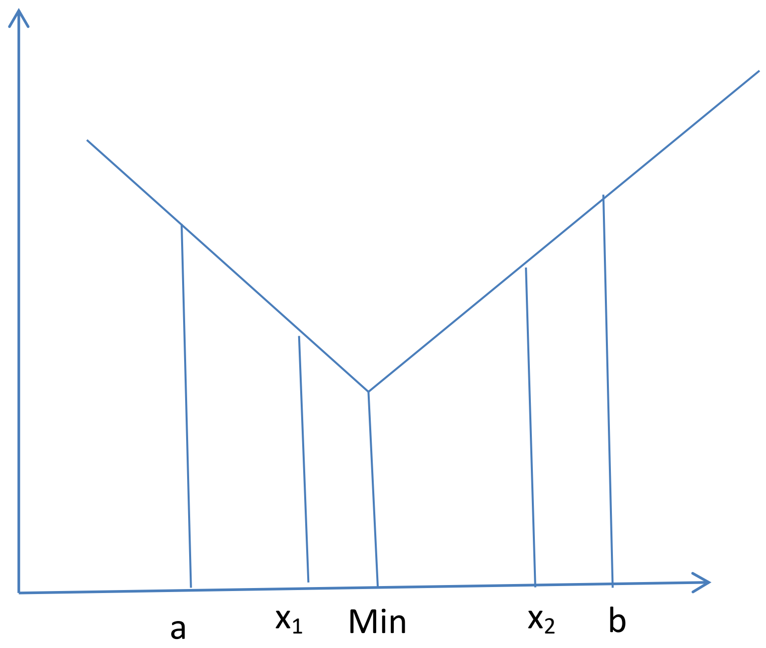
If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then according to the above diagram, the possible intervals in which minimum lies are $[x_1, b]$ and $[x_2, b]$.



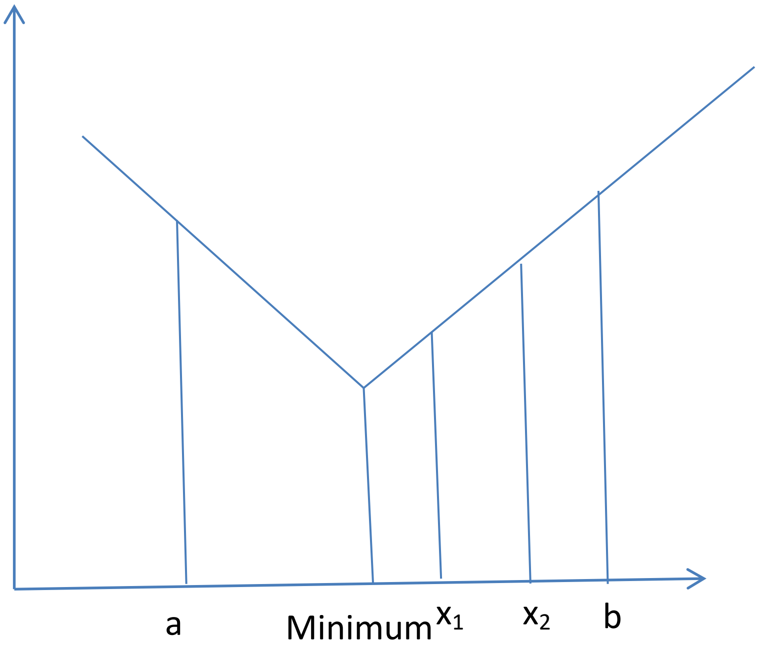
If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then according to the above diagram, the possible intervals in which minimum lies are [a, x_2], [x_1, x_2] and [x_1, b] .

The common for both is [x_1, b].

Hence, if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then the interval in which minimum lies is [x_1, b].



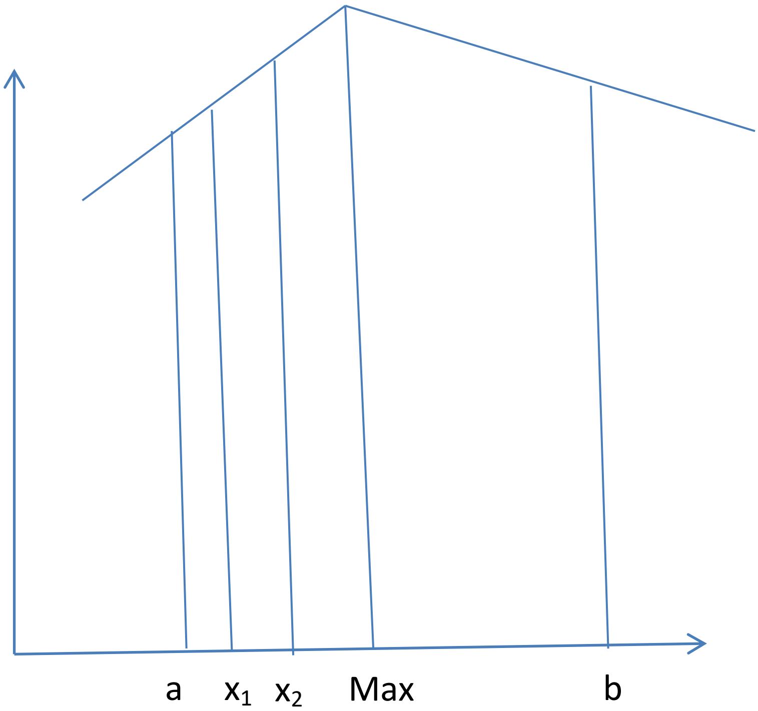
If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then according to the above diagram, the intervals in which minimum lies are [a, x_2], [x_1, x_2] and [x_1, b].



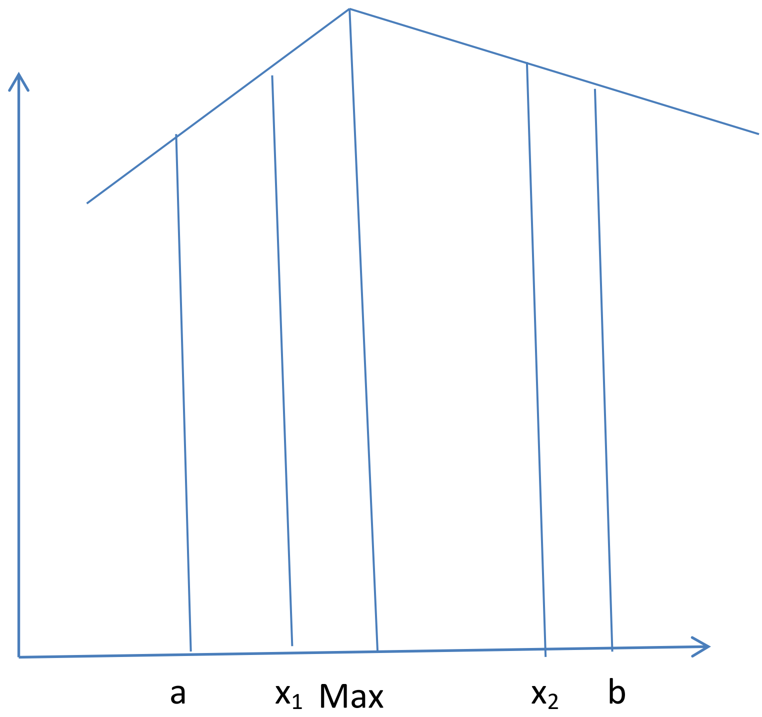
If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then according to the above diagram, the intervals in which minimum lies are [a, x_1] and [a, x_2].

Hence, if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then the interval in which minimum lies is [a, x_2].

Some results for maximum



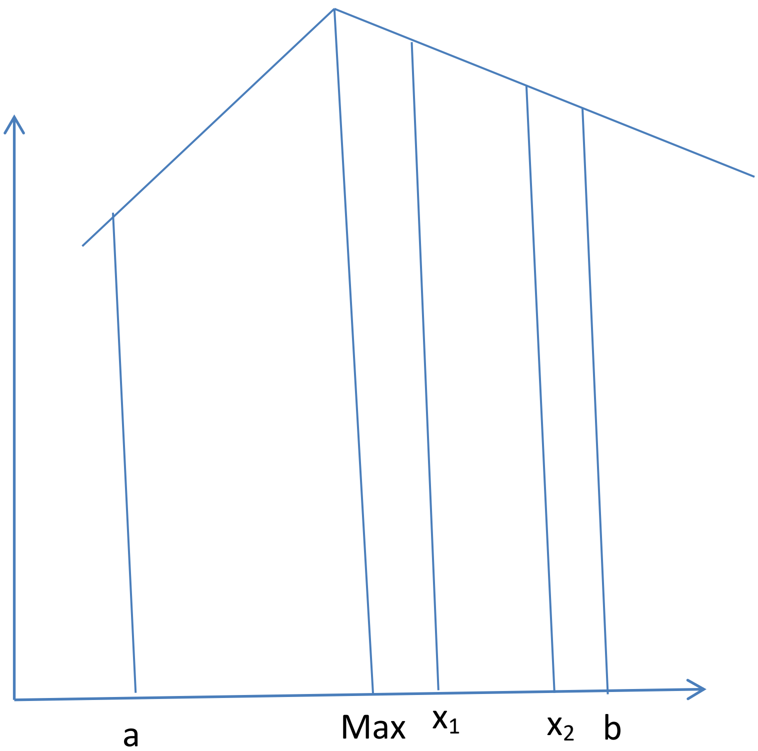
If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then the intervals in which maximum lies are $[x_1, b]$ and $[x_2, b]$.



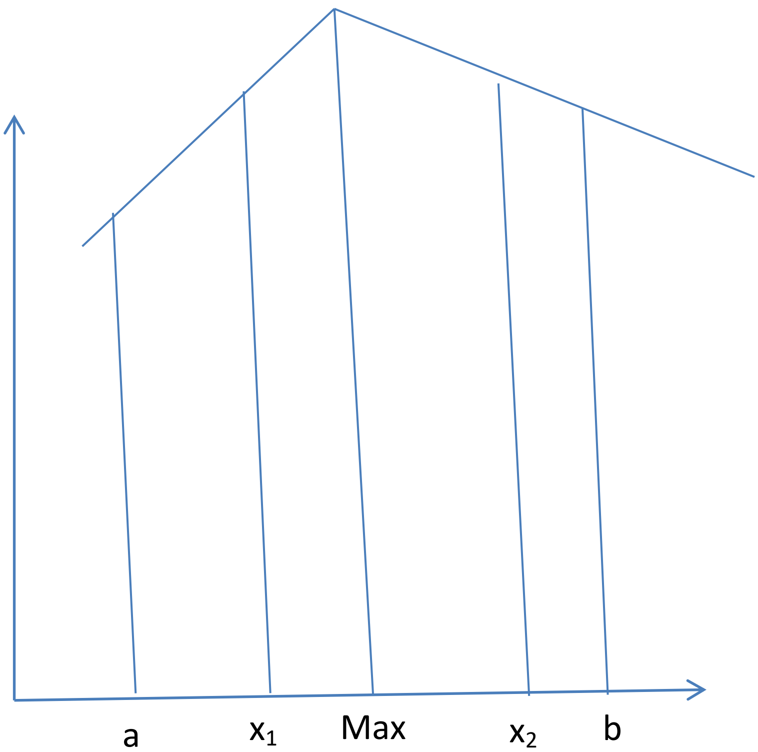
If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then the intervals in which maximum lies are [a, x_2] , [x_1, b] and [x_1, x_2].

The common for both is [x_1, b].

Hence, if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then the interval in which maximum lies is [x_1, b].



If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then the interval in which maximum lies are [a, x_1] and [a, x_2] .



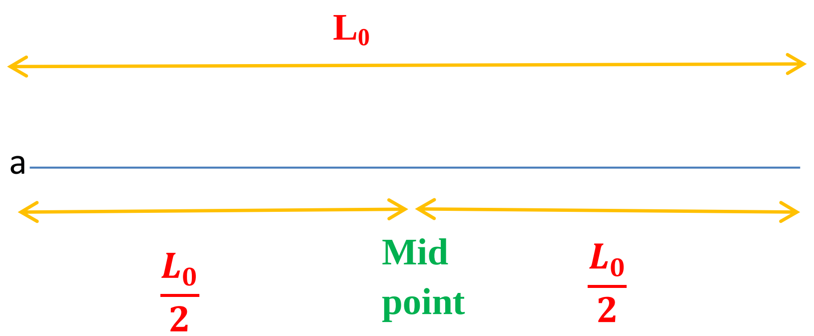
If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then the interval in which maximum lies are [a, x_2], [x_1, x_2] and [x_1, b] .

Hence, if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then the interval in which minimum lies is [a , x_2].

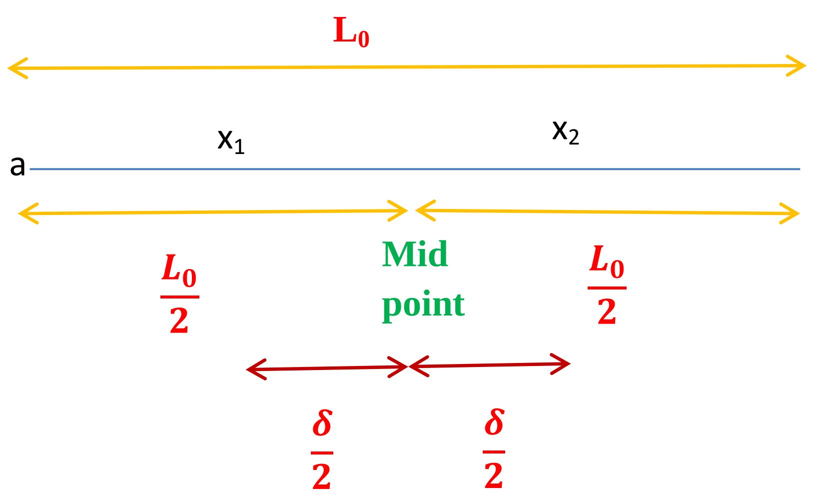
Dichotomous Search Technique

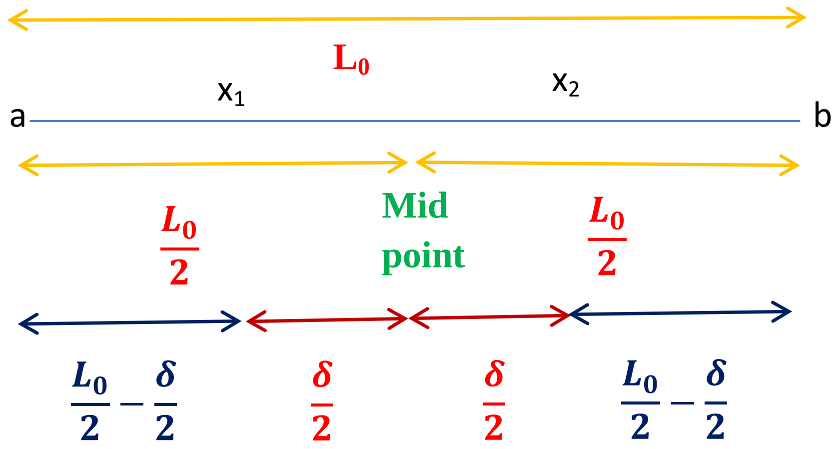
Let [a, b] be the given interval.

Let the length of the interval [a , b] be represented by L_0 i.e., $L_0 = b - a$.



Consider two points x_1 and x_2 at equal distance $\frac{\delta}{2}$ from the midpoint of the line segment.





Then,

$$x_1 = a + \frac{L_0}{2} - \frac{\delta}{2} = a + \frac{L_0 - \delta}{2}$$

and

$$x_2 = a + \frac{L_0}{2} - \frac{\delta}{2} + \frac{\delta}{2} + \frac{\delta}{2} = a + \frac{L_0}{2} + \frac{\delta}{2} = a + \frac{L_0 + \delta}{2}$$

As discussed earlier, the minimum/maximum will lie either in the interval [a, x_2] or [x_1, b].

Length of [a, x_2]= $x_2 - a = a + \frac{L_0+\delta}{2} - a = \frac{L_0+\delta}{2} = \frac{Intial\ length}{2} + \frac{\delta}{2}$

Length of [x_1, b]= $b - x_1 = b - a - \frac{L_0-\delta}{2} = L_0 - \frac{L_0-\delta}{2} = \frac{L_0+\delta}{2} =$

$$\frac{Intial\ length}{2} + \frac{\delta}{2}$$

New length is $\frac{Intial\ length}{2} + \frac{\delta}{2}$**.**

Repeating the procedure second time,

New length will be $\frac{\frac{Intial\ length}{2}+\frac{\delta}{2}}{2}+\frac{\delta}{2}=\frac{Intial\ length}{2^2}+\frac{\delta}{2^2}+\frac{\delta}{2}$

Repeating the procedure third time,

New length will be $\frac{\frac{Intial\ length}{2^2}+\frac{\delta}{2^2}+\frac{\delta}{2}}{2}+\frac{\delta}{2}=\frac{Intial\ length}{2^3}+\frac{\delta}{2^3}+\frac{\delta}{2^2}+\frac{\delta}{2}$

Repeating the procedure fourth time,

New length will be $\frac{\frac{Intial\ length}{2^3}+\frac{\delta}{2^3}+\frac{\delta}{2^2}+\frac{\delta}{2}}{2}+\frac{\delta}{2}=\frac{Intial\ length}{2^4}+\frac{\delta}{2^4}+\frac{\delta}{2^3}+\frac{\delta}{2^2}+\frac{\delta}{2}$

Repeating the procedure n^{th} time,

New length will be $=\frac{Intial\ length}{2^n}+\frac{\delta}{2^n}+\frac{\delta}{2^{n-1}}+\frac{\delta}{2^{n-2}}+\cdots+\frac{\delta}{2}$

$$=\frac{Intial\ length}{2^n}+\frac{\delta}{2}\left(1-\left(\frac{1}{2}\right)^n\right)$$

Measure of effectiveness=

$$=\frac{\frac{Intial\ length}{2^n}+\frac{\delta}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{Initial\ length}$$

$$=\frac{\frac{L_0}{2^n}+\frac{\delta}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{L_0}$$

Using it we will find n.