# Group Theory

Dr. Smita Agrawal
Assistant Professor, CSED
TIET, Patiala
smita.agrawal@thapar.edu

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# Homomorphism and Isomorphism

- A mapping $f$ from a group $(G,*)$ into a group $(G',*')$ is called a **homomorphism** if, $\forall\ a, b \ \in G$

$$f(a * b) = f(a) *' f(b)$$

- In addition, if $f$ is one to one and onto, then $f$ is called an **isomorphism** and $G$ and $G'$ are said to be **isomorphic**.

# Automorphism

- An isomorphism from a group $(G,*)$ to itself is called an automorphism of this group.

$f(a * b) = f(a) *' f(b)$

Prove $\hookrightarrow f(m+n) = f(m) \cdot f(n)$

# Examples

$(G, *')$ $(I, *)$ addition

1. Let $G = \{1, -1, i, -i\}$ which forms a group under multiplication and $I$ be the group of all integers under addition. Prove that the mapping $f$ from $I$ onto $G$ such that $f(n) = i^n$, $\forall n \in I$ is a homomorphism.

$\Rightarrow f(n) = i^n, \quad f(m) = i^m, \quad \forall m, n \in I$

$f(m+n) = i^{m+n} = i^m \cdot i^n$

$= f(m) \cdot f(n)$

$\therefore f$ is a homomorphism.

# Examples

2. The group of all real numbers with addition $(R, +)$is isomorphic to the group of $+ve$ real numbers with multiplication $(R^{+}, \times)$.


Thank you!!! ...