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Thapar University, Patiala

Department of Computer Science and Engineering

AUXILIARY EXAMINATION

B. E. (Second Year): Semester-I (COE/COEM/CAG/CML/ SEM)	Course Code: UCS405 Course Name: Discrete Mathematical Structures
July 25, 2017	Time: 3 Hours, M. Marks: 100

Note: Attempt all questions with proper justification. Assume missing data, if any.

Q.1 a) Determine whether the pair of graphs given in Figure 1 is isomorphic or not? If yes, exhibit an isomorphism. If not, give a suitable reason. (5+5)

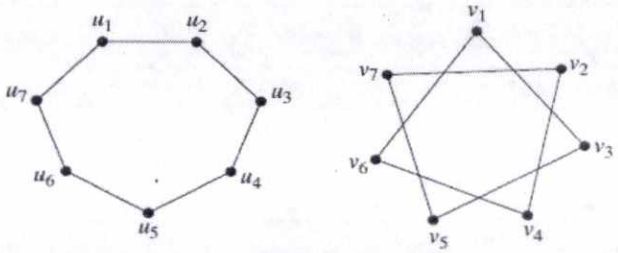

Figure 1

b) Determine whether the directed graphs given in Figure 2 have an Euler circuit or not. If yes, write the Euler circuit. If no, then determine whether they have an Euler path. Write the Euler path if one exists.

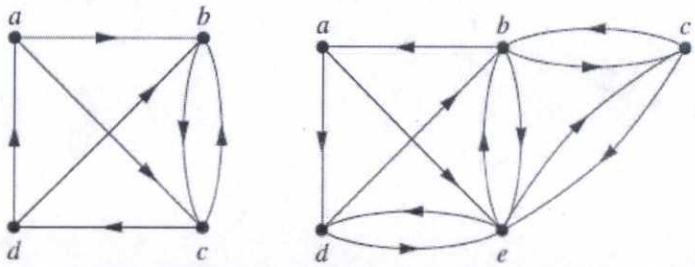

Figure 2

Q.2 a) What is a ring? Mention the conditions necessary to be a ring. (6+4)

b) What do you mean by a monoid? Mention the conditions necessary to be a monoid.

Q.3 Answer the following questions for the poset ({3, 5, 9, 15, 24, 45}, |). (10)

a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {3, 5}.
f) Find the least upper bound of {3, 5}, if it exists.

g) Find all lower bounds of {15, 45}.
h) Find the greatest lower bound of {15, 45}, if it exists.

Q.4 i) Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence. (5+5)

a) $\neg q$ b) $p \wedge q$ c) $\neg p \vee q$ d) $p \rightarrow \neg q$ e) $\neg q \rightarrow p$

ii) Let p and q be the following propositions

p :You drive over 65 miles per hour.
q :You get a speeding ticket.

Write the following propositions using p and q and logical connectives (including negations).
a) You do not drive over 65 miles per hour.
b) You drive over 65 miles per hour, but you do not get a speeding ticket.
c) You will get a speeding ticket if you drive over 65 miles per hour.
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

Q.5 Write the name of rule of inference used in each of the following arguments. (10)
Also mention the rule using propositional variables clearly stating the statements those propositional variables represent.

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Q.6 Show that if A and B are sets, then (5+5)
a) $A - B = A \cap B$.
b) $(A \cap B) \cup (A \cap B) = A$.

Q.7 Suppose that g is a function from A to B and f is a function from B to C. (5+5)

a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

Q.8 Find big-O estimate for the following functions: (10)

a) $(n \log n + n^2)(n^3 + 2)$

b) $(n! + 2^n)(n^3 + \log(n^2 + 1))$

Q.9 A parking lot has 31 visitor spaces, numbered from 0 to 30.Visitors are assigned parking spaces using the hashing function $h(k) = k$ **mod** 31, where k is the number formed from the first three digits on a visitor's license plate. (5+5)

a) Which spaces are assigned by the hashing function to cars that have these first three digits on their license plates: 317, 918, 007, 100, 111, 310?

b)Use linear probing to resolve collisions in part a). Find new spaces assigned to cars after resolving collisions.

Q.10 Let S be the subset of the set of ordered pairs of integers defined recursively by (4+6)

Basis step: $(0, 0) \in S$.

Recursive step: If $(a, b) \in S$, then $(a, b + 1) \in S$, $(a + 1, b + 1) \in S$, and $(a + 2, b + 1) \in S$.

a) List the elements of S produced by the first four applications of the recursive definition.

b) Use structural induction to show that $a \leq 2b$ whenever $(a, b) \in S$.