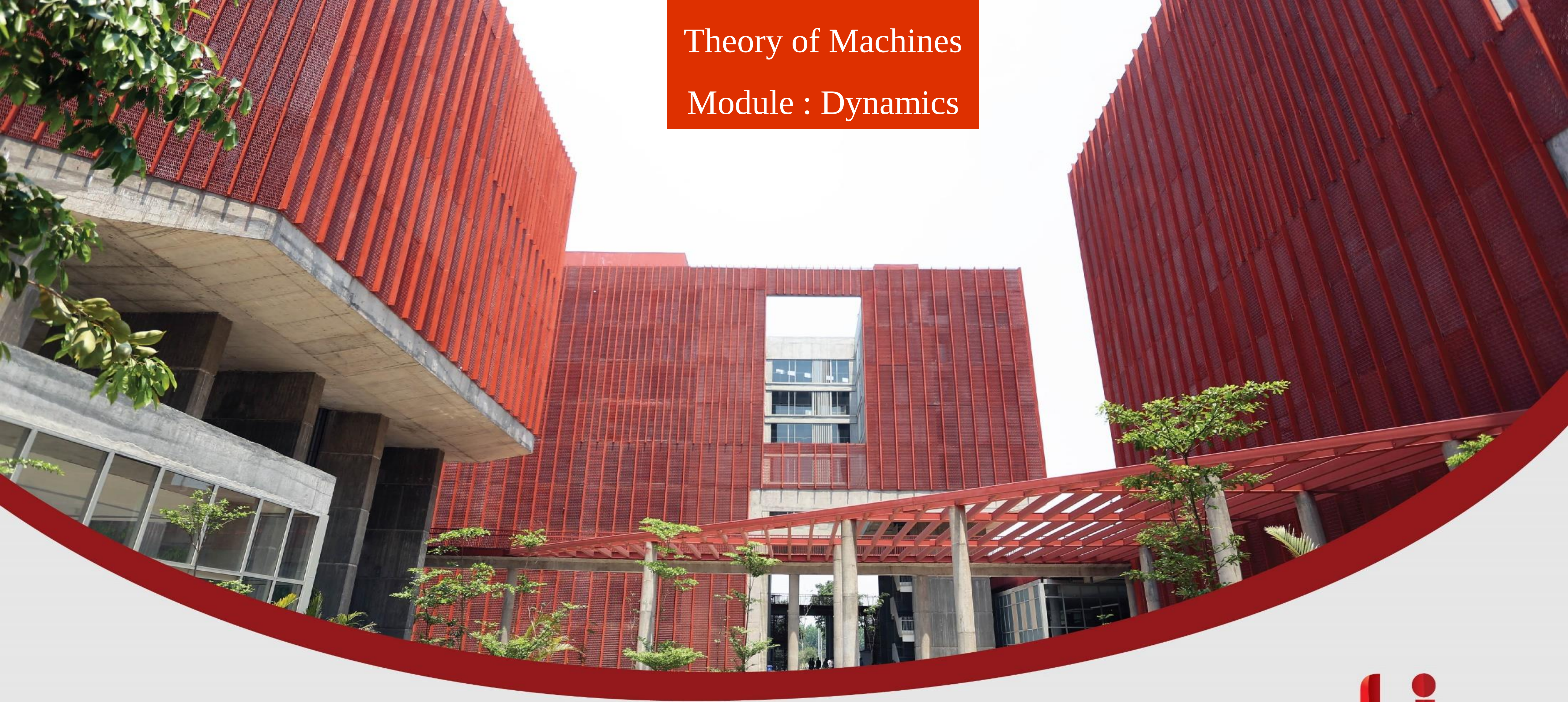
Theory of Machines

Module : Dynamics



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Lecture Contents and Learning Outcomes

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- Equivalent dynamical systems
- Dynamic force analysis in engines

Learning Outcomes

Dynamic Analysis of Slider-Crank Mechanism

Equivalent Dynamical Systems

Dynamic Force Analysis of Engines

References

1. S S Ratan “Theory of Machines” 3rd Edition, Tata Macgraw Hill Publications
2. J. J. Uicker, G. R. Pennock, and J. E. Shigley “Theory of Machines and Mechanisms” Oxford Press (2009)
3. Neil Sclater, Nicholas P. Chironis “Mechanisms and Mechanical Devices Sourcebook” 4th Edition, McGraw Hill Publications
4. R S Khurmi “A text Book of Theory of Machines” S Chand Publications

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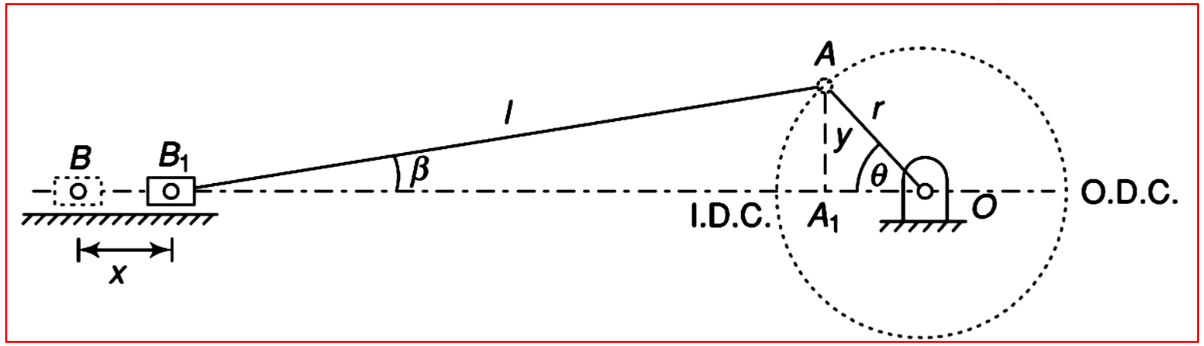
Velocity and Acceleration Analysis of Piston



$$x = B_1B = BO - B_1O = BO - (B_1A_1 + A_1O)$$

$$= (l + r) - (l \cos \beta + \cos \theta)$$

$$= (nr + r) - (nr \cos \beta + r \cos \theta) \qquad \text{Taking } l/r = n$$

$$= r\,[(n + 1) - (n \cos \beta + \cos \theta)] \tag{1}$$

where

$$\cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \frac{y^2}{l^2}} = \sqrt{1 - \frac{(r \sin \theta)^2}{l^2}} = \sqrt{1 - \frac{\sin^2 \theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2 \theta}$$

$$x = r[(n+1) - (\sqrt{n^2 - \sin^2 \theta} + \cos \theta)] = r[(1 - \cos \theta) + (n - \sqrt{n^2 - \sin^2 \theta})] \tag{2}$$

- If the connecting rod is large in comparison with the crank, then

$$x = r\,(1 - \cos \theta) \tag{3}$$

- Thus, the piston executes a simple harmonic motion when the connecting rod is large

Velocity and Acceleration Analysis of Piston

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Velocity: $v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \frac{d}{d\theta}[r\{(1-\cos\theta)+n-(n^2-\sin^2\theta)^{1/2}\}]\frac{d\theta}{dt}$

$$= r[(0+\sin\theta)+0-\frac{1}{2}(n^2-\sin^2\theta)^{1/2}(-2\sin\theta\cos\theta)]\omega = r\omega\left[\sin\theta+\frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right] \rightarrow 4$$

If n^2 is large compared to $sin^2\theta$

$$v = r\omega\left[\sin\theta+\frac{\sin 2\theta}{2n}\right] \rightarrow 5$$

If $(sin2\theta)/2n$ can be neglected (when n is quite large) $v = r\omega\sin\theta \rightarrow 6$

Acceleration:

$$f = \frac{dv}{dt} = \frac{dvd\theta}{d\theta dt} = \frac{d}{d\theta}\left[r\omega\left(\sin\theta+\frac{\sin 2\theta}{2n}\right)\right]\omega = r\omega\left(\cos\theta+\frac{2\cos 2\theta}{2n}\right)\omega = r\omega^2\left(\cos\theta+\frac{\cos 2\theta}{n}\right) \rightarrow 7$$

If n is very large $f = r\omega^2\cos\theta$ As in case of SHM $\rightarrow 8$

When $\theta = 0^\circ$, i. e., at IDC $f = r\omega^2\left(1+\frac{1}{n}\right)$ When $\theta = 180^\circ$, i. e., at ODC $f = r\omega^2\left(-1+\frac{1}{n}\right)$

At $\theta = 180^\circ$, when direction of motion is reversed $f = r\omega^2\left(1-\frac{1}{n}\right) \rightarrow 9$

Angular Velocity and Acceleration Analysis of Connecting Rod

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As $y = l\sin\beta = r\sin\theta$ $\sin\beta = \dfrac{\sin\theta}{n}$

Differentiating with respect to time

$$\cos\beta\frac{d\beta}{dt} = \frac{1}{n}\cos\theta\frac{d\theta}{dt} \Rightarrow \frac{d\beta}{dt} = \frac{\cos\theta}{n\cos\beta}\omega$$

But $\cos\beta = \dfrac{1}{n}\sqrt{n^2 - \sin^2\theta}$

Angular velocity of the connecting rod (ω_c)is given by

$$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \rightarrow 10$$

Angular Acceleration (α_c) of the connecting rod (ω_c) is

$$= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}[\cos\theta(n^2 - \sin^2\theta)^{-1/2}]\omega$$

$$= \omega^2[-\cos\theta\frac{1}{2}(n^2 - \sin^2\theta)^{-3/2}(-2\sin\theta\cos\theta) + (n^2 - \sin^2\theta)^{-1/2}(-\sin\theta)]$$

$$= \omega^2\sin\theta\left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$= -\omega^2\sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right] \rightarrow 11$$

The negative sign indicates that the sense of the angular acceleration of the rod is such that it tends to reduce the angle β.

Engine Force Analysis

- Forces acting in engine: Weight of reciprocating masses and connecting rod, gas forces, forces due to friction and forces due to acceleration and retardation of engine elements, last one is dynamic forces.

Piston Effort (net or effective force on piston):

- The reciprocating masses accelerate during first half of the stroke and inertia force tends to resist the same. Therefore net force decreases during first half of stroke.
- During the later half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of applied gas pressure which results into increase in the net piston effort.
- In vertical engine, the weight of reciprocating masses assists the piston during the outstroke (down stroke) increasing the piston effort by an amount equal to the weight of the piston. During the instroke (upstroke), the piston effort is decreased by the same amount.
- A_1 = area of cover end, A_2 = area of the piston rod end, P_1 = pressure on the cover end, P_2 = pressure on the rod end, m = mass of the reciprocating mass end.

Engine Force Analysis Contd..

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- Force on the piston due to gas pressure, $Fp = P_1A_1 - P_2A_2$ → 12
- Inertia Force, $$F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$ → 13

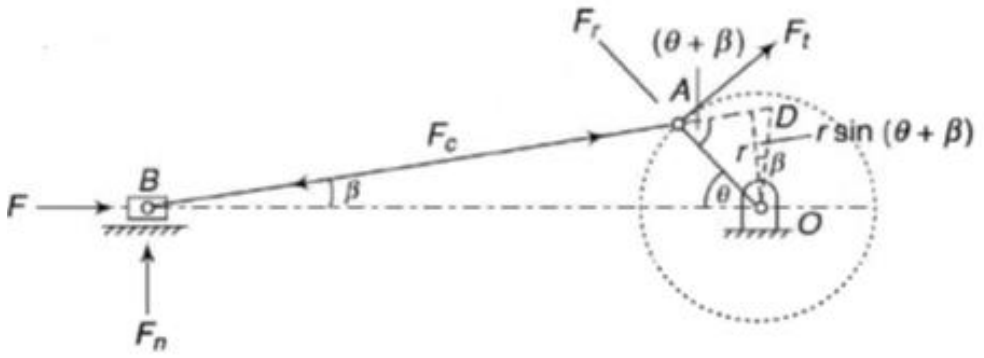
  which is in the opposite direction to that of the acceleration of the piston
- Net (effective) force on the piston $F = F_p - F_b$ → 14
- In case, friction resistance F_f is also taken into account

$$F = F_p - F_b - F_f$$

- In case of vertical engines, the weight of the piston or reciprocating parts also acts as force and thus force on the piston,

$$F = F_p + mg - F_b - F_f$$

Engine Force Analysis contd...



Force (thrust) along connecting rod

- Horizontal component of the forces

$$F_c \times \cos\beta = F \quad \text{or} \quad F_c = \frac{F}{\cos\beta}$$

Thrust on the sides of cylinder

- It is normal reaction on the cylinder walls $F_n = F_c \sin\beta = F\tan\beta$

Crank effort

- *Crank effort* is the net effort (force) applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft.

$$F_t \times r = F_c\, r\sin(\theta+\beta) \qquad F_t = F_c\sin(\theta+\beta) = \frac{F}{\cos\beta}\sin(\theta+\beta) \longrightarrow 15$$

Thrust on the bearing

- The component if Fc along the crank (in the redial direction) produces a thrust on the crankshaft bearings

$$F_r = F_c\cos(\theta+\beta) = \frac{F}{\cos\beta}\cos(\theta+\beta)$$

Turning Moment on Crankshaft

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$$T = F_t \times r = \frac{F}{\cos\beta}\sin(\theta+\beta)\times r$$

$$= \frac{Fr}{\cos\beta}(\sin\theta\cos\beta + \cos\theta\sin\beta)$$

$$= Fr\left(\sin\theta + \cos\theta\sin\beta\frac{1}{\cos\beta}\right)$$

$$= Fr\left(\sin\theta + \cos\theta\frac{\sin\theta}{n}\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{2\sin\theta\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right) \longrightarrow 16$$

Also, as $r\sin(\theta+\beta) = OD\cos\beta$

$$T = F_t \times r = \frac{F}{\cos\beta}r\sin(\theta+\beta)$$ From Eq (15)

$$= \frac{F}{\cos\beta}(OD\cos\beta)$$

$$= F \times OD \longrightarrow 17$$

Worked Example 1

A horizontal gas engine running at 210 rpm has a bore of 220 *mm* and a stroke of 440 *mm*. The connecting rod is 924 *mm* long and the reciprocating parts weigh 20 *kg*. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m^2 and 60 kN/m^2 respectively. Diameter of the piston rod is 40 *mm*. Determine

i. *Turning moment on the crank shaft*
ii. *Thrust on the bearings*
iii. *Acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22kW*

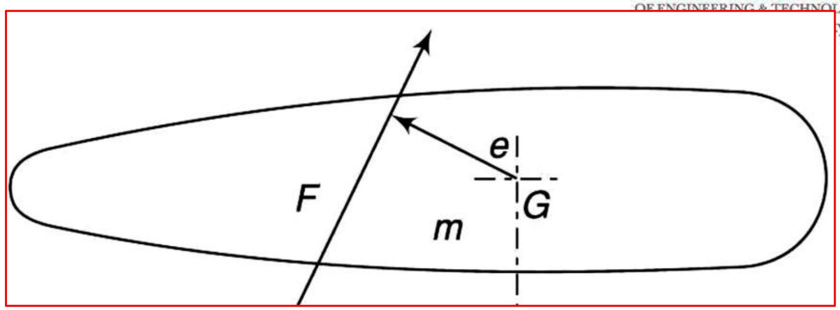
Worked Example 2

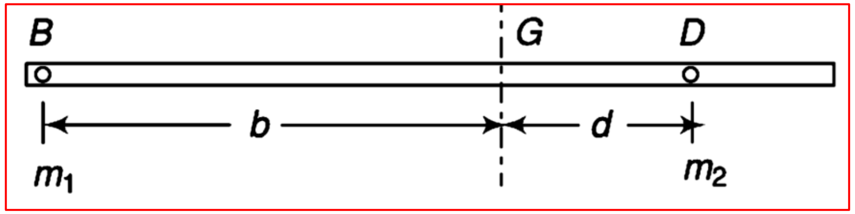
In a vertical double-acting steam engine, the connecting rod is 4.5 times the crank. The weight of the reciprocating parts is 120 *k*g and the stroke of the piston is 440*mm*. The engine runs at 250 rpm. If the net load on the piston due to steam pressure is 25 *kN* when the crank has turned through an angle of 120° from the top dead centre, determine the

i. *Thrust in the connecting rod*
ii. *Pressure on slide bars*
iii. *Tangential force on the crank pin*
iv. *Thrust on the bearings*
v. *Turning moment on the crankshaft*

Dynamically Equivalent System

- The net force F on the piston obtained in previous section may be the gas force with or without considering inertia force acting on the piston
- As neither the mass of the connecting rod is uniformly distributed nor the motion is linear, its inertia cannot be found as such.
- Usually, the inertia of the connecting rod is taken into account by considering a *dynamically-equivalent system*.





- A dynamically equivalent system means that the rigid link is replaced by a link with two point masses in such a way that it has the same motion as the rigid link when subjected to the same force.
- The center of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.

As we know, $F = mf$ and $Fe = I\alpha$ Acceleration of G, $f = F/m$

Angular acceleration of body $\alpha = \dfrac{F.e}{I}$

Dynamically Equivalent System Contd..

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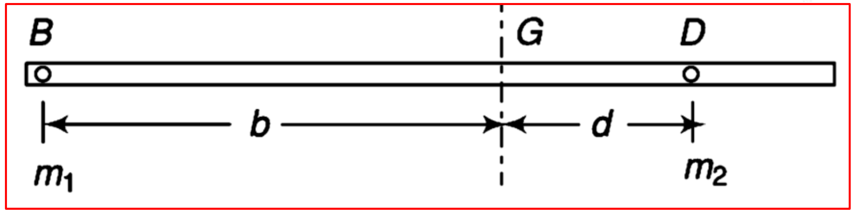
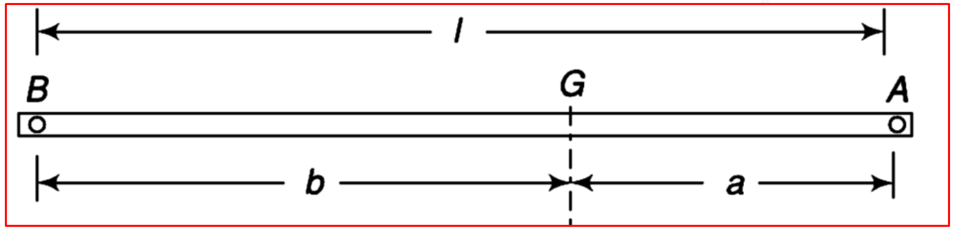
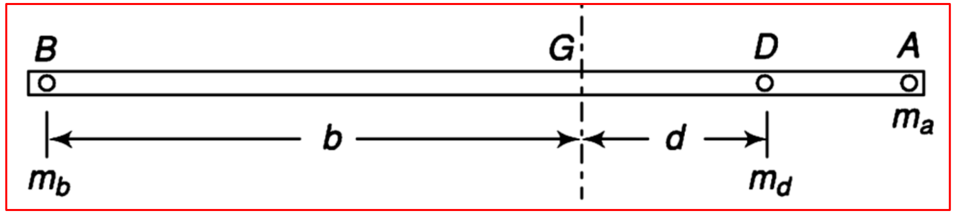
- To satisfy the first condition, as the force *F* is to be same, the sum of the equivalent masses m_1 and m_2 has to be equal to m to have the same acceleration. Thus, $m = m_1 + m_2$.



- To satisfy the second condition, as the numerator *F.e* and the denominator *I* must remain the same. *F* already taken same, thus *e* has to be same which means that the perpendicular distance of *F* from *G* should remain same or combined center of mass of the equivalent system remains at *G*. This is possible if $m_1 b = m_2 d$
- To have the same MI: $I = m_1 b^2 + m_2 d^2$
- Thus, any distributed mass can be replaced by two point masses to have the same dynamical properties if:
  1. The sum of the two masses is equal to the total mass
  2. The combined center of mass coincides with that of the rod.
  3. MI of two point masses about the perpendicular axis through their combined center of mass is equal to that of the rod.

Inertia of Connecting Rod





$$m_b + m_d = m \quad \longrightarrow \quad i$$

and

$$m_b . b = m_d . d \quad \longrightarrow \quad ii$$

From (*i*) and (*ii*)

$$m_b + \left(m_b \frac{b}{d} \right) = m \Rightarrow m_b \left(1 + \frac{b}{d} \right) = m \Rightarrow m_b \left(\frac{b+d}{d} \right) = m \Rightarrow m_b = m \frac{d}{b+d}$$

Similarly $\quad m_d = m \dfrac{b}{b+d}$

Also

$$I = m_b b^2 + m_d d^2 = m \frac{d}{b+d} b^2 + m \frac{b}{b+d} d^2 = mbd \left(\frac{b+d}{b+d} \right) = mbd \quad \longrightarrow \quad 1$$

$$mk^2 = mbd \Rightarrow k^2 = bd \quad \longrightarrow \quad 2$$

Equivalent length of simple pendulum is given by

$$L = \frac{k^2}{b} + b = d + b$$

Inertia of Connecting Rod contd..

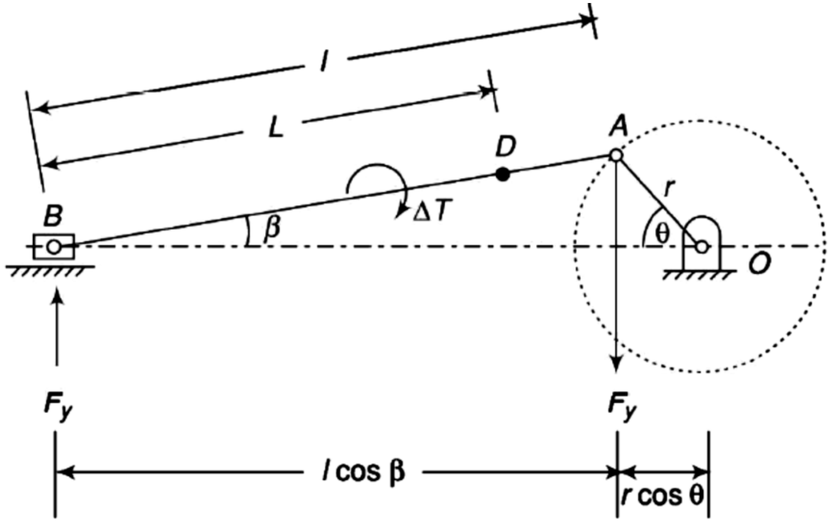
However, in the analysis of the connecting rod, it is much more convenient if the two point masses are considered to be located at the center of the two end bearings, i. e., at *A* and *B*.

$$m_a + m_b = m \qquad m_a = m\frac{b}{a+b} = m\frac{b}{l} \qquad m_b = m\frac{a}{a+b} = m\frac{a}{l} \qquad I' = mab$$

Assuming $a > d$, $I' > I$

Correction couple $\Delta T = \alpha_c\,(mab - mbd) = mb\alpha_c\,(a-d) = mb\alpha_c\,[(a+b)-(b+d)] = mb\alpha_c\,(l-L)$ 3

The correction couple must be applied in the opposite direction to that of the applied inertia torque will be produced by two equal and opposite forces F_y acting at gudgeon pin and crank pin ends.



Turning moment at crankshaft due to force at *A* or correction torque

$$T_c = F_y \times r\cos\theta = \frac{\Delta T}{l\cos\beta}\times r\cos\theta = \frac{\Delta T}{(l/r)}\frac{\cos\theta}{\cos\beta} = \Delta T\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2-\sin^2\theta}} = \Delta T\frac{\cos\theta}{\sqrt{n^2-\sin^2\theta}}$$

4

Inertia of Connecting Rod contd..

This correction torque will be deducted from the inertia torque acting on the crankshaft.

Also, due to the weight of the mass at *A*, a torque is exerted on the crankshaft which is given by

$$T_a = (m_a g) r \cos\theta \longrightarrow 5$$

In case of vertical engine, a torque is also exerted on the crankshaft due to the weight of mass at *B* and the expression will be

$$T_b = (m_b g) r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right) \longrightarrow 4$$

The net torque or turning moment on the crankshaft will be the algebraic sum of the

i. Turning moment due to the force of gas pressure (T)
ii. Inertia torque due to the inertia force at the piston as a result of inertia of the reciprocating mass including the mass of the portion of the connecting rod (T_b)
iii. Inertia torque due to the weight (force) of the mass at the crank pin which is the portion of the mass of the connecting rod taken at the crank pin (T_a)
iv. Inertia torque due to the correction couple (T_c)
v. Turning moment due to the weight (force) of the piston in case of vertical engines

It is convenient to combine the forces at the piston occurring in (*ii*) and (*v*)

Worked Example 3

The piston diameter of an internal combustion engine is 125 *mm* and the stroke is 220 *mm*. The connecting rod is 4.5 times the crank length and has a mass of 50 *kg*. The mass of the reciprocating parts is 30 *kg*. The center of mass of the connecting rod is 170 *mm* from the crank-pin center and the radius of gyration about an axis through the center of mass is 148 *mm*. The engine runs at 320 rpm. Find the magnitude and the direction of the inertia force and the corresponding torque on the crankshaft when the angle turned by the crank is 140° from the inner dead center.

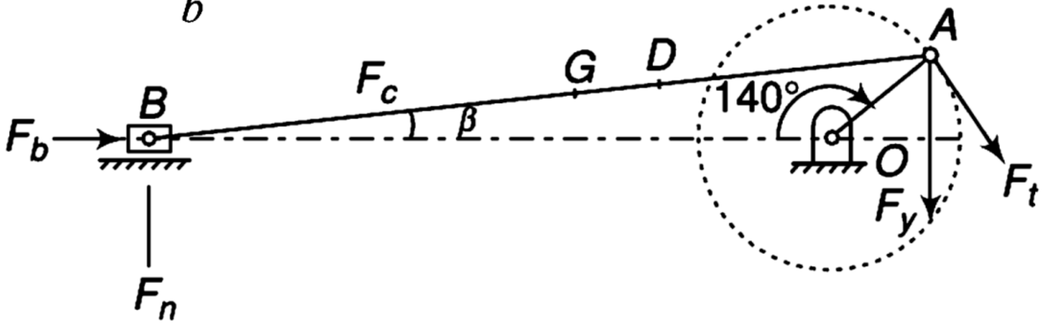
$$\Delta T = m\alpha_c b(l - L)$$

$$L = b + \frac{k^2}{b}$$

$$\alpha_c = -\omega^2 \sin\theta \left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$f = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$T_c = \Delta T \frac{\cos\vartheta}{\sqrt{n^2 - \sin^2\theta}}$$



$$F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$T_a = (m_a g)\, r \cos\theta$$

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

$$T_b - T_c + T_a$$

Thank You

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