Review of Shear Force and Bending Moment Diagrams

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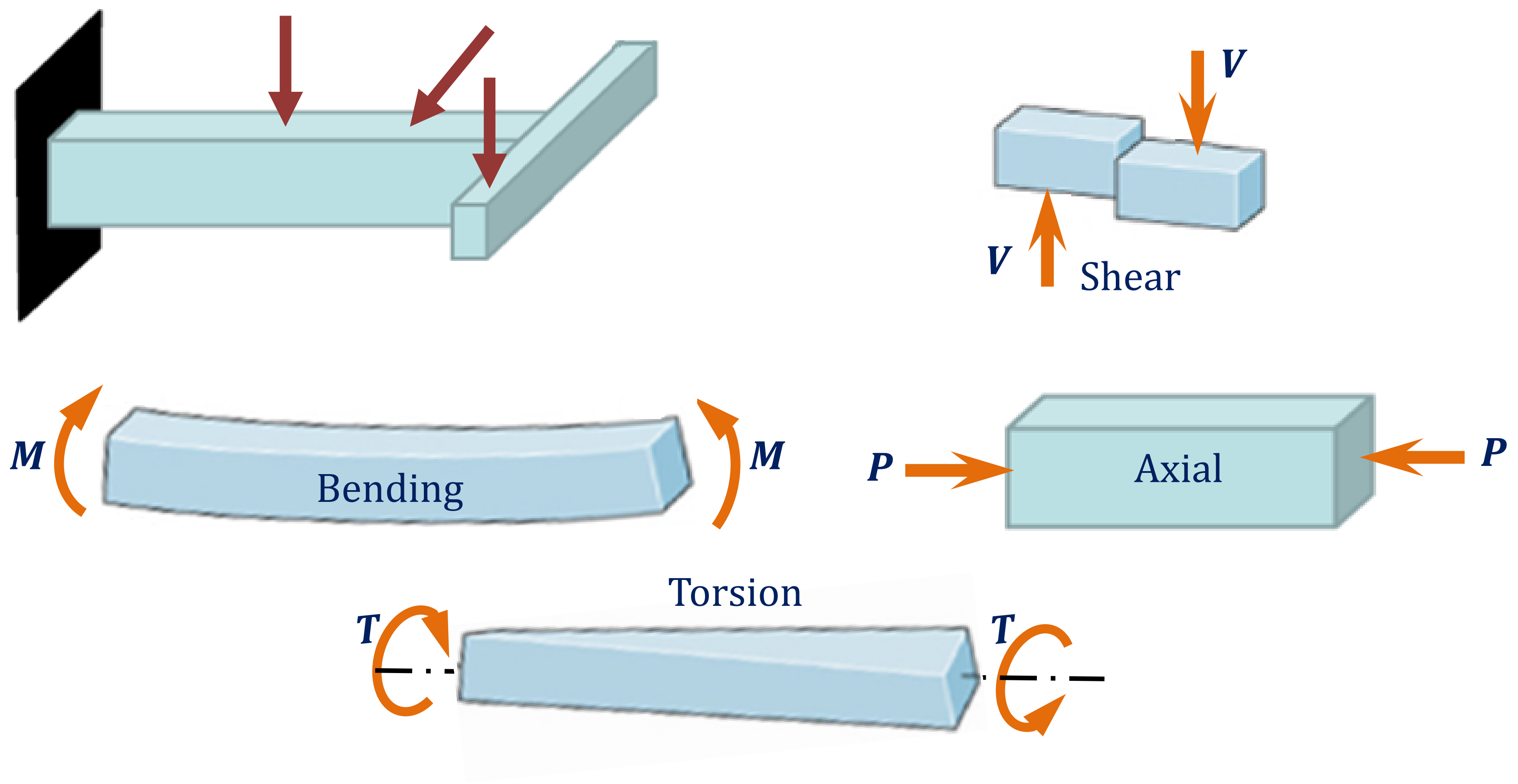
Internal Effects of a Force

Axial Force (P), Shear Force (V), Bending Moment (M), Twisting Moment (T)

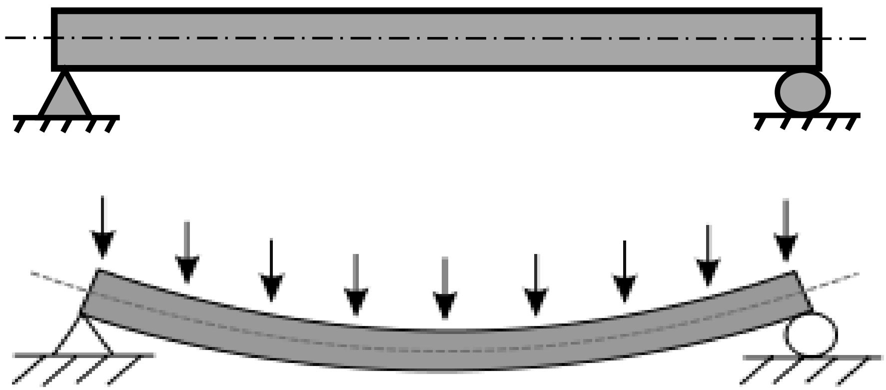


BEAM

A structural member designed to resist forces acting transverse to its axis is called a beam.

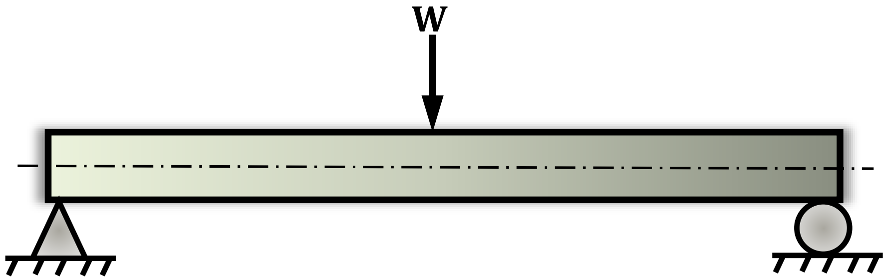
The analysis of beams involves the determination of Shear Force, Bending Moment and the Deflection of beam at various sections.

Bending: The deformation of a member produced by loads acting perpendicular to its axis as well as couples acting in a plane passing through the axis of the members.

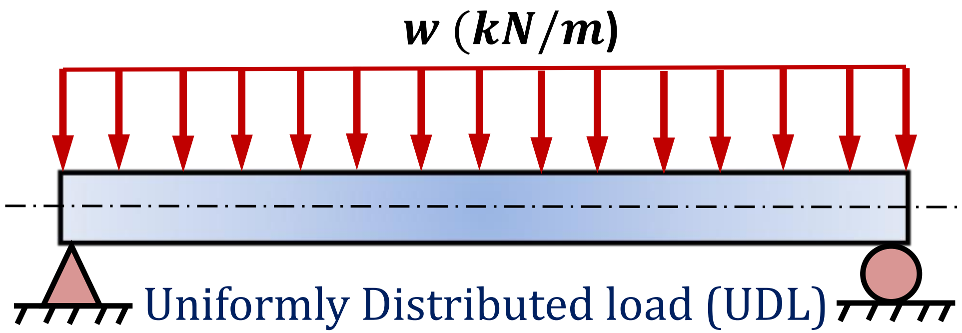


Different Types of Loads

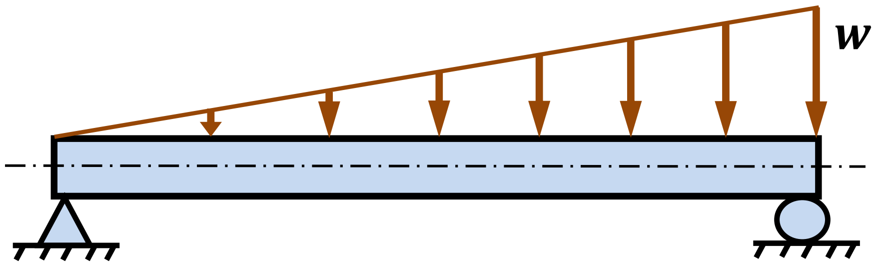
Point load:



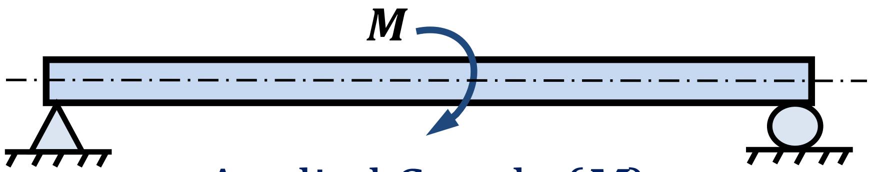
Distributed load



Uniformly Distributed load (UDL)



Uniformly Varying Load(UVL)



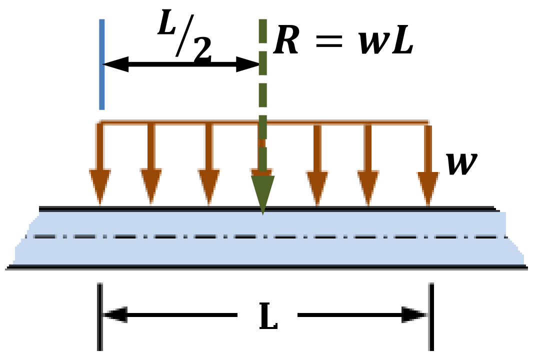
Applied Couple (M)

Applied Couple

Distributed Loads on Beams

How to find the Net Force (R) acting on the beam

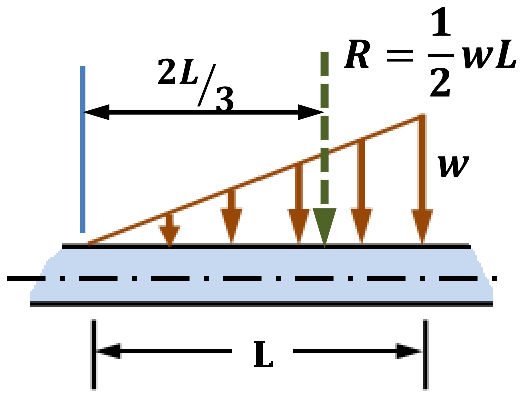
- $R =$ Area under the loading diagram
- R acts through the centroid of the area



Uniformly distributed load (UDL)

Area under the loading diagram, $R = wL$

R, acts at $L/2$, i.e. the centroid of the area



Uniformly varying load (UVL)

Area under the loading diagram,

$R = \frac{1}{2}wL$ and R acts at $2L/3$, i.e. the centroid of the area

Distributed Loads on Beams

Combination of Uniformly Distributed and Varying Loads

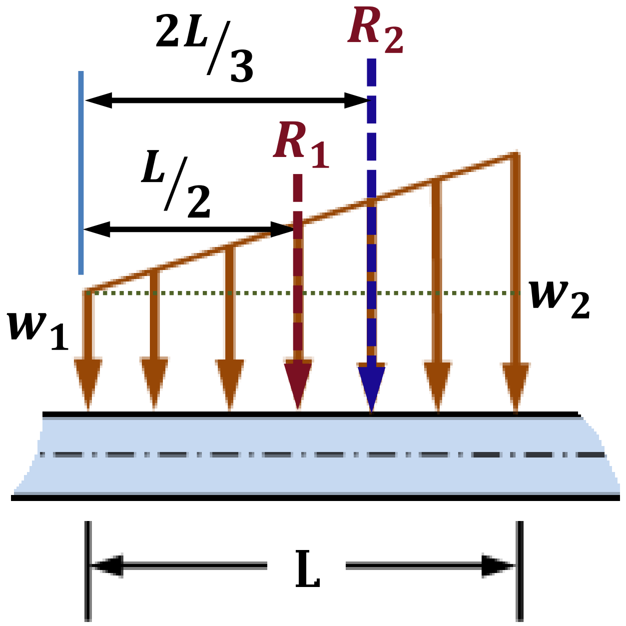
The load divided into two loads, i.e.

R_1 = rectangular load and

R_2 = triangular load

$R_1 = w_1 L$ and R_1 acts at $L/2$,
i.e. the centroid of the rectangular area

$R_2 = \frac{1}{2}(w_2 - w_1)L$ and R_2 acts at $2L/3$,
i.e. the centroid of the triangular area



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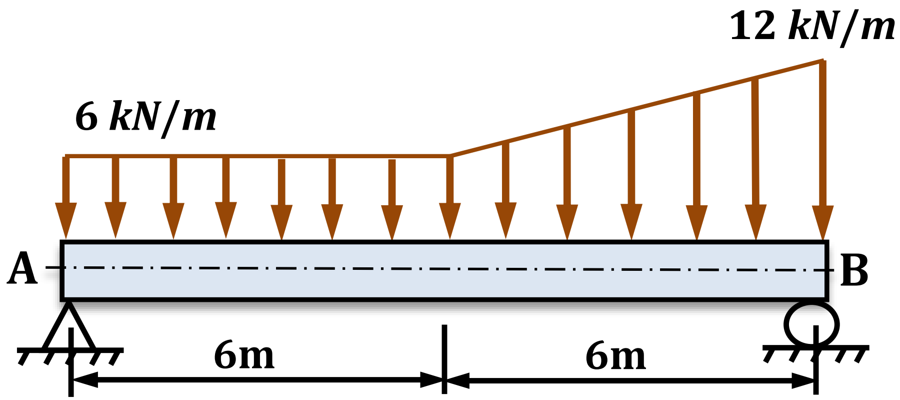
Solved Problem: Distributed Loads



- **Load is divided into two loads, i.e.**

  R_1 = rectangular load, and R_2 = triangular load

  $R_1 = w_1 L = 6 \times 12 = 72\ kN$ and

  $R_2 = \frac{1}{2}(w_2 - w_1)L = \frac{1}{2}\ (12 - 6) \times 6 = 18\ kN$

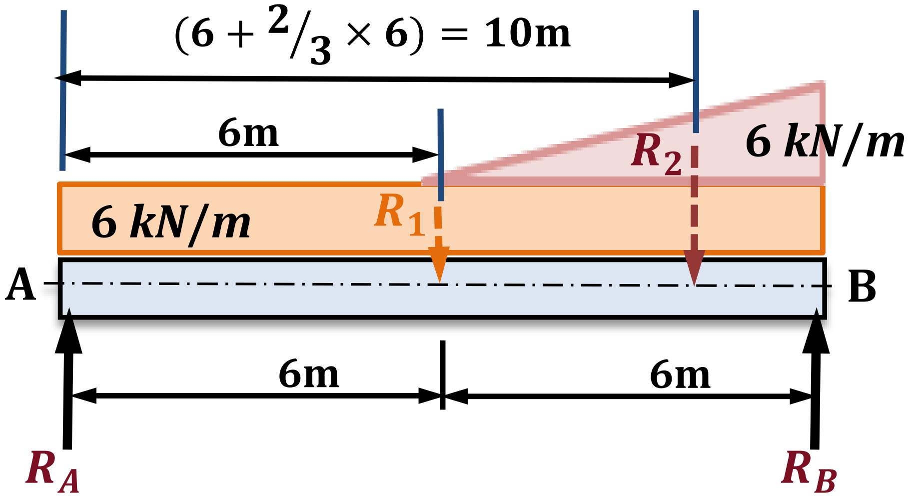
- **Determine the reactions R_A and R_A**

$R_A + R_B = (6 \times 12) + (\frac{1}{2} \times 6 \times 6) = 90\ kN$

- $\Sigma M_A = 0$

$(6 \times 12 \times 6) + \left[\left(\frac{1}{2} \times 6 \times 6\right) \times \left(6 + \frac{2}{3} \times 6\right)\right] - 12R_B\ = 0$

- **$R_B = 51\ kN$, $R_A = 39\ kN$**



Types of Beams on the Basis of Support Conditions

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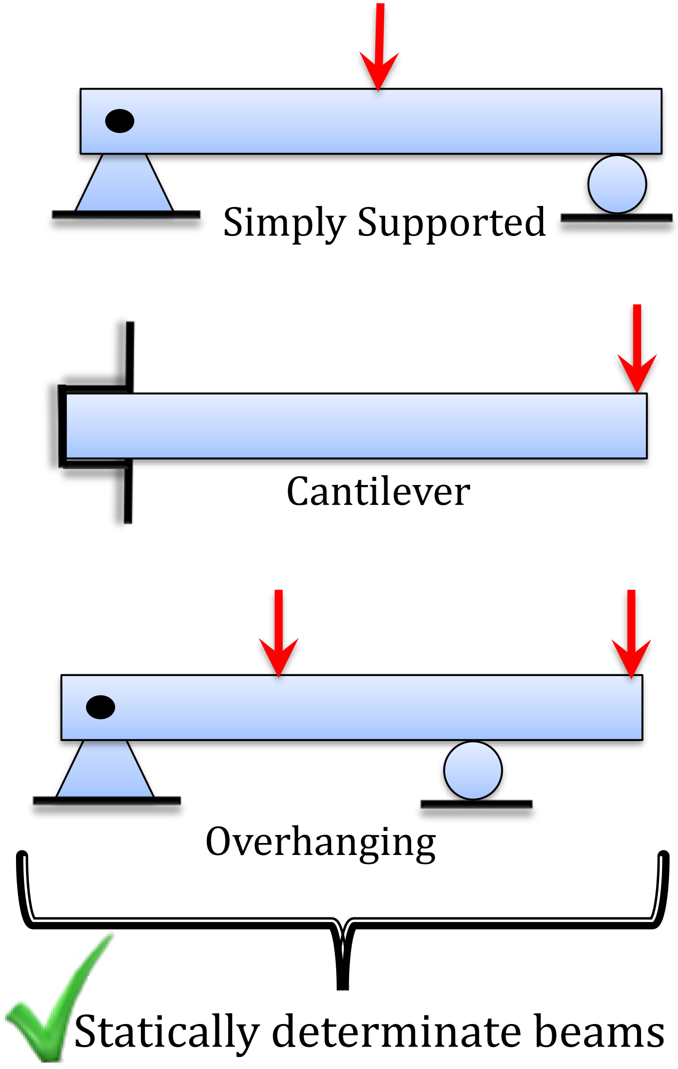
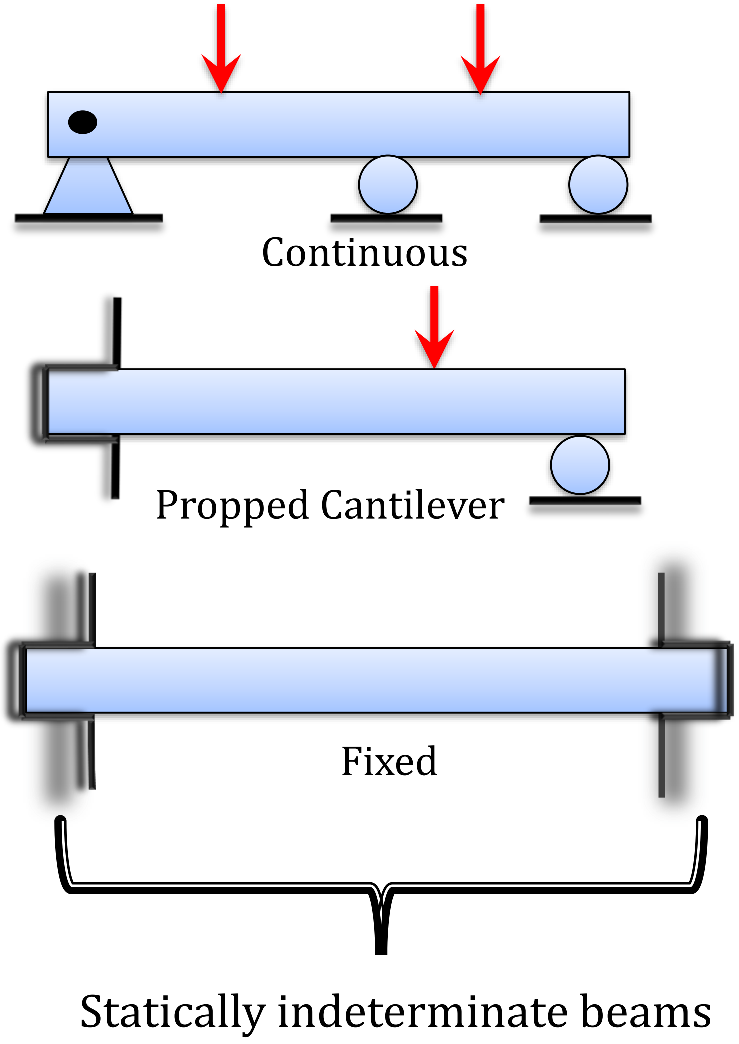
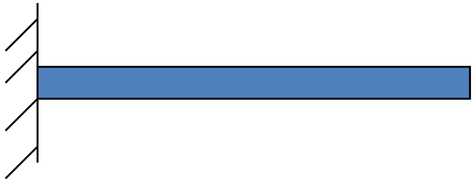




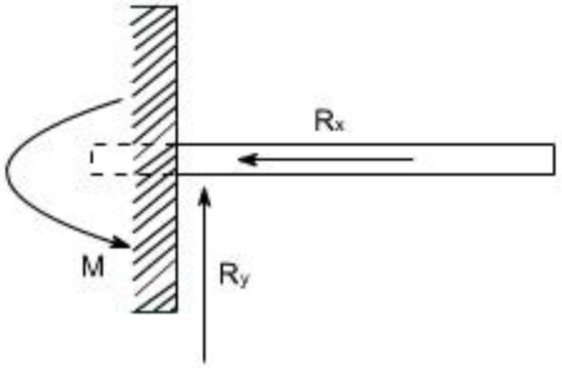
Classification of Beams



Cantilever: A cantilever beam is one whose one end is fixed and other end is free. One end of the cantilever beam is rigidly fixed and other end is free.

Disadvantage:
When the beam is loaded at one end, the moment at the fixed end is higher, if more load is applied, it can break free from the support.

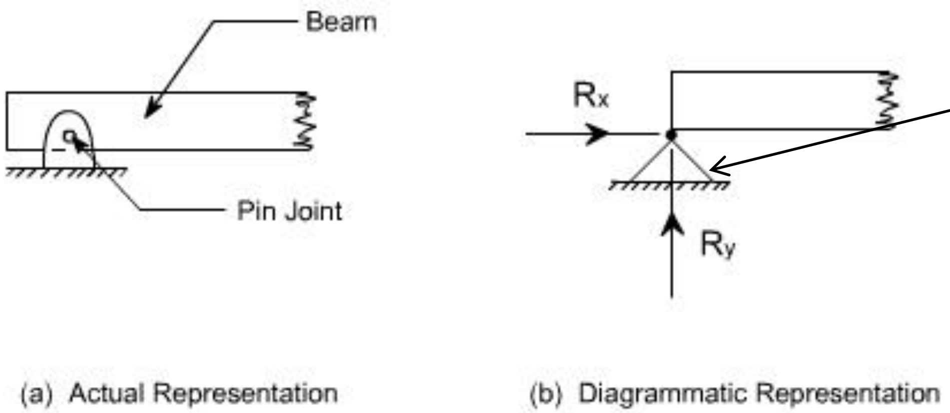
Applications:
- Parking canopies are awesome cantilevers;
- Traffic light cantilevers have a remarkable span;



Classification of Beams

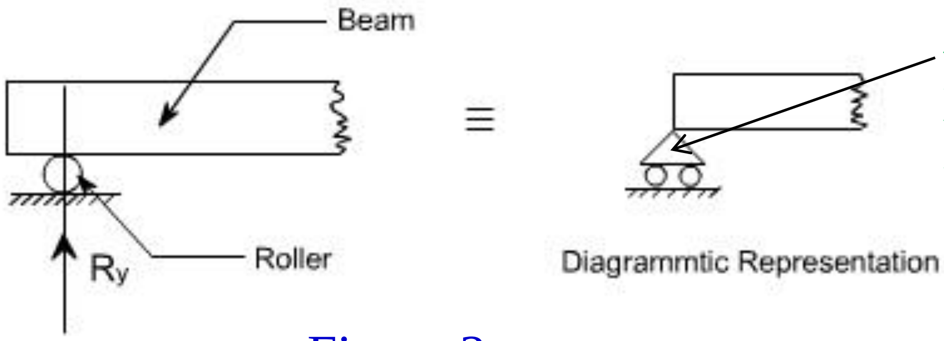
Simply Supported Beam: The beam which has a pin support at one end and a roller support at the other end.

The beams are said to be simply supported if their supports creates only the translational constraints.



The essential feature of pin support is that it restrains the beam from translating both horizontally and vertically, but it does not prevent rotation.

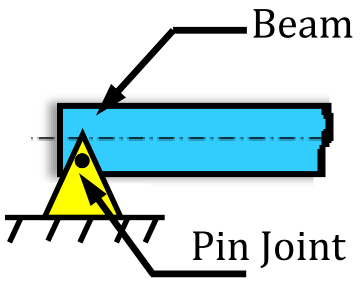
Figure 1

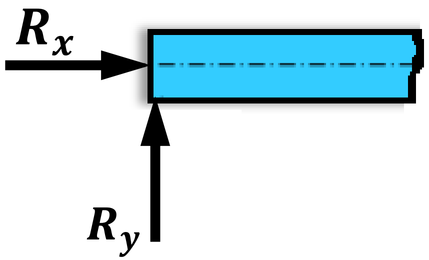


At the roller support, translation is prevented in the vertical direction but not in the horizontal direction.

Figure 2

Simply Supported Beam: The beam which has a pin support at one end and a roller support at the other end.

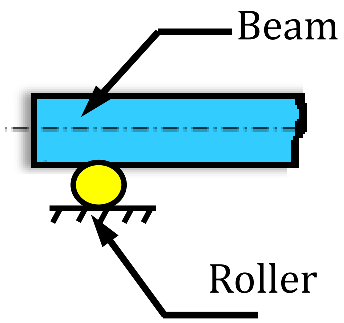


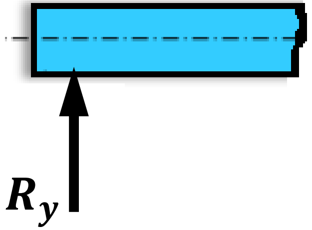


The essential feature of pin support is that it restrains the beam from translating both horizontally and vertically, but it does not prevent rotation.

Actual Representation

Diagrammatic Representation



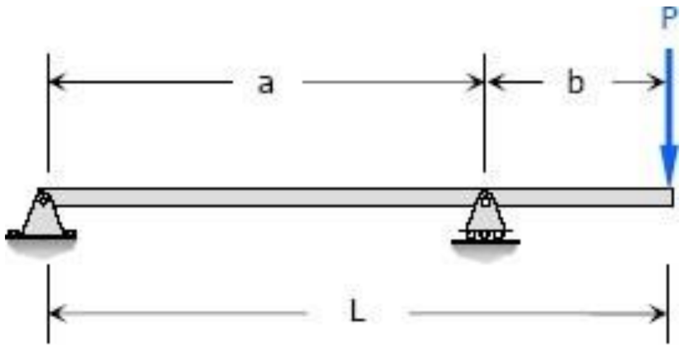


At the roller support, translation is prevented in the vertical direction but not in the horizontal direction.

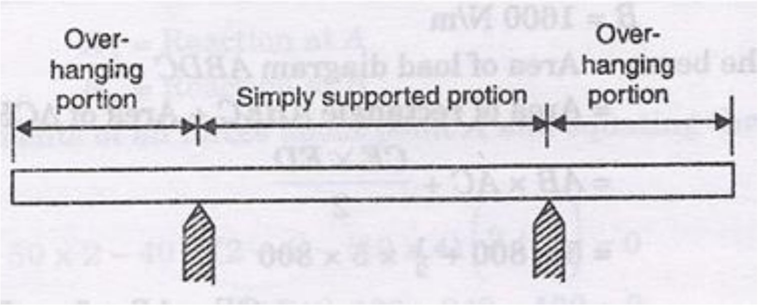
Classification of Beams

Single Over Hanging Beam: Beam freely supported at two points and having one ends extending beyond the supports.

Double Over Hanging Beam: Beam freely supported at two points and having both ends extending beyond these supports.

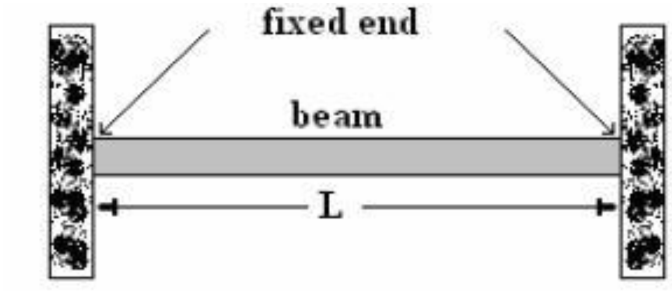


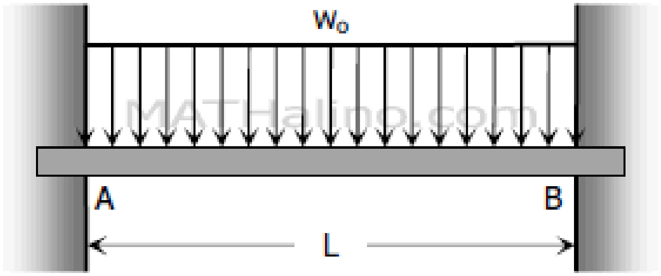
Single overhanging beam

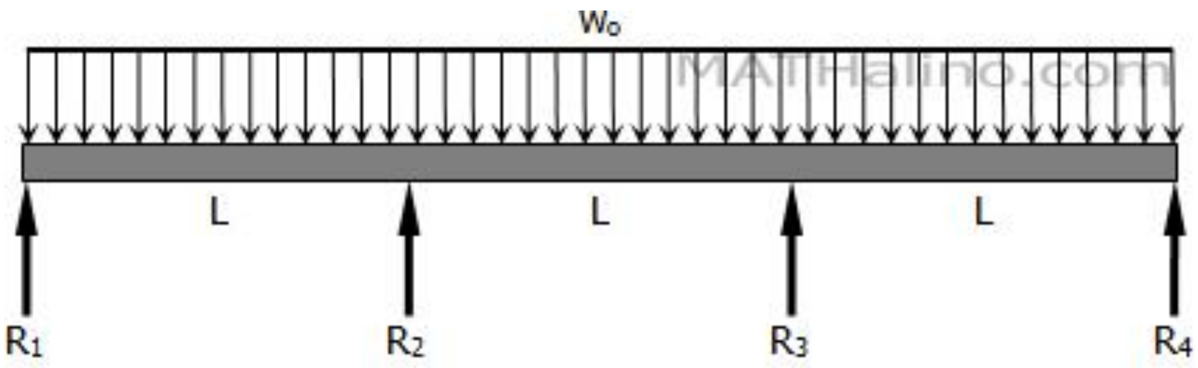


Double overhanging beam

Classification of Beams







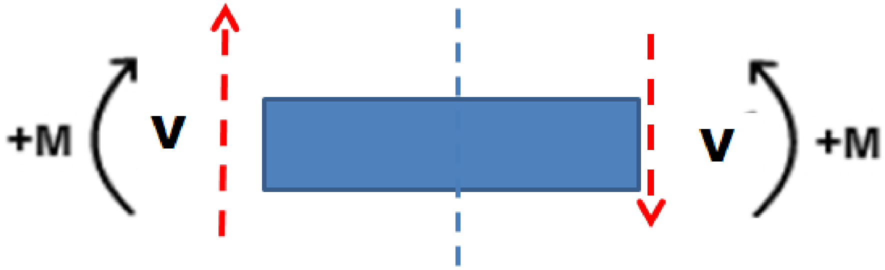
Continuous Beam: A continuous beam is one which has more than two supports. The support at the extreme left and right are called the end supports, except the extreme, are called intermediate supports.

Shear Force and Bending Moment:

Shear Force: It is the algebraic sum of the vertical forces acting to the left or right of cut section along the span of the beam.

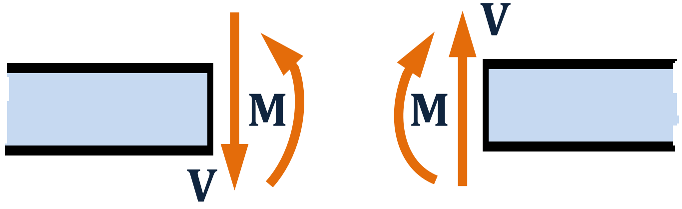
Bending Moment: It is the algebraic sum of the moment of the forces to the left or to the right of the section taken about the section.

Sign Convention

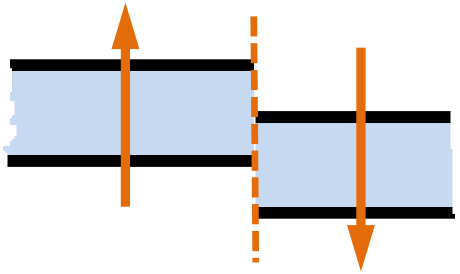


Hogging moment is negative

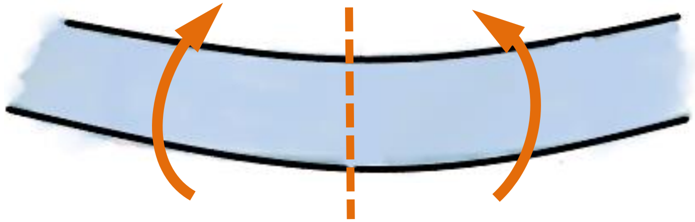
Sagging moment is positive
(beam retains water)



Internal forces at section
(negative shear and positive bending moment)



Effect of external forces
(positive shear)



Effect of external forces
(positive bending moment)

Analysis Procedure

The method of sections can be used to determine the internal loads on the cross section of a member by the procedure as below:

Support Reactions

Determine the support reactions

Free-Body Diagram

- Pass an imaginary section through the member perpendicular to its axis at the point where the internal loadings are to be determined.
- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it, and indicate the components of the internal force and couple moment resultants at the cross section according to positive sign convention.

Analysis Procedure

Equations of Equilibrium

Forces and moments should be summed at the section. This way the equations for shear forces and the moments can be obtained.

Shear force and Bending moment

Substitute values of ‘x’ in the equations to obtain the magnitude of Shear Force and Bending Moment at various points where there is a change in the loading on the beam.

CANTILEVER BEAM

Shear Force and Bending Moment Diagrams

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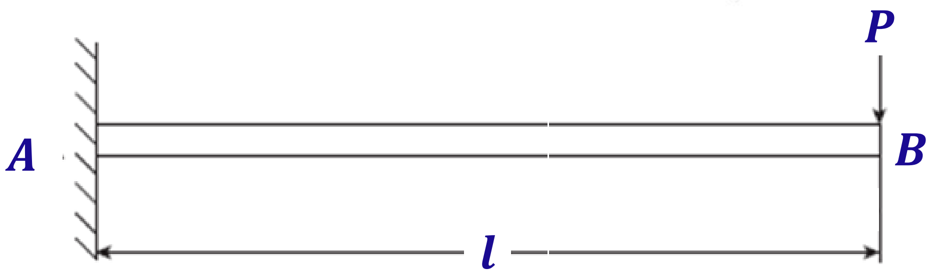
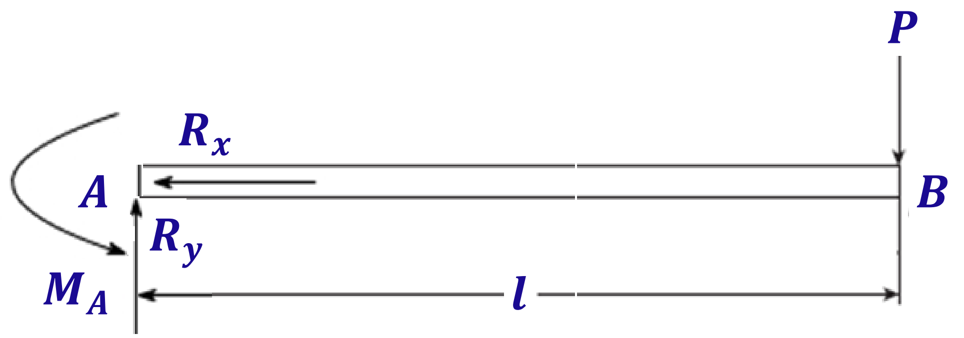
Draw Shear Force and Bending Moment Diagrams for a cantilever beam with a point load ***P*** acting at the free end.



Solution:

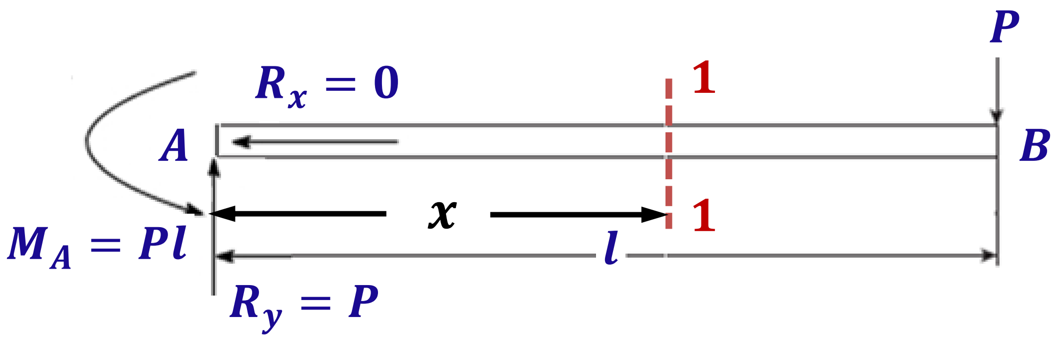
1.Draw FBD of the entire beam, and find reactions.



2. $R_y = P;\ \ R_x = 0;\ \ \ M_A = P.l;$

3.Take a section 1-1 at a distance x somewhere

between **A** and **B**



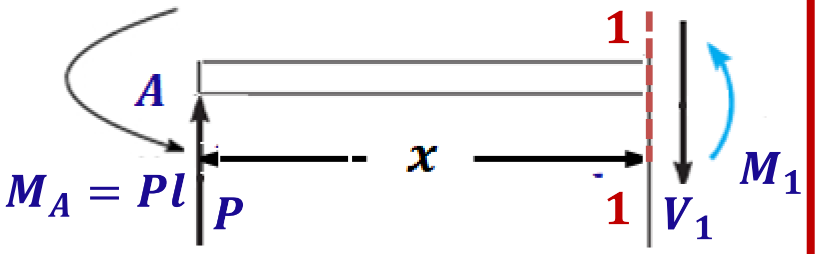
4. Draw FBD of LHS or RHS of the section.

4. $\Sigma F_y = 0$, $\boldsymbol{V_1 = P}$ ......(1)

[Eq. for S.F. for entire length of cantilever beam where $(0 \leq x \leq \boldsymbol{l})$]

5. $\Sigma M_{1-1} = 0$, $P.x - M_A - M_1 = 0$; $\boldsymbol{M_1 = P.x - P.l}$.........(2)

[Eq. for B.M. for entire length of cantilever beam where $(0 \leq x \leq \boldsymbol{l})$]



To find S.F. and B.M.

After obtaining equations for S.F. and B.M. , put values of $\boldsymbol{x}$ in these equations to obtain the magnitude of shear force and bending moment at all the salient points ,

At **A, $\boldsymbol{x}$ = 0**, and at $\boldsymbol{B, x = l}$

From eq. (1), S.F. at A, and B, $\boldsymbol{V_A = V_B = P}$, **since $\boldsymbol{V_1}$ is independent of $\boldsymbol{x}$, it will be constant throughout.**

To plot S.F. and B.M. diagrams

$V_A = V_B = P,$

since V_1 is independent of x,

it will be constant throughout.

$M_1 = P.x - P.l$ ....(2)

From eq. (2), B.M. at A,

$M_A(x = 0) = P.0 - P.l = -Pl$

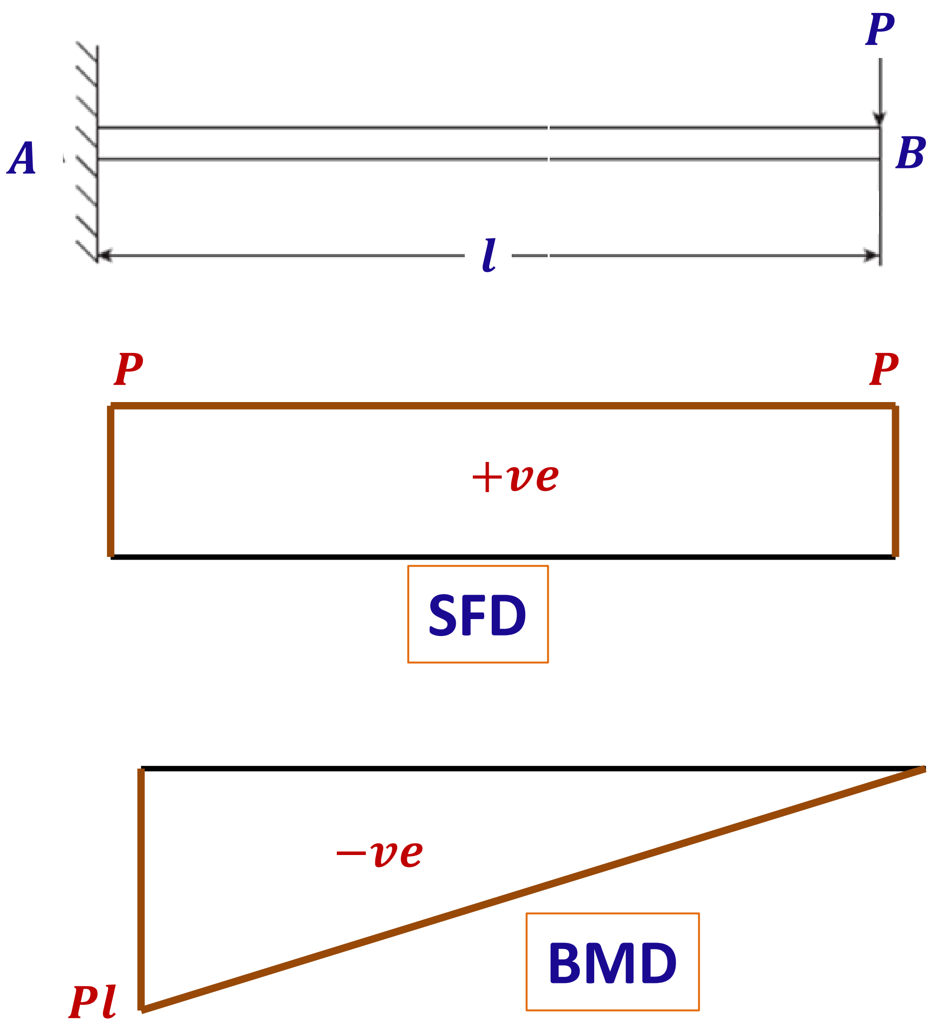
and B.M. at B,

$M_B(x = l) = P.l - P.l = 0,$

$M_A = -Pl$

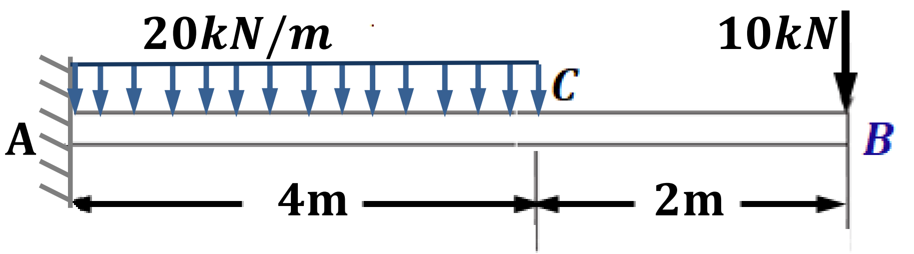
$M_B = 0,$

Variation of B. M. will be linear.



Problem

Draw SF and BM diagrams for the given cantilever beam loaded as shown.

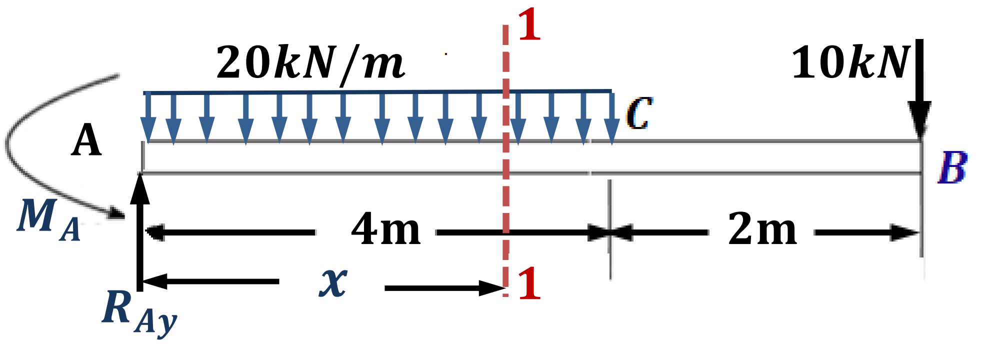


Solution:

1. Draw FBD of the beam, and find reactions.

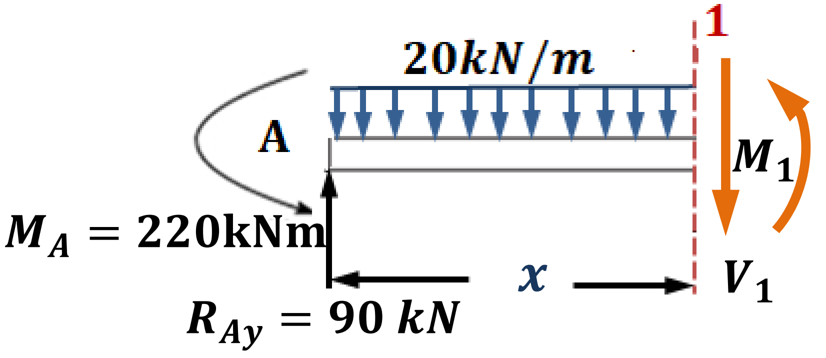
$$R_{Ay} = 10 + 20 \times 4 = 90\ kN;$$

$$M_A = (20 \times 4 \times 2) + (10 \times 6) = 220\ kNm;$$



2. Take a section 1-1 at a distance x somewhere between **A** and **C** and

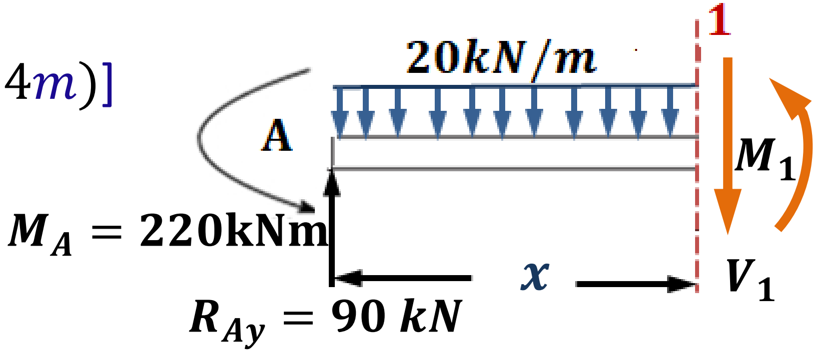
Draw FBD of LHS or RHS of the section.



4. $\Sigma F_y = 0, \quad V_1 - 90 + 20x = 0;$

$\boldsymbol{V_1 = 90 - 20x}$ ....(1)

[Eq. for S.F. for the first segment of the Cantilever beam where ($0 \leq x \leq 4m$)]

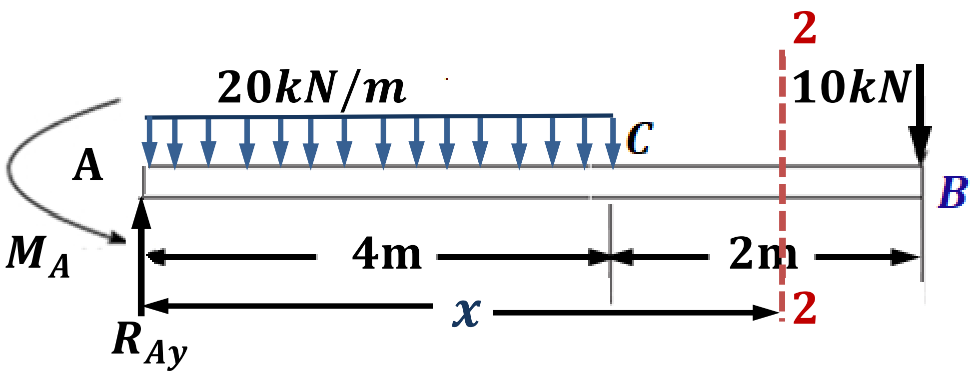


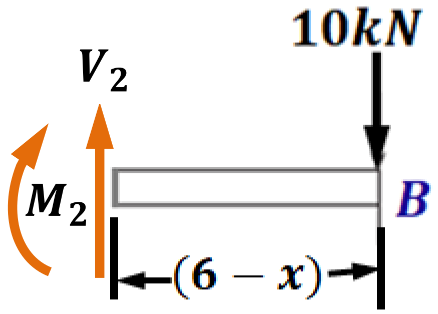
5. $\Sigma M_{1-1} = 0, \quad 90.x - M_A - 20.x.\frac{x}{2} - M_1 = 0;$

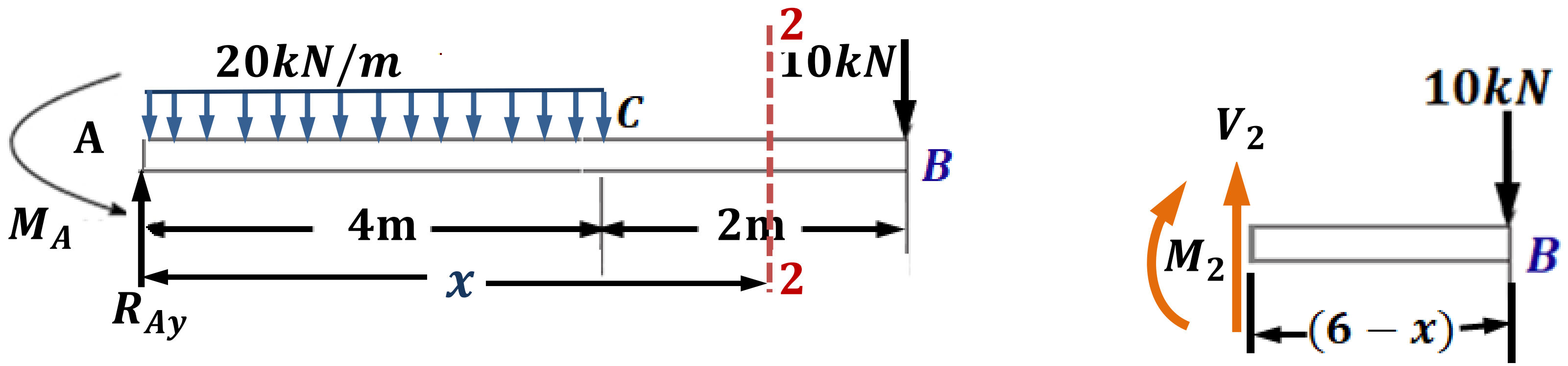
$\boldsymbol{M_1 = 90.x - 220 - 10.x^2}$ ....(2)

[Eq. for B.M. for the first segment of the Cantilever beam where ($0 \leq x \leq 4m$)]

6. Take another section 2-2 between ***C*** and ***B***, at a distance ***x*** from point ***A and draw FBD of the RHS of the section.***







7. $\Sigma F_y = 0,\quad V_2 - 10 = 0;$ $\boldsymbol{V_2 = 10}$ ....(3)

[Eq. for S.F. for the IInd segment of the Cantilever beam where $(4m \le x \le 6m)$]

8. $\Sigma M_{2-2} = 0,\quad M_2 + 10.(6-x) = 0;$ $\boldsymbol{M_2 = -10(6-x)}$ ....(4)

[Eq. for B.M. for the first segment of the Cantilever beam where $(4m \le x \le 6m)$]

To find S.F. and B.M.

After obtaining equations for S.F. and B.M., put values of $\boldsymbol{x}$ in these equations to obtain the magnitude of shear force and bending moment at all the salient points , i.e. at **A**, $\boldsymbol{x}$ **=0**, at **C**, $\boldsymbol{x = 4m}$ and at **B**, $\boldsymbol{x = 6m}$

$V_1 = 90 - 20x$.........(1) $(0 \le x \le 4m)$

$V_2 = 10$ ....(3) $(4m \le x \le 6m)$

$M_1 = 90.x - 220 - 10.x^2 \ldots\ldots\ldots\ldots(2)$ $(0 \le x \le 4m)$

$M_2 = -10(6 - x)$ ....(4) $(4m \le x \le 6m)$

To find S.F. and B.M.

at ***A***, x =0, at ***C***, $x = 4m$ and at ***B***, $x = 6m$

From eq. (1), S.F. at ***A***, x =0, $V_A = 90\ kN$

and ***C***, $x = 4m$, $V_C = 10\ kN$, **variation of S.F. will be linear.**

and ***C*** **and** ***B***, $x = 4m$, $V_C = V_B = 10\ kN$ **S.F. will be constant from C to B.**

From eq. (2), B.M. at ***A***, $M_A(x = 0) = -220\ kNm$;

B.M. at **C**, $M_C(x = 4m) = 90 \times 4 - 220 - 10 \times 4^2 = -20\ kNm$, **variation of B. M. will be parabolic.**

and B.M. at **C**, $M_C(x = 4m) = -10(6 - 4) = -20\ kNm$ **(from eq. 4)**

and B.M. at **B**, $M_B(x = 6m) = -10(6 - 6) = 0$ **variation will be linear.**

To find S.F. and B.M.

$V_A = 90\ kN,\ V_C = 10\ kN,$

variation of S.F. will be linear from A to C.

$V_C = V_B = 10\ kN$ S.F. will be constant from C to B.

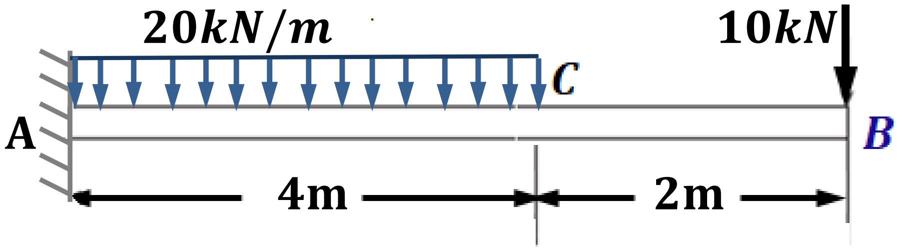
$M_A(x = 0) = -220\ kNm;$

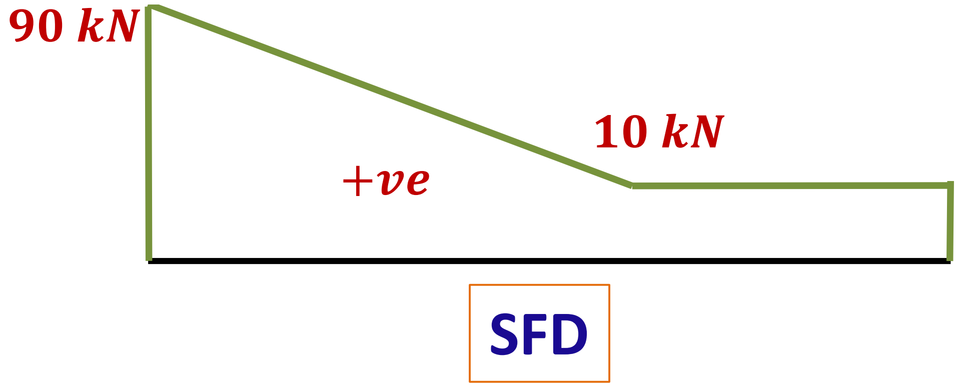
$M_C(x = 4m) = -20kNm,$

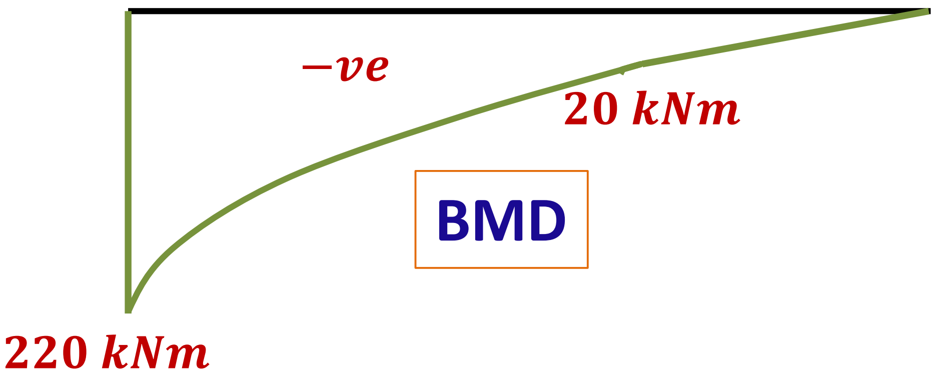
variation will be parabolic from A to C.

$M_C(x = 4m) = -20\ kNm$

$M_B(x = 6m) = 0$ variation will be linear from C to B.







SIMPLY SUPPORTED BEAM

Problem: Draw shear force and bending moment diagrams for a simply supported beam with a point load **P** acting at the mid span.

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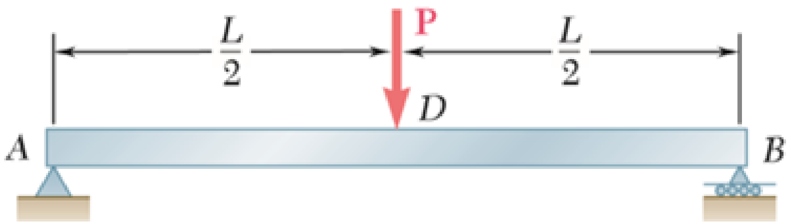
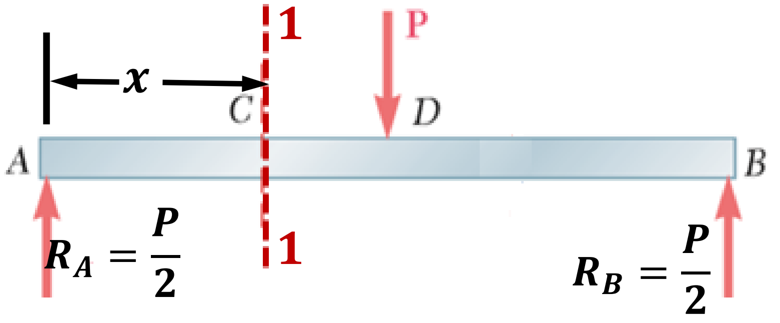


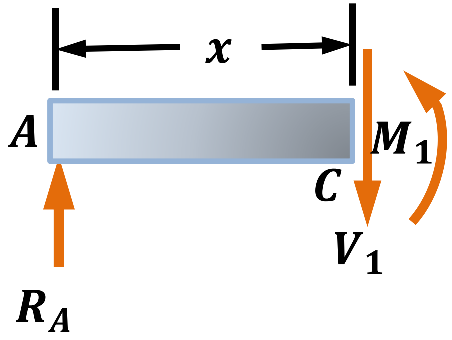
Solution:

1.Draw FBD of the beam, and find reactions.

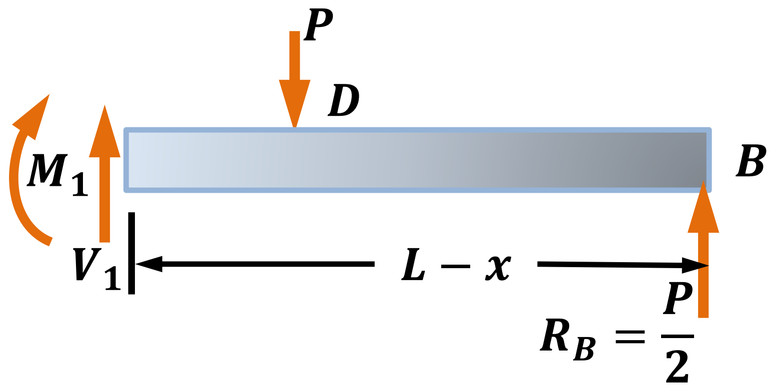
2. $R_A = R_B = \frac{P}{2}$;



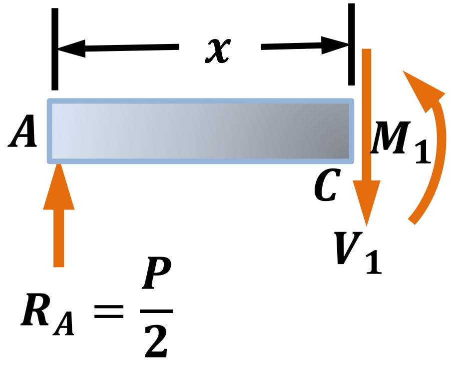
3.Take a section at a distance x somewhere between **A** and **D** and draw FBD of LHS or RHS of the section.



FBD of LHS of the section



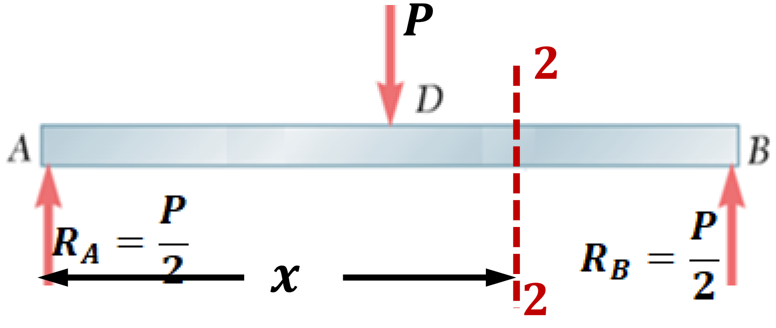
FBD of RHS of the section

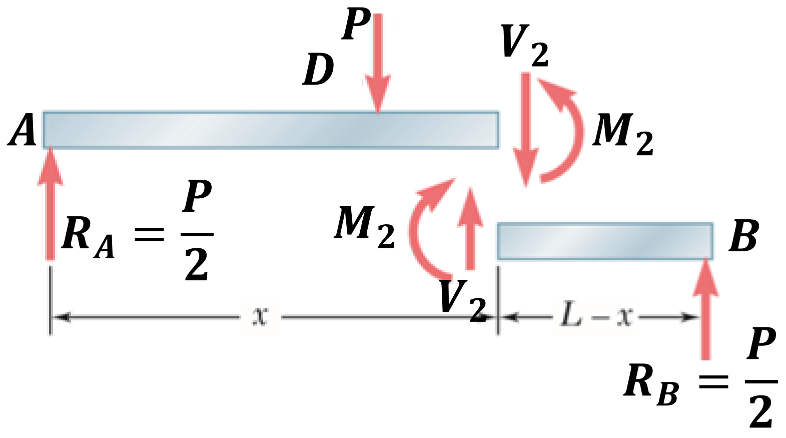


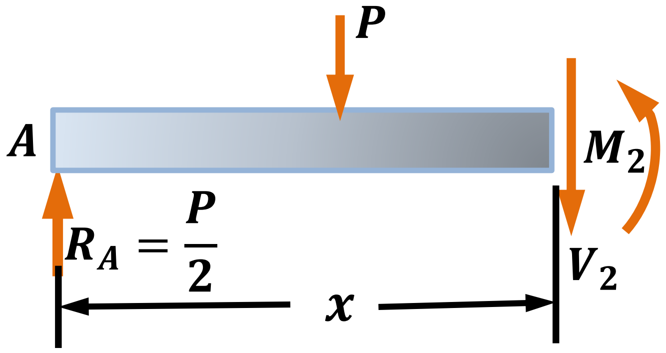
4. $\Sigma F_y = 0,$ $\boldsymbol{V_1 = \frac{P}{2}}$ ......(1) [Eq. for S.F. for the *Ist* segment of beam where $(0 \leq x \leq \frac{L}{2})$]

5. $\Sigma M_C = 0,$ $\frac{P}{2}x - M = 0;$ $\boldsymbol{M_1 = \frac{P}{2}x}$ ....(2) [Eq. for B.M. for *Ist* segment of beam where $(0 \leq x \leq \frac{L}{2})$]

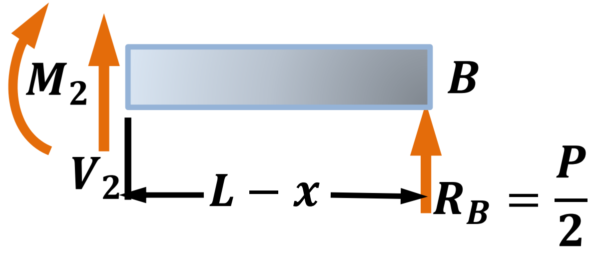
6. Similarly take another section between ***D*** and ***B*** (*IInd* segment) and draw FBD of LHS or RHS of the section.



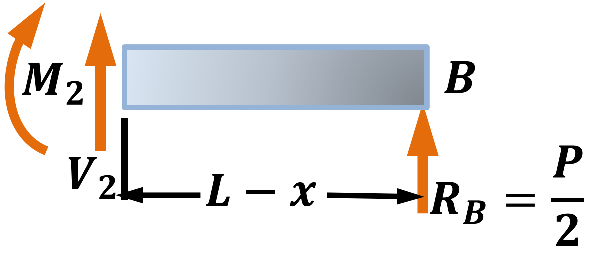




FBD of LHS of the section



FBD of RHS of the section



FBD of RHS of the section

7. $\Sigma F_y = 0,$ $\quad V_2 + \frac{P}{2} = 0;$ $\quad \boldsymbol{V_2 = -\frac{P}{2}}$ ……(3)

[Eq. for S.F. for the *IInd* segment of the beam, where $(\frac{L}{2} \leq x \leq L)$]

8. $\Sigma M_2 = 0,$ $\quad M_2 - \frac{P}{2}(L - x) = 0;$ $\quad \boldsymbol{M_2 = \frac{P}{2}(L - x)}$ ……(4)

[Eq. for B.M. for *IInd* segment of the beam, where $(\frac{L}{2} \leq x \leq L)$]

$V_1 = \frac{P}{2}$ ......(1), $M_1 = \frac{P}{2}x$ ......(2) $(0 \leq x \leq \frac{L}{2})$

$V_2 = -\frac{P}{2}$ ......(3), $M_2 = \frac{P}{2}(L - x)$ ......(4) $(\frac{L}{2} \leq x \leq L)$

To find S.F. and B.M.

After obtaining equations for S.F. and B.M. , put values of x in these equations to obtain the magnitude of shear force and bending moment at all the salient points , i.e. at A, $x = 0$, at D, $x = \frac{L}{2}$ and at B, $x = L$

From eq. (1), S.F. at A and at D, $V_A = V_D = \frac{P}{2}$, and from eq. (3), S.F. at D and at B,

$V_D = V_B = -\frac{P}{2}$, Similarly,

From eq. (2), B.M. at A, $M_A = 0$, at D, $M_D = \frac{PL}{4}$

and from eq. (4), B.M. at D, $M_D = \frac{PL}{4}$, and at B, $M_B = 0$.

$V_1 = \frac{P}{2}$ ……….(1) $V_2 = -\frac{P}{2}$……..(3)

$V_A = V_D = \frac{P}{2}$

$V_D = V_B = -\frac{P}{2}$,

$M_1 = \frac{P}{2}x$……(2), $M_2 = \frac{P}{2}(L - x)$ … (4)

$M_A = 0$,

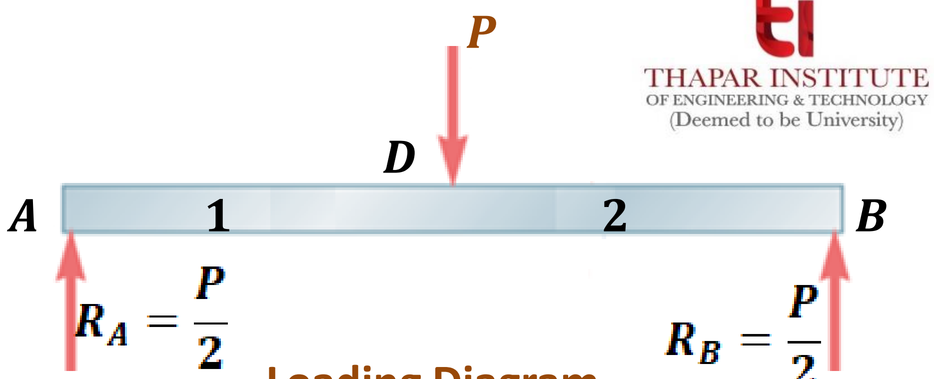
at D, $M_D = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$

B.M. at D, $M_D = \frac{P}{2}(L - \frac{L}{2}) = \frac{PL}{4}$,

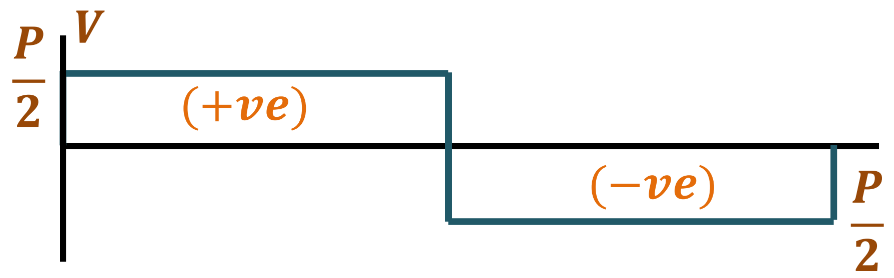
B.M. at B, $M_B = \frac{P}{2}(L - L)$;.

$M_B = 0$

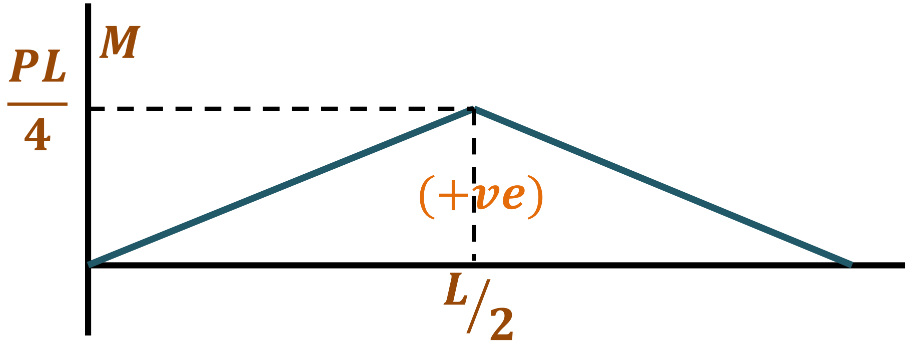
The shear force is of constant between concentrated loads, and the bending moment varies linearly;



Loading Diagram



Shear Force Diagram (SFD)



Bending Moment Diagram (BMD)

Problem: Draw the Shear Force and Bending Moment diagrams for the beam as shown.

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Solution: Draw FBD and find reactions.

$R_A = R_B = 60\ kN;$

Take a section 1-1 at a distance x somewhere between A and B and draw FBD of LHS.

$\Sigma F_y = 0; 60 - 20.x - V_1 = 0;$

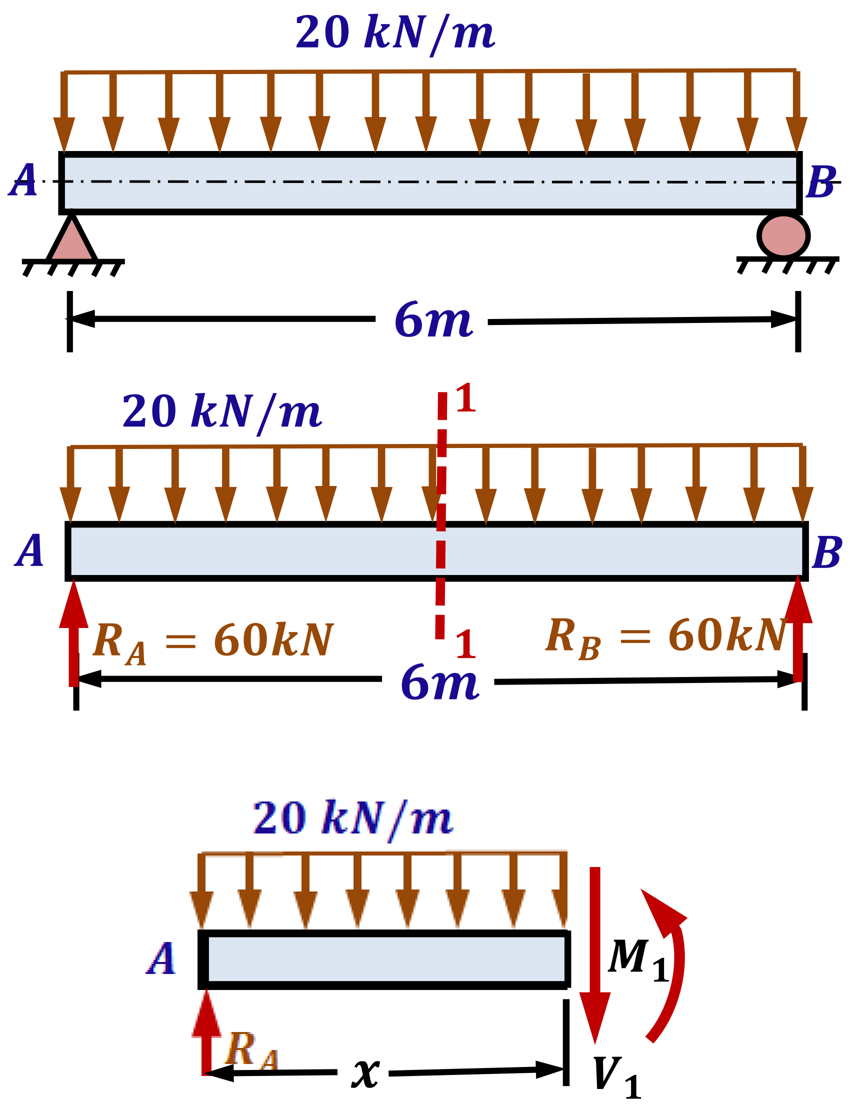
$V_1 = 60 - 20.x$......(1)

[Eq. for S.F. in the beam where ($0 \le x \le 6m$)]

$\Sigma M_{1-1} = 0;\ \ 60.x - 20.x.\frac{x}{2} - M_1 = 0;$

$M_1 = 60.x - 10.x^2$ ....(2)

[Eq. for B.M. in the beam where ($0 \le x \le 6m$)]



$V_1 = 60 - 20.x$

S.F. at A, $x = 0$, $\boldsymbol{V_A = 60\ kN}$,

at B, $x = 6m$, $\boldsymbol{V_B = -60\ kN}$,

Shear force varies linearly between points A and B, and changes from +ve to –ve.

Somewhere it will be zero. So, put

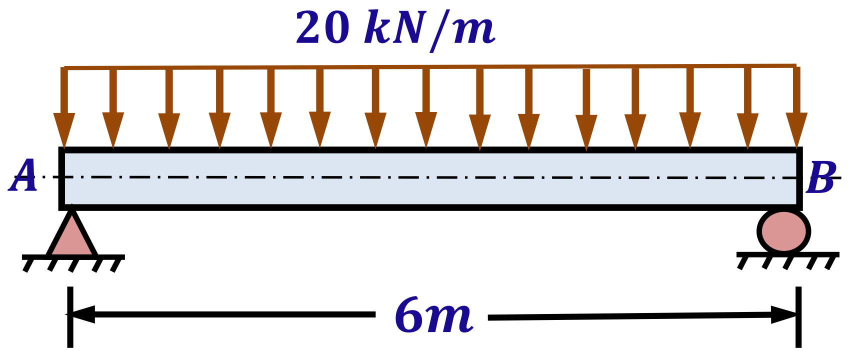
$V_1 = 0;\ 60 - 20.x = 0;$

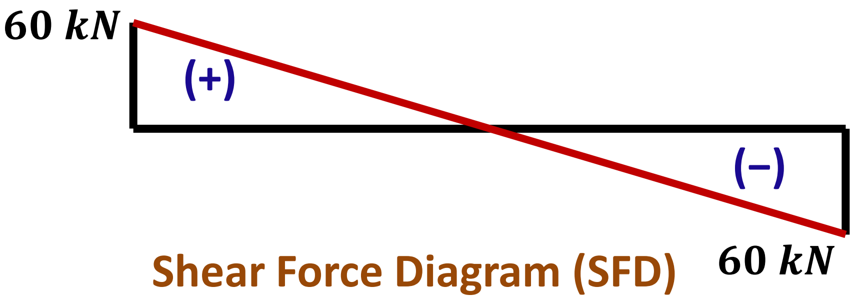
$$\boxed{\boldsymbol{x = 3m}}$$

$M_1 = 60.x - 10.x^2$

B,M. at A, $x = 0$, $\boldsymbol{M_A = 0}$,

at B, $x = 6m$, $\boldsymbol{M_B = 0}$,





Shear Force Diagram (SFD)

To find maxima of B.M., differentiate M_1 w.r.t. $\boldsymbol{x}$

and put it equal to zero. $\frac{dM}{dx} = 0;$

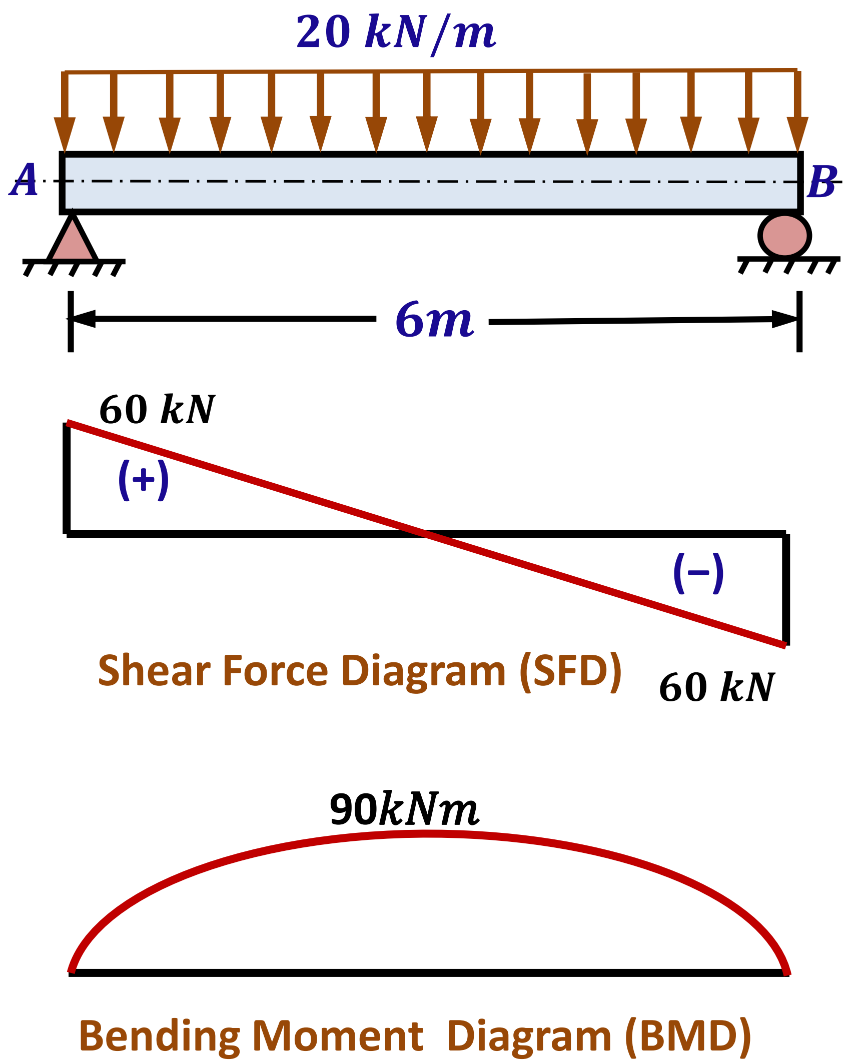
which is equal to shear force, so,

$$\frac{dM}{dx} = V;$$

$$\frac{dM}{dx} = 0; \quad 60 - 20.x = 0; \quad \boxed{x = 3m}$$

$$M_{max}(at\ x = 3m) = 60 \times 3 - 10 \times 3^2$$

$$\boldsymbol{M_{max} = 90kNm}$$



$$\frac{d^2M}{dx^2} = -20;$$

Concavity will be downward.

Relationship Between Loading, Shear Force and Bending Moment

$$w(x) = \frac{dV}{dx}$$

$$v(x) = \frac{dM}{dx}$$

At a point where *dM/dx* = 0 i.e. shear force is zero, the bending moment will have the maximum value.

Point of Contraflexure or Inflexion

In a beam if bending moment changes sign at a point, the point itself having zero bending moment, the beam changes curvature at this point of zero bending moment and this point is called the Point of Contraflexure or Inflexion.

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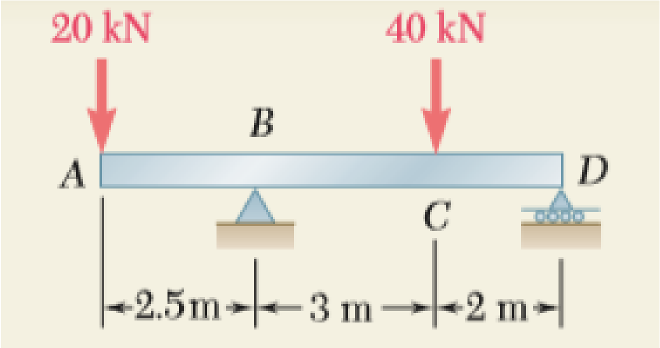
OVER HANG BEAM

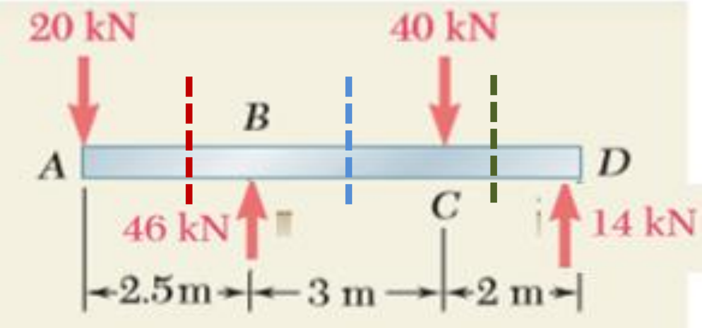
Problem: Draw the SFD and BMD for the beam and loading shown.



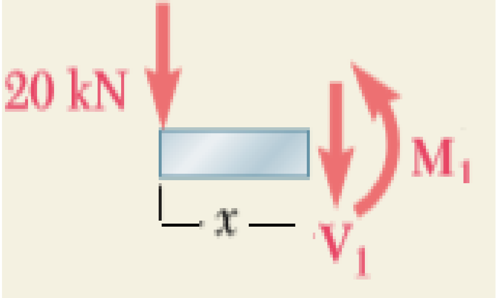
Solution:

From the FBD of the beam, find reactions.

$R_B = 46$ kN; $R_D = 14$ kN



Take a section at a distance x somewhere between A and B and draw FBD of LHS of the section.



$\Sigma F_y = 0, V_1 + 20 = 0; V_1 = -20$ kN ......(1) [Eq. for S.F. for the *Ist* segment of the beam where $(0 \le x \le 2.5m)$]

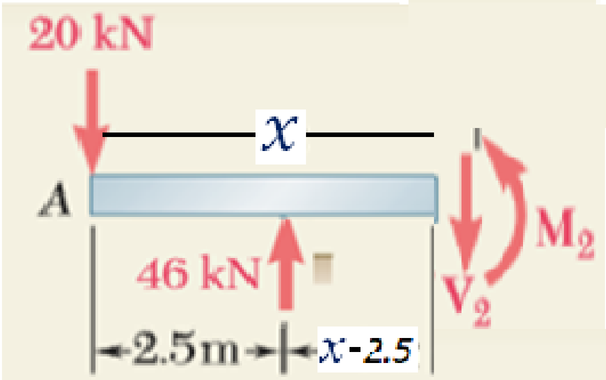
$\Sigma M_1 = 0; \ -20x - M_1 = 0; \ \ M_1 = -20x$ ....(2) [Eq. for B.M. for *Ist* segment of beam where $(0 \le x \le 2.5m)$]

Similarly take another section between *B* and *C* (*II*nd segment) and draw FBD of LHS of the section.

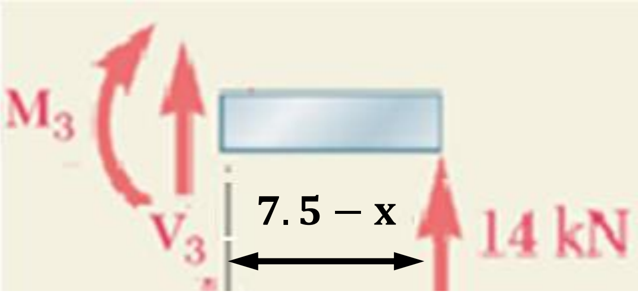
$\Sigma F_y = 0$, **20** + V_2 **–** 46 = 0; $\boldsymbol{V_2}$ **= 26 kN** ……(3)

[Eq. for S.F. for the *IInd* segment of the beam

where $(2.5m \leq x \leq 5.5m)$]



$\Sigma M_2 = 0$; $-20x + 46(x–2.5) – M_2 = 0$; $\boldsymbol{M_2 = -20x + 46(x-2.5)}$**……(4)** [Eq. for B.M. for *IInd* segment of the beam where $(2.5m \leq x \leq 5.5m)$]



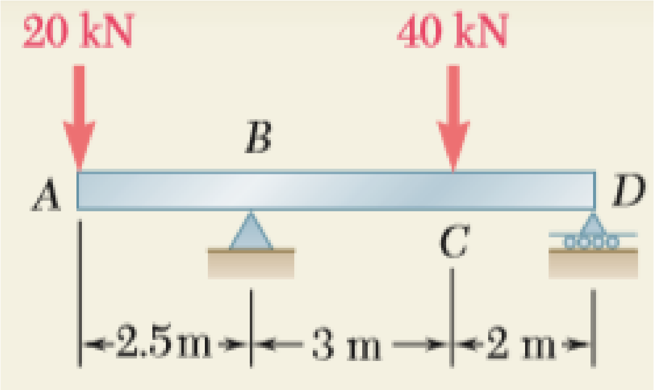
Now take third section between ***C*** and ***D*** (*III*[rd] segment) and draw FBD of RHS of the section.

$\Sigma F_y = 0$, $V_3 + 14 = 0$; $\boldsymbol{V_3}$ **= – 14kN** …(5) [Eq. for S.F. $(5.5m \leq x \leq 7.5m)$]

$\Sigma M_3 = 0$; $-14(7.5 - x) + M_3 = 0$; $\boldsymbol{M_3 = 14(7.5-x)}$**……(6)**

[Eq. for B.M. for *IIIrd* segment of beam where $(5.5m \leq x \leq 7.5m)$]

V_1 = – 20 kN …….(1) : $(0 \leq x \leq 2.5m)$
V_2 = 26 kN …….(3) : $(2.5m \leq x \leq 5.5m)$
V_3 = – 14kN …….(5) : $(5.5m \leq x \leq 7.5m)$



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To find Shear Force

Put values of x in these equations to obtain the magnitude of shear force at all the salient points , i.e. at **A, x = 0**, at **B**, $x = 2.5m$,
at **C**, $x = 5.5m$ and at **D**, $x = 7.5m$.

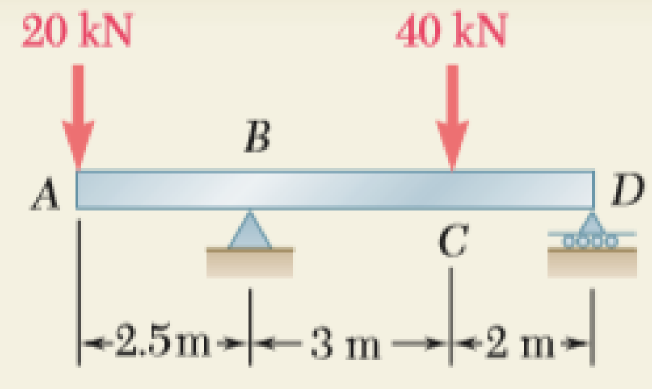
From eq. (1),

S.F. at A and at B, V_A = V_B = – 20 kN,

and from eq. (3), S.F. at B and at C, $V_B = V_C$= +26 kN,

from eq. (5), S.F. at C and at D, $V_C = V_D$= – 14kN.

$M_1 = -20x$ ....(2): $(0 \le x \le 2.5m)$
$M_2 = -20x + 46(x - 2.5)$..(4): $(2.5m \le x \le 5.5m)$
$M_3 = 14(7.5 - x)$ ....(6): $(5.5m \le x \le 7.5m)$



To find Bending Moment

Now put values of x in eqs. (2, 4, 6) to obtain the magnitude of bending moment at all the salient points , i.e. at **A**, $x = 0$,

at **B**, $x = 2.5m$, at **C**, $x = 5.5m$ and at **D**, $x = 7.5$

From eq. (2),

B.M. at A, $M_A = -20 \times 0$, $M_A = 0$, at B, $M_B = -20 \times 2.5 = -50\ kNm$,

eq. (4), B.M. at B, $M_B = -20 \times 2.5 + 46(2.5 - 2.5) = -50\ kNm$

B.M. at C, $M_C = -20 \times 5.5 + 46(5.5 - 2.5) = 28\ kNm$.

from eq. (6), B.M. at C, $M_C = 14(7.5 - 5.5) = 28\ kNm$

B. M. at D, $M_D = 14(7.5 - 7.5) = 0$.

The shear force is of constant between concentrated loads, and the bending moment varies linearly;

we therefore obtain the shear and bending-moment diagrams as shown.

$V_A = V_B = -20$ kN,

$V_B = V_C = +26$ kN,

$V_C = V_D = -14$ kN.

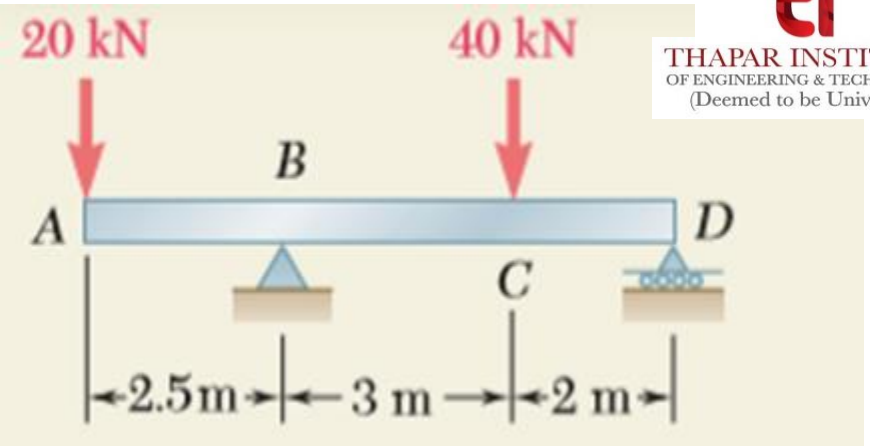
$M_A = 0$, $M_B = -50\ kNm$

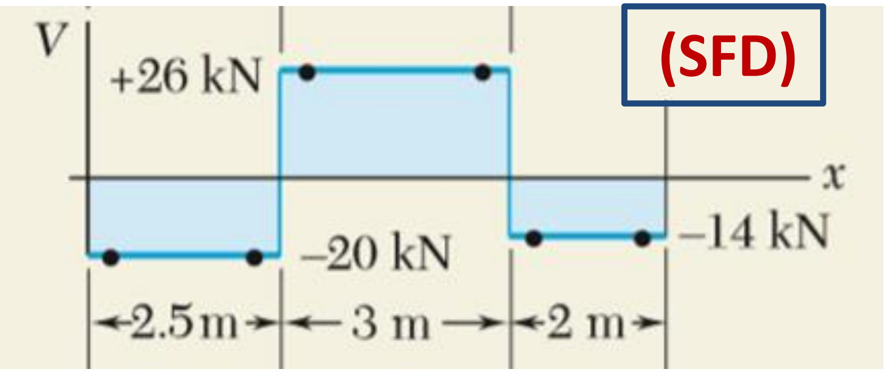
$M_C = 28\ kNm$

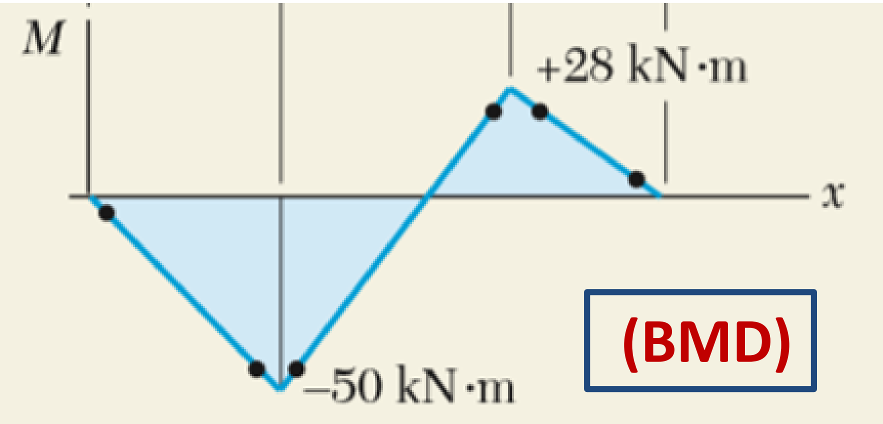
$M_D = 0$

$V_1 = -20$ kN
$V_2 = 26$ kN
$V_3 = -14$ kN

$M_1 = -20x$
$M_2 = -20x + 46(x - 2.5)$
$M_3 = 14(7.5 - x)$







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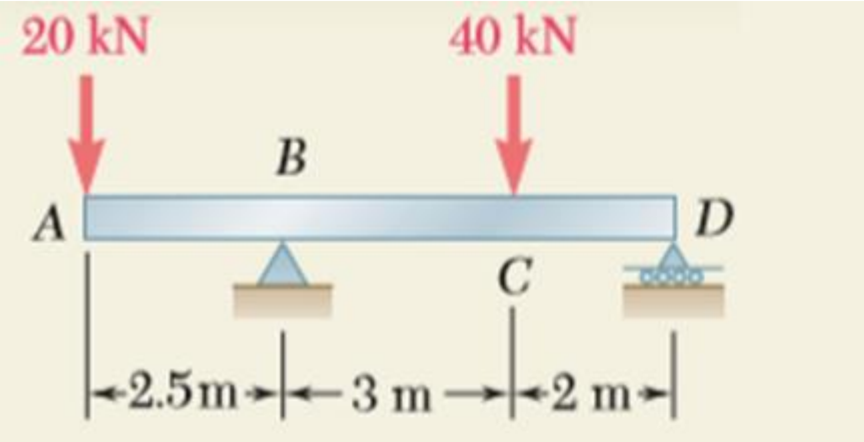
Point of Contra-flexure

In the BMD, it can be noted that bending moment changes its sign i.e. it becomes negative to positive at Point ***E.*** This point in known as *point of contra-flexure*.



To find the location of this point, put

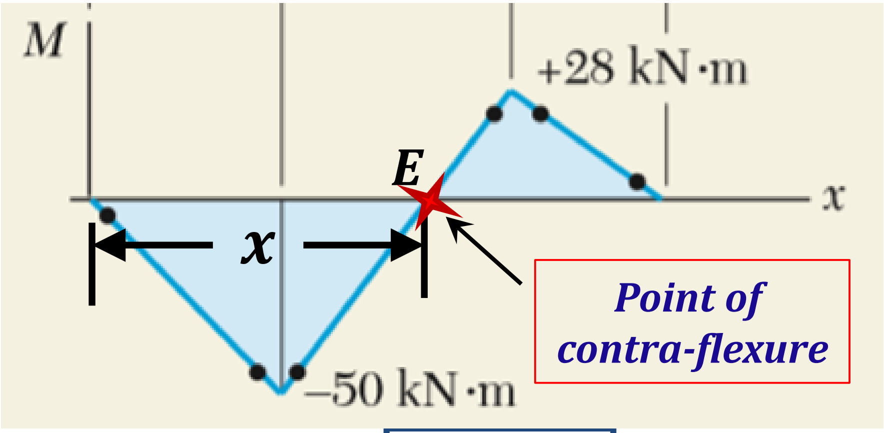
$M_2 = 0$; $M_2 = -20x + 46(x-2.5)=0$

($\because$ It is in 2nd segment)

$-20x + 46(x-2.5) = 0$

$-20x + 46x-115 = 0$

$$\boxed{x = 4.42m}$$



(BMD)

Draw SF and BM diagrams for the given overhang beam loaded as shown.

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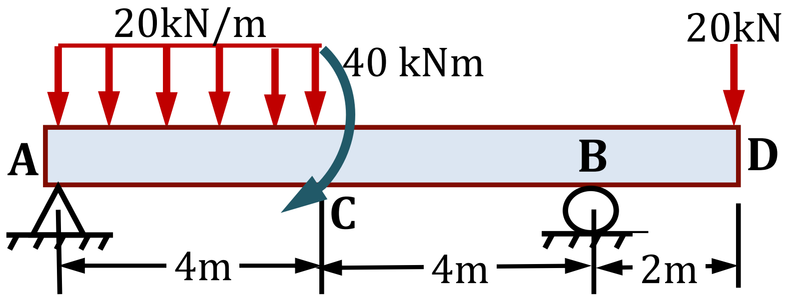


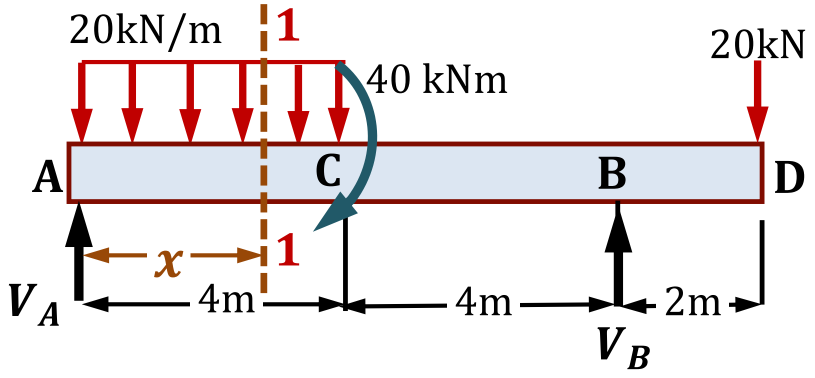
Solution:

$V_A + V_B = 20 \times 4 + 20 = 100kN;$

$\Sigma M_A = 0;$

$(20 \times 4 \times 2) + 40 + (20 \times 10) - 8V_B = 0;$

$\boldsymbol{V_B = 50kN};\qquad \boldsymbol{V_A = 50kN};$



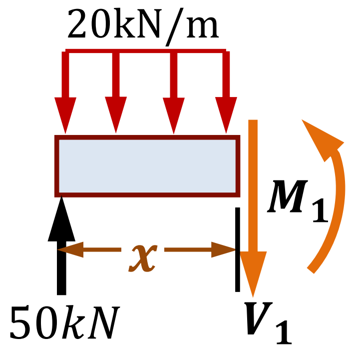
$50 - 20.x - V_1 = 0;$ $\boldsymbol{V_1 = 50 - 20.x}$......(1)

[Eq. for S.F. for the *Ist* segment where ($0 \leq x \leq 4$)]

$\Sigma M_{1-1} = 0,\;\; 50x - 20.x.\frac{x}{2} - M_1 = 0;$

$\boldsymbol{M_1 = 50x - 10x^2}$ ....(2)

[Eq. for B.M. for *Ist* segment where ($0 \leq x \leq 4$)]



FBD of LHS of the section

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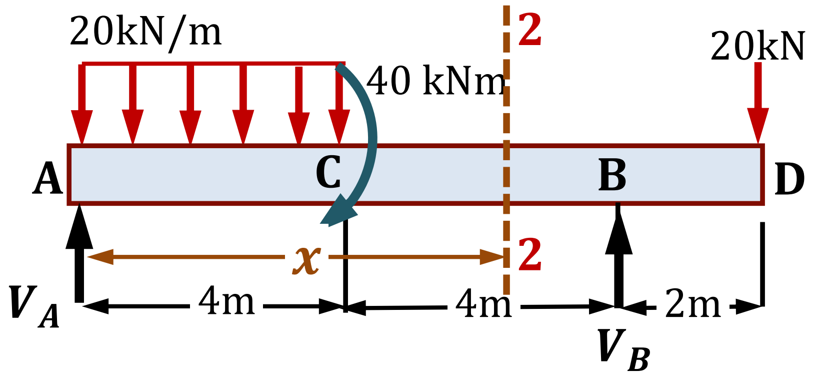
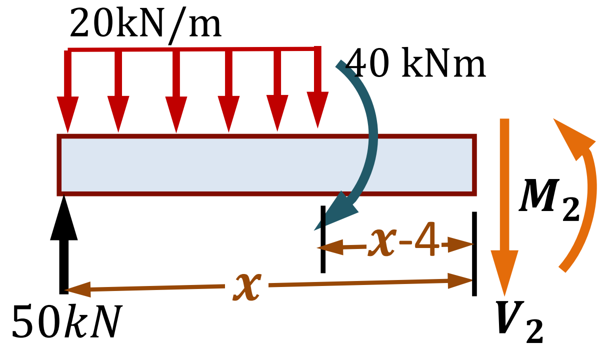




$50 - 20 \times 4 - V_2 = 0$; $\boldsymbol{V_2 = -30kN}$ **......(3)**

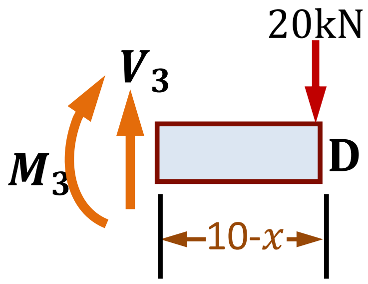
[Eq. for S.F. for the *IInd* segment where ($4 \leq x \leq 8$)]

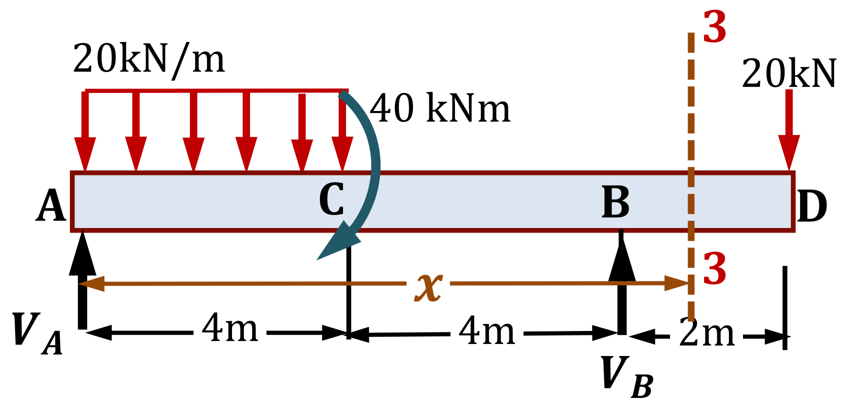
$\Sigma M_{2-2} = 0,$

$50x - 20 \times 4(x - 4 + 2) + 40 - M_2 = 0;$

$\boldsymbol{M_2 = 50x - 80(x - 2) + 40}$ **.........(4)**

[Eq. for B.M. for *IInd* segment where ($4 \leq x \leq 8$)]





$V_3 - 20 = 0$; $\boldsymbol{V_3 = 20kN}$ **......(5)**

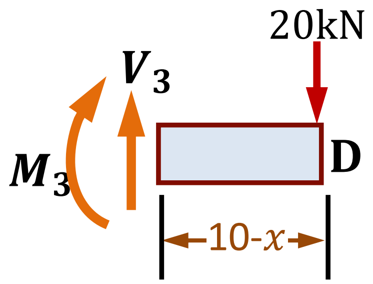
[Eq. for S.F. for the *IIIrd* segment where (8$\leq x \leq 10$)]

$\Sigma M_{3-3} = 0,$

$20(10 - x) + M_3 = 0;$

$\boldsymbol{M_3 = -20(10 - x)}$ **.........(6)**

[Eq. for B.M. for *IIIrd* segment where (8$\leq x \leq 10$)]



$\boldsymbol{V_1 = 50 - 20.x}$ **......(1);** $\boldsymbol{V_2 = -30kN}$ **......(3);** $\boldsymbol{V_3 = 20kN}$ **......(5)**

$\boldsymbol{V_A(x = 0) = 50kN;}$ $\boldsymbol{V_C(x = 4) = -30kN;}$

$\boldsymbol{V_B(x = 8) = -30kN;}$ $\boldsymbol{V_B = V_D = 20kN;}$

$\boldsymbol{M_1 = 50x - 10x^2}$ **.........(2);** $\boldsymbol{M_2 = 50x - 80(x - 2) + 40}$ **.........(4)**

$\boldsymbol{M_3 = -20(10 - x)}$ **.........(6)**

$\boldsymbol{M_A(x = 0) = 0;}$ $\boldsymbol{M_C(x = 4) = 40kNm;}$ $\boldsymbol{M_C(x = 4) = 80kNm; M_B(x = 8) = -40kNm;}$

$\boldsymbol{M_D(x = 10) = 0;}$

$V_A = 50kN;\quad V_C = -30kN;$

$V_B = -30kN;\quad V_B = V_D = 20kN;$

$V_1 = 50 - 20.x = 0;$

$x = 2.5m;$

$M_A = 0;\quad M_C = 40kNm;\quad M_C = 80kNm;$

$M_B = -40kNm;\quad M_D = 0;$

B.M. is maximum where S.F. is zero.

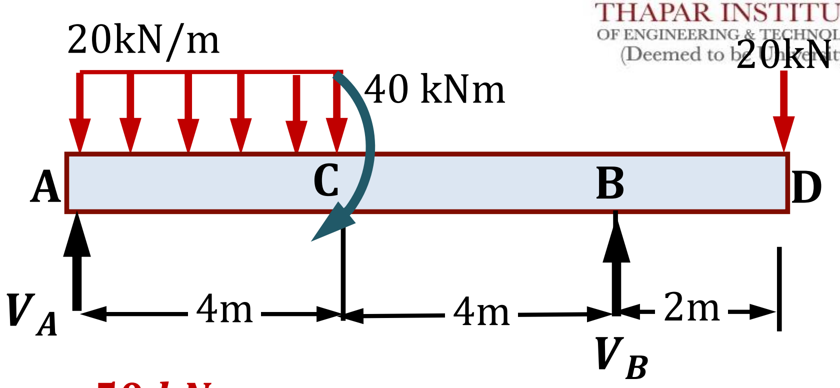
$M_{max}(x = 2.5) = 50 \times 2.5 - 10(2.5)^2$

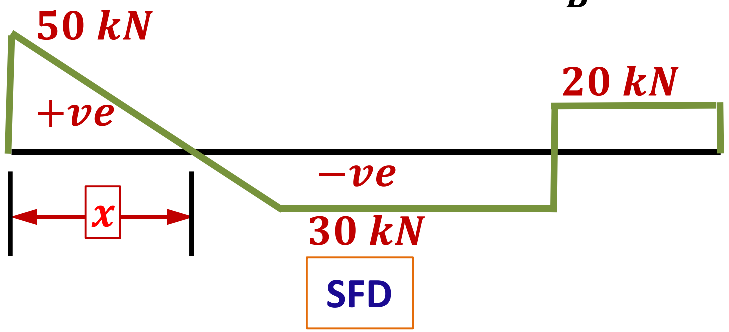
$$\boxed{M_{max} = 62.5kNm}$$

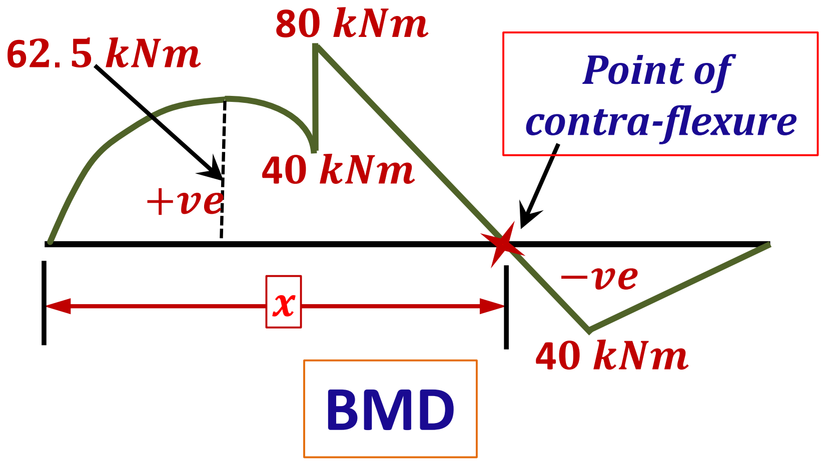
To find location of *Point of contra-flexure*

Put $M_2 = 50x - 80(x - 2) + 40 = 0$

$x = 6.67m$







Relationship Between Load, Shear Force and Bending Moment

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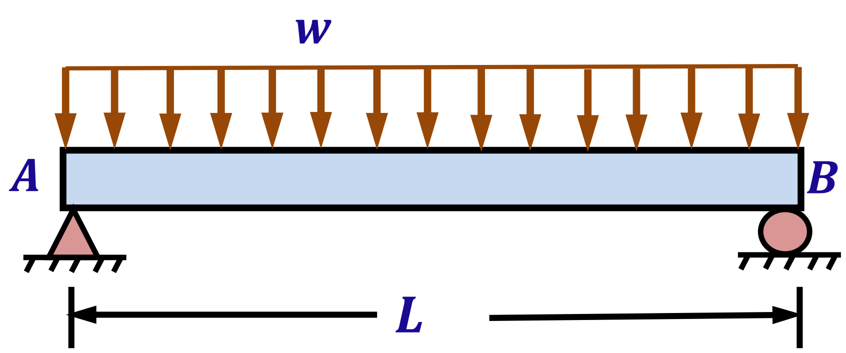
Consider a small element of beam cut out between two sections $'dx'$ distance apart.



$\Sigma F_y = 0;$

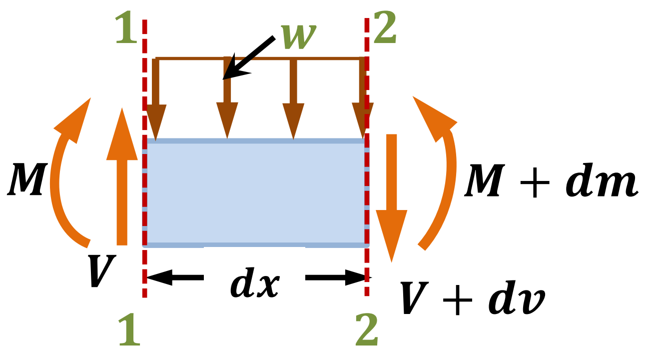
$$V - (V + dv) - wdx = 0;$$

$$dv = -wdx \ldots\ldots\ldots\ldots(1) \qquad \frac{dv}{dx} = -w$$

Integrating eq. (1) between 1 and 2

$$\int_1^2 dv = -\int_1^2 w\,dx$$

$$V_2 - V_1 = -(Area\ under\ loading\ diagram)$$



$\Sigma M_2 = 0;$

$$M - (M + dm) - wdx\frac{dx}{2} + Vdx = 0;$$

$$dm = Vdx \ldots\ldots\ldots\ldots(2) \qquad \frac{dm}{dx} = V$$

Integrating eq. (2) between 1 and 2

$$\int_1^2 dm = \int_1^2 v\,dx$$

$$M_2 - M_1 = (Area\ under\ S.F.\,diagram)$$

Ex: Draw the Shear Force and Bending Moment Diagrams for the beam and loading shown.

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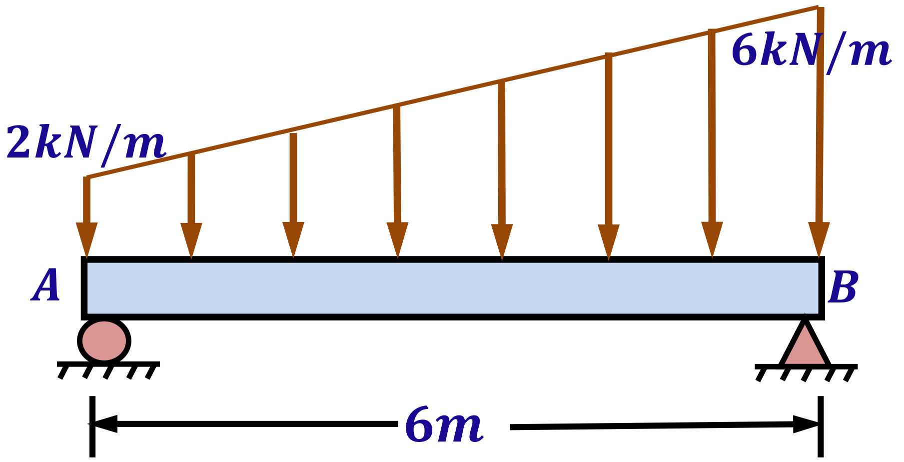


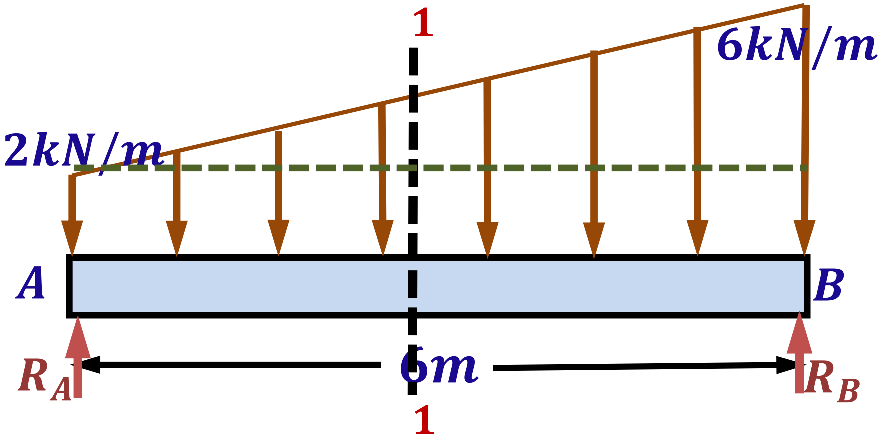
Solution: $R_A + R_B = (2 \times 6) + \left(\frac{1}{2} \times 4 \times 6\right) = \mathbf{24kN}$

$\Sigma M_A = 0;$

$2 \times 6 \times 3 + \left(\frac{1}{2} \times 4 \times 6\right) \times 4 - 6R_B = 0;$

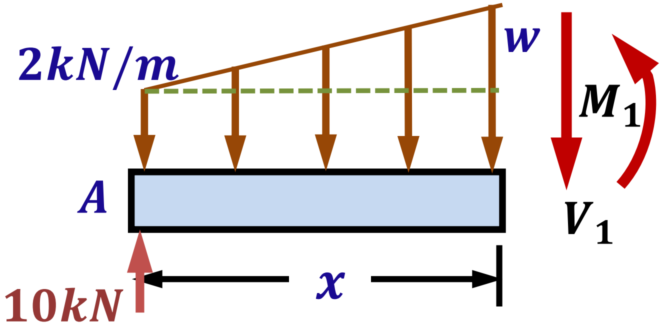
$R_B = 14kN;$ $\quad\quad R_A = 10kN;$

Now take a section **1-1** and draw FBD of LHS.



$\frac{w}{x} = \frac{4}{6}$ or $w = \frac{2}{3}x$

$\Sigma F_y = 0;$ $10 - 2x - \frac{1}{2}.x.\frac{2}{3}x - V_1 = 0$



$\boldsymbol{V_1 = 10 - 2x - \frac{x^2}{3}}$ **...............(1)**

[Eq. for S.F. in the beam where ($0 \leq x \leq 6m$)]

$\boldsymbol{V_A(x = 0) = 10kN;}$ $\boldsymbol{V_B(x = 6) = -14kN}$

$\Sigma M_{1-1} = 0;$ $10.x - 2.x.\frac{x}{2} - \left(\frac{1}{2}.x.\frac{2}{3}x.\frac{x}{3}\right) - M_1 = 0;$

$\boldsymbol{M_1 = 10.x - x^2 - \frac{x^3}{9}}$...........(2)

$\boldsymbol{M_A(x = 0) = 0; M_B(x = 6) = 0;}$

$V_A(x = 0) = 10kN; V_B(x = 6) = -14kN$

S.F. is varying from (10kN) to (-14kN),

so it will be zero somewhere. So,

put $V_1 = 0; 10 - 2x - \frac{x^2}{3} = 0;$

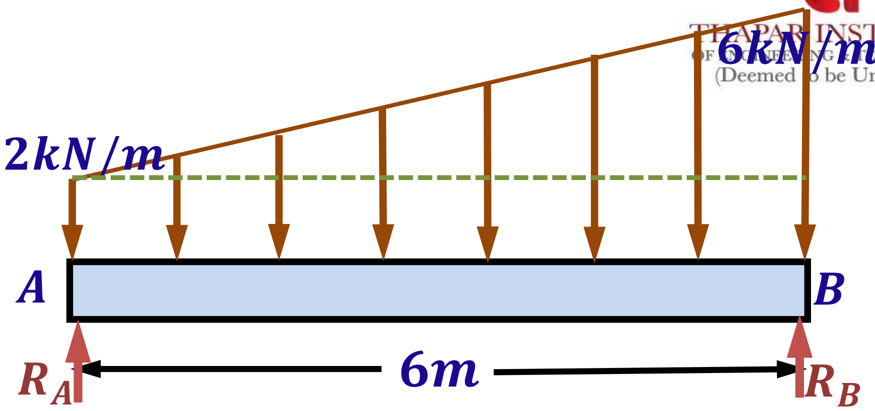
$$\boxed{x = 3.25m}$$

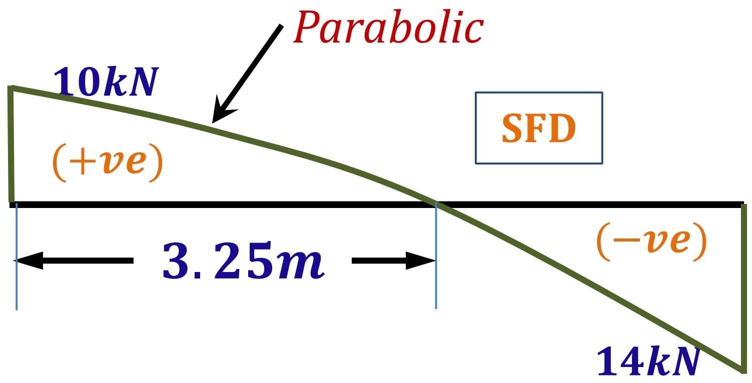
$M_A(x = 0) = 0; M_B(x = 6) = 0;$

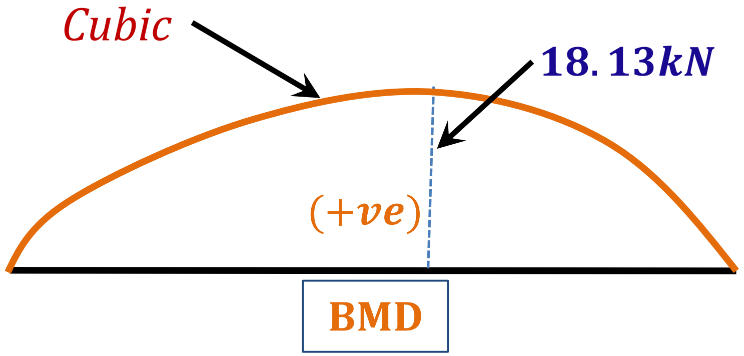
$M_{max}(at\ x = 3.25)$

$= 10 \times 3.25 - (3.25)^2 - \frac{(3.25)^3}{9}$

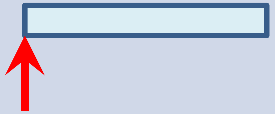
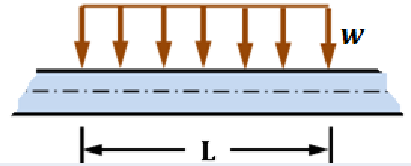
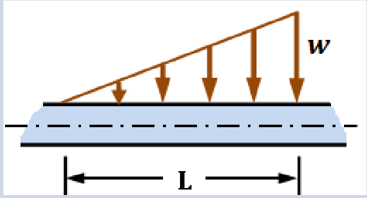
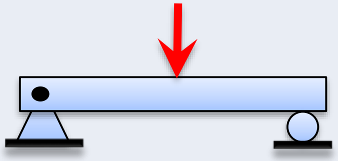
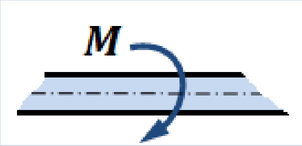
$$\boxed{M_{max} = 18.13kNm}$$







SUMMARY

Effect of Load	Shear Force Variation	Bending Moment Variation
No Load 	Constant	Linear
UDL 	Linear	Parabolic
UVL 	Parabolic	Cubic
Point Load 	Sudden Change	No change
Concentrated Moment 	No change	Sudden change

THANK YOU