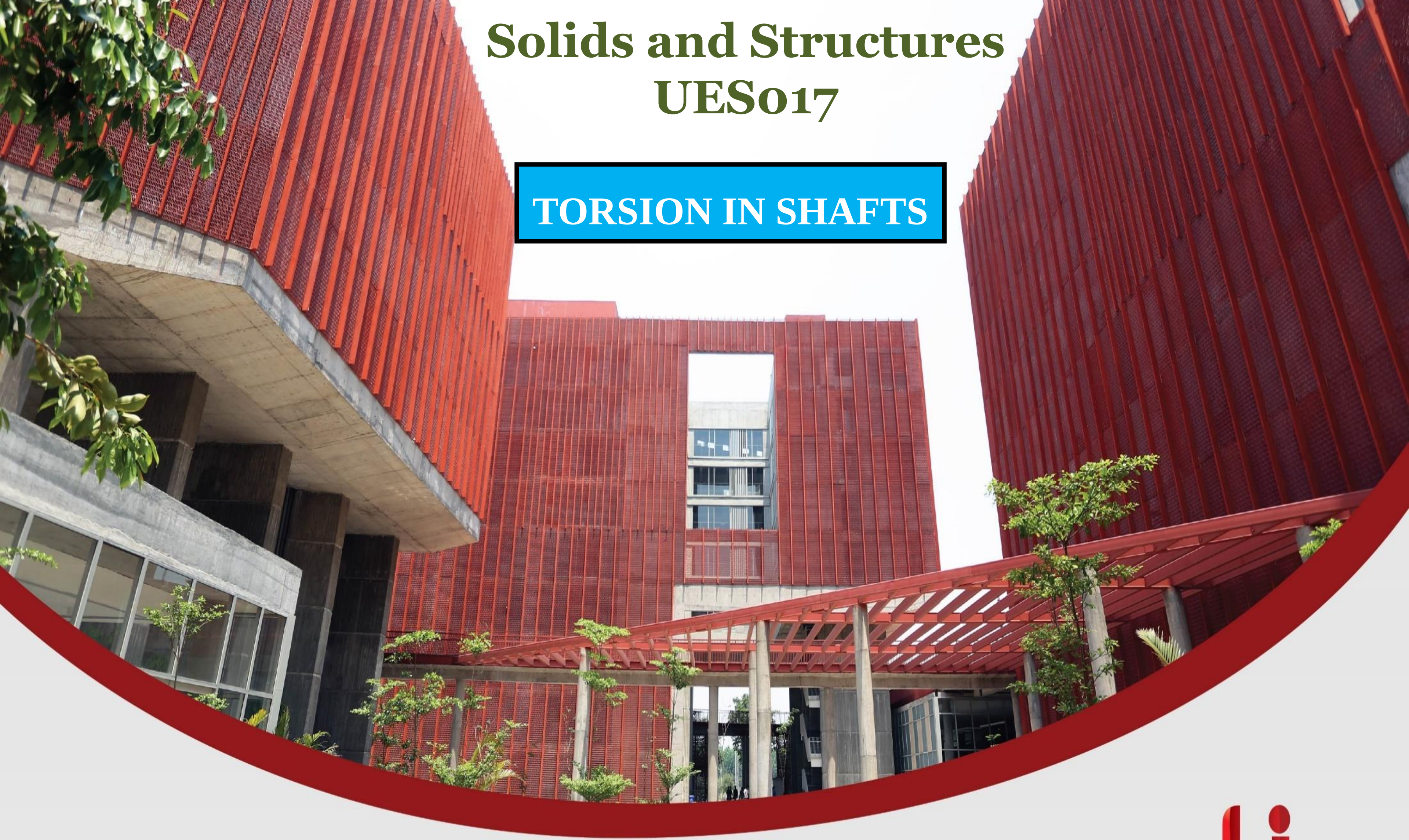
Solids and Structures UES017

TORSION IN SHAFTS



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TORSION IN SHAFTS

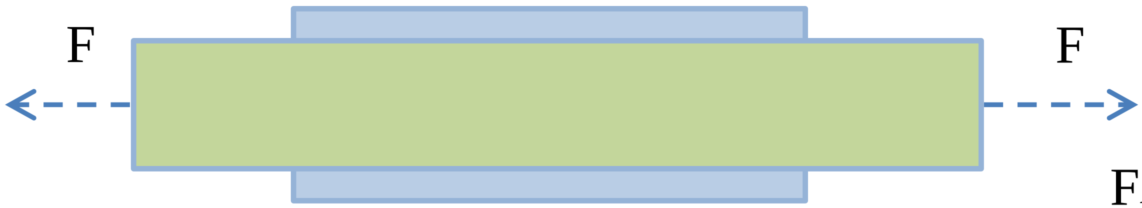
- Torsion refers to the twisting of the structure when it is loaded by couples that produce by rotation about its longitudinal axis.
- A shaft is a rotating machine element, which is of circular in cross section.
- In factories and workshops, a shaft is used to transmit power from one part to another i.e. from a motor to machine tool.
- Another example is to transfer power from an engine to rear axle of an automobile.
- The shafts used for above mentioned purpose used may be of **solid or hollow type.**

Cross section of hollow shaft

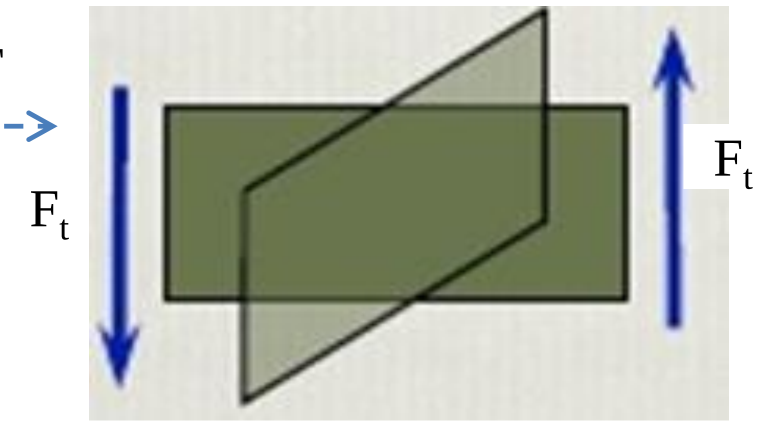
Cross section of solid shaft

Different Types of Loads

Normal Force

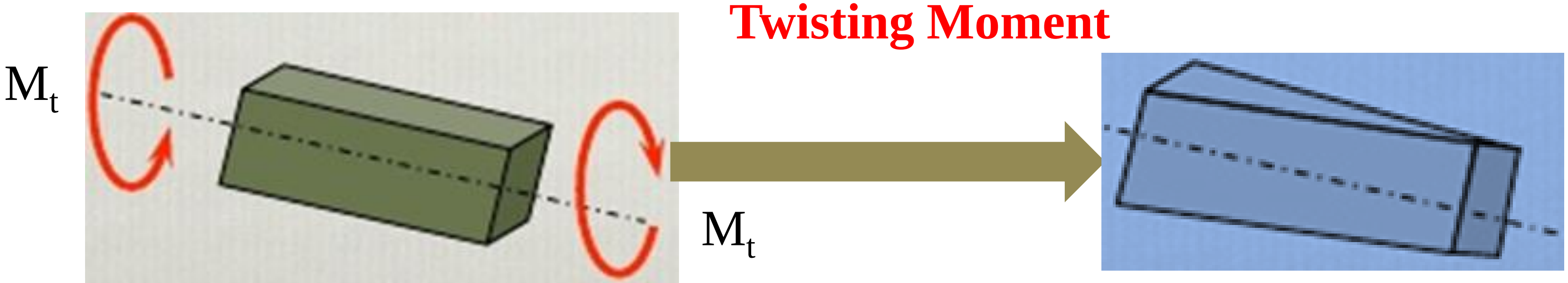


Shear Force



Bending Moment

Twisting Moment



Torsion in Members

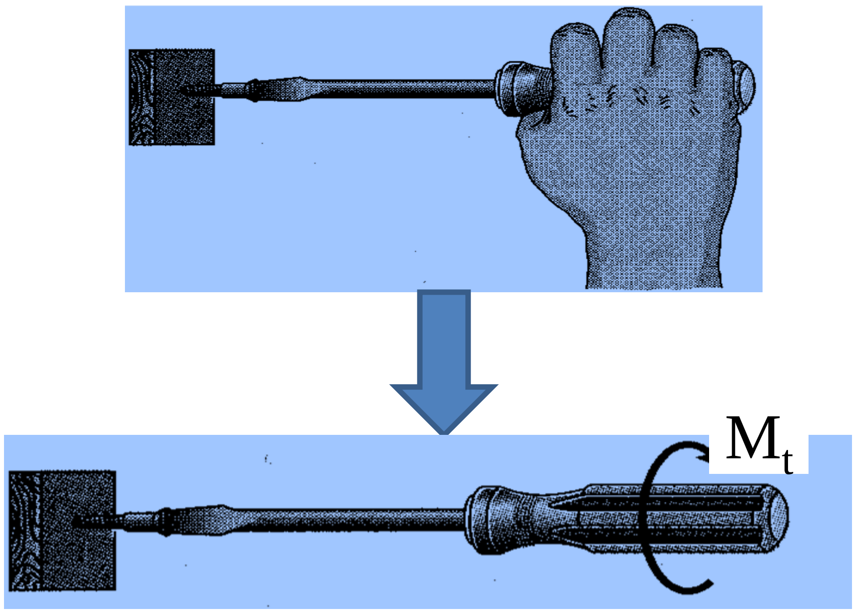
Twisting moments or Torques (Mt) are forces acting through a distance so as to promote rotation.

Example:

- Wrench to tighten a nut in the bolt
- Bolt, wrench and the force are all perpendicular to one another and the moment is the force *F* times the length of the wrench
- Mt = F * L in Nm

CAUSE: Appling Torque/ twisting moment (Mt)

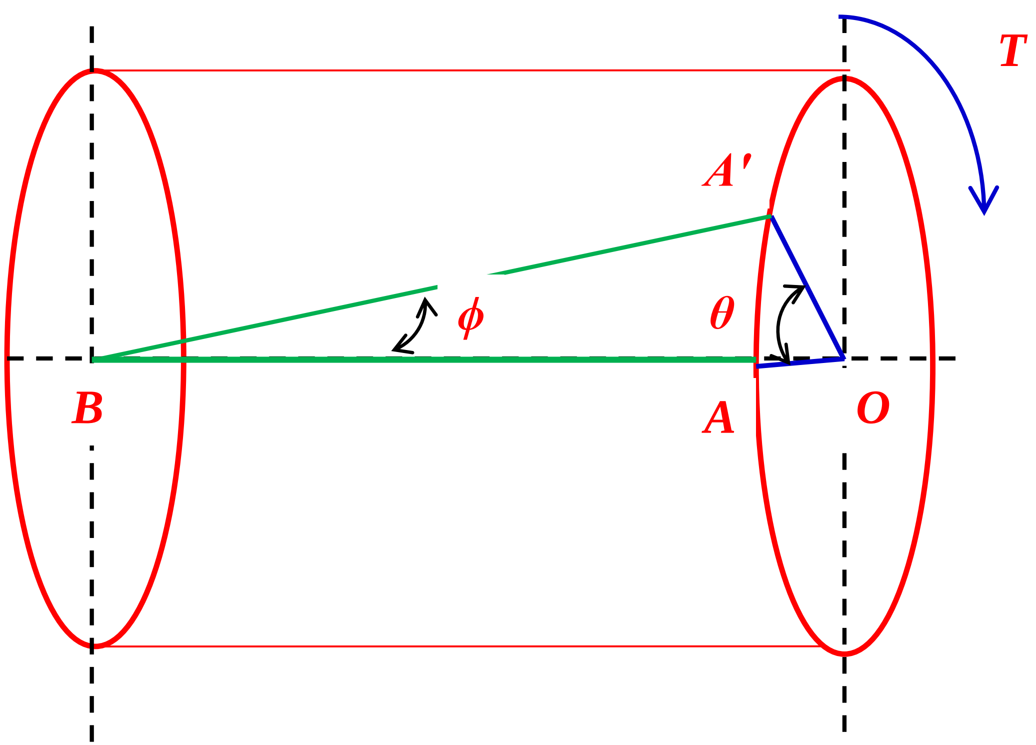
Torsion is the effect of torque that causes twisting of a straight bar when it is loaded by twisting moments or torque that tends to produce rotation about the longitudinal axis of the bar.

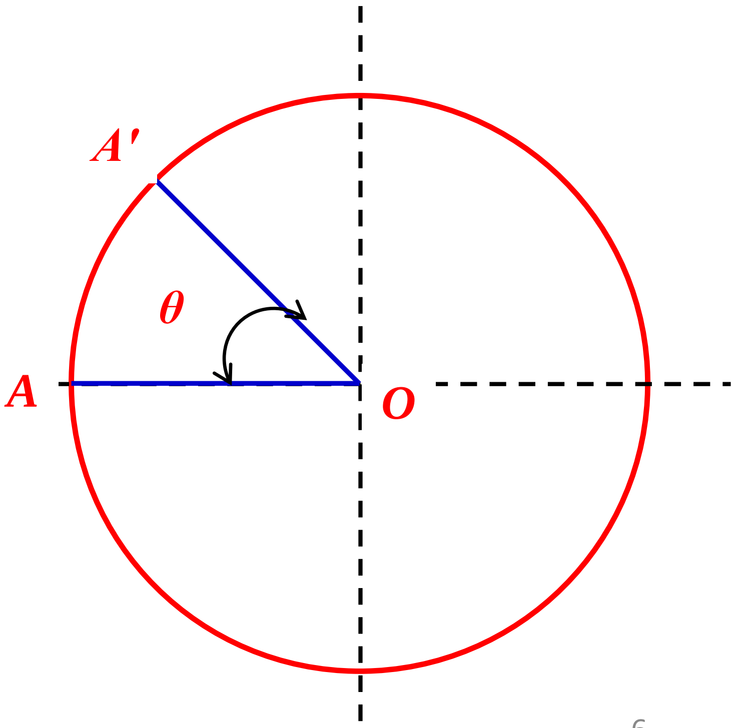


EFFECT: Causes Rotation in the shaft as 'θ'

ANGLE OF TWIST

- When the shaft is subjected to torque (T), the point A on the surface of the shaft moves to A ‘ position.
- The angle AOA' at the centre of the shaft is called the angle of twist.
- $<AOA' = \theta$ = angle of twist
- Cross section is twisted by angle θ and surface by angle ϕ.
- **Angle of twist is measured in radians**.

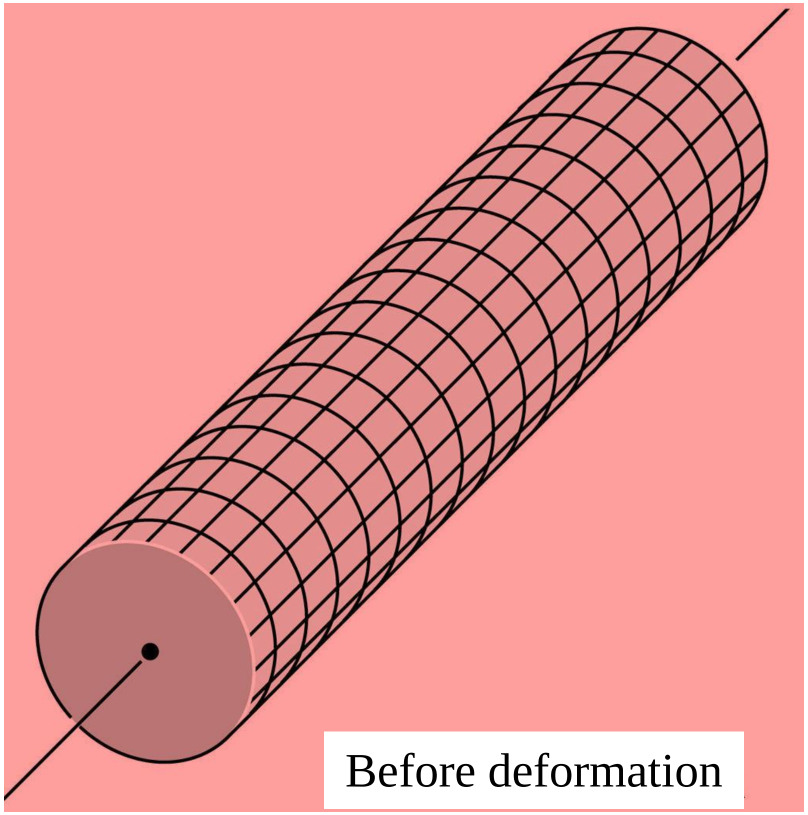


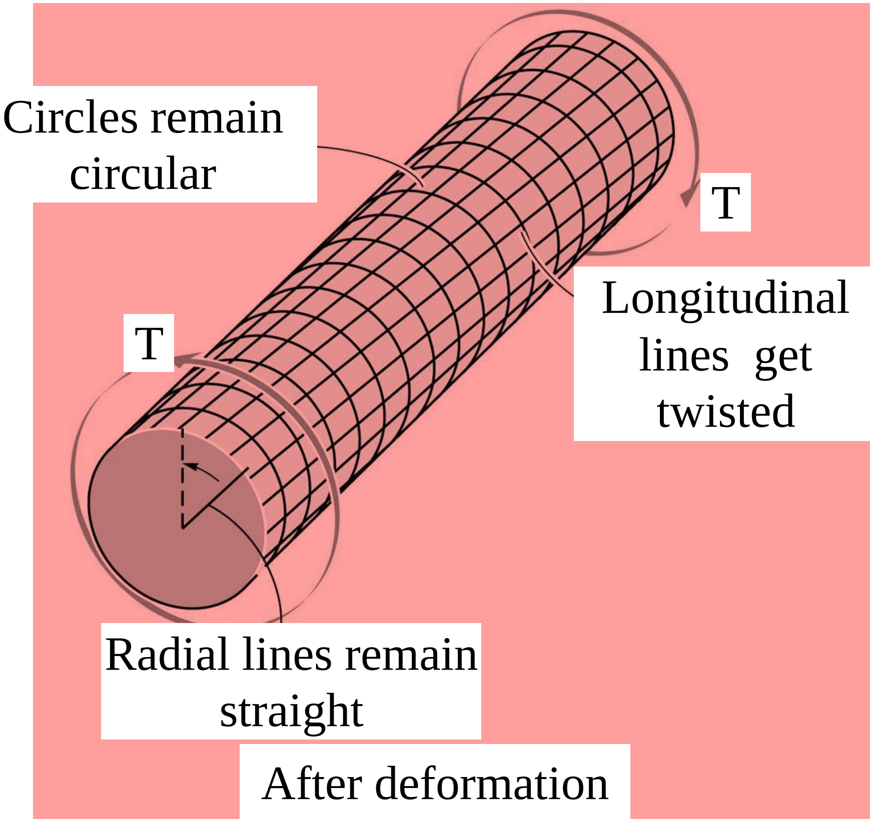


Assumptions in the Theory of Torsion

The following assumptions are assumed while computing the **Shear Stresses in a circular shaft subjected to torsion:**

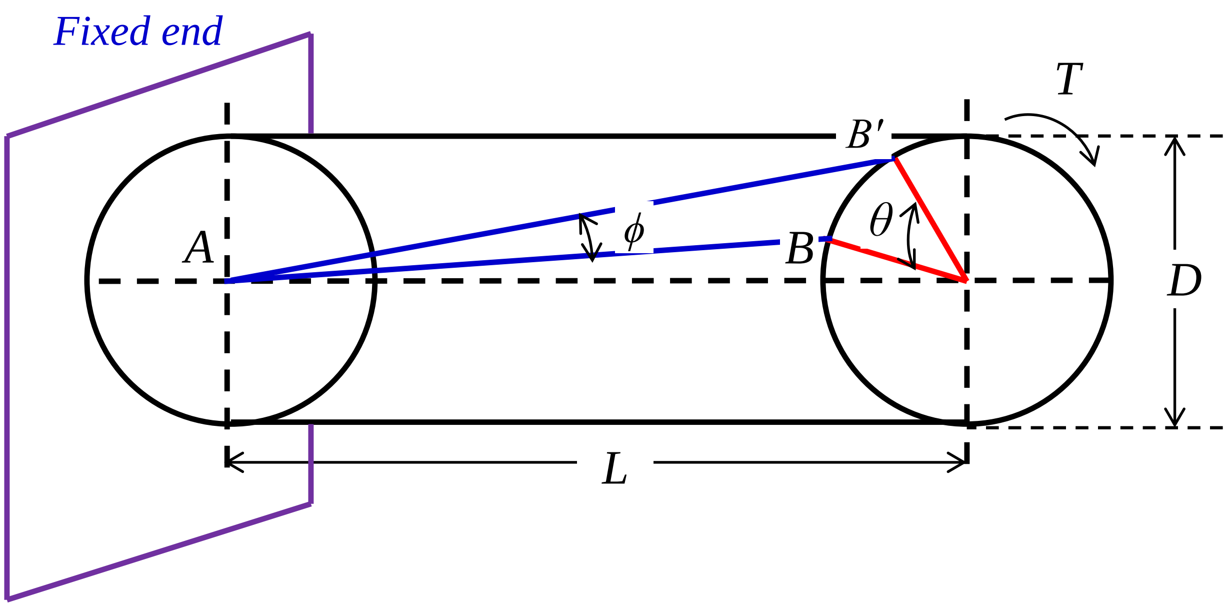
- The material of the **shaft is homogeneous, isotropic and perfectly elastic**;
- The **material obeys Hooke's law** and the stress remains within limit of proportionality;
- The shaft is of circular cross section throughout the length
- The **cross section of shaft which are plane before twist remains plane after twist**.
- **All radii remain straight after torsion**.


Before deformation


Circles remain circular
T
Longitudinal lines get twisted
T
Radial lines remain straight
After deformation

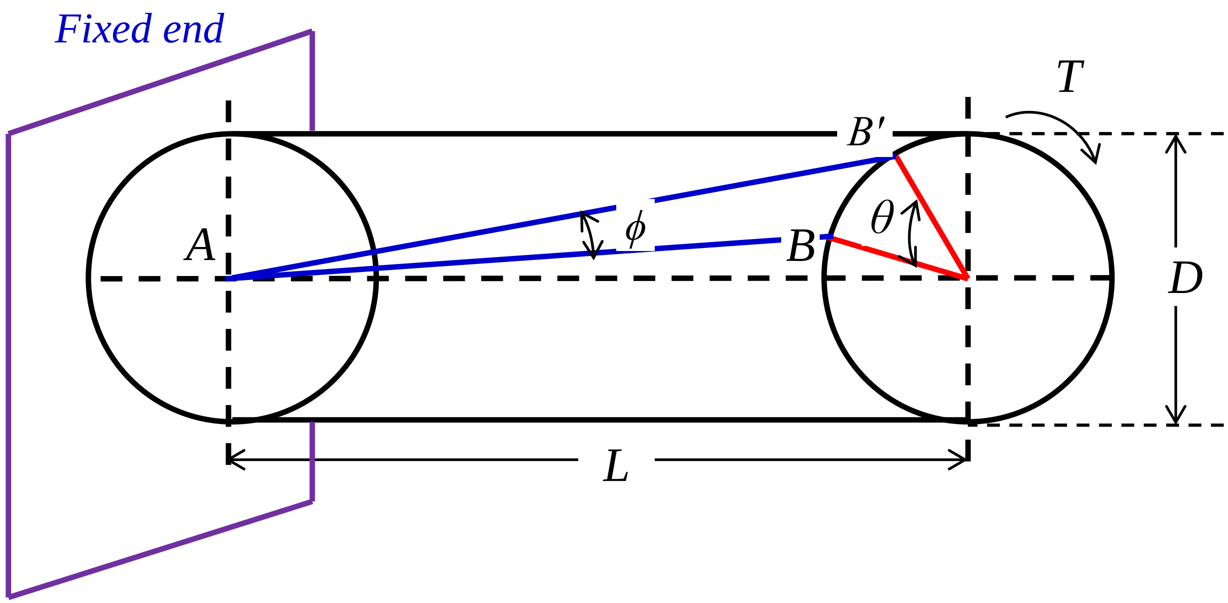
Torsion Equation for Solid Shaft

T – Maximum twisting torque (in N-m)
D – Diameter of the shaft (in m)
I_p – Polar moment of Inertia (in m^4)
τ – Shear stress in shaft (in MPa)
G – Modulus of Rigidity (in MPa)
θ – Angle of twist (in *radians*)
L – Length of the shaft (in m)



Torsion Equation for Solid Shaft

The cross section will be twisted through an angle θ and surface by angle ϕ on the application of torque T.



$$Shear\ strain(\phi) = \frac{BB'}{l} \qquad (a)$$

$$Shear\ strain(\phi) = \frac{\tau}{G} \qquad (b)$$

On comparing *eq. (a)* and *eq. (b)*, we will get:

$$\frac{BB'}{l} = \frac{\tau}{G} \qquad (c)$$

Also, we known that $BB' = R\theta$ *(c1)*

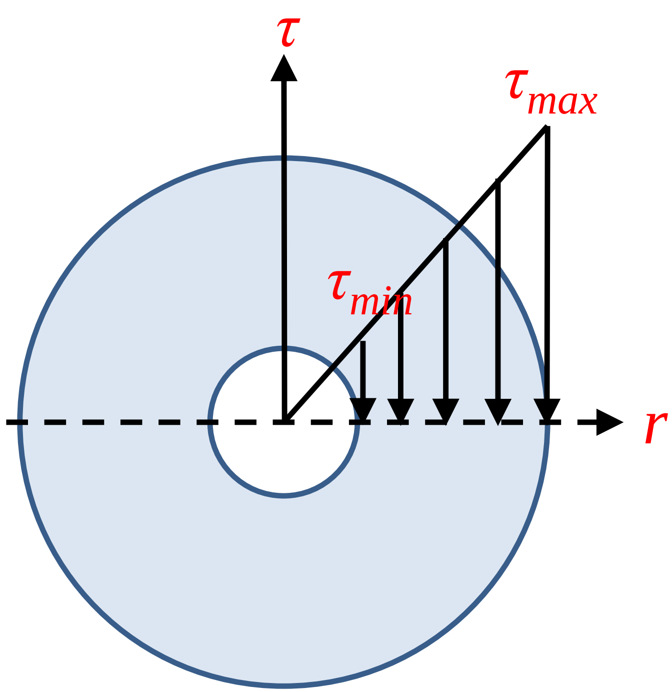
Now, put the value of BB' from *eq. (d)* into *eq. (c)*

$$\boxed{\frac{R\theta}{l} = \frac{\tau}{G}} \qquad (c2)$$

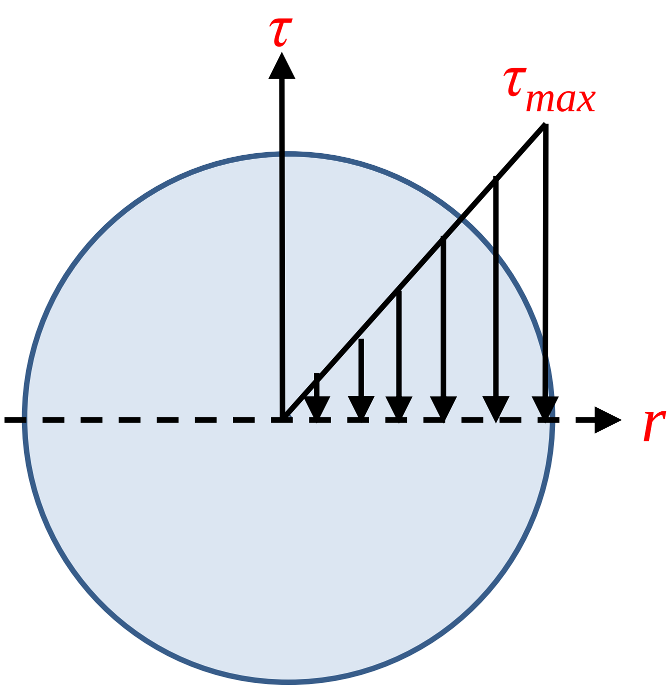
From this equation, ζ is directly proportional to 'R'- Radius of shaft.

Variation of Shear Stress in Torsion

The shearing stress varies linearly with the radial position in the circular section.



Shear stress variation in hollow circular shaft



Shear stress variation in solid circular shaft

- ❑ From this equation, ζ is directly proportional to 'R'- Radius of shaft.
- ❑ Shear stress will be zero at centre and maximum stress occurs at 'R' i.e material at outside is under maximum stress.

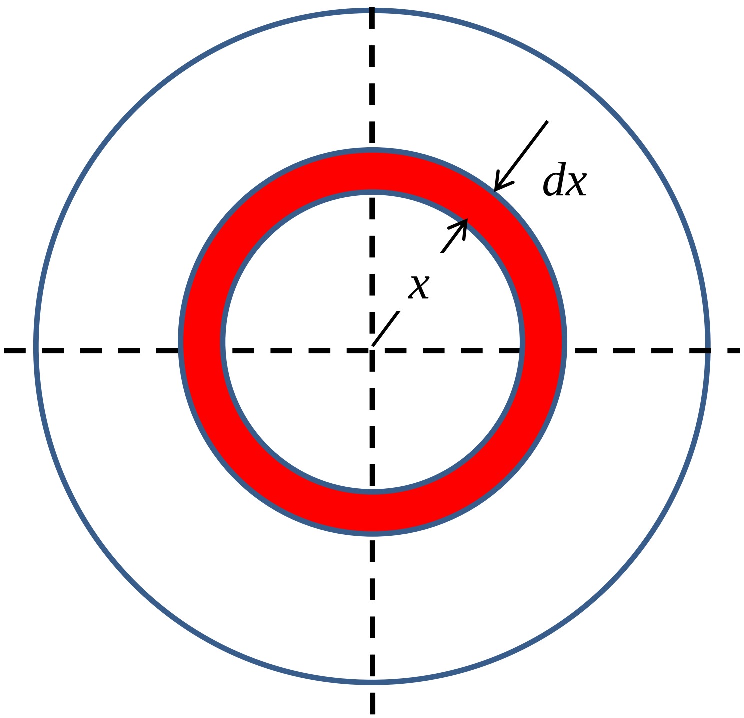
Torsion Equation for Solid Shaft

Consider an elementary ring of thickness *dx* at a radius *x*, and let the shear stress at this radius be τ_x.

The turning force on the elementary ring,

$$=\left(2\pi x.dx\right)\tau_x \qquad (d)$$

The turning moment due to this turning force on the elementary ring,



$$dT=\left(\tau_x.2\pi x.dx\right)\times x \qquad (e)$$

Now we will integrate the *eq.* (*e*) in order to get the total turning moment,

$$\int_0^T dT=2\pi\int_0^R \tau_x x^2 dx \qquad (f)$$

$$\frac{\tau}{R}=\frac{\tau_x}{x}\Rightarrow \tau_x=\frac{\tau x}{R} \qquad (g)$$

Torsion Equation for Solid Shaft

Now, substitute the value of τ_x from *eq. (g)* into *eq. (f)*

$$\int_0^T dT = 2\pi\int_0^R \frac{\tau x}{R} x^2 dx = 2\pi\int_0^R \frac{\tau}{R} x^3 dx \qquad (h)$$

$$T = 2\pi\left[\frac{\tau x^4}{4R}\right]_0^R = 2\pi\frac{\tau R^3}{4} = \tau\frac{\pi R^3}{2} \qquad (i)$$

$$I_p = \frac{\pi D^4}{32} = \frac{\pi R^4}{2}$$

$$T = \frac{\pi}{16}\tau D^3 \qquad (j)$$

Also, we can rewrite *eq. (i)* $T = \frac{\tau}{R}\frac{\pi R^3}{2}R \Rightarrow T = \frac{\pi R^4}{2}\frac{\tau}{R} \Rightarrow \frac{T}{I_p} = \frac{\tau}{R} \qquad (k)$

Now, equating *eq. (e)* and *eq. (k)* $\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}$

Where Ip = Polar Modulus = $I_{XX} + I_{YY}$

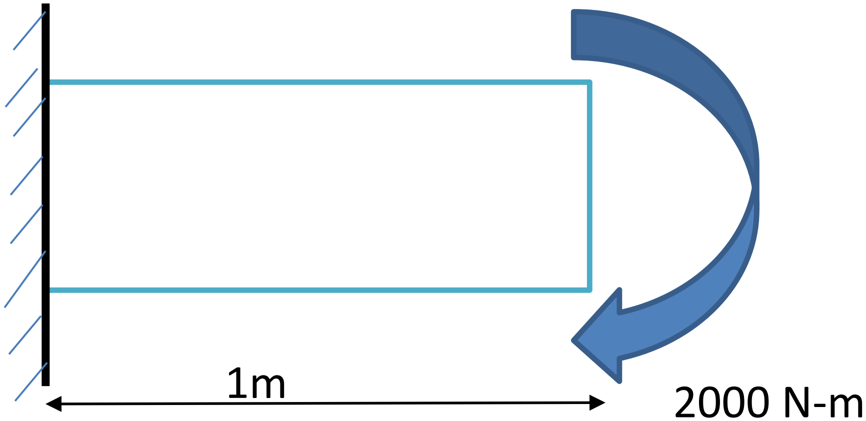
TORSION EQUATION

Numerical Problems

Problem 1: A circular shaft is subjected to a torque of 2000 N-m torque. Diameter of the shaft is 50m. Shaft is made of steel with G =77 GPa with length of 1m. Determine the maximum shear stress in the shaft and the angle of twist.

Solution: Using Torsion Equation,

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}$$



$$\zeta \textbf{ max} = \frac{T * R}{Ip} = \frac{2000 * 1000 * 50/2}{\frac{\Pi * 50^4}{32}} = 326 \text{ MPa}$$

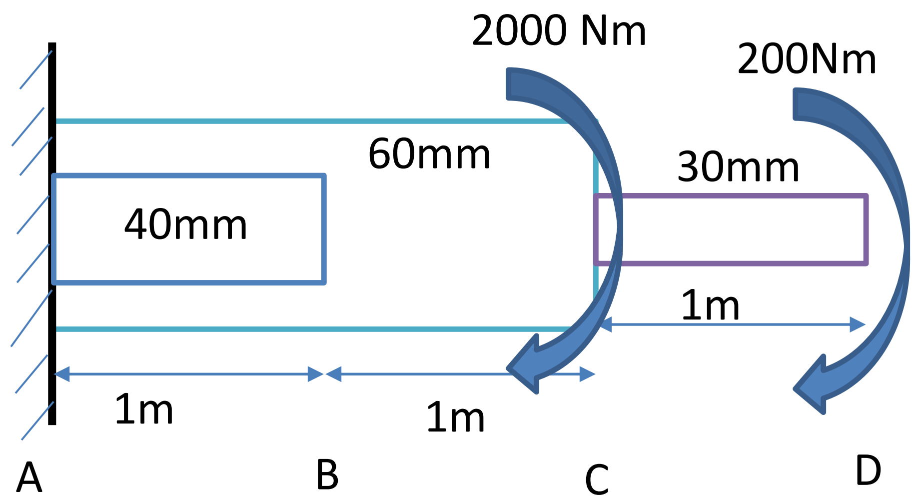
$$\theta = \frac{T * L}{G * Ip} = \frac{2000 * 1000 * 1000}{\frac{\Pi * 50^4}{32} * 77 * 1000} = 42.5 \text{ radians}$$

Problem 2: A composite shaft is subjected to torques as shown. Determine the maximum shear stress in the shaft and the angle of twist at end "D"

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}$$

ζ max → AB, BC, CD

T max → T_{CD} = 200 Nm, T_{BC} = 2200 Nm, T_{AB} = 2200 Nm



$$\zeta \max,\ CD = \frac{T * R}{Ip} = \frac{200 * 1000 * (30/2)}{\frac{\Pi * 30^4}{32}}$$

$$\zeta \max,\ BC = \frac{T * R}{Ip} = \frac{2200 * 1000 * (60/2)}{\frac{\Pi * 60^4}{32}}$$

$$\zeta \max,\ AB = \frac{T * R}{Ip} = \frac{2200 * 1000 * (60/2)}{\frac{\Pi * (60^4 - 40^4)}{32}}$$

$$\theta_D = \theta_{AB} + \theta_{BC} + \theta_{CD}$$

$$= \left(\frac{T * L}{G * Ip}\right)_{AB} + \left(\frac{T * L}{G * Ip}\right)_{BC} + \left(\frac{T * L}{G * Ip}\right)_{CD}$$

Problem 3: A preliminary design of a shaft says that the hollow shaft is used with inner diameter of 100mm and outer diameter of 150mm and the allowable shear stresses is 82 MPa. Determine the torque that can be transmitted by the shaft.

If the solid shaft is used of the same material and same weight, determine the maximum torque transmitted by the solid shaft.

SOLUTION

$$\zeta \max = \frac{T * R}{Ip}$$

$$82 * 10^3 = \frac{T * (150/2)}{\frac{\Pi * (150^4 - 100^4}{32}}$$

Hence, solving for $T_{max} = 43.58$ kN-m

Torque transmitted by Hollow shaft= 43.58 kN-m

Now, according to given, $V_{solid} = V_{hollow}$ shafts

$A_{solid} * Ls = A_{hollow} * Lh$

Shafts are of same material (γ) and same weight ($V * \gamma$)

(If same lengths are used, then , $A_{solid} = A_{hollow}$

$$\frac{\pi Ds^2}{4} = \frac{\pi(150^{2-}100^{2)}}{4}$$

On solving, we get, $D_s = \sqrt{150^2 - 100^2}\, mm = 112\, mm$

$$\zeta\ \mathbf{max} = \frac{\boldsymbol{T * R}}{\boldsymbol{Ip}}$$

$$82 * 10^3 = \frac{T * (112/2)}{\frac{\Pi * (\mathbf{112^4})}{\mathbf{32}}}$$

Hence, solving for T_{max} = 22.4 kN-m

Torque transmitted by Hollow shaft= 43.58 kN-m

Torque transmitted by Solid shaft= 22.4 kN-m

Problem 4: A circular shaft of diameter of 60mm is provided. Calculate the allowable torque that can be transmitted by the shaft, if the allowable shear stresses cannot exceed 40 MPa and the allowable angle of twist is 1deg/m.

Solution: Using Torsion Equation,

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}$$

According to given, $\theta = \frac{\Pi * 1}{180}$ *radians*

$$\zeta \max = \frac{T * R}{Ip} = \frac{T * 60/2}{\frac{\Pi * 60^4}{32}} = 40 \text{ MPa}$$

Solving, T = 1.6956 kN-m

$$\theta = \frac{T * L}{G * Ip} = \frac{\Pi * 1}{180}$$

Solving, we get , T = 1.774 kN-m

T = 1.6956 kN-m , allowable torque that can be transmitted.

Power Transmission by the Shaft

Shafts are the medium to transmit power from the point of power generation to the point of its application.

The power transmitted is the work done per second. Thus,

$$P = T\omega$$

$$P = T\frac{2\pi N}{60} \Rightarrow P = \frac{2\pi NT}{60} \ \ watts$$

$$P = \frac{2\pi NT}{60 \times 1000} \ \ kW$$

T - Applied torque in *N-m*

N - Revolutions of the shaft in *rpm*.

- ❖ A shaft may transmit power at more than one section to different machines.
- ❖ Also, the shafts may be subjected to different torques in different sections.
- ❖ In such cases the principle of conservation of energy can be applied.

Numerical Problem

Problem 5

A hollow shaft is to transmit 300 kW at 80 r.p.m. If the shear stress is not to exceed 60 MN/m^2 and the internal diameter is 0.6 of the external diameter, **find the external and internal diameters** assuming that the **maximum torque is 1.4 times the mean.**

Solution:

Power = 300 kW; N= 80 r.p.m; τ = 60 MN/m^2= 60×10^6 N/m^2;

$T_{max} = 1.4\times T_{mean}$; d = 0.6×D

$$P=\frac{2\pi NT}{60\times1000}\ kW \Rightarrow 300=\frac{2\pi(80)T}{60\times1000} \Rightarrow T=35809\,N-m$$

$$T_{max}=1.4\times35809\,Nm=50132\ Nm$$

$$T_{max}=\frac{\pi}{16}\tau\frac{\left(D^4-d^4\right)}{D}\Rightarrow 50132=\frac{3.14}{16}60\times10^6\times\frac{\left(D^4-(0.6D)^4\right)}{D}$$

$$D=0.169\,m\,or\,170\,mm$$

$$d=0.6\times170=102\,mm$$

Numerical Problem

Problem 6: A hollow shaft of diameter ratio 3/8 is required to transmit 600 kW at 110 r.p.m, the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MN/m^2 and the twist in a length of 3 m not to exceed 1.4 degrees. **Calculate the maximum external diameter satisfying these conditions.**

Take G = 84 GN/m^2

Given:

Power (P) = 600 kW; N = 110 $r.p.m$; $\tau = 63\ MN/m^2 = 63 \times 10^6\ N/m^2$; $l = 3m$

$\theta = (3.14/180) \times 1.4\ rad$; $G = 84 \times 10^9\ N/m^2$; $T_{max} = 1.2 \times T_{mean}$; $d = 0.375 \times D$

$$P = \frac{2\pi NT}{60\times1000}\ kW \Rightarrow 600 = \frac{2\pi(110)T}{60\times1000} \Rightarrow T = 52087\ N-m$$

$$T_{\max} = 1.2\times52087\ N-m = 62504\ N-m$$

***Case I*: When the shear stress is not to exceed 63 *MN/m*^{2}**

$$T_{max} = \frac{\pi}{16}\tau\frac{\left(D^4-d^4\right)}{D} \Rightarrow 62504 = \frac{3.14}{16}\times63\times10^6\times\frac{\left(D^4-(0.375D)^4\right)}{D}$$

$$D = 172.7\ mm$$

Numerical Problem

***Case II*: When the angle of twist is not to exceed 1.4°**

$$\frac{T}{I_p} = \frac{G\theta}{l} \Rightarrow T = \frac{G\theta}{l} I_p \Rightarrow T = \frac{G\theta}{l} \times \frac{\pi}{32}\left(D^4 - d^4\right)$$

$$T = \frac{G\theta}{l} \times \frac{\pi}{32}\left(D^4 - d^4\right)$$

$$\Rightarrow 62504 = \frac{\pi}{32} \times \frac{84 \times 10^9}{3} \times \frac{1.4 \times 3.14}{180} \times \left(D^4 - (0.375D)^4\right)$$

$$D = 175.5\,mm$$

Selected diameter will be 175.5 mm because this diameter satisfies both the criteria's.

Composite Shafts (Shafts in Series)

If two or more shaft of different material (Value of *G* is different), diameter and length are connected together in such a way that **each shaft carries the same torque, then the shafts are said to be connected in series.**
The composite shaft so produced is defined as series connected.

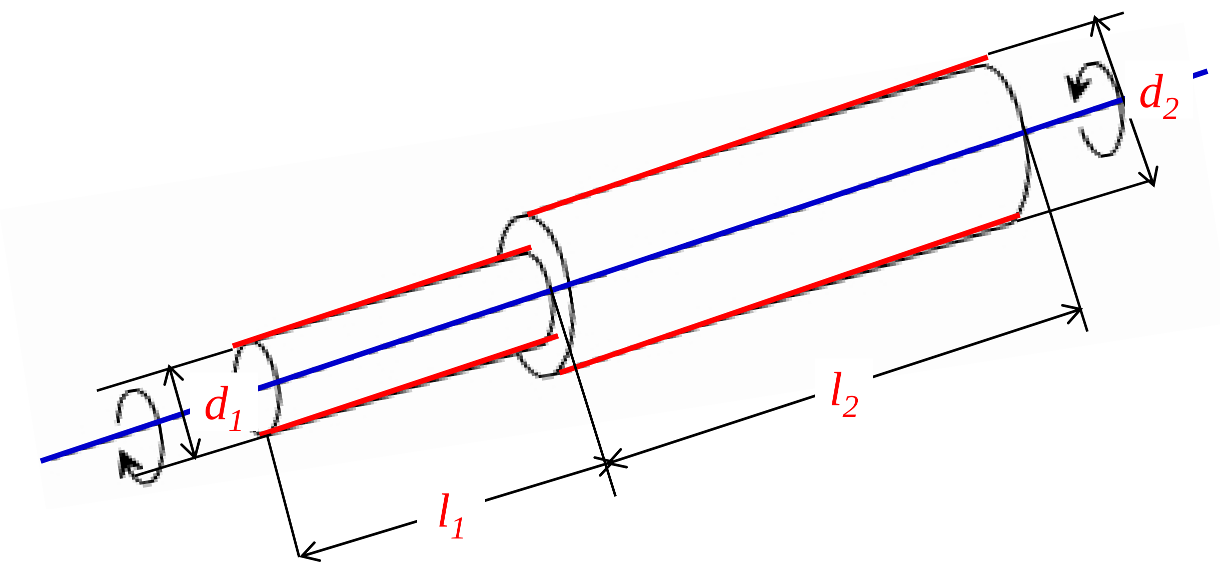
OR

When two shafts are joined in a series, **a single torque is applied**. Therefore, both shafts are subjected to the same torque.

Angle of twist is the sum of the angle of twist of the two shafts connected in series.

$$\theta = \theta_1 + \theta_2$$

$$\theta = \frac{T}{G}\left[\frac{l_1}{I_{p1}} + \frac{l_2}{I_{p2}}\right]$$



Composite Shafts (Shafts in Parallel)

When the driving torque is applied at the junction of the shafts and the resisting torque is at the other ends of the shafts. **The angle of twist is same for each shaft. The applied torque is divided between the two shafts.**

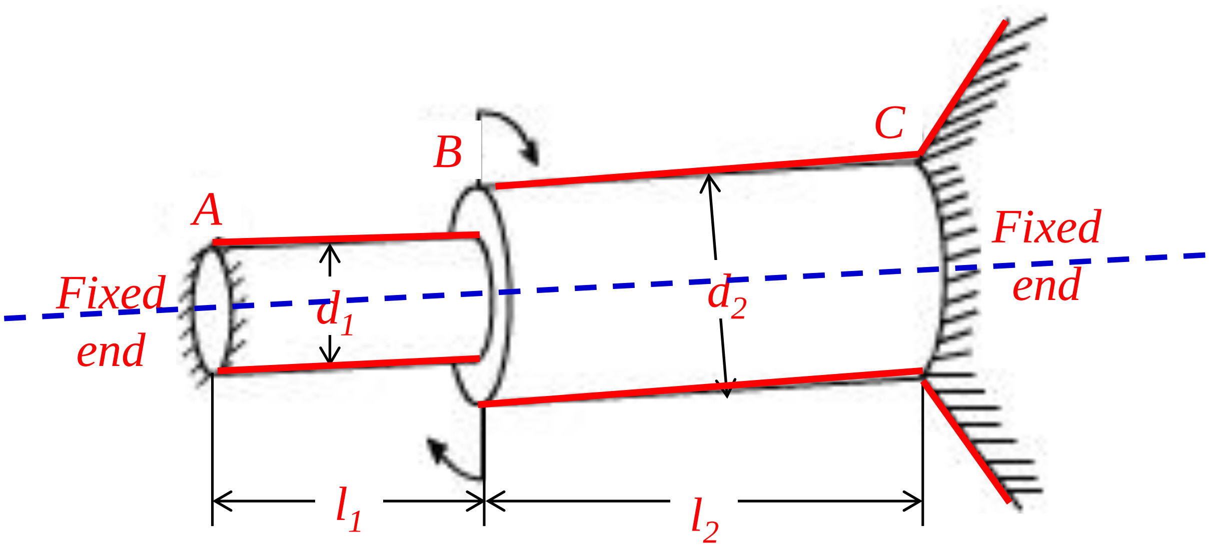
OR

When the two shafts are connected in parallel, the torque applied to the composite shaft is the sum of the torques on the two shafts;

$$\theta_1 = \theta_2$$

$$\frac{T_1 l_1}{G_1 I_{p1}} = \frac{T_2 l_2}{G_2 I_{p2}}$$

$$T = T_1 + T_2$$



Numerical Problem

Problem 7: A steel shaft *ABCD* as shown in Figure. **If equal opposite torques are applied at the end of the shaft**, find the maximum permissible value of d_1 for the maximum shearing stress in *AB* not to exceed that in *CD*. If the torque applied is *10 kNm*, what is the total angle of twist?
Take $G = 80\ GN/m^2$

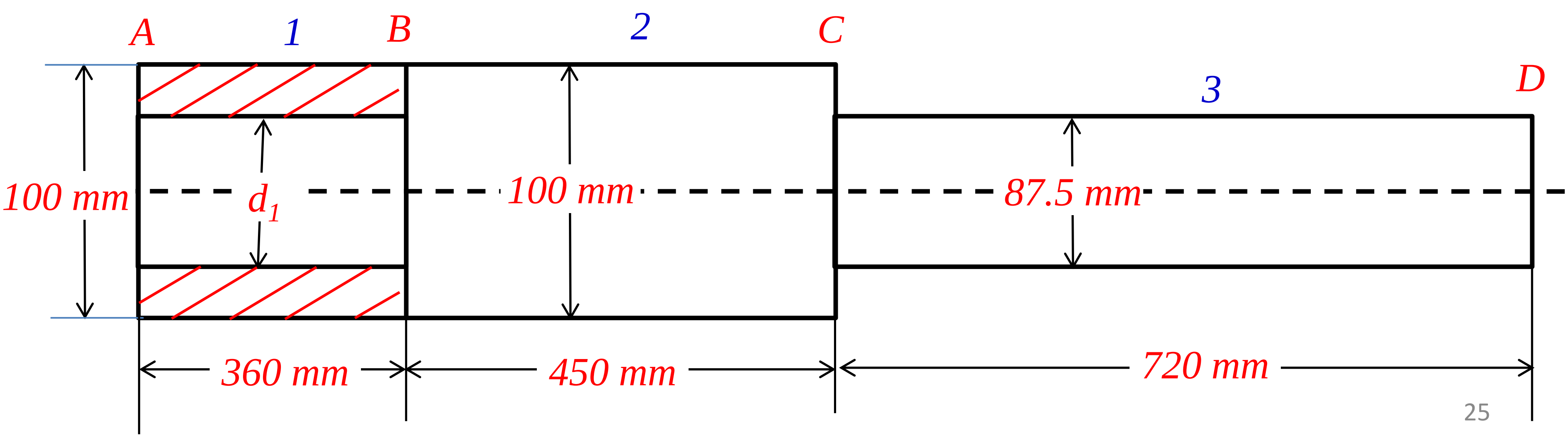
Shafts in Series.
'T' is same in all parts

$$\theta = \theta_1 + \theta_2$$

$$\theta = \frac{T}{G}\left[\frac{l_1}{I_{p1}} + \frac{l_2}{I_{p2}}\right]$$

Shear stress developed in hollow shaft i.e. *AB*

$$T = \frac{\pi}{16}\tau_1\frac{\left(D_1^4 - d_1^4\right)}{D_1} = \frac{3.14}{16}\times\tau_1\times\frac{\left(0.1^4 - (d_1)^4\right)}{0.1} \Rightarrow \tau_1 = \frac{1.6T}{3.14\times\left(0.1^4 - (d_1)^4\right)}$$



Numerical Problem

Shear stress developed in solid shaft i.e. *CD*

$$T = \frac{\pi}{16}\tau_3 D_3^3 = \frac{\pi}{16}\tau_3 \times (0.0875)^3 \Rightarrow \tau_3 = \frac{16T}{\pi \times (0.0875)^3}$$

Since $\tau_1 = \tau_3$

$$\frac{1.6T}{3.14 \times \left(0.1^4 - (d_1)^4\right)} = \frac{16T}{\pi \times (0.0875)^3} \Rightarrow d_1 = 75.8\,mm$$

Total angle of twist:

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$\theta = \frac{T}{G}\left[\frac{l_1}{I_{p1}} + \frac{l_2}{I_{p2}} + \frac{l_3}{I_{p3}}\right] \times \frac{180}{\pi} \deg rees = 1.616^o$$

Numerical Problem

Problem 8: A solid shaft 6 *m* long is securely fixed at each end. A torque of 1250 *N-m* is applied to the shaft at a section of 2.4 *m* from one end. What are the fixing torques set up at the ends of the shaft? If the diameter of the shaft is 40 *mm*, what are the maximum shear stresses in the two portions.? Calculate also the angle of twist for the section where the torque is applied.

Take G = 84 GN/m^2

Shafts in Parallel

$T = T_1 + T_2$

$$\theta_1 = \theta_2$$

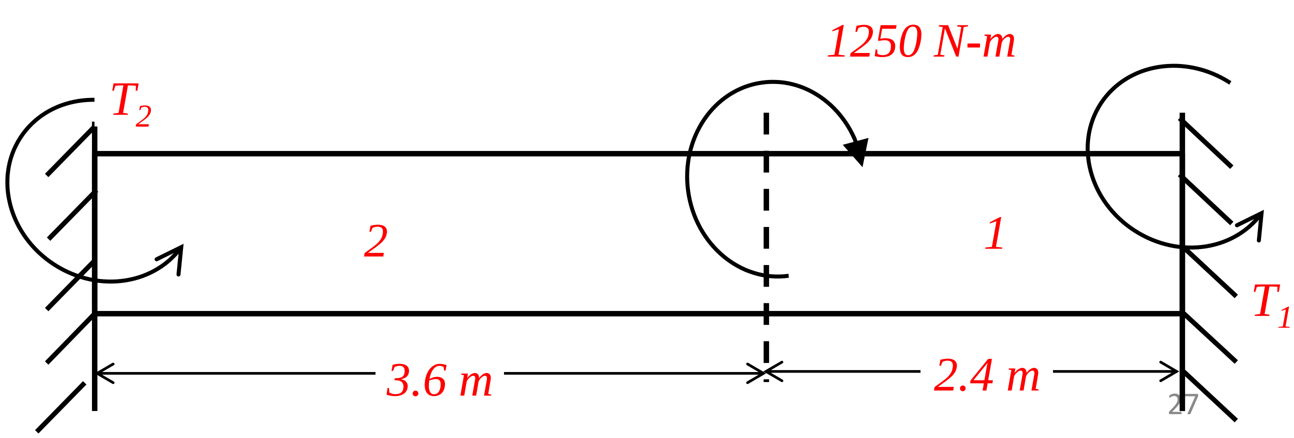
$$\frac{T_1 l_1}{G_1 I_{p1}} = \frac{T_2 l_2}{G_2 I_{p2}}$$

Angle of twist θ

$$\theta_1 = \theta_2 \Rightarrow \frac{T_1 l_1}{G_1 I_{p1}} = \frac{T_2 l_2}{G_2 I_{p2}} \Rightarrow T_1 l_1 = T_2 l_2 \Rightarrow T_2 = \frac{T_1 l_1}{l_2}$$

$$T_2 = \frac{T_1 (2.4)}{(3.6)} = 0.67 T_1$$

$$T_2 + T_1 = 1250$$



Numerical Problem

$$T_2 = 500.2\,N - m$$

$$T_1 = 749.8\,N - m$$

$$\theta = \theta_1 = \theta_2 = \frac{T_1 l_1}{G_1 I_{p1}} = \frac{749.8 \times 2.4 \times 32}{84 \times 10^9 \times \pi \times (0.04)^4} = 0.0852\,rad$$

$$\theta = 0.0852 \times \frac{180}{\pi} = 4.88^o$$

Maximum shear stresses in the two portions

$$\tau_1 = \frac{16T_1}{\pi D^3} = \frac{16 \times 749.8}{\pi \times 0.04^3} = 59.69 \times 10^6\ N/m^2 = 59.66\,MN/m^2$$

$$\tau_2 = \frac{16T_2}{\pi D^3} = \frac{16 \times 500.2}{\pi \times 0.04^3} = 39.8 \times 10^6\ N/m^2 = 39.8\ \mathrm{MN/m^2}$$

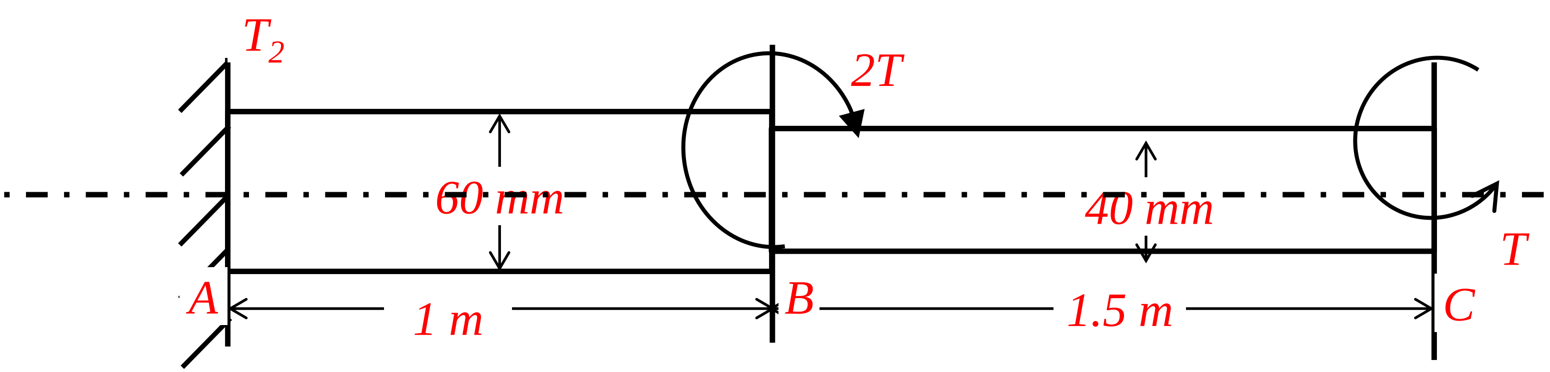
Numerical Problem

Problem 9: A stepped steel shaft is subjected to a torque T at the free end and a torque $2T$ in the opposite direction at the junction of the two sizes. ***Determine the total angle of twist if the maximum shear stress is limited to 80 MPa.*** Take G = 80 GPa.

Given: τ = 80 MPa; G = 80 GPa

Torque in the portion $BC = T$ (counter-clockwise)
Torque in the portion $AB = T\text{-}2T$ (counter-clockwise) or (T clockwise)
Thus the two portions of the shaft are subjected to a torque of same magnitude but in the opposite direction.
Therefore, the maximum stress will reach the maximum value in the thinner portion.

$$T = \frac{\pi}{16}\tau_1 D^3 = \frac{\pi}{16} \times 80 \times (40)^3 = 1005310\, N - mm$$



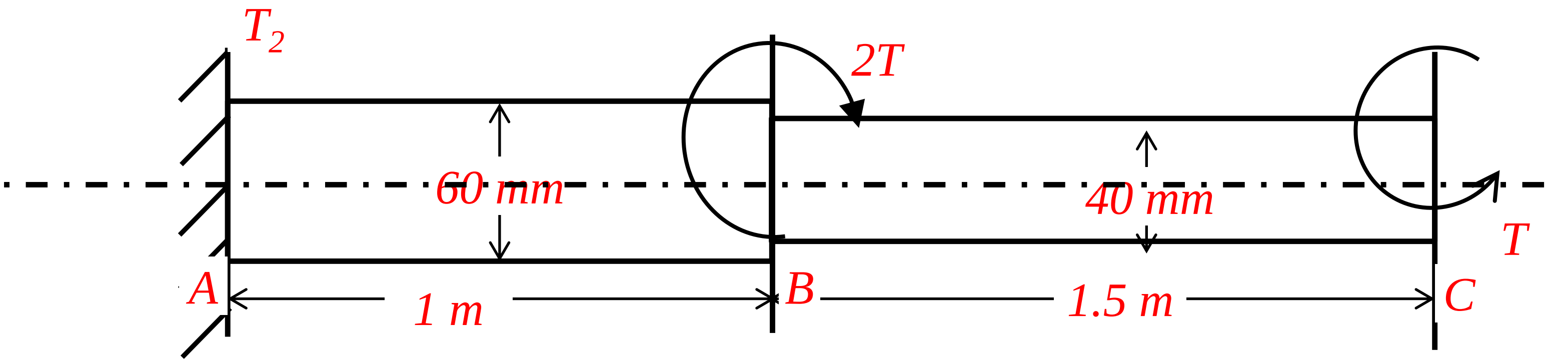
Numerical Problem

Angle of twist ‘θ’

$$\theta = \frac{Tl_{bc}}{G\left(I_p\right)_{bc}} - \frac{Tl_{ab}}{G\left(I_p\right)_{ab}} \Rightarrow \theta = \frac{32T}{G\times\pi}\left(\frac{l_{bc}}{d_{bc}^4} - \frac{l_{ab}}{d_{ab}^4}\right)$$

$$\theta = \frac{32\times 1005310}{80000\times\pi}\left(\frac{1500}{40^4} - \frac{1000}{60^4}\right) = 0.065\,rad$$

$$\theta = 0.065\times\frac{180}{\pi} = 3.73^o$$



Torsional Rigidity

- Torque per radian, Twist is known as the **Torsional Stiffness**.
- **Parameter** ***GI_p*** is known as the **Torsional Rigidity of the Shaft.**
- It is similar to that **Flexural Rigidity of a beam** in case of beam that is ***EI*.**
- **Torsional Rigidity** is also defined as the torque per unit angular twist.

$$\frac{T}{I_p} = \frac{G\theta}{l} \Rightarrow \theta = \frac{Tl}{GI_p} = \frac{T}{k}$$

$$k = \frac{GI_p}{l} = \frac{T}{\theta}$$

Where G, I_p and l are constants for a given shaft

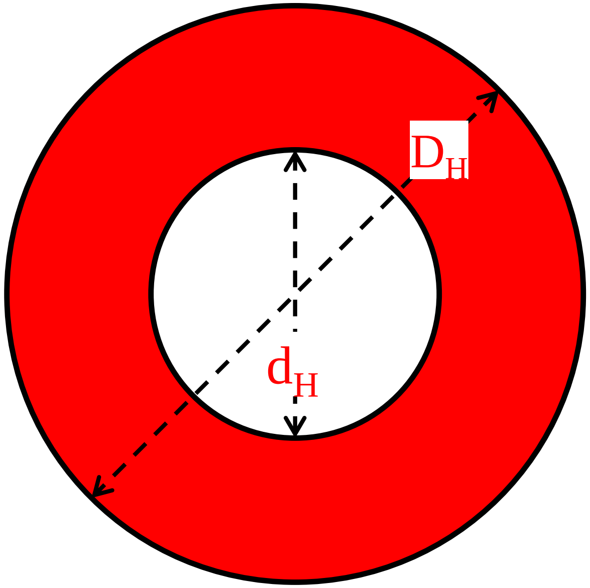
'θ' is the angle of twist is directly proportional to the twisting moment.

Comparison of Solid and Hollow Shaft

Comparison by Strength: Let us consider both the shafts have same length, material, weight and shear stress

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}$$

D_S – Diameter of solid shaft;
D_H – Outer diameter of hollow shaft;
A_S – Area of solid shaft;
A_H – Area of hollow shaft;
T_S – Torque transmitted by solid shaft;
T_H – Torque transmitted by hollow shaft;

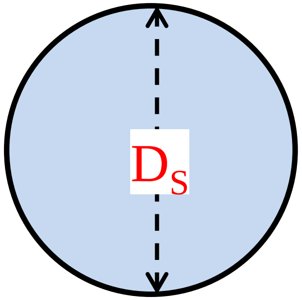


Hollow Shaft

$$T_S = \frac{\pi}{16}\tau D_S^3 \qquad \text{(a)}$$

$$T_H = \frac{\pi}{16}\tau\left[\frac{D_H^4 - d_H^4}{D_H}\right] \qquad \text{(b)}$$

$$\frac{T_H}{T_S} = \frac{\pi}{16}\tau\left[\frac{D_H^4 - d_H^4}{D_H}\right] \bigg/ \frac{\pi}{16}\tau D_S^3 = \left(D_H^4 - d_H^4\right) \Big/ \left(D_S^3 D_H\right) \qquad \text{(c)}$$



Solid Shaft

Comparison of Solid and Hollow Shaft

$$\frac{D_H}{d_H} = n \Rightarrow D_H = n d_H \qquad \text{(d)}$$

Now, rewrite equation (c)

$$\frac{T_H}{T_S} = \frac{\left(D_H^4 - d_H^4\right)}{\left(D_S^3 D_H\right)} \qquad \text{(e)}$$

Now, substitute the value of D_H from *eq. (d)* into *eq. (e)*

$$\frac{T_H}{T_S} = \frac{\left(n^4 d_H^4 - d_H^4\right)}{n\left(D_S^3 d_H\right)} = \frac{\left(n^4 - 1\right) d_H^3}{n\left(D_S^3\right)} \qquad \text{(f)}$$

Since the length, material and weight of both the shafts are same. Therefore,

$$A_H = A_S$$

$$\frac{\pi}{4}\left(D_H^2 - d_H^2\right) = \frac{\pi}{4} D_S^2 \qquad \text{(g)}$$

$$D_S = \sqrt{D_H^2 - d_H^2}$$

$$D_s^3 = \left(D_H^2 - d_H^2\right)\sqrt{D_H^2 - d_H^2} \qquad \text{(h)}$$

Now, substitute the value of D_H from *eq. (d)* into *eq. (h)*

$$D_s^3 = \left(n^2 d_H^2 - d_H^2\right)\sqrt{n^2 d_H^2 - d_H^2}$$

$$D_s^3 = d_H^3\left(n^2 - 1\right)\sqrt{n^2 - 1} \qquad \text{(i)}$$

Comparison of Solid and Hollow Shaft

Now substitute the value of D_S^3 from *eq. (i)* into *eq. (f)*

$$\frac{T_H}{T_S} = \frac{\left(n^4-1\right)d_H^3}{n\left(d_H^3\left(n^2-1\right)\sqrt{n^2-1}\right)} = \frac{\left(n^2+1\right)\left(n^2-1\right)}{n\left(n^2-1\right)\sqrt{n^2-1}} = \frac{\left(n^2+1\right)}{n\sqrt{n^2-1}} \qquad (j)$$

Since the value of D_H is always greater than 1. Therefore, the value of n is always greater than unity.

Let us assume that n = 2

$$\frac{T_H}{T_S} = \frac{\left(2^2+1\right)}{2\sqrt{2^2-1}} = \frac{5}{2\sqrt{3}} = 1.44$$

Therefore, the torque transmitted by hollow shaft is greater than the solid shaft.

Hence, it means that hollow shaft is more stronger than solid shaft.

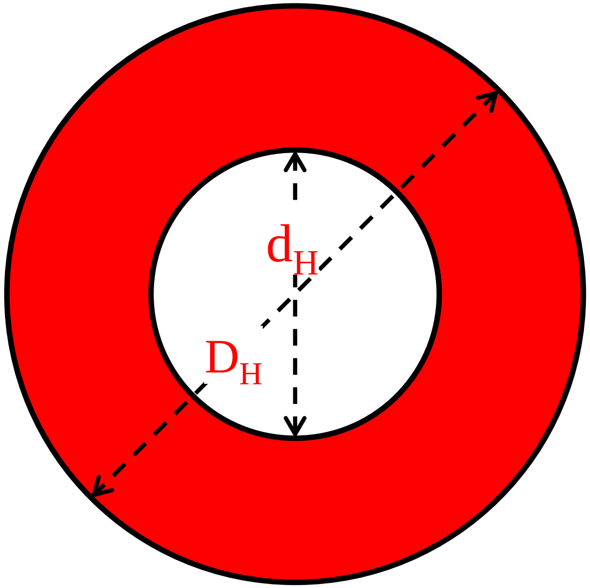
Comparison of Solid and Hollow Shaft

Comparison by Weight: let us consider that both the shafts have same length and material. If the torque applied to both the shafts is same, then the maximum shear stress will also be same in solid and hollow shaft.

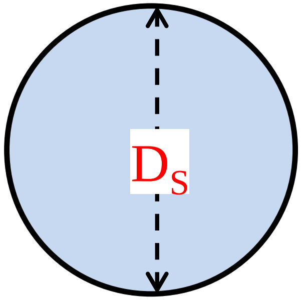
D_S – Diameter of solid shaft;
D_H – Outer diameter of hollow shaft;
A_S – Area of solid shaft;
A_H – Area of hollow shaft;
T_S – Torque transmitted by solid shaft;
T_H – Torque transmitted by hollow shaft;

$$\frac{W_H}{W_S} = \frac{A_H}{A_S} = \frac{\frac{\pi}{4}\left(D_H^2 - d_H^2\right)}{\frac{\pi}{4}\left(D_S^2\right)} = \frac{\left(D_H^2 - d_H^2\right)}{D_S^2} \qquad \text{(a)}$$

$$\frac{D_H}{d_H} = n \Rightarrow D_H = n d_H \qquad \text{(b)}$$



Hollow Shaft



Solid Shaft

Comparison of Solid and Hollow Shaft

Now, substitute the value of D_H from *eq. (b)* into *eq. (a)*

$$\frac{W_H}{W_S} = \frac{\left(n^2 d_H^2 - d_H^2\right)}{D_S^2} = \frac{d_H^2\left(n^2 - 1\right)}{D_S^2} \qquad \text{(c)}$$

Since the torque applied in same in both the shafts i.e. solid and hollow shaft

$$T_H = T_S$$

$$\frac{\pi}{16}\tau\left[\frac{D_H^4 - d_H^4}{D_H}\right] = \frac{\pi}{16}\tau D_S^3 \qquad \text{(d)}$$

Now, substitute the value of D_H from *eq. (b)* into *eq. (d)*

$$\frac{\pi}{16}\tau\left[\frac{n^4 d_H^4 - d_H^4}{n d_H}\right] = \frac{\pi}{16}\tau D_S^3$$

$$D_S^3 = \left[\frac{n^4 - 1}{n}\right] d_H^3$$

$$D_S = d_H\left[\frac{n^4 - 1}{n}\right]^{1/3}$$

$$D_S^2 = d_H^2\left[\frac{n^4 - 1}{n}\right]^{2/3} \qquad \text{(e)}$$

Comparison of Solid and Hollow Shaft

Now substitute the value of D_S^2 from *eq. (e)* into *eq. (c)*

$$\frac{W_H}{W_S}=\frac{\left(n^2 d_H^2-d_H^2\right)}{D_S^2}=\frac{d_H^2\left(n^2-1\right)}{d_H^2}\left[\frac{n}{n^4-1}\right]^{2/3} \qquad \text{(f)}$$

$$\frac{W_H}{W_S}=\frac{\left(n^2 d_H^2-d_H^2\right)}{D_S^2}=\left(n^2-1\right)\left[\frac{n}{n^4-1}\right]^{2/3} \qquad \text{(g)}$$

Since the value of D_H is always greater than 1. Therefore, the value of n is always greater than unity.

Let us assume that n = 2

$$\frac{W_H}{W_S}==\left(2^2-1\right)\left[\frac{2}{2^4-1}\right]^{2/3}=0.7829$$

Hence, hollow shaft is more economical as compared to solid shaft.

Numerical Problem

Problem 9 : Two shafts of the same material and same length are subjected to the same torque. If the first shaft is of solid circular section, and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in the each shaft is same, compare the weights of two shafts.

$$d_H = \frac{2}{3} D_H \Rightarrow D_H = \frac{3}{2} d_H \Rightarrow n = \frac{3}{2}$$

$$\frac{W_H}{W_S} = \frac{\left(n^2 d_H^2 - d_H^2\right)}{D_S^2} = \left(n^2 - 1\right)\left[\frac{n}{n^4 - 1}\right]^{2/3} = \left(1.5^2 - 1\right)\left[\frac{1.5}{1.5^4 - 1}\right]^{2/3}$$

$$\frac{W_H}{W_S} = 0.64$$