

Course: UMA 035 (Optimization Techniques)

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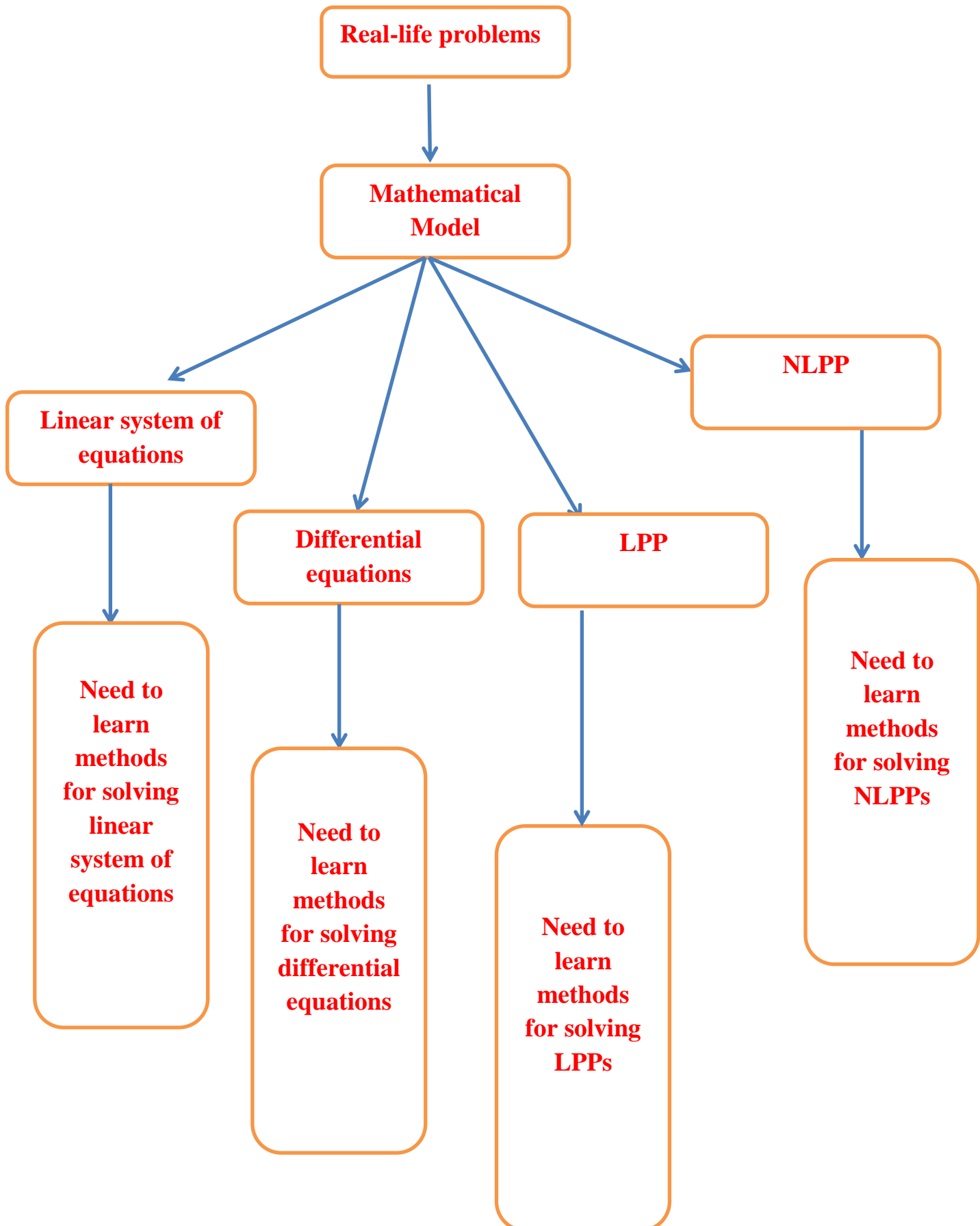
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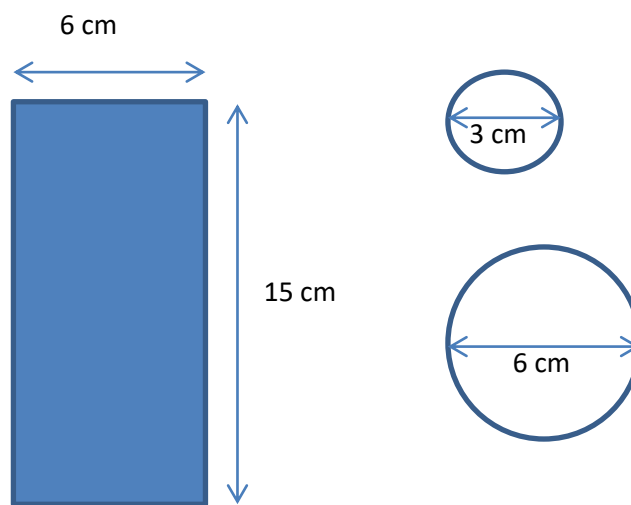
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Need of LPP and NLPP in real-life problems



Some real-life problems

Material Science department of TIET needs circular metallic plates of diameter 3 cm and 6 cm to perform experiments on heat treatment studies and requires minimum 2500 and 1500 units respectively. These are to be cut from parent metallic sheets of dimension $6 \times 15 \text{ cm}^2$. Construct a mathematical model so that minimum number of parent metallic sheets are used.



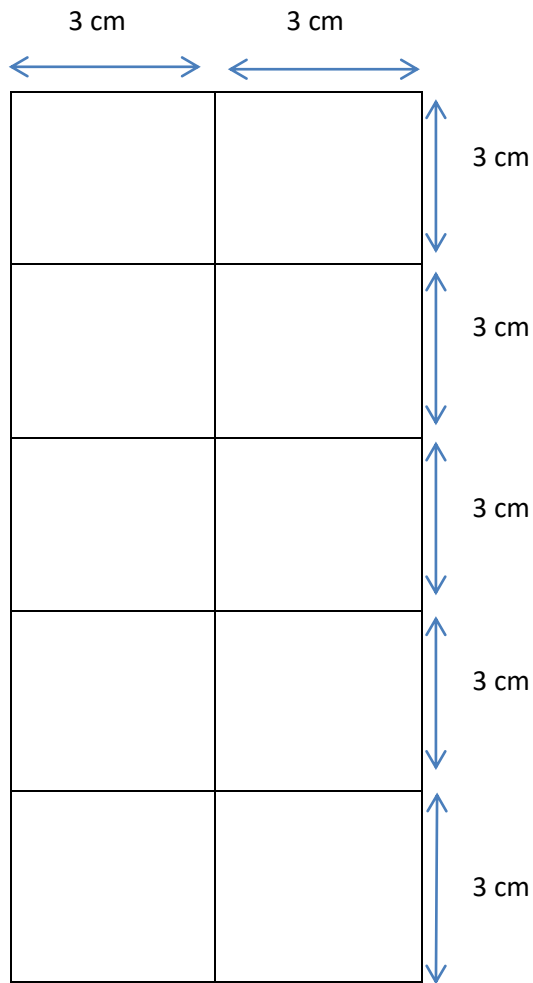
Minimize (Total number of used rectangular metallic sheets)

Subject to

Circular plates of diameter 3 cm ≥ 2500

Circular plates of diameter 6 cm ≥ 1500

Pattern 1

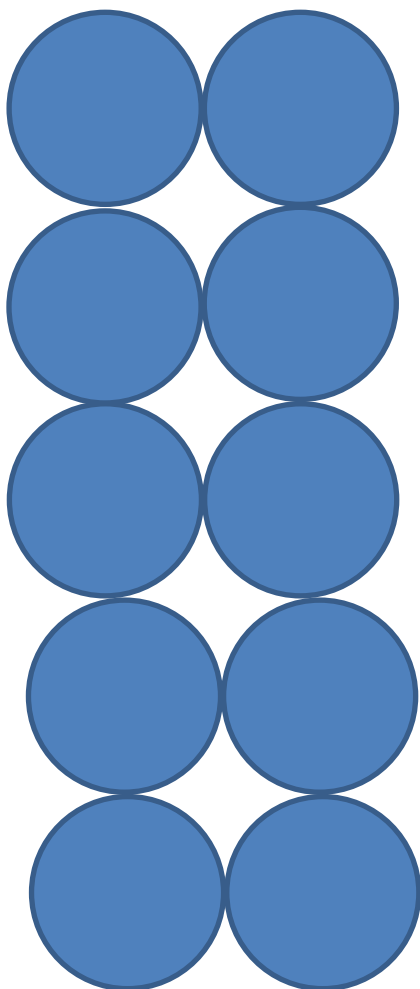


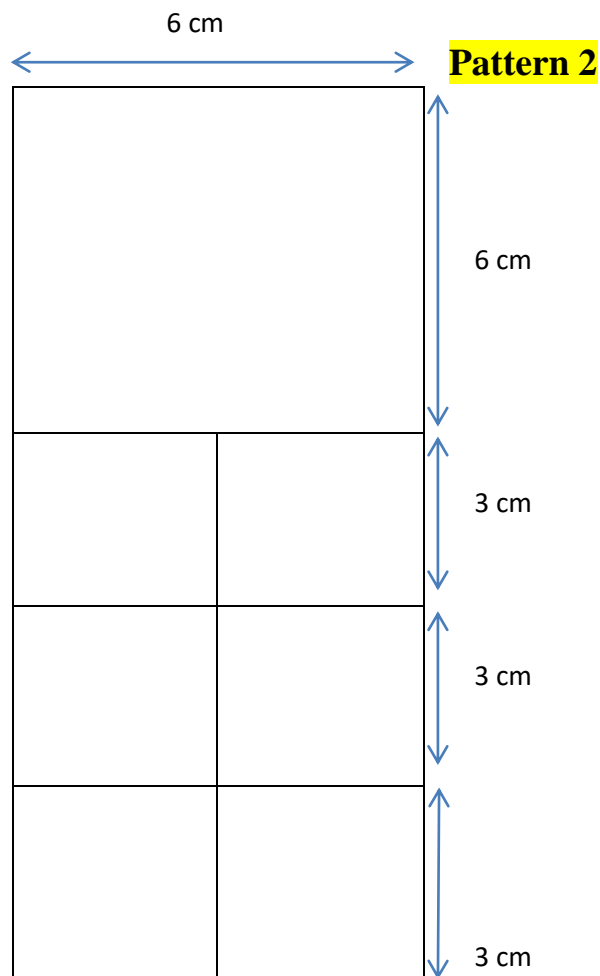
One sheet → **6 cm** → **0 circular plates**

x1 sheet → **6 cm** → **0x1 circular plates**

One sheet → **3 cm** → **10 circular plates**

x1 sheet → **3 cm** → **10x1 circular plates**



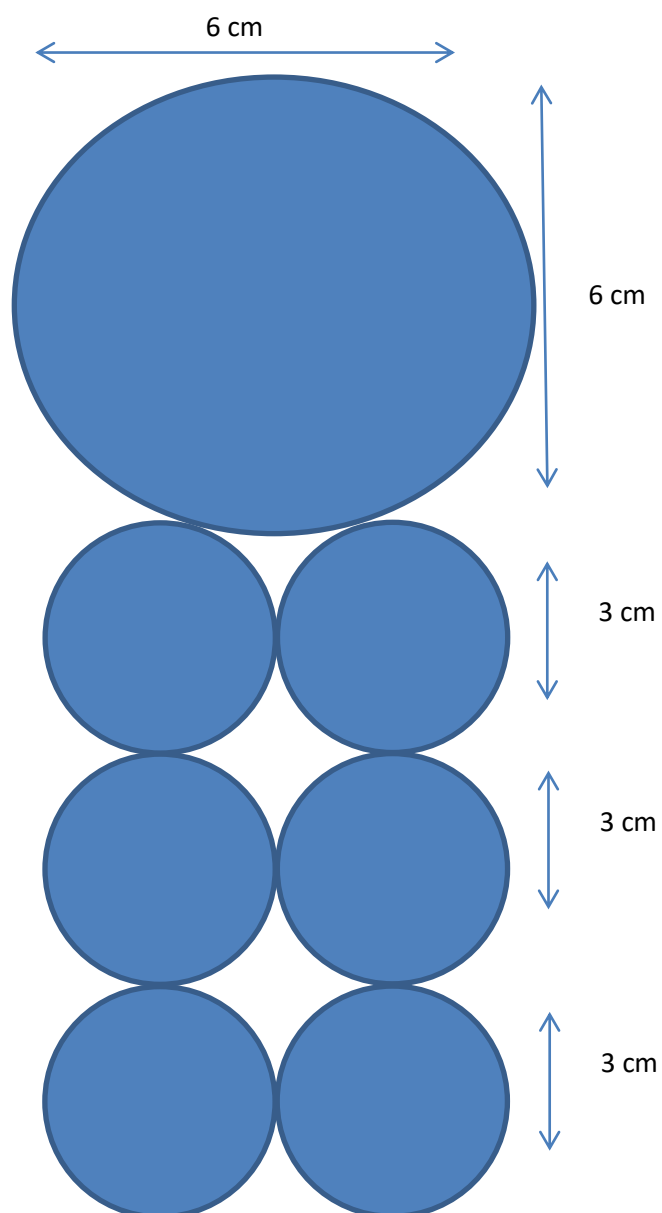


One sheet → **6 cm** → **1 circular plates**

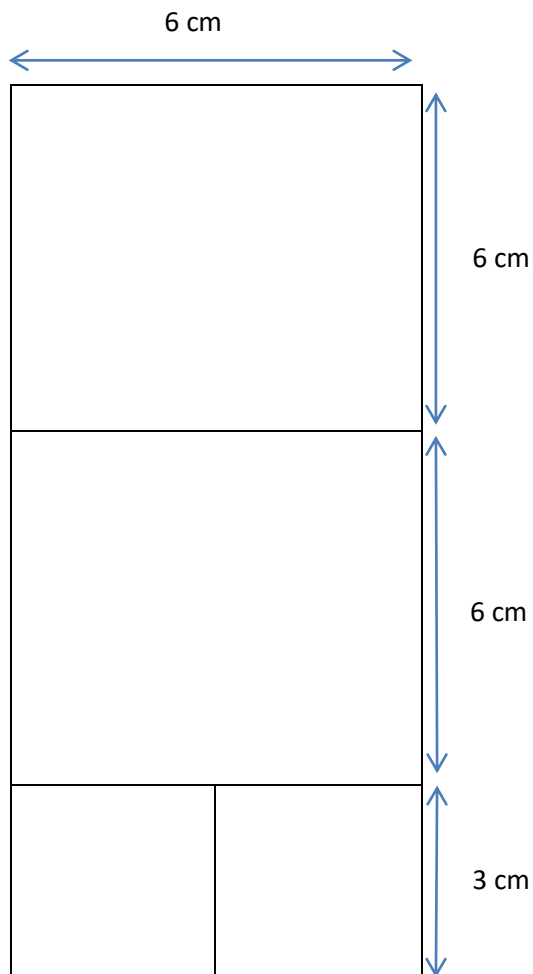
x2 sheet → **6 cm** → **1x2 circular plates**

One sheet → **3 cm** → **6 circular plates**

x2 sheet → **3 cm** → **6x2 circular plates**



Pattern 3



One sheet → 6 cm → 2 circular plates

x3 sheet → 6 cm → 2x2 circular plates

One sheet → 3 cm → 2 circular plates

x3 sheet → 3 cm → 2x3 circular plates

	Number of circular plates having diameter 3 cm	Number of circular plates having diameter 6 cm	Number of rectangular sheets
Pattern 1	10x1	0x1	x1
Pattern 2	6x2	1x2	x2
Pattern 3	2x3	2x3	x3
Total	10x1+6x2+2x3	0x1+1x2+2x3	x1+ x2+ x3

x1, x2 and x3 physically represents the number of rectangular sheets.

If sheets will not be used then number of sheets will be 0.

If sheets will be used then number of sheets will be positive.

Number of sheets cannot be negative.

Therefore, $x1 \geq 0$, $x2 \geq 0$, $x3 \geq 0$.

Minimize (Total number of used rectangular metallic sheets)

Subject to

Circular plates of diameter 3 cm ≥ 2500

Circular plates of diameter 6 cm ≥ 1500

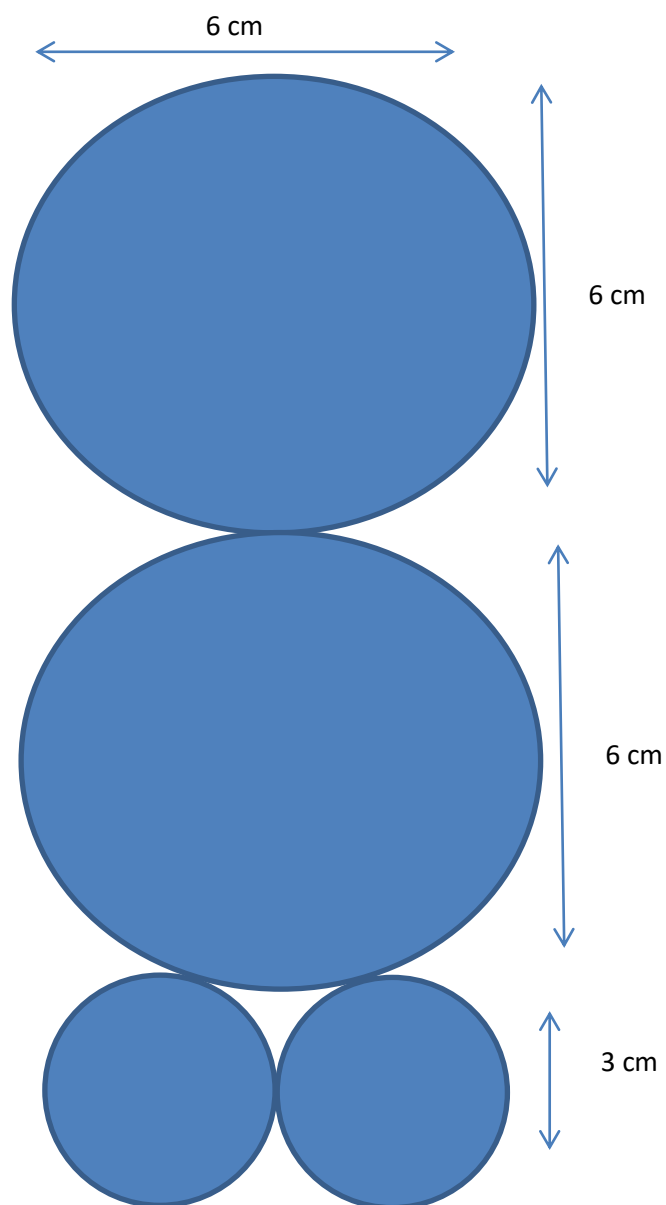
Minimize ($x1 + x2 + x3$)

Subject to

$10x1 + 6x2 + 2x3 \geq 2500$

$0x1 + 1x2 + 2x3 \geq 1500$

$x1 \geq 0$, $x2 \geq 0$, $x3 \geq 0$



A metal slitting company cuts master rolls with width 200 centimeters into subrolls of small width. Customer specifies that they need subrolls of different widths given in the following table:

Width of subroll (in cm)	Numbers required
35	200
80	90
90	350
120	850

The objective is to use a minimum number of master rolls to satisfy set of customers' orders. Construct the mathematical model.

Minimize (Number of master rolls)

Subject to

Number of subrolls of width 35 = 200

Number of subrolls of width 80 = 90

Number of subrolls of width 90 = 350

Number of subrolls of width 120 = 850

Pattern 1

All of width 35

$200/35 = 5.71...$

This indicates maximum 5 subrolls of width 35 can be obtained

Used material is $35 \times 5 = 175$

Wastage = $200 - 175 = 25$

Pattern 2

If we reduce the subrolls of width 35 from 5 to 4 (Fixed) then

Used material is $35 \times 4 = 140$

Remaining = $200 - 140 = 60$

Only possibility to cut from 60 is 35.

60 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this Pattern.

Pattern 3

If we reduce the subrolls of width 35 from 5 to 3 then

Used material is $35 \times 3 = 105$

Remaining = $200 - 105 = 95$

95 is not a wastage as it is greater than 35.

The possibilities are one subroll of 80 or one subroll of 90

Pattern 3a

One subroll of 80

If we cut one subroll of 80 from 95 then remaining is $95 - 80 = 15$.

15 is wastage as it is less than 35

Pattern 3b

One subroll of 90

If we cut one subroll of 90 from 95 then wastage is $95 - 90 = 5$.

5 is wastage as it is less than 35

Pattern 4

If we reduce the subrolls of width 35 from 5 to 2 then

Used material is $35 \times 2 = 70$

Remaining = $200 - 70 = 130$

130 is not a wastage as it is greater than 35.

The possibilities are one subroll of 80 or one subroll of 90 or one subroll of 120

Pattern 4a

One subroll of 80

If we cut one subroll of 80 from 130 then remaining is $130 - 80 = 50$.

50 is not wastage as we can cut subroll of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this case.

Pattern 4b

One subroll of 90

If we cut one subroll of 90 from 130 then remaining is $130-90=40$.

40 is not wastage as we can cut subroll of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this pattern.

Pattern 4c

One subroll of 120

If we cut one subroll of 120 from 130 then remaining is $130-120=10$.

10 is a wastage as it is less than 35.

Pattern 5

If we reduce the subrolls of width 35 from 5 to 1 then

Used material is $35*1=35$

Remaining = $200-35=165$

The possibilities are two subrolls of 80 or one subroll of 90 or one subroll of 120

Pattern 5a

Two subrolls of 80

If we cut two subrolls of 80 from 165 then remaining is $165-80*2=5$.

5 is a wastage as it is less than 35.

Pattern 5b

One subroll of 90

If we cut one subroll of 90 from 165 then remaining is $165-90=75$.

75 is not wastage as we can cut two subrolls of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this pattern.

Pattern 5c

One subroll of 120

If we cut one subroll of 120 from 165 then remaining is $165-120=45$.

45 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this case.

Pattern 6

If we reduce the subrolls of width 35 from 5 to 0 then

Used material is $35*0=0$

Remaining = $200-0=200$

The possibilities are two subrolls of 80 or two subrolls of 90 or one subroll of 120 and one subroll of 80 or one subroll of 80 and one of 90

Pattern 6a

Two subrolls of 80

If we cut two subrolls of 80 from 200 then remaining is $200-80*2=40$.

40 is not wastage as we can cut one subroll of 35 from it. But we have fixed subrolls of width 35.

So we will not consider this case.

Pattern 6b

Two subrolls of 90

If we cut two subrolls of 90 from 200 then remaining is $200-90*2=20$.

20 is a wastage as it is less than 35.

Pattern 6c

One subroll of 120 and one subroll of 80

If we cut one subroll of 120 and one subroll of 80 from 200 then remaining is $200-120-80=0$.

0 is wastage as it is less than 35.

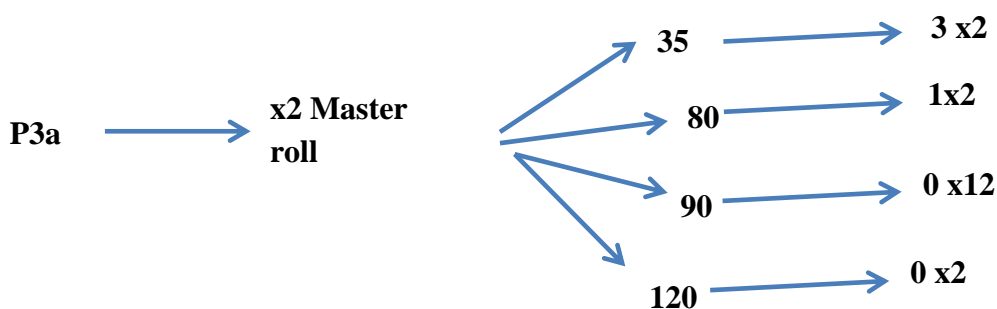
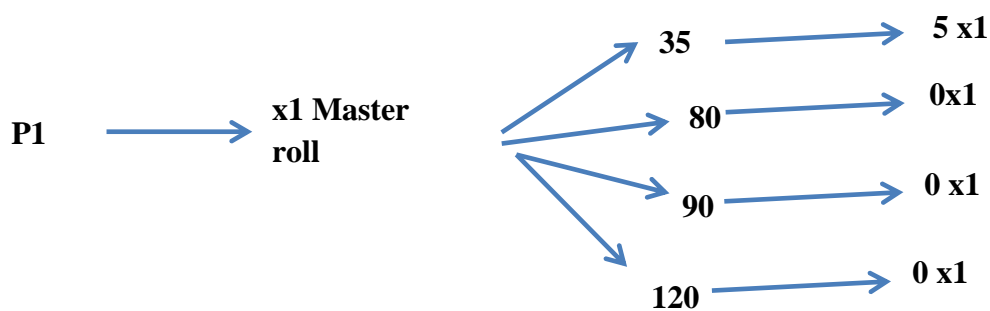
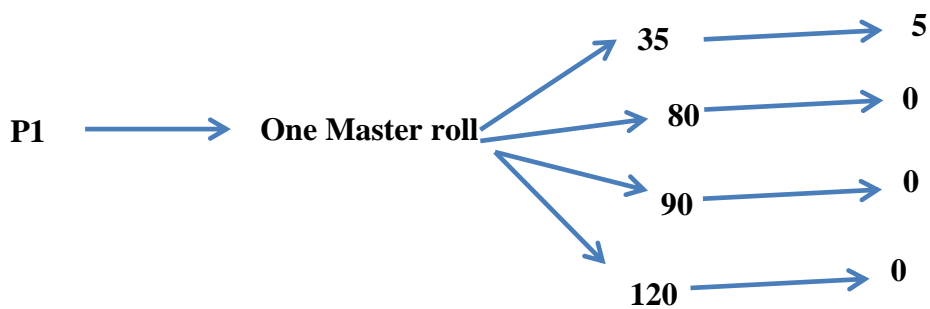
Pattern 6d

One subroll of 80 and one subroll of 90

If we cut one subroll of 80 and one subroll of 90 from 200 then remaining is $200 - 90 - 80 = 30$.

30 is wastage as it is less than 35.

	P1	P3a	P3b	P4a	P5a	P6b	P6c	P6d
35	5	3	3	2	1	0	0	0
80	0	1	0	0	2	0	1	1
90	0	0	1	0	0	2	0	1
120	0	0	0	1	0	0	1	0



	P1	P3a	P3b	P4a	P5a	P6b	P6c	P6d	Total
Master Rolls	x1	x2	x3	x4	x5	x6	x7	x8	$x1 + x2 + \dots + x8$
35	5x1	3 x2	3 x3	2 x4	1 x5	0 x6	0 x7	0 x8	$5x1 + 3 x2 + 3 x3 + 2 x4 + x5$
80	0 x1	1 x2	0 x3	0 x4	2 x5	0 x6	1 x7	1 x8	$2 x5 + x7 + x8$
90	0 x1	0 x2	1 x3	0 x4	0 x5	2 x6	0 x7	1 x8	$1 x3 + 2 x6 + x8$
120	0 x1	0 x2	0 x3	1 x4	0 x5	0 x6	1 x7	0 x8	$1 x4 + 1 x7$
Waste	25 x1	15 x2	5x3	10x4	5x5	20x6	0x7	30x8	$25 x1 + 15 x2 + 5x3 + 10x4 + 5x5 + 20x6 + 30x8$

Minimize (Number of master rolls)

Subject to

Number of subrolls of width 35 = 200

Number of subrolls of width 80 = 90

Number of subrolls of width 90 = 350

Number of subrolls of width 120 = 850

Minimize ($x1 + x2 + \dots + x8$)

Subject to

$5x1 + 3 x2 + 3 x3 + 2 x4 + x5 = 200$

$2 x5 + x7 + x8 = 90$

$1 x3 + 2 x6 + x8 = 350$

$1 x4 + 1 x7 = 850$

$x1, x2, \dots, x8 \geq 0$

Minimize (Wastage)

Subject to

Number of subrolls of width 35 = 200

Number of subrolls of width 80 = 90

Number of subrolls of width 90= 350

Number of subrolls of width 120 = 850

Minimize $(25x_1 + 15x_2 + 5x_3 + 10x_4 + 5x_5 + 20x_6 + 30x_8)$

Subject to

$5x_1 + 3x_2 + 3x_3 + 2x_4 + x_5 = 200$

$2x_5 + x_7 + x_8 = 90$

$x_3 + 2x_6 + x_8 = 350$

$x_4 + x_7 = 850$

$x_1, x_2, \dots, x_8 \geq 0$