Course: UMA 035 (Optimization Techniques)

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Basic solutions and Non-basic solutions of a LPP

Use the following steps to find basic and non-basic solutions of a LPP.

Step 1

Transform the given LPP in standard form.

Step 2

If in the standard form, there are m linearly independent equations having n where, n>m. Then, consider n–m variables as 0 at a time.

There may be ${}^{n}C_{m}$ cases to consider n–m variables as 0 at a time.

Consider all the possible cases.

Step 3

In each case, a system of linear equations will be obtained (number of variables will be equal to number of equations).

Step 4

If solution exist for the obtained system of linear equation (in any case). Then, there may exist unique solution (if determinant of the coefficient matrix is not 0) or there may exist infinite number of solutions (if determinant of the coefficient matrix is 0) for the obtained system of linear equations.

Case (1) If a unique solution exist for the obtained system of linear equations then that unique solution is called basic solution.

Case (2) If infinite number of solutions exist for the obtained system of linear equations then all the infinite number of solutions are called non-basic solutions.

Basic variables and non-basic variables

In each case n–m variables will be considered 0 at a time. These n–m variables are called non-basic variables for the considered case.

The remaining m variables are called basic variables for the considered case.

Degenerate solution and Non-degenerate solution

If value of any basic variable is 0 in the obtained solution then such a solution is called degenerate solution otherwise non-degenerate solution.

Basic feasible solution

A solution which is basic as well as feasible is called basic feasible solution.

Non-basic feasible solution

A solution which is non-basic as well as feasible is called non-basic feasible solution.

Basic infeasible solution

A solution which is basic as well as infeasible is called basic infeasible solution.

Non-basic infeasible solution

A solution which is non-basic as well as infeasible is called non-basic infeasible solution.

Degenerate basic solution

A solution which is basic as well as degenerate is called degenerate basic solution.

Non-degenerate basic solution

A solution which is basic as well as non-degenerate is called non-degenerate basic solution.

Degenerate non-basic solution

A solution which is non-basic as well as degenerate is called degenerate basic solution.

Non-degenerate non-basic solution

A solution which is non-basic as well as non-degenerate is called non-degenerate non-basic solution.

Degenerate basic feasible solution

A solution which is basic, degenerate and feasible is called degenerate basic feasible solution.

Non-degenerate basic feasible solution

A solution which is basic non-degenerate and feasible is called non-degenerate basic feasible solution.

Degenerate basic infeasible solution

A solution which is basic, degenerate and infeasible is called degenerate basic infeasible solution.

Non-degenerate basic infeasible solution

A solution which is basic non-degenerate and infeasible is called non-degenerate basic infeasible solution.

Example: **Find all the basic and non-basic solutions for the following LPP. Also, check that the obtained solution is feasible/infeasible and degenerate/non-degenerate.**

Maximize $(3x_1 - 2x_2)$

Subject to

$x_1 - 4x_2 <= 5,$

$2x_1 - 8x_2 >= 10,$

$x_1 >= 2, x_2 >= -8.$

Step 1: Transform the LPP in standard form

First variable

$x_1 >= 2$

$x_1 - 2 >= 0$

Assume

$x_1 - 2 = y_1$

i.e.,

$x_1 = y_1 + 2$

Replace the variable x_1 with $y_1 + 2$

Second variable

$x_2 >= -8$

$x_2 + 8 >= 0$

Assume

$x_2 + 8 = y_2$

i.e.,

$x_2 = y_2 - 8$

Replace the variable x_2 with $y_2 - 8$

Transformed LPP

Maximize $(3(y_1+2) - 2(y_2-8))$

Subject to

$(y_1+2) - 4(y_2-8) <= 5,$

$2(y_1+2) - 8(y_2-8) >= 10,$

$(y_1+2) >= 2, (y_2-8) >= -8.$

Maximize $(3y_1 + 6 - 2y_2 + 16)$

Subject to

$y_1 + 2 - 4y_2 + 32 <= 5,$

$2y_1 + 4 - 8y_2 + 64 >= 10,$

$y_1 >= 2 - 2, \quad y_2 >= -8 + 8.$

Maximize $(3y_1 - 2y_2 + 22)$

Subject to

$y_1 - 4y_2 <= 5-34,$

$2y_1 - 8y_2 >= 10-68,$

$y_1 >= 0, \quad y_2 >= 0.$

Maximize $(3y_1 - 2y_2 + 22)$

Subject to

$y_1 - 4y_2 <= -29,$

$2y_1 - 8y_2 >= -58,$

$y_1 >= 0, \quad y_2 >= 0.$

Maximize $(3y_1 - 2y_2 + 22)$

Subject to

$-y_1 + 4y_2 >= 29,$

$-2y_1 + 8y_2 <= 58,$

$y_1 >= 0, \quad y_2 >= 0.$

Maximize $(3y_1 - 2y_2 + 22)$

Subject to

$-y_1 + 4y_2 - S_1 = 29,$

$-2y_1 + 8y_2 + S_2 = 58,$

$y_1 >= 0, \quad y_2 >= 0, S_1 >= 0, S_2 >= 0$

Step 2

Number of linearly independent equations = m=2

Number of variables=n=4

Extra variables to be considered as 0 at a time = n–m=4–2=2

Number of possible cases to consider 2 variables as 0 at a time = $^{n}C_{m}$ = $^{4}C_{2}$

=4*3/2*1=6

Case (1):

y_1=0 and y_2=0

(y_1 and y_2 are non-basic variables and remaining S_1 and S_2 are basic variables)

$-y_1 + 4y_2 - S_1 = 29$ implies $\quad 0+0-S_1 = 29$

$-2y_1 + 8y_2 + S_2 = 58$ implies $\quad 0+0+S_2 = 58$

Unique solution: $S_1 = -29$ and $S_2 = 58$

Final unique solution (Basic solution)

y_1=0 and y_2=0, $S_1 = -29$ and $S_2 = 58$

Since, $S_1 = -29$ is not satisfying the restriction $S_1 >= 0$. So, it is an infeasible solution.

Since, value of none of the basic variables S_1 and S_2 is 0. So, the solution is non-degenerate

Hence, $y_1=0$ and $y_2=0$, $S_1= -29$ and $S_2= 58$ is a non-degenerate basic infeasible solution.

Case (2):

$y_1=0$ and $S_1=0$

(y_1 and S_1 are non-basic variables and remaining y_2 and S_2 are basic variables)

$-y_1 + 4y_2 - S_1 = 29$ implies $\quad 0+4y_2 - 0 = 29$

$-2y_1 + 8y_2 + S_2 = 58$ implies $\quad 0+8y_2 + S_2 = 58$

Unique solution: $y_2=29/4$ and $S_2= 58-(8*29/4)=0$

Final unique solution (Basic solution)

$y_1=0$ and $y_2=29/4$, $S_1=0$ and $S_2= 0$

Since, all constraints are satisfying. So, it is a feasible solution.

Since, value of the basic variable S_2 is 0. So, the solution is Degenerate

Hence, $y_1=0$ and $y_2=29/4$, $S_1=0$ and $S_2= 0$ is a Degenerate basic feasible solution.

Case (3):

$y_1=0$ and $S_2=0$

(y_1 and S_2 are non-basic variables and remaining y_2 and S_1 are basic variables)

$-y_1 + 4y_2 - S_1 = 29$ implies $\quad$ **$0+4y_2 - S_1 = 29$**

$-2y_1 + 8y_2 + S_2 = 58$ implies $\quad$ **$0+8y_2 + 0 = 58$**

Unique solution: $y_2=58/8=29/4$ and $S_1= 4*(29/4) - 29=0$

Final unique solution (Basic solution)

$y_1=0$ and $y_2=29/4$, $S_1=0$ and $S_2= 0$

Since, all constraints are satisfying. So, it is a feasible solution.

Since, value of the basic variable S_1 is 0. So, the solution is Degenerate

Hence, $y_1=0$ and $y_2=29/4$, $S_1=0$ and $S_2= 0$ is a Degenerate basic feasible solution.

Case (4):

$y_2=0$ and $S_1=0$

(y_2 and S_1 are non-basic variables and remaining y_1 and S_2 are basic variables)

$-y_1 + 4y_2 - S_1 = 29$ implies $\quad$ **$-y_1 + 0 - 0 = 29$**

$-2y_1 + 8y_2 + S_2 = 58$ implies $\quad$ **$-2y_1 + 0 + S_2 = 58$**

Unique solution: $y_1 = -29$ **and** $S_2 = 58+2*(-29) = 0$

Final unique solution (Basic solution)

$y_1 = -29$ **and** $y_2=0$, $S_1=0$ **and** $S_2 = 0$

Since, $y_1 >=$ **is not satisfying. So, it is not a feasible solution.**

Since, value of the basic variable S_2 **is 0. So, the solution is Degenerate**

Hence, $y_1 = -29$ **and** $y_2=0$, $S_1=0$ **and** $S_2 = 0$ **is a Degenerate basic infeasible solution.**

Case (5):

$y_2=0$ **and** $S_2=0$

(y_2 **and** S_2 **are non-basic variables and remaining** y_1 **and** S_1 **are basic variables)**

$-y_1 + 4y_2 - S_1 = 29$ **implies** $\quad -y_1 - S_1 = 29$

$-2y_1 + 8y_2 + S_2 = 58$ **implies** $\quad -2y_1 + 0 + 0 = 58$

Unique solution: $y_1 = -29$ **and** $S_1 = -(-29) - 29 = 0$

Final unique solution (Basic solution)

$y_1 = -29$ **and** $y_2=0$, $S_1=0$ **and** $S_2 = 0$

Since, $y_1 >=$ **is not satisfying. So, it is not a feasible solution.**

Since, value of the basic variable S_2 **is 0. So, the solution is Degenerate**

Hence, $y_1 = -29$ and $y_2=0$, $S_1=0$ and $S_2= 0$ is a Degenerate basic infeasible solution.

Case (6):

$S_1=0$ and $S_2=0$

(S_1 and S_2 are non-basic variables and remaining y_1 and y_2 are basic variables)

$-y_1 + 4y_2 - S_1 = 29$ implies $\quad -y_1 + 4y_2 = 29$

$-2y_1 + 8y_2 + S_2 = 58$ implies $\quad -2y_1 + 8y_2 = 58$ implies $-y_1 + 4y_2 = 29$

Since, $\begin{vmatrix} -1 & 4 \\ -2 & 8 \end{vmatrix} = 0$. So, infinite solution.

Infinite number of solutions (Non-basic solutions):

$y_1 = a$ (any real number)

$y_2 = (29+a)/4$

$y_1 = a$ and $y_2 = (29+a)/4$, $S_1=0$ and $S_2= 0$ is non-basic solution.

$y_1 = a$ and $y_2=(29+a)/4$, $S_1=0$ and $S_2= 0$ will be non-basic feasible solution if

$a>=0$ and $(29+a)/4>=0$

i.e., $a>=0$ and $a>= -29$

i.e., $a>=0$

$y_1 = a$ and $y_2=(29+a)/4$, $S_1=0$ and $S_2= 0$ will be non-basic infeasible solution if

$a<0$

If $a=0$ then the value of the basic variable y_1 will be 0.

Hence, $y_1= a$ and $y_2=(29+a)/4$, $S_1=0$ and $S_2= 0$ will be non-basic degenerate feasible solution

If $a = –29$ then the value of the basic variable y_2 will be 0 and the basic variable y_1 will be $–29$.

Hence, $y_1= a$ and $y_2=(29+a)/4$, $S_1=0$ and $S_2= 0$ will be non-basic degenerate infeasible solution