

Course: UMA 035 (Optimization Techniques)

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Need of LPP and NLPP in real-life problems

A company has two grades of inspectors, I and II, who are to be assigned for a quality Control inspection. It is required that at least 2000 pieces be inspected per 8 hour day. Grade I inspectors can check pieces at the rate of 50 per hour with an accuracy of 97%. Grade II inspectors can check pieces at the rate of 40 per hour with an accuracy of 95%. The wage rate of grade I inspector is Rs. 4.50 per hour and that of grade II is Rs. 2.50 per hour. Each time an error is made by an inspector, the cost to the company is Rs. 2.00. The company has available for inspection job, 10 grade I and 5 grade II inspectors. Formulate the problem (DO NOT SOLVE) to minimize the total cost of inspection.

Let x_1 inspectors of grade I and x_2 inspectors of grade II should be assigned.

Then,

First Constraint is : $x_1 \leq 10$ and $x_2 \leq 5$, $x_1 \geq 0$, $x_2 \geq 0$, x_1 is an integer, x_2 is an integer

No. of pieces checked by one inspector of grade I in one hour = 50

No. of pieces checked by one inspector of grade I in 8 hour = $50 \times 8 = 400$

No. of pieces checked by x_1 inspectors of grade I in 8 hour = $400 \times x_1$

No. of pieces checked by one inspector of grade II in one hour = 40

No. of pieces checked by one inspector of grade II in 8 hour = $40 \times 8 = 320$

No. of pieces checked by x_2 inspectors of grade II in 8 hour = $320 \times x_2$

Total pieces checked in 8 hour = $400 \times x_1 + 320 \times x_2$

Second Constraint is : $400 \times x_1 + 320 \times x_2 \geq 2000$

Number of pieces wrongly checked by x_1 inspectors of grade I in 8 hour = $(400 * x_1) * (3/100)$

Number of pieces wrongly checked by x_2 inspectors of grade I in 8 hour = $(320 * x_2) * (5/100)$

Total pieces checked wrongly = $(400 * x_1) * (3/100) + (320 * x_2) * (5/100)$

Extra cost for one wrongly checked piece = Rs. 2

Extra cost for $(400 * x_1) * (3/100) + (320 * x_2) * (5/100)$ wrongly checked piece = $2 * ((400 * x_1) * (3/100) + (320 * x_2) * (5/100))$

Wage of one inspector of grade I for one hour = Rs 4.50

Wage of one inspector of grade I for 8 hour = $4.50 * 8$

Wage of x_1 inspectors of grade I for 8 hour = $4.50 * 8 * x_1$

Wage of one inspector of grade II for one hour = Rs 2.50

Wage of one inspector of grade II for 8 hour = $2.50 * 8$

Wage of x_2 inspectors of grade II for 8 hour = $2.50 * 8 * x_2$

Total Cost = $4.50 * 8 * x_1 + 2.50 * 8 * x_2 + 2 * ((400 * x_1) * (3/100) + (320 * x_2) * (5/100))$

Objective function

Minimize $(4.50 * 8 * x_1 + 2.50 * 8 * x_2 + 2 * ((400 * x_1) * (3/100) + (320 * x_2) * (5/100)))$

LPP

Minimize $(4.50 \times 8 \times x_1 + 2.50 \times 8 \times x_2 + 2 \times ((400 \times x_1) \times (3/100) + (320 \times x_2) \times (5/100)))$

Subject to

$400 \times x_1 + 320 \times x_2 \geq 2000$

$x_1 \leq 10$ and $x_2 \leq 5$, $x_1 \geq 0$, $x_2 \geq 0$, x_1 is an integer, x_2 is an integer

A company manufacturing TV and Radio sets has four major departments, chasis, cabinet, assembly and final testing. Monthly capacities are:

Capacity →	TV		Radio
Departments ↓			
Chasis	2500	or	4500
Cabinet	2000	or	8000
Assembly	3000	or	4000
Final Testing	4500	or	9000

The profit per TV set is Rs. 250 and that of a radio set is Rs. 50. Assuming that the company can sell any quantity of either product, determine the optimal combination of output. Formulate it as Linear programming problem.

Let x_1 TV and x_2 radio.

Then,

First Constraint: $x_1 \geq 0$ and $x_2 \geq 0$, x_1 is an integer, x_2 is an integer.

Time required for chasis for 2500 Tv = 1 month

Time required for chasis for 1 Tv = $1/2500$ month

Time required for chasis for x_1 Tv = $x_1/2500$ month

Time required for chasis for 4500 Radio = 1 month

Time required for chasis for 1 Radio = $1/4500$ month

Time required for chasis for x_2 Radio = $x_2/4500$ month

Total time for chasis for TV and Radio ≤ 1 month

$$x_1/2500 \text{ month} + x_2/4500 \text{ month} \leq 1 \text{ month}$$

$$\text{First constraint: } x_1/2500 + x_2/4500 \leq 1$$

$$\text{Second constraint: } x_1/2000 + x_2/8000 \leq 1$$

$$\text{Third constraint: } x_1/3000 + x_2/4000 \leq 1$$

$$\text{Fourth constraint: } x_1/4500 + x_2/9000 \leq 1$$

Profit on one TV = Rs 250

$$\text{Profit on } x_1 \text{ TV} = 250 x_1$$

Profit on one Radio = Rs 50

$$\text{Profit on } x_2 \text{ Radio} = 50 x_2$$

$$\text{Total Profit} = 250 x_1 + 50 x_2$$

$$\text{Objective : Maximize } (250 x_1 + 50 x_2)$$

LPP

$$\text{Maximize } (250 x_1 + 50 x_2)$$

Subject to

$$x_1/2500 + x_2/4500 \leq 1$$

$$x_1/2000 + x_2/8000 \leq 1$$

$$x_1/3000 + x_2/4000 \leq 1$$

$$x_1/4500 + x_2/9000 \leq 1$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0, x_1 \text{ is an integer, } x_2 \text{ is an integer}$$

A company produces two types of hats. Type 1 requires twice as much labour time as type 2 alone. If all labour time is dedicated to type 2 alone, the company can produce a total of 400 hats of type 2 per day. Respective market limits for two types are 150 and 200 per day. The profit is Rs. 8 per type 1 hat and Rs. 5 per type 2 hat. Find the optimum number of hats to be prepared so that it maximizes company's profits.

Let x_1 hats of type 1 and x_2 hats of type 2 Then

First Constraint: $x_1 \geq 0, x_2 \geq 0, x_1$ is an integer, x_2 is an integer

Time required for producing one hat of type 1 = 2* time required for producing one hat of type 2

Let time required for producing one hat of type 2 = t

Then,

Time required for producing one hat of type 1 = $2t$

Time for producing 400 hats of type 2 = $400*t$ (Time for whole day)

Time required for producing one hat of type 1 + time required for producing one hat of type 2 \leq Time for whole day

$x_1*2t + x_2*t \leq 400t$ i.e., Second constraint: $2x_1 + x_2 \leq 400$

Profit on one hat of type I = 8

Profit on x_1 hats of type I = $8 x_1$

Profit on one hat of type II=5

Profit on x_2 hats of type I = 5 x_2

Total Profit= 8 x_1 +5 x_2

Objective: Maximize (8 x_1 +5 x_2)

LPP

Maximize (8 x_1 +5 x_2)

Subject to

2 x_1 + x_2 <=400

x_1 <=150

x_2 <=200

x_1 >=0, x_2 >=0, x_1 is an integer, x_2 is an integer