

School of Mathematics, Thapar University, Patiala**UMA031: Optimization Techniques****Mid-Term Examination: B.E. (III Sem)****M.M. 30; Time: 02 Hours****Date: September 24, 2016**

Instructors: Dr. Amit Kumar, Dr. Jolly Puri, Dr. Mahesh Kumar Sharma, Dr. Meenakshi Rana, Dr. Navdeep Kailey, Dr. Ram Niwas and Dr. Vikas Sharma.

1. (a) A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X , Y and Z), it is necessary to buy two additional products, say A and B . One unit of product A contains 36 units of X , 3 units of Y and 20 units of Z . One unit of product B contains 6 units of X , 12 units of Y and 10 units of Z . The minimum requirement of X , Y and Z is 40 units, 50 units and 60 units respectively. The product A costs rupees 20 per unit and product B rupees 40 per unit. Formulate this problem as a liner programming problem to minimize the total cost. (4)
- (b) Find the optimal solution of the following linear programming problem (LPP) using Two- Phase method. Also, find alternative solution, if they exist.

$$\text{Min } Z = 2x_1 + x_2 \text{ subject to } -2x_1 + x_2 \leq 2, -2x_1 + 3x_2 \leq 6, 2x_1 + x_2 \geq 4, x_1 \geq 0, x_2 \geq 0. \quad (6)$$

- (c) Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. One unit of exterior paint uses one unit of M1 and 2 units of M2, and one unit of interior paint uses one unit of M1 and one unit of M2. The maximum daily availability of the raw material M1 is 8 units and that of M2 is 10 units. The profit per unit of exterior paint is rupees 5 and for interior paint is rupees 4. The linear programming model of this problem which aims at maximizing total profit is given below, where x and y represents the number of units of exterior and interior paint respectively.

$$\text{Max } 4x+3y \text{ subject to } x + y \leq 8, 2x + y \leq 10, x, y \geq 0.$$

Using the graphical sensitivity answer the following:

- (i) If a choice is made to increase the availability of M1 and M2 at an additional cost of rupees 1.5 per unit. Is it advisable?
- (ii) Suppose the unit profits of exterior and interior paints are changed to rupees 5 and 10 respectively. Will the current optimum remain the same? Justify your answer. (5)

2. Consider the following primal LPP

$$\text{Min } Z = 3x_1 + x_2 \text{ subject to } x_1 + x_2 \geq 1, 2x_1 + 3x_2 \geq 2, x_1 \geq 0, x_2 \geq 0.$$

- (a) Find the optimal solution of the given primal LPP using dual simplex method. (4)
- (b) Write the dual of the above LPP and using the optimal table of primal find the optimal solution of dual problem. (3)

3. Consider the following primal LPP

$$\text{Max } Z = 16x_1 + 15x_2 \text{ subject to } 2x_1 + 3x_2 \leq 12, -x_1 + x_2 \leq 1, x_1 \leq 3, x_1 \geq 0, x_2 \geq 0.$$

- (a) Find the optimal solution using graphical method. (2)
- (b) Find all the basic feasible solutions and show its correspondence with corner points obtained graphically in part (a). (3)
- (c) Construct the simplex table corresponding to optimal solution obtained graphically in part (a). (3)