THAPAR INSTITUTE OF ENGINEERING AND TECHNOLOGY, PATIALA

Department of Electronics and Communication Engineering

UEC310 – Information and Communication Theory

TUTORIAL - 2

Q1 If the universal set is given by $S = \{1,2,3,4,5,6\}$, and $A = \{1,2\}, B = \{2,4,5\}, C = \{1,5,6\}$ are three sub-sets, find the following sets:

a) $A \cup B$

b) $A \cap B$

c) $\bar{A}$

d) $\bar{B}$

e) Check De Morgan's law by finding $(A \cup B)^c$ and $A^c \cap B^c$.

f) Check the distributive law by finding $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$.

Q2 In a party,

- there are 10 people with white shirts and 8 people with red shirts;
- 4 people have black shoes and white shirts;
- 3 people have black shoes and red shirts;
- the total number of people with white or red shirts or black shoes is 21.

How many people have black shoes?

Q3 Let $S = \{1,2,3\}$. Write all the possible partitions of S.

Q4 In a presidential election, there are four candidates. Call them A, B, C, and D. Based on the polling analysis, it has been estimated that A has a 20 percent chance of winning the election, while B has a 40 percent chance of winning. What is the probability that A or B win the election?

Q5 In the experiment of rolling a fair die, what is the probability of $E = \{1,5\}$?

Solution of tutorial-2

A 1	a. $A \cup B = \{1,2,4,5\}$. b. $A \cap B = \{2\}$. c. $\overline{A} = \{3,4,5,6\}$ ($\overline{A}$ consists of elements that are in S but not in A). d. $\overline{B} = \{1,3,6\}$. e. We have $$(A \cup B)^c = \{1,2,4,5\}^c = \{3,6\},$$ which is the same as $$A^c \cap B^c = \{3,4,5,6\} \cap \{1,3,6\} = \{3,6\}.$$ f. We have $$A \cap (B \cup C) = \{1,2\} \cap \{1,2,4,5,6\} = \{1,2\},$$ which is the same as $$(A \cap B) \cup (A \cap C) = \{2\} \cup \{1\} = \{1,2\}.$$

A
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Let W, R, and B, be the number of people with white shirts, red shirts, and black shoes respectively. Then, here is the summary of the available information:

$$|W| = 10$$

$$|R| = 8$$

$$|W \cap B| = 4$$

$$|R \cap B| = 3$$

$$|W \cup B \cup R| = 21.$$

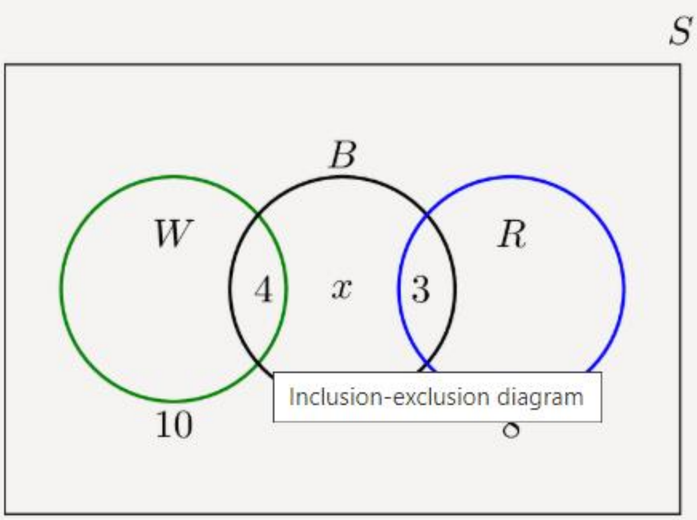
Also, it is reasonable to assume that W and R are disjoint, $|W \cap R| = 0$. Thus by applying the inclusion-exclusion principle we obtain

$$\begin{aligned} |W \cup R \cup B| = 21 \\ &= |W| + |R| + |B| - |W \cap R| - |W \cap B| - |R \cap B| + |W \cap R \cap B| \\ &= 10 + 8 + |B| - 0 - 4 - 3 + 0. \end{aligned}$$

Thus

$$|B| = 10.$$

Note that another way to solve this problem is using a Venn diagram



$$21 = 10 + 8 + x \Rightarrow x = 3$$

$$\Rightarrow |B| = 4 + x + 3 = 10$$

A 3	Remember that a partition of S is a collection of nonempty sets that are disjoint and their union is S. There are 5 possible partitions for $S = \{1, 2, 3\}$: 1. $\{1\}, \{2\}, \{3\}$; 2. $\{1, 2\}, \{3\}$; 3. $\{1, 3\}, \{2\}$; 4. $\{2, 3\}, \{1\}$; 5. $\{1, 2, 3\}$.
A 4	Notice that the events that $\{\text{A wins}\}$, $\{\text{B wins}\}$, $\{\text{C wins}\}$, and $\{\text{D wins}\}$ are disjoint since more than one of them cannot occur at the same time. For example, if A wins, then B cannot win. From the third axiom of probability, the probability of the union of two disjoint events is the summation of individual probabilities. Therefore, $$\begin{aligned} P(\text{A wins or B wins}) &= P\big(\{\text{A wins}\} \cup \{\text{B wins}\}\big) \\ &= P(\{\text{A wins}\}) + P(\{\text{B wins}\}) \\ &= 0.2 + 0.4 \\ &= 0.6 \end{aligned}$$
A 5	Let's first use the specific information that we have about the random experiment. The problem states that the die is fair, which means that all six possible outcomes are equally likely, i.e., $$P(\{1\}) = P(\{2\}) = \cdots = P(\{6\}).$$ Now we can use the axioms of probability. In particular, since the events $\{1\}, \{2\}, \cdots, \{6\}$ are disjoint we can write $$\begin{aligned} 1 &= P(S) \\ &= P\Big(\{1\} \cup \{2\} \cup \cdots \cup \{6\}\Big) \\ &= P(\{1\}) + P(\{2\}) + \cdots + P(\{6\}) \\ &= 6P(\{1\}). \end{aligned}$$ Thus, $$P(\{1\}) = P(\{2\}) = \cdots = P(\{6\}) = \frac{1}{6}.$$ Again since $\{1\}$ and $\{5\}$ are disjoint, we have $$P(E) = P(\{1, 5\}) = P(\{1\}) + P(\{5\}) = \frac{2}{6} = \frac{1}{3}.$$