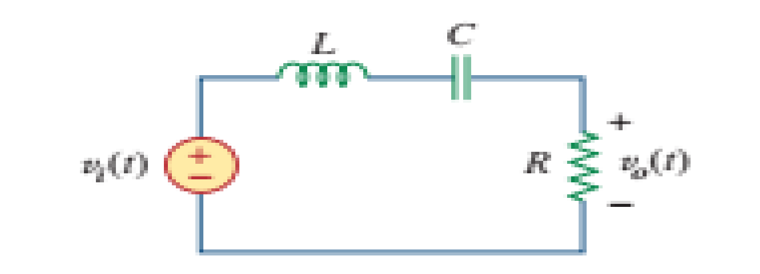
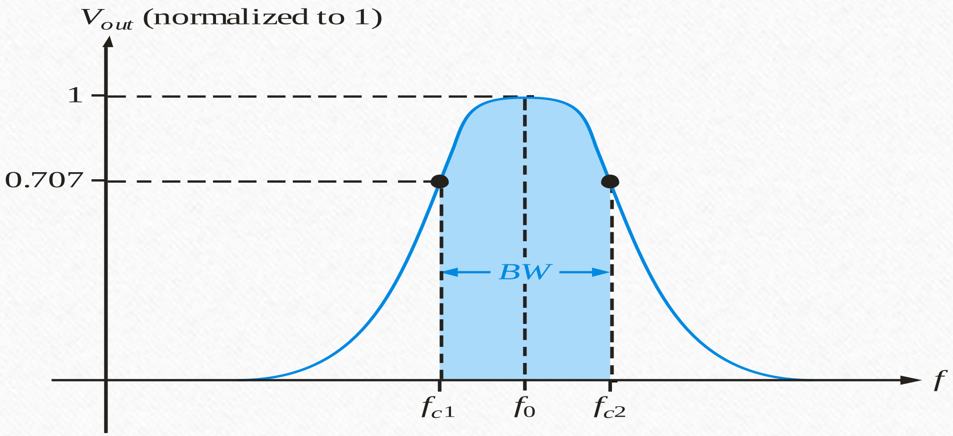
Active Filters

Bandpass & Low Pass Butterworth Filters

The Band-Pass Filter

- A **band-pass filter** passes all frequencies between two critical frequencies.
- The **bandwidth** is defined as the difference between the two critical frequencies f_{c1} and f_{c2}.
- Bandwidth=fc_2-fc_1
- *Centre freq*= $(fc_2\ X\ fc_1)^{0.5}$
- The simplest band-pass filter is an *RLC* circuit.





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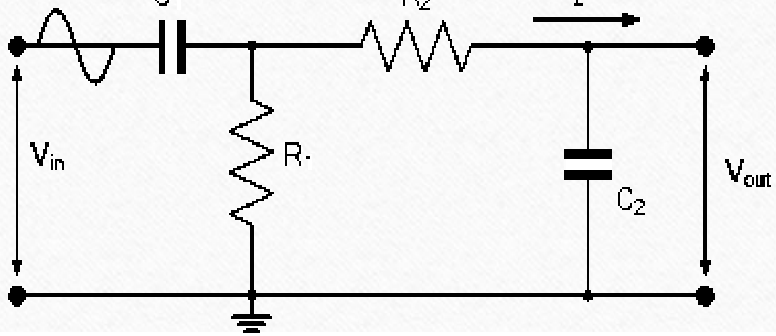
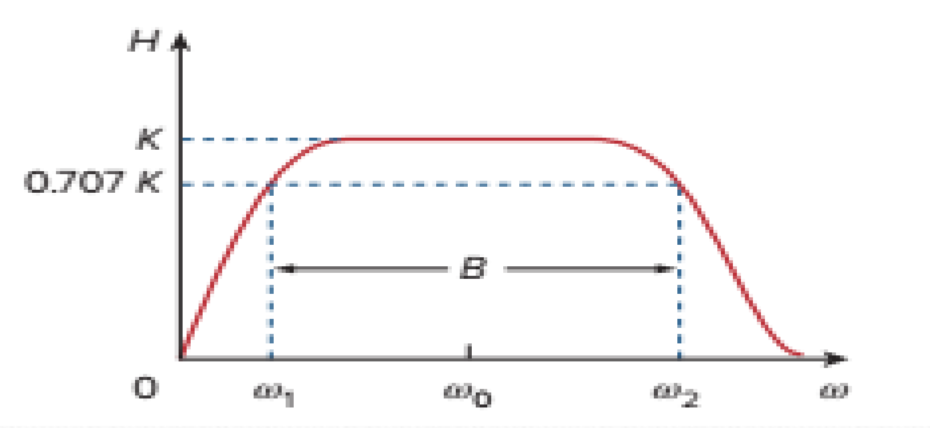
- A band-pass filter that will have a gain *K* over the required range of frequencies.
- By cascading a unity-gain low-pass filter, a unity-gain high-pass filter, and an inverter with gain $-R_f/R_i$.
- Band pass filters are known generally as second-order filters, (two-pole) because they have “two” reactive component, the capacitors, within their circuit design.

$$fc = \frac{1}{2\pi RC}\ Hz$$





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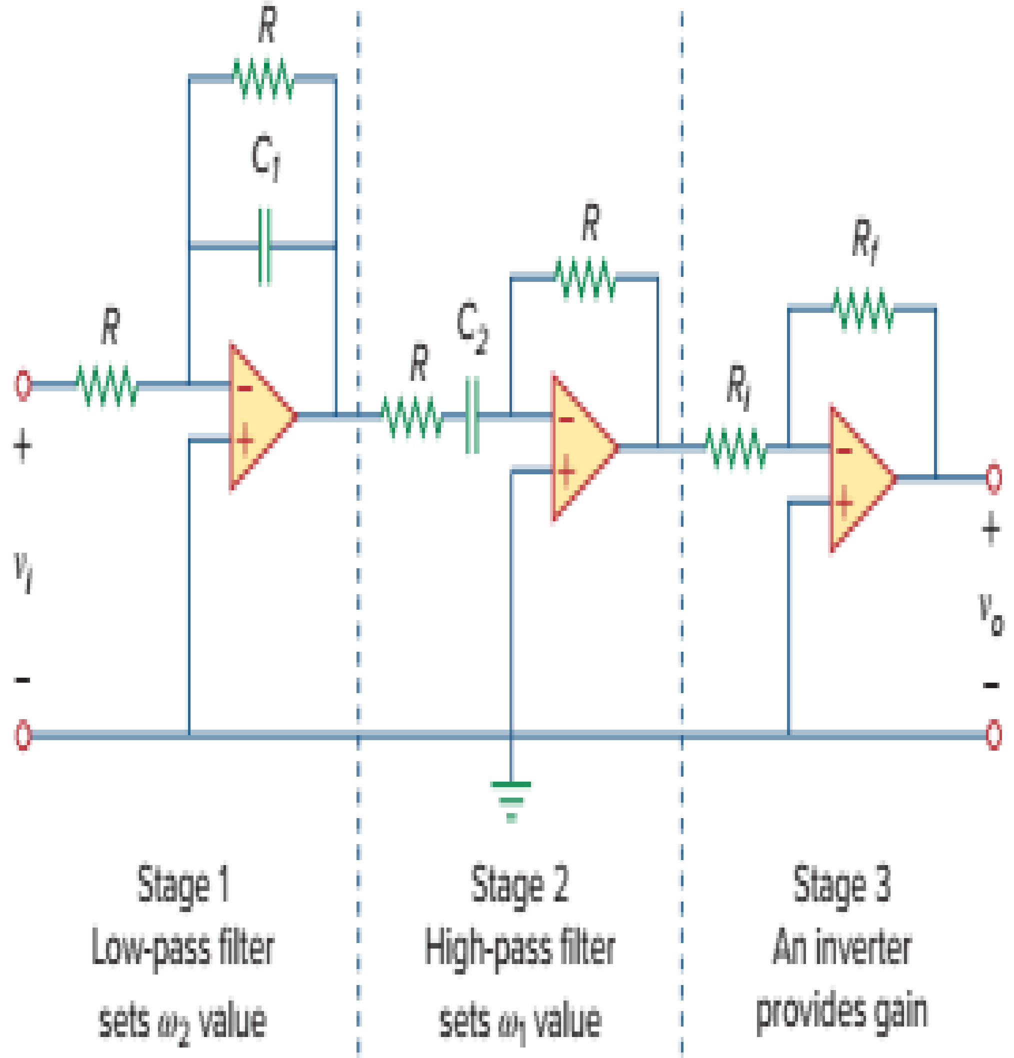
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$
$$= -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

The upper and lower cut off frequency is given by

$$\omega_2 = \frac{1}{RC_1} \qquad \omega_1 = \frac{1}{RC_2}$$



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- The bandwidth, quality factor can be calculated as

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$B = \omega_2 - \omega_1$$

$$Q = \frac{\omega_0}{B}$$

- Pass band gain be calculated by

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)} = -\frac{Rf}{R_i}\frac{j\omega\omega_2}{(\omega_1+j\omega)(\omega_2+j\omega)}$$

$$|\mathbf{H}(\omega_0)| = \left|\frac{R_f}{R_i}\frac{j\omega_0\omega_2}{(\omega_1+j\omega_0)(\omega_2+j\omega_0)}\right| = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1+\omega_2}$$

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1+\omega_2}$$

Butterworth Filter

- Flat response in the pass band & stop band and called flat-flat filter .
- The Butterworth filter has frequency response as flat as mathematically possible, hence it is also called as a maximally flat magnitude filter (from 0Hz to cut-off frequency at -3dB without any ripples).
- The quality factor for this type is just Q=0.707 and thus, all high frequencies above the cut-off point band rolls down to zero at 20dB per decade or 6dB per octave in the stop band.

Transfer Function

The transfer function provides a basis for determining important system response characteristics

The transfer function is a rational function in the complex variable $s = \sigma + j\omega$, *that is*

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

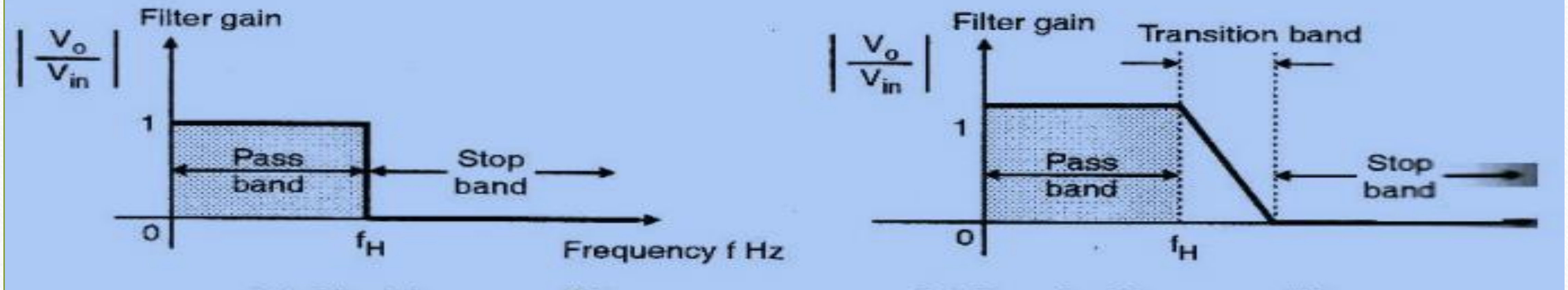
$$H(s) = \frac{N(s)}{D(s)} = K \frac{(s - z_1)(s - z_2) \ldots (s - z_{m-1})(s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_{n-1})(s - p_n)}$$

z_i's are the roots* of the equation *$N(s) = 0$*, and are defined to be the system *zeros, and the p_i's are the roots of the equation $D(s) = 0$,* and are defined to be the system *poles.

$N(s) = 0$; Zeros. $D(s) = 0$; Poles.

FREQUENCY RESPONSE OF FILTERS

- Gain of a filter is given as, G=Vo/Vin.
- Ideal & practical frequency responses of different types of filters are shown below.

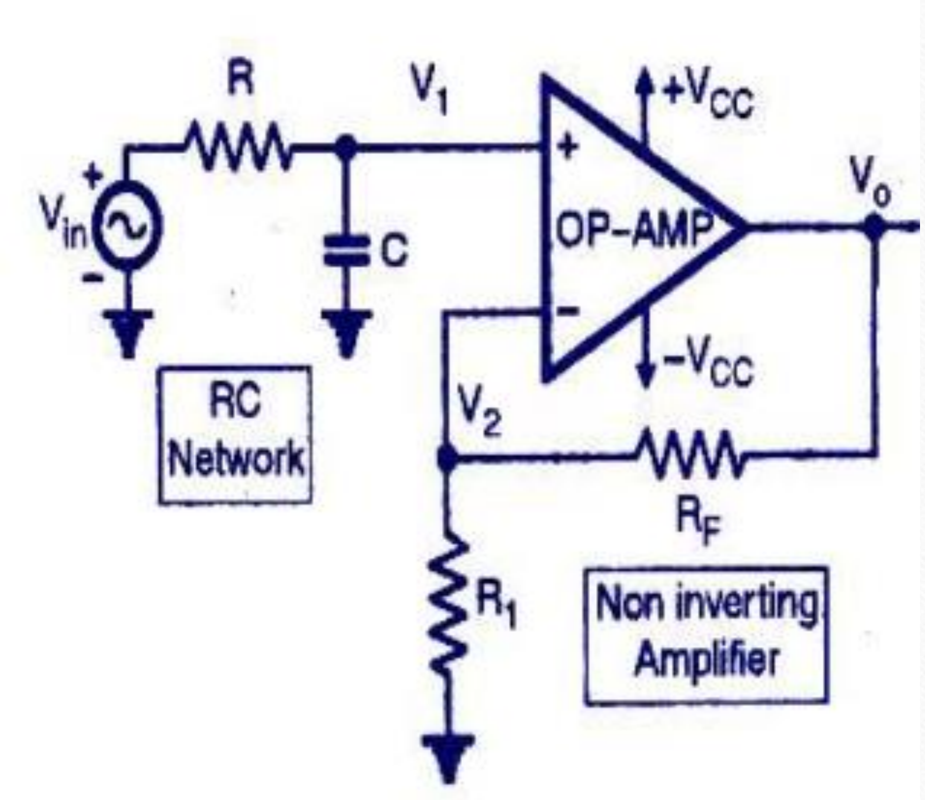


(a) Ideal low-pass filter (b) Practical low-pass filter

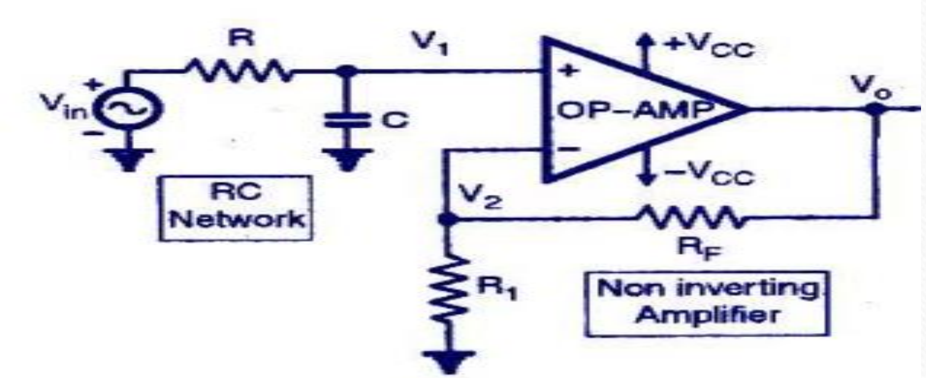
First Order Low-Pass Butterworth Filter

- In 1st. order LPF which is also known as *one pole LPF.*
- RC values decide the cut-off frequency of the filter.
- Resistors R_1 & R_F will decide it's gain in pass band.
- As the OP-AMP is used in the non-inverting configuration, the closed loop gain of the filter is given by

$$A_{VF} = 1 + \frac{R_F}{R_1}$$



Mathematical Analysis



- The Voltage at non inverting input is

$$v_1 = \frac{-jX_c}{R - jX_c} V_{in} = \frac{V_{in}}{1 + j2\Pi fCR}$$

$$V_{out} = \left(1 + \frac{R_F}{R_1}\right) v_1 = \left(1 + \frac{R_F}{R_1}\right) \frac{V_{in}}{1 + j2\Pi fCR}$$

- $$\frac{V_{out}}{V_{in}} = \frac{A_F}{1 + j2\Pi fCR}$$

$$where\ A_F = \left(1 + \frac{R_F}{R_1}\right)$$

- is a Pass band gain of filter

- Substitute $f_H = \frac{1}{2\Pi RC}$ $\quad \frac{V_{out}}{V_{in}} = \frac{A_F}{1 + j\frac{f}{f_H}}$

- The gain magnitude can be calculated using

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{A_F}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

- The phase angle is

$$\phi = -\tan{-1}\left(\frac{f}{f_H}\right)$$

DESIGN PROCEDURE

- *Step1: Choose the cut-off frequency f_H*
  Step2: Select a value of 'C' ≤ 1μF (Approximately between .001 & 0.1μF)
  Step3: Calculate the value of R using

$$R = \frac{1}{2\pi f_H C}$$

- *Step4: Select resistors R1 & R2 depending on the desired pass band gain.*

$$A_{VF} = 1 + \frac{R_F}{R_1}$$

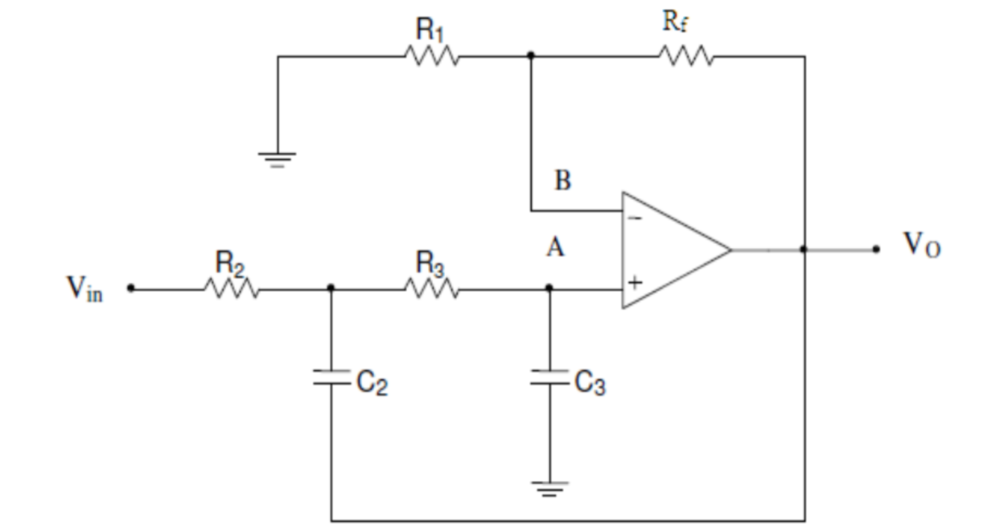
Q1. For a first order Butterworth LPF, calculate the cut –off frequency if R=10Kohm & C=0.001µF.Also calculate the pass band voltage gain if R_1=10Kohm* and *R_F =100Kohm

$$f_H = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10 \times 10^3 \times 0.001 \times 10^{-6}} = 15.915KHz$$

$$A_{VF} = 1 + \frac{R_F}{R_1} \qquad 1+100K/10K = 11$$

2^{nd} Order **Low-Pass Butterworth Filter**

- As the order of the filter increase the gain roll off also increases.
- Hence for a second order filter the roll off will be -40dB / decade.
- Hence the gain reduces sharply compared to first ordered filters.
- A first order filter can be converted to a second order by using an additional RC network as shown in the figure.
- The closed loop gain A_f must be 1.586 for Butterworth response.



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_H}\right)^4}}$$

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

References

- *Coughlin, R.F., Operational Amplifiers and Linear Integrated Circuits, Pearson Education (2006).*
- *Gayakwad, R.A., Op-Amp and Linear Integrated Circuits, Pearson Education (2002).*
- Franco, S., Design with Operational Amplifier and Analog Integrated circuit, McGraw Hill (2016).
- *Terrell, D., Op Amps Design Application and Troubleshooting, Newness (1996).*