Course: UMA 035 (Optimization Techniques)

Instructor: Dr. Amit Kumar,

Associate Professor,

School of Mathematics,

TIET, Patiala

Email: amitkumar@thapar.edu

Mob: 9888500451

Method to transform a special type of non-linear programming problem into its equivalent LPP (Non-linear due to presence of modulus function)

Replace each $|a_1x_1 + a_2x_2 + \cdots + a_nx_n|$ with sum of two non-negative variables as well as add an additional constraint "Value inside mode = difference of the considered non-negative variables"

Example: **Transform the following NLPP into its equivalent LPP and hence, in standard form.**

Maximize/Minimize $(3x_1 + 2x_2 - x_3)$

Subject to

$2x_1 + 4|2x_2 - 3x_3| <= 8$,

$3|4x_1 - 3x_2| + 5x_3 >= 15$,

$x_1 >= 0$, $x_2 >= 0$, $x_3 >= 0$.

Solution: There are two modulus $|2x_2 - 3x_3|$ and $|4x_1 - 3x_2|$.

Assuming

$|2x_2 - 3x_3| = y_1 + y_2$ with the additional constraint $2x_2 - 3x_3 = y_1 - y_2$, $y_1 \geq 0$, $y_2 \geq 0$

and

$|4x_1 - 3x_2| = y_3 + y_4$ with the additional constraint $4x_1 - 3x_2 = y_3 - y_4$, $y_3 \geq 0$, $y_4 \geq 0$,

The considered NLPP may be transformed into its equivalent LPP

Maximize/Minimize ($3x_1 + 2x_2 - x_3$)

Subject to

$2x_1 + 4(y_1 + y_2) <= 8,$

$2x_2 - 3x_3 = y_1 - y_2$

$3(y_3 + y_4) + 5x_3 >= 15,$

$4x_1 - 3x_2 = y_3 - y_4$

$x_1>=0,\ x_2>= 0,\ x_3>=0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0.$

Standard form

Maximize/Minimize ($3x_1 + 2x_2 - x_3$)

Subject to

$2x_1 + 4(y_1 + y_2) + S_1 = 8,$

$2x_2 - 3x_3 - y_1 + y_2 = 0$

$3(y_3 + y_4) + 5x_3 - S_2 = 15,$

$4x_1 - 3x_2 - y_3 + y_4 = 0$

$x_1>=0,\ x_2>= 0,\ x_3>=0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0,\ S_1>=0,\ S_2>=0.$

Example: **Transform the following NLPP into its equivalent LPP and hence, in standard form.**

Maximize/Minimize ($3x_1 + 2x_2 - x_3$)

Subject to

$2x_1 + 4|x_2 + x_3| <= 8,$

$3|x_1 - 3x_2| + 5x_3 >= -15,$

$x_1>=0, x_2<= 0.$

Solution: There are two modulus $|x_2 + x_3|$ and $|x_1 - 3x_2|$.

Assuming

$|x_2 + x_3| = y_1 + y_2$ with the additional constraint $x_2 + x_3 = y_1 - y_2$, $y_1 \geq 0, y_2 \geq 0$

and

$|x_1 - 3x_2| = y_3 + y_4$ with the additional constraint $x_1 - 3x_2 = y_3 - y_4$, $y_3 \geq 0, y_4 \geq 0,$

The considered NLPP may be transformed into its equivalent LPP

Maximize/Minimize $(3x_1 + 2x_2 - x_3)$

Subject to

$2x_1 + 4(y_1 + y_2) <= 8,$

$x_2 + x_3 = y_1 - y_2$

$3(y_3 + y_4) + 5x_2 >= -15,$

$x_1 - 3x_2 = y_3 - y_4$

$x_1>=0, x_2<= 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0.$

Standard form

$x_2<= 0$

$0 <= 0 - x_2$

Assume

$0 - x_2 = y_5$ i.e., $x_2 = -y_5$

Replace x_2 by $-y_5$ in the obtained LPP.

x_3 is an unrestricted variable. As discussed earlier, it may be replaced by $y_6 - y_7$

Replacing x_2 by $-y_5$ and x_3 by $y_6 - y_7$ in the obtained LPP,

Maximize/Minimize $(3x_1 + 2(-y_5) - (y_6 - y_7))$

Subject to

$2x_1 + 4(y_1 + y_2) <= 8,$

$(-y_5) + (y_6 - y_7) = y_1 - y_2$

$3(y_3 + y_4) + 5(-y_5) >= -15,$

$x_1 - 3(-y_5) = y_3 - y_4$

$x_1 >= 0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0,\ y_5 \geq 0,\ y_6 \geq 0,\ y_7 \geq 0$

Maximize/Minimize $(3x_1 + 2(-y_5) - (y_6 - y_7))$

Subject to

$2x_1 + 4(y_1 + y_2) <= 8,$

$(-y_5) + (y_6 - y_7) = y_1 - y_2$

$-3(y_3 + y_4) - 5(-y_5) <= 15,$

$x_1 - 3(-y_5) = y_3 - y_4$

$x_1 >= 0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0,\ y_5 \geq 0,\ y_6 \geq 0,\ y_7 \geq 0$

Maximize/Minimize $(3x_1 + 2(-y_5) - (y_6 - y_7)$

Subject to

$2x_1 + 4(y_1 + y_2) + S_1 = 8$,

$(-y_5) + (y_6 - y_7) - y_1 + y_2 = 0$

$-3(y_3 + y_4) - 5(-y_5) + S_2 = 15$,

$x_1 - 3(-y_5) - y_3 + y_4 = 0$

$x_1 >= 0$, $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$, $y_4 \geq 0$, $y_5 \geq 0$, $y_6 \geq 0, y_7 \geq 0$ $S_1 >= 0$, $S_2 >= 0$.

Why the above method?

$$|a_1x_1 + a_2x_2 + \cdots + a_nx_n|$$

$$= \begin{cases} (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \ if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \geq 0 \\ -(a_1x_1 + a_2x_2 + \cdots + a_nx_n) if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \leq 0 \end{cases}$$

$$= \begin{cases} (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \ if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \geq 0 \\ 0 \qquad if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \leq 0 \end{cases}$$

$$+$$

$$\begin{cases} 0 \qquad if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \geq 0 \\ -(a_1x_1 + a_2x_2 + \cdots + a_nx_n) \ if \ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \leq 0 \end{cases}$$

$|a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n|$=$\mathrm{Y}_1$+ Y_2

where,

$$\mathrm{Y}_1=\begin{cases}(a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \geq 0 \\ 0 & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \leq 0\end{cases}$$

and

Y_2=

$$\begin{cases}0 & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \geq 0 \\ -(a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \leq 0\end{cases}$$

Now,

$$\mathrm{Y}_1 - \mathrm{Y}_2 = \begin{cases}(a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \geq 0 \\ 0 & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \leq 0\end{cases}$$

$$-$$

$$\begin{cases}0 & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \geq 0 \\ -(a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) & if\ (a_1\mathrm{x}_1 + a_2\mathrm{x}_2 + \cdots + a_n\mathrm{x}_n) \leq 0\end{cases}$$

$\mathbf{Y_1 - Y_2 =}$

$$\begin{cases} (a_1x_1 + a_2x_2 + \cdots + a_nx_n) - 0 & if\ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \geq 0 \\ 0 - (-(a_1x_1 + a_2x_2 + \cdots + a_nx_n)) & if\ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \leq 0 \end{cases}$$

$$\mathbf{Y_1 - Y_2 =} \begin{cases} (a_1x_1 + a_2x_2 + \cdots + a_nx_n) & if\ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \geq 0 \\ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) & if\ (a_1x_1 + a_2x_2 + \cdots + a_nx_n) \leq 0 \end{cases}$$

$\mathbf{Y_1 - Y_2 =}(a_1x_1 + a_2x_2 + \cdots + a_nx_n)$

Hence,

$|a_1x_1 + a_2x_2 + \cdots + a_nx_n| = Y_1 + Y_2$

and

$Y_1 - Y_2 = (a_1x_1 + a_2x_2 + \cdots + a_nx_n)$

Graphical method cannot be used if there are more than two variables. In such a case, we need to apply some other methods.

To understand these methods, there is a need to understand the following basic concepts.

If there are "n" sets $X_1, X_2, \ldots, X_n$ such that $x_1 \in X_1$, $x_2 \in X_2$, …, $x_n \in X_n$. Then,

- $(x_1, x_2) \in X_1 \times X_2$
- $(x_1, x_3) \in X_1 \times X_3$
- $(x_2, x_5) \in X_2 \times X_5$

- $(x_2, x_3, x_5) \in X_2 \times X_3 \times X_5$
- $(x_1, x_4, x_5) \in X_1 \times X_4 \times X_5$
- $(x_1, x_7, x_8) \in X_1 \times X_7 \times X_8$

- $(x_1, x_2, x_3, \ldots, x_n) \in X_1 \times X_2 \times X_3 \times \ldots \times X_n$

If $X_1 = X_2 = \ldots = X_n = R$

- **Then,**

- $(x_1, x_2, x_3, \ldots, x_n) \in R \times R \times R \times \ldots \times R$

  OR

- $(x_1, x_2, x_3, \ldots, x_n) \in R^n$
- $(x_1, x_2) \in R^2$
- $(x_1, x_2, x_3) \in R^3$

- **Write some elements of R^2**

  $(2, 3)$, $(-2, 3)$, $(-2, -3)$

- **Write some elements of R^3**

  $(2, 3, 4)$, $(-2, 3, -4)$, $(-2, -3, 4)$

- **Write some elements of R^5**

  $(2, 3, 4, -2, -1)$, $(1, -2, 3, -4, 5)$, $(-2, -3, 4, 0, 6)$

- **Write any three general elements of R^5**

  $(x_1, x_2, x_3, x_4, x_5)$, $(y_1, y_2, y_3, y_4, y_5)$, $(z_1, z_2, z_3, z_4, z_5)$

- **Write any three general elements of R^3**

  (x_1, x_2, x_3), (y_1, y_2, y_3), (z_1, z_2, z_3)

- **Write any three general elements of R^3**

  (x_1, x_2, x_3), (y_1, y_2, y_3), (z_1, z_2, z_3)

- **Write any three general elements of R^2**

  (x_1, x_2), (y_1, y_2), (z_1, z_2)

- **Write the set R^3**

  $R^3 = \{ (x_1, x_2, x_3): x_1, x_2, x_3 \in R\}$

- **Write the set R^2**

  $R^2 = \{ (x_1, x_2): x_1, x_2 \in R\}$

- **Write the set R^n**

  $R^n = \{ (x_1, x_2 , x_3,\ldots,x_n): x_1, x_2,x_3,\ldots,x_n \in R\}$

Coordinates of a point (say, (z_1, z_2)) which divides the line segment joining (x_1, x_2) and (y_1, y_2) internally into m:n.

$$z_1 = \frac{my_1 + nx_1}{m+n}$$

and

$$z_2 = \frac{my_2 + nx_2}{m+n}$$

z_1 and z_2 may also be written as

$$z_1 = \frac{my_1 + nx_1}{m+n} = \frac{my_1}{m+n} + \frac{nx_1}{m+n} = \left(\frac{m}{m+n}\right)y_1 + \left(\frac{n}{m+n}\right)x_1$$

$$z_2 = \frac{my_2 + nx_2}{m+n} = \frac{my_2}{m+n} + \frac{nx_2}{m+n} = \left(\frac{m}{m+n}\right)y_2 + \left(\frac{n}{m+n}\right)x_2$$

Assuming

$\left(\frac{m}{m+n}\right) = a_2$ **and** $\left(\frac{n}{m+n}\right) = a_1$

$z_1 = (a_2)y_1 + (a_1)x_1$ **and** $z_2 = (a_2)y_2 + (a_1)x_2$

where,

- $a_1 \geq 0$
- $a_2 \geq 0$
- $a_1 + a_2 = 1$

Coordinates of a point (say, (z_1, z_2, …, z_n) which divides the line segment joining (x_1, x_2, x_3,…, x_n) and (y_1, y_2, y_3 …,y_n) internally into m:n.

$$z_1 = \frac{my_1+nx_1}{m+n}$$

$$z_2 = \frac{m\mathrm{y}_2 + n\mathrm{x}_2}{m+n}$$

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$$z_n = \frac{m\mathrm{y}_n + n\mathrm{x}_n}{m+n}$$

$z_1, z_2, \ldots, z_n$ **may also be written as**

$$z_1 = \frac{m\mathrm{y}_1+n\mathrm{x}_1}{m+n} = \frac{m\mathrm{y}_1}{m+n} + \frac{n\mathrm{x}_1}{m+n} = \left(\frac{m}{m+n}\right)\mathrm{y}_1 + \left(\frac{n}{m+n}\right)\mathrm{x}_1$$

$$z_2 = \frac{m\mathrm{y}_2+n\mathrm{x}_2}{m+n} = \frac{m\mathrm{y}_2}{m+n} + \frac{n\mathrm{x}_2}{m+n} = \left(\frac{m}{m+n}\right)\mathrm{y}_2 + \left(\frac{n}{m+n}\right)\mathrm{x}_2$$

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$$z_n = \frac{m\mathrm{y}_n + n\mathrm{x}_n}{m+n} = \frac{m\mathrm{y}_n}{m+n} + \frac{n\mathrm{x}_n}{m+n} = \left(\frac{m}{m+n}\right)\mathrm{y}_n + \left(\frac{n}{m+n}\right)\mathrm{x}_n$$

Assuming

$\left(\frac{m}{m+n}\right) = a_2$ **and** $\left(\frac{n}{m+n}\right) = a_1$

$$z_1 = (a_2)y_1 + (a_1)x_1$$

$$z_2 = (a_2)y_2 + (a_1)x_2$$

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$$z_n = (a_2)y_n + (a_1)x_n$$

where,

- $a_1 \geq 0$
- $a_2 \geq 0$
- $a_1 + a_2 = 1$

Linear combination of two numbers $(x_{11}, x_{12},\ldots,x_{1n})$ and $(x_{21}, x_{22},\ldots,x_{2n})$

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ a_2(x_{21}, x_{22},\ldots,x_{2n})=(a_1x_{11}+ a_2x_{21}, a_1x_{12}+ a_2x_{22},\ldots, a_1x_{1n}+ a_2x_{2n})$

where,

a_1 and a_2 are any real numbers.

Linear combination of three numbers $(x_{11}, x_{12},\ldots,x_{1n})$, $(x_{21}, x_{22},\ldots,x_{2n})$ and $(x_{31}, x_{32},\ldots,x_{3n})$

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ a_2(x_{21}, x_{21},\ldots,x_{2n})+ a_3(x_{31}, x_{32},\ldots,x_{3n})=(a_1x_{11}+ a_2x_{21}+ a_3x_{31}, a_1x_{12}+ a_2x_{22}+ a_3x_{32},\ldots, a_1x_{1n}+ a_2x_{2n}+ a_3x_{3n})$

where,

a_1 a_2 and a_3 are any real numbers.

Linear combination of "m" numbers $(x_{11}, x_{12},\ldots,x_{1n})$, $(x_{21}, x_{22},\ldots,x_{2n})$ … $(x_{m1}, x_{m2},\ldots,x_{mn})$

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ a_2(x_{21}, x_{21},\ldots,x_{2n})+\ldots+ a_m(x_{m1}, x_{m2},\ldots,x_{mn})=(a_1x_{11}+ a_2x_{21}+\ldots+ a_mx_{m1}, a_1x_{12}+ a_2x_{22}+\ldots+ a_mx_{m2},\ldots, a_1x_{1n}+ a_2x_{2n}+\ldots+ a_mx_{mn})$

where,

$a_1, a_2 , \ldots, a_m$ are any real numbers.

Convex linear combination of two numbers $(x_{11}, x_{12},\dots,x_{1n})$ and $(x_{21}, x_{22},\dots,x_{2n})$

$a_1(x_{11}, x_{12},\dots,x_{1n})+ a_2(x_{21}, x_{21},\dots,x_{2n})=(a_1x_{11}+ a_2x_{21}, a_1x_{12}+ a_2x_{22},\dots, a_1x_{1n}+ a_2x_{2n})$

where,

- $a_1 >=0$
- $a_2 >=0$
- $a_1+ a_2=1$

Linear combination of three numbers $(x_{11}, x_{12},\dots,x_{1n})$, $(x_{21}, x_{22},\dots,x_{2n})$ and $(x_{31}, x_{32},\dots,x_{3n})$

$a_1(x_{11}, x_{12},\dots,x_{1n})+ a_2(x_{21}, x_{21},\dots,x_{2n})+ a_3(x_{31}, x_{32},\dots,x_{3n})=(a_1x_{11}+ a_2x_{21}+ a_3x_{31}, a_1x_{12}+ a_2x_{22}+ a_3x_{32},\dots, a_1x_{1n}+ a_2x_{2n}+ a_3x_{3n})$

where,

- $a_1 >=0$
- $a_2 >=0$
- $a_3 >=0$
- $a_1+ a_2+ a_3=1$

Linear combination of "m" numbers $(x_{11}, x_{12},\dots,x_{1n})$, $(x_{21}, x_{22},\dots,x_{2n})$ … $(x_{m1}, x_{m2},\dots,x_{mn})$

$a_1(x_{11}, x_{12},\dots,x_{1n})+ a_2(x_{21}, x_{21},\dots,x_{2n})+\dots+ a_m(x_{m1}, x_{m2},\dots,x_{mn})=(a_1x_{11}+ a_2x_{21}+\dots+ a_mx_{m1}, a_1x_{12}+ a_2x_{22}+\dots+ a_mx_{m2},\dots, a_1x_{1n}+ a_2x_{2n}+\dots+ a_mx_{mn})$

where,

- $a_1 >=0$
- $a_2 >=0$

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- $a_m >= 0$
- $a_1 + a_2 + \ldots + a_m = 1$

Example:

2*(3,4) −5*(6,7) is a convex linear combination of (3,4) and (6, 7) or not.

Ans: On comparing $a_1 = 2$ and $a_2 = -5$

Neither the condition $a_2 >= 0$ nor the condition $a_1 + a_2 = 1$ is satisfying. So, it is not a convex linear combination.

Example:

2*(3,4)+5*(6,7) is a convex linear combination of (3,4) and (6, 7) or not.

Ans: On comparing $a_1 = 2$ and $a_2 = 5$

The condition $a_1 + a_2 = 1$ is not satisfying. So, it is not a convex linear combination.

Example:

$\frac{4}{3}$*(3,4)$-\frac{1}{3}$*(6,7) is a convex linear combination of (3,4) and (6, 7) or not.

Ans: On comparing $a_1 = \frac{4}{3}$ and $a_2 = -\frac{1}{3}$

The condition $a_2 >= 0$ is not satisfying. So, it is not a convex linear combination.

Example:

$\frac{2}{3}*(3,4)+\frac{1}{3}*(6,7)$ is a convex linear combination of (3,4) and (6, 7) or not.

Ans: On comparing $a_1 = \frac{2}{3}$ and $a_2 = \frac{1}{3}$

All the conditions $a_1>=0$, $a_2>=0$ and $a_1+a_2=1$ aresatisfying. So, it is a convex linear combination.

Example:

$\frac{2}{3}*(3,4)+\frac{1}{3}*(6,7)+\frac{1}{3}*(1,7)$ is a convex linear combination of (3,4), (6, 7) and (1,7) or not.

Ans: On comparing $a_1 = \frac{2}{3}$, $a_2 = \frac{1}{3}$ and $a_3 = \frac{1}{3}$

The condition $a_1+a_2+a_3=1$ is not satisfying. So, it is not a convex linear combination.

Example:

$\frac{1}{3}*(3,4)+\frac{1}{3}*(6,7)+\frac{1}{3}*(1,7)$ is a convex linear combination of (3,4), (6, 7) and (1,7) or not.

Ans: On comparing $a_1 = \frac{2}{3}$, $a_2 = \frac{1}{3}$ and $a_3 = \frac{1}{3}$

All the conditions $a_1>=0$, $a_2>=0$, $a_2>=0$ and $a_1+a_2+a_3=1$ are satisfying. So, it is a convex linear combination.

Convex linear combination of two numbers $(x_{11}, x_{12},\ldots,x_{1n})$ and $(x_{21}, x_{22},\ldots,x_{2n})$

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ a_2(x_{21}, x_{22},\ldots,x_{2n})=(a_1x_{11}+ a_2x_{21}, a_1x_{12}+ a_2x_{22},\ldots, a_1x_{1n}+ a_2x_{2n})$

where,

- **$a_1 >=0$**
- **$a_2 >=0$**
- **$a_1+ a_2=1$**

$a_1+ a_2=1$ implies $a_2=1- a_1$

So, convex linear combination can also be written as

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ (1- a_1)(x_{21}, x_{22},\ldots,x_{2n})=(a_1x_{11}+ (1- a_1)x_{21}, a_1x_{12}+ (1- a_1)x_{22},\ldots, a_1x_{1n}+ (1- a_1)x_{2n})$

where,

- **$a_1 >=0$**
- **$1- a_1 >=0$ implies $a_1<=1$**

$a_1(x_{11}, x_{12},\ldots,x_{1n})+ (1- a_1)(x_{21}, x_{22},\ldots,x_{2n})=(a_1x_{11}+ (1- a_1)x_{21}, a_1x_{12}+ (1- a_1)x_{22},\ldots, a_1x_{1n}+ (1- a_1)x_{2n})$

where,

- **$0<=a_1 <=1$**

Convex set

Theoretically

A set S is said to a convex set if the convex linear combination of every two arbitrary points of the set S also belong to the same set S.

Mathematically

If $a_1X_1 + a_2X_2 \in S$ for all $X_1, X_2 \in S$

where,

- $a_1 >= 0$
- $a_2 >= 0$
- $a_1 + a_2 = 1$

Then, the set S will be convex.

OR

If $a_1X_1 + (1-a_1)X_2 \in S$ for all $X_1, X_2 \in S$

where,

- $0 <= a_1 <= 1$

Then, the set S will be convex.

Graphically

Draw the region for the set S.

If it is possible to find any two distinct points inside the region of the set S such that some of the portion of the line segment joining the points lies

outside the region of the set S. Then the set S will not be a convex set otherwise the set S will be a convex set.