Course: UMA 035 (Optimization Techniques)

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Example:

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

Solve the problem (Apply Least Cost method to find initial basic feasible solution).

Solution:

Assume the problem is of minimization.

30+50+35=15+30+25+45

Balanced Transportation problem.

Initial Basic Feasible solution by Least cost method

Minimum cost in the table is 1.

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

x_{11}=Minimum {30, 15}=15

Cut the first column and reduce the availability of the first source by 15 units.

Minimum cost in Table is 2. Consider any 2 out of the three 2's.

	D_2	D_3	D_4	Availability
S_1	2	3	4	$30-15=15$
S_2	6	2	5	50
S_3	3	2	7	35
Demand	30	25	45	

x_{33}=Minimum {25, 35}=25

Cut the second column and reduce the availability of the third source by 25 units.

Minimum cost in Table is 2.

	D_2	D_4	Availability
S_1	2	4	30−15=15
S_2	6	5	50
S_3	3	7	35−25=10
Demand	30	45	

x_{12}=Minimum {30, 15}=15

Cut the first row and reduce the demand of the second destination by 30 units.

Minimum cost is 3

	D_2	D_4	Availability
S_2	6	5	50
S_3	3	7	35−25=10
Demand	30−15 = 15	45	

x_{32}=Minimum {15, 10}=10

Cut the second row and reduce the demand of the second destination by 10 units.

Minimum cost is 5.

	D_2	D_4	Availability
S_2	6	5	50
Demand	$15-10=5$	45	

x_{24}=Minimum {45, 50}=45

Cut the second column and reduce the availability of the second source by 45 units.

	D_2	Availability
S_2	6	$50-45=5$
Demand	$15-10=5$	

x_{22}=Minimum {5, 5}=5

	D_1	D_2	D_3	D_4	Availability
S_1	1 (15)	2 (15)	3	4	30
S_2	7	6 (5)	2	5 (45)	50
S_3	4	3 (10)	2(25)	7	35
Demand	15	30	25	45	

Initial transportation cost=1*15+2*15+6*5+5*45+3*10+2*25=380

Check solution is optimal or not

For basic variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=2$

$x_{22} \Rightarrow u_2+v_2=c_{22} \Rightarrow u_2+v_2=6$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{33} \Rightarrow u_3+v_3=c_{33} \Rightarrow u_3+v_3=2$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=2$

$u_3=1$

$v_3=1$

$v_4=1$

$u_2=4$

For non-basic variables

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+1)=2$

$x_{14} \Rightarrow c_{14}-(u_1+v_4)= 4-(0+1)=3$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(4+1)=2$

$x_{23} \Rightarrow c_{23}-(u_2+v_3)= 2-(4+1)= -3$

$x_{31} \Rightarrow c_{31}-(u_3+v_1)= 4-(1+1)= 2$

$x_{34} \Rightarrow c_{34}-(u_3+v_4)= 7-(1+1)= 5$

x_{23} is entering variable.

	D_1	D_2	D_3	D_4	Availability
S_1	15	15			30
S_2		$5-\theta$	θ	45	50
S_3		$10+\theta$	$25-\theta$		35
Demand	15	30	25	45	

θ = minimum {5, 25}=5

New basic feasible solution

$x_{11}=15$

$x_{12}=15$

$x_{22}=5-\theta=5-5=0$ (Leaving Variable)

$x_{23}=\theta=5$

$x_{24}=45$

$x_{32}=10+\theta=15$

$x_{33}=25-\theta=25-5=20$

Check that the new solution is optimal or not

For Basic Variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{12} \Rightarrow u_1+v_2=c_{12} \Rightarrow u_1+v_2=2$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{33} \Rightarrow u_3+v_3=c_{33} \Rightarrow u_3+v_3=2$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_2=2$

$u_3=1$

$v_4=4$

$u_2=1$

$v_3=1$

For Non-basic Variables

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+1)=2$

$x_{14} \Rightarrow c_{14}-(u_1+v_4)= 4-(0+4)=0$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(1+1)= 5$

$x_{22} \Rightarrow c_{22}-(u_2+v_2)= 6-(1+2)=3$

$x_{31} \Rightarrow c_{31}-(u_3+v_1)= 4-(1+1)= 2$

$x_{34} \Rightarrow c_{34}-(u_3+v_4)= 7-(1+4)= 2$

All values are >=0. Solution is optimal.

	D_1	D_2	D_3	D_4	Availability
S_1	1 (15)	2 (15)	3	4	30
S_2	7	6	2 (5)	5 (45)	50
S_3	4	3 (15)	2(20)	7	35
Demand	15	30	25	45	

Transportation cost=1*15+2*15+2*5+5*45+3*15+2*20=365

Alternative solution

x_{14} is a non-basic variable and $c_{14}-(u_1+v_4)=0$. So alternative solution may exist.

Enter x_{14} to find alternative solution.

	D_1	D_2	D_3	D_4	Availability
S_1	15	$15-\theta$		θ	30
S_2			$5+\theta$	$45-\theta$	50
S_3		$15+\theta$	$20-\theta$		35
Demand	15	30	25	45	

θ=minimum {45, 20, 15}=15

New solution is

$x_{11}=15$

$x_{12}=15-\theta=0$

$x_{14}=15$

$x_{23}=20$

$x_{24} = 30$

$x_{32} = 30$

$x_{33} = 5$

Example:

	D_1	D_2	D_3	D_4	Availability
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	

Solve the problem (Apply Vogel's approximation method to find initial basic feasible solution).

Solution:

Assume the problem is of minimization.

30+50+35=15+30+25+45

Balanced Transportation problem.

Initial Basic Feasible solution by Vogel's approximation method

	D_1	D_2	D_3	D_4	Availability	Penalty (Second lowest−Lowest)
S_1	1	2	3	4	30	2−1=1
S_2	7	6	2	5	50	5−2=3
S_3	4	3	2	7	35	3−2=1
Demand	15	30	25	45		
Penalty (Second lowest−Lowest)	4−1=3	3−2=1	3−2=1	5−4=1		

Maximum penalty out of all the penalties

	D_1	D_2	D_3	D_4	Availability	Penalty (Second lowest−Lowest)
S_1	1	2	3	4	30	2−1=1
S_2	7	6	2	5	50	5−2=3
S_3	4	3	2	7	35	3−2=1
Demand	15	30	25	45		
Penalty (Second lowest−Lowest)	4−1=3	3−2=1	3−2=1	5−4=1		

Maximum penalty is 3

Minimum cost in the row corresponding to max penalty =2

Minimum cost in the column corresponding to max penelty =1

minimum {1,2}=1

	D_1	D_2	D_3	D_4	Availability	Penalty (Second lowest−Lowest)
S_1	1	2	3	4	30	2−1=1
S_2	7	6	2	5	50	5−2=3
S_3	4	3	2	7	35	3−2=1
Demand	15	30	25	45		
Penalty (Second lowest−Lowest)	4−1=3	3−2=1	3−2=1	5−4=1		

x_{11}=Minimum {30, 15}=15

Cut the first column and reduce the availability of the first source by 15 units.

	D_2	D_3	D_4	Availability	Penalty
S_1	2	3	4	30−15=15	3−2=1
S_2	6	2	5	50	5−2=3
S_3	3	2	7	35	3−2=1
Demand	30	25	45		
Penalty	3−2=1	3−2=1	5−4=1		

	D_2	D_3	D_4	Availability	Penalty
S_1	2	3	4	30−15=15	3−2=1
S_2	6	2	5	50	5−2=3
S_3	3	2	7	35	3−2=1
Demand	30	25	45		
Penalty	3−2=1	3−2=1	5−4=1		

x_{23}=Minimum {25, 50}=25

Cut the second column and reduce the availability of the second source by 25 units.

	D_2	D_4	Availability	Penalty
S_1	2	4	30−15=15	2
S_2	6	5	50−15=25	1
S_3	3	7	35	4
Demand	30	45		
Penalty	1	1		

x_{32}=Minimum {30, 35}=30

Cut the first column and reduce the availability of the third source by 30 units.

	D_4	Availability	Penalty
S_1	4	15	4
S_2	5	25	5
S_3	7	35−30=5	7
Demand	45		
Penalty	1		

x_{34}=Minimum {5, 45}=5

Cut the third row and reduce the demand of the fourth destination by 5 units.

	D_4	Availability	Penalty
S_1	4	15	4
S_2	5	25	5
Demand	$45-5=$ 40		
Penalty	1		

x_{24}=Minimum {40, 25}=25

Cut the second row and reduce the demand of the fourth destination by 25 units.

	D_4	Availability	Penalty
S_1	4	15	4
Demand	$40-25=$ 15		
Penalty	1		

x_{14}=Minimum {15, 15}=15

	D_1	D_2	D_3	D_4	Availability
S_1	1 (15)	2 (15)	3	4 (15)	30
S_2	7	6	2(25)	5 (25)	50
S_3	4	3 (30)	2	7 (5)	35
Demand	15	30	25	45	

Initial transportation cost=1*15+4*15+2*25+5*25+3*30+7*5=375

Check solution is optimal or not

For basic variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{14} \Rightarrow u_1+v_4=c_{14} \Rightarrow u_1+v_4=4$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{34} \Rightarrow u_3+v_4=c_{34} \Rightarrow u_3+v_4=7$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_4=4$

$u_3=3$

$v_3=1$

$v_2=0$

$u_2=1$

For non-basic variables

$x_{12} \Rightarrow c_{12}-(u_1+v_2)= 2-(0+1)=1$

$x_{13} \Rightarrow c_{13}-(u_1+v_3)= 3-(0+1)=2$

$x_{21} \Rightarrow c_{21}-(u_2+v_1)= 7-(1+1)=5$

$x_{22} \Rightarrow c_{22}-(u_2+v_2)= 6-(1+0)= 5$

$x_{31} \Rightarrow c_{31}-(u_3+v_1)= 4-(3+1)=0$

$x_{33} \Rightarrow c_{33}-(u_3+v_3)= 2-(3+0)= -1$

x_{33} is entering variable.

	D_1	D_2	D_3	D_4	Availability
S_1	15			15	30
S_2			$25-\theta$	$25+\theta$	50
S_3		30	θ	$5-\theta$	35
Demand	15	30	25	45	

θ = minimum {5, 25}=5

New basic feasible solution

$x_{11}=15$

$x_{14}=15$

$x_{23}=25-\theta=25-5=20$

$x_{24}=25+\theta=30$

$x_{32}=30$

$x_{33}=\theta=5$

$x_{34}=5-\theta=5-5=0$ **(Leaving Variable)**

Check that the new solution is optimal or not

For Basic Variables

$x_{11} \Rightarrow u_1+v_1=c_{11} \Rightarrow u_1+v_1=1$

$x_{14} \Rightarrow u_1+v_4=c_{14} \Rightarrow u_1+v_4=4$

$x_{23} \Rightarrow u_2+v_3=c_{23} \Rightarrow u_2+v_3=2$

$x_{24} \Rightarrow u_2+v_4=c_{24} \Rightarrow u_2+v_4=5$

$x_{32} \Rightarrow u_3+v_2=c_{32} \Rightarrow u_3+v_2=3$

$x_{33} \Rightarrow u_3+v_3=c_{33} \Rightarrow u_3+v_3=2$

Six equations in Seven variables

Assuming $u_1=0$, we have

$v_1=1$

$v_4=4$

$u_2=1$

$v_3=1$

$u_3=1$

$v_2=2$

For Non-basic Variables

$x_{12} \Rightarrow c_{12} - (u_1+v_2) = 2-(0+2)=0$

$x_{13} \Rightarrow c_{13} - (u_1+v_3) = 3-(0+1)=2$

$x_{21} \Rightarrow c_{21} - (u_2+v_1) = 7-(1+1)= 5$

$x_{22} \Rightarrow c_{22} - (u_2+v_2) = 6-(1+2)=4$

$x_{31} \Rightarrow c_{31} - (u_3+v_1) = 4-(1+1)= 2$

$x_{34} \Rightarrow c_{34} - (u_3+v_4) = 7-(1+4)= 2$

All values are >=0. Solution is optimal.

	D_1	D_2	D_3	D_4	Availability
S_1	1 (15)	2	3	4(15)	30
S_2	7	6	2 (20)	5 (30)	50
S_3	4	3 (30)	2(5)	7	35
Demand	15	30	25	45	

Transportation cost=1*15+4*15+2*20+5*30+3*30+2*5=365

Alternative solution

x_{12} is a non-basic variable and $c_{12}-(u_1+v_2)=0$. So alternative solution may exist.

Enter x_{12} to find alternative solution.

	D_1	D_2	D_3	D_4	Availability
S_1	15	θ		$15-\theta$	30
S_2			$20-\theta$	$30+\theta$	50
S_3		$30-\theta$	$5+\theta$		35
Demand	15	30	25	45	

θ=minimum {15, 20, 30}=15

New solution is

$x_{11}=15$

$x_{12}=15$

$x_{14}=15-\theta=0$

$x_{23}=20-\theta=5$

$x_{24}=30+\theta=45$

$x_{32}=30-\theta=15$

$x_{33}=5+\theta=20$

Problem of degeneracy

Example:

	D_1	D_2	D_3	Availability
S_1	8	7	3	60
S_2	3	8	9	70
S_3	11	3	5	80
Demand	50	80	80	

Solve the problem (Apply North West Corner method to find initial basic feasible solution).

Solution:

Assume the problem is of minimization.

60+70+80=50+80+80

Balanced Transportation problem.

Initial Basic Feasible solution by North West Corner method

	D_1	D_2	D_3	Availability
S_1	8	7	3	60
S_2	3	8	9	70
S_3	11	3	5	80
Demand	50	80	80	

x_{11}=minimum {50, 60}=50

Cut the first column and reduce the availability of the first source by 50 units.

	D_2	D_3	Availability
S_1	7	3	$60-50=$ 10
S_2	8	9	70
S_3	3	5	80
Demand	80	80	

x_{12}=minimum {10, 80}=10

Cut the first row and reduce the demand of the second destination by 10 units.

	D_2	D_3	Availability
S_2	8	9	70
S_3	3	5	80
Demand	$80-10=70$	80	

x_{22}=minimum {70, 70}=70

Since, both are equal so cut either row or column not both and reduce the availability/demand by 70 units

	D_2	D_3	Availability
S_3	3	5	80
Demand	$70-70=0$	80	

x_{32}=minimum {0, 70}=0 **(Value of basic variable is 0. Solution is degenerate)**

Cut the first column and reduce the availability of the third source by 0 units.

	D_3	Availability
S_3	5	$80-0=80$
Demand	80	

x_{33}=minimum {80, 80}=80

Check yourself that solution is optimal or not. If not then find an optimal solution.