Discrete Mathematical Structures (UCS405)

Tutorial Sheet-7

1. What rule of inference is used in each of these arguments?
a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

2. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

3. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."
b) "If I work, it is either sunny or partly sunny." "I worked last Monday or I worked last Friday." "It was not sunny on Tuesday." "It was not partly sunny on Friday."
c) "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies."
d) "Every student has an Internet account." "Homer does not have an Internet account." "Maggie has an Internet account."
e) "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat."
f) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road."

4. State the value of x after the statement **if** $P(x)$ **then** $x := 1$ is executed, where $P(x)$ is the statement "$x > 1$," if the value of x when this statement is reached is
a) $x = 0$.　　　　**b)** $x = 1$.　　　　**c)** $x = 2$.

5. Let $N(x)$ be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists xN(x)$　　　　　**b)** $\forall xN(x)$　　　　　**c)** $\neg \exists xN(x)$　　　　　**d)** $\exists x \neg N(x)$

e) $\neg \forall xN(x)$　　　　　**f)** $\forall x \neg N(x)$

6. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.
a) $\forall x(R(x) \rightarrow H(x))$　　　**b)** $\forall x(R(x) \wedge H(x))$　　　**c)** $\exists x(R(x) \rightarrow H(x))$　　　**d)** $\exists x(R(x) \wedge H(x))$

7. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
a) A student in your class has a cat, a dog, and a ferret.
b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.
d) No student in your class has a cat, a dog, and a ferret.
e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

8. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$ **d)** $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$

g) $\forall x \neg Q(x)$

9. Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y" and where the domain for x and y consists of all systems and all possible states, respectively.
a) $\exists x S(x, \text{open})$
b) $\forall x(S(x, \text{malfunctioning}) \lor S(x, \text{diagnostic}))$
c) $\exists x S(x, \text{open}) \lor \exists x S(x, \text{diagnostic})$

d) $\exists x \neg S(x, \text{available})$

e) $\forall x \neg S(x, \text{working})$

10. Express each of these system specifications using predicates, quantifiers, and logical connectives.
a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
b) No directories in the file system can be opened and no files can be closed when system errors have been detected.
c) The file system cannot be backed up if there is a user currently logged on.
d) Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.