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School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

End-Semester Examination, May 2018

B.E. IV Semester | UMA007 : Numerical Analysis
Time Limit: 03 Hours | Maximum Marks: 100
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Instructions: This question paper has two printed pages. You are expected to answer all the questions. Organize your work in a reasonably neat, organized, and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode is permitted.

1. (a) How many multiplications and additions are required to determine a sum of the form

$$\sum_{i=1}^{n}\sum_{j=1}^{i} a_i\, b_j \ ?$$

Modify the above sum to an equivalent form that reduces the number of computations. [10 marks]

(b) Use the secant method to find the reciprocal of square root of number 15. Using initial approximations $x_0 = 0.1$ and $x_1 = 0.2$, compute four iterations. [10 marks]

2. (a) Suppose that $\tilde{x}$ is an approximation to the solution of linear system $Ax = b$, A is a nonsingular matrix, and r is the residual vector for $\tilde{x}$. For any natural norm, prove that

$$\frac{\|x-\tilde{x}\|}{\|x\|} \le K(A)\frac{\|r\|}{\|b\|},$$

provided $x \neq 0$ and $b \neq 0$, where $K(A)$ denotes the condition number for matrix A. [10 marks]

(b) Let λ be the smallest eigen-value in magnitude of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}.$$

Using four-digit rounding arithmetic and $\mathbf{x^{(0)}} = [\mathbf{1}, \mathbf{0}, \mathbf{0}]^{\mathbf{t}}$, perform two iterations of the inverse power method to find approximate value of λ. Use the LU factorization to solve the system of linear equations that originate during the process. [10 marks]

3. (a) Suppose that $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$ and $f \in C^{n+1}[a, b]$. Let $P_n(x)$ be the unique polynomial of degree $\le n$ that passes through $n + 1$ given points. Prove that for every $x \in [a, b]$, there exists $\xi = \xi(x) \in (a, b)$ such that

$$f(x) - P_n(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(n+1)!}\, f^{(n+1)}(\xi).$$

[10 marks]

(b) The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2	3
$P(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1. [10 marks]

4. (a) The area A inside the closed curve $y^2 + x^2 = \cos x$ is given by

$$A = 4\int_0^{\alpha} \left(\cos x - x^2\right)^{1/2} dx$$

where α is the positive root of the equation $\cos x = x^2$.

(i) Compute α with three correct decimals by using Newton's method.

(ii) Use trapezoidal rule to compute the area A by taking two subintervals. [10 marks]

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(b) Determine constants c_0, c_1 and x_1 that will produce a quadrature formula

$$\int_0^1 x\, f(x)\, dx = c_0\, f(0) + c_1\, f(x_1)$$

that has the highest degree of precision. [10 marks]

5. (a) Use the modified Euler's method to approximate the solution to the following initial-value problem with step-size $h = 0.5$.

$$\frac{dy}{dt} = t - 3y, \quad 0 \leq t \leq 1, \quad y(0) = 1.$$

[10 marks]

(b) A simple model to account for the way in which two different animal species sometimes interact is the predator-prey model. If $u(t)$ is the number of individuals in the predator species and $v(t)$ the number of individuals in the prey species, then under suitable simplifying assumptions, the model is given by

$$\begin{aligned} \frac{du}{dt} &= 0.25u - 0.01uv, \quad u(0) = 80 \\ \frac{dv}{dt} &= 0.01uv - v, \quad v(0) = 30. \end{aligned}$$

Use the fourth-order Runge-Kutta method with step-size $h = 1$ to approximate the solution at $t = 1$.

[10 marks]