Deterministic and Random Signals

The signals which can be described uniquely by a mathematical expression, table, graph or a well defined rule are known as deterministic signals. Therefore, it is possible to model a deterministic signal by a known function of time t. On the other hand, if the signals cannot be described by formula or graph, they are known as random signals. These signals take random values at a given time. They should be characterized statistically. The sound signal in a radio, data signals in a computer and picture signal in TV are treated as random signals. Noise signals are also examples of random signals.

Sr No	Deterministic signals	Random signals
1	Deterministic signals can be represented or described by a mathematical equation or lookup table.	Random signals that cannot be represented or described by a mathematical equation or lookup table.
2	Deterministic signals are preferable because for analysis and processing of signals we can use mathematical model of the signal.	Not Preferable. The random signals can be described with the help of their statistical properties.
3	The value of the deterministic signal can be evaluated at time (past, present or future) without certainty.	The value of the random signal can not be evaluated at any instant of time.
4	Example Sine or exponential waveforms.	Example Noise signal or Speech signal

Multichannel Signals

If the different signals are recorded from the same source, they are known as Multichannel signals. ECG signals recorded form in 3 leads or 12 leads for the same person results in 3 channel or 12 channel signal.

Multidimensional Signals

Generally, most of the signals are function of time only, i.e., they are function of single variable. The brightness of a picture during scanning is a function of x and y co–ordinate. Therefore, the picture can be described by a function f(x,y) depending on two variables. The intensity of a TV signal also varies from frame to fame. Therefore, it becomes a function of f(x,y,t). There may be many signals at input and output of a real system. Each of these are termed as a channel. The signal in a black and white TV picture tube is a function of I(x,y,t). The three signals comes out from RED, BLUE and GREEN channels of the picture tube of a colour TV. Therefore, total signal of colour TV can be written as

$$I(x,y,t) = \begin{bmatrix} I_R(x,y,t) \\ I_B(x,y,t) \\ I_G(x,y,t) \end{bmatrix}$$

Energy and Power Signals

If a voltage v(t) is applied across a resistor R and produces a current i(t) through it, the instantaneous power per Ohm is given by

$$p(t) = v(t)i(t) = i(t)i(t)R = i^2(t) \quad [R=1] \qquad (1)$$

We can write the total energy (E) and average power (P) on a per Ohm basis as follows:

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{Joules} \qquad (2)$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i^2(t)\,dt \quad \text{Watts} \qquad (3)$$

The normalized energy content E of an arbitrary continuous–time signal x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt \qquad (4)$$

We can define the normalized average power P of x(t) as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left| x(t) \right|^2 dt \tag{5}$$

A signal x(t) is said to be an energy signal if $0<E<\infty$ and P = 0. The signal x(t) is said to be power signal if $0<P<\infty$ and $E = \infty$. If the signal |x(t)| does not satisfy any of the above conditions, the signal is neither energy signal nor power signal.

All short time or transient signals are energy signals whereas power signals are $\cos\omega_0 t$ and $\sin\omega_0 t$ etc.

$t^{-\frac{1}{2}}, \left(t^2 + a^2\right)^{-\frac{1}{2}}$ are neither energy nor power signals.

1. $x(t)$ (or $x[n]$) is said to be an *energy* signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. $x(t)$ (or $x[n]$) is said to be a *power* signal (or sequence) if and only if $0 < P < \infty$, thus implying that $E = \infty$.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Example Find whether the following signals are energy or power signals or not.

(a) $y(t) = A\sin(\omega_0 t + \phi)$

(b) $y(t) = e^{-bt}u(t)$ and

(c) $y(t) = t^n u(t)$, $n > 0$.

(a) The signal $y(t) = A\sin(\omega_0 t + \phi)$ is a periodic signal with period

$$T_0 = \frac{2\pi}{\omega_0}$$

The average power of y(t) is given by

$$P_{av} = \underset{T_0\to\infty}{Lt}\ \frac{1}{2T_0}\int_{-T_0}^{T_0}\left[y(t)\right]^2 dt$$

$$= \underset{T_0\to\infty}{Lt}\ \frac{1}{2T_0}\int_{-T_0}^{T_0} A^2\sin^2\left(\omega_0 t + \phi\right)dt = \underset{T_0\to\infty}{Lt}\ \frac{1}{2T_0}\int_{-T_0}^{T_0}\frac{1}{2}\left[1-\cos 2\left(\omega_0 t + \phi\right)\right]dt$$

$$= \frac{A^2}{2} < \infty$$

Energy of the signal is given by

$$E = \underset{T_0 \to \infty}{Lt} \int_{-T_0}^{T_0} \left[y(t)\right]^2 dt$$

$$= \underset{T_0 \to \infty}{Lt} \int_{-T_0}^{T_0} A^2 \sin^2\left(\omega_0 t + \phi\right) dt = \underset{T_0 \to \infty}{Lt} \int_{-T_0}^{T_0} \frac{1}{2}\left[1 - \cos 2\left(\omega_0 t + \phi\right)\right] dt$$

$$= \underset{T_0 \to \infty}{Lt} \frac{A^2}{2} \times 2T_0 = \infty$$

The signal is power signal as P is finite and E is infinite.

$y(t) = e^{-bt}u(t)$

$$E = \underset{T\to\infty}{Lt}\int_0^{\infty}\left[y(t)\right]^2 dt = \int_0^{T} e^{-2bt}dt = \frac{1}{2b} < \infty$$

$$P = \underset{T\to\infty}{Lt}\frac{1}{T}\int_0^{T}\left[y(t)\right]^2 dt = \underset{T\to\infty}{Lt}\frac{1}{T}\int_0^{T} e^{-2bt}dt$$

$$= \underset{T\to\infty}{Lt}\frac{1}{T}\times\frac{1}{2b}\times\frac{T}{1} = 0$$

Therefore, y(t) is an energy signal as E is finite as P is zero.

y(t) = $t^n u(t)$

$$E = \underset{T \to \infty}{Lt} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[y(t)\right]^2 dt = \underset{T \to \infty}{Lt} \int_{0}^{\frac{T}{2}} t^{2n} dt = \underset{T \to \infty}{Lt} \left[\frac{t^{2n+1}}{2n+1}\right]_0^{\frac{T}{2}} = \underset{T \to \infty}{Lt} \frac{\left(\frac{T}{2}\right)^{2n+1}}{2n+1} = \infty$$

and

$$P = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[y(t)\right]^2 dt = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{0}^{\frac{T}{2}} t^{2n} dt = \underset{T \to \infty}{Lt} \frac{1}{T} \left[\frac{t^{2n+1}}{2n+1}\right]_0^{\frac{T}{2}} = \underset{T \to \infty}{Lt} \frac{1}{T} \left[\frac{\left(\frac{T}{2}\right)^{2n+1}}{2n+1}\right] = \infty$$

This is neither an energy nor a power signal as P and E both are infinite.