Functions

Ms. Rajanpreet Kaur Chahal

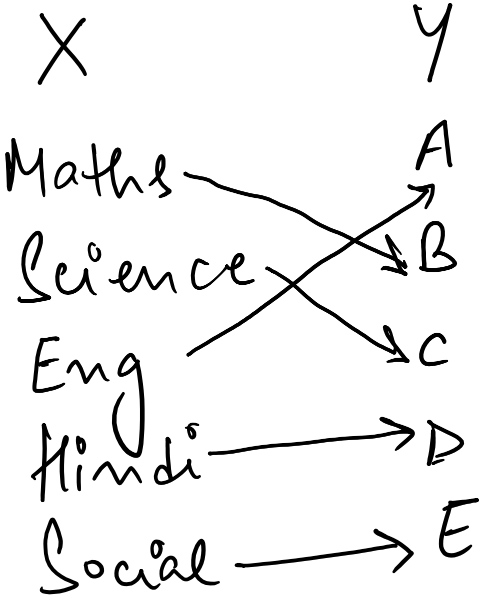
Assistant Professor, CSE

Thapar Institute of Engineering & Technology, Patiala

Email: rajanpreetchahal@gmail.com

Contents

- Definition of function
- Domain, Codomain and Range of function
- Equal functions
- Real-valued and integer-valued functions
- Function addition and multiplication
- Image of a subset

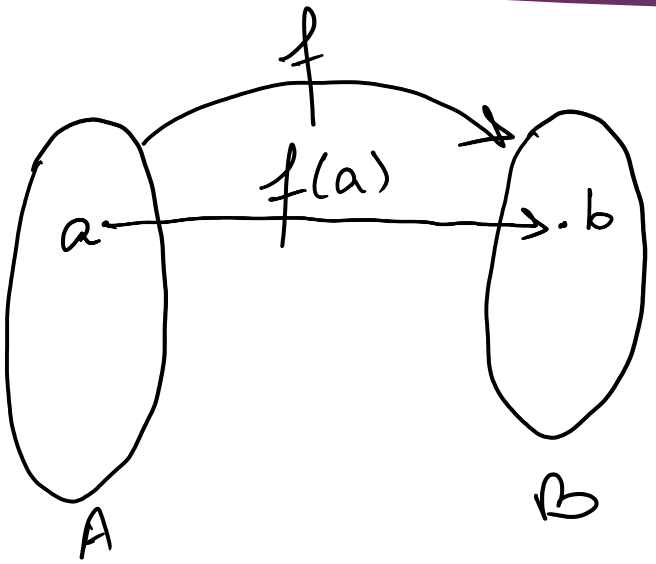


X : set of courses taken by a student

Y : set of grades

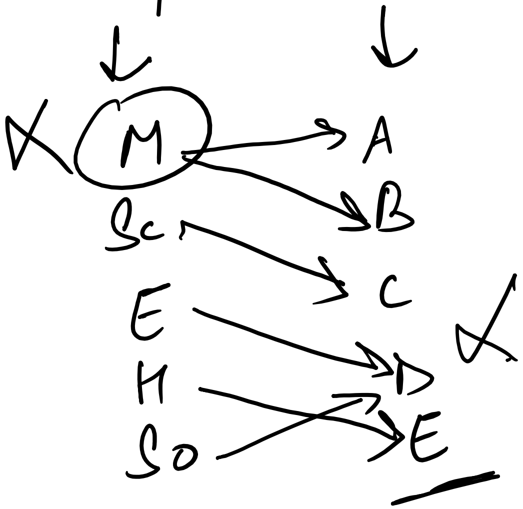
$f : X \to Y$

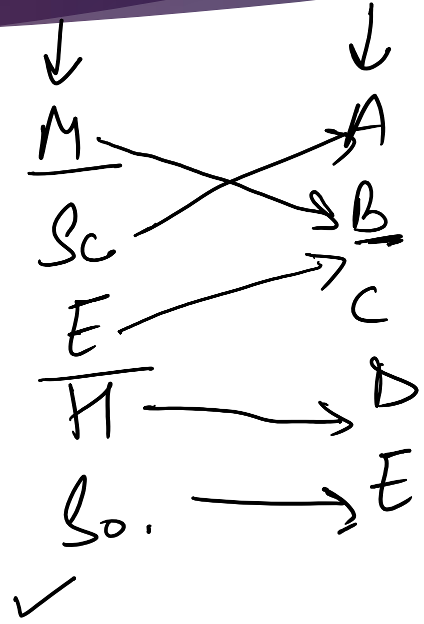
Function



$f: A \to B$

$f(a) = b$





Function definition

- Let A and B be nonempty sets.
- A *function* f from A to B is an assignment of exactly one element of B to each element of A.
- We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.
- If f is a function from A to B, we write
  - $f : A \to B$

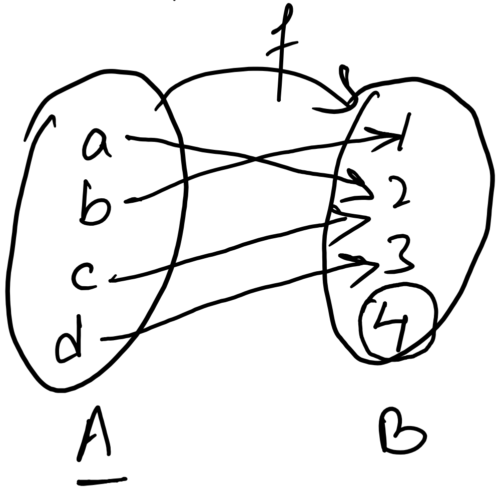
Domain and Range

- If f is a function from A to B, we say that A is the *domain* of f and B is the *codomain* of f.
- If $f(a) = b$, we say that b is the *image* of a and a is a *preimage* of b.
- The *range*, or *image*, of f is the set of all images of elements of A.
- Also, if f is a function from A to B, we say that f *maps* A to B.

Function / Mapping / Transformation

Example

$f : \underline{A} \to \underline{B}$ where $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4\}$.
Also, $f(a) = 2$, $f(b) = 1$, $f(c) = 2$, $f(d) = 3$. Find domain, codomain and range of f.



Domain : A or Domain : $\{a, b, c, d\}$
Codomain : B or Codomain : $\{1, 2, 3, 4\}$ ✓
Range / Image : $\{1, 2, 3\}$ ✓

Equal functions

$f(x): \mathbb{Z} \to \mathbb{Z}$ ✗

$g(x): \mathbb{Z} \to R$ ✗

- Two functions are **equal** when:
  - They have the same domain,
  - They have the same codomain,
  - They map each element of their common domain to the same element in their common codomain.

$$f: \begin{array}{ccc} \mathbb{Z} & & \mathbb{Z} \\ 0 & \to & 1 \\ 1 & \to & 2 \\ 2 & \to & 3 \\ \vdots & & \vdots \end{array} \qquad g: \begin{array}{ccc} \mathbb{Z} & & \mathbb{Z} \\ 0 & \to & 2 \\ 1 & \to & 3 \\ 2 & \to & 4 \\ \vdots & & \vdots \end{array} \quad ✗$$

$f(x) = x + 1$

Example

Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function defined by $f(x) = x^2$. Find domain, codomain & range of function f.

$f(0) = 0^2 = 0$

$f(1) = 1^2 = 1$

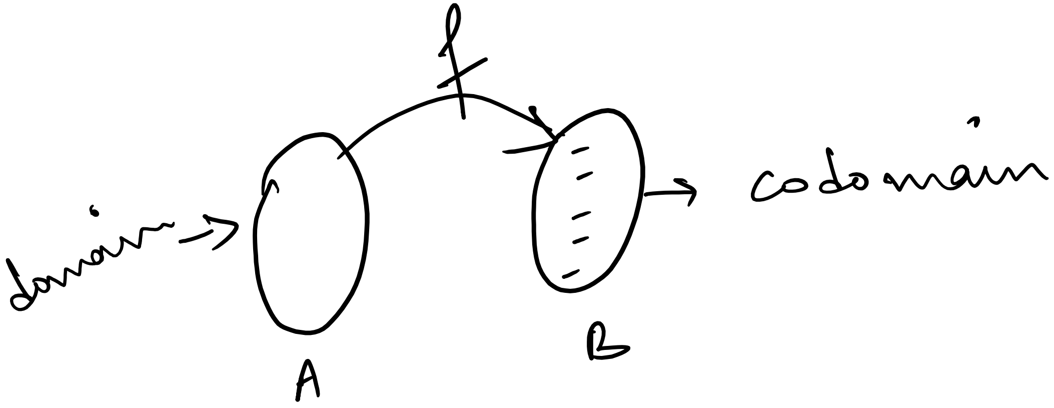
Domain : $\mathbb{Z}$

Codomain : $\mathbb{Z}$

Range : $\{0, 1, 4, 9, \ldots\ldots\}$

Real valued/Integer valued functions

- A function is called **real-valued** if its codomain is the set of real numbers.
- A function is called **integer-valued** if its codomain is the set of integers.
- Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.



Function addition/multiplication

- Let f1 and f2 be functions from A to **R**. Then f1 + f2 and f1f2 are also functions from A to **R** defined for all $x \in A$ by
  - (f1 + f2)(x) = f1(x) + f2(x),
  - (f1f2)(x) = f1(x)f2(x).

Example

Question: Let f1 and f2 be functions from **R** to **R** such that

$f1(x) = x^2$ and $f2(x) = x - x^2$.

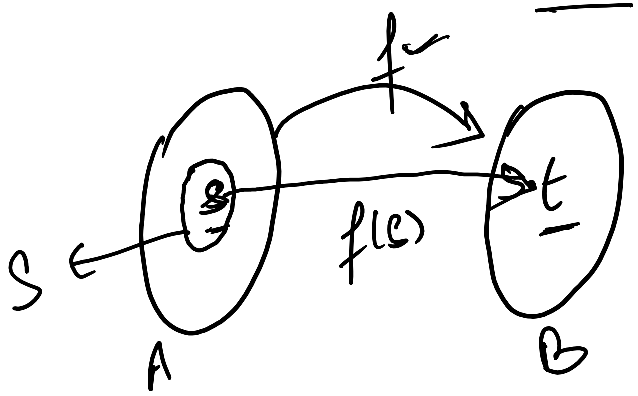
What are the functions f1 + f2 and f1f2?

Answer: $(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + x - x^2 = x$

$(f_1 f_2)(x) = f_1(x) f_2(x) = x^2(x - x^2) = x^3 - x^4$

Image of a subset

- Let f be a function from A to B and let S be a subset of A.
- The *image* of S under the function f is the subset of B that consists of the images of the elements of S.
- The image of S is denoted by $f(S)$ where $f(S) = \{t \mid \exists s \in S\ (t = f(s))\}$.



$f : A \to B$

$f(S) = \{t \mid \exists s \in S\ (t = f(s))\}$

Example

- Question: Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. Consider subset $S = \{b, c, d\}$.

  What is image of S?

  Answer:

$f(b) = 1$

$f(c) = 4$

$f(d) = 1$

$f(S) = \{1, 4\}$

Summary

- Concept of Functions ✓
- Domain, Codomain and Range of functions ✓
- Equal Functions ✓
- Real-valued and integer-valued functions ✓
- Function addition/multiplication ✓
- Image of a subset ✓

Thanks