Course: UMA 035 (Optimization Techniques)

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	New interval	
	$f(x_1) > f(x_2)$	$f(x_1) < f(x_2)$
Minimum	$[x_1, b]$	$[a, x_2]$
Maximum	$[a, x_2]$	$[x_1, b]$

Dichotomous Search Technique

$$\text{Measure of effectiveness} = \frac{\frac{Intial\ length}{2^n} + \frac{\delta}{2}\left(1 - \left(\frac{1}{2}\right)^n\right)}{Initial\ length}$$

$$= \frac{\frac{L_0}{2^n} + \frac{\delta}{2}\left(1 - \left(\frac{1}{2}\right)^n\right)}{L_0}$$

Using it we will find n (if not given).

Example:

Find the maximum of $f(x) = x(1.5 - x)$ in the interval $[0, 1]$ to within 10% of the exact value. Take $\delta = 0.001$.

Solution

If the middle point of the final interval is taken as optimal solution then

$$\frac{\text{Measure of effectiveness}}{2} \le 10\%$$

i.e.,

$$\frac{\frac{L_0}{2^n} + \frac{\delta}{2}\left(1 - \left(\frac{1}{2}\right)^n\right)}{2L_0} \le \frac{10}{100}$$

Since, the given interval is $[0, 1]$. So, $L_0 = 1 - 0 = 1$. Therefore,

$$\frac{\frac{1}{2^n}+\frac{.001}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{2}\leq\frac{1}{10}$$

$$\frac{1}{2^n}+\frac{1}{2000}\left(1-\left(\frac{1}{2}\right)^n\right)\leq\frac{2}{10}$$

$$\frac{1}{2^n}+\frac{1}{2000}-\frac{1}{2000}\left(\frac{1}{2}\right)^n\leq\frac{1}{5}$$

$$\frac{1}{2^n}\left(1-\frac{1}{2000}\right)\leq\frac{1}{5}-\frac{1}{2000}$$

$$\frac{1}{2^n}\left(\frac{1999}{2000}\right)\leq\frac{399}{2000}$$

$$\frac{1}{2^n}\left(\frac{1999}{2000}\right)\leq\frac{2000}{399}$$

$$2^n\left(\frac{2000}{1999}\right)\geq\frac{2000}{399}$$

$$2^n\geq\left(\frac{1999}{399}\right)$$

$$2^n\geq 5.01253133$$

$$n\log 2\geq\log 5.01253133$$

$$n\geq\frac{\log 5.01253133}{\log 2}$$

$$n\geq 2.32812739$$

Since, n is positive integer. So, $n=3$.

This indicates that there is a need to perform 3 steps.

First iteration

$$[a,b]=[0,1]$$

$$L_0=1-0=1$$

$$x_1 = a + \frac{L_0}{2} - \frac{\delta}{2} = 0 + \frac{1}{2} - \frac{.001}{2} = 0.4995$$

$$x_2 = a + \frac{L_0}{2} + \frac{\delta}{2} = 0 + \frac{1}{2} + \frac{.001}{2} = 0.5005$$

$$f(x_1) = x_1(1.5 - x_1) = 0.4995(1.5 - 0.4995) = 0.49975$$

$$f(x_2) = x_2(1.5 - x_2) = 0.5005(1.5 - 0.5005) = 0.50025$$

Since, $f(x_1) < f(x_2)$ and the problem is of maximum. So, new interval is [x_1, b]= $[0.4995, 1]$.

Second iteration

$$[a, b] = [0.4995, 1]$$

$$L_0 = 1 - 0.4995 = 0.5005$$

$$x_1 = a + \frac{L_0}{2} - \frac{\delta}{2} = 0.4995 + \frac{0.5005}{2} - \frac{.001}{2} = 0.74925$$

$$x_2 = a + \frac{L_0}{2} + \frac{\delta}{2} = 0.4995 + \frac{0.5005}{2} + \frac{.001}{2} = 0.75025$$

$$f(x_1) = x_1(1.5 - x_1) = 0.74925(1.5 - 0.74925) = 0.49975$$

$$f(x_2) = x_2(1.5 - x_2) = 0.75025(1.5 - 0.75025) = 0.50025$$

Since, $f(x_1) < f(x_2)$ and the problem is of maximum. So, new interval is [x_1, b]= $[0.74925, 1]$.

Third iteration

$$[a, b] = [0.74925, 1]$$

$$L_0 = 1 - 0..74925 = 0.25075$$

$$x_1 = a + \frac{L_0}{2} - \frac{\delta}{2} = 0.74925 + \frac{0.25075}{2} - \frac{.001}{2} = 0.874125$$

$$x_2 = a + \frac{L_0}{2} + \frac{\delta}{2} = 0.74925 + \frac{0.25075}{2} + \frac{.001}{2} = 0.875125$$

$$f(x_1) = x_1(1.5 - x_1) = 0.874125(1.5 - 0.874125) = 0.545403122$$

$$f(x_2) = x_2(1.5 - x_2) = 0.875125(1.5 - 0.875125) = 0.546843734$$

Since, $f(x_1) < f(x_2)$ and the problem is of maximum. So, new interval is [x_1, b]= $[0.874125, 1]$.

The center of the interval can be considered as an optimal solution.

Approximate optimal solution is $\frac{0.874125+1}{2} = 0.9370625$

Approximate optimal value is $x(1.5 - x) = 0.9370625(1.5 - 0.9370625) = 0.527507621$

Fibonacci Search Technique

Fibonacci numbers

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
1	1	2	3	5	8	13	21	34

$F_0 = 1, F_1 = 1, F_n = F_{n-1} + F_{n-2}, n > 1.$

$$x_1 = a + \frac{F_{n-2}}{F_n} L_0$$

$$x_2 = a + \frac{F_{n-1}}{F_n} L_0$$

Measure of effectiveness

$$= \frac{1}{F_n}$$

Example:

Minimize the function $x(x-2), 0 \le x \le 1.5$ within the interval of uncertainty $0.25L_0$.

Solution:

Measure of effectiveness $= \frac{Interval\ of\ uncertainty}{L_0} = \frac{0.25L_0}{L_0} = 0.25$

$$\frac{1}{F_n} \le 0.25$$

$$\frac{1}{F_n} \le \frac{1}{4}$$

$$F_n \ge 4$$

From Table $F_4 = 5$

Therefore, n=4.

First iteration

n=4

$$[a, b] = [0, 1.5]$$

$$L_0 = 1.5 - 0 = 1.5$$

$$x_1 = a + \frac{F_{n-2}}{F_n} L_0 = 0 + \frac{F_{4-2}}{F_4} L_0 = 0 + \frac{F_2}{F_4} L_0 = 0 + \frac{2}{5}(1.5) = 0.6$$

$$x_2 = a + \frac{F_{n-1}}{F_n} L_0 = 0 + \frac{F_{4-1}}{F_4} L_0 = 0 + \frac{F_3}{F_4} L_0 = 0 + \frac{3}{5}(1.5) = 0.9$$

$$f(x_1) = x_1(x_1 - 2) = 0.6(0.6 - 2) = -0.84$$

$$f(x_2) = x_2(x_2 - 2) = 0.9(0.9 - 2) = -0.99$$

Since, $f(x_1) > f(x_2)$ and the problem is of minimum. So, new interval is $[x_1, b]$= $[0.6, 1.5]$.

Second iteration

n=3

$$[a, b] = [0.6, 1.5]$$

$$L_0 = 1.5 - 0.6 = 0.9$$

$$x_1 = a + \frac{F_{n-2}}{F_n} L_0 = 0.6 + \frac{F_{3-2}}{F_3} L_0 = 0.6 + \frac{F_1}{F_3} L_0 = 0.6 + \frac{1}{3}(0.9) = 0.9$$

$$x_2 = a + \frac{F_{n-1}}{F_n} L_0 = 0.6 + \frac{F_{3-1}}{F_3} L_0 = 0.6 + \frac{F_2}{F_3} L_0 = 0.6 + \frac{2}{3}(0.9) = 1.2$$

$$f(x_1) = x_1(x_1 - 2) = 0.9(0.9 - 2) = -0.99$$

$$f(x_2) = x_2(x_2 - 2) = 1.2(1.2 - 2) = -0.96$$

Since, $f(x_1) < f(x_2)$ and the problem is of minimum. So, new interval is [a, x_2]= $[0.6, 1.2]$.

Third iteration

n=2

$$[a, b] = [0.6, 1.2]$$

$$L_0 = 1.2 - 0.6 = 0.6$$

$$x_1 = a + \frac{F_{n-2}}{F_n} L_0 = 0.6 + \frac{F_{2-2}}{F_2} L_0 = 0.6 + \frac{F_0}{F_2} L_0 = 0.6 + \frac{1}{2}(0.6) = 0.9$$

$$x_2 = a + \frac{F_{n-1}}{F_n} L_0 = 0.6 + \frac{F_{2-1}}{F_2} L_0 = 0.6 + \frac{F_1}{F_2} L_0 = 0.6 + \frac{1}{2}(0.6) = 0.9$$

Since, $x_1 = x_2$. So, consider an arbitrary point near to $x_1 = x_2$ and greater than $x_1 = x_2$ (say, $x_2{}'$=0.91).

Now, $f(x_1) = x_1(x_1 - 2) = 0.9(0.9 - 2) = -0.99$

$$f(x_2{}') = x_2{}'(x_2' - 2) = 0.91(0.91 - 2) = -0.981$$

Since, the problem is of minimum and $f(x_1) < f(x_2{}')$. So, new interval is [a, x_2]= $[0.6, 0.9]$.

Approximate optimal solution is $\frac{0.6+0.9}{2} = 0.75$

Approximate optimal value is $x(x-2) = 0.75(0.75-2) = -0.9375$