School of Mathematics

Thapar Institute of Engineering and Technology, Patiala

Optimization Techniques (UMA035)

Practice sheet No. 6

1. Consider the LPP

   Maximize $z = 3x_1 + 5x_2 + 4x_3$

   Subject to $2x_1 + 3x_2 \leq 8$, $2x_2 + 5x_3 \leq 10$, $3x_1 + 2x_2 + 4x_3 \leq 15$, $x_1, x_2, x_3 \geq 0$.

   (a) Solve the LPP.

   (b) Find the range over which b_2 can be changed maintaining the feasibility of the solution.

2. Consider the problem, Max z = 3x1 + 2x2 + 5x3

   s/t x1 + 2x2 + x3 ≤ 430, 3x1 + 2x3 ≤ 460,

   x1 + 4x2 ≤ 420, x1, x2, x3 ≥ 0.

Given that x2, x3, x6 (slack variable corresponding to constraint 3) form the optimal basis and inverse of the optimal basis is, row-wise; ½, -1/4, 0; 0, ½ 0; -2, 1, 1. Form the optimal table based on this information.

3. In problem 2, find the optimal solution if the objective function is changed to

   (i) z = 4x1 + 2x2 + x3 (ii) z = 3x2 + x3

4. In problem 2, a fourth variable is added with the technological (constraint) coefficients as 3, 2 and 4. Determine the optimal solution if the profit per unit of the new variable is given as 5 and 10.

5. Consider the following LPP, Max z = 5x1 + 2x2 + 3x3 s/t x1 + 5x2 + 3x3 = 30, x1 – 5x2 – 6x3 ≤40, x1, x2, x3 ≥ 0. Solve this problem using M-method.

6. In problem 5, find the optimal solution, using sensitivity analysis if the objective function is changed to

   (i) max z = 12x1 + 5x2 + 2x3 (ii) min z = 2x2 – 5x3

7. In problem 5, suppose that the technological coefficients of x2 are (5 – a, -5 + a) instead of (5, -5), where a is a nonnegative parameter. Find the value of a so that the solution remains optimal.

8. In problem 5, suppose that the right hand side of the constraint becomes (30 + a, 40 – a), a is nonnegative parameter. Determine the values of a so that the solution of the problem remain optimal.

9. Solve the LPP: Maximize $z=3x_1+x_2+5x_3$

   Subject to $6x_1+3x_2+5x_3 \leq 25$, $3x_1+4x_2+5x_3 \leq 20$, $x_1, x_2, x_3 \geq 0$.

Also, discuss the effect on the optimal solution if a new constraint

$2x_1-3x_2+4x_3 = 15$ is added.

10. Consider the LPP: Maximize $z=3x_1+4x_2+x_3+7x_4$

    Subject to $8x_1+3x_2+4x_3+x_4 \leq 7$, $2x_1+6x_2+x_3+5x_4 \leq 3$

    $x_1+4x_2+5x_3+2x_4 \leq 8$, $x_1, x_2, x_3, x_4 \geq 0$.

a) Solve the LPP.
b) What will be the optimal solution if a new constraint $2x_1+3x_2+x_3+5x_4 \leq 4$ is added?