

Note: Strictly attempt all questions sequentially & Assume missing data, if any, suitably

Q.1 Find the z-transform and corresponding ROC for each of the following (10) signals:

✓ 1. $y(n) = \delta(n-1)$ ✓ 2. $y(n) = \delta(n+1)$ ✓ 3. $y(n) = a^n u(n) - a^n u(n-1)$

✓ 4. $y_k(n) = \begin{cases} y[n/k], & \text{if } n \text{ is an integer multiple of } k \\ 0, & \text{if } n \text{ is not an integer multiple of } k \end{cases}$

Q.2 Consider an LTI system, for which, the input $x(n]$ and output $y(n]$ satisfy (10) the following linear constant-coefficient difference equation:

$y(n) - (1/2)y(n-1) = x(n) + (1/3)x(n-1)$

Find the system impulse response $h(n]$ and verify its stability based on

- its poles and zeros, all values of z for which it has finite value
- i) if the ROC corresponding to $H[z]$ is $|z| > 1/2$
 - ii) if the ROC corresponding to $H[z]$ is $|z| < 1/2$

Q.3 Obtain the discrete-time Fourier-transform (DTFT) of the discrete-time (10) signal $x(n) = a^n$ with $|a| < 1$. Plot the magnitude spectrum $|X(e^{j\omega})|$ vs. ω and the phase spectrum $\angle X(e^{j\omega})$ vs. ω .

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

Q.4 Consider the continuous-time signal $x(t)$, whose continuous-time (10) Fourier-transform (CTFT) is $X(j\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$

$x(t) = \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

Determine the signal $x(t)$ using the inverse-CTFT, and also plot it.

Q.5 Let the discrete-time composite periodic signal be (10)

$x(n) = 1 + \sin(2\pi n/N) + 3\cos(2\pi n/N) + \cos(4\pi n/N + \pi/2)$

with period N . Calculate the Discrete-time Fourier-series (DTFS) spectral coefficients a_k , and also plot the magnitude $|a_k|$ vs. k .

$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kn/N}$

$\frac{1}{1-a^2}$

$\frac{1}{1-a^2}$

z^{-n}

Q.5

The continuous-time periodic square-wave $x(t)$ is defined

$$x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & T/2 < |t| < T \end{cases}$$

with the fundamental period T and the fundamental angular frequency $\omega_0 = 2\pi/T$. Plot this periodic square-wave $x(t)$; and calculate the Continuous-time Fourier-series (CTFS) spectral coefficients a_n .

Q.7(a)

If the input signal to an LTI system is unit-step function $u(n)$ in the discrete-time domain and the impulse response of concerned LTI system is also unit-step function $u(n)$, then obtain the output signal $y(n)$ using the linear convolution sum formula.

Q.7(b)

Consider the z-transform $X(z) = 1/(1 - az^{-1})$ with $ROC \rightarrow |z| < |a|$ of an unknown signal $x(n]$. Determine $x(n]$ using the power-series-expansion method based on long division.

Q.8

Compute the discrete-Fourier-transform (DFT) of the four-point sequence $x(n) = [0 \ 1 \ 2 \ 3]$, using the matrix \mathcal{W}_4 of the linear transformation (i.e., by using the matrix method).

Q.9

Perform the circular convolution of the following two sequences:

(10)

$$x_1(n) = \begin{bmatrix} 2 & 1 & 2 & 1 \end{bmatrix} \text{ and } x_2(n) = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$



Q.10(a)

How the basic butterfly computation in the decimation-in-frequency fast-Fourier-transform (FFT) algorithm is different from the basic butterfly computation in the decimation-in-time FFT algorithm? (Only plot comparison diagrams).

(04)

Q.10(b)

Draw the signal flow graph for an $N = 8$ point decimation-in-frequency FFT algorithm, using the butterfly computation scheme (clearly indicating its different stages).

(06)

Note:

Use $\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{(\pi\theta)}$, wherever it is required.