UEC-404 Signals & Systems

Tutorial #2

Read the following:

B.2-1 Addition of Sinusoids

Two sinusoids having the same frequency but different phases add to form a single sinusoid of the same frequency. This fact is readily seen from the well-known trigonometric identity

$$\begin{aligned} C\cos(\omega_0 t+\theta) &= C\cos\theta\cos\omega_0 t - C\sin\theta\sin\omega_0 t \\ &= a\cos\omega_0 t + b\sin\omega_0 t \end{aligned} \tag{B.23a}$$

in which

$$a = C\cos\theta, \qquad b = -C\sin\theta$$

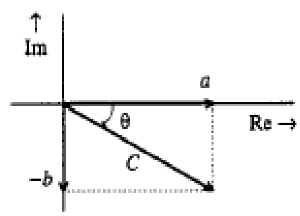

Fig. B.7 Phasor addition of sinusoids.

Therefore,

$$C = \sqrt{a^2+b^2} \tag{B.23b}$$

$$\theta = \tan^{-1}\left(\frac{-b}{a}\right) \tag{B.23c}$$

Equations (B.23b) and (B.23c) show that C and θ are the magnitude and angle, respectively, of a complex number $a - jb$. In other words, $a - jb = Ce^{j\theta}$. Hence, to find C and θ, we convert $a - jb$ to polar form and the magnitude and the angle of the resulting polar number are C and θ, respectively.

To summarize,

$$a\cos\omega_0 t + b\sin\omega_0 t = C\cos(\omega_0 t + \theta)$$

in which C and θ are given by Eqs. (B.23b) and (B.23c), respectively. These happen to be the magnitude and angle, respectively, of $a - jb$.

The process of adding two sinusoids with the same frequency can be clarified by using **phasors** to represent sinusoids. We represent the sinusoid $C\cos(\omega_0 t+\theta)$ by a phasor of length C at an angle θ with the horizontal axis. Clearly, the sinusoid $a\cos\omega_0 t$ is represented by a horizontal phasor of length a ($\theta = 0$), while $b\sin\omega_0 t = b\cos(\omega_0 t - \frac{\pi}{2})$ is represented by a vertical phasor of length b at an angle $-\pi/2$ with the horizontal (Fig. B.7). Adding these two phasors results in a phasor of length C at an angle θ, as depicted in Fig. B.7. From this figure, we verify the values of C and θ found in Eqs. (B.23b) and (B.23c), respectively.

	Proper care should be exercised in computing θ. Recall that $\tan^{-1}(\frac{-b}{a}) \neq \tan^{-1}(\frac{b}{-a})$. Similarly, $\tan^{-1}(\frac{-b}{-a}) \neq \tan^{-1}(\frac{b}{a})$. Electronic calculators cannot make this distinction. When calculating such an angle, it is advisable to note the quadrant where the angle lies and not to rely exclusively on an electronic calculator. A foolproof method is to convert the complex number $a - jb$ to polar form. The magnitude of the resulting polar number is C and the angle is θ. The following examples clarify this point.
[1]	Express the following signals $x(t)$ as a single sinusoid: **[a]** $x(t) = \cos(\omega_0 t) - \sqrt{3}\sin(\omega_0 t)$ **[b]** $x(t) = -3\cos(\omega_0 t) + 4\sin(\omega_0 t)$ **ANSWER:** **[a]** $x(t) = 2\cos(\omega_0 t + 60^\circ)$ **[b]** $x(t) = 5\cos(\omega_0 t - 126.9^\circ)$
[2]	The relations considered in this problem will be used on many occasions throughout your course and engineering curriculum. **(a)** Prove the validity of the following expression: $\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N, & \alpha = 1 \\ \frac{1-\alpha^N}{1-\alpha}, & \text{for any complex number } \alpha \neq 1 \end{cases}$. This is often referred to as the *finite sum formula.* **(b)** Show that if $\lvert\alpha\rvert < 1$, then $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$. This is often referred to as the *infinite sum formula.*
[3]	Determine all complex number z that satisfy the equation $z + 3z^* = 5 - 6i$, where z^* is the complex conjugate of z.
[4]	The complex number $2 + 4i$ is one of the root to the quadratic equation $x^2 + bx + c = 0$, where b and c are real numbers. **(a)** Find b and c? **(b)** Write down the second root and check it.
[5]	Consider the complex-valued exponential signal $x(t) = Ae^{\alpha t + j\omega t}$, $a > 0$. Evaluate the real and imaginary components of $x(t)$ for the following cases: **(a)** α is real, $\alpha = \alpha_1$ **(b)** α imaginary, $\alpha = j\omega_1$ **(c)** α complex, $\alpha = \alpha_1 + j\omega_1$