Roll No. : ....

School of Mathematics, Thapar University, Patiala

Mid-Term Examination, September 2016

B.E. (III Sem) UMA007 : Numerical Analysis

Time Limit: 02 Hours Maximum Marks: 25

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Instructions: 1. You are expected to answer all the questions. Organize your work, in a reasonably neat and coherent way. Mysterious or unsupported answers will not receive full credit.
2. Scientific calculator without graphing mode and alphanumeric memory is permitted.

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1. (a) Let floating point representation of a real number x is $x = (0.a_1a_2\ldots a_na_{n+1}\ldots)_\beta \times \beta^e$, $a_1 \neq 0$. Let $fl(x)$ be its machine approximation with n digits by rounding then obtain a bound for absolute relative error. [3.0 marks]

(b) Write an algorithm to calculate the expression $f(x) = \sin(a+x) - \sin a$, when $x = 0.0001$. By considering the condition number κ of the subproblem of evaluating the function, show that such a function evaluation is not stable. Suggest a modification which makes it stable. [3.0 marks]

2. (a) The sum of two numbers is 20. If each number is added to its square root, then the product of the resulting sums is 155.55. Perform five iterations of bisection method to determine the two numbers.

[3.0 marks]

(b) Use Newton method to approximate, to within 10^{-4}, the value of x that produces the point on the graph of $y = x^2$ that is closest to the point $(1, 0)$. [3.0 marks]

3. (a) Let A be a given positive constant and $g(x) = 2x - Ax^2$. Show that if fixed-point iteration converges to a nonzero limit, then the limit is $\alpha = 1/A$, so the inverse of a number can be found using only multiplications and subtractions. Also find an interval about $1/A$ for which fixed-point iteration converges, provided x_0 is in that interval. [4.0 marks]

(b) Use the secant method to find a solution accurate to within 10^{-4} for $\ln(x) + x - 5 = 0$ on $[3, 4]$.

[3.0 marks]

4. Suppose that $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a, b]$ and $f \in C^{n+1}[a, b]$. Let $P(x)$ be the unique interpolating polynomial of degree $\leq n$ then prove that for each $x \in [a, b]$, there exists a point $\xi \in (a, b)$ such that

$$f(x) - P(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_n)}{(n+1)!} f^{(n+1)}(\xi).$$

Apply this result to find error bound with $f(x) = e^{2x} - x^2$, $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$.

[(4.0+2.0) marks]
