School of Mathematics

Thapar Institute of Engineering and Technology, Patiala

Optimization Techniques (UMA035)

Practice sheet No. 4

1. For the given LPP Maximize $z = 2x1 + 3x2$, s.t. $x1 + 3x2 \leq 6$, $3x1 + 2x2 \leq 6$, $x1, x2 \geq 0$.

(a) Express the problem in standard form.

(b) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(c) Use direct substitution in the objective function to determine the optimum basic feasible solution.

(d) Verify graphically that the solution obtained in (c) is the optimum LP solution hence, conclude that the optimum solution can be determined algebraically by considering the basic feasible solutions only.

2. Consider the LPP Maximize $z = 2x1 + 3x2 + 5x3$, s.t. $-6x1 + 7x2 - 9x3 \geq 4$, $x1 + x2 + 4x3 = 10$, $x1, x3 \geq 0$, x2 unrestricted. Use $x2 = x4 - x5$; $x4 , x5 \geq 0$ and show that a basic solution cannot include both x4 and x5 simultaneously.

3. Find an optimal solution to the following linear program using (a) simplex method (b) algebraic method.

Min. $z = x1 + 2x2 + 3x3$, s.t. $x1 + 3x2 + 6x3 = 6$, $x1, x2, x3 \geq 0$.

4. Use the Simplex method to solve Max. $z = 5x1 + 4x2$, S.T. $6x1 + 4x2 \leq 24$, $x1 + 2x2 \leq 6$, $-x1 + x2 \leq 1$, $x2 \leq 6$, $x1, x2 \geq 0$.

5. Consider the following Linear programming problem Max. $z = x1 + x2$, s.t. $x1 + 2x2 \leq 3$, $2x1 + x2 \leq 3$, $x1, x2 \geq 0$. Answer the following questions:

(a) Construct the simplex table corresponding to the corner point (3/2, 0) of the feasible region.

(b) Using the simplex table obtained in Part-(a) above, find optimal Basic feasible solution of LPP.

(c) Identify all the simplex tables obtained in Part-(b) above on the graph of the feasible region.

6. The following table represents a specific simplex iteration.

Basic Variables	x1	x2	x3	x4	x5	x6	x7	x8	Solution
z-row→	0	−5	0	4	−1	−10	0	0	620
x8	0	3	0	−2	−3	−1	5	1	12
x3	0	1	1	3	1	0	3	0	6
x1	1	−1	0	0	6	−4	0	0	0

(a) Can the given table be an optimal table? Justify.

(b) Assuming that the problem is of the maximization type, identify the nonbasic variables that have the potential to improve the value of z. If each such variable enters the basic solution, determine the associated leaving variable, if any, and the associated change in z.

(c) Which nonbasic variable(s) will not cause a change in the value of z when selected to enter the solution?

7. Use the simplex method to solve Min. $z = -x1 - x2$, S.T. $x1 + 2x2 \leq 3$, $x1 + x2 \leq 2$, $3x1 + 2x2 \leq 6$, $x1, x2 \geq 0$. Find an alternate optimal solution if one exists.

8. Formulate the LPP from the following Optimal table;

CB	B.V↓	x1	x2	x3	s1	s2	Soln.
z-row→		0	0	17/7	6/7	4/7	2
4	x2	0	1	1/7	2/7	−1/7	0
2	x1	1	0	17/7	−1/7	4/7	1

9. Two consecutive simplex tableaus of a LPP are

B.V↓	x1	x2	x3	x4	x5	Soln
z-row→	A	−1	3	0	0	
x4	B	C	D	I	0	6
x5	−1	2	E	0	1	1

B.V↓	x1	x2	x3	x4	x5	Soln.
z-row→	0	−4	J	K	0	
x1	G	2/3	2/3	1/3	0	F
x5	H	8/3	−1/3	1/3	1	3

Find the values of “A to K”.

10. Use the Simplex method to show that the following problem has unbounded solution:

Max. $z = x1 + x2$, S.T. $3x1 - 4x2 \geq -3$, $x1 - x2 - x3 = 0$, $x1, x2, x3 \geq 0$.

11. Solve the following systems of equations using Simplex method:

(a) $2x_1 + x_2 - x_3 = 1$, $-2x_1 + 2x_2 - x_3 = -2$, $x_1 + x_3 = 3$, $x_1, x_2, x_3 \geq 0$.

(b) $x_1 - x_2 + x_3 = 1$, $x_1 + x_3 = 2$, $2x_1 + x_2 + 2x_3 = 3$

12. Solve the following by the Simplex method, without using artificial variables:

(a) Min. $z = -5x_1 - 3x_2$, S.T. $x_1 + x_2 + x_3 = 2$, $5x_1 + 2x_2 + x_4 = 10$, $3x_1 + 8x_2 + x_5 = 12$, $x_1, x_2, x_3, x_4, x_5 \geq 0$.

(b) Max. $z = 3x_1 + x_2 + 2x_3$, S.T. $12x_1 + 3x_2 + 6x_3 + 3x_4 = 9$, $8x_1 + x_2 - 4x_3 + 2x_5 = 10$, $3x_1 - x_6 = 0$, $x_1, \ldots, x_6 \geq 0$.

13. Solve the following system of equations using both Big M method and two phase method.

(a) Min. $z = 3x_1 + 5x_2$, S.T. $x_1 + 3x_2 \geq 3$, $x_1 + x_2 \geq 2$, $x_1, x_2 \geq 0$.

(b) Max. $z = 4x_2 - 3x_1$, S.T. $x_1 - x_2 \geq 0$, $2x_1 - x_2 \geq 2$, $x_1, x_2 \geq 0$.

(c) Max. $z = 3x_1 + 2x_2$, S.T. $2x_1 + x_2 \leq 2$, $3x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$.