FLUID STATIC

- Fluid static is the study of fluids at rest.
- Topics: Measurement of fluid pressure, thrust on submerged surfaces, buoyancy.

FLUID PRESSURE

- A fluid always has pressure due to molecular activity such as intermolecular collisions.
- Thus, fluid in a container exerts force at all points.
- Force so exerted per unit area is called fluid pressure or intensity of pressure, ***p****i.e.****p = F/A.***
- ***F*** is the <u>pressure force</u> or total pressure and ***A*** is the area on which fluid pressure acts.
- Fluid pressure at rest always acts normal to the surface.

SI Units: N/m^2 or Pa

CONTINUUM CONCEPT

- A complete analysis of the behavior of fluids requires the action of each individual molecule.
- In most engineering applications, we are interested only in the average conditions of fluid.
- We regard fluids as a 'continuum' *i.e.* a continuous distribution of matter with no empty spaces.
- In continuum, overall properties and behavior of fluids are studied, without considering their atomic and molecular structure.

CONTROL VOLUME

- For the analysis of fluid flow, usually a definite volume with fixed boundary shape is chosen.
- This definite volume is called the <u>control volume</u> and boundary of this volume is known as the <u>control surface.</u>

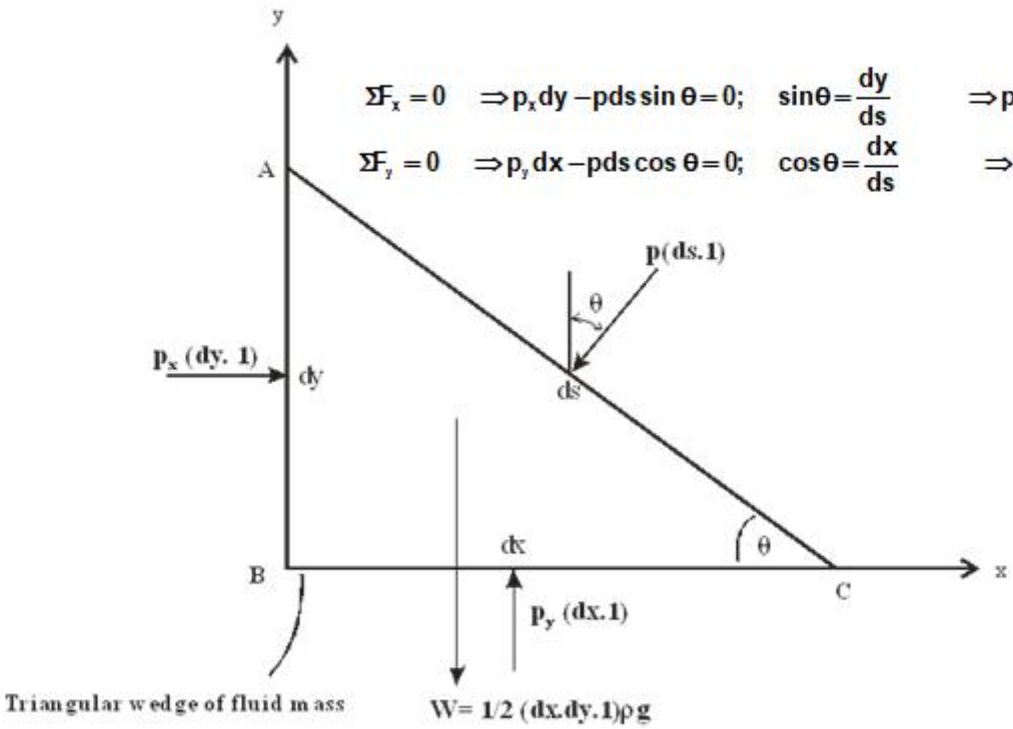
Examples: Stream line, stream tube, triangle, rectangle, cuboid (rectangular paralleopipe) etc.

- ✓ In the analysis, we are generally required to express the change in value of a parameter.
- Assume that within element of a fluid, velocity, pressure and area change only with distance **s** along the streamline, then
  Area **A**, pressure **p** and velocity **V** can be written as:

$$\left(A+\frac{dA}{ds}\delta s\right)=(A+\delta A);\qquad \left(p+\frac{dp}{ds}\delta s\right)=(p+\delta p);\qquad \left(V+\frac{dV}{ds}\delta s\right)=(V+\delta V)$$

PASCAL'S LAW (Variation of fluid pressure at a point)

- Intensity of pressure at any point in a fluid at rest is same in all the directions (**p_x = p_y = p_z**) i.e. when a certain pressure is applied at any point, it is equally transmitted in all the directions.
- Pressure at a point is independent of direction! (is it scalar?).
- Pascal law has applications in hydraulic - press, jack, crane.
- In all these cases, application of relatively small forces develops considerably large forces.
- To prove Pascal law, a control volume of fluid mass in the shape of triangular prism is analysed.



- Prism is first analysed in **xy**-plane and the equations of ≡^{m} are applied **i.e.**

  ΣFx = 0 and **ΣF_y** = 0.

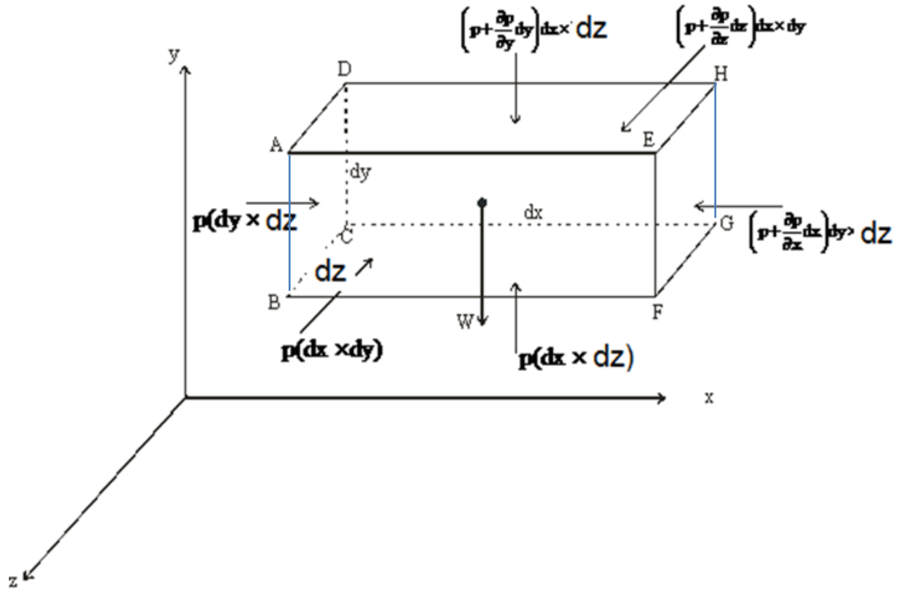
- It can be shown that **p_x = p_y= p**.
- Prism is then analysed in **xz**-plane by applying **ΣFx = 0** and **ΣF_z =** 0.
- It is then shown that **p_x = p_z= p.**

❖ Weight of fluid mass in triangular wedge is neglected (being small)

DIFFERENTIAL EQUATION OF PRESSURE VARIATION

(Variation of fluid pressure from point to point in a fluid mass)

- Consider a **CV** in the shape of a small rectangular paralleopipe element of fluid mass of size dx, dy and dz.



- Forces acting on the element are:
- <u>Gravity force</u> due to weight of fluid, **W = ρg (dx × dy ×dz)**
- <u>Pressure forces</u> acting normal to the surfaces/faces.
- For pressure forces, three pair of faces are considered viz. faces **ABCD** and **EFGH** (⊥ to x-axis), faces **AEHD** and **BFGC** (⊥ to y-axis), faces **ABFE** and **CGHD** (⊥ to z-axis).
- Pressure force on **ABCD** is **pdydz**, ***p*** is the intensity of pressure.
- Pressure force on **EFGH** is written using Taylor expansion series and is given by the expression $\left(p+\frac{\partial p}{\partial x}dx\right)dy \times dz$
- Pressure forces on the other faces are shown in **Figure.**
- As element is in equilibrium under the action of these forces

∴ Applying equations of equilibrium along x-, y- and z-directions i.e.

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$

- $\Sigma F_x = 0$; $(\partial p/\partial x) = 0$
- $\Sigma F_y = 0$; $(\partial p/\partial y) = -\rho g$,

-ve sign signifies that pressure decreases with increase in ***y*** *i.e.* elevation

- $\Sigma F_z = 0$; $(\partial p/\partial z) = 0$

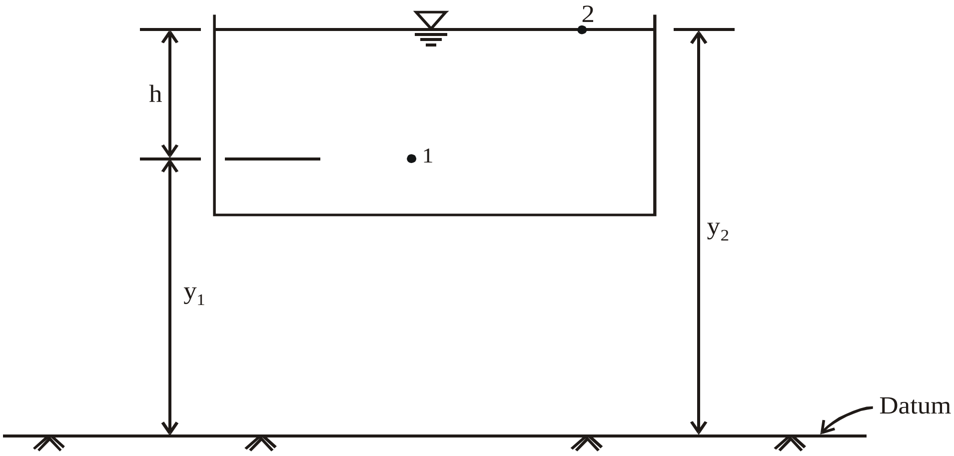
- **Eqs.** show that pressure intensity ***p*** does not vary along **x-** and **z-** directions but varies only along **y-** direction *i.e.* vertical direction

$$\therefore \frac{dp}{dy} = -\rho g$$

Observations

(i) To find the difference in pressure between two points of a fluid in a container.

- Consider two points **1** and **2** as shown in **Figur**e



- Separating the variables and integrating , to write $\int_1^2 dp = -\int_1^2 \rho g . dy$

- For incompressible fluids, ρ = constant $(p_2 - p_1) = -\rho g (y_2 - y_1)$

- Rearranging the terms, to write

$$\left(\frac{p_1}{\rho g} + y_1\right) = \left(\frac{p_2}{\rho g} + y_2\right) \quad \text{or} \quad \left(\frac{p}{\rho g} + y\right) = \text{constant}$$

- Each term has the unit of length, therefore each term is also known as head.
  - **p/ρg** is called pressure head or static head (pressure energy per unit weight) and ***y*** is called elevation head or datum head (potential energy per unit weight).
- Summation of ***p/ρg*** and ***y*** is called piezometric head.
  - ❖ ∴ Piezometric head at every point in a static fluid is constant.

(ii) To deduce hydrostatic law of pressure variation
If ***h*** is the difference in elevation between 1 and 2, then $\mathbf{h = (y_2 - y_1)}$

$\therefore \mathbf{(p_1 - p_2) = \gamma h}$ **or** $\mathbf{p_1 = (\gamma h + p_2)}$; $\quad (\gamma = \rho g)$

- $\mathbf{p_2}$ is the pressure at point **2** *i.e.* atmospheric pressure (**AP**).

- Since **AP** at a place is constant, usually pressure in excess of **AP** is considered, say **p.**

$$\therefore\ \mathbf{p = \gamma h \quad or \quad (p/\gamma) = h}$$

i.e. pressure head at any point in a liquid is equal to the height of the point from the free surface of liquid**.**

- **Eq.** describes the law of variation of static pressure,known as hydrostatic law of pressure variation.
- It states that pressure intensity at any point in a static fluid depends on the vertical height of point below the free surface and specific weight of fluid.
- ❖ In other words,pressure intensity varies linearly with depth from the free surface. It does not depend upon the shape and size of container.

(iii) Eq., $\mathbf{p = \gamma h}$ can be used to obtain a relationship between heights of columns of different liquids, which would develop same pressure at any point

$$\mathbf{p = \gamma_1 h_1 = \gamma_2 h_2 \quad OR \quad S_1 h_1 = S_2 h_2}$$

- ➢ $\mathbf{h_1}$ and $\mathbf{h_2}$ are the heights of columns of liquids having specific weights γ_1 and γ_2 ***or*** specific gravities $\mathbf{S_1}$ and $\mathbf{S_2}$*.*

(iv) If height, $\mathbf{dy = 0}$, then $\mathbf{dp = 0}$ or **p =** constant *i.e.* pressure intensity remains constant over any horizontal plane in a fluid.

- A surface where pressure is same at all points is called an isobaric or equipotential surface (free surface).

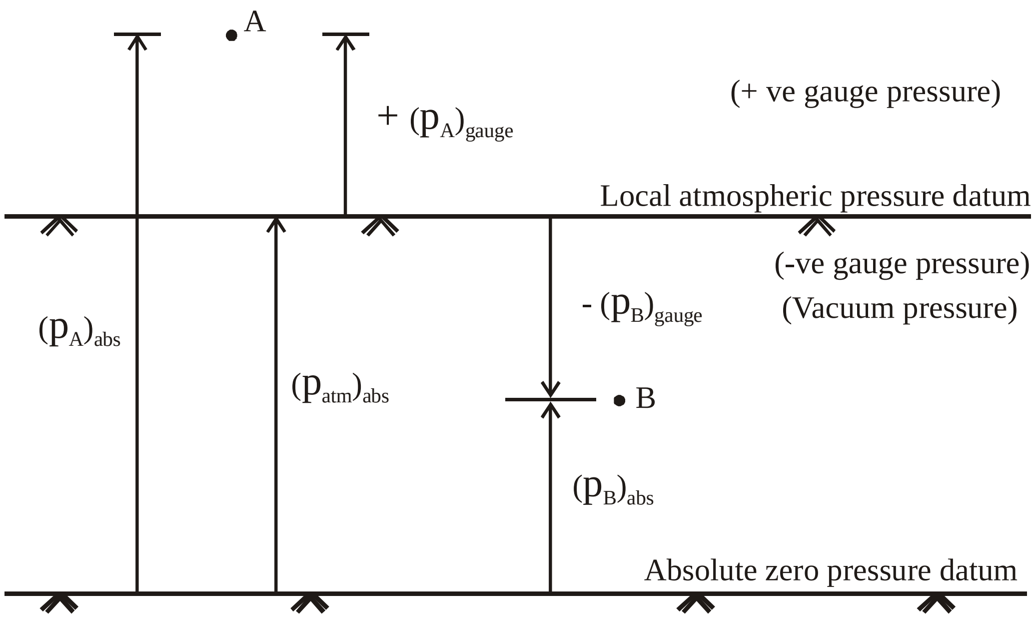
Absolute Pressure

- As atmospheric pressure varies with altitude. Thus,it is necessary to establish a scale/datum which is independent of changes in atmospheric pressure.
- Such a datum is known as absolute zero pressure datum.
- Absolute pressure is the pressure measured from absolute zero pressure datum (vacuum state).

Gauge Pressure

- is the pressure measured using pressure measuring devices in which local atmospheric pressure is taken as datum i.e. gauge value of atmospheric pressure is zero.
- Gauge pressure can be + ve or – ve.
- If pressure of fluid is
- ➢ above local atm. pressure, it is called + ve gauge pressure
- ➢ below local atm. pressure, it is called - ve gauge pressure or vacuum or suction pressure.

- A schematic diagram showing relationship among the various pressures is shown in **Fig.**



Observations:

- $\mathbf{(p_A)_{abs} = (p_A + p_{atm})}$; $\mathbf{(p_B)_{abs} = (- p_B + p_{atm})}$
- If local atmospheric pressure datum is not given, then standard atmospheric pressure may be taken as datum.
- Standard atmospheric pressure has a value of 760 mm of mercury column or 10.33 m of water column measured from absolute zero pressure datum or 101.325 kN/m^2 or 1.013 bar *i.e.* $\mathbf{p_{atm}}$ = 101.325 kN/m^2(abs)

MEASUREMENT OF FLUID PRESSURE

- commonly measured bymanometers and mechanical gauges.
- ❖ Both devices measure only gauge pressures.

Manometers

- Fluid columns can be used to measure pressure (pressure varies with height).
- A device based on the principle of balancing a column of liquid by the same or another column of liquid is called manometer.
- Manometers are classified as simple and differential manometers:

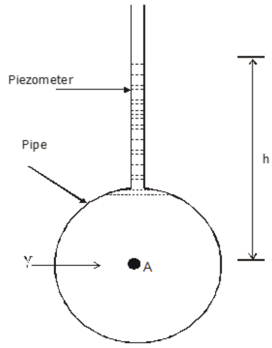
Simple manometers measure pressure at a point of a fluid contained in a pipe or vessel

Differential manometers measure difference of pressure between two points in the same pipe or different pipes.

- Some of the common types of manometers are
- ➢ Piezometer

- U-tube manometer
- Single column manometer

Piezometer is the simplest form of a manometer



Piezometer

- used for measuring moderate pressures of liquids which are greater than the atmospheric pressure.
- consists of a glass tube connected to the gauge point and the other end remains open to the atmosphere.
- tube is extended vertically upward to such a height that liquid can rise in it without overflowing.
- point of a pipe wall where manometer is connected is known as gauge point.

❖ Fluid pressure at any point **A** is given by, $\mathbf{p_A = \gamma h}$

Observations:

- Piezometer should not penetrate into the flow
- As intensity of pressure is same in all the directions (**PL**), location of gauge point makes no difference to the level of rise of liquid in the piezometer.

Limitations of piezometer

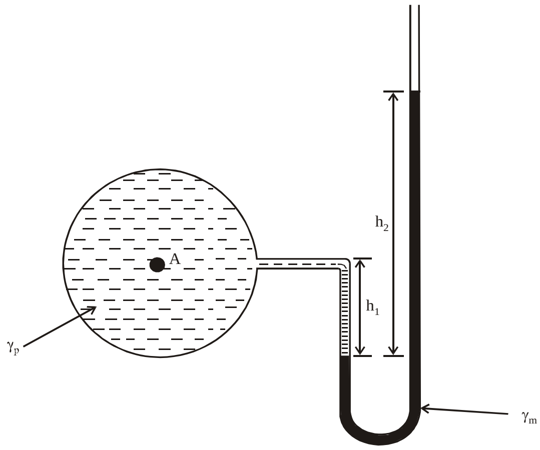
- Large pressures cannot be conveniently measured.
- Negative pressures and gas pressures cannot be measured.
- ❖ Limitations are overcome by using U-tube manometers.

U-tube manometeris the modified form of piezometer.

- consists of a glass tube bent in U-shape.
- one end of U-tube is connected to the gauge point and the other end remains open to the atmosphere.

Different cases for measuring pressures:

(a) Measurement of large positive pressure



- Manometer contains manometric liquid of greater specific weight (**γ_m**) as compared to the specific weight of liquid in pipe (γ_p) i.e. **$\gamma_m > \gamma_p$**.
- Point **A** is the centre of pipe where fluid pressure is to be measured.

❖ It is always convenient to write down the gauge equation keeping in mind the facts about the variation of fluid pressure i.e.

➢ Pressure decreases will increase in elevation (+ or – sign)

➢ on a horizontal plane in the same expanse of fluid, the fluid pressure remains constant.

❖ Gauge equation is generally written by starting from one end point (usually centre of pipe) and reaching to another end point *i.e.* level of liquid in the U-tube.

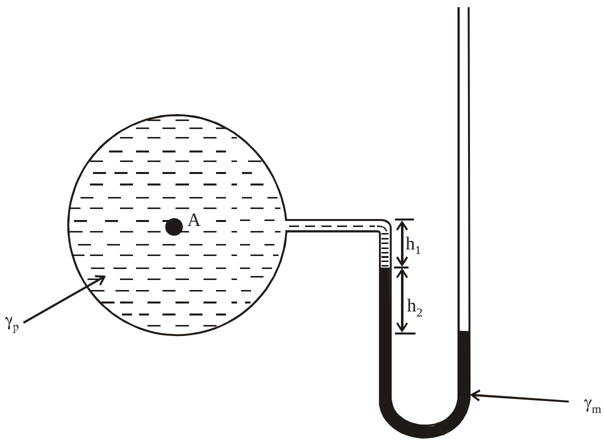
- Writing gauge equation, to get

$$\mathbf{p_A + \gamma_p h_1 - \gamma_m h_2 = 0 \therefore p_A = (\gamma_m h_2 - \gamma_p h_1)}$$

- Absolute pressure at **A** i.e. **$(p_A)_{abs}$** is given by

$$\mathbf{(p_A)_{abs} = (p_A + p_{atm})}$$

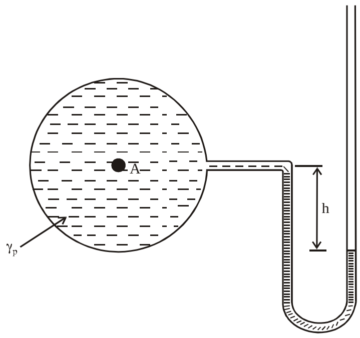
(b) Measurement of large -ve pressure (<p_{atm})



$$p_A + \gamma_p h_1 + \gamma_m h_2 = 0 \qquad \Rightarrow p_A = -(\gamma_p h_1 + \gamma_m h_2)$$

(c) Measurement of small +ve or -ve pressures

- Manometer may not contain any man uid or $\gamma_m<\gamma_p$

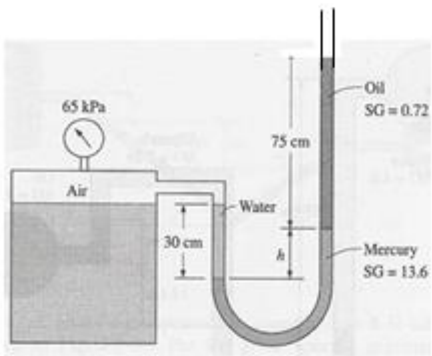


$$p_A + \gamma_p h = 0 \qquad \Rightarrow p_A = -\gamma_p h$$

(d) Measurement of <u>very small</u> +ve or -ve pressures

- Right limb of the manometer is inclined
- For the previous case, $p_A = -\gamma_p (h \sin\theta)$
  - θ is the angle made by right limb with horizontal
- vertical height of manometric liquid level is considered

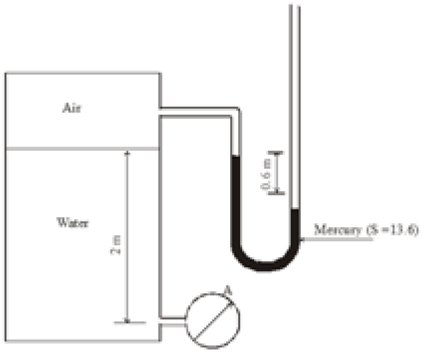
Problem 1: In the arrangement shown in **Fig,** find the value of **h.**



$$65 + 9.810 \times 0.3 - 13.6 \times 9.810 \times h - 0.72 \times 9.810 \times 0.75 = 0 \qquad \Rightarrow h = 0.47\text{m}$$

Problem 2:

For the system shown in **Fig.**, determine (i) Absolute and gauge pressures of air in the tank (ii) Gauge reading at **A**. Given, atmospheric pressure = 755 mm of Hg.



Solution:

Writing gauge equation for the manometer, to get

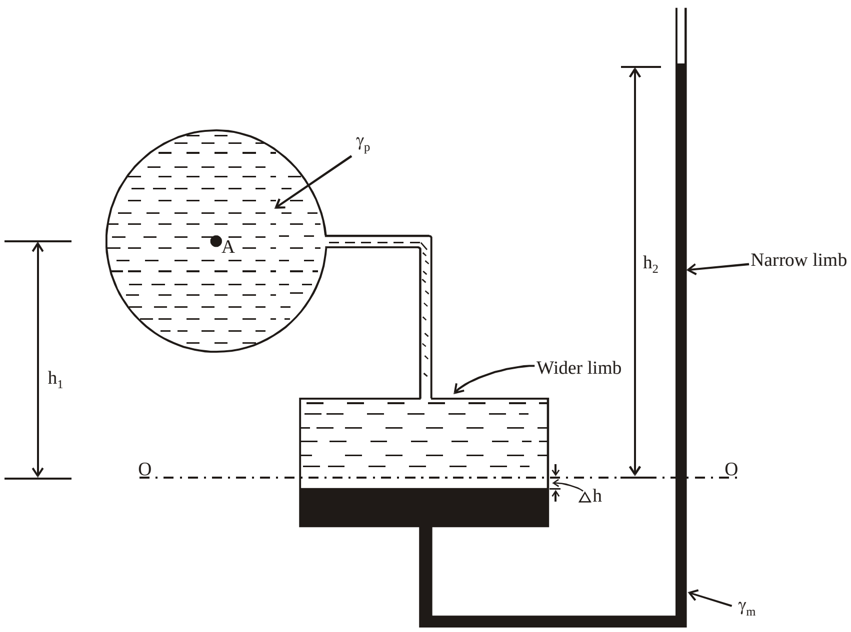
$$p_{air} + 13.6 \times 9.810 \times 0.6 = 0 \qquad \Rightarrow p_{air} = -80.05 \ \text{kN/m}^2 \ \textbf{(Vacuum pressure)}$$

$$\therefore (p_{air})_{abs} = -80.05 + 13.6 \times 9.810 \times 0.755 = 20.68 \ \text{kN/m}^2 \ \text{(abs)}$$

$$p_A - 9.810 \times 2 + 13.6 \times 9.810 \times 0.6 = 0 \qquad \Rightarrow p_A = -\ 60.43 \ \text{kN/m}^2$$

Single Column Manometer

- modified form of a U-tube manometer
- consists of a shallow reservoir having large area (known as a wider limb) as compared to the area of other limb (known as narrow limb)
- due to large area of wider limb, for any pressure in the pipe, change of liquid level in the wider limb would be so small that it may be neglected



Single column manometer

- Let **O-O** be the levels of manometric liquid in the wider and narrow limbs (will happen when wider limb is not connected to the gauge point of pipe or there is no liquid in pipe)
- When wider limb is connected to the gauge point of a pipe containing fluid at pressure greater than the **AP.**
  - Manometric liquid will fall by Δh in the wider limb and there will be a corresponding rise of level h_2 in the narrow limb.
- ❖ Conservation of volume and gauge equations are used
- By conservation of volume, $\Delta h\, A = h_2\, a$
  - A = area of wider limb and a = area of narrow limb

Gauge equation

$$p_A + \gamma_p(h_1 + \Delta h) - \gamma_m(h_2 + \Delta h) = 0 \qquad \therefore p_A = \gamma_m(h_2 + \Delta h) - (h_1 + \Delta h)\gamma_p$$

- Term $\gamma_p(h_1 + \Delta h)$represents pressure change in the wider limb due to pressure in the pipe
- Neglecting this pressure change as being small, $\therefore p_A = \gamma_m . h_2\left(1 + \frac{a}{A}\right)$
- As **A >> a**, $\therefore a/A \sim 0 \therefore p_A = \gamma_m h_2$
- ❖ Pressure is approximately indicated by the height of liquid in the narrow limb (single reading is required)known as single column manometer.

Differential Manometers

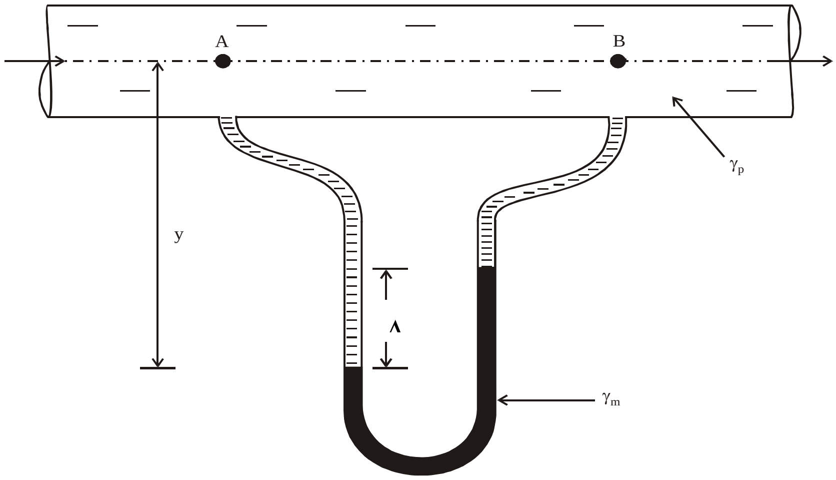
- common types of differential manometers
  - Two-piezometers manometer
  - U-tube differential manometer
  - Inverted U-tube manometer
  - Micromanometer

Two-piezometers manometer consists of two different piezometers connected at two gauge points

- Difference of levels of liquid in the two piezometers gives the differential pressure head between the two points.
- ❖ Used for measuring small pressure difference.

U-tube differential manometer consists of a U-tube containing a heavier liquid whose two ends are connected to the points where difference of pressure is be measured

- **Figure** shows an arrangement for measuring the pressure difference between two points in the same pipe due to flow in the pipe.



Differential manometer (Points in the same pipe)

$$p_A + \gamma_p y - \gamma_m \Delta x - \gamma_p (y - \Delta x) = p_B \qquad \therefore (p_A - p_B) = \Delta x(\gamma_m - \gamma_p)$$

$$\Rightarrow \left(\frac{p_A - p_B}{\gamma_p}\right) = \Delta x\left(\frac{\gamma_m}{\gamma_p} - 1\right) = \Delta x\left(\frac{S_m}{S_p} - 1\right)$$

$$\therefore \Delta h = \Delta x\left(\frac{S_m}{S_p} - 1\right), \qquad \text{where } \Delta h = \left(\frac{p_A - p_B}{\gamma_p}\right)$$

- **Δh** is known as differential pressure head.

If manometric liquid is mercury and liquid in pipe is water (mercury-water manometer), i.e. $\gamma_p = \gamma_w$ and $\gamma_m = \gamma_{Hg}$, then

$$\therefore \left(\frac{p_A - p_B}{\gamma_w}\right) = \Delta x\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) = \Delta x\left(\frac{S_{Hg}}{S_w} - 1\right)$$

$$\therefore \Delta h = \Delta x\left(\frac{S_{Hg}}{S_w} - 1\right)$$

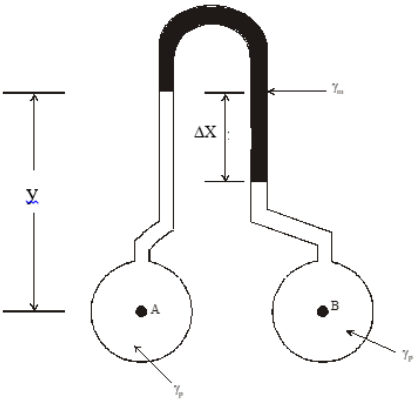
❖ S_{Hg} = 13.6 and $S_w = 1$

$$\Rightarrow \Delta h = \Delta x \times 12.6$$

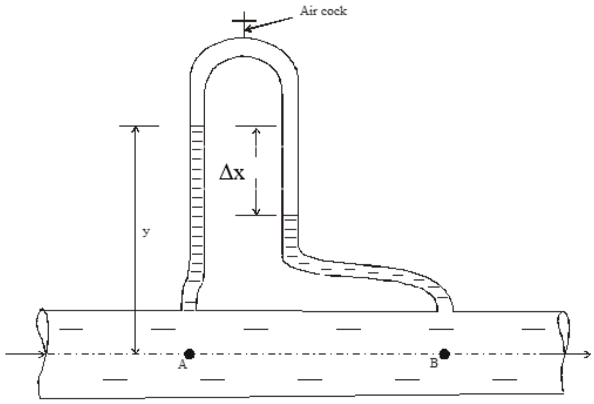
Inverted U-tube manometer

- consists of a **U**-tube held in the inverted position and connected to the gauge points
- used for measuring small pressure difference
- liquid in the manometer is lighter than the liquid in pipe

- **Figures (a) and (b)** show two different arrangements for measuring pressure



(a) **Two pipes at same elevation**



(a) Two points in the same pipe (No manometric liquid)

Gauge equation for **Figure (a)**

$$p_A - \gamma_p y + \gamma_m \Delta x + \gamma_p (y - \Delta x) = p_B \qquad \therefore p_A - p_B = \Delta x(\gamma_p - \gamma_m)$$

$$\Rightarrow \left(\frac{p_A - p_B}{\gamma_p}\right) = \Delta x\left(1 - \frac{\gamma_m}{\gamma_p}\right) = \Delta x\left(1 - \frac{S_m}{S_p}\right)$$

$$\therefore \Delta h = \Delta x\left(1 - \frac{S_m}{S_p}\right), \qquad \Delta h = \left(\frac{p_A - p_B}{\gamma_p}\right)$$

- For oil-water manometer, $\gamma_p = \gamma_w$ and $\gamma_m = \gamma_{oil}$

$$\therefore \left(\frac{p_A - p_B}{\gamma_w}\right) = \Delta x\left(1 - \frac{\gamma_{oil}}{\gamma_w}\right) = \Delta x\left(1 - \frac{S_{oil}}{S_w}\right)$$

$$\therefore \Delta h = \Delta x\left(1 - \frac{S_{oil}}{S_w}\right)$$