Tutorial Sheet-5
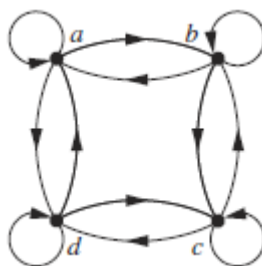
1. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations?
   a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
   b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
   c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
   d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
   e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

2. Determine whether the relation with the directed graph is an equivalence relation or not?

3. Determine whether the relations represented by the zero–one matrices is equivalence relation

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Which of these relations on $\{0, 1, 2, 3\}$ are partial orderings?
   a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
   b) $\{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
   c) $\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}$
   d) $\{(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
   e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0),(2, 2), (3, 3)\}$

5. Draw the Hasse diagram for divisibility on the set
   a) $\{1, 2, 3, 4, 5, 6, 7, 8\}$.
   b) $\{1, 2, 3, 5, 7, 11, 13\}$.
   c) $\{1, 2, 3, 6, 12, 24, 36, 48\}$.
   d) $\{1, 2, 4, 8, 16, 32, 64\}$.

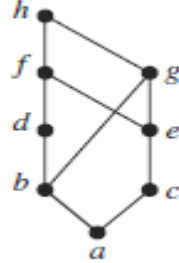6. Answer these questions for the poset $(\{3, 5, 9, 15, 24, 45\}, |)$.
   a) Find the maximal elements.
   b) Find the minimal elements.
   c) Is there a greatest element?
   d) Is there a least element?
   e) Find all upper bounds of $\{3, 5\}$.
   f) Find the least upper bound of $\{3, 5\}$, if it exists.
   g) Find all lower bounds of $\{15, 45\}$.
   h) Find the greatest lower bound of $\{15, 45\}$, if it exists.

7. Draw the Hasse diagram for inclusion on the set P(S), where S = {a, b, c, d}.

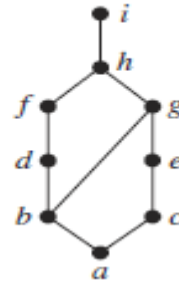8. Determine whether the posets with these Hasse diagrams are lattices
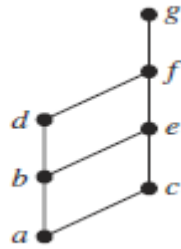
a)   **b)**   **c)**

9. Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆).
   a) Find the maximal elements.
   b) Find the minimal elements.
   c) Is there a greatest element?
   d) Is there a least element?
   e) Find all upper bounds of {{2}, {4}}.
   f) Find the least upper bound of {{2}, {4}}, if it exists.
   g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
   h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.