PAPER · 1

5/12/16

Roll No. : ....

School of Mathematics, Thapar University, Patiala

End-Semester Examination, December 2016

B.E. III Semester | UMA007 : Numerical Analysis

Time Limit: 03 Hours | Maximum Marks: 100

Instructor(s): Arvind Kumar Lal, Navdeep Kailey, Paramjeet Singh, Raj Nandkeolyar, Rajvir Singh, Sanjeev Kumar, Sapna Sharma

Instructions: You are expected to answer all the questions. Organize your work, in a reasonably neat and coherent way. Mysterious or unsupported answers will not receive full credit. Calculator without graphing mode and alphanumeric memory is permitted.

1. (a) Use four-digit rounding arithmetic and the formula for the roots of a quadratic equation, to find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute and relative errors.

$$1.002x^2 + 11.01x + 0.01265 = 0.$$

[10 marks]

(b) Let $f \in C^2[a,b]$. If α is a simple root of $f(x) = 0$ and $f'(\alpha) \neq 0$, then prove that the Newton's method generates a sequence $\{x_n\}$ converging quadratically to root α for any initial approximation x_0 near to α. [10 marks]

2. (a) In Neville's method, suppose $x_i = i$, for $i = 0, 1, 2, 3$ and it is known that $P_{0,1}(x) = x+1$, $P_{1,2}(x) = 3x-1$, and $P_{1,2,3}(1.5) = 4$. Find $P_{2,3}(1.5)$ and $P_{0,1,2,3}(1.5)$. [7 marks]

(b) Show that the cubic polynomials

$$P(x) = 3 - 2(x+1) + 0(x+1)(x) + (x+1)(x)(x-1)$$

and

$$Q(x) = -1 + 4(x+2) - 3(x+2)(x+1) + (x+2)(x+1)(x)$$

both interpolate the given data. Why does this not violate the uniqueness property of interpolating

x	-2	-1	0	1	2
$f(x)$	-1	3	1	-1	3

polynomials? [7 marks]

(c) Let $f(x) = x^n$ for some integer $n \geq 0$. Let $x_0, x_1, \cdots, x_m$ be $m+1$ distinct numbers. What is $f[x_0, x_1, \cdots, x_m]$ for $m = n$? For $m > n$? [6 marks]

3. (a) We require to solve the following system of linear equations using LU decomposition.

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 3 \\ 3x_1 + 8x_2 + 14x_3 &= 13 \\ 2x_1 + 6x_2 + 13x_3 &= 4. \end{aligned}$$

(i) Find the matrices L and U using Gauss elimination.

(ii) Using those values of L and U, solve the system of equations. [12 marks]

(b) The linear system $Ax = b$ given by

$$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}$$

has solution $(1,1)^t$. Use five-digit rounding arithmetic to find the solution of the perturbed system

$$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}.$$

Is matrix A ill-conditioned? [8 marks]

CONTINUED

4. (a) Determine the values of subintervals n and step-size h required to approximate

$$\int_0^2 \frac{1}{x+4}\, dx$$

to within 10^{-5} and hence compute the approximation using composite Simpson's rule. [10 marks]

(b) Find the degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)dx = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

and then derive the formula for

$$\int_a^b f(x)dx.$$

[10 marks].

5. (a) Show that the following initial-value problem has a unique solution.

$$y' = t^{-2}\left(\sin 2t - 2ty\right), \quad 1 \le t \le 2, \ y(1) = 2.$$

Find $y(0.1)$ and $y(0.2)$ with step-size $h = 0.1$ using modified Euler's method.

[10 marks]

(b) Consider the following Lotka-Volterra system in which u is the number of prey and v is the number of predators.

$$\begin{aligned} \frac{du}{dt} &= 2u - uv, \quad u(0) = 1.5 \\ \frac{dv}{dt} &= -9v + 3uv, \quad v(0) = 1.5. \end{aligned}$$

Use the fourth-order Runge-Kutta method with step-size $h = 0.2$ to approximate the solution at $t = 0.2$.

[10 marks]