LECTURE-17

UEI407

Compute the convolution of $x(n) = \{1,2,\underset{\uparrow}{0},2,1\}$ and $h(n) = \{1,2,\underset{\uparrow}{3},4,1\}$ using tabulation method.

h(n) → x(n) ↓	1	2	3	4	1
1	1	2	3	4	1
2	2	4	6	8	2
0	0	0	0	0	0
2	2	4	6	8	2
1	1	2	3	4	1

$$\therefore\ y(n) = \{1,4,7,12,\underset{\uparrow}{14},10,11,6,1\}$$

The impulse response of a linear time invariant system is .
$h(n) = \{1, \underset{\uparrow}{3}, 1, -1\}$. Obtain the response of the system due to the input signal $x(n) = \{\underset{\uparrow}{1}, 3, 2, 1\}$ using the method of multiplication.

The output y(n) is the convolution of x(n) and h(n).

$$\therefore y(n) = x(n) * h(n)$$

There is one sample before $\uparrow$ in h(n) and no sample before $\uparrow$ in x(n). The total number of samples before $\uparrow$ in x(n)*h(n) is 1 + 0 i.e., 1. Hence there will be one sample before $\uparrow$ in y(n). The result of convolution is shown below.

$$\begin{array}{lcccc} & & \downarrow & & \\ h(n) \Rightarrow & 1 & 3 & 1 & -1 \\ x(n) \Rightarrow & 1 & 3 & 2 & 1 \\ & \uparrow & & & \end{array}$$

			$1\times1=1$	$1\times3=3$	$1\times1=1$	$1\times-1=-1$
		$2\times1=2$	$2\times3=6$	$2\times1=2$	$2\times-1=-2$	$\times$
	$3\times1=3$	$3\times3=9$	$3\times1=3$	$3\times-1=-3$	$\times$	$\times$
$1\times1=1$	$1\times3=3$	$1\times1=1$	$1\times-1=-1$	$\times$	$\times$	$\times$
1	$\underset{\uparrow}{6}$	12	9	2	-1	-1

$$\therefore\ y(n)=\{1,\ \underset{\uparrow}{6},\ 12,\ 9,\ 2,\ -1,\ -1\}$$

Properties of Convolution

1. Commutative Property for Convolution: The linear convolution is expressed by $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$$x(n) * h(n) = h(n) * x(n) = y(n)$$

2. Associative Property :The associative property of linear convolution states that $[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$

3. Distributive Property of Convolution: The distributive property satisfied by the linear convolution states that

$$y(n) = x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

Correlation

This technique is always computed for two sequences and it is very much similar to convolution. For digital signal extraction correlation technique has handy usage. These applications are digital communication systems, radars, spectrum communication and mobile communications etc. where the incoming and received signals are compared with the standard set of signals. The signal from the set having maximum correlation with incoming/received signal is selected. This is the prime application of correlation.

Cross–correlation and Auto–correlation

There are two types of correlation. One is called cross–correlation and another is called auto–correlation. The correlation is said to be cross–correlation if correlation of two different sequences x(n) and y(n) is done. On the other hand, if sequences are same, the correlation is said to be auto–correlation. The cross–correlation is denoted by $r_{xy}(l)$ and expressed by

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \qquad , l = 0, \pm 1, \pm 2, \ldots\ldots\ldots \qquad (1)$$

i.e. $$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)y(n) \qquad , l = 0, \pm 1, \pm 2, \ldots\ldots\ldots \qquad (2)$$

In Eq. (1) y(n) is delayed with respect to x(n) whereas in Eq. (2) x(n) is advanced with respect to y(n). These two operations are equivalent and provide identical cross–correlation sequences. The cross–correlation sequence $r_{yx}(l)$ is defined below.

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)x(n-l) \qquad , l = 0, \pm 1, \pm 2, \ldots\ldots\ldots \qquad (3)$$

i.e., $$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n+l)x(n) \qquad l = 0, \pm1, \pm2, \ldots \tag{4}$$

The auto−correlation x(n) denoted by $r_{xx}(l)$ is defined as follow:

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l) \qquad l = 0, \pm1, \pm2, \ldots \tag{5}$$

i.e., $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n+l)x(n) \qquad l = 0, \pm1, \pm2, \ldots \tag{6}$$

Here $r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$

$$= \sum_{n=-\infty}^{\infty} x(n)y(-l+n)$$

$$= \sum_{n=-\infty}^{\infty} x(n)y(-(l-n))$$

$$= x(l) * y(-l)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(l) = \sum_{k=-\infty}^{\infty} x(k)\, h(l-k)$$

Obtain the cross correlation sequence $r_{xy}(l)$ of the following sequences: $x(n) = \{3, -2, 4, 6, \underset{\uparrow}{1}, 3, -5\}$ and $y(n) = \{2, -2, 3, -3, \underset{\uparrow}{5}, 1, -3, 6\}$

The sequences are

$$
\begin{array}{ll}
x(-4) = 3 & y(-4) = 2 \\
x(-3) = -2 & y(-3) = -2 \\
x(-2) = 4 & y(-2) = 3 \\
x(-1) = 6 & y(-1) = -3 \\
x(0) = 1 \leftarrow & y(0) = 5 \leftarrow \\
x(1) = 3 & y(1) = 1 \\
x(2) = -5 & y(2) = -3 \\
 & y(3) = 6
\end{array}
$$

The correlation of x(n) and y(n) can be expressed as

$$r_{xy}(l) = x(l) * y(-l)$$

Here $x(l) = \{3, -2, 4, 6, \underset{\uparrow}{1}, 3, -5\}$ and $y(l) = \{2, -2, 3, -3, \underset{\uparrow}{5}, 1, -3, 6\}$

$$\therefore y(-l) = \{6, -3, 1, \underset{\uparrow}{5}, -3, 3, -2, 2\}$$

$x(l) \Rightarrow$								3	−2	4	6	1	3	−5
												$\uparrow$		
$y(-l) \Rightarrow$							6	−3	1	5	−3	3	−2	2
										$\uparrow$				
								6	−4	8	12	2	6	−10
							−6	4	−8	−12	−2	−6	10	×
						9	−6	12	18	3	9	−15	×	×
					−9	6	−12	−18	−3	−9	15	×	×	×
				15	−10	20	30	5	15	−25	×	×	×	×
			3	−2	4	6	1	3	−5	×	×	×	×	×
		−9	6	−12	−18	−3	−9	15	×	×	×	×	×	×
	18	−12	24	36	6	18	−30	×	×	×	×	×	×	×
	18	−21	33	37	−27	56	−32	27	13	−35	34	−19	16	−10
								$\uparrow$						

$$\therefore r_{xy}(l) = \{18, -21, 33, 37, -27, 56, -32, \underset{\uparrow}{27}, 13, -35, 34, -19, 16, -10\}$$

Properties of cross-correlation

1. The cross–correlation is not commutative.
2. The auto–correlation is an even function.
3. The cross–correlation is equivalent to convolution of one sequence with another folded sequence.
4. The cross–correlation satisfies the following:
$$\left|r_{xy}(l)\right| \le \sqrt{r_{xx}(0)r_{yy}(0)} = \sqrt{E_x E_y}$$
5. The auto–correlation sequence attains maximum value at zero lag ($l = 0$), i.e., $\left|r_{xx}(l)\right| \le r_{xx}(0) = E_x$