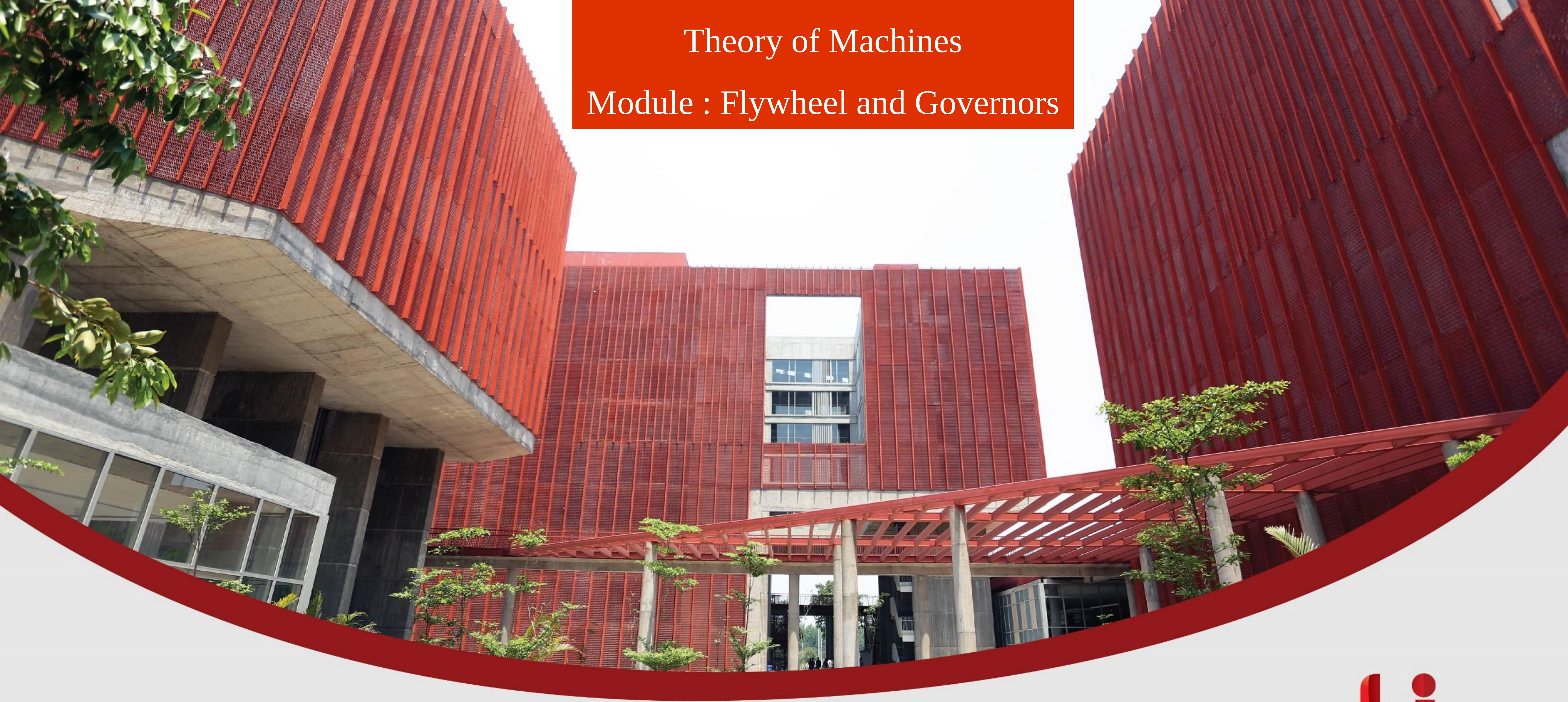

Theory of Machines

Module : Flywheel and Governors

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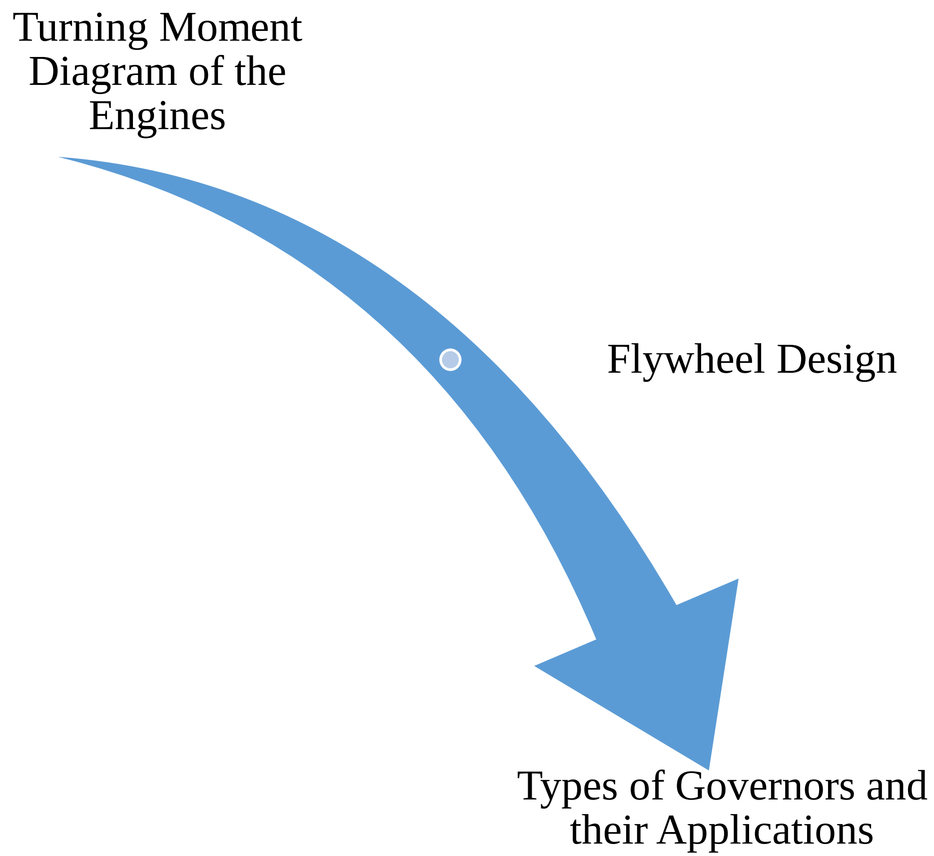
Lecture Contents and Learning Outcomes

- Turning moment diagram of the engines
- Flywheel design
- Types of governors and their applications

Learning Outcomes



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3. Neil Sclater, Nicholas P. Chironis "Mechanisms and Mechanical Devices Sourcebook" 4th Edition, McGraw Hill Publications
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Turning Moment Diagrams

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- During one revolution of the crankshaft of a steam or IC engine, the torque on it varies and is given by

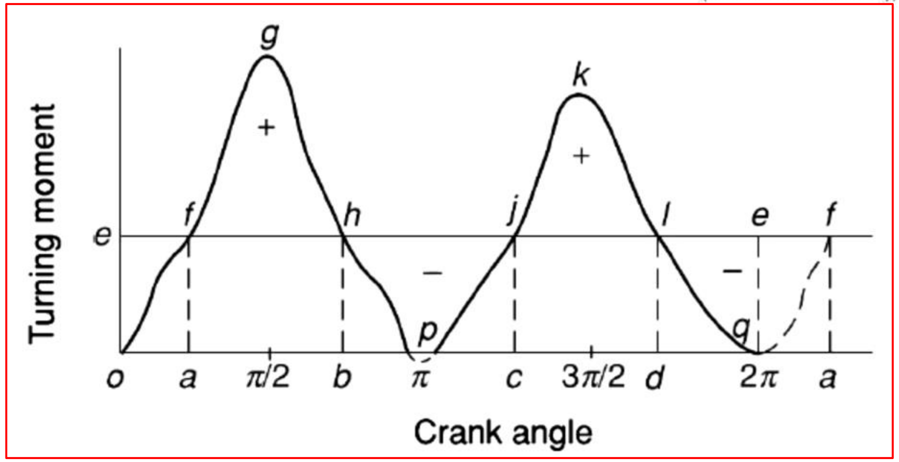
$$T = F_t \times r \;= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

where F is the net piston effort

- A plot of T vs θ is known as the turning-moment diagram.
- The inertia effect of the connecting rod is, usually ignored while drawing these diagrams, but can be taken into account if required.
- As $T = F_t \times r$, a plot of F_t vs θ is (known as crank effort diagram) is identical to a turning-moment diagram.

Single Cylinder Double-acting Steam Engine

- Turning Moment Diagram



- The mean torque against which the engine works is given by

$$oe = \frac{\text{Area } ogpkp}{2\pi}$$

where *oe* is the mean torque and it is mean height of the turning moment diagram

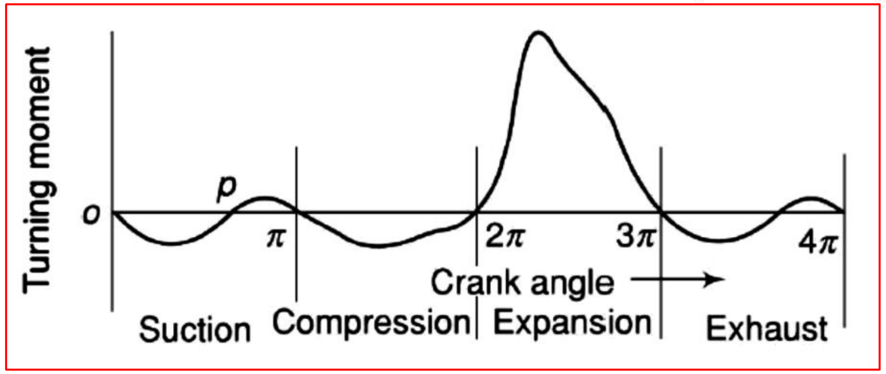
- When the crank turns from the angle *oa* to *ob*, the work done by the engine is represented by the area *afghb*.
- But the work done against the resisting torque is represented by the area *afhb*. Thus, the engine has done more work than what has been taken from it.
- The excess work is represented by the area *fgh*. This excess work increases the speed of the engine and is stored in the flywheel.
- During the crank travel from *ob* or *oc*, the work needed for the external resistance is proportional to *bhjc* whereas the work produced by the engine is represented by the area under *hpj*.

Single Cylinder Double-acting Steam Engine

- During this period, more work has been taken from the engine that is produced. The loss is made up by the flywheel which gives up some of its energy and the speed decreases during this period.
- Similarly, during the period of crank travel from *oc* to *od*, excess work is again developed and is stored in the flywheel and the speed of the engine increases. During the crank travel from *od* to *oa*, the loss of work is made up by the flywheel and the speed again decreases.
- The areas *fgh*, *hpj*, *jkl* and *lqf* represent fluctuations of energy of the flywheel. When the crank is at *b*, the flywheel has absorbed energy while the crank has moved from *a* to *b* and thereby, the speed of the engine is maximum.
- At *c*, the flywheel has given out energy while the crank has moved from *b* to *c* and thus the engine has minimum speed. Similarly, the engine speed is again maximum at *d* and minimum at *a*. Thus, there are two maximum and two minimum speeds for the turning-moment diagram.
- The greatest speed is the greater of the two maximum speeds and the least speed is the lesser of the two minimum speeds. Difference between the greatest and the least speeds of the engine over one revolution is known as the fluctuation of speed.

Single Cylinder Four Stroke Engine

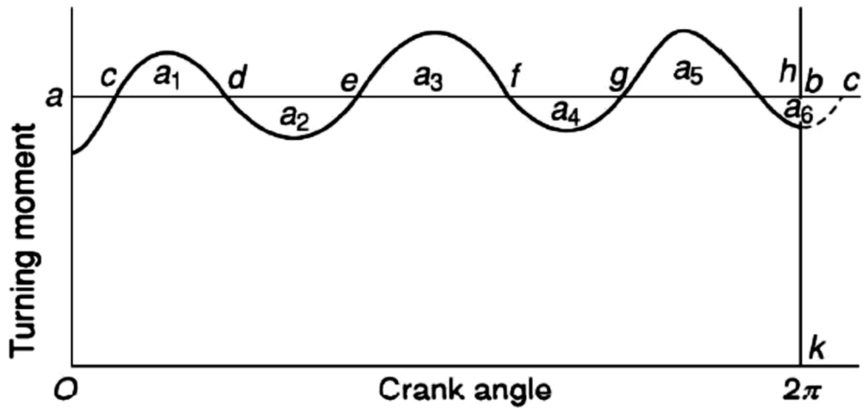
- In 4-stroke IC engine, the diagram repeats itself after every two revolutions instead of one revolution as for a steam engine.
- For majority of the suction stroke, the turning moment is negative but becomes positive after the point *p*.



- During the compression stroke, it is totally negative.
- It is positive throughout the expansion stroke and again negative for most of the exhaust stroke.

Multi Cylinder Engine

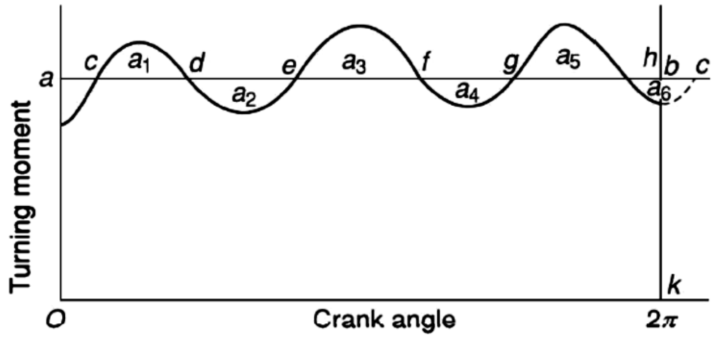
- Turning moment diagram for a single-cylinder engine varies considerably and a greater variation of the same is observed in case of four-stroke, single-cylinder engine.
- For multi cylinder engine, the total torque at any instant is given by the sum of the torques developed by each cylinder at the instant.



- If an engine has two cylinders with crank at 90˚, the resultant turning moment diagram has a less variation than that for a single cylinder. In a three-cylinder engine having its cranks at 120˚, the variation still less.
- The mean torque line *ab* intersects the turning moment curve at *c*, *d*, *e*, *f*, *g* and *h*. The area under wavy curve is equal to the area *oabk*.
- The speed of the engine will be maximum when the crank positions correspond to *d*, *f* and *h*, and minimum corresponding to *c*, *e* and *g*.

Fluctuation of Energy

- Let a_1, a_3, and a_5 be the areas in work units of the portions above the mean torque *ab* of the turning-moment diagram. These areas represent quantities of energies added to the flywheel. Similarly, areas a_2, a_4 and a_6 below ab represent quantities of energies taken from flywheel.
- The energies of the flywheel corresponding to crank positions are as follows



Crank Position	Flywheel Energy
c	*E*
d	$E + a_1$
e	$E + a_1 - a_2$
f	$E + a_1 - a_2 + a_3$
g	$E + a_1 - a_2 + a_3 - a_4$
h	$E + a_1 - a_2 + a_3 - a_4 + a_5$
c	$E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

- From the two values of the energies of the flywheel corresponding to the position *c*, it is concluded that $a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = 0$
- Greatest of these energies is the maximum kinetic energy and for the corresponding crank position speed is *maximum*.
- Least of these energies is the least kinetic energy and for the corresponding crank position speed is *minimum*.
- Difference between max and min KEs is known as the *maximum fluctuation of energy* whereas the ratio of the maximum fluctuation of energy to the work done per cycle is defined as the *coefficient of fluctuation of energy*.
- Difference between the greatest speed and the least speed is known as the *maximum fluctuation of speed* and the ratio of the maximum fluctuation of speed to the mean speed is the *coefficient of fluctuation of speed*.

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Flywheels

- A flywheel of suitable dimensions attached to the crankshaft, makes the moment of inertia of the rotating parts quite large and thus, acts as a reservoir of energy.
- During the periods when the supply of energy is more than required, it stores energy and during the periods the requirements is more than the supply, it releases energy.

K = Coefficient of fluctuation of speed = $\dfrac{\omega_1 - \omega_2}{\omega}$ where ω_1, ω_2, ω are max, min and mean speed

Maximum fluctuation of energy = $e = \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2 = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = I\left(\frac{\omega_1 + \omega_2}{2}\right)(\omega_1 - \omega_2)$

$$= I\omega(\omega_1 - \omega_2)$$

$$= I\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) = I\omega^2 K$$

OR

$$K = \frac{e}{I\omega^2} = \frac{e}{2 \times \frac{1}{2} I\omega^2} = \frac{e}{2E}$$

where E is kinetic energy of the flywheel at mean speed

Worked example 1

A flywheel with a mass of 3 *kN* has radius of gyration of 1.6 *m*. Find the energy stored in the flywheel when its speed increases from 315 rpm to 340 rpm.

$$= \frac{1}{2} I(\omega_1^2 - \omega_2^2)$$

$$= 684\ 900 \text{ N.m or } 684.9 \text{ kN.m}$$

or $\quad 684.9$ kJ

Worked example 2

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A flywheel absorbs 24 *kJ* of energy on increasing its speed of 210 rpm to 214 rpm. Determine its kinetic energy at 250 rpm.

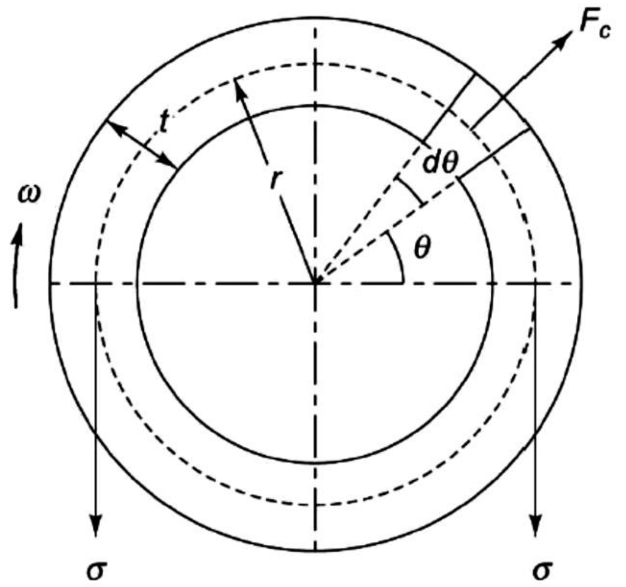
$$24\,000 = \frac{1}{2} I\,(\omega_1^2 - \omega_2^2)$$

$$E = \frac{1}{2} I\omega^2$$

Dimensions of Flywheel Rims

- The inertia of a flywheel is provided by the hub, spokes and the rim. As the inertia due to hub and spokes is very small, usually it is ignored.
- Centrifugal force on the element / unit length $= [\rho(r.d\theta)t].r\omega^2$

  where ω = angular velocity, r = mean radius, t = thickness of rim, ρ = density of the material of rim



- Total vertical force / unit length $= \int_0^{\pi} \rho.r^2.d\theta.t.\omega^2 \sin\theta = \rho.r^2.t.\omega^2 \int_0^{\pi} \sin\theta.d\theta$

$$= \rho.r^2.t.\omega^2(-\cos\theta)\,_0^{\pi} = 2\rho.r^2.t.\omega^2$$

- Let σ = circumferential stress/hoop stress induced in the rim. Then for equilibrium

$$\sigma(2t).1 = 2\rho.r^2.t.\omega^2 \Rightarrow \boxed{\sigma = \rho.r^2\omega^2 = \rho.v^2}$$

- The above relation provided the limiting tangential velocity at the mean radius of the rim of the flywheel. The diameter can be calculated from the relation $v = \pi dN/60.$
- Also, $m = \rho.\pi d.b.t$ The relation can be used to find the width and the thickness of the rim.

Worked example 3

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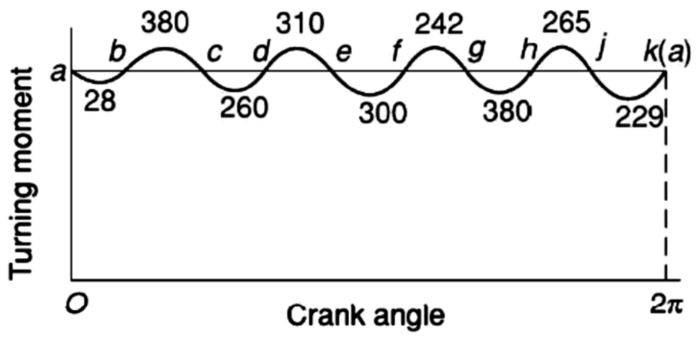
The turning moment diagram for a multicylinder engine has been drawn to a vertical scale of 1 mm = 650 *N.m* and a horizontal scale of 1 mm = 4.5°. The areas above and below the mean torque line are -28, +380, -260, +310, -300, +242, -380, +265 and -229 mm^2. The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m^3 and width of the rim is 4.5 times its thickness. The centrifugal stress (hoop stress) in the rim material is limited to 6 N/mm^2. Neglecting the effect of the boss and arms, determine the diameter and cross section of the flywheel rim.

$$\sigma = \rho v^2 \qquad \frac{\pi d n}{60}$$

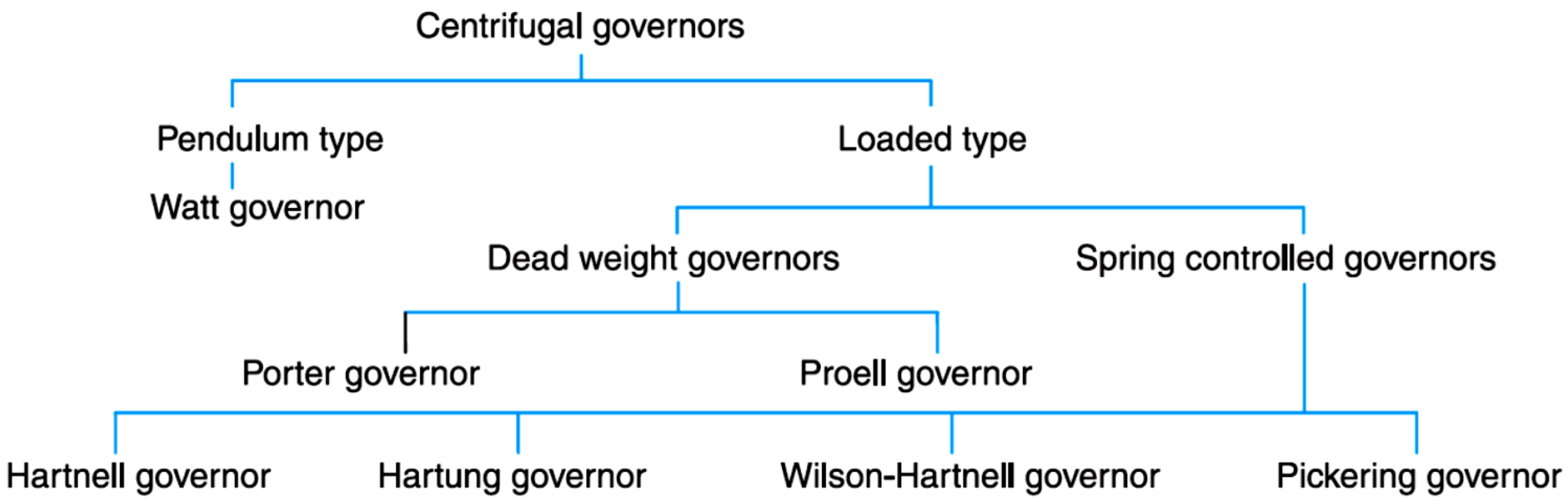
$$e_{max} = (E+402) - (E-36) \times \text{hor. scale} \times \text{vert. scale}$$

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

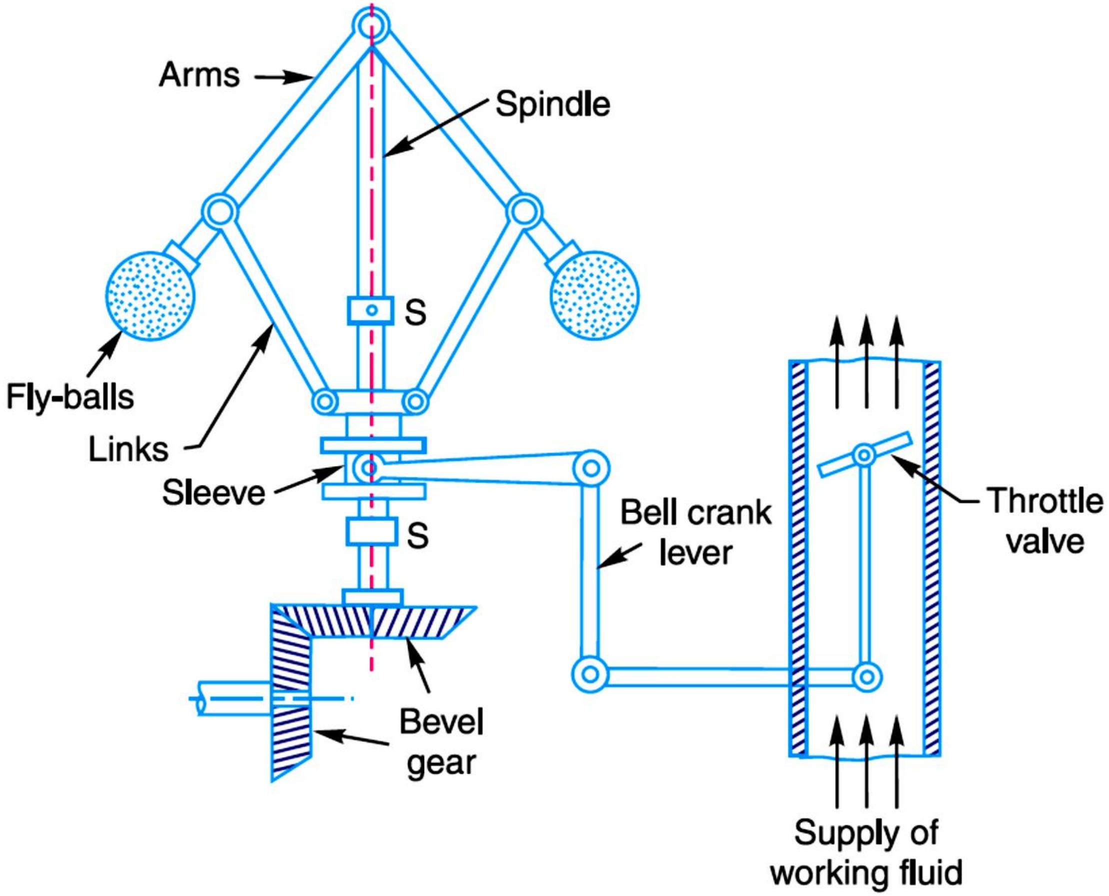


Governors

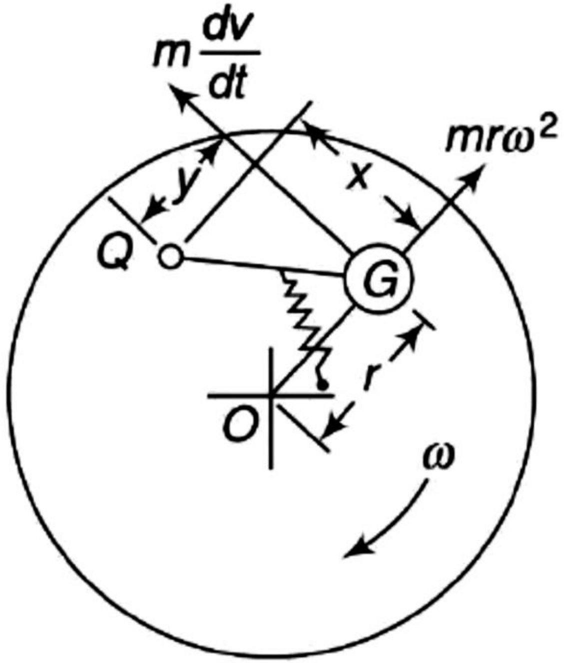
- In contrast to the flywheel, however, governors are used to regulate speed over a much longer interval of time; in fact, they are intended to maintain a balance between the energy supplied to a moving system and the external load or resistance applied to that system.
- Broadly, governors are of two types: Centrifugal Governor and Inertia Governor
- Centrifugal force plays the important role in centrifugal governors.
- In inertia governors it is more the angular acceleration, or change in speed, that dominates the regulating action.
- Compared to the centrifugal governor, the inertia type is more sensitive because it acts at the very beginning of a speed change



Centrifugal Governors



Inertia Governors



Thank You

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