

Que:9

Compute the planar density for the BCC (100), (111) and (110) planes in terms of atomic radius r.



THE PLANAR DENSITY  $\rho_P = \frac{N_e}{A}$ , WHERE  $N_e$  IS THE NO. OF EFFECTIVE ATOMS IN THE PLANE AND A IS THE AREA OF THE PLANE.

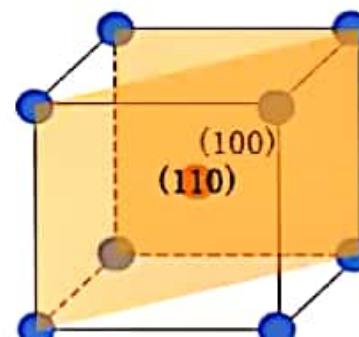
(100)

$$N_e = \frac{1}{4} \times 4 = 1$$

*Area of the plane* =  $a^2$

$$\rho_P = \frac{1}{a^2} = \frac{1}{(\frac{4r}{\sqrt{3}})^2}$$

$$= \frac{3}{16r^2}$$



(110)

$$N_e = \frac{1}{4} \times 4 + 1 = 2$$

*Area of the plane* =  $\sqrt{2}a^2$

$$\rho_P = \frac{2}{\sqrt{2}a^2} = \frac{2}{\sqrt{2}(\frac{4r}{\sqrt{3}})^2}$$

$$= \frac{3}{8\sqrt{2}r^2}$$

(111) plane

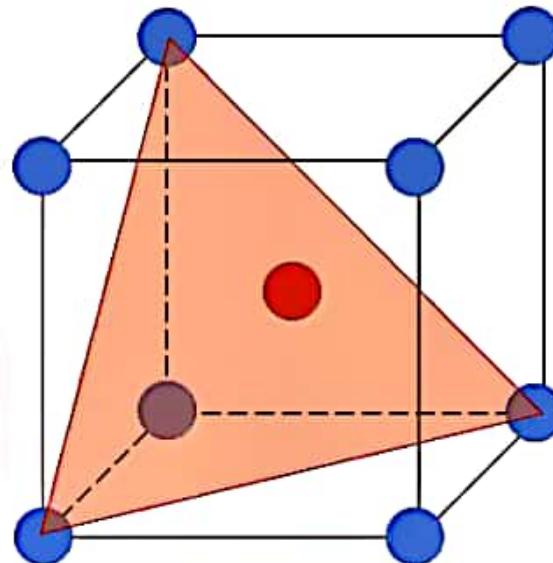
$$N_e = \frac{1}{6} \times 3 = \frac{1}{2}$$

$$\text{Area of the plane} = \frac{\sqrt{3}(\sqrt{2}a)^2}{4}$$

$$= \frac{\sqrt{3}}{2} a^2$$

$$\rho_P = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} a^2} = \frac{1}{\sqrt{3}(\frac{4r}{\sqrt{3}})^2}$$

$$= \frac{\sqrt{3}}{16r^2}$$



**Que:10**

Calculate the planar density for (110) plane of BCC iron lattice in atoms per square millimeter. The lattice constant of iron is 0.287 nm.

Que:11

From an X - Ray powder diffraction of a pure element, peaks at the following  $2\theta$  values in degrees were obtained 38.7, 45.4, 65.7, 78.8, 83.0, 99.6, 112.5, 117.0, 138.1, and 164.2. Copper K $\alpha$  radiation was used. Find the lattice parameter and the crystal structure.

2 0	0	$\sin \theta$	$\sin^2\theta$	$A = \sin^2\theta/\sin^2\theta_1$	A*2	A*3
38.7	19.35	0.331	0.110	0.996	2	3
45.4	22.7	0.386	0.149	1.355	2.709	$4.064 \approx 4$
65.7	32.85	0.542	0.294	2.671	5.341	$8.012 \approx 8$
78.8	39.4	0.635	0.403	3.666	7.331	$10.997 \approx 11$
83	41.5	0.663	0.440	3.996	7.992	$11.988 \approx 12$
99.6	49.8	0.764	0.584	5.306	10.613	$15.919 \approx 16$
112.5	56.25	0.832	0.692	6.293	12.586	$18.879 \approx 19$
117	58.5	0.853	0.728	6.615	13.229	$19.844 \approx 20$
138.1	69.05	0.934	0.872	7.931	15.861	$23.792 \approx 24$
164.2	82.1	0.991	0.982	8.928	17.856	$26.784 \approx 27$

From the above calculations we found that the crystal structure is FCC.

- For  $\theta = 19.35$ ,

$$(hkl) = (111)$$

We know

$$2d \sin\theta = n\lambda$$

Assuming  $n=1$  and  $\lambda=1.54 \text{ \AA}$  (for Cu K<sub>α</sub> radiation)

$$2 \times d \sin 19.35 = 1.54$$

$$d = 2.32 \text{ \AA}$$

The lattice parameter 'a' can be calculated using the equation

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$2.32 = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$a = 4.026 \text{ \AA}$$

**Que:12**

A BCC crystal is used to measure the wavelength of some X - rays. The Bragg angle for reflection from (110) plane is  $20.2^\circ$  . What is the wavelength? The lattice parameter of the crystal is 3.15 Å.

✓ BRAGG'S LAW IS GIVEN AS:

$$2d \sin\theta = n\lambda$$

✓ INTERPLANAR DISTANCE "d" IS

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

✓  $a = 3.15 \text{ \AA}$ ,  $h=1$ ,  $k=1$ ,  $l=0$  FOR (110)

$$\begin{aligned} d &= \frac{3.15}{\sqrt{1^2 + 1^2 + 0^2}} \\ &= 2.227 \text{ \AA} \end{aligned}$$

✓ TAKING  $\theta = 20.2^\circ$ ,  $n=1$ , FROM BRAGG'S LAW

$$2 \times 2.227 \times \sin 20.2 = \lambda$$

$$\lambda = 1.5236 \text{ \AA}$$

Que:13

Determine the Miller indices of cubic crystal plane that intersects the position coordinates  $(1, 1/4, 0)$ ,  $(1, 1, 1/2)$ , and  $(3/4, 1, 1/4)$ .

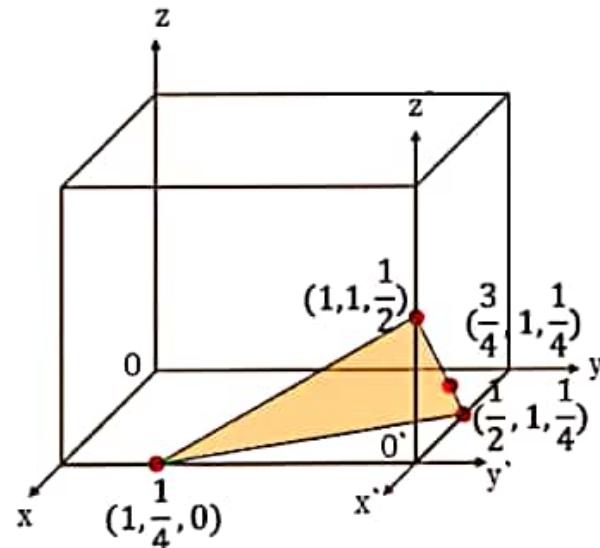
The given position coordinate are  $(1, 1/4, 0)$ ,  $(1, 1, 1/2)$ , and  $(3/4, 1, 1/4)$ .

THE INTERCEPTS ON THE X, Y AND Z  
AXIS ARE RESPECTIVELY  $\frac{-1}{2}, \frac{-3}{4}, \frac{1}{2}$

TAKING THE RECIPROCALES  $-2, \frac{-4}{3}, 2$

THE MILLER INDICES WILL BE

$$(\bar{6}, \bar{4}, 6)$$



Que:14

NaCl has the FCC lattice with  $a = 5.63 \text{ \AA}$ . What is the spacing of {100} plane?

✓ THE FORMULA FOR INTERPLANAR SPACING IS

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

✓  $a = 5.63 \text{ \AA}$ ,  $h=1$ ,  $k=0$ ,  $l=0$ .

✓ SUBSTITUTING VALUES IN ABOVE FORMULA

$$d = \frac{5.63}{\sqrt{1^2 + 0^2 + 0^2}}$$

$$= 5.63 \text{ \AA}$$

Que:15

Gold has atomic weight 197 and the density 19.3 gm/cc. What is the spacing between atoms in solid gold?



- Gold has FCC crystal structure.
- Density,

$$\rho = \frac{M}{V} = \frac{m \times N_e}{a^3}$$

M is mass of an atom,  $N_e$  is no. of effective atoms in fcc and  $a^3$  is volume of fcc.

$$m = \frac{\text{atomic weight } (m_A)}{\text{Avagadro's number } (N_A)}$$

$$\rho = \frac{m_A \times N_e}{a^3 \times N_A}$$

$$19.3 = \frac{197 \times 4}{a^3 \times 6.022 \times 10^{23}}$$

$$a = (6.78 \times 10^{-23})^{\frac{1}{3}}$$

$$a = 4.077 \text{ \AA}$$

Que:16

Compare packing fraction for SC  
and FCC lattice.

## *Atomic Packing Factor (APF)*

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}^*}{\text{Volume of unit cell}}$$

\*assume hard spheres

- APF for a simple cubic structure = 0.52



close-packed directions  
contains  $8 \times 1/8 =$   
 $1 \text{ atom/unit cell}$

$$\text{APF} = \frac{\frac{\text{atoms}}{\text{unit cell}} \times \frac{4}{3} \pi (0.5a)^3}{a^3}$$

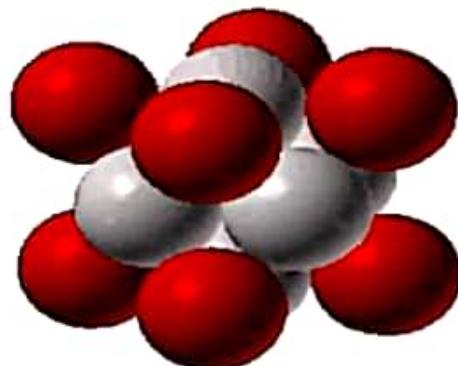
volume  
atom

$\frac{\text{volume}}{\text{unit cell}}$

## Face Centered Cubic (FCC)

- Close packed directions are face diagonals.

—Note: All atoms are identical; the face-centered atoms are shaded differently only for ease of viewing.



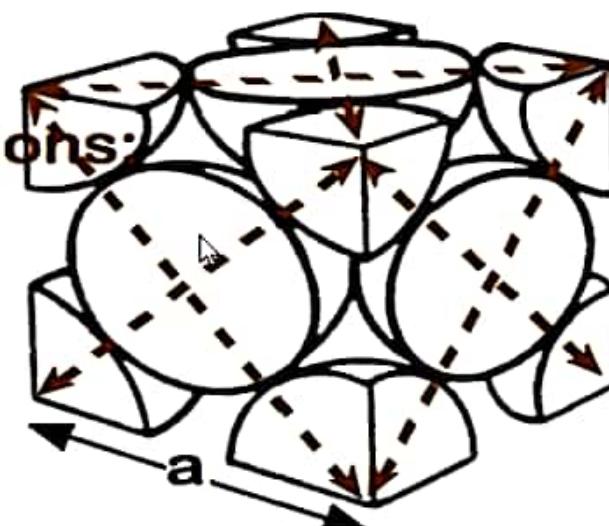
- Coordination # = 12

**Close-packed directions:**

$$\text{length} = 4R \\ = \sqrt{2} a$$

**Unit cell contains:**

$$6 \times 1/2 + 8 \times 1/8 \\ = 4 \text{ atoms/unit ce}$$



$$\text{atoms} \over \text{unit cell} = 4 \quad \frac{4}{3} \pi (\sqrt{2}a/4)^3 \quad \frac{\text{volume}}{\text{atom}}$$

$$\text{APF} = \frac{4 \frac{4}{3} \pi (\sqrt{2}a/4)^3}{a^3} \quad \frac{\text{volume}}{\text{unit cell}}$$

- APF for a FCC = 0.74

**Que:17**

In powder diffraction pattern for lead with radiation of  $\lambda = 1.54 \text{ \AA}$  the (220) Bragg reflection angle is  $\theta = 32^\circ$ . What is the radius of atom?

- Lead has FCC structure. We know

$$\begin{aligned}2d \sin\theta &= n\lambda \\2 \times \frac{a}{\sqrt{2^2 + 2^2 + 0^2}} \sin 32^\circ &= 1.54 \\a &= 4.11 \text{ \AA}\end{aligned}$$

Now, for FCC,

$$\begin{aligned}\sqrt{2}a &= 4r \\r &= 1.45 \text{ \AA}\end{aligned}$$