Solids and Structures

BENDING OF BEAMS

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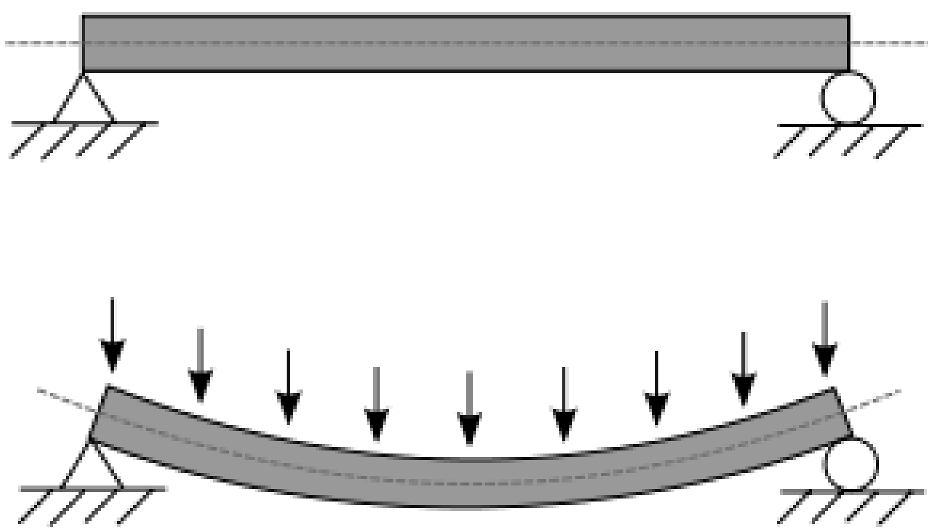
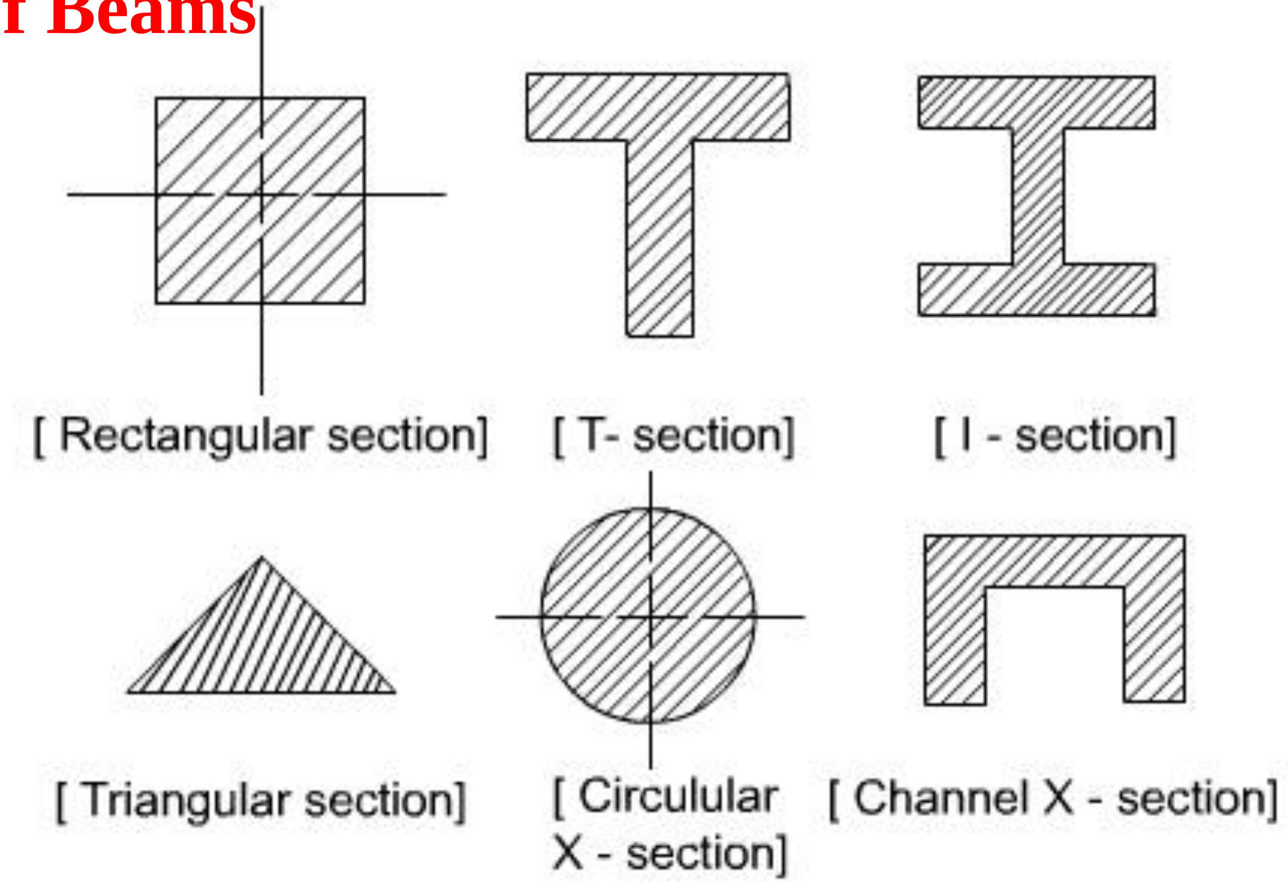
BEAMS

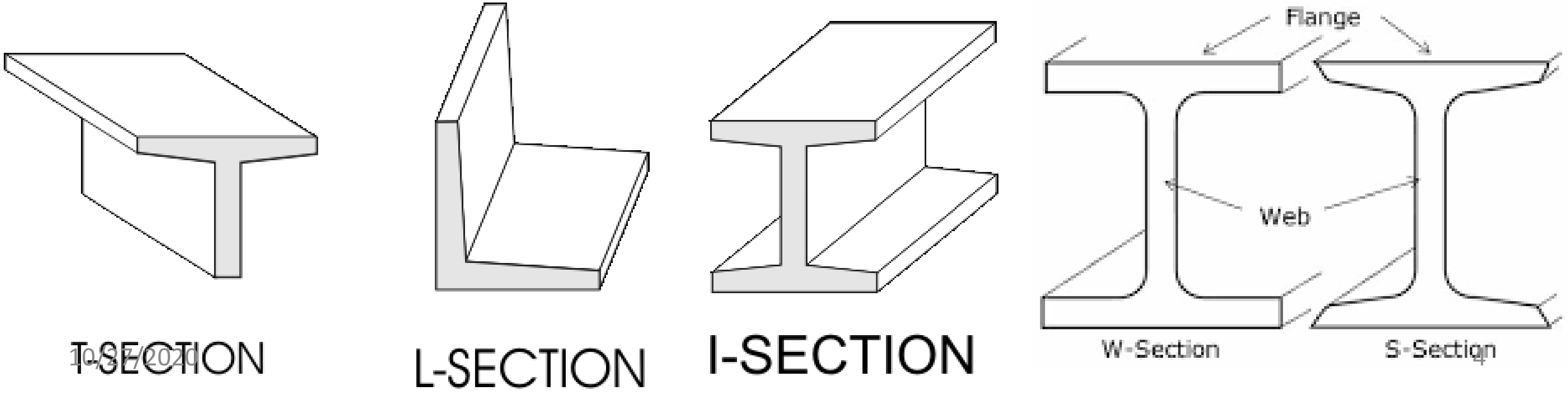
Beams are usually long (compared with their cross section dimensions), straight, prismatic members that supports transverse loads, which are load that acts perpendicular to the longitudinal axis of the member.

The applied load causes the initially straight member to deform into a curved shape, which is called the <u>deflection curve or the elastic curve.</u>



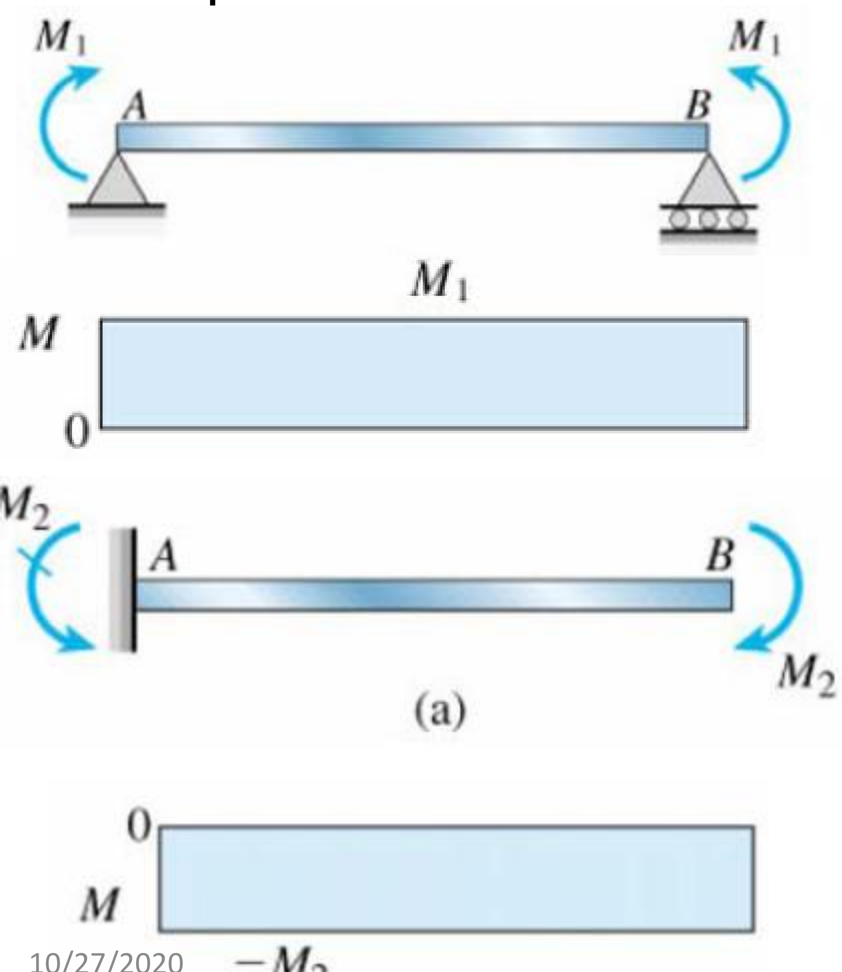
Different Sections of Beams

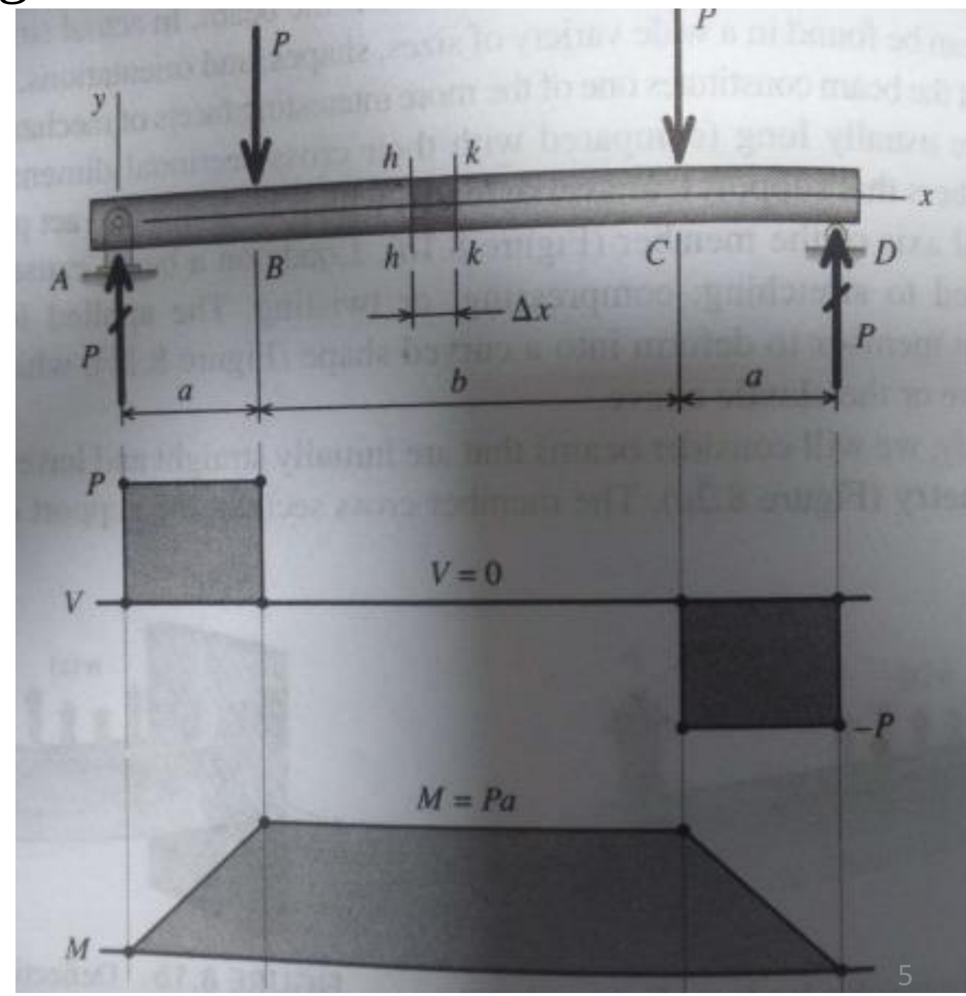




PURE BENDING

- Pure Bending – When M = Constant and V = dM/dx = 0
- Pure bending refers to flexure of beam in response to Constant Bending Moments.
- Pure bending occurs only in regions where the transverse shear force is equal to zero.





Non-Uniform Bending

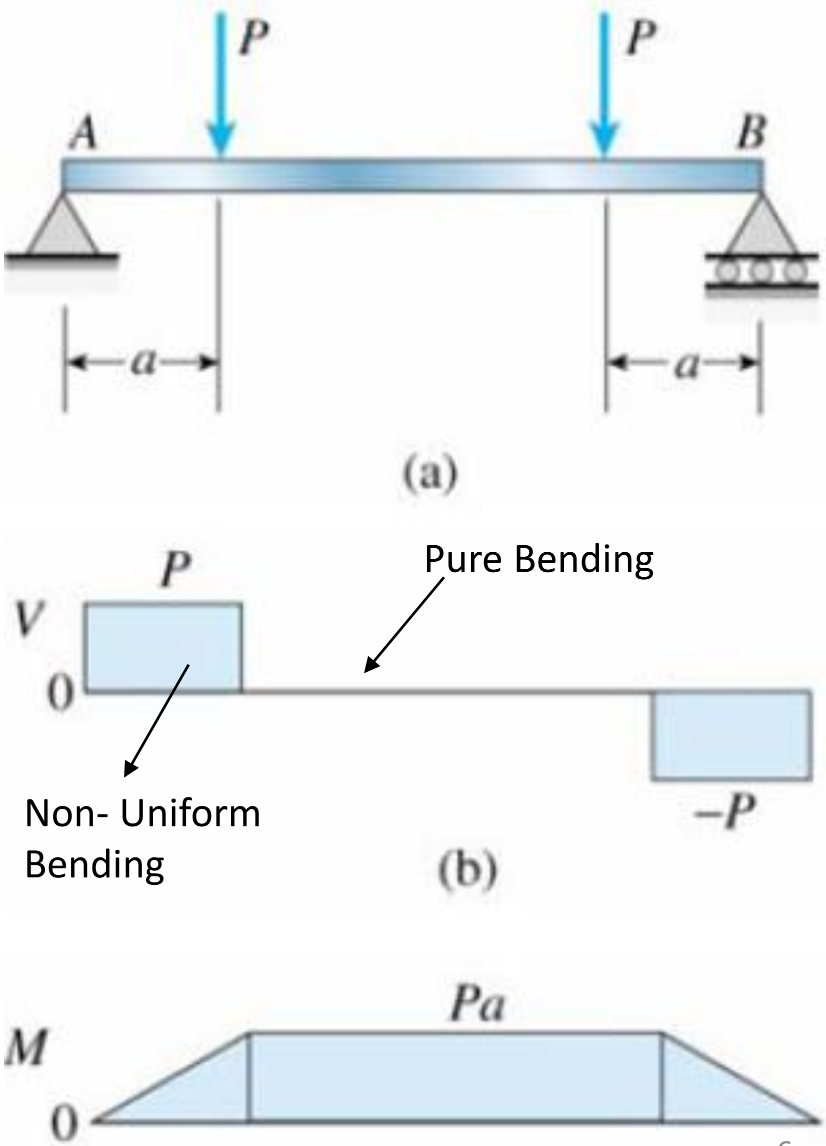
Non-Uniform Bending: V is not constant.

The non uniform bending refers to flexure where the shear force is not equal to zero.

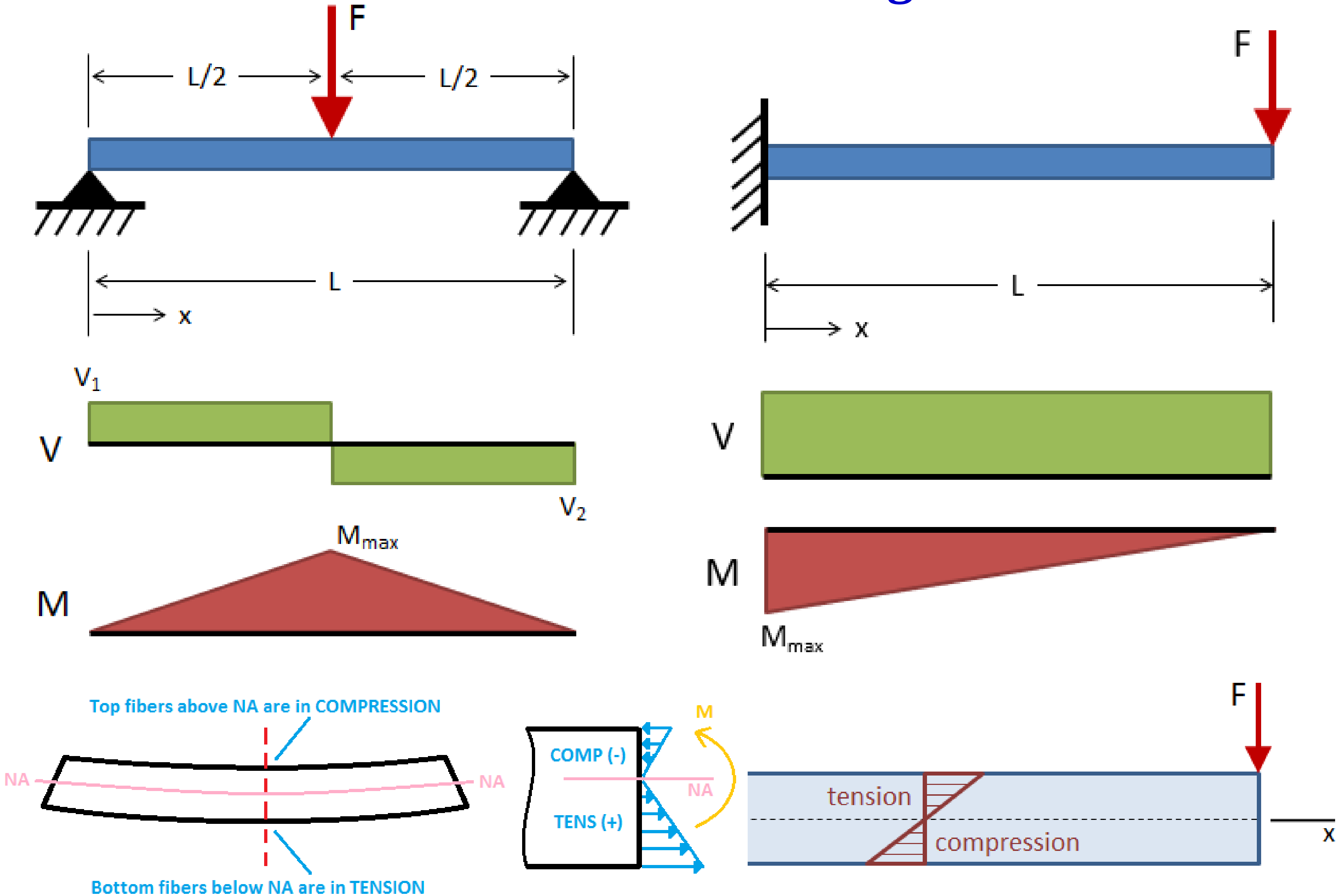
Or

The bending moment changes along the length of span.

In the simply supported beam with central region in pure bending and end regions in non-uniform bending.



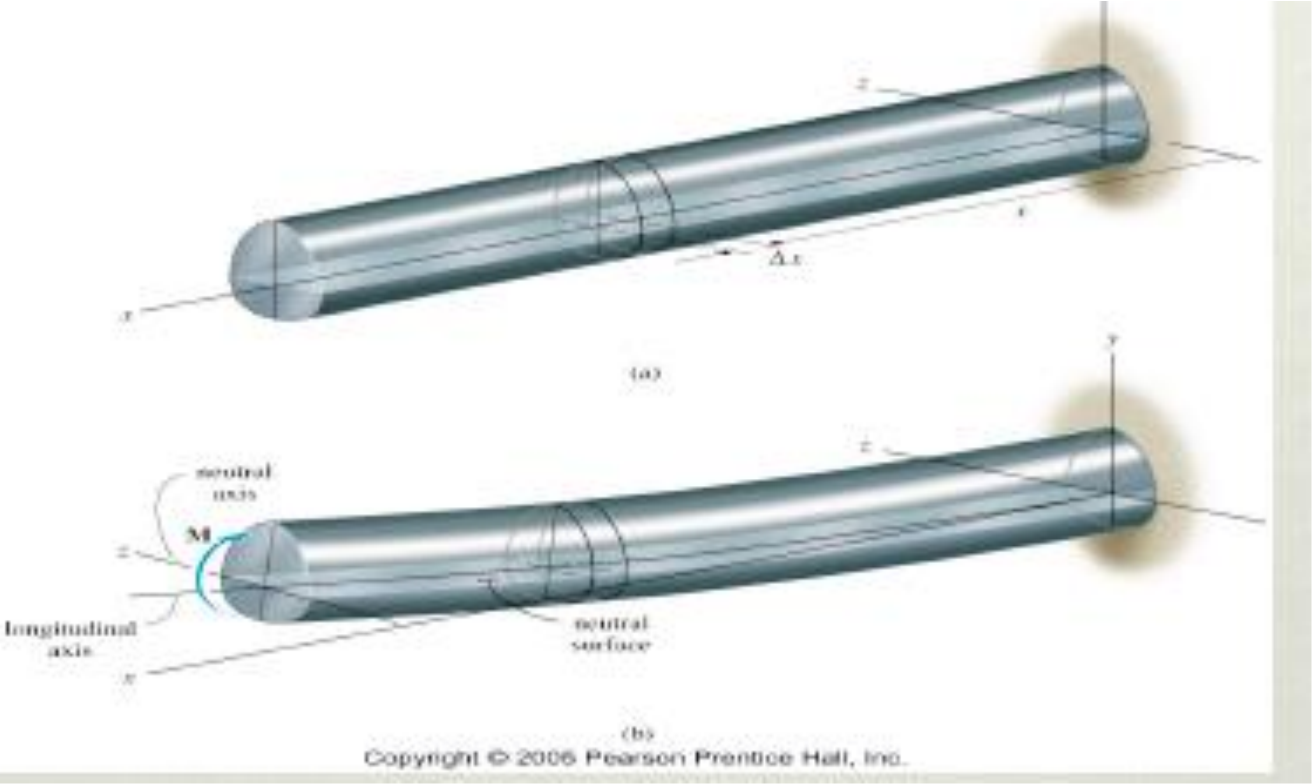
Non-Uniform Bending



Assumptions in Theory of Simple Bending

- Material of the beam is homogenous and isotropic (Constant *E* in all directions)
- The beam is initially straight, and there is no curvature in the beam
- Young's modulus of material is constant in tension and compression (to simplify analysis).
- Each layer of the beam is independent to contract or expand irrespective of the layer above or below it;
- Transverse section which are plane before bending remains plane after bending (Eliminate effects of strains in other direction).
- The radius of curvature of the beam before bending is very large compared with other dimension of cross-sections.

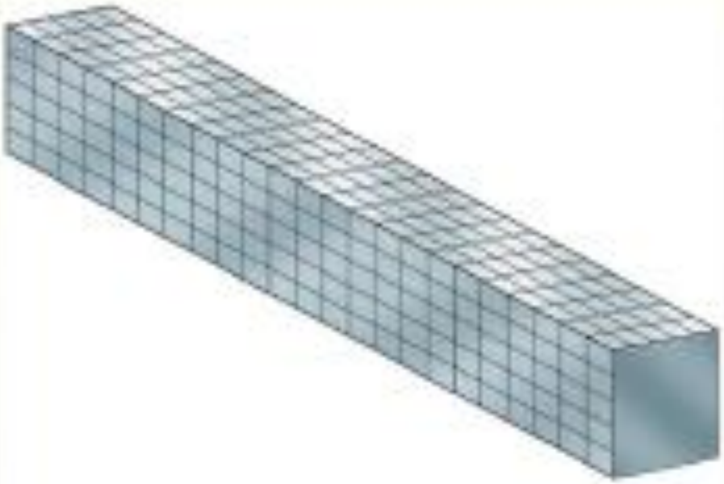
Bending in Beams



Key Points:

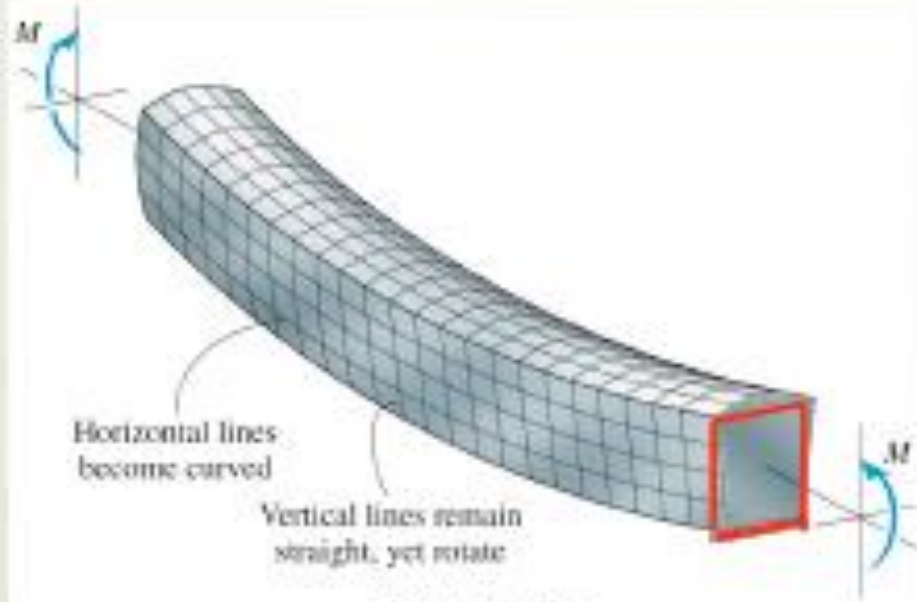
1. **Neutral surface** – no change in length.
2. **Neutral Axis** – Line of intersection of neutral surface with the transverse section.
3. All cross-sections remain plane and perpendicular to longitudinal axis.

Bending in Beams



Before deformation

(a)



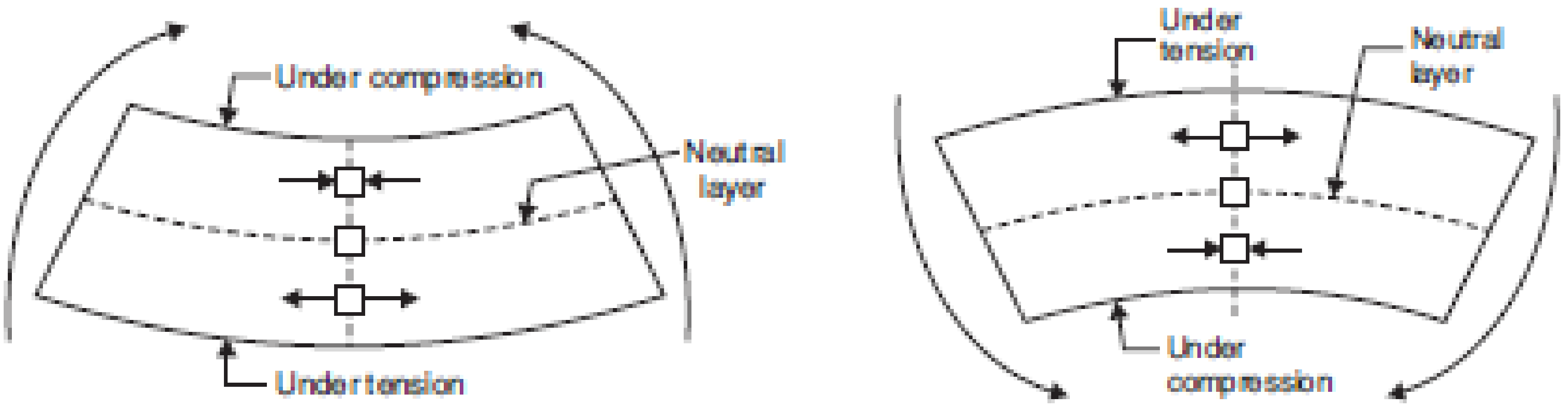
After deformation

(b)

Key Points:

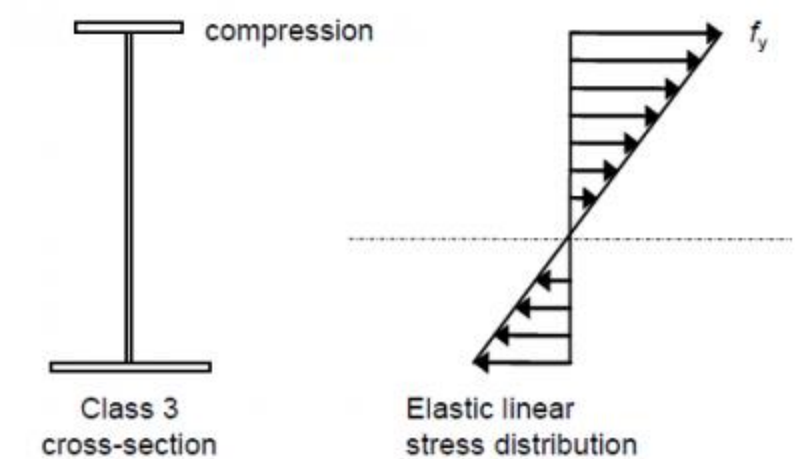
1. Internal bending moment causes beam to deform.
2. For this case, top fibers in compression, bottom in tension.

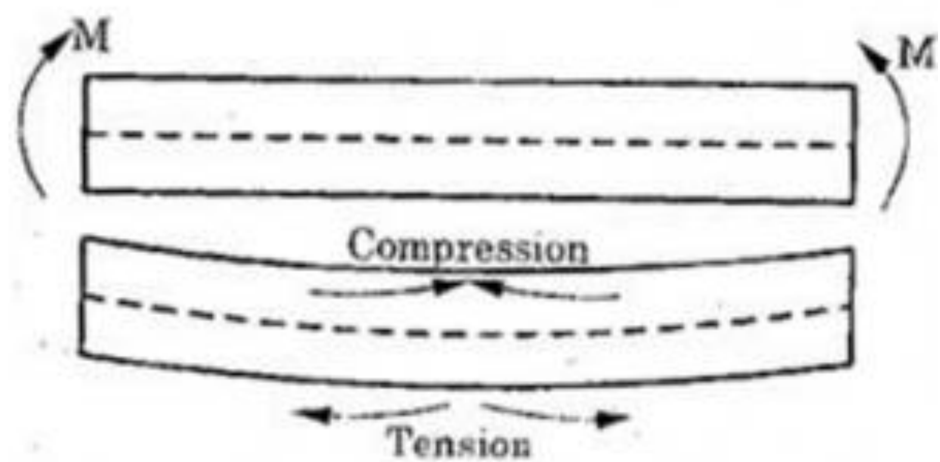
Variation of Bending Stresses in Beams



(a) Sagging moment case

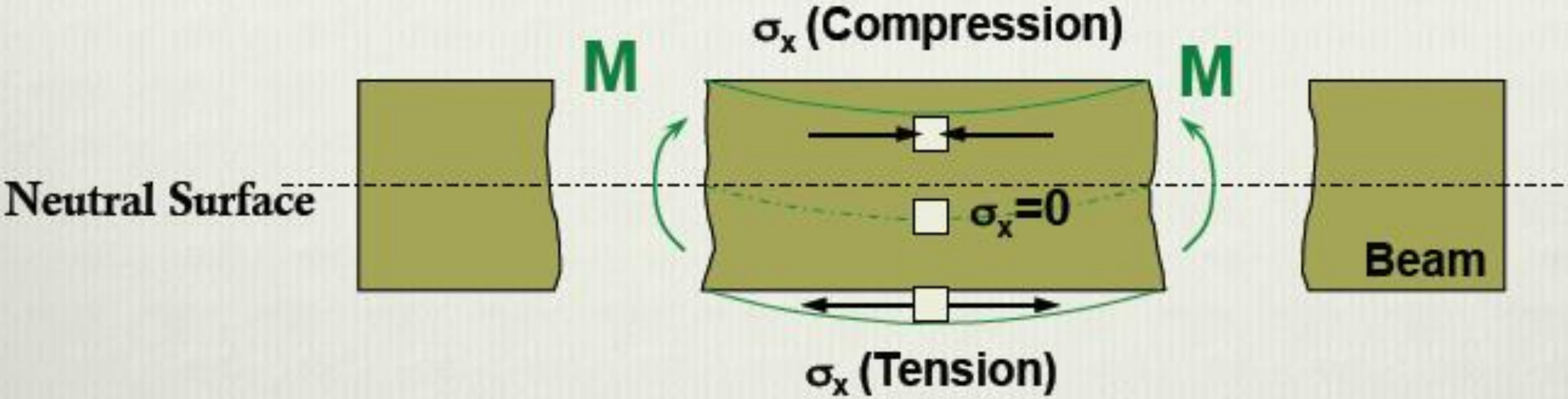
(b) Hogging moment case





Axial Stress Due to Bending:

M=Bending Moment



stress generated due to bending:

- **σ_x is NOT UNIFORM through the section depth**
- **σ_x DEPENDS ON:**

(i) Bending Moment, M

(ii) Geometry of Cross-section

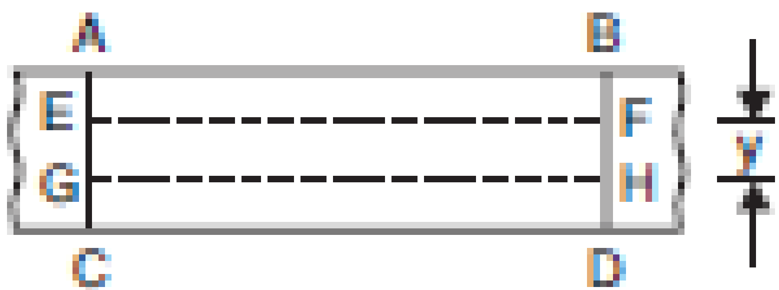
Bending or Flexure Equation

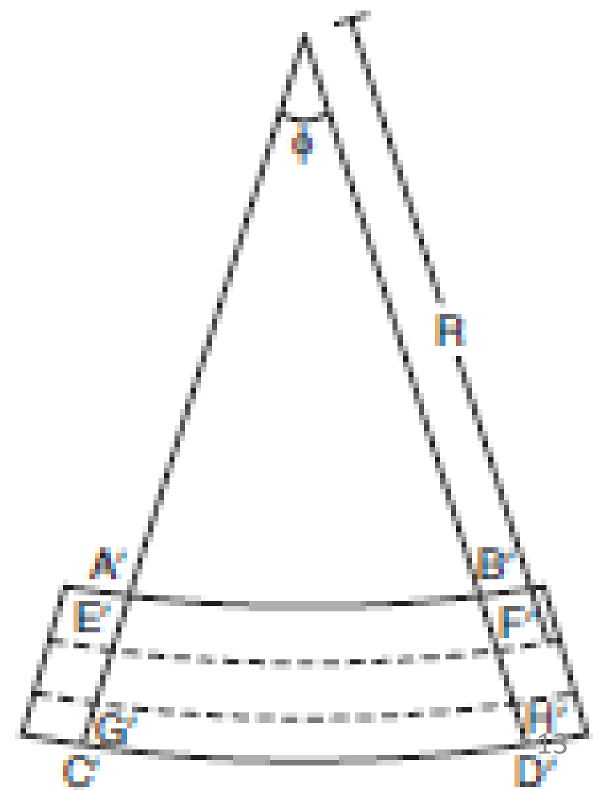
- There exists a definite relation between Applied Moments, Resulting Bending Stresses and Deformation due to Bending (Radius of Curvature, R).
- Relation between Bending Stress and Radius of Curvature
- Relation between Applied Moment and Radius of Curvature

$$\frac{\sigma}{y} = \frac{E}{R}$$

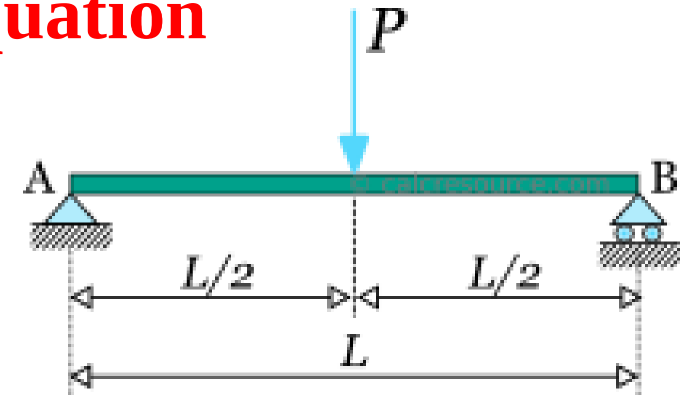
$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$





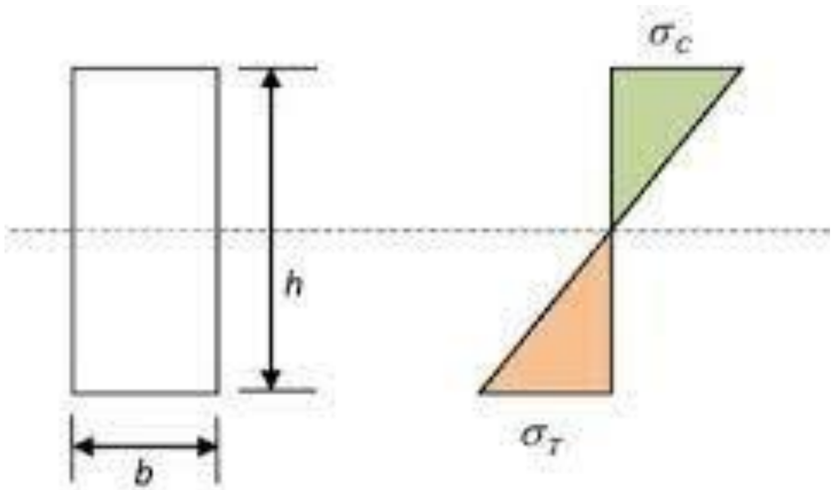
Bending or Flexure Equation



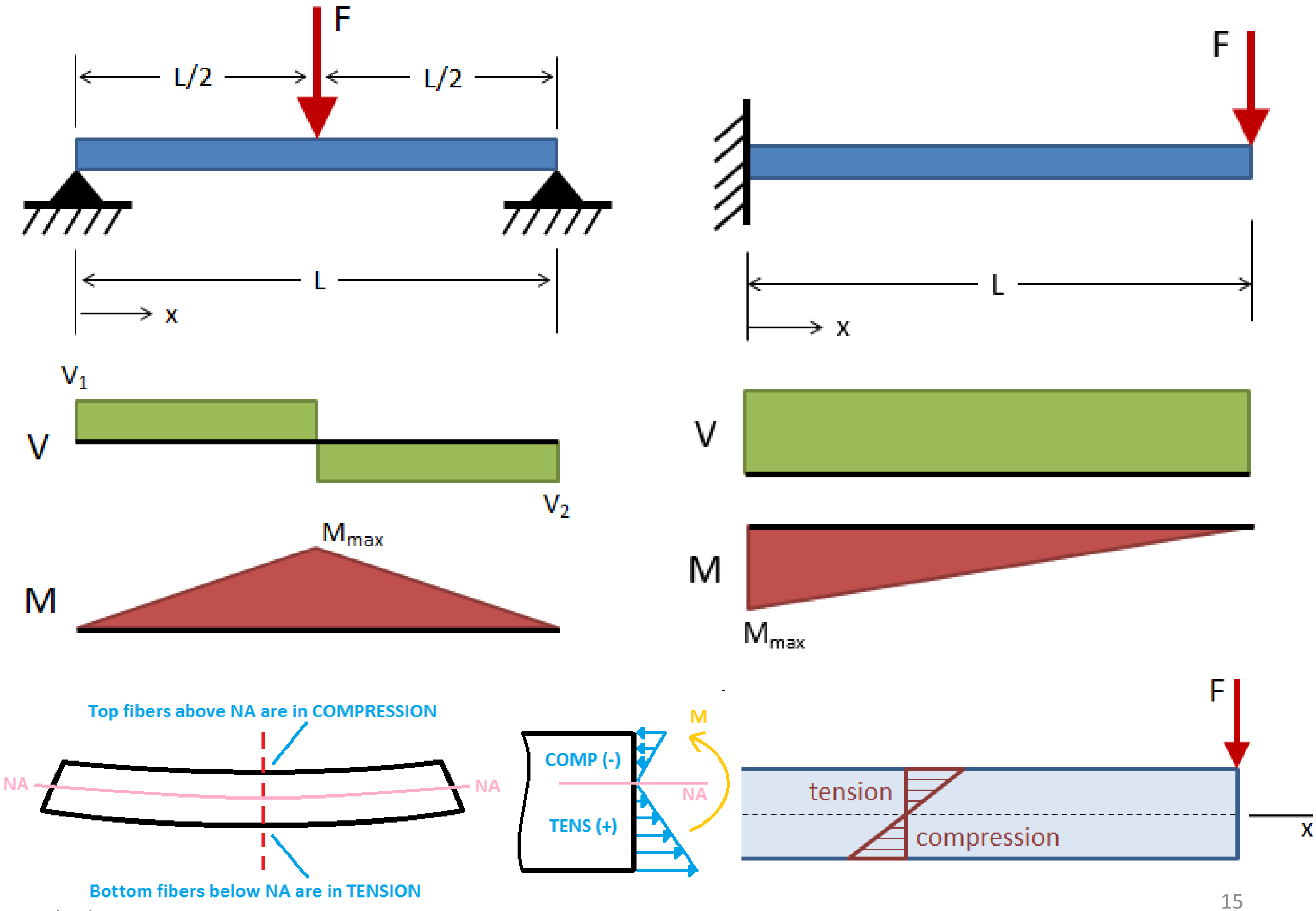
E – Young's modulus of elasticity;
σ – Bending stress in beam;
R – Radius of curvature;
y – Distance of any layer from neutral axis;
M – Applied Bending moment;
I – Moment of inertia of section about neutral axis;
EI – Flexural rigidity of material

Stress is proportional to the distance of layer under consideration from the neutral layer

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$



Stresses in Different Types of Beams



Bending or Flexure Formula

- From the bending equation, stress is proportional to the distance from the neutral axis.
- Neutral axis passes through the centroid of the section where bending stresses are zero.
- For the sake of weight reduction and economy, it is always advisable to make the cross section of beams such that the most of the material is concentrated at the greatest distance from the neutral axis.
- There is universal adoption of I-section for the steel beams

Section Modulus

Section modulus is defined as ratio of moment of inertia about the neutral axis to the distance of the outermost layer from the neutral axis

$$\frac{\mathrm{M}}{\mathrm{I}} = \frac{\sigma}{y}; \quad \mathrm{M} = \sigma \frac{I}{y} = \sigma Z$$

- Moment of Inertia (MOI) and extreme fibre distance from neutral axis are the properties of section. Hence, $\mathbf{I/y}$ is the property of the section of the beam.

- This term is known as *Modulus of Section* and it is denoted by Z.

- Units of section modulus are mm^3.

Section Modulus of Rectangular Section

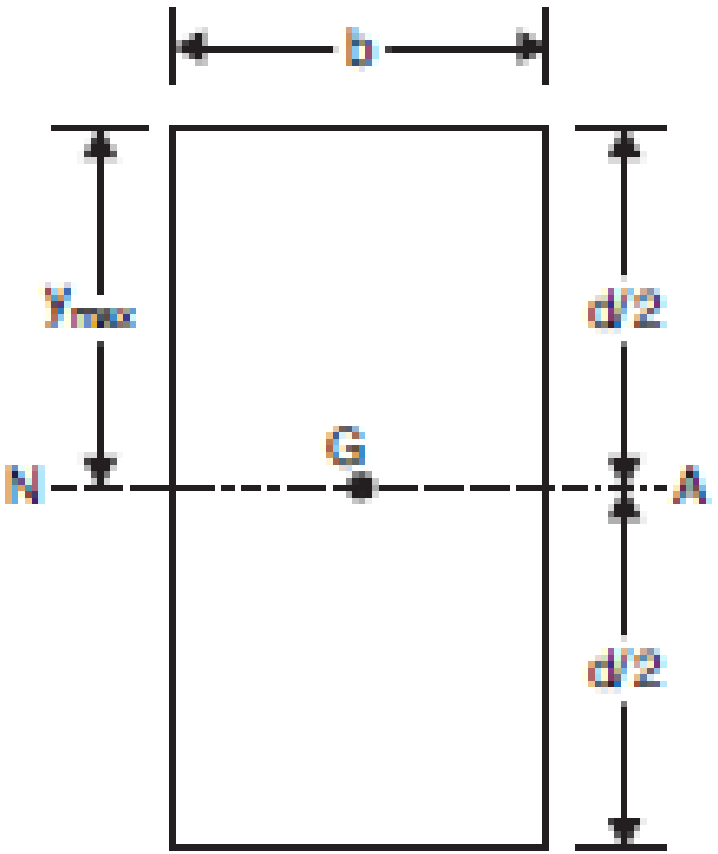
Let width be ‘b’ and depth ‘d’ of rectangular section

$$y_{max} = d/2$$

$$I = \frac{1}{12} bd^3$$

$$Z = \frac{I}{y_{max}} = \frac{1/12\, bd^3}{d/2}$$

$$Z = 1/6\ bd^2$$



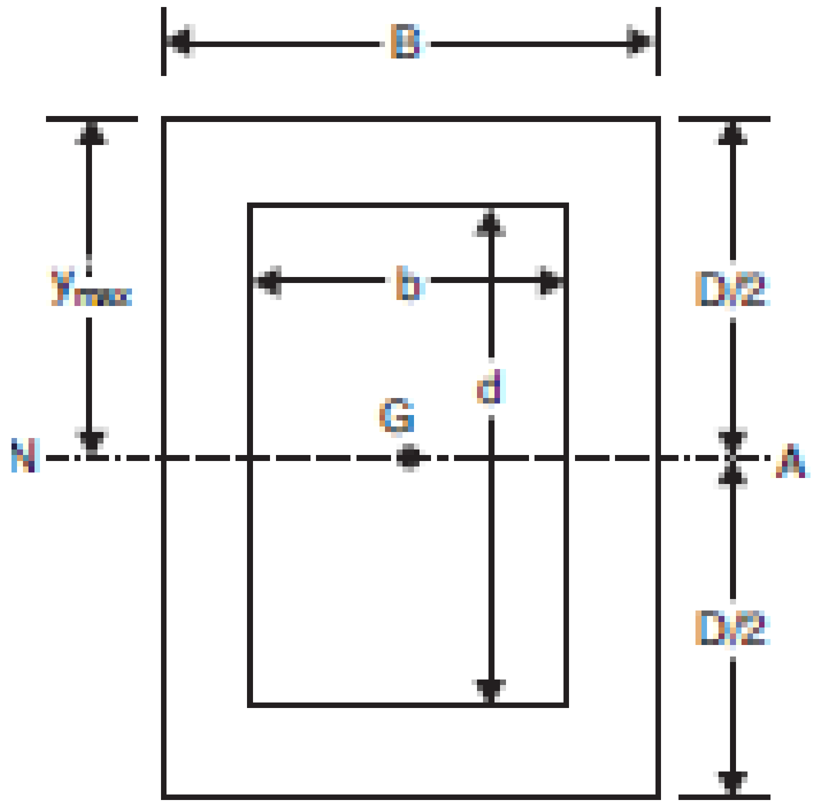
Section Modulus of Hollow Rectangular Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}(BD^3 - bd^3)$$

$$y_{max} = D/2$$

$$Z = \frac{I}{y_{max}} = \frac{1}{12}\frac{(BD^3 - bd^3)}{D/2}$$

$$Z = \frac{1}{6}\frac{BD^3 - bd^3}{D}$$



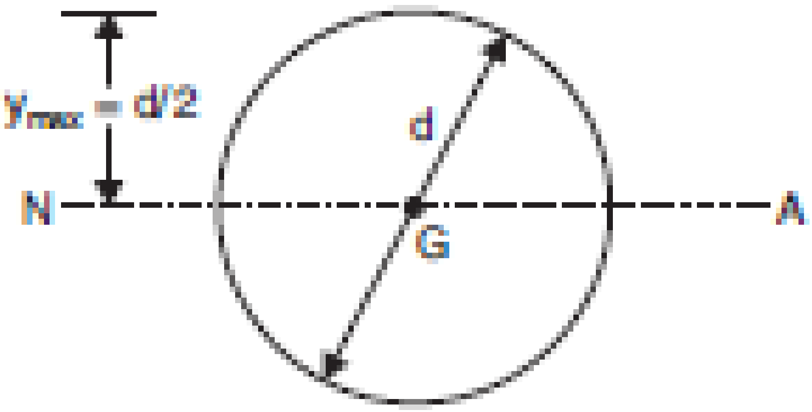
Section Modulus of Circular Section

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = d/2$$

$$Z = \frac{I}{y_{max}} = \frac{\pi/64\ d^4}{d/2}$$

$$Z = \frac{\pi}{32} d^3$$



Section Modulus of Hollow Circular section

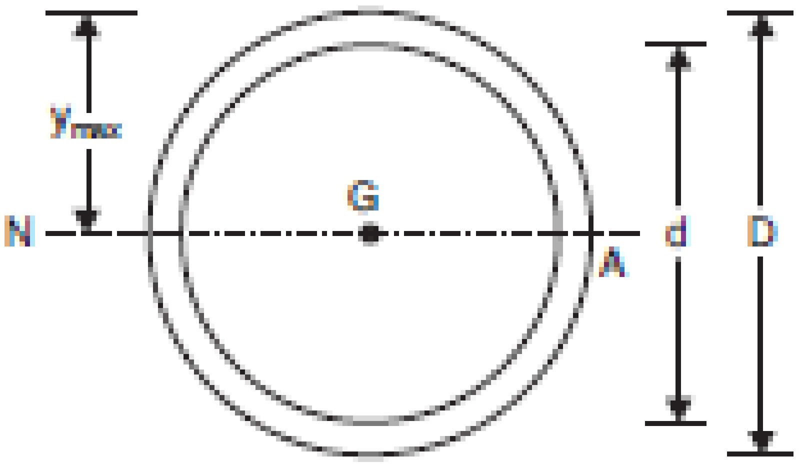
$$I = \frac{\pi}{64} D^4 - \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (D^4 - d^4)$$

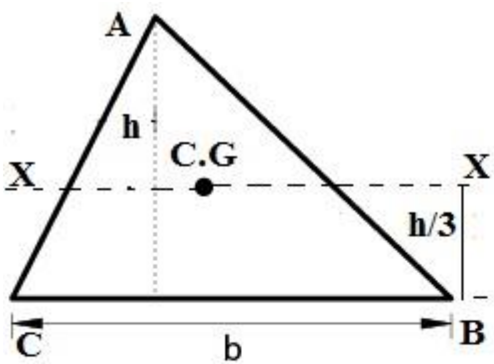
$$y_{max} = D/2$$

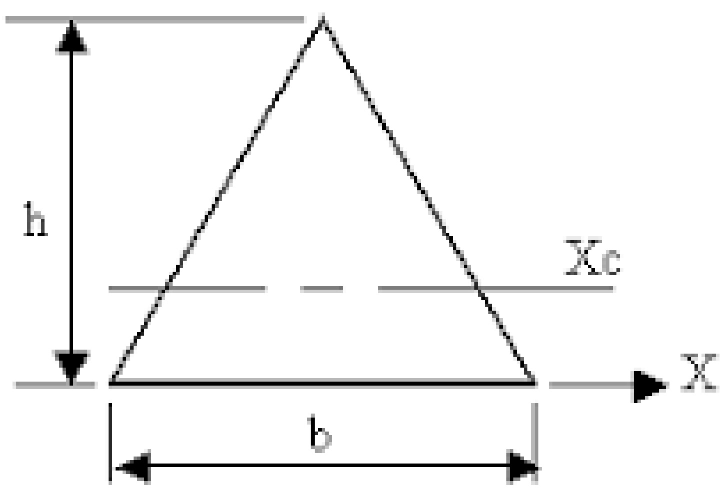
$$Z = \frac{I}{y_{max}} = \frac{\pi}{64} \frac{(D^4 - d^4)}{D/2}$$

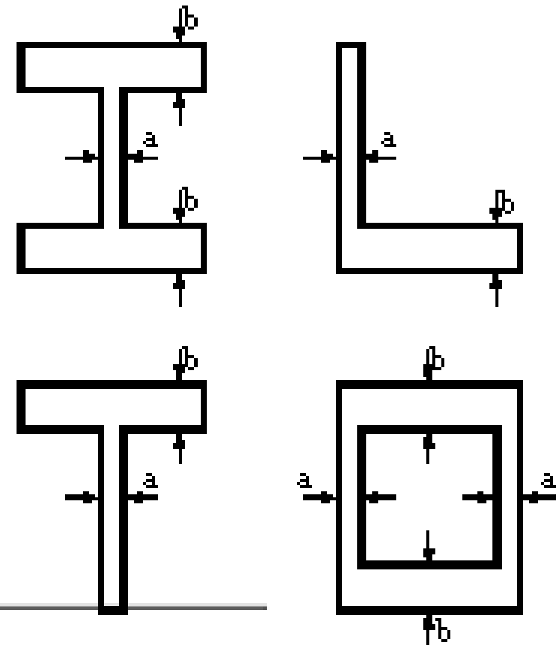
$$Z = \frac{\pi}{32} \frac{D^4 - d^4}{D}$$



MOI of Different Sections

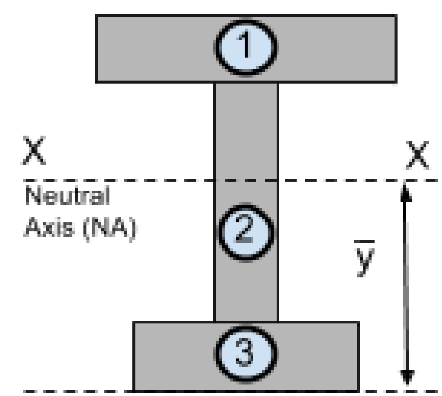




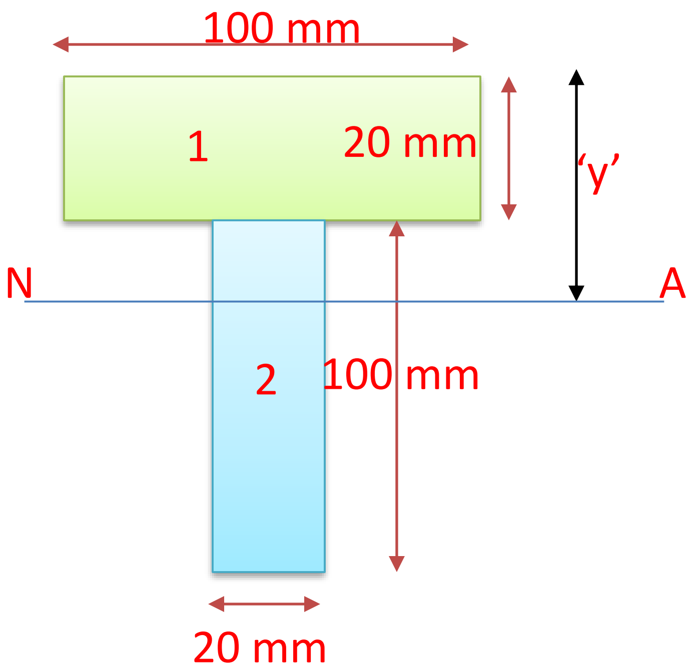


$$I_{xc} = \frac{bh^3}{36}$$

$$I_x = \frac{bh^3}{12}$$



M.O.I of Un-Symmmetrical Sections



$$J = \frac{\sum ay}{\sum a} = \frac{\sum a_1 y_1 + a_2 y_2 + a_3 y_3}{\sum a_1 + a_2 + a_3}$$

$$y = \frac{(100\,X20\,X10)(100\,X20X70)}{100\,X20)(100\,X20)}$$

y = 40 mm from top

$$I_{NA} = I_1 + I_2$$

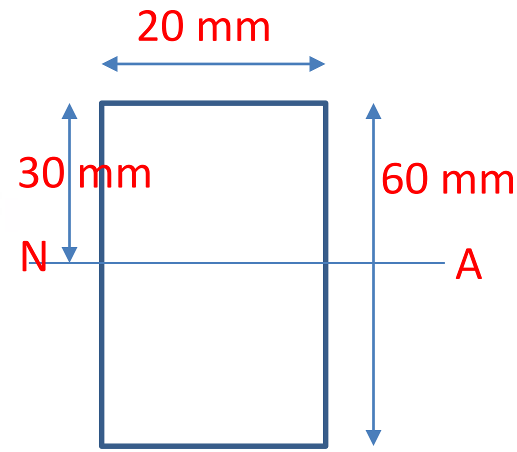
$$= (I_{CG1} + A_1\, h1^2) + (I_{CG2} + A_2\, h2^2)$$

$$= [\frac{(100)\ X\ 20^3}{12} + (100\ X20)\ (30)^2 \quad] +$$

$$[\frac{(20)\ X\ 100^3}{12} + (100\ X20)\ (30)^2 \quad] = 53.33\ X\ 10^5 \ \text{mm4}$$

Problem 1: A steel beam of 20mm by 60mm in crossection carries a maximum bending moment of 3 kN-m. Calculate the maximum bending stress induced in the beam.

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$



Maximum BM = 3 kNm = 3 X 1000 X 1000 N-mm = 3 X 10 ^{6} N=mm

$$I_{NA} = \frac{(20) \times 60^3}{12} \text{ mm}^4$$
$$= 36 \times 10^5 \text{ mm}^4$$

Maximum Bending stress = $\frac{M}{I_{NA}} \times y_{max} = \frac{3 \times 10^6}{36 \times 10^5{}_{A}} \times 30 = 250 \text{ N/mm}^2$

Problem 2: A simply supported beam of span 5.0 m has a cross section 150 mm x 250 mm. If the permissible stress in the material of the beam is 10 N/mm^2. Evaluate:

(a) Maximum UDL it can carry

(b) Maximum concentrated load at a point 2 m from support it can carry;

Solution:

(a) $M = \sigma Z = 10 (150 X 250^2)/6 = 10 X 1562500$ N mm

For a simply supported beam of Span 'L' subjected to a UDL of 'w' kN/m,

Maximum Bending moment occurs at mid-span = $wL^2/8$

Substituting $M_{max} = wL^2/8 = 10 X 1562500$

$w X 5000^2/8 = 1562000$

Solve for 'w' = 5000 N/m= 5kN/m

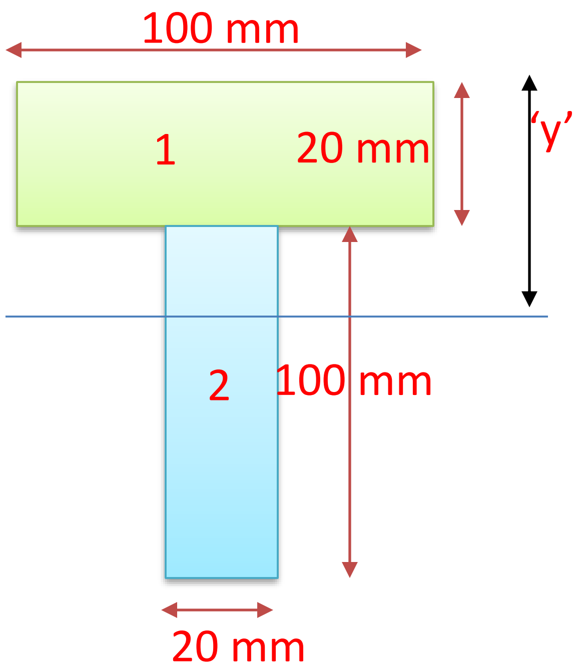
(b) Maximum moment occurs in a beam with a point load under the load = Pab/L

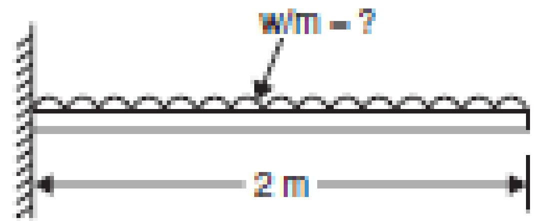
$M_{max} = P (2) (3)/5$

$M_{max} = \sigma Z = 10 (150 X 250^2)/6 = 10 X 1562500$

Equating the two, P = 13020.83 N = 13.02 kN.

Problem 3: The cross-section of a cantilever beam of 2.0 m span is shown. Material used is steel for which maximum permissible stress is 150 N/mm^2. What is the maximum uniformly distributed load this beam can carry?





$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

y = 40 mm from top

y_{max} = 120-40mm = 80mm

I_{NA} = 53.33 X 10^5 mm^4

$M = \sigma \times I_{NA}/y_{max}$

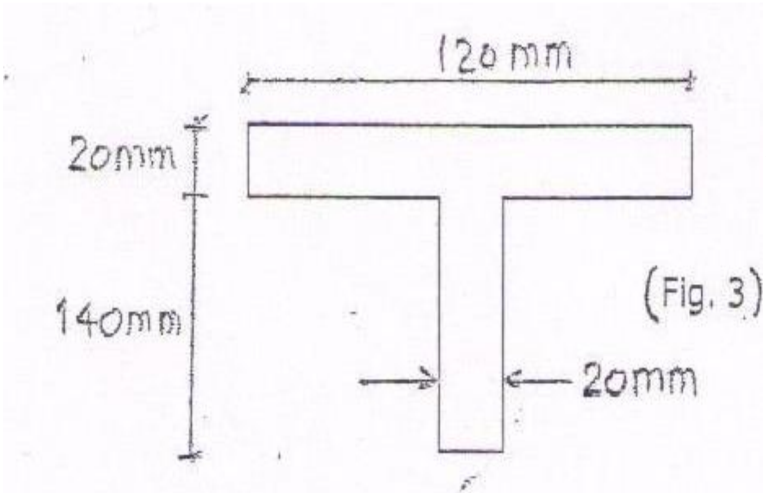
= 150 x 53.33 X 10^5 /80

= N-mm

Equating to Maximum BM in the cantilever at the fixed end = wL^2/2

Calculate 'w'.

Problem 4: Locate the neutral axis and calculate the value of I_{zz} for T section as shown in Figure. Determine the stresses at top and bottom if the applied moment is 60 kN-m. (Assume tension at bottom.)



$$J = \frac{\sum ay}{\sum a} = \frac{((120*20)*10)+((140*20)*90)}{(120*20)+(20*140)}$$

y = 53.07 mm from top

$$I = (120*20^3)/12+(120*20)*(53.07-10)^2$$
$$+(20*140^3)/12+(20*140)*(90-53.07)^2$$

$$\frac{60}{I} = \frac{\sigma_c}{53.07}$$

$$\frac{60}{I} = \frac{\sigma_t}{106.93}$$