RECAP

Periodic Signal:

$$x(t) = x(t+T)$$

Non-periodic (aperiodic) Signal:

$$x(t) \neq x(t+T)$$

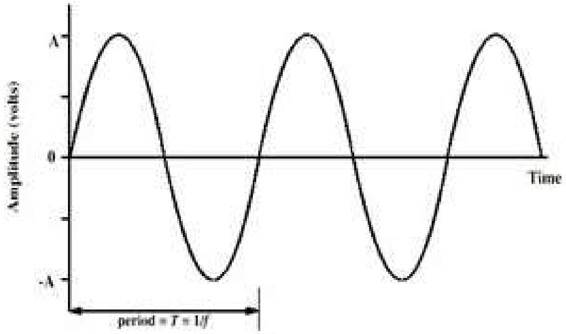
Also

$$\omega = 2\pi f$$

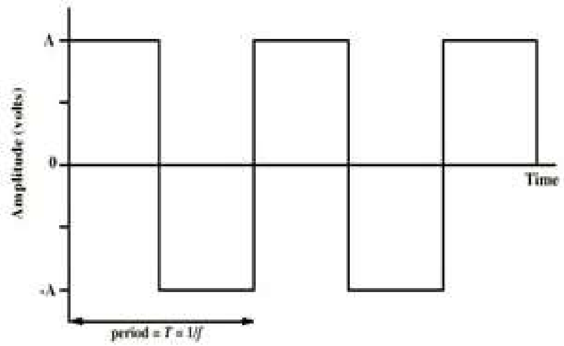
$$f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Periodic signals

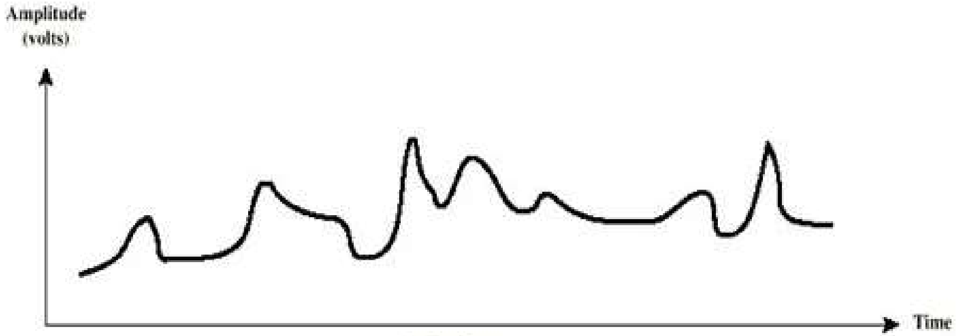


(a) Sine wave

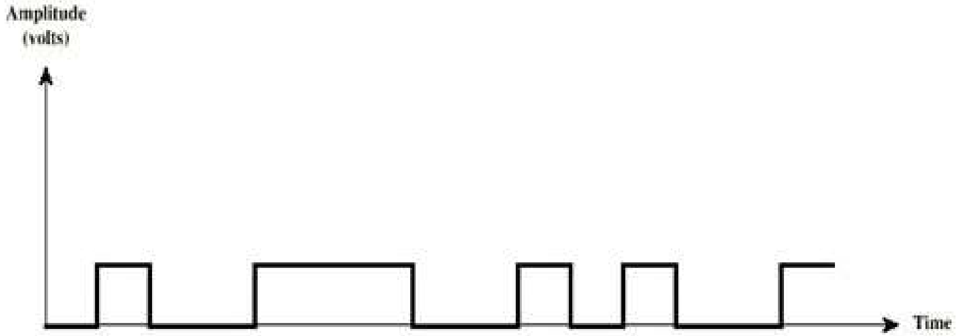


(b) Square wave

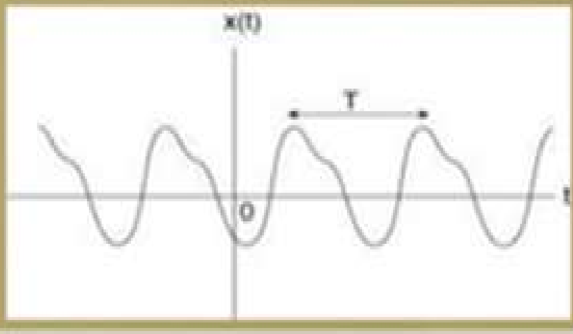
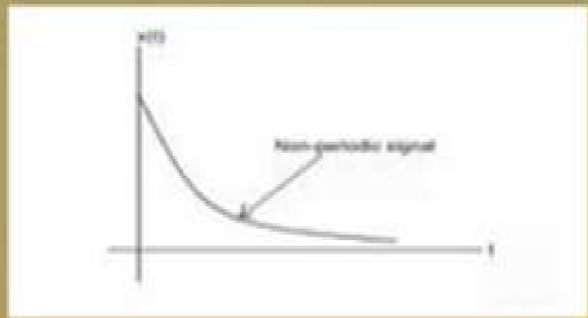
Nonperiodic signals



(a) Continuous



(b) Discrete

Periodic Signal	Aperiodic Signal
❑ A signal which repeats itself after a specific interval of time is called periodic signal.	❑ A signal which does not repeat itself after a specific interval of time is called aperiodic signal.
❑ A signal that repeats its pattern over a period is called periodic signal	❑ A signal that does not repeats its pattern over a period is called aperiodic signal or non periodic.
❑ Figure: 	❑ Figure: 

Prove that $x(t)=\sin^2 t$ is periodic.

Here $x(t)=\sin^2 t$

$$x(t)=\sin^2 t = \frac{1}{2}(1-\cos 2t)$$

$$\cos \omega t = \cos 2t$$

$$\omega = 2$$

$$2\pi f = 2$$

$$f = \frac{1}{\pi}$$

$$T = \frac{1}{f} = \pi$$

$$
\begin{aligned}
\mathrm{x(t+T)} &= \frac{1}{2}[1-\cos 2(t+T)] \\
&= \frac{1}{2}[1-\cos 2(t+\pi)] \\
&= \frac{1}{2}[1-\cos 2t\cos 2\pi+\sin 2t\sin 2\pi] \\
&= \frac{1}{2}[1-\cos 2t\cos 2\pi] \\
&= \frac{1}{2}(1-\cos 2t) \\
&= \mathrm{x(t)}
\end{aligned}
$$

Prove that $x(t) = \cos^2 2\pi t$ is periodic.

Here $x(t) = \cos^2 2\pi t = \frac{1}{2}(1 + \cos 4\pi t)$

$$\cos \omega t = \cos 4\pi t$$

$$\omega = 4\pi$$

$$T = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5$$

Also,

$$\omega = 2\pi f$$

$$\therefore f = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2$$

$$\therefore T = \frac{1}{f} = \frac{1}{2} = 0.5$$

$$
\begin{aligned}
x(t+T) &= \frac{1}{2}[1+\cos 4\pi(t+T)] \\
&= \frac{1}{2}[1+\cos 4\pi(t+0.5)] \\
&= \frac{1}{2}[1+\cos(4\pi t+2\pi)] \\
&= \frac{1}{2}[1+\cos 4\pi t\cos 2\pi-\sin 4\pi t\sin 2\pi] \\
&= \frac{1}{2}[1+\cos 4\pi t\cos 2\pi] \\
&= \frac{1}{2}(1+\cos 4\pi t) \\
&= \mathrm{x}(\mathrm{t})
\end{aligned}
$$

If $x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}$ and $y(t) = \sin \pi t$ are two periodic functions , prove that z(t)=x(t)y(t) is also a periodic function. Find the fundamental period of z(t).

$$
\begin{aligned}
z(t) &= x(t)y(t) \\
&= \left(\cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}\right)\sin \pi t \\
&= \cos\frac{2\pi t}{3}\sin \pi t + 2\sin\frac{16\pi t}{3}\sin \pi t \\
&= \frac{1}{2}\sin\frac{5\pi t}{3} + \frac{1}{2}\sin\frac{\pi t}{3} + \cos\frac{13\pi t}{3} - \cos\frac{19\pi t}{3}
\end{aligned}
$$

$$For \quad \sin\frac{5\pi t}{3}, \omega_1 = \frac{5\pi}{3}, T_1 = \frac{2\pi}{\omega_1} = \frac{6}{5}$$

$$For \quad \sin\frac{\pi t}{3}, \omega_2 = \frac{\pi}{3}, T_2 = \frac{2\pi}{\omega_2} = \frac{6}{1}$$

$$For \quad \cos\frac{13\pi t}{3}, \omega_3 = \frac{13\pi}{3}, T_3 = \frac{2\pi}{\omega_3} = \frac{6}{13}$$

$$For \quad \cos\frac{19\pi t}{3}, \omega_4 = \frac{19\pi}{3}, T_4 = \frac{2\pi}{\omega_4} = \frac{6}{19}$$

$$For \quad \mathrm{z(t)}, \quad \mathrm{T} = \frac{LCM\,(6,6,6,6)}{HCF\,(5,1,13,19)} = \frac{6}{1} = 6$$

Other Way,

Also

$$z(t) = \frac{1}{2}\sin\frac{5\pi t}{3} + \frac{1}{2}\sin\frac{\pi t}{3} + \cos\frac{13\pi t}{3} + \cos\frac{19\pi t}{3}$$

$$= \frac{1}{2}\sin\frac{2\pi t}{6} + \frac{1}{2}\sin\frac{2\pi t \times 5}{6}$$

$$+ \cos\frac{2\pi t \times 13}{6} - \cos\frac{2\pi t \times 19}{6}$$

$$z(t) = \sum_k a_k \sin \frac{2\pi t \times k}{T} + \sum_k b_k \cos \frac{2\pi t \times k}{T}$$

$$a_1 = \frac{1}{2}, T = 6$$

$$a_5 = \frac{1}{2}, T = 6$$

$$b_{13} = 1, T = 6$$

$$b_{19} = -1, T = 6$$

$$Fundamental\ Period\ of\ z(t)$$

$$= LCM(6, 6, 6, 6) = 6$$

Prove $x(t) = \cos(3t + \frac{\pi}{4})$ is a periodic function.

$$x(t) = \cos(3t + \frac{\pi}{4})$$

$$\cos(\omega t + \phi) = \cos(3t + \frac{\pi}{4})$$

$$\omega = 3$$

$$2\pi f = 3$$

$$f = \frac{3}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{3}$$

$$
\begin{aligned}
x(t+T) &= \cos[3(t+\frac{2\pi}{3})+\frac{\pi}{4}] \\
&= \cos[(3t+\frac{\pi}{4})+2\pi] \\
&= \cos(3t+\frac{\pi}{4}) \\
&= \mathrm{x(t)}
\end{aligned}
$$

Piecewise Continuous Signals

If a signal x(t) does not have any definite values at certain points for certain values of t but has definite value at remaining points, the signal is said to be piecewise continuous signal. The following figure shows piecewise continuous signals.

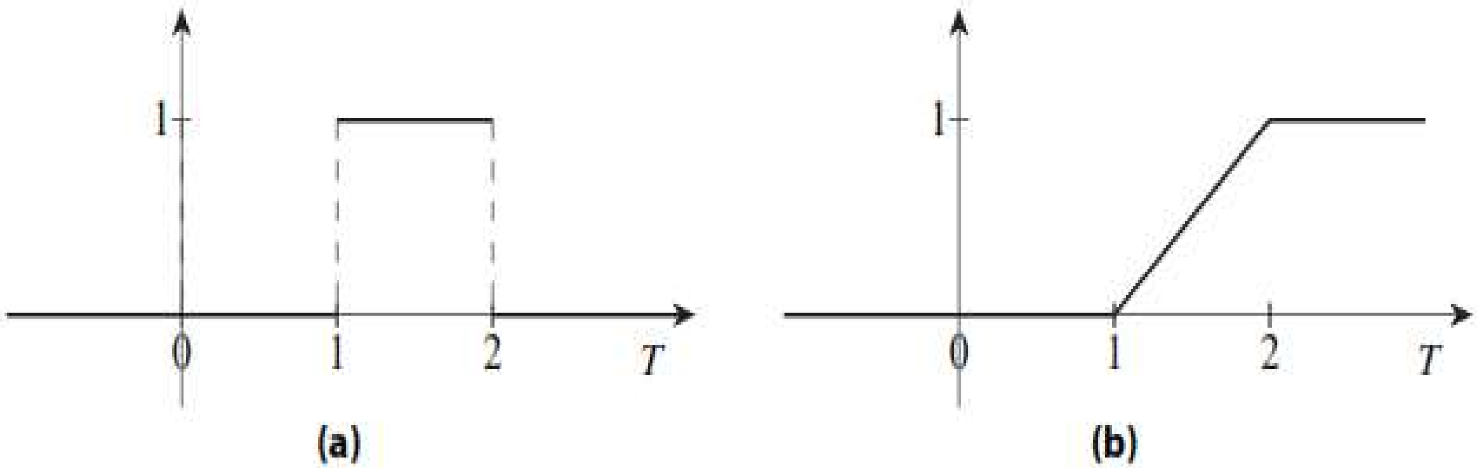


Figure 5 : Piecewise Continuous Function