

Note: Strictly attempt all questions sequentially & Assume missing data, if any, suitably

Q.1 Find the z-transform and corresponding ROC for each of the following (10) signals:

i). $y(n) = \delta(n-1)$ ii). $y(n) = \delta(n+1)$ iii). $y(n) = a^n u(n) - a^n u(n-1)$

iv). $y_k(n) = \begin{cases} y[n/k], & \text{if } n \text{ is an integer multiple of } k \\ 0, & \text{if } n \text{ is not an integer multiple of } k \end{cases}$

Q.2 Consider an LTI system, for which, the input $x(n)$ and output $y(n)$ satisfy (10) the following linear constant-coefficient difference equation:

$$y(n) - (1/2)y(n-1) = x(n) + (1/3)x(n-1)$$

long 2 marks

Find the system impulse response $h(n)$ and verify its stability based on its poles and zeros, i) if the ROC corresponding to $H[z]$ is $|z| > |1/2|$ and
 ii) if the ROC corresponding to $H[z]$ is $|z| < |1/2|$

Q.3 Obtain the discrete-time Fourier-transform (DTFT) of the discrete-time (10) signal $x(n) = a^n$ with $|a| < 1$. Plot the magnitude spectrum $|X(e^{j\omega})|$ vs. ω

and the phase spectrum $\angle X(e^{j\omega})$ vs. ω .

$$x(n) = \sum_{n=-\infty}^{\infty} x(n)e^{jn\omega}$$

Q.4 Consider the continuous-time signal $x(t)$, whose continuous-time (10)

Fourier-transform (CTFT) is $X(f\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$

$$X(f\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Determine the signal $x(t)$ using the inverse-CTFT, and also plot it.

Let the discrete-time composite periodic signal be

$$x(n) = 1 + \sin(2\pi n/N) + 3 \cos(2\pi n/N) + \cos(4\pi n/N + \pi/2) \quad \text{for } n = 0, 1, 2, \dots, N-1 \quad (10)$$

with period N . Calculate the Discrete-time Fourier-series (DTFS) spectral

coefficients a_k , and also plot the magnitude $|a_k|$ vs. k .

(75) $\frac{1}{6} \sum_{k=1}^{N-1}$

$$\left| \sum_{k=1}^{N-1} \right|^2$$

$$\sum_{k=1}^{N-1}$$

$$\sqrt{1.73}$$

$$\frac{1}{1-a_2}$$

$$\frac{1}{1-a_2}$$

$$L = 1$$

$$2^{-2} \quad 2^{-1}$$

$$2^{-2} \quad 2^{-1}$$

Q.5

The continuous-time periodic square-wave $x(t)$ is defined with one period as $x(t) = \begin{cases} 1, & |t| < T, \\ 0, & T_i < |t| < T/2 \end{cases}$ (10)

with the fundamental period T and the fundamental angular frequency $\omega_0 = 2\pi/T$. Plot this periodic square-wave $x(t)$; and calculate the Continuous-time Fourier-series (CTFS) spectral coefficients a_i .

Q.7(a)

If the input signal to an LTI system is unit-step function $u(n)$ in the discrete-time domain and the impulse response of concerned LTI system is also unit-step function $u(n)$, then obtain the output signal $y(n)$ using the linear convolution sum formula.

Q.7(b)

Consider the z-transform $X(z) = 1/(1 - az^{-1})$ with $ROC \rightarrow |z| < |a|$ of an unknown signal $x(n)$. Determine $x(n)$ using the power-series-expansion method based on long division.

$$a^n u(n)$$

Q.8

Compute the discrete-Fourier-transform (DFT) of the four-point sequence $x(n) = [0 \ 1 \ 2 \ 3]$, using the matrix \bar{W}_4 of the linear transformation (i.e., by using the matrix method). $\omega_N = e^{-j\frac{\pi}{2}}$

Q.9

Perform the circular convolution of the following two sequences: (10)

$$x_1(n) = [2 \ 1 \ 2 \ 1] \text{ and } x_2(n) = [1 \ 2 \ 3 \ 4] \quad (4, 1, 1, 1, 4, 1)$$



Q.10(a)

How the basic butterfly computation in the decimation-in-frequency fast-Fourier-transform (FFT) algorithm is different from the basic butterfly computation in the decimation-in-time FFT algorithm? (Only plot comparison diagrams).

DF
Bpw opn yu
wht $\sum_{m=0}^{N-1} w_m x_m$

Q.10(b)

Draw the signal flow graph for an $N = 8$ point decimation-in-frequency FFT algorithm, using the butterfly computation scheme (clearly indicating its different stages).

Note:

Use $\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{(\pi\theta)}$, wherever it is required.