

## School of Mathematics, Thapar University, Patiala

## UMA031: Optimization Techniques

Mid-Term Examination: B.E. (III Sem)

M.M. 30; Time: 02 Hours

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1. (a) A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them  $X$ ,  $Y$  and  $Z$ ), it is necessary to buy two additional products, say  $A$  and  $B$ . One unit of product  $A$  contains 36 units of  $X$ , 3 units of  $Y$  and 20 units of  $Z$ . One unit of product  $B$  contains 6 units of  $X$ , 12 units of  $Y$  and 10 units of  $Z$ . The minimum requirement of  $X$ ,  $Y$  and  $Z$  is 40 units, 50 units and 60 units respectively. The product  $A$  costs rupees 20 per unit and product  $B$  rupees 40 per unit. Formulate this problem as a linear programming problem to minimize the total cost. (4)
- (b) Find the optimal solution of the following linear programming problem (LPP) using Two-Phase method. Also, find alternative solution, if they exist.

$$\text{Min } Z = 2x_1 + x_2 \quad \text{subject to} \quad -2x_1 + x_2 \leq 2, \quad -2x_1 + 3x_2 \leq 6, \quad 2x_1 + x_2 \geq 4, \quad x_1 \geq 0, \quad x_2 \geq 0. \quad (6)$$

- (c) Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. One unit of exterior paint uses one unit of M1 and 2 units of M2, and one unit of interior paint uses one unit of M1 and one unit of M2. The maximum daily availability of the raw material M1 is 8 units and that of M2 is 10 units. The profit per unit of exterior paint is rupees 5 and for interior paint is rupees 4. The linear programming model of this problem which aims at maximizing total profit is given below, where  $x$  and  $y$  represents the number of units of exterior and interior paint respectively.

$$\text{Max } 4x + 3y \quad \text{subject to} \quad x + y \leq 8, \quad 2x + y \leq 10, \quad x, y \geq 0.$$

Using the graphical sensitivity answer the following:

- (i) If a choice is made to increase the availability of M1 and M2 at an additional cost of rupees 1.5 per unit. Is it advisable?
- (ii) Suppose the unit profits of exterior and interior paints are changed to rupees 5 and 10 respectively. Will the current optimum remain the same? Justify your answer. (5)

2. Consider the following primal LPP

$$\text{Min } Z = 3x_1 + x_2 \quad \text{subject to} \quad x_1 + x_2 \geq 1, \quad 2x_1 + 3x_2 \geq 2, \quad x_1 \geq 0, \quad x_2 \geq 0.$$

- (a) Find the optimal solution of the given primal LPP using dual simplex method. (4)
- (b) Write the dual of the above LPP and using the optimal table of primal find the optimal solution of dual problem. (3)

3. Consider the following primal LPP

$$\text{Max } Z = 16x_1 + 15x_2 \quad \text{subject to} \quad 2x_1 + 3x_2 \leq 12, \quad -x_1 + x_2 \leq 1, \quad x_1 \leq 3, \quad x_1 \geq 0, \quad x_2 \geq 0.$$

- (a) Find the optimal solution using graphical method. (2)
- (b) Find all the basic feasible solutions and show its correspondence with corner points obtained graphically in part (a). (3)
- (c) Construct the simplex table corresponding to optimal solution obtained graphically in part (a). (3)