

**Course: UMA 035 (Optimization Techniques)**

**Instructor: Dr. Amit Kumar,**

**Associate Professor,**

**School of Mathematics,**

**TIET, Patiala**

**Email: [amitkumar@thapar.edu](mailto:amitkumar@thapar.edu)**

**Mob: 9888500451**

### Change in Right hand side

$$\text{Max } (3x_1 + 2x_2 + 5x_3)$$

Subject to

$$x_1 + 2x_2 + x_3 + S_1 = 430$$

$$3x_1 + 2x_3 + S_2 = 460$$

$$x_1 + 4x_2 + S_3 = 420$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0.$$

The optimal table for this LPP is as follows:

		3	2	5	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	Solution	Minimum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		
2	$x_2$	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100	
5	$x_3$	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230	
0	$S_3$	2	0	0	-2	1	1	20	

- (i) Find the new optimal solution if the RHS elements are 60, 64 and 59 instead of 430, 460 and 420 respectively.

- (ii) Find the new optimal solution if the RHS elements are 45, 46 and 40 instead of 430, 460 and 420 respectively.
- (iii) Within what range the RHS of first constraint can be changed without affecting the feasibility.
- (iv) Within what range the coefficient of  $x_2$  in the objective function can be changed without affecting the optimality.

**Solution:**

If the given problem is solved by the simplex method then in the starting table

- $S_1$  will be the first basic variable
- $S_2$  will be the second basic variable
- $S_3$  will be the third basic variable

$B^{-1} = [\text{Column of } S_1 \text{ in the optimal table} \quad \text{Column of } S_2 \text{ in the optimal table} \quad \text{Column of } S_3 \text{ in the optimal table}]$

$$B^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

**Column of solution**

**B<sup>-1</sup>\*RHS matrix**

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix} = \begin{bmatrix} 100 \\ 230 \\ 20 \end{bmatrix}$$

**(i) New column of solution is**

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 60 \\ 64 \\ 59 \end{bmatrix} = \begin{bmatrix} 14 \\ 32 \\ 3 \end{bmatrix}$$

**Since, all calculated values are  $\geq 0$ . So, no need to apply Dual Simplex method.**

**New optimal solution is**

$$x_2=14$$

$$x_3=32$$

$$S_3=3$$

**(ii) New column of solution is**

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ 46 \\ 40 \end{bmatrix} = \begin{bmatrix} 11 \\ 23 \\ -4 \end{bmatrix}$$

Since, one value is negative so need to apply dual simplex method.

Since, all calculated values are  $\geq 0$ . So, no need to apply Dual Simplex method.

		3	2	5	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	Solution	Minimum Ratio
$Z_j - C_j =$		4	0	0	1	2	0		
2	$x_2$	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	11	
5	$x_3$	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23	
0	$S_3$	2	0	0	-2	1	1	-4	

		<b>3</b>	<b>2</b>	<b>5</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
<b>Z<sub>j</sub>– C<sub>j</sub> =</b>		<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>0</b>		
<b>2</b>	<b>x<sub>2</sub></b>	$-\frac{1}{4}$	<b>1</b>	<b>0</b>	$\frac{1}{2}$	$-\frac{1}{4}$	<b>0</b>	<b>11</b>	
<b>5</b>	<b>x<sub>3</sub></b>	$\frac{3}{2}$	<b>0</b>	<b>1</b>	<b>0</b>	$\frac{1}{2}$	<b>0</b>	<b>23</b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>–2</b>	<b>1</b>	<b>1</b>	<b>–4</b>	

		<b>3</b>	<b>2</b>	<b>5</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
<b>Z<sub>j</sub>– C<sub>j</sub> =</b>		<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>0</b>		<b>Only one negative value in fourth row</b>
<b>2</b>	<b>x<sub>2</sub></b>	$-\frac{1}{4}$	<b>1</b>	<b>0</b>	$\frac{1}{2}$	$-\frac{1}{4}$	<b>0</b>	<b>11</b>	
<b>5</b>	<b>x<sub>3</sub></b>	$\frac{3}{2}$	<b>0</b>	<b>1</b>	<b>0</b>	$\frac{1}{2}$	<b>0</b>	<b>23</b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>–2</b>	<b>1</b>	<b>1</b>	<b>–4</b>	

		<b>3</b>	<b>2</b>	<b>5</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
<b>Z<sub>j</sub>– C<sub>j</sub> =</b>		<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>0</b>		<b>Only one negative value in fourth row</b>
<b>2</b>	<b>x<sub>2</sub></b>	<b><math>-\frac{1}{4}</math></b>	<b>1</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b><math>-\frac{1}{4}</math></b>	<b>0</b>	<b>11</b>	
<b>5</b>	<b>x<sub>3</sub></b>	<b><math>\frac{3}{2}</math></b>	<b>0</b>	<b>1</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>23</b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>–2</b>	<b>1</b>	<b>1</b>	<b>–4</b>	

		<b>3</b>	<b>2</b>	<b>5</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
<b>Z<sub>j</sub>– C<sub>j</sub> =</b>		<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>0</b>		<b>Only one negative value in fourth row</b>
<b>2</b>	<b>x<sub>2</sub></b>	<b><math>\frac{1}{4}</math></b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b><math>\frac{1}{4}</math></b>	<b>10</b>	
<b>5</b>	<b>x<sub>3</sub></b>	<b><math>\frac{3}{2}</math></b>	<b>0</b>	<b>1</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>23</b>	
<b>0</b>	<b>S<sub>1</sub></b>	<b>–1</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b><math>-\frac{1}{2}</math></b>	<b><math>-\frac{1}{2}</math></b>	<b>2</b>	

**New optimal solution is**

$$x_2=10$$

$$x_3=23$$

$$S_1=2$$

**(ii) New column of solution is**

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} b \\ 46 \\ 40 \end{bmatrix} = \begin{bmatrix} \frac{b}{2} - \frac{23}{2} \\ 23 \\ -2b + 88 \end{bmatrix}$$

**No need to apply dual simplex method, if all calculated values are  $\geq 0$ .**

**Hence,**

$$\frac{b}{2} - \frac{23}{2} \geq 0 \text{ and } -2b + 88 \geq 0$$

$$b \geq 23 \text{ and } b \leq 44$$

$$23 \leq b \leq 44$$

## Addition of a variable

### Example:

$$\text{Min } (2x_1 + x_2 + MA_1 + MA_2)$$

Subject to

$$3x_1 + x_2 + A_1 = 3$$

$$4x_1 + 3x_2 - S_2 + A_2 = 6$$

$$x_1 + 2x_2 + S_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, S_2 \geq 0, S_3 \geq 0.$$

### Optimal Table

		2	1	0	M	M	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$S_2$	$A_1$	$A_2$	$S_3$	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$-\frac{1}{5}$	$\frac{2}{5} - M$	$\frac{1}{5} - M$	0		
2	$x_1$	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{3}{5}$	
1	$x_2$	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{6}{5}$	
0	$S_3$	0	0	1	1	-1	1	0	

**Find the new optimal solution if a non-negative variable  $x_3$  having the coefficients  $\frac{1}{2}$  in the objective function, 0 in the first constraint, 5 in the second constraint and 2 in the third constraint is added in the original problem.**

**Solution:**

**New LPP is**

$$\text{Min } (2x_1 + x_2 + MA_1 + MA_2 + \frac{1}{2}x_3)$$

**Subject to**

$$3x_1 + x_2 + A_1 + 0x_3 = 3$$

$$4x_1 + 3x_2 - S_2 + A_2 + 5x_3 = 6$$

$$x_1 + 2x_2 + S_3 + 2x_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, S_2 \geq 0, S_3 \geq 0.$$

		2	1	0	M	M	0	$\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
<b>Z<sub>j</sub> – C<sub>j</sub> =</b>		<b>0</b>	<b>0</b>	$-\frac{1}{5}$	$\frac{2}{5} - M$	$\frac{1}{5} - M$	<b>0</b>			
<b>2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b>1</b>	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	<b>0</b>		$\frac{3}{5}$	
<b>1</b>	<b>x<sub>2</sub></b>	<b>0</b>	<b>0</b>	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	<b>0</b>		$\frac{6}{5}$	
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>–1</b>	<b>1</b>		<b>0</b>	

The given optimal table is for minimum as all values of  $Z_j - C_j$  are  $\leq 0$ .

Transform it into maximization by changing the sign of  $Z_j - C_j$  and sign of objective function coefficients.

		– 2	– 1	0	–M	–M	0	– $\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variable s</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Solutio n</b>	<b>Minimu m Ratio</b>
<b>Z<sub>j</sub>– C<sub>j</sub> =</b>		<b>0</b>	<b>0</b>	<b><math>\frac{1}{5}</math></b>	<b><math>-\frac{2}{5} +</math> M</b>	<b><math>-\frac{1}{5} +</math> M</b>	<b>0</b>			
<b>– 2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b>1</b>	<b><math>\frac{1}{5}</math></b>	<b><math>\frac{3}{5}</math></b>	<b><math>-\frac{1}{5}</math></b>	<b>0</b>		<b><math>\frac{3}{5}</math></b>	
<b>– 1</b>	<b>x<sub>2</sub></b>	<b>0</b>	<b>0</b>	<b><math>-\frac{3}{5}</math></b>	<b><math>-\frac{4}{5}</math></b>	<b><math>\frac{3}{5}</math></b>	<b>0</b>		<b><math>\frac{6}{5}</math></b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>–1</b>	<b>1</b>		<b>0</b>	

If the given problem is solved by the Big-M method then in the starting table

- A<sub>1</sub> will be the first basic variable
- A<sub>2</sub> will be the second basic variable
- S<sub>3</sub> will be the third basic variable

**B<sup>-1</sup>**=[Column of A<sub>1</sub> in the optimal table Column of A<sub>2</sub> in the optimal table  
Column of S<sub>3</sub> in the optimal table]

$$\mathbf{B}^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Column of  $x_3$

$\mathbf{B}^{-1}$ (Coefficients of  $x_3$  in constraints)

$$= \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}$$

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
$C_B$	Basic Variables	$x_1$	$x_2$	$S_2$	$A_1$	$A_2$	$S_3$	$x_3$	Solution	Minimum Ratio
$Z_j - C_j =$		0	0	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	0			
-2	$x_1$	1	1	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	-1	$\frac{3}{5}$	
-1	$x_2$	0	0	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	3	$\frac{6}{5}$	
0	$S_3$	0	0	1	1	-1	1	-3	0	

		$-2$	$-1$	$0$	$-M$	$-M$	$0$	$-\frac{1}{2}$		
$C_B$	Basic Variables	$x_1$	$x_2$	$S_2$	$A_1$	$A_2$	$S_3$	$x_3$	Solution	Minimum Ratio
$Z_j - C_j =$		$0$	$0$	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	$0$	$-\frac{1}{2}$		
$-2$	$x_1$	$1$	$1$	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	$0$	$-1$	$\frac{3}{5}$	
$-1$	$x_2$	$0$	$0$	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	$0$	$3$	$\frac{6}{5}$	
$0$	$S_3$	$0$	$0$	$1$	$1$	$-1$	$1$	$-3$	$0$	

**Solution is not optimal. The variable  $x_3$  is entering variable.**

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
<b>Z<sub>j</sub> - C<sub>j</sub> =</b>		<b>0</b>	<b>0</b>	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	<b>0</b>	$-\frac{1}{2}$		
<b>-2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b>1</b>	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	<b>0</b>	<b>-1</b>	$\frac{3}{5}$	
<b>-1</b>	<b>x<sub>2</sub></b>	<b>0</b>	<b>0</b>	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	<b>0</b>	<b>3</b>	$\frac{6}{5}$	
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>-1</b>	<b>1</b>	<b>-3</b>	<b>0</b>	

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
<b>Z<sub>j</sub> - C<sub>j</sub> =</b>		<b>0</b>	<b>0</b>	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	<b>0</b>	$-\frac{1}{2}$		
<b>-2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b>1</b>	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	<b>0</b>	<b>-1</b>	$\frac{3}{5}$	$\frac{3}{5} / -$
<b>-1</b>	<b>x<sub>2</sub></b>	<b>0</b>	<b>0</b>	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	<b>0</b>	<b>3</b>	$\frac{6}{5}$	$\frac{6}{5} / 3$
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>-1</b>	<b>1</b>	<b>-3</b>	<b>0</b>	<b>0 / -</b>

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
<b>Z<sub>j</sub> - C<sub>j</sub> =</b>		<b>0</b>	<b>0</b>	$\frac{1}{5}$	$-\frac{2}{5} + M$	$-\frac{1}{5} + M$	<b>0</b>	$-\frac{1}{2}$		
<b>-2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b>1</b>	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	<b>0</b>	<b>-1</b>	$\frac{3}{5}$	$\frac{3}{5} / -$
<b>-1</b>	<b>x<sub>2</sub></b>	<b>0</b>	<b>0</b>	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	<b>0</b>	<b>3</b>	$\frac{6}{5}$	$\frac{6}{5} / 3$
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>-1</b>	<b>1</b>	<b>-3</b>	<b>0</b>	<b>0/-</b>

		-2	-1	0	-M	-M	0	$-\frac{1}{2}$		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>S<sub>2</sub></b>	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>x<sub>3</sub></b>	<b>Soluti on</b>	<b>Minim um Ratio</b>
<b>Z<sub>j</sub> - C<sub>j</sub> =</b>		<b>0</b>	<b><math>\frac{1}{6}</math></b>	<b><math>\frac{1}{5}</math></b>	<b><math>-\frac{8}{15} + M</math></b>	<b><math>-\frac{1}{10} + M</math></b>	<b>0</b>	<b>0</b>		
<b>-2</b>	<b>x<sub>1</sub></b>	<b>1</b>	<b><math>\frac{1}{3}</math></b>	<b>0</b>	<b><math>\frac{1}{3}</math></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	
<b><math>-\frac{1}{2}</math></b>	<b>x<sub>3</sub></b>	<b>0</b>	<b><math>\frac{1}{3}</math></b>	<b><math>-\frac{1}{5}</math></b>	<b><math>-\frac{4}{5}</math></b>	<b><math>\frac{1}{5}</math></b>	<b>0</b>	<b>1</b>	<b><math>\frac{2}{5}</math></b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>0</b>	<b>1</b>	<b><math>\frac{2}{5}</math></b>	<b><math>\frac{1}{5}</math></b>	<b><math>-\frac{2}{5}</math></b>	<b>1</b>	<b>0</b>	<b><math>\frac{6}{5}</math></b>	

**New optimal solution:**

$$x_1 = 1$$

$$x_3 = \frac{2}{5}$$

$$S_3 = \frac{6}{5}$$

**Remaining are 0 i.e.,  $x_2 = S_2 = A_1 = A_2 = 0$**

## Addition of a constraint

### Example:

$$\text{Min } (x_1 - 2x_2 + x_3)$$

Subject to

$$x_1 + 2x_2 - 2x_3 \leq 4$$

$$x_1 - x_3 \leq 3$$

$$2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

### Optimal Table

		1	-2	1	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	Solution	Minimum Ratio
$Z_j - C_j =$		$-\frac{9}{2}$	0	0	$-\frac{3}{2}$	0	-1		
-2	$x_2$	3	1	0	1	0	1	6	
0	$S_2$	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	7	
1	$x_3$	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	4	

Find the new optimal solution if

- (i) The constraint  $x_2 + x_3 \geq 9$  is added.
- (ii) The constraint  $x_2 + x_3 = 10$  is added.
- (iii) The constraint  $x_1 + x_2 = 4$  is added.
- (iv) The constraint  $x_1 + x_2 = 7$  is added.
- (v) The constraint  $x_1 + x_2 \leq 4$  is added.
- (vi) The constraint  $x_1 + x_2 \geq 7$  is added.

### Solution

The given optimal table is for minimization as all values of  $Z_j - C_j$  are  $\leq 0$ .

Transform it for maximization by changing the sign of values of  $Z_j - C_j$  and objective function coefficients.

		-1	2	-1	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	Solution	Minimum Ratio
$Z_j - C_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1		
2	$x_2$	3	1	0	1	0	1	6	
0	$S_2$	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	7	
-1	$x_3$	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	4	

It is obvious from the provided table that

$x_1=0$ ,  $x_2=6$ ,  $x_3=4$ .

(i)

The value of  $x_2 + x_3 = 6 + 4 = 10$ .

**Since, the obtained value is  $> 9$ . So the constraint  $x_2 + x_3 \geq 9$  is satisfying.**

**Hence, on adding the constraint  $x_2 + x_3 \geq 9$ , there will be no change in the optimal solution.**

**(ii)**

**The value of  $x_2 + x_3 = 6 + 4 = 10$ .**

**Since, the obtained value is  $= 10$ . So the constraint  $x_2 + x_3 = 10$  is satisfying.**

**Hence, on adding the constraint  $x_2 + x_3 = 10$ , there will be no change in the optimal solution.**

**(iii)**

**The value of  $x_1 + x_2 = 0 + 6 = 6$ .**

**Since, the obtained value is  $6$ . So the constraint  $x_1 + x_2 = 4$  is not satisfying.**

**Hence, on adding the constraint  $x_1 + x_2 = 4$ , there will be change in the optimal solution.**

**$x_1 + x_2 = 4$  is equivalent to  $x_1 + x_2 \geq 4$  and  $x_1 + x_2 \leq 4$ .**

**Since, the constraint  $x_1 + x_2 \geq 4$  is satisfying and the constraint  $x_1 + x_2 \leq 4$  is not satisfying.**

**So to add  $x_1 + x_2 = 4$  is equivalent to add the constraint  $x_1 + x_2 \leq 4$ .**

Now to add  $x_1 + x_2 \leq 4$  is equivalent to  $x_1 + x_2 + S_4 = 4$

		-1	2	-1	0	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Minimum Ratio
$Z_j - C_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	$x_2$	3	1	0	1	0	1	0	6	
0	$S_2$	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	$x_3$	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	$S_4$	1	1	0	0	0	0	1	4	

In the table,

The first basic variable is  $x_2$ . So, its column should be

1

0

**0**

**0**

**The second basic variable is  $S_2$ . So, its column should be**

**0**

**1**

**0**

**0**

**The third basic variable is  $x_3$ . So, its column should be**

**0**

**0**

**1**

**0**

**The fourth basic variable is  $S_4$ . So, its column should be**

**0**

**0**

**0**

**1**

It is obvious that the column of the first basic variable  $x_2$  is

1

0

0

1

instead of

1

0

0

0

Therefore, we need to apply a row operation for the fifth row such that the last element of this column is 0 and no change in columns of other basic variables.

$$R_5 \rightarrow R_5 - R_2$$

		-1	2	-1	0	0	0	0		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>S<sub>4</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
$Z_j - C_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	x <sub>2</sub>	3	1	0	1	0	1	0	6	
0	S <sub>2</sub>	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x <sub>3</sub>	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	S <sub>4</sub>	1-3 =-2	1 -1 =0	0	0-1 =-1	0	0-1= -1	1-0=0	4-6= -2	

Since, the RHS corresponding to the variable S<sub>4</sub> is negative. So, there is a need to apply dual simplex method.

S<sub>4</sub> is leaving variable.

		-1	2	-1	0	0	0	0		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>s<sub>1</sub></b>	<b>s<sub>2</sub></b>	<b>s<sub>3</sub></b>	<b>s<sub>4</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
$z_j - c_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		<b>Maximum</b> <b>m{</b> $\frac{9}{2}, \frac{3}{2}$ $-\frac{1}{-2}, \frac{1}{-1}$ $\frac{1}{-1}$ <b>}</b> = $\frac{1}{-1}$
2	x <sub>2</sub>	3	1	0	1	0	1	0	6	
0	s <sub>2</sub>	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	x <sub>3</sub>	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	s <sub>4</sub>	-2	0	0	-1	0	-1	1	-2	

		$-1$	$2$	$-1$	$0$	$0$	$0$	$0$		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Maximum Ratio
$Z_j - C_j =$		$\frac{9}{2}$	$0$	$0$	$\frac{3}{2}$	$0$	$1$	$0$		Maximum $\min\left\{\frac{9}{2}, \frac{3}{2}, \frac{1}{-1}\right\} = \frac{1}{-1}$
$2$	$x_2$	$3$	$1$	$0$	$1$	$0$	$1$	$0$	$6$	
$0$	$S_2$	$\frac{7}{2}$	$0$	$0$	$\frac{1}{2}$	$1$	$1$	$0$	$7$	
$-1$	$x_3$	$\frac{5}{2}$	$0$	$1$	$\frac{1}{2}$	$0$	$1$	$0$	$4$	
$0$	$S_4$	$-2$	$0$	$0$	$-1$	$0$	$-1$	$1$	$-2$	

		<b>-1</b>	<b>2</b>	<b>-1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>S<sub>4</sub></b>	<b>Solution</b>	<b>Maximum Ratio</b>
<b><math>z_j - C_j =</math></b>		<b><math>\frac{5}{2}</math></b>	<b>0</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>2</b>	<b>x<sub>2</sub></b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>4</b>	
<b>0</b>	<b>S<sub>2</sub></b>	<b><math>\frac{3}{2}</math></b>	<b>0</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>5</b>	
<b>-1</b>	<b>x<sub>3</sub></b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>1</b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>	
<b>0</b>	<b>S<sub>3</sub></b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>-1</b>	<b>2</b>	

**New optimal solution is**

$$x_2=4$$

$$S_2=5$$

$$x_3=2$$

$$S_3=2$$

**Remaining are 0 i.e.,  $x_1=S_1=S_4=0$ .**

(iv)

**The value of  $x_1 + x_2 = 0 + 6 = 6$ .**

**Since, the obtained value is 6. So the constraint  $x_1 + x_2 = 7$  is not satisfying.**

**Hence, on adding the constraint  $x_1 + x_2 = 7$ , there will be change in the optimal solution.**

**$x_1 + x_2 = 7$  is equivalent to  $x_1 + x_2 \geq 7$  and  $x_1 + x_2 \leq 7$ .**

**Since, the constraint  $x_1 + x_2 \leq 7$  is satisfying and the constraint  $x_1 + x_2 \geq 7$  is not satisfying.**

**So to add  $x_1 + x_2 = 7$  is equivalent to add the constraint  $x_1 + x_2 \geq 7$ .**

**Now to add  $x_1 + x_2 \geq 7$  is equivalent to  $x_1 + x_2 - S_4 = 7$  or  $-x_1 - x_2 + S_4 = -7$**

		<b>-1</b>	<b>2</b>	<b>-1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>		
<b>C<sub>B</sub></b>	<b>Basic Variables</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	<b>S<sub>4</sub></b>	<b>Solution</b>	<b>Minimum Ratio</b>
<b>z<sub>j</sub> - C<sub>j</sub> =</b>		<b><math>\frac{9}{2}</math></b>	<b>0</b>	<b>0</b>	<b><math>\frac{3}{2}</math></b>	<b>0</b>	<b>1</b>	<b>0</b>		
<b>2</b>	<b>x<sub>2</sub></b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>6</b>	
<b>0</b>	<b>S<sub>2</sub></b>	<b><math>\frac{7}{2}</math></b>	<b>0</b>	<b>0</b>	<b><math>\frac{1}{2}</math></b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>7</b>	
<b>-1</b>	<b>x<sub>3</sub></b>	<b><math>\frac{5}{2}</math></b>	<b>0</b>	<b>1</b>	<b><math>\frac{1}{2}</math></b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>4</b>	
<b>0</b>	<b>S<sub>4</sub></b>	<b>-1</b>	<b>-1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>-1</b>	<b>-7</b>	

**In the table,**

**The first basic variable is x<sub>2</sub>. So, its column should be**

**1**

**0**

**0**

**0**

**The second basic variable is S<sub>2</sub>. So, its column should be**

**0**

**1**

**0**

**0**

**The third basic variable is  $x_3$ . So, its column should be**

**0**

**0**

**1**

**0**

**The fourth basic variable is  $S_4$ . So, its column should be**

**0**

**0**

**0**

**1**

**It is obvious that the column of the first basic variable  $x_2$  is**

**1**

**0**

**0**

**-1**

**instead of**

**1**

**0**

**0**

**0**

**Therefore, we need to apply a row operation for the fifth row such that the last element of this column is 0 and no change in columns of other basic variables.**

$$\mathbf{R_5 \rightarrow R_5 + R_2}$$

		-1	2	-1	0	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Minimum Ratio
$Z_j - C_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	$x_2$	3	1	0	1	0	1	0	6	
0	$S_2$	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	$x_3$	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	$S_4$	-1+ 3 =2	-1 +1 =0	0	0+ 1=1	0	0+ 1=1	1+0=1	-7+6= -1	

Since, the RHS corresponding to the variable  $S_4$  is negative. So, there is a need to apply dual simplex method.

$S_4$  is leaving variable.

		-1	2	-1	0	0	0	0		
$C_B$	Basic Variables	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Maximum Ratio
$Z_j - C_j =$		$\frac{9}{2}$	0	0	$\frac{3}{2}$	0	1	0		
2	$x_2$	3	1	0	1	0	1	0	6	
0	$S_2$	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	0	7	
-1	$x_3$	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	0	4	
0	$S_4$	2	0	0	1	0	1	1	-1	

Since, all the values in the row corresponding to leaving variable are  $\geq 0$ .

So, it is not possible to find entering variable.

Hence, no feasible solution exists for the transformed LPP.

(v) Same solution as in (iii)

(vi) Same solution as in (iv)