Anelastic and Plastic behavior

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Anelastic Deformation

Recoverable deformation that occur as a function of time.

Elastic deformation continues after the application of load due to some relaxation process within the material.

This is a time-dependent process.

$$X = 1 - e^{-t/t_r}$$

X = fraction of relaxation process that is completed till time 't'

t_r = relaxation time. 10^{-13} (atomic) to 10^6 (grain boundaries sliding) seconds
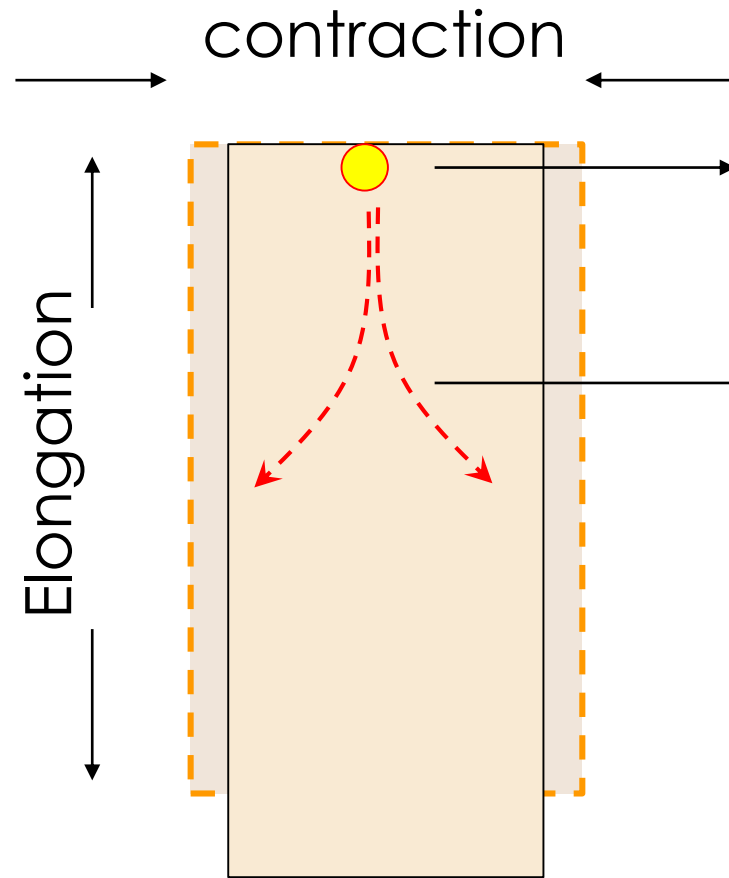
What will happen if we apply load for a time duration

t << tr

t >>tr

or t ~ tr

Define relaxation time t_r.

contraction

Elongation

Carbon atom will experience stress
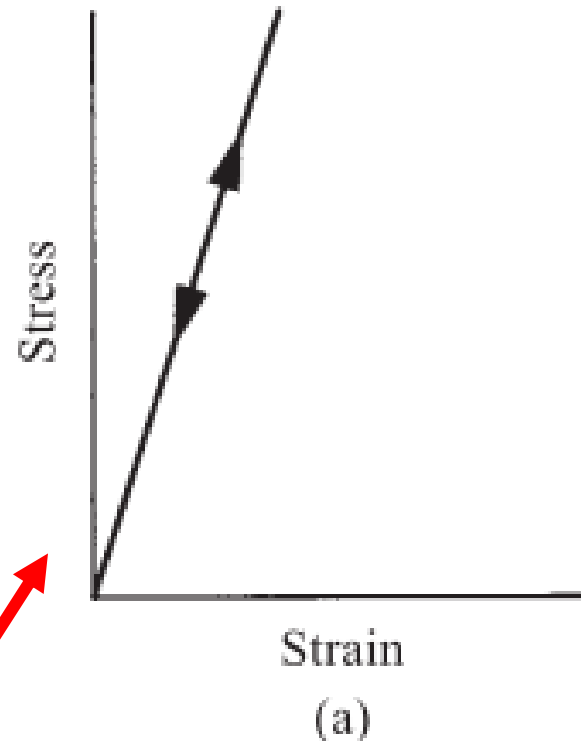
It will jump to places where stress is lower

- "Jumping" of carbon atom from one place to other place is via diffusion.
- Diffusion is a time dependent process, usually ~100 s at room temperature.

Applied force
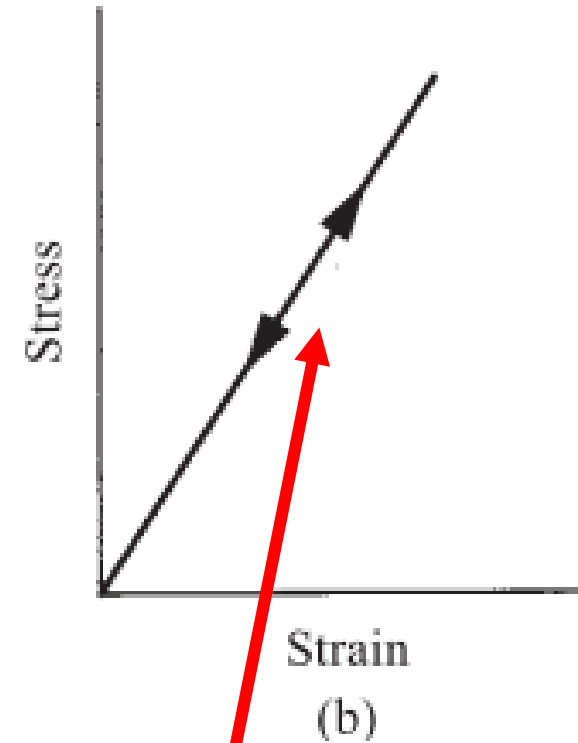or
Tensile stress

What will happen if we apply load for

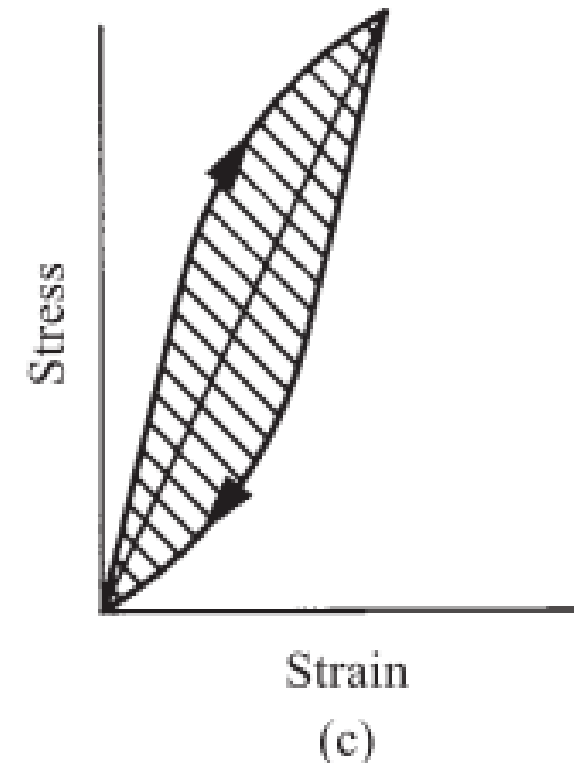(i)   2 s

(ii)   10,000 s

(iii)   120 s

$t << t_r$
2 sec

$t >> t_r$
10000 sec

$t \sim t_r$
120 sec

(a)

(b)

(c)

Strain due to bond stretching only

Strain due to bond stretching + carbon atom redistribution

Higher strain
Strain due to applied force + strain due to jump of carbon atoms

Relaxation process

1. Strongly temperature dependent.

2. Temperature variation varies the relaxation time t_r.

3. Decreases with increase in temperature.

Fraction of energy dissipated

1. Materials with good damping capacity are required to make base pads of machines to damp out the vibrations.

Materials with good damping capacity : grey cast iron, polymer, rubber.

2. Materials with low damping capacity are used in temple bells and piezoelectric device.
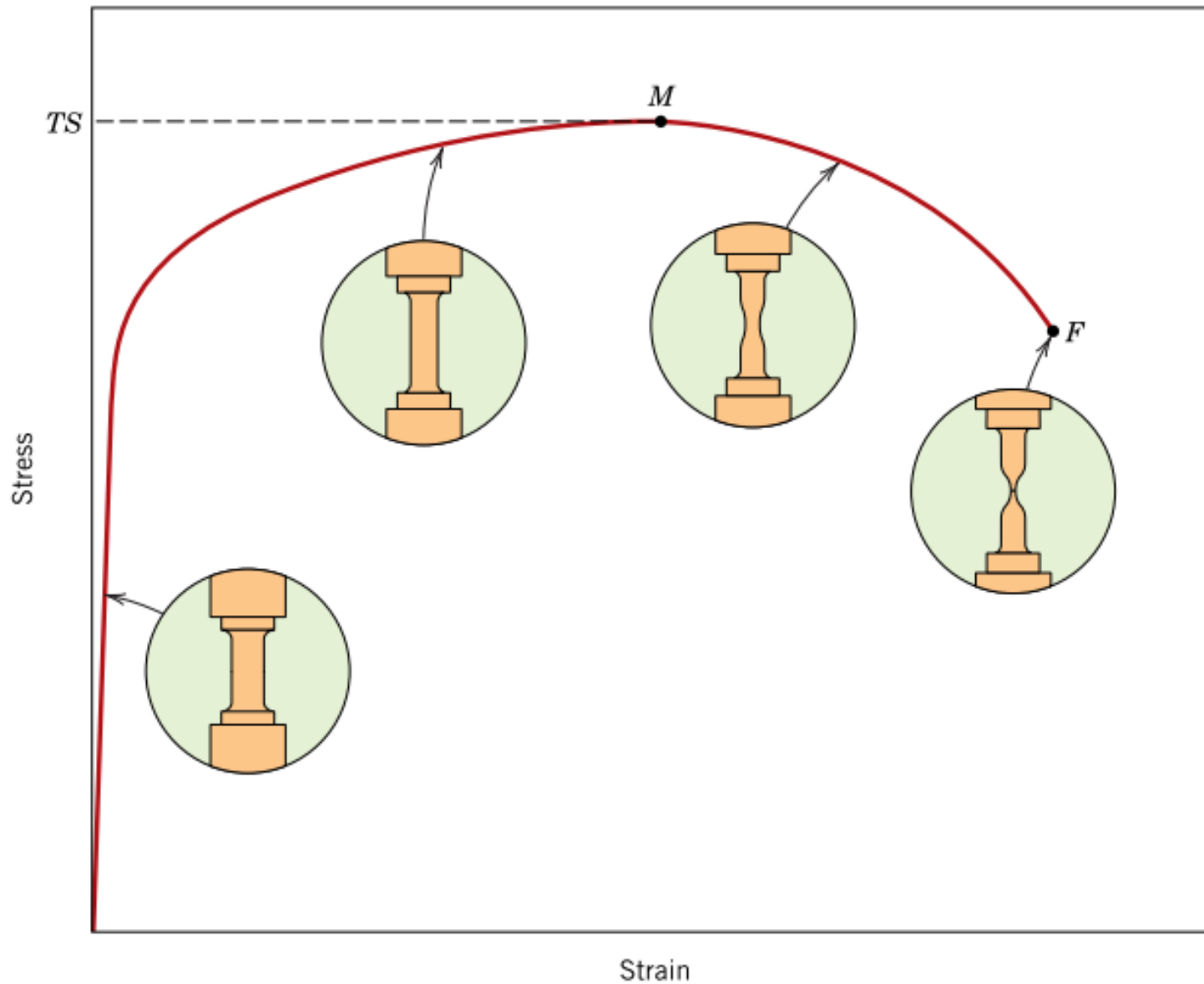
Energy loss during a cycle is frequency dependent.

In common tyres the energy loss is high at speed ~ 1000 kmph.

Plastic deformation

This is a permanent deformation or change in shape of a solid body without fracture under the action of a sustained force.

In Plastic deformation

1. Stress is not proportional to strain

2. Deformation is not reversible

3. Deformation occurs by breaking a re-arrangement of atomic bonds
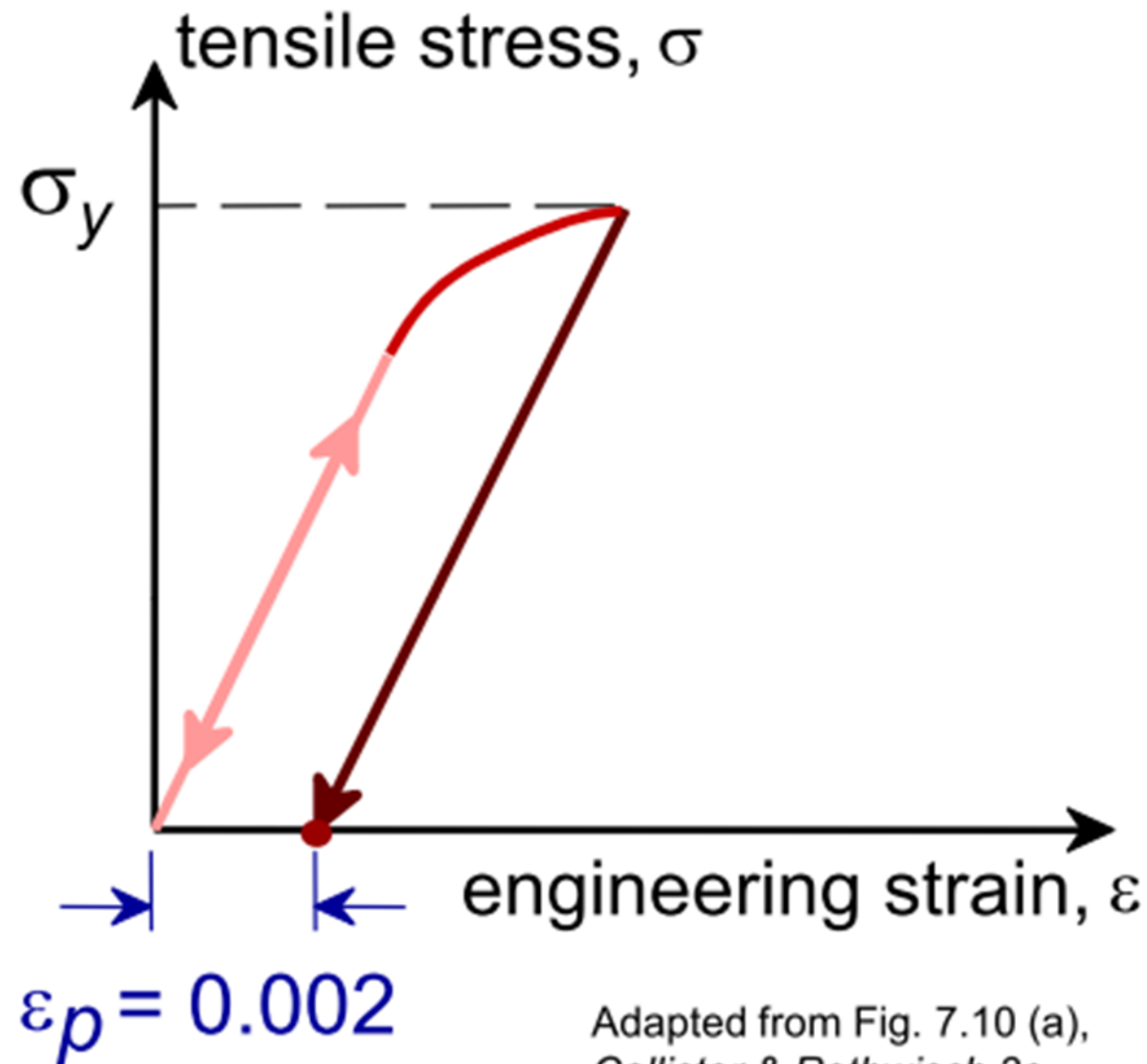
After point M, stress decreases.

Does this mean that the material is becoming weaker?

Necking leads to smaller cross sectional area!

Stress at which noticeable plastic deformation occurred.

when $\varepsilon_p = 0.002$

σ_y = yield strength

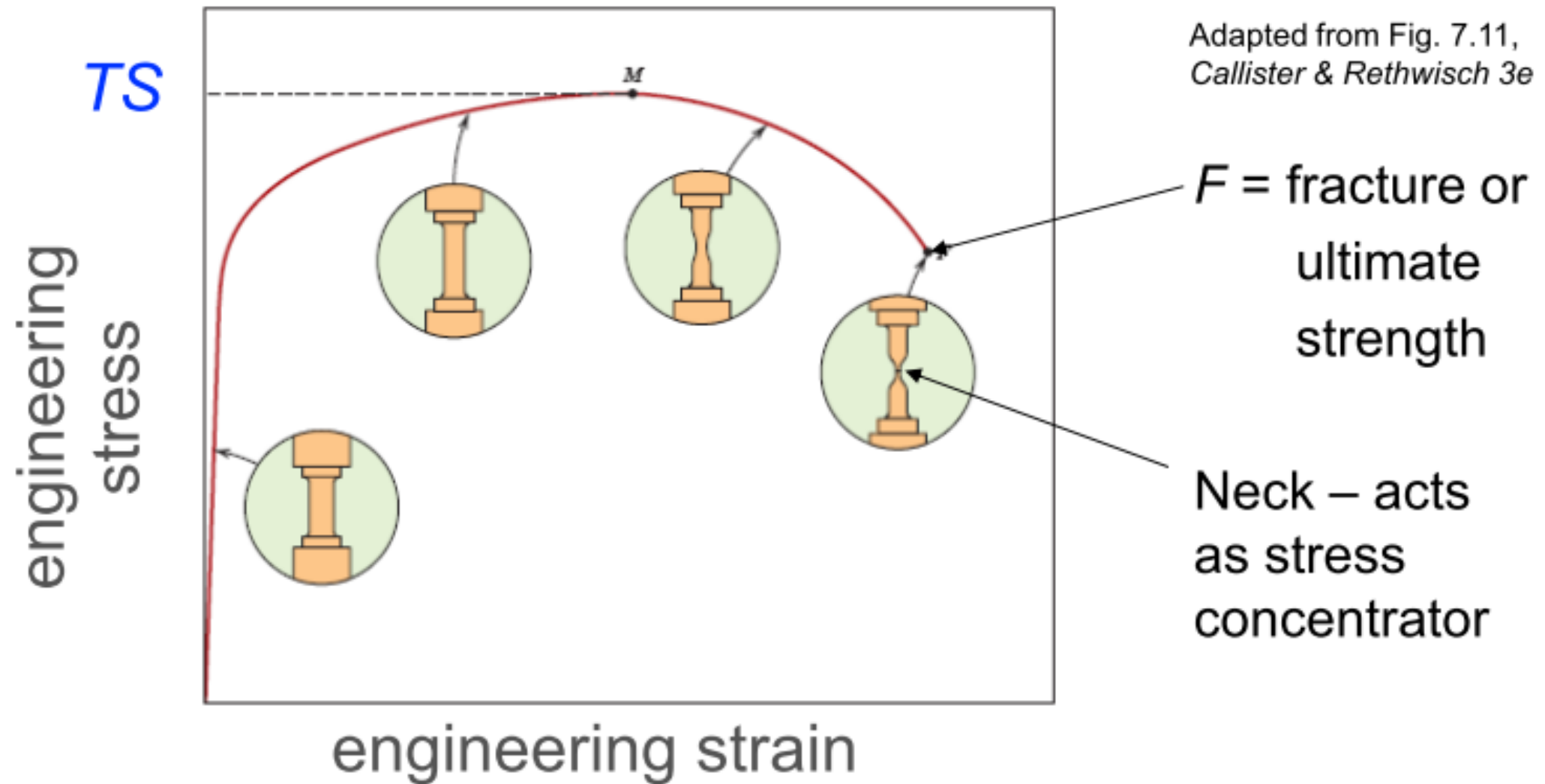Note: for 2 inch sample

$$\varepsilon = 0.002 = \Delta z/z$$

$$\therefore \Delta z = 0.004 \text{ in}$$

tensile stress, σ

σ_y

engineering strain, ε

$\varepsilon_p = 0.002$

Adapted from Fig. 7.10 (a),
Callister & Rethwisch 3e.

Maximum stress on engineering stress-strain curve

Adapted from Fig. 7.11,
Callister & Rethwisch 3e

TS

M

F = fracture or ultimate strength

Neck – acts as stress concentrator

engineering stress

engineering strain
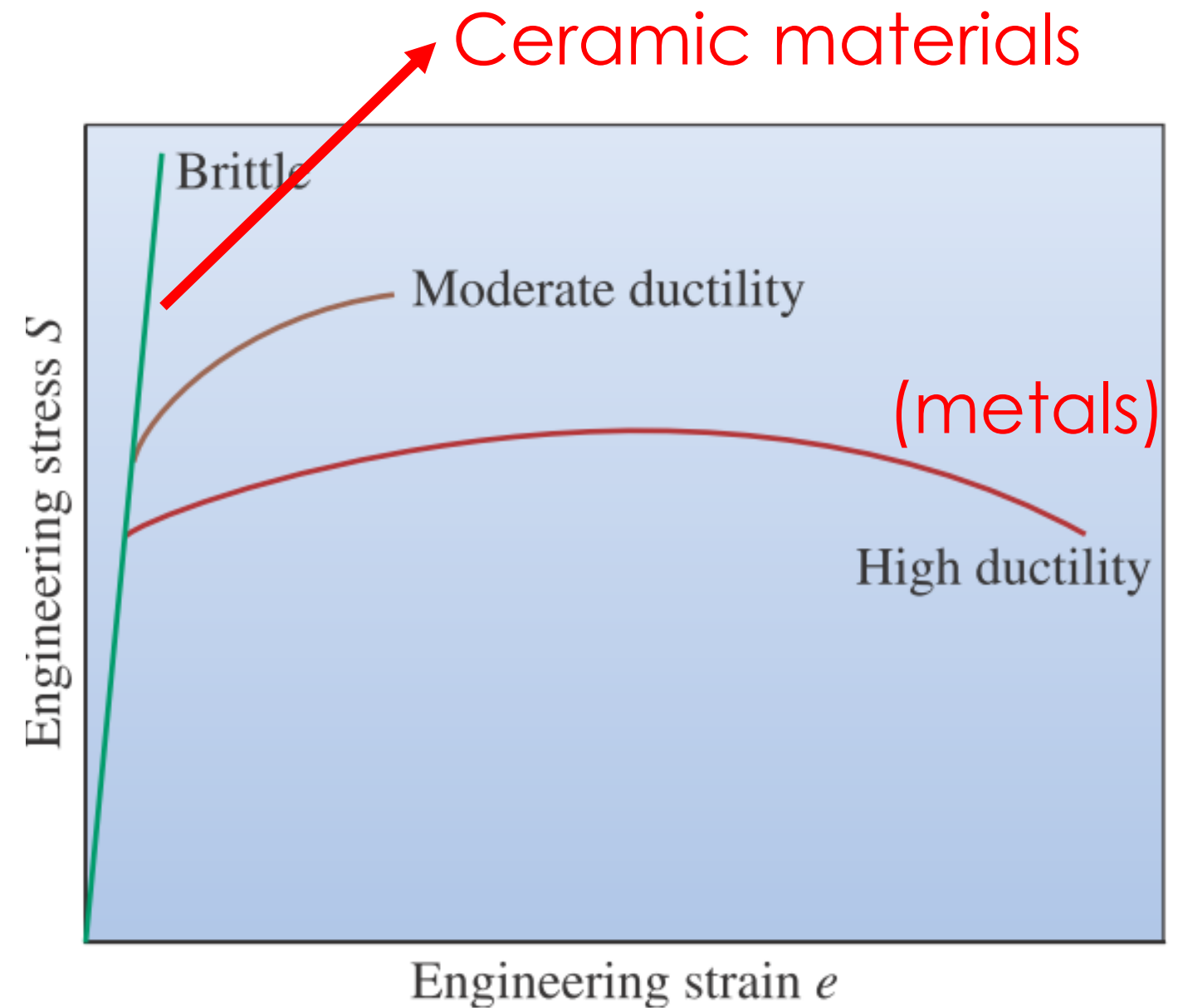
- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Ductility

Measure of degree of plastic deformation that has been sustained at fracture.

Ductile materials can undergo significant plastic deformation before fracture.
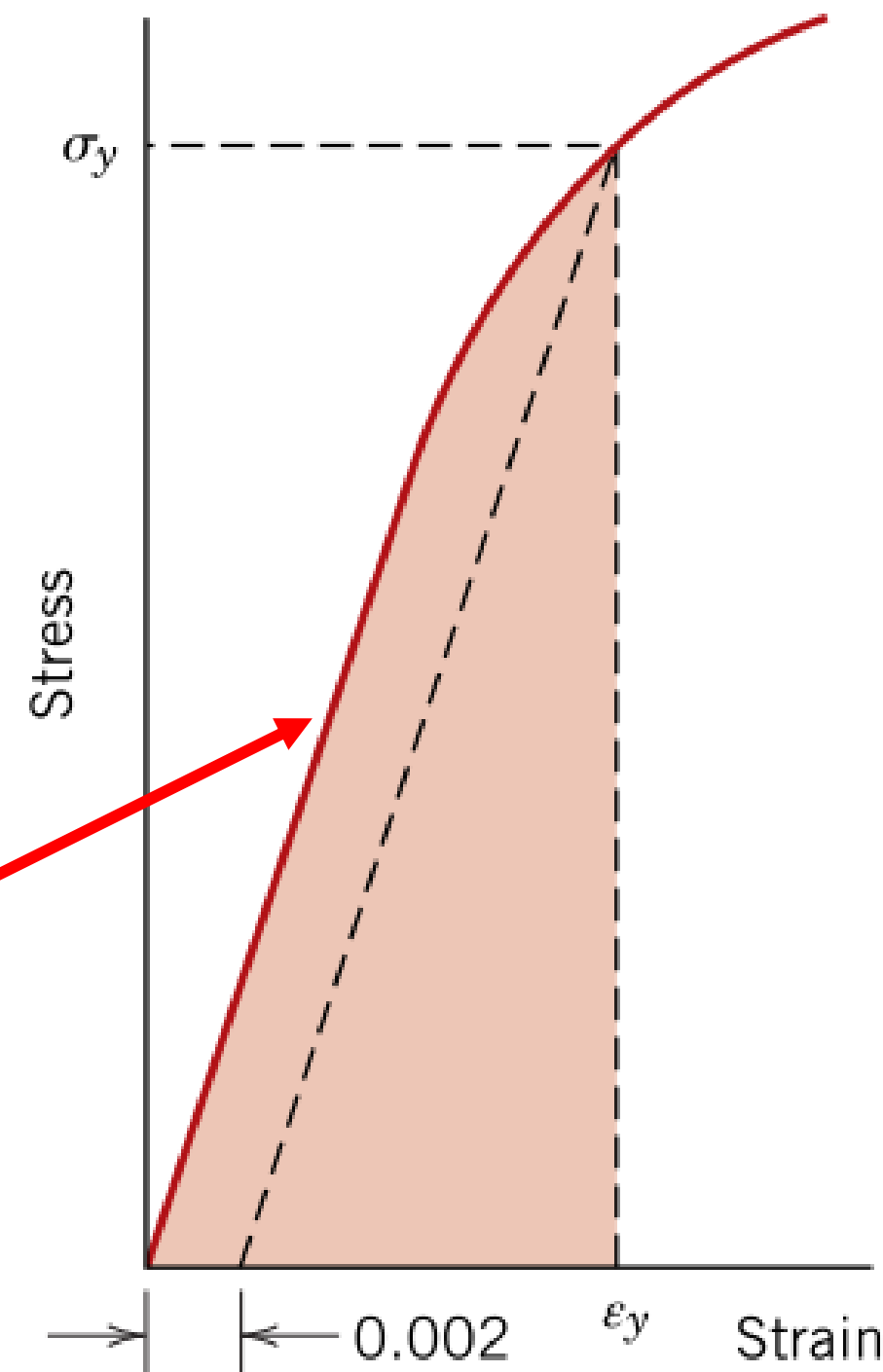
Brittle materials can tolerate only very small plastic deformation.

Stiffness

Stiffness is the extent to which an object resists deformation in response to an applied force. The complementary concept is flexibility or pliability.
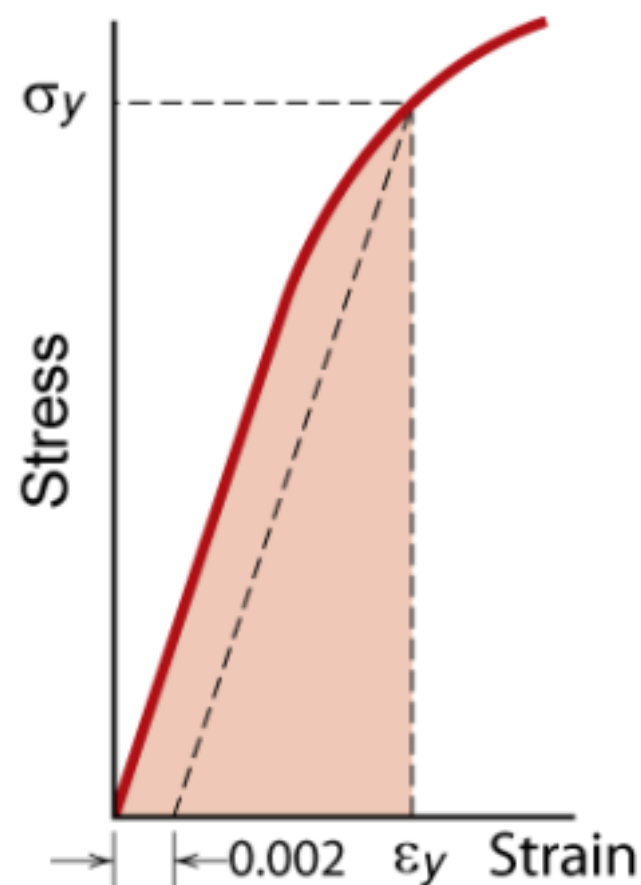
The more flexible an object is, the less stiff it is.

Slope = Stiffness

Ability of a material to store energy

Energy stored best in elastic region

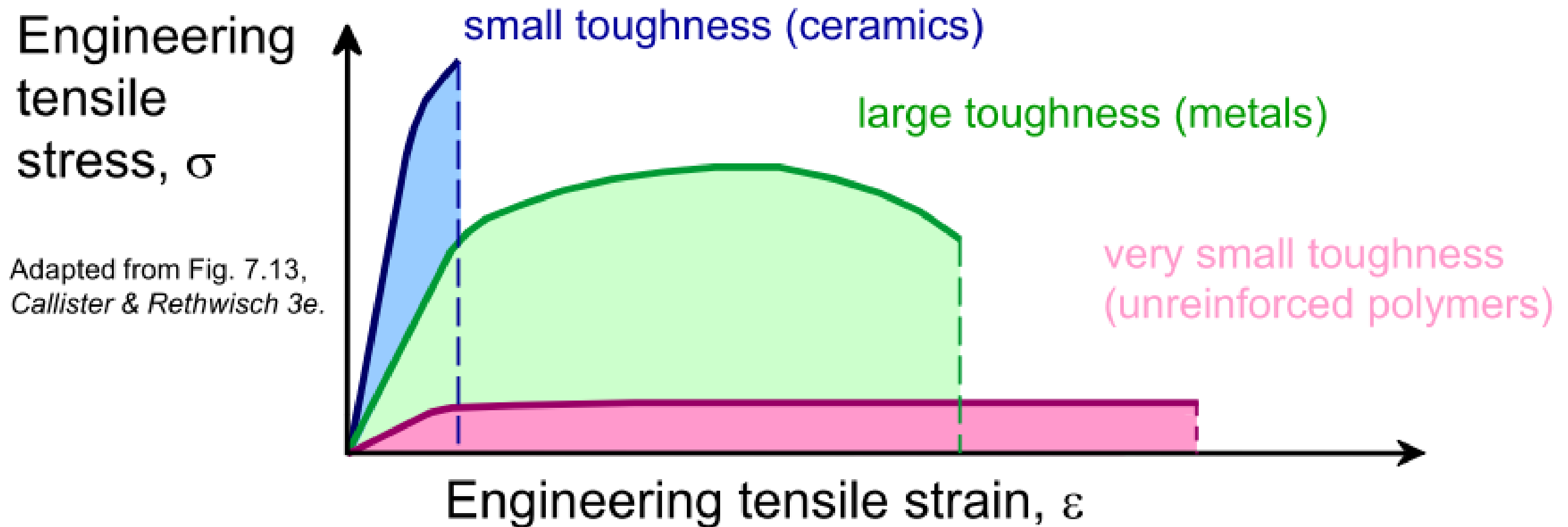$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2} \sigma_y \varepsilon_y$$

Stress

σy

←0.002 εy Strain

Energy to break a unit volume of a material

Approximate by the area under the *stress-strain curve*.

Engineering tensile stress, σ

small toughness (ceramics)

large toughness (metals)

very small toughness (unreinforced polymers)

Adapted from Fig. 7.13, Callister & Rethwisch 3e.

Engineering tensile strain, ε
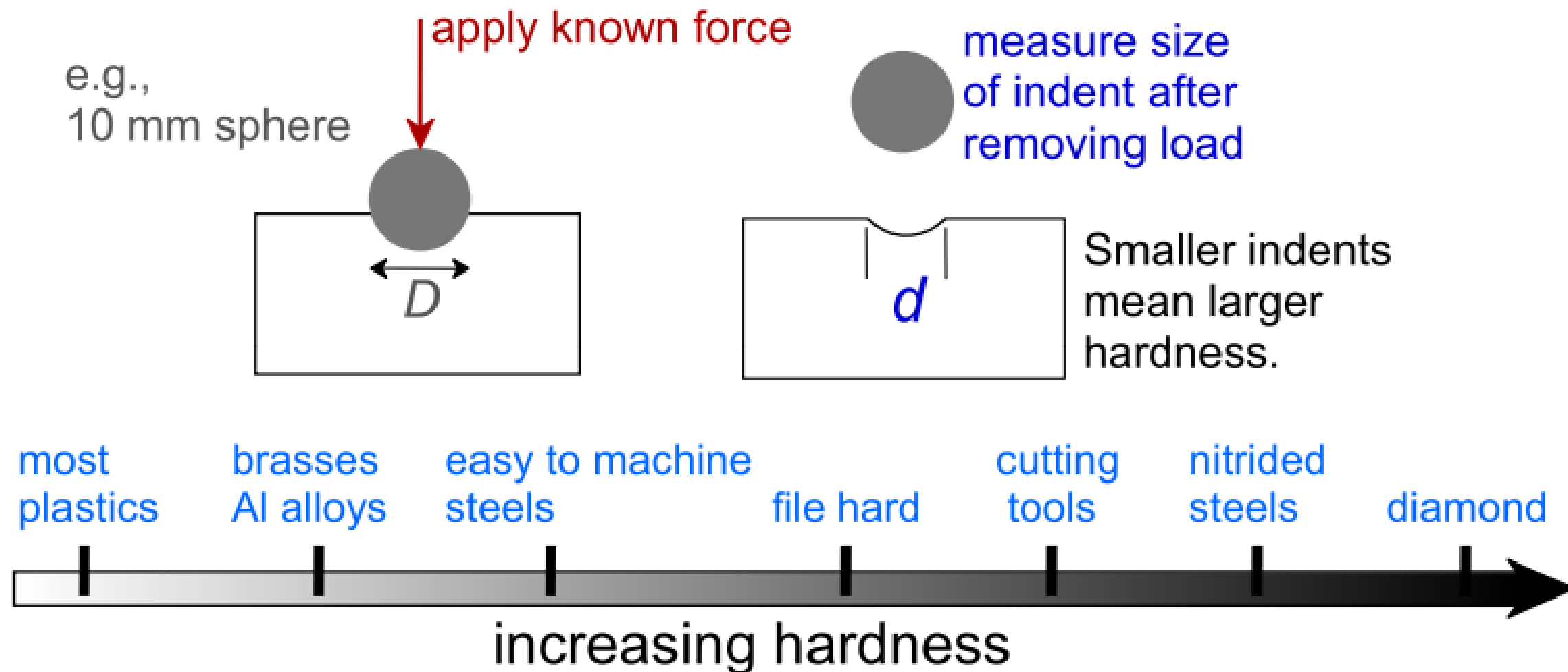
Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resistance to permanent deformation on the surface.

Large hardness means

Resistance to plastic deformation or cracking in the compression.

Better wear properties.

e.g.,
10 mm sphere

apply known force

measure size
of indent after
removing load

D

d

Smaller indents
mean larger
hardness.

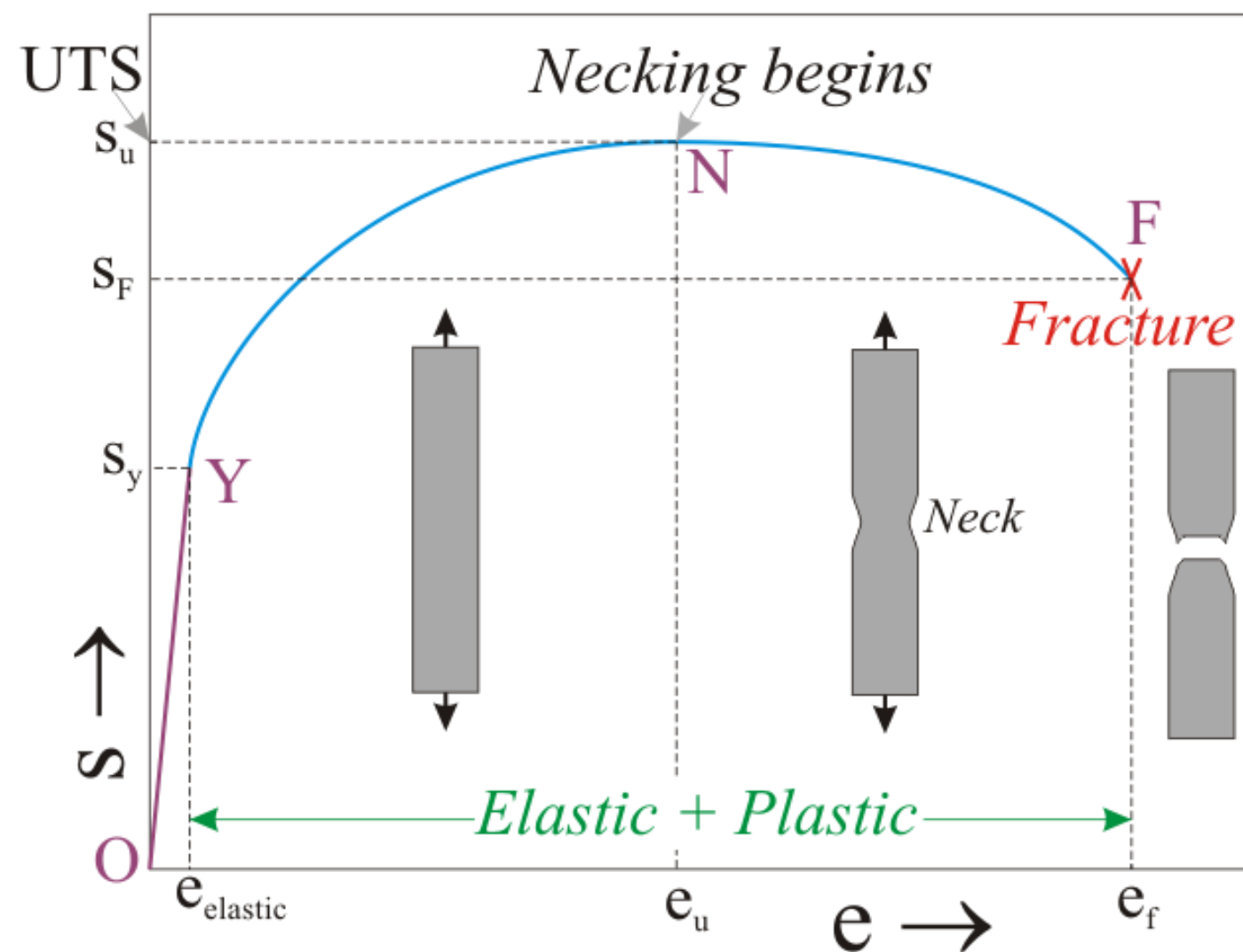| most plastics | brasses Al alloys | easy to machine steels | file hard | cutting tools | nitrided steels | diamond |

increasing hardness

Engineering stress

$$\sigma(S) = \frac{F}{A_0} = \frac{\text{Applied stress}}{\text{Initial cross} - \text{sectional area}}$$
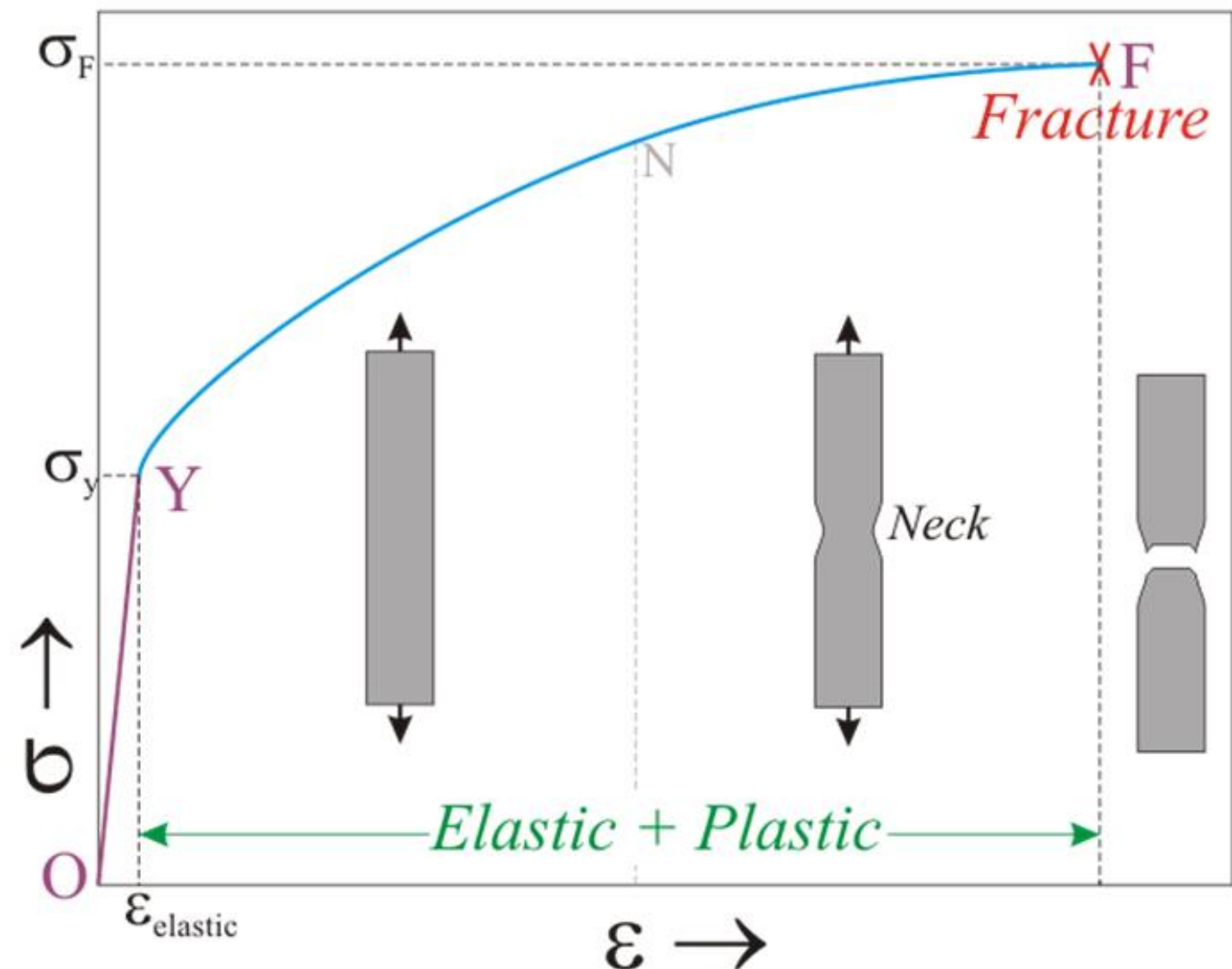
Engineering strain

$$\epsilon(e) = \frac{\Delta l}{l_0} = \frac{\text{Change in length}}{\text{Original length}}$$

True stress $\qquad \sigma_T = \dfrac{F}{A_i} = \dfrac{\text{Applied stress}}{\text{Actual (instantaneous)area}}$

True strain $\qquad \epsilon_T = \displaystyle\int_{l_0}^{l} \dfrac{dl}{l}$

True stress $\sigma_T = \dfrac{F}{A_i} = \dfrac{\text{Applied stress}}{\text{Actual (instantaneous)area}}$

True strain $\epsilon_T = \displaystyle\int_{l_0}^{l} \dfrac{dl}{l}$

For an applied force F,

   cross-sectional area = A_i

   change in length = $l_i - l_o$

Engineering strain

$$\epsilon = \dfrac{l_i - l_0}{l_0} = \dfrac{l_i}{l_0} - 1$$

$$\Rightarrow 1 + \epsilon = \dfrac{l_i}{l_0}$$

$$\therefore \boldsymbol{\epsilon_T} = \int_{l_0}^{l_i} \dfrac{\boldsymbol{dl}}{\boldsymbol{l}} = \boldsymbol{\ln}\dfrac{\boldsymbol{l_i}}{\boldsymbol{l_0}} = \boldsymbol{\ln(1 + \epsilon)}$$

$$\boldsymbol{\sigma_T} = \dfrac{\boldsymbol{F}}{\boldsymbol{A_i}} = \dfrac{\boldsymbol{F}}{\dfrac{\boldsymbol{A_0 l_0}}{\boldsymbol{l_i}}} = \dfrac{\boldsymbol{F}}{\boldsymbol{A_0}}\dfrac{\boldsymbol{l_i}}{\boldsymbol{l_0}} = \boldsymbol{\sigma(1 + \epsilon)}$$

Assuming that volume remains same, i.e., $A_0 l_0 = A_i l_i$

True

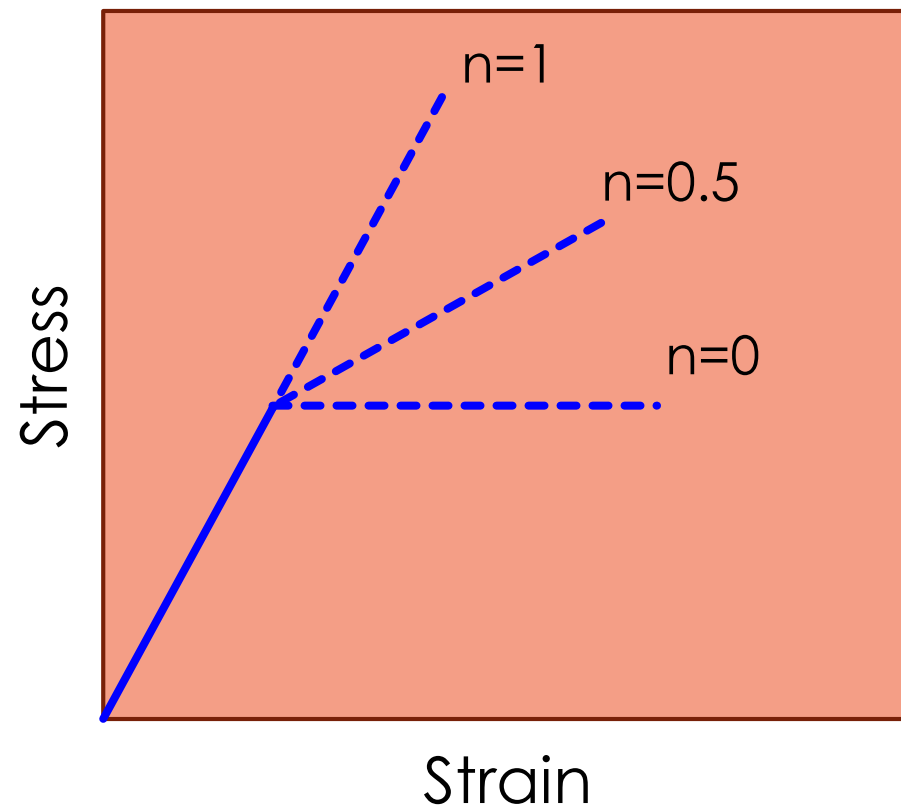Engineering

Stress

Strain

Similar till yield point

$$\sigma_T = K\epsilon_T^n$$

K = strength coefficient
n = work hardening exponent
n = 0.5 → Large plastic strain (copper and brass)
n = 0.15 → less strain (heat treated steel)

Material	n	K MPa	psi
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered at 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

$$\sigma_T = A \left(\frac{d\epsilon}{dt}\right)^m$$

A = Constant
m = strain rate sensitivity

m = 0 → stress is strain rate independent
m = 0.2 for common metals
m = 0.4 – 0.9 → super-plastic behavior (fine grained steel)
- material can deform without necking
m=1 → material behave like viscous liquid

1. Anelastic process is a time dependent elastic deformation process.

2. For may practical applications the relaxation time should be larger than the load unload cycle.

3. Relaxation is temperature dependent process.

4. True stress-strain curve is the best to find out the mechanical properties of materials.