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SET THEORY

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Topics covered

- Set Identities
- Proving set identities

Set Identities

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Proving Set Identities

- Subset Method
- Set-builder notation and propositional logic
- Membership Tables

Proof using subset method

◦ Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

We will prove this identity by showing that

i) $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$, and

ii) $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Proof using subset method

i) To Prove: $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

(We do this by showing that if *x* is in $\overline{A \cap B}$, then it must also be in $\overline{A} \cup \overline{B}$.)

$x \in \overline{A \cap B}$	by assumption
$x \notin A \cap B$	defn. of complement
$\neg((x \in A) \wedge (x \in B))$	defn. of intersection
$\neg(x \in A) \vee \neg(x \in B)$	1st De Morgan Law for Prop Logic
$x \notin A \vee x \notin B$	defn. of negation
$x \in \overline{A} \vee x \in \overline{B}$	defn. of complement
$x \in \overline{A} \cup \overline{B}$	defn. of union

Hence, $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

Proof using subset method

ii) To Prove: $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

(We do this by showing that if *x* is in $\overline{A} \cup \overline{B}$, then it must also be in $\overline{A \cap B}$.)

$x \in \overline{A} \cup \overline{B}$	by assumption
$(x \in \overline{A}) \vee (x \in \overline{B})$	defn. of union
$(x \notin A) \vee (x \notin B)$	defn. of complement
$\neg(x \in A) \vee \neg(x \in B)$	defn. of negation
$\neg((x \in A) \wedge (x \in B))$	by 1st De Morgan Law for Prop Logic
$\neg(x \in A \cap B)$	defn. of intersection
$x \in \overline{A \cap B}$	defn. of complement

Hence, $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

Therefore, $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Proof using set-builder notation and propositional logic

- Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

$$
\begin{aligned}
\overline{A \cap B} &= \{x \mid x \notin A \cap B\} && \text{by defn. of complement} \\
&= \{x \mid \neg(x \in (A \cap B))\} && \text{by defn. of does not belong symbol} \\
&= \{x \mid \neg(x \in A \wedge x \in B\} && \text{by defn. of intersection} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} && \text{by 1st De Morgan law} \\
& && \text{for Prop Logic} \\
&= \{x \mid x \notin A \vee x \notin B\} && \text{by defn. of not belong symbol} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B}\} && \text{by defn. of complement} \\
&= \{x \mid x \in \overline{A} \cup \overline{B}\} && \text{by defn. of union} \\
&= \overline{A} \cup \overline{B} && \text{by meaning of notation}
\end{aligned}
$$

Proof using membership table

◦ Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**1**	**1**	**0**	**0**	**0**	**0**
0	**1**	**0**	**1**	**0**	**0**	**0**	**0**
0	**1**	**1**	**1**	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**1**	**1**	**0**	**1**	**1**
1	**1**	**0**	**1**	**1**	**1**	**0**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**

LHS

RHS

Since both columns (LHS & RHS are identical, therefore,
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exercise…

- Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$

  a) by showing each side is a subset of the other side.

  b) using a membership table.

  c) using set-builder notation and propositional logic.

References

◦ Rosen H. K., Discrete Mathematics and its Applications, McGraw Hill (2011) 7th ed.

Questions ?