Roll Number:________________

Thapar University, Patiala

Department of Computer Science and Engineering

END SEMESTER EXAMINATION

B. E. (Second Year): Semester-III (2017-18)
Branch: COE

Course Code: UCS405
Course Name: Discrete Mathematical Structures

Date: 13^{th} December, 2017
Time: 3 Hours, M. Marks: 100

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Note: Attempt all questions with proper justification. Assume missing data, if any, suitably.

Q1(a) Let S = {a, b, c, d, e, f, g}. Determine which of the following are partitions of S. Provide explanation for each case. (4)

(i) P_1 = [{a, c, e}, {b}, {d, g}] (iii) P_3 = [{a, b, e, g}, {c}, {d, f}]

(ii) P_2 = [{a, e, g}, {c, d}, {b, e, f}] (iv) P_4 = [{a, b, c, d, e, f, g}]

Q1(b) The formula $A/B = A \cap B^C$ defines the difference operation in terms of the operations of intersection and complement. (2)
Find a formula that defines the union $A \cup B$ in terms of the operations of intersection and complement.

Q1(c) Each of the following defines a relation on the positive integers *N*: (4)

(1) "x is greater than y" (3) x + y = 10

(2) "xy is the square of an integer" (4) x + 4y = 10

Determine which of the relations are: (i) reflexive (ii) symmetric (iii) antisymmetric (iv) transitive.

Q2(a) Let a and b be positive integers, and suppose Q is defined recursively as follows: (3)

$$Q(a,b) = \begin{cases} 0, & \text{if } a<b \\ Q(a-b,b)+1, & \text{if } b \le a \end{cases}$$

(i) Find: (i) Q(2, 5) (ii) Q(12, 5)

(ii) What does this function Q do? Find Q(5861, 7).

Q2(b) Let k be a positive integer. Show that $1^k + 2^k + \cdot \cdot \cdot + n^k$ is $O(n^{k+1})$. (3)

Q2(c) Find an ordering of the tasks of a software project if the Hasse diagram for the tasks of the project is as shown in Figure 1. Show and explain all the steps used to arrive at the ordering. (5)

---------------P.T.O.---------------

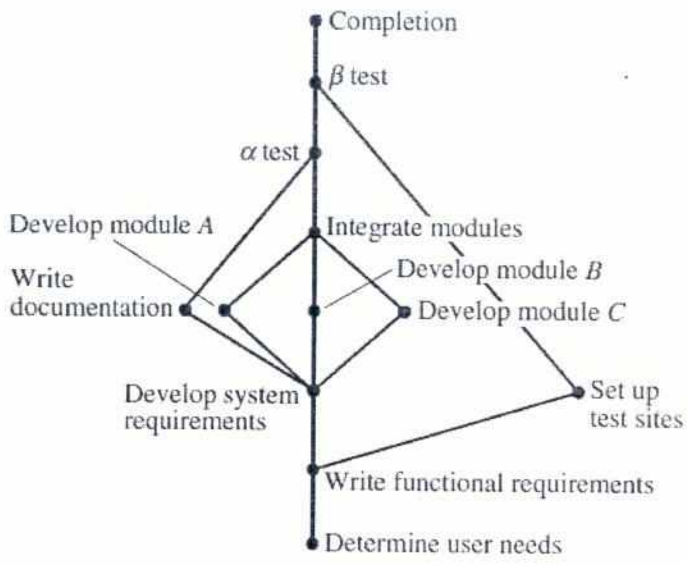

Figure 1

Q3(a) How many students do you need in a school to guarantee that there are at least 2 students who have the same first two initials? (2)

Q3(b) Prove that if n is an integer and $3n + 2$ is even, then n is even using (6)
(i) a proof by contraposition.
(ii) a proof by contradiction.

Q3(c) Consider the second-order homogeneous recurrence relation (8)
$a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1, a_1 = 6$,
(i) Find the next two terms of the sequence.
(ii) Find the general solution.
(iii) Find the unique solution using the initial conditions.

Q4(a) Let G = {1,2,4,7,8,11,13,14} be a set under multiplication modulo 15. Then (10)
(i) Find the multiplication table of G.
(ii) Find inverses of 2, 7, 11.
(iii) Find the orders and subgroups generated by 2 and 7.
(iv) Is G cyclic?

Q4(b) Let * be the operation on a set R of real numbers defined by $a * b = a + b + 2ab$. (8)
(i) Find 2*3, 3*(-5) and 7*(1/2).
(ii) Is (R,*) a semigroup? Is it commutative?
(iii) Find the identity element.
(iv) Which elements have inverses and what are they?

Q5(a) Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people. (4)
(i) No professors are ignorant.
(ii) All ignorant people are vain.

(iii) No professors are vain.

(iv) Does (iii) follow from (i) and (ii)? Explain.

Q5(b) Show that $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$. (5)

Q6(a) The mathematics department has six committees, each meeting once a month. How many different meeting times must be used to ensure that no member is scheduled to attend two meetings at the same time if the committees are $C1$ = {Arlinghaus, Brand, Zaslavsky}, $C2$ = {Brand, Lee, Rosen}, $C3$ = {Arlinghaus, Rosen, Zaslavsky}, $C4$ = {Lee, Rosen, Zaslavsky}, $C5$ = {Arlinghaus, Brand}, and $C6$ = {Brand, Rosen, Zaslavsky}? Also tell which committees can hold meeting at the same time. Draw the graph and explain the process used to arrive at the answer. (10)

Q6(b) Determine whether the graphs given in Figure 2 are isomorphic or not. Explain your answer. If they are isomorphic, then establish an isomorphism f from vertices of graph G to vertices of graph H. (10)

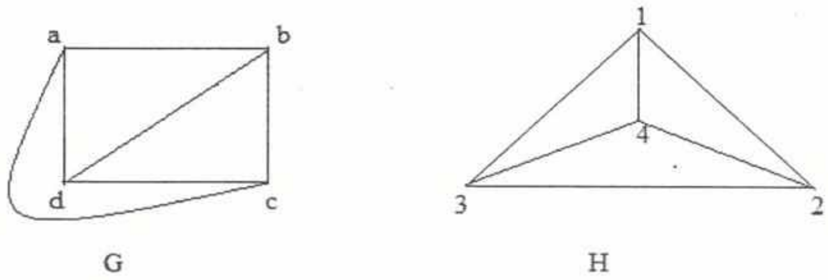

Figure 2

Q7(a) Use Warshall's algorithm to find the transitive closure of the relation (12)
R = {*(a, e), (b, a), (b, d), (c,d), (d,a), (d, c), (e,a), (e,b), (e, c), (e, e)*} on {a,b,c,d,e}.

Q7(b) Let A={1,2,3}, B={a,b,c}, and C={x,y,z}. Consider the following relations R and S from A to B and from B to C respectively. (4)

R = {(1,b), (2,a), (2,c)} and S = {(a,y), (b,x), (c,y), (c,z)}

(i) Find the composition relation R o S.

(ii) Find the matrices M_R, M_S, and $M_{R \circ S}$ of the respective relations R, S and R o S. Compare $M_{R \circ S}$ to the product $M_R M_S$ and draw the conclusion.

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