Theory of Machines

Module : Balancing

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Lecture Contents and Learning Outcomes

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- Balancing of Rotating Masses
- Balancing of Reciprocating Masses
- Single Cylinder in-line Engine
- Multi-cylinder in-line Engine
- Field Balancing of Rotors

Learning Outcomes

Balancing

Static and Dynamic Balancing

Single and Multicylinder in-line engine

References

1. S S Ratan “Theory of Machines” 3rd Edition, Tata Macgraw Hill Publications
2. J. J. Uicker, G. R. Pennock, and J. E. Shigley “Theory of Machines and Mechanisms” Oxford Press (2009)
3. Neil Sclater, Nicholas P. Chironis “Mechanisms and Mechanical Devices Sourcebook” 4th Edition, McGraw Hill Publications
4. R S Khurmi “A text Book of Theory of Machines” S Chand Publications

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Balancing

- Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses.
- Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.
- Serious problems encountered in high-speed machinery are the direct result of unbalanced forces.
- These forces exerted on the frame by the moving machine members are time varying, impart vibratory motion to the frame and produce noise.
- Also, there are human discomfort and detrimental effects on the machine performance and the structural integrity of the machine foundation.
- There are two basic types of unbalance – rotating and reciprocating unbalance- which may occur separately or in combination.

Rotary Balancing: Static

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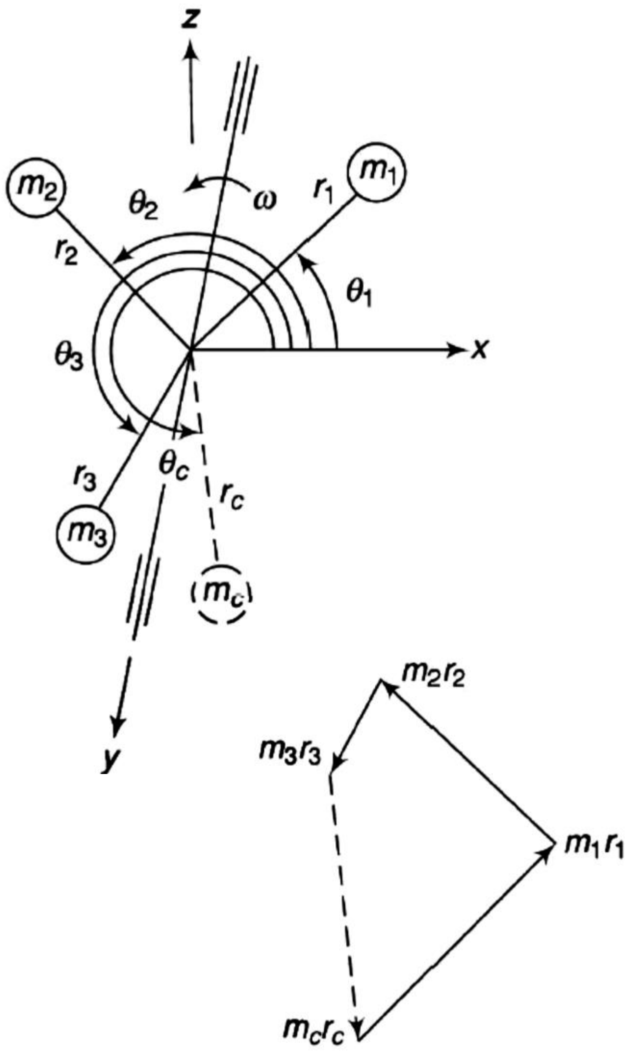
- A system of rotating masses is said to be in static balance if the combined mass center of the system lies on the axis of rotation.
- Let F be the sum of all the forces acting on the rotor. The rotor is said to be statically balanced if the vector sum F is zero.

$$\mathbf{F} = m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2$$

- If F is not zero, i. e., the rotor is unbalanced, then introduce a counterweight m_c, at radius r_c to balance the rotor so that

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 + m_c\mathbf{r}_c\omega^2 = 0 \Rightarrow m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 = 0 \longrightarrow 1$$

- The magnitude of either m_c or r_c may be selected and of the other may be calculated.
- In general: $\Sigma m\mathbf{r} + m_c\mathbf{r}_c = 0 \longrightarrow 2$
- The equation can be solved either mathematically or graphically.



Rotary Balancing: Static contd…

- To solve it mathematically, divide each force into its *x* and *z* components.

$$\sum mr\cos\theta + m_c r_c \cos\theta_c = 0 \quad \text{and} \quad \sum mr\sin\theta + m_c r_c \sin\theta_c = 0$$

or $$m_c r_c \cos\theta_c = -\sum mr\cos\theta \longrightarrow i$$

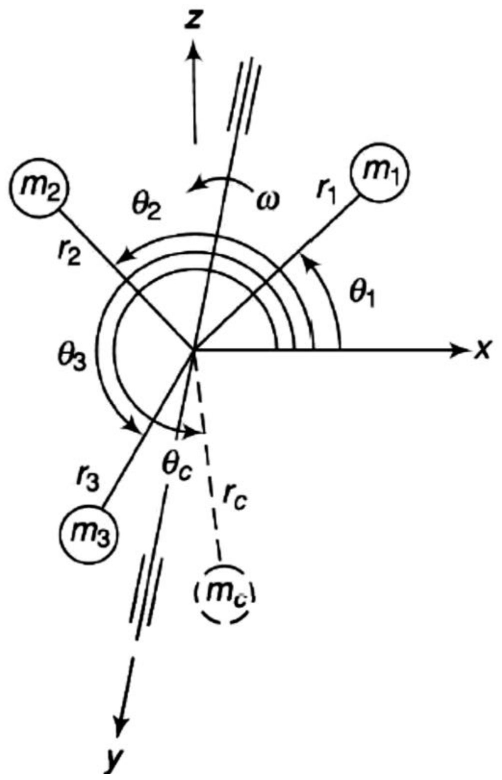
and $$m_c r_c \sin\theta_c = -\sum mr\sin\theta \longrightarrow ii$$

- Squaring and adding (*i*) and (*ii*)

$$m_c r_c = \sqrt{\left(\sum mr\cos\theta\right)^2 + \left(\sum mr\sin\theta\right)^2} \longrightarrow 3$$

- Dividing (*ii*) by (*i*)

$$\tan\theta_c = \frac{-\sum mr\sin\theta}{-\sum mr\cos\theta} \longrightarrow 4$$



- The signs of the numerator and denominator of this function identify the quadrant of the angle.

Worked Example 1

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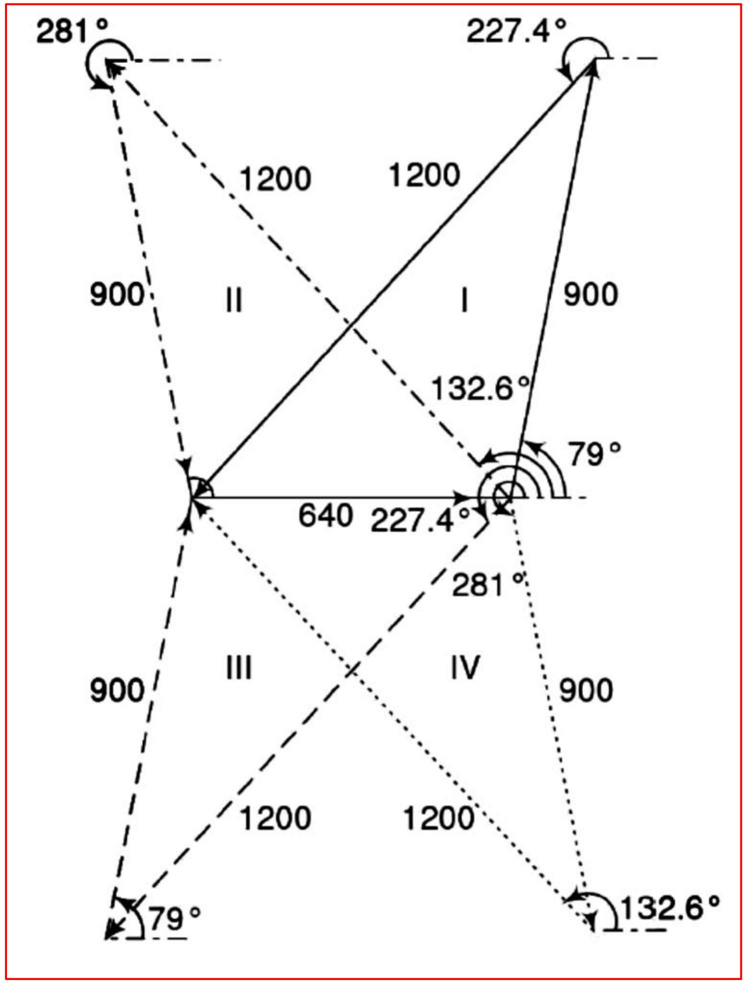
1. Three masses of 8 *kg*, 12 *kg* and 15 *kg* attached at radial distances of 80 *mm*, 100 *mm* and 60 *mm* respectively to a disc on a shaft are in complete balance. Determine the angular positions of the masses of 12 *kg* and 15 *kg* relative to the 8-*kg* mass.

Solution: $m_1r_1 = 640$, $m_2r_2 = 1200$, $m_3r_3 = 900$

Graphical Method:



Analytical Method: $\sum m\mathbf{r} = 0$

$$640 \cos 0° + 1200 \cos\theta_2 + 900 \cos\theta_3 = 0$$

$$1200 \cos\theta_2 = -(640 + 900 \cos\theta_3) \rightarrow i$$

$$640 \sin 0° + 1200 \sin\theta_2 + 900 \sin\theta_3 = 0$$

$$1200 \sin\theta_2 = -900 \sin\theta_3 \rightarrow ii$$

Worked Example 1

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Squaring and adding (*i*) and (*ii*)

$$1200^2 = 640^2 + 900^2 \cos^2\theta_3 + 2 \times 640 \times 900 \times \cos\theta_3 + 900^2 \sin^2\theta_3$$

$$= 640^2 + 900^2 + 2 \times 640 \times 900 \times \cos\theta_3$$

$$\cos\theta_3 = 0.1913$$

$$\theta_3 = 79^{\circ} \text{ or } 281^{\circ}$$

When $\theta_3 = 79^{\circ}$, $1200 \sin \theta_2 = -900 \sin 79^{\circ}$

$$\sin \theta_2 = -0.736$$

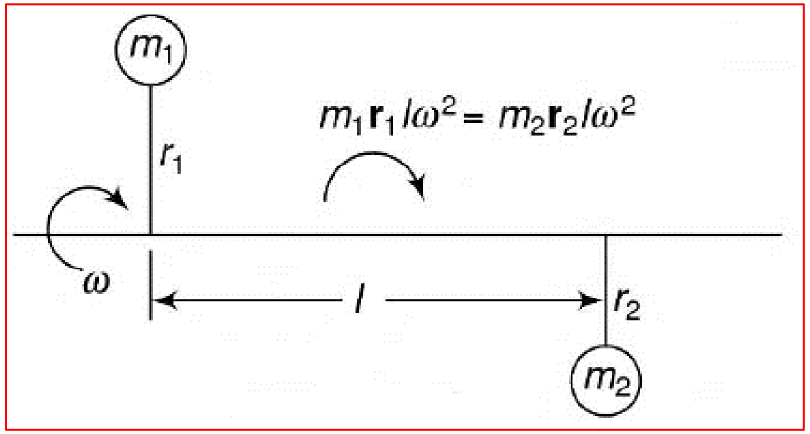
$$\theta_2 = -47.4^{\circ} \text{ or } 132.6^{\circ} \text{ or } 227.4^{\circ}$$

But as $\sin\theta_2$ is negative and $\cos\theta_2$ is also negative which can be found from equation (i), the corresponding angle $\theta_2 = 227.4^{\circ}$

In a similar way by taking $\theta_3 = 281^{\circ}$, θ2 can be found to be 132.6°

Dynamic Balancing

- A system of rotating masses is in dynamic balance when there does not exists any resultant centrifugal force as well as resultant couple.
- Force = mr, Couple = mrl
- Centrifugal forces are balanced ($m_1r_1 = m_2r_2$), but couple of magnitude ($m_1r_1l = m_2r_2l$) is introduced.
- The couple acts in a plane that contains the axis of rotation and the two masses.
- Thus, the couple is of constant magnitude but variable direction.



Force Transfer

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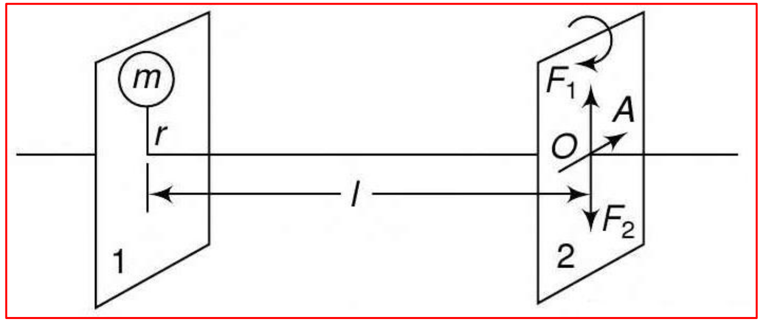
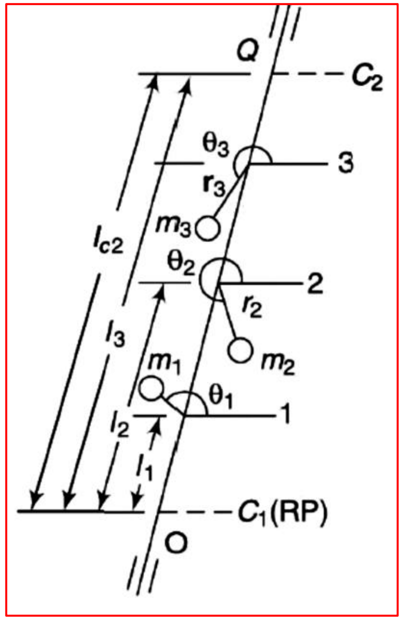
- The equilibrium of the system does not change if two equal and opposite forces $F_1 = F_2$ (= mr) are added in the latter plane.

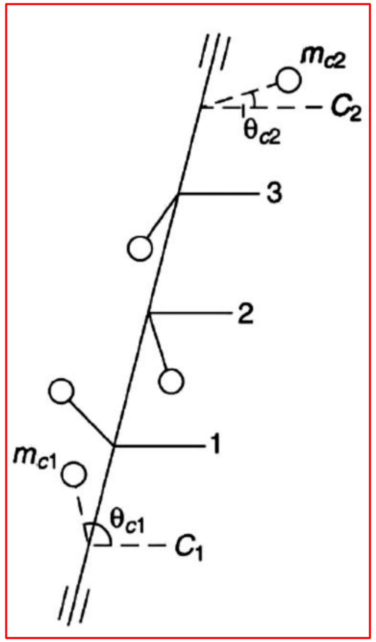


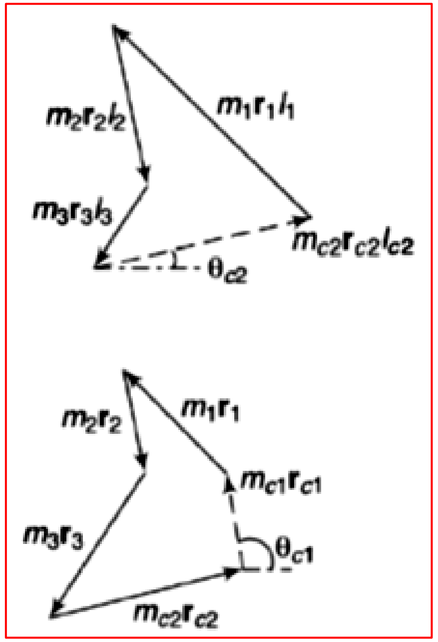
- The net effect would be a single force F_1 (= mr) in the second plane having the direction of the original force along with a couple mrl formed by the force mr and F_2 in a plane containing these forces and the shaft
- As the moment of a couple is the same about any point in its plane, the couple may be assumed to rotate the shaft about O. The axis od rotation of the couple is line OA drawn perpendicular to shaft through OA
- A line with suitable scale drawn parallel to the axis represent couple vectorially, the sense of rotation is given by right hand corkscrew rule. However in balancing problem, it is convenient to draw couple vectors through 90˚, by drawing them parallel to force vectors. This does not affect their relative positions.
- A plane passing through a point such as O and perpendicular to the axis of the shaft is called a *reference plane*.

Dynamic Balancing

- The unbalanced forces in RP are $m_1r_1\omega^2$, $m_2r_2\omega^2$ and $m_3r_3\omega^2$ acting radially outwards.
- The unbalanced couples in RP are $m_1r_1\omega^2l_1$, $m_2r_2\omega^2l_2$ and $m_3r_3\omega^2l_3$ which may be represented by vectors parallel to the respective force vectors, i. e. parallel to respective radii of m_1, m_2 and m_3. radially outwards.
- For complete balancing of rotor, the resultant force and resultant couple both should be zero







$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 = 0 \longrightarrow 1$$

and

$$m_1\mathbf{r}_1 l_1\omega^2 + m_2\mathbf{r}_2\, l_2\omega^2 + m_3\mathbf{r}_3\, l_3\omega^2 = 0 \longrightarrow 2$$

- A mass placed in the RP may satisfy the force equation but the couple equation is satisfied only by two equal forces in different transverse planes.

Dynamic Balancing

- Therefore, in order to satisfy Eqs (1) and (2), introduce two counter-masses m_{c1} and m_{c2} at radii r_{c1} and r_{c2} respectively. Then Eq (1) may be written as

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 + m_{c1}\mathbf{r}_{c1}\omega^2 + m_{c2}\mathbf{r}_{c2}\omega^2 = 0 \longrightarrow 3$$

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \longrightarrow 4$$

In general

$$\Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0 \longrightarrow 5$$

- Let the two countermasses be placed in transverse planes at axial locations O and Q, i. e. the countermass m_{c1} be placed in the RP and the distance of the plane of m_{c2} be l_{c2} from RP.
- Eq (2) modifies to (taking moments about O)

$$m_1\mathbf{r}_1 l_1\omega^2 + m_2\mathbf{r}_2\, l_2\omega^2 + m_3\mathbf{r}_3\, l_3\omega^2 + m_{c2}\mathbf{r}_{c2}\, l_{c2}\omega^2 = 0 \longrightarrow 6$$

$$m_1\mathbf{r}_1\, l_1 + m_2\mathbf{r}_2\, l_2 + m_3\mathbf{r}_3\, l_3 + m_{c2}\mathbf{r}_{c2}\, l_{c2} = 0 \longrightarrow 7$$

In general

$$\Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2}\, l_{c2} = 0 \longrightarrow 8$$

Dynamic Balancing

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- Equation (5) and (8) can be solved mathematically or graphically. Mathematical solution is as follows:
- Dividing Eq (8) into component form

$$\Sigma m\mathbf{r}l\cos\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = 0 \quad \text{and} \quad \Sigma m\mathbf{r}l\sin\theta + m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = 0$$

$$m_{c2}\mathbf{r}_{c2}l_{c2}\cos\theta_{c2} = -\Sigma m\mathbf{r}l\cos\theta \longrightarrow (i)$$

and

$$m_{c2}\mathbf{r}_{c2}l_{c2}\sin\theta_{c2} = -\Sigma m\mathbf{r}l\sin\theta \longrightarrow (ii)$$

- Squaring and adding (i) and (ii)

$$m_{c2}r_{c2}l_{c2} = \sqrt{(\Sigma\, mrl\cos\theta)^2 + (\Sigma mrl\sin\theta)^2} \longrightarrow 9$$

- Dividing (ii) by (i)

$$\tan\theta_{c2} = \frac{-\Sigma\, mrl\sin\theta}{-\Sigma\, mrl\cos\theta} \longrightarrow 10$$

- After obtaining values of m_{c2} and θ_{c2} from the above equations, solve Eq (5) by taking its components

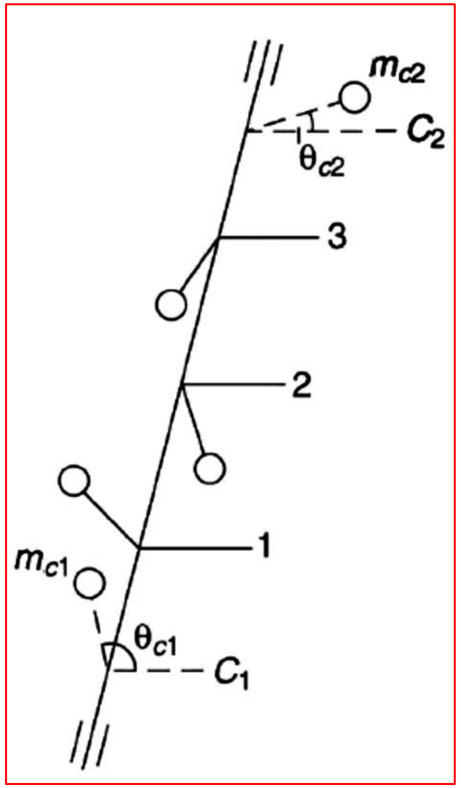
$$m_{c1}r_{c1} = \sqrt{(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})^2 + (\Sigma\, mrl\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})^2} \longrightarrow 11$$

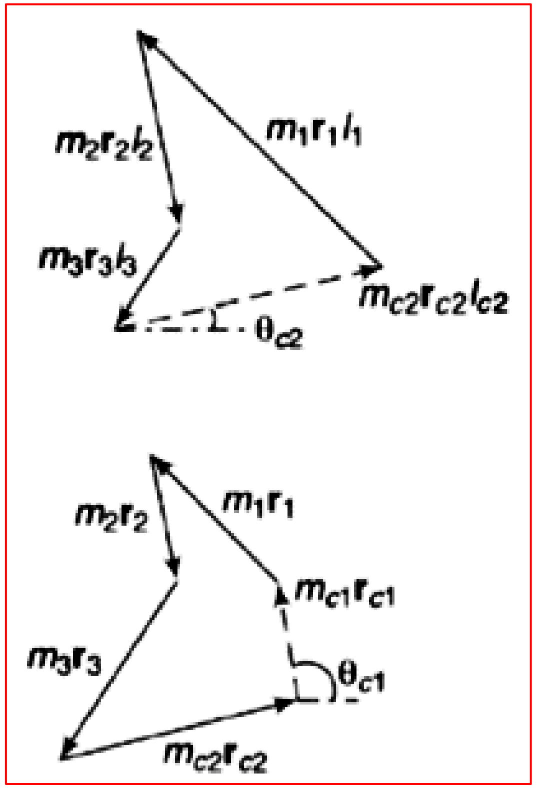
and

$$\boxed{\tan\theta_{c1} = \frac{-(\Sigma\, mr\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})}{-(\Sigma\, mr\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})}} \longrightarrow 12$$

Dynamic Balancing : Graphical Method

- Eq (5) and (8) can be solved graphically. Eq (8) is solved first and a couple polygon is made by adding the known vectors and considering each vector parallel to the radial line of the mass.
- The closing vector will be $m_{c2}r_{c2}l_{c2}$, the direction of which specifies the angular position of the countermass m_{c2} in the plane at point Q.
- Solve Eq (5) and make a force polygon by adding the known vectors (along with the vector $m_{c2}r_{c2}$).





- The closing vector is $m_{c1}r_{c1}$, identifying the magnitude and the direction of the countermass m_{c1}.

Worked Example 2

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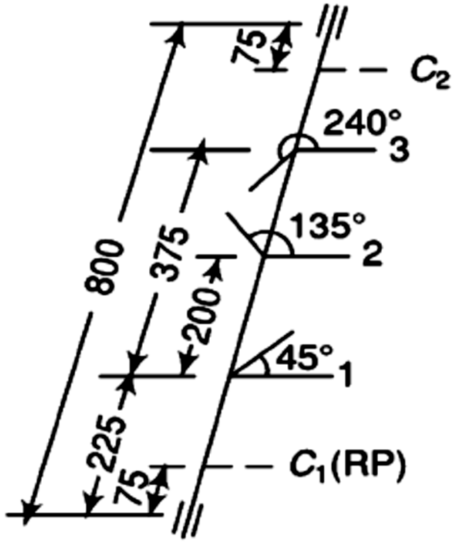
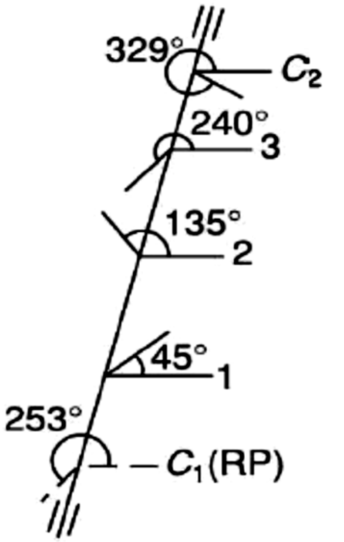
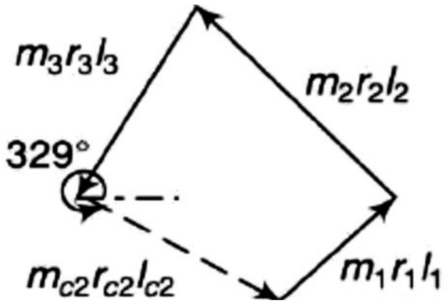
A rotating shaft carries three unbalanced masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The second and third masses are in the places at 200 mm and 375 mm from the plane of the first mass. The angular positions are measured counter-clockwise from the reference line along x-axis and viewing the shaft from the first mass end.

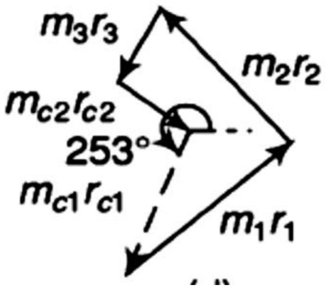
The shaft is 800 mm between bearings and the distance between the plane of the first mass and the bearing at that end is 225 mm.

Determine the amount of the counter masses in planes at 75 mm from the bearings for the complete balance of the shaft. The first counter mass is to be in a plane between the first mass and the bearing and the second mass in a plane between the third mass and the bearing at that end.









Worked Example 3

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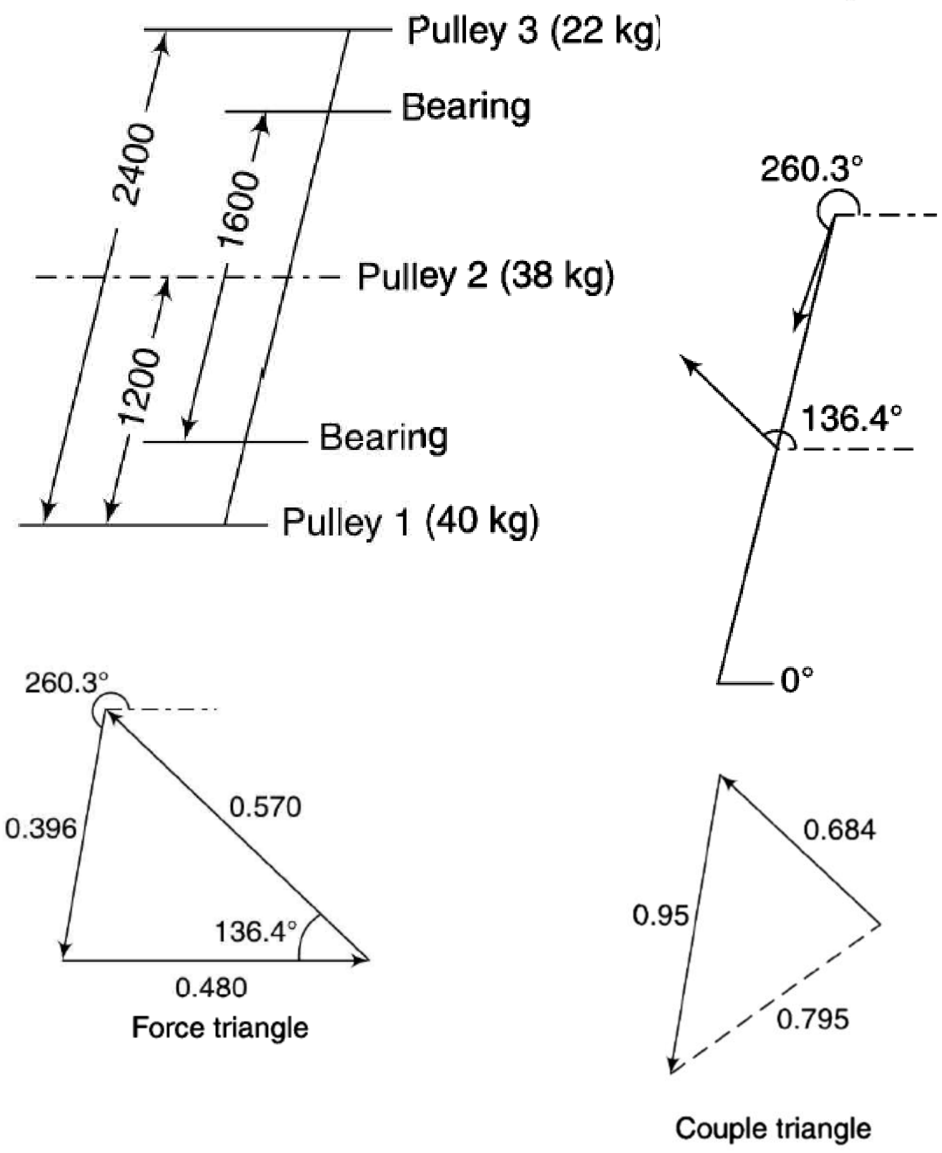
A shaft supported in bearings that are 1.6 m apart projects 400 mm beyond bearings at each end. It carries three pulleys one at each end and one at the centre of its length. The masses of the end pulleys are 40 kg and 22 kg and their centres of mass are at 12 mm and 18 mm respectively from the shaft axes. The mass of the centre pulley is 38 kg and its centre of mass is 15 mm from the shaft axis. The pulleys are arranged in a manner that they give static balance. Determine the

i. Relative angular positions of the pulleys

ii. Dynamic forces developed on bearings when the shaft rotates at 210 rpm.



Worked Example 4

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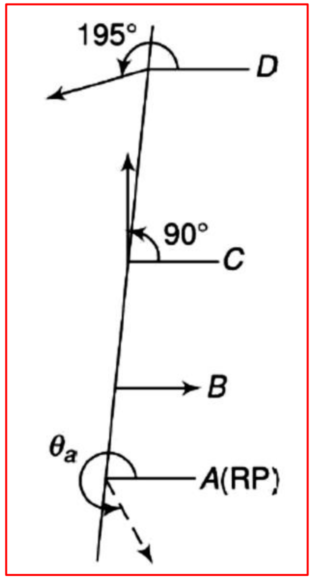
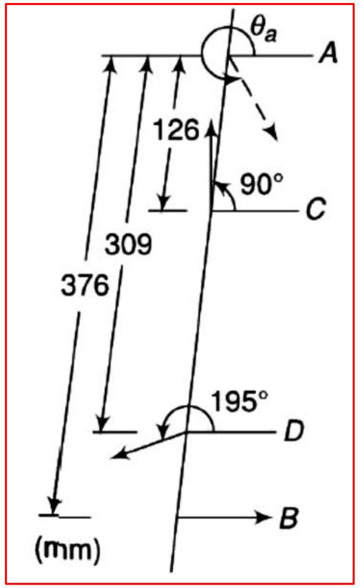
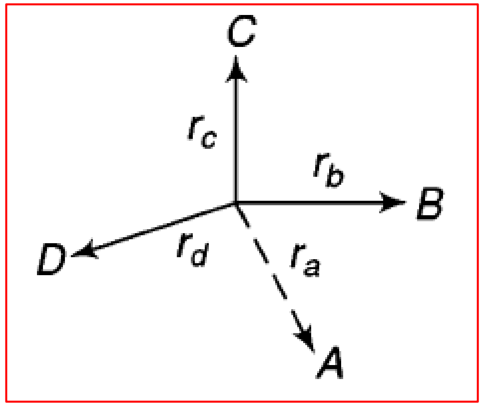
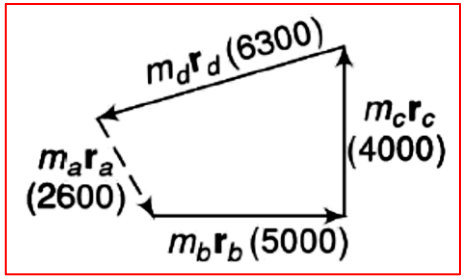
Four masses *A*, *B*, *C* and *D* are completely balanced. Masses *C* and *D* make angles of 90° and 195° respectively with that of mass *B* in the counter-clockwise direction. The rotating masses have the following properties: m_b = 25kg, m_c = 40 kg, m_d = 35 kg, r_a = 150 mm, r_b = 200 mm, r_c = 100 mm, r_d = 180 mm. Planes *B* and *C* are 250 mm apart. Determine the

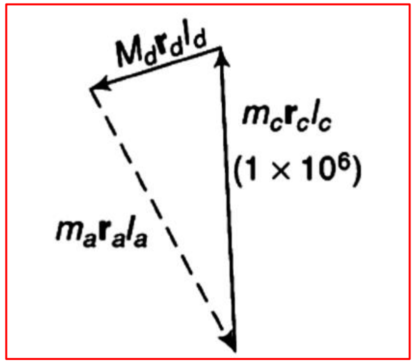
i. mass *A* and its angular position with that of mass *B*
ii. Positions of all the planes relative to plane of mass *A*











Balancing of Reciprocating Mass

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- Acceleration of the reciprocating mass of a slider crank mechanism is given by

$$f = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

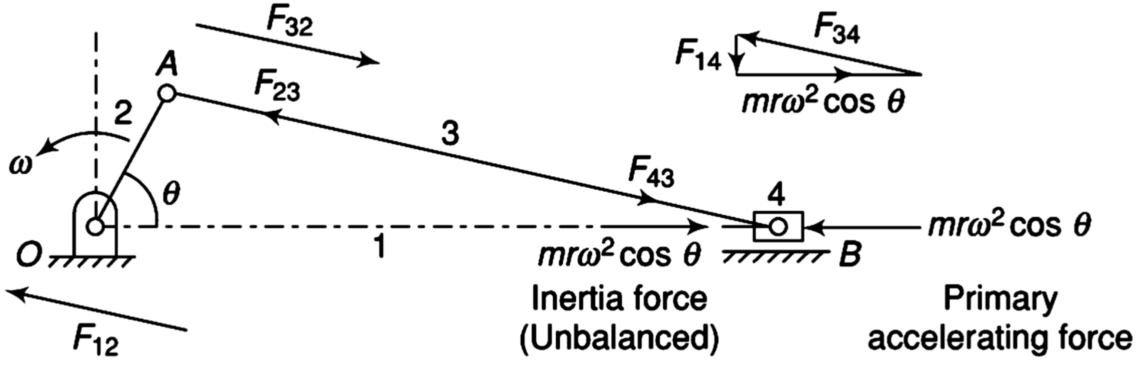
- Therefore, the force required to accelerate mass *m* is

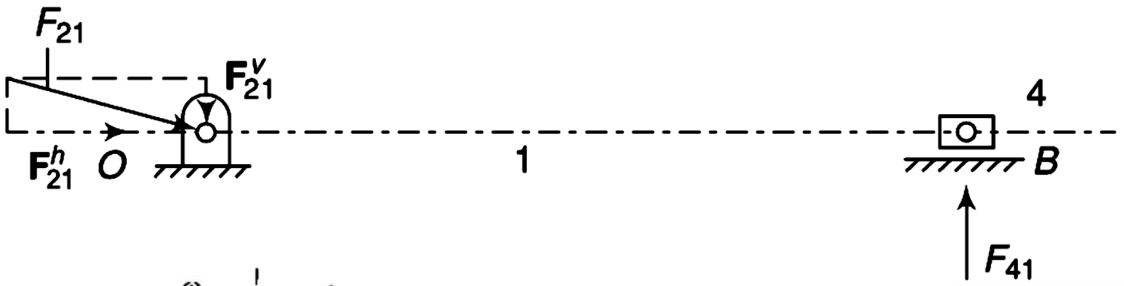
$$F = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

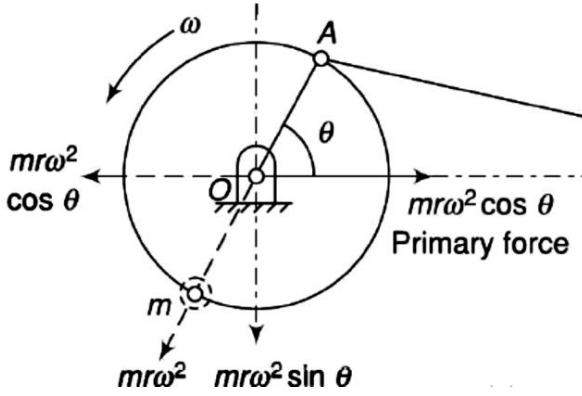
$$= mr\,\omega^2\cos\theta + mr\omega^2\frac{\cos 2\theta}{n} \rightarrow 1$$

- First term $mr\omega^2\cos\theta$ is called the primary accelerating force and the second term $mr\omega^2(\cos 2\theta/n)$ is called the secondary accelerating force.

- Maximum value of the primary force = $mr\omega^2$
- Maximum value of the secondary force = $mr\omega^2/n$







Balancing of Reciprocating Mass

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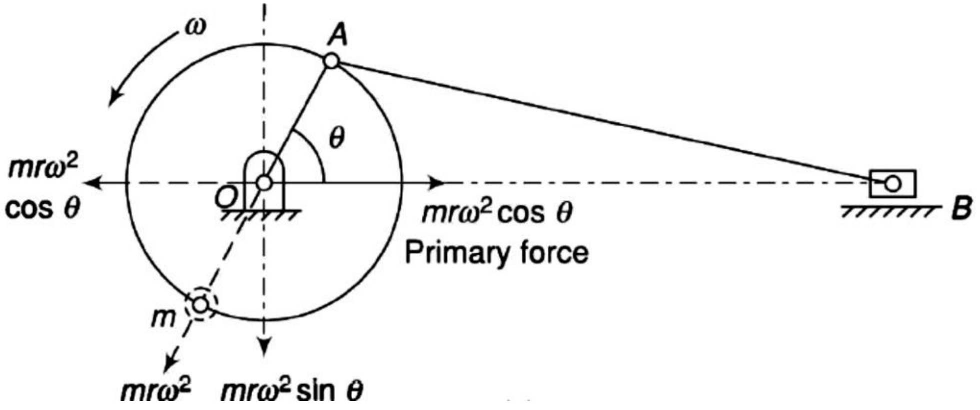
- The horizontal component of the centrifugal force due to the balancing mass is $mr\omega^2\cos\theta$ in the line of stroke. It neutralizes the unbalanced reciprocating force.
- But the rotating mass also has a component $mr\omega^2\sin\theta$ perpendicular to the line of stroke which remains unbalanced
- The unbalanced force is zero at $\theta = 0^\circ$ or 180° and maximum at the middle when $\theta = 90^\circ$ and Max Value = $mr\omega^2$



- To minimize this effect a parameter c is considered as a fraction of reciprocation mass then

  Primary force balanced by the mass = $cmr\omega^2\cos\theta$ Primary force unbalanced by the mass = (1- c) $cmr\omega^2\cos\theta$

  Vertical component of centrifugal force which remains unbalanced = $cmr\omega^2\sin\theta$
- Resultant un balanced force at any instant $= \sqrt{[(1-c)\,mr\omega^2\cos\theta]^2 + [cmr\omega^2\sin\theta]^2}$ → 2

Resultant un balanced force is minimum when c = 1/2

If m_p is the mass at the crankpin and c is the fraction of the reciprocating mass m to be balanced, the mass at the crankpin may be considered as (cm + m_p) which is to be completely balanced

Worked Example-5

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Following data relate to a single-cylinder reciprocating engine:

Mass of the reciprocating parts = 40 kg, Mass of the revolving parts = 30 kg at crank radius, Speed = 150 rpm, Stroke = 350 mm. If 60% of the reciprocating parts and all the revolving parts are to be balanced, determine the

i. Balance mass required at a radius of 320 mm

ii. Unbalanced force when crank has turned 45° from the top dead centre

Solution:

$$\omega = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s} \qquad r = \frac{350}{2} = 175 \text{ mm}$$

Mass to be balanced at the crank pin: $= cm + m_p = 0.6 \times 40 + 30 = 54 \text{ kg}$

$$m_c r_c = mr \qquad m_c \times 320 = 54 \times 175 \qquad m_c = 29.53 \text{ kg}$$

Unbalanced force (at θ = 45°):

$$= \sqrt{[(1-c)\, mr\omega^2 \cos\theta]^2 + (cmr\omega^2 \sin\theta)^2} = \sqrt{\begin{array}{l}[(1-0.6)\times 40\times 0.175\times (15.7)^2 \cos 45°]^2 \\ +[0.6\times 40\times 0.175\times (15.7)^2 \sin 45°]^2\end{array}} = \underline{880.7 \text{ N}}$$

Secondary Balancing

- The secondary acceleration force is given by

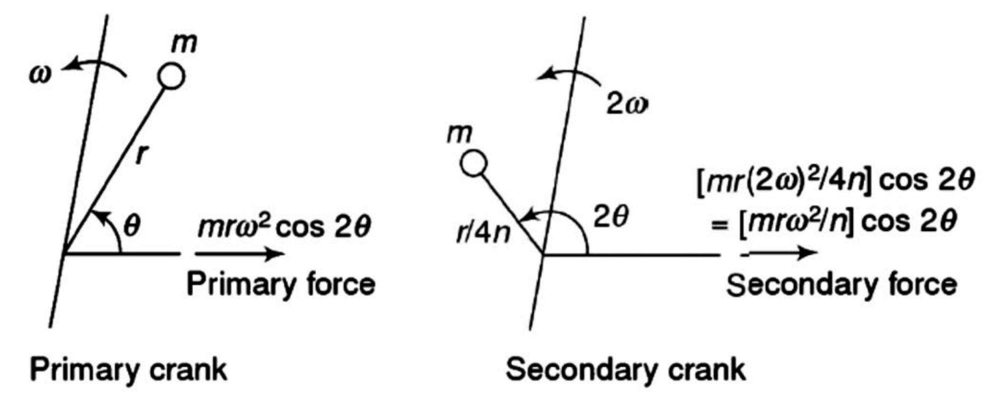
Secondary force = $mr\omega^2 \dfrac{\cos 2\theta}{n}$

- Its frequency is twice that of the primary force and the magnitude 1/*n* times the magnitude of the primary force
- The expression can also be written as

Secondary force = $mr\,(2\omega)^2 \dfrac{\cos 2\theta}{4n}$

- Consider two cranks of an engine one actual and the other imaginary, with following specifications

	Actual	Imaginary
Angular Velocity	ω	2ω
Length of crank	r	$r/4n$
Mass at the crank pin	m	m



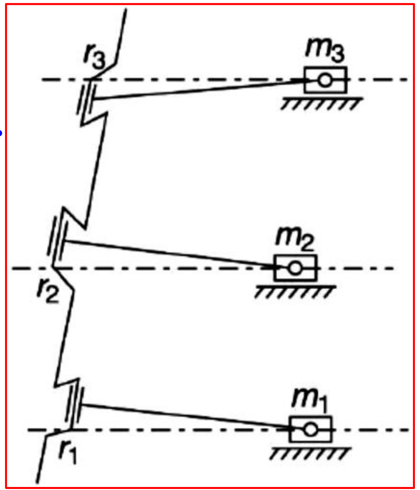
Centrifugal force induced in the imaginary crank = $\dfrac{mr(2\omega)^2}{4n}$

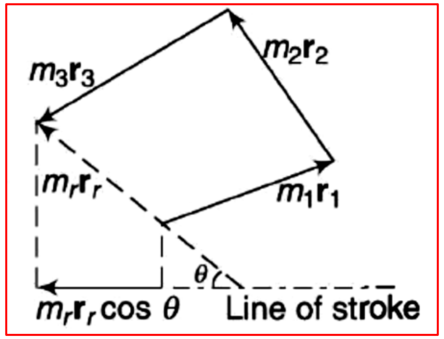
Component of this force along the line of stroke = $\dfrac{mr(2\omega)^2}{4n}\cos 2\theta$

- Thus, the effect of the secondary force is equivalent to an imaginary crank of length *r*/4*n* rotating at double the angular velocity, i. e. twice the engine speed.

Balancing of Inline Engines

- If a reciprocating mass is transferred to the crankpin, the axial component parallel to the cylinder axis of the resulting centrifugal force represents the primary unbalanced force.
- Consider a shaft consisting of three equal cranks unsymmetrically spaced. The crankpins carry equivalence of three unequal reciprocating masses.





Primary force = $\Sigma mr\omega^2 \cos\theta$ Primary couple = $\Sigma mr\omega^2 l \cos\theta$

Secondary force = $\Sigma mr \frac{(2\omega)^2}{4n} \cos 2\theta = \Sigma mr \frac{\omega^2}{n} \cos 2\theta$

Secondary couple = $\Sigma mr \frac{(2\omega)^2}{4n} l \cos 2\theta = \Sigma mr \frac{\omega^2}{n} l \cos 2\theta$

- In order to solve above equations graphically, first draw the $\sum mr\, cos\theta$ polygon. Then the axial component of resultant force ($F_r cos\theta$) multiplied by ω^2 provides the primary unbalanced force on the system at that moment. This unbalanced force is zero when $\theta = 90^\circ$ and a maximum when $\theta = 0^\circ$
- Primary balancing: Polygon encloses and $\sum F_{ph} = 0$ and $\sum F_{pv} = 0$
- Secondary balancing: First find the positions of the imaginary secondary cranks and then transfer the reciprocating masses and multiply the same by $(2\omega^2)/4n$ or ω^2/n to get the secondary force

Four Stroke Engines

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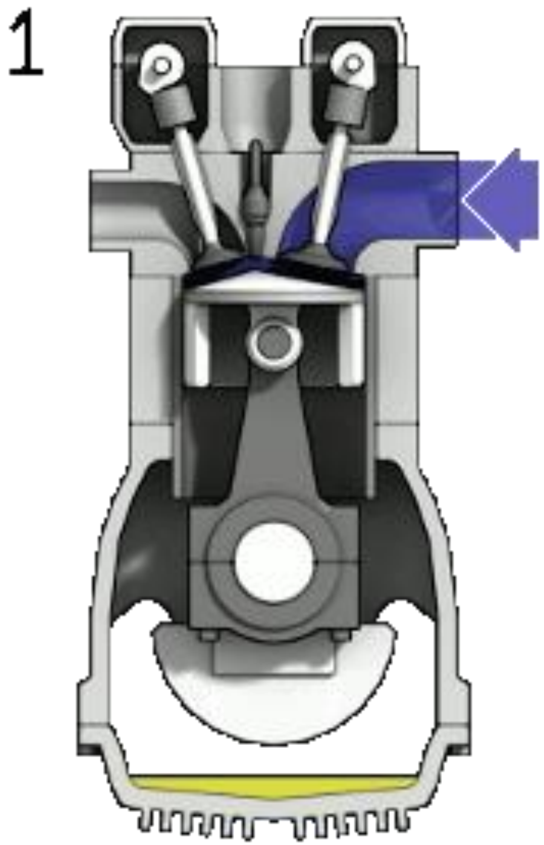

Balancing of Inline Two-cylinder Engines

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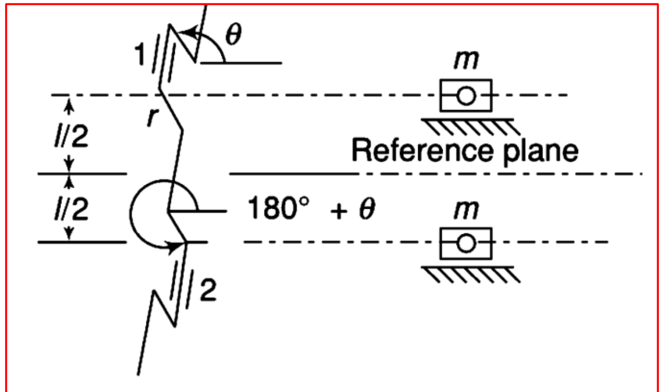
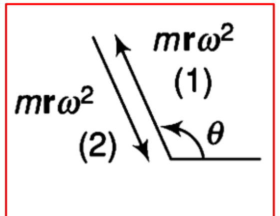
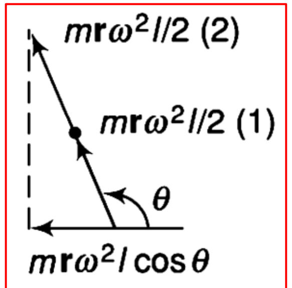
- Cranks are 180˚ apart and have equal reciprocating masses. Taking a plane through the centre line as RP



Primary cranks



Force

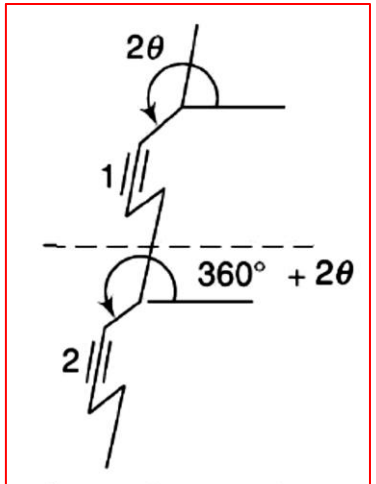


Couple

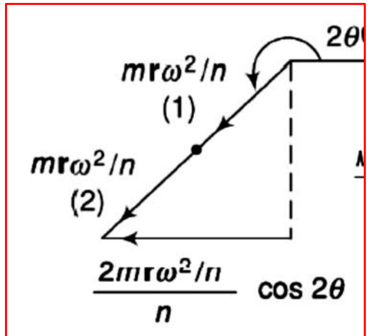
Primary force = $mr\omega^2\,[\cos\theta + \cos(180° + \theta)] = 0$

Primary couple = $mr\omega^2\left[\frac{l}{2}\cos\theta + \left(-\frac{l}{2}\right)\cos(180° + \theta)\right] = mr\omega^2 l\cos\theta$

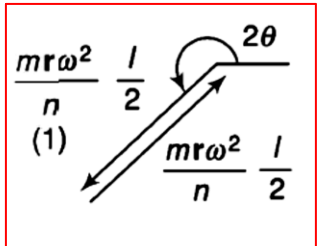
Maximum values are $mr\omega^2 l$ at $\theta = 0˚$ and 180˚



Secondary cranks



Force



Couple

Secondary force =

$$\frac{mr\omega^2}{n}[\cos 2\theta + \cos(360° + 2\theta)] = 2\frac{mr\omega^2}{n}\cos 2\theta$$

Maximum values are $(2mr\omega^2)/n$ at $2\theta = 0˚, 180˚,$ 360˚ and 540˚

or $\theta = 0˚, 90˚$, 180˚ and 270˚

Secondary couple =

$$\frac{mr\omega^2}{n}\left[\frac{l}{2}\cos 2\theta + \left(-\frac{l}{2}\right)\cos(360° + 2\theta)\right] = 0$$

Balancing of Inline Two-cylinder Engines

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- As it is a rotating system, the maximum values or magnitudes of these forces and couples vary instant to instant and are equal to the values as given by the equivalent rotating masses at the crank pin.
- If a particular position of the crankshaft is considered, the above expressions may not give the maximum value.
- In case of any particular position of the crankshaft is considered then both x- and y- components of the force and couple can be taken to find the maximum values, e. g., if the positions of the cranks are considered at 120° and 300°, the primary couple can be obtained as

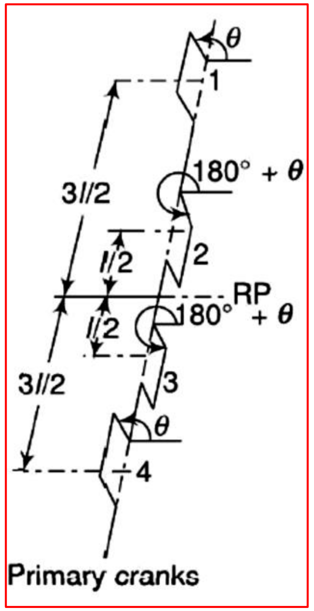
$$x\text{-component} = mr\omega^2\left[\frac{l}{2}\cos 120^\circ + \left(-\frac{l}{2}\right)\cos(180^\circ + 120^\circ)\right] = -\frac{1}{2}mr\omega^2 l$$

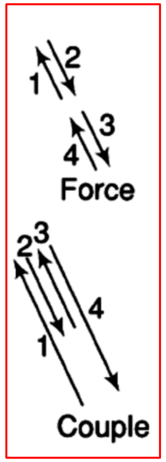
$$y\text{-component} = mr\omega^2\left[\frac{l}{2}\sin 120^\circ + \left(-\frac{l}{2}\right)\sin(180^\circ + 120^\circ)\right] = \frac{\sqrt{3}}{2}mr\omega^2 l$$

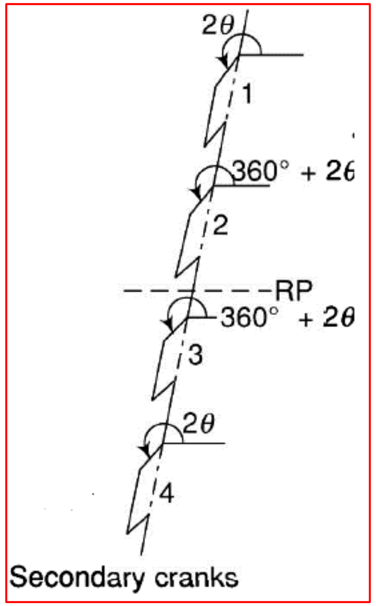
$$\text{Primary Couple} = \sqrt{\left(-\frac{1}{2}mr\omega^2 l\right)^2 + \left(\frac{\sqrt{3}}{2}mr\omega^2 l\right)^2} = mr\omega^2 l$$

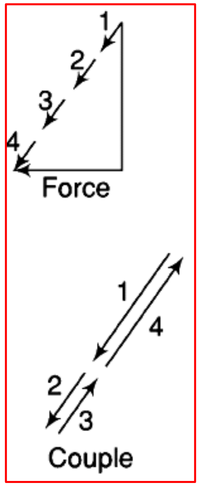
Inline Four-cylinder Four-stroke Engines

- Two outer and inner cranks (throws) in line separated inner throws by 180° to the outer throws. The angular positions: First (θ), second (180°+ θ), third (180°+ θ) and fourth (θ).









Primary force = $mr\omega^2\,[\cos\theta + \cos(180° + \theta) + \cos(180° + \theta) + \cos\theta] = 0$

Primary couple =

$$mr\omega^2\left[\frac{3l}{2}\cos\theta + \frac{l}{2}\cos(180° + \theta) + \left(-\frac{l}{2}\right)\cos(180° + \theta) + \left(-\frac{3l}{2}\right)\cos\theta\right] = 0$$

Secondary force =

$$\frac{mr\omega^2}{n}[\cos 2\theta + \cos(360° + 2\theta) + \cos(360° + 2\theta) + \cos 2\theta] = \frac{4mr\omega^2}{n}\cos 2\theta$$

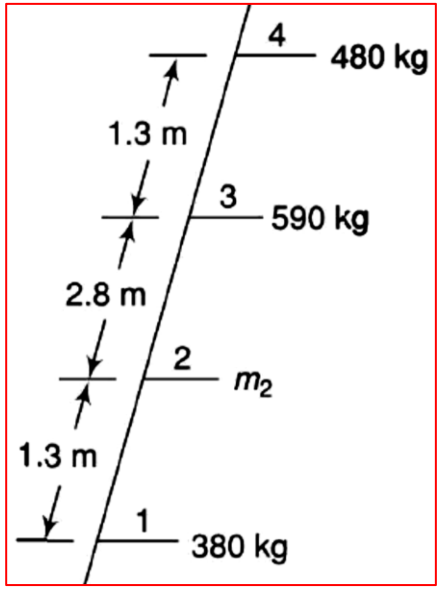
Maximum value = $(4mr\omega^2)/n$ at 2θ = 0°,180°, 360° and 540°or θ = 0°,90°, 180° and 270°

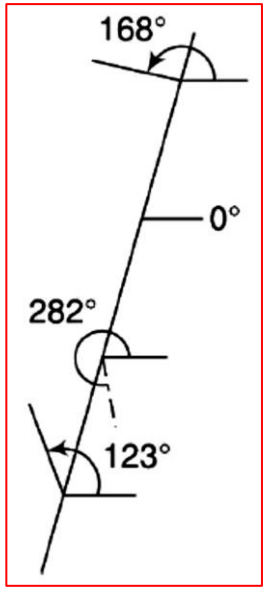
Secondary couple = $\dfrac{mr\omega^2}{n}\left[\dfrac{3l}{2}\cos 2\theta + \dfrac{l}{2}\cos(360° + 2\theta) + \left(-\dfrac{l}{2}\right)\cos(360° + 2\theta) + \left(-\dfrac{3l}{2}\right)\cos 2\theta\right] = 0$

- Graphical solution has been shown in figure. This engine is not balanced in secondary forces.

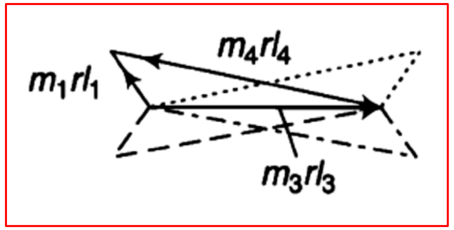
Worked Example 6

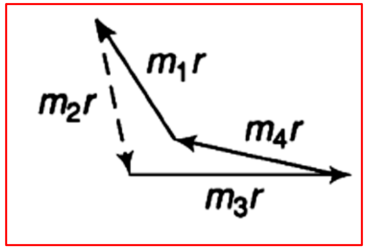
A four-cylinder oil engine is in complete primary balance. The arrangement of the reciprocating masses in planes is shown in figure. The stroke of each piston is 2*r* mm. Determine the reciprocating mass of the cylinder 2 and the relative crank positions.





Crank length = $2r/2 = r$ Take 2 as RP and $\theta_3 = 0^\circ$





$m_1r_1l_1 = 380 \times r \times (-1.3) = -494r$

$m_3r_3l_3 = 590 \times r \times 2.8 = 1652r$

$m_4r_4l_4 = 480 \times r \times (2.8 + 1.3) = 1968r$

$m_1r_1 = 380r$, $m_3r_3 = 590r$, $m_4r_4 = 480r$

Writing couple equations: $494\ cos\theta_1 = 1652 + 1968\theta_4$ and $494\ sin\theta_1 = 1968\theta_4$

Squaring and adding above equations: $cos\theta_4 = -0.978$ $\boxed{\theta_4 = 167.9^\circ \text{ or } 192.1^\circ}$

Dividing equations (*ii*) by (*i*): $\boxed{\theta_1 = 123.4^\circ}$

Writing force equations: $m_2cos\theta_2 = 88.5$ and $m_2sin\theta_2 = -417.9$

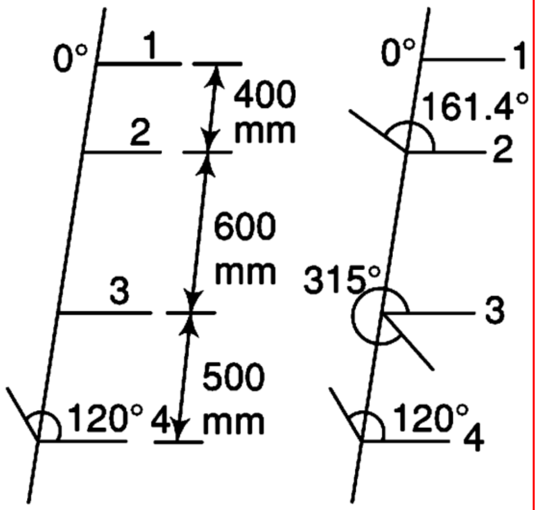
Squaring and adding above equations: $\boxed{m_2 = 427.1 \text{ kg}}$

Dividing equations (*iii*) by (*iv*): $\boxed{\theta_2 = 282^\circ}$

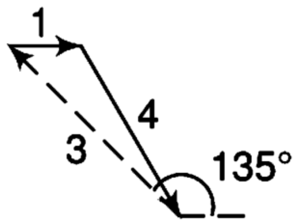
If we choose $\theta_4 = 192.1^\circ$, a different set of m_2, θ_4 θ_1 and θ_2

Worked Example 7

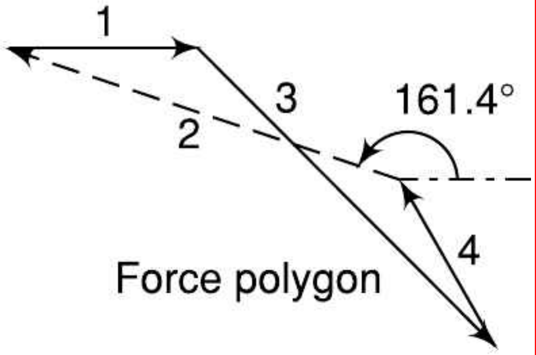
Each crank and the connecting rod of a four-crank in-line engine are 200 mm and 800 mm respectively. The outer cranks are set a 120˚ to each other and each has reciprocating mass of 200 kg. The spacing between adjacent planes of cranks are 400 mm, 600 mm and 500 mm. If the engine is in complete primary balance, determine the reciprocating masses of the inner cranks and their relative angular positions. Also find the secondary unbalanced force if the engine speed is 210 rpm.



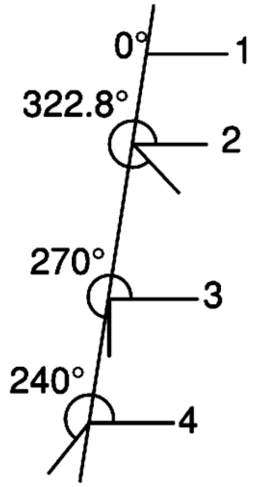
(a) Primary cranks



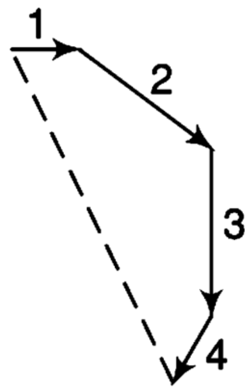
Couple polygon



Force polygon



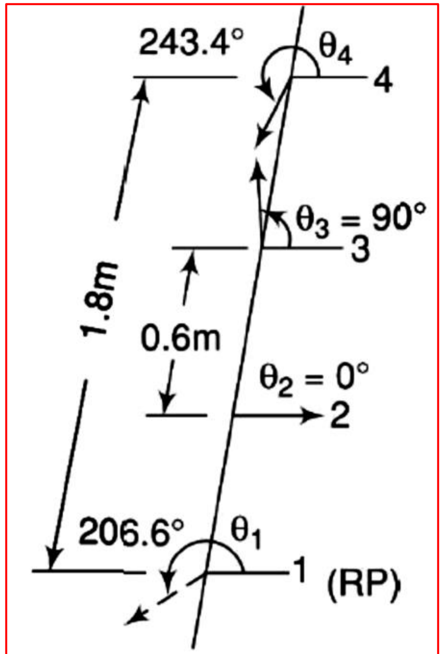
(b) Secondary cranks



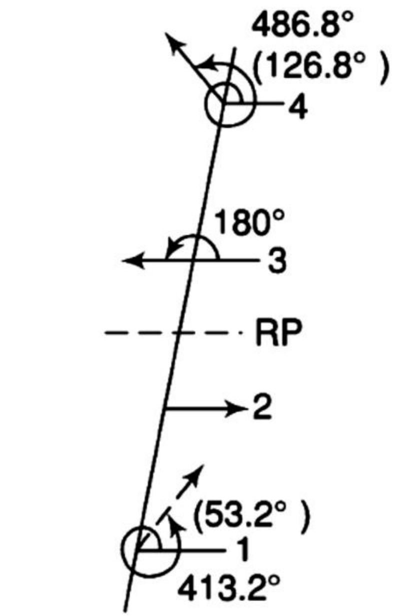
Force polygon

Worked Example 8

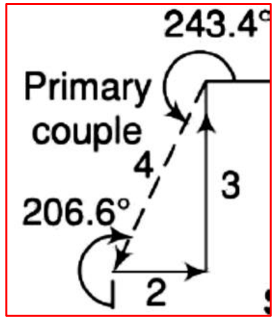
The intermediate cranks of four-cylinder symmetrical engine, which is in complete primary balance, are at 90° to each other and each has a reciprocating mass of 400 kg. The centre distance between intermediate cranks is 600 mm and between extreme cranks, it is 1800 mm. Lengths of the connecting rods and the cranks are 900 mm and 200 mm respectively. Calculate the masses fixed to the extreme cranks with their relative angular positions. Also, find the magnitude of the secondary forces and couples about the centre line of the system if the engine speed is 500 rpm.

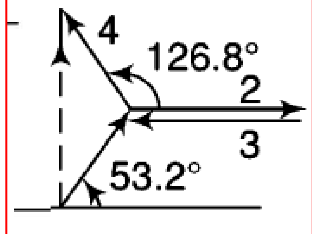


Primary cranks

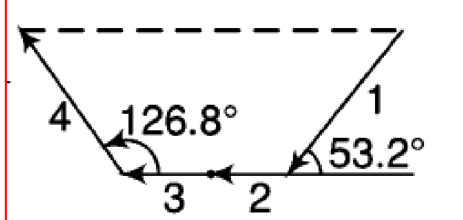


Secondary cranks





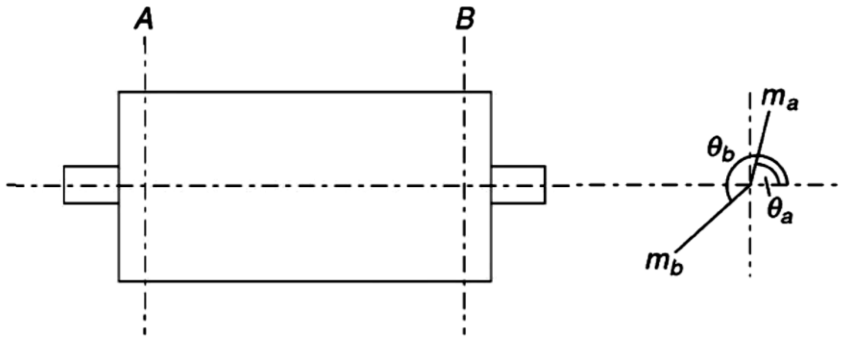
Secondary force



Secondary couple

Field Balancing

- Assume two balancing planes *A* and *B*
- First, the rotor is rotated at a speed which provides measurable amplitudes at planes A and B. Let the vectors ***A*** and ***B*** represent the amplitudes due to the unbalance of the rotor in planes *A* and *B* respectively.



- Attach a trial mass m_a in the plane *A* at a known radius and known angular position and run the rotor at the same speed as in the first case. Measure the amplitudes in the two planes *A* and *B*. Let $\boldsymbol{A_1}$ and $\boldsymbol{B_1}$ represent the amplitudes of the rotor in planes *A* and *B* respectively. Thus

  Effect at *A* of the unbalance + Effect at *A* of trial mass in plane *A* = $\boldsymbol{A_1}$

  Effect at *A* of trial mass in plane *A* = $\boldsymbol{A_1} - \boldsymbol{A}$

  Effect at *B* of the unbalance + Effect at *B* of trial mass in plane *A* = $\boldsymbol{B_1}$

  Effect at *B* of trial mass in plane *A* = $\boldsymbol{B_1} - \boldsymbol{B}$

Field Balancing contd..

- Make a third run of the rotor by attaching a trial mass m_b in the plane B at a known radius and known angular position and run the rotor at the same speed as in the first two cases. Measure the amplitudes in the two planes A and B. Let $\boldsymbol{A_2}$ and $\boldsymbol{B_2}$ represent the amplitudes of the rotor in planes A and B respectively. Thus

  Effect at A of the unbalance + Effect at A of trial mass in plane B = $\boldsymbol{A_2}$

  Effect at A of trial mass in plane B = $\boldsymbol{A_2} - \boldsymbol{A}$

  Effect at B of the unbalance + Effect at B of trial mass in plane B = $\boldsymbol{B_2}$

  Effect at B of trial mass in plane B = $\boldsymbol{B_2} - \boldsymbol{B}$

- Let $m_{ca} = \alpha \times m_a$ and $m_{cb} = \beta \times m_b$

  where $\alpha = a.\ e^{i\theta a}$, i. e. the counter mass in plane A is a times the trial mass located at an angle θ_a with its direction, m_{ca} and m_{cb} be the counter or balancing masses in planes A and B respectively at the same radii as the trial mass

  and $\beta = b.\ e^{i\theta b}$ *i.e* the counter mass in plane B is b times the trial mass located at an angle θ_b with its direction

Field Balancing contd..

- For complete balancing of rotor, the effect of the balancing masses must be to nullify the unbalance in the two planes, i. e., in the plane A it must be equal to $-\boldsymbol{A}$ and in plane B equal to $-\boldsymbol{B}$.

$$\alpha\,(\boldsymbol{A_1} - \boldsymbol{A}) + \beta\,(\boldsymbol{A_2} - \boldsymbol{A}) = -\boldsymbol{A} \longrightarrow i \qquad \text{and} \qquad \alpha\,(\boldsymbol{B_1} - \boldsymbol{B}) + \beta\,(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{B} \longrightarrow ii$$

- These equations can be solved for α and β. Multiplying (i) with $(\boldsymbol{B_2} - \boldsymbol{B})$ and (ii) with $(\boldsymbol{A_2} - \boldsymbol{A})$,

$$\alpha\,(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) + \beta\,(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{A}\,(\boldsymbol{B_2} - \boldsymbol{B}) \longrightarrow iii$$

$$\alpha\,(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B}) + \beta\,(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{B}\,(\boldsymbol{A_2} - \boldsymbol{A}) \longrightarrow iv$$

- Subtracting (iv) from (iii), $\alpha = \dfrac{\boldsymbol{B}(\boldsymbol{A_2} - \boldsymbol{A}) - \boldsymbol{A}(\boldsymbol{B_2} - \boldsymbol{B})}{(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) - (\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B})} \longrightarrow 1$

- Multiplying (i) with $(\boldsymbol{B_1} - \boldsymbol{B})$ and (ii) with $(\boldsymbol{A_1} - \boldsymbol{A})$,

$$\alpha\,(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B}) + \beta\,(\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B}) = -\boldsymbol{A}\,(\boldsymbol{B_1} - \boldsymbol{B}) \longrightarrow v$$

$$\alpha\,(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B}) + \beta\,(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) = -\boldsymbol{B}\,(\boldsymbol{A_1} - \boldsymbol{A}) \longrightarrow vi$$

- Subtracting (v) from (vi), $\beta = \dfrac{\boldsymbol{A}(\boldsymbol{B_1} - \boldsymbol{B}) - \boldsymbol{B}(\boldsymbol{A_1} - \boldsymbol{A})}{(\boldsymbol{A_1} - \boldsymbol{A})(\boldsymbol{B_2} - \boldsymbol{B}) - (\boldsymbol{A_2} - \boldsymbol{A})(\boldsymbol{B_1} - \boldsymbol{B})} \longrightarrow 2$

Thank You

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