

Numbers in RED are index positions in array

\* The corresponding array is shown below each max-heap visualized as a nearly complete binary tree.

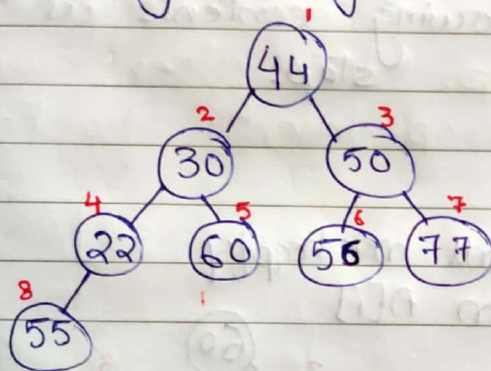
Illustrate stepwise execution of the Heapsort algorithm on an array  $A = [44, 30, 50, 22, 60, 55, 77, 55]$  to arrange its elements in ascending order.

STEP I: Create max-heap.

It can be done in two ways

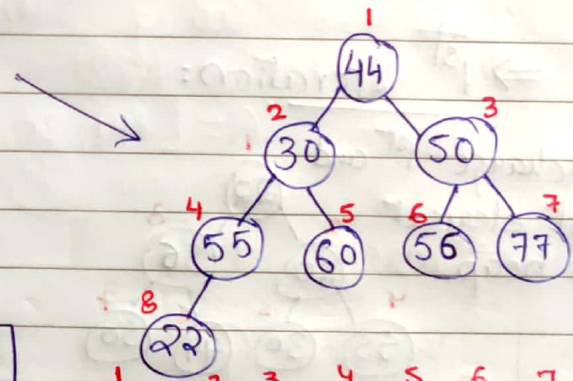
- ① Taking all the elements at once
- ② Taking one element, insert in the existing max-heap.

Using way ①



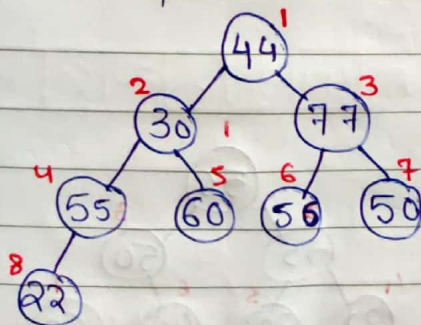
	1	2	3	4	5	6	7	8
A:	44	30	50	22	60	56	77	55

⇒ Execute max-heapify at index 4, i.e.  $A[4] = 22$ .



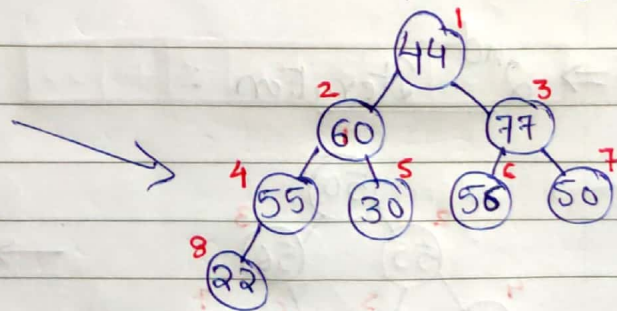
	1	2	3	4	5	6	7	8
A:	44	30	50	55	60	56	77	22

⇒ Execute max-heapify at index 3, i.e.  $A[3] = 50$ .



	1	2	3	4	5	6	7	8
A:	44	30	77	55	60	56	50	22

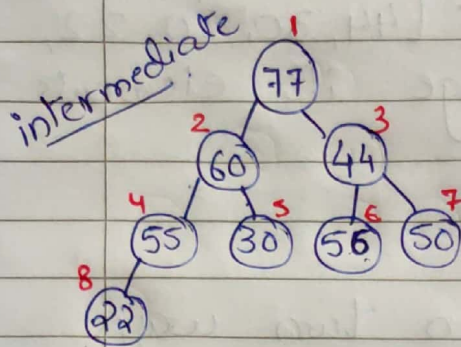
⇒ Execute max-heapify at index 2, i.e.  $A[2] = 30$ .



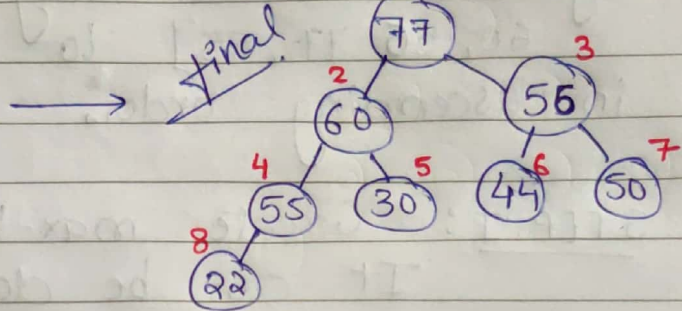
	1	2	3	4	5	6	7	8
A:	44	60	77	55	30	56	50	22



⇒ Execute max-heapify at index 1, i.e.  $A[1] = 44$ .



	1	2	3	4	5	6	7	8
A:	77	60	44	55	30	56	50	22

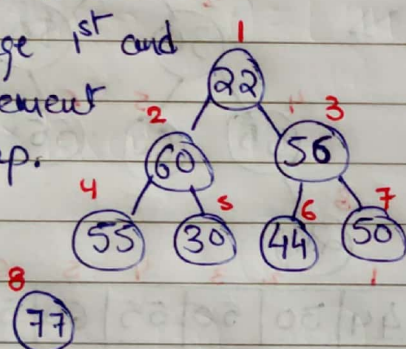


	1	2	3	4	5	6	7	8
A:	77	60	56	55	30	44	50	22

STEP II : Execute rest of the steps of heapsort algorithm showing contents at each intermediate step.

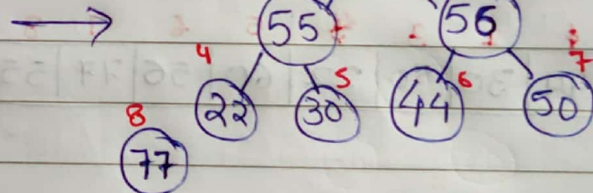
⇒ 1<sup>st</sup> Iteration:

Exchange 1<sup>st</sup> and last element in heap.



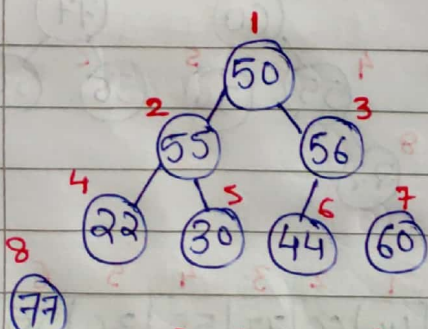
	1	2	3	4	5	6	7	8
A:	22	60	56	55	30	44	50	77

Call max-heapify on  $A[1]$

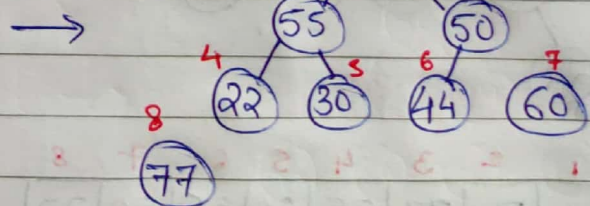


	1	2	3	4	5	6	7	8
A:	60	55	56	22	30	44	50	77

⇒ 2<sup>nd</sup> Iteration:



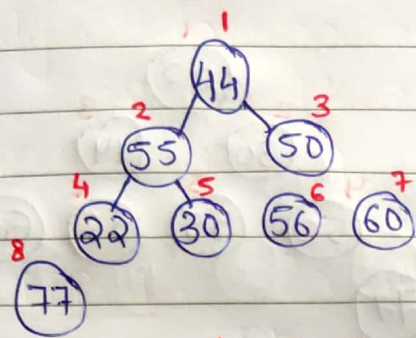
	1	2	3	4	5	6	7	8
A:	50	55	56	22	30	44	60	77



	1	2	3	4	5	6	7	8
A:	56	55	50	22	30	44	60	77

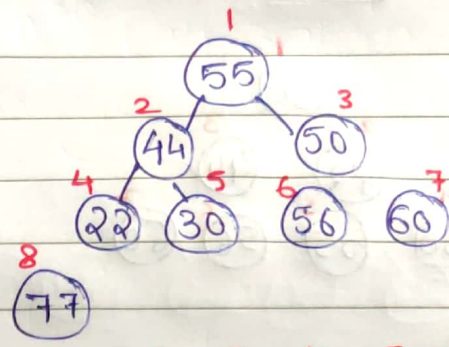


⇒ 3<sup>rd</sup> Iteration



A: 

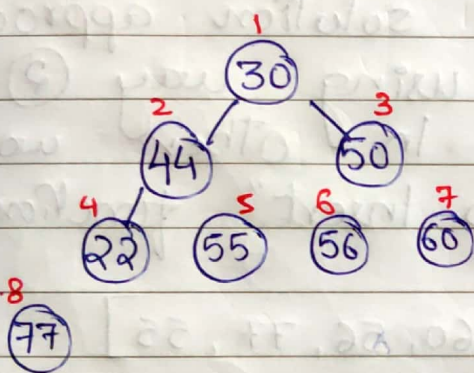
1	2	3	4	5	6	7	8
44	55	50	22	30	56	60	77



A: 

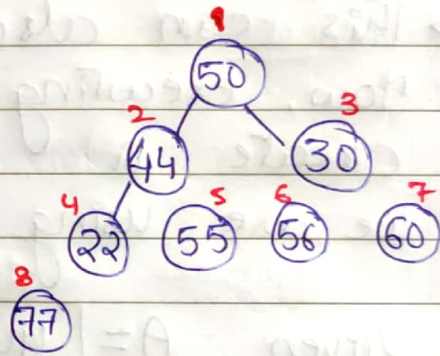
1	2	3	4	5	6	7	8
55	44	50	22	30	56	60	77

⇒ 4<sup>th</sup> Iteration



A: 

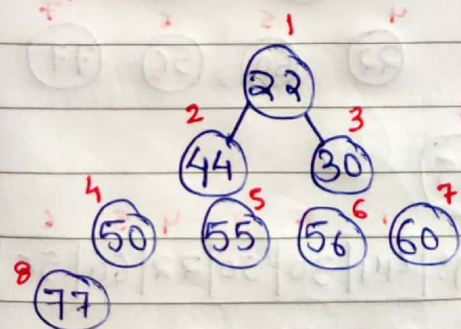
1	2	3	4	5	6	7	8
30	44	50	22	55	56	60	77



A: 

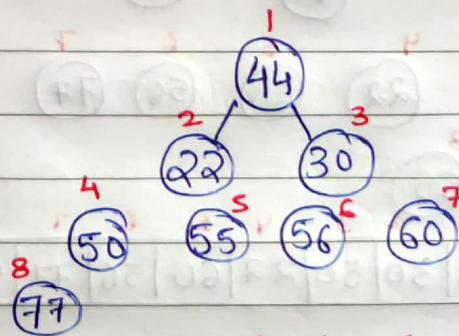
1	2	3	4	5	6	7	8
50	44	30	22	55	56	60	77

⇒ 5<sup>th</sup> Iteration



A: 

1	2	3	4	5	6	7	8
22	44	30	50	55	56	60	77

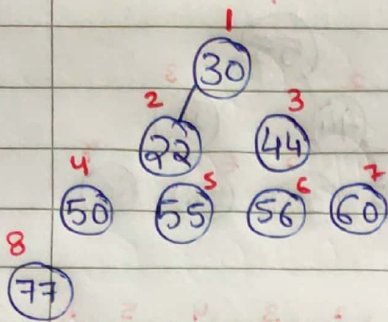


A: 

1	2	3	4	5	6	7	8
44	22	30	50	55	56	60	77



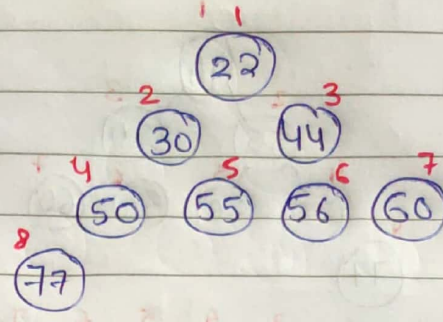
⇒ 6<sup>th</sup> Iteration



A: 

1	2	3	4	5	6	7	8
30	22	44	50	55	56	60	77

⇒ 7<sup>th</sup> Iteration



A: 

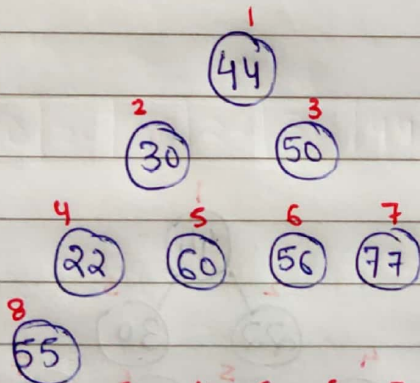
1	2	3	4	5	6	7	8
22	30	44	50	55	56	60	77

"SORTED"

⇒ This can also be the solution approach. Now, executing STEP 1 using way (2) to create a max-heap. In other words we are using "Max-heap-Insert" function.

Given  $A = [44, 30, 50, 22, 60, 56, 77, 55]$

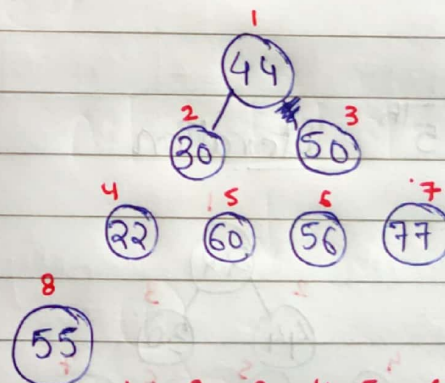
→ Insert 44



A: 

1	2	3	4	5	6	7	8
44	30	50	22	60	56	77	55

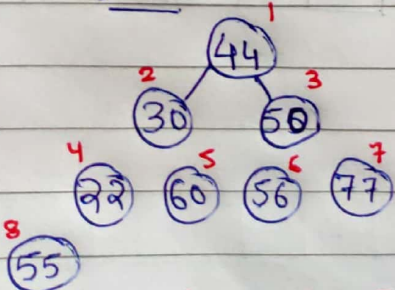
→ Insert 30



A: 

1	2	3	4	5	6	7	8
44	30	50	22	60	56	77	55

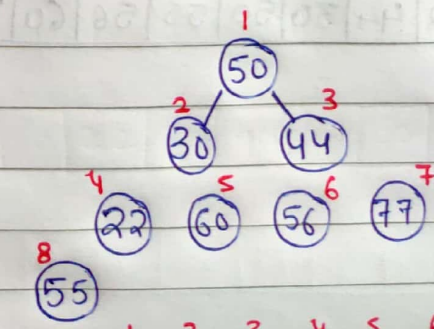
→ Insert 50



A: 

1	2	3	4	5	6	7	8
44	30	50	22	60	56	77	55

→

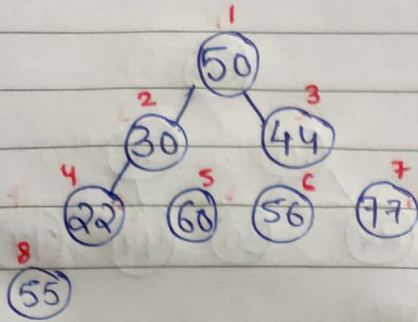


A: 

1	2	3	4	5	6	7	8
50	30	44	22	60	56	77	55

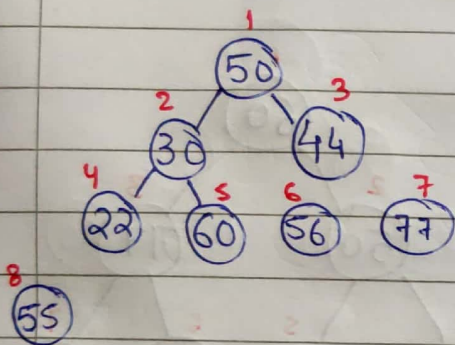


→ Insert 22.

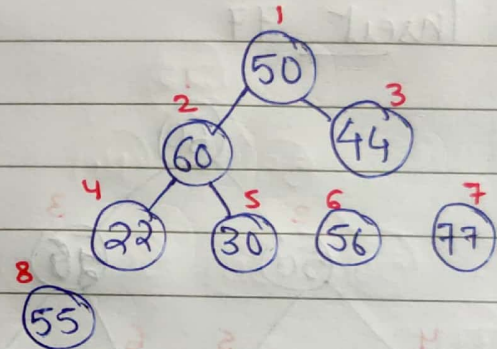


A: 50 30 44 22 60 56 77 55

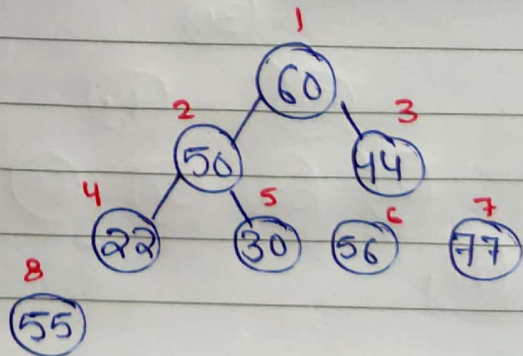
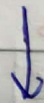
→ Insert 60



A: 50 30 44 22 60 56 77 55

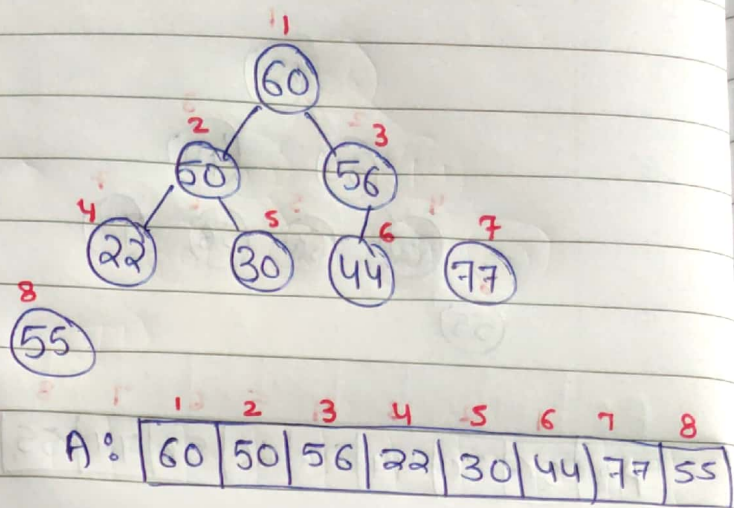
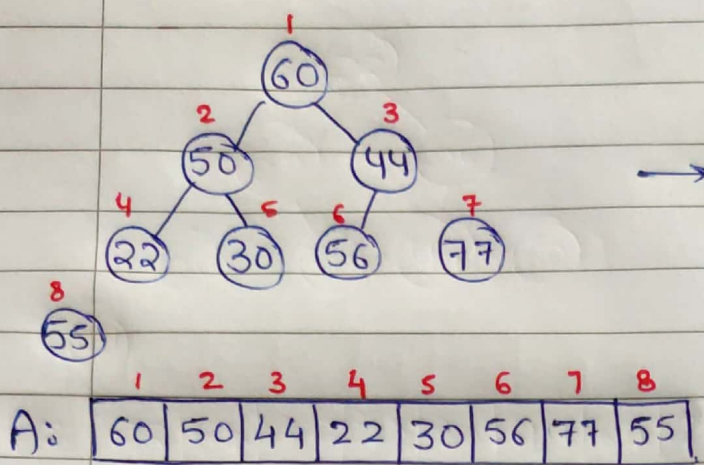


A: 50 60 44 22 30 56 77 55

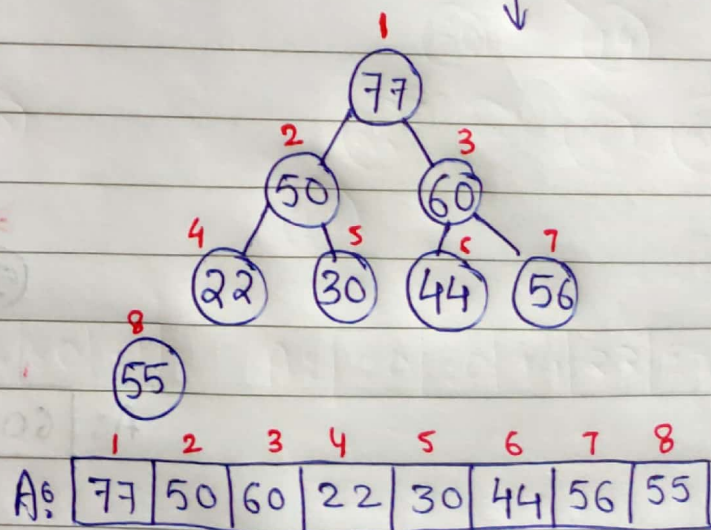
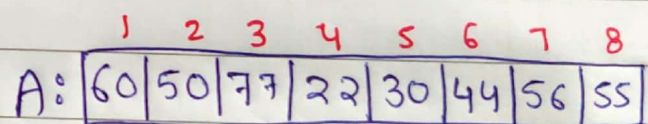
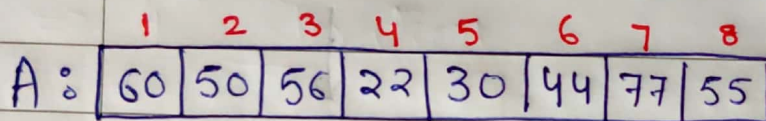
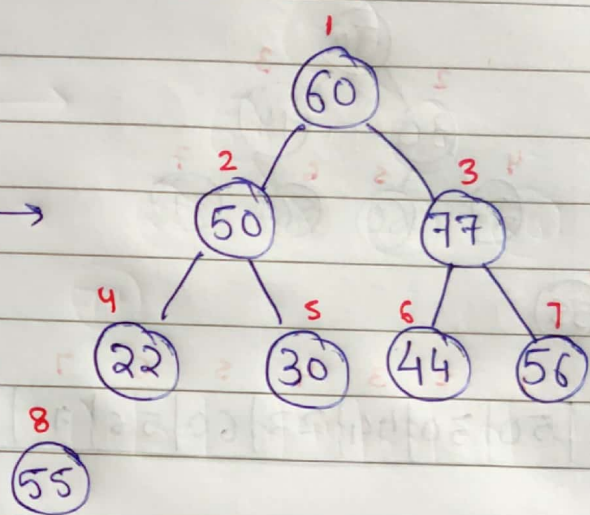
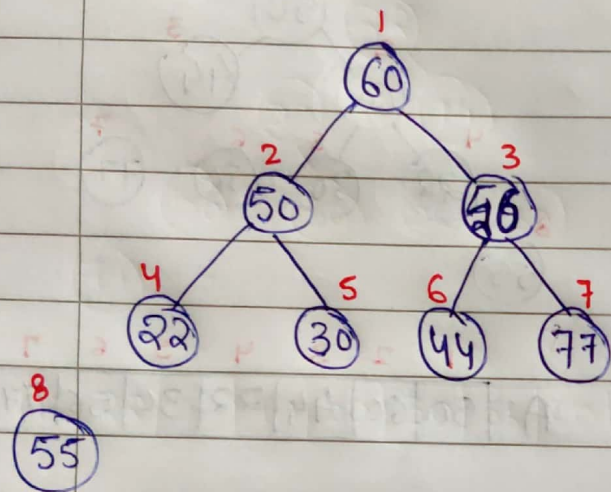


A: 60 50 44 22 30 56 77 55

→ Insert 56

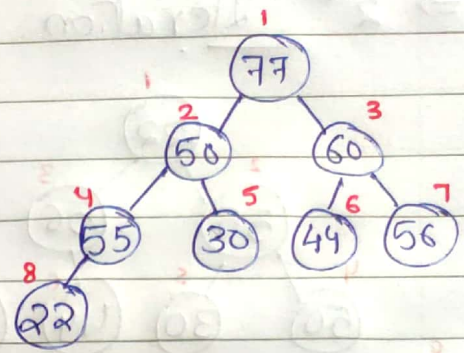
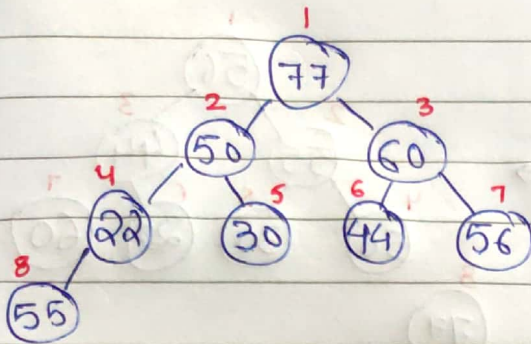


→ Insert 77





→ Insert 55

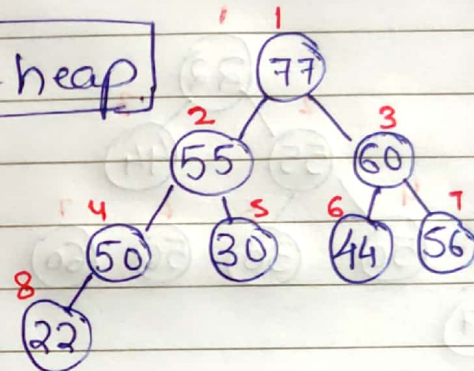


A: 77 50 60 22 30 44 56 55

A: 77 50 60 55 30 44 56 22



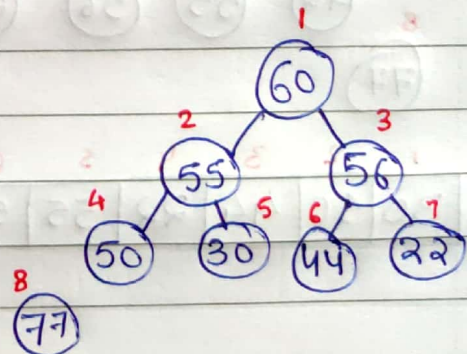
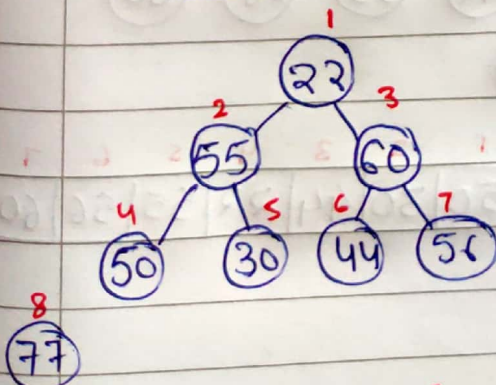
Max-heap



A: 77 55 60 50 30 44 56 22

STEP II : Execute rest of the steps of heapsort algorithm showing contents at each intermediate step.

⇒ 1<sup>st</sup> Iteration

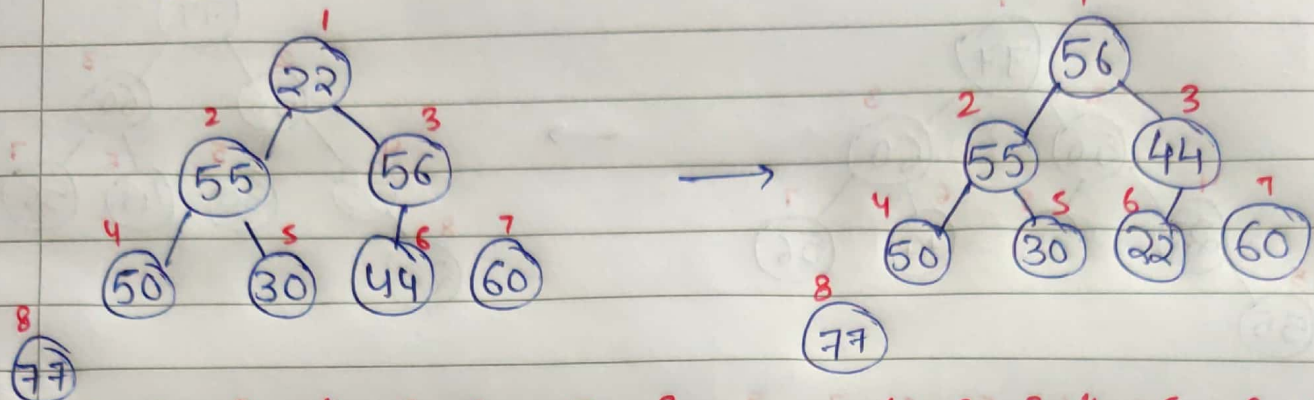


A: 22 55 60 50 30 44 56 77

A: 60 55 56 50 30 44 22 77



⇒ 2<sup>nd</sup> Iteration



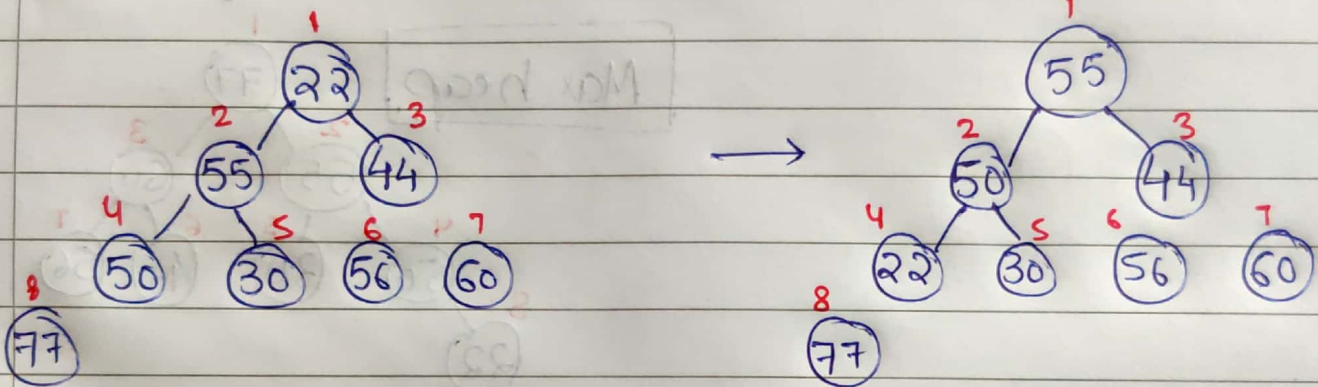
A: 

22	55	56	50	30	44	60	77
----	----	----	----	----	----	----	----

A: 

56	55	44	50	30	22	60	77
----	----	----	----	----	----	----	----

⇒ 3<sup>rd</sup> Iteration



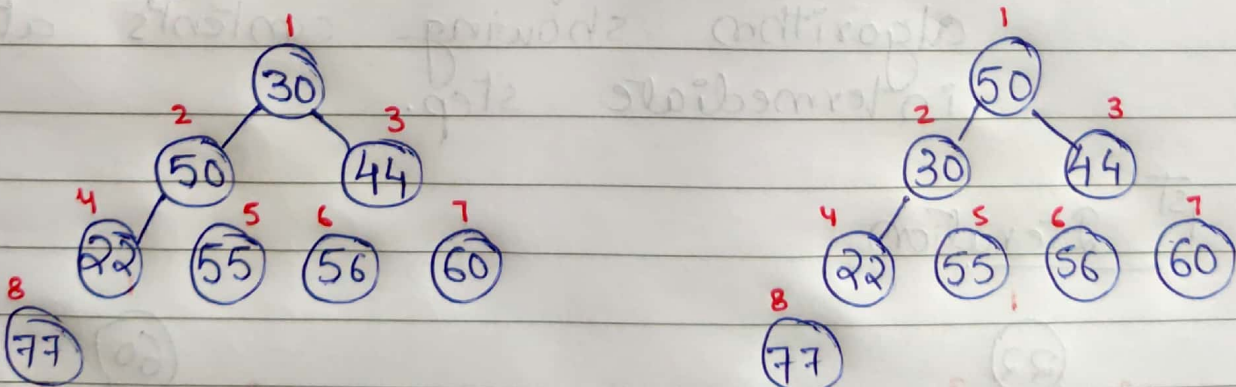
A: 

22	55	44	50	30	56	60	77
----	----	----	----	----	----	----	----

A: 

55	50	44	22	30	56	60	77
----	----	----	----	----	----	----	----

⇒ 4<sup>th</sup> Iteration



A: 

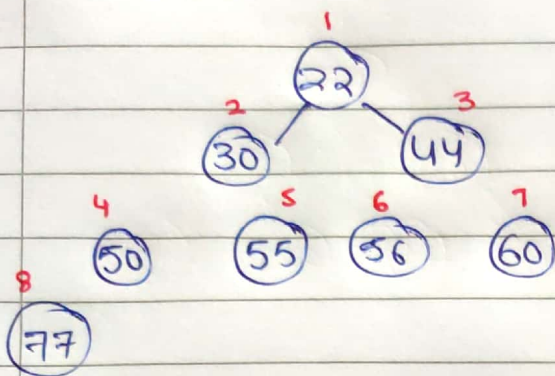
30	50	44	22	55	56	60	77
----	----	----	----	----	----	----	----

A: 

50	30	44	22	55	56	60	77
----	----	----	----	----	----	----	----

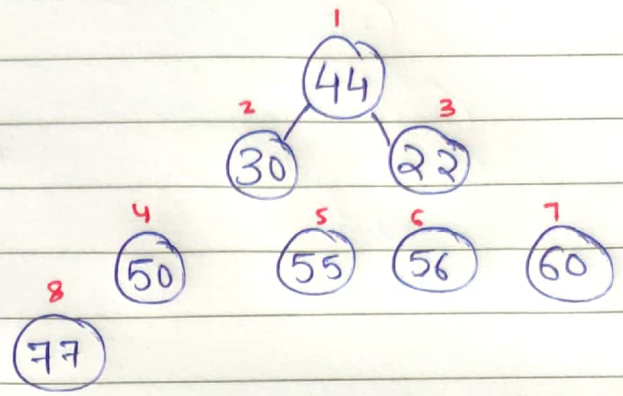


⇒ 5<sup>th</sup> Iteration



A:

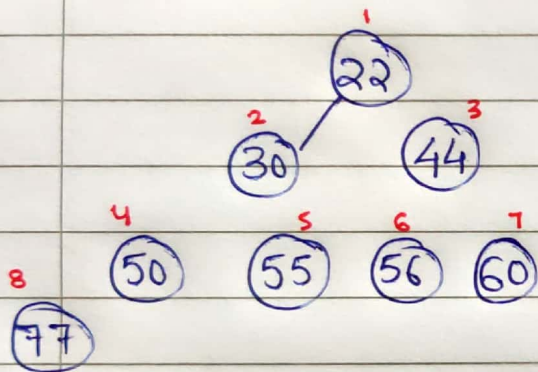
1	2	3	4	5	6	7	8
22	30	44	50	55	56	60	77



A:

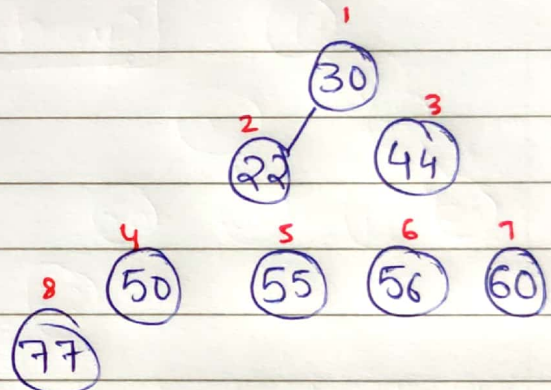
1	2	3	4	5	6	7	8
44	30	22	50	55	56	60	77

⇒ 6<sup>th</sup> Iteration



A:

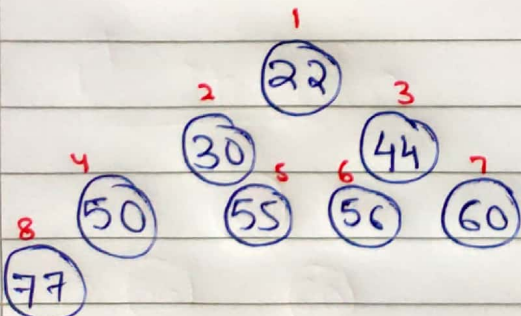
1	2	3	4	5	6	7	8
22	30	44	50	55	56	60	77



A:

1	2	3	4	5	6	7	8
30	22	44	50	55	56	60	77

⇒ 7<sup>th</sup> Iteration



A:

1	2	3	4	5	6	7	8
22	30	44	50	55	56	60	77

"SORTED"