Mass Transfer-I

Stages and Cascades (Continue...)

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Counter-current cascade of ideal stages

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Let's consider three ideal stages.

The solvent stream enters the cascade by the first stage. The solvent can be pure or can have a certain concentration of solute, represented by x_o.
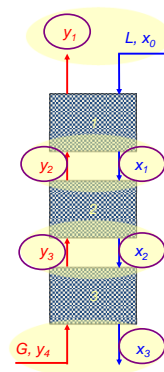
The gas enters the cascade by the last stage and thus, flows in counter-current to the solvent flow.

The operating line is defined by points of compositions of gas and liquid phases at the same level of the process. Points on the operating line are:

$$x_0 , y_1 \qquad x_1 , y_2 \qquad x_2 , y_3 \qquad x_3 , y_4$$

Because the stages are ideal, the equilibrium is reached in each of them. This means that the gas and the liquid phase going out from a stage are at equilibrium, and thus lie on the equilibrium line.

$$x_1 , y_1 \qquad x_2 , y_2 \qquad x_3 , y_3$$

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Cont...

Mass balances

Local partial mass balance: $G\,y_{j+1} + L\,x_0 = G\,y_1 + L\,x_j$

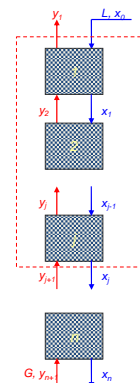dotted area   solute

The equilibrium equation: $y = m\,x$

Dividing by G: $y_{j+1} + \dfrac{L}{G}\,x_0 = y_1 + \dfrac{L}{G}\,x_j$

The operating line is: $y_{j+1} = \dfrac{L}{G}\,x_j + \left(-\dfrac{L}{G}\,x_0 + y_1 \right)$

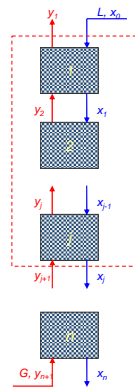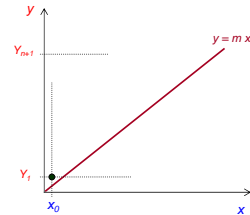The slope of the operating line is positive *(L/G)*

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Cont...

Data are the initial gas composition, y_{n+1}, as well as the initial solvent composition, x_o. On the diagram:

When the final gas composition, y_1, is known because it is restricted to be less than a certain minimum value (specification), the first point of the operating line can be set.
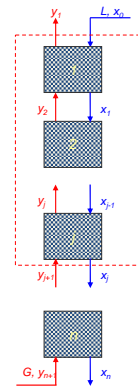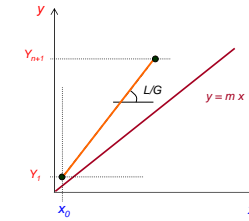

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For the specific slope (L/G), we can set the operating line:

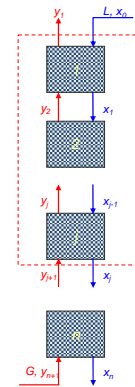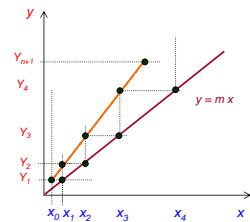$$y_{j+1} = \frac{L}{G} x_j + \left(-\frac{L}{G} x_0 + y_1 \right)$$

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Cont...

The graphical resolution consists on a step by step construction.

We know that x_1 is at equilibrium with y_1. It is then easy to set x_1 on the diagram.

We also know that x_1 and y_2 are the same point on the operating line, since they are both at the same level in the cascade.

And so on, until the specification is reached...

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Cont...

A sequential resolution is also possible without the graphical representation, alternating the use of the operating line equation and the equilibrium equation.

Introducing the absorption factor in the mass balance equation obtained before...

$$y_{j+1} = \frac{L}{G} x_j + \left(-\frac{L}{G} x_0 + y_1 \right) = A y_j + \left(y_1 - A y_0^* \right)$$

Comparing this equation and the general Difference Partial equation, we get

$$\gamma = A \qquad \delta = y_1 - A y_0^*$$

And applying the general solution we get

$$y_j = \left(y_0^* - \frac{y_1 - A y_0^*}{1 - A} \right) A^j + \frac{y_1 - A y_0^*}{1 - A} \qquad \forall \ A \neq 1$$

And for the n stage:

$$y_{n+1} = \left(y_0^* - \frac{y_1 - A y_0^*}{1 - A} \right) A^{n+1} + \frac{y_1 - A y_0^*}{1 - A} \Rightarrow y_{n+1} = y_0^* A^{n+1} + \left(y_1 - A y_0^* \right) \frac{1 - A^{n+1}}{1 - A}$$

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Cont...

The fractional absorption can now be calculated, by substituting the value of y_{n+1}:

$$\alpha = \frac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = 1 - \frac{A-1}{A^{n+1} - 1} \qquad \forall \ A \neq 1$$

This is the *Kremser equation*.

And solving for n, the number of stages can be calculated:

$$n = \frac{Ln\left(\frac{1 - \alpha/A}{1 - \alpha}\right)}{Ln\ A}$$

When *A=1*, the *Kremser equation* cannot be used.

The fraction of absorption is then calculated as

$$\alpha = \frac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = 1 - \frac{1}{1+A} \qquad \forall \ A = 1$$

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Cross-current cascade of ideal stages

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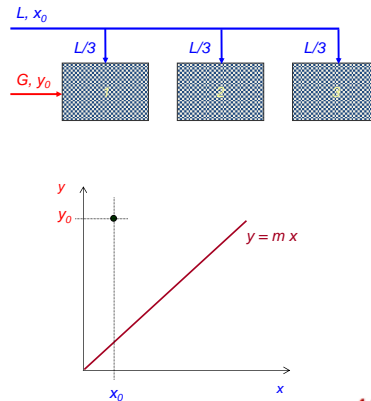
Cross-current cascade of ideal stages

Let's consider three ideal stages.

The solvent stream is divided and each new stream is driven to a stage. The solvent can be pure or can have a certain concentration of solute, represented by x_0. Every stage is receiving solvent with the same composition, x_0.

Let's consider that every stage receives the same amount of solvent ($L/3$)

The equilibrium between phases can be represented in a x-y diagram for a given temperature and pressure. Let's consider again that it is a straight line, $y = m\,x$.

The inlet compositions of the two phases can be represented in the diagram

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Cont...

The content of solute in the gas is now lower than at the entrance: $y_1 < y_0$

The solvent is now charged in solute, so that $x_1 > x_0$.

Because this is an ideal equilibrium stage, the equilibrium between phases is reached.

Repeating what we saw before in the case of ideal single stage, the change on composition in the two phases can be represented on the diagram...
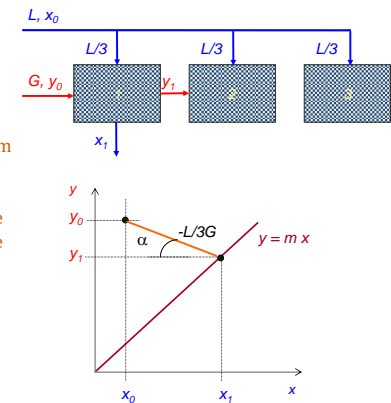
The equation is:

$$y = \left(y_0 + \frac{L/3}{G}\cdot x_0\right) - \frac{L/3}{G}\cdot x$$

The slope of the operating line is then ($-L/3G$)

And in terms of the Absorption factor, A:

$$y_1 = \frac{y_0 + (A/3)\,y_0^*}{1 + (A/3)}$$

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Cont...

Now we have a second ideal stage. We can try to set the new point in the diagram.

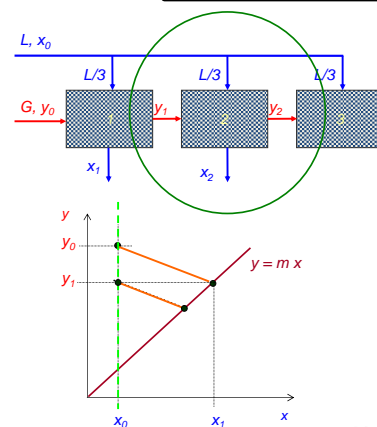The gas at the entrance of the second stage has a concentration equal to y_1...

Because the solvent has the same composition that had before, the new operating point must be located in the same vertical.

The outlet of the second stage is again at equilibrium and because the gas flow is approximately constant and the solvent flow is constant, the operating line has the same slope: L/3G

Because the equilibrium is again reached, we can easily draw the new operating line

And the concentration in terms of A:

$$y_2 = \frac{y_1 + (A/3)\, y_0^*}{1 + (A/3)}$$

The derivation of this equation is shown in the next slide and is similar to the one we did for the ideal single stage.

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Cont...

Derivation of the outlet composition of the gas in the second stage

Material Balance to the solute: $G\, y_1 + L\, x_0 = G\, y_2 + L\, x_2$

The equilibrium equation: $y = m\, x$

Dividing by G: $y_1 + \dfrac{L}{G} x_0 = y_2 + \dfrac{L}{G} x_2$

Using the equilibrium equation, x_2 can be expressed as y_2/m: $y_1 + \dfrac{L}{G} x_0 = y_2 + \dfrac{L}{G}\dfrac{y_2}{m}$

Where we find the Absorption factor: $y_1 + \dfrac{L}{G} x_0 = y_2 + A\, y_2 = y_2\,(1 + A)$

Multiplying and dividing by m in the first term: $y_1 + \dfrac{L\, m}{G\, m} x_0 = y_1 + A\, m\, x_0 = y_1 + A\, y_0^* = y_2\,(1 + A)$

And finally, we obtain the composition at the outlet of the second stage:

$$y_2 = \frac{y_0 + A\, y_0^*}{1 + A}$$

We can obtain the next stage gas compositions by analogy. In the case of the cross-current flow, we always find y_0* in the numerator because the inlet solvent composition is always x_0.
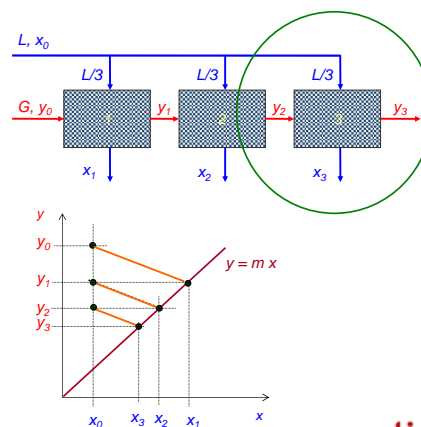
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Cont...

So we have set a new point on the diagram

We arrive now to the last stage and we have to proceed exactly in the same way.

And the concentration in terms of A:

$$y_3 = \frac{y_2 + (A/3)\, y_0^*}{1 + (A/3)}$$
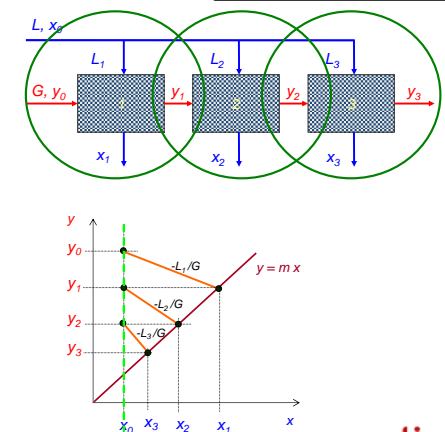

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Cont...

Sometimes the solvent flows are chosen to be different in each stage.

This makes sense because the composition of the gas decreases in each stage and thus, the driving force decreases as well as the solvent requirements.

Then, the slopes of the operating lines are different and must be calculated:

$$-\frac{L_1}{G} \qquad -\frac{L_2}{G} \qquad -\frac{L_3}{G}$$

In principle, the solvent flow-rate will increase and thus, the absolute value of the slope will also increase.

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Cont...
Solving a cross-current cascade

Solving a cross-current cascade is to find the final composition of the gas coming out from the last stage: y_n

We have just seen the graphical resolution of the problem.

Another way to solve the problem is using the equations of type...

$$y_i = \frac{y_{i-1} + (A/N)y_0^*}{1 + (A/N)}$$

A sequential resolution of the equations allows us to find the last gas composition:

$$y_1 = \frac{y_0 + (A/N)y_0^*}{1 + (A/N)} \quad y_2 = \frac{y_1 + (A/N)y_0^*}{1 + (A/N)} \quad y_3 = \frac{y_2 + (A/N)y_0^*}{1 + (A/N)} \quad \cdots \quad y_n = \frac{y_{n-1} + (A/N)y_0^*}{1 + (A/N)}$$

Depending on the number of stages, this resolution can be quite long. In those cases, a mathematical resolution can be used.

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Cont...

Mathematical resolution

In the general equation:
$$y_i = \frac{y_{i-1} + (A/N)y_0^*}{1 + (A/N)}$$

We arrange the terms in a different way:
$$y_i = \frac{1}{1 + A/N}y_{i-1} + \frac{(A/N)}{1 + A/N}y_0^*$$

This equation is a First Order Difference Equation. For a general equation...
$$y_i = \gamma\, y_{i-1} + \delta$$

... the solution has the following form:
$$y_i = y_0\,\gamma^i + \delta\,\frac{1 - \gamma^i}{1 - \gamma}$$

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Cont...

If we look into our equation we can identify the coefficients as:
$$\delta = y_0^*\frac{(A/N)}{1 + (A/N)} \quad \gamma = \frac{1}{1 + (A/N)}$$

And using those coefficients in the general solution:
$$y_i = y_0\frac{1}{(1 + A/N)^i} + y_0^*\left(1 - \frac{1}{(1 + A/N)^i}\right)$$

So, at the last stage, when i =n:
$$y_n = y_0\frac{1}{(1 + A/N)^n} + y_0^*\left(1 - \frac{1}{(1 + A/N)^n}\right)$$

And applying the definition of fractional absorption:
$$\alpha = \frac{y_0 - y_n}{y_0 - y_0^*} = 1 - \frac{1}{(1 + A/N)^n}$$

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Comparison between co-, cross- and counter-current

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Comparison between co-, cross- and counter-current

In order to compare the three cascade configuration, co-, cross- and counter-current, we use the fraction of absorption, α

The fraction of absorption is defined as: $\alpha = \dfrac{y_0 - y_1}{y_0 - y_0^*}$

... and represents the amount of solute absorbed divided by the maximum amount of solute that can be absorbed, which is given by the equilibrium. The fraction of absorption represents therefore the extension of the absorption process.

The fraction of absorption increases when the number of stages increases. When an infinity number of stages is used the fraction of absorption is maximum.

$$\max \ \alpha = \alpha_{n \to \infty}$$

This value of a when the number of stages tends to infinity, will be used to compare the different configurations.

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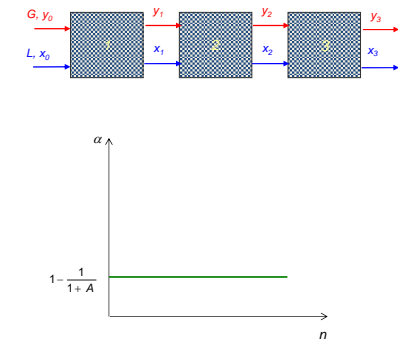### Cont...

Co-current cascade

$$\alpha = \frac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = 1 - \frac{1}{1 + A}$$

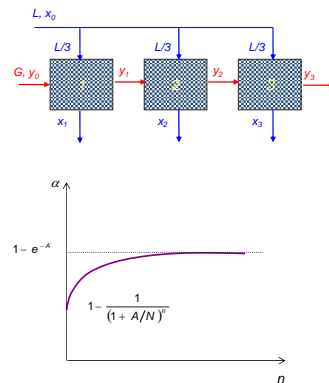$$\alpha_{n \to \infty} = 1 - \frac{1}{1 + A}$$

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Cont...

Cross-current cascade

$$\alpha = \frac{y_0 - y_n}{y_0 - y_0^*} = 1 - \frac{1}{(1 + A/N)^n}$$
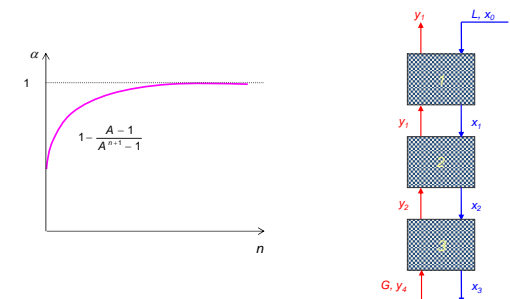
$$\alpha_{n \to \infty} = 1 - e^{-A}$$

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Cont...

Counter-current cascade

$$\alpha = \frac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = 1 - \frac{A - 1}{A^{n+1} - 1}$$
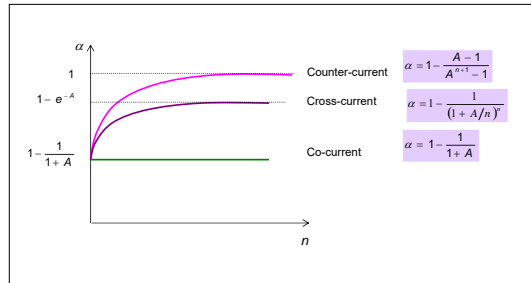
$$\alpha_{n \to \infty} = 1$$

Cont...

Comparison

The graphical representation gives the picture of the maximum absorption factor for each case. The absorption factor is always bigger for the counter-current case.

Counter-current: $\alpha = 1 - \dfrac{A - 1}{A^{n+1} - 1}$

Cross-current: $\alpha = 1 - \dfrac{1}{(1 + A/n)^n}$

Co-current: $\alpha = 1 - \dfrac{1}{1 + A}$

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References

http://www.hyper-tvt.ethz.ch/tvt_links.php

- Lecture notes/ppt of Dr. Yahya Banat (ybanat@qu.edu.qa)

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