Lecture 18: Numerical Analysis (UMA011)

Dr. Meenu Rani

School of Mathematics
TIET, Patiala
Punjab-India

System of linear equations

"This example shows that why Pivot strategies are required in Gauss elimination process"

Example

Apply Gaussian elimination to the system:

$$0.003000x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

using four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution:

$$[A:b] = \begin{array}{c} E_1 \\ E_2 \end{array} \left[\begin{array}{cc:c} 0.003 & 59.14 & 59.17 \\ 5.291 & -6.130 & 46.78 \end{array}\right] \sim$$

$$E_2 \to E_2 - \frac{5.291}{0.003} E_1 \sim E_2 \to E_2 - 1764 E_1$$

$$[A:b] \sim \begin{matrix} E_1 \\ \\ \end{matrix} \left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 0 & -1.043 \times 10^5 & -1.044 \times 10^5 \end{array}\right]$$

$E_2 \to E_2 - 1764 E_1$

$46.78 - 1764 * 59.17$

$46.78 - 104375.88$

$46.78 - 1.0437588 \times 10^5$

$0.0004678 \times 10^5 - 1.044 \times 10^5$

-1.044×10^5

$E_2 \to E_2 - 1764 E_1$

$-6.130 - 1764 * 59.14$

$= -6.130 - 104322.96$

$= -6.130 - 1.0432296 \times 10^5$

$= -6.130 - 1.043 \times 10^5$

$= -0.00006130 \times 10^5 - 1.043 \times 10^5$

$= -1.043 \times 10^5$

Use backward substitution

$-1.043 \times 10^5 x_2 = -1.044 \times 10^5$

$x_2 = 1.001$

$0.003 x_1 + 59.14 x_2 = 59.17$

$0.003 x_1 + 59.14 * 1.001 = 59.17$

$$0.003x_1 + 59.20 = 59.17$$

$$0.003x_1 = -0.03$$

$$x_1 = \frac{-0.03}{0.003} = -10$$

$$X = \begin{bmatrix} -10 \\ 1.001 \end{bmatrix}$$ Ans

not good app.

System of linear equations

$a_{11} \neq 0$

$a_{22} \neq 0$

$a_{33} \neq 0$

Pivot element

In the elimination process, we divide with diagonal element a_{ii} at each stage and assume that $a_{ii} \neq 0$. These elements are known as pivot element.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & & \end{bmatrix}$$

$$E_2 \to E_2 - \frac{a_{21}}{a_{11}} E_1$$

Pivot Strategies

1. Partial Pivoting
2. Scaled Partial Pivoting

System of linear equations

Partial Pivoting

If at any stage of elimination, one of the pivot becomes small (or zero) then we bring other element as pivot by interchanging the rows. This process is called Gauss elimination with partial pivoting.

$a_{11} = 0$

or $a_{11} \approx 0$

or a_{11} smaller than a_{p1}

$$a_{p1} = \max\{|a_{21}|, |a_{31}|, |a_{41}| \cdots |a_{n1}|\}$$

$$\begin{bmatrix} a_{11} & & \\ 0 & a_{22} & \\ & 0 & \end{bmatrix}$$

$$E_1 \leftrightarrow E_p$$

Repeat the process

System of linear equations

Example

Apply Gaussian elimination to the system:

$$0.003000x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

using partial pivoting and four-digit arithmetic with rounding, and compare the results to the exact solution $x_1 = 10.00$ and $x_2 = 1.000$.

Solution:

$$[A:b] = \begin{array}{c} E_1 \\ E_2 \end{array}\left[\begin{array}{cc:c} 0.003 & 59.14 & 59.17 \\ 5.291 & -6.130 & 46.78 \end{array}\right]$$

$$\max\{|0.003|, |5.291|\} = 5.291 = a_{21}$$

$E_1 \leftrightarrow E_2$

$$[A:b] = \begin{array}{c} E_1 \\ E_2 \end{array} \begin{bmatrix} 5.291 & -6.130 & : & 46.78 \\ 0.003 & 59.14 & : & 59.17 \end{bmatrix}$$

$E_2 \to E_2 - \dfrac{0.003}{5.291} E_1 \sim E_2 \to E_2 - 0.0005670 E_1$

$59.14 - 0.0005670 * (-6.130)$

$59.14 + 0.003476$

$= 59.14$

$$\sim \begin{array}{c} E_1 \\ E_2 \end{array} \begin{bmatrix} 5.291 & -6.130 & : & 46.78 \\ 0 & 59.14 & : & 59.14 \end{bmatrix}$$

$59.17 - 0.0005670 * 46.78$

$59.17 - 0.02652 = 59.14$

Using back sub, $59.14 x_2 = 59.14$

$\Rightarrow x_2 = 1$

and

$$5.291 x_1 - 6.130 x_2 = 46.78$$

$$5.291 x_1 - 6.130 * 1 = 46.78$$

$$5.291 x_1 = 46.78 + 6.130$$

$$5.291 x_1 = 52.91$$

$$x_1 = 10$$

$$X = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$ Ans

System of linear equations:

Exercise:

1. Use Gaussian elimination with partial pivoting and three-digit chopping arithmetics to solve the following linear system, and compare the approximations with the actual solution $[0, 10, 1/7]^T$.

$$\begin{aligned}
3.03x_1 - 12.1x_2 + 14x_3 &= -119 \\
-3.03x_1 + 12.1x_2 - 7x_3 &= 120 \\
6.11x_1 - 14.2x_2 + 21x_3 &= -139.
\end{aligned}$$