THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

Mass Transfer-I

Mass Transfer Analogies

DEPARTMENT OF CHEMICAL ENGINEERING
Thapar Institute of Engineering & Technology
Patiala (Punjab), INDIA-147004

Dr. Avinash Chandra
(Ph.D., IIT Kanpur)
Associate Professor

Web: http://www.thapar.edu/faculties/view/Dr.-Avinash-Chandra/ODU=/Mg==

Analogies among Mass, Heat, and Momentum Transfer

Analogies

Heat ⇔ Mass ⇔ *(sometimes)* Momentum

Analogies are useful tools

1. An aid to understand transfer phenomena
2. A sound means to predict behavior of systems for which limited quantitative data are available

Molecular Transport Equations

$$\text{Rate of transport} = \frac{\text{Driving force}}{\text{Resistance}}$$

$$\tau_{yx} = -\nu \frac{d(v_x \rho)}{dy}$$

MOMENTUM
Newton's law

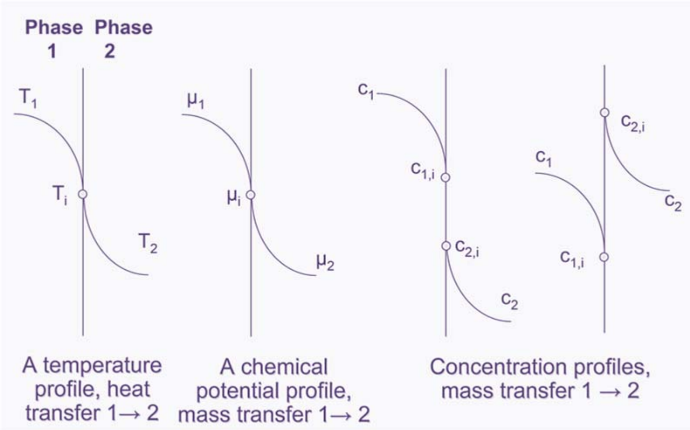
$$\frac{q_y}{A} = -\alpha \frac{d(\rho\, c_p\, T)}{dy}$$

HEAT
Fourier's law

$$J^*_{Ay} = -D_{AB} \frac{dc_A}{dy}$$

MASS
Fick's law

Cont…



Analogous quantities in transport phenomena

	Momentum	Mass	Energy
Transport quantity per volume	ρu_x	C_A	$\rho C_p T$
Transport coefficient	μ [g·cm^{-1}s^{-1}]	D_{AB} [cm^2·s^{-1}]	k [cal·cm^{-1}s^{-1}K^{-1}]
Diffusivity [cm^2·s^{-1}]	$\nu = \mu/\rho$	D_{AB}	$\alpha = k/\rho C_p$
Flux law	$\tau_{xz} = -\nu \frac{d}{dz}(\rho u_x)$	$J_z = -D_{AB}\frac{d}{dz}C_A$	$q_z = -\alpha \frac{d}{dz}(\rho C_p T)$
Dimensionless transport groups	$Re = \frac{UL}{\nu}$	$Sc = \frac{\nu}{D_{AB}}$	$Pr = \frac{\nu}{\alpha}\left(= \frac{C_p \mu}{k}\right)$

Reynolds Analogy

Assumptions

- Only turbulent core is present
- Velocity, temperature and concentration profiles are perfectly matching.
- All diffusivities (turbulent diffusivities) are same
- The molecular diffusivities are negligible.

$$(\alpha) = (D_{AB}) = \left(\frac{\mu}{\rho}\right)$$

When all the three diffusivities are equal, then

Prandtl Number (N_{Pr}) = Schmidt number (N_{Sc}) = 1.

Reynolds analogy equation is

$$\boxed{\left(\frac{f}{2}\right) = \frac{h}{\rho C_p u_0} = \frac{k_c}{u_0}}$$

Where,

f = Friction factor
h = Heat transfer coefficient
α = Thermal diffusivity
C_p = Heat capacity
D_{AB} = Mass diffusivity
k_c = Mass transfer coefficient
u_0 = Characteristic velocity
ρ = Density
St = Stanton number
Pr = Prandtl number
Sc = Schmidt number
Re = Reynolds number
Sh = Sherwood number

$$\boxed{St = \frac{Sh}{Re.Sc} = \frac{f}{2}}$$

$$\boxed{St = \frac{Nu}{Re.Pr} = \frac{f}{2}}$$

Cont…

The Reynolds analogy in not applicable for following cases

- For other fluids, where ***Pr* ≠ *Sc* ≠ 1**; usually the case for liquids
- We **CANNOT** neglect molecular diffusivities in case of boundary layer, where diffusion, conduction, and viscosity are important

Experimental results show that the above equation

- **Correlate data approximately for gases in turbulent flow for *0.6 < Pr (for gases) < 2.5***
- DOES NOT correlate experimental data for liquids in turbulent flow
- DOES NOT correlate experimental data for any fluids in laminar flow

CONCLUSIONS

1. At *Pr = Sc = 1*, the mechanisms for mass, heat, and momentum are identical
2. For other fluids, transfer processes differ in some manner functionally related to the ***Pr*** and ***Sc*** numbers.
3. The Reynolds analogy is valid ONLY at ***Pr = Sc =1***

Chilton-Colburn Analogy

Assumptions

- Only turbulent core is present
- Velocity, temperature and concentration profiles are perfectly matching.
- $N_{Pr} \neq N_{Sc} \neq 1$

Analogy equation is

$$\left(\frac{f}{2}\right) = \left(\frac{k_c}{u_0}\right)(\mathrm{Sc})^{2/3} = \frac{h}{(\rho C_P u_0)}(\mathrm{Pr})^{2/3}$$

$$St \approx \left(\frac{f}{2}\right).(Sc)^{-2/3}$$

Taylor-Prandtl Analogy

Assumptions

- turbulent and laminar sublayer are present
- $N_{Pr} \neq N_{Sc} \neq 1$

Analogy equation is

$$\frac{k_c}{u_0} = \frac{h}{\rho C_P u_0} = \frac{\left(\frac{f}{2}\right)}{\left[1 + 5\sqrt{\frac{f}{2}}(\mathrm{Sc}-1)\right]} = \frac{\left(\frac{f}{2}\right)}{\left[1 + 5\sqrt{\frac{f}{2}}(\mathrm{Pr}-1)\right]}$$

$$St \cong \frac{\frac{1}{2}f}{1 + 5(\frac{1}{2}f)^{1/2}(Pr-1)}$$

Von-Karman Analogy

Assumptions

- Turbulent, laminar sublayer and buffer layers are present
- Universal velocity profile equations are applicable
- $N_{Pr} \neq N_{Sc} \neq 1$

Analogy equation is

$$\frac{k_c}{u_0} = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}\left[(\mathrm{Sc}-1)+\ln\left(\frac{5\mathrm{Sc}+1}{6}\right)\right]} = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}\left[(\mathrm{Pr}-1)+\ln\left(\frac{5\mathrm{Pr}+1}{6}\right)\right]}$$

$$St_m \approx 0.08.\left(\frac{c_f}{2}\right)^{1/2}.(Sc)^{-0.704}$$

Example

A 1 m^2 thin plate of solid naphthalene is oriented parallel to a stream of air flowing at 30 cm/s. The air is at 300 K and 1 atm pressure. The plate is also at 300 K. Determine the rate of sublimation from the plate. The diffusivity of naphthalene in air at 300 K and 1 atm is 5.9×10^{-4} m^2/s. Vapour pressure of naphthalene at 300 K is 0.2 mm Hg.

Solution

$$u_0 = 30 \text{ cm/s},\ T = 300 \text{ K},\ P_t = 1 \text{ atm},\ D_{AB} = 5.9 \times 10^{-4} \text{ m}^2/\text{s},$$

$$p_A = 0.2 \text{ mm Hg}$$

$$\rho_{air} = 1.15 \times 10^{-3} \text{ g/cc},\ \mu_{air} = 0.0185\ cp,\ D = 1 \text{ m} \quad \text{(Length)}$$

Cont…

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{0.0185 \times 10^{-3}}{1.15 \times 10^{-3} \times 10^{3} \times 5.9 \times 10^{-4}} = 0.0273 \neq 1$$

So we will use Chilton analogy,

$$N_{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{1.15 \times 10^{-3} \times 10^{3} \times 1 \times 0.3}{0.0185 \times 10^{-3}} = 18648.65$$

So flow is turbulent, so Chilton–Colburn analogy can be used.

$$\frac{f}{2} = \frac{k_c}{u_0}(N_{Sc})^{2/3}$$

Cont…

$$f = 0.072 \times (N_{Re})^{-0.25}$$

$$f = 6.161 \times 10^{-3}$$

$$k_c = \frac{f \times u_0}{2(N_{Sc})^{2/3}}$$

$$k_c = \frac{6.161 \times 10^{-3} \times 0.3}{2(0.0273)^{2/3}} = 0.0102 \text{ m/s}$$

$$N_A = k_c\,(C_{A1} - C_{A2}) = k_c \frac{(p_{A1} - p_{A2})}{RT}$$

$$= \frac{0.0102 \times 1.0133 \times 10^{5}}{8314 \times 300}\left[\left(\frac{0.2}{760}\right) - 0\right]$$

$$N_A = 1.09 \times 10^{-7} \text{ kmol/m}^2\text{ s.} \quad \textbf{Ans.}$$

References

ETH

Mass Transfer

Theories for Mass Transfer Coefficients

Lecture 9, 15.11.2017, Dr. K. Wegner

- Lecture notes/ppt of Dr. Yahya Banat **(ybanat@qu.edu.qa)**