Solids and Structures

Topic: Shear Stresses in Beam

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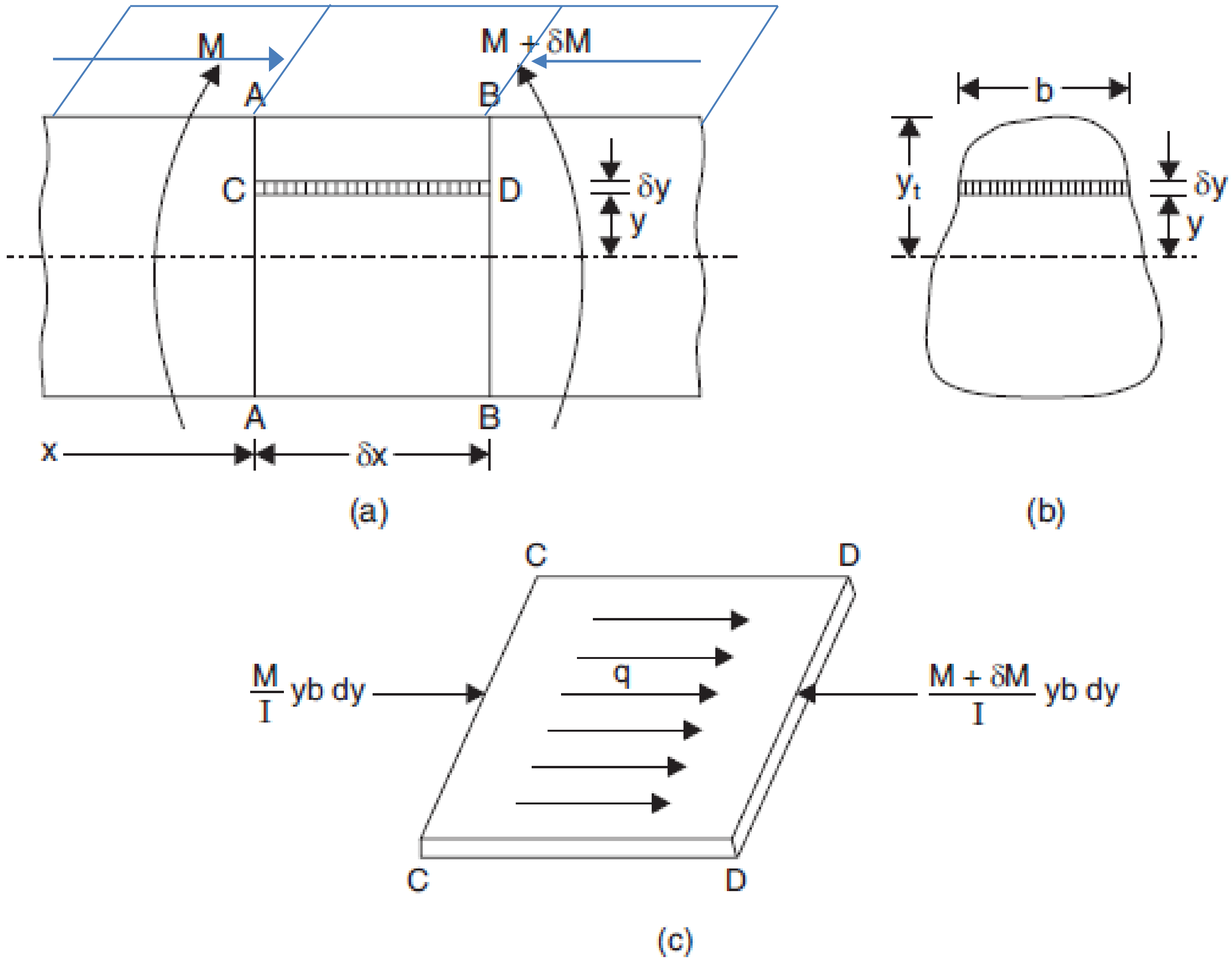
Expression For Shear Stresses in Beams

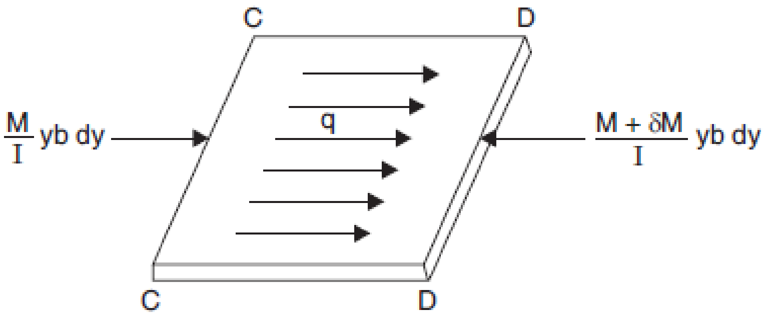
Consider an elemental length ‘δx’ of beam shown in Figure. Let bending moment at section *A-A* be *M* and that at section *B-B* be $M + \delta M$. Let CD be an elemental fibre at distance y from neutral axis and its thickness be δy.



(a) (b) (c)

Bending stress on left side of elemental fibre

$$= \frac{M}{I} y$$



(c)

∴ The force on left side of element

$$= \frac{M}{I} y\ b\ \delta y$$

Similarly, force on right side on elemental fibre

$$= \frac{M + \delta M}{I} y\ bdy$$

∴ Unbalanced horizontal force on right side of elemental fibre

$$= \frac{M + \delta M}{I} y\ b\delta y - \frac{M}{I} y\ b\delta y \quad = \frac{\delta M}{I} yb\ \delta y$$

There are a number of such elemental fibres above CD. Hence unbalanced horizontal force on section CD

$$= \int_y^{y_t} \frac{dM}{I} y\ b\ \delta y \quad = \int_y^{y_t} \frac{dM}{I} y\ b\ dy = \frac{\delta M}{I} \int_y^{y_t} yb\ dy$$

Let intensity of shearing stress on element CD be q. Then equating resisting shearing force to unbalanced horizontal force, we get

$$q\ b\ \delta x = \frac{\delta M}{I}\int_{y}^{y_t} yb\ dy$$

$$\therefore \qquad q = \frac{\delta M}{\delta x}\frac{1}{bI}\int_{y}^{y_t} yb\ dy$$

As $\delta x \to 0$,

$$q = \frac{dM}{dx}\frac{1}{bI}\,(a\overline{y})$$

where $a\overline{y}$ = Moment of area above the section under consideration about neutral axis.

But we know

$$\frac{dM}{dx} = F$$

$$\therefore \qquad q = \frac{F}{bI}\,(a\overline{y})$$

The above expression gives shear stress at any fibre y distance above neutral axis.

Expression for Shear Stresses in Beams

$$q = \frac{F}{bI}\,(a\bar{y})$$

Where,

F or S – shear force;

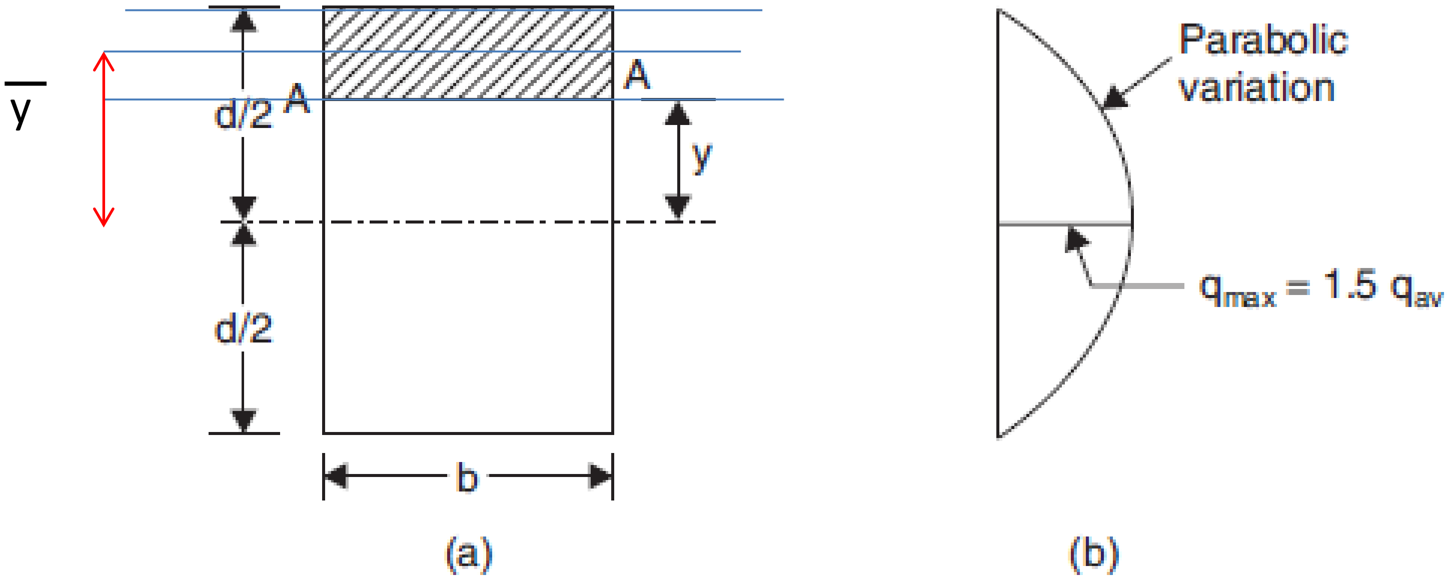
A – area of cross section under consideration;

$\bar{y}$ – distance of the C.G of the area above layer;

b – width of layer;

Variation of Shear Stresses across Rectangular Section

$$q = \frac{F}{bI}(a\bar{y})$$



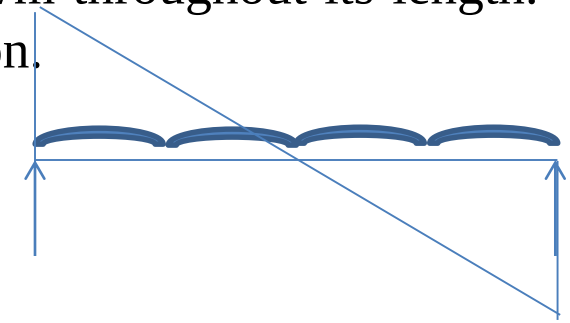
Shaded Area = a = b(d/2-y)

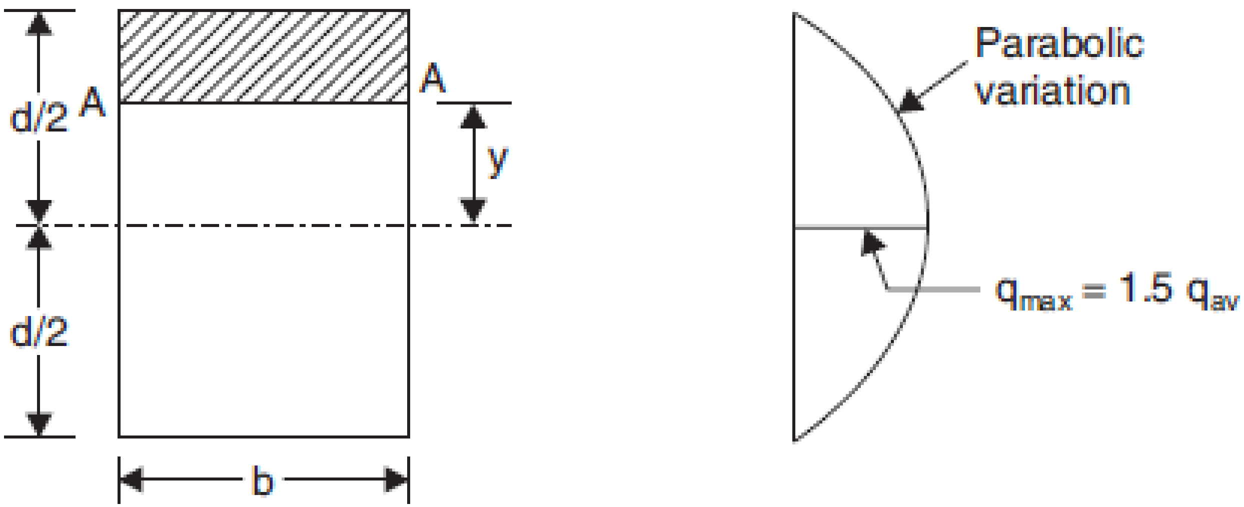
Centroid of shaded area about NA = y + ½ (d/2 - y)

= ½ (d/2 + y)

$$q = \frac{6F}{bd^3}(d^2/4 - y^2)$$

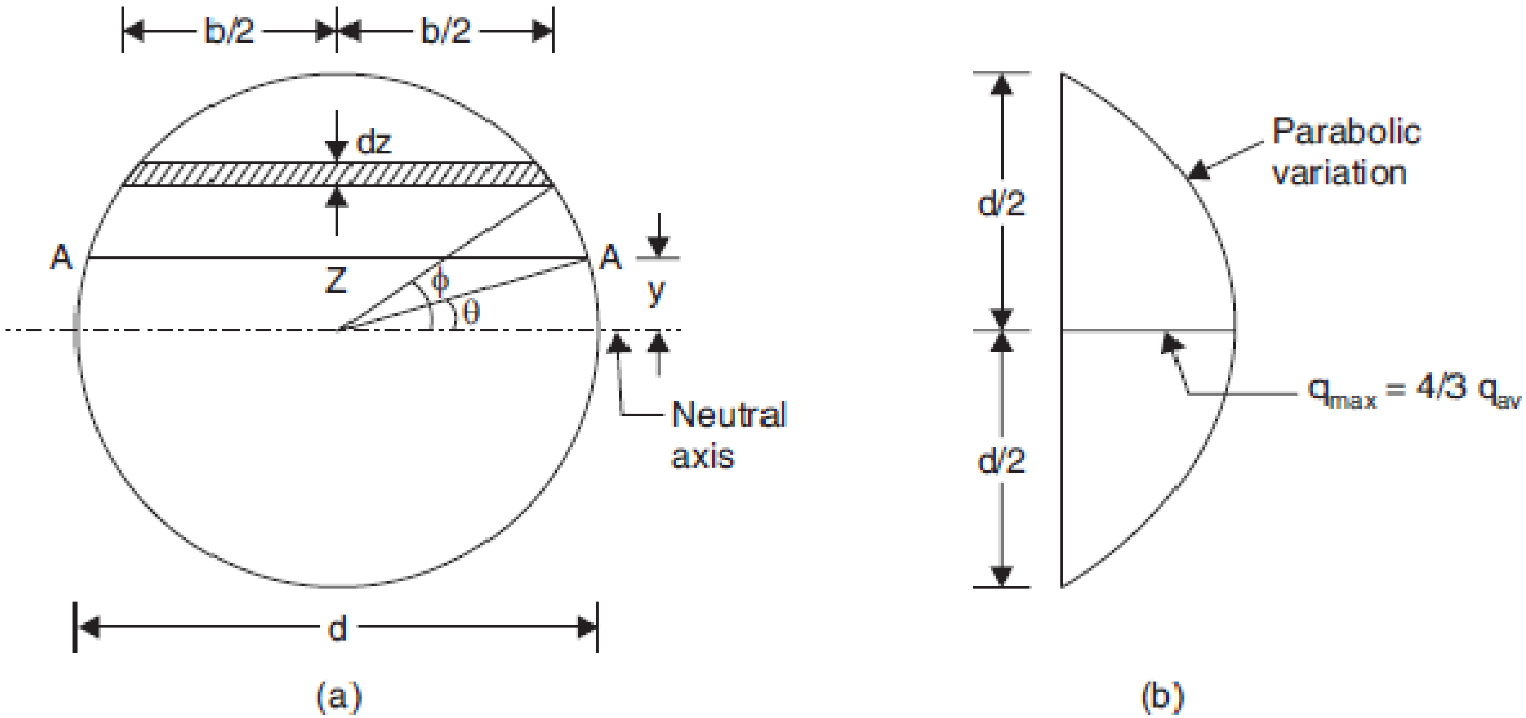
Problem 1: A wooden beam of rectangular section 150mm x 300 mm is simply supported over a length of 4 m. It carries a UDL of 4 kN/m throughout its length. What is the maximum shear stress developed in beam section.





(a) (b)

Variation of Shear Stresses across Circular Section

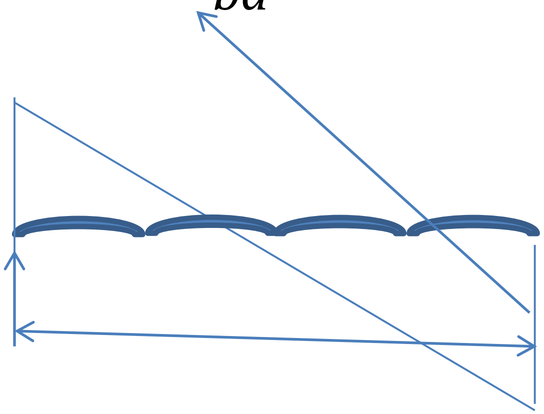


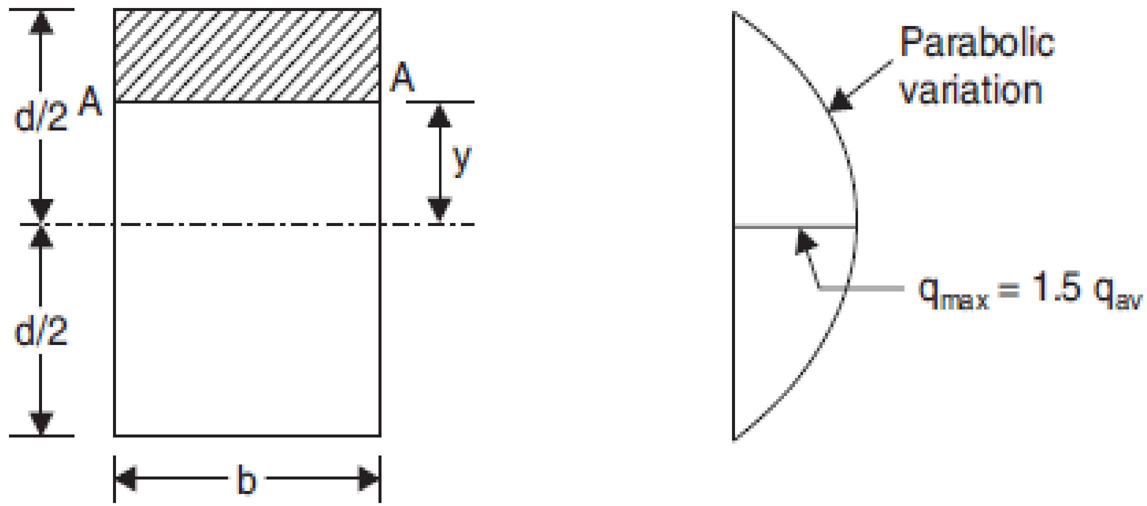
$$q = \frac{F}{bI}(a\bar{y}) \quad = \frac{16}{3}\frac{F}{\pi d^2}\left[1 - \frac{4y^2}{d^2}\right]$$

Problem 2: A timber beam 150 mm x 250 mm in cross section is simply supported at its ends and has a span of 3.5 m. The maximum safe allowable stress in bending is 7500 kN/m^2. Find the maximum safe U.D.L which the beam can carry. What is the maximum shear stress in the beam for U.D.L calculated.

$$M=\frac{wl^2}{8};\frac{M}{I}=\frac{\sigma}{y};I=\frac{bd^3}{12};y=\frac{d}{2}$$

$$\tau_{\max}=1.5\frac{F}{bd}$$



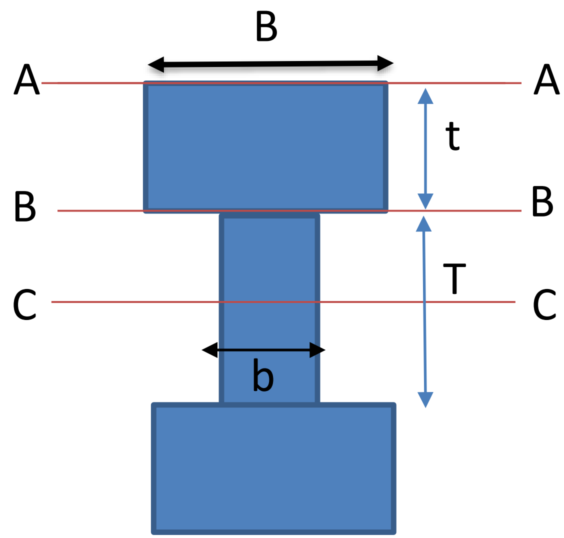


(a) (b)

SHEAR STRESSES IN BUILT-UP SECTIONS

- In sections like I, T and channel, shear stresses at various salient points are calculated and the shear stress variation diagram across depth is plotted.
- It may be noted that at extreme fibres, the value of shear stress is zero.
- However it may be noted that the procedure explained is for built up section with at least one symmetric axis.
- If there is no symmetric axis along the depth, the analysis for shear stress is complex.

$$q = \frac{F}{bI}\,(a\bar{y})$$



- F is $V_{maximum}$ or V at a section
- 'b' is the width at the section considered
- $I = I_{NA}$

Section	b	A y	q
AA	B	0	0
BB (Just Above Junction)	B	Bt (T/2 + t/2)	X
BB (Just Below the junction)	b	Bt (T/2 + t/2)	X * B/b
CC (NA)	b	Bt (T/2 + t/2) + B X T/2 (T/4)	Y

Variation of Shear Stresses in I - section

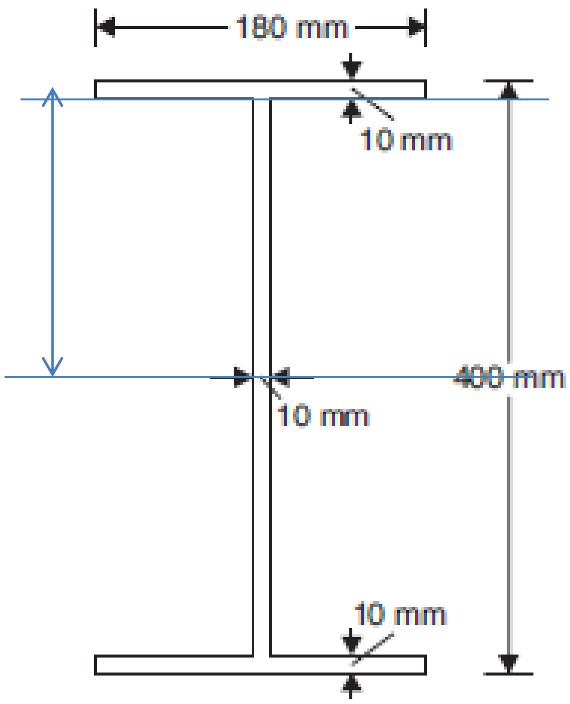
Problem 3: Draw the shear stress variation diagram for the I-section shown in Fig. if it is subjected to a **shear force of 100 kN**.

$$I = \frac{1}{12} \times 180 \times 10^3 + 180 \times 10 \times (200 - 5)^2$$

$$+ \frac{1}{12} \times 10 \times 380^2 + 10 \times 380 \times (200 - 200)^2$$

$$+ \frac{1}{12} \times 180 \times 10^3 + 180 \times 10 \times (200 - 5)^2$$

$$= 182.646666 \times 10^6 \text{ mm}^4$$



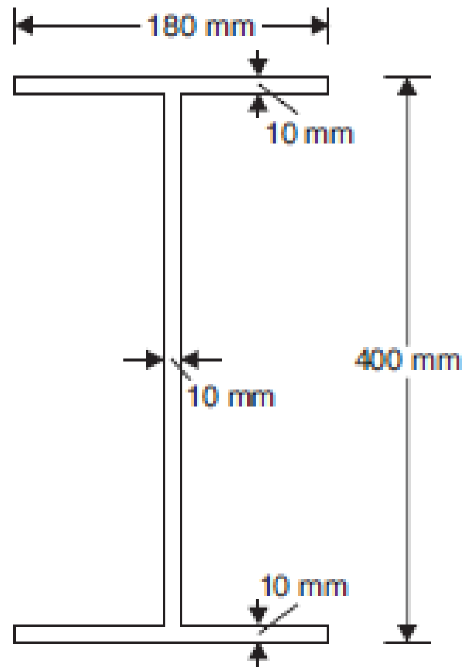
Shear stress at $y = 200$ mm is zero since $a\bar{y} = 0$.

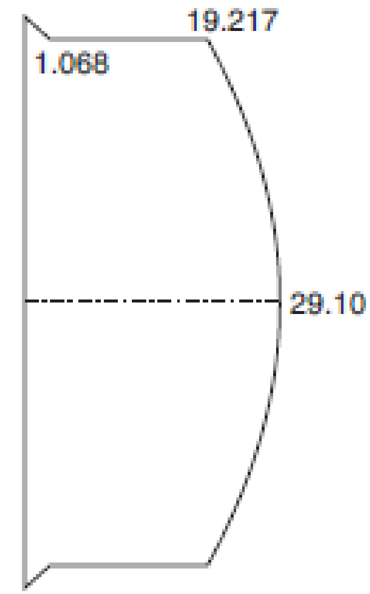
Shear stress at bottom of top flange

$$= \frac{F}{bI}(a\bar{y})$$

$$= \frac{100 \times 1000}{180 \times 182.646666 \times 10^6} \times (180 \times 10 \times 195) = 1.068 \text{ N/mm}^2$$

- $I_{NA} = 182.64 \times 10^6\ mm^4$
- $F = \quad 100\ kN = 100 \times 1000\ N$





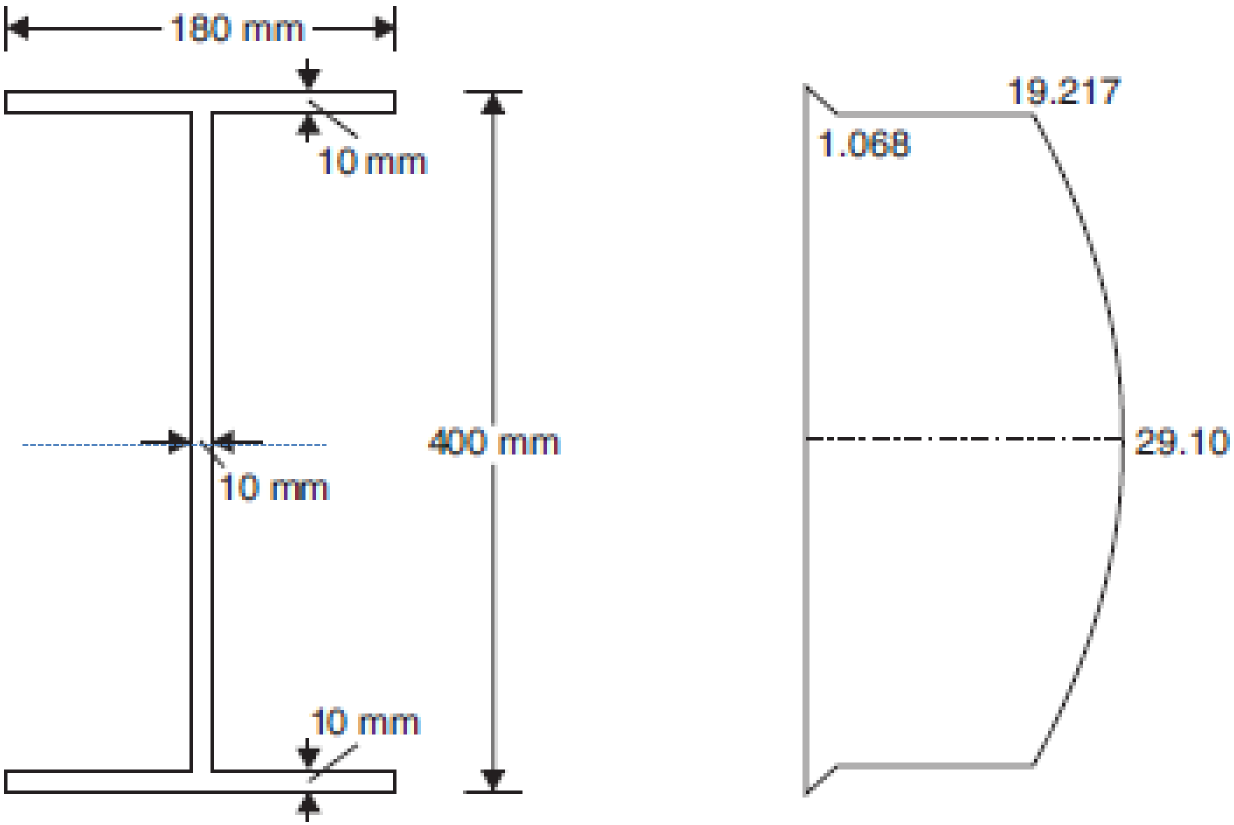
Section	b (mm)	A y	q (N/mm^2)
Top of Flange	180	0	0
BB (Just Above Junction)- Bottom of Top Flange	180	(180 X10) (190 +5)	1.068
BB (Just Below the junction)	10	(180 X10) (190 +5)	1.068 X 180/10 = 19.217
CC (NA)	10	(180 X10) (190 +5) + 10 X 190 X 190/2	29.10

Shear stress in the web at the junction with flange

$$= \frac{100 \times 1000}{10 \times 182.646666 \times 10^6} (180 \times 10 \times 195) = 19.217 \text{ N/mm}^2$$

Shear stress at *N-A*

$$= \frac{100 \times 1000}{10 \times 182.646666} \times \left[180 \times 10 \times 195 + 10 \times (200 - 10) \times \frac{190}{2}\right] = 29.10 \quad \text{N/mm}^2.$$



Variation of Shear Stresses in T-Section

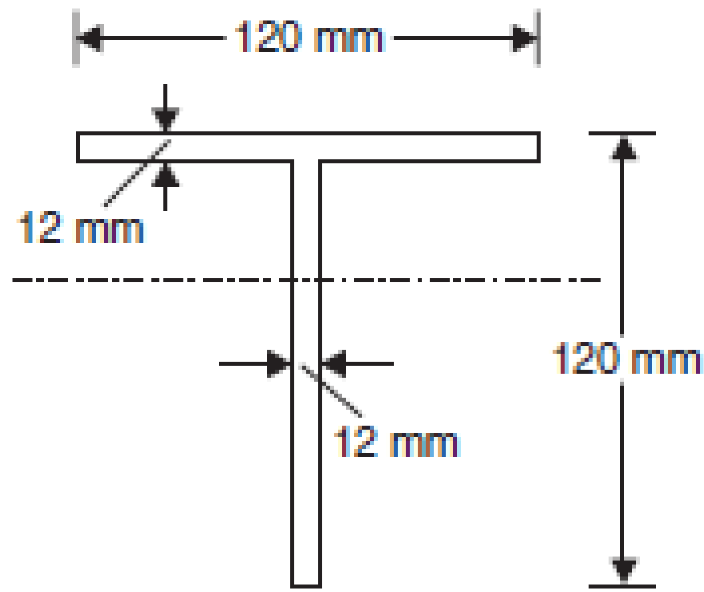
Problem 4: A beam has cross-section as shown in Fig. If the shear force acting on **this is 25 kN**, draw the shear stress distribution diagram across the depth.

$$y' = \frac{\text{Moment of area about top fibre}}{\text{Total area}}$$

$$= \frac{120 \times 12 \times 6 + (120 - 12) \times 12 \times \left(12 + \frac{120 - 12}{2}\right)}{120 \times 12 + (120 - 12) \times 12} = 34.42 \text{ mm}$$

Moment of inertia about centroid

$$= 2936930 \text{ mm}^4$$



- $I_{NA} = 2.93 \text{ X}10^6 \text{ mm}^4$
- $F = 25 \text{ kN} = 25 \text{ X } 1000 \text{ N}$

Shear stresses are zero at extreme fibres.

Shear stress at bottom of flange:

Area above this level, $a = 120 \times 12 = 1440 \text{ mm}^2$

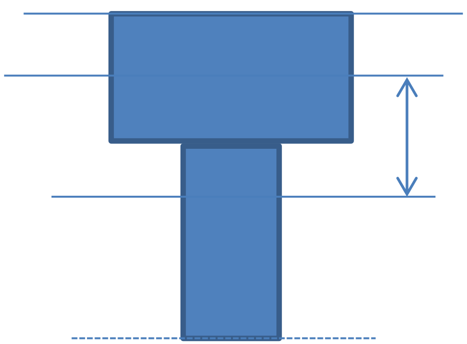
Centroid of this area above *N-A*

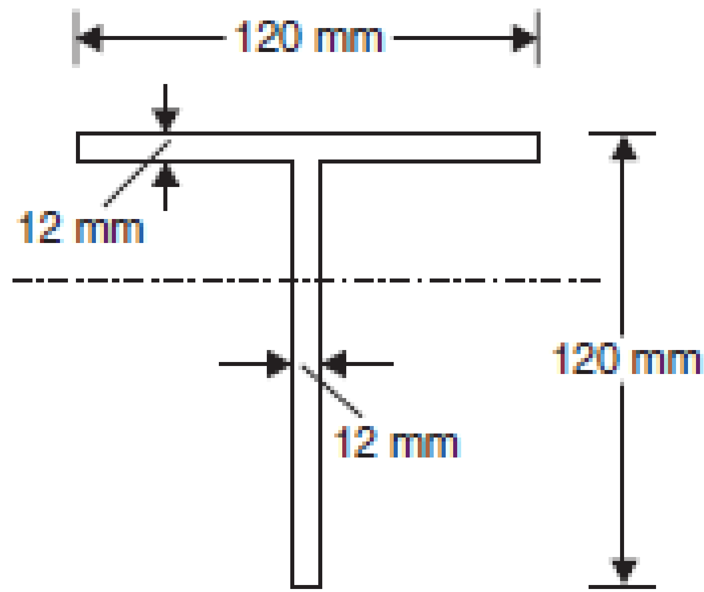
$$\bar{y} = 34.42 - 6 = 28.42 \text{ mm}$$

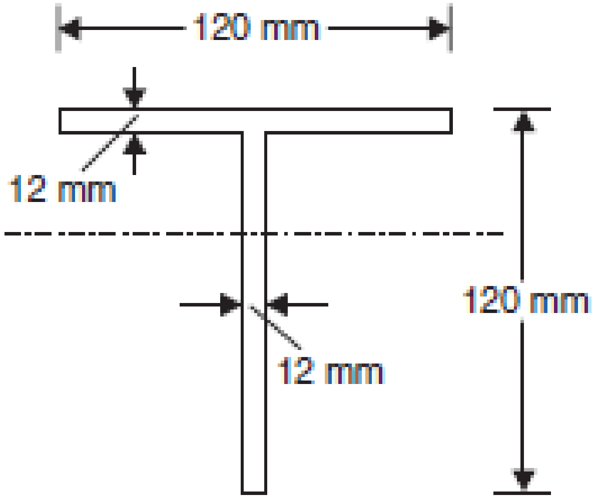
Width at this level $b = 120$ mm.

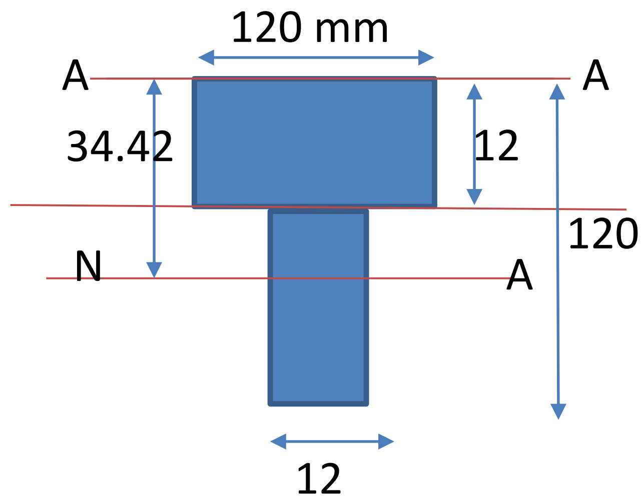
$$\therefore \quad q_{\text{bottom of flange}} = \frac{25 \times 1000}{120 \times 2936930} \times 1440 \times 28.42$$

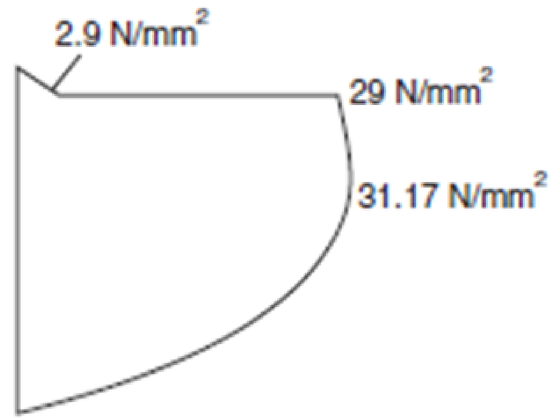
$$= 2.90 \text{ N/mm}^2$$





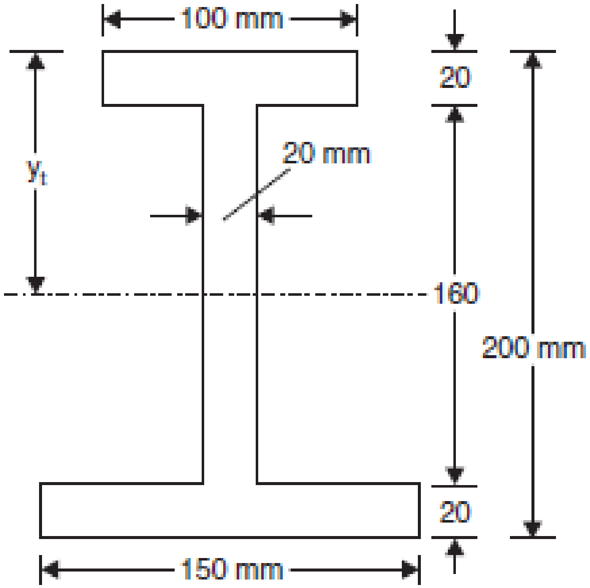






Section	b	A y	q
AA	120	0	0
BB (Just Above Junction)	120	120 X12 X (34.42-6)	2.9
BB (Just Below the junction)	12	120 X12 X (34.42-6)	29
CC (NA)	12	120 X12 X (34.42-6) + [(34.42-12)X 12] X (34.42-12)/2	31.17
At bottom	12	0	0

Problem 4: The unsymmetrical I-section shown in *Figure*, The cross-section of a beam, which is subjected to a shear force of 60 kN. Draw the shear stress variation diagram across the depth.



Distance of neutral axis (centroid) of the section from top fibre be y_t. Then

$$y_t = \frac{100 \times 20 \times 10 + (200 - 20 - 20) \times 20 \times \left(20 + \frac{160}{2}\right) + 150 \times 20 \times (200 - 10)}{100 \times 20 + 160 \times 20 + 150 \times 20}$$

$$= 111 \text{ mm}$$

$$I = \frac{1}{12} \times 100 \times 20^3 + 100 \times 20\,(111 - 10)^2$$

$$+ \frac{1}{12} \times 20 \times 160^3 + 160 \times 20\,(111 - 100)^2$$

$$+ \frac{1}{12} \times 150 \times 20^3 + 150 \times 20\,(111 - 190)^2$$

$$= 46505533 \text{ mm}^4$$

Shear stress at bottom of top flange

$$= \frac{F}{bI}\, a\bar{y}$$

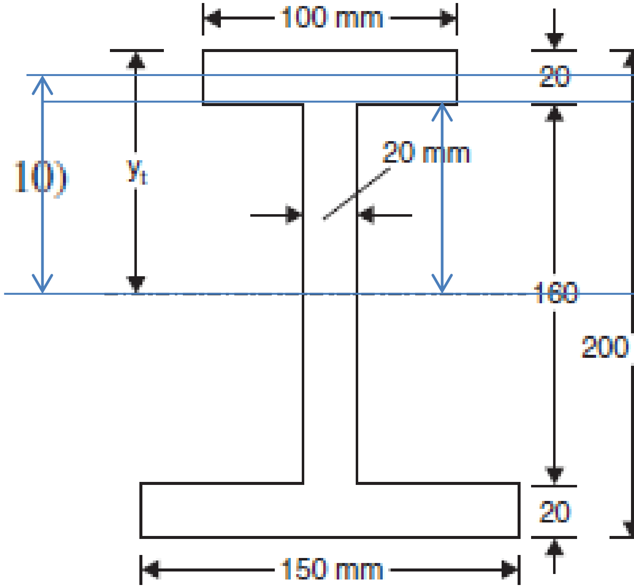
$$= \frac{60 \times 1000}{100 \times 46505533} \times 100 \times 20 \times (111 - 10)$$

$$= 2.61 \text{ N/mm}^2$$

∴ Shear stress at the same level, but in web

$$= \frac{60 \times 1000}{20 \times 46505533} \times 100 \times 20\ (111 - 10)$$

$$= 13.03 \text{ N/mm}^2$$

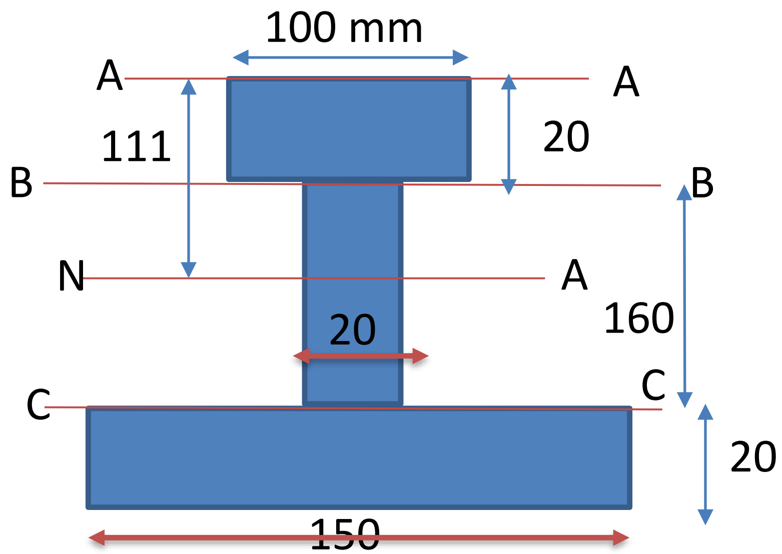


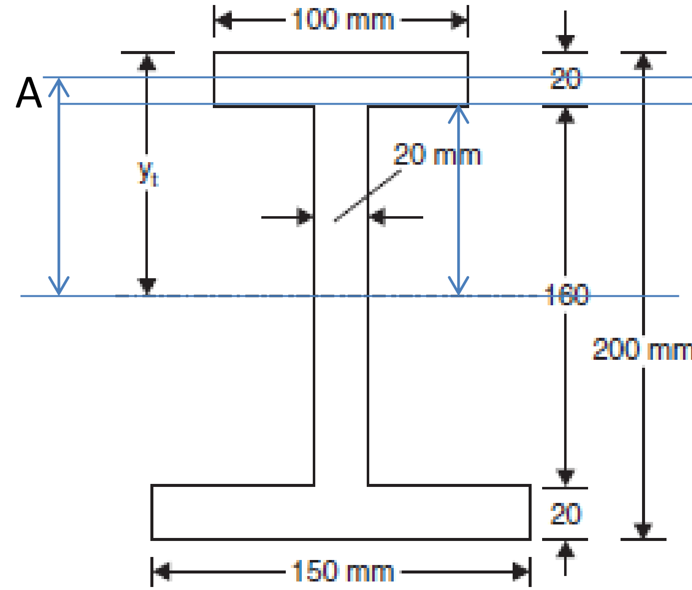
Shear stress at neutral axis:

$a\bar{y} = a\bar{y}$ of top flange + $a\bar{y}$ of web above N-A

$$= 100 \times 20 \times (111 - 10) + 20 \times (111 - 20) \times \frac{111 - 20}{2}$$

$$= 284810 \text{ mm}^3.$$





Section	b	A y	q
AA	100	0	0
BB (Just Above Junction)	100	100 X20 X (111-10)	2.61
BB (Just Below the junction)	20	120 X12 X (111-10)	13.03
NA	20	120 X12 X (111-10) + [(111-20)X 20] X (111-20)/2	18.37
CC- At junction at top	20	(150 X20) X (180-111 +10)	15.24
CC- At junction at bottom	150	(150 X20) X (180-111 + 10)	2.04

Shear stress at neutral axis

$$= \frac{F}{bI}(a\bar{y}) = \frac{60 \times 1000}{20 \times 46505533} \times 284810 = 18.37 \text{ N/mm}^2.$$

Shear stress at junction of web and lower flange:

Considering the lower side of the section for finding $a\bar{y}$, we get

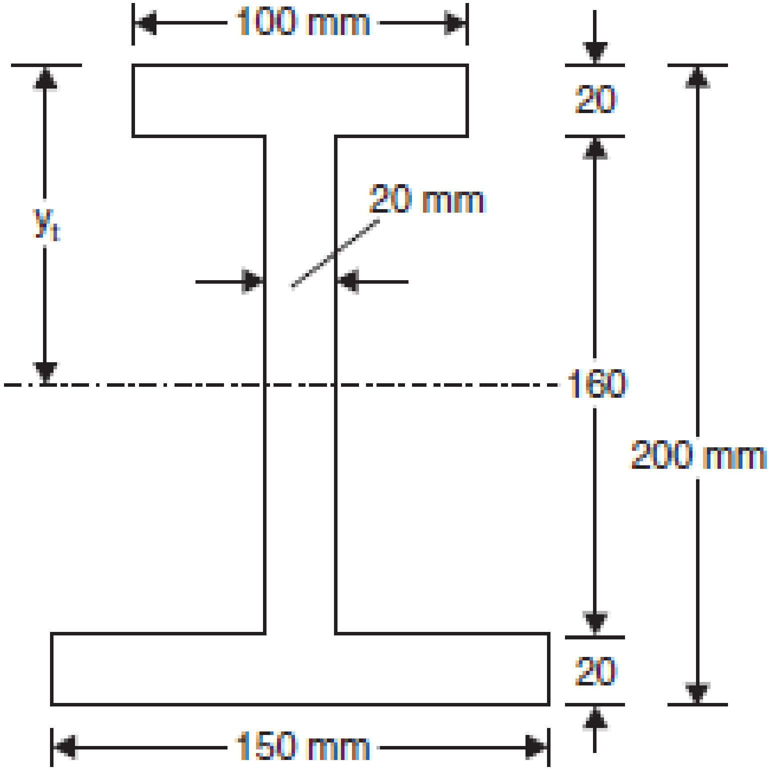
$$a\bar{y} = 150 \times 20 \times (190 - 111) = 237000 \text{ mm}^3$$

$$\therefore \quad q = \frac{60 \times 1000}{20 \times 46505533} \times 237000 = 15.28 \text{ N/mm}^2$$

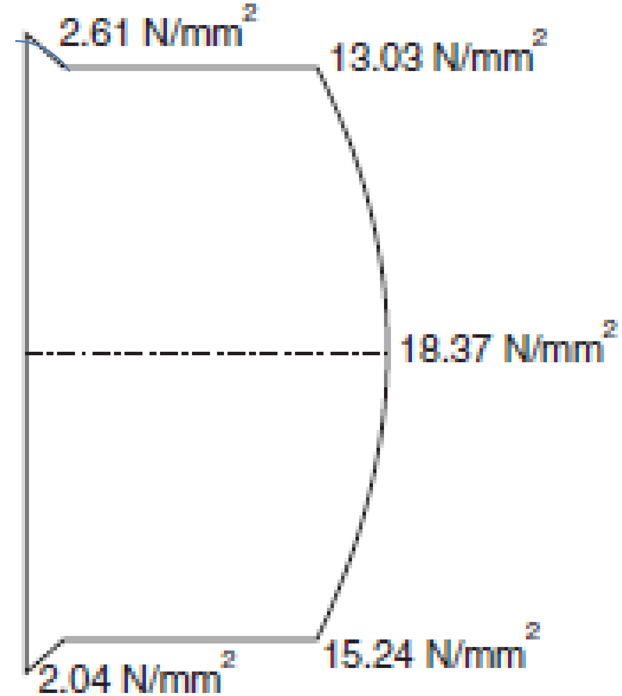
At the above level but in web, shear stress

$$= \frac{60 \times 1000}{150 \times 46505533} \times 237000$$

$$= 2.04 \text{ N/mm}^2$$


100 mm
20
y_t
20 mm
160
200 mm
20
150 mm

(a)

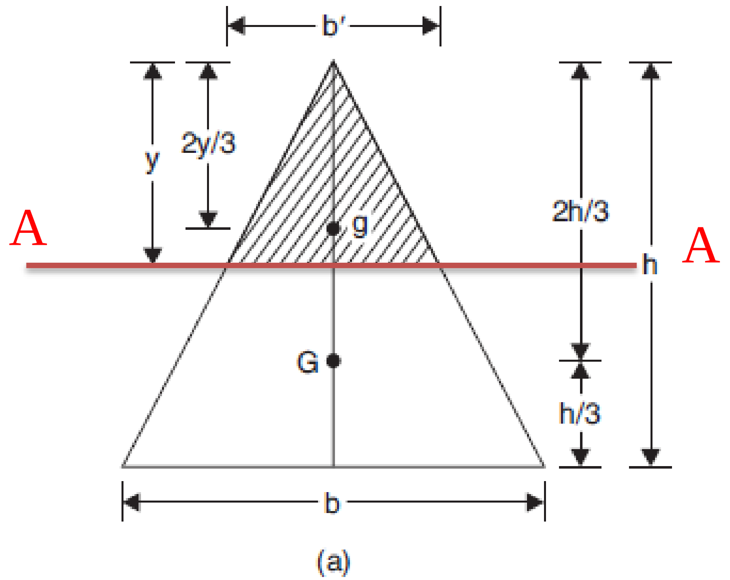

2.61 N/mm²
13.03 N/mm²
18.37 N/mm²
15.24 N/mm²
2.04 N/mm²

(b)

Variation of Shear Stresses Across Triangular Section

Consider the isosceles triangular section of width '*b*' and height '*h*' as shown in Fig. Its centroid and hence neutral axis is at *2h/3* from top fibre.

Now shear stress is to be found at section A-A which is at a depth '*y*' from top fibre.



Isosceles Triangular Section

At A-A width $b' = \dfrac{y}{h} b$

Area above A-A $\quad a = \dfrac{1}{2} b'y = \dfrac{1}{2}\dfrac{b}{h} y^2$

Its centroid from top fibre is at $\dfrac{2y}{3}$.

Distance of shaded area above the section A-A from neutral axis

$$\bar{y} = \frac{2h}{3} - \frac{2y}{3}$$

$$a\bar{y} = \frac{1}{2}\frac{b}{h}y^2\left(\frac{2h}{3} - \frac{2y}{3}\right) = \frac{1}{3}\frac{b}{h}y^2(h-y)$$

Moment of inertia of the section

$$I = \frac{bh^3}{36}$$

Shear stress at A-A

$$q = \frac{F}{bI}a\bar{y} = \frac{F}{\frac{y}{h}b \times \frac{bh^3}{36}} \times \frac{1}{3}\frac{b}{h}y^2(h-y)$$

$$q = \frac{12F}{bh^3}y(h-y)$$

$$q = \frac{F}{bI} a\bar{y} = \frac{12\,F}{bh^3} y(h - y)$$

Hence at $y = 0$, $q = 0$

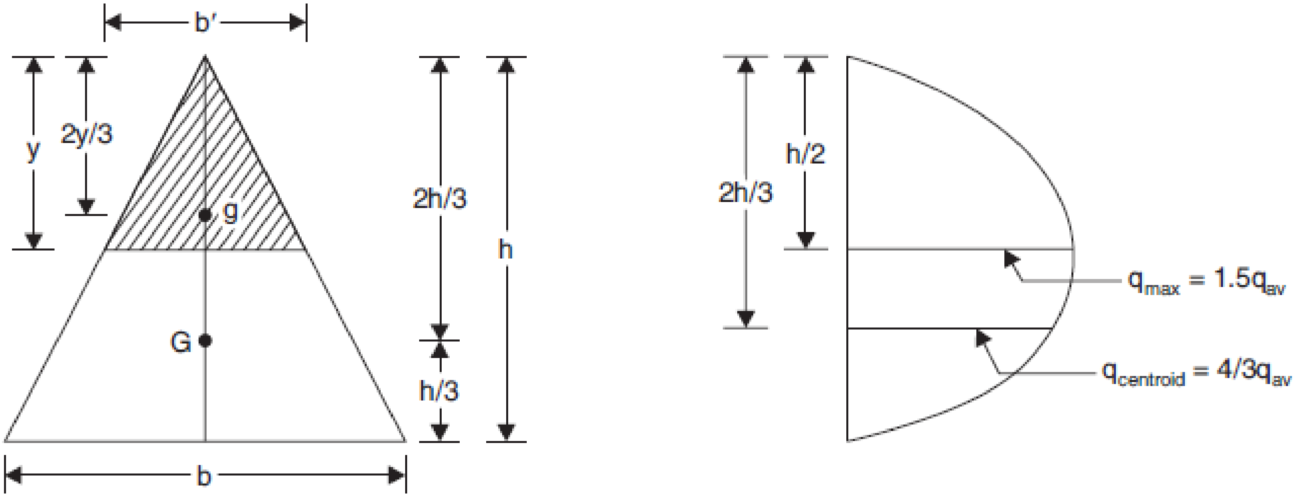
At $y = h$, $q = 0$

At centroid, $y = \frac{2h}{3}$

$$q = \frac{12\,F}{bh^3} \frac{2h}{3} (h - 2h/3)$$

$$= \frac{8}{3} \frac{F}{bh} = \frac{4}{3} \frac{F}{1/2\,bh}$$

$$= \frac{4}{3} q_{av}$$



(a) (b)

Problem 6: The laminated beam is composed of seven 200 mm x 50 mm wooden planks that are glued together. The beam carries a uniformly distributed load of intensity 6 kN/m over its 6 m simply supported span. Calculate the shear stress in glue at various levels and maximum shear stress in wood.