Lecture 13: Numerical Analysis (UMA011)

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Secant method:

$f(x) = 0$

In N·M,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

need to evaluate derivative of function at each iteration.

Importance:

requires only evaluation of function

Secant method:

Derivation:

$f(x)=0$

By N.M. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, $f'(x_n) \neq 0$

$\lim_{x\to y} \frac{f(x)-f(y)}{x-y} = f'(y)$

Now, $f'(x_n) = \lim_{x_{n-1}\to x_n} \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$

$$\Rightarrow \boxed{x_{n+1} = x_n - \frac{f(x_n) * (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$ Secant Method.

$$x_{n+1} = \frac{x_n f(x_n) - x_n f(x_{n-1}) - x_n f(x_n) + x_{n-1} f(x_n)}{f(x_n) - f(x_{n-1})}$$

$$\boxed{x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}} \checkmark \rightarrow \text{S.M.} \qquad f(x_n) \neq f(x_{n-1}) \quad \forall n.$$

Take x_0, x_1 as initial guesses

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Stopping criteria
$|x_n - x_{n-1}| < tol$

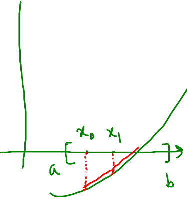
$$x_3 = \left(x_1 f(x_2) - x_2 f(x_1)\right) / \left(f(x_2) - f(x_1)\right)$$

Secant method:

Graphical representation:

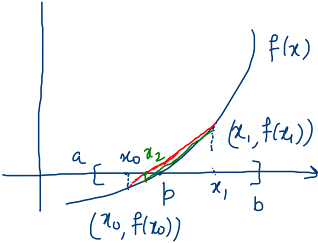
$f(x)=0$

$x_0 \quad x_1$



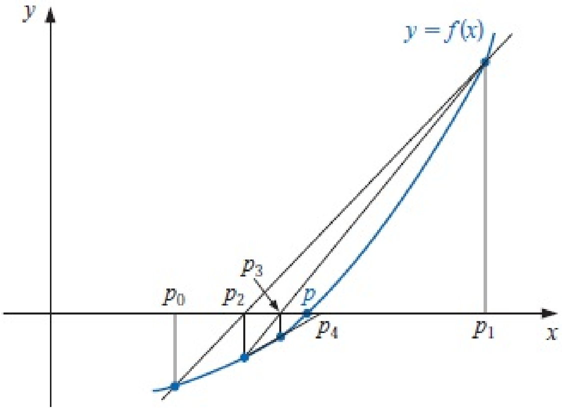
Eqn of secant line

$$y-f(x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_1)$$



If we take x_0, x_1 only one side of pt p then extend the secant joining x_0 & x_1 to x-axis.

At x-axis ie $y=0$



y
y = f(x)
p3
p0
p2
p
p4
p1
x

$$-f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1)$$

$$\frac{-f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = x - x_1$$

$$\Rightarrow x = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$(= x_2)$

|ly $$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$ - - - - continue

- . . . . . - - -

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$ → secant method.

Secant method:

Example:

Find the root of an equation $f(x) = \cos(x) - x = 0$ by using secant method.

Solution

$f(0) = 1 - 0 = +ve$, $f(\pi/2) = -\frac{\pi}{2} = -ve$.

By IVT, the root of $f(x) = 0$ lie in betn $[0, \pi/2]$

By using Secant method $x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$

Take $x_0 = 0.5$, $x_1 = \frac{\pi}{4}$, $n = 1$

$$x_2 = x_1 - \frac{f(x_1)\ (x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{\pi}{4} - \frac{f(\pi/4) - \left(\frac{\pi}{4} - 0.5\right)}{f(\pi/4) - f(0.5)} = 0.785398$$

$$x_3 = 0.785398 - \frac{f(0.785398)\ (0.785398 - \pi/4)}{f(0.785398) - f(\pi/4)}$$

$$= 0.73638$$

$x_4 = 0.739058$, $x_5 = 0.739085$ Ans

Numerical methods:

Examples:

① To compute $\sqrt{17}$ by using Numerical methods

$\downarrow$

Non-linear eqn.

$\cancel{x - \sqrt{17} = 0}$ Linear eqn.

$x^2 - 17 = 0$ ✓

By N.M.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n - \sqrt{17}}{1} = \sqrt{17}$$

get the 1st iteration as $\sqrt{17}$ only.

② Compute $(23)^{1/3}$ $\rightarrow$ $x^3 - 23 = 0$

③ Compute $\frac{1}{17}$ $\quad 17x - 1 = 0$ $\quad \frac{1}{x} - 17 = 0$ ✓ $\rightarrow$ non-linear eqns.

Root-finding problems:

Exercise:

1. Find the root of an equation $3x - e^x = 0$ by using secant method for $1 \leq x \leq 2$ with the accuracy of 10^{-2}.
2. Use Newton's and secant methods to find $1/1.732$ correct to 4 decimal places.