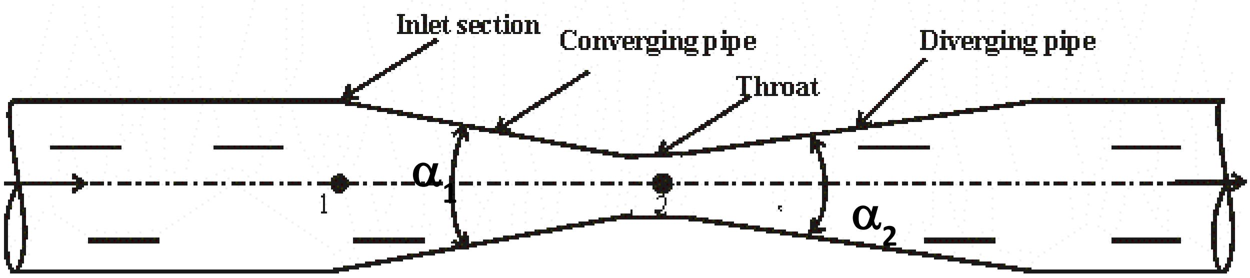
FLOW MEASURING DEVICES

Topics: Venturimeter, Orificemeter, Pitot tube, orifice, Time of emptying tanks of different cross-sections.

- Measurement of flow (discharge) through any system is very important as efficiency of the system depends on the discharge flowing through the system viz. pipes, tanks, channels etc.
- Devices used for the measurement of discharge in pipes are: venturimeter, orificemeter, bend meter, flow nozzle etc.
- Orifices and mouthpieces are usually to measure discharge from tanks.
- Notches, weirs, venturiflume etc. are used to measure discharge of open channel flows.
- Discharge can also be computed from the observations of velocity which can be measured using Pitot tube, current meter, floats etc.

VENTURIMETER

- is a device used for measuring flow in pipelines.
- consists of an inlet section, a converging pipe, a throat and a diverging pipe.



- Inlet section is of the same diameter as that of the pipe.
- Converging pipe is a pipe of short length, which tapers from the dia. of the pipe to the dia. of throat.
- Diverging pipe is a pipe of long length with its diameter increasing from throat to the diameter of pipe (diverging pipe is also known as diffuser).
- Included angle, α_1 of converging pipe is (21 ± 2^o) and length parallel to axis is **2.7(D_1 - D_2).**

➢ **D_1** is the diameter of pipe and **D_2** is the dia. of throat

- Length of throat = **D_2** and **D_2** = (1/3 to 3/4) **D_1**
- Included angle α_2 of diverging pipe is **5^o to 7^o**

Principle

- By reducing the cross-sectional area of flow passage, a pressure difference is created and the measurement of this pressure difference enables calculation of flow rate or discharge.

Location of pressure tappings

- Two pressure tappings are provided, one at inlet section and the other at throat section so that for a given discharge, a maximum possible pressure difference exists.
- The pressure difference is measured by connecting a differential **U-**tube manometer between the gauge points.

Discharge Equation

- Assuming **1D** flow, apply Bernoulli's eq. between **1** (or 1-1) and **2** (2-2), (neglecting losses of energy), to get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2$$

- Simplifying, to get

$$\left(\frac{p_1}{\rho g}-\frac{p_2}{\rho g}\right)=\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$$

- ($p_1/\rho g - p_2/\rho g$) is the difference of pressure heads between sections **1-1** and **2-2** and is known as venturi head or differential pressure head, may be denoted by Δh

$$\therefore \Delta h=\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$$

If **Q** is the discharge through pipe, then by continuity eq.

$$\Rightarrow Q=A_1V_1=A_2V_2 \qquad \Rightarrow V_1=\frac{Q}{A_1} \text{ and } V_2=\frac{Q}{A_2}$$

- Substituting V_1 and V_2, to get

$$Q=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\Delta h}$$

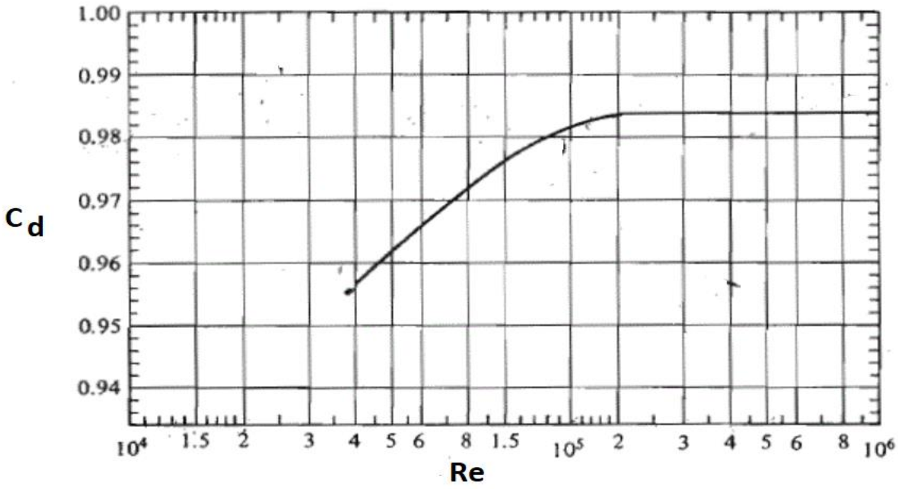
- This eq. gives only the theoretical discharge as losses of energy have not been considered.
- For real fluids, flow is not **1D** *.e.* velocity across any section is not uniform.

  $\therefore$ Actual discharge will be less than the theoretical discharge.
- Two discharges are related by a constant, called coefficient of discharge, may be denoted by C_d, defined as the ratio of actual discharge to theoretical discharge.

  $\therefore$ Actual discharge is given by

$$Q=C_d\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\Delta h}$$

❖ Value of C_d varies from 0.95 to 0.98 and is a function of **Re** as shown in **Figure.**



Observations:

(i) For measuring large pressure differences, heavier liquid is used in the manometer (e.g. mercury).

- **Δh** is given by the expression;

$$\Delta h = \Delta x\left(\frac{S_m}{S_p} - 1\right)$$

➢ Δx is the difference of mercury levels in the manometer.

❖ For mercury-water manometer, $S_m = S_{Hg} = 13.6$ and $S_p = S_w = 1$

$$\therefore \Delta h = (12.6 \times \Delta x)$$

(ii) For measuring small pressure differences, lighter liquid with inverted U-tube manometer is used (e.g. oil). For this case, **Δh** is given by;

$$\Delta h = \Delta x\left(1 - \frac{S_m}{S_p}\right)$$

(i) Venturimeter may be vertical or inclined

Δh in this case is equal to the difference of piezometric heads at the inlet and throat sections *i.e.*

$$\Delta h = \left(\frac{p_1}{\rho g} + y_1\right) - \left(\frac{p_2}{\rho g} + y_2\right)$$

ORIFICE METER

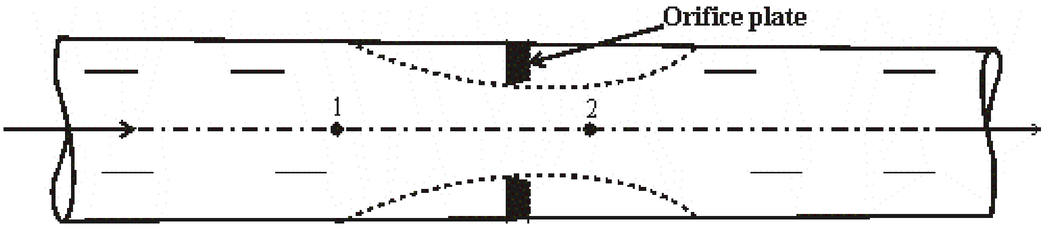
- consists of a flat circular plate with a hole called orifice.
- may be circular or sector of a circle, generally circular.
- Circular orifice is concentric with the pipe axis.
- Diameter varies from **0.2 to 0.85 D** (Generally **0.5D**), ***D*** is the diameter of pipe.

- **u/s** face of orifice is bevelled at an angle varying from 30^o to 45^o, so that liquid has minimum contact with the orifice.
- Orifice plate is clamped between the pipe flanges with bevelled surface facing **d/s.**

❖ Works on the same principle as that of venturimeter

Difference

- In venturimeter, there is a gradual reduction of flow area, whereas in orificemeter, there is a sudden reduction of flow area.
- Energy losses in orifice meter are more than that of a venturimeter.
- Installation of orificemeter requires a smaller length as compared to a venturimeter
- When space is limited, an orifice meter may be used.
- Line diagram of an orifice meter is shown.



Location of pressure tappings

- Two pressure tappings are provided viz. one on the u/s side and the other on the d/s side of orifice plate.
- Flow approaching orifice starts converging. The effect of convergence extends upto a certain distance u/s.
- u/s pressure tapping is provided where effect of convergence is minimum or zero
- Generally taken as **0.9 to 1.1 D** (Section **1-1**)
- On the d/s side, flow converges to a minimum area known as vena-contracta (Section **2-2**)

❖ d/s pressure tapping is provided at vena-contracta.

Discharge Equation:

- Assuming **1D** flow, applying Bernoulli's equation between sections **1-1** and **2-2** (neglecting losses of energy)
- Differential pressure head **Δh** and continuity equation is same as in case of venturimeter i.e. $\Delta h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right)$ and $Q = A_1V_1 = A_2V_2$
- Area of flow at section **2-2** *i.e.* at vena contracta (A_2) is related to the area of orifice, A_o by the relation; $A_2 = C_c A_o$, C_c = coefficient of contraction.

- Substituting **V1**and**V2** and solving for **Q**, to get $$Q = C_c \frac{A_1 A_o}{\sqrt{A_1^2 - C_c^2 A_o^2}} \sqrt{2g\Delta h}$$

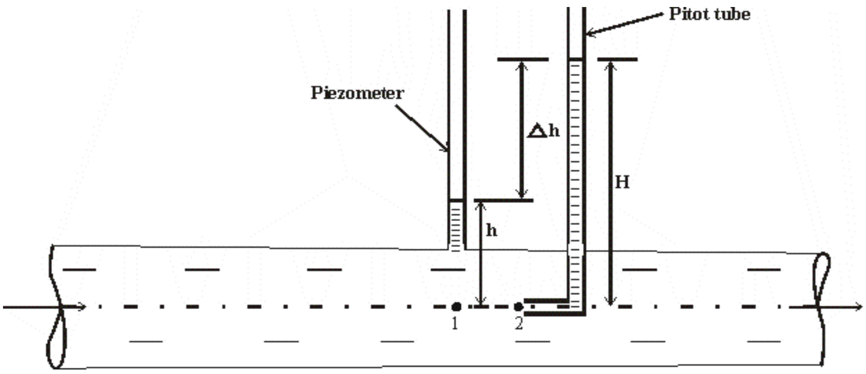
- Introducing a constant **C** (can be called **Cd**), Eq. can be expressed as:

$$Q = C_d A_o \sqrt{2g\Delta h}, \qquad \text{where } C_d = \frac{C_c}{\sqrt{1 - C_c^2\left(\frac{d_o}{D}\right)^4}}$$

❖ **Cd** varies from 0.60 to 0.70.

PITOT TUBE

- device used for measuring the velocity of flow at a point (local velocity) in a pipe or a channel.
- consists of a tube bent at right angle.
- tube is vertically dipped in the flow with open end pointing against the direction of flow.
- For measuring velocity of flow in a pipe, two tubes are to be used *i.e.* a piezometer and a Pitot tube.



Principle

- If velocity of flow at a point becomes zero, pressure at that point is increased due to conversion of kinetic energy into pressure energy. The point where velocity is zero, is known as stagnation point.

❖ Stagnation pt. is the pt. in a fluid where velocity is reduced to zero.

- Consider flow through a horizontal pipe as shown: Let **1** and **2** are the two closely spaced points at the same level
- Piezometer is connected to a point at a surface of pipe exactly above point **1** and Pitot tube is connected to point **2.**
- Applying B. Eq. between 1 and 2 (neglecting losses of energy), to get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

- Here, $p_1/\rho g$ = **h**, V_1 = Local velocity = u_1, $p_2/\rho g$ = **H** and $V_2 = 0$.

$$\therefore \left(h + \frac{u_1^2}{2g}\right) = H \;\Rightarrow (H - h) = \frac{u_1^2}{2g} \;\Rightarrow \Delta h = \frac{u_1^2}{2g}$$

- Here, **h** is the static pressure head and **H** is called the stagnation pressure head and **Δh = (H – h) =** Difference of levels in the piezometer and Pitot tube.

❖ **Δh** is called velocity head, also called dynamic pressure head.

√ In general, if velocity of flow at point 1 is **u**, then $u = \sqrt{2g\Delta h}$

- Eq. gives only the theoretical velocity, as losses of energy have not been considered
- Actual velocity will be less than the theoretical velocity. The two velocities are related to each other by a constant, called coefficient of velocity, C_v, defined as the ratio of actual velocity to theoretical velocity

$\therefore$ Actual velocity = Coefficient of velocity × Theoretical velocity

❖ C_v varies from 0.97 to 0.99.

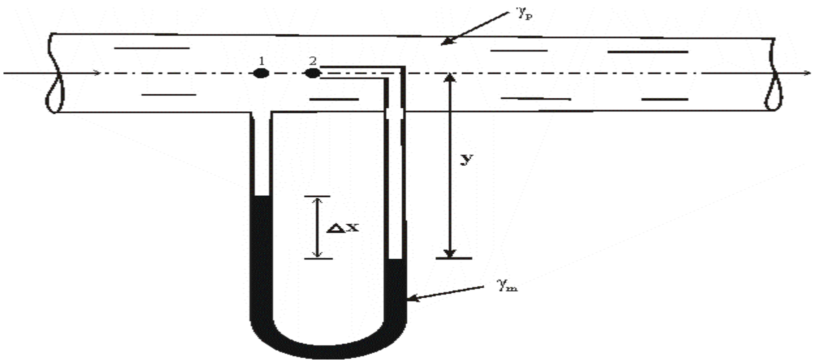
OBSERVATIONS:

(i) For an enclosed stream of fluid (pipe flow), Pitot tube measures stagnation pressure or total pressure which is the sum of static pressure and dynamic pressure *i.e.*

Stagnation pressure = (Static pressure + Dynamic pressure)

❖ Static pressure has to be measured separately by using a piezometer

(ii) High velocity of flow in a pipe may also be determined directly by connecting a **U**-shaped Pitot tube as shown



- Applying **B. E.** between **1** and **2** (neglecting losses), to get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

- Pressure at pt. **2** is the stagnation pressure ($V_2 = 0$); $V_1 = u_1$

$$\therefore \left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g}\right) = \frac{u_1^2}{2g}$$

- Write gauge equation between points **1** and **2**, to get

$$p_2 + \gamma_p y - \gamma_m \Delta x - \gamma_p (y - \Delta x) = p_1$$

$$\therefore \frac{(p_2 - p_1)}{\gamma_p} = \Delta x\left(\frac{\gamma_m}{\gamma_p} - 1\right) \qquad \Rightarrow \left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g}\right) = \Delta x\left(\frac{S_m}{S_p} - 1\right); \quad \gamma_p = \rho g$$

- Equating Eqs., to get

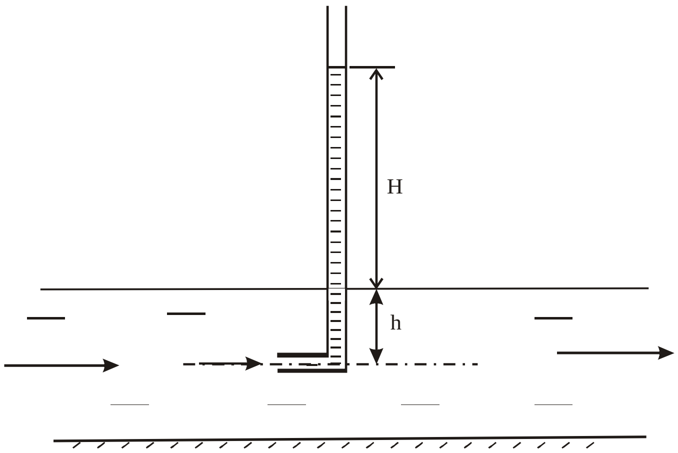
$$u_1 = \sqrt{2g\Delta x\left(\frac{S_m}{S_p} - 1\right)}$$

In general, $u = \sqrt{2g\Delta h}$; $\quad \Delta h = \Delta x\left(\dfrac{S_m}{S_p} - 1\right)$

(ii) For measuring moderate velocities of flow, an inclined manometer may be used.

$u = \sqrt{2g\Delta h \sin\theta}$, θ is the inclination of manometer with horizontal.

(iii) Pitot tube (non-enclosed system) [channels]



- In this case, **h** is the static pressure head and **H** is the dynamic pressure head (**$u^2/2g$**).

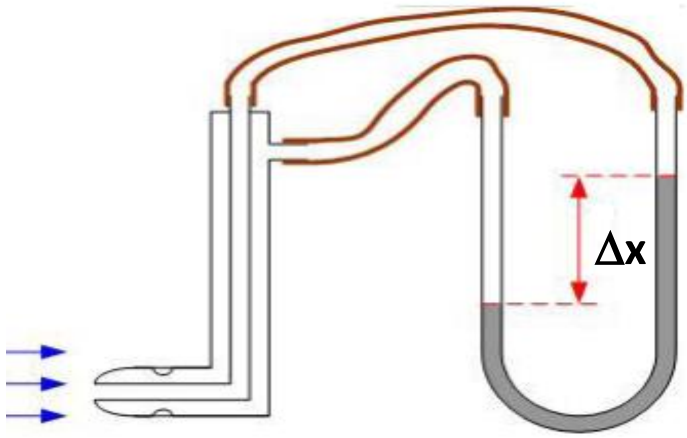
$$\therefore \left(h + \frac{u^2}{2g}\right) = (H + h) \qquad\qquad \therefore u = \sqrt{2gH}$$

(iv) Pitot tube which is most commonly used for measuring flow velocity is called **Prandtl Pitot tube** or **Pitot-static tube.**

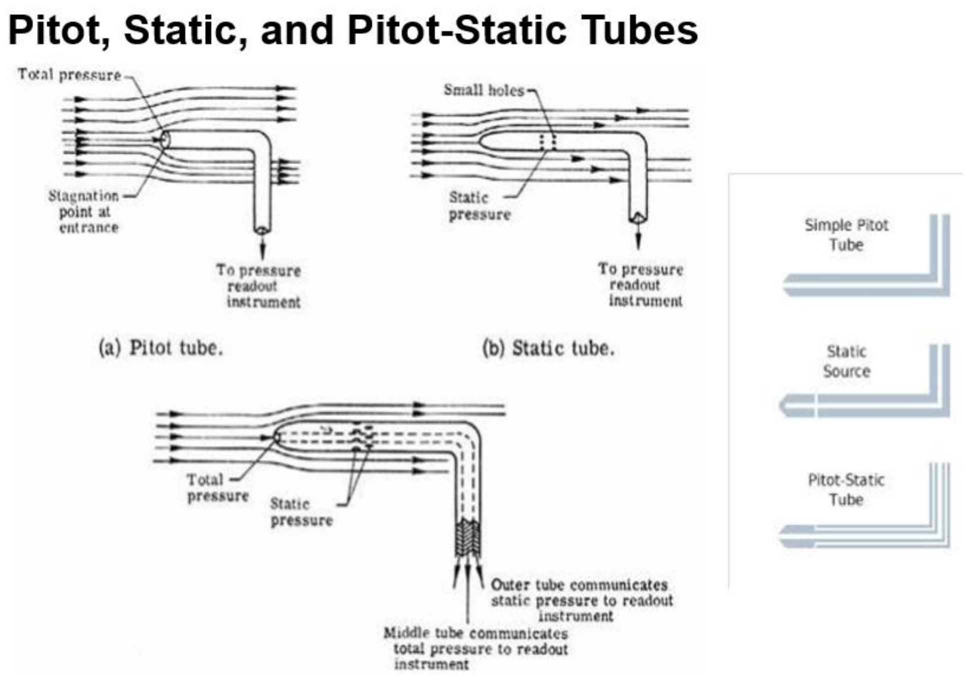
- Consists of two concentric tubes bent at right angle.
- Inner tube measures stagnation pressure head or total head.
- Outer tube surrounds the inner tube and have a number of holes (two or more) drilled radially on the surface - measures static pressure head.

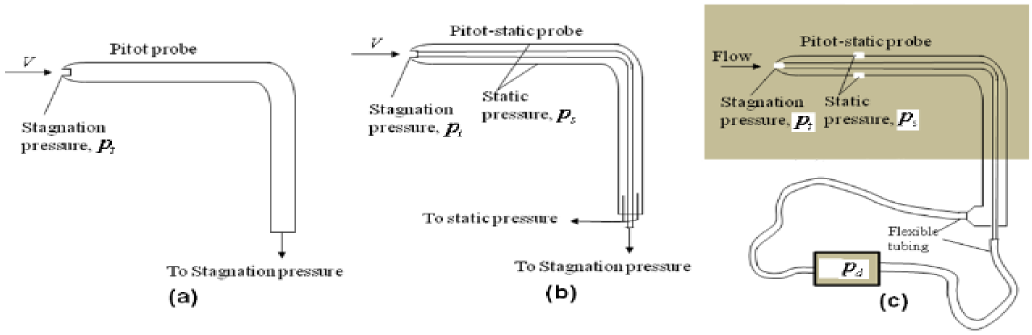
❖ Front portion of tube is rounded so as to prevent separation of flow.

- If datum is taken at the centre of Pitot-static tube, then **y = 0.**
- Pressure difference can be visualised by connecting to a U-tube differential manometer.
- In other words, when two tappings of Prandtl tube are connected to a U-tube differential manometer, difference of levels in the manometer is equal to velocity head.



Prandtl Pitot tube or Pitot-static tube





(a) Pitot probe; (b) Pitot-static probe; (c) Measuring flow velocities with a Pitot-static probe

Problems:

Q1: A 300 x 150 mm venturimeter is provided in a vertical pipe carrying oil of specific gravity 0.90, the flow being upward. The difference in elevation of the throat and the entrance is 300 mm. A differential U-tube mercury manometer shows a gauge deflection of 250 mm. Calculate (i) discharge (ii) pressure difference between entrance and throat sections. Take C_d = 0.98.

Solution: D_1 = 300 mm, D_2 = 150 mm, Δx = 250 mm

- Discharge is given by the equation:

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\Delta h}$$

$$\therefore A_1 = \frac{\pi}{4}(0.3)^2 = 0.0707\,\text{m}^2, \qquad A_2 = \frac{\pi}{4}(0.15)^2 = 0.0177\,\text{m}^2$$

$$\Delta h = \Delta X\left(\frac{S_m}{S_p} - 1\right) \qquad \therefore \Delta h = 0.25\left(\frac{13.6}{0.90} - 1\right) = 3.53\text{ m of oil}$$

- Substituting the various values and solve for **Q**, to get
- **Q** = 0.15 m^3/s (Ans)
- Also, for inclined or vertical venturimeter,

$$\Delta h = \left(\frac{p_1}{\rho g} + y_1\right) - \left(\frac{p_2}{\rho g} + y_2\right) \qquad \Rightarrow \left(\frac{p_1 - p_2}{\rho g}\right) = \Delta h + (y_2 - y_1)$$

- Here, (y_1 = 0, y_2 = 0.3 m)

$$\Rightarrow \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = 3.53 + 0.30 = 3.83\text{ m of oil}$$

$$\Rightarrow (p_1 - p_2) = 3.83 \times 0.9 \times 9.810 = 33.8\,\text{kN/m}^2$$

Q2: A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm is used to measure discharge of water. The pressure at inlet is 176 kN/m^2 and the vacuum pressure at the throat is 300 mm of mercury. Find discharge. Take C_d = 0.98.

Solution: D_1 = 200 mm, D_2 = 100 mm, p_1 = 176 kN/m^2

Pressure head at throat, $\frac{p_2}{\rho g} = -300$ mm of Hg

$$\Rightarrow p_2 = -0.30 \times 13.6 \times 9.810 = -40.02 \text{ kN/m}^2 \text{ of water}$$

$$\therefore \Delta h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) \qquad \Rightarrow \Delta h = \left(\frac{176}{9.810} - \left\{-\frac{40.02}{9.810}\right\}\right) = 22.02 \text{ m of water}$$

- Using, $Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\Delta h}$; $A_1 = \frac{\pi}{4}(0.2)^2 = 0.0314$ $\quad A_2 = \frac{\pi}{4}(0.1)^2 = 0.007874$
- Substituting various values and solving, to get
- **$Q = 0.165$ m³/s (Ans)**

Q3: A venturimeter with **d - D** ratio as ½ is fitted in a pipe of diameter, **D** 200 mm. The head loss between the inlet and the throat is 10% of the velocity head at the throat. Calculate the discharge when an inverted U- tube differential manometer connected to the inlet and the throat shows a reading of 300 mm. Also, find the coefficient of discharge. Assume relative density of the manometer fluid as 0.75.

Solution: d/D = 1/2, **D** = 200 mm $\qquad \therefore$ d = 100 mm

- Applying Bernoulli's eq. between inlet and throat, to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0.1\frac{V_2^2}{2g} \qquad (1)$$

- Also, for an inverted U- tube differential manometer,

$$\Delta h = \Delta X\left(1 - \frac{S_m}{S_p}\right) \qquad \Rightarrow \Delta h = 0.3\left(1 - \frac{0.75}{1}\right) = 0.075 \text{ m} \qquad \Rightarrow \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = 0.075 \text{ m of water}$$

- From continuity equation, $A_1 V_1 = A_2 V_2 \quad \Rightarrow \pi/4(0.2)^2 \times V_1 = \pi/4(0.1)^2 \times V_2$

$$\therefore 4V_1 = V_2$$

- Substituting the values in Eq. (1), to get

$$0.075 + \frac{V_1^2}{2g} = 1.1\frac{(4V_1)^2}{2g}$$

- Solve for V_1, to get
- $V_1 = 0.297$ m/s

$$\therefore Q = A_1 V_1 \Rightarrow Q = \pi/4(0.2)^2 \times 0.297 = 9.33 \times 10^{-3} \text{ m}^3/\text{s}$$

To find C_d

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\Delta h} \qquad \Rightarrow C_d = 0.95$$

Q4: An orificemeter with orifice diameter 150 mm is inserted in a pipe of 300 mm diameter. The water pressure on the u/s and d/s of the meter are 147 kN/m^2 and 98.1 kN/m^2, respectively. Find the discharge if C_d = 0.60. Also, find the coefficient of contraction of orifice.

Solution: D_o = 150 mm, **D** = 300 mm, p_1 =147 kN/m^2, p_2 = 98.1 kN/m^2

$$\therefore \Delta h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) \qquad \Rightarrow \Delta h = \left(\frac{147 - 98.1}{9.810}\right) = 4.985 \text{m/s}$$

- As, $Q = C_d A_o \sqrt{2g\Delta h}$

$$\therefore Q = 0.60 \times \frac{\pi}{4}(0.15)^2 \times \sqrt{2 \times 9.81 \times 4.985} \qquad = 0.105 \text{ m}^3/\text{s}$$

- Also, $C_d = \dfrac{C_c}{\sqrt{1 - C_c^2\left(\dfrac{d_o}{D}\right)^4}}$
- Solve for C_c, to get $C_c = 0.575$

Q5: A Pitot tube is used to measure velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate velocity of flow. Take C_v = 0.98.

Solution: Stagnation pressure head, **H** = 6 m

- Static pressure head, **h** = 5 m

$$\therefore \Delta h = (H - h) = 1 \text{m}$$

- Using, $u = C_v \times \sqrt{2g\Delta h}$

$\therefore$ Velocity of flow, **u** = 4.34 m/s (**Ans**)

Q6: A submarine moves horizontally in seawater and has its axis 15 m below the surface of water. A Pitot tube placed just in front of the submarine and along its axis is connected to the two-limbs of a U-tube manometer containing mercury. The difference of mercury levels is found to be 170 mm. Find the speed of submarine in kmph. Take specific gravity of sea water as 1.026.

Solution:

- Using, $\Delta h = \Delta x\left(\frac{S_m}{S_w} - 1\right) = 0.17\left(\frac{13.6}{1.026} - 1\right) = 2.08 \text{ m of sea water}$

$\therefore u = C_v\sqrt{2g\Delta h} = 6.4 \text{ m/s}$ (Take C_v = 1)

$\therefore$ Speed of submarine = 6.4 × 3.6 = 23 kmph

ORIFICE

- An opening either in one of the walls or at the bottom of a tank.

- Used for discharging liquid from the tank.

CLASSIFICATION OF ORIFICES

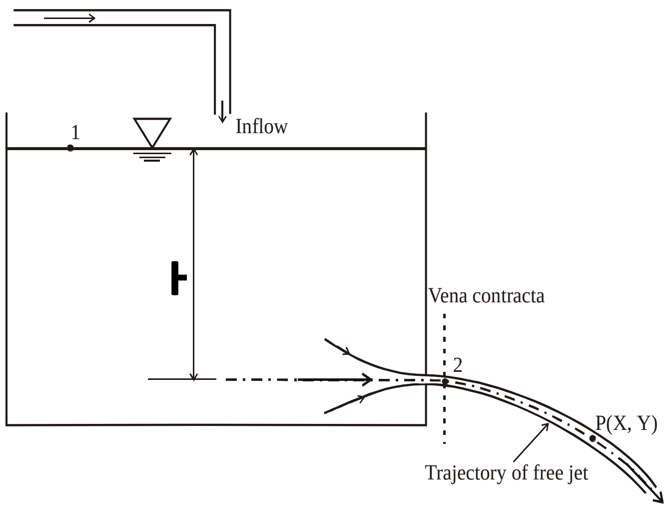
- Classified according to different considerations such as:
- ➢ shape, entrance to flow, small or large, discharging conditions
- **Shape** - circular, square, rectangular, triangular
- **Entrance to flow -** sharp or rounded
- ❖ Generally, circular and have sharp entrance with bevelled surface facing downstream.
- **Small or large -** depending on the head or head causing flow (**H**).
- ➢ Small if **H > D** and large if **H ≤ D**, **D** is the diameter of orifice.

Discharging conditions - free and drowned or submerged orifice.

- may discharge into; the atmosphere or another tank of liquid.
- ➢ If discharging into the atmosphere - free orifice.
- ➢ If discharging into another tank of liquid- submerged orifice.
- Submerged orifice is further of two types - fully submerged and partially submerged.
- If depth of liquid in the discharging tank is above the top of orifice - fully submerged otherwise partially submerged.
- ❖ Orifice can also be used for the measurement of flow/discharge of liquid from a tank.

Flow from a Sharp Edged Small Circular Orifice

- Provided in one of walls of a water tank and discharging freely.
- Water comes out from the orifice as a free jet.
- Jet continues to converge till streamlines are <u>parallel</u> to each other.
- Cross-section of jet at this section is minimum and is known as <u>vena-contracta.</u>
- Beyond vena contracta, jet expands (diverges) and is directed downward.
- ❖ Generally, vena contracta occurs at a distance of **0.5D** measured from the outer face of orifice.
- ✓ Experimental studies have indicated that pressure across jet at vena-contracta is atmospheric.



- Contraction and expansion of jet results in loss of energy and therefore it is necessary to know the characteristics of free jet, known as <u>hydraulic coefficients</u> or constants of orifice.
- Three - coefficient of; discharge, contraction and velocity (**C_d, C_c, C_v**)

$$C_d = \frac{Q}{Q_{th}}, \qquad C_c = \frac{A_c}{A_o}, \qquad C_v = \frac{V}{V_{th}}$$

Discharge equation

- Applying Bernoulli's equation between points **1** and **2** (**2** is at vena contracta)**,** neglecting losses of energy, to get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2 \qquad \Rightarrow 0 + 0 + H = 0 + \frac{V_2^2}{2g} + 0 \qquad \Rightarrow V_2 = \sqrt{2gH}$$

❖ Gives only the theoretical velocity at vena contracta.

- Actual velocity will be less than the theoretical velocity, given by $V_2 = C_v\sqrt{2gH}$

In general, actual velocity of jet at vena-contracta, $V = C_v\sqrt{2gH}$

- Actual discharge = Area of jet at vena contracta × Actual velocity of jet at vena contracta *i.e.*
- $Q = A_c \times V$; $A_c = C_c A_o$ $\therefore Q = C_c A_o \times C_v \sqrt{2gH}$
- Also, theoretical discharge, $Q_{th} = A_o \sqrt{2gH}$; Dividing

$$C_d = C_c C_v; \qquad Q = C_d A_o \sqrt{2gH}$$

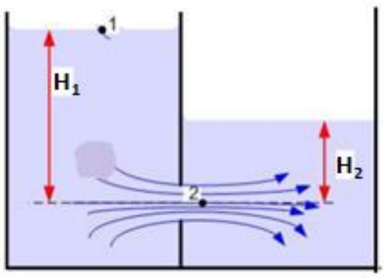
Determination of C_v

- Consider any point **P(X, Y)** at time ***t*** on the trajectory of jet from vena-contracta.
- Horizontal distance of **P**, **X = V × t**
- Vertical distance of **P**, $Y = \frac{1}{2} g t^2$
- Eliminate **t**, to get
- Actual velocity, $V = \sqrt{\frac{gX^2}{2Y}}$
- Also, $V_{th} = \sqrt{2gH}$

$\therefore$ Coefficient of velocity, $C_v = \frac{V}{V_{th}} = \frac{X}{\sqrt{4YH}}$

Submerged Orifice

- Consider a system where fluid discharges through an orifice into fluid of the same type:



- Apply Bernoulli's Equation between **1** and **2** (assuming datum is passing through the centre of orifice)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2$$

$$\therefore H_1 + 0 + 0 = H_2 + \frac{V_2^2}{2g} + 0 \qquad \Rightarrow V_2 = \sqrt{2g(H_1 - H_2)}$$

UNSTEADY FLOW THROUGH ORIFICE

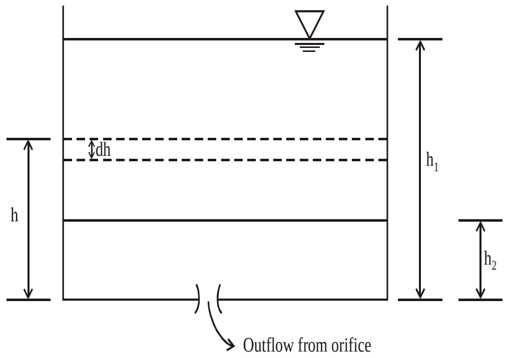
(Time of emptying tanks of different cross-sections)

- In the analysis of flow through orifice, the head acting on the orifice is constant. This happens when inflow = outflow and this condition is known as steady condition.

- If head on the orifice is not constant, the flow becomes unsteady. This condition will occur when the inflow is stopped or inflow and outflows are different.
- For these unsteady conditions, it may be required to determine the time required for the free liquid surface to fall or rise from its initial position to some other position or the time required to empty the tank.
- Let us discuss the cases of different shapes of tanks:

Rectangular/Square/Vertical circular Tank:

- Consider a tank which is provided with an orifice at its bottom as shown in **Figure.**



A tank fitted with an orifice at

- Let **A** = Plan area of tank, A_o = Area of orifice, h_1 = Initial liquid level in the tank, h_2 = Final liquid level in the tank, **T** = Time for the liquid to fall from h_1 to h_2.
- Let at any instant **h** be the height of the liquid above the orifice and let it decreases by **dh** in a small time **dt**, then
- Volume of liquid leaving orifice in time **dt** = $C_d A_o \sqrt{2gh}\, dt$
- Also, corresponding decrease in the volume of liquid in the tank = $(A \times dh)$

$\therefore$ Equating the two equations, to get

$$C_d A_o \sqrt{2gh}\, dt = -A \times dh \qquad (1)$$

- -ve sign signifies that with the increase of time, head acting on the orifice decreases.

$$\therefore dt = -\frac{A \times dh}{C_d A_o \sqrt{2gh}}$$

$\therefore$ Total time required by liquid level to fall from $\mathbf{h_1}$ to $\mathbf{h_2}$ is given by:

$$\int_0^T dt = \int_{h_1}^{h_2} -\frac{A \times dh}{C_d A_o \sqrt{2gh}}$$

- Solving, to get

$$T = \frac{2A}{C_d A_o \sqrt{2g}} \left(\sqrt{h_1} - \sqrt{h_2}\right) \qquad (2)$$

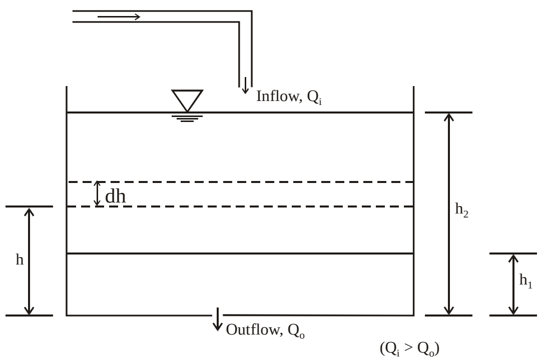
- If tank is to be completely emptied, $\mathbf{h_2 = 0}$ and Eq. (2) reduces to:

$$T = \frac{2A}{C_d A_o \sqrt{2g}} \sqrt{h_1}$$

❖ **Note**: Eq. (2) can be applied to a rectangular/square/vertical circular tank.

Discharge From a Tank Through an Orifice with Inflow

- Consider a tank having an orifice at its bottom and having inflow as shown in **Figure.**



Discharge from a tank

- Let $\mathbf{Q_i}$ = Inflow rate and $\mathbf{Q_o}$ = Outflow rate through the orifice
- To start with, let us assume that $\mathbf{Q_i > Q_o}$

$\therefore$ There will be an increase in the level of liquid in the tank at any instant of time.

- Let at any instant **h** be the height of liquid above the orifice and let it increases by **dh** in a small time **dt**, then
- Inflow volume of liquid in time **dt** = $Q_i dt$
- Outflow volume through orifice in time **dt** = $Q_o dt$ $= C_d A_o \sqrt{2gh}\, dt$

$$= k\sqrt{h}dt, \qquad k = C_d A_o \sqrt{2g}$$

$\therefore$ Increase in volume of liquid in the tank = $\left(Q_i - k\sqrt{h}\right)dt$ **(1)**

- Also, corresponding increase in the volume of liquid in time **dt** = $(A \times dh)$ **(2)**

$\therefore$ Equating Eqs. (1) and (2), to get

$$dt = \frac{A}{(Q_i - k\sqrt{h})} dh$$

- Put $(Q_i - k\sqrt{h}) = z$ $\quad \therefore \sqrt{h} = \frac{Q_i - z}{k}$ $\quad \Rightarrow h = \left(\frac{Q_i - z}{k}\right)^2$ $\quad \therefore dh = -\frac{2}{k^2}(Q_i - z)dz$

$$dt = -\frac{A}{z}\left[\frac{2}{k^2}(Q_i - z)dz\right] \qquad = -\frac{2A}{k^2}\left[\left(\frac{Q_i}{z} - 1\right)dz\right]$$

$\therefore$ Total time required by the liquid level to rise from h_1 to h_2 is given by:

$$T = \int_{h_1}^{h_2} -\frac{2A}{k^2}\left[\left(\frac{Q_i}{z} - 1\right)dz\right]$$

- On solving, one gets

$$T = -\frac{2A}{k^2}\left[Q_i \log_e\left(\frac{Q_i - k\sqrt{h_2}}{Q_i - k\sqrt{h_1}}\right) + k(\sqrt{h_2} - \sqrt{h_1})\right]$$

- This is the required expression.