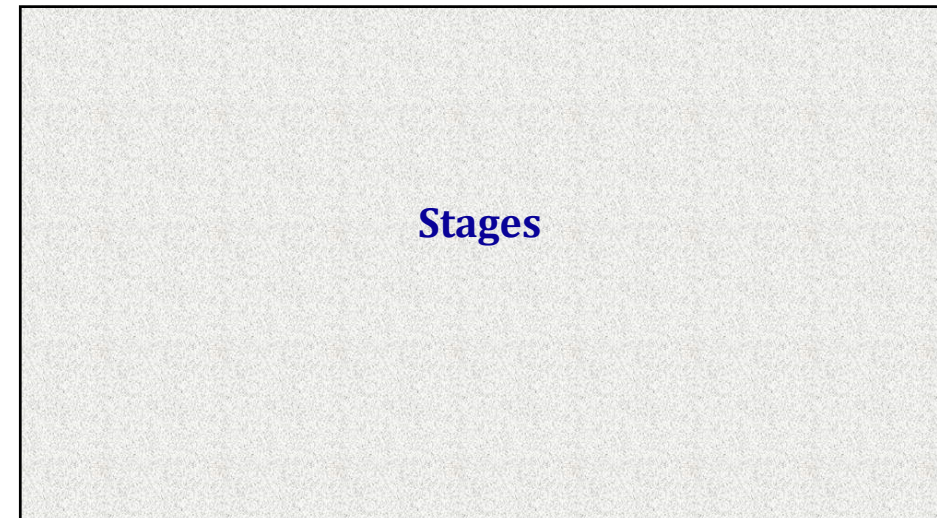# Mass Transfer-I

Stages and Cascades

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Stages

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Equilibrium stages

Stage is defined as any device or combination of devices in which two insoluble phases are brought in to intimate contact, where mass transfer occurs between the phases are tending to bring them to equilibrium, and where the phases are then mechanically separated . A process carried out in this manner is a **single-stage process**.

OR

Any device or combination of devices in which two immiscible phases are brought into intimate contact in order to achieve mass transfer from one phase to the other is called **a stage**.

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Ideal stages

An **equilibrium**, **ideal** or **theoretical stage** is one where the time of contact between phases is sufficient for the effluents indeed to be in equilibrium.

Or

An **ideal / theoretical stage** is one in which the leaving streams are in equilibrium.

Or

If the contacting is so efficient that the phases reach equilibrium when they leave, the stage is called an **ideal stage**.

> ➢ The stage efficiency gives a measure of how close to equilibrium the phases may reach in a stage. An ideal stage has 100% efficiency.
> ➢ Although in principle this can not be attained, in practice be can frequently approach so close to equilibrium if the cost warrants it, that the difference is unimportant.
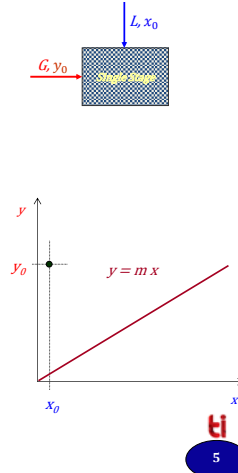
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Slide 5

Single equilibrium stage

This is the gas inlet. The gas has a certain concentration of solute, represented by the molar fraction, y_0

The solvent is put in contact with the gas phase. The solvent can be pure or can have a certain concentration of solute, represented by x_0.

The equilibrium between phases can be represented in a x-y diagram for a given temperature and pressure. Sometimes the equilibrium is a straight line, $y = m\,x$

The inlet compositions of the two phases can be represented in the diagram

L, x_0

G, y_0

$y = m\,x$

Slide 6

Cont...

The content of solute in the gas is now lower than at the entrance: $y < y_o$

The solvent is now charged in solute, so that $x > x_o$.

Because this is an ideal equilibrium stage, the equilibrium between phases is reached.

The mass balance equation is:

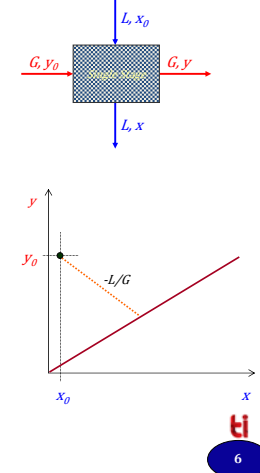Total mass of solute at the inlet:
$$= L \cdot x_o + G \cdot y_0$$

Total mass of solute at the outlet:
$$= L \cdot x + G \cdot y$$

Solving with respect to y yields:
$$y = \left(y_0 + \frac{L}{G} \cdot x_0 \right) - \frac{L}{G} \cdot x$$

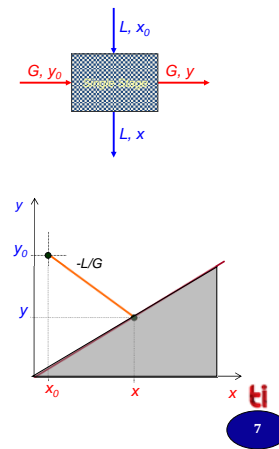The slope of the operating line is then ($-L/G$)

L, x_0

G, y_0 G, y

L, x

$-L/G$

Slide 7

Cont...

Because this is an equilibrium stage, the equilibrium between phases is reached, so the operating line goes to the equilibrium line.

$$y = \left(y_0 + \frac{L}{G} \cdot x_0 \right) - \frac{L}{G} \cdot x$$

The equilibrium line represents a barrier and determines a region of values (x, y) that cannot be reached for any ratio (L/G)

L, x_0

G, y_0 G, y

L, x

$-L/G$

Slide 8

Cont...

The equilibrium equation and the operating line,
$$y = m\,x$$
$$y = \left(y_0 + \frac{L}{G} x_0 \right) - \frac{L}{G} x$$

One can get the value of y corresponding to the intersection.
$$y = \left(y_0 + \frac{L}{G} x_0 \right) - \frac{L}{G} \left(\frac{y}{m} \right)$$
$$y = \left(y_0 + \frac{L}{G} \left(\frac{m x_0}{m} \right) \right) - \frac{L}{G} \frac{y}{m}$$

By definition the ratio (L/Gm) is called absorption factor A.
$$A = \frac{L}{G\,m}$$

The inverse of the absorption factor (A) is defined as Stripping factor S
$$S = \frac{1}{A} = \frac{Gm}{L}$$

Cont...

So the expression can be written in terms of A.

$$y = (y_0 + A \, m \, x_0) - A \, y$$

Using the equilibrium equation, the product $m.x_o$ can be replaced by y_o^*, or composition of a gas at equilibrium with a liquid of composition x_o.

$$y_0^* = m \, x_0$$

$$y = (y_0 + A \, y_0^*) - A \, y$$

So the outlet composition of the gas, y is now given in terms of A.
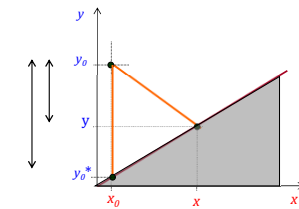
$$y = \frac{y_0 + A \, y_0^*}{1 + A}$$

Cont...

The fraction of absorption α is defined as the ratio:

$$\alpha = \frac{y_0 - y}{y_0 - y_0^*}$$

Where the numerator is the distance between y_o and y and the denominator is the distance between y_o and the composition of gas at equilibrium with x_o. This represents the maximum solute exchange

The fraction of absorption can also be written in terms of the absorption factor, A

$$\alpha = \frac{y_0 - y}{y_0 - y_0 \, ^*} = 1 - \frac{1}{1 + A}$$

y_0^* represents the minimum concentration of solute in the gas phase, so the maximal absorption (A tends to infinity)

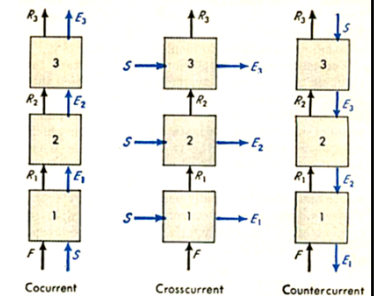|		
$y \rightarrow y_0$	$A \rightarrow 0$	$\alpha \rightarrow 0$
$y \rightarrow y_0 \, ^*$	$A \rightarrow \infty$	$\alpha \rightarrow 1$

Cascade

Cascade

A group of stages interconnected to each other so that the various streams/phases flow from one to the other is called cascade.

Its purpose is to increase the extent of mass transfer over and above that which is possible with single stage.

The fractional overall stage efficiency of a cascade is then defined as the number of equilibrium stages to which the cascade is equivalent divided by the number of real stages.

Two or more stages can be connected as per the following cascade arrangements

➢ Co-current Cascades

➢ Counter-current Cascades

➢ Cross-currentCascades

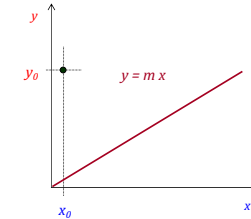# Co-current cascade of ideal stages

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Co-current cascade of ideal stages

Let's consider two ideal stages.

The solvent is put in contact with the gas phase in the first stage. The solvent can be pure or can have a certain concentration of solute, represented by x_o.

The equilibrium between phases can be represented in a x-y diagram for a given temperature and pressure. Let's consider again that it is a straight line, $y = m\,x$.

The inlet compositions of the two phases can be represented in the diagram

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Cont...

The content of solute in the gas is now lower than at the entrance: $y_1 < y_0$

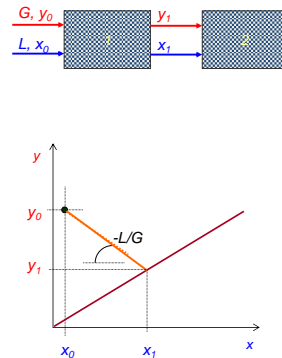The solvent is now charged in solute, so that $x_1 > x_o$.

Because this is an ideal equilibrium stage, the equilibrium between phases is reached.

Repeating what we saw before in the case of ideal single stage, the change on composition in the two phases can be represented on the diagram...

The equation is:

$$y = \left(y_0 + \frac{L}{G} \cdot x_0 \right) - \frac{L}{G} \cdot x$$

The slope of the operating line is then $(-L/G)$

Cont...

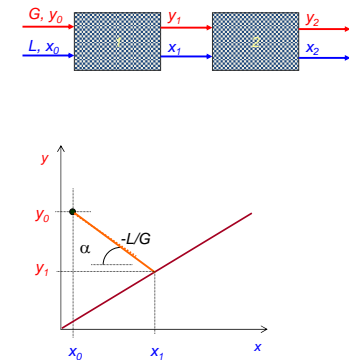In the single ideal stage we have derived the next equation for y_1:

$$y_1 = \frac{y_0 + A\,y_0^*}{1 + A}$$

Where **A** is the absorption factor.

Now we have a second ideal stage, in which the gas composition at the outlet can be expressed by analogy:

$$y_2 = \frac{y_1 + A\,y_1^*}{1 + A}$$

You find the derivation of these equations in the next slide

Slide 1

Cont...

Derivation of the outlet composition of the gas

Material Balance to the solute: $G\,y_0 + L\,x_0 = G\,y_1 + L\,x_1$

The equilibrium equation: $y = m\,x$

Dividing by G: $y_0 + \dfrac{L}{G}\,x_0 = y_1 + \dfrac{L}{G}\,x_1$

Using the equilibrium equation, x_1 can be expressed as y_1/m: $y_0 + \dfrac{L}{G}\,x_0 = y_1 + \dfrac{L}{G\,m}\,y_1$

Where we find the Absorption factor: $y_0 + \dfrac{L}{G}\,x_0 = y_1 + A\,y_1 = y_1\,(1+A)$

Multiplying and dividing by m in the first term: $y_0 + \dfrac{L\,m}{G\,m}\,x_0 = y_0 + A\,m\,x_0 = y_0 + A\,y_0^* = y_1\,(1+A)$

Finally,

we obtain the composition at the outlet of the first stage: $y_1 = \dfrac{y_0 + A\,y_0^*}{1+A}$

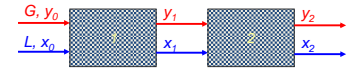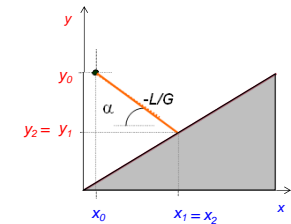The next outlet compositions can be calculated by analogy as... $y_i = \dfrac{y_{i-1} + A\,y_{i-1}^*}{1+A}$

Slide 2

Cont...

The aim of the second stage is to have y_2 smaller than y_1, so that we improve the operation by decreasing the gas concentration of pollutant.

But we already know that the equilibrium line represents a barrier and determines a region of values (x, y) that cannot be reached for any ratio (L/G) and any number of stages...

Since in the first stage we have already arrived to the equilibrium, we cannot go further and y_2 must be equal to y_1.

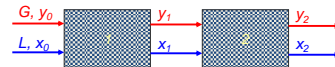This result can be obtained looking at the equations...

$$y_2 = \dfrac{y_1 + A\,y_1^*}{1+A}$$

Slide 3

Cont...

Since the equilibrium is reached...

$$y_1^* = y_1$$

So in the equation of y_2, we get:

$$y_2 = \dfrac{y_1 + A\,y_1}{1+A} \Rightarrow y_2 = \dfrac{y_1(1+A)}{(1+A)}$$

Thus, as we have already deduced:

$$y_2 = y_1$$

The composition equation for the last stage can be written as follows:

$$y_n = \dfrac{y_0 + A\,y_0^*}{1+A}$$

And then, the fractional absorption...

$$\alpha = \dfrac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = 1 - \dfrac{1}{1+A}$$

Slide 4

References

http://www.hyper-tvt.ethz.ch/tvt_links.php

- Lecture notes/ppt of Dr. Yahya Banat (ybanat@qu.edu.qa)

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