**Course: UMA 035 (Optimization Techniques)**

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## Simplex in $R^n$

**Let $X_1, X_2, \ldots, X_n, X_{n+1}$ be (n+1) points in $R^n$. Then the convex set $\{a_1X_1 + a_2X_2 + \ldots + a_nX_n + a_{n+1}X_{n+1} : a_1 >= 0, a_2 >= 0, \ldots, a_n >= 0, a_{n+1} >= 0$ and $a_1 + a_2 + \ldots + a_n + a_{n+1} = 1\}$ is called a simplex in $R^n$.**

## Simplex in R

**Putting n =1,**

**Let $X_1$ and $X_2$ be two points in R. Then the convex set $\{a_1X_1 + a_2X_2 : a_1 >= 0, a_2 >= 0$ and $a_1 + a_2 = 1\}$ is called a simplex in R.**

## Simplex in $R^2$

**Putting n =2,**

**Let $X_1$, $X_2$ and $X_3$ be three points in $R^2$. Then the convex set $\{a_1X_1 + a_2X_2 + a_3X_3 : a_1 >= 0, a_2 >= 0, a_3 >= 0$ and $a_1 + a_2 + a_3 = 1\}$ is called a simplex in $R^2$.**

## Simplex method

**Step 1:**

**Convert the considered LPP in standard form.**

**Step 2:**

**If the problem is of minimization then convert it into maximization by changing the sign of all the coefficients in the objective function.**

**Step 3:**

**Construct the following Table.**

Coefficients from objective function

| Coefficient of Basic Variables in Objective function ($C_B$) | Basic Variables | $x_1$ $x_2$ … $x_n$ $S_1$ $S_2$ … $S_m$ | Solution | Minimum Ratio |
|---|---|---|---|---|
| $Z_j - C_j =$ | | | | |
| Coefficient of first basic variable in objective function | first basic variable | Coefficients from first constraint | Right hand side of first constraint | |
| Coefficient of second basic variable in objective function | second basic variable | Coefficients from second constraint | Right hand side of second constraint | |
| ⋮ | ⋮ | ⋮ | ⋮ | |
| Coefficient of $m^{th}$ basic variable in objective function | $m^{th}$ basic variable | Coefficients from $m^{th}$ constraint | Right hand side of $m^{th}$ constraint | |

**First Basic variable will be that variable corresponding to which the**

**column** $\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ **will exist in the Table.**

**Second Basic variable will be that variable corresponding to which the**

**column** $\begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ **will exist in the Table.**

**Third Basic variable will be that variable corresponding to which the**

**column** $\begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ **will exist in the Table.**

$\vdots$

**$m^{th}$ Basic variable will be that variable corresponding to which the column**

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ **will exist in the Table.**

**Step 4:**

**Calculate $Z_j - C_j$ corresponding to each variable**

**For $x_1$**

**[(Coefficient of first basic variable in objective function)(First element of column of $x_1$)+ (Coefficient of second basic variable in objective function)(Second element of column of $x_1$)+…+ (Coefficient of $m^{th}$ basic variable in objective function)($m^{th}$ element of column of $x_1$)]−[element lying outside table of column of $x_1$]**

**For $x_2$**

**[(Coefficient of first basic variable in objective function)(First element of column of $x_2$)+ (Coefficient of second basic variable in objective function)(Second element of column of $x_2$)+…+ (Coefficient of $m^{th}$ basic variable in objective function)($m^{th}$ element of column of $x_2$)]−[element lying outside table of column of $x_2$]**

⋮

**For $x_n$**

**[(Coefficient of first basic variable in objective function)(First element of column of $x_n$)+ (Coefficient of second basic variable in objective function)(Second element of column of $x_n$)+…+ (Coefficient of $m^{th}$ basic variable in objective function)($m^{th}$ element of column of $x_n$)]−[element lying outside table of column of $x_n$]**

**For $S_1$**

**[(Coefficient of first basic variable in objective function)(First element of column of $S_1$)+ (Coefficient of second basic variable in objective function)(Second element of column of $S_1$)+…+ (Coefficient of $m^{th}$ basic variable in objective function)($m^{th}$ element of column of $S_1$)]−[element lying outside table of column of $S_1$]**

⋮

**For $S_m$**

**[(Coefficient of first basic variable in objective function)(First element of column of $S_m$)+ (Coefficient of second basic variable in objective function)(Second element of column of $S_m$)+…+ (Coefficient of $m^{th}$ basic variable in objective function)($m^{th}$ element of column of $S_m$)]−[element lying outside table of column of $S_m$]**

**Step 5:**

**Check that all the calculated values of $Z_j - C_j$ are >=0 or not**

**Case (i)**

**If all the calculated values of $Z_j - C_j$ are >=0 then the solution is optimal.**

**Case (ii)**

**If one or more values of $\mathrm{Z_j - C_j}$ is negative. Then the solution is not optimal. Go to Step 6.**

**Step 6:**

**Find minimum {negative values of $\mathrm{Z_j - C_j}$}. The variable corresponding to which the minimum exist is called entering variable.**

**Step 7:**

**Find minimum {negative values of $\mathrm{Z_j - C_j}$}. The variable corresponding to which the minimum exist is called entering variable.**

**Step 8:**

**In the last column of the table, find minimum**

$$\left\{\frac{number\ corresponding\ to\ the\ first\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ first\ basic\ variable\ in\ the\ column\ of\ entering\ variable^{*}},\right.$$

$$\frac{number\ corresponding\ to\ the\ second\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ second\ basic\ variable\ in\ the\ column\ of\ entering\ variable^{*}}, \ldots,$$

$$\left.\frac{number\ corresponding\ to\ the\ m^{th}\ basic\ variable\ in\ the\ solution\ column}{number\ corresponding\ to\ the\ m^{th}\ basic\ variable\ in\ the\ column\ of\ entering\ variable^{*}}\right\}.$$

***If the element is negative or zero then do not consider it. Only positive numbers in the denominator.**

**The variable corresponding to which the minimum exist is called leaving variable.**

**Step 8:**

**Construct a new table by replacing the leaving basic variable with the entering variable in the second column (Basic variables) of the table.**

**Step 9:**

**Apply the following row operations to find the elements of the table.**

- $R_1 \longrightarrow R_1 - (a_{1i}) * R_p / (a_{pi})$
- $R_2 \longrightarrow R_2 - (a_{2i}) * R_p / (a_{pi})$
- $\vdots$
- $R_p \longrightarrow R_p / (a_{pi})$
- $\vdots$
-
- $R_m \longrightarrow R_m - (a_{mi}) * R_p / (a_{pi})$

**These operations have been obtained as follows:**

- **Write m rows:**

  $R_1$

  $R_2$

  $\vdots$

  $R_m$

- **Insert arrow in front of each row:**

$R_1 \longrightarrow$

$R_2 \longrightarrow$

$\vdots$

$R_m \longrightarrow$

- **Insert same row after arrow**

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2$

$\vdots$

$R_m \longrightarrow R_m$

- **Insert division sign in that row corresponding to which leaving variable exist (let $R_p$)**

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2$

$\vdots$

$R_p \longrightarrow R_p/$

$\vdots$

$R_m \longrightarrow R_m$

- **Insert negative sign in remaining rows**

$R_1 \longrightarrow R_1 -$

$R_2 \longrightarrow R_2 -$

⋮

$R_p \longrightarrow R_p/$

⋮

$R_m \longrightarrow R_m -$

- **Insert the elements of that column corresponding to which entering variable exist (let ith column)**

  $R_1 \longrightarrow R_1 - (a_{1i})$

  $R_2 \longrightarrow R_2 - (a_{2i})$

  ⋮

  $R_p \longrightarrow R_p/(a_{pi})$

  ⋮

  $R_m \longrightarrow R_m - (a_{mi})$

- **Multiply to all with $R_p/(a_{pi})$**
- $R_1 \longrightarrow R_1 - (a_{1i})*R_p/(a_{pi})$
- $R_2 \longrightarrow R_2 - (a_{2i})*R_p/(a_{pi})$
- ⋮
- $R_p \longrightarrow R_p/(a_{pi})$
- ⋮
-

- $R_m \longrightarrow R_m - (a_{mi}) * R_p / (a_{pi})$

**Step 10:**

**Go to Step 5 and repeat the procedure until an optimal solution is obtained.**

**Example:**

**Solve the following LPP by the Simplex method.**

**Minimize** $(x_1 - 3x_2 + 2x_3)$

**Subject to**

$3x_1 - x_2 + 3x_3 <= 7,$

$-2x_1 + 4x_2 <= 12,$

$-4x_1 + 3x_2 + 8x_3 <= 10$

$x_1 >= 0,$

$x_2 >= 0,$

$x_3 >= 0.$

**Solution**

**Minimize** $(x_1 - 3x_2 + 2x_3)$

**Subject to**

$3x_1 - x_2 + 3x_3 + S_1 = 7,$

$-2x_1 + 4x_2 + S_2 = 12,$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1 >= 0,$

$x_2 >= 0$,

$x_3 >= 0$,

$S_1 >= 0$,

$S_2 >= 0$,

$S_3 >= 0$.

**Maximize ($-x_1+3x_2-2x_3$)**

**Subject to**

$3x_1-x_2+3x_3+S_1=7$,

$-2x_1+4x_2+S_2=12$,

$-4x_1+3x_2+8x_3+S_3=10$

$x_1 >= 0$,

$x_2 >= 0$,

$x_3 >= 0$,

$S_1 >= 0$,

$S_2 >= 0$,

$S_3 >= 0$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $-1$ | 3 | $-2$ | 0 | 0 | 0 | | |
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| | | 3 | $-1$ | 3 | 1 | 0 | 0 | | |
| | | $-2$ | 4 | 0 | 0 | 1 | 0 | | |
| | | $-4$ | 3 | 8 | 0 | 0 | 1 | | |

| | | $-1$ | $3$ | $-2$ | $0$ | $0$ | $0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| | $S_1$ | $3$ | $-1$ | $3$ | $1$ | $0$ | $0$ | | |
| | $S_2$ | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | | |
| | $S_3$ | $-4$ | $3$ | $8$ | $0$ | $0$ | $1$ | | |

| | | $-1$ | $3$ | $-2$ | $0$ | $0$ | $0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| | $S_1$ | $3$ | $-1$ | $3$ | $1$ | $0$ | $0$ | $7$ | |
| | $S_2$ | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | $12$ | |
| | $S_3$ | $-4$ | $3$ | $8$ | $0$ | $0$ | $1$ | $10$ | |

| | | $-1$ | 3 | $-2$ | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| 0 | $S_1$ | 3 | $-1$ | 3 | 1 | 0 | 0 | 7 | |
| 0 | $S_2$ | $-2$ | 4 | 0 | 0 | 1 | 0 | 12 | |
| 0 | $S_3$ | $-4$ | 3 | 8 | 0 | 0 | 1 | 10 | |

[(0)(0)+(0)(0)+ (0)(1)] –(0) =0

| | | $-1$ | 3 | $-2$ | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | | | | | | | | |
| 0 | $S_1$ | 3 | $-1$ | 3 | 1 | 0 | 0 | 7 | |
| 0 | $S_2$ | $-2$ | 4 | 0 | 0 | 1 | 0 | 12 | |
| 0 | $S_3$ | $-4$ | 3 | 8 | 0 | 0 | 1 | 10 | |

[(0)(0)+(0)(1)+ (0)(0)] –(0) =0

[(0)(3)+(0)(−2)+ (0)(−4)] –(

−1) =1

[(0)(1)+(0)(0)+ (0)(0)] –(3) =−3

[(0)(−1)+(0)(4)+ (0)(3)] –(−2) =2

[(0)(3)+(0)(0)+ (0)(8)] –(0) =0

Entering variable

| | | 3 | −2 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | 1 | −3 | 2 | 0 | 0 | 0 | | |
| 0 | $S_1$ | 3 | −1 | 3 | 1 | 0 | 0 | 7 | |
| 0 | $S_2$ | −2 | 4 | 0 | 0 | 1 | 0 | 12 | |
| 0 | $S_3$ | −4 | 3 | 8 | 0 | 0 | 1 | 10 | |

| | | 3 | −2 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | 1 | −3 | 2 | 0 | 0 | 0 | | |
| 0 | $S_1$ | 3 | −1 | 3 | 1 | 0 | 0 | 7 | 7/- |
| 0 | $S_2$ | −2 | 4 | 0 | 0 | 1 | 0 | 12 | 12/4=3 |
| 0 | $S_3$ | −4 | 3 | 8 | 0 | 0 | 1 | 10 | 10/3=3.33 |

**Leaving Variable**

| | | 3 | −2 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | 1 | −3 | 2 | 0 | 0 | 0 | | |
| 0 | $S_1$ | 3 | −1 | 3 | 1 | 0 | 0 | 7 | 7/- |
| 0 | $S_2$ | −2 | 4 | 0 | 0 | 1 | 0 | 12 | 12/4=3 |
| 0 | $S_3$ | −4 | 3 | 8 | 0 | 0 | 1 | 10 | 10/3=3.33 |

**Remark: In the next table there is a need to replace the second basic variable $S_2$ with $x_2$. Therefore, the column of $x_2$ should be 0, 1, 0 in the next table as well as the value of $Z_j - C_j$ should be 0 corresponding to $x_2$.**

**Use the following operations for the same.**

**$R_1 \longrightarrow R_1$**

**$R_2 \longrightarrow R_2$**

**$R_3 \longrightarrow R_3$**

**$R_4 \longrightarrow R_4$**

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2$

$R_3 \longrightarrow R_3\,/$ (Minimum Ratio in this row)

$R_4 \longrightarrow R_4$

$R_1 \longrightarrow R_1 -$

$R_2 \longrightarrow R_2 -$

$R_3 \longrightarrow R_3\,/$

$R_4 \longrightarrow R_4 -$

$R_1 \longrightarrow R_1 - (-3)$

$R_2 \longrightarrow R_2 - (-1)$

$R_3 \longrightarrow R_3\,/(4)$

$R_4 \longrightarrow R_4 - (3)$

$R_1 \longrightarrow R_1 - (-3)(\,R_3\,/(4))$

$R_2 \longrightarrow R_2 - (-1)(\,R_3\,/(4))$

$R_3 \longrightarrow R_3\,/(4)$

$R_4 \longrightarrow R_4 - (3)(\,R_3\,/(4))$

$R_1 \longrightarrow R_1 + \frac{3}{4}R_3$

$R_2 \longrightarrow R_2 + \frac{1}{4}R_3$

$R_3 \longrightarrow R_3\,/(4)$

$R_4 \longrightarrow R_1 - \frac{3}{4} R_3$

**How to apply operations?**

$R_1 \longrightarrow R_1 + \frac{3}{4} R_3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_1$: | $1$ | $-3$ | $2$ | $0$ | $0$ | $0$ | - |
| $R_3$: | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | $12$ |
| $\frac{3}{4} R_3$: | $-\frac{3}{2}$ | $3$ | $0$ | $0$ | $\frac{3}{4}$ | $0$ | $9$ |
| $R_1 + \frac{3}{4} R_3$: | $1 - \frac{3}{2}$ | $-3+3$ | $2+0$ | $0+0$ | $0+\frac{3}{4}$ | $0+0$ | -+9 |
| | $= -\frac{1}{2}$ | $=0$ | $=2$ | $=0$ | $=\frac{3}{4}$ | $=0$ | = - |

$R_2 \longrightarrow R_2 + \frac{1}{4} R_3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_2$: | $3$ | $-1$ | $3$ | $1$ | $0$ | $0$ | $7$ |
| $R_3$: | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | $12$ |
| $\frac{1}{4} R_3$: | $-\frac{1}{2}$ | $1$ | $0$ | $0$ | $\frac{1}{4}$ | $0$ | $3$ |
| $R_2 + \frac{1}{4} R_3$: | $3 - \frac{1}{2}$ | $-1+1$ | $3+0$ | $1+0$ | $0+\frac{1}{4}$ | $0+0$ | $7+3$ |
| | $=\frac{5}{2}$ | $=0$ | $=3$ | $=1$ | $=\frac{1}{4}$ | $=0$ | $=10$ |

$R_3 \longrightarrow \frac{1}{4} R_3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_3$: | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | $12$ |
| $\frac{1}{4} R_3$: | $-\frac{1}{2}$ | $1$ | $0$ | $0$ | $\frac{1}{4}$ | $0$ | $3$ |

$R_4 \longrightarrow R_4 - \frac{3}{4} R_3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_4$: | $-4$ | $3$ | $8$ | $0$ | $0$ | $1$ | $10$ |
| $R_3$: | $-2$ | $4$ | $0$ | $0$ | $1$ | $0$ | $12$ |
| $-\frac{3}{4} R_3$: | $\frac{3}{2}$ | $-3$ | $0$ | $0$ | $-\frac{3}{4}$ | $0$ | $-9$ |
| $R_4 - \frac{3}{4} R_3$: | $-4 + \frac{3}{2}$ | $3 - 3$ | $8+0$ | $0+0$ | $0 - \frac{3}{4}$ | $1+0$ | $10-9$ |
| | $= -\frac{5}{2}$ | $=0$ | $=8$ | $=0$ | $= -\frac{3}{4}$ | $=1$ | $= 1$ |

| | | 3 | $-2$ | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | | |
| 0 | $S_1$ | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 | 7/- |
| $-2$ | $x_2$ | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 | 12/4=3 |
| 0 | $S_3$ | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 | 10/3=3.33 |

**Entering variable**

| | | 3 | $-2$ | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | | |
| 0 | $S_1$ | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 | |
| $-2$ | $x_2$ | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 | |
| 0 | $S_3$ | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 | |

| | | 3 | $-2$ | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | | |
| 0 | $S_1$ | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 | 10/(5/2)=4 |
| $-2$ | $x_2$ | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 | 3/- |
| 0 | $S_3$ | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 | 1/- |

**Leaving Variable**

| | | 3 | $-2$ | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | | |
| 0 | $S_1$ | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 | 10/(5/2)=4 |
| $-2$ | $x_2$ | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 | 3/- |
| 0 | $S_3$ | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 | 1/- |

**Remark:** **In the next table there is a need to replace the first basic variable $S_1$ with $x_1$. Therefore, the column of $x_1$ should be 1, 0, 0 in the next table as well as the value of $Z_j - C_j$ should be 0 corresponding to $x_1$.**

**Use the following operations for the same.**

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2$

$R_3 \longrightarrow R_3$

$R_4 \longrightarrow R_4$

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2\,/$ **(Minimum Ratio in this row)**

$R_3 \longrightarrow R_3$

$R_4 \longrightarrow R_4$

$R_1 \longrightarrow R_1-$

$R_2 \longrightarrow R_2/$

$R_3 \longrightarrow R_3-$

$R_4 \longrightarrow R_4-$

$R_1 \longrightarrow R_1-(-\frac{1}{2})$

$R_2 \longrightarrow R_2/(\frac{5}{2})$

$R_3 \longrightarrow R_3-(-\frac{1}{2})$

$R_4 \longrightarrow R_4-(-\frac{5}{2})$

$R_1 \longrightarrow R_1 - (-\frac{1}{2})\left(\frac{R_2}{\frac{5}{2}}\right)$

$R_2 \longrightarrow R_2/(\frac{5}{2})$

$R_3 \longrightarrow R_3 - (-\frac{1}{2})\left(\frac{R_2}{\frac{5}{2}}\right)$

$R_4 \longrightarrow R_4 - (-\frac{5}{2})\left(\frac{R_2}{\frac{5}{2}}\right)$

$R_1 \longrightarrow R_1 + \frac{1}{5} R_2$

$R_2 \longrightarrow \frac{2}{5} R_2$

$R_3 \longrightarrow R_3 + \frac{1}{5} R_2$

$R_4 \longrightarrow R_4 + R_2$

**How to apply operations?**

$R_1 \longrightarrow R_1 + \frac{1}{5} R_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_1$: | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | - |
| $R_2$: | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 |
| $\frac{1}{5}R_2$: | $\frac{1}{2}$ | 0 | $\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{1}{20}$ | 0 | 2 |
| $R_1 + \frac{1}{5}R_2$: | $-\frac{1}{2} + \frac{1}{2}$ | $0 + 0$ | $2 + \frac{3}{5}$ | $0 + \frac{1}{5}$ | $\frac{3}{4} + \frac{1}{20}$ | 0+0 | -+2 |
| | =0 | =0 | $= \frac{13}{5}$ | $= \frac{1}{5}$ | $= \frac{4}{5}$ | =0 | = - |

$R_2 \longrightarrow \frac{2}{5}R_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_2$: | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 |
| $\frac{2}{5}R_2$: | 1 | 0 | $\frac{6}{5}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | 0 | 4 |

$R_3 \longrightarrow R_3 + \frac{1}{5}R_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_3$: | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 |
| $R_2$: | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 |
| $\frac{1}{5}R_2$: | $\frac{1}{2}$ | 0 | $\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{1}{20}$ | 0 | 2 |
| $R_3 + \frac{1}{5}R_{22}$: | $-\frac{1}{2}+\frac{1}{2}$ | $1+0$ | $0+\frac{3}{5}$ | $0+\frac{1}{5}$ | $\frac{1}{4}+\frac{1}{20}$ | $0+0$ | $3+2$ |
| | $=0$ | $=1$ | $=\frac{3}{5}$ | $=\frac{1}{5}$ | $=\frac{3}{10}$ | $=0$ | $=5$ |

$R_4 \longrightarrow R_4 + R_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $R_4$: | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 |
| $R_2$: | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 |
| $R_4 + R_2$: | $-\frac{5}{2}+\frac{5}{2}$ | $0+0$ | $8+3$ | $0+1$ | $-\frac{3}{4}+\frac{1}{4}$ | $1+0$ | $1+10$ |
| | $=0$ | $=0$ | $=11$ | $=1$ | $=-\frac{1}{2}$ | $=1$ | $=11$ |

|  |  | 3 | −2 | 0 | 0 | 0 | 0 |  |  |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = |  | 0 | 0 | $\frac{13}{5}$ | $\frac{1}{5}$ | $\frac{4}{5}$ | 0 |  |  |
| 3 | $x_1$ | 1 | 0 | $\frac{6}{5}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | 0 | 4 |  |
| −2 | $x_2$ | 0 | 1 | $\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{3}{10}$ | 0 | 5 |  |
| 0 | $S_3$ | 0 | 0 | 11 | 1 | $-\frac{1}{2}$ | 1 | 11 |  |

**Since, all values of $Z_j - C_j$ are greater than or equal to 0. So, the obtained solution is optimal.**

**Optimal solution is**

**$x_1$=4**

**$x_2$=5**

**$S_3$=11**

**Remaining are 0 i.e., $x_3=S_1=S_2=0$.**

**Putting these values in the objective function of the given LPP ($x_1-3x_2+2x_3$), the minimum value is $4-3*5+2*0=-11$.**

**Pattern for Examination:**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 3 | $-2$ | 0 | 0 | 0 | 0 | | |
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | 1 | $-3$ | 2 | 0 | 0 | 0 | | |
| 0 | $S_1$ | 3 | $-1$ | 3 | 1 | 0 | 0 | 7 | 7/- |
| 0 | $S_2$ | $-2$ | 4 | 0 | 0 | 1 | 0 | 12 | 12/4=3 |
| 0 | $S_3$ | $-4$ | 3 | 8 | 0 | 0 | 1 | 10 | 10/3=3.33 |
| $Z_j - C_j$ = | | $-\frac{1}{2}$ | 0 | 2 | 0 | $\frac{3}{4}$ | 0 | | |
| 0 | $S_1$ | $\frac{5}{2}$ | 0 | 3 | 1 | $\frac{1}{4}$ | 0 | 10 | 10/(5/2)=4 |
| $-2$ | $x_2$ | $-\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{4}$ | 0 | 3 | 3/- |
| 0 | $S_3$ | $-\frac{5}{2}$ | 0 | 8 | 0 | $-\frac{3}{4}$ | 1 | 1 | 1/- |
| $Z_j - C_j$ = | | 0 | 0 | $\frac{13}{5}$ | $\frac{1}{5}$ | $\frac{4}{5}$ | 0 | | |
| 3 | $x_1$ | 1 | 0 | $\frac{6}{5}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | 0 | 4 | |
| $-2$ | $x_2$ | 0 | 1 | $\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{3}{10}$ | 0 | 5 | |
| 0 | $S_3$ | 0 | 0 | 11 | 1 | $-\frac{1}{2}$ | 1 | 11 | |

**Optimal solution is**

$x_1=4$

$x_2=5$

$S_3=11$

**Remaining are 0 i.e., $x_3=S_1=S_2=0$.**

**Putting these values in the objective function of the given LPP ($x_1-3x_2+2x_3$), the minimum value is $4-3*5+2*0=-11$.**

**Example:**

**Solve the following LPP by the Simplex method.**

**Maximize ($3x_1+2x_2$)**

**Subject to**

$x_1+x_2<=4$,

$x_1-x_2<=2$,

$x_1>=0$,

$x_2>=0$

**Solution**

**Maximize ($3x_1+2x_2$)**

**Subject to**

$x_1+x_2+S_1=4$,

$x_1-x_2+S_2=2$,

$x_1>=0$,

$x_2>=0$,

$S_1>=0$, $S_2>=0$.

| | | 3 | 2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| $C_B$ | Basic Variables | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Solution | Minimum Ratio |
| $Z_j - C_j$ = | | $-3$ | $-2$ | 0 | 0 | | |
| 0 | $S_1$ | 1 | 1 | 1 | 0 | 4 | 4/1=4 |
| 0 | $S_2$ | 1 | $-1$ | 0 | 1 | 2 | 2/1=2 |
| $Z_j - C_j$ = | | 0 | $-5$ | 0 | 3 | | |
| 0 | $S_1$ | 0 | 2 | 1 | $-1$ | 2 | 2/2=1 |
| 3 | $x_1$ | 1 | $-1$ | 0 | 1 | 2 | 2/- |
| $Z_j - C_j$ = | | 0 | 0 | $\frac{5}{2}$ | $\frac{1}{2}$ | | |
| 2 | $x_2$ | 1 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | |
| 3 | $x_1$ | 0 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 3 | |

**Optimal solution is**

$x_2=1$

$x_1=3$

**Remaining are 0 i.e., $S_1=S_2=0$.**

**Putting these values in the objective function of the given LPP ($3x_1+2x_2$), the maximum value is 3*3+2*1=11.**

**Row operations used to obtain Second Table**

$R_1 \longrightarrow R_1 - (-3) * (R_3/(1)) \Longrightarrow R_1 \longrightarrow R_1 + 3 * R_3$

$R_2 \longrightarrow R_2 - (1) * (R_3/(1)) \Longrightarrow R_2 \longrightarrow R_2 - R_3$

$R_3 \longrightarrow R_3/(1) \Longrightarrow R_3 \longrightarrow R_3$

**Row operations used to obtain Third Table**

$R_1 \longrightarrow R_1 - (-5) * (R_2/(2)) \Longrightarrow R_1 \longrightarrow R_1 + \frac{5}{2} * R_2$

$R_2 \longrightarrow R_2/(2) \Longrightarrow R_2 \longrightarrow R_2/(2)$

$R_3 \longrightarrow R_3 - (-1) * (R_2/(2)) \Longrightarrow R_3 \longrightarrow R_3 + \frac{1}{2} * R_2$