

Roll Number: 101703501

**Thapar Institute of Engineering & Technology, Patiala**  
Department of Computer Science and Engineering

**END SEMESTER EXAMINATION**

B. E. (Second Year): Semester-III (2018/19)  
(COE)

Course Code: UCS405

Course Name: Discrete Mathematical Structures

December 1, 2018

Time: 09:00 A.M. - 12:00 P.M.

Time: 3 Hours, M. Marks: 100

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**Note:** Attempt all questions in a proper sequence with justification.  
Assume missing data, if any, suitably.

Q1(a) Let  $N = \{1, 2, 3, \dots\}$  and, for each  $n \in N$ , Let  $A_n = \{n, 2n, 3n, \dots\}$ . Find: (3)

i.  $A_3 \cap A_5$

ii.  $A_4 \cap A_5$

iii.  $\bigcup_{i \in Q} A_i$  where  $Q = \{2, 3, 5, 7, 11, \dots\}$  is the set of prime numbers.

Q1(b) Suppose that  $A$  is the multiset that has as its elements the types of computer equipment needed by one department of a university and the multiplicities are the number of pieces of each type needed, and  $B$  is the analogous multiset for a second department of the university. For instance,  $A$  could be the multiset {107 personal computers, 44 routers, 6 servers} and  $B$  could be the multiset {14 personal computers, 6 routers, 2 mainframes}. (4)

i. What combination of  $A$  and  $B$  represents the equipment the university should buy assuming both departments use the same equipment?

ii. What combination of  $A$  and  $B$  represents the equipment that will be used by both departments if both departments use the same equipment?

iii. What combination of  $A$  and  $B$  represents the equipment that the second department uses, but the first department does not, if both departments use the same equipment?

iv. What combination of  $A$  and  $B$  represents the equipment that the university should purchase if the departments do not share equipment?

Q1(c) From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examination? (6)

Q2(a) Show that the function  $f(x) = e^x$  from the set of real numbers to the set of real numbers is not invertible, but if the codomain is restricted to the set of positive real numbers, the resulting function is invertible. (4)

Q2(b) Consider the function  $f: \mathbb{N} \rightarrow \mathbb{N}$ , where  $\mathbb{N}$  is the set of natural numbers, defined by  $f(n) = n^2 + n + 1$ . Show that the function  $f$  is one-to-one but not onto. (6)

Q2(c) Suppose  $P(n) = a_0 + a_1 n + a_2 n^2 + \dots + a_m n^m$ . Prove that  $P(n) = O(n^m)$ . (4)

Q2(d) Calculate the Big-O complexity of the following codes with proper explanation: (6)

<p>i.</p> <pre> void printAllNumbers(int arr[ ], int size) {     for (int i = 0; i &lt; size; i++)     {         printf("%d\n", arr[i]);     }     for (int i = 0; i &lt; size; i++)     {         for (int j = 0; j &lt; size; j++)         {             printf("%d\n", arr[i] + arr[j]);         }     } } </pre>	<p>ii.</p> <pre> void method4(int [ ] arr) {     for(int i = 0; i &lt; arr.length; i++)     {         for(int k = arr.length - 1; k &gt; 0; k = k / 3)         {             print(arr[i]);         }     } } </pre>
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Q3(a) A partition of a positive integer  $m$  is a set of positive integers whose sum is  $m$ . Draw the Hasse diagram of the partitions of  $m$  where  $m = 6$ . Find all minimal and maximal elements of  $m$ . (4)

Q3(b) Schedule the tasks needed to build a house, by specifying their order, if the Hasse diagram representing these tasks is as shown in the Figure 1. Show and explain all the steps used to arrive at the scheduling. (5)

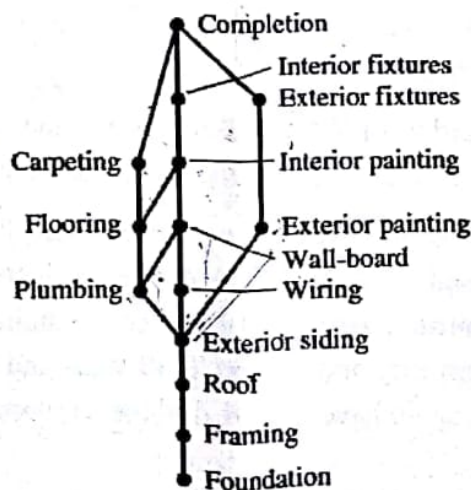


Figure 1

Q3(e) Consider the second-order homogeneous recurrence relation (6)

$$a_n = a_{n-1} + a_{n-2} \text{ with initial conditions } a_0 = 0, a_1 = 1$$

- i. Find the general solution.
- ii. Find the unique solution using the initial conditions.

Q4(a) Consider the following open propositions over the universe  $U = \{-4, -2, 0, 1, 3, 5, 6, 8, 10\}$  (4)

$$P(x) : x \geq 4$$

$$Q(x) : x^2 = 25$$

$$R(x) : x \text{ is a multiple of } 2$$

Find the truth values of

- i.  $P(x) \wedge R(x)$
- ii.  $[\sim Q(x)] \wedge P(x)$

Q4(b) Proof by contrapositive that for every real number  $x \in [0, \pi/2]$ , we have (5)

$$\sin x + \cos x \geq 1.$$

Q4(c) Using truth table show that  $(p \leftrightarrow q) \leftrightarrow r$  and  $p \leftrightarrow (q \leftrightarrow r)$  are logically equivalent. (5)

Q5(a) Define isomorphism. Determine whether the graphs given in Figure 2 are isomorphic (5) or not? Justify your answer.

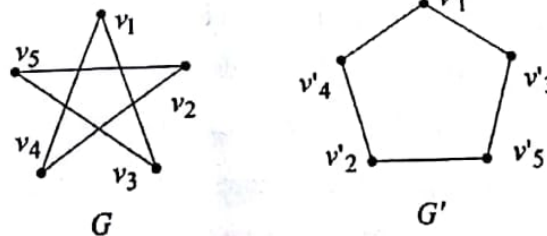


Figure 2

Q5(b) Using Floyd Warshall's algorithm find the distance between all pair of vertices for (12) Figure 3:

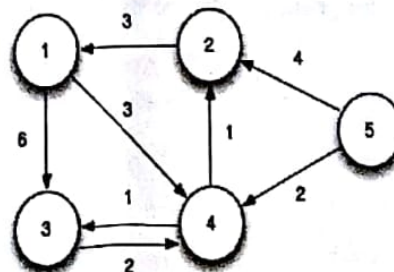


Figure 3

- Q5(c) In a king's miraculous garden, there is a miraculous lake on which, exactly once every year, seven miraculous lotus flowers blossom. Because they are miraculous, the lotus flowers bloom in an improbably straight and evenly spaced line, as one can see in Figure 4: (5)



Figure 4

The garden becomes even more miraculous when one learns of the existence of the king's frog. When the lotuses bloom, the frog appears, as if out of nowhere, and lands on one of the flowers. The frog will then start jumping to other lotus flowers, always jumping by either three or five flowers. For instance, if the frog lands on the second lotus, then it might jump from there to the fifth or seventh lotus, and so on. According to the customs and the everlasting tradition, the frog's duty and privilege is to first land on a lotus from which it can embark on a journey, proceeding as indicated above, to visit each lotus once and once only. This means, of course, that the starting point and the finishing point will be different.

Which lotuses can serve as starting points for the king's frog? Justify your answer with proper explanation.

- Q6(a) Let  $G = \{x \in \mathbf{R} \mid x > 1\}$  be the set of all real numbers greater than 1. Define (8)
- $$x * y = xy - x - y + 2, \text{ for } x, y \text{ in } G.$$

- i. Show that the operation  $*$  is closed on  $G$ .
- ii. Show that the associative law holds for  $*$ .
- iii. Show that 2 is the identity element for the operation  $*$ .
- iv. Show that for element  $a$  in  $G$  there exists an inverse  $a^{-1}$  in  $G$ .

(8)

- Q6(b) Consider  $G = \{1, 5, 7, 11, 13, 17\}$  under multiplication modulo 18.

- i. Construct the multiplication table of  $G$ .
- ii. Find inverse of 5, 7 and 17.
- iii. Find the order and group generated by 5 and 13.
- iv. Is  $G$  cyclic?