Thapar University, Patiala

Computer Science and Engineering Department

B. Tech. (Second Year): Semester-IV	Course Code: <mark>UCS405/UCS101</mark>
COE	Course Name: Discrete Mathematical Structures
May 13, 2016	Friday
Time: 3 Hours, M. Marks: 100	Name of Faculty: Dr A. K. Loura, Mr. S.
Weightage: 45	Modi, Dr H. S. Pannu, Ms. R. K. Chahal

*Note: Attempt all questions with **proper Justification**. Without Justification zero marks will be awarded. Assume missing data, if any, suitably.*

1. a) Check whether $p \rightarrow (p \vee q)$ is a Tautology or not. (5+5)

    b) Convert $p \leftrightarrow q$ into Conjunctive normal form.

2. Consider the hash function $h(x) = x \bmod 13$, Insert the data in the order given in the initially empty cells indexed 0 to 12. Resolve any collision with linear probing. Show step by step. **Construct a table to show the final results.** (5)

    784, 281, 1141, 18, 1, 329, 620, 43, 31, 684

3. Given the recurrence relation $T(n) = T(n-1) + n$ where $n \geq 1$ and $T(1) = 1$. Determine its complexity in Big-Theta notation. (5)

4. The Paper Street Soap Company has 6 employees; Bob, Cornelius, Jack, Marla, Rupert, and Tyler. The boss needs to meet with groups of employees to discuss a new overtime policy, but unfortunately some employees can't stand to be in the same room with others. In the table below, an X means that those two people can't stand each other, and can't be in the same room together. (10)

	B	C	J	M	R	T
B	--	X			X	
C	X	--	X	X		X
J		X	---		X	
M		X		---	X	X
R	X		X	X	---	
T		X		X		---

    Draw the graph using appropriate reasoning and use appropriate technique of graph theory to determine the number of different separate meetings the boss must make with the employees (**Write clearly the name of the appropriate technique**).

5. Check whether $G = \{1, -1, i, -i\}$ is a monoid or not under addition operation? Prove it. Check whether it is a group or not? (5)

6. a) For any integers k and m, Check whether $(Z, \oplus, \otimes)$ is a ring or not? Binary operations $\oplus$ and $\otimes$ are defined by $x \oplus y = x + y - k$, $x \otimes y = x + y - mxy$. (5)

    b) Given set $S = \{1, 2, 3, 4\}$, check whether S is cyclic group or not under multiplication modulo 5. Find the generator of the group. (5)

7. Draw the Hasse's diagram for inclusion relation on the power set $P(S)$, where $S = \{a, b, c, d\}$. Check whether it is a lattice or not. Give Justification for the same. (5)

[P.T.O.]

8. Using **appropriate proof technique**, to prove the following mathematical   (5)
    statement:
        "If n = ab, where a and b are positive integers, then a $\leq \sqrt{n}$ or b $\leq \sqrt{n}$"

9. a) Express the negation of the following statement in English   (5)
        i)   All Swedish movies are serious.
        ii) Some birds can talk.
    b) Using propositional logic check whether premises leads to the
    conclusion or not:
    " It is not sunny this afternoon and it is colder than yesterday", "We will go
    swimming only if it is sunny," "If we do not go swimming, then we will   (5)
    take a canoe trip," and "if we take a canoe trip, then we will be home by
    sunset".
    Conclusion: "We will be home by sunset".

10. a) Consider the following statements determine Converse, Inverse and   (6)
    Contrapositive of the following statements:
        i)   "If we are on vacation we go fishing" .
        ii) $\neg P \rightarrow Q$
    b) Translate the sentence into quantified expressions of predicate logic
        All students that study discrete math are good at logic.   (4)

11. Apply Floyd-Warshall Algorithm on the following weighted Graph (Fig.   (10)
    1). Explain the procedure step by step.

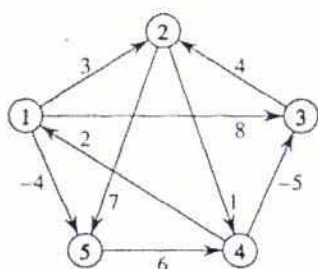
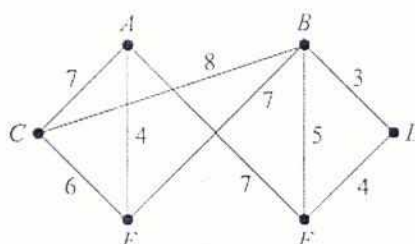Fig. 1   Fig. 2

12. Determine the minimum spanning tree (Explain step by step) using   (5)
    Kruskal algorithm for Fig. 2.

13. a) Determine the edge connectivity of the following graph (Fig. 3). Justify   (5+5)
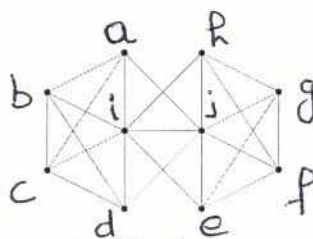   your answer.

Fig. 3

    b) Determine the Chromatic number of the Graph given in Fig. 3.