Lecture-13

To plot $y(n) = x(n/2)$, the following calculations are carried out.

$y(0) = x(0) = 3.5$, $y(2) = x(1) = 1.5$, $y(4) = x(2) = 2.5$, $y(6) = x(3) = 4$, $y(8) = x(4) = 3$, $y(10) = x(5) = 2$, $y(12) = x(6) = 0$ and so on.

Similarly, $y(-2) = x(-1) = 1$, $y(-4) = x(-2) = 2$, $y(-6) = x(-3) = 3$, $y(-8) = x(-4) = 4$, $y(-10) = x(-5) = 5$, $y(-12) = x(-6) = 0$ and so on.

Figure 8 shows that the signal x(n/2) obtained by increasing the sampling on the continuous time signal by a factor 2. This process of increasing the sampling rate is known as **up sampling**, which is the reverse of down sampling.

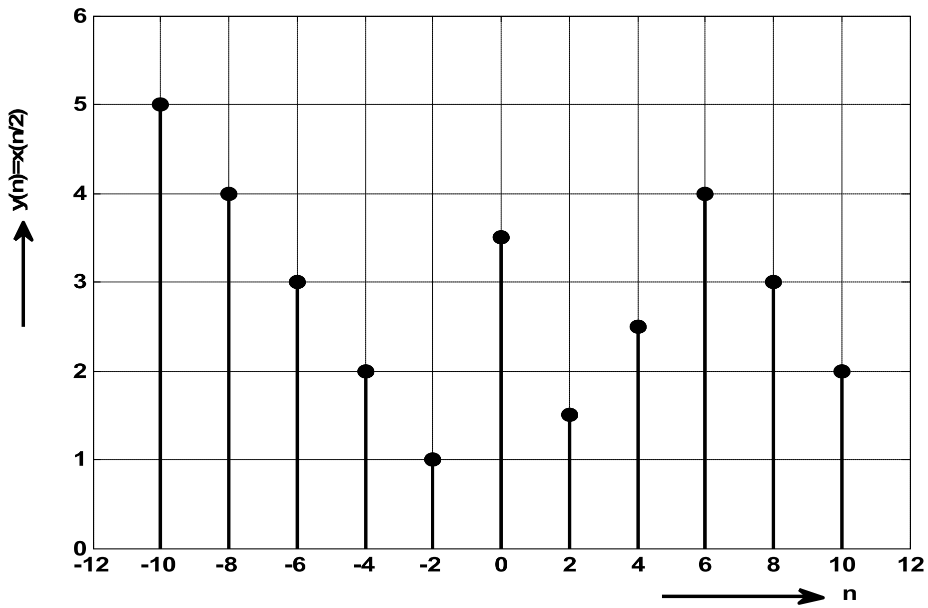

Figure 8: Plot of $y(n) = x(n/2)$

Scalar Multiplication

Figure 9 shows the multiplication of a signal by a scale factor 'a', which is known as scalar multiplication.

For example, let $x(n) = \{1,2,3,4\}$ and a = 3.

$\therefore ax(n) = \{3,6,9,12\}$

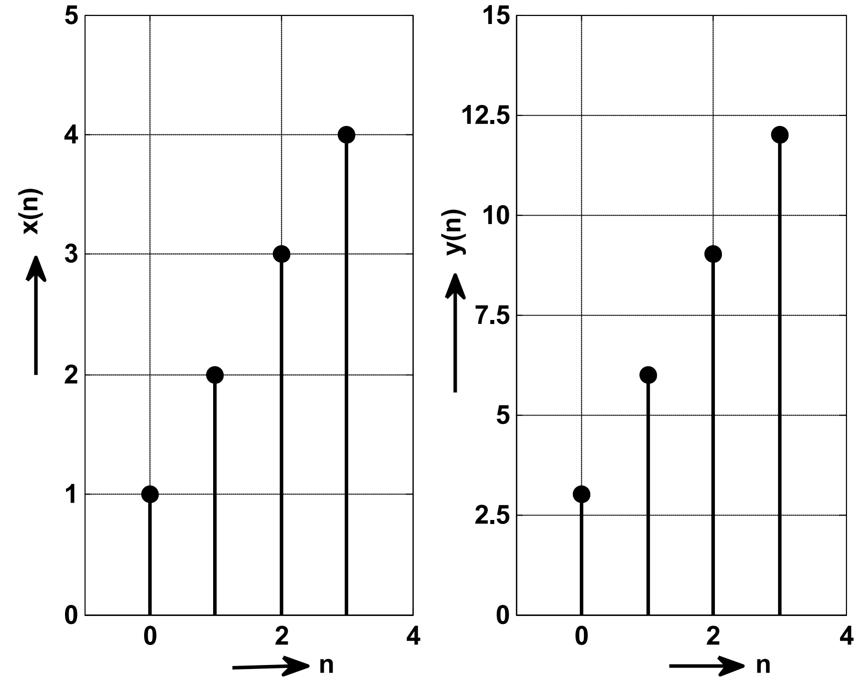

Figure 9 A scalar multiplier

Multiplication Operation

Figure 10 shows the multiplication of two signal sequences $x_1(n)$ and $x_2(n)$, which forms another sequence y(n).

In Fig. 10, let $x_1(n) = \{1,2,3,4\}$ and $x_2(n) = \{2,3,2,1\}$

$$\therefore y(n) = x_1(n) \cdot x_2(n) = \{1 \times 2, 2 \times 3, 3 \times 2, 4 \times 1\} = \{2,6,6,4\}$$

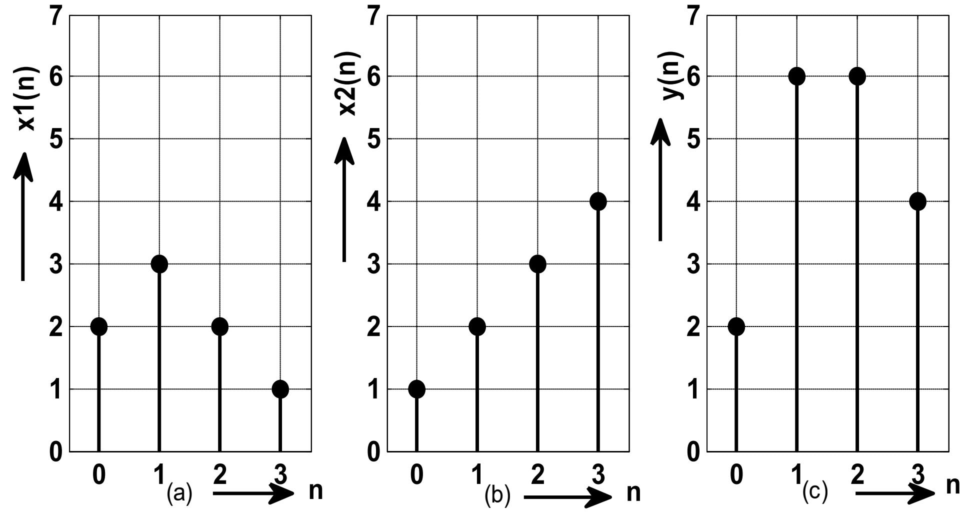

Figure 10 A signal multiplier

Addition Operation

Figure 11 shows the addition operation of two signal sequences, which forms another sequence y(n).

For example, let $x_1(n) = \{1,2,3,4\}$ and $x_2(n) = \{2,3,2,1\}$

$$\therefore\ y(n) = x_1(n) + x_2(n) = \{1+2, 2+3, 3+2, 4+1\} = \{3,5,5,5\}$$

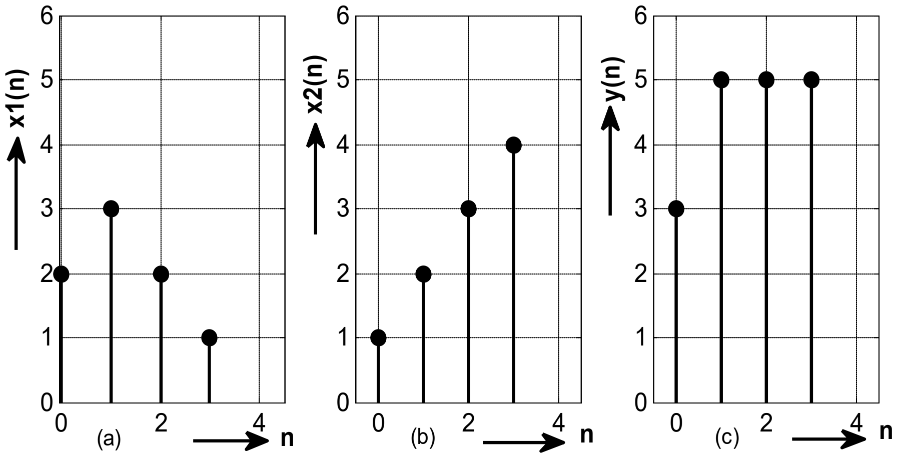

Figure 11 Addition operation

If $x_1(n)=\{2,1,\underset{\uparrow}{3},4,3,4\}$ *and* $x_2(n)=\{2,\underset{\uparrow}{3},1,4,3,4\}$ find $y_1(n) = x_1(n)+x_2(n)$, $y_2(n)= x_1(n)\, x_2(n)$.

$$x_1(n)=\{2,1,\underset{\uparrow}{3},4,3,4\} \text{ and } x_2(n)=\{2,\underset{\uparrow}{3},1,4,3,4\}$$

$$y_1(n)=x_1(n)$$

$$x_1(n)=\{2,1,\underset{\uparrow}{3},4,3,4,0\}$$

$$x_2(n)=\{0,2,\underset{\uparrow}{3},1,4,3,4\}$$

$$y_1(n)=x_1(n)+x_2(n)$$

$$=\{2+0,1+2,3\underset{\uparrow}{+}3,4+1,3+4,4+3,0+4\}$$

$$=\{2,3,\underset{\uparrow}{6},5,7,7,4\}$$

$$y_2(n)=x_1(n)\times x_2(n)$$

$$=\{2\times 0,1\times 2,3\underset{\uparrow}{\times} 3,4\times 1,3\times 4,4\times 3,0\times 4\}$$

$$=\{0,2,\underset{\uparrow}{9},4,12,12,0\}$$

Classification of Discrete Time Signals

The discrete time signals can be classified as follows:

- Even and Odd Signals
- Periodic and Non–periodic Signals
- Deterministic and Random Signals
- Energy Signals and power Signals
- Multichannel and Multidimensional Signals
- Causal and Anti-causal Signals

Even and Odd Signals

A discrete signal is said to be even or symmetric if

$$x(n) = x(-n)$$

A discrete signal is said to be odd or asymmetric if

$$x(n) = -x(-n)$$

Figure 12 and Figure 13 show the even and odd signals.

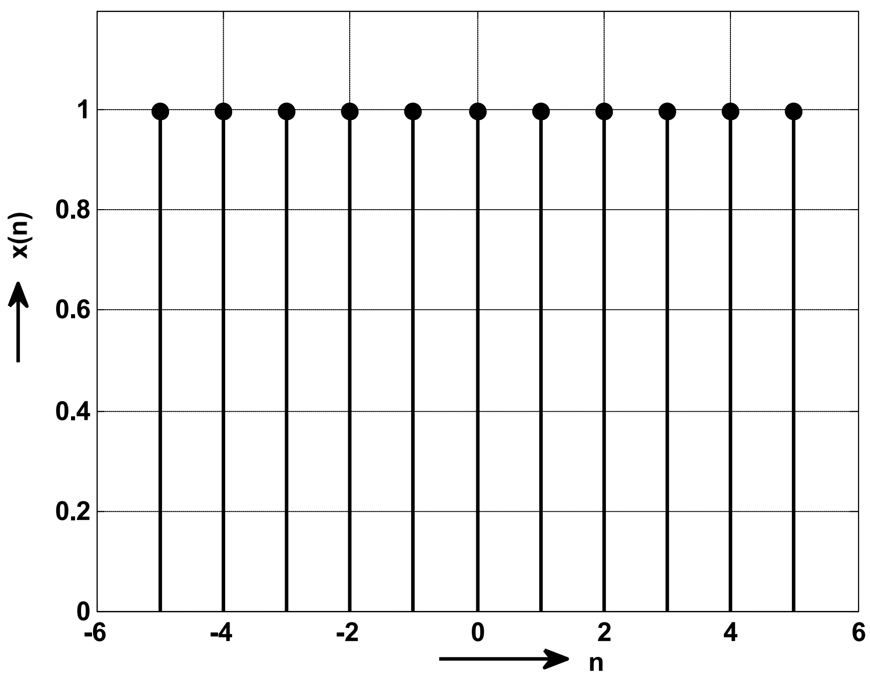

Figure 12 : Even Signal

From Figure 12

$$\begin{aligned} x(-1) &= x(1) \\ x(-2) &= x(2) \\ x(-3) &= x(3) \\ \vdots \quad & \quad \vdots \\ x(-n) &= x(n) \end{aligned}$$

A real valued signal x(n) is said to be odd or antisymmetric if

$$x(-n) = -x(n)$$

for all values of n.

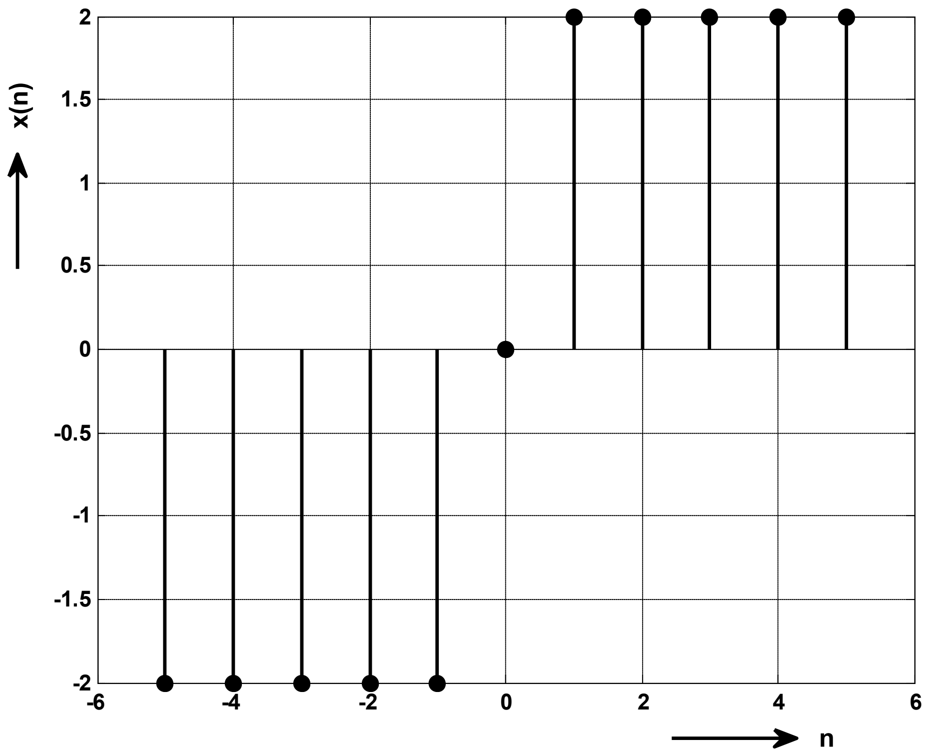

Figure 13: Odd Signal

From Figure 13

$$
\begin{aligned}
x(-1) &= -x(1) \\
x(-2) &= -x(2) \\
x(-3) &= -x(3) \\
\vdots \quad & \quad \vdots \\
x(-n) &= -x(n)
\end{aligned}
$$

A complex signal is said to be complex conjugate symmetric if

$$x(n) = x^*(-n)$$

for all values of n.

A complex signal is said to be complex conjugate antisymmetric if

$$x(n) = -x^*(-n)$$

for all values of n.

Periodic Signals and Non–periodic Signals

A discrete signal x(n) is said to be periodic if

$$x(n + N) = x(n) \qquad \text{for all } n \tag{1}$$

where ‘N’ is the period of the signal.

The signal x(n) is said to be non–periodic if it does not satisfy the relation of equation (1).

A discrete time sinusoid is periodic for all values of ‘n’ only if its frequency is rational i.e., $f = m/N$ where N is the fundamental period and ‘m’ is some integer.

The signal becomes aperiodic or non–periodic if the signal x(n) does not satisfy the Eq. (2).

If $x_1(n)$ and $x_2(n)$ are the periodic sequences with period N_1 and N_2 respectively, the sum $x(n) = x_1(n) + x_2(n)$ will be periodic having fundamental period

$$N = \frac{N_1 N_2}{\gcd(N_1, N_2)} \qquad (2)$$

where **gcd(N_1,N_2)** is the greatest common divisor of N_1 and N_2. Similarly, the product $x(n) = x_1(n)\ x_2(n)$ will be periodic having fundamental period ‘N’ expressed by Eq. (2).

Example: If $\Omega_0/2\pi$ is a rational number then the sequence x(n) = $e^{j\Omega_0 n}$ will be periodic ($\Omega_0 \neq 0$).

x(n) will be periodic if the following relation holds good:

$$e^{j\,\Omega_0(n+N)} = e^{j\,\Omega_0 n}$$

$$\text{i.e., } e^{j\,\Omega_0 n}\, e^{j\,\Omega_0 N} = e^{j\,\Omega_0 n}$$

$$\text{i.e., } e^{j\,\Omega_0 N} = 1 \qquad \text{(E1)}$$

Equation (1) holds good for $\Omega_0 = 0$. Here $\Omega_0 \neq 0$. Therefore, equation (E1) will hold good if $\Omega_0 N = p2\pi$ where ‘p’ is a positive number.

$$i.e., \frac{\Omega_0}{2\pi} = \frac{m}{N} = \text{rational number}$$

Hence we can conclude that if $\frac{\Omega_0}{2\pi}$ is a rational number then x(n) = $e^{j\Omega_0 n}$ will be periodic.

Determine whether the following signals are periodic or not:

(i) $\cos(0.1\pi n)$, (ii) $\cos\left(\frac{n}{10}\right)$ $\cos\left(\frac{n\pi}{10}\right)$

(i) $\cos(0.1\pi n)$

$$\omega = 0.1\pi = 2\pi f$$

$$\therefore f = \frac{\omega}{2\pi} = \frac{0.1\pi}{2\pi} = \frac{1}{20}$$

$$\therefore N = \frac{1}{f} = 20$$

Also, $$\cos(0.1\pi n) = \cos\left(\frac{\pi n}{10}\right) = \cos\left(\frac{\pi}{10}(n+20)\right)$$

Therefore, N = 20

(ii) $\cos\left(\frac{n}{10}\right)\cos\left(\frac{n\pi}{10}\right)$

Let the sequence be as $\cos\omega_1\cos\omega_2$.

Here $\omega_1 = \frac{1}{10}$ and $\omega_2 = \frac{\pi}{10}$. We have $f_1 = \frac{\omega_1}{2\pi} = \frac{\frac{1}{10}}{2\pi} = \frac{1}{20\pi} = \frac{m_1}{N_1}$

Here N_1 is not an integer. Therefore, $\cos\left(\frac{n}{10}\right)$ is not periodic.

We have $f_2 = \frac{\omega_2}{2\pi} = \frac{\frac{\pi}{10}}{2\pi} = \frac{1}{20} = \frac{m_2}{N_2}$. Here f_2 is the ratio of two integers and

hence $\cos\left(\frac{n\pi}{10}\right)$ is periodic. Therefore, the given sequence $\cos\left(\frac{n}{10}\right)\cos\left(\frac{n\pi}{10}\right)$

Therefore, the given sequence is aperiodic.

Deterministic and Random Signals

The signals which can be described uniquely by a mathematical expression, table, graph or a well defined rule are known as deterministic signals. On the other hand, if the signals cannot be described by formula or graph, they are known as random signals.

Energy signals and Power signals

The energy of discrete time signal x(n) can be represented by

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

If the energy of the signal x(n) is finite i.e., $0<E<\infty$, the signal is called energy signal. The average power (P) of discrete time signal is represented by $P = \underset{N \to \infty}{Lt} \frac{1}{(2N+1)} \left\{ \sum_{n=-N}^{N} |x(n)|^2 \right\}$.

The signal x(n) is said to be power signal if $0<P<\infty$.

Table 1: Comparison of power and energy signals

Power Signal	Energy Signal
$0 < P < \infty$	$0 < E < \infty$
Almost all the periodic signals are power signal.	Almost all the non periodic signals are energy signal.
Power signals are not time limited because they can exist over an infinite time.	Energy signals are time limited because they exist over a short period of time.
Energy of power signal is infinite.	Power of an energy signal is zero.

Find whether the following signals are energy signal , power signal or neither of them:

(a) $x(n) = (-0.3)^n u(n)$ and

(b) $x(n) = 3e^{j2n}$

(a) $x(n) = (-0.3)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left|(-0.3)^n u(n)\right|^2 = \sum_{n=0}^{\infty} 0.09^n = \frac{1}{1-0.09} = \frac{1}{0.91} < \infty$$

Therefore, x(n) is an energy signal.

(b) $x(n) = 3e^{j2n}$

$$\therefore |x(n)| = \left|3e^{j2n}\right| = 3\left|e^{j2n}\right| = 3$$

$$P = \underset{N \to \infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \underset{N \to \infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} 3^2 = \underset{N \to \infty}{\mathrm{Lt}} \frac{9(2N+1)}{2N+1} = 9 < \infty$$

Therefore, x(n) is a power signal.

Causal and Anti-causal Signal

If the value of the signal is zero for $n < 0$, the signal is said to be causal signal. Otherwise, it will be anti-causal signal.

For example $x_1(n) = a^n u(n)$ and $x_2(n) = \{\underset{\uparrow}{1}, 2, 3, -1, -2\}$ are the examples of causal signal.

For example $x_1(n) = a^n u(-n-1)$ and $x_2(n) = \{1, 2, 3, 6, \underset{\uparrow}{-2},\}$ are the examples of anti-causal signal.

If a signal has the values for $n < 0$ and $n > 0$, the signal is known as noncausal signal. For $x(n) = \{1, 2, 3, 6, \underset{\uparrow}{2}, 3, 5, 4, 7\}$ is the example of noncausal signal.