Course: UMA 035 (Optimization Techniques)

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Example:

Check that the following LPP can be solved by the Simplex method or not.

If not then solve by Big-M method.

Maximize $(3x_1+2x_2)$

Subject to

$2x_1+x_2 \quad <=2,$

$3x_1+4x_2>=12,$

$x_1>=0,$

$x_2>=0.$

Solution

Maximize $(3x_1+2x_2)$

Subject to

$2x_1+x_2+S_1 \quad =2,$

$3x_1+4x_2-S_2 \quad =12,$

$x_1>=0, x_2>=0, S_1>=0, S_2>=0.$

		3	2	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =							
	S_1	2	1	1	0		
		3	4	0	-1		

Not possible to find second basic variable.

Maximize $(3x_1+2x_2-MA_1)$

Subject to

$2x_1+x_2 \quad +S_1 \quad =2,$

$3x_1+4x_2-S_2 \quad +A_1=12,$

$x_1>=0,\ x_2>=0,\ S_1>=0,\ S_2>=0.$

		3	2	0	0	$-M$		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		$-3M-3$	$-4M-2$	0	M	0		
0	S_1	2	1	1	0	0	2	2/1=2
$-M$	A_1	3	4	0	-1	1	12	12/4=3
$Z_j - C_j$ =		1+5M	0	2+ 4M	M	0		
2	x_2	2	1	1	0	0	2	
$-M$	A_1	-5	0	-4	-1	1	4	

All $Z_j - C_j$ are greater than or equal to 0.

Since, $A_1=4\neq0$. So, the problem has no solution.

Row operations

$R_1 \longrightarrow R_1 - (-4M-2)*(R_2/(1)) \Longrightarrow$ Not apply it due to presence of M

$R_2 \longrightarrow R_2/(1) \Longrightarrow R_2 \longrightarrow R_2$

$R_3 \longrightarrow R_3 - (4)_* (R_2/(1)) \Longrightarrow R_3 \longrightarrow R_3 - 4\, R_2$

Two Phase method

Phase 1:

Do the following changes in the Big-M method and solve the problem.

- **Consider 1 instead of M**

- **Replace the coefficients of $x_1, x_2, \ldots, x_m$ in the objective function with 0.**

  If any of the artificial variable is not 0 in the last table. Then, no solution otherwise go to Phase 2.

Phase 2:

In the last table of Phase-I, consider the given coefficients of $x_1, x_2, \ldots, x_m$ and calculate $Z_j - C_j$.

If solution is not optimal then apply Simplex method to find the optimal solution.

Example:

Check that the following LPP can be solved by the Simplex method or not.

If not then solve by Two-Phase method.

Maximize $(3x_1+2x_2)$

Subject to

$2x_1+x_2 \quad <=2,$

$3x_1+4x_2>=12,$

$x_1>=0,$

$x_2>=0.$

Solution

Maximize $(3x_1+2x_2)$

Subject to

$2x_1+x_2+S_1 \quad =2,$

$3x_1+4x_2-S_2 \quad =12,$

$x_1>=0, x_2>=0, S_1>=0, S_2>=0.$

		3	2	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =							
	S_1	2	1	1	0		
		3	4	0	-1		

Not possible to find second basic variable.

Maximize ($\cancel{3}\ 0x_1+\cancel{2}\ 0x_2-\cancel{M}\ 1A_1$) or Maximize ($0x_1+0x_2-1A_1$)

Subject to

$2x_1+x_2 \quad +S_1 \quad =2,$

$3x_1+4x_2-S_2 \quad +A_1=12,$

$x_1>=0,\ x_2>=0,\ S_1>=0,\ S_2>=0.$

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =								
	S_1	2	1	1	0	0		
	A_1	3	4	0	−1	1		

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =								
0	S_1	2	1	1	0	0		
−1	A_1	3	4	0	−1	1		

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =								
0	S_1	2	1	1	0	1	2	
−1	A_1	3	4	0	−1	0	12	

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =				0		0		
0	S_1	2	1	1	0	0	2	
−1	A_1	3	4	0	−1	1	12	

		0	0	0	0	-1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =				0		0		
0	S_1	2	1	1	0	0	2	
-1	A_1	3	4	0	-1	1	12	

$(0)(2)+(-1)(3)-0=-3$

$(0)(1)+(-1)(4)-0=-4$

$(0)(0)+(-1)(-1)-0=1$

		0	0	0	0	-1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		-3	-4	0	1	0		
0	S_1	2	1	1	0	0	2	
-1	A_1	3	4	0	-1	1	12	

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		−3	−4	0	1	0		
0	S_1	2	1	1	0	0	2	
−1	A_1	3	4	0	−1	1	12	

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		−3	−4	0	1	0		
0	S_1	2	1	1	0	0	2	2/1=2
−1	A_1	3	4	0	−1	1	12	12/4=3

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		−3	−4	0	1	0		
0	S_1	2	1	1	0	0	2	2/1=2
−1	A_1	3	4	0	−1	1	12	12/4=3

Row operations

$R_1 \longrightarrow R_1 - (-4)*(R_2/(1)) \Longrightarrow R_1 + 4*R_2$

$R_2 \longrightarrow R_2/(1) \Longrightarrow R_2 \longrightarrow R_2$

$R_3 \longrightarrow R_3 - (4)_* (R_2/(1)) \Longrightarrow R_3 \longrightarrow R_3 - 4 R_2$

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		5	0	4	1	4		
0	x_2	2	1	1	0	0	2	
−1	A_1	−5	0	−4	−1	1	4	

All $Z_j - C_j$ are greater than or equal to 0.

Since, $A_1=4\neq0$. So, the problem has no solution.

Example:

Check that the following LPP can be solved by the Simplex method or not.

If not then solve by Two-Phase method.

Minimize $(x_1 - 2x_2 - 3x_3)$

Subject to

$-2x_1+x_2+3x_3=2,$

$2x_1+3x_2+4x_3=1,$

$x_1>=0,$

$x_2>=0,$

$x_3>=0.$

Solution

Maximize $(-x_1+2x_2+3x_3)$

Subject to

$-2x_1+x_2+3x_3=2,$

$2x_1+3x_2+4x_3=1,$

$x_1>=0,$

$x_2>=0,$

$x_3>=0.$

		-1	2	3		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
Z_j-C_j =						
		-2	1	3		
		2	3	4		

Not possible to find first and second basic variable.

Maximize $(-\cancel{x_1}\ 0x1+\cancel{2}\ 0x_2+\cancel{3}\ 0x_3-A_1-A_2)$

or

Maximize $(-0x1+0x_2+0x_3-A_1-A_2)$

Subject to

$-2x_1+x_2+3x_3+A_1=2,$

$2x_1+3x_2+4x_3+A_2=1,$

$x_1>=0,$

$x_2>=0,$

$x_3>=0.$

		0	0	0	−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =								
	A_1	−2	1	3	1	0		
	A_2	2	3	4	0	1		

Pattern of examination

		0	0	0	0	−1		
C_B	Basic Variables	x_1	x_2	S_1	S_2	A_1	Solution	Minimum Ratio
$Z_j - C_j$ =		−3	−4	0	1	0		
0	S_1	2	1	1	0	0	2	2/1=2
−1	A_1	3	4	0	−1	1	12	12/4=3
$Z_j - C_j$ =		5	0	4	1	4		
0	x_2	2	1	1	0	1	2	
−1	A_1	−5	0	−4	−1	−4	4	

All $Z_j - C_j$ are greater than or equal to 0.

Since, $A_1=4\neq0$. So, the problem has no solution.

Example:

Check that the following LPP can be solved by the Simplex method or not.

If not then solve by Two-Phase method.

Minimize $\left(\frac{15}{2}x_1 - 3x_2\right)$

Subject to

$3x_1 - x_2 - 3x_3 >= 3,$

$x_1 - x_2 + x_3 >= 2,$

$x_1 >= 0,$

$x_2 >= 0,$

$x_3 >= 0.$

Solution

Maximize $\left(-\frac{15}{2}x_1 + 3x_2\right)$

Subject to

$3x_1 - x_2 - 3x_3 - S_1 = 3,$

$x_1 - x_2 + x_3 - S_2 = 2,$

$x_1 >= 0,$

$x_2>=0$,

$x_3>=0$.

		$-\frac{15}{2}$	3	0				
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =								
		3	-1	-1	-1	0		
		1	-1	1	0	-1		

Not possible to find first and second basic variable.

Maximize ($0x1+ 0x_2+0x_3-A_1-A_2$)

Subject to

$3x_1-x_2-x_3- S_1+A_1=3$,

$x_1-x_2+x_3- S_2+A_2=2$,

$x_1>=0$, $x_2>=0$, $x_3>=0$, $S_1>=0$, $S_2>=0$.

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =										
	A_1	3	−1	−1	−1	0	1	0		
	A_2	1	−1	1	0	−1	0	1		

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =										
−1	A_1	3	−1	−1	−1	0	1	0		
−1	A_2	1	−1	1	0	−1	0	1		

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =										
−1	A_1	3	−1	−1	−1	0	1	0	3	
−1	A_2	1	−1	1	0	−1	0	1	2	

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =							0	0		
−1	A_1	3	−1	−1	−1	0	1	0	3	
−1	A_2	1	−1	1	0	−1	0	1	2	

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		−4	2	0	1	1	0	0		
−1	A_1	3	−1	−1	−1	0	1	0	3	
−1	A_2	1	−1	1	0	−1	0	1	2	

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		−4	2	0	1	1	0	0		
−1	A_1	3	−1	−1	−1	0	1	0	3	
−1	A_2	1	−1	1	0	−1	0	1	2	

		0	0	0			−1	−1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		−4	2	0	1	1	0	0		
−1	A_1	3	−1	−1	−1	0	1	0	3	3/3=1
−1	A_2	1	−1	1	0	−1	0	1	2	2/1=2

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		-4	2	0	1	1	0	0		
-1	A_1	3	-1	-1	-1	0	1	0	3	3/3=1
-1	A_2	1	-1	1	0	-1	0	1	2	2/1=2

Row operations

$R_1 \longrightarrow R_1 - (-4)*(R_2/(3)) \Longrightarrow R_1 + \frac{4}{3}*R_2$

$R_2 \longrightarrow R_2/(3) \Longrightarrow R_2 \longrightarrow \frac{1}{3}R_2$

$R_3 \longrightarrow R_3 - (1)_* (R_2/(3)) \Longrightarrow R_3 \longrightarrow R_3 - \frac{1}{3}R_2$

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	1	*	0		
0	x_1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	*	0	1	
-1	A_2	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-1	*	1	1	

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	1	*	0		
0	x_1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	*	0	1	
-1	A_2	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-1	*	1	1	

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	1	*	0		
0	x_1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	*	0	1	1/-
-1	A_2	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-1	*	1	1	$1/\frac{4}{3} = \frac{3}{4}$

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	1	*	0		
0	x_1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	*	0	1	1/-
-1	A_2	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-1	*	1	1	$1/\frac{4}{3} = \frac{3}{4}$

Row operations

$R_1 \longrightarrow R_1 - (-\frac{4}{3})*(R_3/\left(\frac{4}{3}\right)) \implies R_1 \longrightarrow R_1 + R_3$

$R_2 \longrightarrow R_2 - (-\frac{1}{3})*(R_3/\left(\frac{4}{3}\right)) \implies R_2 \longrightarrow R_2 + \frac{1}{4}*R_3$

$R_3 \longrightarrow R_3/\left(\frac{4}{3}\right) \implies R_3 \longrightarrow \frac{3}{4}R_3$

		0	0	0			-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	0	0	0	0	*	*		
0	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
0	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

All $Z_j - C_j$ are greater than or equal to 0 as well as $A_1 = A_2 = 0$. So, go to Phase 2.

Phase 2

							*	*		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =							*	*		
	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

Maximize $\left(-\frac{15}{2}x_1 + 3x_2\right)$

		$-\frac{15}{2}$	3	0	0	0	*	*		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =							*	*		
$-\frac{15}{2}$	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
3	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

		$-\frac{15}{2}$	3	0	0	0	*	*		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{3}{4}$	0	$\frac{15}{8}$	$\frac{15}{8}$	*	*		
$-\frac{15}{2}$	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
3	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

All $Z_j - C_j$ are greater than or equal to 0. So, optimal solution is:

$x_1 = \frac{5}{4}$

$x_3 = \frac{5}{4}$

Remaining are 0 i.e., $A_1 = A_2 = x_2 = S_1 = S_2 = 0$.

Pattern of examination

		0	0	0	0	0	-1	-1		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		-4	2	0	1	1	0	0		
-1	A_1	3	-1	-1	-1	0	1	0	3	3/3=1
-1	A_2	1	-1	1	0	-1	0	1	2	2/1=2
$Z_j - C_j$ =		0	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	1	*	0		
0	x_1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	*	0	1	1/-
-1	A_2	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-1	*	1	1	$1/\frac{4}{3}=\frac{3}{4}$
$Z_j - C_j$ =		0	0	0	0	0	*	*		
0	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
0	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

All $Z_j - C_j$ are greater than or equal to 0 as well as $A_1 = A_2 = 0$. So, go to Phase 2.

Phase 2

		$-\frac{15}{2}$	3	0	0	0	*	*		
C_B	Basic Variables	x_1	x_2	x_3	S_1	S_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	$\frac{3}{4}$	0	$\frac{15}{8}$	$\frac{15}{8}$	*	*		
$-\frac{15}{2}$	x_1	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	*	*	$\frac{5}{4}$	
3	x_3	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	*	*	$\frac{3}{4}$	

All $Z_j - C_j$ are greater than or equal to 0. So, optimal solution is:

$x_1 = \frac{5}{4}$

$x_3 = \frac{5}{4}$

Remaining are 0 i.e., $A_1 = A_2 = x_2 = S_1 = S_2 = 0$.

Example:

Solve the following problem by Simplex method.

Maximize $(2x_1+x_2+x_3)$

$x_1-x_2 \qquad =0,$

$2x_1+x_3=3,$

$x_1>=0,\ x_2>=0,\ x_3>=0.$

Solution

		2	1	1		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
$Z_j - C_j$ =						
		1	-1	0		
	x_3	2	0	1		

Not possible to find first basic variable. But as the RHS of the first constraint is 0. So even after dividing the first constraint by -1 the problem will remain in standard from.

Maximize $(2x_1+x_2+x_3)$

Subject to

$-x_1+x_2 \quad =0,$

$2x_1+x_3=3,$

$x_1>=0,\ x_2>=0,\ x_3>=0.$

		2	1	1		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
$Z_j - C_j$ =		-1	0	0		
1	x_2	-1	1	0	0	0/-
1	x_3	2	0	1	3	3/2=1.5
$Z_j - C_j$ =		0	0	$\frac{1}{2}$		
1	x_2	0	1	$\frac{1}{2}$	$\frac{3}{2}$	
2	x_1	1	0	$\frac{1}{2}$	$\frac{3}{2}$	

Optimal Solution:

$x_1=\frac{3}{2}$, $x_2=\frac{3}{2}$, $x_3=0$

Optimal Value: $2x_1+x_2+x_3=2*\frac{3}{2}+\frac{3}{2}+0=\frac{9}{2}$

Example:

Solve the following problem by Simplex method.

Maximize $(2x_1+x_2+2x_3)$

$x_1-3x_2 \qquad =0,$

$2x_1+5x_3=3,$

$x_1>=0,\ x_2>=0,\ x_3>=0.$

Solution

		2	1	1		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
Z_j-C_j =						
		1	-3	0		
		2	0	5		

Not possible to find second basic variable. But as the RHS of the first constraint is 0 . So even after dividing the first constraint by -3 the problem will remain in standard from.

Maximize ($2x_1+x_2+x_3$)

Subject to

$-\frac{1}{3}x_1+x_2 \quad =0,$

$2x_1+5x_3=3,$

$x_1>=0,\ x_2>=0,\ x_3>=0.$

		2	1	1		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
$Z_j - C_j$ =						
	x_2	$-\frac{1}{3}$	1	0		
		2	0	5		

Not possible to find first basic variable. But as the coefficient of x_3 is positive. So even after dividing the second constraint by 5 the problem will remain in standard from.

Maximize $(2x_1+x_2+x_3)$

Subject to

$-\frac{1}{3}x_1+x_2 \qquad =0,$

$\frac{2}{5}x_1+x_3=\frac{3}{5},$

$x_1>=0,\ x_2>=0,\ x_3>=0.$

		2	1	1		
C_B	Basic Variables	x_1	x_2	x_3	Solution	Minimum Ratio
$Z_j - C_j$ =						
1	x_2	$-\frac{1}{3}$	1	0	0	
1	x_3	$\frac{2}{5}$	0	1	$\frac{3}{5}$	

DO YOURSELF

Problem of Degeneracy

If minimum ratio is same corresponding to more than one basic variables. Then, out of such variables, any basic variable may be considered as leaving variable. But in the next table the value of remaining basic variables will be zero. Such problems are called problems of degeneracy.

Example:

Solve the following problem by Simplex method.

Maximize $(2x_1+x_2)$

$4x_1+x_2 <= 8$,

$4x_1-x_2<=8$,

$x_1>=0$, $x_2>=0$.

Solution

		2	1	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =		-2	-1	0	0		
0	S_1	4	1	1	0	8	8/4=2
0	S_2	4	-1	0	1	8	8/4=2
$Z_j - C_j$ =		0	$-\frac{3}{2}$	0	$\frac{1}{2}$		
0	S_1	0	2	$\frac{1}{2}$	-1	0	0/2=0
2	x_1	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	2	2/-
$Z_j - C_j$ =		0	0	$\frac{3}{8}$	$-\frac{1}{4}$		
1	x_2	0	1	$\frac{1}{4}$	$-\frac{1}{2}$	0	
2	x_1	1	0	$\frac{1}{16}$	$\frac{1}{8}$	2	

Optimal Solution:

$x_1=2$, $x_2=0$, $S_1=S_2=0$

Optimal Value: $2x_1+x_2=2*2+0=4$

Problems without objective function

Example:

Solve the following problem by Big-M method.

$x_1 - x_2 = 1,$

$2x_1 - x_2 = 3,$

$x_1 >= 0,\ x_2 >= 0.$

Solution

Maximize $(0x_1 + 0x_2)$

Subject to

$x_1 - x_2 = 1,$

$2x_1 - x_2 = 3,$

$x_1 >= 0,\ x_2 >= 0.$

		0	0		
C_B	Basic Variables	x_1	x_2	Solution	Minimum Ratio
$Z_j - C_j$ =					
		1	-1		
		2	-1		

Not possible to find first and second basic variables.

Maximize $(0x_1+0x_2-MA_1-MA_2)$

Subject to

$x_1-x_2 \quad +A_1 = 1,$

$2x_1-x_2+A_2=3,$

$x_1>=0, x_2>=0.$

		0	0	$-M$	$-M$		
C_B	Basic Variables	x_1	x_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		$-3M$	$2M$	0	0		
$-M$	A_1	1	-1	1	0	1	1/1=1
$-M$	A_2	2	-1	0	1	3	3/2=1.5
$Z_j - C_j$ =		0	$-M$	*	0		
0	x_1	1	-1	*	0	1	1/-
$-M$	A_2	0	1	*	1	1	1/1
$Z_j - C_j$ =		0	0	*	*		
0	x_1	1	0	*	*	2	
0	x_2	0	1	*	*	1	

Solution:

$x_1=2$

$x_2=1$

Example:

Solve the following problem by Two-Phase method.

$x_1 - x_2 \quad = 1,$

$2x_1 - x_2 = 3,$

$x_1 >= 0,\ x_2 >= 0.$

Solution

Maximize $(0x_1 + 0x_2)$

Subject to

$x_1 - x_2 \quad = 1,$

$2x_1 - x_2 = 3,$

$x_1 >= 0,\ x_2 >= 0.$

		0	0		
C_B	Basic Variables	x_1	x_2	Solution	Minimum Ratio
$Z_j - C_j$ =					
		1	-1		
		2	-1		

Not possible to find first and second basic variables.

Since, the coefficients of x_1 and x_2 are 0. So, the solution, obtained in Phase-1, will be final solution.

Phase 1

Maximize $(0x_1+0x_2-A_1-A_2)$

Subject to

$x_1 - x_2 \quad +A_1 = 1,$

$2x_1 - x_2 + A_2 = 3,$

$x_1 >= 0,\ x_2 >= 0.$

		0	0	-1	-1		
C_B	Basic Variables	x_1	x_2	A_1	A_2	Solution	Minimum Ratio
$Z_j - C_j$ =		-3	2	0	0		
-1	A_1	1	-1	1	0	1	1/1=1
-1	A_2	2	-1	0	1	3	3/2=1.5
$Z_j - C_j$ =		0	-1	*	0		
0	x_1	1	-1	*	0	1	1/-
-1	A_2	0	1	*	1	1	1/1
$Z_j - C_j$ =		0	0	*	*		
0	x_1	1	0	*	*	2	
0	x_2	0	1	*	*	1	

Solution:

x_1=2

x_2=1

Construct a Simplex Table corresponding to given Basic Variables

Step 1: Construct a matrix having

- **First column as coefficients of first basic variable in constraints**
- **Second column as coefficients of second basic variable in constraints**
- **Third column as coefficients of third basic variable in constraints**

  ⋮

- **mth column as coefficients of mth basic variable in constraints**

Step 2: Find the multiplicative inverse of the matrix.

Step 3: Multiply the inverse matrix with

- **Coefficients of first variable in constraints to obtain the column corresponding to the first variable.**
- **Coefficients of second variable in constraints to obtain the column corresponding to the second variable.**

  ⋮

- **Coefficients of mth variable in constraints to obtain the column corresponding to the mth variable.**
- **RHS elements to obtain the column of solution.**

Example:

Construct a Simplex table for the following LPP by considering S_3, x_1 and x_2 as first, second and third basic variables respectively. Check that the obtained solution is optimal or not. If not then find the optimal solution.

Max $(4x_1+10x_2)$

Subject to

$2x_1+x_2+S_1=10$

$2x_1+5x_2+S_2=20$

$2x_1+3x_2+S_3=18$

$x_1, x_2, S_1, S_2, S_3 >= 0$.

Solution:

$$\begin{array}{cc} & \begin{array}{ccc} S_3 & x_1 & x_2 \end{array} \\ B = & \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 5 \\ 1 & 2 & 3 \end{bmatrix} \end{array}$$

$$B^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{5}{8} & -\frac{1}{8} & 0 \\ -\frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix}$$

Since, S_3 is first basic variable so its column in table will be

1

0

0

and the corresponding value of $Z_j - C_j$ will be 0.

Since, x_1 is second basic variable so its column in table will be

0

1

0

and the corresponding value of $Z_j - C_j$ will be 0.

Since, x_2 is third basic variable so its column in table will be

0

0

1

and the corresponding value of $Z_j - C_j$ will be 0.

		4	10	0	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j$ =		0	0			0		
0	S_3	0	0			1		
4	x_1	1	0			0		
10	x_2	0	1			0		

Column of S_1

$B^{-1}*$ **Coefficients of S_1 in constraints**

$$\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{5}{8} & -\frac{1}{8} & 0 \\ -\frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{5}{8} \\ -\frac{1}{4} \end{bmatrix}$$

Column of S_2

$B^{-1}*$ **Coefficients of S_2 in constraints**

$$\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{5}{8} & -\frac{1}{8} & 0 \\ -\frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{8} \\ \frac{1}{4} \end{bmatrix}$$

Column of Solution

B^{-1}* RHS

$$\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{5}{8} & -\frac{1}{8} & 0 \\ -\frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 18 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{15}{4} \\ \frac{5}{2} \end{bmatrix}$$

		4	10	0	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j$ =		0	0			0		
0	S_3	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	3	
4	x_1	1	0	$\frac{5}{8}$	$-\frac{1}{8}$	0	$\frac{15}{4}$	
10	x_2	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{5}{2}$	

		4	10	0	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	S_3	Solution	Minimum Ratio
$Z_j - C_j$ =		0	0	0	2	0		
0	S_3	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	3	
4	x_1	1	0	$\frac{5}{8}$	$-\frac{1}{8}$	0	$\frac{15}{4}$	
10	x_2	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{5}{2}$	

Optimal solution:

$S_3=3$

$x_1=\frac{15}{4}$

$x_2=\frac{5}{2}$

Remaining are 0 i.e., $S_1= S_2=0$

Optimal Value:

$4x_1+10x_2=4*\frac{15}{4}+10*\frac{5}{2}=40$

Example:

Construct a Simplex table for the following LPP by considering x_1 and S_2 as first and second basic variables respectively. Check that the obtained solution is optimal or not. If not then find the optimal solution.

Max $(2x_1+x_2)$

Subject to

$x_1 - x_2 + S_1 = 10$

$2x_1 - x_2 + S_2 = 40$

$x_1, x_2, S_1, S_2 >= 0.$

Solution:

$$\begin{array}{cc} x_1 & S_2 \end{array}$$

$$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Since, x_1 is first basic variable so its column in table will be

1

0

and the corresponding value of $Z_j - C_j$ will be 0.

Since, S_2 is second basic variable so its column in table will be

0

1

and the corresponding value of $Z_j - C_j$ will be 0.

		2	1	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0			0		
1	x_1	1			0		
0	S_2	0			1		

Column of x_2

B^{-1}* **Coefficients of x_2 in constraints**

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Column of S_1

B^{-1}* **Coefficients of S_1 in constraints**

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Column of Solution

B^{-1}* **RHS**

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 40 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

		2	1	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0			0		
2	x_1	1	-1	1	0	10	
0	S_2	0	1	-2	1	20	

		2	1	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	-3	2	0		
2	x_1	1	-1	1	0	10	
0	S_2	0	1	-2	1	20	

Solution is not optimal.

		2	1	0	0		
C_B	Basic Variables	x_1	x_2	S_1	S_2	Solution	Minimum Ratio
$Z_j - C_j$ =		0	−3	2	0		
2	x_1	1	−1	1	0	10	10/
0	S_2	0	1	−2	1	20	20/1=20
$Z_j - C_j$ =		0	0	−4	3		
2	x_1	1	0	−1	1	30	30/-
1	x_2	0	1	−2	1	20	20/-

Unbounded solution