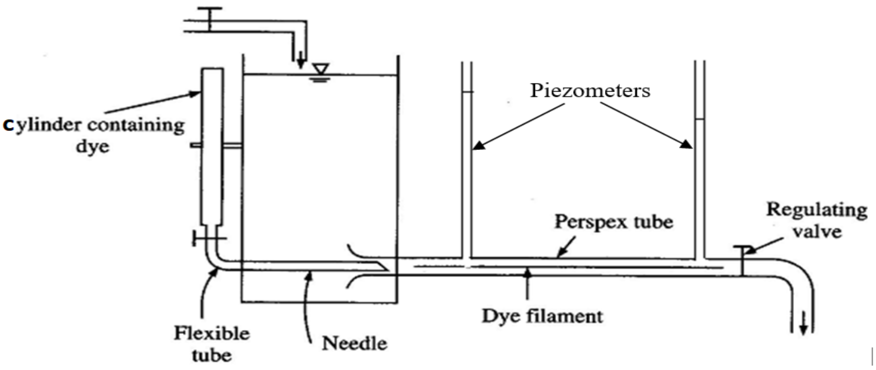
FLOW THROUGH PIPES

- Pipe is a closed conduit through which fluid flows under pressure.
- Pipes are used in water distribution networks, conveyance of gas, heating and cooling applications etc.

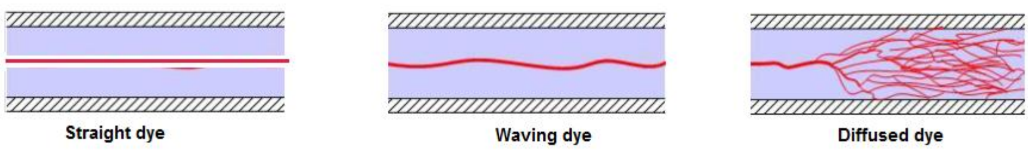
❖ Flow in a pipe can be either laminar or turbulent.

- Osborne Reynolds was the first who demonstrated that two types of flow exist and established a criteria by which the two flows can be distinguished.
- Reynolds injected dye as filament at the centre of a transparent tube carrying water and studied its characteristics.



Reynolds apparatus

- Velocity of water through the tube was controlled by a valve at the downstream end.
- At low flow velocities, dye filament would pass straight down the tube, not mixing with water and move so steadily that it hardly seemed to be in motion.
- Further opening of valve (increasing the velocity of flow) resulted in dye showing signs of irregularities and began to waver.
- Still further increase in velocity of flow resulted in dye completely diffusing over the entire cross-section of tube and mixing with water.



- First type of flow which occurred at low velocities (when dye remained in the form straight filament), particles of fluid are moving entirely in straight lines.
- Fluid may be considered to be moving in layers or laminae; this type of flow is referred to as laminar flow.
- When dye showed signs of irregularities and began to waver, shows that flow is no longer laminar but in transitional state.

- Second type of flow which occurred at high velocities (when dye completely diffused over the entire cross-section of tube) is called turbulent flow.
- In **TF,** paths of fluid particles are no longer orderly but random in nature
- In engineering practice, most of the fluid flows are turbulent.

√ Reynolds' experiments showed that velocity at which transition from laminar to turbulent flow takes place depends on the fluid density, fluid viscosity, flow velocity and the pipe diameter.

- Derived a dimensionless no. which represent the ratio of magnitude of inertia force in the fluid to the viscous force, known as Reynolds No. **Re.**

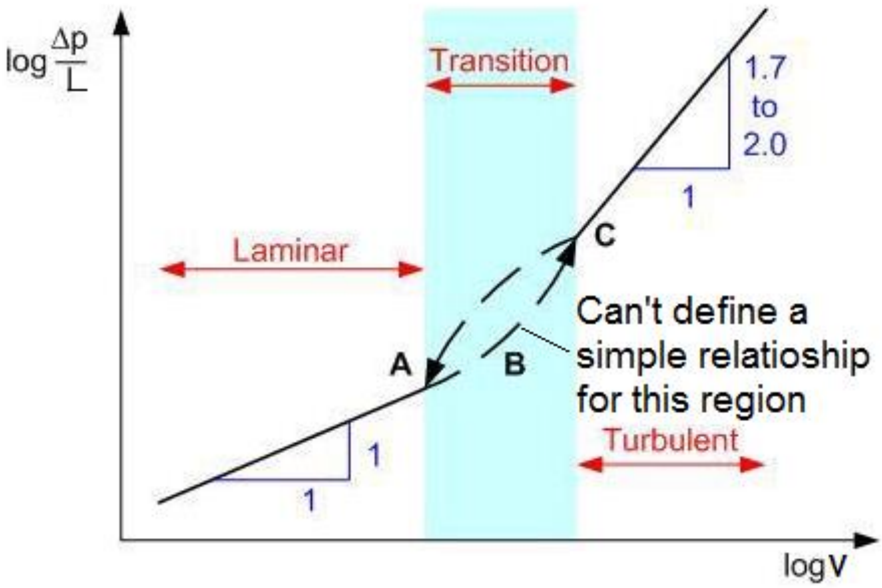
❖ **Re** represents fundamental characteristic of flow in which inertia and viscous forces are present.

- At high Reynolds Number $\Rightarrow$ Inertia forces predominate
- At low Reynolds Number $\Rightarrow$ Viscous forces predominate
- Laminar flow occurs at low velocities (low Re).
- Turbulent flow occurs at high velocities (high Re).
- Viscous forces dominate in laminar flow.
- Inertia forces dominate in turbulent flow.

Lower and Upper Critical Velocities

- Reynolds further conducted experiments to measure pressure drop (Δp) over a length of pipe (L) as a function of velocity (V).
- Results obtained from these experiments i.e. pressure drop unit length ($\Delta p/L$) and vel. (V) were plotted using log scales.

❖ For laminar flows, $(\Delta p/L) \propto V$ and for turbulent flows, $(\Delta p/L) \propto V^n$



- Point at which flow changes from laminar to turbulent (point **B**) is known as the upper critical velocity and the corresponding Re No. as the upper critical Reynolds no.
- When flow velocity is reduced, flow changes from turbulent to laminar at a slightly lower velocity (point **A**) known as the lower critical velocity and the corresponding Reynolds number as the lower critical Reynolds number.

✓ Upper critical Reynolds number is not a fixed quantity.

- Depends upon a number of factors such as initial disturbance to flow, shape of entry to the tube etc.

➢ In normal engineering practice, disturbances will always be present.

❖ Usual value of the upper critical **Re** no. is 2500 – 4000.

✓ Lower critical **Re no**. is well established.

- For smooth and long pipes, lower critical Re no. is approximately taken equal to 2300 and for rough pipes - 2000.

TYPES OF RESISTANCES AND LOSSES OF ENERGY

- Fluid flows through a pipe is subjected to two types of hydraulic resistances viz. viscous-frictional resistance and local resistances.
- Viscous resistance is between the fluid particles.
- Frictional resistance is between the fluid particles and the surface of pipe.
- Local resistances are due to change of velocity either in magnitude or direction or both.
- To overcome hydraulic resistances, some energy of the flow gets dissipated as heat energy i.e. there is loss of energy in the direction of flow.
- Viscous-frictional resistance is called major loss of energy.
- Local resistances are called minor losses of energy.

Observations

(i) In turbulent flow, viscous resistance is negligible i.e. major loss of energy in turbulent flow is due to friction only.

(ii) In long pipes, minor losses can be neglected as compared to the major loss.

(iii) In short pipes with number of fittings, minor losses are quite significant and therefore need to be considered.

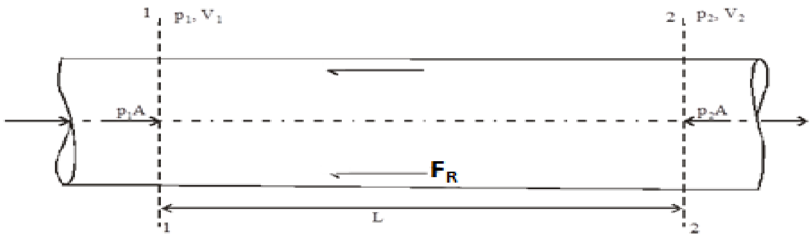
(iv) For flow through pipes, one need to determine the viscous-frictional resistance to flow as it is related to the head loss which is used to determine the power of pump (**$P = \gamma Q h_L$**).

(v) Major loss of energy can be calculated using Darcy Weisbach eq. and Hagen-Poiseuille eq.

❖ Darcy Weisbach equation is used for both laminar as well as turbulent flows whereas Hagen-Poiseuille equation is used for lamianr flow only.

DARCY-WEISBACH EQUATION FOR MAJOR HEAD LOSS

- Consider flow through a horizontal pipe of area **A** as shown in **Figure.**



- Applying Bernoulli's equation between **1-1** and **2-2**, to write

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2+h_f$$

- **h_f** is the head loss due to <u>viscous-frictional resistance to flow.</u>
- As diameter of pipe is same (**$V_1 = V_2$**) and pipe is horizontal (**$y_1 = y_2$**)

$$\therefore h_f=\left(\frac{p_1}{\rho g}-\frac{p_2}{\rho g}\right) \qquad \textbf{(1)}$$

❖ Head loss due to viscous-frictional resistance = Differential pressure head in the direction of flow

- Experimental studies show that viscous-frictional resistance **F_R** offered to the flowing fluid depends upon length of pipe **L,** wetted perimeter, **P** and velocity **V** i.e. **F_R** is proportional **L**, **P** and **V^n**, **n** is an index.

➢ **n** depends on the type of flow *viz.* laminar or turbulent.

- For laminar flow **n = 1** and for turbulent flow, ***n*** varies from 1.7 to 2.
- Darcy considered the value of **n = 2**

  ∴ Frictional resistance can be expressed as; $F_R = f' LPV^2$

➢ **f'** is a constant of proportionality

- Frictional resistance can also be expressed as; $F_R = (p_1A - p_2A)$

$$\therefore (p_1-p_2)A = f' LPV^2$$ (Dividing both sides by **ρg**)

$$\Rightarrow \left(\frac{p_1-p_2}{\rho g}\right)=\frac{f'}{\rho g}L\left(\frac{P}{A}\right)V^2$$

- (**A/P**) is called hydraulic radius or hydraulic mean depth, may be denoted by **R_h**.

$$\therefore h_f = \frac{f'}{\rho g} L\left(\frac{1}{R_h}\right)\frac{V^2}{2g}(2g) = \left(\frac{2f'}{\rho}\right)\left(\frac{L}{R_h}\right)\left(\frac{V^2}{2g}\right) \qquad \textbf{(Using Eq. 1)}$$

- Term **$(L/R_h)(V^2/2g)$** has the dimension of length and therefore **$(2f'/\rho)$** is a dimensionless quantity, can be replaced by another constant ***f***, known as coefficient of friction.

$$\therefore h_f = f\left(\frac{L}{R_h}\right)\left(\frac{V^2}{2g}\right)$$

- For flow in circular pipes, **$R_h = D/4$**, **D** is the dia. of pipe.

$$h_f = \frac{(4f)LV^2}{D(2g)} \quad \textbf{OR} \quad h_f = \frac{f_1 LV^2}{D(2g)} \qquad \textbf{(}\mathbf{f_1 = 4f}\textbf{, } \boldsymbol{f_1} \textbf{ is known as friction factor)}$$

❖ known as Darcy-Weisbach equation.

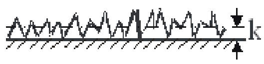
- can be expressed in different forms ; $h_f = \dfrac{fLQ^2}{3D^5} = \dfrac{f_1LQ^2}{12D^5}$
- Generally, **f_1** lies between 0.02 for new pipes to 0.04 for old pipes.
- **f** thus lies between 0.005 for new pipes to 0.01 for old pipes.

❖ No clear distinction in books between **f** and **f_1**.

Determination of friction factor

- To calculate **h_f**, one needs to first determine **f_1**
- Experimental studies indicate that **f_1** is not constant but depends upon Reynolds no. (**Re**) of flow and the type of pipe surface *i.e.* rough or smooth

$$\mathbf{f_1 = \phi\,(Re, k/D)}$$

➢ **k** is the average height of roughness projections, **ϕ** is any function 

- **(k/D)** is known as relative roughness.
- **(R/k)** is called relative smoothness, **R** is the radius of pipe.

f_1 for laminar and turbulent flows can be calculated using following empirical relations

<u>*Laminar flow*</u>: ***f_1*** depends only on **Re**

$$f_1 = \frac{64}{Re}, \qquad\qquad Re = \frac{\rho VD}{\mu}$$

❖ Obtained by equating Hagen-Poiseuille and Darcy-Weisbach equations for **h_f**.

<u>Turbulent flow</u>

❖ Analysed with respect to smooth and rough pipes.

<u>For smooth pipes</u>, **f_1** is a function of only **Re** and can be calculated using, $f_1 = \dfrac{0.316}{Re^{0.25}}$

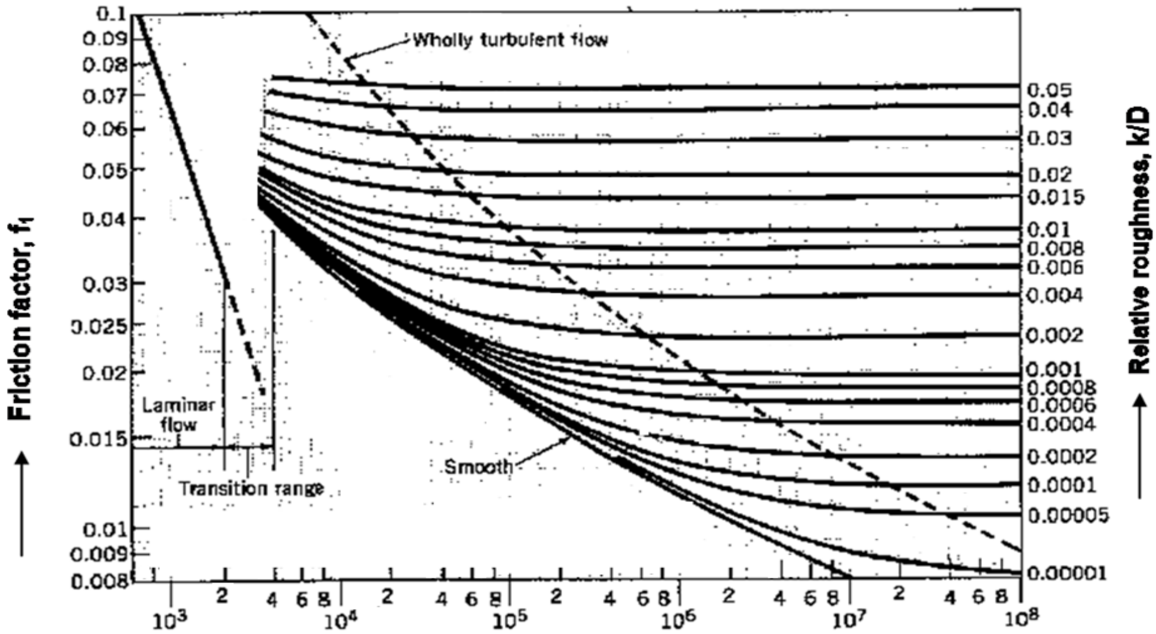
➢ known as Blasius equation.

- valid for Reynolds number varying from 4000 to 10^5
- can also be calculated using, $f_1 = 0.0032 + \frac{0.221}{Re^{0.237}}$ (Known as Nikuradse's equation)
- valid for Reynolds number varying from 50000 to 4×10^7

For rough pipes, f_1 depends only on the roughness of pipe

$$\frac{1}{\sqrt{f_1}} = 2\log\left(\frac{R}{k}\right) + 1.74$$

- Moody diagram can also be used to determine f_1



Friction factor as a function of Reynolds number and relative roughness for pipes – Moody diagram

Observation:

Hagen-Poiseuille equation for calculating lead loss is laminar flow is given by the expression:

$$h_f = \frac{32\mu VL}{\rho g D^2}$$ μ is the dynamic viscosity of liquid.

Problems:

Q1: Estimate the pressure drop in a 3 m long and 12 mm diameter pipe delivering 0.50 lpm of water. Given, density of water = 1000 kg/m^3 and viscosity of water = 1cP.

Solution: L = 3 m, **D =** 12 mm, **Q =** 0.50 lpm, ρ = 1000 kg/m^3, μ = 1cP

$$Q = 0.50\,\text{lpm} = \frac{0.50 \times 10^{-3}}{60} = 8.33 \times 10^{-6}\ \text{m}^3/\text{s}$$

$$\therefore V = \frac{Q}{A} = \frac{8.33 \times 10^{-6}}{\frac{\pi}{4}(0.012)^2} = 0.074\ \text{m/s}$$

- To determine the nature of flow

$$Re = \frac{\rho V D}{\mu} = \frac{1000 \times 0.074 \times 0.012}{0.001} = 884.4 \qquad < 2000$$

$\therefore$ Flow is laminar.

$$\therefore f_1 = \frac{64}{Re} \qquad \Rightarrow f_1 = \frac{64}{884.4} = 0.072$$

- Head loss is given by:

$$h_f = \frac{fLV^2}{D(2g)} \qquad \Rightarrow h_f = \frac{0.072 \times 3 \times 0.074^2}{0.012 \times 2 \times 9.81} = 5.02 \times 10^{-3}\ \text{m}$$

- Also,

$$h_f = \left(\frac{p_1 - p_2}{\rho g}\right) \qquad \Rightarrow (p_1 - p_2) = h_f \times \rho g = 5.02 \times 10^{-3} \times 9810 = 49.25\ \text{N/m}^2 \quad \text{(Ans)}$$

Observation:

- Hagen-Poiseuille equation can also be used, $h_f = \dfrac{32\mu VL}{\rho g D^2}$

Q2: Oil of specific gravity 0.80 and viscosity 0.3 Stoke is flowing through a pipe at the rate of 0.45 m^3/s. The length and diameter of the pipe are 800 m and 300 mm, respectively. Determine the head loss due to friction and the power required to maintain the flow.

Solution:

SG = 0.80, **ν** = 0.3 Stoke, **Q** = 0.45 m^3/s, **L** = 800 m, **D** = 300 mm

$$V = \frac{0.45}{\pi/4(0.3)^2} = 6.4\ m/s$$

<u>To determine the nature of flow</u>

Reynolds number of flow, $Re = \frac{VD}{\nu} = \frac{6.4 \times 0.3}{0.3 \times 10^{-4}} = 64000 \quad > 4000$

∴ Flow in the pipe is smooth turbulent and f_1 can be calculated using:

$$f_1 = \frac{0.316}{Re^{0.25}}$$ •Valid for; $4000 < Re < 10^5$

Or $$f_1 = 0.0032 + \frac{0.221}{Re^{0.237}}$$ • Valid for; $50000 < Re < 4 \times 10^7$

- Calculate f_1 using the first equation:

$$\therefore f_1 = 0.0198$$

$$\therefore h_f = \frac{f_1 L V^2}{D(2g)} \qquad \Rightarrow h_f = \frac{0.0198 \times 800 \times (6.4)^2}{0.3(2 \times 9.81)} = 110.23\ m \quad \text{(Ans)}$$

- Power required to maintain the flow is given by

$$P = \gamma Q h_f \qquad = 0.8 \times 9810 \times 0.45 \times 110.23 = 389.3\ kW$$

DETERMINATION OF MINOR LOSSES OF ENERGY/HEAD

- Minor losses are the localised losses frequently occur in pipeline systems.
- Most of these losses are caused by the sudden changes in velocity, due to separation of flow/shear layer from the wall of pipe.
- Also, sudden changes in velocity generate large scale turbulence due to the formation of eddies or vortices, resulting in the dissipation/loss of energy as heat energy.

❖ Minor losses are also known as form losses.

- Found to vary as the square of mean velocity of flow
- Generally expressed as a function of velocity head i.e. $h_L = K_L \dfrac{V^2}{2g}$

➢ h_L is the loss of energy per unit weight/head loss

➢ K_L is called loss coefficient.

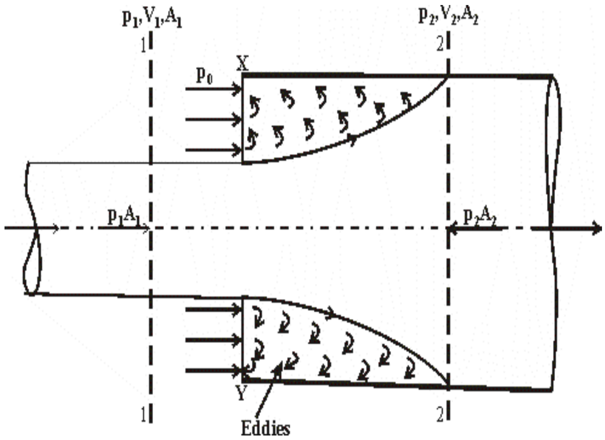
❖ K_L is constant for a given device at high Reynolds Numbers.

Main physical causes of minor losses are

- Sudden expansion/increase in diameter of pipe.
- Sudden contraction/decrease in diameter of pipe.
- Entry to a pipe
- Exit from the pipe
- Pipe fittings (bends, elbows, tees, valves, sockets)

Loss due to sudden expansion of a pipe section

- Consider the effect on the flow in a pipe when pipe diameter suddenly increases.



- Flow is suddenly decelerated resulting in the formation of eddies, causing loss of energy due to separation of flow from boundary.
- Loss of energy can be determined using Bernoulli's, continuity and momentum eqs.
- Consider two sections **1-1** and **2-2** (**1-1** is just before the expansion and **2-2** after flow stabilises again)
- Applying Bernoulli's equation between **1-1** and **2-2**, to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2 + (h_L)_{exp}$$

➢ $(h_L)_{exp}$ is the head loss due to sudden expansion

$$\therefore (h_L)_{exp} = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \textbf{(1)}$$

- As $V_1 > V_2$, this change in velocity gives rise to a change in momentum of fluid, resulting in <u>force</u> acting on the annular face **XY** due to eddies.
- Eddies will exert pressure of intensity p_o (say) on **XY** *i.e.* on the area $(A_2 - A_1)$ a shown in **Figure.** Similarly, face **XY** will exert pressure on the eddies in the direction of flow.
- Write momentum equation in the direction of flow, to get

$$p_o(A_2 - A_1) + p_1A_1 - p_2A_2 = \rho Q(V_2 - V_1)$$

- Experimentally, it has been found that $p_o \sim p_1$ (radial accelerations over the annular face **XY** are very small)
- Simplify, to get $(p_1 - p_2)A_2 = \rho Q(V_2 - V_1)$;
- Eliminate Q using continuity equation, $Q = A_1V_1 = A_2V_2$

$$\frac{(p_1 - p_2)}{\rho g} = \frac{V_2}{g}(V_2 - V_1);$$ Put in Eq. (1)

$$\therefore (h_L)_{exp} = \frac{V_2}{g}(V_2 - V_1) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

- Simplifying, to get; $$(h_L)_{exp} = \frac{(V_1 - V_2)^2}{2g}$$

❖ Required expression, known as Borda's Carnot equation.

- can also be expressed as: $$(h_L)_{exp} = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2 = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

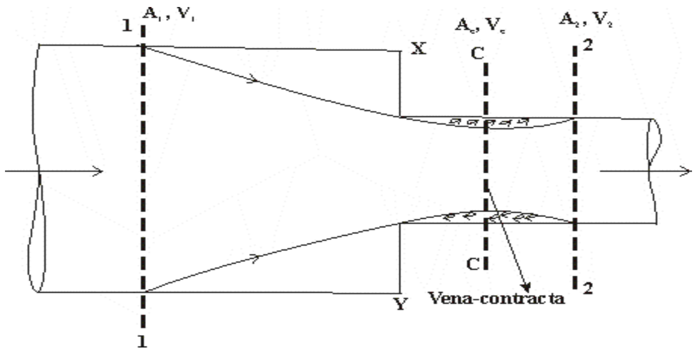
$$\Rightarrow (h_L)_{exp} = (K_L)_{exp}\frac{V_1^2}{2g}$$

- $(K_L)_{exp}$ is the loss coefficient of expansion = $(1 - A_1/A_2)^2$
- Value of $(K_L)_{exp}$ depends upon (D_1/D_2)

D_1/D_2	$\cong 0$	0.5	1.0
$(K_L)_{exp}$	1.0	0.25	0

Loss due to sudden contraction of pipe Section

- Consider flow through a pipe which has a sudden contraction



- As liquid flows from larger pipe to the smaller pipe, it starts converging to a minimum area, thereby forming a vena-contracta just d/s of **XY,** after which flow widens to fill the pipe.
- Consider three sections **1-1**, **C-C** and **2-2** as shown.
- Geometrically, sudden contraction is the reverse of sudden expansion. However, loss of head cannot be determined just by applying Carnot equation.

This is due to

(i) Momentum equation cannot be applied between **1-1** and **2-2**.

- Acceleration of fluid into the contraction section has unknown effects on the pressure distribution u/s (Face **XY**).

(ii) Flow being accelerated, no loss of energy occurs between **1-1** & **C-C.**

- Eddies are formed between **C-C** and **2-2** due to expansion of flow beyond the vena contracta, which cause loss of energy and the flow pattern between **C-C** and **2-2** is similar to that of sudden expansion.
- Apply expansion Eq. between **C-C** and **2-2,** to write

$$(h_L)_{con} = \frac{(V_c - V_2)^2}{2g}$$

➢ **$(h_L)_{con}$** is the head loss due to sudden contraction

- Continuity equation - $A_cV_c = A_2V_2$; $A_c = C_cA_2$ $\therefore V_c = \dfrac{V_2}{C_c}$

  ➢ **C_c** is the coefficient of contraction

$$(h_L)_{con} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = (K_L)_{con}\frac{V_2^2}{2g}$$

➢ **$(K_L)_{con}$** is the loss coefficient of contraction = **$[(1/C_c) - 1]^2$**

- **$(K_L)_{con}$** depends upon **(D_2/D_1)**

D_2/D_1	$\cong 0$	0.5	1.0
C_c	0.586	0.671	1.0
$(K_L)_{conc}$	0.50	0.24	0

Loss at pipe outlet/exit

- Situation is encountered when a pipe exits to a large reservoir or discharging freely.

- Energy is lost in the reservoir or in the form of a free jet.
- Flow pattern in both cases is similar to that of a sudden expansion

$$\therefore (h_L)_o = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

- As **A_2>> A_1**, $\quad \therefore (A_1/A_2) \sim 0 \;\therefore (h_L)_o = \dfrac{V_1^2}{2g}$
- In general, **V_1 = V, V** is the mean velocity of flow in the pipe

Loss at Pipe Entrance

- Liquid enters a pipe from a tank or reservoir.
- Flow pattern at the pipe entrance is similar to the case of a sudden contraction

$$\therefore (h_L)_i = (K_L)_i \frac{V^2}{2g}$$

- **$(K_L)_i$** is known as inlet loss coefficient whose value depends on the shape of entrance.
- For sharp edged entrance (**Fig. 1**); **$(K_L)_i$ = 0.50,** (**$A_2/A_1 \cong 0$)**
- For bell shape or rounded entrance (**Fig. 2**); **$(K_L)_i$ = 0.05 to 0**

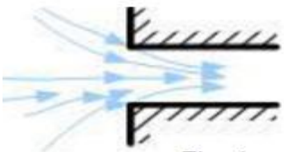


Fig. 1

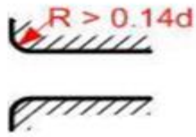


Fig. 2

Losses in fittings (Bends, Tees, Elbows, **Sockets, Valves**)

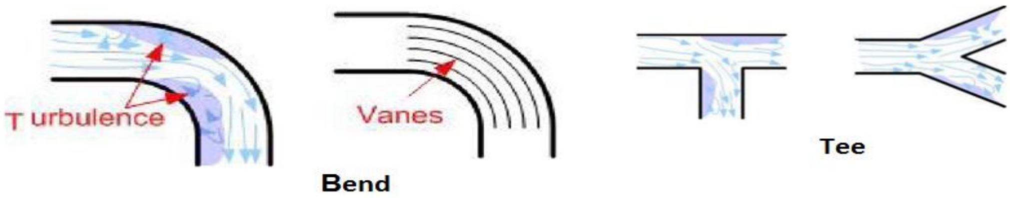
- Bends, tees, elbows - provided in the pipe for changing the direction of flow
  - ❖ Change in direction of flow causes loss of energy.

<u>Bends</u> - loss takes place due to the separation of flow from the surface of bend and consequent formation of eddies

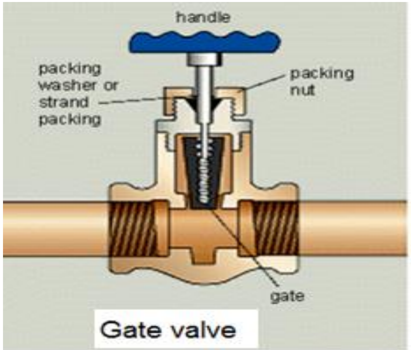
<u>Fittings</u> - due to the rough interior surfaces of fittings, which produce turbulence, resulting in the formation of eddies.

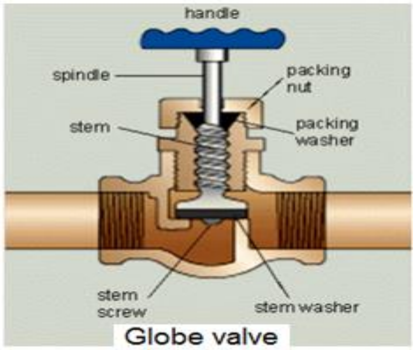
$$(h_L)_b = (K_L)_b \frac{V^2}{2g}, \qquad h_L = K_L \frac{V^2}{2g}$$

- **$(K_L)_b$** is the loss coefficient of bend - depends on the angle of bend, radius of curvature, diameter of bend etc.
- **K_L** is the loss coefficient of a particular fitting.
- Bend losses can be minimised by the use of large radii and by using vanes.



<u>Valves</u> - provided for regulating the flow (gate valve and globe valve)



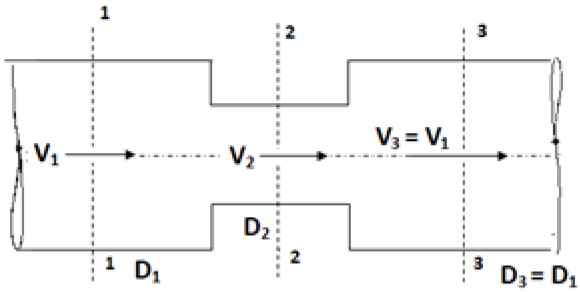


Loss coefficient K_L for some fittings

Type of fittings/valves	K_L
90° Elbow	0.90
45° Elbow	0.35 to 0.45
Bend	2.20
Globe valve fully open	10
Gate valve fully open	0.19
Gate valve ¾ open	1.15
Gate valve ½ open	5.60
Gate valve ¼ open	24

Q: When a sudden contraction is introduced in a horizontal pipe from 500 mm diameter to 250 mm diameter, pressure changes from 105 to 69 kN/m^2. If $\mathbf{C_c}$ = 0.65, calculate discharge of water. Following this, if there is a sudden enlargement from 250 mm to 500 mm and if pressure at 250 mm section is same, what is the pressure at the other section?

Solution:



- $\mathbf{D_1}$ = 500 mm, $\mathbf{D_2}$ = 250 mm, $\mathbf{p_1}$ = 105 kN/m^2, $\mathbf{p_2}$ = 69 kN/m^2
- Let **1-1**, **2-2** and **3-3** are the three sections as shown in **Figure.**
- From continuity equation

$$A_1V_1 = A_2V_2 \Rightarrow \frac{\pi}{4}(0.5)^2 \times V_1 = \frac{\pi}{4}(0.25)^2 \times V_2 \quad \Rightarrow V_2 = 4V_1$$

- Apply Bernoulli's eq. between **1-1** and **2-2,** to write

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g} \qquad \Rightarrow \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}\left[1 + \left(\frac{1}{C_c} - 1\right)^2\right]$$

- Substituting the values, to get

$$\frac{105 \times 10^3}{9810} + \frac{V_1^2}{2 \times 9.81} = \frac{69 \times 10^3}{9810} + \frac{(4V_1)^2}{2 \times 9.81}\left[1 + \left(\frac{1}{0.65} - 1\right)^2\right]$$

- Solve for $\mathbf{V_1}$, to get
- $\mathbf{V_1}$ = 1.915 m/s

$$\therefore Q = A_1V_1 \Rightarrow Q = \frac{\pi}{4}(0.5)^2 \times 1.915 = 0.376 \text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

- Applying Bernoulli's eq. between 2-2 and 3-3 , to write

$$\frac{p_2}{\rho g}+\frac{V_2^2}{2g}=\frac{p_3}{\rho g}+\frac{V_3^2}{2g}+\frac{(V_2-V_3)^2}{2g}$$

- Here, $p_2 = 69\ \text{kN/m}^2, V_2 = 4V_1 = 7.66\ \text{m/s}, V_3 = V_1 = 1.915\ \text{m/s}$
- Substituting the values, to get

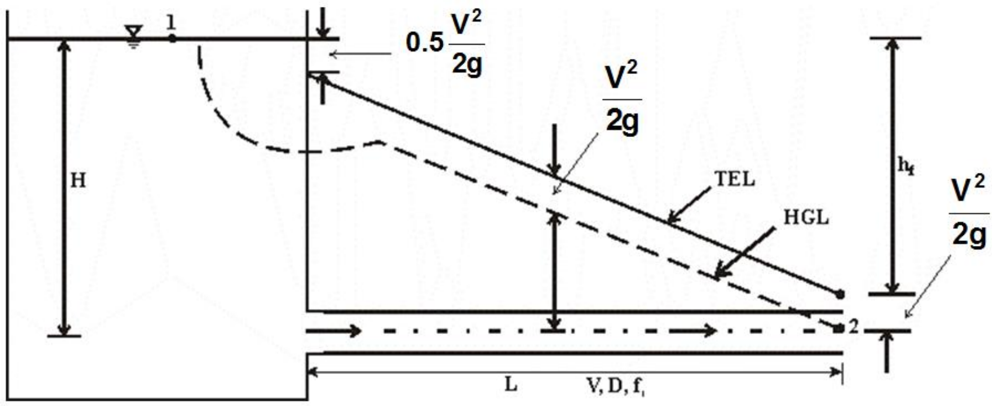
$$\frac{69\times10^3}{9810}+\frac{(7.66)^2}{2\times9.81}=\frac{p_3}{9810}+\frac{(1.915)^2}{2\times9.81}+\left(\frac{7.66-1.915}{2\times9.81}\right)^2$$

- Solve for p_3, to get
- $p_3 = 80006\ \text{N/m}^2 = 80.1\ \text{kN/m}^2$

TOTAL ENERGY LINE (TEL) AND HYDRAULIC GRADIENT LINE (HGL)

- **TEL** is the graphical representation of longitudinal variation in total head.
- **HGL** - representation of longitudinal variation in piezometric head.
- ✓ If total heads and piezometric heads at salient sections of a pipe flow (to scale) are plotted as ordinates and joined by straight lines, known as **TEL** and **HGL**, respectively.
- Total head at any section is the sum of pressure head, velocity head and datum head.
- Piezometric head is the sum of pressure head and datum head.
- Thus, for a pipe of constant dia. of, **TEL** will be parallel to HGL and the vertical distance between TEL and HGL is equal to velocity head.

HGL and TEL for horizontal pipe carrying water from a reservoir and discharging freely



Apply **B. E.** between **1** and **2 (**datum through **B),**

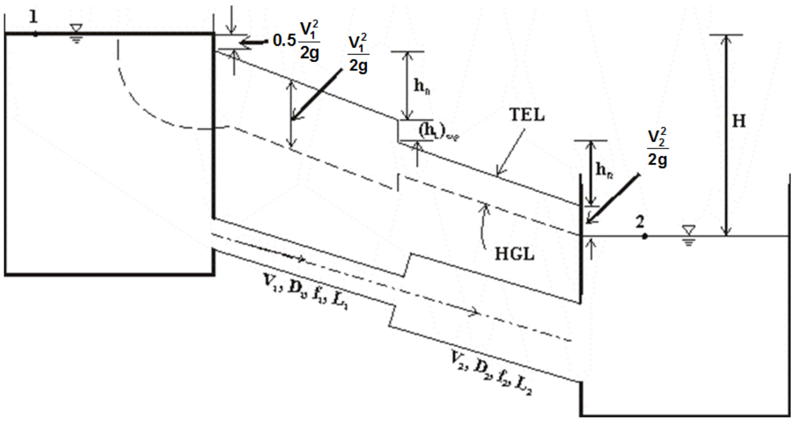
$$0+0+H=0+0+0+0.5\frac{V^2}{2g}+\frac{f_1LV^2}{D(2g)}+\frac{V^2}{2g}$$

<u>HGL</u>: At the entrance of pipe, pressure head is equal to the height of liquid in the tank from the centre of pipe i.e.

- **HGL** should start from the top surface level
- At the start **HGL** is not well defined
- As liquid just enters the pipe, <u>vena-contracta</u> is formed and a sudden drop in pressure takes place in this portion.
- **HGL** in this portion is thus shown by a dotted curve.
- At exit, pressure is atmospheric and thus **HGL** coincides with the centre of pipe

<u>TEL</u>: At entrance, inlet loss (**h_i = 0.5 $V^2/2g$**) occurs and hence **TEL** at this section will lie at a vertical distance equal to the inlet loss below the liquid surface in the reservoir

- At exit, there occurs exit loss (**h_e = $V^2/2g$**) and therefore, **TEL** at this section will lie at a vertical distance equal to exit loss
- Vertical distance from the point where **TEL** starts sloping down, up to any section is **h_f** and (**h_f/L**) is the slope of **TEL**



HGL and TEL for an inclined pipe connecting two reservoirs

Observations:

(i) If piezometers are installed along the pipeline of constant diameter, then water will rise in each piezometer to some height corresponding to the pressure at that section. Due to viscous-friction, the pressure will decrease from section to section in the direction of flow i.e. water levels in the piezometers will fall in the direction of flow. The line joining the water levels in each piezometer is **HGL**.

(ii) **TEL** is drawn parallel to **HGL**, with vertical distance between the two lines equal to (**$V^2/2g$**).

(iii) **TEL** always slopes <u>down</u> in the direction of flow whereas **HGL** may <u>rise or fall</u> depending upon the variation of pressure due to change of velocity in the direction of flow.

(iv) Sudden rise in **HGL** will occur when flow velocity suddenly reduces i.e. diameter of pipe suddenly expands.

(v) Sudden fall in **HGL** will take place when flow velocity suddenly increases *i.e.* diameter of pipe suddenly reduces.

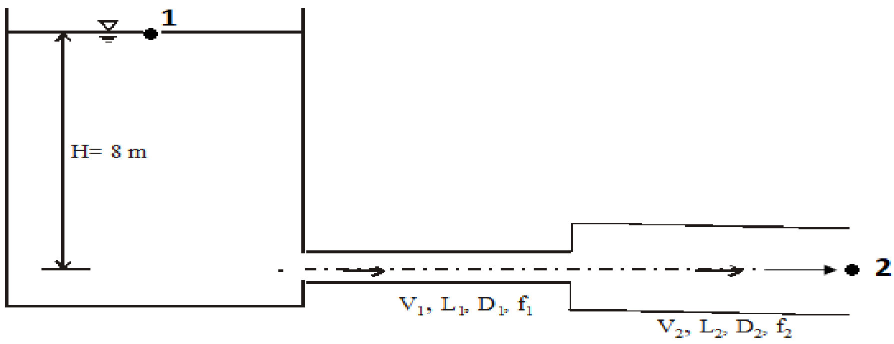
(vi) Sudden rise in both **TEL** and **HGL** will occur if mechanical energy is supplied to the liquid - a pump is installed in the flow system.

(vii) Sudden drop in both **TEL** and **HGL** will occur, when energy is extracted from the liquid - a turbine is installed in the flow system.

Q: A horizontal pipe 40 m long is connected to a water tank at one end and discharges freely into atmosphere at the other end. For the first 25 m of length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of pipe. Calculate the rate of flow and plot HGL and TEL. The friction factor for both sections of pipe is 0.04.

Solution:

- $\mathbf{D_1}$ = 150 mm, $\mathbf{D_2}$ = 300 mm, $\mathbf{L_1}$ = 25 mm, $\mathbf{L_2}$ = 15 m, $f_1 = f_2 = 0.04$



- From continuity equation,

$$A_1V_1 = A_2V_2 \quad \Rightarrow \frac{\pi}{4}(0.15)^2 \times V_1 = \frac{\pi}{4}(0.3)^2 \times V_2 \quad \Rightarrow V_1 = 4V_2$$

- Apply Bernoulli's eq. between points **1** and **2** (datum through **2**), to write

$$H = 0.5\frac{V_1^2}{2g} + \frac{f_1L_1V_1^2}{D_1(2g)} + \frac{(V_1 - V_2)^2}{2g} + \frac{f_2L_2V_2^2}{D_2(2g)} + \frac{V_2^2}{2g}$$

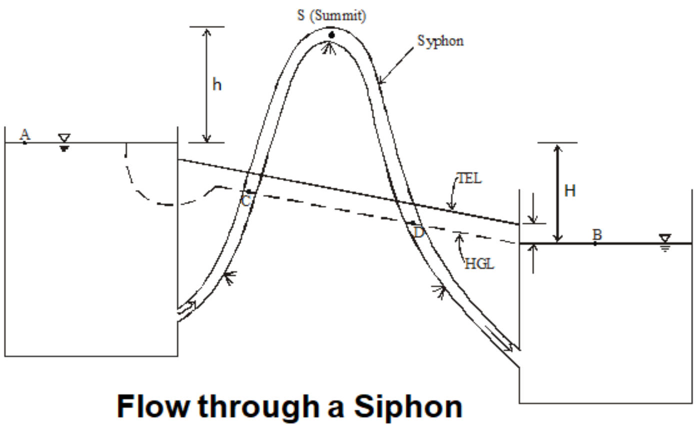
- Substitute the values and solve for $\mathbf{V_2}$, to get

$$V_2 = 1.113 \text{ m/s}$$

$$\therefore Q = A_2V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.0787 \text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

FLOW THROUGH A SIPHON

- A siphon is a long bent pipe which is used to transfer liquid from a reservoir at higher elevation to another reservoir at a lower elevation, when the two reservoirs are separated by a high level ground or a hill as shown in **Figure**.



Flow through a Siphon

- Another use of siphon is to take out liquid from a tank, which is not having any outlet.
- The highest point of siphon is known as summit (**S**).
- **HGL** and **TEL** for a siphon may be drawn in the same manner as in the case of an ordinary pipe connecting the two reservoirs.
- **HGL** and **TEL** are thus shown in **Figure**.
- Apply Bernoulli's equation between points **A** and **B** (assuming datum is passing through **B)**, to write

$$H = 0.5\frac{V^2}{2g} + \frac{f_1LV^2}{D(2g)} + \frac{V^2}{2g} \qquad \textbf{(1)}$$

- **H** is the difference of liquid levels between the two reservoirs, **V** is the mean velocity of flow in siphon, **D** is the diameter and f_1 is the friction factor and **L** is the length of siphon pipe.

- If Bernoulli's equation is applied between points **A** and **S** (assuming datum is passing through **A)**, to write

$$0 = \frac{p_s}{\rho g} + \frac{V^2}{2g} + h + 0.5\frac{V^2}{2g} + \frac{f_1 L_1 V^2}{D(2g)} \qquad (2)$$

- p_s is the pressure at summit, **h** is the height of summit from upper reservoir and L_1 is the length of siphon pipe from inlet up to summit.

- Using Eqs. (1) and (2), discharge through the siphon and pressure at summit can be calculated.

WORKING OF SYPHON

- The working of siphon can be explained by making observations from **HGL**:
- **HGL** cuts siphon at points **C** and **D**. Thus, some part of siphon is above **HGL** and the some part is below **HGL**. In other words, in siphon flow, a part of pipe is above **HGL.**
- Since, vertical distance between **HGL** and the centre of pipe at any section represents pressure head at that section.
- Thus, part of siphon which is above **HGL** (*i.e.* part above points **C** and **D**), pressure is below atmospheric (-ve) and the part below points **C** and **D**, the pressure is above atmospheric (+ve).
- Pressure at points **C** and **D** is equal to the atmospheric pressure.
- From the above observations, it is thus concluded that as liquid flows through the siphon, pressure changes from positive to zero and then it becomes negative at the summit *i.e.*
- pressure decreases from inlet to the summit of siphon.
- This enables the liquid to flow from inlet to summit and afterward the flow takes place under gravity.
- It may be noted that summit has maximum negative pressure.

Observation:

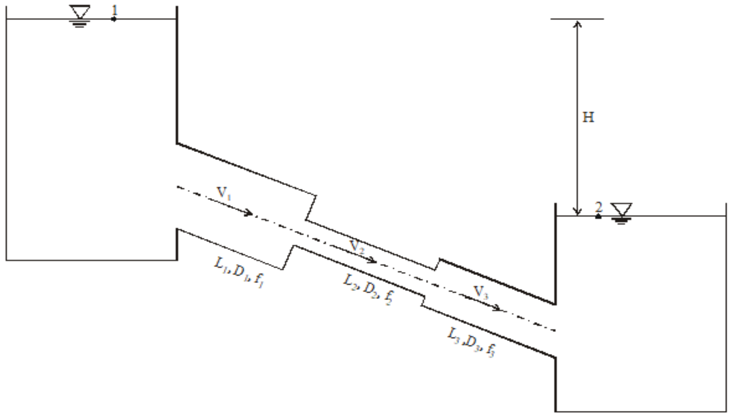
- Theoretically, for water the pressure at summit may be reduced to **- 10.3 m** of water or (**0 m** of water absolute) but in actual practice, this pressure is allowed to reduce <u>upto -</u> <u>**7.8 m**</u> of water or **2.5 m** of water absolute.

- This is because if pressure is allowed to reduce further; it may approach the vapour pressure of liquid.
- As a result, water vaporizes and gases dissolved in it are liberated and get deposited near the summit.
- These gases may block the flow leading to either discontinuity of flow or its complete stoppage. This phenomenon is known as **cavitation** and should be avoided.

FLOW THROUGH PIPES IN SERIES AND PARALLEL

Flow Through Pipes in Series

- When pipes of different lengths, diameters and materials are connected end to end, then the pipes are said to be connected in series.
- Pipes in series are also known as a compound pipe.
- Consider flow through a compound pipe connected between two reservoirs as shown in **Figure**.



Flow through pipes in series

- Applying B.E. between points **1** and **2** (datum through **2**) as shown.

$$H = 0.5\frac{V_1^2}{2g} + \frac{f_1L_1V_1^2}{D_1(2g)} + \left(\frac{1}{C_c} - 1\right)^2\frac{V_2^2}{2g} + \frac{f_2L_2V_2^2}{D_2(2g)} + \frac{(V_2 - V_3)^2}{2g} + \frac{f_3L_3V_3^2}{D_3(2g)} + \frac{V_3^2}{2g} \qquad (1)$$

$\therefore$ Difference of levels between two reservoirs = Total head loss

- If minor losses are neglected, then Eq. (1) reduces to:

$$H = \left(\frac{f_1L_1V_1^2}{D_1} + \frac{f_2L_2V_2^2}{D_2} + \frac{f_3L_3V_3^2}{D_3}\right)\left(\frac{1}{2g}\right) \qquad (2)$$

- Also, from continuity equation $Q = A_1V_1 = A_2V_2 = A_3V_3$

$$\Rightarrow V_1 = \frac{Q}{\frac{\pi}{4}D_1^2}, V_2 = \frac{Q}{\frac{\pi}{4}D_2^2} \text{ and } V_3 = \frac{Q}{\frac{\pi}{4}D_3^2}$$

- Substituting these values of V_1, V_2 and V_3 in Eq. (2), to get

$$H = \frac{Q^2}{(\pi/4)^2}\left(\frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5}\right)\left(\frac{1}{2g}\right) \qquad (3)$$

- If pipes are of the same material, then $f_1 = f_2 = f_3 = f_1$ (say)

$$\therefore H = \frac{f_1}{2g}\frac{Q^2}{(\pi/4)^2}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}\right)$$

- Required equation.

Equivalent pipe in series

- If a compound pipe is replaced by a pipe of uniform diameter, then this pipe of uniform diameter is known as equivalent pipe.
- ❖ Equivalent pipe would carry the same discharge as that of a compound pipe if head loss is same in both the pipes. In other words, two pipe systems are said to be equivalent when same head loss produces the same discharge in both the systems.
- ✓ Generally, it is required to determine the diameter of equivalent pipe which may be determined as follows:
- If D_e is the diameter, L_e is the length and f_e is the friction factor of equivalent pipe, then

$$H = \frac{f_eL_eV^2}{D_e(2g)} = \frac{Q^2}{(\pi/4)^2}\left(\frac{f_eL_e}{D_e^5}\right)\left(\frac{1}{2g}\right) \qquad (4)$$

- Equating Eqs. (3) and (4) under the condition of same head loss and simplify, to get

$$\frac{f_eL_e}{D_e^5} = \left(\frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5}\right)$$

- This eq. has three variables viz. D_e, L_e and f_e. Knowing any two, the third can be calculated.
- If equivalent pipe and the compound pipes are of the same material, then $f_e = f_1 = f_2 = f_3$

$$\therefore \frac{L_e}{D_e^5} = \left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + --- \right)$$

- This Eq. known as Dupit's equation.
- This eq. has two variables viz. $\mathbf{D_e}$ and $\mathbf{L_e}$. Knowing any one, the second can be calculated.

Observation:

- Minor losses due to fittings can also be expressed in terms of equivalent length ($\mathbf{L_e}$) which is the length of an unobstructed pipe having loss of head due to friction equal to the sum of loss of head due to various fittings i.e.

$$\frac{f_e L_e V^2}{D_e(2g)} = \Sigma K_L \frac{V^2}{2g} \qquad \Rightarrow L_e = \Sigma K_L \frac{D_e}{f_e}$$

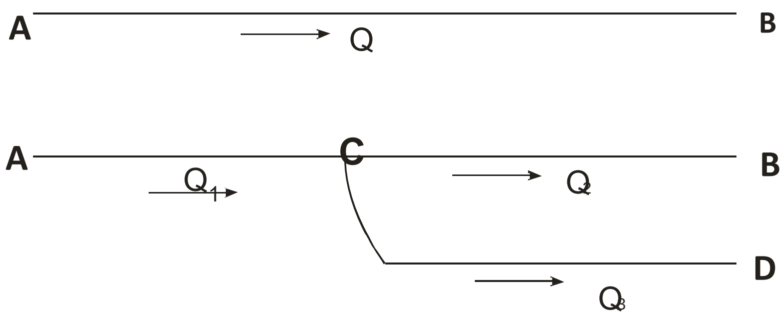
- Equivalent length due to sudden expansion and sudden contraction can be calculated using:

$$\frac{f_e L_e V_e^2}{D_e(2g)} = \frac{(V_1 - V_2)^2}{2g} \qquad \text{and} \qquad \frac{f_e L_e V_e^2}{D_e(2g)} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

❖ The total length of equivalent pipe is the sum of all the equivalents lengths.

Pipes in Parallel

- When a main pipe divides into two or more parallel pipes, the pipes are said to be in parallel.
- Pipes are connected in parallel for increasing discharge passing through the main pipe.



- As shown in **Figure**, **AB** is the initial main pipe carrying discharge **Q**.
- For increasing the discharge, a pipe **CD** is laid parallel to the pipe **AB** for some length and connected to it.
- If $\mathbf{Q_1}$ is the new discharge in pipe **AB**, then $\mathbf{Q_1 > Q}$

<u>To find increase in discharge</u>

- Continuity equation, $Q_1 = Q_2 + Q_3$
- Flow in parallel pipes **CB** and **CD** takes place due to the difference of heads at the inlet and outlet of these pipes (head loss in parallel pipes is same) i.e.
  Head loss/drop in pipe **CB** = Head loss/drop in pipe **CD**
- Neglecting minor losses, to get

$$(h_f)_{CB} = (h_f)_{CD}$$

- Further, total head loss = Head loss in **AC** + Head loss in pipe **CB** or **CD**
- Using the above Eqs., increase in discharge can be determined.

Observation:

- Head loss for pipes in series is equal to the sum of the head losses in all the pipes whereas head loss in parallel pipes is same.

Equivalent pipe in parallel

- Consider a set of pipes **(D_1, L_1, f_1)**, (D_2, L_2, f_2) and **(D_3, L_3, f_3)** ------ are connected in parallel between the two points.
- If an equivalent pipe, **(D_e, L_e, f_e)** is required to replace this set of parallel pipes.
- As, head loss is equal in the parallel pipes.

$$\therefore h_f = \frac{f_1 L_1 V_1^2}{D_1(2g)} = \frac{f_2 L_2 V_2^2}{D_2(2g)} = \frac{f_3 L_3 V_3^2}{D_3(2g)} \qquad (1)$$

- Also, Total discharge = Sum of discharges in the parallel pipes

$$\therefore Q = Q_1 + Q_2 + Q_3 \qquad (2)$$

- From **Eq. (1)**

$$V_1 = \left(\frac{D_1}{f_1 L_1}\right)^{\frac{1}{2}} \sqrt{2gh_f}, \qquad Q_1 = \frac{\pi}{4} D_1^2 V_1, \qquad \therefore Q_1 = \frac{\pi}{4}\left(\frac{D_1^5}{f_1 L_1}\right)^{\frac{1}{2}} \sqrt{2gh_f}$$

- Similarly,

$$Q_2 = \frac{\pi}{4}\left(\frac{D_2^5}{f_2 L_2}\right)^{\frac{1}{2}} \sqrt{2gh_f} \qquad \text{and} \qquad Q_3 = \frac{\pi}{4}\left(\frac{D_3^5}{f_3 L_3}\right)^{\frac{1}{2}} \sqrt{2gh_f}$$

- For equivalent pipe;

$$Q = \frac{\pi}{4} D_e^2 V_e, \qquad \text{and} \qquad h_f = \frac{f_e L_e V_e^2}{D_e (2g)}$$

$$\therefore Q = \frac{\pi}{4}\left(\frac{D_e^5}{f_e L_e}\right)^{\frac{1}{2}} \sqrt{2gh_f}$$

$\therefore$ From Eq. (2) [Substituting **Q, Q₁, Q₂** and **Q₃**] , to get

$$\left(\frac{D_e^5}{f_e L_e}\right)^{\frac{1}{2}} = \left(\frac{D_1^5}{f_1 L_1}\right)^{\frac{1}{2}} + \left(\frac{D_2^5}{f_2 L_2}\right)^{\frac{1}{2}} + \left(\frac{D_3^5}{f_3 L_3}\right)^{\frac{1}{2}}$$

- This is the required expression.

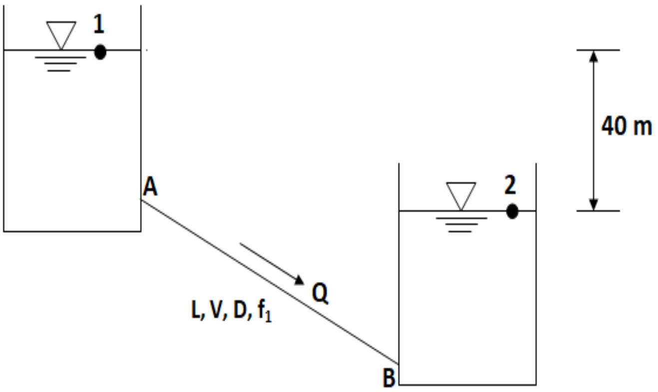
Observation:

- If equivalent pipe and the parallel pipes are of the same material, then $f_e = f_1 = f_2 = f_3$

$$\therefore \left(\frac{D_e^5}{L_e}\right)^{\frac{1}{2}} = \left(\frac{D_1^5}{L_1}\right)^{\frac{1}{2}} + \left(\frac{D_2^5}{L_2}\right)^{\frac{1}{2}} + \left(\frac{D_3^5}{L_3}\right)^{\frac{1}{2}}$$

Problems:

Q1: A pipeline of diameter 250 mm and length 5 km is laid between two reservoirs having 40 m difference in water levels. In order to increase the discharge, an additional 2.5 km long and 250 mm diameter pipe line is laid parallel from the first reservoir to the mid-point of the original pipe. Determine the percentage increase in discharge due to the installation of new pipe. Given friction factor = 0.025 for both the pipes. Neglect minor losses.



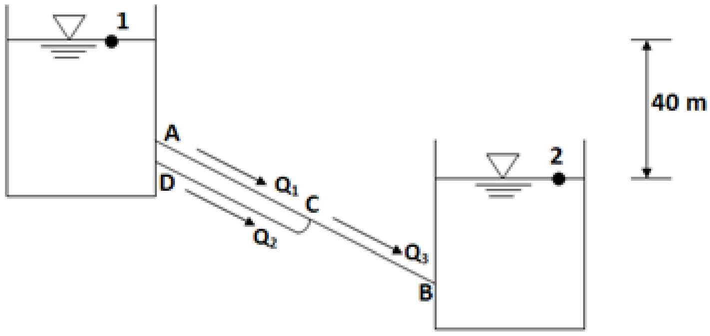
Solution:

- <u>Initial case (First case)</u>
  - Apply Bernoulli's eq. between **1** and **2** (datum through **2**), to get

$$0 + 0 + 40 = 0 + 0 + \frac{f_1 L V^2}{D(2g)} \qquad \Rightarrow 40 = \frac{0.025 \times 5000 \times V^2}{0.25(2 \times 9.81)} \qquad \Rightarrow V = 1.253 \text{ m/s}$$

$$\therefore Q = \frac{\pi}{4}(0.25)^2 \times 1.253 = 0.0615 \text{ m}^3/\text{s}.$$

Second case



(i) From continuity equation

$$Q_3 = Q_1 + Q_2 \qquad \Rightarrow Q_3 = 2Q_1 \qquad [Q_1 = Q_2 = Q_1 \text{ (say)} \qquad (D_1 = D_2)]$$

(ii) Head loss due to friction in pipe **AC** = Head loss in pipe **CD** *i.e.*

$$(h_f)_{AC} = (h_f)_{CD} \qquad \Rightarrow \frac{f_1 L_1 V_1^2}{D_1(2g)} = \frac{f_2 L_2 V_2^2}{D_2(2g)}$$

- Here, $f_1 = f_2$; $D_1 = D_2$; $L_1 = L_2$; $V_1 = V_2$

(iii) Apply Bernoulli's eq. between **1** and **2** (datum through **2**), to get

40 = Head loss in pipe **AC** or pipe **CD** + Head loss in pipe **CB**

$$= \frac{f_1 L_1 V_1^2}{D_1(2g)} + \frac{f_3 L_3 V_3^2}{D_3(2g)} \qquad = \frac{f_1 L_1 Q_1^2}{12 D_1^5} + \frac{f_3 L_3 Q_3^2}{12 D_3^5}$$

- Here, $f_1 = f_3$; $D_1 = D_3$; $L_1 = L_3$

$$\therefore 40 = \frac{f_1 L_1}{12 D_1^5}\left(Q_1^2 + Q_3^2\right) \qquad Q_1 = \frac{Q_3}{4}$$

- Solve for Q_3, to get
- **$Q_3 = 0.0774$ m³/s**

$$\therefore \% \text{ increase in discharge} = \frac{(0.0774 - 0.0615)}{0.0615} \times 100 = 26\%$$

Q2: A 300 mm diameter pipe is required for a town's water supply. As pipes of this diameter are not available in the market, it is decided to lay two parallel pipes of equal diameter. Find the diameter of the parallel pipes, assuming same coefficient of friction for all the pipes.

Solution:

- Head loss due to friction in 300 mm diameter pipe, $h_f = \frac{(4f)LV^2}{D(2g)}$
- Let **d** = Diameter of parallel pipes, then

$$h_f = h_{f_1} = h_{f_2} \qquad \Rightarrow \frac{(4f)LV^2}{D(2g)} = \frac{(4f)LV_1^2}{d(2g)} = \frac{(4f)LV_1^2}{d(2g)}$$

$$\Rightarrow \frac{V^2}{D} = \frac{V_1^2}{d} \qquad \Rightarrow \left(\frac{V}{V_1}\right) = \left(\frac{0.3}{d}\right)^{1/2} \qquad (1)$$

- From continuity equation, $Q = Q_1 + Q_2 \Rightarrow Q = 2Q_1$

$$\therefore \frac{\pi}{4}(0.3)^2 \times V = 2 \times \frac{\pi}{4}(d)^2 \times V_1 \qquad \Rightarrow \left(\frac{V}{V_1}\right) = \frac{2d^2}{0.09} \qquad (2)$$

- Equating Eqs. (1) and (2), to get

$$d = 227\,\text{mm} \sim 230\,\text{mm}$$

Q3: A pipeline consists of following fittings:

Fitting	Value of loss coefficient
Standard Tee	1.8
Elbow	0.9
90° bend	1.2
Gate valve	0.19

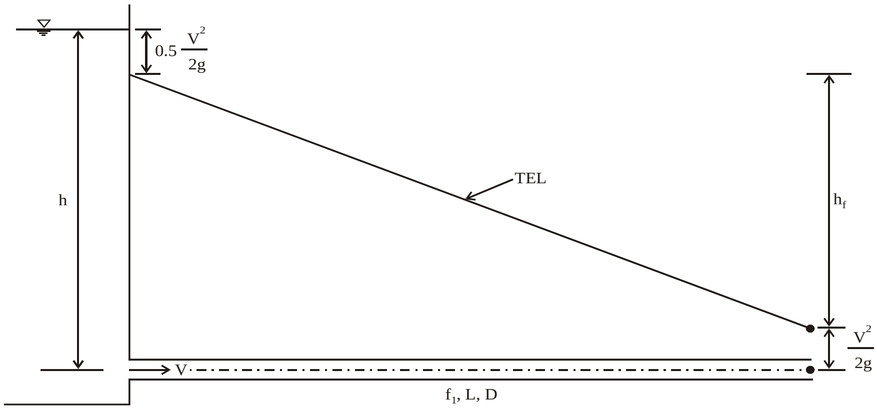
Determine the equivalent length of a 400 mm diameter pipe (friction factor = 0.02) for the above fittings.

Solution:

$$\frac{f_e L_e V^2}{D_e(2g)} = \Sigma K_L \frac{V^2}{2g} \Rightarrow L_e = \Sigma K_L \frac{D_e}{f_e} \qquad \therefore L_e = (1.8 + 0.9 + 1.2 + 0.19)\frac{0.4}{0.02} = 81.8\,\text{m}$$

TRANSMISSION OF POWER THROUGH PIPES AND NOZZLES

- Pipes carrying water (or any other liquid) under high pressure may be used to transmit hydraulic power which can be used for the operation of hydraulic machines such as penstocks carrying water to turbines, running of flour mill etc.
- Consider a horizontal pipe (penstock) connected to a high level reservoir as shown in **Figure:**



Transmission of power through a pipe

- If **h** = Inlet head
- $\mathbf{h_f}$ = Head loss due to friction, then
- Head available at outlet of pipe/or net head = $\mathbf{(h - h_f)}$ (Neglecting the inlet loss)
- Also, power available at the outlet of pipe, **P** (also called power transmitted through the pipe) is given by:

$$\mathbf{P} = \text{Weight of water per sec} \times \text{outlet head}$$
$$= \rho\mathbf{g} \times \text{Volume of water per sec} \times \text{outlet head}$$
$$= \rho\mathbf{g} \times \mathbf{Q} \times \mathbf{(h - h_f)}$$
$$\therefore \mathbf{P} = \rho\mathbf{g} \times \frac{\pi}{4}\mathbf{D}^2 \times \mathbf{V}\left(\mathbf{h} - \frac{\mathbf{f_1 L V^2}}{\mathbf{D(2g)}}\right) \qquad \mathbf{(1)}$$

Condition for maximum transmission of power

- may be obtained by differentiating Eq. (1) with respect to **V** and equating the expression to zero *i.e.*

$$\frac{dP}{dV} = \rho g \times \frac{\pi}{4} D^2 \left(h - \frac{f_1}{D(2g)} L(3V^2) \right) = 0 \qquad \Rightarrow h - 3\frac{f_1 L V^2}{D(2g)} = 0$$

$$\therefore (h - 3h_f) = 0 \qquad \Rightarrow h_f = \frac{h}{3}$$

i.e. power transmission through the pipe is maximum when head loss due to friction is one-third of the inlet head.

✓ Efficiency of power transmission, $\eta = \frac{\text{Power available at the outlet}}{\text{Power supplied at inlet}} = \frac{\rho g Q(h - h_f)}{\rho g Q h}$

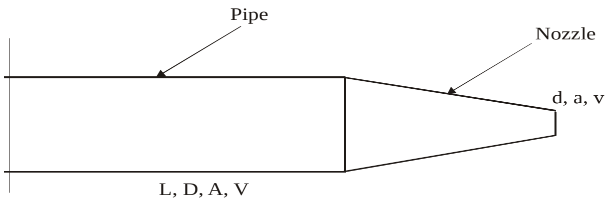
$$\therefore \eta = \frac{h - h_f}{h}$$

$$\therefore \eta_{max} = \frac{(h - h/3)}{h} = 66.7\% \qquad \textbf{(Using, } h_f = h/3\textbf{)}$$

❖ Maximum efficiency of power transmission is 66.7%.

TRANSMISSION OF POWER THROUGH A NOZZLE

- A nozzle is a small tapering pipe, fitted at outlet of a pipe for obtaining a high velocity jet.
- This high velocity jet emerging from the nozzle can be used for various operations such as fire fighting, mining etc.
- **Figure** shows a nozzle connected at the end of a pipe:



- Let **L**, **D**, **A** and **V** are the length, diameter, cross-sectional area and velocity of flow through the pipe, respectively and **d**, **a** and **v** are the diameter, cross-sectional area and velocity of flow through the nozzle at exit, respectively.

✓ Total energy at the end of a horizontal pipe consists of pressure energy and kinetic energy.

- By connecting nozzle at the end of pipe, whole of the energy is converted into kinetic energy at exit of nozzle i.e. as water flows through the nozzle; head available at the base of nozzle is converted into the velocity head at exit of nozzle.
- If **h** = Total head at the inlet of pipe,
- h_f = Head loss due to friction in pipe
- Head available at the base of nozzle is converted into the velocity head at exit of nozzle, which is equal to $(v^2/2g)$

∴ Inlet head = (Head loss due to friction + Velocity head at exit of nozzle)

$$\therefore h = h_f + \frac{v^2}{2g} \qquad \Rightarrow h = \frac{f_1 L V^2}{D(2g)} + \frac{v^2}{2g} \qquad \textbf{(1)}$$

- Eliminate **V** using continuity equation, $AV = av \Rightarrow V = \frac{a}{A}v$

$$\therefore h = \frac{f_1 L}{D(2g)} \times \frac{a^2}{A^2} v^2 + \frac{v^2}{2g} \qquad \textbf{(2)}$$

Expression for velocity of jet:

- Solving the above Eq. for **v**, to get

$$v = \sqrt{\frac{2gh}{\left(1 + \frac{f_1 L}{D}\frac{a^2}{A^2}\right)}}$$

- This is the required expression for velocity of jet or velocity of flow at the exit of nozzle.

Expression for power:

- Power available at exit of nozzle, $P = \rho g \times Q \times (h - h_f)$

(Neglecting inlet loss and head loss in the nozzle)

$$\therefore P = \rho g \times a \times v \times \left(h - \frac{f_1 L}{D(2g)}\frac{a^2}{A^2}v^2\right)$$

Maximum transmission of power:

- May be obtained by differentiating the above Eq. with respect to **v** and equating the expression to zero.
- On simplification, one gets **$h_f = h/3$**

$\therefore$ Power transmission through the nozzle is maximum when head loss due to friction is one-third of the inlet head.

Diameter of nozzle for maximum transmission of power:

- For maximum transmission of power, **$h_f = h/3$**

$\therefore$ Using Eq. (1), one gets

$$3h_f = \frac{f_1 L V^2}{D(2g)} + \frac{v^2}{2g} \qquad \Rightarrow 3\frac{f_1 L V^2}{D(2g)} = \frac{f_1 L V^2}{D(2g)} + \frac{v^2}{2g} \qquad \Rightarrow \frac{2f_1 L V^2}{D(2g)} = \frac{v^2}{2g}$$

$$v^2 = \frac{2f_1 L V^2}{D} \qquad \Rightarrow v^2 = \frac{2f_1 L}{D}\frac{a^2}{A^2}v^2 \qquad \left(\because V = \frac{a}{A}v\right)$$

$$\Rightarrow v^2 = \frac{2f_1 L}{D}\frac{d^4}{D^4}v^2$$

- Solve for **d**, to get

$$d = \left(\frac{D^5}{2f_1L}\right)^{1/4}$$

- This is the required expression for diameter of nozzle.

OBSERVATIONS:

(i) Power available at outlet of nozzle can also be calculated using kinetic energy concept.

- Power, **P** = Kinetic energy supplied by the jet per sec = $\frac{1}{2}(\rho a v)v^2$
- Using Eq. (2) i.e.

$$h = \frac{f_1L}{D(2g)} \times \frac{a^2}{A^2}v^2 + \frac{v^2}{2g} \qquad \Rightarrow v^2 = 2g\left(h - \frac{f_1L}{D(2g)}\frac{a^2}{A^2}v^2\right)$$

$$\therefore P = \frac{1}{2}(\rho a v)v^2 = \frac{1}{2}\rho a v \times 2g\left(h - \frac{f_1L}{D(2g)}\frac{a^2}{A^2}v^2\right)$$

$$\Rightarrow P = \rho g \times a \times v \times \left(h - \frac{f_1L}{D(2g)}\frac{a^2}{A^2}v^2\right)$$

(ii) Head loss in nozzle

- In the analysis, loss of head in the nozzle is neglected.
- If head loss in the nozzle is to be considered, then actual velocity of flow at exit of nozzle is given by, $v = C_v\sqrt{2gh_b}$, C_v is the coefficient of velocity and h_b is the head at the base of nozzle.
- Head at the base of nozzle, $h_b = \frac{1}{C_v^{\,2}}\frac{v^2}{2g}$

$\therefore$ Head loss in nozzle, $h_n = \left(\frac{1}{C_v^{\,2}}\frac{v^2}{2g} - \frac{v^2}{2g}\right) = \left(\frac{1}{C_v^{\,2}} - 1\right)\frac{v^2}{2g}$

$\therefore$ Inlet head (considering head loss in the nozzle) is given by:

$$h = \frac{f_1LV^2}{D(2g)} + \left(\frac{1}{C_v^{\,2}} - 1\right)\frac{v^2}{2g} + \frac{v^2}{2g}$$

(iii) In the case of nozzles, there is a formation of vena contracta just downstream of the nozzle and velocity of jet is actually (represents) the velocity of jet at vena contracta *i.e.* $\mathbf{v = v_c}$.

- By continuity equation at the base of nozzle and at vena-contracta, one can write

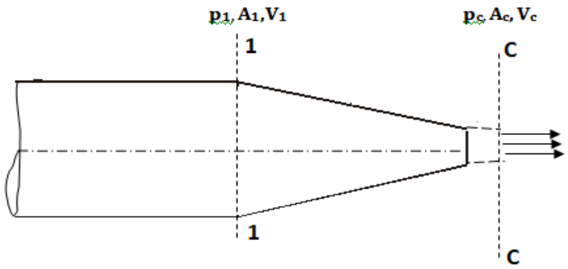
$AV = A_cV_c = (C_ca)\,V_c$, $\quad$ **C_c** is the coefficient of contraction.

$\therefore$ In the above analysis, **v** is to be replaced by **V_c**.

Q: A nozzle attached to the end of 100 mm diameter pipe has 30 mm diameter at the other end. If nozzle issues out a jet of water at 25 m/s into the atmosphere, calculate pressure at the base of nozzle. Given, C_c = 0.80 and C_v = 0.96.

Solution: **D** = 100 mm, **d** = 30 mm

- When jet of water emerges out of nozzle, there is a formation of vena contracta just downstream of the nozzle and velocity of jet is actually the velocity of jet at vena contracta.



∴ Apply continuity eq. at the base of nozzle (Section **1-1**) and at vena-contracta (Section **C-C**), to write

$$A_1V_1 = A_cV_c = (C_c a)V_c; \qquad (\mathbf{V_c} = 25 \text{ m/s}, \mathbf{C_c} = 0.80)$$

$$\therefore \frac{\pi}{4}(0.1)^2 \times V_1 = 0.80 \times \frac{\pi}{4}(0.03)^2 \times 25 \qquad \Rightarrow V_1 = 1.8 \text{ m/s}$$

- Apply Bernoulli's eq. between **1-1** and **C-C, to write**

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + \left(\frac{1}{C_V} - 1\right)^2 \frac{V_c^2}{2g}$$

- Here, $(p_c = 0, V_1 = 1.8 \text{ m/s}, C_v = 0.96, V_c = 25 \text{ m/s})$
- Solve for **p_1**, to get
- **p_1 = 336.8 kPa (Ans)**

Non-Circular ducts

- In the analysis of flow in pipes, all the equations are valid only for circular pipes/cross-sections.
- The relationships/equations for pipe flow can also be used for non-circular cross-sections (ducts) such as square or rectangular provided ratio of width to height of such sections should be less than **4**.

- For non-circular ducts, diameter of circular pipe (**D**) is replaced by $\mathbf{D_h}$, known as hydraulic diameter and is given as: $\mathbf{D_h = \frac{4A}{P}}$

➢ **A** is the cross-sectional area and **P** is the wetted perimeter of non-circular duct.

For example: Consider a rectangular duct of width **b** and height **h.**

A = bh, P = 2(b + h)

$$\therefore D_h = \frac{4bh}{2(b+h)} = \frac{2b}{(1+b/h)},$$ (**b/h** is known as aspect ratio)

- For a square duct, **h = b** $\therefore D_h = h$

Observation:

- For a circular duct, $\mathbf{A = (\pi/4)\, D^2, P = \pi D}$

$$\therefore D_h = \frac{4A}{P} = D$$

❖ (Factor **4** is introduced in the equation so that hydraulic diameter is equal to duct diameter for circular cross-section).