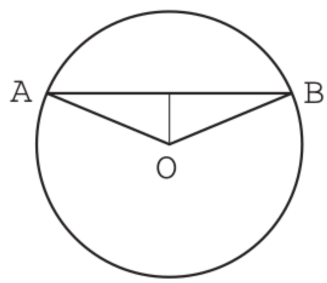
School of Mathematics, Thapar Institute of Engineering and Technology, Patiala
UMA007: Numerical Analysis
Assignment 3

1. Solve the equation $x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$ using secant method with initial guesses 0.02 and 0.05. Use the stopping criterion that the relative error is less than 0.5%.

2. Newton's method for solving the equation $f(x) = c$, where c is a real valued constant, is applied to the function
$$f(x) = \begin{cases} \cos x, & |x| \geq 1 \\ \cos x + (x^2 - 1)^2, & |x| < 1. \end{cases}$$
For what value of c, the Newton's method gives us a sequence $p_n = (-1)^n$ starting with $p_0 = 1$.

3. It costs a firm $C(q)$ dollars to produce q grams per day of certain chemical, where $C(q) = 1000+2q+3q^{2/3}$. The firm can sell any amount of chemical at \$4 a gram. Find the break even point of the firm, i.e, how much it should produce per day in order to have neither a profit nor a loss. Use Newton's method and give the answer to nearest gram.

4. The circle below has radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to four decimal places. Use Newton's method.



5. (i) Apply Newton's method to the function
$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ -\sqrt{-x}, & x < 0 \end{cases}$$
with the root $\alpha = 0$. What is the behavior of the iterates? Do they converge, and if so, at what rate?

   (ii) Do the same but with
$$f(x) = \begin{cases} \sqrt[3]{x^2}, & x \geq 0 \\ -\sqrt[3]{x^2}, & x < 0 \end{cases}$$

6. Using modified Newton method obtain a root of multiplicity two of the equation $ln(x+1) - x = 0$ starting with an initial guess $x_0 = 1$. To the same equation apply Newton's method. Compare the order of convergence in the two approaches.

7. Suppose p is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing p. Show that $g'(p) = 0$ for the fixed-point method $x = g(x)$ with the following choice of g:
$$g(x) = x - m\frac{f(x)}{f'(x)}.$$
What is the implication of having $g'(p) = 0$?

8. Let $f(x) = \frac{1}{2} + \frac{1}{4}x^2 - x\sin x - \frac{1}{2}\cos 2x$. Verify that $x = 0$ is a zero of $f(x)$. Find the multiplicity m of the zero at $x = 0$. Use modified Newton's method with $p_0 = -0.5$ to obtain a sequence which converges to the zero $x = 0$ of $f(x)$.

9. Show that the equation $1 - xe^{1-x} = 0$ has a double root at $x = 1$. Obtain the root by using (i) Newton method and (ii) modified Newton method with $m = 2$, starting with $x_0 = 0$. Verify that the order of convergence is linear in (i) and quadratic in (ii).

10. What is the order of convergence of the iterations

$$x_{n+1} = \frac{1}{2}x_n\left(1 + \frac{a}{x_n^2}\right)$$

as it converges to the fixed point $\sqrt{a}$.

11. For turbulent flow through pipes, the friction factor is a function of Reynolds number and relative roughness ϕ. The Colebrook correlation is a widely used equation, which is given by

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\phi}{3.7} + \frac{2.51}{Re\sqrt{f}}\right).$$

Using relative roughness $\phi = 2 \times 10^{-3}$ and the Reynolds number $Re = 5 \times 10^5$, compute the friction factor f. Use the Newton-Raphson method to obtain the solution. The initial estimate may be obtained from the Miller equation

$$f_0 = 0.25\left[\log\left(\frac{\phi}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}.$$

Report the number of iterations required to reach the solution if the error tolerance is 10^{-10}.