Solids and Structures

Buckling of Column

Euler’s Theory of Buckling

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Column

- The columns are long slender structural members loaded axially in compression.
- If a compression member is relatively slender, it may deflect laterally and fail rather than failing by direct compression of the material and this lateral deflection is called buckling.
- You can demonstrate this behavior by compressing a plastic ruler/plastic straw or other slender object.
- When lateral deflection occurs, the column has *buckled*.
- Under an increasing axial load, the lateral deflections will increase too, and eventually the column will collapse completely.

Buckled R.C.C. Columns

Buckled steel columns

Column: Buckling Mechanism

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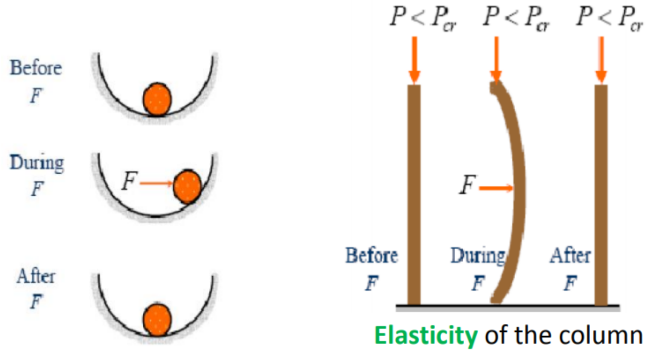
1. Stable equilibrium:

If the load P is sufficiently small, when the force F is removed, the column will go back to its original straight condition .

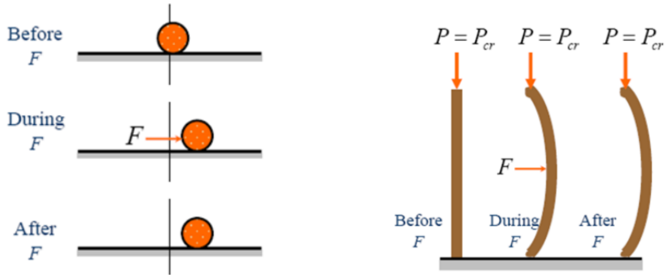


Gravity is the restoring force

Elasticity of the column is the restoring force.

2. Neutral equilibrium:

When the column carries critical load P_{cr}(Increased value of the load P) and a lateral force F is applied and removed, the column will remain in the slightly deflected position.

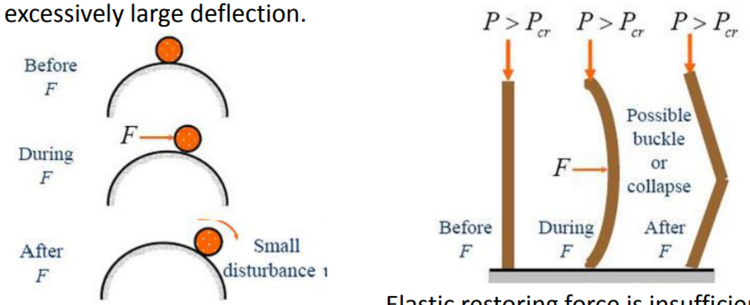


Deflection amount depends on magnitude of force.

Elastic restoring force is sufficient to prevent excessive deflection.

3. Unstable equilibrium:

When the column carries a load which is more than critical load P_{cr} (Increased value of the load P) and a lateral force F is applied and removed, the column will bend considerably and it grows into excessively large deflection.



Even small disturbance causes unstable.

Elastic restoring force is insufficient to prevent excessive deflection.

- The critical load represents the boundary between stable and unstable conditions.
- If the axial load is less than P_{cr}, the structure returns to the vertical position after a slight disturbance
- if the axial load is larger than P_{cr}, the structure buckles and hence fails.

Column: Euler's Theory of Buckling

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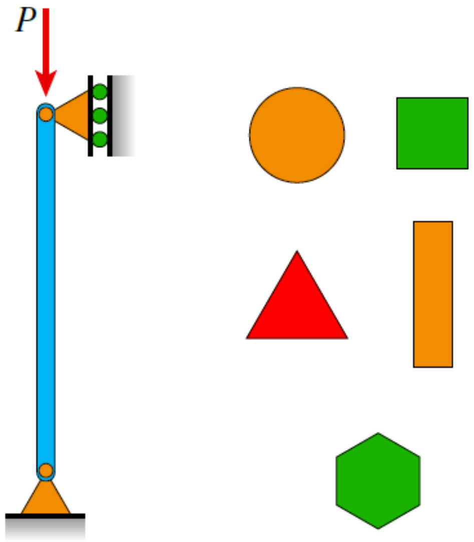
Assumptions in the Euler's theory



1. The column is initially straight.
2. The cross section is uniform throughout.
3. The ends of the column are frictionless.

4. The material is homogeneous and isotropic.
5. The self weight of the column is neglected.

6. The line of thrust coincides exactly with the axis of the column.
7. The shortening of column due to axial compression is negligible.
8. The column failure occurs due to buckling only.

Column: Column based on End Conditions

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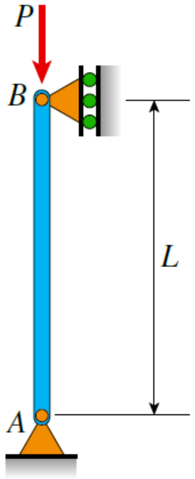
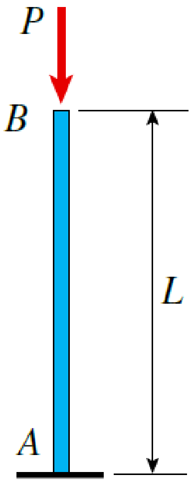
Cases of long columns based on end conditions

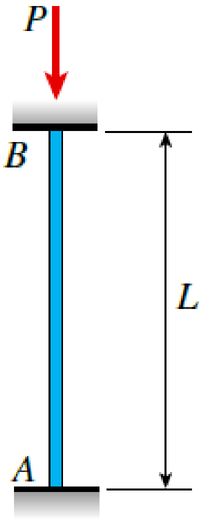
- 1. Both end pinned
- 2. Both ends fixed
- 3. One end fixed and the other end pinned
- 4. One end fixed and the other end free



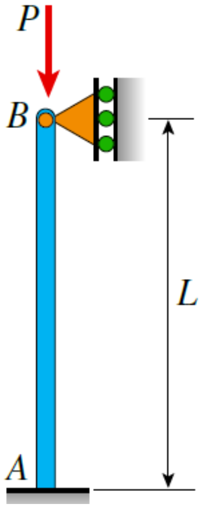
Pinned-Pinned



Fixed-Free



Fixed-Fixed

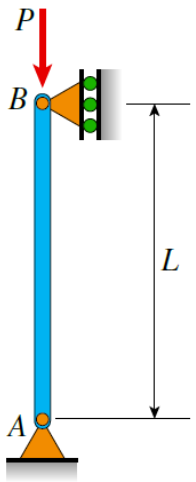


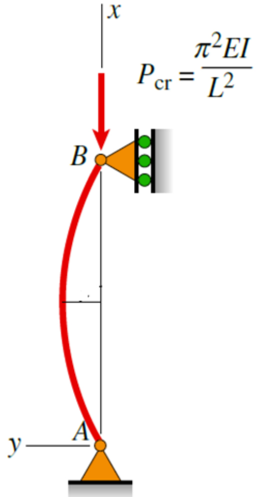
Fixed-Pinned

Euler's Buckling Load

Pinned-Pinned Column

- Consider a column of length L
- Young's modulus of material is E
- Minimum moment of Inertia is I



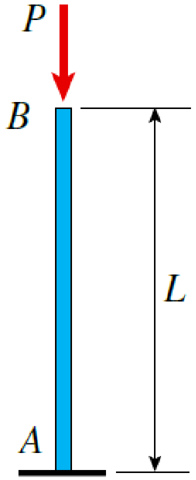


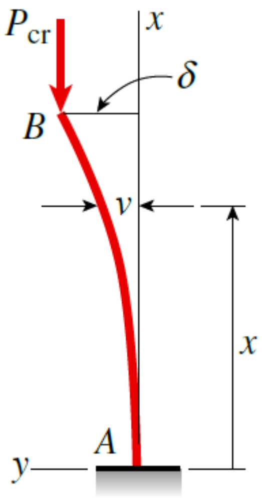
Euler's buckling load , P_{cr} is

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

Fixed-Free Column

- Consider a column of length L
- Young's modulus of material is E
- Minimum moment of Inertia is I





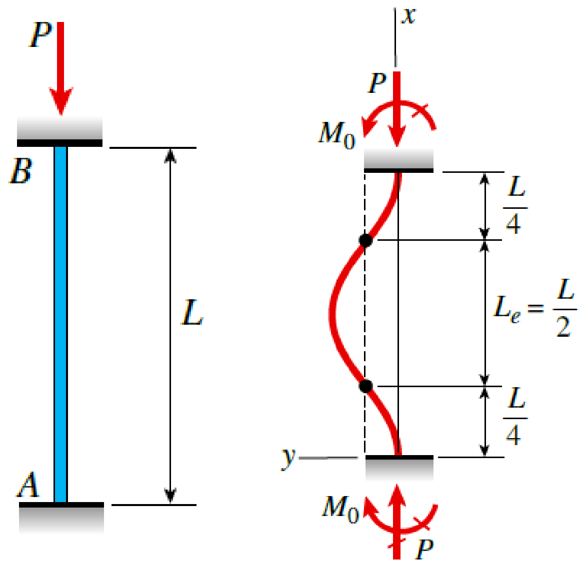
Euler's buckling load , P_{cr} is

$$\boxed{P_{cr} = \frac{\pi^2 EI}{4L^2}}$$

Euler's Buckling Load

Fixed-Fixed Column

- Consider a column of length L
- Young's modulus of material is E
- Minimum moment of Inertia is I

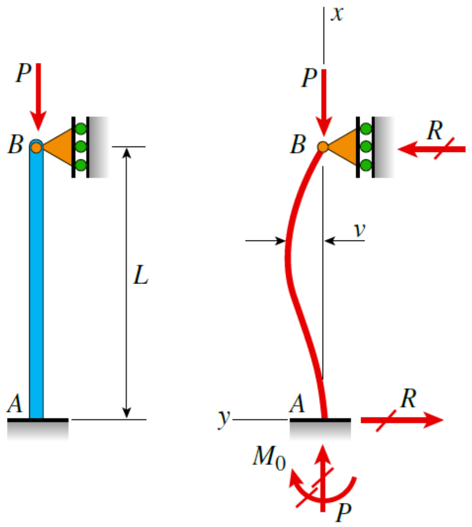


Euler's buckling load , P_{cr} is

$$\boxed{P_{cr} = \frac{4\pi^2 EI}{L^2}}$$

Fixed-Pinned Column

- Consider a column of length L
- Young's modulus of material is E
- Minimum moment of Inertia is I



Euler's buckling load , P_{cr} is

$$\boxed{P_{cr} = \frac{2\pi^2 EI}{L^2}}$$

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Equivalent Length of a column

We can write the general equation for Euler’s critical load as

$$P = \frac{\pi^2 EI}{{l_e}^2}$$

Where, l_e is the equivalent length of a column, the distance between points on the column where the moment is zero, corresponding to the end conditions of the standard pinned–pinned column.

Column: Euler's Theory of Buckling

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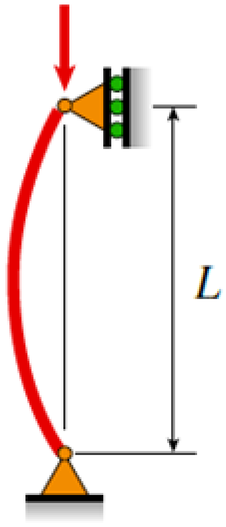
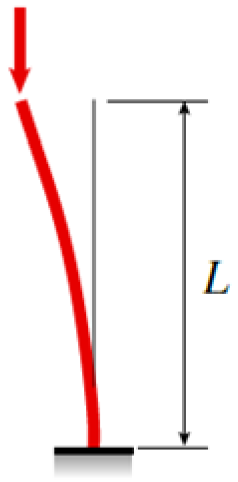
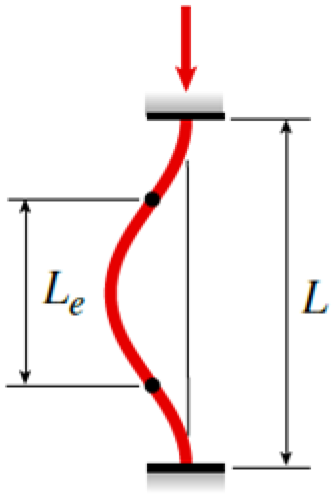
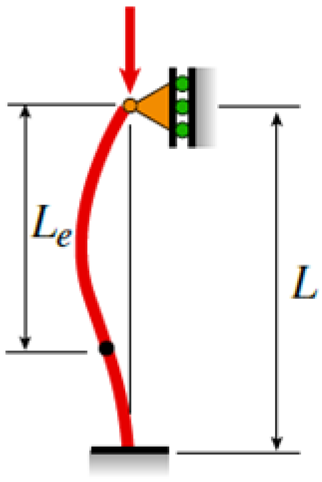
(a) Pinned-pinned column	(b) Fixed-free column	(c) Fixed-fixed column	(d) Fixed-pinned column
$P_{cr} = \dfrac{\pi^2 EI}{L^2}$	$P_{cr} = \dfrac{\pi^2 EI}{4L^2}$	$P_{cr} = \dfrac{4\pi^2 EI}{L^2}$	$P_{cr} = \dfrac{2\,\pi^2 EI}{L^2}$
			
$L_e = L$	$L_e = 2L$	$L_e = 0.5L$	$L_e = 0.699L$

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Limitations of Euler's formula

- 1. It is applicable to an ideal strut only and in practice, there is always crookedness in the column and the load applied may not be exactly co-axial.

- 2. It takes no account of direct stress. It means that it may give a buckling load for struts, far in excess of load which they can be withstand under direct compression.

Column: Numerical Illustrations

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Problem. A hollow circular column of internal diameter 20 mm and external diameter 40 mm has a total length of 5m. One end of the column is fixed and the other end is hinged. Find out the crippling stress of the column if $E = 2 \times 10^5$ N/mm^2. Also findout the shortest length of this column for which Euler's formula is valid taking the yield stress equal to 250 N/mm^2.

Column: Numerical Illustrations

Solution.

d=20 mm; D=40 mm; $l = 5\ mm$; E=2×10^5N/mm².

Euler's crippling load for one end fixed and the

Hinged, $P = \frac{2\pi^2 EI}{l^2}$

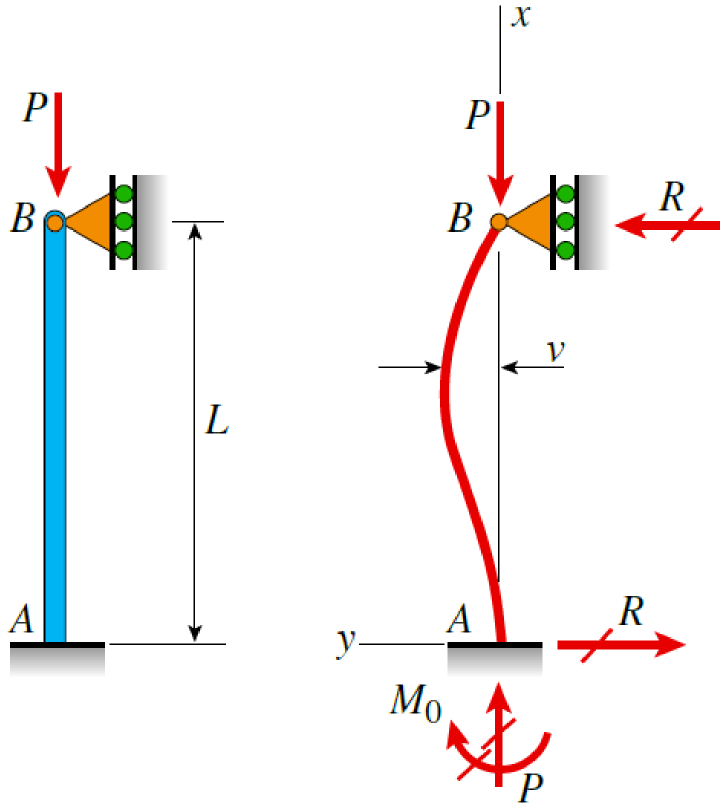
Euler's crippling stress, $p_c = \frac{P_c}{A} = \frac{2\pi^2 EI}{Al^2}$

Area of the column, $A = \frac{\pi(D^2-d^2)}{4} = \frac{\pi(40^2-20^2)}{4}$

$= 942.48\ mm^2$

Moment of inertia, $I = \frac{\pi(D^4-d^4)}{64} = \frac{\pi(40^4-20^4)}{64}$

$= 117809.75\ mm^4$



Column: Numerical Illustrations

Euler's crippling stress, $p_c = \frac{P_c}{A} = \frac{2\pi^2 EI}{Al^2} = \frac{2\pi^2 \times 2 \times 10^5 \times 117809.73}{942.48 \times 5000^2}$

$$= 19.74 \text{ N}/mm^2$$

Yield stress= 250 N/mm² .

$$\frac{l}{k} = \sqrt{\frac{2\pi^2 E}{250}} = \sqrt{\frac{2\pi^2 \times 2 \times 10^5}{250I}} = 125.66$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{117809.73}{942.48}} = 11.18$$

$$l = 125.66 \times k = 125.66 \times 11.18 = 1404.9 \; mm$$

∴Shortest length of this column= 1.4 m.

Column: Numerical Illustrations

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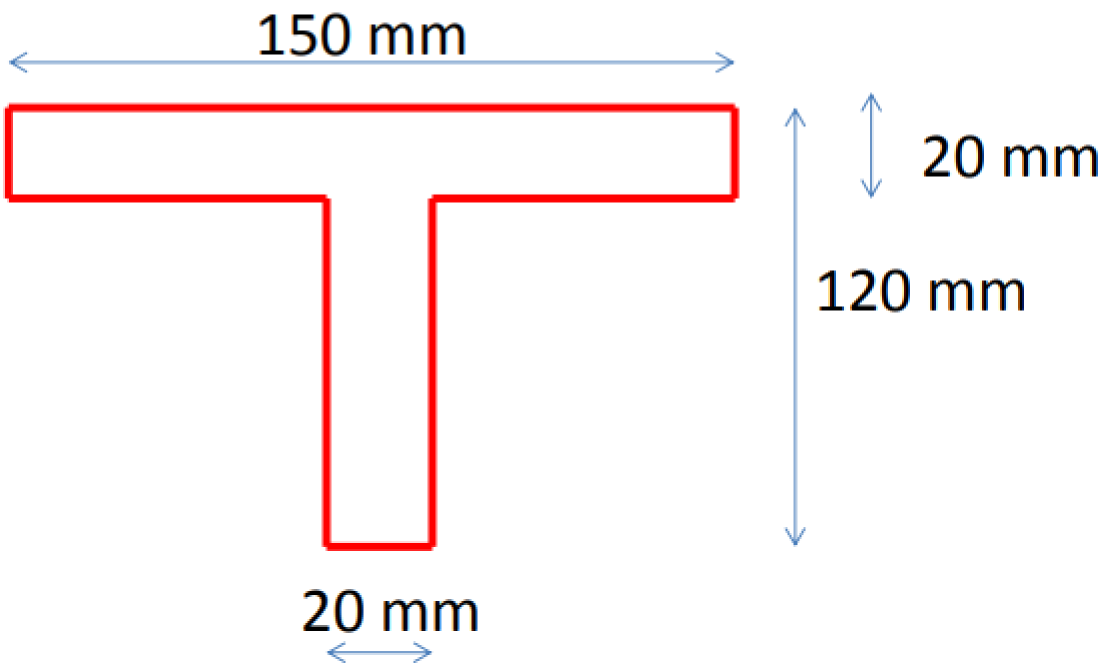
Problem A T-section 150 mm x 120 mm x 20 mm is used as a strut of 4 m long with hinged at its both ends. Calculate the crippling load if modulus of elasticity for the material be 2.0×10^5 N/mm^2.



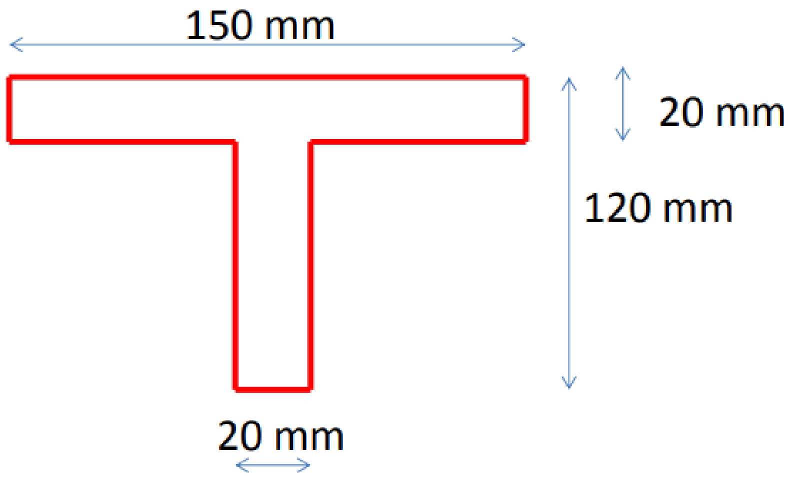
- **Solution :**

First of all, let us find out the C.G of the section. Let $\bar{y}$ be the distance between C.G of the section from top of the flange

By geometry of the figure, $\bar{x} = 0$

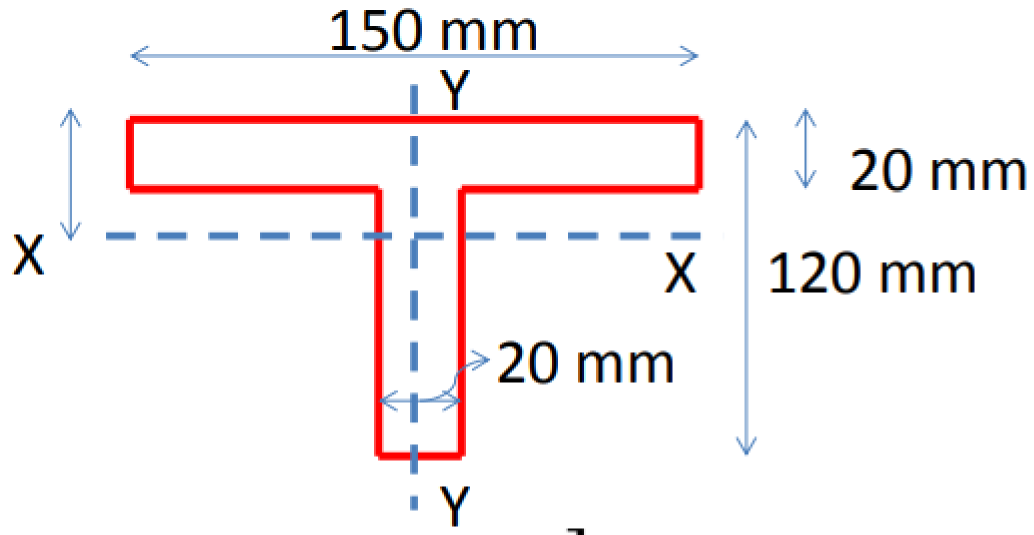
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(150 \times 20) \times 10 + (100 \times 20) \times 70}{(150 \times 20) + (100 \times 20)} = 34\, mm$$



Column: Numerical Illustrations

Solution (contd…)



$$I_{XX} = \left[\frac{1}{12} \times 150 \times 20^3 + (150 \times 20) \times 24^2\right] + \left[\frac{20 \times 100^3}{12} + 2000 \times 36^2\right]$$

$$= 6.0867 \times 10^6 mm^4$$

And

$$I_{YY} = \frac{20 \times 150^3}{12} + \frac{100 \times 20^3}{12}$$ = 5.69 X 10 ^{6} mm^4

Column: Numerical Illustrations

Since I_{YY} is less than I_{XX}.

$\therefore$ The column will tend to buckle in y-y direction

Given End condition : both ends hinged

$$\therefore \qquad l_e = l$$

$$\therefore \qquad P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2.0 \times 10^5 \times 5.6917 \times 10^6}{4000^2}$$

$$= 702185\ N$$

Self Assessment

Q1 A 9m long steel (E = 200 GPa) pipe column has an outside diameter 220 mm and wall thickness of 8 mm. the column is supported only at its ends. Calculate the critical load for the following end conditions;

(i) Pinned-pinned
(ii)Fixed-free
(iii)Fixed-pinned
(iv)Fixed-fixed

(Answers: 731 kN, 182.6 kN, 1491 kN, 2920 kN)

Q2 A column of rectangular cross section of dimensions 40 mm x 60 mm, having a length of 4m is fixed at one end and hinged at the other end. Determine the safe load this column can carry using Euler's formula. Take E=210 GPa and factor of safety =2.5.

Self Assessment

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Q3	A rectangular column 80 mmx100mm is braced at the mid point along its weaker direction. Determine the critical load the column can support with pin ended conditions over a length of 3.25 m. E= 20 GPa. (Answer: 124.64 kN)
Q4	A timber column of rectangular cross-section of dimensions 50 mm x 100 mm, is used to support an axial compressive load. Determine the limiting length till which the column will behave as a short column. Given σ_y = 30 MPa and E=10 GPa. (Answer : 1.66 m)