Maths 4 Real

PERPLEXING PRICES

Imagine you are in your local supermarket, shopping for supplies for a party. You need 7 to 8 litres of soft drink and some lollies, so you move to the soft drink aisle first. A vast array of products and prices confronts you. You see that your favourite brand has three different-sized bottles:

1.25 L for $2.25

1.5 L $2.78

2 L for $3.56

"Hmmm," you think. "The 2 L bottle gives me half a litre more than the 1.5 L, for only 78 cents extra. I could get two 1.5 L bottles, that's 3 L, for $5.56. Maybe I should get a couple of 1.25 L bottles instead. Which do I choose?"

1 At first glance, which bottle size looks to be the best value for money to you?

You remember that soft drink also comes in 375 mL cans. You move further down the aisle and see the following:

6-pack: $7.00 12-pack: $13.59 18-pack: $19.34 30-pack: $26.34

You begin to wonder: "Should I buy cans instead? How many cans are equivalent to one bottle? Are they better value for money? Help!"

2 What is it that makes your 'value for money' decision a difficult one? What information would be useful to help you decide?

In December 2009, the Australian government introduced a new, compulsory pricing system in supermarkets, designed to make decisions easier for shoppers. The system is called 'unit pricing'. As well as the regular price, supermarkets are required to display the price for a particular 'unit' of the product, such as 1 L, 1 kg, 100 g or 100 mL. Comparing the prices of different products is much easier when you are comparing the same amount.

We can calculate a unit price for the bottles of soft drink by calculating the price *per litre*. To do this, we divide the price by the capacity of the bottle, in litres:

For example, the unit price for the 1.25 L bottle is: $2.25 ÷ 1.25 L = $1.80 per L.

3 (a) Calculate the price per litre of the other two bottle sizes.

(b) Which size bottle is the best value for money?

(c) Would it always be better to purchase this size? Explain your answer.

(d) A bit further down the aisle, you notice that a 1.5 L bottle of a rival brand of cola is on special for $2.39. Calculate the price per litre of this discounted bottle.

(e) You also notice that you can buy a 'multipack' of twelve 300 mL bottles for $11.86. Calculate the price per litre of the multipack. (300 mL is 0.3 L)

(f) Why is the unit price of the multipack so different to the unit prices for individual bottles?

4 (a) Calculate the price per can of each of the packs.

(b) Which pack represents the best value for money? Did you find anything surprising in your results?

(c) The price of a single can of soft drink is $1.35. Why is there such a difference between the price of a single can and the price per can in the can packs?

5 (a) Calculate the price per litre of each of the can packs, remembering that each can contains 375 mL (0.375 L).

(b) Suggest a possible explanation for the difference in the price per litre between the cans and the bottles.

(c) When might people prefer to buy multipacks of cans instead of bottles?

You move down to the confectionery aisle, and spot a 190 g 'Party Mix' bag of lollies for $2.90, and a 'Jumbo' size 570 g bag for $5.82.

6 The unit price for products like bags of lollies is the price per 100 g.

(a) Calculate the unit price for the 190 g bag and the 570 g bag. (Hint: Work out what you need to divide 190 and 570 by to get 100, and divide the prices by this same amount.)

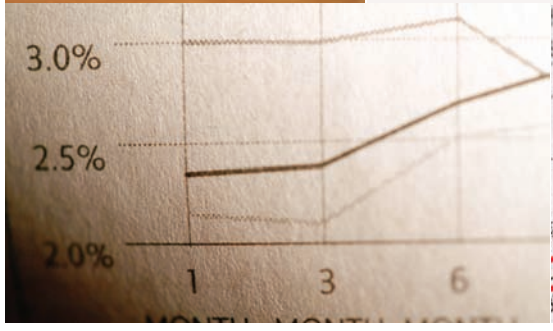
(b) Which is the better buy in this case?

RESEARCH

- Visit the supermarket, use catalogues or go shopping online to investigate the cost of goods that come in different sizes, such as coffee, Vegemite™, washing powder and pet food. Use a suitable unit, such as 100 g or 1 kg, to calculate the unit prices.
- Comment on your findings. Is there a general rule or conclusion you can make? Discuss, with examples, why different people might buy different-sized products.

4.7 Percentages, fractions and decimals

Percentages are used in many everyday situations. Shopping discounts, survey results, business statistics and sporting league ladders all use percentages to communicate information. Percentages make it easier for us to visualise and compare quantities, especially when comparing parts from different-sized wholes.



The word **'per cent'** literally means 'for every hundred' or 'out of one hundred'. The percentage symbol is %. Percentages can be thought of and written as fractions with a denominator of 100. Percentages can also be thought of as division by 100.

For example, 25% means 25 out of 100, and 9% means 9 out of 100. We can write percentages as fractions, some of which can be simplified, and some which can't.

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 9\% = \frac{9}{100}$$

If we perform the division, we get the percentage in decimal form.

$$\frac{25}{100} = 25 \div 100 = 0.25 \qquad \frac{9}{100} = 9 \div 100 = 0.09$$

Writing percentages as fractions and decimals

To write a percentage as a fraction …

- Write the percentage value as the numerator of a fraction with a denominator of 100.
- Simplify the fraction if possible.

To write a percentage as a decimal …

- Divide by 100.

Worked Example 17 WE17

Write 12% as a fraction in simplest form.

Thinking	Working
1 Write the value of the percentage as the numerator over a denominator of 100.	$12\% = \frac{12}{100}$
2 Simplify the fraction if possible. (Here, we have cancelled a common factor of 4.)	$= \frac{3}{25}$

Worked Example 18 WE18

Write 45% as a decimal.

Thinking	Working
Divide the value of the percentage by 100. Write your answer without the percentage symbol.	$45\% = 45 \div 100$ $= 0.45$

Writing fractions as percentages

To write a fraction as a percentage, we can use one of two methods:

Method 1: Write the fraction as an equivalent fraction with a denominator of 100. Use the numerator of this new fraction as the percentage.

Method 2: Multiply the fraction by 100%.

Method 1 is convenient, but only works with fractions that have denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2 can be used with any fraction.

Note: As Worked Example 19 shows, not all percentages have whole number values. These percentage numbers are usually written as decimals. If it is necessary to round them, one decimal place usually gives an answer of acceptable accuracy. These percentages can also be written in fractional form; for example, $16.7\% = 16\frac{7}{10}\%$, but decimal form is usually more convenient to work with. Remember that you can use the $a\frac{b}{c}$ or S⇔D key on your calculator to convert answers from decimal to fraction form, and vice versa.

100% is equivalent to 1 whole, $\frac{1}{1}$ and 1.0.

Worked Example 19 WE19

Write each of the following fractions as percentages.

(a) $\frac{4}{5}$ **(b)** $\frac{27}{40}$

Thinking	Working
(a) 1 Is the denominator a factor of 100? (Yes) Determine what number the denominator must be multiplied by to get 100.	(a) $\frac{4}{5} \times \frac{\quad}{20} = \frac{\quad}{100}$
2 Multiply the numerator by the same number.	$\frac{4}{5} \times \frac{20}{20} = \frac{80}{100}$
3 Write the numerator of the new fraction with the percentage symbol.	$= 80\%$
(b) 1 Is the denominator a factor of 100? (No)	(b) $\frac{27}{40} \times \frac{100}{1}\%$
2 Multiply the fraction by 100. (Remember that we can write a whole number as an improper fraction with a denominator of 1.) Cancel common factors first to simplify the multiplication. (Here, a common factor of 20 is cancelled.)	$= \frac{27}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$ $= \frac{135}{2}\%$
3 Divide the numerator of the answer by the denominator. Write your answer as a decimal (rounded if necessary).Write the answer with a percentage symbol.	$= 67.5\ \%$

Writing decimals as percentages

To write a decimal as a percentage, multiply it by 100.

Worked Example 20 WE20

Write each of the following decimals as percentages.

(a) 0.04 **(b)** 0.125

Thinking	Working
(a) Multiply the decimal by 100%.	(a) $0.04 = 0.04 \times 100\%$ $= 4\%$
(b) Multiply the decimal by 100%.	(b) $0.125 = 0.125 \times 100\%$ $= 12.5\%$

4.7 Percentages, fractions and decimals

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q15, Q16, Q18	Q1, Q2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18	Q1, Q2, Q3 Columns 3 & 4, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18

Answers page 644

Fluency

1 Write each of the following percentages as fractions in simplest form. WE17

(a) 15% **(b)** 24% **(c)** 18% **(d)** 64%

(e) 110% **(f)** 122% **(g)** 140% **(h)** 198%

2 Write each of the following percentages as decimals. WE18

(a) 34% **(b)** 13% **(c)** 78% **(d)** 92%

(e) 120% **(f)** 123% **(g)** 254% **(h)** 320%

3 Write each of the following fractions as percentages. WE19

(a) $\frac{2}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{20}$

(e) $\frac{11}{10}$ **(f)** $\frac{3}{2}$ **(g)** $\frac{28}{25}$ **(h)** $\frac{6}{5}$

(i) $\frac{3}{8}$ **(j)** $\frac{1}{3}$ **(k)** $\frac{5}{6}$ **(l)** $\frac{4}{9}$

(m) $\frac{13}{15}$ **(n)** $\frac{19}{30}$ **(o)** $\frac{33}{40}$ **(p)** $\frac{44}{60}$

4 Write each of the following decimals as percentages. WE20

(a) 0.87 **(b)** 0.34 **(c)** 0.65 **(d)** 0.96

(e) 1.34 **(f)** 3.58 **(g)** 5.32 **(h)** 1.21

5 Complete the following table.

Percentage	Fraction (simplest form)	Decimal
40%		
	$\frac{7}{10}$	
		0.05
	$\frac{1}{50}$	
75%		
	$\frac{5}{4}$	

Understanding

6 $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{5}$ are all common fractions.

(a) Change each of these fractions into decimals.

(b) Change each of these decimals into percentages.

7 95% of all the animal species in the world are insects. Write this as a fraction in simplest form.

8 At Valley High School, $\frac{5}{8}$ of the students are studying Chinese. Write this fraction as a percentage.

9 Place the following in ascending order (smallest to largest).

(a) 82%, 0.03, $\frac{7}{50}$, $\frac{3}{5}$, 0.8, 57%

(b) $\frac{1}{2}$, 73%, 1.05, $\frac{3}{10}$, 43%, 0.05

10 The Petronas Towers in Kuala Lumpur are approximately 0.6 times the size of the tallest building in the world, the Burj Khalifa in Dubai.

(a) Write the height of the Petronas Towers as a fraction of the height of the Burj Khalifa.

(b) Now, write the equivalent percentage.

Reasoning

11 A rugby stadium contains 43 500 people. 37% of the crowd supports England and the rest supports Australia. What percentage of the crowd supports Australia?

12 Allison is a journalist. She has surveyed 150 people and found that 6 believe that aliens exist.

(a) Write the survey result:

(i) as a fraction in simplest form

(ii) as a percentage

(iii) as a decimal.

(b) Which of the three forms of the result would you recommend that Allison uses as the headline for the article she is writing? Give a reason for your answer.

(c) Use your answer from **(b)** to write a potential headline.

13 $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, or $3 \times \frac{1}{4}$. How is this fact reflected in the percentage values for $\frac{1}{4}$ and $\frac{3}{4}$?

14 13% of a class have red hair, 62% of the class have black hair.

(a) What is the total percentage of the class that have red or black hair?

(b) What is this total as a decimal?

(c) What percentage of the class have a hair colour that is not red or black?

(d) Write your answer to **(c)** as a decimal.

Open-ended

15 Write down at least three fractions that have equivalent percentage values between 10 and 40 per cent.

16 Find at least three percentages that have decimal values between 0.34 and 0.35.

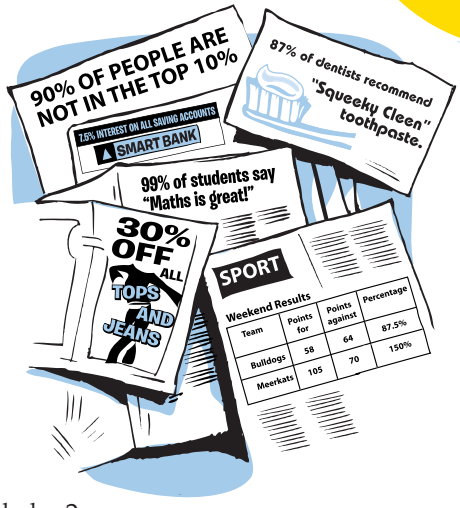
17 Estimate the percentage of one weekday that you would spend:

(a) sleeping **(b)** eating **(c)** studying **(d)** socialising.

Using percentages

4.8

In this section you will learn two important skills that will enable you to understand and work with two common uses of percentages.



Finding a percentage of a quantity

Statements involving percentages are common:

'32% of the students in the school have blue eyes.'
'48% of voters voted for a particular candidate.'
'The television is discounted by 20% of the original price.'

Knowing the fraction equivalents of some of the common percentages is very useful.

For example:

50% = $\frac{1}{2}$, so to find 50% of something, we can simply divide by 2.

25% = $\frac{1}{4}$, so we can find 25% easily by dividing by 4 (or by dividing by 2 twice).

10% = $\frac{1}{10}$, so we divide by 10 to find 10%.

20% = $\frac{1}{5}$, so we divide by 5 to find 20% (or divide by 10, then multiply by 2).

We can use these facts to find a percentage of a quantity. This is shown in Method 1 of the following Worked Example.

Another way to find a percentage of a quantity is to use the fact that in mathematics, the word 'of' means 'multiply'. This is shown in Method 2 of the following Worked Example.

Worked Example 21

WE 21

Calculate the following percentage: 15% of 70.

Method 1: Use basic percentages

Thinking	Working
1 Break the percentage up into basic percentages, such as 50%, 10%, 5% or 1%.	15% = 10% + 5%
2 Calculate each of the basic percentages.	10% of 70 = 70 ÷ 10 = 7 5% of 70 = (10% of 70) ÷ 2 = 7 ÷ 2 = 3.5
3 Use combinations of the basic percentages calculated to obtain the required percentage.	15% of 70 = 7 + 3.5 = 10.5

Method 2: Fraction or decimal multiplication

Thinking	Working
1 Replace the word 'of' with the multiplication symbol, ×.	15% of 70 $= 15\% \times 70$
2 Convert the percentage to a fraction or decimal.	$= \frac{15}{100} \times 70$ or 0.15×70
3 Perform the multiplication.	$= \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{100}} \times \frac{\cancel{70}}{1}$ $= \frac{3 \times 7}{2 \times 1}$ $= \frac{21}{2}$ $= 10.5$

Writing one amount as a percentage of another

Writing fractions of a whole as percentages makes them easier to visualise, compare and order. The results of surveys, tests and sports teams are often reported using percentages.

To write one amount as a percentage of another:

1. Write the amount we are interested in as the numerator of a fraction, with the other amount as the denominator.
2. Convert this fraction to a percentage. If the percentage is not a whole number, then rounding to one decimal place gives an accurate enough answer in most cases.

Worked Example 22 WE22

Write the following as percentages. Round your answers to one decimal place, if necessary.

(a) Melissa got 21 out of 25 questions on her grammar test correct.

(b) 13 of the 24 students in class 7C have brown hair.

Thinking	Working
(a) 1 Write the result as a fraction. Is the denominator a factor of 100? (Yes)	(a) $\frac{21}{25}$
2 Determine what number the denominator must be multiplied by to get 100.	$\frac{21}{25} \times \frac{\_}{4} = \frac{\_\_}{100}$
3 Multiply the numerator by the same number.	$\frac{21}{25} \times \frac{4}{4} = \frac{84}{100}$
4 Write the numerator of the new fraction with a percentage symbol.	84%
5 Write your answer in a sentence.	Melissa got 84% of the test questions correct.

(b) 1 Write the first amount as the numerator, and the second amount as the denominator of a fraction. Is the denominator a factor of 100? (No)

(b) $\frac{13}{24}$

2 Convert the fraction to a percentage by multiplying by 100. Cancel common factors first to simplify the multiplication. (Here, a common factor of 4 has been cancelled.)

$\frac{13}{\cancel{24}_{6}} \times \frac{\cancel{100}^{25}}{1}\%$

$= \frac{325}{6}\%$

3 Divide the numerator of the answer by the denominator. Write your answer as a decimal, rounded to one decimal place.

54.2%

4 Write your answer in a sentence.

54.2% of the students in 7C have brown hair.

4.8 Using percentages

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q18, Q20	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q20	Q1 Columns 2 & 3, Q2 (d)–(i), Q3, Q4, Q5, Q8, Q9, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

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Equipment required: A calculator may be used for Questions **2**, **7–12**, **15–17**

Fluency

1 Calculate each of the following percentages.

WE21

(a) 50% of $36
(b) 10% of 450 m
(c) 20% of 840 L
(d) 80% of 120 m
(e) 90% of 500 kg
(f) 2% of $8400
(g) 13% of 72 kg
(h) 27% of $354
(i) 21% of 876 m
(j) 28% of 231.25 m
(k) 34% of 210.4 L
(l) 4% of $32.65

2 Write the following as percentages. Round your answers to one decimal place, if necessary.

WE22

(a) 17 out of 20 on a spelling quiz.
(b) 5 out of 22 students in the class have blond hair.
(c) There are 100 kittens at the animal shelter, 40 of which are female.
(d) 4 out of 5 students surveyed own an mp3 player.
(e) 50 pairs of shoes were sold, of which 38 pairs were black.
(f) 54 out of 70 cars counted on a freeway had only 1 person in them.
(g) 15 of the 27 cars in the staff carpark are white or silver.
(h) 3 out of every 4 people on the bus said they were travelling to work.
(i) 28 out of 34 people surveyed said they were non-smokers.

3 Calculate the following using the 'basic percentages method' or any other mental strategy.

(a) 50% of $25 **(b)** 10% of 75 kg **(c)** 25% of 60 km

(d) 20% of 40 L **(e)** 1% of 360 m **(f)** 5% of $120

(g) 60% of 350 kg **(h)** 75% of 800 mL **(i)** 35% of $400

4 Mehmet successfully shot 13 basketball goals out of 18 attempts. The calculation that converts this result to a percentage would be set out as follows.

A $\frac{13}{100} \times \frac{18}{1}$ **B** $\frac{13}{18} \times \frac{100}{1}$ **C** $\frac{18}{13} \times \frac{100}{1}$ **D** $\frac{100}{13} \times \frac{18}{1}$

5 20% of $420 is:

A $8.40 **B** $20 **C** $84 **D** $200

Understanding

6 40% of the 25 students in class 7A have black hair. How many students is this?

7 16 out of the 28 biscuits in a packet have cream centres. What percentage of the packet are cream biscuits?

8 A 2 litre bottle of orange drink contains 35% orange juice. Work out how many litres of orange juice is contained in the bottle. (It may help to work in mL.)

9 A lion can sleep for up to 20 hours a day. What percentage of a whole day is this?

10 There are 11 boys and 15 girls in class 7B. Write the number of boys and the number of girls as percentages of the total number of students in the class.

11 Damian kicked 7 goals from 10 attempts. Franco kicked 11 goals from 16 attempts.

(a) Express the boys' results as percentages.

(b) Who is the more accurate goalkicker?

12 Australia scored 381 runs in a cricket match against India. If Shane Watson scored 126 of those runs, what percentage of the team's total did he score?

13 Shops often uses percentages to advertise savings during sales. The percentage is used to calculate the amount that is removed from the price of the goods. This amount is known as the 'discount'.

(a) Calculate the dollar value of the discount on the following.

(i) $40 shirt, 20% discount **(ii)** $125 jacket, 25% discount

(iii) $65 jeans, 50% discount **(iv)** $239 suitcase, 10% discount

(v) $799 camera, 40% discount **(vi)** $1299 sofa, 5% discount

(b) Now, find the sale price of the above items by subtracting the discount amount from the original price.

Reasoning

14 8% of the apples in a crate were rotten. What percentage of the apples were not rotten?

15 Chelsea and Sonia are netballers. In one game, Chelsea scored 17 goals from 25 attempts, and Sonia scored 15 goals from 20 attempts. In the next game, Chelsea scored 19 goals from 27 attempts, and Sonia scored 17 goals from 23 attempts. Use percentages to show which one of the girls improved the accuracy of her goal shooting. Round your figures to one decimal place, if necessary.

16 The 180 students at St John's College all completed an online survey. The results showed that:

22% of the students had blue eyes

34% spoke two or more languages fluently

7% of them were left handed.

(a) Calculate the number of students in each category using the above three percentages.

(b) Considering that we are dealing with numbers of students, what is strange about your answer to **(a)**?

(c) What might be the actual numbers of students represented by each of the percentages?

17 The energy in food is measured in kilojoules (kJ). The National Heart Foundation of Australia recommends that no more than 30% of our daily energy intake should come from eating fats.

(a) A healthy, active woman should eat up to 9000 kJ of food a day. Up to how many kJ in the form of fats can she eat in 1 day?

(b) 1 gram of fat gives the body 37 kJ of energy. How many grams of fat would provide a woman with the maximum daily amount of kilojoules allowed from fat? Round your answer to the nearest whole number.

Open-ended

18 Suggest some reasons as to why some stores use percentages to advertise savings, rather than dollar amounts.

19 Mark saved $5.00 when a store had a sale. Determine at least three different percentage discounts and original prices for the item that Mark purchased.

20 Fedora and Tanya were asked to show their working to the following question. 'Calculate 15% of 80'.

Here is Tanya's working:

$$\frac{15}{80} \times \frac{100}{1}$$

$$= \frac{150}{8}$$

$$= 18.75$$

Here is Fedora's working:

$$15\% \text{ of } 80$$
$$= 0.15 \times 80$$
$$= 12$$

Which of the girls' working is correct? Give a piece of advice to the other girl to help her avoid similar mistakes in the future.

Outside the Square — Problem solving

The great half-price sale

Tricia and Carly were out shopping. They headed towards their favourite clothes shop. On the shop window was a sign that read: 'Take 50% off all marked prices!' Stuck over the top of that sign was another sign, saying: 'Take another 50% off reduced prices!'

'Eh?' said Carly. '50% is one half. If I take half off the price, then take off another half, there won't be any price left! That means everything in the shop is free!'

'I don't think so', said Tricia. 'How can you run a business like that?'

'Well', said Carly, 'If I pick up a pair of $100 jeans and take two lots of 50% off the price, what do they cost now?'

'Errr...', said Tricia.

Can you work it out? How much would the jeans cost?

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

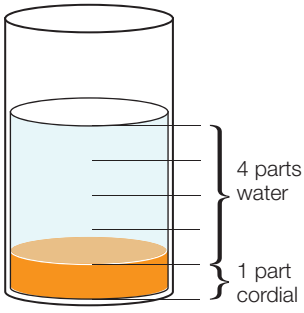
Ratio

4.9

A **ratio** is a comparison of amounts of the same type of quantity.

When we mix a cordial drink, we add different amounts of cordial and water.

The amount of cordial compared to the amount of water may be written in many ways.



1 part cordial to 4 parts water

or

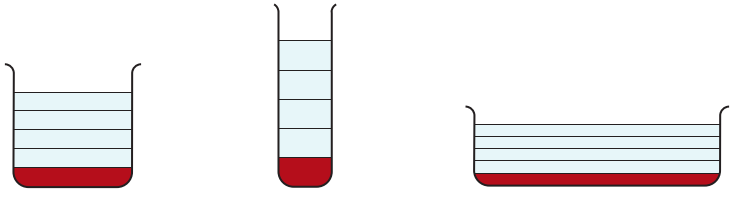
1 to 4

or using a ratio symbol,

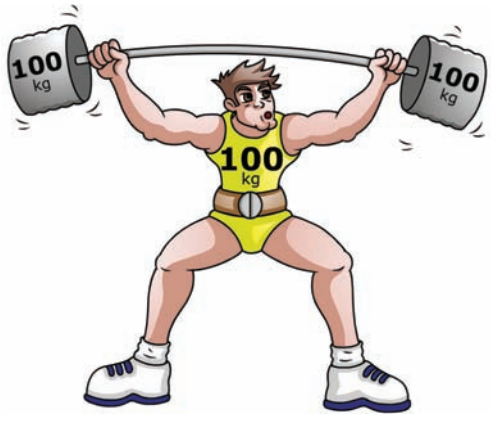
1 : 4

(The ':' can be read as 'to'.)

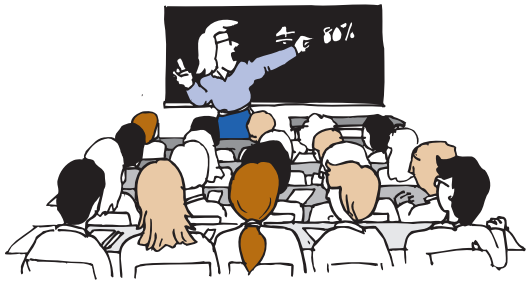
The actual volumes (the number of millilitres) of cordial and water are not given. It is the ratio, or proportion, of cordial compared to water that is important in getting the right taste. Specifying actual quantities would not be very useful, as cordial is made up in glasses and jugs of many different sizes. A sip of cordial from each of the vessels below would taste the same after the mixtures are stirred.



Some more examples of ratios are given below.



The ratio of the mass being lifted to the mass of the lifter is 2 : 1.



The teacher to student ratio in the class is 1 : 20.

A ratio is a comparison of two or more amounts, written using a ':' symbol.

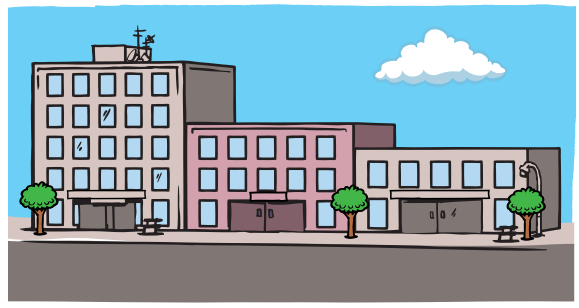
The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

Worked Example 23 WE23

Write a ratio for each of the following.

(a) the number of windows in the middle building compared to the number of windows in the smallest building

(b) the number of floors in the buildings, from left to right



Thinking			Working
(a)	**1**	Write the quantities in the correct order.	(b) Windows in middle building: 12 Windows in smallest building: 7
	2	Write as a ratio.	12 : 7
(b)	**1**	Write the quantities in the correct order.	(a) 5 floors to 3 floors to 2 floors
	2	Write as a ratio.	5 : 3 : 2

We can see from the Worked Example above that a ratio can compare more than two amounts.

Part : part and part : whole ratios

Suppose we had a class of 9 girls and 11 boys. We could write a ratio comparing the number of boys to girls as 11 : 9. This **part : part ratio** compares two parts of a whole. We can add the parts in a ratio to find the total number of parts. Here, 11 + 9 = 20 students in total. We can write another ratio that compares the number of boys to the total number of students in the class as 11 : 20. This **part : whole ratio** compares one part to the whole.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

Ratios, fractions and percentages

The numbers in part : whole ratios can be written as fractions. For example, the above boys : total students ratio of 11 : 20 could be written as the fraction $\frac{11}{20}$ or as the percentage 55%. The fraction of girls would therefore be $\frac{9}{20}$ of the class, or 45%.

Worked Example 24 WE24

A large enclosure at the zoo contains 5 giraffes and 12 zebras.

(a) Write a ratio comparing the number of zebras to the number of giraffes.

(b) Write a ratio comparing the number of giraffes to the total number of animals.

(c) Write the number of giraffes as a fraction of the total number of animals.

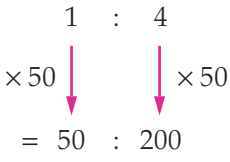
(d) Write the number of zebras as a percentage of the total number of animals.

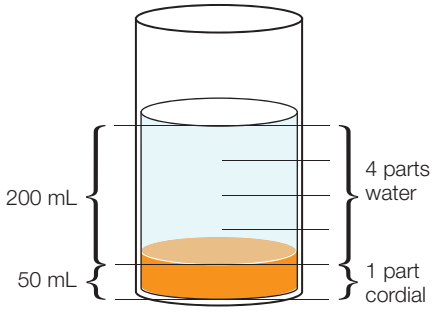
Thinking	Working
(a) Write the ratio in words, in the correct order, then write the numbers underneath.	(a) zebras : giraffes = 12 : 5
(b) **1** Find the number of parts in the whole (the total number of animals).	(b) 5 + 12 = 17
2 Write the ratio in words, in the correct order, then write the numbers underneath.	giraffes : total animals = 5 : 17
(c) Write one number in the part : whole ratio as a fraction of the other.	(c) $\frac{5}{17}$
(d) Write a new part : whole ratio as a fraction, then convert the fraction to a percentage, rounding to a sensible value.	(d) zebras : total animals = 12 : 17 $\frac{12}{17} \times \frac{100}{1}$ % = 71% (to nearest whole number)

Equivalent ratios

Imagine that you wanted to mix up a large quantity of a cordial drink using the cordial : water ratio of 1 : 4 that we considered at the beginning of this section. Mixing 1 mL of cordial and 4 mL would only give 5 mL of cordial; however, we can make 500 mL of cordial drink that would taste exactly the same if we mixed 100 mL of cordial with 400 mL of water.

1 : 4 and 100 : 400 are **equivalent ratios**. Equivalent ratios are created in the same way as equivalent fractions. We multiply each part in the ratio by the same number. Here, we have multiplied 1 : 4 by 100 to obtain the ratio 100 : 400. Multiplying by 50 would give a ratio of 50 : 200.





Simplifying ratios

Imagine that you have mixed a cordial drink using the above ratio of 50 : 200, which is equivalent to 1 : 4. Your friend Jack has made his drink by mixing 60 mL of cordial with 300 mL of water. He says his drink is stronger than yours, but it is not easy to tell by comparing the ratios of 50 : 200 and 60 : 300. It would be easier if we could compare the ratios in simplest form.

We can simplify a ratio in the same way we would simplify a fraction—by dividing each part in the ratio by a common factor. If we divide by the HCF, we will obtain the ratio in simplest form.

The HCF of the ratio 60 : 300 is 60.

$$\begin{array}{ccccc} & 60 & : & 300 & \\ \div 60 & \downarrow & & \downarrow & \div 60 \\ = & 1 & : & 5 & \end{array}$$

(we could also have obtained this ratio by doing a series of smaller divisions, such as ÷10, ÷6)

Now, instead of comparing 50 : 200 and 60 : 300, we can compare 1 : 4 with 1 : 5. The ratio 1 : 5 has one extra part of water to the same amount (1 part) of cordial, so Jack's drink is actually weaker than yours. (You may be able to tell this by looking at the colour, or by tasting it.)

Equivalent ratios are created by multiplying or dividing each part in the ratio by the same number.

Ratios can be simplified by dividing each part in the ratio by a common factor. Dividing by the highest common factor (HCF) will give the ratio in simplest form.

4.9 Ratio

Navigator

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Fluency

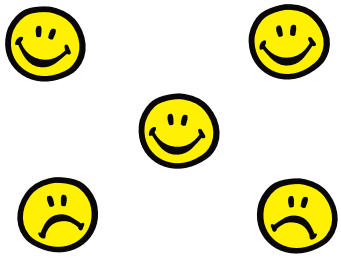
WE23

1 Write a ratio for each of the following.

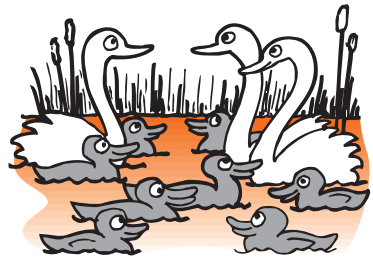
(a) circles to squares

(b) blue circles to red circles

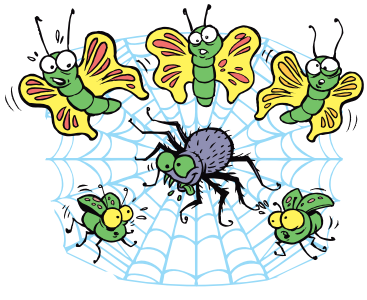
(c) happy faces to sad faces



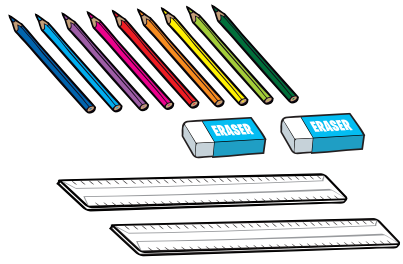
(d) ducks to swans



(e) spider to flies to butterflies

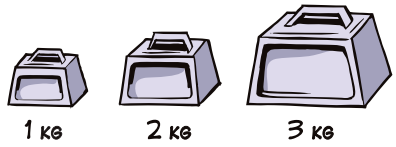


(f) rulers to pencils to erasers



(g) bees to flowers to leaves

(h) large mass to medium mass to small mass



2 Sumi has 3 canaries and 2 mice as pets.

WE24

(a) Write a ratio comparing the number of mice to the number of canaries.

(b) Write a ratio comparing the number of canaries to the total number of pets.

(c) Write the number of canaries as a fraction of the total number of pets.

(d) Write the number of mice as a percentage of the total number of pets.

3 Create pairs of equivalent ratios by multiplying or dividing each part in the ratio by the number given in brackets beside it.

(a) 1 : 3 (×2)	**(b)** 1 : 8 (×2)	**(c)** 1 : 11 (×2)	**(d)** 1 : 33 (×2)
(e) 4 : 5 (×3)	**(f)** 3 : 7 (×4)	**(g)** 6 : 9 (÷3)	**(h)** 8 : 12 (÷4)
(i) 10 : 8 (÷2)	**(j)** 18 : 9 (÷9)	**(k)** 8 : 5 (×7)	**(l)** 12 : 11 (×5)
(m) 48 : 18 (÷6)	**(n)** 20 : 6 (×5)	**(o)** 60 : 54 (÷3)	**(p)** 15 : 13 (×5)

4 Write the following ratios in simplest form, by dividing by the highest common factor (HCF).

(a) 2 : 10	**(b)** 3 : 12	**(c)** 4 : 18	**(d)** 6 : 21
(e) 21 : 14	**(f)** 24 : 15	**(g)** 32 : 20	**(h)** 48 : 36
(i) 54 : 64	**(j)** 56 : 68	**(k)** 72 : 81	**(l)** 38 : 95
(m) 75 : 50	**(n)** 120 : 80	**(o)** 105 : 60	**(p)** 240 : 66

5 There are 14 girls and 9 boys in a Year 7 maths class.

(a) The ratio of boys to girls is:

A 14 : 9 **B** 9 : 14 **C** 14 : 23 **D** 9 : 23

(b) The fraction of the whole class that are girls is:

A $\frac{9}{23}$ **B** $\frac{9}{14}$ **C** $\frac{14}{23}$ **D** $\frac{14}{9}$

Understanding

6 A packet of M&M's contains three colours: 12 red, 7 blue and 16 brown.

(a) Write the ratio of red to blue to brown M&M's.

(b) How many M&M's are in the packet?

(c) What fraction of the M&M's in the packet are red?

(d) What percentage of the M&M's are blue?

7 During a cricket season, Erika's team won 15 matches, drew 2 and lost 7.

(a) Write the won : lost : drew ratio for Erika's team for that season.

(b) What fraction of their games did Erika's team lose?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

8 The students at Hillview High School are voting for a school captain. The results show that for every 4 students who voted for Guy, 5 students voted for Dominic and 3 for Melissa.

(a) Write the ratio of the number who voted for Dominic to the number who voted for Guy, to the number who voted for Melissa.

(b) What fraction of the total vote did Dominic receive?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

9 There must be 1 instructor for every 6 students on a kayaking course.

(a) Write the comparison of students to instructors as a ratio.

(b) If 24 students want to do the course, use an equivalent ratio to determine how many instructors will be required.

(c) If 5 instructors are available, what is the maximum number of students that can do the course?

10 Tom's recipe for making concrete is to add gravel, sand and cement in the ratio 3 : 2 : 1 before adding water. Tom has 4 buckets of cement. Use an equivalent ratio to show how many buckets of sand and gravel he should add to get the correct concrete mixture.

11 'White gold' is commonly made by mixing gold and nickel in the ratio 9 : 1.

(a) Write the amount of nickel as a percentage of the total mixture.

(b) A jeweller wants to make 20 g of white gold. How much nickel will he require?

12 A survey found that 75% of Year 7 students were studying music.

(a) What percentage were not studying music?

(b) Use the two percentages to write a ratio that compares the group studying music to the group not studying music, in simplest form.

Reasoning

13

Which point divides the above line so that the ratio shorter part : longer part is:

(a) 1 : 7 **(b)** 5 : 3 **(c)** 1 : 3 **(d)** 1 : 1

14 There must be at least 1 teacher for every 20 students on a school excursion.

(a) If 68 students are going on an excursion, how many teachers are required? Give reasons for your answer.

(b) Use your answer to **(a)** to write the actual ratio of teachers : students, in simplest form.

15 Chloe made a cordial drink by mixing 30 mL of concentrate with 150 mL of water. Her friend Hannah made a cordial drink by mixing 80 mL of concentrate with 200 mL of water. Hannah said her cordial drink was 'stronger' than Chloe's. Chloe could not tell by looking.

(a) Write the ratio of concentrate : water for each girl's drink, in simplest form.

(b) Use the ratios to state whether Hannah is correct, giving a reason for your answer.

16 Mia has three cats and five dogs as pets.

(a) What fraction of her pets are dogs?

(b) What fraction of her pets are cats?

(c) If Mia adopts another cat, what fraction of her pets are cats now?

(d) After adopting the new cat, Mia moves into a share house with Shona who also has some cats and dogs. If the final ratio of cats to dogs is now 1 : 1, what is the smallest number of cats and dogs that Shona brings to the house?

Open-ended

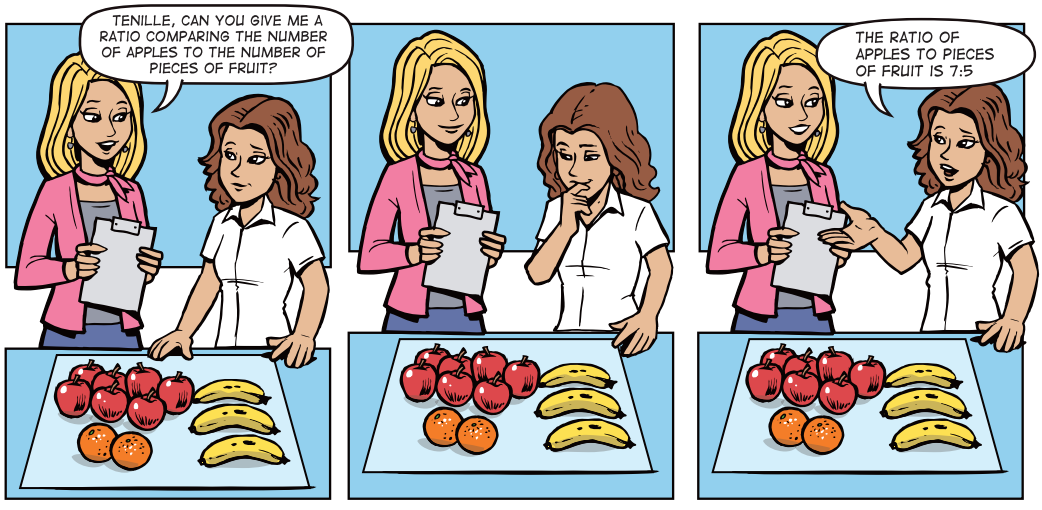
17 **(a)** What is the ratio of girls to boys in your class?

(b) Would the ratio be the same in all the Year 7 classes in your school? Explain your answer by giving examples.

(c) Would the ratio be the same for all school classes in Australia? Give some reasons for your answer.

18 Write three ratios that are equivalent to 4 : 7.

19



What mistake has Tenille made? What is the correct answer?