10

Transformation and visualisation 10

Why is The Pentagon a pentagon?

In 1941, a radical new design for a government building impressed the President of the USA.

Located near Washington, DC, the building known as 'The Pentagon' was built during World War II to house the War Department, now the United States Department of Defense. It is one of the largest buildings in the world, despite being only five storeys high (the steel required for extra storeys was needed for warships and weapons). The five-sided shape was chosen because it was the best fit to the shape of the land originally chosen for the location of the building. President Roosevelt changed the location, but kept the shape because, 'nothing like it has ever been done that way before'. The Pentagon is a regular pentagon. Each of the five sides and the five angles formed where the sides meet are equal. Because it is a regular shape, the Pentagon has a high degree of reflectional and rotational symmetry.

Forum

How do you think the builders of the Pentagon ensured that each of the five sides and angles were equal?

Looking at the Pentagon from above:

How many shapes can you identify within the main pentagon shape?

How many straight lines can you draw that divide the shape into two identical halves?

Many buildings and structures that are considered 'beautiful' are symmetrical. Do you think we are naturally 'drawn' towards symmetry?

Why learn this?

Everyday we see and work with geometrical figures. We get food and drink from cylindrical cans, pack things into prism-shaped boxes, and play sport on rectangular courts with spherical balls. Each of these objects has been designed so that its shape suits its purpose. Many careers involve design, from architecture (designing buildings), to industrial design (designing products for a mass market) to graphic design (designing the images used in advertising, books, magazines and websites). Knowledge of geometry and appreciation of symmetry is important in these and many other careers.

After completing this chapter you will be able to:

- translate, reflect and rotate shapes
- describe the process and compare the results of different transformations
- identify lines of symmetry
- identify the order of reflectional symmetry and the order of rotational symmetry
- recognise symmetry in design and nature
- visualise and draw 3D shapes
- draw elevations and plan views of 3D shapes.

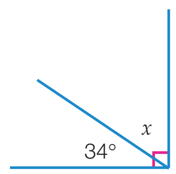
Recall 10

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

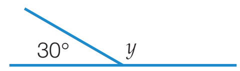
Worksheet R10.1

1 Find the size of the unknown angle in each of the following.

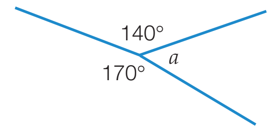
(a)



(b)



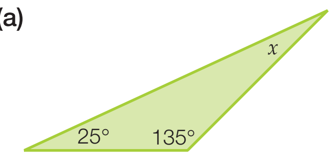
(c)



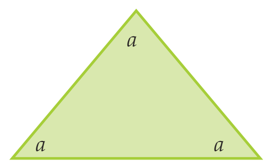
Worksheet R10.2

2 Find the size of the unknown angle in each of the following.

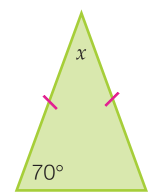
(a)



(b)



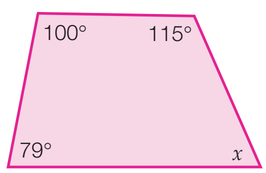
(c)



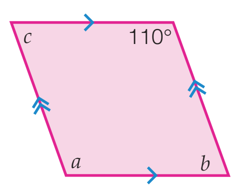
Worksheet R10.3

3 Find the size of the unknown angle in each of the following.

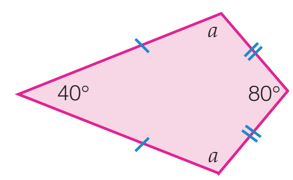
(a)



(b)



(c)



Worksheet R10.4

4 Draw an example of:

(a) a rhombus **(b)** a trapezium **(c)** an equilateral triangle

(d) a kite **(e)** an isosceles triangle **(f)** a parallelogram.

Mark all sides that are equal and/or parallel. Use letter names to name your figures.

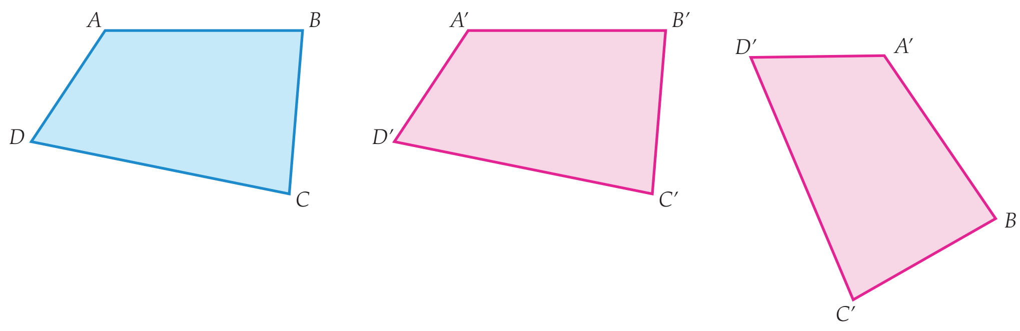
Key Words

asymmetrical	isometric	perpendicular distance	rotation
axis of symmetry	line of reflection	plan views	rotational symmetry
centre of rotation	order of reflectional symmetry	reflection	transformation
elevations	order of rotational symmetry	reflectional symmetry	translation
image	original	rotate	

Translations

10.1

A **transformation** changes a particular shape (or figure) or set of points according to a defined rule. A transformation may move, turn, flip or change the shape of a figure. The transformations we will deal with here are those that move a shape but retain its original shape and size. The first figure is called the **original** and the transformed shape is called the **image** of the transformation. We give the original shape letter names such as A, B, C, D and the image letter names such as A', B', C', D'. Transformations may change the position only, or they may involve a change in orientation such as a turn or a reflection.



Original

Image 1
Change in position only

Image 2
Change in position and orientation

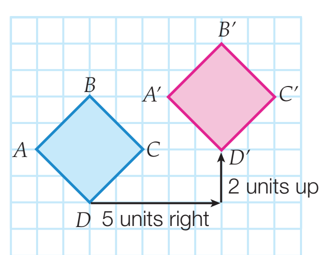
There are three types of transformations in which the shape does not change:

- translation
- reflection
- rotation.

Each type of transformation moves a figure in a particular way. Transformations can be combined to make a complex move and can be reversed to move the image back to its original position and orientation.

Translations

A **translation** occurs when a figure is shifted or slid across the page to change its position. When a figure undergoes translation, the image is moved without turning. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both. The example at right shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as [5, 2].

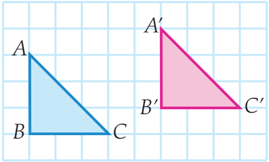


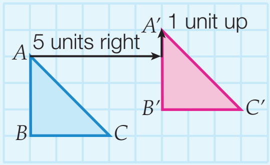
- Vertices of an image are denoted by a dash; e.g. A' or B'. We say 'A dash' or 'A prime'.
- To distinguish between the original figure and the translated image, you can use a different colour or broken lines.
- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

Worked Example 1

WE1

Describe the translation shown in the diagram.

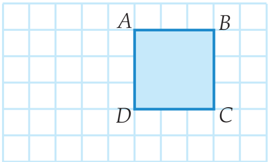


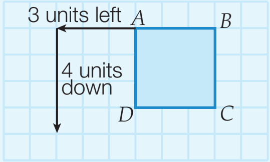
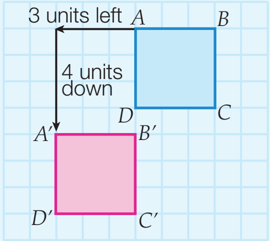
Thinking	Working
1 Start by selecting one of the vertices and count the units in the horizontal and vertical directions to get to the corresponding translated vertex.	
2 Write the translation in words or as an ordered pair.	5 units right and 1 unit up or [5, 1].

Worked Example 2

WE2

Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.



Thinking	Working
1 Start by selecting one of the vertices (A) and move according to the given translation. **2** Label this vertex with a letter using image notation (A').	
3 Repeat with the remaining vertices and then draw the resulting image in a different colour. **4** Label all vertices of the image using image notation.	

If our shape is drawn on the Cartesian plane, we can identify the coordinates of the vertices and then find the coordinates of the transformed shape after a given transformation. In this section, we will identify the coordinates of a shape after a given translation.

Worked Example 3

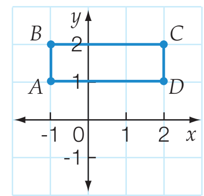
WE3

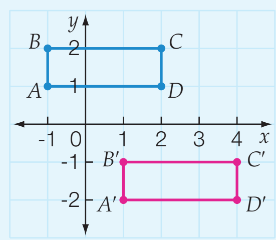
(a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (-1, 1)$, $B = (-1, 2)$, $C = (2, 2)$, $D = (2, 1)$
(b) Translate each vertex by the given translation, labelling each vertex with image notation. (Here, we move each point 2 units right and 3 units down. This is a translation of [2, -3].)	(b) 
(c) Identify the coordinates of each vertex on the resulting image.	(c) $A' = (1, -2)$, $B' = (1, -1)$, $C' = (4, -1)$, $D' = (4, -2)$
(d) Look for the connection between the original coordinates, the transformation and the image coordinates.	(d) Adding 2 to the x-coordinate of the original and subtracting 3 from the y-coordinate of the original will give the image coordinates.

10.1 Translations

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18

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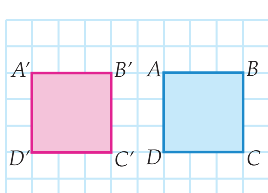
Equipment required: Grid paper for Questions **2**, **3**, **9**, **13(b)**, **15(c)**, **16** and **17**

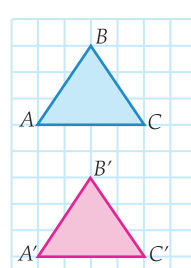
Fluency

1 Describe the translation shown in each of the following diagrams.

(a)

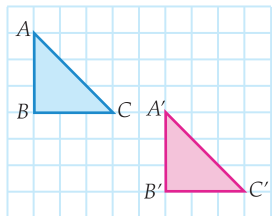


(b)

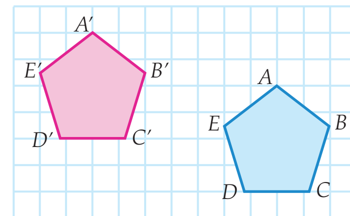


WE1

(c)



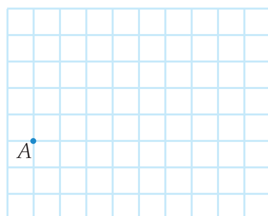
(d)



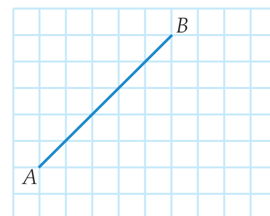
WE2

2 Copy each of the following onto grid paper and draw the resulting image after the translation.

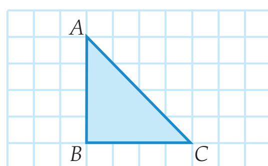
(a) 6 units right and 2 units up



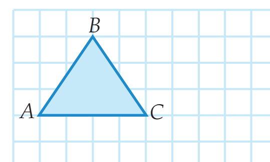
(b) 6 units left and 6 units down



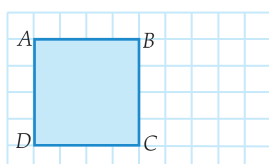
(c) 6 units left and 1 unit down



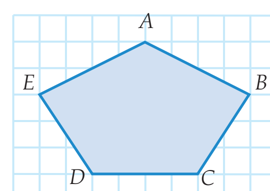
(d) 1 unit right and 6 units down



(e) [7, -5]



(f) [-6, 4]



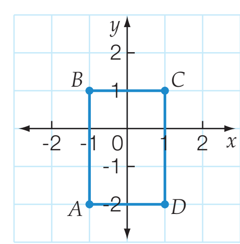
WE3

3 (a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 1 unit to the left and 2 units up.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



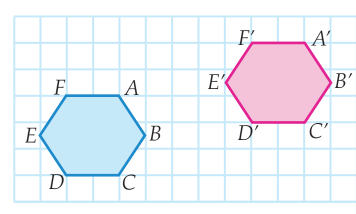
4 How has the hexagon in the diagram been translated?

A 7 units up and 2 units right

B 7 units right and 2 units up

C 7 units left and 2 units down

D 6 units right and 2 units up



5 What would be the reverse translation of 4 units left and 8 units down?

A 4 units up and 8 units right

B 4 units left and 8 units up

C 4 units left and 8 units down

D 4 units right and 8 units up

6 Write the translation [-2, 6] using direction instructions (i.e. right, left, up, down).

7 Write the translation of 3 units left and 4 units down using square brackets.

8 Write the reverse translation of 10 units right and 4 units up using square brackets.

Understanding

9 **(a)** Plot the points (-1, 4), (2, 3) and (0, 2) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form triangle *ABC*, then translate the shape 2 units right and 1 unit down, labelling the image with image notation.

(c) Write the coordinates of the translated points.

10 A figure is translated 4 units left, 3 units down, 6 units left and 7 units up. What would be the final position of the image compared to that of the original figure?

11 What single translation would be equivalent to 12 units down, 5 units right, 7 units left, 8 units up and 2 units right?

12 What single translation would be equivalent to a translation of [2, 3] followed by a translation of [-2, 6]?

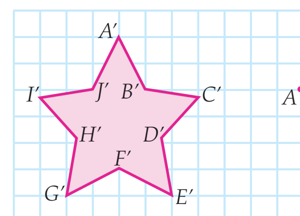
Reasoning

13 **(a)** Without plotting points, write the coordinates of the image after the line joining *A*(4, 2) and *B*(6, -1) is translated 2 units left and 2 units up.

(b) Plot the line *AB* and its image *A′B′* on a Cartesian plane to check your answer to part **(a)**.

14 If a particular translation was 2 units right, what would be the single translation that would be equivalent to repeating 2 units right a hundred times?

15 The following diagram is of a translated image and a single point of the original figure.



(a) What translation has occurred to the original figure?

(b) What is the reverse translation?

(c) Use this to reverse the translation and draw the original figure.

Open-ended

16 Plot three points on a Cartesian plane and write their coordinates. Decide upon a particular translation and use this translation to translate your shape. Draw the image after the translation and write down the coordinates of the translated points using image notation.

17 On grid paper, draw a figure that has 4 vertices. Then, translate that figure in any direction and identify the reverse translation required to get the image back to the original figure.

18 Provide an explanation either for or against the following statement:

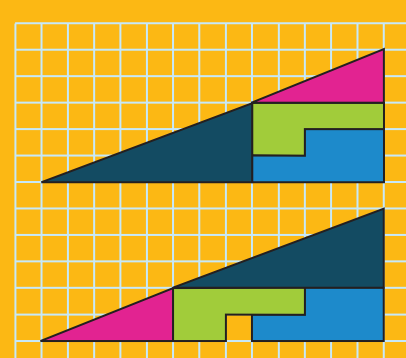
'If a figure undergoes more than one translation, the order in which you translate a figure does not affect the final position of the image.'

Outside the Square Puzzle

Jigsaw paradox

This Jigsaw Paradox orginates from the famous Curry's Paradox and was modified by Martin Gardner in 1956. This paradox or puzzle shows two arrangements of four identical shapes to create two right-angled triangles, but one has a 1×1 hole in it. Both triangles have 13 units for the base and 5 units for the height.

How can this be true? Can you explain what happened to the missing 1×1 square?



10.2 Reflections

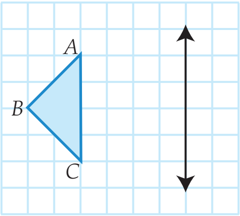
A **reflection** is a transformation that creates an image of the original figure in the same way that a mirror creates an image when you look into it. A reflection is therefore often called a mirror image. It is a reversed or 'flipped over' version of the original about a **line of reflection**. A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed.

Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side. The distance used is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection. To reflect a figure, reflect each vertex on the figure and then join the points in order.

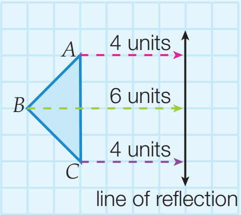
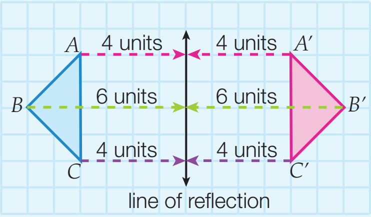
Worked Example 4

WE4

Copy the following figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



A point on the mirror line or line of reflection doesn't move when you reflect it.

Thinking	Working
1 Measure the perpendicular distance between each vertex and the line of reflection.	
2 Use the distance between the vertices on the line of reflection to plot the points on the opposite side of the line of reflection and reproduce the original figure.	
3 Label the image vertices with image notation.	

- A reflection always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same; for example, the size of the side lengths or angles will stay the same.

We can find the image of a shape drawn on a Cartesian plane after it is reflected in either the x- or y-axis.

Worked Example 5

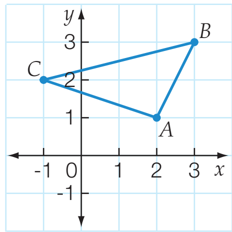
WE5

(a) **(i)** Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

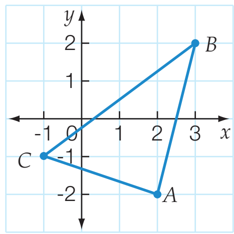


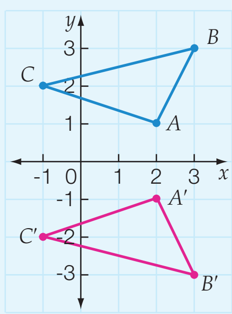
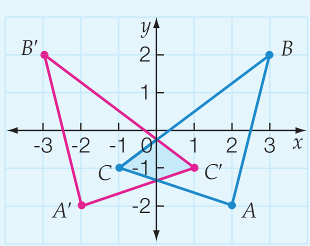
(b) **(i)** Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.



Thinking	Working
(a) **(i)** Use the x- and x-axes to identify the Cartesian coordinates of the points.	(a) (i) $A = (2, 1)$, $B = (3, 3)$, $C = (-1, 2)$
(ii) Reflect each point in the x-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	(ii) 
(iii) Identify the coordinates of each vertex on the resulting image.	(iii) $A' = (2, -1)$, $B' = (3, -3)$, $C' = (-1, -2)$
(iv) Look for the connection between the coordinates of the original and the image.	(iv) *The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original.*
(b) **(i)** Use the x- and y-axes to identify the Cartesian coordinates of the points.	(b) (i) $A = (2, -2)$, $B = (3, 2)$, $C = (-1, -1)$
(ii) Reflect each point in the y-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	(ii) 

(iii) Identify the coordinates of each vertex on the resulting image. | (iii) $A' = (-2, -2)$, $B' = (-3, 2)$, $C' = (1, -1)$

(iv) Look for the connection between the coordinates of the original and the image. | (iv) The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.

10.2 Reflections

Navigator

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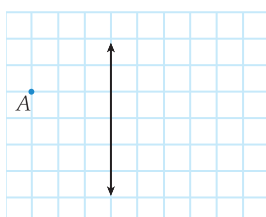
Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13

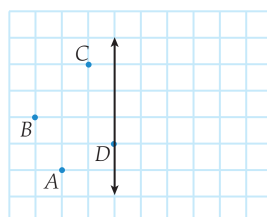
Equipment required: Grid paper for Questions **1–3**, **5–8** and **10–13**

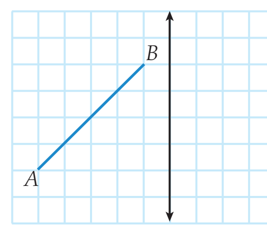
Fluency

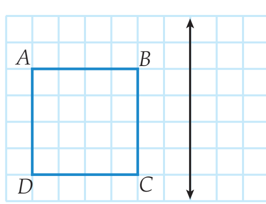
WE4

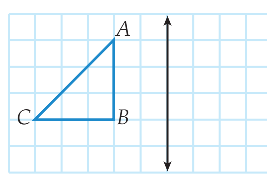
1 Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection shown.

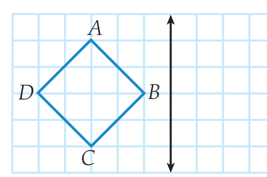
(a)


(b)


(c)


(d)


(e)


(f)


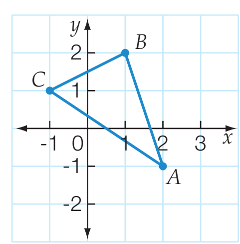
WE5

2 **(a)** **(i)** Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

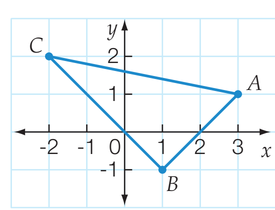


(b) **(i)** Write down the coordinates of each vertex on the given shape.

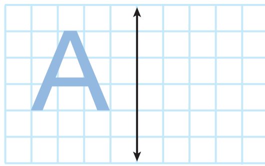
(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

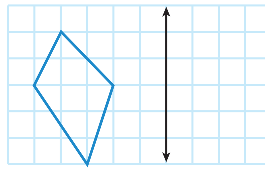
(iii) Write the coordinates of the reflected vertices.

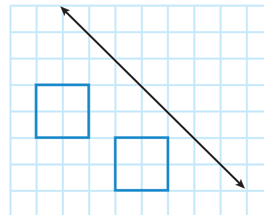
(iv) Explain how the coordinates of the image could be found without drawing the shape.

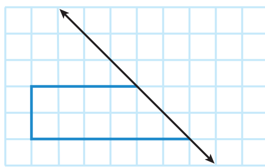


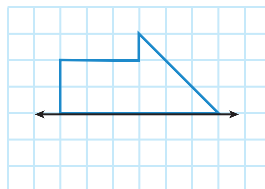
3 Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection shown.

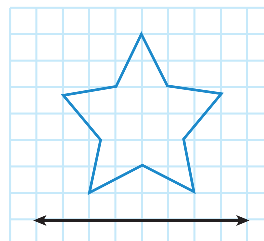
(a)


(b)


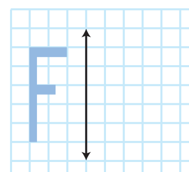
(c)


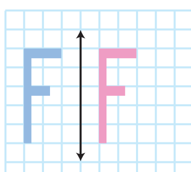
(d)


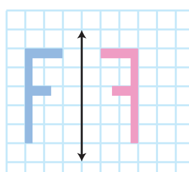
(e)


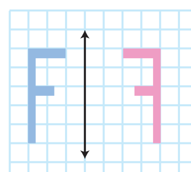
(f)


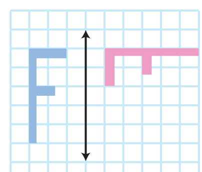
4 Which of the following options is the correct reflection of the capital F in the vertical line of reflection?



A


B


C


D


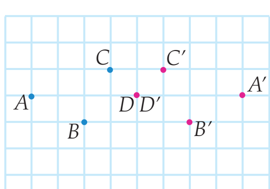
Understanding

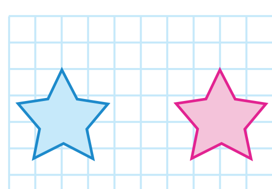
5 (a) Plot the points (-1, 5), (3, 1) and (2, -1) on a Cartesian plane and label them *A*, *B* and *C*.

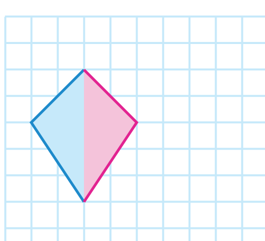
(b) Join the points to form triangle *ABC*, then reflect the shape first in the x-axis and then in the y-axis.

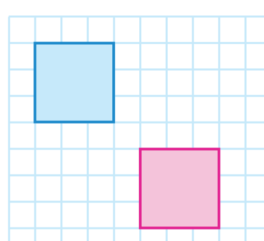
(c) Write the coordinates of the points after the second reflection.

6 Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

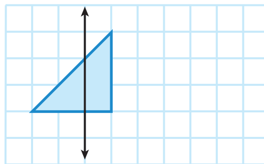
(a)


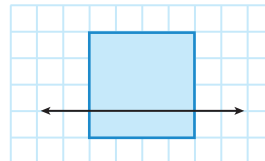
(b)


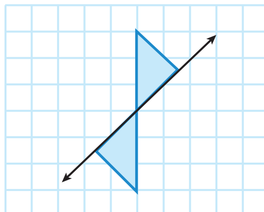
(c)


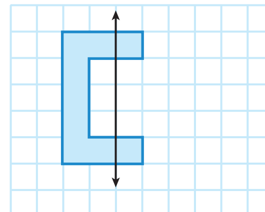
(d)


7 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.

(a) 

(b) 

(c) 

(d) 

Reasoning

8 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the x-axis.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

(c) Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the y-axis.

(d) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(c)**.

9 Bruno has a digital clock on his glass bedside table. When he looked at the time, he noticed that at that particular instance, the time 8.30 was reflected perfectly on the table. Thinking this to be a little odd, he decided to find out the following.

(a) List the number of minutes past the hour when the minute part of the time reflects the correct time (include 00).

(b) In a 12-hour mode, list the hours when the hour part of the time reflects the correct time.

(c) In a 12-hour period, use your answers to parts **(a)** and **(b)** to find how many times the reflection on the table gives the correct time in a 12-hour mode.

(d) In a 24-hour period, how many times does the reflection in the table give the correct time in a 24-hour mode?

(e) Is your answer to part **(d)** double your answer to part **(c)**? Explain your answer.

Open-ended

10 Plot three points on a Cartesian plane and write their coordinates. Reflect this shape in either the x- or y-axis. Draw the image after the reflection and write down the coordinates of the reflected points using image notation.

11 On grid paper, draw a figure with four vertices. Reflect this figure in:

(a) a horizontal line of reflection

(b) a vertical line of reflection.

12 Repeat Question **11** with a figure with five, six, or seven vertices.

13 On grid paper, draw a figure with four vertices.

(a) Reflect this figure in a horizontal line of reflection and then in a vertical line of reflection.

(b) Reflect the same figure in the vertical line of reflection and then in the horizontal line of reflection.

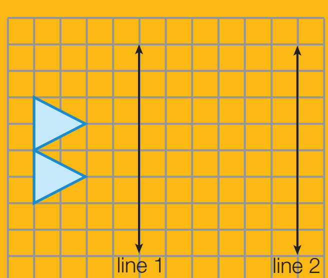
(c) Comment on the images found in parts **(a)** and **(b)**.

Outside the Square — Problem solving

Mirror, mirror on the wall, does it matter where I stand at all?

Equipment required: 1 brain, grid paper

The diagram below consists of two blue triangles and two vertical lines of reflection.



To begin, reflect the two triangles in the line of reflection labelled 'line 1' and then in the line of reflection labelled 'line 2' and note the final location of the image. Next, switch the order in which the two triangles are reflected so that they are reflected in line 2 first, and then in line 1, and note the final location of the image.

Does the order in which these triangles are reflected in either line affect the final location and orientation of the final image? What did you notice about the final image? Can this transformation be described another way?

To justify your decision, experiment with different situations, such as drawing the two triangles originally on the right-hand-side of line 2 or in-between the lines. You can also alter the position of the lines of reflection. For example, line 1 and line 2 could be two horizontal lines or one could be horizontal and the other vertical.

Strategy options

- Draw a diagram.
- Test all possible combinations.

10.3 Rotations

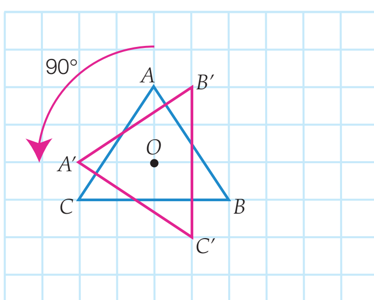
We also transform a figure when we **rotate** it around a fixed point. This is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter *O*.

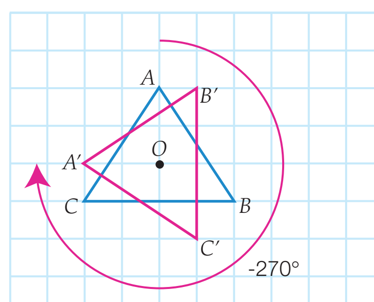
When rotating any figure you need to know:

- the location of the centre of rotation
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The centre of rotation can be located inside or outside the figure and is the only point that does not rotate. The size of the angle of rotation is generally in multiples of 30° or 45°. Common angles of rotation are 30°, 45°, 60°, 90°, 180° and 270°. A rotation of 360° will rotate the image of the figure through a full revolution about the centre of rotation and will return the figure to its original position.

- The vertices of a rotated image are labelled in the same order as the original figures.
- Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.
- Two rotations that result in an image in the same position will always add to 360°. For example, 90° clockwise is equivalent to 270° anticlockwise as 90° + 270° = 360°.
- A rotation in an anticlockwise direction is a positive rotation.
- A rotation in a clockwise direction is a negative rotation.

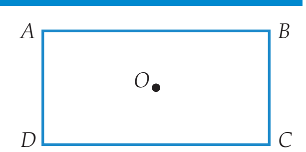


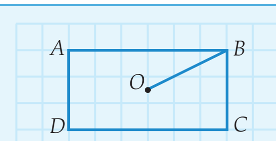


Worked Example 6

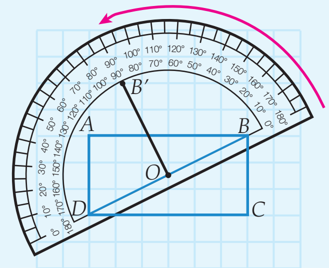
WE6

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about *O*.

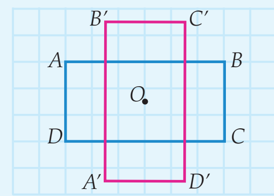


Thinking	Working
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.	

2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex. Plot the resulting point. The new point must be the same distance from O as the original point.



3 Continue the process with the other vertices and connect the resulting points to produce the original figure in its rotated position.

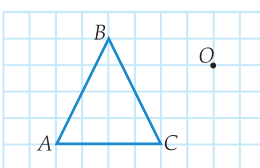


The same steps are used when the centre of rotation is outside the shape. The distance between the vertices and the centre of rotation is the same before and after the rotation, similar to reflection of an object in a line of reflection. Measuring this distance and checking that it is the same before and after the rotation is an easy way to check that the rotation is correct.

Worked Example 7

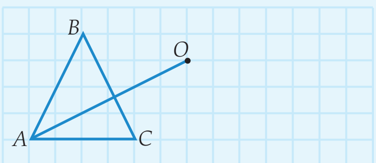
WE7

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (-180°) about O.

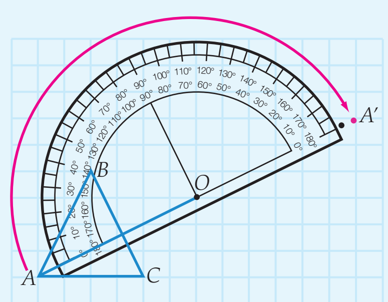


Thinking	Working

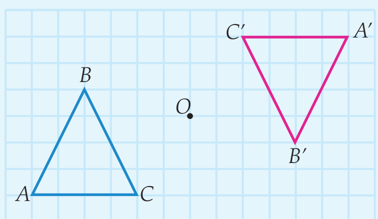
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.



2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex and plot a point. Extend AO through this point and mark the point A' so that OA' is the same length as OA.



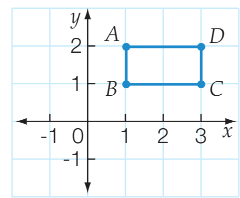
3 Continue the process with the other vertices and connect the resulting points to reproduce the original figure in its rotated position.



We can find the coordinates of an image after a rotation of 90°, 180° or 270° in a clockwise or anticlockwise direction about the origin.

Worked Example 8

WE8



(a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.

(b) **(i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) **(i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

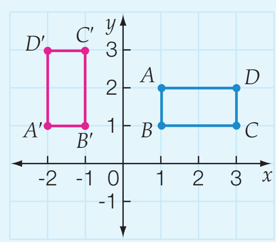
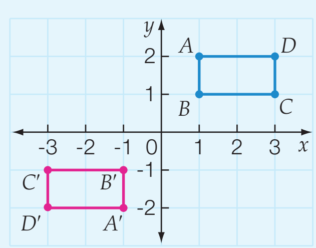
(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) **(i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

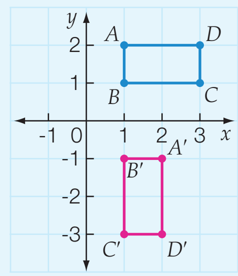
Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (1, 2)$, $B = (1, 1)$, $C = (3, 1)$, $D = (3, 2)$
(b) **(i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 90° in an anticlockwise direction about the origin.)	(b) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-2, 1)$, $B' = (-1, 1)$, $C' = (-1, 3)$, $D' = (-2, 3)$
(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.	(iii) The x-coordinates of the image vertices are the negative of the y-coordinates of the original vertices and the y-coordinates of the image vertices are the x-coordinates of the original vertices.
(c) **(i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 180° in an anticlockwise direction about the origin.)	(c) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-1, -2)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, -2)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) *All the coordinates of the image vertices are the negative of the original coordinates of the vertices.*

(d) **(i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 270° or -90°.)

(d) (i)



(ii) Identify the coordinates of each vertex on the resulting image.

(ii) $A' = (2, -1)$, $B' = (1, -1)$, $C' = (1, -3)$, $D' = (2, -3)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) *The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.*

10.3 Rotations

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 (a), Q11, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12 Column 2, Q13, Q14, Q15, Q16

Answers page 689

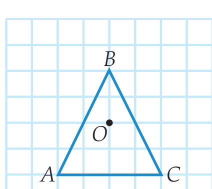
Equipment required: Protractor and grid paper for Questions **1**, **2**, **3**, **6**, **9 (b)**, **10**, **14** and **16**

Fluency

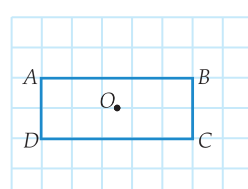
1 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

WE6

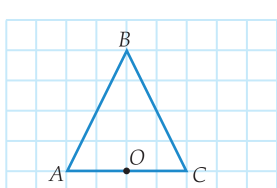
(a) Rotate 180° in an anticlockwise direction (180°) about *O*.



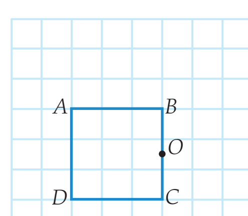
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.



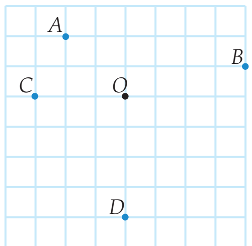
(d) Rotate 90° in a clockwise direction (-90°) about *O*.



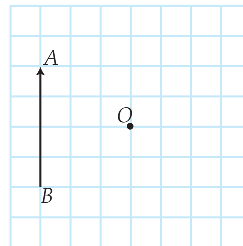
WE7

2 Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

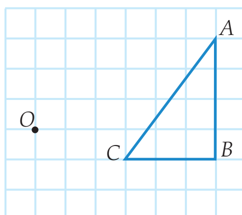
(a) Rotate each of the points 90° in a clockwise direction (-90°) about *O*.



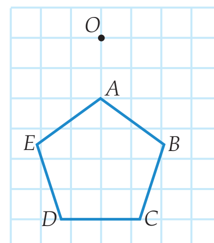
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.

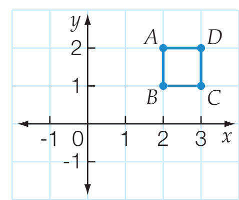


(d) Rotate 90° in an anticlockwise direction (90°) about *O*.



WE8

3 (a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) (i) Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) (i) Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) (i) Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

4 Which of the following statements about rotation is incorrect?

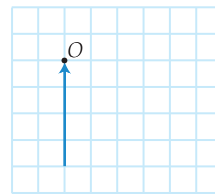
A Rotation can be in either a clockwise or an anticlockwise direction.

B A rotation of 360° will result in the image located in the same position as the original figure.

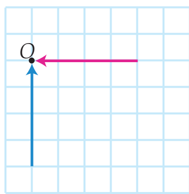
C A rotation will always produce an image that is a perfect replica and in the same orientation as the original figure.

D A rotation of 90° in a clockwise direction is equivalent to a rotation of 270° in an anticlockwise direction about the same centre of rotation.

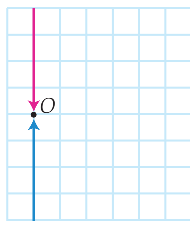
5 This arrow turns about the centre of rotation O. What does the shape look like after a turn of 180° anticlockwise?



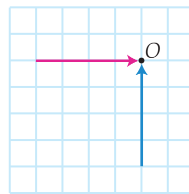
A



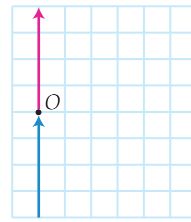
B



C



D



Understanding

6 (a) Plot the points (1, 4), (2, 4), (2, 2) and (1, 2) on a Cartesian plane and label them A, B, C and D.

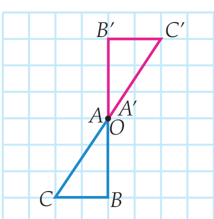
(b) Join the points to form a figure $ABCD$, then rotate the figure 90° in a clockwise direction about point A.

(c) Write the coordinates of the rotated points using image notation.

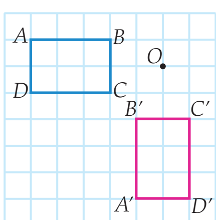
7 A clockwise rotation of 120° is the same as an anticlockwise rotation of how many degrees?

8 Find the rotation (the size of the rotation and the direction) that has taken place to produce the following images. Identify more than one rotation that will achieve the same result.

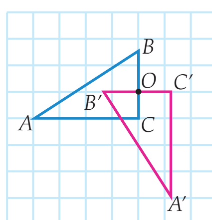
(a)



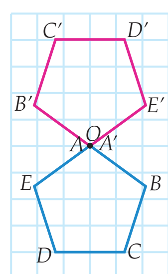
(b)



(c)



(d)



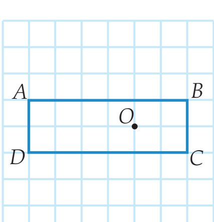
Reasoning

9 (a) Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(3, -2)$ is rotated 180° clockwise about the origin.

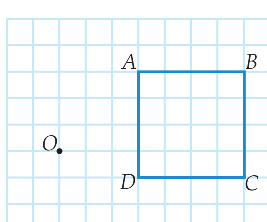
(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

10 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

(a) Rotate 90° in an anticlockwise direction about O.



(b) Rotate 90° in a clockwise direction about O.



11 The following are the capital letters of the alphabet.

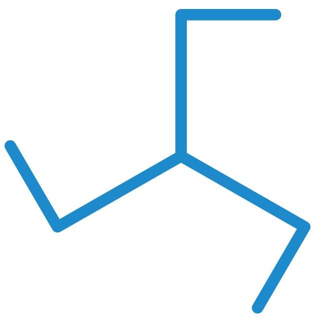
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which letters of the alphabet are able to be rotated about a centre of rotation in either direction and at an angle of less than 360° to produce an identical image in the same orientation as the original letter?

(b) Four of these letters have something else in common with each other. Can you identify these four letters? Describe the similarity you have noticed.

12 Estimate the smallest angle through which the following figures need to be rotated so the resulting image will match the original figure.

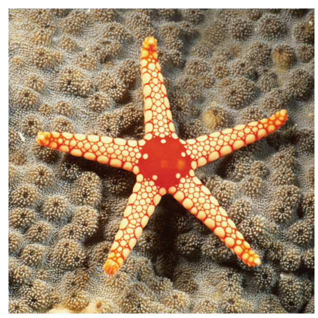
(a)



(b)

(c)

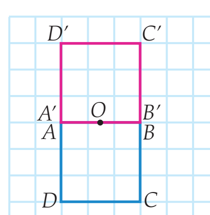
(d)



(e)

(f)

13 Is the following transformation a rotation of 180° around point *O*, a reflection along the line *AB* or both? Provide an explanation.



Open-ended

14 Plot four points on a Cartesian plane and write their coordinates. Decide upon a particular rotation and use this rotation to rotate your shape. Draw the image after the rotation and write down the coordinates of the rotated points using image notation.

15 Explain whether you agree or disagree with the following statement.

'A rotation of 180° in either direction and about any point of rotation can be equivalent to reflecting the same figure in a line of reflection through that point.'

16 On grid paper, draw two figures, a regular and an irregular polygon with the same number of sides. Decide upon a centre of rotation for each polygon that is on a vertex of that polygon.

(a) Rotate each of the figures by 90° about the centre of rotation in a clockwise direction and draw the image.

(b) Repeat this four times.

(c) Rotate the figures 45° about the centre of rotation.

(d) Repeat this eight times.

(e) Comment on the final images in parts **(b)** and **(d)**.

Outside the Square — Puzzle

Tangram magic

Equipment required: 1–2 brains, 2 pieces of grid paper, blank paper, ruler, pens and scissors

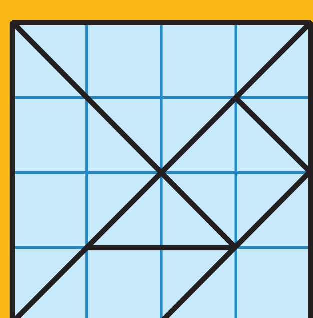
Tangram is an ancient Chinese puzzle. The object of the puzzle is to rearrange the pieces of a square (the puzzle pieces) to form as many different shapes using all seven pieces.

Before you start this puzzle you will need to make the seven pieces required to form the shapes.

Step 1 On a piece of grid paper, rule a square. The larger the square, the larger the pieces will be to work with. An 8 cm × 8 cm square is recommended.

Step 2 Use the tangram template below and divide your square into seven pieces as shown.

Step 3 Cut out your pieces carefully.



The tangram pieces will be:

- one square
- two small congruent (identical) isosceles triangles
- two large congruent isosceles triangles
- one medium isosceles triangle
- one parallelogram.

Once the pieces are cut out, it is time to begin. Form as many of the 12 shapes given below with your tangram pieces. You can translate, reflect and rotate the pieces, but remember to use all seven pieces.

When you have formed a shape, draw it carefully on blank paper, showing how the pieces fit together. Keep a list of the shapes you have made.

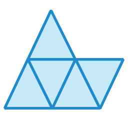
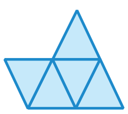
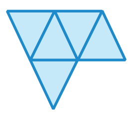
Investigation

Equipment required: 1 or 2 brains, isometric grid paper, isometric dot paper. (A computer drawing program may be used to produce the required shapes.)

Diamonds are forever!

A frieze is a decorative horizontal strip that consists of a repeating pattern. The repeating pattern is formed by combining shapes with translations, reflections and rotations. A frieze contains a high degree of symmetry.

Throughout this investigation you will be asked for different ways to connect your shapes. Mirror reflections or rotations are not considered to be different.

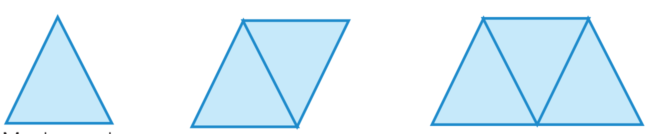
 and  and  are not different ways to connect five equilateral triangles.

The Big Question

Can you make a frieze with at least five different shapes made from equilateral triangles, diamonds and trapeziums that includes translations, reflections and rotations?

Engage

If an equilateral triangle or moniamond is reflected along an edge it becomes a diamond (rhombus). If the triangular shape is reflected along another edge it becomes a triamond (trapezium).



Moniamond Diamond Triamond

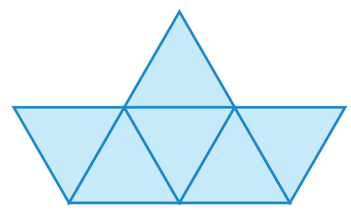
1 On isometric grid paper, draw the three different ways that four equilateral triangles can be connected to make a new shape. These are called tetriamonds.

2 On isometric grid paper, draw the four different ways that five equilateral triangles can be connected to make a new shape. These are called pentiamonds.

Explore

3 Six equilateral triangles can be connected to make a new shape. These are called hexiamonds. There are 12 different hexiamond shapes that can be made. Here is an example:

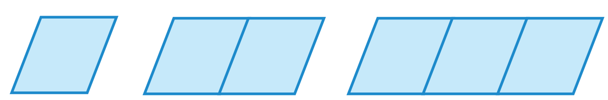
On isometric grid paper, copy the hexiamond on the right and find the 11 other hexiamond shapes.



The 12 names given below might help you find the shape. Can you match each name to a shape?

- bar (parallelogram)
- crook (club)
- crown
- sphinx
- snake
- yacht
- chevron (bat)
- signpost (pistol)
- lobster
- hook (shoe)
- hexagon
- butterfly

If a diamond shape (rhombus) is translated by a full side length and then joined to the original shape, it makes a shape we can call a straight bidiamond. If the diamond shape is translated again to make three diamond shapes joined together in a row, the result is a shape we can call a straight tridiamond.



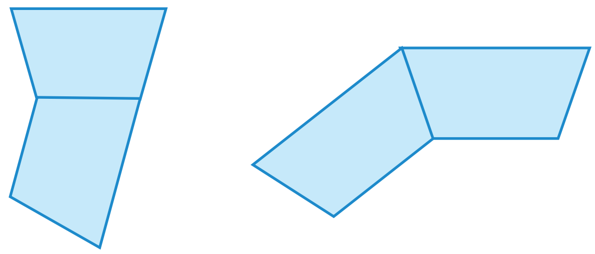
Diamond Straight bidiamond Straight tridiamond

Three diamonds can be connected to make tridiamonds of different shapes, including the straight tridiamond given above.

4 Nine of the 12 hexiamonds found in Question **3** can be made by reflecting, translating and rotating diamonds to make these tridiamonds. On isometric dot paper, draw the nine tridiamonds showing the diamonds that have been used to form them.

5 One of the hexiamond shapes when reflected along one of its sides will form a six-pointed star. Find the shape, reflect it to form the star and then translate the shape to make a row of three six-pointed stars. Draw your pattern on isometric dot paper.

6 There are nine ways that two triamonds (trapeziums made from three equilateral triangles) can be joined exactly edge to edge to make different shapes. On isometric dot paper, draw the nine triamonds (including the two given below) showing the triamonds that have been used to form them.



7 In how many ways can three triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

8 In how many ways can four triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

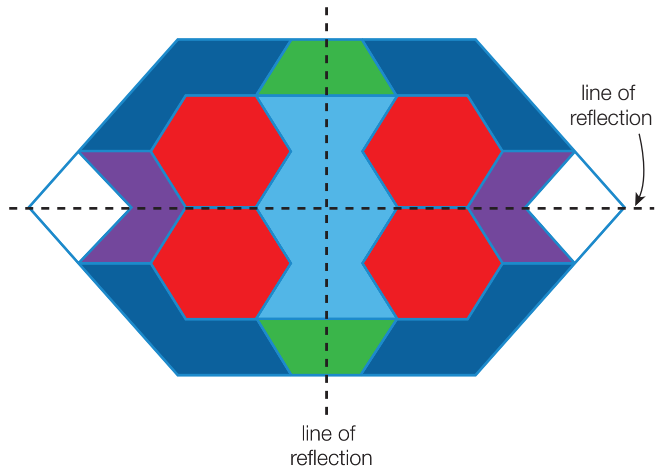
Strategy options
- Draw a diagram.
- Guess and check.
- Look for a pattern.

Explain

9 For two hexiamonds found in Question **3**, two tridiamonds found in Question **4** and two shapes found in Question **7**, explain how they were formed in terms of translation, reflection and rotation.

Elaborate

10 Answer the Big Question by making a frieze with at least five different shapes made from equilateral triangles, diamonds and trapeziums that includes translations, reflections and rotations. Your frieze pattern needs to be at least four triangles high and nine triangle lengths long. An example is given below. Indicate any lines of reflection.



Evaluate

11 Did you find it hard to find all the different shapes? What made it difficult?

12 Did the names help you to find some of the shapes in Question **3**?

Extend

There are at least 40 ways of using the 12 different shapes found in Question **3** to cover a rhombus of side 6 units. Can you find one?

10.4 Combined transformations

We can combine translations, reflections and rotational transformations into one transformation of a figure.

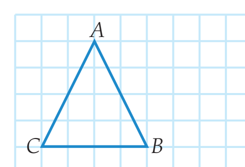
When combining transformations, the transformations are performed in sequence (one after the other) and, after each transformation, the vertices are labelled with an additional dash. For example, the vertex A''' has been transformed by three different transformations.

- Translations, reflections and rotations can be combined into a sequence of transformations.
- A side length can act as the line of reflection during a combined transformation.
- Different colours can be used to distinguish between several transformations.
- The properties of the three different types of transformations still hold true for combined transformations.

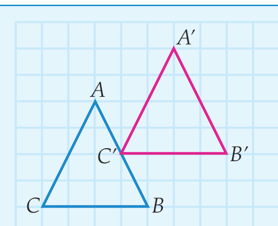
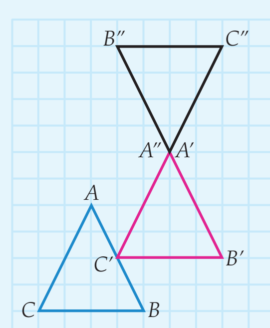
Worked Example 9

WE 9

Copy the following figure onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.



A translation of 3 units right and 2 units up, followed by a rotation of 180° in a clockwise direction about the vertex A'.

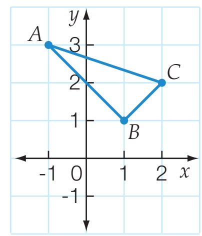
Thinking	Working
1 Perform the first identified transformation in the instructions. Make sure you use the correct notation with the transformed vertices.	
2 Perform each subsequent transformation on the next transformed image. You may want to distinguish each transformation by using a different colour.	

If a figure is drawn on a Cartesian plane, we can perform more than one type of transformation on it. For example, we can perform a rotation, then a translation. We could then reflect the figure in the x- or y-axis. We can then identify the coordinates of the final image.

Worked Example 10

WE10

For the figure given opposite:



(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

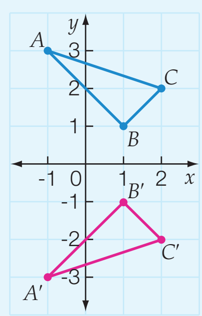
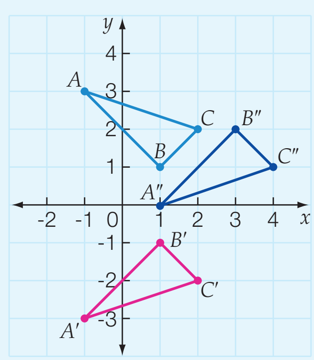
(c) write the coordinates of the reflected vertices

(d) translate this image 2 units right and 3 units up

(e) write the coordinates of the translated vertices

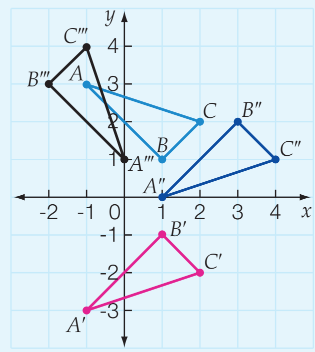
(f) rotate this translated image 90° in an anticlockwise direction about the origin

(g) write the coordinates of the vertices of the final image.

Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (-1, 3)$, $B = (1, 1)$, $C = (2, 2)$
(b) Reflect each vertex in the x-axis.	(b) 
(c) Identify the coordinates of each vertex on the resulting image.	(c) $A' = (-1, -3)$, $B' = (1, -1)$, $C' = (2, -2)$
(d) Translate each vertex with the given translation. (Here, we translate each vertex 2 units to the right and 3 units up.)	(d) 
(e) Identify the coordinates of each vertex on the resulting image.	(e) $A'' = (1, 0)$, $B'' = (3, 2)$, $C'' = (4, 1)$

(f) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 90°.)

(f)



(g) Identify the coordinates of each vertex on the resulting image.

(g) $A''' = (0, 1)$, $B''' = (-2, 3)$, $C''' = (-1, 4)$

10.4 Combined transformations

Navigator

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Q1, Q2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8 Column 1, Q9 Column 1, Q10, Q11, Q13, Q15	Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q15	Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q14, Q15

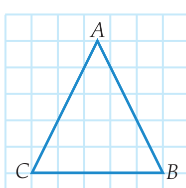
Equipment required: Protractor and grid paper for Questions **1–3**, **6**, **10 (b)** and **13–15**

Fluency

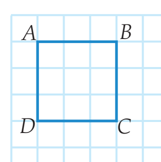
WE9

1 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.

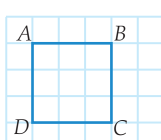
(a) A translation of 3 units left and 1 unit down, followed by a reflection in the line $B'C'$.



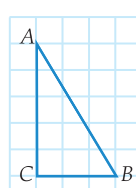
(b) A translation of 5 units right and 2 units down, followed by a reflection in the line $D'C'$.



(c) A translation of 5 units right, followed by a rotation of 90° about B' in a clockwise direction.



(d) A translation of 3 units left and 4 units down, followed by a rotation of 180° about C' in a clockwise direction.



2 For the figure given opposite:

WE10

(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

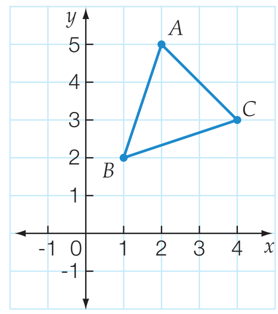
(c) write the coordinates of the reflected vertices

(d) translate this image 5 units left and 3 units up

(e) write the coordinates of the translated vertices

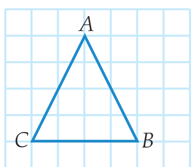
(f) rotate this translated image 180° in a clockwise direction about the origin

(g) write the coordinates of the vertices of the final image.

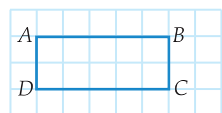


3 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each of the vertices appropriately after each transformation (for **(e)–(h)**, point O is a point on or inside the figure and is reflected with the shape).

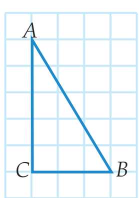
(a) Reflect along the line CB and translate 6 units right and 2 units up.



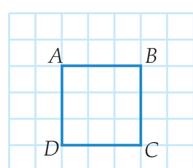
(b) Reflect along the line BC and translate 1 unit left and 3 units down.



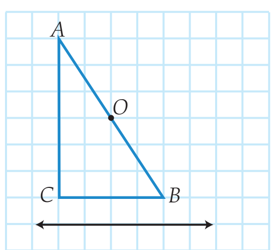
(c) Rotate 90° in a clockwise direction about vertex A and translate 8 units down.



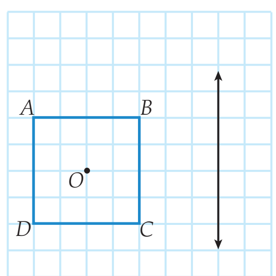
(d) Rotate 180° in an anticlockwise direction about vertex D and move 2 units left and 6 units up.



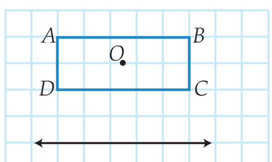
(e) Reflect along the line of reflection and then rotate 90° in an anticlockwise direction about the centre of rotation O'.



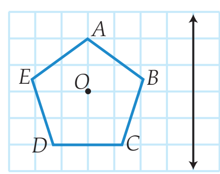
(f) Reflect along the line of reflection and then rotate 90° in a clockwise direction about the centre of rotation O'.



(g) Rotate 90° in an anticlockwise direction about the centre of rotation O and then reflect in the line of reflection.



(h) Rotate 180° in a clockwise direction about the centre of rotation O and then reflect in the line of reflection.



4 Which transformation reverses the order in which the vertices are labelled?

A rotation and translation
B reflection
C translation
D rotation

5 Which transformation can result in an image in the same position and orientation as the original figure?

A rotation and translation
B translation
C reflection
D rotation

Understanding

6 (a) Plot the points (-1, 2), (2, 3) and (3, -1) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form a triangle *ABC*, then translate the shape 3 units right and 1 unit down.

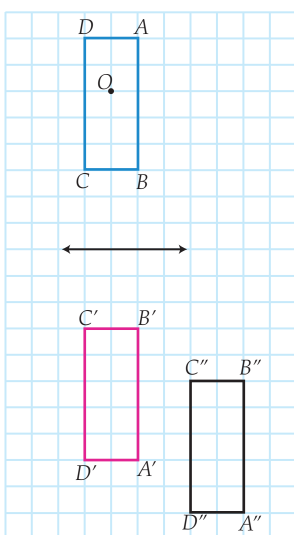
(c) Now, rotate the figure 90° in a clockwise direction about the origin.

(d) Write the coordinates of the vertices of the final image using image notation.

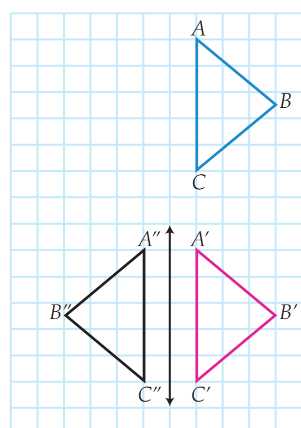
7 For the combined transformations in Question **1**, identify the necessary combined transformation that would move the image back to the position and orientation of the original figure.

8 Describe the transformations that have taken place to move the original figure in blue to the position in pink, then to its final position in black. List the relevant names of each transformation.

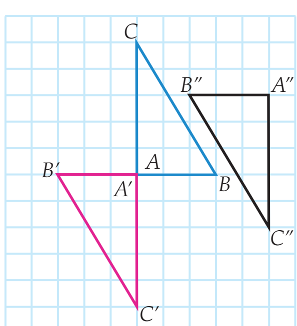
(a)



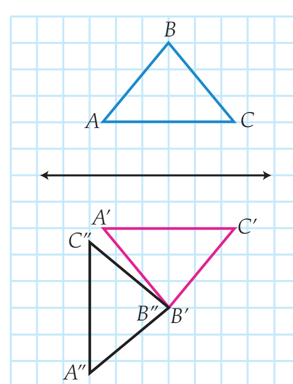
(b)



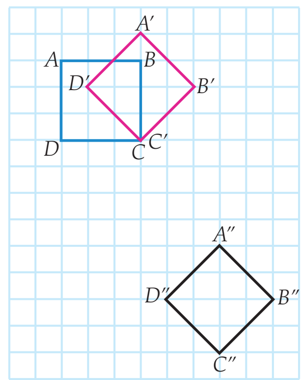
(c)



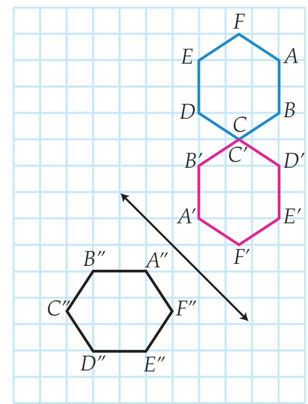
(d)



(e)



(f)



9 For the combined transformations stated in each part of Question **8**, identify the transformations required to move the final image back to the original position and orientation of the figure.

Reasoning

10 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 3)$ and $B(4, -1)$ is translated 1 unit right and 5 units down, then reflected in the y-axis.

(b) Plot the line AB, the first image $A'B'$ and the final image $A''B''$ on a Cartesian plane to check your answer to part **(a)**.

11 Sarah has written a computer program to transform pictures of tiles. There are only two instructions in her program.

1 Reflect vertically in the line of reflection through the centre of each tile.

2 Rotate 90° clockwise around the given centre of rotation.

Sarah wants to transform the first pattern to the second pattern.

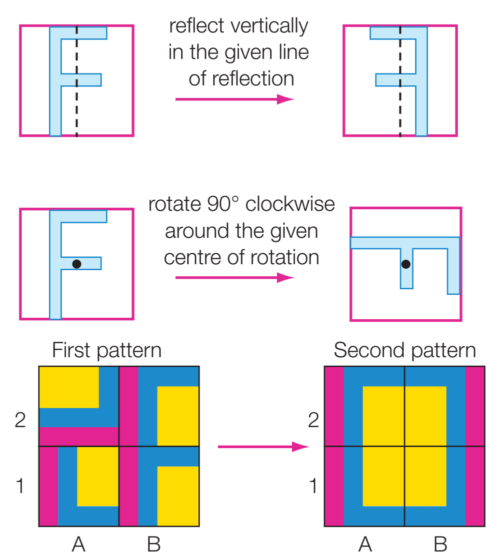
Copy and complete the following instructions to transform the tiles B1 and B2. You must only reflect vertically and rotate 90° clockwise through the centre of each tile.

A1 – Tile is in the correct position.

A2 – Reflect vertically, and then rotate 90° clockwise.

B1 – Rotate 90° clockwise and then …

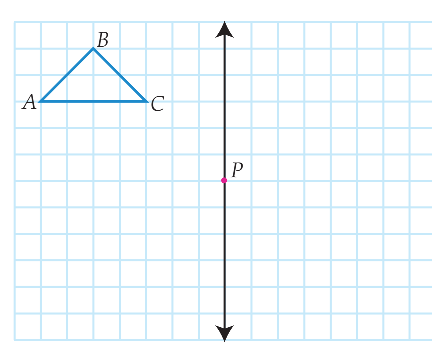
B2 – …



12 The following combined transformation is performed on triangle ABC.

- A reflection in the given line of reflection.
- A rotation of 180° about point P.
- A reflection in the given line of reflection.

Find a single transformation that will move the image $A'''B'''C'''$ back to the original position of the figure. Draw a diagram to show each required transformation.



Open-ended

13 Plot three points on a Cartesian plane and write their coordinates. Decide upon two particular transformations and use them to transform your figure. Draw the image after both transformations and write down the coordinates of the transformed vertices using image notation.

14 Select one of the following capital letters of the alphabet: *B, C, D, E, F, G*.

(a) Perform a translation and then a reflection in a vertical line of reflection. Draw the resulting images on grid paper and label each image appropriately.

(b) Find a single transformation that will move the final image back to the original position. Show this transformation on your diagram.

(c) Use your answer for part **(b)** to find a single transformation that would produce the same result as the two transformations in part **(a)**.

15 A shape is rotated through 180° about a point *O* and then its image is reflected in a horizontal line of reflection passing through *O*.

(a) Choose any shape and, on grid paper, carry out the transformations described above.

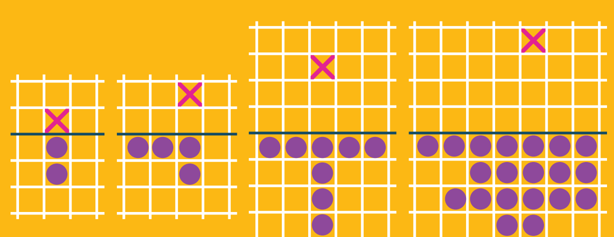
(b) Find a single transformation that would have the same results as the two transformations described above.

Outside the Square — Problem solving

Conway's army

Equipment required: 1–2 brains, grid paper, approximately 40 small counters

Conway's army is a one-person puzzle that was created by mathematician John Horton Conway in 1961. It is a variant of the peg solitaire game and uses a checkerboard of any dimension. The board is divided by a horizontal line; above it are empty cells and below it are a random number of game pieces, which are referred to as the 'soliders'. Examples of different boards and arrangements of soldiers is as follows.



How to play:

Play proceeds by jumping horizontally or vertically (not diagonally) over other pieces onto an empty space, where jumped-over pieces are then removed.

How to win:

The goal is to place a soldier as far above the horizontal line as possible. The cross in each of the diagrams at left is the maximum distance that a soldier can be placed from the horizontal line for the given number of original soldiers.

Questions

1 Create your own checkerboards with the horizontal line drawn. Begin by investigating how far four counters can be moved beyond the horizontal line. Try at least three different arrangements of the four counters. Is it possible to move a soldier past the second row?

2 Continue to investigate with 4, 5, 6, 7, 8, 9, 10, 15 and 20 counters to see how far past the horizontal line the counters can be moved.

3 Use your observations thus far to predict if it is possible to move a soldier to the fifth row above the horizontal line. Explain your answers.

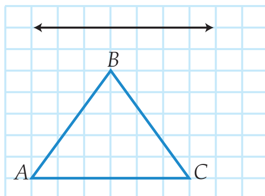
Strategy options

- Act it out.
- Test all possible combinations.

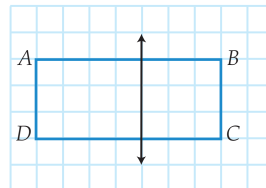
Half-time 10

1 Reflect each of the following figures in the line of reflection shown. Ex 10.2

(a)

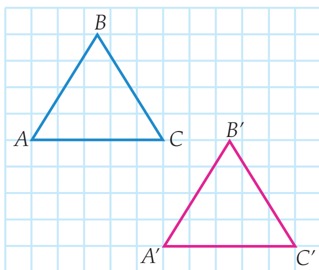


(b)

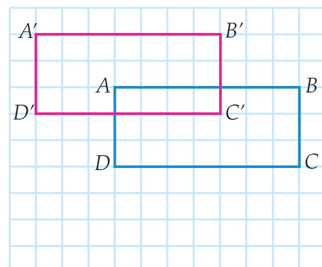


2 Describe the translation shown in each of the following diagrams. Ex 10.1

(a)



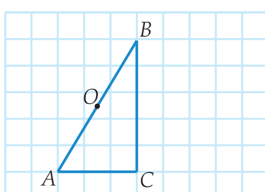
(b)



3 Show the image of each of the following figures after it is rotated about the given centre of rotation. Ex 10.3

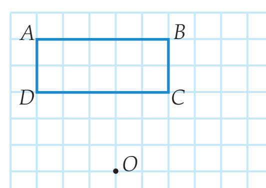
Rotate 180° in an anticlockwise direction about *O*.

(a)

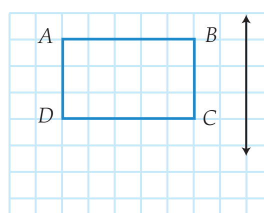


Rotate 90° in a clockwise direction about *O*.

(b)



4 Reflect the following shape in the line of reflection shown, then translate it 2 units right and 1 unit down. Ex 10.4



5 **(a)** Plot the points (1, 2), (3, 4) and (5, -2) on a Cartesian plane and join them to form a triangle. Ex 10.4

(b) Draw the image after a translation of 6 units left and 1 unit up.

(c) Write the coordinates of the vertices of the translated points.

(d) Draw the image after the image drawn in part **(b)** is reflected in the x-axis.

(e) Write the coordinates of the vertices of this new image.

(f) Finally, rotate the image drawn in part **(d)** 180° clockwise about the origin.

(g) Write the coordinates of your final image.

Transformations with GeoGebra

Versions of this Exploration for other technologies are available in Pearson Reader.

Equipment required: 1 brain, 1 computer with GeoGebra

When you open the GeoGebra program, you will see seven menu options (File, Edit etc.) at the top of the screen. Below these are eleven icons called tools. By clicking on the small arrow in the bottom right-hand corner, a drop-down list of more tools appears. If you hover over a tool the name and how to use it will appear in the top right-hand corner of the screen.

1 Click on the View menu. Deselect 'Axes' and 'Algebra View' by clicking on them. Select the 'Grid View' (the tick on the left of the word will appear or disappear when you select or deselect an option).

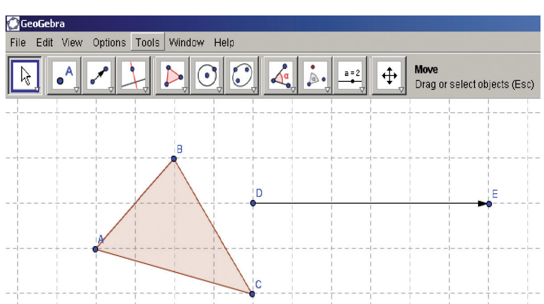
2 Click on the Options menu. Select 'Labelling', then 'New Points only'. Click on Options again, then select 'Point Capturing' and 'On (grid)'.

3 If a larger font is required, click on the Options menu and select 'Font Size'. Choose an appropriate size from the list provided.

Translating an object

A translation is a slide. We will construct a triangle and translate it along a vector (a line with length and direction).

4 Click on the 'Polygon' tool, . Click on three points on the grid as shown in the diagram below. Notice that a line joins the points as you move around the triangle. To complete the triangle, click on the starting point again. We will be translating (sliding) the triangle ABC 6 units to the right. Can you picture where it will go?



5 Turn off the labelling by clicking on the Options menu, then selecting 'Labelling', then 'No New Objects'. Use the 'Polygon' tool, , to construct a triangle where you think the translated shape will appear.

6 Click on the drop-down arrow of the third tool from the left. Select 'Vector Between two Points'. Click on any grid point, then move the cursor 6 units to the right and click again to complete the vector.

7 Click on the drop-down arrow of the ninth tool from the left. Select the 'Translate Object by Vector' tool . Click anywhere inside the original triangle and then on the vector you have just made.

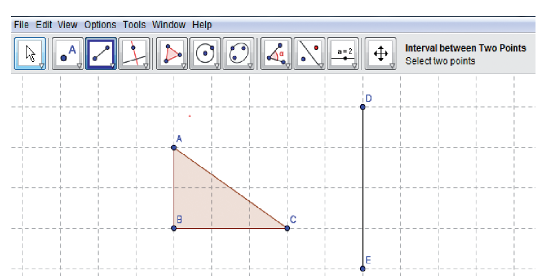
Does your translated image appear on top of the triangle you drew in step **5**? Notice that the points on the translated object are labelled as A′, B′ and C′. What is the relationship between a point on the original triangle and the translated point in the image?

8 Using the 'Select' tool , click on one of the end points of the vector (it does not matter which one). Drag the point about the page and notice how the translated image moves.

Reflecting an object in a line

A reflection is a flip. We will now construct a right triangle, and reflect it in a line.

9 Click on the File menu. Select 'New'. Unless instructed by your teacher, click 'Don't Save'. Construct a triangle as instructed in step **4**.

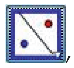


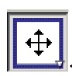
10 Construct an interval by clicking on the 'Line through Two Points' tool (third tool from the left). Click on the drop-down arrow to select 'Interval between Two Points'. Construct a straight line segment 2 units to the right of your triangle by clicking on two points, one underneath the other.

We will be reflecting (flipping) the triangle ABC in the line segment DE. Can you picture where it will go?

11 Repeat step **5** to turn off the points labelling, then use the 'Polygon' tool to construct a triangle where you think the reflection will go.

12 To turn the labelling back on, click on the Options menu. Select 'Labelling', then 'New Points only'.

13 Select 'Reflect Object in Line' tool , the ninth tool from the left. Click anywhere inside the triangle and anywhere along the interval DE to reflect your object. Did your reflected image match up on top of the triangle you drew in step **11**? Notice that the points on the reflected object are labelled as A′, B′ and C′. What is the relationship between the points on the original triangle and the reflected points in the image?

14 Using the 'Select' tool  (you can get to this tool easily by pressing the 'Escape' key), click and drag one of the end points of the interval DE about the page and notice what happens to the reflected triangle A′B′C′. Keep dragging your point until you find another reflection that has all of the reflected points A′B′C′ on the grid. (Note: If your reflected object disappears from the page you can move the page using the 'Move Graphics View' tool . Press Escape to return to the 'Select' tool.)

Rotating an object

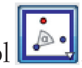
A rotation is a turn. We will now construct a triangle and rotate it by 180° in a clockwise direction.

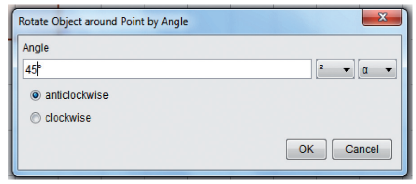
15 Click on the file menu. Select 'New'. Unless instructed by your teacher, click 'Don't Save'. Use the 'Polygon' tool to construct the same triangle you constructed in step **4**.

16 Select the 'New Point' tool , second from the left. Make a new point that is 1 unit to the right and 1 unit down from point C. This will be your 'centre of rotation'.

We will be rotating (turning) the triangle ABC by 180° about this centre of rotation. Can you picture where it will go?

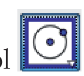
17 Repeat step **5** to turn off the points labelling, then use the 'Polygon' tool to construct a triangle where you think the rotated triangle will appear.

18 Click on the drop-down arrow of the ninth tool from the left. Select the 'Rotate Object around Point by Angle' tool . Click anywhere inside the original triangle and then on the point you have just made. A pop-up window will appear. Delete the number 45 and type 180, but be sure to leave the degree symbol there. (If you delete it, select it again from the drop-down menu immediately to the right of the box.) Click clockwise. Click OK.



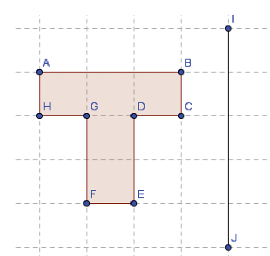
Does your rotated image match up on top of the triangle you drew in step **17**? Notice that the points on the rotated object are labelled as A′, B′ and C′. What is the relationship between a point on the original triangle and the rotated point in the image?

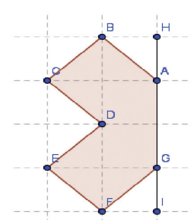
19 Repeat step **18** using angle sizes of 90°, 270° and 360°.

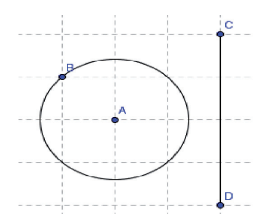
20 You should now have four triangles on the page (not including the unlabelled one you drew). To better understand how the points of the original figure are related to the rotated figures we will now use the 'Circle with Centre Through Point' tool  located as the sixth tool from the left. Click on the centre of rotation and then on one of the original vertices. Repeat this for all three of the original vertices so that you have three circles. What do you notice about the points on each circle?

Taking it further

21 Perform reflections, translations and rotations with various other shapes such as those shown. Be creative—see if you can create a pattern by performing a series of transformations on a shape, or combining it with other shapes.



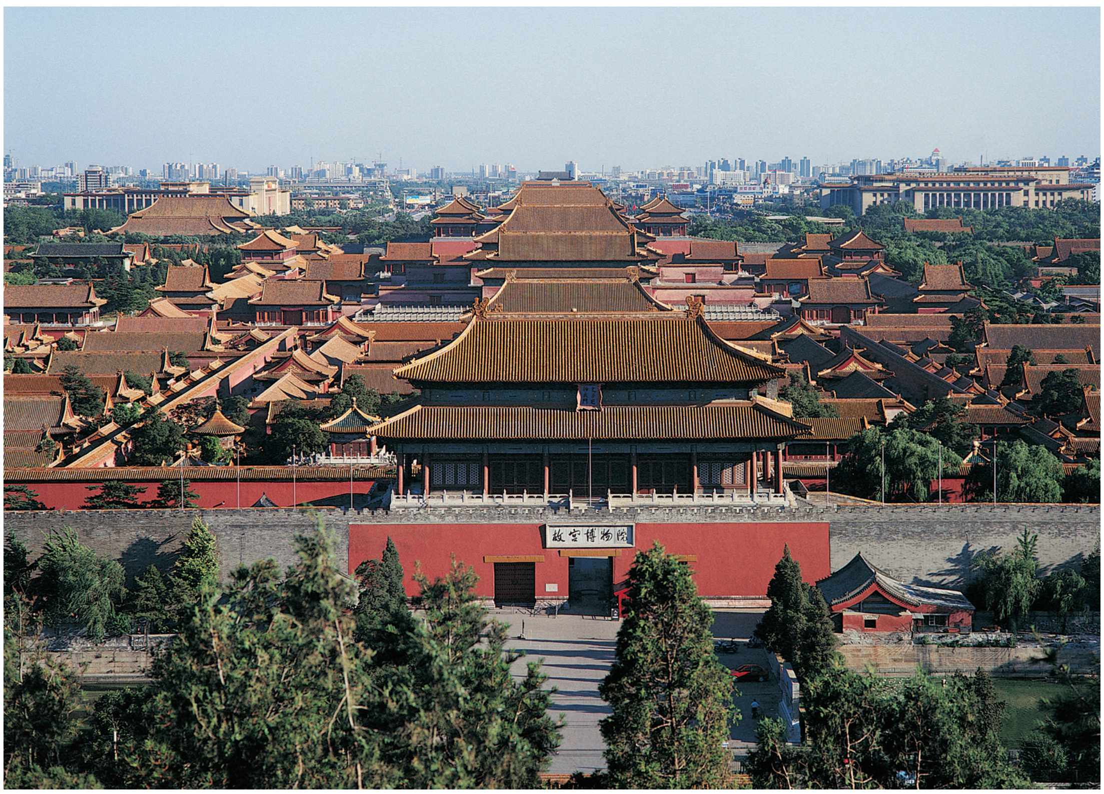




10.5 Symmetry

Reflectional symmetry

We have seen how a line of reflection can produce a reversed or mirror image of an object, but many objects in our world, from flowers and insects in nature to company logos, flags and letters of the alphabet, have **reflectional symmetry** or mirror symmetry by themselves. Reflectional symmetry occurs when a line can be drawn through an object to divide it into two identical but reversed parts. Each part is the mirror image of the other. This line is called the **axis of symmetry**. In the photo below, a line can be drawn to divide the building into two identical but reversed parts.



Some objects have multiple axes of symmetry. The **order of reflectional symmetry** is the number of axes of reflectional symmetry that can be drawn through a shape. The Pentagon, as shown on the chapter title page, has five axes of symmetry, so a pentagon has symmetry of order 5. Objects with no line of symmetry are called **asymmetrical**. The letter R has no axes of symmetry and is therefore asymmetrical.

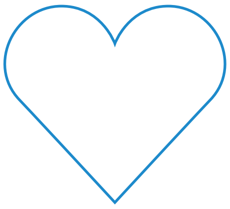
Is a human face symmetrical? Although we think our faces are symmetrical, if we reflect each side of our face separately, we get two different outcomes, neither of which is a perfect reproduction of our own face, one thinner and the other fatter. The face you see in a mirror is the mirror image or reversed view of your actual face.

Worked Example 11

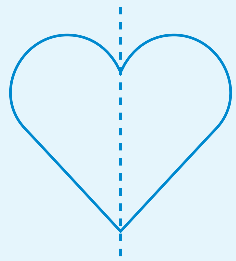
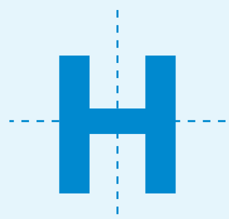
WE11

Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

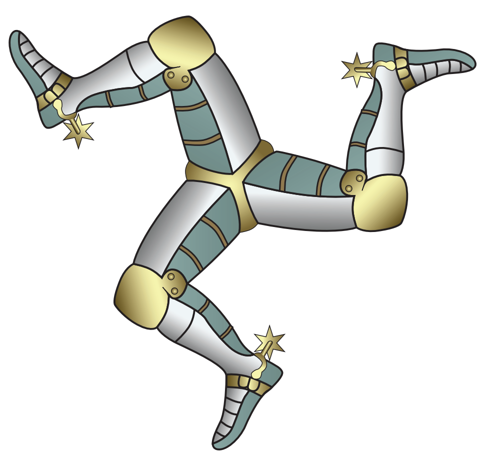
(a)



(b)

Thinking	Working
(a) 1 Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves. **2** Count the number of axes of symmetry. This gives us the order of reflectional symmetry.	(a) (i)  (ii) Order of reflectional symmetry = 1.
(a) 1 Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves. **2** Count the number of axes of symmetry. This gives us the order of reflectional symmetry.	(b) (i)  (ii) Order of reflectional symmetry = 2.

Rotational symmetry

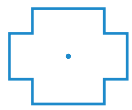


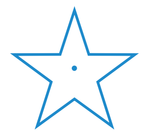
An object can also have **rotational symmetry**. If an object can be turned about a point less than 360 degrees and still look like the original object, that object is said to have rotational symmetry. The point about which the object is rotated is called the centre of rotation. An example of an object with rotational symmetry is the armoured 'triskelion', which appears on the Isle of Man's flag. The triskelion has three rotational symmetries referred to as 'three-fold symmetry', because every rotation of 120 degrees produces the original image.

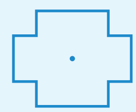
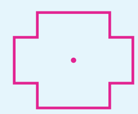
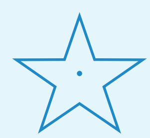
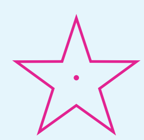
The **order of rotational symmetry** is the number of times in a complete 360° rotation that the original orientation of the shape appears. A pentagon has a rotational order of 5 because every rotation of 72° about the centre of the pentagon produces exactly the same image as the original. This occurs five times in a complete 360° rotation. A pentagon is therefore an interesting shape, as it has an order of 5 in both reflectional and rotational symmetry.

Worked Example 12 WE12

Determine **(i)** which of the following figures has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

(a) 

(b) 

Thinking	Working
(a) Find the centre of rotation and place your pen/pencil on it.	(a) 
Rotate the image around this point to determine whether the original image is reproduced.	
Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation.	(i) Yes, it has rotational symmetry. (ii) Order of rotational symmetry = 2. (iii) $\frac{360}{2}$ $= 180°$
(b) Find the centre of rotation and place your pen/pencil on it.	(b) 
Rotate the image around this point, to determine whether the original image is reproduced.	
Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation.	(i) Yes, it has rotational symmetry. (ii) Order of rotational symmetry = 5 (iii) $\frac{360}{5}$ $= 72°$

- An axis of symmetry is a mirror line that can be drawn through a shape to divide it into two identical but reversed halves.
- The order of reflectional symmetry is related to the number of lines of reflectional symmetry.
- An asymmetrical object has no reflectional symmetry.
- If a figure can be turned less than 360° so that it matches the original figure, it has rotational symmetry.
- The order of rotational symmetry is the number of times in a complete 360° rotation that an identical image of the original would be produced.

10.5 Symmetry

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q16	Q1, Q2, Q3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Answers page 694

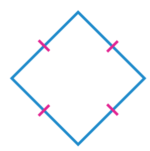
Equipment required: Grid paper for Question **12**

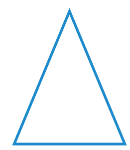
Fluency

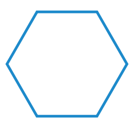
1 Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

WE11

(a) 

(b) 

(c) 

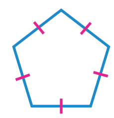
(d) 

(e) **W**

(f) **J**

(g) **F**

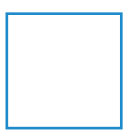
(h) **X**

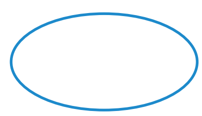
(i) 

2 Determine **(i)** which of the figures from Question **1** has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

WE12

3 How many axes of symmetry do the following objects have?

(a) 

(b) 

(c) 

For Questions **4**, **5** and **6** refer to the letters of the word M A T H S.

4 Which letter does not have an axis of symmetry?

A M **B** A **C** H **D** S

5 How many of the letters have one axis of symmetry?

A 5 **B** 2 **C** 3 **D** 4

6 Which letter has two axes of symmetry?

A H **B** M **C** A **D** S

Understanding

7 Copy the lower case letters of the alphabet into your book.

a b c d e f g h i j k l m
n o p q r s t u v w x y z

(a) Which lower case letters are asymmetrical?

(b) Which lower case letters have one axis of symmetry?

(c) Which lower case letters have two or more axes of symmetry?

8 Copy the upper case letters of the alphabet into your book.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which upper case letters are asymmetrical?

(b) Which upper case letters have one axis of symmetry?

(c) Which upper case letters have two or more axes of symmetry?

9 Do any of the capital letters from Question **8** have rotational symmetry? If so, which ones are they?

Reasoning

10 Symmetry is present everywhere in nature. For each of the following images, determine whether it has almost perfect reflectional and/or rotational symmetry and state the order of each.

(a)


(b)

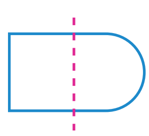

(c)

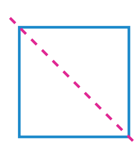

(d)

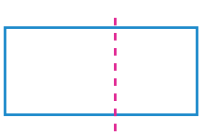

(e)

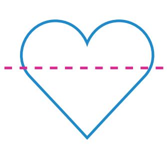

(f)

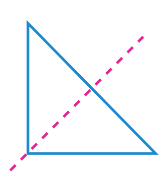

11 For each of the following shapes, state whether or not the dotted line is an axis of symmetry.

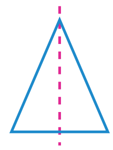
(a)


(b)


(c)


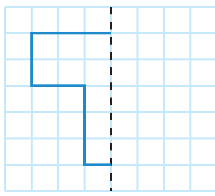
(d)


(e)


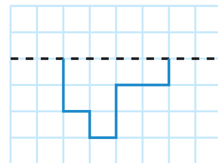
(f)


12 For each of the following diagrams, use the dotted line as an axis of symmetry to complete the diagram using grid paper.

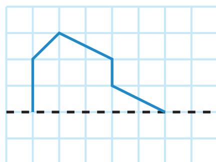
(a)



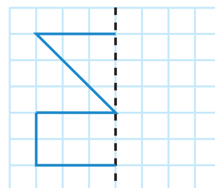
(b)



(c)



(d)



13 Which two quadrilaterals have exactly two lines of reflectional symmetry?

14 Which regular polygon has exactly three lines of reflectional symmetry?

Open-ended

15 WOW is a symmetrical word. Locate the axis of symmetry and find three more words that are also symmetrical.

16 The dates of some years have reflectional and/or rotational symmetry. An example of a year that has rotational symmetry is l96l and a year that has reflectional symmetry is l88l.

Identify an example of a year that has:

(a) rotational symmetry

(b) reflectional symmetry

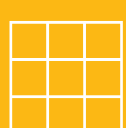
(c) both reflectional and rotational symmetry.

17 Describe when the following statement is true and when it is false.

'This parallelogram has reflectional symmetry.'

Outside the Square — Problem solving

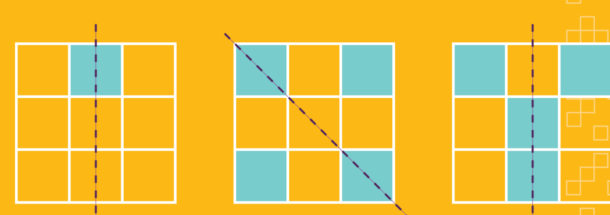
Symmetry challenge



How many symmetrical designs of shaded squares can you find in a blank 3 by 3 square grid?

Hint: Before you shade any squares, can you identify the lines of symmetry present in this 3 by 3 grid shown at left? What symmetries are possible if you shade one square only?

Here are three examples of possible symmetrical designs. Note the position of the line of symmetry.



Don't include any rotations; e.g.  is considered the same as .

Strategy options

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

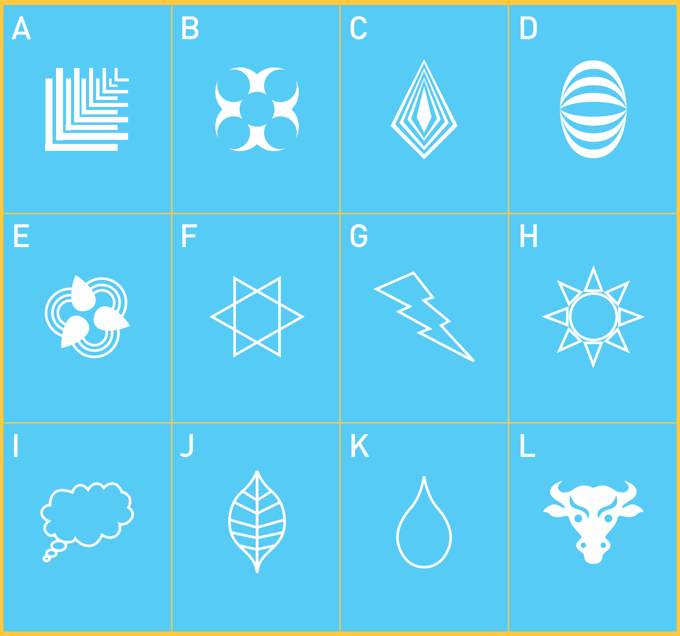
Maths 4 Real

SNOW LOGO

Symmetry is used in art, architecture, company logos and even in the design of national flags. We seem to be drawn to symmetrical objects, and designers and artists take advantage of our preference for symmetry.

Imagine you are a young marketing executive for a company that manufactures and sells snowboards. You know that many companies succeed or fail on the recognition of their brand by customers. The easiest method for achieving customer recognition is by integrating elements of symmetry into a company logo. Your manager, who is a symmetry fanatic, has given you the task of selecting the most symmetrical (reflectional and rotational symmetry) logo from a collection of sample logos developed by your company's graphic artists.

Sample logos



1 To assist you with this task, use the following table as a template and note down in your book the order of reflectional and rotational symmetry for each of the sample logos.

Logo	Order of reflectional symmetry	Order of rotational symmetry
A		
B		
C		

2 At the end of the selection process, you think that you may be able to develop a better-looking, more symmetrical logo for your company. Your task is to develop your own version of your new company's logo and identify all elements of its symmetry to show the CEO of the company.

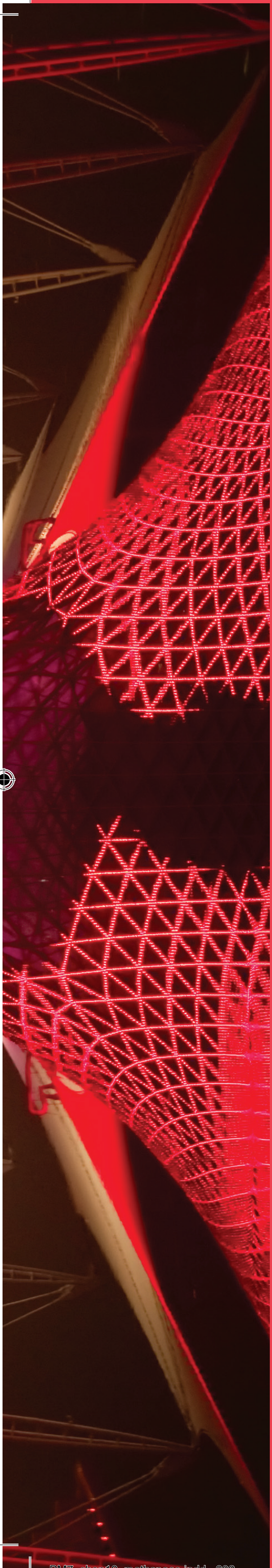
3 Apart from the symmetry elements, write down two or three other criteria you would use when deciding which is the best logo for a company.

Research

- Find as many different examples as you can of well-known company logos or brands that have reflectional and/or rotational symmetry. (Car manufacturers are a good place to start.) Find some examples that are asymmetric (have no symmetry).
- Large companies spend a lot of money on getting the right design for their logo—the symbol by which the company is identified. Why do you think such importance is placed on the logo design?
- Which logo do you think is the most recognised throughout the world? Try to find out.

Mathspace

Ambigrams and symmetry



Equipment required: 1 brain, a small mirror

Letter ambigrams

An ambigram is a word that you can read in more than one direction or orientation. The word 'ambigram' is derived from *ambi* (meaning 'both', or 'on both sides') and *gram* (meaning 'something written down or drawn').

Ambigrams can have either reflectional symmetry or rotational symmetry.

Vertical reflectional symmetry

Use your mirror to check

Horizontal reflectional symmetry

COOKBOOK

Rotational symmetry

dollop

Turn your book to check the rotational symmetry

1 For each of the following ambigrams, complete the word (if required) and state whether it has rotational, vertical reflectional, or horizontal reflectional symmetry. (Use your mirror to check.)

(a) SWIMS (d) BID
(b) HIDE (e) bid
(c) MUM (f) su____

2 (a) Some pairs of letters, such as 'b' and 'q', have rotational symmetry when used together. (Rotate 'b q' or 'q b' to see how this works.) What other pairs of letters have rotational symmetry when used together?
(b) What single letters have rotational symmetry when used by themselves?
(c) Create your own ambigram with rotational symmetry or vertical reflectional symmetry.

Number ambigrams

Some numbers have symmetrical properties. For instance, the number 88 has vertical reflectional and horizontal reflectional symmetry, as well as rotational symmetry.

3 Find some number ambigrams that have the following symmetries.
(a) vertical reflectional symmetry
(b) horizontal reflectional symmetry
(c) rotational symmetry

4 Digital clocks often display numbers using a figure-of-eight digital number grid. This means that some digital numbers look the same upside down.

0123456789

(a) Which of these digital numbers have rotational symmetry?
(b) Which digital square numbers between 1 and 100 are ambigrams? What types of symmetry do they have?
(c) What is the largest 4-digit ambigram you can make that:
(i) uses only one of the digits 0 to 9
(ii) uses two of the digits 0 to 9
(d) What is the largest 5-digit ambigram you can make that:
(i) uses only one of the digits 0 to 9
(ii) uses three of the digits 0 to 9

Grid symmetry

Draw up a Cartesian grid that goes from 0 to 4 on the horizontal axis, and from 0 to 3 on the vertical axis.

Using the clues below, place the letters R, C, K, H, S, U, Y, M, and B at their correct places on the Cartesian grid.

Clues

- The letter at (1, 2) is not symmetrical in any way.
- The letters at (0, 0), (1, 1), (2, 2) and (3, 3) all have horizontal reflectional symmetry.
- The letters at (0, 2) and (3, 3) have rotational symmetry.
- The letters at (3, 0), (3, 1) and (3, 2) all have vertical reflectional symmetry.
- The letters at (1, 1) and (3, 1) contain only curved lines.
- The letters at (2, 2) and (3, 2) contain only straight lines.
- The letters at (3, 0), (3, 1) and (3, 2), in that order, spell a word that means 'tasty'.

What word is spelt by the letters at (1, 2), (3, 1), (0, 0) and (3, 0), in that order?

10.6 Drawing and visualising 3D shapes

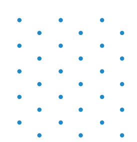
Objects such as buildings, machinery, car parts, people and play stations have three dimensions: length, width and height. It is often necessary to draw three-dimensional objects on a two-dimensional (2D) plane, such as a piece of paper or a computer screen.

The drawing of the house in this diagram is an isometric drawing used by an architect.

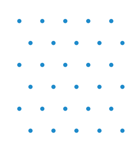
In Greek, 'iso' means 'equal' and 'metric' refers to 'measuring'. In isometric dot paper, the distances between each dot and the dots surrounding it are identical, be it a diagonal or a vertical direction.

Triangular dot paper, also called **isometric** paper can help to overcome the difficulties of drawing the three dimensions of real objects on a 2D plane. Objects represented on triangular dot paper are also known as isometric drawings.

It is recommended that isometric dot paper be held in this orientation:



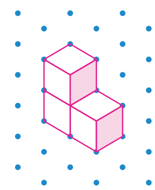
This orientation is very difficult to use (not recommended).

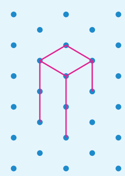
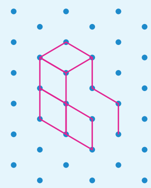
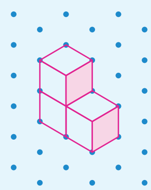


Worked Example 13

WE13

Use isometric dot paper to copy the following shape.



Thinking	Working
18 Join the points of a parallelogram that will represent the top face. Draw in all the necessary vertical lines from the corners of the face.	
19 Draw in all the lines from the bottom of the verticals you have just drawn in step **1** that represent horizontal lines. Draw in all additional required vertical lines from the ends of the lines you have just drawn.	
20 Join the ends of the lines you have just drawn. To create the illusion of 3D, choose one direction (front, side or top) and shade in all faces you would see from that direction.	

10.6 Drawing and visualising 3D shapes

Navigator

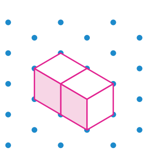
Q1, Q2 (a)–(c), Q3, Q4, Q5, Q6 (a)–(c), Q8, Q9 (a), Q10, Q11, Q13	Q1, Q2 (b),(d),(f), Q4, Q5, Q6 (c)–(e), Q7, Q8, Q9 (b), Q10, Q11, Q14	Q1, Q2 (d)–(f), Q4, Q6 (d)–(f), Q7, Q8, Q9 (c), Q10, Q11, Q12, Q14

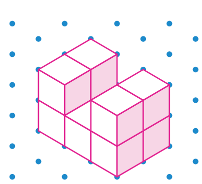
Answers page 696

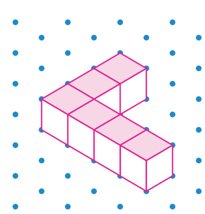
Equipment required: Isometric dot paper for Questions **1**, **5**, **8**, **9** and **12**–**14**

Fluency

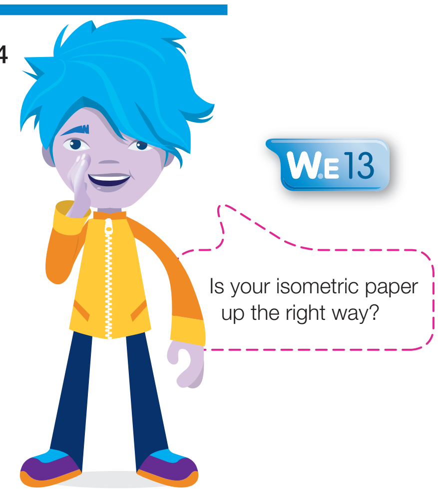
1 Use isometric dot paper to copy the following shapes.

(a) 

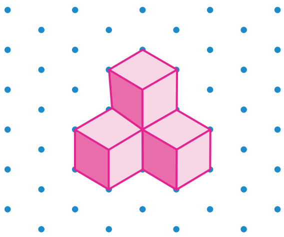
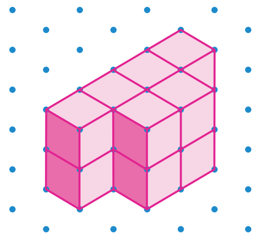
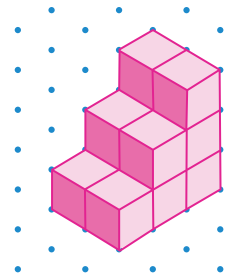
(b) 

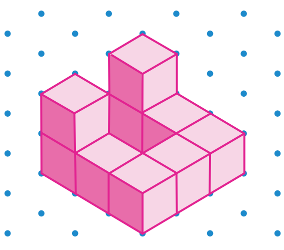
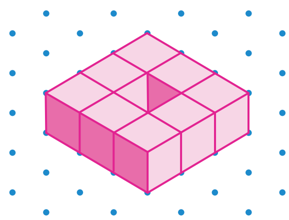
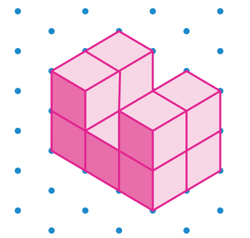
(c) 

WE13

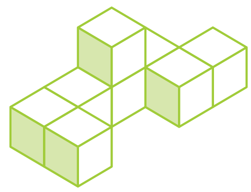


2 How many cubes would be required to build these solids? (Assume that there are no cubes missing at the back of the solids where you cannot see.)

(a)

(b)

(c)


(d)

(e)

(f)


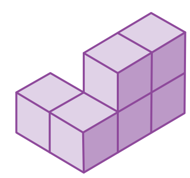
3 The number of cubes required to build this solid is:

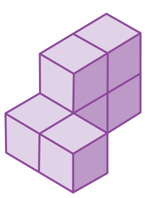
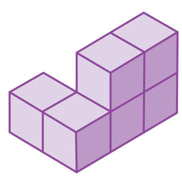
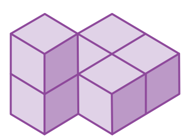
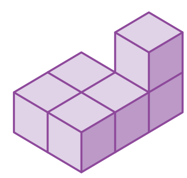


A 7 B 8

C 9 D 10

4 Which of the shapes below is *not* a rotation of the one shown?

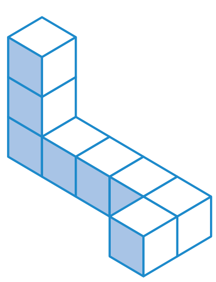
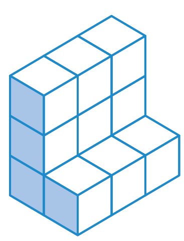
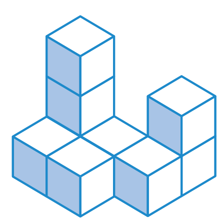


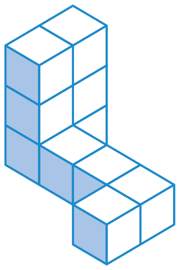
A

B

C

D


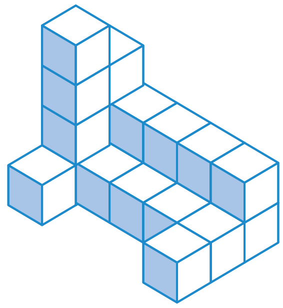
Understanding

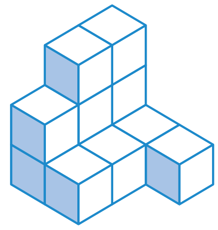
5 Using isometric dot paper, draw a 3D sketch of a Rubik's cube. (A Rubik's cube is made of 27 cubes stacked three layers high, three layers wide and three layers deep.)

6 How many cubes are in each solid?

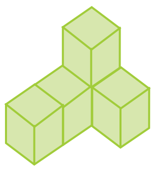
(a)

(b)

(c)


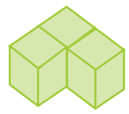
(d)


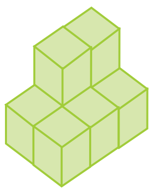
(e)


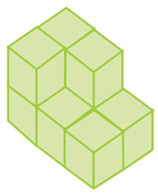
(f)


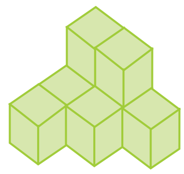
7 Which of the following shapes could result from joining the two shapes below?

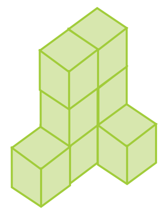




A


B


C


D


Reasoning

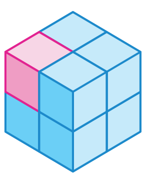
8 Using the aerial photograph on page 560 on the opening page of this chapter:

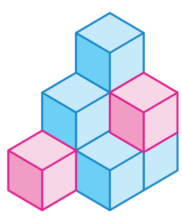
(a) draw a top view of the Pentagon

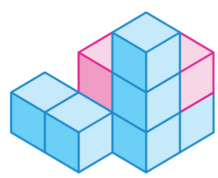
(b) draw a side view of the Pentagon.

(c) Does it matter which side you choose? Explain.

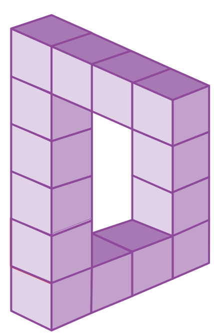
9 Draw the solid after the pink shaded blocks have been removed.

(a)


(b)


(c)


10 This shape is known as the impossible rectangle.



(a) Why is this shape known as the *impossible* rectangle? (You may like to try to make it with blocks.)

(b) Copy the impossible rectangle onto isometric dot paper.

(c) Draw a similar rectangle that is possible. Colour your drawing using three colours.

11 Draw the top view of the house drawn on page 604, the first page of this section.

Open-ended

12 A sculptor is designing a sculpture based on an arrangement of 24 cubes. Each cube has a width of 50 cm and the maximum height of the sculpture is to be 2.5 m. The sculptor wants to have a maximum of six cubes at the front and a maximum of four cubes in depth. Each cube must have at least one whole face touching another cube. Use isometric dot paper to design some sculptures that meet all these requirements.

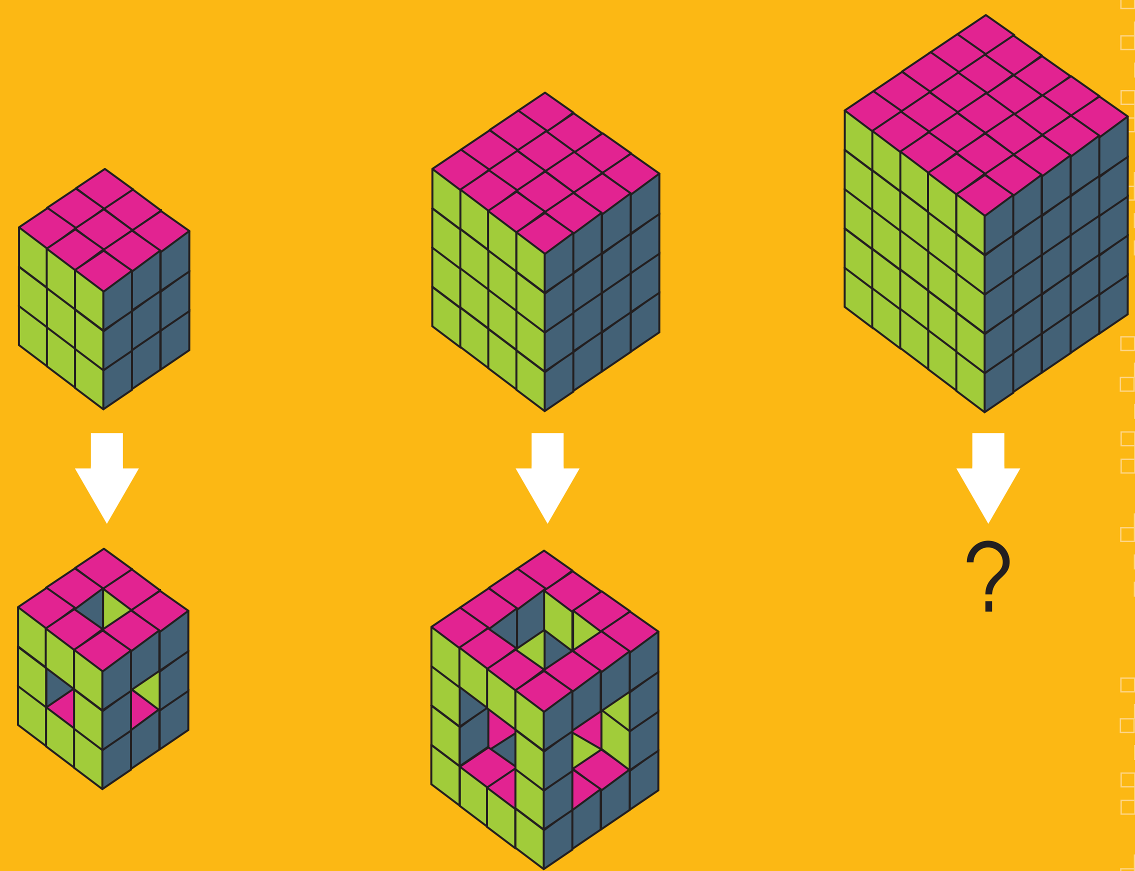
13 Using isometric dot paper, draw three different solids that can be built from 10 cubes that have whole faces touching.

14 Draw three shapes on isometric dot paper that, when joined together, form a $3 \times 3 \times 3$ cube.

Outside the Square — Problem solving

What comes next?

Look at the pattern below.



A solid $3 \times 3 \times 3$ cube is made using 27 blocks. When one block is removed from the centre of each of the six faces and one block from the centre of the cube, a frame of the same cube is made using 20 cubes.

- Draw the next shape in the pattern.
- How many cubes are needed to make the frame of a $5 \times 5 \times 5$ cube?
- How many cubes are needed to make the frame of a $6 \times 6 \times 6$ cube?

Strategy options

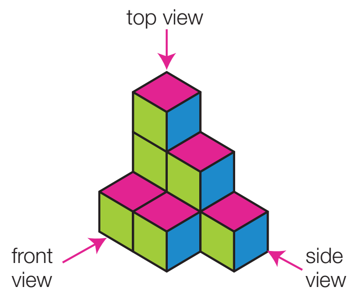
- Make a table.
- Look for a pattern.

Plan views and elevations

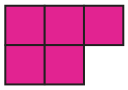
10.7

Three-dimensional objects can be viewed from various positions, or views. Objects looked at from the top, front and side can be drawn in two dimensions, and are known as **plan views**, or orthogonal projections.

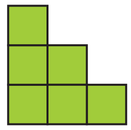
A 3D shape can look different when viewed from different points of view. A top view, or 'bird's eye view' is what you see when looking directly down on the shape, a side view is what you see when looking directly at the side and the front view is what you see when looking at the front.



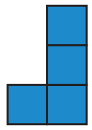
top view



front view



side view



Worked Example 14

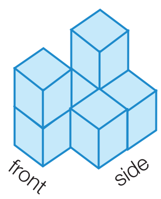
WE 14

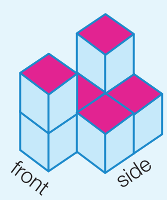
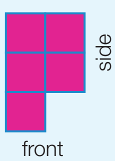
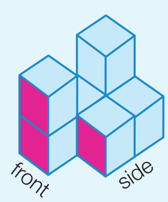
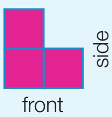
For the following solid, draw the:

(a) top view

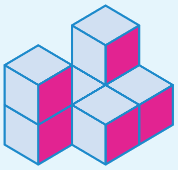
(b) front view

(c) side view.

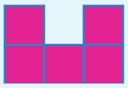


Thinking	Working
(a) 1 Highlight the faces seen from the top.	(a) 
2 Draw squares to make the shape you would see if you were looking down on the solid.	
(b) 1 Highlight the faces seen from the front. Do not highlight front faces that are behind another block.	(b) 
2 Draw squares to make the shape you would see if you were standing in front of the solid.	

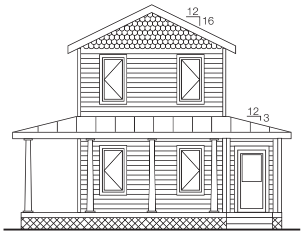
(c) 1 Highlight the faces seen from the side.

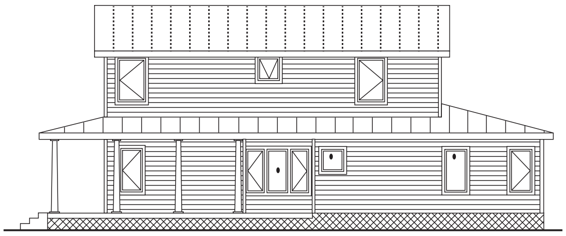

(c)

2 Draw squares to make the shape you would see if you were standing at the side of the solid.



Drawings of houses and buildings viewed from a side are known as **elevations**. Elevations may be referred to as front, rear and side elevations, or by compass direction, such as an east elevation.


FRONT ELEVATION


SIDE ELEVATION

10.7 Plans views and elevations

Navigator

Answers page 696

Q1, Q2, Q3, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: Isometric dot paper for Questions **5**, **7**, **8**, **9**, and **10**

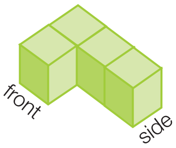
Fluency

WE14

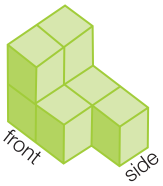
1 For each of the following solids, draw the:

(i) top view **(ii)** front view **(iii)** side view.

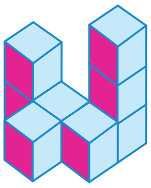
(a)



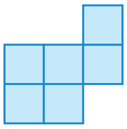
(b)



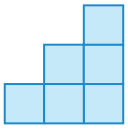
2 The front view of the solid shown has been shaded. The top view of this solid looks like:



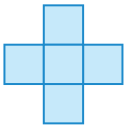
A



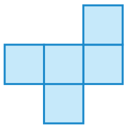
B



C

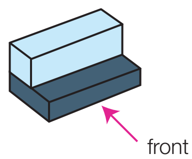


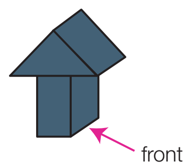
D

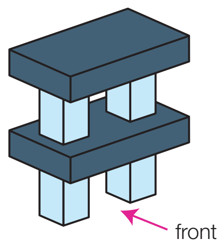


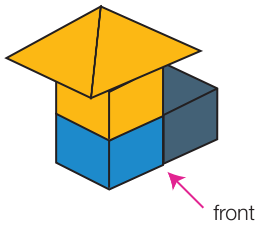
Understanding

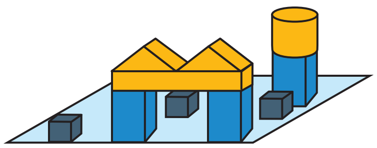
3 The following structures are made with coloured blocks. Draw the front elevation of each structure by imagining you are standing looking from the position of the arrow, then move 90° to the right and draw the side elevation.

(a)


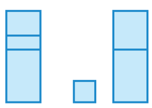
(b)


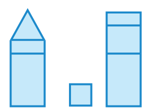
(c)


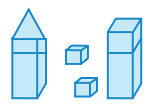
(d)


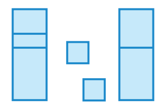
4


(a) Which diagram below shows a possible side elevation of the blocks above?

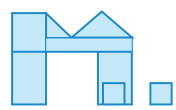
A


B


C


D


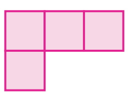
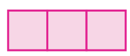
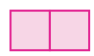
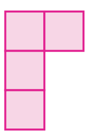
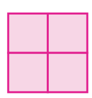
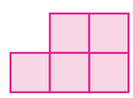
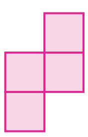
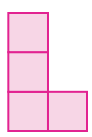
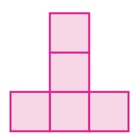
(b) Which elevation of the block structure above is shown opposite?



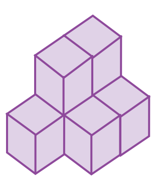
A front B rear

C side D top

5 For each set of the plan views below, draw the 3D solid on isometric dot paper.

	top view	front view	side view
(a)			
(b)			
(c)			

6 A *mat plan* shows the top view of a shape and the number of cubes in each position.

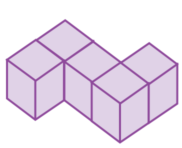


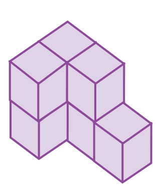
Isometric drawing

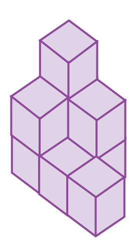
2	1
2	1
1	

mat plan

Draw the mat plan for each of the shapes below.

(a)


(b)


(c)


Reasoning

7 Draw each of the following mat plans as 3D shapes on isometric dot paper.

(a)

1	1
1	

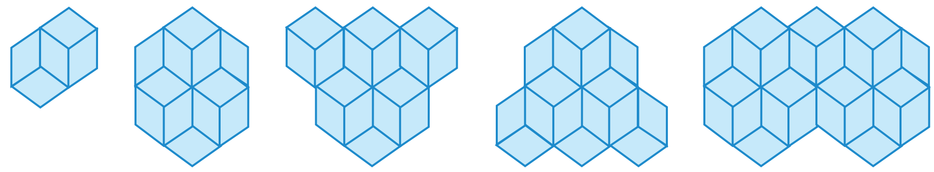
(b)

3	2
1	

(c)

3	3	3
2	2	
1		

8 (a) Copy the following diagrams onto isometric dot paper.



(b) On your drawings, use three colours to shade the different faces of the cubes.

(c) How many cubes are in each of the diagrams?

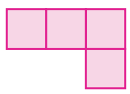
(d) There is more than one way to colour the diagrams. If the cubes were coloured differently, would each diagram still look like it has the same number of cubes?

(e) Draw the top view of each shape.

Open-ended

9 A shape made with small cubes looks like this from above.

Draw three possible 3D drawings on isometric dot paper, stating the number of cubes in each diagram.



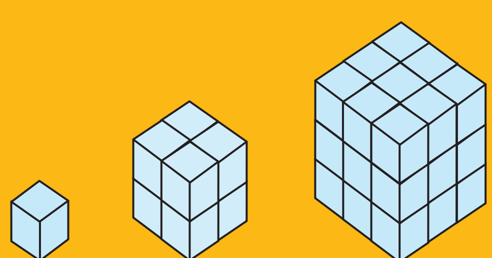
top view

10 Select a three-dimensional object and draw the front, rear and side elevations.

Outside the Square — Problem solving

Bird's eye squares

Ezra glued some 1 cm cube blocks together to form different-sized cubes. He made four cubes of side length 2 cm, four cubes of side length 3 cm and had four 1 cm cubes left over.



Ezra made a solid with his different-sized cubes. The solid's top view was a square.

(a) What is the minimum number of cubes Ezra would need to build his solid?

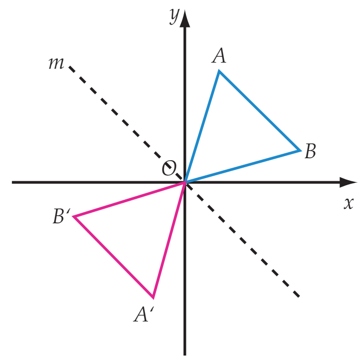
(b) How many different solids can be made with the cubes he has made that have a top view of a square and are made with more than one cube? Draw the top view of each solid.

Strategy options

- Draw a diagram.
- Make a model.

Challenge 10

1 The equilateral triangle OAB has been transformed into the equilateral triangle $OA'B'$.



(a) Which single transformation changes OAB into $OA'B'$?

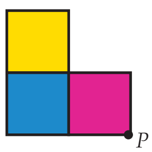
(b) Which two different transformations, applied one after the other, would change OAB into $OA'B'$?

(c) OAB is reflected in the line m. What is the name of the new position of A?

2 Jemima looks in a mirror and sees the clockface behind her. The hands appear to show 2:45. What will the time appear to be in 15 minutes?

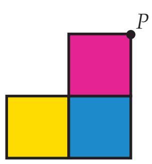
3 A horizontal line CD is copied three times, and each copy appears on top of the original. Using the terms 'rotation' and 'translation', describe how each copy must be moved in order to create a square.

4 If the shape

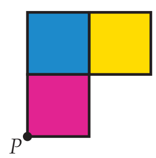


is rotated 90° clockwise about the point P, the resulting figure could be:

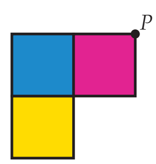
A



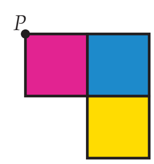
B



C

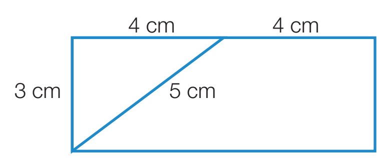


D



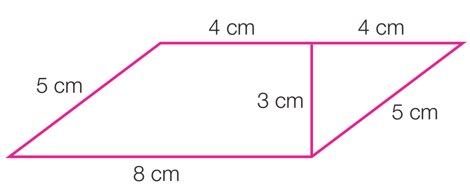
5 To be decoded, a message has to be read by looking in a mirror. Decode the following message: 'TAƎB OT DЯAH ƧI ƆƆƎ DƎ⅃IOB A'. To make sense of the message, what did you have to do?

6 A line is drawn on the 8 cm by 3 cm rectangle opposite to make a triangle and a trapezium.

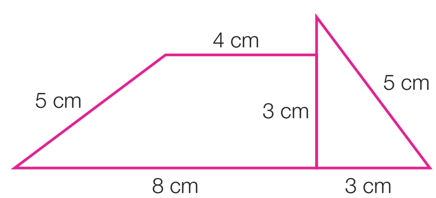


What transformation(s) need to be done to the triangle to create each of the following shapes?

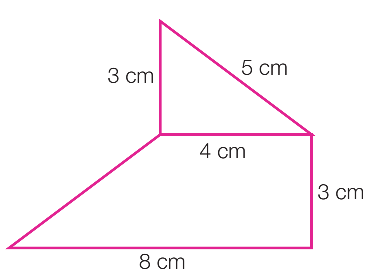
(a)



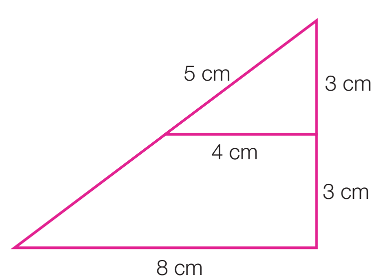
(b)



(c)



(d)



Chapter review 10

D.I.Y. Summary

Key Words

asymmetrical	isometric	perpendicular distance	rotation
axis of symmetry	line of reflection	plan views	rotational symmetry
centre of rotation	order of reflectional symmetry	reflection	transformation
elevations	order of rotational symmetry	reflectional symmetry	translation
image	original	rotate	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 Reflections, rotations and translations are three types of __________s.
2 When an object is turned about a point, a __________ has been performed.
3 When a figure is moved in a horizontal and vertical direction, a __________ has been performed.
4 When a figure is a reversed image of the original, a __________ has been performed.
5 The transformed version of a shape or diagram is called an __________.
6 A __________ acts as a mirror to reflect an object.
7 A point about which a figure is turned is called the __________.
8 The __________ indicates how many times the image is identical to the original in a complete 360° rotation.
9 A 2D representation of a 3D solid can be drawn on __________ paper.

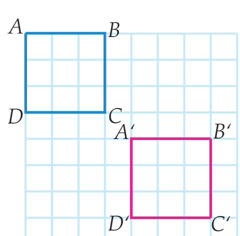
Equipment required: Grid paper for Questions **2**, **4**, **5**, **6**, **7**, **9**, **23** and **28**, isometric dot paper for Questions **14**, **16** and **17**, and a protractor for Questions **7**, **9**, **23** and **28**

Fluency

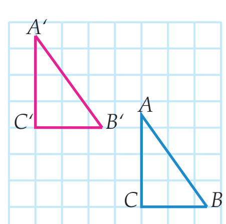
1 Describe the translation shown in each of the following diagrams.

Ex. 10.1

(a)



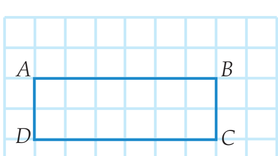
(b)



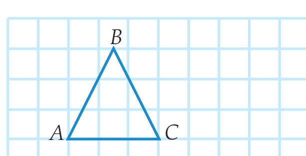
2 Copy each figure below and draw the resulting image after the translation required.

Ex. 10.1

(a) 3 units right and 1 unit up



(b) 3 units left and 3 units up

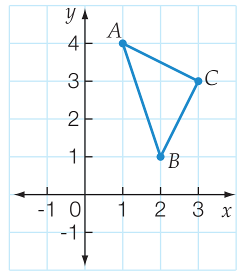


3 What would be the reverse translation of 7 units right and 4 units down? Ex. 10.1

- **A** 7 units up and 4 units right
- **B** 7 units left and 4 units up
- **C** 7 units left and 4 units down
- **D** 4 units left and 7 units up

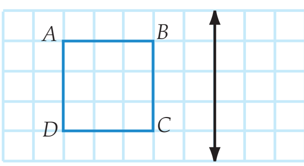
4 For the figure in the diagram opposite: Ex. 10.1

- **(a)** write the coordinates of the vertices
- **(b)** copy the figure and translate it 4 units left and 3 units down, marking the image vertices with image notation
- **(c)** write the coordinates of the vertices of the image.

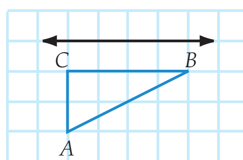


5 Copy each of the following figures onto grid paper and draw the resulting image when the figure is reflected in the line shown. Ex. 10.2

(a)

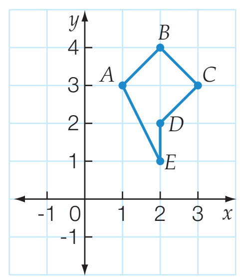


(b)



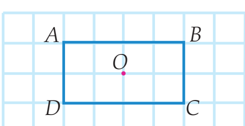
6 For the figure in the diagram opposite: Ex. 10.2

- **(a)** write the coordinates of the vertices
- **(b)** copy the figure and reflect it in the x-axis, marking the image vertices with image notation
- **(c)** write the coordinates of the vertices of this image
- **(d)** copy and reflect the original figure in the y-axis, marking the image vertices with image notation
- **(e)** write the coordinates of the vertices of this image.

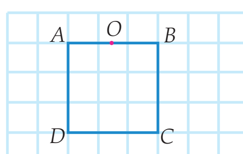


7 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Ex. 10.3

(a) Rotate 90° in a clockwise direction about O.



(b) Rotate 180° in an anticlockwise direction about O.

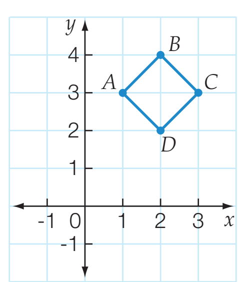


8 A clockwise rotation of 90° is equivalent to an anticlockwise rotation of how many degrees? Ex. 10.3

- **A** 180°
- **B** 360°
- **C** 90°
- **D** 270°

9 For the figure in the diagram opposite: Ex. 10.3

- **(a)** write the coordinates of the vertices
- **(b)** copy the figure and rotate it 90° in an anticlockwise direction about the origin, marking the image vertices with image notation
- **(c)** write the coordinates of the vertices of this image
- **(d)** copy and rotate the original figure 180° in an anticlockwise direction about the origin, marking the image vertices with image notation
- **(e)** write the coordinates of the vertices of this image



(f) copy and rotate the original figure 270° in an anticlockwise direction about the origin, marking the image vertices with image notation

(g) write the coordinates of the vertices of this image.

10 What are the reverse transformations for a translation of 5 units right and 8 units down followed by a clockwise rotation of 90°? **Ex. 10.4**

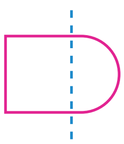
A An anticlockwise rotation of 90° and a translation of 5 units right and 8 units down

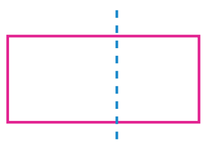
B A clockwise rotation of 180° and a translation of 5 units right and 8 units down

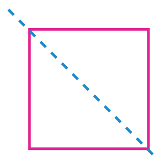
C An anticlockwise rotation of 90° and a translation of 5 units left and 8 units up

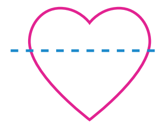
D A clockwise rotation of 90° and a translation of 5 units left and 8 units down

11 In which of the following shapes is the dotted line an axis of symmetry? **Ex. 10.5**

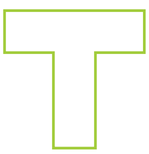
A


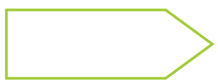
B


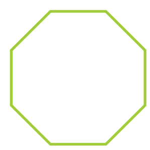
C


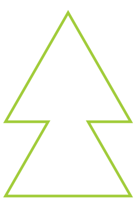
D


12 Copy the following shapes and draw lines of symmetry. Some shapes may have more than one line of symmetry. **Ex. 10.5**

(a)


(b)


(c)


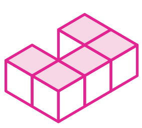
(d)


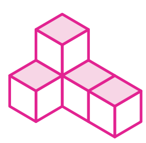
(e)


(f)


13 Identify the shapes from Question **12** that have rotational symmetry. **Ex. 10.5**

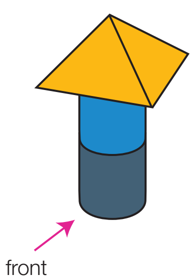
14 Two solids have been made using wooden cubes, as shown. **Ex. 10.6**

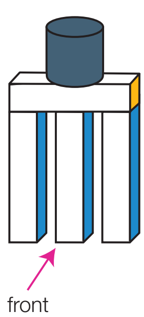
(a)


(b)


Use isometric dot paper to copy the shapes.

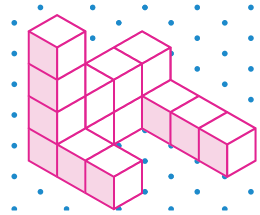
15 For each of the three-dimensional structures below, draw the front and side elevations. **Ex. 10.7**

(a)


(b)


Understanding

16 Use isometric dot paper to draw the shape shown.

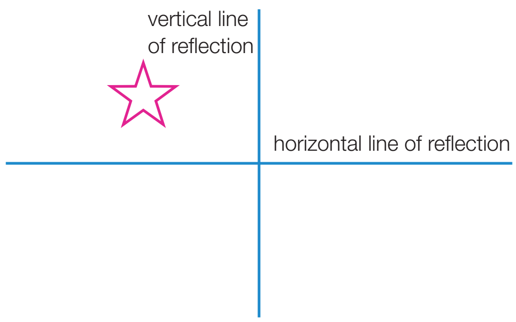


17 Use the shape above to draw:

(a) a front view **(b)** a top view **(c)** a side view.

18 What single translation would be equivalent to 6 units down, 7 units right, 2 units up, 8 units left, 4 units up and 5 units right?

19 **(a)** Using the diagram below, reflect the star in the vertical and then the horizontal lines of reflection.



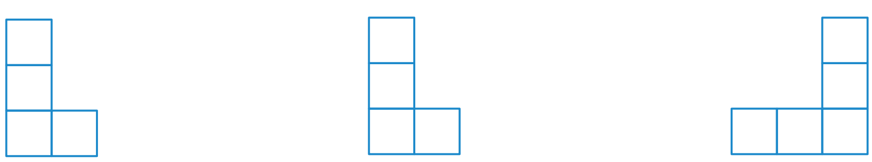
(b) Using the same starting position, reflect the star in the horizontal and then the vertical lines of reflection. Does it result in the same outcome as in **(a)**?

20 What rotation is equivalent to reflecting an object along horizontal, then vertical, lines of reflection?

21 How many letters of the word 'MATHEMATICS' do not have reflectional or rotational symmetry?

22 The plan views of a three-dimensional shape made with cubes are shown. Draw the solid on isometric dot paper.

top view front view side view



23 On grid paper:

(a) draw the line joining the points $A(-1, 3)$ and $B(2, 5)$

(b) translate this line 3 units right and 2 units down

(c) reflect this image in the x-axis to form a new image

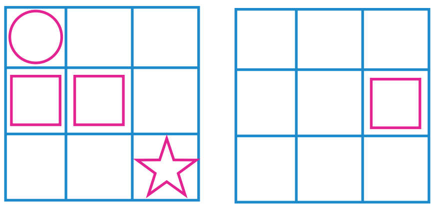
(d) rotate this image 90° in a clockwise direction (about the origin) to form a final image

(e) write the coordinates of the final image.

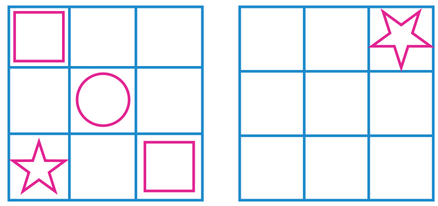
Reasoning

24 The following 3 by 3 squares comprise identical symbols, but each one has been reflected horizontally, vertically or along the diagonal and is missing three shapes. Your task is to draw the missing three shapes.

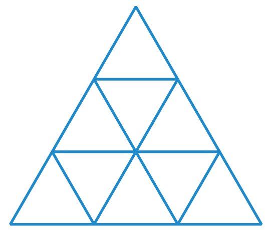
(a)



(b)



25 The following equilateral triangle comprises nine smaller equilateral triangles.

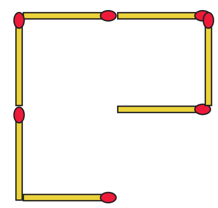


By shading the smaller equilateral triangles, create three different designs that have:

(a) one line of symmetry

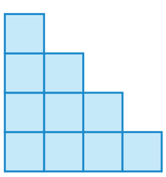
(b) three lines of symmetry.

26 The diagram at right shows a pattern made from sticking seven matchsticks of equal length to a piece of paper. What is the smallest number of matchsticks that need to be added so that the resulting image has at least one line of reflectional symmetry?

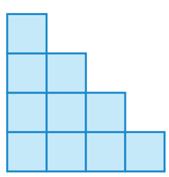


27 A solid is built using cubed blocks. The front, side and top views of the solid are shown. What is the minimum number of blocks this solid can have?

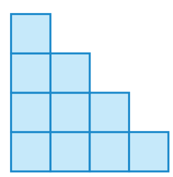
top view



front view



side view



28 Matt is asked to reflect a particular figure in the y-axis. This image is then rotated 180° in a clockwise direction.

Matt believes he can replace these two combined transformations with one transformation.

Choose a shape of your own and perform the two transformations on it that Matt was required to perform. Is he correct? Can one transformation replace the two transformations? If so, write the required transformation.

NAPLAN practice 10

Numeracy: Non-calculator

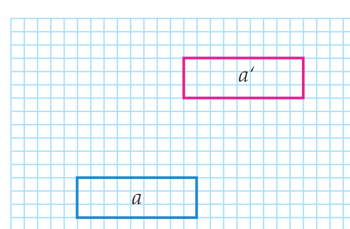
1 The translation shown is:

A 6 units right, 8 units up

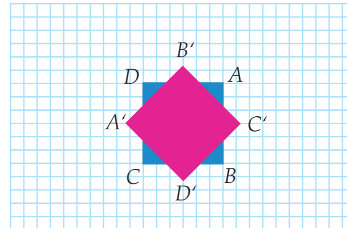
B 8 units right, 9 units up

C 8 units down, 6 units left

D 6 units down, 8 units left



2 The rotation needed to transform the original shape (blue) to the new rotated image (pink) is:



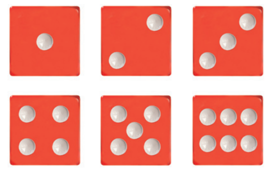
A 45° clockwise

B 135° anticlockwise

C 90° clockwise

D 90° anticlockwise

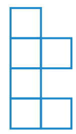
3 How many of the six faces of a die have fewer than three lines of symmetry?



A 1   **B** 2

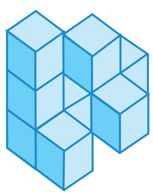
C 3   **D** 4

4 The top view of a shape is shown at right.

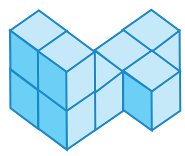


Which solid has this top view?

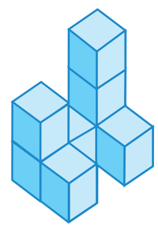
A



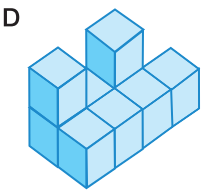
B



C

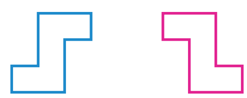


D

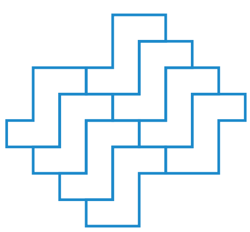


Numeracy: Calculator allowed

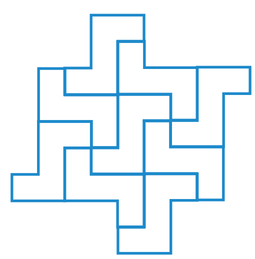
5 Mia has these S-shaped tiles.
They are blue on one side and red on the other side.
Mia wants to make a pattern with all the tiles blue side up.
Which one of the patterns can Mia not make?



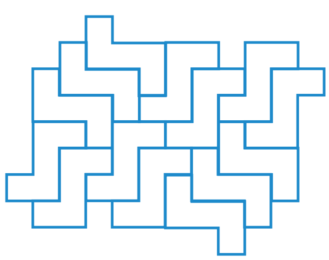
A



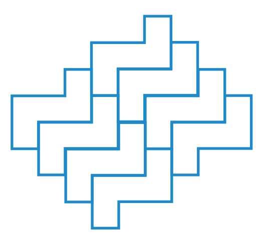
B



C



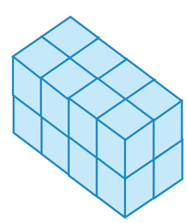
D



6 Which letter in the word SAME does not have an axis of reflectional symmetry?

A S   **B** A   **C** M   **D** E

7 Janita glued small wooden cubes together to form the solid shown below.
She then painted the solid on all surfaces.



How many wooden cubes are painted on just one face?

A 0   **B** 2   **C** 4   **D** 8

Mixed review E

Equipment required: Grid paper and protractor for Questions **11**, **17** and **18**

Fluency

1 Simplify: Ex. 1.5

(a) $44 - 8 \times 4$ (b) $8 \times 6 \div 3 \times 4$ (c) $28 \div 4 + 3 \times 6$

2 Simplify: Ex. 1.2

(a) $2^4 - 3^2$ (b) $3^3 \times 10^2$ (c) $10^5 - 10^3$

3 Calculate: Ex. 4.4–4.6

(a) $15.006 + 2.45 + 0.059$ (b) 0.2×0.036 (c) $0.597 \div 0.3$

4 Use the following rules to complete the tables. Ex. 5.3

(a) $n = 3m$

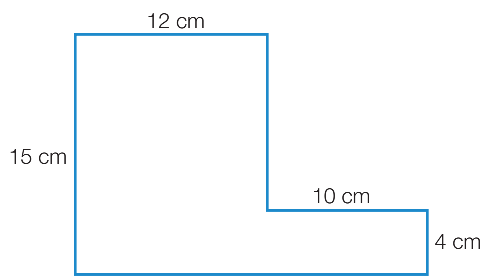
m	4	0	11	25	15
n					

(b) $t = s - 12$

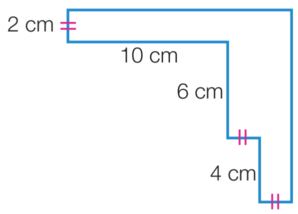
s	18	100	12	22	312
t					

5 Find the area of each of the following shapes. (All angles are right angles.) Ex. 6.5

(a)



(b)



6 Find the following probabilities. Ex. 9.7

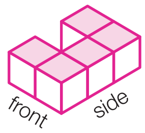
(a) getting a 6 on one roll of a normal die

(b) getting a red king on a single draw from a normal pack of cards

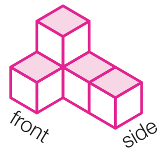
(c) getting two heads when tossing two coins

7 Two shapes have been made using wooden cubes, as shown. Ex. 10.7

(a)



(b)



For each shape, draw: (i) the front elevation (ii) the top elevation.

8 Solve each of the following equations using the balance method. Check your answer by substitution. Ex. 7.4

(a) $3(x - 1) = 6$ (b) $\frac{x}{4} + 1 = 7$ (c) $2(x + 3) = 12$

9 Find the supplementary angles for each of the following. Ex. 8.3

(a) 35° (b) 127° (c) 90°

10 The following numbers represent the heights of students in a class, to the nearest centimetre. Ex. 9.1

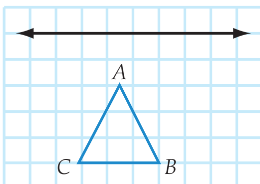
148, 163, 158, 162, 151, 153, 170, 167, 165, 162, 172, 173, 157, 162, 158, 155, 163, 159, 160, 161

Draw the frequency table showing the information. You will need to group the results. Use 145–149, 150–154, 155–159 etc.

11 Show the image of each of the following figures after each of the following combined transformations. Use a protractor to help you. Ex. 10.4

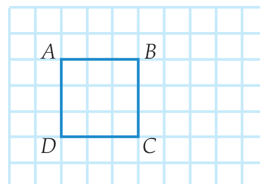
Reflect in the horizontal line of reflection and translate 4 units left and 3 units down.

(a)



Reflect in the line BC and rotate 90° in a clockwise direction about point C'.

(b)



12 For each of the following, identify the number of axes of symmetry. Ex. 10.5

(a) isosceles triangle **(b)** square **(c)** rhombus

13 Find the mean, mode, median and range of the following data, noting any possible outliers. Ex. 9.2

23, 34, 45, 26, 35, 42, 6, 34, 22, 36, 21, 29, 32, 34, 41, 39, 25, 27, 31, 30

Understanding

14 A figure is translated 7 units left, 4 units down, 8 units right and 3 units up. What would be the final position of the image compared to that of the original figure?

15 **(a)** Find the prime factors of 24.

(b) Find the prime factors of 30.

(c) Use the prime factors to find the highest common factor of 24 and 30.

(d) Use the prime factors to find the lowest common multiple of 24 and 30.

16 **(a)** What is the size of each angle in an equilateral triangle?

(b) A parallelogram has one angle that measures 126°. What size is each of the three remaining angles?

17 Which capital letters of the alphabet that have both reflectional and rotational symmetry also have reflectional and rotational symmetry in lower case form?

Reasoning

18 A point (1, 1) is translated 2 units right and 1 unit up. It is then reflected in the y-axis and then reflected in the x-axis. It is then translated 2 units right and 1 unit up. Finally, it is rotated 180° in a clockwise direction about the origin. What are the coordinates of the final image?

19 **(a)** Find a data set with at least six different values that has a median of 6 and a mean of 8.

(b) Describe the process you followed to find these values.

(c) Why might it be easier to use an odd number of values in a question of this type?

20 A game is played in which three different coins are being tossed. It costs 50 cents to play this game. You win $1 (plus your original 50 cents) if more than one of the coins shows heads, otherwise you lose your money.

(a) Write down the set of results that are possible.

(b) Is the game fair? Explain your reasoning.