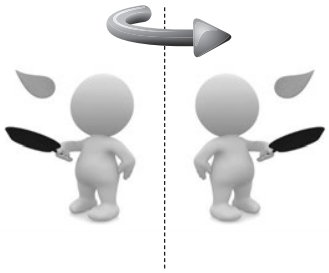
Transformations

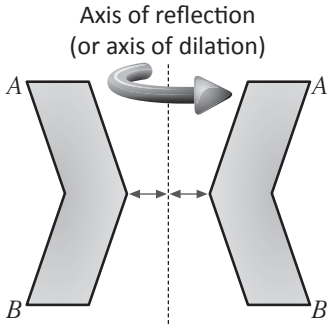
Transformations are all about re-positioning shapes without changing any of their dimensions.

There are three main types:

Reflections (Flip) Reflecting an object about a fixed line called the axis of reflection.

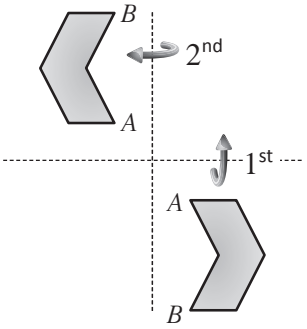


object (before) image (after)



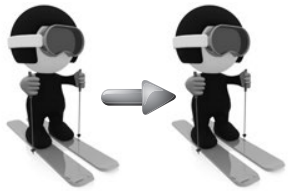
Keep equal spacing from axis.

Horizontal reflection to the right.

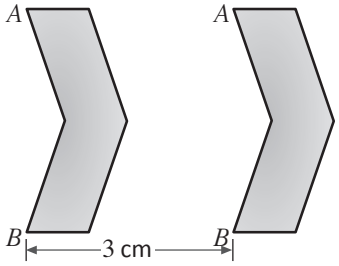


Vertical reflection up followed by a horizontal reflection left.

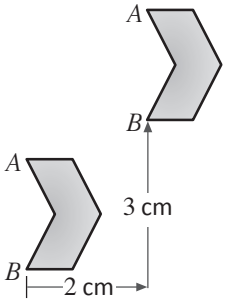
Translations (Slide) This transformation involves sliding an object either horizontally, vertically or both. Every part of the object is moved the same distance.



object (before) image (after)

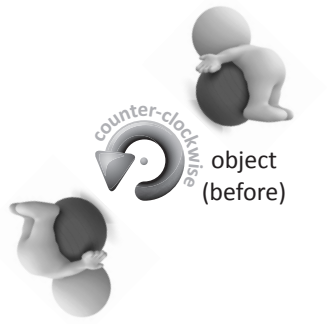


3 cm translation horizontally to the right



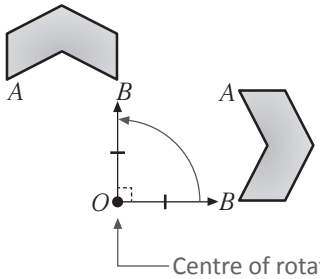
Two translations: 2 cm horizontally right, and then 3 cm vertically up

Rotations (Turn) A transformation of turning an object about a fixed point **counter-clockwise**.

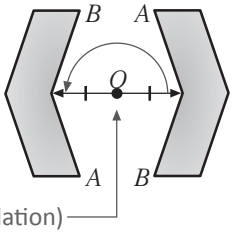


object (before)

image (after)

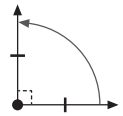


90° rotation (or $\frac{1}{4}$ turn)



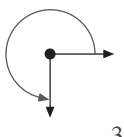
Centre of rotation (or centre of dilation)

180° rotation (or $\frac{1}{2}$ turn)



90° rotation (or $\frac{1}{4}$ turn)

180° rotation (or $\frac{1}{2}$ turn)

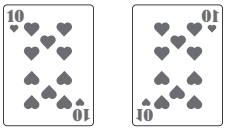


270° rotation (or $\frac{3}{4}$ turn)

Transformations

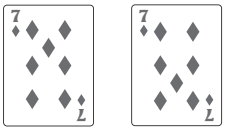
1 Identify which type of transformation each of these playing cards has undergone:

a



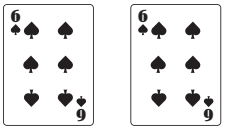
- ☐ Reflection
- ☐ Translation
- ☐ Rotation

b



- ☐ Reflection
- ☐ Translation
- ☐ Rotation

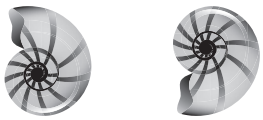
c



- ☐ Reflection
- ☐ Translation
- ☐ Rotation

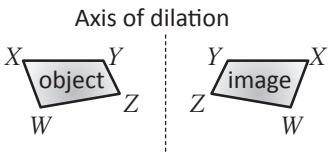
2 Each of these objects has undergone two different transformations. Tick them both.

a



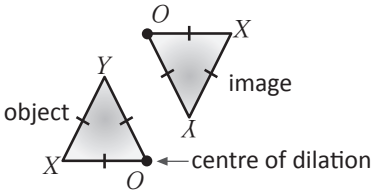
- ☐ Reflection
- ☐ Translation
- ☐ Rotation

b



- ☐ Reflection
- ☐ Translation
- ☐ Rotation

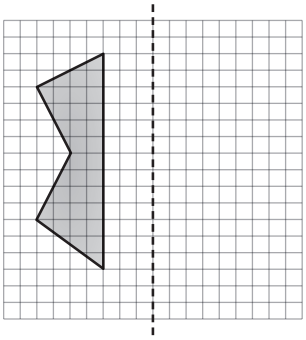
c



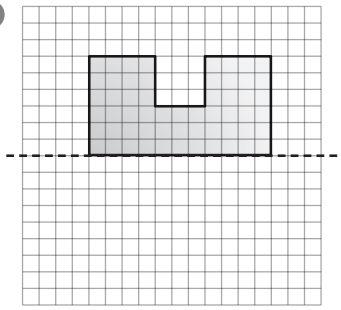
- ☐ Reflection
- ☐ Translation
- ☐ Rotation

3 Draw the image on the grids below when each of these objects are **reflected** about the given axis.

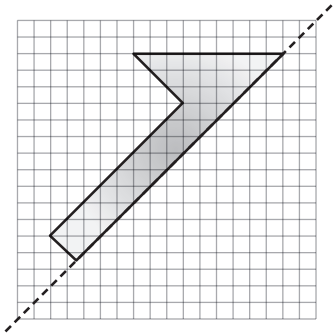
a



b

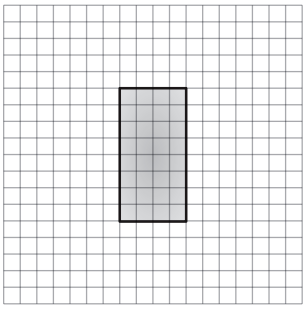


c

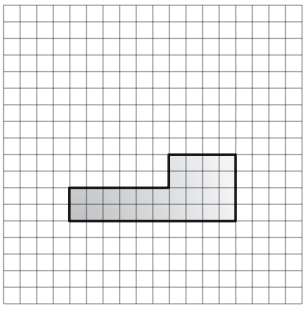


4 Draw the image on the grids below when each of these objects are **translated** by the given amounts.

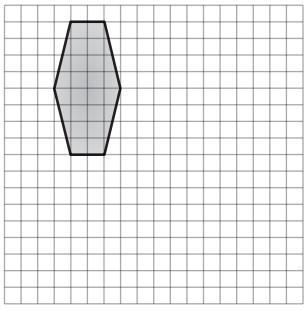
a Five squares horizontally to the left.



b Four squares vertically up.



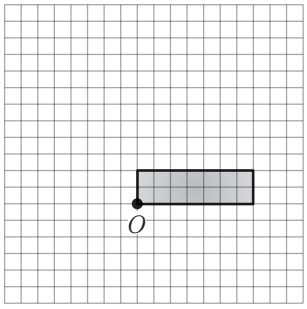
c Eight squares to the right, then six squares down.



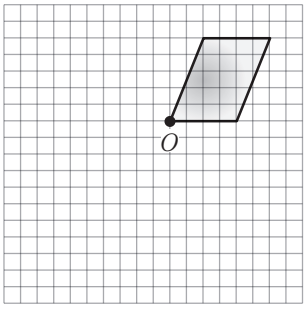
Transformations

5 Draw the image on the grids below when each of these objects are **rotated** by the given amounts.

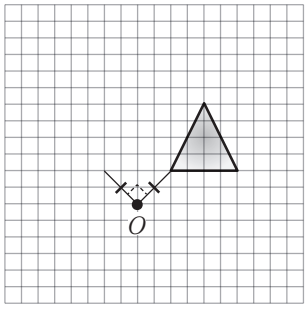
a One half turn (180° rotation).



b Three quarter turn (270° rotation).

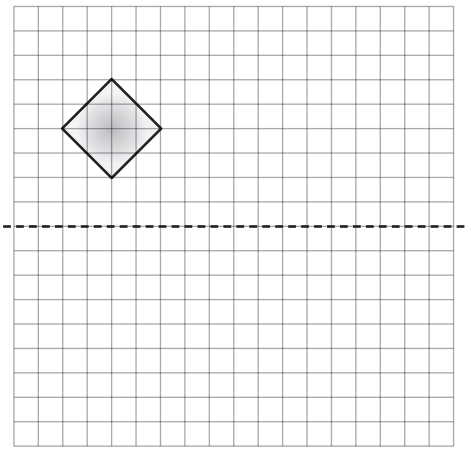


c One quarter turn (90° rotation).

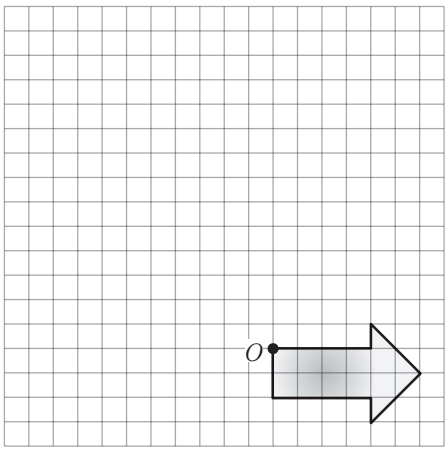


6 Draw the image on the grids below when each of these objects undergo the transformations given.

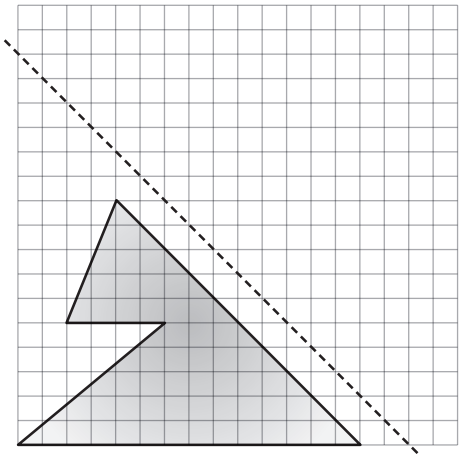
a Translate ten units to the right first then reflect down about the given axis of reflection.



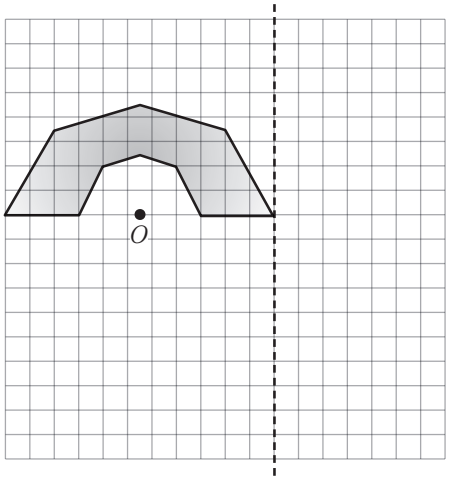
b Rotate 180° about the centre of rotation O, then translate six units up.



c Reflect about the given axis first, then tranlsate two units to the left.



d Three quarter turn (270° rotation) first, then reflect about the given axis of dilation.

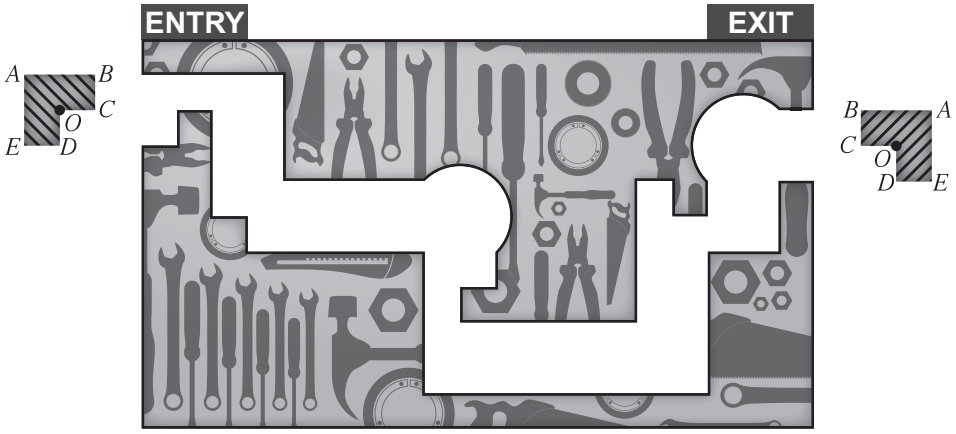


Transformations

7 Earn yourself an awesome passport stamp with this one.
The object ($ABCODE$) requires thirteen transformations to move along the white production line below. It needs to leave in the position shown at the exit for the next stage of production.

- The object must not overlap the shaded part around the production line path.
- Any of the sides AB, BC, DE and AE can be used as an axis of reflection.
- The vertex O is the only centre of rotation used at the two circle points along the path.

Describe the thirteen transformation steps used to navigate this object along the path, including the direction of transformation and the sides/points used as axes of dilation where appropriate.



(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

(viii)

(ix)

(x)

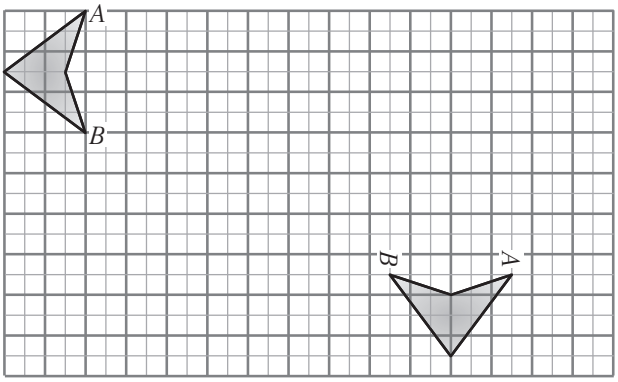
(xi)

(xii)

(xiii)

Transformations

8 For the diagram shown below, describe four different ways the final image of the object can be achieved using different transformations.



a Method 1

b Method 2

c Method 3

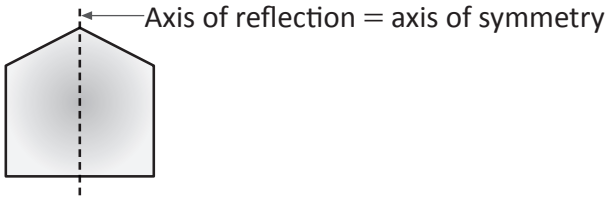
d Method 4

Reflection symmetry

There are many types of symmetry and in this booklet we will just be focusing on three of them.

If the axis of reflection splits a shape into two identical pieces, then that shape has reflection symmetry.

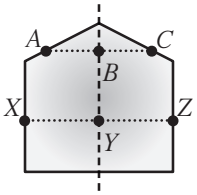
The axis of reflection is then called the "axis of symmetry".



Symmetric
Shape has reflection symmetry

Asymmetric
Shape does not have reflection symmetry

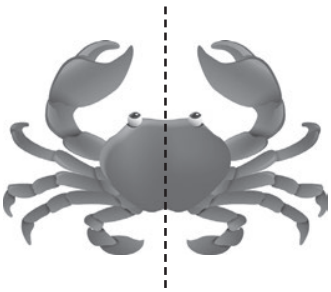
The distances from the edge of the shape to the axis of symmetry are the same on both sides of the line.



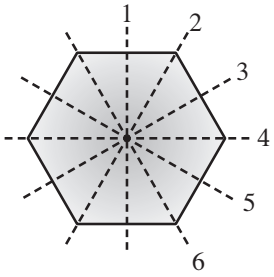
$AB = BC$ and $XY = YZ$

This shape has only one axis of symmetry. When this happens, we say the shape has bilateral symmetry.

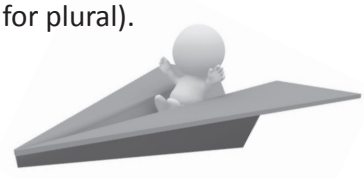
Many animals/plants or objects in nature have nearly perfect bilateral symmetry.



Other shapes can have more than one axis of symmetry (axes of symmetry for plural).



Regular Hexagon



There are 6 different ways this shape can be folded in half with both sides of the fold fitting over each other exactly. So we can say it has six-fold symmetry.

Reflection symmetry

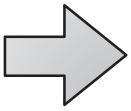
1 (i) Identify which of these shapes have reflection symmetry by ticking symmetric or asymmetric.

(ii) Draw all the axes of symmetry for those that do.

a

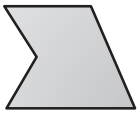
☐ Symmetric
☐ Asymmetric

b



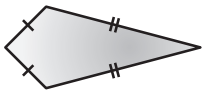
☐ Symmetric
☐ Asymmetric

c



☐ Symmetric
☐ Asymmetric

d



☐ Symmetric
☐ Asymmetric

e

☐ Symmetric
☐ Asymmetric

f

☐ Symmetric
☐ Asymmetric

g

☐ Symmetric
☐ Asymmetric

h

☐ Symmetric
☐ Asymmetric

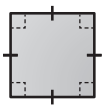
i

☐ Symmetric
☐ Asymmetric

j

☐ Symmetric
☐ Asymmetric

k



☐ Symmetric
☐ Asymmetric

l

☐ Symmetric
☐ Asymmetric

2 How many axes of reflection symmetry would these nature items have if perfectly symmetrical?

a

☐

b

☐

c

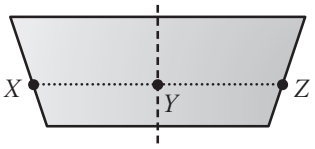
☐

d

☐

3 These shapes all have reflection symmetry. Calculate the distance between X and Y.

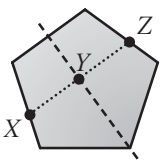
a



$YZ = 5$ cm

Distance from X to $Y =$ ☐

b



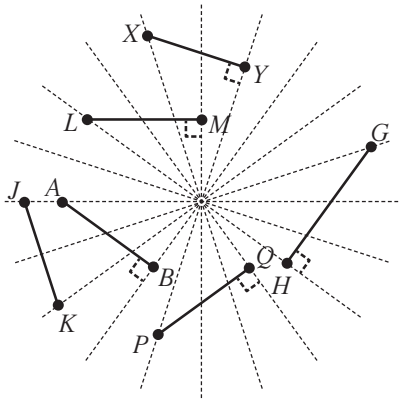
$XZ = 14$ cm

Distance from X to $Y =$ ☐

Reflection symmetry

/20

4 Answer these questions about the symmetric web below:



a How many axes of symmetry does the web have?

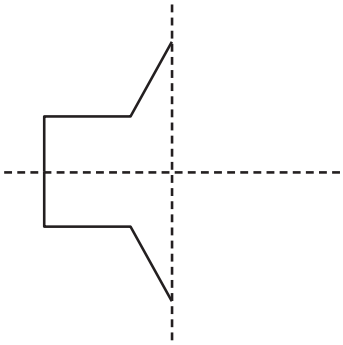
b What pair of points are equidistant to LM?
Psst: equidistant means the 'same distance'

c Briefly explain below how you decided this was the correct answer.

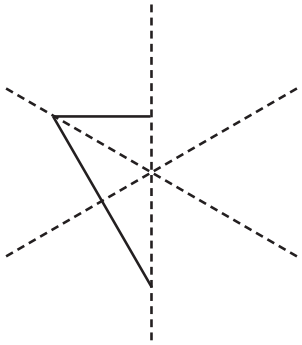
5 Complete these diagrams to produce an image with as many axes of reflective symmetry as indicated.

a Bilateral symmetry.

b Two fold symmetry.

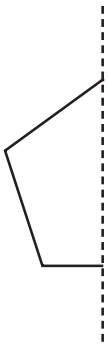


c Three axes of symmetry.



d Two axes of symmetry.

e Five-fold symmetry.
(show the other four axes)



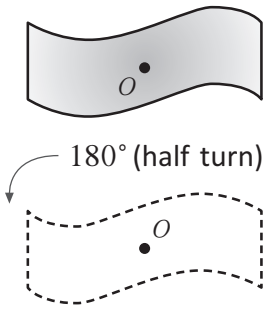
f Eight-fold symmetry.
(show the other seven axes)

Rotational symmetry

When an object is rotated $360°$ (a full circle), it looks the same as it was before rotating.

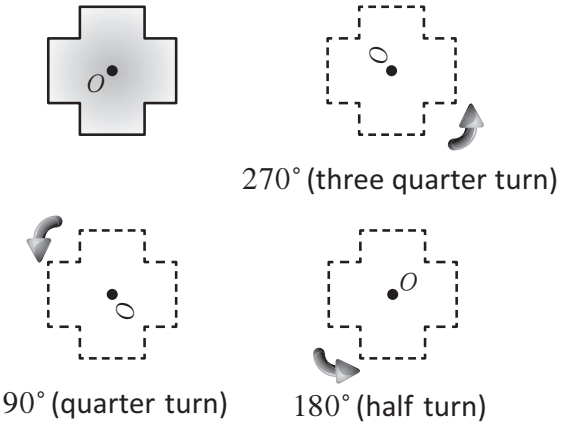
If the object looks the same again before completing a full circle, it has **rotational symmetry**.

The number of times the object 'repeats' before completing the full circle tells us the **order of rotational symmetry**.



Rotational Symmetry of order 2

i.e. it looks the same 2 times in one full rotation.



Rotational Symmetry of order 4

i.e. it looks the same 4 times in one full rotation.

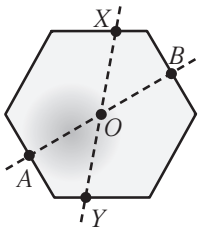
Point symmetry

This is when an object has parts the same distance away from the centre of symmetry in the opposite direction.

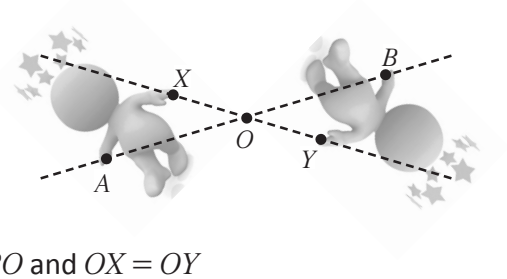
A straight line through the centre of symmetry will cross at least two points on the object.

Each pair of points crossed on opposite sides of the centre of symmetry are an equal distance away from it.

Point symmetry for one object



Point symmetry for a picture with two objects



For both diagrams: $AO = BO$ and $OX = OY$

These both have point symmetry because for every point on them, there is another point opposite the centre of symmetry (O) the same distance away.

Objects and pictures can often have both rotational and point symmetry.

Rotational and point symmetry

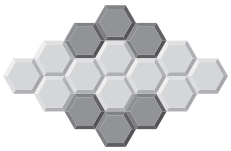
1 Identify which of these objects are rotationally symmetric or asymmetric.

a 

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

b 

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

c 

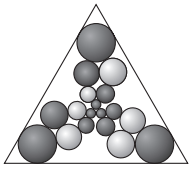
- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

d 

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

e 

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

f 

- ☐ Rotationally symmetric
- ☐ Rotationally asymmetric

2 Write the order of rotational symmetry each of these mathematical symbols have:

a  ☐

b  ☐

c  ☐

d  ☐

3 a All these propellers have rotational symmetry. Identify which ones also have point symmetry.

(i) 

- ☐ Has point symmetry
- ☐ No point symmetry

(ii) 

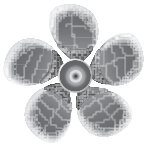
- ☐ Has point symmetry
- ☐ No point symmetry

(iii) 

- ☐ Has point symmetry
- ☐ No point symmetry

(iv) 

- ☐ Has point symmetry
- ☐ No point symmetry

(v) 

- ☐ Has point symmetry
- ☐ No point symmetry

(vi) 

- ☐ Has point symmetry
- ☐ No point symmetry

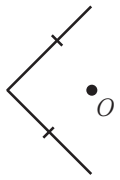
b Describe the relationship between the number of blades and the point symmetry of these propellers.

c Describe the relationship between the number of blades and the order of point symmetry for the symmetric blades.

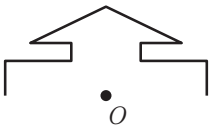
Rotational and point symmetry

4 Complete each of the half drawn shapes below to match the given symmetries.

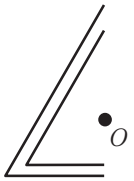
a Rotational symmetry of order 4 and also point symmetry.



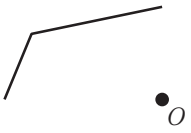
b Rotational symmetry of order 2 and also point symmetry.



c Rotational symmetry of order 3 and no point symmetry.



d Rotational symmetry of order 2 and also point symmetry.

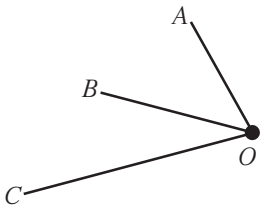


5 All the vertices shown below represent half of all the vertices of shapes which have point symmetry about the centre of rotation (O).

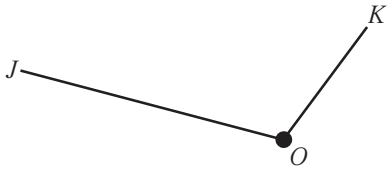
(i) Mark in the other vertices.

(ii) Draw the boundary of the whole shape.

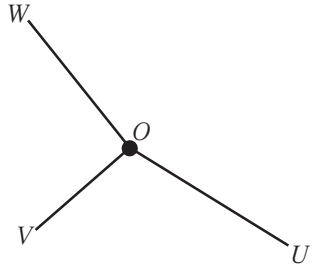
a



b



c



d