Chapter

3 Number properties and patterns

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

Investigate and use square roots of perfect square numbers (ACMNA150)

Linear and non-linear relationships

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

Investigate, interpret and analyse graphs from authentic data (ACMNA180)

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Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Number patterns around us: Architecture

The Louvre Palace in Paris is the world's largest museum and is visited by over 8 million people a year. Visitors enter the museum through a giant glass pyramid that has a square base of length 35.4 metres and is 21.6 metres in height. It is said that the pyramid contains 666 glass panels.

To carefully count the number of glass panels, we can observe how the sides of the pyramid are constructed. Each triangular side of the pyramid has 17 rows of rhombus-shaped glass panels. The base row is joined to the ground by triangular-shaped glass panels. The trapezium-shaped entry has a height of one rhombus and a width of six triangles. The number of glass panels used in the Louvre Pyramid can be determined using these facts and the related properties and patterns.

Each rhombus panel is supported by four segments of steel. Adjacent rhombuses share the ame steel segment for support. The number of steel segments per row can be calculated, as well as the total number of steel segments used.

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3A Factors and multiples CONSOLIDATING

Interactive
Widgets
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Walkthroughs

Number patterns are fascinating. Factors and multiples are key building blocks for a broad field known as Number Theory. Many famous mathematicians have studied number patterns in an attempt to better understand our world and to assist with new scientific discoveries. Around 600 BCE, the Greeks built on the early work of the Egyptians and Babylonians. Thales of Miletus, the 'father of Greek mathematics', is credited for significant advances in Number Theory. One of his students, Pythagoras of Samos, went on to become one of the most well-known mathematicians to have lived. Pythagoras was primarily a religious leader, but he believed that the understanding of the world could be enhanced through the understanding of numbers. We start this chapter on Number Patterns by explaining the concepts of factors and multiples.


How many factors are there in a set of 12?

One dozen doughnuts are generally packed into bags with 3 rows of 4 doughnuts each. Since $3 \times 4 = 12$, we can say that 3 and 4 are **factors** of 12.

Purchasing 'multiple' packs of one dozen doughnuts could result in buying 24, 36, 48 or 60 doughnuts, depending on the number of packs. These numbers are known as **multiples** of 12.

Let's start: The most factors, the most multiples

Which number that is less than 100 has the most factors?

Which number that is less than 100 has the most multiples less than 100?

Key ideas

- **Factors** of a particular number are numbers that divide exactly into that number.
  - For example: The factors of 20 are pairs of numbers that multiply to give 20 which are 1×20, 2×10 and 4×5.
    Therefore, written in **ascending** order, the factors of 20 are 1, 2, 4, 5, 10, 20.
  - Every whole number is a factor of itself and also 1 is a factor of every whole number.
- **Multiples** of a particular number are numbers created by multiplying the particular number by any whole number.
  - For example: The multiples of 20 are 20, 40, 60, 80, 100, 120, …
    Multiples of 20 are also 480, 2000, 68 600. There is an infinite number of multiples!
- Given the statements above, it follows that factors are less than or equal to the particular number being considered and multiples are greater than or equal to the number being considered.

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Example 1 Finding factors

Find the complete set of factors for each of these numbers.

a 15 **b** 40

SOLUTION	EXPLANATION
a Factors of 15 are 1, 3, 5, 15.	$1 \times 15 = 15$, $3 \times 5 = 15$
b Factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40.	$1 \times 40 = 40$, $2 \times 20 = 40$ $4 \times 10 = 40$, $5 \times 8 = 40$ The last number you need to check is 7.

Example 2 Listing multiples

Write down the first six multiples for each of these numbers.

a 11 **b** 35

SOLUTION	EXPLANATION
a 11, 22, 33, 44, 55, 66	The first multiple is always the given number. Add on the given number to find the next multiple. Repeat this process to get more multiples.
b 35, 70, 105, 140, 175, 210	Start at 35, the given number, and repeatedly add 35 to continue producing multiples.

Example 3 Finding factor pairs

Express 195 as a product of two factors, both of which are greater than 10.

SOLUTION	EXPLANATION
$195 = 13 \times 15$	Systematically divide 195 by numbers greater than 10 in an attempt to find a large factor.

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Exercise 3A

1–2	2	—

UNDERSTANDING

1 For each of the following numbers, state whether they are factors (F), multiples (M) or neither (N) of the number 60.

a 120	b 14	c 15	d 40
e 6	f 5	g 240	h 2
i 22	j 600	k 70	l 1

2 For each of the following numbers, state whether they are factors (F), multiples (M) or neither (N) of the number 26.

a 2	b 54	c 52	d 4
e 210	f 27	g 3	h 182
i 1	j 26 000	k 13	l 39

3–5(½)	3–6(½)	3–6(½)

FLUENCY

Example 1

3 List the complete set of factors for each of the following numbers.

a 10	b 24	c 17
d 36	e 60	f 42
g 80	h 12	i 28

Example 2

4 Write down the first six multiples for each of the following numbers.

a 5	b 8	c 12
d 7	e 20	f 75
g 15	h 100	i 37

5 Fill in the gaps to complete the set of factors for each of the following numbers.

a 18 1, 2, __, 6, 9, __

b 25 1, __, 25

c 72 __, 2, 3, __, __, 8, __, __, 18, __, 36, 72

d 120 1, 2, __, __, __, 6, __, 10, __, __, 20, __, 30, __, 60, __

6 Which number is the incorrect multiple for each of the following sequences.

a 3, 6, 9, 12, 15, 18, 22, 24, 27, 30

b 43, 86, 129, 162, 215, 258, 301, 344

c 11, 21, 33, 44, 55, 66, 77, 88, 99, 110

d 17, 34, 51, 68, 85, 102, 117, 136, 153, 170

7–8	8–10	8–10

PROBLEM-SOLVING

7 Consider the set of whole numbers from 1 to 25 inclusive.

a Which number has the most factors?

b Which number has the fewest factors?

c Which numbers have an odd number of factors?

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3A

PROBLEM-SOLVING

Example 3

8 Express each of the following numbers as a product of two factors, both of which are greater than 10.

a 192　　b 315　　c 180
d 121　　e 336　　f 494

9 Zane and Matt are both keen runners. Zane takes 4 minutes to jog around a running track and Matt takes 5 minutes. They start at the same time and keep running until they both cross the finish line at the same time.

a How long do they run for?
b How many laps did Zane run?
c How many laps did Matt run?

10 Anson is preparing for his 12th birthday party. He has invited 12 friends and is making each of them a 'lolly bag' to take home after the party. To be fair, he wants to make sure that each friend has the same number of lollies. Anson has a total of 300 lollies to share among the lolly bags.

a How many lollies does Anson put in each of his friends' lolly bags?
b How many lollies does Anson have left over to eat himself?

Anson then decides that he wants a lolly bag for himself also.

c How many lollies will now go into each of the 13 lolly bags?

After much pleading from his siblings, Anson prepares lolly bags for them also. His sister Monique notices that the total number of lolly bags is now a factor of the total number of lollies.

d What are the different possible number of sibling(s) that Anson could have?
e How many siblings do you expect Anson has?

11	11–13	13–15

REASONING

11 Are the following statements true or false?

a A multiple of a particular number is always smaller than that number.
b 2 is a factor of every even number.
c 3 is a factor of every odd number.
d A factor is always greater than or equal to the given number.
e When considering a particular number, that number is both a factor and a multiple of itself.

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3A

REASONING

12 60 is a number with many factors. It has a total of 12 factors and, interestingly, it has each of the numbers 1, 2, 3, 4, 5, 6 as a factor.

- a What would be the smallest number that could boast having 1, 2, 3, 4, 5, 6, 7 and 8 as factors?
- b What would be the smallest number that could boast having 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 as factors?

13 a What numbers can claim the number 100 to be a multiple?
- b What are the factors of 100?

14 All Australian AM radio stations have frequencies that are multiples of 9. For example, a particular radio station has a frequency of 774 (kilohertz or kHz). Find three other AM radio stations and show their frequencies are, indeed, multiples of 9.

15 Two numbers are chatting with one another when one number asks the other, 'Are you a multiple of mine?' The reply comes back, 'Well, I have always considered you to be one of my factors'. Explain why this response is enough to help the first number answer her question. Which number is the larger number?

Factors and multiples with computers

—	—	16

ENRICHMENT

16 a Design a spreadsheet that will enable a user to enter any number between 1 and 100 and it will automatically list the first 30 multiples of that number.
- b Design a spreadsheet that will enable a user to enter any particular number between 1 and 100 and it will automatically list the number's factors.
- c Improve your factor program so that it finds the sum of the factors and also states the total number of factors for the particular number.
- d Use your spreadsheet program to help you find a pair of **amicable numbers**. A pair of numbers is said to be amicable if the sum of the factors for each number, excluding the number itself, is equal to the other number. Each number that makes up the first such pair of amicable numbers falls between 200 and 300.

  An example of a non-amicable pair of numbers:

  12 – factor sum $= 1 + 2 + 3 + 4 + 6 = 16$

  16 – factor sum $= 1 + 2 + 4 + 8 = 15$

  The factor sum for 16 would need to be 12 for the pair to be amicable numbers.

  Helpful Excel formulas

  INT(number) – Rounds a number down to the nearest integer (whole number).

  MOD(number, divisor) – Returns the remainder after a number is divided by its divisor.

  IF(logical test, value if true, value if false) – Checks whether a condition is met and returns one value if true and another value if false.

  COUNTIF(range, criteria) – Counts the number of cells within a range that meet the given condition.

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3B Highest common factor and lowest common multiple

CONSOLIDATING

Interactive
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In the previous exercise, factors and muliples of a number were explained. Remember that factors are less than or equal to a given number and that multiples are greater than or equal to a given number.

given number
e.g. 12

factors	**multiples**
≤ 12	≥ 12
e.g. 1, 2, 3, 4, 6, 12	e.g. 12, 24, 36, 48, . . .

There are many applications in Mathematics for which the highest common factor (HCF) of two or more numbers must be determined. In particular, the skill of finding the HCF is required for the future topic of factorisation, which is an important aspect of Algebra.

Similarly, there are many occasions for which the lowest common multiple (LCM) of two or more numbers must be determined. Adding and subtracting fractions with different denominators requires the skill of finding the LCM.

Let's start: You provide the starting numbers!

For each of the following answers, you must determine possible starting numbers. On all occasions, the numbers involved are less than 100.

1. The HCF of two numbers is 12. Suggest two possible starting numbers.
2. The HCF of three numbers is 11. Suggest three possible starting numbers.
3. The LCM of two numbers is 30. Suggest two possible starting numbers.
4. The LCM of three numbers is 75. Suggest three possible starting numbers.
5. The HCF of four numbers is 1. Suggest four possible numbers.
6. The LCM of four numbers is 24. Suggest four possible numbers.

Key ideas

- **HCF** stands for **highest common factor**.
- As the name suggests, it refers to the highest (i.e. largest) factor that is common to the numbers provided in the question.
  - For example: Find the HCF of 24 and 40.
    Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
    Factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.
    Therefore, common factors of 24 and 40 are 1, 2, 4 and 8.
    Therefore, the highest common factor of 24 and 40 is 8.

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Key ideas

- **LCM** stands for **lowest common multiple**.
- As the name suggests, it refers to the lowest (i.e. smallest) multiple that is common to the numbers provided in the question.
  - For example: Find the LCM of 20 and 12.
    Multiples of 20 are 20, 40, 60, 80, 100, 120, 140, …
    Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, …
    Therefore, common multiples of 20 and 12 are 60, 120, 180, …
    Therefore, the lowest common multiple of 20 and 12 is 60.
- The LCM of two numbers can always be found by multiplying the two numbers together and dividing by their HCF.
  - For example: Find the LCM of 20 and 12.
    The HCF of 20 and 12 is 4.
    Therefore, the LCM of 20 and 12 is $20 \times 12 \div 4 = 60$.

Example 4 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 36 and 48.

SOLUTION	EXPLANATION
Factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.	$1 \times 36 = 36, 2 \times 18 = 36, 3 \times 12 = 36,$ $4 \times 9 = 36, 6 \times 6 = 36$
Factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.	$1 \times 48 = 48, 2 \times 24 = 48, 3 \times 16 = 48,$ $4 \times 12 = 48, 6 \times 8 = 48$
The HCF of 36 and 48 is 12.	Common factors are 1, 2, 3, 4, 6 and 12, of which 12 is the highest.

Example 5 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of the following pairs of numbers.

a 5 and 11 **b** 6 and 10

SOLUTION	EXPLANATION
a The LCM of 5 and 11 is 55.	Note that the HCF of 5 and 11 is 1. $5 \times 11 \div 1 = 55$
b The LCM of 6 and 10 is 30.	Note that the HCF of 6 and 10 is 2. The LCM of 6 and 10 is $6 \times 10 \div 2 = 30$. Multiples of 6 are 6, 12, 18, 24, 30, 36, … Multiples of 10 are 10, 20, 30, 40, …

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Exercise 3B

1–4	4	—

UNDERSTANDING

1 The factors of 12 are 1, 2, 3, 4, 6 and 12, and the factors of 16 are 1, 2, 4, 8 and 16.
- a What are the common factors of 12 and 16?
- b What is the HCF of 12 and 16?

2 Fill in the missing numbers to find out the HCF of 18 and 30.
Factors of 18 are 1, __, 3, __, __ and 18.
Factors of __ are 1, __, __, 5, __, 10, __ and 30.
Therefore, the HCF of 18 and 30 is __.

3 The first 10 multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72 and 80.
The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54 and 60.
- a What are two common multiples of 8 and 6?
- b What is the LCM of 8 and 6?

4 Fill in the missing numbers to find out the LCM of 9 and 15.
Multiples of 9 are 9, 18, __, 36, __, __, __, __, 81 and __.
Multiples of 15 are __, 30, __, 60, 75, __, __ and 120.
Therefore, the LCM of 9 and 15 is __.

5–8(½)	5–9(½)	5–9(½)

FLUENCY

Example 4

5 Find the HCF of the following pairs of numbers.
- a 4 and 5
- b 8 and 13
- c 2 and 12
- d 3 and 15
- e 16 and 20
- f 15 and 60
- g 50 and 150
- h 48 and 72
- i 80 and 120
- j 75 and 125
- k 42 and 63
- l 28 and 42

6 Find the HCF of the following groups of numbers.
- a 20, 40, 50
- b 6, 15, 42
- c 50, 100, 81
- d 18, 13, 21
- e 24, 72, 16
- f 120, 84, 144

Example 5

7 Find the LCM of the following pairs of numbers.
- a 4 and 9
- b 3 and 7
- c 12 and 5
- d 10 and 11
- e 4 and 6
- f 5 and 10
- g 12 and 18
- h 6 and 9
- i 20 and 30
- j 12 and 16
- k 44 and 12
- l 21 and 35

8 Find the LCM of the following groups of numbers.
- a 2, 3, 5
- b 3, 4, 7
- c 2, 3, 4
- d 3, 5, 9
- e 4, 5, 8, 10
- f 6, 12, 18, 3

9 Find the HCF of the following pairs of numbers and then use this information to help calculate the LCM of the same pair of numbers.
- a 15 and 20
- b 12 and 24
- c 14 and 21
- d 45 and 27

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3B

	10, 11	11, 12	11–13

PROBLEM-SOLVING

10 Find the LCM of 13 and 24.

11 Find the HCF of 45 and 72.

12 Find the LCM and HCF of 260 and 390.

13 Andrew runs laps of 'the circuit' in 4 minutes. Bryan runs laps of the same circuit in 3 minutes. Chris can run laps of the same circuit in 6 minutes. They all start together on the starting line and run a 'race' that goes for 36 minutes.
   a What is the first time, after the start, that they will all cross over the starting line together?
   b How many laps will each boy complete in the race?
   c How many times does Bryan overtake Andrew during this race?

	14	14, 15	15, 16

REASONING

14 Given that the HCF of a pair of different numbers is 8, find the two numbers:
   a if both numbers are less than 20
   b when one number is in the 20s and the other in the 30s

15 Given that the LCM of a pair of different numbers is 20, find the seven possible pairs of numbers.

16 The rule for finding the LCM of two numbers x and y is $\dfrac{x \times y}{\text{HCF }(x, y)}$. Is the rule for the LCM of three numbers x, y and z $\dfrac{x \times y \times z}{\text{HCF }(x, y, z)}$?

LCM of large groups of numbers	—	—	17

ENRICHMENT

17 a Find the LCM of these single-digit numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9.
   b Find the LCM of these first 10 natural numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
   c Compare your answers to parts **a** and **b**. What do you notice? Explain.
   d Find the LCM of the first 11 natural numbers.

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3C Divisibility EXTENDING

Interactive
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Walkthroughs

It is useful to know whether a large number is exactly divisible by another number. Although we can always carry out the division algorithm, this can be a difficult and tedious process for large numbers. There are simple divisibility tests for each of the single-digit numbers, with the exception of 7. These divisibility tests determine whether or not the number is divisible by the chosen divisor.

Let's start: Five questions in 5 minutes

In small groups, attempt to solve the following five questions in 5 minutes.

1. Some numbers are only divisible by 1 and themselves. What are these numbers called?
2. Is 21 541 837 divisible by 3?
3. What two-digit number is the 'most divisible' (i.e. has the most factors)?
4. Find the smallest number that is divisible by 1, 2, 3, 4, 5 and 6.
5. Find a number that is divisible by 1, 2, 3, 4, 5, 6, 7 and 8.

Key ideas

- A number is said to be **divisible** by another number if there is **no remainder** after the division has occurred.
- If the **divisor** divides into the **dividend** exactly, then the divisor is said to be a **factor** of that number.
- **Division notation**

  Example: $27 \div 4 = 6$ remainder 3

  remainder

  dividend →
  divisor →

  $$\frac{27}{4} = 6 \text{ rem. } 3 = 6\frac{3}{4}$$

  quotient

  Another way of representing this information is $27 = 4 \times 6 + 3$.
- **Key terms**

  Dividend The starting number; the total; the amount you have

  Divisor The number doing the dividing; the number of groups

  Quotient The number of times the divisor went into the dividend, also known as 'the answer'

  Remainder The number left over; the number remaining (sometimes written as 'rem.')
- **Divisibility tests**
  1. All numbers are divisible by 1.
  2. All even numbers are divisible by 2. Last digit must be a 0, 2, 4, 6 or 8.
  3. The sum of the digits must be divisible by 3.
  4. The number formed from the last two digits must be divisible by 4.
  5. The last digit must be a 0 or 5.

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Key ideas

6 Must pass the divisibility tests for 2 and 3.
7 There is no easy divisibility test for the numeral 7.
8 The number formed from the last three digits must be divisible by 8.
9 The sum of the digits must be divisible by 9.
10 The last digit must be 0.

Example 6 Applying divisibility tests

Determine whether or not the following calculations are possible without leaving a remainder.

a $54\,327 \div 3$ **b** $765\,146 \div 8$

SOLUTION

a Digit sum = 21
Yes, 54 327 is divisible by 3.

b $8\overline{)14{}^{6}6}$ = 18 rem. 2
No, 765 146 is not divisible by 8.

EXPLANATION

a $5 + 4 + 3 + 2 + 7 = 21$
21 is divisible by 3.

b Check whether the last three digits are divisible by 8.

Example 7 Testing divisibility

Carry out divisibility tests on the given number and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412								

SOLUTION

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412	☑	☑	☑	✗	☑	✗	✗	✗

EXPLANATION

48 569 412 is an even number and therefore is divisible by 2.
48 569 412 has a digit sum of 39 and therefore is divisible by 3, but not by 9.
48 569 412 is divisible by 2 and 3, therefore it is divisible by 6.
The last two digits are 12, which is divisible by 4.
The last three digits are 412, which is not divisible by 8.
The last digit is a 2 and therefore is not divisible by 5 or 10.

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Exercise 3C

	1–4	4	—

UNDERSTANDING

1 Give a reason why:

a 8631 is not divisible by 2
b 31 313 is not divisible by 3
c 426 is not divisible by 4
d 5044 is not divisible by 5
e 87 548 is not divisible by 6
f 214 125 is not divisible by 8
g 3 333 333 is not divisible by 9
h 56 405 is not divisible by 10

2 Give the remainder when:

a 326 is divided by 3
b 21 154 is divided into groups of four
c 72 is divided into six groups
d 45 675 is shared into five groups

3 Which three divisibility tests involve calculating the sum of the digits?

4 If you saw only the last digit of a 10-digit number, which three divisibility tests (apart from 1) could you still apply?

	5–7	5–6(½), 7	5–6(½), 7

FLUENCY

Example 6

5 a Determine whether the following calculations are possible without leaving a remainder.

i $23\,562 \div 3$
ii $39\,245\,678 \div 4$
iii $1\,295\,676 \div 9$
iv $213\,456 \div 8$
v $3\,193\,457 \div 6$
vi $2\,000\,340 \div 10$
vii $51\,345\,678 \div 5$
viii $215\,364 \div 6$
ix $9543 \div 6$
x $25\,756 \div 2$
xi $56\,789 \div 9$
xii $324\,534\,565 \div 5$
xiii $2\,345\,176 \div 8$
xiv $329\,541 \div 10$
xv $225\,329 \div 3$
xvi $356\,781\,276 \div 9$
xvii $164\,567 \div 8$
xviii $2\,002\,002\,002 \div 4$

b Repeat the process using a calculator. Which way is quicker?

6 Write down five two-digit numbers that are divisible by:

a 5
b 3
c 2
d 6
e 8
f 9
g 10
h 4

Example 7

7 Carry out divisibility tests on the given numbers and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567								
28 080								
189 000								
1 308 150								
1 062 347								

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3C

8, 9	9–11	11–13

PROBLEM-SOLVING

8 a Can Julie share $41.75 equally among her three children?
b Julie finds one more dollar on the floor and realises that she can now share the money equally among her three children. How much do they each receive?

9 The game of 'clusters' involves a group getting into smaller-sized groups as quickly as possible once a particular cluster size has been called out. If a year level consists of 88 students, which 'cluster' sizes would ensure no students are left out of a group?

10 How many of the whole numbers between 1 and 250 inclusive are not divisible by 5?

11 How many two-digit numbers are divisible by 2 and 3?

12 Find the largest three-digit number that is divisible by both 4 and 5.

13 Find the largest three-digit number that is divisible by both 6 and 7.

14	14, 15	15–17

REASONING

14 a Is the number 968 362 396 392 139 963 359 divisible by 3?
b Many of the digits in the number above can actually be ignored when calculating the digit sum. Which numbers can be ignored and why?
c To determine if the number above is divisible by 3, only five of the 21 digits actually need to be added together. Find this 'reduced' digit sum.

15 The divisibility test for the numeral 4 is to consider whether the number formed by the last two digits is a multiple of 4. Complete the following sentences to make a more detailed divisibility rule.
a If the second-last digit is even, the last digit must be either a ___, ___or ___.
b If the second-last digit is odd, the last digit must be either a ___or ___.

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REASONING

16 Blake's age is a two-digit number. It is divisible by 2, 3, 6 and 9. How old is Blake if you know that he is older than 20 but younger than 50?

17 Find the smallest number that satisfies each of the conditions below. The number must be larger than the divisor and leave:

a a remainder of 5 when divided by 6
b a remainder of 4 when divided by 5
c a remainder of 3 when divided by 4
d a remainder of 2 when divided by 3
e a remainder of 1 when divided by 2

Divisible by 11?

—	—	18

ENRICHMENT

18 a Write down the first nine multiples of the numeral 11.
b What is the difference between the two digits for each of these multiples?
c Write down some three-digit multiples of 11.
d What do you notice about the sum of the first digit and the last digit?

The following four-digit numbers are all divisible by 11:
1606, 2717, 6457, 9251, 9306

e Find the sum of the odd-placed digits and the sum of the even-placed digits. Then subtract the smaller sum from the larger. What do you notice?
f Write down a divisibility rule for the number 11.
g Which of the following numbers are divisible by 11?

i 2 594 669	ii 45 384 559	iii 488 220
iv 14 641	v 1 358 024 679	vi 123 456 789 987 654 321

An alternative method is to alternate adding and subtracting each of the digits.
For example: 4 134 509 742 is divisible by 11.
Alternately adding and subtracting the digits will give the following result:

$$4 - 1 + 3 - 4 + 5 - 0 + 9 - 7 + 4 - 2 = 11$$

h Try this technique on some of your earlier numbers.

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3D Prime numbers

Interactive
Widgets
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Walkthroughs

It is believed that prime numbers (i.e. positive whole numbers with two factors) were first studied by the ancient Greeks. More recently, the introduction of computers has allowed for huge developments in this field. Computers have allowed mathematicians to determine which large numbers are primes. Programs have also been written to automatically generate huge prime numbers that could not be calculated previously by hand.


Computers allow mathematicians to examine and work with extremely large numbers.

There continues to be much debate as to whether or not 1 is a prime number. The current thinking is that 1 should not be considered a prime number, the basic reason being that it does not have two distinct factors.

Remarkable fact: There are some interesting prime numbers that have patterns in their digits; for example, 12 345 678 901 234 567 891. This is known as an ascending prime.

You can also get palindromic primes, such as 111 191 111 and 123 494 321.

Below is a palindromic prime number that reads the same upside down or when viewed in a mirror.

1888081808881

Let's start: How many primes?

How many numbers from 1 to 100 are prime?

You and a classmate have 4 minutes to come up with your answer.

Key ideas

- A **prime number** is a positive whole number that has only two factors: 1 and itself.
- A number that has more than two factors is called a **composite number**.
- 0 and 1 are neither prime nor composite numbers.

Example 8 Determining whether a number is a prime or composite

State whether each of these numbers is a prime or composite: 22, 35, 17, 11, 9, 5.

SOLUTION	EXPLANATION
Prime: 5, 11, 17	5, 11, 17 have only two factors (1 and itself).
Composite: 9, 22, 35	$9 = 3 \times 3, 22 = 2 \times 11, 35 = 5 \times 7$

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Example 9 Finding prime factors

Find the prime numbers that are factors of 30.

SOLUTION	EXPLANATION
Factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30	Find the entire set of factors first.
Prime numbers from this list of factors are 2, 3 and 5.	Determine which factors are prime according to the given definition.

Exercise 3D

1–6	4, 6	—

UNDERSTANDING

1 The factors of 12 are 1, 2, 3, 4, 6 and 12. Is 12 a prime number?

2 The factors of 13 are 1 and 13. Is 13 a prime number?

3 List the first 10 prime numbers.

4 List the first 10 composite numbers.

5 What is the first prime number greater than 100?

6 What is the first prime number greater than 200?

7(½), 8	7(½), 8, 9	7–8(½), 9

FLUENCY

Example 8

7 State whether each of the following is a prime (P) or composite (C) number.

a 14	b 23	c 70	d 37
e 51	f 27	g 29	h 3
i 8	j 49	k 99	l 59
m 2	n 31	o 39	p 89

Example 9

8 Find the prime numbers that are factors of:

a 42	b 39	c 60
d 25	e 28	f 36

9 List the composite numbers between:

a 30 and 50	b 50 and 70	c 80 and 100

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3D

	10	11, 12	11–13

PROBLEM-SOLVING

10 The following are not prime numbers, yet they are the product (×) of two primes. Find the two primes for each of the following numbers.

a 55 b 91 c 143
d 187 e 365 f 133

11 Which one of these numbers has factors that are only prime numbers, itself and 1?
12, 14, 16, 18, 20

12 Twin primes are pairs of primes that are separated from each other by only one even number; for example, 3 and 5 are twin primes. Find three more pairs of twin primes.

13 13 and 31 are known as a pair of ‘reverse numbers’. They are also both prime numbers. Find any other two-digit pairs of prime reverse numbers.

	14	14, 15	15, 16

REASONING

14 Find three different prime numbers that are less than 100 and which sum to a fourth different prime number. Can you find more than five sets of such numbers?

15 Many mathematicians believe that every even number greater than 2 is the sum of two prime numbers. Show this is true for even numbers between 30 and 50.

16 Give two examples of a pair of primes that add to a prime number. Explain why all possible pairs of primes that add to a prime must contain the number 2.

Prime or not prime?

	—	—	17

ENRICHMENT

17 Design a spreadsheet that will check whether or not any number entered between 1 and 1000 is a prime number.
If your spreadsheet is successful, someone should be able to enter the number 773 and very quickly be informed whether or not this is a prime number.
You may choose to adapt your factor program (Enrichment activity **Exercise 3A**, Question 16).

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3E Powers

Interactive
Widgets
HOTsheets
Walkthroughs

When repeated multiplication of the same factor occurs, the expression can look quite cumbersome. Mathematicians have a method for simplifying such expressions by writing them as **powers**. This involves writing the repeated factor as the base number and then including an index number to indicate how many times this factor must be multiplied by itself. This is also known as writing a number in **index form**.

Powers are also used to represent very large and very small numbers. For example, 400 000 000 000 000 would be written as 4×10^{14}. This way of writing a number is called standard form or scientific notation, and you will come across this concept in future years.

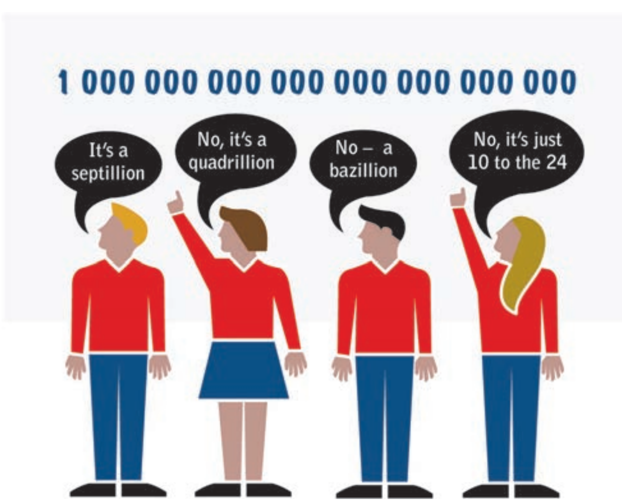
Let's start: A better way…

- What is a better way of writing $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$ (that is not the answer, 20)?
- What is a better way of writing $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (that is not the answer, 1024)?

You may need to access the internet to find out some of the following answers.

Computers have the capacity to store a lot of information. As you most likely know, computer memory is given in bytes.

- How many bytes (B) are in a kilobyte (kB)?
- How many kilobytes are in a megabyte (MB)?
- How many megabytes are in a gigabyte (GB)?
- How many gigabytes are in a terabyte (TB)?
- How many bytes are in a gigabyte?
  Hint: It is over 1 billion and it is far easier to write this number as a power!
- Why do computers frequently use base 2 (binary numbers)?



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Key ideas

- **Powers** are used to help write expressions involving repeated multiplication in a simplified form using indices
  For example: $8 \times 8 \times 8 \times 8 \times 8$ can be written as 8^5
- When writing a **basic numeral** as a power, you need a base number and an index number. This is also known as writing an expression in **index form**.

$$\underbrace{8}_{\text{base number}}{}^{\overset{\text{index number}}{5}} = \underbrace{32\,768}_{\text{basic numeral}}$$

- a^b reads as 'a to the power of b'. In expanded form it would look like:

$$\underbrace{a \times a \times a \times a \times a \ldots\ldots \times a}_{a \text{ is repeated } b \text{ times}}$$

- Powers take priority in the order of operations.
  For example: $3 + 2 \times 4^2 = 3 + 2 \times 16$
  $= 3 + 32$
  $= 35$
- Note: $2^3 \neq 2 \times 3$, therefore $2^3 \neq 6$. This is a common mistake that must be avoided.
  Instead: $2^3 = 2 \times 2 \times 2 = 8$.

Example 10 Converting to index form

Simplify the following expressions by writing them in index form.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **b** $3 \times 3 \times 2 \times 3 \times 2 \times 3$

SOLUTION	EXPLANATION
a $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$	The number 5 is the repeated factor and it appears six times.
b $3 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^2 \times 3^4$	2 is written two times. 3 is written four times.

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Example 11 Expanding a power

Expand and evaluate the following terms.

a 2^4 **b** $2^3 \times 5^2$

SOLUTION	EXPLANATION
a $2^4 = 2 \times 2 \times 2 \times 2$ $= 16$	Write 2 down four times and multiply.
b $2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5$ $= 8 \times 25$ $= 200$	Write the number 2 three times, and the number 5, two times.

Example 12 Evaluating expressions with powers

Evaluate:

a $7^2 - 6^2$ **b** $2 \times 3^3 + 10^2 + 1^7$

SOLUTION	EXPLANATION
a $7^2 - 6^2 = 7 \times 7 - 6 \times 6$ $= 49 - 36$ $= 13$	Write in expanded form (optional). Powers are evaluated before the subtraction occurs.
b $2 \times 3^3 + 10^2 + 1^7$ $= 2 \times 3 \times 3 \times 3 + 10 \times 10 + 1 \times 1 \times 1$ $\times 1 \times 1 \times 1 \times 1$	Write in expanded form (optional). Follow order of operation rules.
$= 54 + 100 + 1$ $= 155$	Carry out the multiplication first, then carry out the addition.

Exercise 3E

1–3	3	—

UNDERSTANDING

1 Select the correct answer from the following alternatives.
3^7 means:

A 3×7 **B** $3 \times 3 \times 3$ **C** $7 \times 7 \times 7$
D $3 \times 7 \times 3 \times 7 \times 3 \times 7 \times 3$ **E** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **F** 37

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3E

UNDERSTANDING

2 Select the correct answer from the following alternatives.
$9 \times 9 \times 9 \times 9 \times 9$ can be simplified to:
A 9×5 **B** 5×9 **C** 5^9
D 9^5 **E** 99 999 **F** 95

3 Copy and complete the table.

Index form	Base number	Index number	Basic numeral
2^3	2	3	8
5^2			
10^4			
2^7			
1^{12}			
12^1			
0^5			

4–5(½), 6, 7–10(½) 4–5(½), 6, 7–10(½) 4–5(½), 6, 7–10(½)

FLUENCY

Example 10a
4 Simplify the following expressions by writing them as powers.
a $3 \times 3 \times 3$ **b** $2 \times 2 \times 2 \times 2 \times 2$ **c** $15 \times 15 \times 15 \times 15$
d $10 \times 10 \times 10 \times 10$ **e** 6×6 **f** $20 \times 20 \times 20$
g $1 \times 1 \times 1 \times 1 \times 1 \times 1$ **h** $4 \times 4 \times 4$ **i** 100×100

Example 10b
5 Simplify the following expressions by writing them as powers.
a $3 \times 3 \times 5 \times 5$ **b** $7 \times 7 \times 2 \times 2 \times 7$ **c** $12 \times 9 \times 9 \times 12$
d $8 \times 8 \times 5 \times 5 \times 5$ **e** $6 \times 3 \times 6 \times 3 \times 6 \times 3$ **f** $13 \times 7 \times 13 \times 7 \times 7 \times 7$
g $4 \times 13 \times 4 \times 4 \times 7$ **h** $10 \times 9 \times 10 \times 9 \times 9$ **i** $2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 2$

6 Simplify by writing using powers.
$2 \times 3 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2 \times 2 \times 5 \times 3 \times 2 \times 2 \times 5 \times 3$

Example 11a
7 Expand these terms. (Do not evaluate.)
a 2^4 **b** 17^2 **c** 9^3 **d** 3^7
e 14^4 **f** 8^8 **g** 10^5 **h** 54^3

Example 11b
8 Expand these terms. (Do not evaluate.)
a $3^5 \times 2^3$ **b** $4^3 \times 3^4$ **c** $7^2 \times 5^3$ **d** $4^6 \times 9^3$
e 5×7^4 **f** $2^2 \times 3^3 \times 4^1$ **g** $11^5 \times 9^2$ **h** $20^3 \times 30^2$

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3E

FLUENCY

9 Evaluate:

a 2^5 b 8^2 c 10^3 d $3^2 \times 2^3$
e 10^4 f $2^3 \times 5^3$ g $1^6 \times 2^6$ h $11^2 \times 1^8$

Example 12

10 Evaluate:

a $3^2 + 4^2$ b $2 \times 5^2 - 7^2$ c $8^2 - 2 \times 3^3$
d $(9 - 5)^3$ e $2^4 \times 2^3$ f $2^7 - 1 \times 2 \times 3 \times 4 \times 5$
g $1^4 + 2^3 + 3^2 + 4^1$ h $10^3 - 10^2$ i $(1^{27} + 1^{23}) \times 2^2$

11, 12	12, 13	12–14

PROBLEM-SOLVING

11 Determine the index number for the following basic numerals.

a $16 = 2^?$ b $16 = 4^?$ c $64 = 4^?$ d $64 = 2^?$
e $27 = 3^?$ f $100 = 10^?$ g $49 = 7^?$ h $625 = 5^?$

12 Write one of the symbols <, = or > in the box to make the following statements true.

a $2^6 \square 2^9$ b $8^3 \square 8^2$ c $2^4 \square 4^2$ d $3^2 \square 4^2$
e $6^4 \square 5^3$ f $12^2 \square 3^4$ g $11^2 \square 2^7$ h $1^8 \square 2^3$

13 A text message is sent to five friends. Each of the five friends then forwards it to five other friends and each of these people also sends it to five other friends. How many people does the text message reach, not including those who forwarded the message?

14 Jane writes a chain email and sends it to five friends. If each person who receives the email reads it within 5 minutes of the email arriving and then sends it to five other people:

a How many people, including Jane, will have read the email 15 minutes after Jane first sent it?
b If the email always goes to a new person, and assuming every person in Australia has an email address and access to email, how long would it take until everyone in Australia has read the message? (Australian population is approx. 25 million people.)
c How many people will read the email within 1 hour?
d Using the same assumptions as above, how long would it take until everyone in the world has read the message? (World population is approx. 7 billion people.)
e How many people will have read the email in 2 hours?

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3E

	15	15, 16	16, 17

REASONING

15 Write the correct operation (+, –, ×, ÷) in the box to make the following equations true.

a $3^2 \square 4^2 = 5^2$
b $2^4 \square 4^2 = 4^4$
c $2^7 \square 5^3 = 3^1$
d $9^2 \square 3^4 = 1^{20}$
e $10^2 \square 10^2 = 10^4$
f $10^2 \square 8^2 = 6^2$

16 A chain email is initiated by an individual and sent to x number of recipients. This process is repeated (i.e. is forwarded to x new recipients) y times including the first sending. How many people receive the email, not including those who forwarded the message?

17 Find a value for a and for b such that $a \neq b$ and $a^b = b^a$.

Investigating factorials

	—	—	18

ENRICHMENT

18 In mathematics, the exclamation mark (!) is the symbol for factorials.

$4! = 4 \times 3 \times 2 \times 1 = 24$

$n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

a Evaluate 1!, 2!, 3!, 4!, 5! and 6!

Factorials can be written in prime factor form, which involves powers.

For example:
$$\begin{aligned} 6! &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= (2 \times 3) \times 5 \times (2 \times 2) \times 3 \times 2 \times 1 \\ &= 2^4 \times 3^2 \times 5 \end{aligned}$$

b Write these numbers in prime factor form.

i 7!
ii 8!
iii 9!
iv 10!

c Write down the last digit of 12!

d Write down the last digit of 99!

e Find a method of working out how many consecutive zeros would occur on the right-hand end of each of the following factorials if they were evaluated. Hint: Consider prime factor form.

i 5!
ii 6!
iii 15!
iv 25!

f $10! = 3! \times 5! \times 7!$ is an example of one factorial equal to the product of three factorials. Express 24! as the product of two or more factorials.

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3F Prime decomposition

Interactive Widgets HOTsheets Walkthroughs

All composite numbers can be broken down (i.e. decomposed) into a unique set of prime factors. A common way of performing the decomposition into prime factors is using a factor tree. Starting with the given number, 'branches' come down in pairs, representing a pair of factors that multiply to give the number above it.This process continues until prime factors are reached.

Let's start: Composition of numbers from prime factors

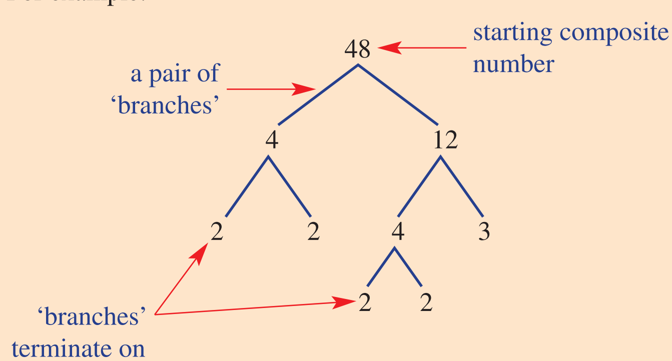
'Compose' composite numbers from the following sets of prime factors. The first one has been done for you.

a $2 \times 3 \times 5 = 30$
b $2 \times 3 \times 7 \times 3 \times 2$
c $3^2 \times 2^3$
d $5 \times 11 \times 2^2$
e $13 \times 17 \times 2$
f $2^2 \times 5^2 \times 7^2$
g $2^5 \times 3^4 \times 7$
h $11 \times 13 \times 17$

Note that this process is the reverse of **decomposition**.

Key ideas

- Every **composite number** can be expressed as a product of its **prime factors**.
- A **factor tree** can be used to show the prime factors of a composite number.
- Each 'branch' of a factor tree eventually terminates in a prime factor.
- Powers are often used to efficiently represent composite numbers in prime factor form.
  For example:



$$\therefore\ 48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$= 2^4 \times 3$$ expressed with powers

- It does not matter with which pair of factors you start a factor tree. The final result of prime factors will always be the same.
- It is conventional to write the prime factors in **ascending** (i.e. increasing) order.
  For example: $600 = 2^3 \times 3 \times 5^2$

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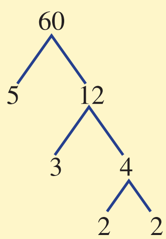
Example 13 Expressing composites in prime factor form

Express the number 60 in prime factor form.

SOLUTION



$\therefore\ 60 = 2 \times 2 \times 3 \times 5$

$60 = 2^2 \times 3 \times 5$

EXPLANATION

A pair of factors for 60 are 5×12.

The 5 branch terminates since 5 is a prime factor.

A pair of factors for 12 are 3×4.

The 3 branch terminates since 3 is a prime factor.

A pair of factors for 4 are 2×2.

Both these branches are now terminated.

Hence, the composite number, 60, can be written as a product of each terminating branch.

Exercise 3F

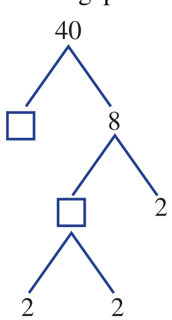
1–4	4	—

UNDERSTANDING

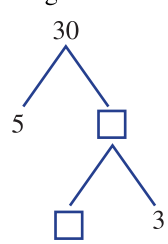
1 Sort the following list of numbers into two groups: composite numbers and prime numbers.
15, 13, 7, 5, 8, 9, 27, 23, 11, 4, 12, 2

2 Fill in the gaps to complete the following factor trees.

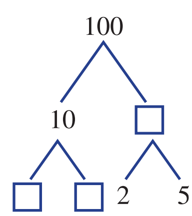
a



b

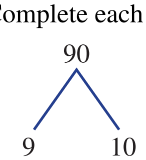


c

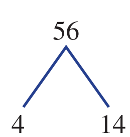


3 Complete each of the following factor trees.

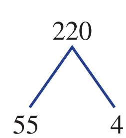
a



b



c



4 Write the following prime factors, using powers.

a $2 \times 3 \times 3 \times 2 \times 2$

b $5 \times 3 \times 3 \times 3 \times 3 \times 5$

c $7 \times 2 \times 3 \times 7 \times 2$

d $3 \times 3 \times 2 \times 11 \times 11 \times 2$

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5(½)	5–6(½)	5–6(½)

FLUENCY

Example 13

5 Express the following numbers in prime factor form.
- a 72
- b 24
- c 38
- d 44
- e 124
- f 80
- g 96
- h 16
- i 75
- j 111
- k 64
- l 56

6 Express these numbers in prime factor form.
- a 600
- b 800
- c 5000
- d 2400
- e 1 000 000
- f 45 000
- g 820
- h 690

7, 8	7–9	8–10

PROBLEM-SOLVING

7 Match the correct composite number (**a** to **d**) to its set of prime factors (**A** to **D**).
- a 120 — A $2 \times 3 \times 5^2$
- b 150 — B $2^2 \times 3^2 \times 5$
- c 144 — C $2^4 \times 3^2$
- d 180 — D $2 \times 3 \times 2 \times 5 \times 2$

8 Find the smallest composite number that has the five smallest prime numbers as factors.

9 a Express 144 and 96 in prime factor form.
b By considering the prime factor form, determine the HCF of 144 and 96.

10 a Express 25 200 and 77 000 in prime factor form.
b By considering the prime factor form, determine the HCF of 25 200 and 77 000.

11	11, 12	12–14

REASONING

11 Represent the number 24 with four different factor trees, each resulting in the same set of prime factors. Note that simply swapping the order of a pair of factors does not qualify it as a different form of the factor tree.

12 Only one of the following is the correct set of prime factors for 424.
- A $2^2 \times 3^2 \times 5$
- B $2 \times 3^2 \times 5^2$
- C 53×8
- D $2^3 \times 53$

i Justify why you can eliminate alternatives **A** and **B** straight away.
ii Why can option **C** be discarded as an option?
iii Show that option **D** is the correct answer.

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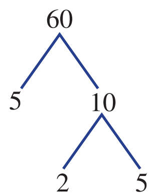
3F

REASONING

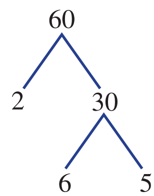
13 **a** State the error in each of the following prime factor trees.

i



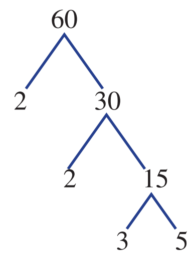
$60 = 2 \times 5^2$

ii



$60 = 2 \times 5 \times 6$

iii



$60 = 2 \times 3 \times 5$

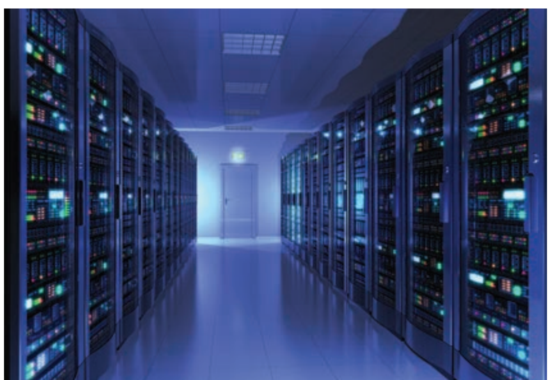
b What is the correct way to express 60 in prime factor form?

14 Write 15 different (i.e. distinct) factor trees for the number 72.

Four distinct prime factors	—	—	15–17

ENRICHMENT

15 There are 16 composite numbers that are smaller than 1000 which have four distinct (i.e. different) prime factors. For example: $546 = 2 \times 3 \times 7 \times 13$.
By considering the prime factor possibilities, find all 16 composite numbers and express each of them in prime factor form.



Supercomputers like this have been used to search for prime numbers with millions of digits.

16 A conjecture is a statement that may appear to be true but has not been proved conclusively. Goldbach's conjecture states: 'Every even number greater than 2 is the sum of two prime numbers.' For example, $52 = 47 + 5$
Challenge: Try this for every even number from 4 to 50.

17 Use the internet to find the largest-known prime number.

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Progress quiz

3A **1** Find the complete set of factors for each of these numbers.
a 16 **b** 70

3A **2** Write down the first four multiples for each of these numbers.
a 7 **b** 20

3B **3** Find the HCF of the following groups of numbers.
a 15 and 10 **b** 36, 54 and 72

3B **4** Find the LCM of the following groups of numbers.
a 8 and 12 **b** 3, 5 and 9

3C Ext **5** Use the divisibility rules to determine whether the following calculations are possible without leaving a remainder. Give a reason for each answer.
a $34\,481 \div 4$ **b** $40\,827 \div 3$ **c** $824\,730 \div 6$ **d** $5\,247\,621 \div 9$

3C Ext **6** The game of 'clusters' involves a group getting into smaller-sized groups as quickly as possible once a particular cluster size has been called out. If a year level consists of 120 students, which 'cluster' sizes (of more than one person) would ensure no students are left out of a group?

3D **7** State whether each of the following is a prime (P) or composite (C) number or neither (N). Give reasons.
a 60 **b** 1 **c** 13 **d** 0

3D **8** Find the prime numbers that are factors of:
a 35 **b** 36

3E **9** Simplify the following expressions by writing them as powers.
a $5 \times 5 \times 5 \times 5$ **b** $7 \times 3 \times 7 \times 7 \times 3 \times 7 \times 7$

3E **10** Expand and evaluate the following terms.
a 3^4 **b** $1^4 \times 3^2$ **c** $5^1 \times 10^4$ **d** $(12 - 8)^2$ **e** $9^2 - 3^3 \times 2$

3F **11** Express the following numbers in prime factor form, writing them in ascending order.
a 24 **b** 180

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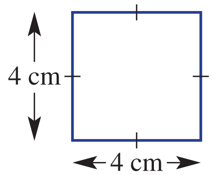
3G Squares and square roots

Interactive Widgets HOTsheets Walkthroughs

A square number can be illustrated by considering the area of a square with a whole number as its side length.

For example:

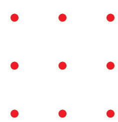


Area of square $= 4\text{ cm} \times 4\text{ cm} = 16\text{ cm}^2$

Therefore, 16 is a square number.

Another way of representing square numbers is through a square array of dots.

For example:



$$\begin{aligned}\text{Number of dots} &= 3\text{ rows of }3\text{ dots}\\ &= 3 \times 3\text{ dots}\\ &= 3^2\text{ dots}\\ &= 9\text{ dots}\end{aligned}$$

Therefore, 9 is a square number.

To produce a square number you must multiply the number by itself. All square numbers written in index form will have a power of 2.

Finding a square root of a number is the opposite of squaring a number.

For example: $4^2 = 16$ and therefore $\sqrt{16} = 4$.

To find square roots we use our knowledge of square numbers. A calculator is also frequently used to find square roots. Geometrically, the square root of a number is the side length of a square whose area is that number.

Old Town Square in Prague. The square root of its area is the length of one of its sides.

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Let's start: Speed squaring tests

In pairs, test one another's knowledge of square numbers.

- Ask 10 quick questions, such as '3 squared', '5 squared' etc.
- Have two turns each. Time how long it takes each of you to answer the 10 questions.
- Aim to be quicker on your second attempt.

Write down the first 10 square numbers.

- Begin to memorise these important numbers.
- Time how quickly you can recall the first 10 square numbers without looking at a list of numbers.
- Can you go under 5 seconds?

Key ideas

- Any whole number multiplied by itself produces a **square number**.
  For example: $5^2 = 5 \times 5 = 25$. Therefore, 25 is a square number.
  - Square numbers are also known as **perfect squares**.
  - The first 12 square numbers are:

Index form	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
Basic numeral	1	4	9	16	25	36	49	64	81	100	121	144

  - All square numbers have an odd number of factors.
  - The symbol for squaring is $(\ )^2$. The brackets are optional, but can be very useful when simplifying more difficult expressions.
- The **square root** of a given number is the 'non-negative' number that, when multiplied by itself, produces the given number.
  - The symbol for square rooting is $\sqrt{\ }$.
  - Finding a square root of a number is the opposite of squaring a number.
    For example: $4^2 = 16$; hence, $\sqrt{16} = 4$
    We read this as: '4 squared equals 16, therefore, the square root of 16 equals 4.'
  - Squaring and square rooting are 'opposite' operations.
    $(\sqrt{x})^2 = x$ also $\sqrt{(x)^2} = x$
  - A list of common square roots are:

Square root form	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$
Basic numeral	1	2	3	4	5	6	7	8	9	10	11	12

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Example 14 Evaluating squares and square roots

Evaluate:

a 6^2 **b** $\sqrt{64}$ **c** $\sqrt{1600}$

SOLUTION	EXPLANATION
a $6^2 = 36$	$6^2 = 6 \times 6$
b $\sqrt{64} = 8$	$8 \times 8 = 64$ $\quad \therefore \sqrt{64} = 8$
c $\sqrt{1600} = 40$	$40 \times 40 = 1600$ $\quad \therefore \sqrt{1600} = 40$

Example 15 Evaluating expressions involving squares and square roots

Evaluate:

a $3^2 - \sqrt{9} + 1^2$ **b** $\sqrt{8^2 + 6^2}$

SOLUTION

a
$$\begin{aligned} 3^2 - \sqrt{9} + 1^2 &= 9 - 3 + 1 \\ &= 7 \end{aligned}$$

EXPLANATION

$3^2 = 3 \times 3, \sqrt{9} = 3, 1^2 = 1 \times 1$

SOLUTION

b
$$\begin{aligned} \sqrt{8^2 + 6^2} &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

EXPLANATION

$8^2 = 8 \times 8, 6^2 = 6 \times 6$

$\sqrt{100} = 10$

Exercise 3G

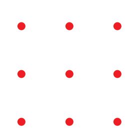
1–5	5	—

UNDERSTANDING

1 Draw a square of side length 6 cm. What would be the area of this shape? What special type of number is your answer?

2 Write down the first 15 square numbers in index form and as basic numerals.

3 We can confirm that 9 is a square number by drawing the diagram shown at right.



a Show, using dots, why 6 is not a square number.

b Show, using dots, why 16 is a square number.

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3G

UNDERSTANDING

4 Evaluate:

- **a** 6^2
- **b** 5 squared
- **c** $(11)^2$
- **d** 10 to the power of 2
- **e** 7^2
- **f** 12×12

5 Evaluate:

- **a** $\sqrt{25}$
- **b** square root of 16
- **c** $\sqrt{100}$
- **d** the length of a square that has an area of 49 cm^2

FLUENCY 6–9(½) 6–9(½) 6–9(½)

Example 14a

6 Evaluate:

- **a** 8^2
- **b** 7^2
- **c** 1^2
- **d** 12^2
- **e** 3^2
- **f** 15^2
- **g** 5^2
- **h** 0^2
- **i** 11^2
- **j** 100^2
- **k** 17^2
- **l** 33^2

Example 14b

7 Evaluate:

- **a** $\sqrt{25}$
- **b** $\sqrt{9}$
- **c** $\sqrt{1}$
- **d** $\sqrt{121}$
- **e** $\sqrt{0}$
- **f** $\sqrt{81}$
- **g** $\sqrt{49}$
- **h** $\sqrt{16}$
- **i** $\sqrt{4}$
- **j** $\sqrt{144}$
- **k** $\sqrt{400}$
- **l** $\sqrt{169}$

Example 14c

8 Evaluate:

- **a** $\sqrt{2500}$
- **b** $\sqrt{6400}$
- **c** $\sqrt{8100}$
- **d** $\sqrt{729}$

Example 15

9 Evaluate:

- **a** $3^2 + 5^2 - \sqrt{16}$
- **b** 4×4^2
- **c** $8^2 - 0^2 + 1^2$
- **d** $1^2 \times 2^2 \times 3^2$
- **e** $\sqrt{5^2 - 3^2}$
- **f** $\sqrt{81} - 3^2$
- **g** $6^2 \div 2^2 \times 3^2$
- **h** $\sqrt{9} \times \sqrt{64} \div \sqrt{36}$
- **i** $\sqrt{12^2 + 5^2}$

PROBLEM-SOLVING 10, 11 11, 12 11–13

10 List all the square numbers between 50 and 101.

11 List all the square numbers between 101 and 200. Hint: There are only four.

12 **a** Find two square numbers that add to 85.
b Find two square numbers that have a difference of 85.

13 Find three different square numbers that sum to 59.

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3G

	14	14, 15	15–17

REASONING

14 a Evaluate $3^2 \times 4^2$.
b Evaluate 12^2.
c The rule $a^2 \times b^2 = (a \times b)^2$ can be used to link $3^2 \times 4^2$ and 12^2. What are the values of a and b if $3^2 \times 4^2 = 12^2$?
d Check this formula using other numbers.

15 a Show that $3^2 + 4^2 = 5^2$.
b What does $6^2 + 8^2$ equal? Write the answer using a power of 2.
c What does $9^2 + 12^2$ equal?
d What does $30^2 + 40^2$ equal?

16 a Evaluate 11^2 and 111^2.
b Predict an answer for 1111^2.
c Evaluate 1111^2 and test your prediction.

17 Stuart decides there are no odd square numbers. His justification is that 'because an even number multiplied by an even number produces an even number, and that an odd number multiplied by an odd number also produces an even number, then there are no odd square numbers'. Do you agree with Stuart's claim? If not, give an example to explain your answer.

Properties of square roots	—	—	18

ENRICHMENT

18 Trial different numbers in the following formulas to determine whether these algebraic statements involving square roots are true or false.
a $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$
b $\sqrt{a} - \sqrt{b} = \sqrt{a-b}$
c $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$
d $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
e $\sqrt{a^2} = a$
f $a^3 = a\sqrt{a}$
g $\sqrt{a^2 + b^2} = a + b$
h $\sqrt{a^2 - b^2} = a - b$

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3H Number patterns CONSOLIDATING

Interactive
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Mathematicians commonly look at lists of numbers in an attempt to discover a pattern. They also aim to find a rule that describes the number pattern to allow them to predict future numbers in the sequence.

Here is a list of professional careers that all involve a high degree of mathematics and, in particular, involve looking at data so that comments can be made about past, current or future trends:

Statistician, economist, accountant, market researcher, financial analyst, cost estimator, actuary, stock broker, data analyst, research scientist, financial advisor, medical scientist, budget analyst, insurance underwriter and mathematics teacher!




There are many careers that involve using mathematics and data.

Let's start: What's next?

A number sequence consisting of five terms is placed on the board. Four gaps are placed after the last number.

20, 12, 16, 8, 12, ___, ___, ___, ___,

- Can you work out and describe the number pattern?
  This number pattern involves a repeated process of subtracting 8 and then adding 4.
- Make up your own number pattern and test it on a class member.

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Key ideas

- Number patterns are also known as **sequences**, and each number in a sequence is called a **term**.
  - Each number pattern has a particular starting number and terms are generated by following a particular rule.
- Strategies to determine the pattern involved in a number sequence include:
  - Looking for a common difference
    Are terms increasing or decreasing by a constant amount?
    For example: 2, 6, 10, 14, 18, … Each term is increasing by 4.
  - Looking for a common ratio
    Is each term being multiplied or divided by a constant amount?
    For example: 2, 4, 8, 16, 32, … Each term is being multiplied by 2.
  - Looking for an increasing/decreasing difference
    Is there a pattern in the difference between pairs of terms?
    For example: 1, 3, 6, 10, 15, … The difference increases by 1 each term.
  - Looking for two interlinked patterns
    Is there a pattern in the odd-numbered terms, and another pattern in the even-numbered terms?
    For example: 2, 8, 4, 7, 6, 6, … The odd-numbered terms increase by 2, the even-numbered terms decrease by 1.
  - Looking for a special type of pattern
    Could it be a list of square numbers, prime numbers, Fibonacci numbers etc.?
    For example: 1, 8, 27, 64, 125, … This is the pattern of cube numbers: $1^3, 2^3, 3^3, \ldots$

Example 16 Identifying patterns with a common difference

Find the next three terms for these number patterns that have a common difference.

a 6, 18, 30, 42, ___, ___, ___ **b** 99, 92, 85, 78, ___, ___, ___

SOLUTION	EXPLANATION
a 54, 66, 78	The common difference is 12. Continue adding 12 to generate the next three terms.
b 71, 64, 57	The pattern indicates the common difference is 7. Continue subtracting 7 to generate the next three terms.

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Example 17 Identifying patterns with a common ratio

Find the next three terms for the following number patterns that have a common ratio.

a 2, 6, 18, 54, ___, ___, ___

b 256, 128, 64, 32, ___, ___, ___

SOLUTION	EXPLANATION
a 162, 486, 1458	The common ratio is 3. Continue multiplying by 3 to generate the next three terms.
b 16, 8, 4	The common ratio is $\frac{1}{2}$. Continue dividing by 2 to generate the next three terms.

Exercise 3H

1–3	3	—

UNDERSTANDING

1 Generate the first five terms of the following number patterns.

- **a** starting number of 8, common difference of adding 3
- **b** starting number of 32, common difference of subtracting 1
- **c** starting number of 52, common difference of subtracting 4
- **d** starting number of 123, common difference of adding 7

2 Generate the first five terms of the following number patterns.

- **a** starting number of 3, common ratio of 2 (multiply by 2 each time)
- **b** starting number of 5, common ratio of 4
- **c** starting number of 240, common ratio of $\frac{1}{2}$ (divide by 2 each time)
- **d** starting number of 625, common ratio of $\frac{1}{5}$

3 State whether the following number patterns have a common difference (+ or –), a common ratio (× or ÷) or neither.

- **a** 4, 12, 36, 108, 324, …
- **b** 19, 17, 15, 13, 11, …
- **c** 212, 223, 234, 245, 256, …
- **d** 8, 10, 13, 17, 22, …
- **e** 64, 32, 16, 8, 4, …
- **f** 5, 15, 5, 15, 5, …
- **g** 2, 3, 5, 7, 11, …
- **h** 75, 72, 69, 66, 63, …

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3H

	4–7(½)	4–8(½)	4–8(½)

FLUENCY

Example 16

4 Find the next three terms for the following number patterns that have a common difference.

a 3, 8, 13, 18, __, __, __
b 4, 14, 24, 34, __, __, __
c 26, 23, 20, 17, __, __, __,
d 106, 108, 110, 112, __, __, __
e 63, 54, 45, 36, __, __, __
f 9, 8, 7, 6, __, __, __
g 101, 202, 303, 404, __, __, __
h 75, 69, 63, 57, __, __, __

Example 17

5 Find the next three terms for the following number patterns that have a common ratio.

a 2, 4, 8, 16, __, __, __
b 5, 10, 20, 40, __, __, __
c 96, 48, 24, __, __, __
d 1215, 405, 135, __, __, __
e 11, 22, 44, 88, __, __, __
f 7, 70, 700, 7000, __, __, __
g 256, 128, 64, 32, __, __, __
h 1216, 608, 304, 152, __, __, __

6 Find the missing numbers in each of the following number patterns.

a 62, 56, __, 44, 38, __, __
b 15, __, 35, __, __, 65, 75
c 4, 8, 16, __, __, 128, __
d 3, 6, __, 12, __, 18, __
e 88, 77, 66, __, __, __, 22
f 2997, 999, __, __, 37
g 14, 42, __, __, 126, __, 182
h 14, 42, __, __, 1134, __, 10 206

7 Write the next three terms in each of the following sequences.

a 3, 5, 8, 12, __, __, __
b 1, 2, 4, 7, 11, __, __, __
c 1, 4, 9, 16, 25, __, __, __
d 27, 27, 26, 24, 21, __, __, __
e 2, 3, 5, 7, 11, 13, __, __, __
f 2, 5, 11, 23, __, __, __
g 2, 10, 3, 9, 4, 8, __, __, __
h 14, 100, 20, 80, 26, 60, __, __, __

8 Generate the next three terms for the following number sequences and give an appropriate name to the sequence.

a 1, 4, 9, 16, 25, 36, __, __, __
b 1, 1, 2, 3, 5, 8, 13, __, __, __
c 1, 8, 27, 64, 125, __, __, __
d 2, 3, 5, 7, 11, 13, 17, __, __, __
e 4, 6, 8, 9, 10, 12, 14, 15, __, __, __
f 121, 131, 141, 151, __, __, __

	9, 10	10, 11	10–12

PROBLEM-SOLVING

9 Complete the next three terms for the following challenging number patterns.

a 101, 103, 106, 110, __, __, __
b 162, 54, 108, 36, 72, __, __, __
c 3, 2, 6, 5, 15, 14, __, __, __
d 0, 3, 0, 4, 1, 6, 3, __, __, __

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10 When making human pyramids, there is one less person on each row above, and it is complete when there is a row of only one person on the top.
Write down a number pattern for a human pyramid with 10 students on the bottom row. How many people are needed to make this pyramid?

11 The table below represents a seating plan with specific seat number for a section of a grandstand at a soccer ground. It continues upwards for another 20 rows.

Row 4	25	26	27	28	29	30	31	32
Row 3	17	18	19	20	21	22	23	24
Row 2	9	10	11	12	13	14	15	16
Row 1	1	2	3	4	5	6	7	8

a What is the number of the seat directly above seat number 31?
b What is the number of the seat on the left-hand edge of row 8?
c What is the third seat from the right in row 14?
d How many seats are in the grandstand?

12 Find the next five numbers in the following number pattern.
1, 4, 9, 1, 6, 2, 5, 3, 6, 4, 9, 6, 4, 8, 1, __, __, __, __, __

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3H

13	13, 14	14, 15

REASONING

13 Jemima writes down the following number sequence: 7, 7, 7, 7, 7, 7, 7, …
Her friend Peta declares that this is not really a number pattern. Jemima defends her number pattern, stating that it is most definitely a number pattern as it has a common difference and also has a common ratio. What are the common difference and the common ratio for the number sequence above? Do you agree with Jemima or Peta?

14 Find the sum of the following number sequences.
- a $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$
- b $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$
- c $1 + 2 + 3 + 4 + 5 + \cdots + 67 + 68 + 69 + 70$
- d $5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 + 38$

15 The great handshake problem. There are a certain number of people in a room and they must all shake one another's hand. How many handshakes will there be if there are:
- a 3 people in the room?
- b 5 people in the room?
- c 10 people in the room?
- d 24 people in a classroom?
- e n people in the room?

What number am I?

—	—	16

ENRICHMENT

16 Read the following clues to work out the mystery number.
- a I have three digits.
  I am divisible by 5.
  I am odd.
  The product of my digits is 15.
  The sum of my digits is less than 10.
  I am less than 12×12.
- b I have three digits.
  The sum of my digits is 12.
  My digits are all even.
  My digits are all different.
  I am divisible by 4.
  The sum of my units and tens digits equals my hundreds digit.
- c I have three digits.
  I am odd and divisible by 5 and 9.
  The product of my digits is 180.
  The sum of my digits is less than 20.
  I am greater than 30^2.
- d Make up two of your own mystery number puzzles and submit your clues to your teacher.

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3I Spatial patterns

Patterns can also be found in geometric shapes. Mathematicians examine patterns carefully to determine how the next term in the sequence is created. Ideally, a rule is formed that shows the relationship between the geometric shape and the number of objects (e.g. tiles, sticks, counters) required to make such a shape. Once a rule is established it can be used to make predictions about future terms in the sequence.

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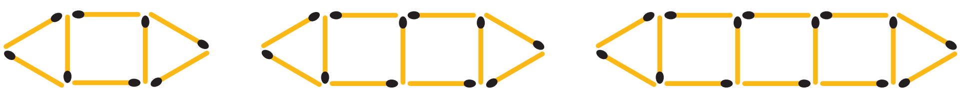
A pattern rule can be created to show how these shapes can be constructed.

Let's start: Stick patterns

Materials required: One box of toothpicks/matches per student.

- Generate a spatial pattern using your sticks.
- You must be able to make at least three terms in your pattern.
  For example:

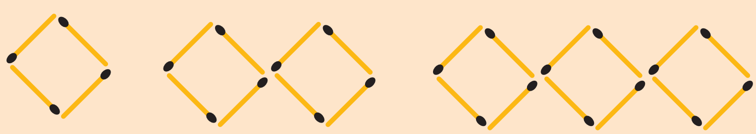


- Ask your partner how many sticks would be required to make the next term in the pattern.
- Repeat the process with a different spatial design.

Key ideas

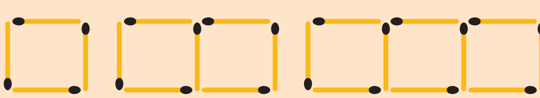
- A **spatial pattern** is a sequence of geometrical shapes that can be described by a **number pattern**.
  For example:

  spatial pattern

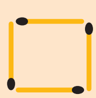


  number pattern 4 8 12

- A spatial pattern starts with a simple geometric design. Future terms are created by adding on repeated shapes of the same design. If designs connect with an edge, the repetitive shape added on will be a subset of the original design, as the connecting edge does not need to be repeated.
  For example:



  starting design



  repeating design

- To help describe a spatial pattern, it is generally converted to a number pattern and a common difference is observed.

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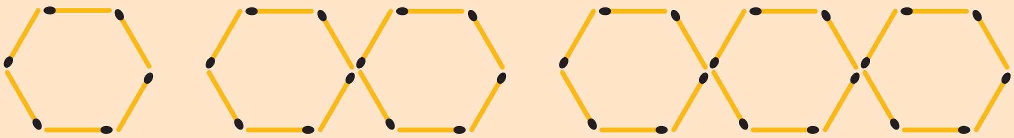
Key ideas

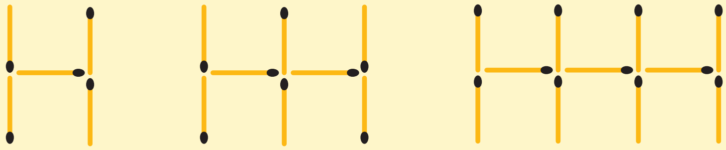
- The common difference is the number of objects (e.g. sticks) that need to be added on to create the next term.
- Rules can be found that connect the number of objects (e.g. sticks) required to produce the number of designs.
  For example: hexagon design



Rule is: Number of sticks used = 6 × number of hexagons formed

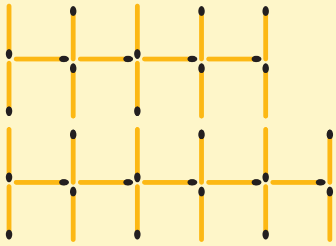
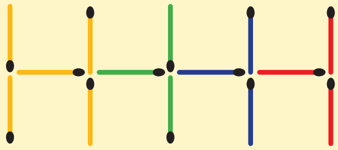
Example 18 Drawing and describing spatial patterns

a Draw the next two shapes in the spatial pattern shown.



b Write the spatial pattern above as a number pattern in regard to the number of sticks required to make each shape.

c Describe the pattern by stating how many sticks are required to make the first term, and how many sticks are required to make the next term in the pattern.

SOLUTION	EXPLANATION
a 	Follow the pattern.
b 5, 8, 11, 14, 17	Count the number of sticks in each term. Look for a pattern.
c 5 matches are required to start the pattern, and an additional 3 matches are required to make the next term in the pattern.	

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Example 19 Finding a general rule for a spatial pattern

a Draw the next two shapes in this spatial pattern.

b Complete the table.

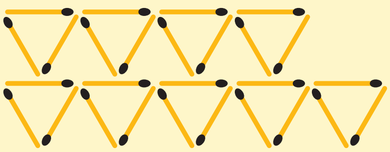
Number of triangles	1	2	3	4	5
Number of sticks required	3				

c Describe a rule connecting the number of sticks required to the number of triangles produced.

d Use your rule to predict how many sticks would be required to make 20 triangles.

SOLUTION | **EXPLANATION**

a



Follow the pattern by adding one triangle each time.

b

No. of triangles	1	2	3	4	5
No. of sticks	3	6	9	12	15

An extra 3 sticks are required to make each new triangle.

c Number of sticks = 3 × number of triangles

3 sticks are required per triangle.

d Number of sticks = 3 × 20 triangles
= 60 sticks

20 triangles × 3 sticks each

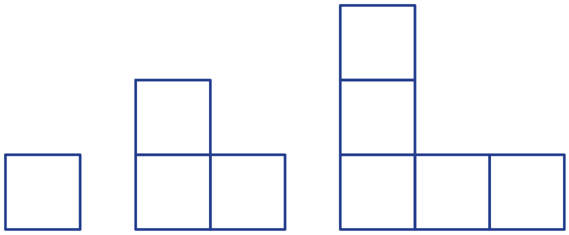
Exercise 3I

1–3	3	—

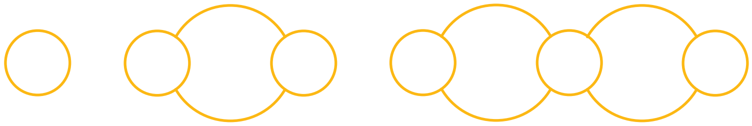
UNDERSTANDING

1 Draw the next two terms for each of these spatial patterns.

a



b



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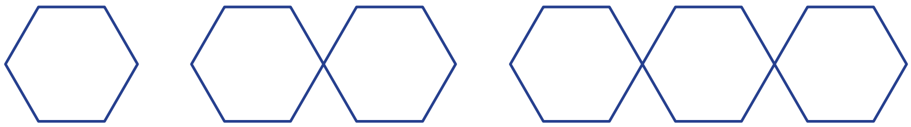
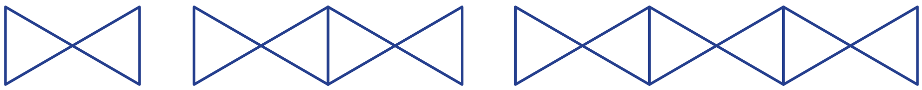
3I

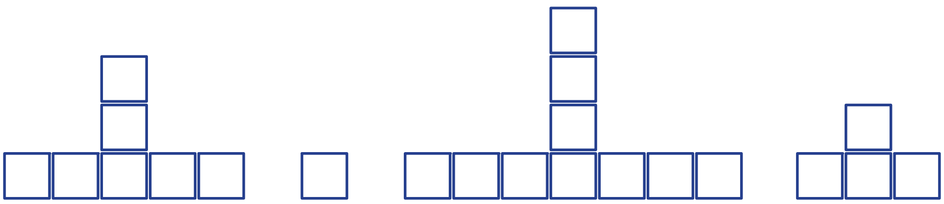
UNDERSTANDING

c 

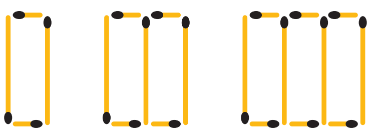
d 

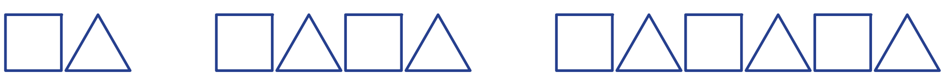
e 

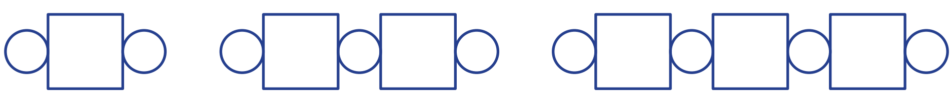
2 Draw the following geometrical designs in sequential ascending (i.e. increasing) order and draw the next term in the sequence.

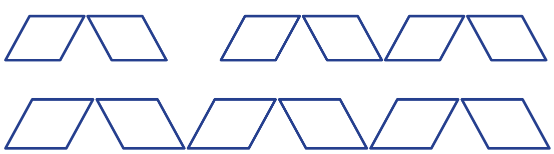


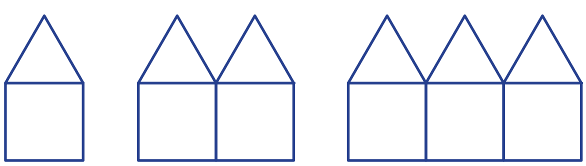
3 For each of the following spatial patterns, draw the starting geometrical design and also the geometrical design added on repetitively to create new terms. (For some patterns the repetitive design is the same as the starting design.)

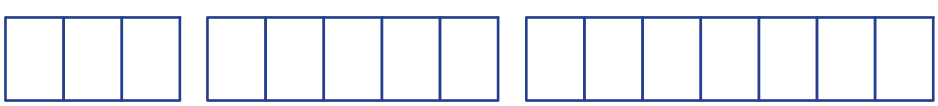
a 

b 

c 

d 

e 

f 

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	4, 5	4(½), 5, 6	4(½), 5, 6

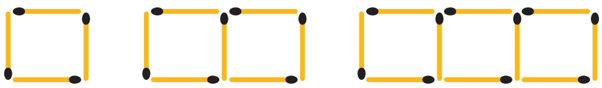
FLUENCY

Example 18

4 For each of the spatial patterns below:

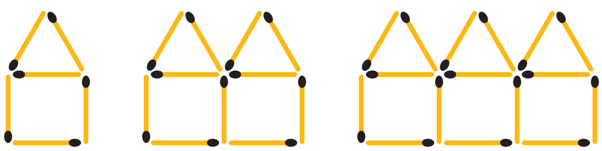
i Draw the next two shapes.
ii Write the spatial pattern as a number pattern.
iii Describe the pattern by stating how many sticks are required to make the first term and how many more sticks are required to make the next term in the pattern.

a

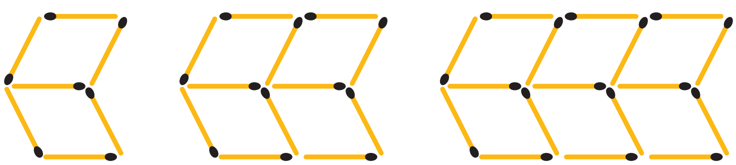


b

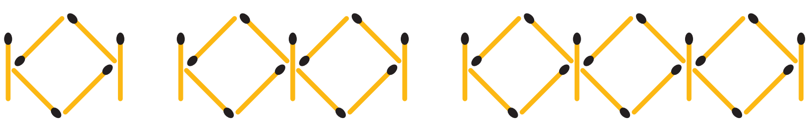
c



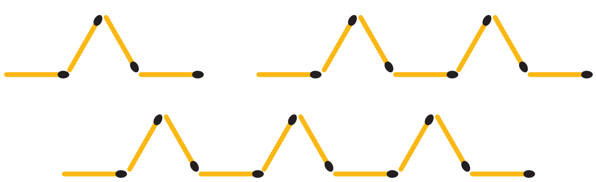
d



e

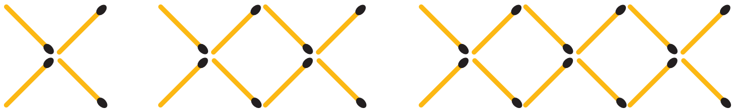


f



Example 19

5 **a** Draw the next two shapes in this spatial pattern.



b Copy and complete the table.

Number of crosses	1	2	3	4	5
Number of sticks required					

c Describe a rule connecting the number of sticks required to the number of crosses produced.

d Use your rule to predict how many sticks would be required to make 20 crosses.

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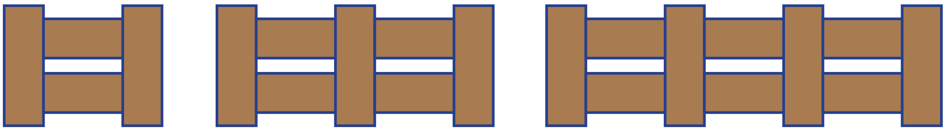
3I

FLUENCY

6 a Draw the next two shapes in this spatial pattern.



b Copy and complete the table. Planks are vertical and horizontal.

Number of fence sections	1	2	3	4	5
Number of planks required					

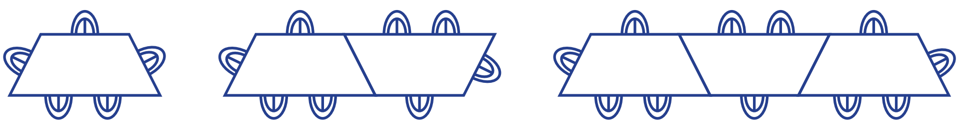
c Describe a rule connecting the number of planks required to the number of fence sections produced.

d Use your rule to predict how many planks would be required to make 20 fence sections.

7, 8 | 8, 9 | 8–10

PROBLEM-SOLVING

7 At North Park Primary School, the classrooms have trapezium-shaped tables. Mrs Greene arranges her classroom's tables in straight lines, as shown.



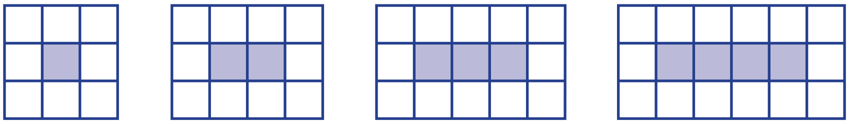
a Draw a table of results showing the relationship between the number of tables in a row and the number of students that can sit at the tables. Include results for up to five tables in a row.

b Describe a rule that connects the number of tables placed in a straight row to the number of students that can sit around the tables.

c The room allows seven tables to be arranged in a straight line. How many students can sit around the tables?

d There are 65 students in Grade 6 at North Park Primary School. Mrs Greene would like to arrange the tables in one straight line for an outside picnic lunch. How many tables will she need?

8 The number of tiles required to pave around a spa is related to the size of the spa. The approach is to use large tiles that are the same size as that of a small spa.



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PROBLEM-SOLVING

A spa of length 1 unit requires 8 tiles to pave around its perimeter, whereas a spa of length 4 units requires 14 tiles to pave around its perimeter.

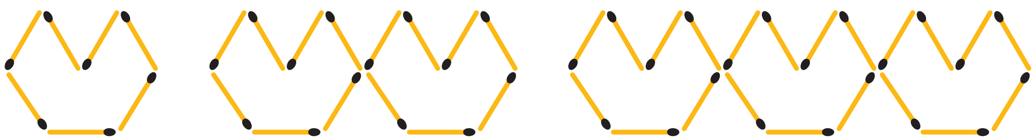
a Complete a table of values relating length of spa and number of tiles required, for values up to and including a spa of length 6 units.

b Describe a rule that connects the number of tiles required for the length of the spa.

c The largest size spa manufactured is 15 units long. How many tiles would be required to pave around its perimeter?

d A paving company has only 30 tiles left. What is the largest spa they would be able to tile around?

9 Which rule correctly describes this spatial pattern?



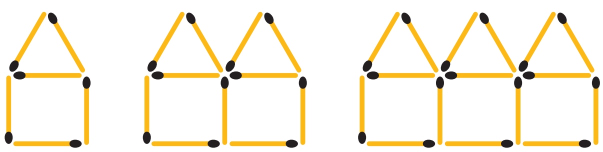
A Number of sticks = 7 × number of 'hats'

B Number of sticks = 7 × number of 'hats' + 1

C Number of sticks = 6 × number of 'hats' + 2

D Number of sticks = 6 × number of 'hats'

10 Which rule correctly describes this spatial pattern?



A Number of sticks = 5 × number of houses + 1

B Number of sticks = 6 × number of houses + 1

C Number of sticks = 6 × number of houses

D Number of sticks = 5 × number of houses

11	11, 12	12, 13

REASONING

11 Design a spatial pattern to fit the following number patterns.

a 4, 7, 10, 13, …

b 4, 8, 12, 16, …

c 3, 5, 7, 9, …

d 3, 6, 9, 12, …

e 5, 8, 11, 14, …

f 6, 11, 16, 21, …

12 A rule to describe a special window spatial pattern is written as $y = 4 \times x + 1$, where y represents the number of 'sticks' required and x is the number of windows created.

a How many sticks are required to make one window?

b How many sticks are required to make 10 windows?

c How many sticks are required to make g windows?

d How many windows can be made from 65 sticks?

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3I

REASONING

13 A rule to describe a special fence spatial pattern is written as $y = m \times x + n$, where y represents the number of pieces of timber required and x represents the number of fencing panels created.

- a How many pieces of timber are required to make one panel?
- b What does m represent?
- c Draw the first three terms of the fence spatial pattern for $m = 4$ and $n = 1$.

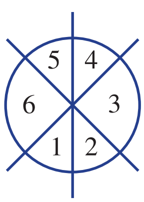
Cutting up a circle

—	—	14

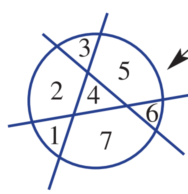
ENRICHMENT

14 What is the *greatest* number of sections into which you can divide a circle, using only a particular number of straight line cuts?

- a Explore the problem above.
  Note: The greatest number of sections is required and, hence, only one of the two diagrams below is correct for three straight line cuts.



Incorrect.
Not the maximum number of sections.



Correct.
The maximum number of sections.

- b Copy and complete this table of values.

Number of straight cuts	1	2	3	4	5	6	7
Number of sections created			7				

- c Can you discover a pattern for the maximum number of sections created? What is the maximum number of sections that could be created with 10 straight line cuts?
- d The formula for determining the maximum number of cuts is quite complex:

  $$\text{sections} = \frac{1}{2}\ \text{cuts}^2 + \frac{1}{2}\ \text{cuts} + 1$$

  Verify that this formula works for the values you listed in the table above.
  Using the formula, how many sections could be created with 20 straight cuts?

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3J Tables and rules

Interactive

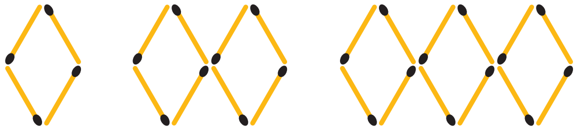
Widgets

HOTsheets

Walkthroughs

In the previous section on spatial patterns, it was observed that rules can be used to connect the number of objects (e.g. sticks) required to make particular designs.

A table of values can be created for any spatial pattern. Consider this spatial pattern and the corresponding table of values.



Number of diamonds (*input*)	Number of sticks (*output*)
1	4
2	8
3	12

What values would go in the next row of the table?
A rule that produces this table of values is:

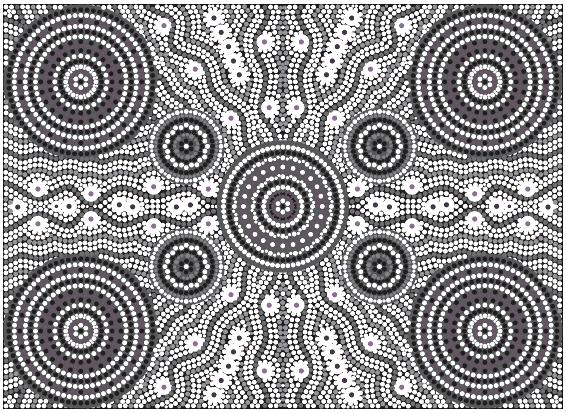
Number of sticks = 4 × number of diamonds

Alternatively, if we consider the number of diamonds as the *input variable* and the number of sticks as the *output variable*, then the rule could be written as:

output = 4 × *input*

If a rule is provided, a table of values can be created.

If a table of values is provided, often a rule can be found.



Let's start: Guess the output

- A table of values is drawn on the board with three completed rows of data.
- Additional values are placed in the *input* column. What *output* values should be in the *output* column?
- After adding *output* values, decide which rule fits (models) the values in the table and check that it works for each *input* and *output* pair.
  Four sample tables are listed below.

input	*output*
2	6
5	9
6	10
1	?
8	?

input	*output*
12	36
5	15
8	24
0	?
23	?

input	*output*
2	3
3	5
9	17
7	?
12	?

input	*output*
6	1
20	8
12	4
42	?
4	?

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Key ideas

- A **rule** shows the relation between two varying quantities.
  For example: *output* = *input* + 3 is a rule connecting the two quantities *input* and *output*.
  The values of the *input* and the *output* can vary, but we know from the rule that the value of the *output* will always be 3 more than the value of the *input*.
- A **table of values** can be created from any given rule. To complete a table of values, the *input* (one of the quantities) is replaced by a number. This is known as **substitution**. After substitution the value of the other quantity, the *output*, is calculated.
  For example: If *input* = 4, then

$$\begin{aligned} output &= input + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

- Often, a rule can be determined from a table of values. On close inspection of the values, a relationship may be observed. Each of the four operations should be considered when looking for a connection.

input	1	2	3	4	5	6
output	6	7	8	9	10	11

  By inspection, it can be observed that every *output* value is 5 more than the corresponding *input* value. The rule can be written as: *output* = *input* + 5.

Example 20 Completing a table of values

Complete each table for the given rule.

a *output* = *input* – 2

input	3	5	7	12	20
output					

b *output* = (3 × *input*) + 1

input	4	2	9	12	0
output					

SOLUTION

a *output* = *input* – 2

input	3	5	7	12	20
output	1	3	5	10	18

b *output* = (3 × *input*) + 1

input	4	2	9	12	0
output	13	7	28	37	1

EXPLANATION

Replace each *input* value in turn into the rule.
e.g. When *input is* 3:
output = 3 – 2 = 1

Replace each *input* value in turn into the rule.
e.g. When *input* is 4:
output = (3 × 4) + 1 = 13

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Example 21 Finding a rule from a table of values

Find the rule for each of these tables of values.

a

input	3	4	5	6	7
output	12	13	14	15	16

b

input	1	2	3	4	5
output	7	14	21	28	35

SOLUTION

EXPLANATION

a $output = input + 9$

Each *output* value is 9 more than the *input* value.

b $output = input \times 7$ or $output = 7 \times input$

By inspection, it can be observed that each *output* value is 7 times bigger than the *input* value.

Exercise 3J

1–4 | 4 | —

UNDERSTANDING

1 State whether each of the following statements is true or false.

a If $output = input \times 2$, then when $input = 7$, $output = 14$.

b If $output = input - 2$, then when $input = 5$, $output = 7$.

c If $output = input + 2$, then when $input = 0$, $output = 2$.

d If $output = input \div 2$, then when $input = 20$, $output = 10$.

2 Which table of values matches the rule $output = input - 3$?

A

input	10	11	12
output	13	14	15

B

input	5	6	7
output	15	18	21

C

input	8	9	10
output	5	6	7

D

input	4	3	2
output	1	1	1

3 Which table of values matches the rule $output = input \div 2$?

A

input	20	14	6
output	18	12	4

B

input	8	10	12
output	4	5	6

C

input	4	5	6
output	8	10	12

D

input	4	3	2
output	6	5	4

4 Match each rule (A to D) with the correct table of values (**a** to **d**).

Rule A: $output = input - 5$

Rule B: $output = input + 1$

Rule C: $output = 4 \times input$

Rule D: $output = 5 + input$

a

input	20	14	6
output	15	9	1

b

input	8	10	12
output	13	15	17

c

input	4	5	6
output	5	6	7

d

input	4	3	2
output	16	12	8

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3J

	5–7	5–7	5–7(½)

FLUENCY

Example 20a

5 Copy and complete each table for the given rule.

a $output = input + 3$

input	4	5	6	7	10
output					

b $output = input \times 2$

input	5	1	3	21	0
output					

c $output = input - 8$

input	11	18	9	44	100
output					

d $output = input \div 5$

input	5	15	55	0	100
output					

Example 20b

6 Copy and complete each table for the given rule.

a $output = (10 \times input) - 3$

input	1	2	3	4	5
output					

b $output = (input \div 2) + 4$

input	6	8	10	12	14
output					

c $output = (3 \times input) + 1$

input	5	12	2	9	0
output					

d $output = (2 \times input) - 4$

input	3	10	11	7	50
output					

Example 21

7 State the rule for each of these tables of values.

a

input	4	5	6	7	8
output	5	6	7	8	9

b

input	1	2	3	4	5
output	4	8	12	16	20

c

input	10	8	3	1	14
output	21	19	14	12	25

d

input	6	18	30	24	66
output	1	3	5	4	11

	8, 9	8, 9	9, 10

PROBLEM-SOLVING

8 Copy and complete the missing values in the table and state the rule.

input	4	10	13	24			5	11	2
output			39		42	9	15		6

9 Copy and complete the missing values in the table and state the rule.

input	12	93	14	17		10			
output	3			8	12	1	34	0	200

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PROBLEM-SOLVING

10 Copy and complete each table for the given rule.

a $output = input \times input - 2$

input	3	6	8	12	2
output					

b $output = (24 \div input) + 1$

input	6	12	1	3	8
output					

c $output = input^2 + input$

input	5	12	2	9	0
output					

d $output = 2 \times input \times input - input$

input	3	10	11	7	50
output					

REASONING — 11 | 11, 12 | 12, 13

11 Copy and complete each table for the given rule.

a $output = input + 6$

input	c	d	$2p$	b^2	www
output					

b $output = 3 \times input - 2$

input	t	k	p^2	$2f$	ab
output					

12 Copy and complete the missing values in the table and state the rule.

input	b		e	g^2		x	c		1
output		cd			cmn	xc		0	c

13 It is known that for an *input* value of 3 the *output* value is 7.

a State two different rules that work for these values.

b How many different rules are possible? Explain.

Finding harder rules

ENRICHMENT — — | — | 14

14 **a** The following rules all involve two operations. Find the rule for each of these tables of values.

i

input	4	5	6	7	8
output	5	7	9	11	13

ii

input	1	2	3	4	5
output	5	9	13	17	21

iii

input	10	8	3	1	14
output	49	39	14	4	69

iv

input	6	18	30	24	66
output	3	5	7	6	13

v

input	4	5	6	7	8
output	43	53	63	73	83

vi

input	1	2	3	4	5
output	0	4	8	12	16

b Write three of your own two-operation rules and produce a table of values for each rule.

c Swap your tables of values with those of a classmate and attempt to find one another's rules.

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3K The number plane and graphs

Interactive

Widgets

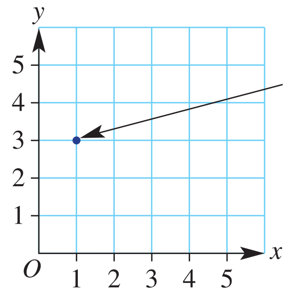
HOTsheets

Walkthroughs

We are already familiar with number lines.

A number line is used to locate a position in one dimension (i.e. along the line).

A number plane is used to locate a position in two dimensions (i.e. within the plane). A number plane uses two number lines to form a grid system, so that points can be located precisely. A rule can then be illustrated visually using a number plane by forming a graph.



What is the position of this point on the number plane?

Let's start: Estimate your location

Consider the door as 'the origin' of your classroom.

- Describe the position you are sitting in within the classroom in reference to the door.
- Can you think of different ways of describing your position? Which is the best way? Submit a copy of your location description to your teacher.

Can you locate a classmate correctly when location descriptions are read out by your teacher?

Key ideas

- A **number plane** is used to represent position in two dimensions, therefore it requires two coordinates.
- In mathematics, a number plane is generally referred to as a **Cartesian plane**, named after the famous French mathematician, René Descartes (1596–1650).
- A number plane consists of two straight perpendicular number lines, called axes.
  - The horizontal number line is known as the x-axis.
  - The vertical number line is known as the y-axis.
- For a rule describing a pattern with *input* and *output*, the x value is the *input* and the y value is the *output*.
- The point at which the two axes intersect is called the **origin**, and is often labelled O.

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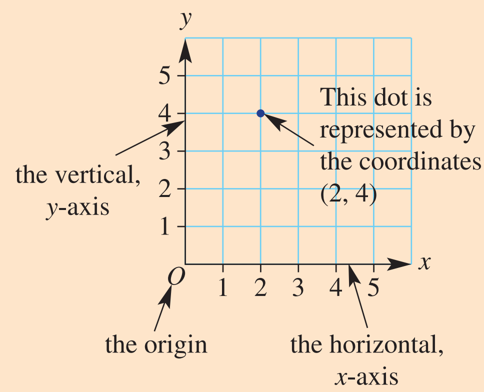
Key ideas

- The position of a point on a number plane is given as a pair of numbers, known as the **coordinates** of the point. Coordinates are always written in brackets and the numbers are separated by a comma.
  For example: (2, 4).
  - The x-coordinate (*input*) is always written first. The x-coordinate indicates how far to go from the origin in the horizontal direction.
  - The y-coordinate (*output*) is always written second. The y-coordinate indicates how far to go from the origin in the vertical direction.

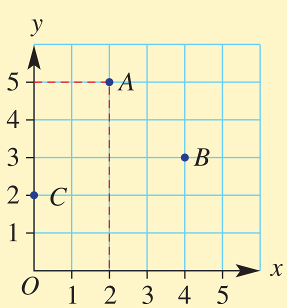


Example 22 Plotting points on a number plane

Plot these points on a number plane.

$A(2, 5)$ $B(4, 3)$ $C(0, 2)$

SOLUTION



EXPLANATION

Draw a Cartesian plane, with both axes labelled from 0 to 5.
The first coordinate is the x-coordinate.
The second coordinate is the y-coordinate.
To plot point A, go along the horizontal axis to the number 2, then move vertically up 5 units. Place a dot at this point, which is the intersection of the line passing through the point 2 on the horizontal axis and the line passing through the point 5 on the vertical axis.

Example 23 Drawing a graph

For the given rule *output* = *input* + 1:

a Complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	1
1	
2	
3	

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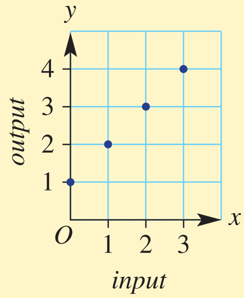
SOLUTION | **EXPLANATION**

a

input (x)	*output* (y)
0	1
1	2
2	3
3	4

Use the given rule to find each *output* value for each *input* value. The rule is:
output = *input* + 1, so add 1 to each *input* value.

b



Plot each (x, y) pair.
The pairs are $(0, 1)$, $(1, 2)$, $(2, 3)$ and $(3, 4)$.

Exercise 3K

1–5	5	—

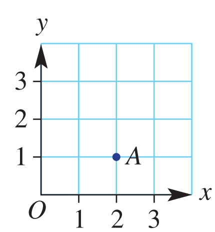
UNDERSTANDING

1 Draw a number plane, with the numbers 0 to 6 marked on each axis.

2 Draw a Cartesian plane, with the numbers 0 to 4 marked on both axes.

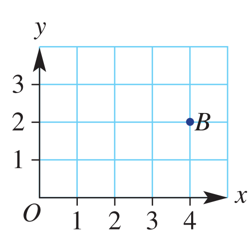
3 Which of the following is the correct way to describe point A?

A 2, 1 **B** 1, 2 **C** (2, 1)
D (x2, y1) **E** $(2_x, 1_y)$



4 Which of the following is the correct set of coordinates for point B?

A (2, 4) **B** 4, 2 **C** (4, 2)
D (24) **E** $x = 4, y = 2$



5 Copy and complete the following sentences.

a The horizontal axis is known as the________.
b The________is the vertical axis.
c The point at which the axes intersect is called the________.
d The x-coordinate is always written________.
e The second coordinate is always the ________.
f ________ comes before ________ in the dictionary, and the ________ coordinate comes before the ________ coordinate on the Cartesian plane.

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6–11	6(½), 7–12	6(½), 7–12

FLUENCY

Example 22

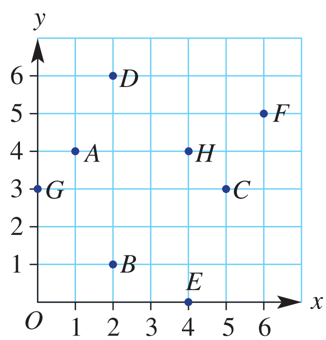
6 Plot the following points on a number plane.

a $A(4, 2)$ **b** $B(1, 1)$ **c** $C(5, 3)$ **d** $D(0, 2)$

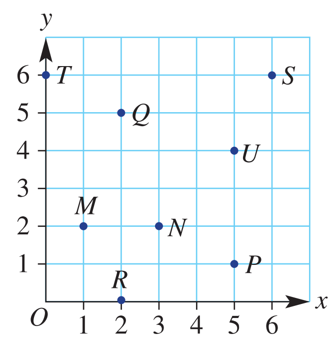
e $E(3, 1)$ **f** $F(5, 4)$ **g** $G(5, 0)$ **h** $H(0, 0)$

7 Write down the coordinates of each of these labelled points.

a



b



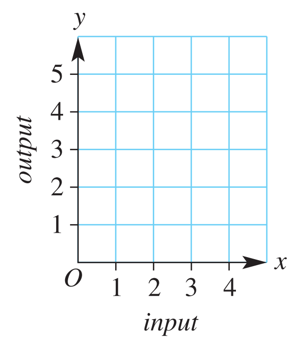
Example 23

8 For the given rule $output = input + 2$:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	2
1	
2	
3	

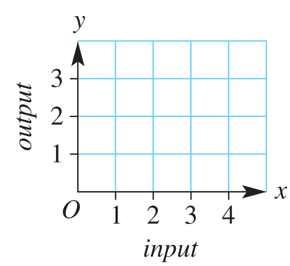


9 For the given rule $output = input - 1$:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
1	
2	
3	
4	

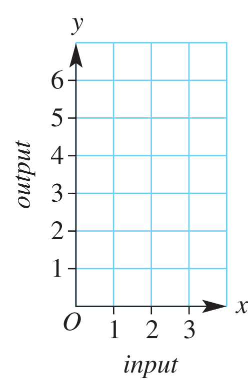


10 For the given rule $output = input \times 2$:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	
1	
2	
3	



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3K

FLUENCY

11 Draw a Cartesian plane from 0 to 5 on both axes. Place a cross on each pair of coordinates that have the same x and y value.

12 Draw a Cartesian plane from 0 to 8 on both axes. Plot the following points on the grid and join them in the order they are given.
(2, 7), (6, 7), (5, 5), (7, 5), (6, 2), (5, 2), (4, 1), (3, 2), (2, 2), (1, 5), (3, 5), (2, 7)

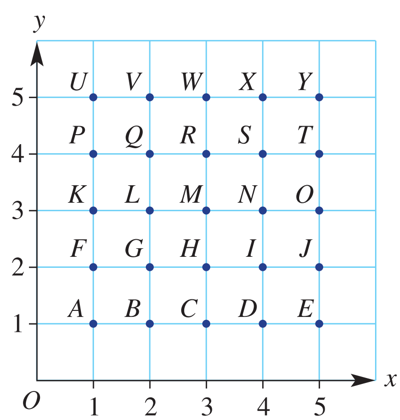
13, 14	14, 15	14–16

PROBLEM-SOLVING

13 **a** Plot the following points on a Cartesian plane and join the points in the order given, to draw the basic shape of a house.
(1, 5), (0, 5), (5, 10), (10, 5), (1, 5), (1, 0), (9, 0), (9, 5)
b Describe a set of four points to draw a door.
c Describe two sets of four points to draw two windows.
d Describe a set of four points to draw a chimney.

14 Point $A(1, 1)$ is the bottom left-hand corner of a square of side length 3.
a State the other three coordinates of the square.
b Draw the square on a Cartesian plane and shade in half of the square where the x-coordinates are greater than the y-coordinates.

15 A grid system can be used to make secret messages. Jake decides to arrange the letters of the alphabet on a Cartesian plane in the following manner.



a Decode Jake's following message:
(3, 2), (5, 1), (2, 3), (1, 4)
b Code the word 'secret'.
c To increase the difficulty of the code, Jake does not include brackets or commas and he uses the origin to indicate the end of a word.
What do the following numbers mean?
13515500154341513400145354001423114354.
d Code the phrase: 'Be here at seven'.

16 $ABCD$ is a rectangle. The coordinates of A, B and C are given below. Draw each rectangle on a Cartesian plane and state the coordinates of the missing corner, D.
a $A(0, 5)$ $B(0, 3)$ $C(4, 3)$ $D(?, ?)$
b $A(4, 4)$ $B(1, 4)$ $C(1, 1)$ $D(?, ?)$
c $A(0, 2)$ $B(3, 2)$ $C(3, 0)$ $D(?, ?)$
d $A(4, 1)$ $B(8, 4)$ $C(5, 8)$ $D(?, ?)$

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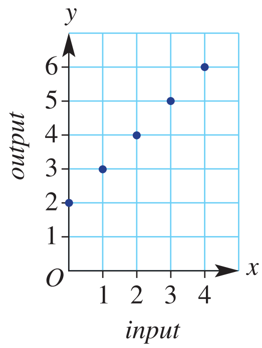
17	17	17, 18

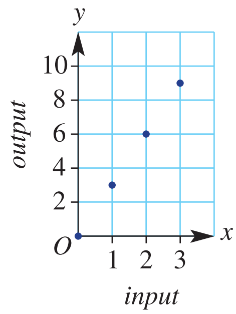
REASONING

17 Write a rule (e.g. *output* = *input* × 2) that would give these graphs.

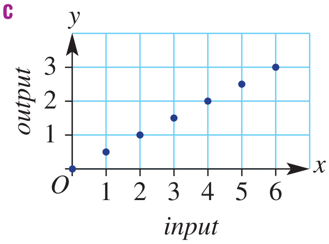
a



b



c



18 $A(1, 0)$ and $B(5, 0)$ are the base points of an isosceles triangle.

a Find the coordinates of a possible third vertex.

b Show on a Cartesian plane that there are infinite answers for this third vertex.

c The area of the isosceles triangle is 10 square units. State the coordinates of the third vertex.

Locating midpoints

—	—	19

ENRICHMENT

19 **a** Plot the points $A(1, 4)$ and $B(5, 0)$ on a Cartesian plane. Draw the line segment AB. Find the coordinates of M, the midpoint of AB, and mark it on the grid.

b Find the midpoint, M, of the line segment AB, which has coordinates $A(2, 4)$ and $B(0, 0)$.

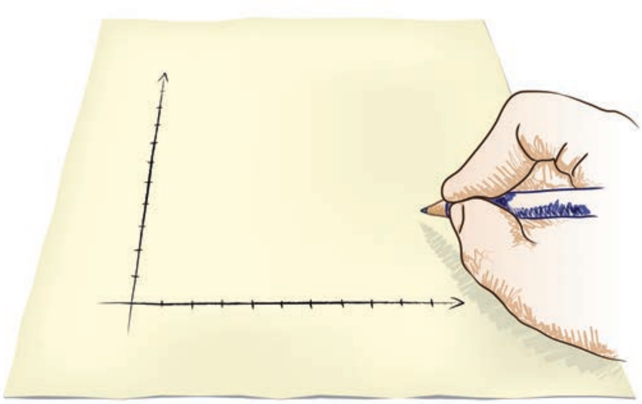
c Determine a method for locating the midpoint of a line segment without having to draw the points on a Cartesian plane.

d Find the midpoint, M, of the line segment AB, which has coordinates $A(6, 3)$ and $B(2, 1)$.

e Find the midpoint, M, of the line segment AB, which has coordinates $A(1, 4)$ and $B(4, 3)$.

f Find the midpoint, M, of the line segment AB, which has coordinates $A(–3, 2)$ and $B(2, –3)$.

g $M(3, 4)$ is the midpoint of AB and the coordinates of A are $(1, 5)$. What are the coordinates of B?



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Investigation

Fibonacci sequences

Leonardo Fibonacci was a famous thirteenth century mathematician who discovered some very interesting patterns of numbers that are found in nature.

Fibonacci's rabbits

These rules determine how fast rabbits can breed in ideal circumstances.

- Generation 1: One pair of newborn rabbits is in a paddock. A pair is one female and one male.
- Generation 2: When it is 2 months old, the female produces another pair of rabbits.
- Generation 3: When it is 3 months old, this same female produces another pair of rabbits.
- Every female rabbit always produces one new pair *every month* from age 2 months.

a Using the 'rabbit breeding rules', complete a drawing of the first five generations of rabbit pairs. Use it to complete the table opposite.

Month	1	2	3	4	5
Number of rabbits	2				
Number of pairs					

b Write down the numbers of pairs of rabbits at the end of each month for 12 months. This is the Fibonacci sequence.

c How many rabbits will there be after 1 year?

d Explain the rule for the Fibonacci sequence.

Fibonacci sequence in plants

a Count the clockwise and anticlockwise spiralling 'lumps' of some pineapples and show how these numbers relate to the Fibonacci sequence.

b Find three examples of flowers that have two terms of the Fibonacci sequence as the ratio of the numbers of clockwise and anticlockwise spirals of petals.

c On many plants, the number of petals is a Fibonacci number. Research the names and images of some of these 'Fibonacci' flowers.

Fibonacci sequence and the golden ratio

a Write down the next 10 terms of the Fibonacci sequence: 1, 1, 2, 3, 5, …

Fibonacci sequence	1	1	2	3	5	…
Ratio	1	1	2	1.5	…	…
		$1 \div 1$	$2 \div 1$	$3 \div 2$		

b Write down a new set of numbers that is one Fibonacci number divided by its previous Fibonacci number. Copy and complete this table.

c What do you notice about the new sequence (ratio)?

d Research the *golden ratio* and explain how it links to your new sequence.

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Problems and challenges

1 Matches are arranged by a student such that the first three diagrams in the pattern are:

How many matches are in the 50th diagram of the pattern?

2 A number is said to be a 'perfect number' if the sum of its factors equals the number. For this exercise, we must exclude the number itself as one of the factors.
The number 6 is the first perfect number.
Factors of 6 (excluding the numeral 6) are 1, 2 and 3.
The sum of these three factors is $1 + 2 + 3 = 6$. Hence, we have a perfect number.

- **a** Find the next perfect number. Hint: It is less than 50.
- **b** The third perfect number is 496. Find all the factors for this number and show that it is a perfect number.

3 Anya is a florist who is making up bunches of tulips with every bunch having the same number of tulips. Anya uses only one colour in each bunch. She has 126 red tulips, 108 pink tulips and 144 yellow tulips. Anya wants to use all the tulips.

- **a** What is the largest number of tulips Anya can put in each bunch?
- **b** How many bunches of each colour would Anya make with this number in each bunch ?

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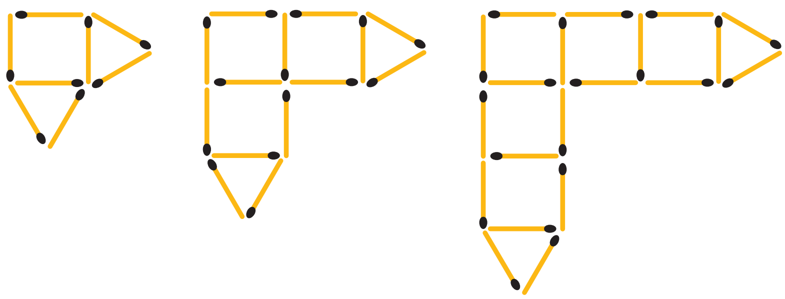
4 Mr and Mrs Adams have two teenage children. If the teenagers' ages multiply together to give 252, find the sum of their ages.

5 Complete this sequence.
$2^2 = 1^2 + 3$
$3^2 = 2^2 + 5$
$4^2 = 3^2 + 7$
$5^2 =$ _______
$6^2 =$ _______

6 Use the digits 1, 9, 7 and 2, in any order, and any operations and brackets you like, to make as your answers the whole numbers 0 to 10. For example:
$1 \times 9 - 7 - 2 = 0$
$(9 - 7) \div 2 - 1 = 0$

7 The first three shapes in a pattern made with matchsticks are:



How many matchsticks make up 100th shape?

8 Two numbers have a highest common factor of 1. If one of the numbers is 36 and the other number is a positive integer less than 36, find all possible values for the number that is less than 36.

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Chapter summary

Number properties

Composite numbers
10: factors 1, 2, 5, 10
62: factors 1, 2, 31, 62
Composite numbers are not prime.

Multiples
Multiples are $\geq$ number
20: 20, 40, 60, 80, ...
15: 15, 30, 45, 60, 75, ...
Lowest common multiple
LCM $= 60$

Factor trees
90
9 10
3 3 2 5
$90 = 2 \times 3 \times 3 \times 5$
$90 = 2 \times 3^2 \times 5$

Factors
Factors are $\leq$ number
20: 1, 2, 4, 5, 10, 20
15: 1, 3, 5, 15
Highest common factor
HCF $= 5$

Powers
$$\underset{\text{basic numeral}}{243} = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{expanded form}} = \underbrace{3^5}_{\text{index form}}$$
base number: 3
index number: 5

Square number
$16 = 4 \times 4 = 4^2$
$25 = 5 \times 5 = 5^2$

First 10 square numbers
1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Square root
$\sqrt{16} = 4$
$\sqrt{25} = 5$

Prime numbers
A prime number only has two factors: 1 and itself.
5: factors 1 and 5
17: factors 1 and 17

1 is not a prime number.

Division
$$\text{dividend} \rightarrow \frac{52}{3} = 17\frac{1}{3}$$
divisor: 3
quotient: 17
remainder: 1
$52 = 3 \times 17 + 1$

Divisibility tests (Ext)
2: last digit even (0, 2, 4, 6, 8)
3: sum digits $\div$ 3
4: number from last two digits $\div$ 4
5: last digit 0 or 5
6: $\div$ by 2 and 3
8: number from last 3 digits $\div$ 8
9: sum digits $\div$ 9
10: last digit 0

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Chapter summary

Number patterns

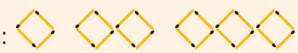
Sequence
2, 4, 6, 8, ...
↑ ↑ ↑ ↑
terms

Common difference of minus 3
100, 97, 94, ...

Common ratio of times 2
3, 6, 12, 24, ...

Pair differences increase by 3
6, 7, 11, 18, 28, ...

Common difference of an increase of 4

Spatial pattern:  ...

Number pattern: 4 8 12 ...

Rule: Number of sticks
= 4 times the number of diamonds

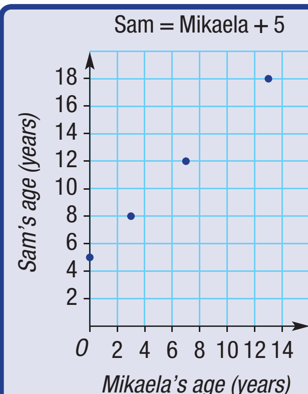
Rules, graphs and tables show a relation between two quantities that can vary.

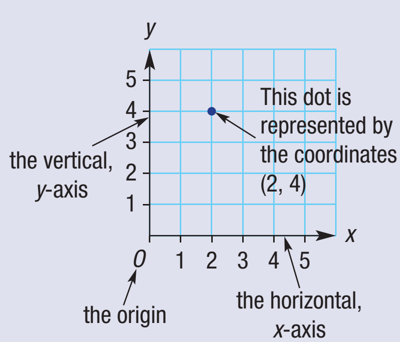
input	20	10	5
output	37	17	7

output = 2 × *input* − 3

Sam is 5 years older than his sister Mikaela

Mikaela's age	0	3	7	13
Sam's age	5	8	12	18





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Multiple-choice questions

3A **1** Which number is the incorrect multiple for the following sequence?
3, 6, 9, 12, 15, 18, 22, 24, 27, 30
A 18 **B** 22 **C** 30 **D** 6 **E** 3

3A **2** Which group of numbers contains every factor of 60?
A 2, 3, 4, 5, 10, 12, 15, 60
B 2, 3, 4, 5, 10, 12, 15, 20, 30
C 1, 2, 3, 4, 5, 10, 12, 15, 20, 30
D 2, 3, 4, 5, 10, 15, 20, 30, 60
E 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

3C Ext **3** Which of the following numbers is *not* divisible only by prime numbers, itself and 1?
A 21 **B** 77 **C** 110 **D** 221 **E** 65

3D **4** Which of the following groups of numbers include one prime and two composite numbers?
A 2, 10, 7 **B** 54, 7, 11 **C** 9, 32, 44 **D** 5, 17, 23 **E** 18, 3, 12

3E **5** $7 \times 7 \times 7 \times 7 \times 7$ can be simplified to:
A 5^7 **B** 7^5 **C** 7×5 **D** 75 **E** 77 777

3G **6** Evaluate $\sqrt{3^2 + 4^2}$.
A 7 **B** 5 **C** 14 **D** 25 **E** 6

3B **7** The HCF and LCM of 12 and 18 are:
A 6 and 18 **B** 3 and 12 **C** 2 and 54 **D** 6 and 36 **E** 3 and 18

3F **8** The prime factor form of 48 is:
A $2^4 \times 3$ **B** $2^2 \times 3^2$ **C** 2×3^3 **D** 3×4^2 **E** $2^3 \times 6$

3E **9** Evaluate $4^3 - 3 \times (2^4 - 3^2)$.
A 427 **B** 18 **C** 43 **D** 320 **E** 68

3A **10** Factors of 189 are:
A 3, 7, 9, 18, 21, 27
B 3, 9, 18, 21
C 3, 9, 18
D 3, 7, 9, 17, 21
E 3, 7, 9, 21, 27, 63

3C Ext **11** Which number is *not* divisible by 3?
A 25 697 403
B 31 975
C 7 297 008
D 28 650 180
E 38 629 634 073

3K **12** Which set of points is in a horizontal line?
A (5, 5), (6, 6), (7, 7)
B (3, 2), (3, 4), (3, 11)
C (2, 4), (3, 6), (4, 8)
D (5, 4), (6, 4), (8, 4), (12, 4)
E (1, 5), (5, 1), (1, 1), (5, 5)

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Chapter review

Short-answer questions

3A/D

1 a Find the complete set of factors of 120 and circle those that are composite numbers.
b Determine three numbers between 1000 and 2000 that each have factors 1, 2, 3, 4, 5 and itself.

3A/D

2 a Write down the first 12 multiples for each of 8 and 7 and circle the odd numbers.
b Which two prime numbers less than 20 have multiples that include both 1365 and 1274?

3B

3 a Find the HCF of the following pairs of numbers.
i 15 and 40 ii 18 and 26 iii 72 and 96
b Find the LCM of the following pairs of numbers.
i 5 and 13 ii 6 and 9 iii 44 and 8

3D

4 a State whether each of these numbers is a prime or composite number.
21, 30, 11, 16, 7, 3, 2
b How many prime multiples are there of 13?

3D/F

5 a State the prime factors of 770.
b Determine three composite numbers less than 100, each with only three factors that are all prime numbers less than 10.

3E

6 Simplify these expressions by writing them in index form.
a $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
b $5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2$

3F

7 Write these numbers as a product of prime numbers. Use a factor tree and then index form.
a 32 b 200 c 225

3F

8 Determine which number to the power of 5 equals each of the following.
a 100 000 b 243 c 1024

3F

9 Evaluate each of the following.
a $5^2 - 3^2$ b $2 \times 4^2 - 5^2$
c $5 \times 3^4 - 3^2 + 1^6$ d $12^2 - (7^2 - 6^2)$

3C Ext

10 Determine whether the following calculations are possible without leaving a remainder.
a $32\,766 \div 4$ b $1136 \div 8$ c $2417 \div 3$

3C Ext

11 a Carry out divisibility tests on the given number and fill in the table with ticks or crosses. State the explanation for each result.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
84 539 424								

b Use divisibility rules to determine a 10-digit number that is divisible by 3, 5, 6 and 9.
c Determine a six-digit number that is divisible by 2, 3, 5, 6, 9 and 10.

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3G

12 Evaluate:

a $\sqrt{25}$ b $\sqrt{2500}$ c $\sqrt{5^2 + 12^2}$

d $4^2 - \sqrt{25} + \sqrt{7^2}$ e $\sqrt{16 \times 49} \div \sqrt{4}$ f $10^2 \div \sqrt{3^2 + 4^2}$

3H

13 Find the next three terms for the following number patterns that have a common difference.

a 27, 30, 33, … b 67, 59, 51, … c 238, 196, 154, …

3H

14 Find the next three terms for the following number patterns that have a common ratio.

a 35, 70, 140, … b 24 300, 8100, 2700, … c 64, 160, 400, …

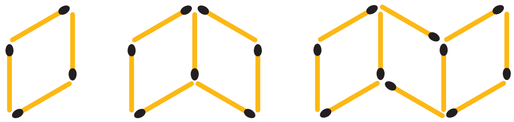
3H

15 Find the next six terms for each of these number patterns.

a 21, 66, 42, 61, 84, 56, … b 22, 41, 79, 136, …

3I

16 a Draw the next two shapes in this spatial pattern of sticks.



b Copy and complete this table.

Number of rhombuses	1	2	3	4	5
Number of sticks required					

c Describe the pattern by stating how many sticks are required to make the first rhombus and how many sticks must be added to make the next rhombus in the pattern.

3I

17 A rule to describe a special window spatial pattern is:

Number of sticks = 3 × number of windows + 2

a How many sticks are required to make one window?

b How many sticks are required to make 10 windows?

c How many sticks are required to make g windows?

d How many windows can be made from 65 sticks?

3J

18 Copy and complete each table for the given rule.

a $output = input + 5$

input	3	5	7	12	20
output					

b $output = 2 \times input + 7$

input	4	2	9	12	0
output					

3J

19 Find the rule for each of these tables of values.

a

input	3	4	5	6	7
output	12	13	14	15	16

b

input	1	2	3	4	5
output	20	32	44	56	68

c

input	0	1	2	3	4
output	1	3	5	7	9

d

input	3	4	5	6	7
output	7	6	5	4	3

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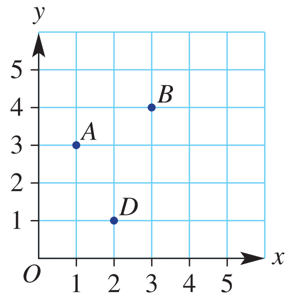
3K

20 a State the coordinates of each point plotted on this number plane.



b State the coordinates on this grid of a point C so that $ABCD$ is a square.

c State the coordinates on this grid of a point E on the x-axis so that $ABED$ is a trapezium (i.e. has only one pair of parallel sides).

Extended-response questions

1 For the following questions, write the answers in index notation (i.e. b^x) and simplify where possible.

a A rectangle has width 27 cm and length 125 cm. Determine power expressions for its area
and perimeter.

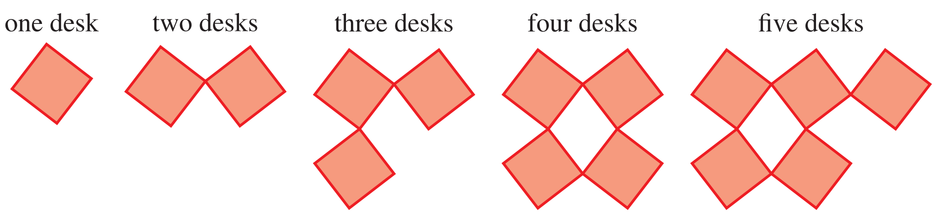
b A square's side length is equal to 4^3. Determine three power expressions for each of the area and perimeter of this square.

c $a \times a \times a \times a \times c \times c$

d $4^3 + 4^3 + 4^3 + 4^3$

e $\dfrac{3^x + 3^x + 3^x}{3}$

2 A class arranges its square desks so that the space between their desks creates rhombuses of identical size, as shown in this diagram.



a How many rhombuses are contained between:

i four desks that are in two rows (as shown in the diagram above)?

ii six desks in two rows?

b Draw 12 desks in three rows arranged this way.

c Rule up a table with columns for the number of:

- rows
- desks per row
- total number of desks
- total number of rhombuses

If there are four desks per row, complete your table for up to 24 desks.

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d If there are four desks per row, write a rule for the number of rhombuses in n rows of square desks.

e Using a computer spreadsheet, complete several more tables, varying the number of desks per row.

f Explain how the rule for the number of rhombuses changes when the number of desks, d, per row varies and also the number of rows, n, varies.

g If the number of rows of desks equals the number of desks per row, how many desks would be required to make 10 000 rhombuses?

3 Determine the next three terms in each of these sequences and explain how each is generated.

a 1, 4, 9, 16, 25, …

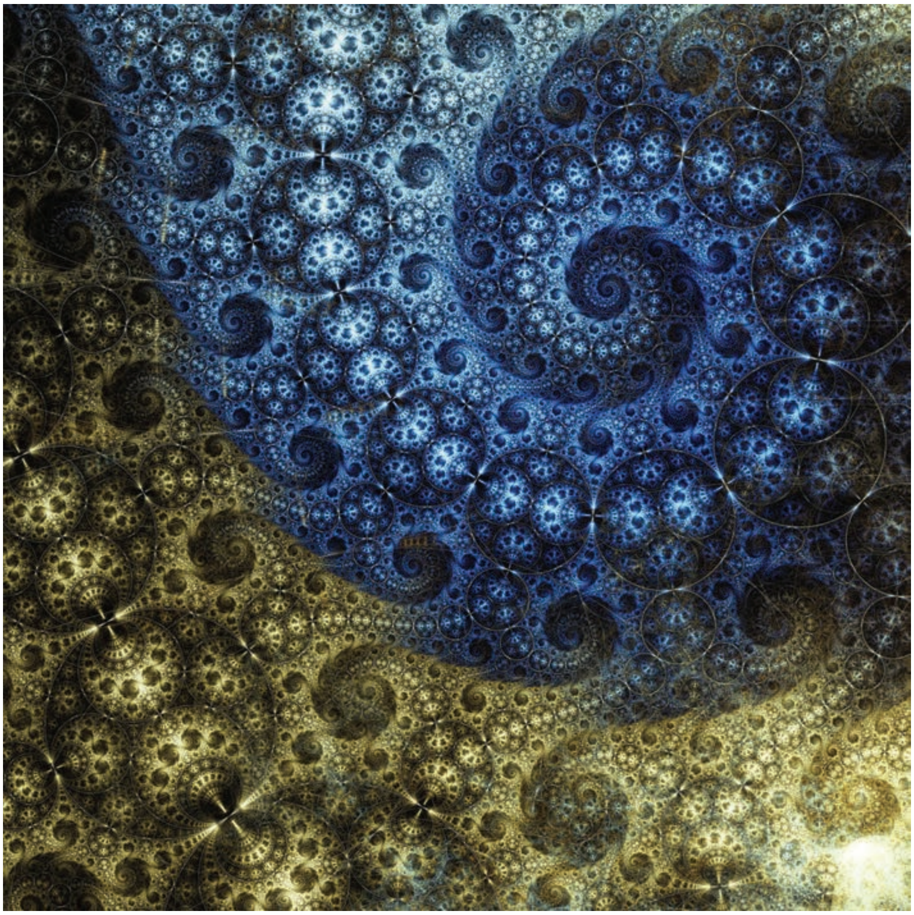
b 1, 8, 27, 64, …

c (1, 3), (2, 4), (3, 5), …

d 31, 29, 31, 30, 31, 30, …

e $1, \sqrt{2}, \sqrt{3}, 2$, …

f 1, 1, 2, 2, 3, 4, 4, 8, 5, 16, 6, 32, …



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