
3

Fractions

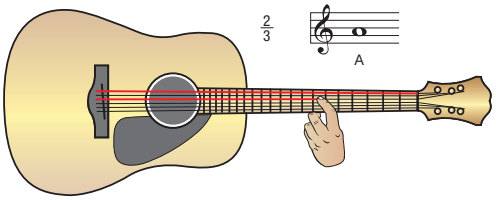
3

Frets make fractions.

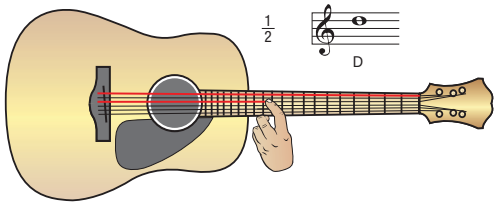
Learning the guitar or violin could help your understanding of fractions.

Guitarists and violinists practising their scales know that shortening a string on their instrument produces a higher sounding note. What they may not realise is that for each note of a scale, the shortened string length is an exact fraction of the full length, or 'open' string. Plucking the third string on a guitar gives the note 'D':

Shortening the string to $\frac{2}{3}$ of its length gives the note 'A'.



Shortening the string to $\frac{1}{2}$ of its length gives the note 'D' an octave higher.



Instrument makers use this information to work out where to place the frets on the necks of guitars. Violins do not have frets, so the player must learn exactly where to place his or her fingers to produce different notes. You will find out other ways that fractions are involved in music later in this chapter.

Forum

Some studies claim that there is a link between musical and mathematical ability—that people who are good at maths are also good at music. What do you think? What could you do to investigate this theory?

Why learn this?

Whether it be money, measurement or mixing a cake, we do not always work with whole numbers. Fractions are numbers that enable us to work with parts of wholes, and we need to be able to estimate and calculate with them.

After completing this chapter you will be able to:

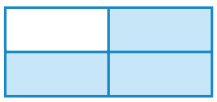
- identify and calculate fractions of a whole
- write one amount as a fraction of another
- simplify fractions and find equivalent fractions
- convert between fractions, whole numbers and mixed numbers
- use strategies to visualise, estimate and order fractions
- add, subtract, multiply and divide fractions
- solve problems involving fractions.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from the Online Student Book.

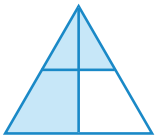
1 Which of these have three-quarters of the whole shape shaded in?

Worksheet R3.1

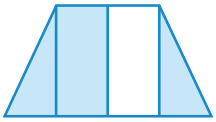
(a) 

(b) 

(c) 

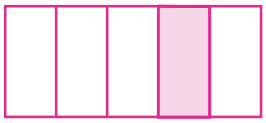
(d) 

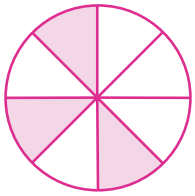
(e) 

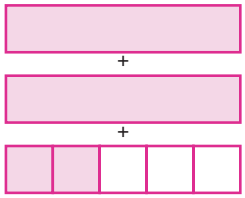
(f) 

Worksheet R3.2

2 Write down a fraction to represent each of the following diagrams or descriptions.

(a) 

(b) 

(c) 

(d) One person's share of a birthday cake divided equally between ten people.

(e) Joseph eats five out of eight equal-sized slices of a pizza.

(f) two thirds (g) twelve fifths (h) seven sixteenths

Worksheet R3.3

3 (a) Write these in order from largest to smallest: $\frac{3}{4}$, 1, 0, 2, $1\frac{1}{4}$

(b) Write these in order from smallest to largest: $\frac{3}{8}$, $\frac{1}{8}$, 0, $\frac{7}{8}$, $\frac{11}{8}$, 1, $\frac{9}{8}$

Worksheet R3.4

4 Calculate:

(a) $\frac{4}{7}+\frac{1}{7}$ (b) $\frac{8}{11}-\frac{2}{11}$ (c) $\frac{2}{3}+\frac{2}{3}$

Worksheet R3.5

5 (a) Write the first five multiples of 8. (b) Write the first five multiples of 12.

(c) Write the lowest common multiple (LCM) of 8 and 12.

(d) Write all the factors of 24. (e) Write all the factors of 36.

(f) Write the highest common factor (HCF) of 24 and 36.

Key Words

cancelling	improper fraction	mixed number	simplest form
denominator	inverse	numerator	simplify
equivalent fractions	Lowest Common Denominator (LCD)	proper fraction	unit fraction
fraction			

Understanding fractions

3.1

What is a fraction?

A **fraction** is a number that is used to show parts of a whole.

A whole might be an object, a collection of objects or a section of a number line.

If we are dividing a whole into parts to show a fraction, each of the parts must be equal in size.

If we are dividing up a collection of objects into groups to show a fraction, there must be the same number of objects in each group.

To write a fraction, two numbers are required.

$\frac{3}{5}$ ← The **numerator** tells us how many equal parts we have.

$\frac{3}{5}$ ← The **denominator** tells us how many equal parts one whole has been divided into.

Worked Example 1

WE1

How many smileys are there in $\frac{4}{5}$ of this collection?

Thinking	Working
1 The denominator shows us how many equal parts to divide the collection into. (Dividing 15 into 5 groups gives 3 smileys in each group.)	$15 \div 5 = 3$ $\frac{1}{5}$ is 3 smileys 
2 The numerator tells us how many of these 'equal parts' we should count for our total. ($\frac{4}{5}$ means take 4 of the 5 groups.)	$4 \times 3 = 12$ 
3 Write the answer.	$\frac{4}{5}$ is 12 smileys

Types of fractions

Proper fractions have a numerator that is less than the denominator. They have values less than 1. Examples: $\frac{1}{2}$ and $\frac{3}{5}$.

Improper fractions have a numerator that is greater than or equal to the denominator. They have values greater than or equal to 1. Examples: $\frac{3}{2}$, $\frac{9}{9}$ and $\frac{17}{6}$.

Mixed numbers have whole number parts and fraction parts written separately. Examples: $1\frac{1}{2}$, $2\frac{5}{6}$ and $34\frac{2}{7}$.

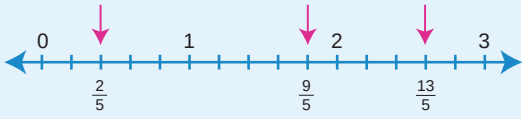
Using a number line to represent fractions

Placing fractions on a number line helps us to understand their size when compared to other numbers.

Drawing up a number line is easier if you can make the distance between whole numbers the same as the denominator. For example, if dividing into thirds, make the distance 3 cm. Then, you can mark off a third for every cm.

Worked Example 2 WE2

Copy this number line and show the positions of these fractions: $\frac{2}{5}$, $\frac{9}{5}$ and $\frac{13}{5}$.

Thinking	Working
1 Copy the number line.	
2 The denominator tells us how to divide up the spaces between the whole numbers (in this case, counting up in fifths). Make sure the distance between each marked division is the same.	
3 For each fraction, look at the numerator and count that many parts along from zero. Indicate the location of the fraction with an arrow.	

Fractions and division

There is an improper fraction that corresponds to each whole number on the number line.

If we fill in the rest of the fractions on the answer line in the Worked Example able, we can see that: $\frac{5}{5} = 1$, $\frac{10}{5} = 2$, $\frac{15}{5} = 3$ and so on.

We also know that: $5 \div 5 = 1$, $10 \div 5 = 2$, $15 \div 5 = 3$.

Writing $\frac{5}{5}$ is the same as writing $5 \div 5$.

We can write any whole number as an improper fraction, by writing the numerator as a multiple of the denominator. For example, $3 = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$, because $6 \div 2 = 3$, $9 \div 3 = 3$ and $12 \div 4 = 3$.

The simplest way of writing a whole number as an improper fraction is to write it with a denominator of 1. For example, $3 = \frac{3}{1}$ $(3 \div 1 = 3)$.

Drawing the line between the numerator and the denominator is equivalent to writing the division sign, ÷.

Worked Example 3

WE3

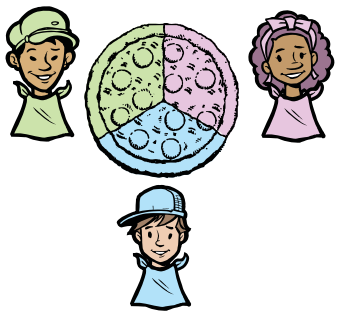
Write the whole number 9 as an improper fraction with a denominator of 4.

Thinking	Working
1 Which number divided by the denominator gives the required whole number? (Which number divided by 4 gives 9?)	$9 = \frac{\square}{4}$
2 Multiply the whole number you want by the denominator to find this number.	$9 \times 4 = 36$
3 Write the answer.	$9 = \frac{36}{4}$

We can consider all fractions in terms of division. Consider the following situation: 1 pizza is shared equally between 3 students. How much pizza does each student get?

We can show the answer is $\frac{1}{3}$ by drawing lines to divide the pizza into 3 equal pieces.

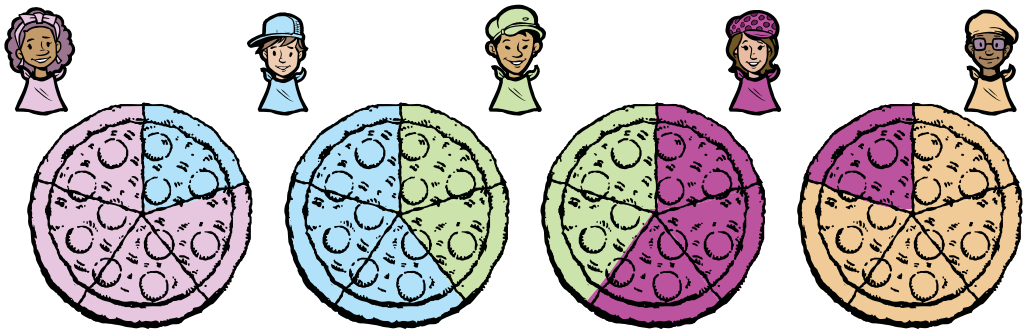
The different colours show which piece goes to which student.



We can write the division of 1 whole pizza among 3 students as $1 \div 3 = \frac{1}{3}$.

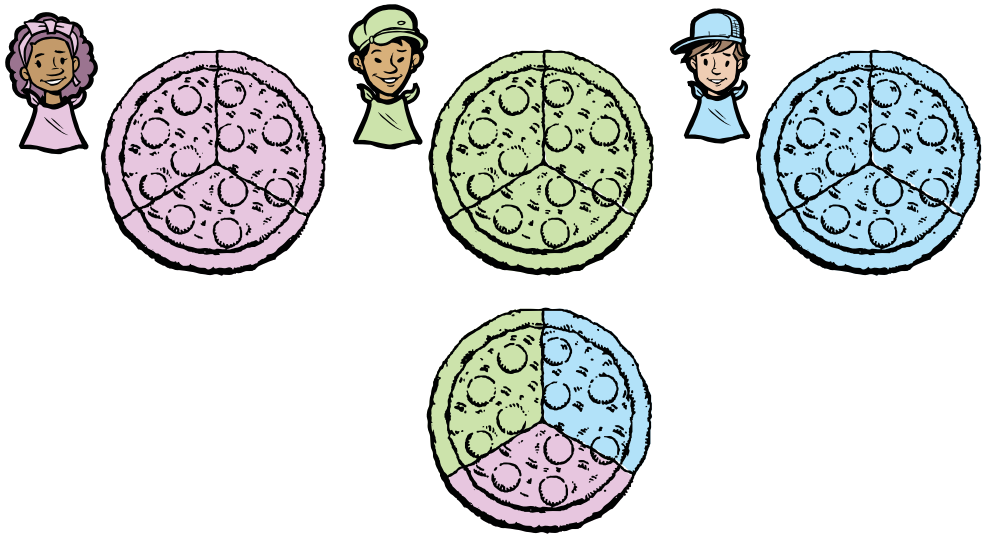
Similarly, if 4 pizzas are shared between 5 students, each student receives $\frac{4}{5}$ of a pizza.

$4 \div 5 = \frac{4}{5}$.



If we have a greater number of pizzas than students, each student receives more than a whole pizza. For example, 4 pizzas shared between 3 students gives $4 \div 3 = \frac{4}{3}$ of a pizza each.

From the diagram, we can see that this is equal to 1 whole and $\frac{1}{3}$ of a pizza each, or $1\frac{1}{3}$.



Worked Example 4

WE4

For the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

5 blocks of chocolate are shared equally between 8 students.

Thinking	Working
1 Identify the whole and how many it is being shared between. (The whole is 5 and it is being shared between 8.) Write a division that shows this.	$5 \div 8$
2 Write this division as a fraction.	$= \frac{5}{8}$

3.1 Understanding fractions

Navigator

Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q11 (a), Q13, Q16, Q17, Q19 (a), Q20 (b), Q22	Q1, Q2, Q3 (b), Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 (d)–(f), Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19 (b), Q20 (b), Q22	Q2 (b), Q3 (b), Q4, Q5, Q7 Column 3, Q8 (d)–(f), Q9, Q10, Q11 (b), Q12, Q13, Q14, Q15, Q17, Q18, Q19 (c), Q20, Q21, Q22

Answers page 624

Fluency

1 (a) How many smileys are there in $\frac{3}{4}$ of this collection?

(b) How many jelly beans are there in $\frac{2}{3}$ of this collection?

WE1

2 (a) Copy this number line and show the positions of these fractions: $\frac{3}{4}$, $\frac{9}{4}$, $\frac{5}{4}$ and $\frac{12}{4}$.

WE2

(b) Copy this number line and show the positions of these fractions: $\frac{5}{6}$, $\frac{8}{6}$, $\frac{1}{6}$ and $\frac{14}{6}$.

3 (a) Write the whole number 5 as an improper fraction with a denominator of:

WE3

(i) 2 (ii) 7 (iii) 11 (iv) 5 (v) 1

(b) Write the whole number 13 as an improper fraction with a denominator of:

(i) 2 (ii) 5 (iii) 8 (iv) 13 (v) 1

4 (a) For each of the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

WE4

(i) 1 pizza is shared equally between 2 students.

(ii) 2 apples are shared equally between 3 students.

(iii) 6 packets of lollies are shared equally between 5 students.

(iv) 10 packets of biscuits are shared equally between 7 students.

(b) In which of the above situations does a student receive more than one whole?

Understanding

5 Here are 16 lollies. How many will you eat if you eat these fractions of the total:

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

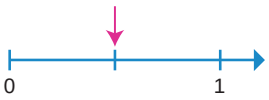
(c) $\frac{1}{8}$

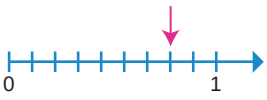
(d) $\frac{5}{8}$?

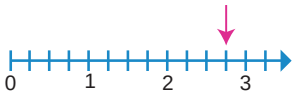
6 (a) What fraction of the bananas in this bunch are rotten?

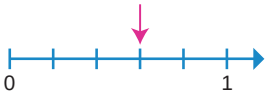
(b) What fraction of the flowers in this vase are red?

7 Write the value of the fraction indicated by the arrow on each of the number lines below.

(a)


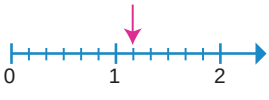
(b)


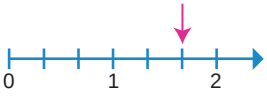
(c)


(d)


(e)


(f)


(g)


(h)


(i)


8 Write a fraction or a mixed number to show each of these:

(a) a numerator of 8 and a denominator of 17

(b) denominator of 4 and a numerator of 15

(c) nine chocolate biscuits in a packet of 20

(d) two wholes and two thirds

(e) 4 complete pairs of socks and one odd sock

(f) 3 whole 24-piece blocks of chocolate, with 7 extra pieces.

9 Write a fraction to show each of these:

(a) the weekend days as a fraction of a whole week

(b) 1 hour out of a whole day

(c) 1 second out of a whole minute

(d) 17 minutes out of a whole hour

(e) 157 mL of cola drunk from a 375 mL can

(f) 421L of water in a 500 L rainwater tank.

10 What fraction of this collection of shapes are:

(a) stars

(b) stars or hearts

(c) not hearts?

11 (a) Draw a diagram to show that if 5 pizzas are shared equally between 6 students, then each student receives $\frac{5}{6}$ of a pizza.

(b) Draw a diagram to show that if 5 blocks of chocolate are shared equally between 4 students, then each student receives 1 full block and $\frac{1}{4}$ of a second block.

12 Tim is 149 cm tall. His dad is 185 cm tall.

(a) Write Tim's height as a fraction of his dad's height.

(b) Write his dad's height as a fraction of Tim's height.

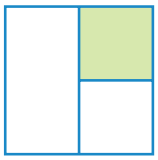
13 (a) After driving 8 km, the Johnson family are $\frac{1}{4}$ of the way to the zoo. How far away from the zoo do the Johnsons live?

(b) Toby and his Mum shared a bag of lollies: $\frac{2}{3}$ for Toby and $\frac{1}{3}$ for Mum. Mum had 6 lollies. How many did Toby have? How many lollies were in the bag?

Reasoning

14 4 girls share 5 pizzas evenly between them. 5 boys share 4 of the same size pizzas evenly between them. Write the fraction of pizza that each boy and each girl receives. Who gets more pizza, a boy or a girl?

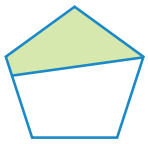
15 To show fractions, a whole must be divided into equal parts. For each of the shapes below, consider whether the shaded section represents one or more equal parts. State the fraction shown. If you don't think it is possible to state a definite fraction, explain why.

(a) 

(b) 

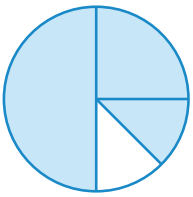
(c) 

(d) 

(e) 

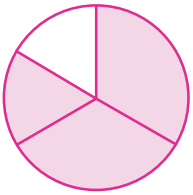
(f) 

16 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ B $\frac{4}{5}$ C $\frac{5}{6}$ D $\frac{7}{8}$

17 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ B $\frac{4}{5}$ C $\frac{5}{6}$ D $\frac{7}{8}$

18 How many whole numbers would appear on the section of the number line between $\frac{1}{2}$ and $\frac{11}{2}$?

Open-ended

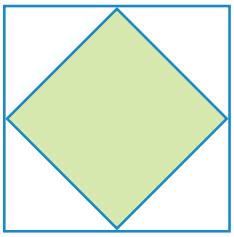
19 Draw a diagram that shows:

(a) three-eighths of an object (b) $\frac{5}{6}$ of a collection of objects (c) $\frac{9}{3}$

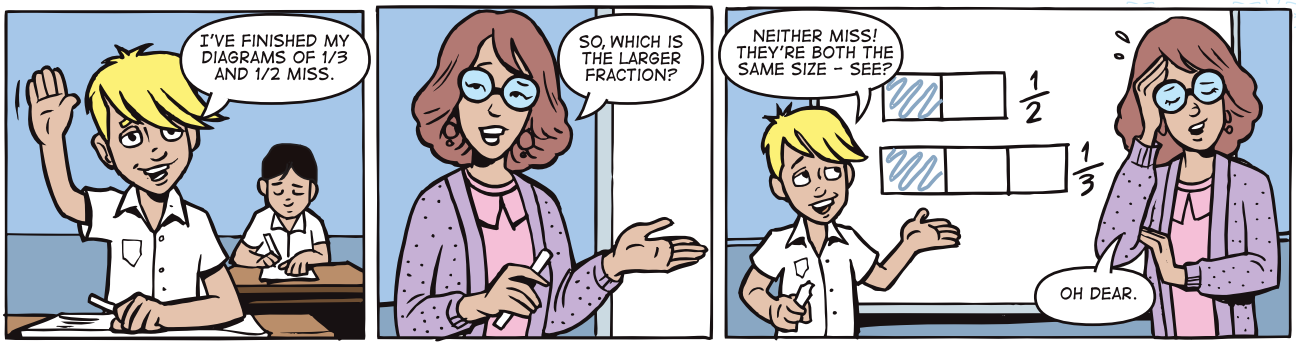
20 (a) This shape  represents $\frac{1}{5}$ of a whole. Draw two examples of what one whole could look like.

(b) This shape  represents $\frac{1}{4}$ of a whole. Draw two examples of what two wholes could look like.

21 Draw a diagram to help you explain why the fraction of this tile that is shaded is $\frac{1}{2}$.



22



Josh's diagram is supposed to show which of the fractions $\frac{1}{2}$ or $\frac{1}{3}$ is the larger fraction.

(a) Explain to Josh what is wrong with his diagram.

(b) Draw a more accurate diagram to help you explain where he has gone wrong.

Outside the Square Puzzle

Fractionally funny fiction

Q: Which English ruler invented fractions?

To find out, write down the letters that correspond to the fractions of each of the following words.

For example, 'the first $\frac{1}{4}$ of 'TERRIFIC' would be the letters 'TE' ('TE-RR-IF-IC')

The letters in each of the 4 boxes form a word.

When you have put together four words, rearrange them to find the answer.

The second $\frac{1}{5}$ of ENTHUSIASM

The middle $\frac{1}{5}$ of WHEEL

The first $\frac{1}{5}$ of KNIFE

The middle $\frac{1}{3}$ of FRINGE

The last $\frac{1}{4}$ of SONG

The last $\frac{1}{6}$ of ENDURE

The third $\frac{1}{6}$ of REFRIGERATOR

The last $\frac{1}{3}$ of HEIGHT

The middle $\frac{1}{7}$ of WASHING

The first $\frac{1}{6}$ of HEAVEN

The middle $\frac{1}{5}$ of CONVENTION

The last $\frac{1}{3}$ of PANTRY

Figured it out?

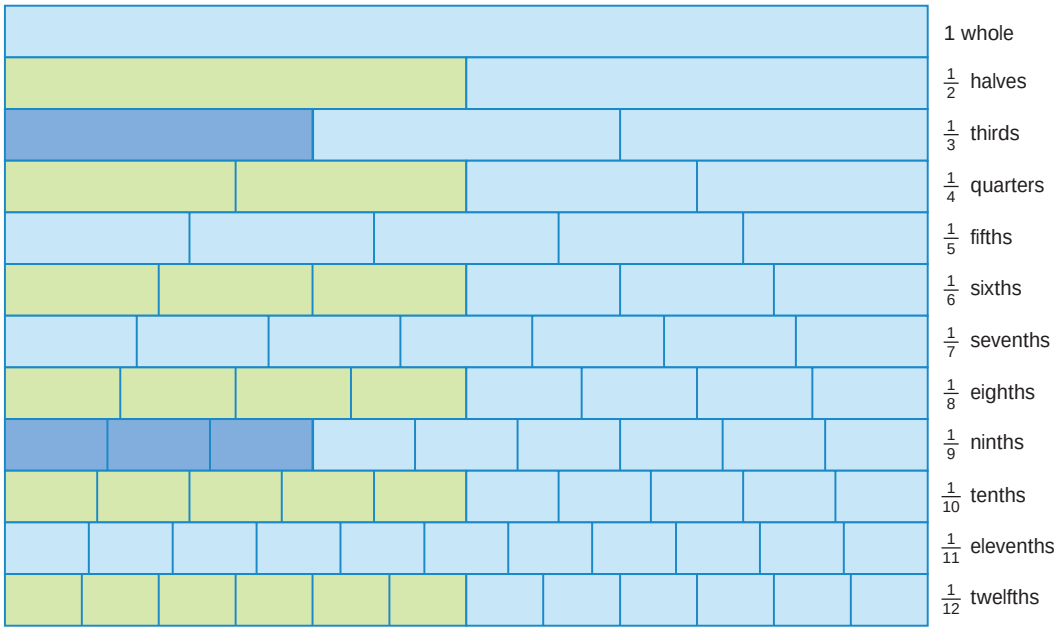
Make up your own puzzle that uses fractions of words to make the answer to a joke or fact.

3.2 Working with fractions

The fraction wall

Discovery Task

This fraction wall is made up of 12 layers of identical rectangles. One layer is left whole at the top, then each layer is divided into halves ($\frac{1}{2}$), thirds ($\frac{1}{3}$), quarters ($\frac{1}{4}$), fifths ($\frac{1}{5}$) … all the way down to twelfths ($\frac{1}{12}$). We can see that the size of the individual fractions (the 'bricks' in the wall) get smaller as we divide the whole into a greater number of pieces.



Equivalent fractions

If we examine the small vertical lines on the fraction wall, we can see that some lie exactly underneath each other (holding a ruler against the lines can help us see this more clearly). These lines in identical positions across the wall indicate equivalent fractions—fractions that represent the same amount, but with different numerators and denominators.

For example, if we consider $\frac{1}{2}$, we can see that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (these fractions are all shaded in the same colour on the wall).

If we consider $\frac{1}{3}$, we can see that $\frac{1}{3} = \frac{3}{9}$ ($\frac{1}{3}$ is also equivalent to $\frac{2}{6}$ and $\frac{4}{12}$).

Can you see the pattern that links the numerators and the denominators of the fractions that are equivalent to $\frac{1}{2}$ and $\frac{1}{3}$?

Multiplying the numerator and the denominator of $\frac{1}{2}$ by 2 gives $\frac{2}{4}$: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

Similarly, multiplying the numerator and the denominator by 3, 4, 5 and 6 gives $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and $\frac{6}{12}$.

Dividing the numerator and the denominator by a common factor also produces equivalent fractions: $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$.

As $\frac{2}{2}$ and $\frac{3}{3}$ are equivalent to 1 whole, multiplying or dividing a fraction by them is the same as multiplying or dividing by 1, which does not change the value of the fraction.

Equivalent fractions are found either by multiplying both the numerator and the denominator by the same number, or by dividing both the numerator and the denominator by the same number.

Worked Example 5 WE5

Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{7}{10} = \frac{\square}{40}$ (b) $\frac{27}{36} = \frac{3}{\square}$

Thinking		Working
(a) 1	Compare the two denominators to determine what you need to multiply the first denominator by to get the second. (To get 40, we multiply 10 by 4.)	(a) $\frac{7}{10} \times \frac{\square}{4} = \frac{\square}{40}$
2	Multiply the numerator by the same number to complete the equivalent fraction.	$\frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$
(b) 1	Compare the two numerators to determine what you need to divide the first numerator by to get the second. (To get 3, we divide 27 by 9.)	(b) $\frac{27}{36} \div \frac{9}{\square} = \frac{3}{\square}$
2	We divide the denominator by the same number to complete the equivalent fraction.	$\frac{27}{36} \div \frac{9}{9} = \frac{3}{4}$

Simplifying fractions

$\frac{1}{4}$ and $\frac{5}{20}$ are equivalent fractions: $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$.
While they both represent the same amount, $\frac{1}{4}$ is the simplest form of this pair of fractions.

A fraction in simplest form is often the easiest to understand and visualise.

To write a fraction in its simplest form, or to simplify it, we divide the numerator and the denominator by their highest common factor (HCF).

Answers to fraction questions should always be written in simplest form.

For example, the HCF of 5 and 20 is 5, so: $\frac{5}{20} \div \frac{5}{5} = \frac{1}{4}$ gives us the simplest form of $\frac{5}{20}$.

The process of simplifying fractions is often called 'cancelling'. To cancel a fraction, we show the division in the following way. Draw a line through the numerator. Divide the numerator by the HCF and write the result of the division next to it. Repeat this for the denominator.

For example: $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} = \frac{1}{4}$

Sometimes, if the HCF is not obvious, we can do several divisions by smaller factors, such as 2 or 3. This is shown in Method 2 of the Worked Example below.

Worked Example 6 WE6

Write the following fraction and mixed number in their simplest form.

(a) $\frac{24}{60}$ (b) $3\frac{12}{28}$

Method 1: Find the HCF

Thinking		Working
(a) 1	Find the HCF of the numerator and the denominator.	(a) HCF of 24 and 60 = 12
2	Divide both the numerator and the denominator by the HCF. (This is shown here using cancelling notation.)	$\overset{2}{\cancel{24}}{\cancel{60}}\ \overset{}{5}$
3	Write the answer.	$= \frac{2}{5}$
(b) 1	Find the HCF of the numerator and the denominator of the fraction part.	(b) HCF of 12 and 28 = 4
2	Divide both the numerator and the denominator by the HCF.	$3\frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{28}}}$
3	Write the answer.	$= 3\frac{3}{7}$

Method 2: Do several small divisions

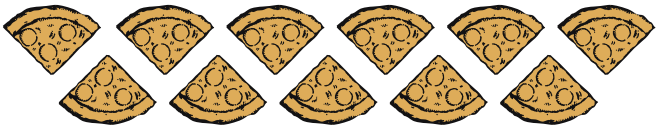
Thinking		Working
(a) 1	Divide by any small common factor such as 2, 3 or 5. (Here, both numerator and denominator are even, so we divide by 2.)	(a) $\overset{12}{\cancel{24}}{\cancel{60}}\ \overset{}{30}$
2	Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again, then by 3.)	$= \overset{6}{\cancel{12}}{\cancel{30}}\ \overset{}{15}$ $= \overset{2}{\cancel{6}}{\cancel{15}}\ \overset{}{5}$
3	Write the answer.	$= \frac{2}{5}$

(b)	1	Divide by any small common factor such as 2, 3 or 5. (Here, both numerator and denominator are even, so we divide by 2.)	(b) $3\frac{\overset{6}{\cancel{12}}}{\underset{14}{\cancel{28}}}$
	2	Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again.)	$= 3\frac{\overset{3}{\cancel{6}}}{\underset{7}{\cancel{14}}}$
	3	Write the answer.	$= 3\frac{3}{7}$

Mixed numbers and improper fractions

Improper fractions that have a value greater than 1 can be written as mixed numbers. To write an improper fraction as a mixed number, we need to see how many 'wholes' we can make, and what fraction we have left over.

For example, here are 11 slices of pizza. Each slice represents $\frac{1}{4}$, so we have $\frac{11}{4}$.



With 11 quarter slices, we can make 2 whole pizzas with 3 slices left over, or $2\frac{3}{4}$.

This is the same as doing $11 \div 4 = 2$ remainder 3. The 3 becomes the numerator of the fraction part of the mixed number with the same denominator, 4.

To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part. We then write the remainder as the numerator of the fraction part.

Worked Example 7

WE7

Write the improper fraction $\frac{34}{7}$ as a mixed number.

Thinking	Working
1 Divide the numerator by the denominator. Include the remainder with your answer.	$34 \div 7 = 4$ rem 6
2 Write down the whole number part. Write the remainder as the numerator of the fraction part.	$\frac{34}{7} = 4\frac{6}{7}$

Writing a mixed number as an improper fraction is the opposite of the above process. Instead of forming wholes from fraction parts, we 'cut up' our whole numbers into fraction parts, and count the total number of parts that we have.

To write a mixed number as an improper fraction, write the whole number part as an improper fraction with the same denominator. Then, add the proper fraction part.

Worked Example 8 WE8

Write the mixed number $7\frac{3}{5}$ as an improper fraction.

Thinking	Working
1 Write the whole number part as an improper fraction. (Here, 7×5 tells us how many fifths there are in 7.)	$7 \times 5 = 35$ $7 = \frac{35}{5}$
2 Add the proper fraction part.	$\frac{35}{5} + \frac{3}{5} = \frac{38}{5}$
3 Write the mixed number with its equivalent proper fraction.	$7\frac{3}{5} = \frac{38}{5}$

You can probably spot a shortcut for these types of questions—multiply the whole number by the denominator, add the numerator, and place the answer over the denominator: $7\frac{3}{5} = \frac{38}{5}$.

3.2 Working with fractions

Navigator

Answers page 625

Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1–3, Q4 Columns 1–3, Q5 (a) & (b), Q6, Q7, Q8, Q9, Q10, Q11, Q15 (a)–(c), Q16, Q17, Q18, Q19, Q22	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q21, Q22	Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q17, Q20, Q21, Q22

Fluency

WE5

1 Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{3}{4} = \frac{\square}{20}$ (b) $\frac{1}{3} = \frac{7}{\square}$ (c) $\frac{1}{2} = \frac{\square}{52}$ (d) $\frac{2}{5} = \frac{16}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{7}{11} = \frac{\square}{99}$

(i) $\frac{4}{3} = \frac{\square}{27}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{24}{16} = \frac{\square}{4}$ (l) $\frac{72}{36} = \frac{\square}{9}$

(m) $\frac{8}{\square} = \frac{88}{55}$ (n) $\frac{\square}{5} = \frac{42}{35}$ (o) $\frac{54}{\square} = \frac{9}{6}$ (p) $\frac{\square}{36} = \frac{4}{9}$

WE6

2 Write the following fractions and mixed numbers in their simplest form.

(a) $\frac{5}{10}$ (b) $\frac{3}{9}$ (c) $\frac{3}{12}$ (d) $\frac{7}{21}$

(e) $\frac{4}{10}$ (f) $\frac{8}{22}$ (g) $\frac{10}{16}$ (h) $\frac{36}{24}$

(i) $2\frac{14}{21}$ (j) $3\frac{5}{30}$ (k) $1\frac{6}{20}$ (l) $12\frac{24}{30}$

(m) $6\frac{20}{45}$ (n) $3\frac{25}{100}$ (o) $2\frac{35}{42}$ (p) $100\frac{12}{84}$

If you need to revise how to find the HCF, go to pages 56 and 57.

WE7

3 Write these improper fractions as mixed numbers.

(a) $\frac{7}{5}$ (b) $\frac{13}{6}$ (c) $\frac{23}{4}$ (d) $\frac{15}{7}$

(e) $\frac{37}{10}$ (f) $\frac{48}{5}$ (g) $\frac{44}{7}$ (h) $\frac{59}{10}$

(i) $\frac{77}{9}$ (j) $\frac{91}{12}$ (k) $\frac{107}{100}$ (l) $\frac{97}{40}$

WE8

4 Write these mixed numbers as improper fractions.

(a) $1\frac{3}{4}$ (b) $3\frac{1}{5}$ (c) $6\frac{2}{3}$ (d) $5\frac{9}{10}$

(e) $4\frac{3}{8}$ (f) $4\frac{7}{11}$ (g) $10\frac{2}{7}$ (h) $6\frac{8}{11}$

(i) $6\frac{5}{12}$ (j) $3\frac{9}{100}$ (k) $6\frac{8}{9}$ (l) $8\frac{7}{9}$

5 (a) Which one of the following is the simplest form of $3\frac{35}{45}$?

A $3\frac{5}{9}$ B $3\frac{3}{4}$ C $3\frac{7}{9}$ D $3\frac{70}{90}$

(b) Which of the following fractions is not equivalent to $\frac{10}{15}$?

A $\frac{2}{3}$ B $\frac{30}{45}$ C $\frac{40}{60}$ D $\frac{20}{25}$

(c) As a mixed number, $\frac{33}{4}$ equals:

A $3\frac{3}{4}$ B $7\frac{4}{5}$ C $8\frac{1}{4}$ D $8\frac{1}{3}$

(d) As an improper fraction, $2\frac{5}{8}$ equals:

A $\frac{7}{8}$ B $\frac{21}{8}$ C $\frac{25}{8}$ D 21

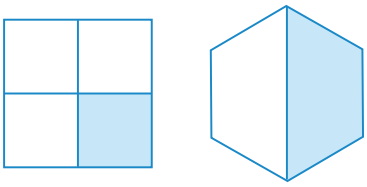
Understanding

6 Use the fraction wall on page 116 to identify the fraction or fractions that are equivalent to:

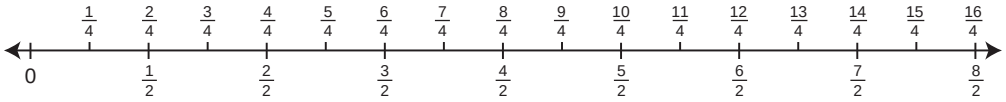
(a) $\frac{3}{5}$ (b) $\frac{2}{8}$ (c) $\frac{5}{6}$ (d) $\frac{9}{12}$

7 (a) Copy each of these figures. Write down the fraction that is shaded.

(b) In each figure, rule two diagonal lines joining the opposite vertices (corners). Name the equivalent fraction of the shaded part that you have created.

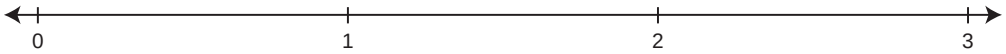


8 (a) The top side of this number line is divided into quarters, and the bottom side into halves.



What do you observe about the positions of the equivalent pairs of fractions $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{2}$ and $\frac{4}{4}$, $\frac{3}{2}$ and $\frac{6}{4}$, and $\frac{4}{2}$ and $\frac{8}{4}$?

(b) Copy the number line below. Divide the top side into thirds and the bottom side into ninths. Use your observation from (a) to identify pairs of equivalent fractions.



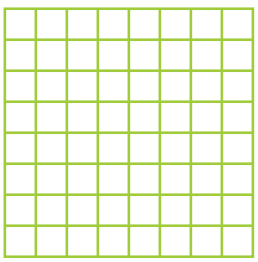
9 Monica, Michelle and Travis are sharing an apple pie that has been cut into 12 equal slices. Monica and Michelle decide to have $\frac{1}{4}$ of the apple pie each and give Travis the remainder. How many slices of pie did each person receive? Draw a diagram that shows this information.

10 Six pizzas were each sliced into 8 equal pieces. If $4\frac{3}{8}$ of the pizzas were eaten, how many slices is that?

11 A block of chocolate has 24 individual squares. How many squares of the chocolate will be gone if $\frac{7}{12}$ of the chocolate is eaten?

12 Talia shaded in a fraction of this long rectangle.

How many squares on this grid should Talia shade to show a fraction equivalent to the fraction she shaded in the rectangle above?



13 Write your answers in simplest form.

(a) What fraction of an hour has passed from 3.49 p.m. to 3.51 p.m.?

(b) What fraction of an hour has passed from 11.03 a.m. to 11.23 a.m.?

(c) What fraction of an hour has passed from 8.13 p.m. to 8.25 p.m.?

(d) What fraction of an hour has passed from 2.36 a.m. to 2.45 a.m.?

14 Imagine that the whole rectangle on the top of the fraction wall on page 116 represents 1 hour. How many minutes would each of the individual fractions on the wall represent? Write any answers that are not whole numbers as mixed numbers.

Reasoning

15 How many whole numbers would appear on the section of the number line between $\frac{4}{3}$ and $\frac{51}{9}$? (Hint: Convert these improper fractions to mixed numbers.)

16 Which of the following fractions could not be the simplified form of an original fraction that had a denominator of 40?

A $\frac{3}{8}$ B $\frac{2}{5}$ C $\frac{3}{7}$ D $\frac{9}{10}$

17 Which rows of fractions in the fraction wall on page 116 have no equivalent fractions on the wall? Explain why.

18 For each of the following problems, it may be useful to draw and label a number line with the known fractions. Then, locate the position of the unknown fraction and identify it. Equivalent fractions can help you do this.

Which fraction lies exactly in between the following fractions on the number line?

(a) $\frac{2}{5}$ and $\frac{3}{5}$ (b) $\frac{3}{7}$ and $\frac{4}{7}$ (c) $\frac{1}{4}$ and $\frac{2}{4}$

(d) $\frac{6}{10}$ and $\frac{7}{10}$ (e) $\frac{1}{9}$ and $\frac{6}{9}$ (f) $\frac{2}{11}$ and $\frac{7}{11}$

Open-ended

19 Write down three fractions that are equivalent to $\frac{4}{5}$.

20 Write three improper fractions that simplify to $1\frac{2}{3}$.

21 For each of these pairs of fractions, draw a diagram that shows they are equivalent.

(a) $\frac{1}{3}, \frac{5}{15}$ (b) $\frac{3}{4}, \frac{12}{16}$

22 Choose three fractions with three different denominators from the fraction wall on page 116 that have no equivalent fractions in the wall.

Outside the Square Puzzle

What fractions are we?

1 I am greater than $\frac{1}{2}$.

My numerator and denominator are both square numbers.

My numerator is an odd number.

The difference between my numerator and denominator is seven.

What fraction am I?

2 My numerator is a single-digit odd number.

My denominator is a two-digit number with both digits the same.

If you add 1 to both my numerator and denominator, you get a fraction equivalent to $\frac{1}{2}$.

What fraction am I?

3 My numerator is a multiple of 5.

My denominator is the LCM of 6 and 9.

I am less than 1.

I am in simplest form.

What fraction am I?

3.3 Estimating and comparing fractions

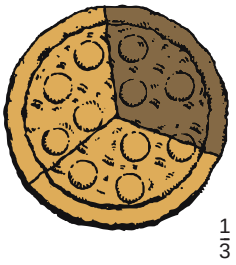
Comparing fractions by using the Lowest Common Denominator (LCD)

If two fractions have the same denominator, it is clear which fraction is bigger.

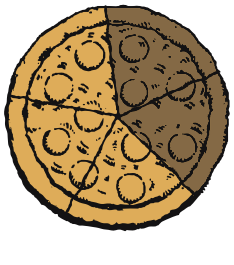
$\frac{2}{8}$ of a pizza is obviously more pizza than $\frac{1}{8}$ of a pizza. (It is twice as big.)

Comparing fractions with different denominators is harder. Having different denominators is like having different-sized pieces of pizza.

For example, which is larger, $\frac{1}{3}$ or $\frac{2}{5}$?



$\frac{1}{3}$

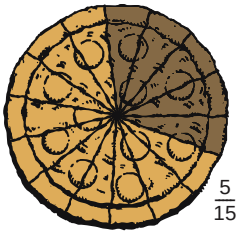


$\frac{2}{5}$

We need to rewrite the fractions so they have the same denominator. This is like cutting the pizza pieces into smaller, but equal-sized pieces. One way to do this is to find the Lowest Common Denominator (LCD). The LCD is the lowest common multiple of the denominators.

Here, the LCD is 15. We cut both pizzas into 15 pieces, then count how many pieces we have of each:

$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$



$\frac{5}{15}$

$\frac{6}{15}$

$\frac{2}{5}$ is the larger fraction, so we write $\frac{2}{5} > \frac{1}{3}$.

To compare fractions by using the lowest common denominator (LCD):

- find the lowest common multiple of the denominators
- rewrite each fraction as an equivalent fraction with this multiple as the new denominator
- write '>' (greater than), '<' (less than) or '=' (equal to) between the fractions.

If working with mixed numbers, write them as improper fractions first.

Worked Example 9 WE9

By finding the LCD, determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{5}$ or $\frac{4}{7}$ (b) $\frac{15}{6}$ or $2\frac{7}{12}$

Thinking		Working
(a) 1	Write the list of multiples for each denominator.	(a) $\frac{3}{5}$ or $\frac{4}{7}$ Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, ... Multiples of 7: 7, 14, 21, 28, 35, 42, 49, ...
2	Find the lowest common multiple of these denominators (LCD).	LCD = 35
3	Multiply each fraction to get an equivalent fraction with the LCD as the denominator.	$\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$
4	Use < or > to show which of the two fractions is larger.	$\frac{21}{35} > \frac{20}{35}$
5	Use < or > to show which of the original fractions is larger.	So, $\frac{3}{5} > \frac{4}{7}$
(b) 1	Write any mixed numbers as improper fractions.	(b) $2\frac{7}{12} = \frac{31}{12}$ $\frac{15}{6}$ or $\frac{31}{12}$
2	Find the LCD.	LCD = 12
3	Multiply to get equivalent fractions with the LCD as the denominator. (Only one fraction needs to be multiplied here.)	$\frac{15}{6} \times \frac{2}{2} = \frac{30}{12}$
4	Use < or > to show which of the two fractions is larger.	$\frac{30}{12} < \frac{31}{12}$
5	Use < or > to show which of the original fractions is larger.	So, $\frac{15}{6} < 2\frac{7}{12}$

Comparing fractions by estimating and visualising

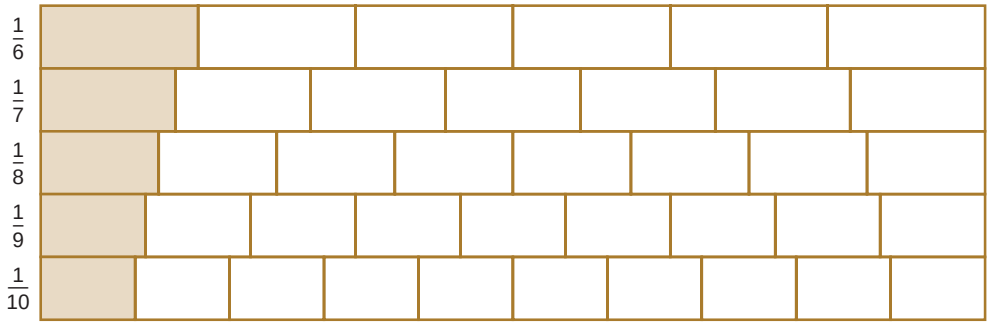
Often, we do not need to know an exact value of a fraction—it is enough to have an estimate. Similarly, when we compare two fractions, we only need to know which fraction is larger, not how much larger it is.

The following strategies can help us visualise what fractions 'look like', so we can estimate and compare them.

Strategy 1 – Compare the denominators

The fraction wall is very useful for comparing fraction. A section of the fraction wall on page 116 is shown here. It shows that as we divide a whole into more parts, the sizes of the parts (the 'bricks' in the wall) get smaller. (You are possibly already aware of this—the more people you share a pizza with, the smaller the amount that each person gets!)

The shaded parts of each layer of the fraction wall demonstrate that: $\frac{1}{6} > \frac{1}{7} > \frac{1}{8} > \frac{1}{9} > \frac{1}{10}$.



Looking at the fraction wall, we can see that:

$\frac{2}{6}$ is slightly larger than $\frac{2}{7}$, and much larger than $\frac{2}{9}$

$\frac{5}{6}$ is larger than $\frac{5}{8}$ and also larger than $\frac{6}{8}$.

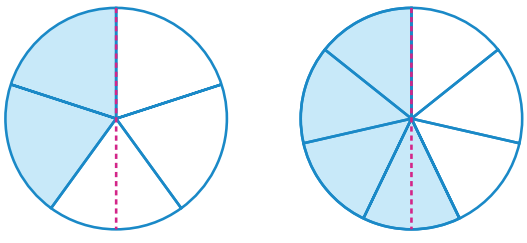
As the denominator numbers get larger, the fraction sizes get smaller.

If two fractions have the same numerator but different denominators, the *larger fraction* is the one with the *smaller number as the denominator*. For example, $\frac{5}{9} > \frac{5}{12}$, $\frac{3}{7} > \frac{3}{8}$.

Strategy 2 – Compare to a common number

Visualise what a common number such as $\frac{1}{2}$, $\frac{1}{4}$, 1 or 2 'looks like' by imagining a shape, such as a circle or a rectangle, with that fraction shaded in. How would your fraction look in comparison? Would it be bigger or smaller?

For example, we can see that $\frac{2}{5} < \frac{4}{7}$ if we compare them to $\frac{1}{2}$.



$\frac{2}{5}$ $\frac{4}{7}$

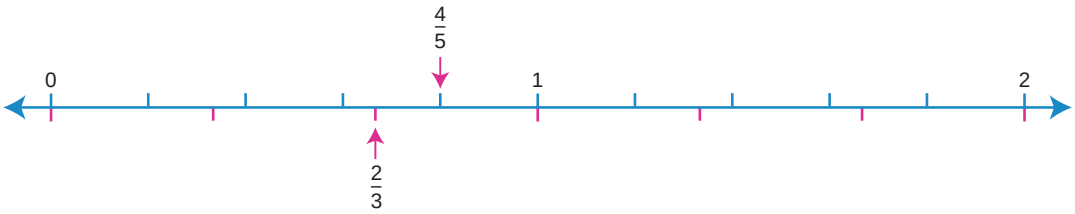
$\frac{2}{5}$ is slightly less than $\frac{1}{2}$ and $\frac{4}{7}$ is slightly greater than $\frac{1}{2}$, so $\frac{2}{5} < \frac{4}{7}$.

Strategy 3 – Compare by using a number line

One number is larger than another if it lies to the right of it on the number line.

It is not always necessary to measure up the number line exactly—sometimes an accurate sketch will be sufficient.

For example, we can show that $\frac{4}{5} > \frac{2}{3}$ by sketching a number line and dividing one side into fifths, and the other side into thirds. We can then show that $\frac{4}{5}$ lies to the right of $\frac{2}{3}$, which means $\frac{4}{5}$ is larger than $\frac{2}{3}$.



3.3 Estimating and comparing fractions

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q17	Q1 Columns 2 & 3, Q2, Q3 Columns 1–3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q17	Q1 Column 4, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17

Answers page 626

Fluency

WE9

1 By finding the LCD, determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{5}, \frac{7}{15}$ (b) $\frac{13}{12}, \frac{5}{4}$ (c) $\frac{7}{8}, \frac{5}{6}$ (d) $\frac{5}{9}, \frac{7}{12}$

(e) $\frac{2}{3}, \frac{5}{7}$ (f) $\frac{1}{6}, \frac{2}{9}$ (g) $\frac{3}{8}, \frac{5}{12}$ (h) $\frac{7}{4}, \frac{11}{6}$

(i) $1\frac{3}{4}, \frac{11}{8}$ (j) $\frac{5}{2}, 2\frac{1}{4}$ (k) $\frac{16}{7}, 2\frac{1}{5}$ (l) $2\frac{2}{7}, 2\frac{4}{9}$

2 Use the fraction wall on page 116 or 126 to determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{7}$ or $\frac{3}{10}$ (b) $\frac{7}{9}$ or $\frac{7}{8}$ (c) $\frac{4}{5}$ or $\frac{5}{6}$ (d) $\frac{1}{3}$ or $\frac{2}{5}$

(e) $\frac{9}{12}$ or $\frac{3}{4}$ (f) $\frac{5}{11}$ or $\frac{4}{9}$ (g) $\frac{6}{7}$ or $\frac{7}{8}$ (h) $\frac{3}{5}$ or $\frac{7}{11}$

3 Copy the following fraction pairs. Use any appropriate strategy to decide which fraction is larger, and place < or > between them. If the fractions are equivalent, write = between them.

(a) $\frac{2}{7}$ _____ $\frac{5}{7}$ (b) $\frac{4}{5}$ _____ $\frac{4}{7}$ (c) $\frac{1}{2}$ _____ $\frac{5}{8}$ (d) $\frac{8}{11}$ _____ $\frac{9}{10}$

(e) $\frac{2}{4}$ _____ $\frac{3}{6}$ (f) $\frac{6}{12}$ _____ $\frac{7}{10}$ (g) $\frac{3}{7}$ _____ $\frac{5}{8}$ (h) $\frac{5}{9}$ _____ $\frac{2}{5}$

(i) $\frac{5}{6}$ _____ $\frac{7}{8}$ (j) $\frac{7}{11}$ _____ $\frac{8}{10}$ (k) $\frac{6}{10}$ _____ $\frac{3}{5}$ (l) $\frac{4}{5}$ _____ $\frac{6}{9}$

(m) $\frac{2}{4}$ _____ $\frac{4}{2}$ (n) $\frac{24}{6}$ _____ $\frac{8}{2}$ (o) $\frac{18}{11}$ _____ $\frac{9}{5}$ (p) $\frac{9}{7}$ _____ $\frac{11}{9}$

Understanding

4 (a) Draw a number line from 0 to 2. Indicate the approximate positions of the following fractions on the number line, then use them to write the fractions in ascending order (smallest to largest). $\frac{3}{4}, \frac{2}{3}, \frac{6}{4}, 1\frac{1}{3}$

(b) Draw a number line from 0 to 3. Indicate the approximate positions of the following fractions on the number line, then use them to write the fractions in descending order (largest to smallest). $\frac{5}{2}, \frac{1}{2}, \frac{4}{10}, 1\frac{7}{10}$

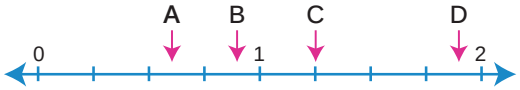
5 Ali gets 7 out of 8 shots at goal in the basket. Joseph gets 10 out of 12 shots in. By comparing fractions, determine who is the more accurate shooter.

6 In a box containing 16 chocolates, 5 of them contain nuts. A larger box with 64 of the same brand of chocolates contains 23 nut chocolates. Which box contains the greater fraction of chocolates with nuts?

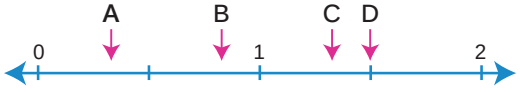
7 Use the fraction wall on page 116 to determine which fraction or fractions on the wall lie in between the following (include all equivalent fractions).

(a) $\frac{9}{10}$ and $\frac{11}{12}$ (b) $\frac{5}{8}$ and $\frac{7}{10}$ (c) $\frac{3}{8}$ and $\frac{4}{9}$ (d) $\frac{2}{11}$ and $\frac{3}{11}$

8 (a) Which arrow is pointing closest to the location of $\frac{9}{10}$ on this number line?



(b) Which arrow is pointing closest to the location of $1\frac{1}{3}$?



9 (a) Which one of the following is closest to $\frac{5}{8}$?

A $\frac{2}{9}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{7}{8}$

(b) Which one of the following is closest to 2?

A $\frac{11}{12}$ B $\frac{3}{2}$ C $\frac{7}{4}$ D $\frac{7}{3}$

Reasoning

10 Write each of these lists in ascending order (from smallest to largest). Explain using a series of brief points, or steps, how you decided the order.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{10}{11}$

(b) $\frac{4}{7}, 1, \frac{7}{4}, \frac{6}{7}, \frac{7}{8}, 0$

11 Write each of these lists in descending order (from largest to smallest). Explain briefly how you decided the order.

(a) $\frac{4}{7}, \frac{2}{9}, \frac{2}{3}, 1, \frac{11}{21}, \frac{45}{63}$

(b) $\frac{21}{16}, 1\frac{3}{80}, \frac{5}{4}, 1\frac{7}{20}, \frac{39}{40}, 1\frac{3}{10}$

12 (a) Estimate the fraction of the Australian flag that is blue.

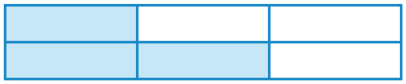
(b) Estimate the fraction of Australia that is taken up by the state of Queensland.

(c) Estimate the fraction of this jug that has cordial in it.

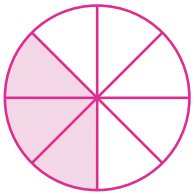
13 For each of the following pairs of identical shapes, state whether the fraction of the shape that is shaded is larger in A or in B. If the fraction shaded is the same in A and B write 'same'.

(a) A

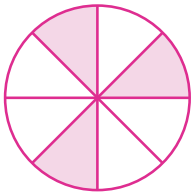
B



(b) A

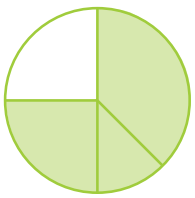


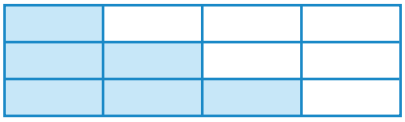
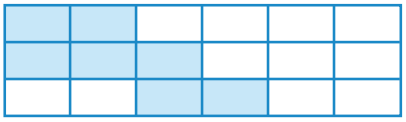
B

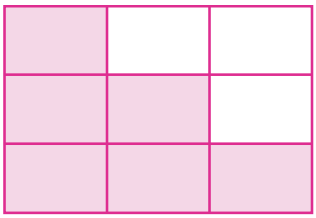
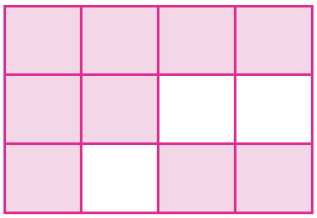


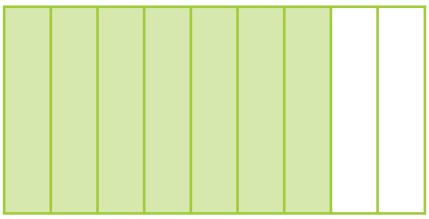
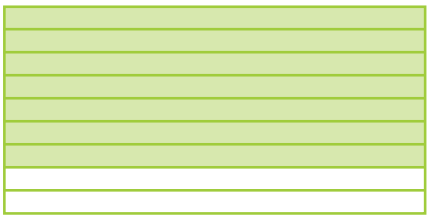
(c) A

B



(d) A  B 

(e) A  B 

(f) A  B 

14 Angus has 100 mL left of his 500 mL bottle of cola. Gianni has 50 mL left in his 300 mL bottle.

(a) Who has drunk more cola?

(b) Who has drunk a greater fraction of the cola in their bottle?

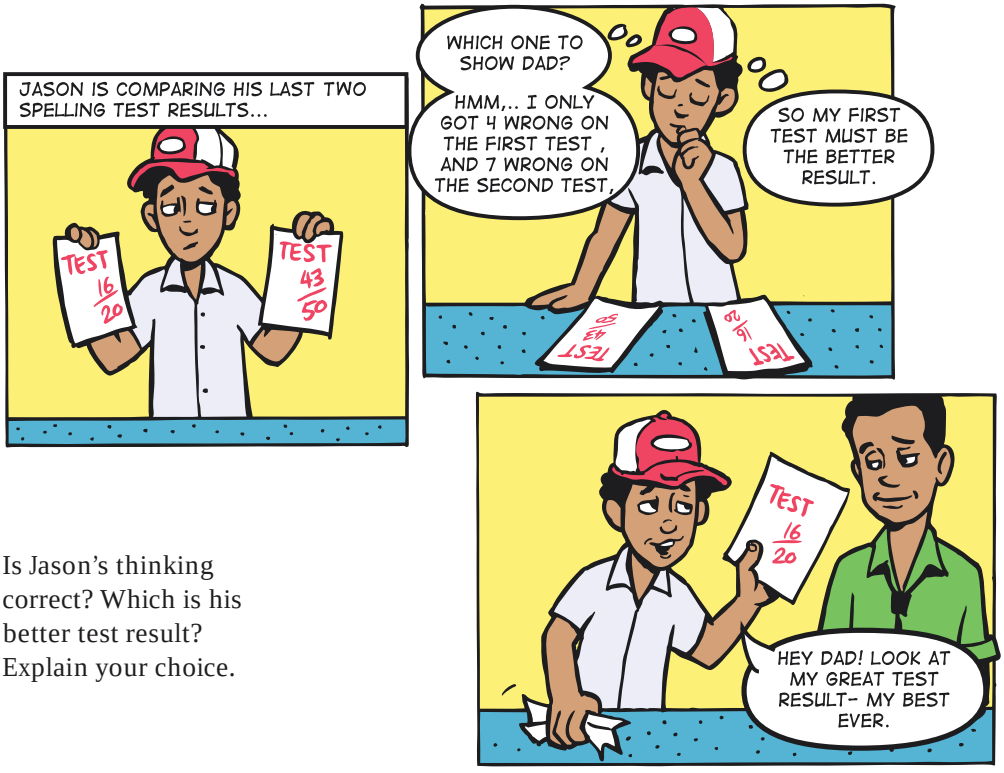
Open-ended

15 (a) A proper fraction is greater than $\frac{3}{4}$ but less than 1. What might the fraction be? Give two possibilities.

(b) An improper fraction is greater than 2 but less than 3. What might the fraction be? Give two possibilities.

16 Write three fractions that are greater than $\frac{3}{5}$ and less than $\frac{4}{5}$.

17



Is Jason's thinking correct? Which is his better test result? Explain your choice.

Outside the Square Game

Flatlining

Equipment required: 2 brains, 1 die, small counters of two different colours or pieces of paper marked with different colours or the players' initials

How to win:
The first player to get 3 circles in a row on the number line is the winner.

How to play:
Decide who will go first.

Either use the number line shown or draw an accurate copy in your book.

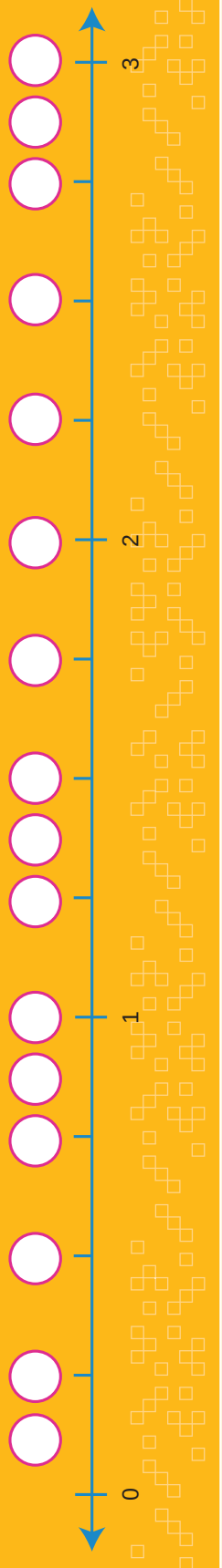
On your turn, roll the die and look up the number you rolled in the table. Choose one of the fractions from the list. Locate that fractions' approximate position on the number line and occupy the circle above it with one of your counters or pieces of paper.

If you can't occupy a circle you miss your turn.

Make sure your opponent locates their fraction correctly.

Once a circle is filled it cannot be taken by another player.

Dice roll	Fractions to choose from		
1	$\frac{1}{4}$	$1\frac{1}{2}$	$\frac{7}{4}$
2	$\frac{1}{2}$	$\frac{5}{4}$	$2\frac{4}{5}$
3	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{9}{4}$
4	3	$\frac{4}{5}$	$\frac{5}{2}$
5	2	$\frac{11}{4}$	$\frac{3}{3}$
6	1	$\frac{1}{5}$	$1\frac{3}{4}$

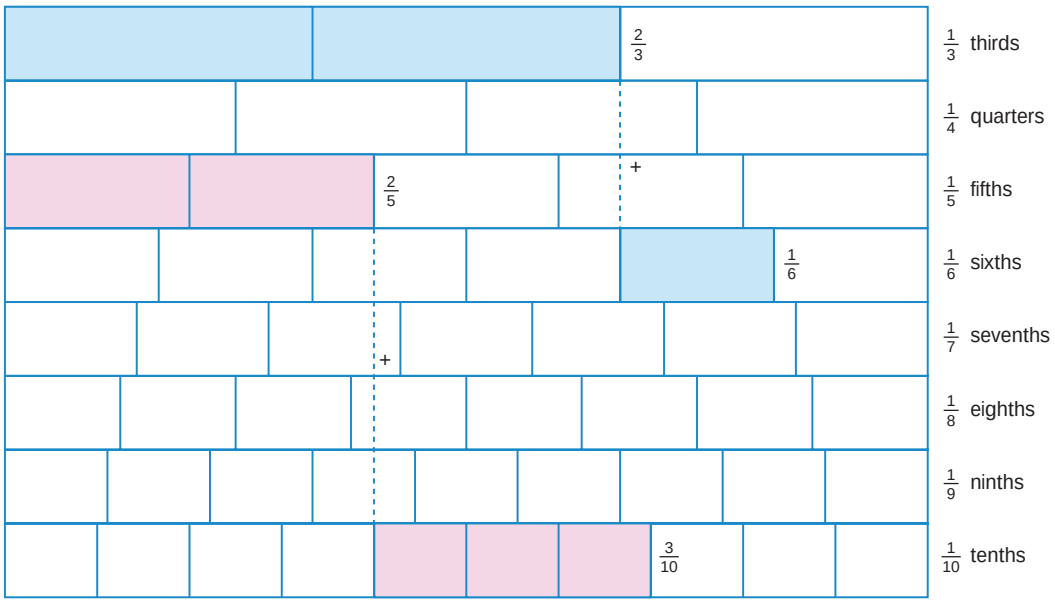


3.4 Adding and subtracting fractions

The fraction wall can be useful for learning to add fractions.

For example, we can show $\frac{2}{3} + \frac{1}{6}$ by moving along the wall to $\frac{2}{3}$, then along $\frac{1}{6}$ to arrive at $\frac{5}{6}$. This sum is shaded in blue on the fraction wall below.

We can also show $\frac{2}{5} + \frac{3}{10}$ by moving along the wall to $\frac{2}{5}$, then along $\frac{3}{10}$ to arrive at $\frac{7}{10}$. This sum is shaded in pink on the fraction wall below.



These additions are quite straightforward as the second fraction joins exactly onto the end of the first. This is because the denominators of the two fractions are related — we can write one of the fractions as an equivalent fraction with the second fraction's denominator.

$$\frac{2}{3} + \frac{1}{6}$$
$$= \frac{4}{6} + \frac{1}{6}$$
$$= \frac{5}{6}$$

$$\frac{2}{5} + \frac{3}{10}$$
$$= \frac{4}{10} + \frac{3}{10}$$
$$= \frac{7}{10}$$

To add or subtract fractions, we must have fractions with the same denominator. If the denominators are the same, the size of the fraction parts is the same, and we can simply count how many of each part we have.

For example: $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$.

Adding or subtracting fractions with different, unrelated denominators requires some additional steps. Before we can add, we need to change both fractions into equivalent fractions with the same denominator. This new denominator will be a multiple of the denominators of the original fractions. The Lowest Common Denominator (LCD) is often the most efficient one to use.

For example: $\frac{1}{3} + \frac{2}{5}$ LCD = 15

$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$

Each fraction is now written in fifteenths, so we can count how many fifteenths there are:

$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

Estimating answers to fraction problems

To check whether your answer to a fraction addition or subtraction is 'reasonable', make an estimate of the answer before doing the calculation.

For example, the answer to $\frac{3}{5} + \frac{3}{7}$ will be approximately 1, as $\frac{3}{5}$ is slightly bigger than $\frac{1}{2}$ and $\frac{3}{7}$ is slightly smaller. (The actual answer is $\frac{36}{35}$ or $1\frac{1}{35}$, which is very close to 1.)

Worked Example 10

WE10

Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{2}{3} + \frac{3}{4}$ (b) $\frac{7}{10} - \frac{1}{5}$

Thinking		Working
(a) 1	Estimate the answer first. (Here, both fractions are greater than $\frac{1}{2}$ but less than 1. The answer should lie between 1 and 2.)	(a) Estimate: between 1 and 2
2	Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (In this case, 4 and 3.)	$\frac{2}{3} + \frac{3}{4}$ LCD = 12 $= \frac{8}{12} + \frac{9}{12}$
3	Add the numerators.	$= \frac{17}{12}$
4	Write the answer as a mixed number if appropriate.	$= 1\frac{5}{12}$
5	Check your answer against your estimate. Is it reasonable?	Reasonable

(b) 1	Estimate the answer first. (Here, we are subtracting $\frac{1}{5}$ from a fraction that is less than 1, so the answer will be about $\frac{1}{2}$.)	(b) Estimate: about $\frac{1}{2}$
2	Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (Here, only one fraction needs to be rewritten.)	$\frac{7}{10} - \frac{1}{5}$ LCD = 10 $= \frac{7}{10} - \frac{2}{10}$
3	Subtract the numerators.	$= \frac{5}{10}$
4	Simplify the answer.	$= \frac{1}{2}$
5	Check your answer against your estimate. Is it reasonable?	Exact

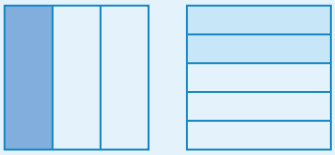
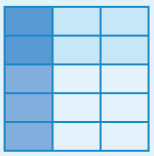
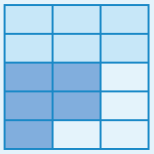
The grid method

The grid method is a visual way of adding fractions demonstrated in the following Worked Example (however, it does not work so well for subtracting fractions)

Worked Example 11

WE11

Add $\frac{1}{3}$ and $\frac{2}{5}$ using the grid method.

Thinking	Working
1 Draw two identical squares or rectangles. Divide them up and shade in each fraction.	$\frac{1}{3} + \frac{2}{5}$ 
2 Lay one square or rectangle on top of the other to form a grid.	
3 Where the grid squares are shaded twice due to overlap, shade an equal number of blank grid squares to 'remove' the overlap. The resultant shaded grid represents the fraction sum.	
4 Write the answer to the fraction sum.	$\frac{11}{15}$

Adding and subtracting mixed numbers

To add or subtract mixed numbers, we can either:

- add or subtract the whole number parts and fraction parts separately (Method 1), or
- convert the mixed numbers to improper fractions, then add or subtract them in the usual way (Method 2).

Worked Example 12 WE12

Calculate $4\frac{5}{6} + 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Method 1: Add wholes and fractions separately

Thinking	Working
1 Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are each bigger than $\frac{1}{2}$. The answer lies between 7 and 8.	Estimate: between 7 and 8 $4\frac{5}{6} + 2\frac{3}{4}$
2 Rearrange the addition by separating the whole numbers from the fractions.	$= 4 + 2 + \frac{5}{6} + \frac{3}{4}$
3 Add the whole numbers.	$= 6 + \frac{5}{6} + \frac{3}{4}$
4 Rewrite the fractions with the LCD as the denominator.	$= 6 + \frac{10}{12} + \frac{9}{12}$
5 Add the fractions and simplify if possible.	$= 6 + \frac{19}{12}$
6 If the fraction sum is an improper fraction, convert it to a mixed number.	$= 6 + 1\frac{7}{12}$
7 Add the whole numbers.	$= 7\frac{7}{12}$
8 Check your answer against your estimate. Is it reasonable?	Reasonable

Method 2: Use improper fractions

Thinking	Working
1 Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are bigger than $\frac{1}{2}$. The answer lies between 7 and 8.	Estimate: between 7 and 8 $4\frac{5}{6} + 2\frac{3}{4}$
2 Write the mixed numbers as improper fractions.	$= \frac{29}{6} + \frac{11}{4}$
3 Rewrite the fractions with the LCD as denominator.	$= \frac{58}{12} + \frac{33}{12}$
4 Add the numerators, simplify if possible.	$= \frac{91}{12}$
5 Convert to a mixed number.	$= 7\frac{7}{12}$

Worked Example 13 WE13

Calculate $6\frac{5}{8} - 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Method 1: Subtract wholes and fractions separately

Thinking	Working
1 Estimate the answer first. Here, we are subtracting $2\frac{3}{4}$ from a number less than $6\frac{3}{4}$, so the answer will be less than 4.	Estimate: less than 4 $6\frac{5}{8} - 2\frac{3}{4}$
2 Rearrange the subtraction by separating the whole numbers from the fractions.	$= 6 - 2 + \frac{5}{8} - \frac{3}{4}$
3 Subtract the whole numbers.	$= 4 + \frac{5}{8} - \frac{3}{4}$
4 Rewrite the fractions with the LCD as denominator.	$= 4 + \frac{5}{8} - \frac{6}{8}$
5 Subtract the fractions. If you cannot do the subtraction without getting a negative answer, 'borrow' 1 whole. (Here, 1 whole is $\frac{8}{8}$.)	$= 3 + \frac{8}{8} + \frac{5}{8} - \frac{6}{8}$ $= 3 + \frac{13}{8} - \frac{6}{8}$
6 Write the answer and simplify it if possible.	$= 3\frac{7}{8}$
7 Check your answer against your estimate. Is it reasonable?	Reasonable

Method 2: Use improper fractions

Thinking	Working
1 Make an estimate of the answer first, as for Method 1.	Estimate: less than 4 $6\frac{5}{8} - 2\frac{3}{4}$
2 Write the mixed numbers as improper fractions.	$= \frac{53}{8} - \frac{11}{4}$
3 Rewrite the fractions with the LCD as the denominator (only one fraction needs to be rewritten here).	$= \frac{53}{8} - \frac{22}{8}$ LCD = 8
4 Subtract the numerators.	$= \frac{31}{8}$
5 Simplify, if possible, and write your answer as a mixed number.	$= 3\frac{7}{8}$
6 Check your answer against your estimate. Is it reasonable?	Reasonable

Working with fractions on a calculator

The key that can be used for entering fractions into your calculator will often look something like this $\square\!\!\square$. To enter a fraction, press $\square\!\!\square$, then key in the numerator and the denominator, using the cursor keys to move between them. To enter a mixed number, press SHIFT $\square\!\!\square$.

On some calculators, the key looks like this: $a\,{}^{b}\!/_{c}$, or this ${}^{a}\!/_{b}$. To enter a fraction, such as $\frac{5}{6}$, type 5 $a\,{}^{b}\!/_{c}$ 6. You may need to press = for the fraction to be displayed on the screen. To enter a mixed number, such as $4\frac{3}{8}$, press 4 $a\,{}^{b}\!/_{c}$ 3 $a\,{}^{b}\!/_{c}$ 8.

The screen will use symbols such as ⌐, ⌙, or _ to separate the whole numbers, numerator and denominator.

Check that you can work with fractions correctly on your calculator by entering some additions and subtractions for which you already know the answers. For example, if you enter $\frac{3}{8} + 1\frac{2}{3}$ correctly, the answer of $2\frac{1}{24}$ will appear on the screen.

If you want to convert a mixed number that you have already entered to an improper fraction, press SHIFT and the $a\,{}^{b}\!/_{c}$ key, or SHIFT and the S⇔D key.

You might need to press = to see the result.

You will also notice that pressing the S⇔D key, or pressing the $a\,{}^{b}\!/_{c}$ key twice converts a fraction to its decimal form.

3.4 Adding and subtracting fractions

Navigator

Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q5 Column 1, Q6, Q7, Q8, Q10, Q13, Q14, Q16, Q17	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17	Q1 Column 4, Q3 Columns 3 & 4,Q4 Columns 3 & 4, Q5 Column 3, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Answers page 627

Fluency

WE10

1 Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{5}{14} + \frac{1}{7}$ (b) $\frac{2}{5} - \frac{7}{20}$ (c) $\frac{5}{8} - \frac{7}{40}$ (d) $\frac{4}{55} + \frac{2}{11}$

(e) $\frac{5}{6} + \frac{1}{3}$ (f) $\frac{15}{49} + \frac{5}{7}$ (g) $\frac{3}{5} + \frac{13}{20}$ (h) $\frac{3}{4} + \frac{1}{5}$

(i) $\frac{1}{4} - \frac{1}{6}$ (j) $\frac{5}{9} + \frac{3}{4}$ (k) $\frac{1}{6} + \frac{3}{7}$ (l) $\frac{11}{15} + \frac{7}{20}$

(m) $\frac{11}{12} + \frac{3}{10}$ (n) $\frac{23}{30} - \frac{7}{20}$ (o) $\frac{3}{5} + \frac{9}{14}$ (p) $\frac{41}{45} - \frac{11}{18}$

WE11

2 Complete these fraction sums by using the grid method.

(a) $\frac{2}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{1}{7}$

WE12

3 Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{1}{7}+4\frac{2}{7}$ (b) $8\frac{1}{9}+5\frac{5}{9}$ (c) $2\frac{3}{8}+7\frac{1}{4}$ (d) $6\frac{7}{10}+1\frac{1}{5}$

(e) $1\frac{3}{4}+1\frac{1}{2}$ (f) $3\frac{1}{5}+2\frac{9}{10}$ (g) $4\frac{1}{2}+2\frac{3}{4}$ (h) $1\frac{7}{12}+7\frac{2}{3}$

(i) $6\frac{4}{5}+5\frac{1}{2}$ (j) $3\frac{8}{9}+7\frac{1}{6}$ (k) $1\frac{4}{11}+3\frac{2}{5}$ (l) $1\frac{4}{25}+2\frac{3}{20}$

WE13

4 Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{5}{7}-2\frac{1}{7}$ (b) $3\frac{2}{9}-2\frac{5}{9}$ (c) $5\frac{7}{9}-1\frac{1}{3}$ (d) $7\frac{11}{12}-6\frac{1}{4}$

(e) $3\frac{1}{12}-1\frac{5}{6}$ (f) $4\frac{7}{10}-2\frac{1}{5}$ (g) $4\frac{1}{10}-2\frac{4}{5}$ (h) $6\frac{1}{18}-4\frac{1}{6}$

(i) $2\frac{1}{9}-1\frac{1}{6}$ (j) $2\frac{1}{4}-1\frac{2}{5}$ (k) $2\frac{1}{8}-1\frac{1}{6}$ (l) $5\frac{1}{8}-2\frac{2}{3}$

Did you know?
5 out of 4 people don't understand jokes about fractions!

5 Find the following. You might like to use your calculator to check your answers.

(a) $\frac{3}{8}+\frac{1}{6}+\frac{1}{3}$ (b) $\frac{1}{2}+\frac{3}{4}-\frac{1}{3}$ (c) $\frac{7}{10}+\frac{4}{5}-\frac{1}{2}$

(d) $1\frac{1}{2}+\frac{2}{5}+2\frac{3}{10}$ (e) $2\frac{1}{3}+5\frac{3}{4}-1\frac{2}{5}$ (f) $3\frac{1}{4}+2\frac{1}{5}-3\frac{1}{3}$

6 (a) $\frac{5}{8}-\frac{1}{3}=$

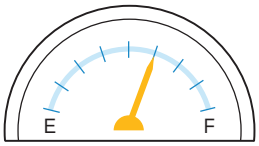
A $\frac{4}{24}$ B $\frac{7}{24}$ C $\frac{4}{5}$ D $\frac{23}{24}$

(b) $1\frac{2}{3}+\frac{3}{20}=$

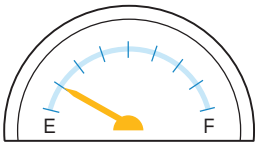
A $1\frac{1}{17}$ B $1\frac{5}{23}$ C $1\frac{5}{20}$ D $1\frac{49}{60}$

Understanding

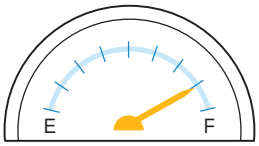
7 When Eric got in his car and started driving, his fuel gauge looked like this:



Eric stopped to get petrol when his fuel gauge looked like this:



After putting some petrol in his car, Eric's fuel gauge looked like this:



Write a fraction addition or a subtraction to answer the following.

(a) What fraction of the tank of fuel did Eric use between the first two gauge readings? Write your answer in simplest form.

(b) What fraction of the tank of petrol did Eric add when he was at the petrol station? Write your answer in simplest terms.

8 What fraction of a large pizza was eaten if Steven had $\frac{5}{12}$ of the pizza and Lara had $\frac{1}{3}$ of it? Write your answer in simplest form.

9 Jaydeep, Corey and Isaac were sharing the driving on a trip along the coast. If Jaydeep drove $\frac{3}{5}$ of the distance and Isaac drove $\frac{3}{20}$, what fraction of the total distance did Corey drive?

10 Belinda worked part-time at a fast food restaurant. Her hours for the three days she worked one week were: $2\frac{1}{4}$, $3\frac{1}{2}$ and $4\frac{1}{3}$. Calculate the total hours she worked for the week.

11 Five large packets of cereal were bought for a Year 7 camp. On the first day $1\frac{1}{3}$ packets were eaten, on the second day $\frac{3}{4}$ of a packet was eaten and on the third day $2\frac{1}{6}$ packets were eaten. How much cereal was available for the fourth day?

Reasoning

12 A fraction is added to $\frac{3}{5}$. The lowest common denominator for the two fractions is 40. Which of the following fractions was added?

A $\frac{2}{6}$ B $\frac{3}{8}$ C $\frac{13}{20}$ D $\frac{11}{15}$

13 A fraction is added to $\frac{3}{10}$. Which of the following would give an improper fraction as the result?

A $\frac{7}{15}$ B $\frac{7}{12}$ C $\frac{3}{5}$ D $\frac{18}{25}$

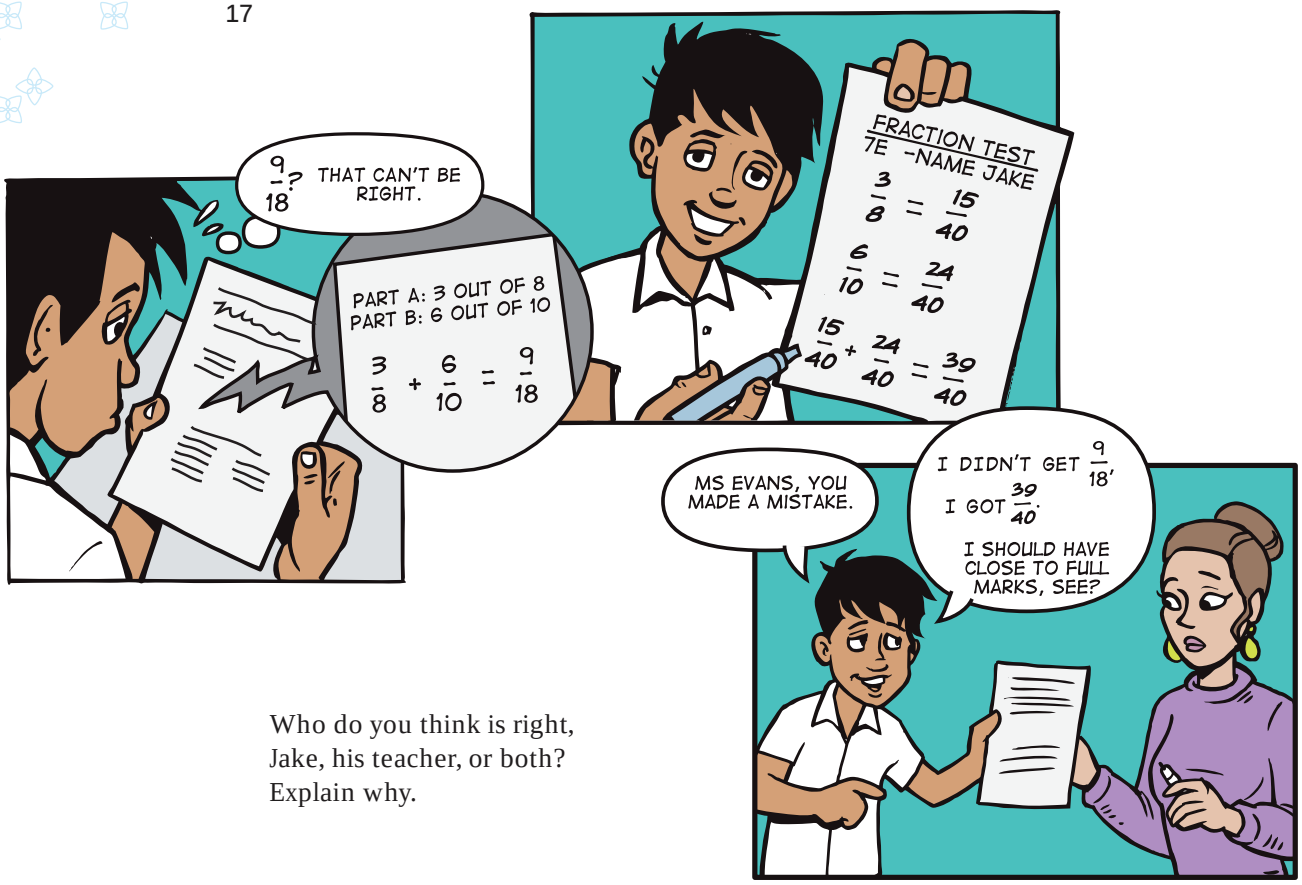
Open-ended

14 This question appeared on a Year 7 maths test: 'What whole number is $\frac{7}{8} + \frac{12}{13}$ approximately equal to?' Several students answered 19 or 21, both of which are incorrect. How might the students have arrived at their incorrect answers? Describe how you could use estimation to arrive at the correct answer.

15 Find two fractions that add to $\frac{3}{4}$ and do not have a denominator of 4.

16 Use a diagram to help you explain why $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$, not $\frac{7}{20}$. Why do we add the numerators, but not the denominators?

17



Who do you think is right, Jake, his teacher, or both? Explain why.

Outside the Square Game

Race to 10

Equipment required: 2 brains, 2 dice

How to win:

The aim is to be the person to get to a total as close to 10 as possible. Beware, because if you go over 10 you are out!

How to play:

On your turn, roll both dice. Choose one number to represent the numerator of a fraction and the other to represent the denominator. It is your choice whether you make a proper or an improper fraction.

For example: Die A rolls 5, Die B rolls 4, so the fraction is either $\frac{5}{4}$ or $\frac{4}{5}$.

Take it in turns to roll and keep a running total by adding your fractions as you go. When your total is as close to 10 as you dare, say 'I'll sit' when it comes to your turn. If you choose to roll and your total goes over 10, you are out.

Play the best of three games

Half-time 3

1 There are 15 cookies in a bag. Ex 3.1

(a) If I eat 7 cookies, what fraction of the total cookies is this?

(b) If I eat $\frac{2}{5}$ of the cookies in the bag, how many cookies do I eat?

2 Simplify the following. Ex 3.2

(a) $\frac{18}{48}$ (b) $\frac{9}{36}$ (c) $3\frac{9}{12}$ (d) $7\frac{35}{50}$

3 Convert the following improper fractions to whole numbers or mixed numbers. Ex 3.2

(a) $\frac{27}{4}$ (b) $\frac{40}{8}$ (c) $\frac{50}{10}$ (d) $\frac{37}{7}$

4 Determine which fraction in each of these pairs is the bigger number, and write a < or > sign between them. If they are equivalent, write a '=' sign between them. Ex 3.3

(a) $\frac{5}{8}$ ____ $\frac{2}{3}$ (b) $\frac{4}{9}$ ____ $\frac{15}{27}$ (c) $\frac{48}{56}$ ____ $\frac{6}{7}$ (d) $\frac{11}{5}$ ____ $\frac{9}{4}$

5 Draw a number line from 0 to 2. Divide the bottom side of the number line into quarters and the top side into eighths. Ex 3.1, 3.2, 3.3

(a) Show the positions of these fractions with a labelled arrow: $\frac{3}{4}, \frac{5}{8}, \frac{5}{4}, \frac{9}{8}, \frac{7}{4}, \frac{14}{8}$.

(b) Identify five pairs of equivalent fractions that are shown on the number line.

(c) Use the number line to determine which fraction out of the following pairs is larger:

(i) $\frac{5}{4}$ or $\frac{9}{8}$ (ii) $\frac{11}{8}$ or $\frac{7}{4}$.

6 Calculate the following, writing your answers in simplest form. Ex 3.4

(a) $\frac{3}{8} + \frac{3}{4}$ (b) $\frac{7}{15} - \frac{3}{10}$ (c) $\frac{7}{9} + \frac{5}{6}$ (d) $\frac{11}{18} - \frac{5}{12}$

7 Place the following in ascending order by first locating them on a number line. Ex 3.3

$\frac{1}{3}, \frac{6}{3}, \frac{9}{4}, \frac{3}{4}, 2\frac{2}{3}$

8 7 pizzas were each sliced into 6 equal pieces. Ex 3.2

(a) If $4\frac{5}{6}$ of the pizzas were eaten, how many slices was that?

(b) If $3\frac{1}{2}$ of the pizzas were eaten, how many slices were left over?

9 Calculate the following. Ex 3.4

(a) $2\frac{7}{12} + 2\frac{1}{10}$ (b) $1\frac{16}{25} + 2\frac{13}{20}$ (c) $2\frac{13}{20} - 1\frac{3}{100}$ (d) $3\frac{9}{10} - 2\frac{8}{25}$

10 Dave is leading a group of 12 people on a bushwalk. He has brought 30 L of water to supply the group. Ex 3.1, 3.2

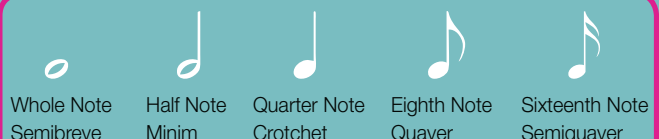
(a) How many litres does each group member receive? Write your answer as both an improper fraction and as a mixed number in simplest form. Ex 3.4

(b) One of the group members has drunk $\frac{3}{4}$ of a litre by lunchtime. How many litres do they have left?

Maths 4 Real

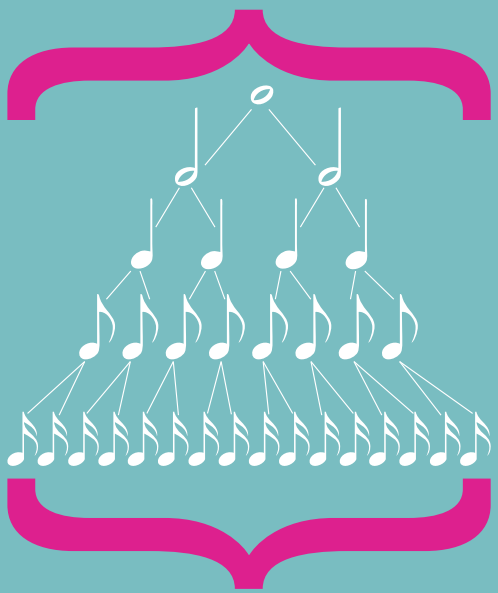
MUSICAL FRACTIONS

In western cultures, music is written using symbols called 'notes'. Music notes have different shapes that indicate sounds of different durations, that is, they tell the musician how long the sound should be played for. The note symbols and their names are:



Whole Note Semibreve	Half Note Minim	Quarter Note Crotchet	Eighth Note Quaver	Sixteenth Note Semiquaver

The note names are based on fractions, because music notes 'add' together in the same way fractions do. This tree diagram shows us how shorter notes add together to give longer notes.



From the diagram, we can see that:

2 half notes are equal in length to 1 whole note:	4 quarter notes are equal in length to 1 whole note:	2 quarter notes are equal in length to 1 half note:
$\frac{1}{2}+\frac{1}{2}=1$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$	$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Note that:

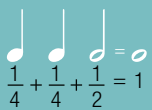
Eighth notes and sixteenth notes are often joined in pairs, like this:

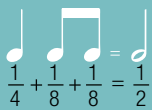
Or groups of four, like this:

1 Using the tree diagram and examples above, copy and complete the questions below by writing one equivalent note after the equal sign. Write the fraction sum underneath.

(a)  (b)  (c)  (d) 

We can also add different types of notes together, e.g.:


$\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$


$\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$

2 Copy and complete the questions below by writing one note in the space. Write the fraction sums underneath.

(a) ♩♫♫ = □ **(b)** ♫♪♩ = □

(c) □♫♫♬♬ = 𝅗𝅥 **(d)** ♪□♩ = 𝅗𝅥

3 Copy and complete the questions below by writing 2 or more different notes in the spaces (there are several ways this can be done).

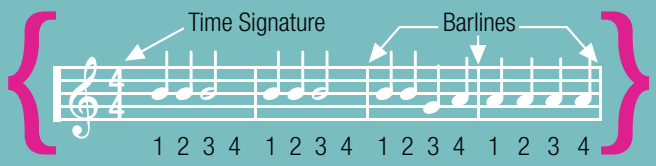
(a) ♫♫♫ = □ **(b)** ♩𝅗𝅥 = ♬□

(c) ♩♫♫□ = 𝅗𝅥𝅗𝅥 **(d)** ♩♩♩□ = 𝅝

Time Signatures

Music is written on 5 parallel lines called a 'stave' and in sections called 'bars'. Each bar contains the same number of 'beats', or counts. 'Barlines' are drawn across the lines of the stave. You can see 4 bars and barlines in the piece of music below.

The 'time signature' is placed at the beginning of the first bar of a piece of music, and looks like a fraction. The top number tells us the number of beats in each bar, while the bottom number tells us what type of note counts as one beat.



A common time signature is $\frac{4}{4}$. There are 4 beats in each bar, and each beat is worth 1 quarter note. In $\frac{4}{4}$ the note values in each bar must add up to 4 quarter notes (1 whole note).

4 (a) Copy these bars, then use quarter notes or half notes to fill in the missing beats. Check that the fraction sum for each bar adds up to $\frac{4}{4}$.

(b) Copy these bars, then use any combination of half, quarter, eighth or sixteenth notes to fill in the missing beats. Make sure you check the fraction sum of each bar.

5 Write your own piece of music in the four bars below. Use any number or combination of notes, just make sure that every bar 'adds up' to 4 quarter notes.

Research

Find out about the music notation of other cultures, such as traditional Chinese or Indian music. Write some simple questions or exercises like the ones in this task to teach others what you have learnt.

Investigation

Tangram teaser

Equipment required: 1 brain, 3A Tangram template, scissors

The tangram is an ancient Chinese puzzle. It has seven pieces that can be arranged to make hundreds of shapes and patterns, including the large square you can see here.

The Big Question

How many different squares can be made by using various combinations of the seven tangram pieces? If the largest square is one whole, what are the fraction values of the smaller squares?

Engage

The tangram has seven individual pieces of five different shapes: two large triangles, two small triangles, one medium triangle, one small square and one parallelogram.

1 If the large square that is made up of all the pieces represents one whole, estimate the fraction that is represented by:

   (a) a large triangle

   (b) the medium triangle

   (c) a small triangle

   (d) the small square

   (e) the parallelogram.

2 Check your estimates by cutting out the parts on your tangram template and using them to visually show what fraction of the large square they cover.

3 Not all seven pieces must be used to make a square. A smaller square can be made using four pieces, for example:

By adding up the fractions represented by each piece, work out what fraction of the large seven-piece square is taken up by this four-piece square. Write your answer in simplest form.

Explore

4 Experiment with different numbers and combinations of pieces to make as many different squares as possible. Make sure you keep a visual record of each different square that you make. (Hint: There is more than one way of making the big, seven-piece square.)

Strategy options

- Draw a diagram.
- Make a table.
- Test all possible combinations.

Explain

5 Present accurate drawings of all the squares you have made, clearly showing the parts in each. Underneath your drawings, write a fraction sum that shows how each of the pieces add to give a fraction of the big, seven-piece square. How many different squares have you found?

Elaborate

6 Sometimes, the same four or five pieces can be used to make more than one square. How did you decide whether two squares were 'different' to each other?

7 Using your definition of 'different,' summarise your findings by answering the Big Question.

Evaluate

8 Consider how you worked on the investigation and the methods you used. Do you think you have the complete set of solutions to this problem? Explain your answer.

9 (a) Did you find this task challenging, or straightforward?

(b) What was the most difficult part of the task?

(c) Would you tackle puzzles like the tangram in your own time?

Extend

10 Try any or all of the following.

(a) Can you form a triangle using:

(i) two tangram pieces

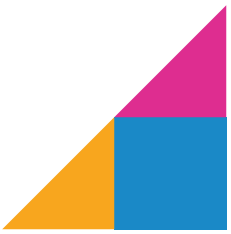
(ii) three tangram pieces

(iii) four tangram pieces

(iv) five tangram pieces

(v) six tangram pieces

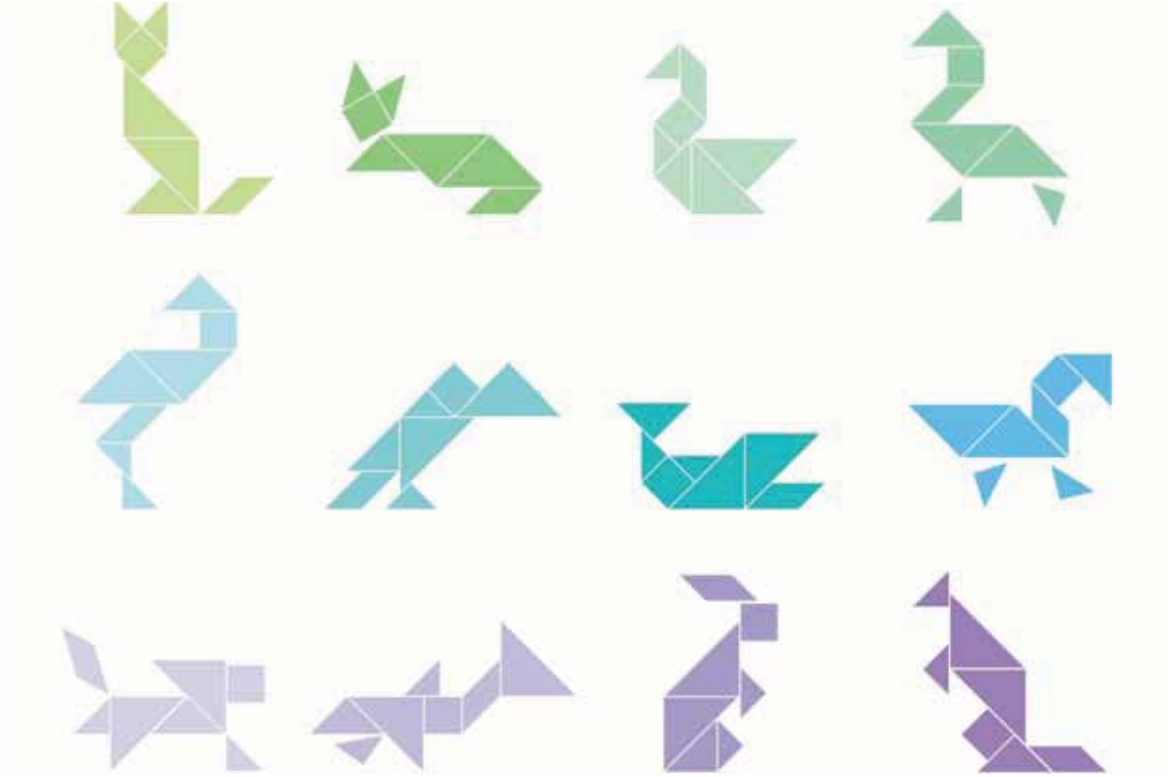
(vi) all seven tangram pieces?



Draw a diagram of each triangle formed and write a fraction sum to show what fraction of the large square each triangle represents.

(b) Use the tangram pieces to make one or both of your initials.

(c) Use the tangram designs below as inspiration to make your own tangram designs.



3.5 Multiplying fractions

The multiplication sign, ×, means '*lots of*', or simply '*of*'.
For example, 8 lots of \$5 = 8 × \$5 = \$40.

Finding a fraction of a whole amount

We know that three-quarters of an hour is 45 minutes. $\frac{3}{4}$ of 60 = 45.

We can show how to calculate this in two ways:

1 We could find one-quarter of an hour, then multiply by three.

$\frac{1}{4}$ of 60 = 60 ÷ 4 = 15

$\frac{3}{4}$ of 60 = 3 × 15 = 45

This method is called the 'Unit fraction method'.
A unit fraction is a fraction with a numerator of 1.
In the above example, the unit fraction is $\frac{1}{4}$.

(This method works well if the denominator of the fraction is a factor of the whole number. This makes the division step straightforward.)

2 We could replace the word 'of' with the multiplication sign, ×, and multiply:

$$\frac{3}{4} \text{ of } 60 = \frac{3 \times 60}{4 \times 1} \quad \text{(writing 60 as an improper fraction with a denominator of 1)}$$

$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{60}^{15}}{1} \quad \text{(multiplying the numerators together and the denominators together, cancelling common factors first)}$$

$$= \frac{45}{1}$$

$$= 45$$

To find a fraction of a whole number, either:

- find the unit fraction (by dividing by the denominator), then multiply by the numerator

or

- write the whole number as an improper fraction and multiply the numerators together and the denominators together. Cancel any common factors between numerators and denominators before multiplying (this avoids having to work with large numbers).

Worked Example 14 WE14

Find $\frac{2}{5}$ of 40.

Method 1: Find the unit fraction, then multiply

Thinking	Working
1 Find the unit fraction (in this case, $\frac{1}{5}$) by dividing the whole number by the denominator.	$40 \div 5 = 8$
2 Multiply the unit fraction amount by the numerator.	$2 \times 8 = 16$
3 Write the answer.	$\frac{2}{5}$ of $40 = 16$

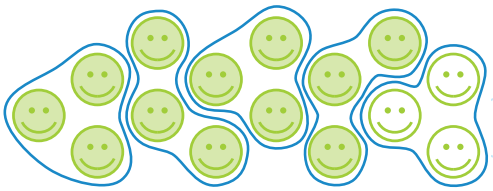
Method 2: Multiply two fractions

Thinking	Working
1 Replace 'of' with '×' and write the whole number as an improper fraction with a denominator of 1.	$\frac{2}{5}$ of 40 $= \frac{2}{5} \times \frac{40}{1}$
2 Multiply the numerators and denominators together.	$= \frac{80}{5}$
3 Simplify if possible (80 ÷ 5).	$= 16$

Finding a fraction of a fraction

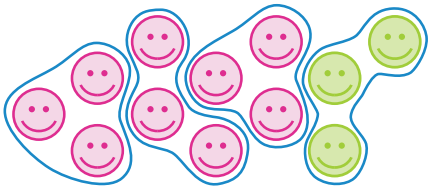
Our 'smiley collection' from page 107 can help us understand what it means to find a fraction of a fraction.

In Worked Example 1, we found $\frac{4}{5}$ of 15 by dividing the collection into five equal groups and counting how many smileys were in four of those groups.



$\frac{4}{5}$ of $15 = 12$

If we wanted to find $\frac{3}{4}$ of $\frac{4}{5}$, we would take the 12 smileys *as our new whole*, ignoring the fifth group. Finding $\frac{3}{4}$ of this new whole means taking three of the four groups, or nine smileys.



$\frac{3}{4}$ of $12 = 9$

These nine of the 15 original smileys represent $\frac{3}{4}$ of $\frac{4}{5}$.

$\frac{3}{4}$ of $\frac{4}{5}$ of $15 = 9$

They also represent $\frac{3}{5}$ of the original collection: $\frac{3}{5}$ of $15 = 9$.

The following Worked Example shows how multiplying $\frac{3}{4}$ and $\frac{4}{5}$ gives $\frac{3}{5}$.

Worked Example 15 WE15

Find $\frac{3}{4}$ of $\frac{4}{5}$.

Thinking	Working
1 Change the 'of' to $\times$.	$\frac{3}{4}$ of $\frac{4}{5}$
2 Multiply the numerators and denominators together, cancelling common factors before multiplying. (Here, the common factor is 4.)	$= \frac{3}{\cancel{4}_1} \times \frac{\cancel{4}^1}{5}$ $= \frac{3}{5}$

To find a fraction of a fraction, multiply the numerators together and the denominators together, then simplify the answer. Mixed numbers should be written as improper fractions before multiplying.

Worked Example 16 WE16

Find $4 \times 2\frac{3}{20}$. Write your answer as a mixed number, if appropriate.

Thinking	Working
1 Write both numbers as improper fractions. (Whole numbers can be written as fractions with a denominator of 1.)	$4 \times 2\frac{3}{20}$ $= \frac{4}{1} \times \frac{43}{20}$
2 Cancel any common factors between numerators and denominators. (Here, the common factor is 4.)	$= \frac{\cancel{4}^1}{1} \times \frac{43}{\cancel{20}_5}$
3 Multiply the simplified numerator and the denominator.	$= \frac{1 \times 43}{1 \times 5} = \frac{43}{5}$
4 Write the answer as a mixed number.	$= 8\frac{3}{5}$

3.5 Multiplying fractions

Navigator

Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Column 1, Q5, Q6, Q8, Q10, Q11, Q13, Q14, Q16, Q19, Q20	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q4 Column 2, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q19, Q20	Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 3, Q5, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

Answers page 628

Fluency

1 Find the following.

WE14

(a) $\frac{1}{3}$ of 18 (b) $\frac{1}{4}$ of 12 (c) $\frac{1}{6}$ of 18 (d) $\frac{1}{9}$ of 36

(e) $\frac{3}{4}$ of 28 (f) $\frac{4}{5}$ of 30 (g) $\frac{6}{7}$ of 56 (h) $\frac{3}{8}$ of 72

(i) $\frac{3}{2}$ of 20 (j) $\frac{6}{5}$ of 100 (k) $\frac{11}{10}$ of 50 (l) $\frac{5}{4}$ of 60

2 Find the following.

WE15

(a) $\frac{1}{2}$ of $\frac{2}{13}$ (b) $\frac{2}{9} \times \frac{1}{2}$ (c) $\frac{4}{5} \times \frac{5}{9}$ (d) $\frac{3}{7} \times \frac{7}{13}$

(e) $\frac{5}{7} \times \frac{1}{10}$ (f) $\frac{3}{5}$ of $\frac{10}{11}$ (g) $\frac{5}{12}$ of $\frac{6}{7}$ (h) $\frac{5}{6}$ of $\frac{1}{11}$

(i) $\frac{6}{7} \times \frac{3}{8}$ (j) $\frac{9}{10} \times \frac{5}{6}$ (k) $\frac{4}{7}$ of $\frac{2}{3}$ (l) $\frac{3}{10} \times \frac{9}{25}$

3 Find the following. Write your answers as mixed numbers, if appropriate.

WE16

(a) $3 \times \frac{2}{9}$ (b) $\frac{3}{4} \times 8$ (c) $\frac{5}{8} \times 2$ (d) $3 \times \frac{5}{12}$

(e) $5 \times 2\frac{3}{10}$ (f) $8 \times 1\frac{3}{4}$ (g) $1\frac{4}{5} \times 15$ (h) $5\frac{1}{12} \times 8$

(i) $5\frac{1}{3} \times \frac{3}{8}$ (j) $3\frac{1}{2} \times 3\frac{1}{5}$ (k) $4\frac{1}{5} \times 4\frac{2}{7}$ (l) $1\frac{1}{8} \times 3\frac{5}{9}$

When cancelling common factors, don't cancel out two things on the same line—cancel something on the bottom with something on the top.

4 Calculate the following, giving your answers in simplest form.

(a) $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ (b) $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ (c) $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$

(d) $1\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ (e) $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ (f) $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$

5 (a) $\frac{2}{5}$ of $2\frac{1}{2}$ cups of sugar is:

A $\frac{1}{5}$ cup B $\frac{4}{10}$ cup C 1 cup D $2\frac{1}{5}$ cups

(b) $\frac{1}{5}$ of $\frac{1}{3}$ of 15 is:

A 1 B 2 C 3 D 5

Understanding

6 A group of 7 workers shared third prize in a local raffle. The prize was worth $2170.

(a) What fraction of the prize money does each worker receive?

(b) How much money does each worker get?

7 The proceeds of a $48 000 inheritance were to be shared between four cousins. Each cousin was to receive a specific fraction of the inheritance. Michael was to receive $\frac{5}{24}$, Marcus $\frac{1}{12}$, Lily $\frac{3}{8}$ and Julie $\frac{1}{3}$. How much money did each cousin receive?

8 A chocolate bar was made up of 45 equal pieces. Ling had to share it equally with her brother and sister, Gao and Chen.

(a) How many pieces did each person receive? Copy this chocolate block into your book and divide it up to show each person's share.

(b) Ling then decided to divide her share of chocolate equally between herself and four of her friends.

(i) How many pieces did each person receive?

(ii) What fraction of the original block is this?

(iii) On your drawing of the chocolate bar show the fraction that Ling and her friends receive.

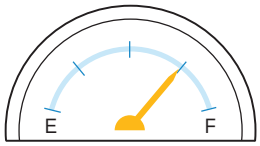
9 Annika cut one-quarter of an apple pie in halves to share with her brother Arjun. What fraction of the original, whole pie do they each now have?

10 Draw a rectangle that is 5 cm long and 4 cm wide. Divide it lengthways into fifths. Shade in $\frac{1}{5}$. Now, take the shaded section and divide it into quarters. Shade in $\frac{3}{4}$ of this section.

(a) What fraction of the original rectangle have you shaded twice?

(b) Write the fraction multiplication that is shown by your diagram.

11 A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.



(a) How much petrol has been used?

(b) How much petrol is still in the tank?

12 Melissa surveyed 75 Year 7 students. $\frac{2}{3}$ said they enjoyed studying mathematics. $\frac{4}{5}$ of those who enjoyed mathematics also said they enjoyed studying science.

(a) How many students said they enjoyed mathematics?

(b) How many students said they enjoyed mathematics and science?

13 Angelina is making Custard Cream cookies. This is her recipe. It makes 24 cookies.

180 g butter

$\frac{1}{2}$ cup icing sugar

$1\frac{1}{4}$ cups self-raising flour

$\frac{1}{3}$ cup custard powder

Rewrite Angelina's recipe so that it makes 60 cookies. (Hint: What do you need to multiply 24 by to get 60?)

Reasoning

14 Without doing the calculation, which is larger: $1\frac{1}{2} \times 12$ or $\frac{1}{3} \times 9$?

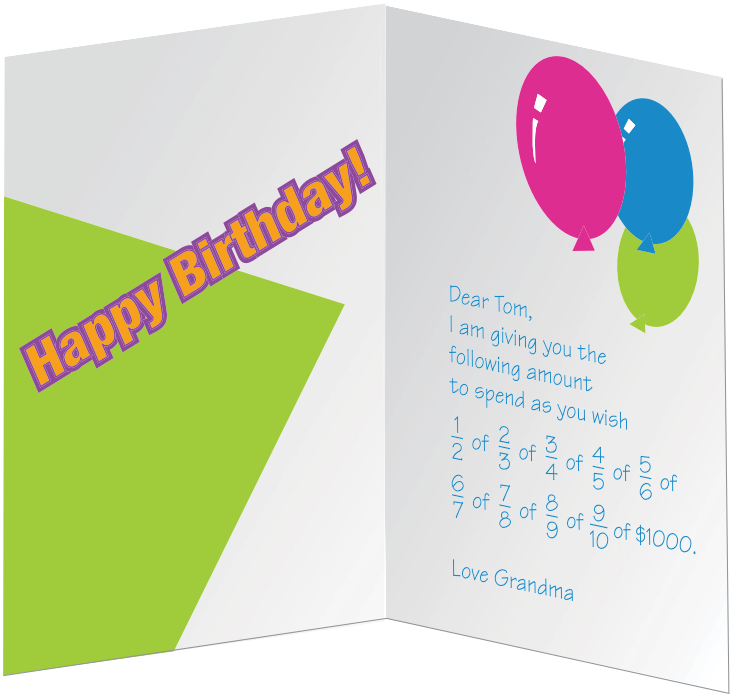
15 Without doing any calculations, say which of the following would give the biggest answer.

A $\frac{1}{2} \times \frac{3}{4}$ B $\frac{2}{5} \times \frac{2}{3}$ C $\frac{3}{4} \times \frac{6}{5}$ D $1\frac{1}{2} \times 1\frac{2}{5}$

16 The missing number in the statement $\frac{5}{12} \times \frac{\square}{10} = \frac{5}{8}$ is:

A 4 B 5 C 10 D 15

17 Tom was born into a strange mathematical family. His grandmother sent him a money voucher for his birthday. The card said:



How much birthday money did Tom receive?

Open-ended

18 Write two fractions that multiply to give $\frac{1}{6}$ as the simplified answer.

19 $\frac{1}{\square} \times 3\square = 1\square$

Fill each of the boxes with a single digit (0–9) to make the statement correct. There are six possible combinations. Try to find at least three of them.

20 Asher was asked to find $\frac{3}{5}$ of 60. His answer was 4. Here is his working:

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{\overset{20}{\cancel{60}}}{\cancel{1}} = \frac{20}{5} = 4$$

Asher knows his answer is not correct, because he knows that $\frac{3}{5}$ is bigger than one half, and 4 is nowhere near half of 60! Explain where Asher has gone wrong. Include the correct working and answer to the problem.

Outside the Square Game

Fringo!

Equipment required: 2 brains, 2 dice

How to win:
The winner is the first person to have four fractions in a row (horizontal, vertical or diagonal) crossed off their grid. When they do, they should call out 'Fringo!'

(HINT: You may realise after playing a few rounds that some answers come up more than others. Thinking about where the best places are to put these on the grid will increase your chance of winning. You could also investigate just how many ways the fractions on the list can be made.)

How to play:
Fringo is fraction bingo! Draw a 4×4 grid into your book. Fill in your grid with any 16 of the numbers, fractions and mixed numbers on this list:

1, 2, 3, 4, 5, 6, $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $1\frac{1}{3}$, $1\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $1\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $1\frac{1}{5}$, $\frac{1}{6}$, $\frac{5}{6}$

Take turns in rolling the dice one at a time. Make the number you roll first into the denominator of a unit fraction and write it down. For example, if you roll a 4, write down $\frac{1}{4}$. Multiply the number you roll second by this unit fraction, and write the answer in simplest form.

For example, Roll 1 = 4, Roll 2 = 2. Write down $\frac{1}{4} \times 2$ and calculate it (in this case, $\frac{2}{4}$, which simplifies to $\frac{1}{2}$).

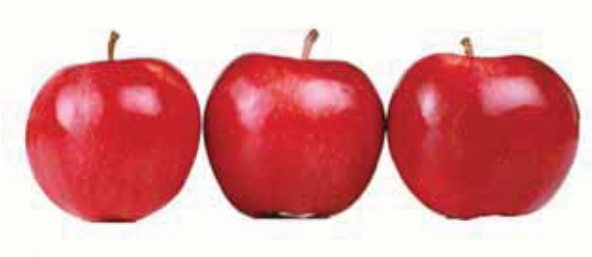
If you have the answer to the fraction multiplication in your grid, cross it off.

Dividing fractions

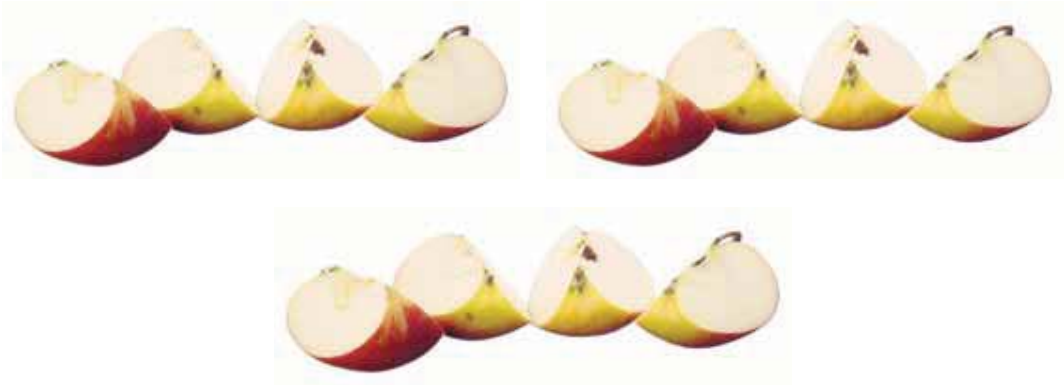
3.6

Whole numbers divided by fractions

Here are three whole apples.



If we cut each apple into quarters, how many apple quarters will there be? We can think of this as 'how many quarters in 3' and write it as $3 \div \frac{1}{4} = ?$



We have 4 quarters in each apple, so there are 12 quarters in 3 apples: $3 \div \frac{1}{4} = 12$.

If we cut the apples into fifths, we would have fifteen fifths: $3 \div \frac{1}{5} = 15$.

If we cut the apples into sixths, we would have eighteen sixths: $3 \div \frac{1}{6} = 18$.

We can get our answer by multiplying the whole number by the denominator of the fraction.

i.e. $3 \div \frac{1}{4}$	$3 \div \frac{1}{5}$	$3 \div \frac{1}{6}$
$= 3 \times \frac{4}{1}$	$= 3 \times \frac{5}{1}$	$= 3 \times \frac{6}{1}$
$= 12$	$= 15$	$= 18$

The fractions have been turned upside down, or 'inverted', to show they are being multiplied like whole numbers.

Inverting a fraction (turning it upside down) is also known as 'finding the inverse'.

The inverse of $\frac{1}{4}$ is $\frac{4}{1}$ or simply 4.

What about dividing by fractions where the numerator is not 1, such as $3 \div \frac{2}{4}$?

If we put our 12 apple quarters on the previous page into groups of 3 quarters, there will be 4 groups. Because $\frac{3}{4}$ is three times as big as $\frac{1}{4}$, the answer to $3 \div \frac{3}{4}$ is one-thrid as big as $3 \div \frac{1}{4}$.

We can achieve the same answer by multiplying 3 by $\frac{4}{2}$, the inverse of $\frac{2}{4}$.

$$3 \div \frac{3}{4}$$
$$= \frac{\overset{1}{\cancel{3}}}{1} \times \frac{4}{\underset{1}{\cancel{2}}}$$
$$= \frac{4}{1}$$
$$= 4$$

To divide a whole number by a fraction, multiply by the inverse of the fraction.

The result of a division calculation is called the 'quotient'.

Another way of dividing a fraction (or a whole number) by a fraction is to convert them both to the same type of fraction; that is, fractions with the same denominator (LCD). A common denominator means we have fractions of the same type, like having two measurements in the same units. Once we have a common denominator, we can perform the division on just the numerators. This is demonstrated in Method 2 of the Worked Example below.

Worked Example 17 WE17

Calculate $9 \div \frac{3}{8}$.

Method 1: Multiply by the inverse

Thinking	Working
1 Write the whole number as an improper fraction with a denominator of 1.	$\frac{9}{1} \div \frac{3}{8}$
2 Find the inverse of the second fraction (turn it upside down) and change the ÷ to ×.	$= \frac{9}{1} \times \frac{8}{3}$
3 Cancel any common factors between numerators and denominators. (Here, the common factor is 3.)	$= \frac{\overset{3}{\cancel{9}}}{1} \times \frac{8}{\underset{1}{\cancel{3}}}$
4 Multiply the simplified numerators and denominators.	$= \frac{3 \times 8}{1 \times 1} = \frac{24}{1}$
5 Write the answer.	$= 24$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Write the whole number as an improper fraction with the same denominator as the fraction (in this case, 8).	LCD = 8 $9 = \frac{72}{8}$
2 Rewrite the division using the two fractions.	$9 \div \frac{3}{8}$ $= \frac{72}{8} \div \frac{3}{8}$
3 Perform the division with just the numerators, and write the answer.	$= 72 \div 3$ $= 24$

Fractions divided by fractions

The 'invert the fraction you are dividing by and multiply' method also applies when dividing one fraction by another.

The 'use equivalent fractions with the LCD' method can also be used.

Convert mixed numbers to improper fractions before dividing.

Worked Example 18 WE18

Calculate $1\frac{1}{6} \div \frac{2}{3}$.

Method 1: Multiply by the inverse

Thinking	Working
1 If there are any mixed numbers, convert them to improper fractions.	$1\frac{1}{6} = \frac{7}{6}$ $\frac{7}{6} \div \frac{2}{3}$
2 Find the inverse of the second fraction (turn it upside down) and change the $\div$ to $\times$.	$\frac{7}{6} \times \frac{3}{2}$
3 Cancel common factors and perform the simplified multiplication.	$= \frac{7}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2}$ $= \frac{7 \times 1}{2 \times 2}$ $= \frac{7}{4}$
4 Write the answer as a mixed number.	$= 1\frac{3}{4}$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Find the LCD. Write the mixed number as an improper fraction with the LCD. Write the fraction as an equivalent fraction with the LCD.	LCD = 6 $1\frac{1}{6} = \frac{7}{6}$ $\frac{2}{3} = \frac{4}{6}$
2 Rewrite the division using the fractions with the LCD.	$1\frac{1}{16} \div \frac{2}{3}$ $= \frac{7}{6} \div \frac{4}{6}$
3 Perform the division with just the numerators, and write the answer.	$= 7 \div 4$ $= 1\frac{3}{4}$

3.6 Dividing fractions

Navigator

Answers page 628

Q1 Columns 1–3, Q2 Columns 1 & 2, Q4, Q5, Q6, Q8, Q9, Q12, Q15	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q15	Q1 Column 4, Q2 Column 4, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency

WE17

1 Calculate the following.

(a) $6 \div \frac{1}{4}$ (b) $2 \div \frac{1}{6}$ (c) $3 \div \frac{1}{7}$ (d) $4 \div \frac{1}{9}$

(e) $4 \div \frac{2}{9}$ (f) $4 \div \frac{4}{5}$ (g) $5 \div \frac{3}{7}$ (h) $7 \div \frac{4}{9}$

(i) $6 \div 1\frac{1}{8}$ (j) $8 \div 2\frac{2}{3}$ (k) $6 \div 3\frac{1}{5}$ (l) $3 \div 2\frac{1}{4}$

WE18

2 Calculate the following.

(a) $\frac{1}{2} \div \frac{1}{4}$ (b) $\frac{1}{4} \div \frac{1}{6}$ (c) $\frac{3}{4} \div \frac{2}{3}$ (d) $\frac{7}{10} \div \frac{1}{2}$

(e) $\frac{1}{3} \div 2$ (f) $\frac{1}{5} \div 4$ (g) $\frac{3}{4} \div 3$ (h) $\frac{9}{10} \div 6$

(i) $\frac{9}{10} \div 1\frac{3}{5}$ (j) $\frac{7}{6} \div 2\frac{1}{12}$ (k) $\frac{4}{5} \div 1\frac{8}{9}$ (l) $\frac{9}{10} \div 2\frac{2}{3}$

(m) $2\frac{2}{3} \div 1\frac{1}{3}$ (n) $5\frac{1}{2} \div 5\frac{3}{8}$ (o) $4\frac{1}{2} \div 1\frac{1}{5}$ (p) $4\frac{5}{7} \div 2\frac{3}{4}$

3 (a) $5 \div \frac{15}{8} =$

A $\frac{8}{75}$ B $\frac{3}{8}$ C $2\frac{2}{3}$ D $\frac{75}{8}$

(b) $\frac{16}{7} \div \frac{20}{49} =$

A $\frac{5}{28}$ B $\frac{320}{343}$ C $\frac{343}{320}$ D $5\frac{3}{5}$

4 Copy and complete the following. Do all working in your head.

(a) There are _____ halves in one, so $1 \div \frac{1}{2} =$ _____.

(b) There are _____ halves in three, so $3 \div \frac{1}{2} =$ _____.

(c) There are _____ quarters in one, so $1 \div \frac{1}{4} =$ _____.

(d) There are _____ quarters in two, so $2 \div \frac{1}{4} =$ _____.

5 The inverse of $\frac{1}{3}$ is:

A $\frac{1}{6}$ B $\frac{2}{3}$ C 1 D 3

Understanding

6 There is $\frac{5}{8}$ of a metre of ribbon in the textiles class cupboard. Students require $\frac{1}{10}$ of a metre each for their projects. How many students will be able to use the ribbon?

7 Scientists in a medical laboratory work in shifts of $3\frac{1}{2}$ hours each. How many whole shifts are to be worked during a period of 42 hours?

8 Hannah is a baker whose specialty is scones. She has a sack containing 80 cups of flour. Her scone recipe uses $2\frac{2}{3}$ of a cup of flour for each batch of scones. How many batches of scones will Hannah get out of her sack of flour?

9 On a 15 km Fun Run course, drinks stations are placed every $2\frac{1}{2}$ km from the start. How many drinks stations will there be on the course? (There are no drinks stations at the start or finish lines.)

10 The hit movie 'Exterminator 5' runs for $1\frac{3}{4}$ hours. A local cinema complex plans to screen it back to back (alternating the screenings between 2 cinemas). The first screening will start at 1:30 p.m. The last screening must finish by 11 p.m. How many times can the movie be screened within this time period?

Reasoning

11 The missing number from the statement $\frac{3}{4} \div \frac{\square}{8} = 2$ is:

A 1 B 2 C 3 D 4

12 Without doing a calculation, state which will give the larger number as a result:
$\frac{4}{5} \div \frac{1}{3}$, or $\frac{4}{5} \div \frac{2}{3}$?

13 Consider this sequence: $5 \div \frac{3}{4} = 6\frac{2}{3}$, $5 \div \frac{3}{5} = 8\frac{1}{3}$, $5 \div \frac{3}{6} = 10$, $5 \div \frac{3}{7} = 11\frac{2}{3}$.

(a) Why does the quotient (the answer to the division) increase as the denominator of the fractions in red is increased?

(b) Predict the answer to the next term in the sequence; i.e. $5 \div \frac{3}{8}$ will be ...

Open-ended

14 $4 \div \frac{2}{\square} = 1\square$

Use digits from 0 to 9 to fill in the boxes to make a correct statement. Find at least two different combinations.

15 Here is Gabi's working on a fraction division problem:

$$3 \div \frac{1}{4} = 3 + 4 = \frac{3}{4}$$

Her friend Natalie has worked it out differently. This is what Natalie wrote:

$$3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$$

'That can't be right', said Gabi—12 is way too big to be the answer!

Which of the girls' working is correct? Explain what the other has done wrong.

Outside the Square — Problem solving

Fill in the blanks

1 Fill in the blanks with positive whole numbers to make mathematically true statements. Do not use the same number twice within a statement. See if you can come up with at least two different combinations.

(a) $\frac{\blacklozenge}{4} + \frac{1}{\blacklozenge} = \frac{\blacklozenge}{20}$ (b) $\frac{\bigstar}{\bigstar} - \frac{\bigstar}{6} = \frac{\bigstar}{12}$

2 Use four of the digits 1, 3, 4, 5, 6, 7 to make two fractions whose sum is close to but less than 1.

$\frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} < 1$

Try to come up with at least three combinations. Which combination is the closest to 1?

Strategy options

- Guess and check.
- Test all possible combinations

Technology Exploration Word

Equipment required: 1 brain, Microsoft Word 2010 (with free Mathematics add-in) or CAS/scientific calculator

Versions of this Exploration are available for other technologies

The evil genie and the curse of the infinite fraction

Once upon a time, there were two ladybirds that were very much in love. One day in an antique store, one of them crawled onto a lamp. With a puff of smoke, a genie appeared. The genie roared "Ha, ha, ha, I am an evil genie. You are currently 1 metre apart. For waking me up I curse you with the following rules: You will only be able to move once per day towards your love, and each day you will only be able to travel half of the distance remaining between you and your love. Your love may not step towards you". The genie then disappeared.

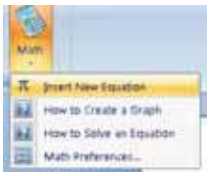
Were the loving ladybirds ever able to be together again?

On the first day, one of the ladybirds could travel half a metre forward, then on the next day a quarter of a metre, then an eighth of a metre on the third day.

The total distance travelled by the ladybird in the first 3 days is represented by the following addition: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$.

1 Enter the above expression into Word 2010.

Step 1: Select Math from the Add-Ins menu. A drop-down menu will appear; select 'Insert New Equation'.



Step 2: Select the Fraction template from the Design ('Equation Tools') menu and enter the necessary fractions.

Step 3: To calculate the answer, select Math> Calculate, and the answer should appear.

2 Explore the distance travelled by the ladybird by copying and pasting the previous fraction sum, and adding the next fraction in the series.

(a) What distance does it travel in:

(i) the first 5 days

(ii) the first 7 days

(iii) the first 11 days?

(b) At this stage, does it look like the distance between the ladybirds will ever be zero?

3 Consider the distances travelled over the first 3, 5, 7 and 11 days. What value are your results getting closer to?

4 To help you visualise the pattern, you can construct a diagram.

Draw a long rectangle to represent the distance between the ladybirds.

Shade in half of the rectangle to represent the distance travelled on the first day, and label it with $\frac{1}{2}$.

Shade in half of the unshaded portion of the rectangle to represent the distance travelled on the second day. Label this $\frac{1}{4}$.

Continue to shade the rectangle according to the pattern.

Continue this process as far as you can.

5 From your diagram, you can see that you are getting closer and closer to shading in the whole rectangle. If you were able to zoom in and continue shading forever, would you ever be able to shade the entire rectangle? Explain your answer.

6 Will the distance between the ladybirds ever be zero? Explain why or why not.

7 Would the situation be different if the ladybirds had started closer together or further apart? Use an example to help you explain.

You considered the problem by calculating distances (in questions 1–3) and the same problem as a diagram (in questions 4, 5). Which of these methods gave you a better 'sense' of the problem? Explain why.

Taking it further

8 When you add together the numbers in a pattern such as this, it is called a series. As you add together more and more terms, you may notice that the series gets closer and closer to a particular value. Series like these are called 'convergent series'. Sometimes, when you add together more and more terms, you may notice that the value just gets larger and larger. Series like these are called 'divergent series'.

(a) For the ladybird problem, did it seem to be a convergent or a divergent series?

(b) For each of the following series:

write down the next two terms in the series.

Use the Word Mathematics add-in to help you calculate the sum of the series for as many terms as you can.

State whether you think the following series will be convergent or divergent. Explain your answer.

(i) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\ldots$

(ii) $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\frac{1}{54}\ldots$

(iii) $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\ldots$

(iv) $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}\ldots$

(c) In English, what does it mean when people say there was 'divergent opinion' about something? How is this similar to what you know about divergent series?

Mixed fraction problems

3.7

Fractions and the order of operations

The order of operations rules that apply to whole numbers also apply to fractions.

The first step in any calculation is to complete any operation within brackets. The next step is to do any multiplication or division as you come to it, working from left to right. Then, do addition or subtraction as you come to it, working from left to right.

Worked Example 19

WE19

Simplify the following using the correct order of operations.

(a) $3+\frac{2}{5}\times\frac{1}{2}$ (b) $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$

Thinking	Working
(a) 1 As there are no brackets, do the multiplication first.	(a) $3+\frac{2}{5}\times\frac{1}{2}$ $=3+\frac{{}^{1}\cancel{2}\times 1}{5\times{}^{1}\cancel{2}}$ $=3+\frac{1}{5}$
2 Do the addition and state your answer in simplest form.	$=3\frac{1}{5}$
(b) 1 Perform the operation in the brackets first.	(b) $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\left(\frac{5}{3}+\frac{1}{4}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\left(\frac{20}{12}+\frac{3}{12}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\frac{23}{12}-\frac{3}{7}$
2 Next, perform any multiplication or division, cancelling any common factors.	$=\frac{{}^{1}\cancel{6}\times 23}{7\times{}^{2}\cancel{12}}-\frac{3}{7}$ $=\frac{23}{14}-\frac{3}{7}$
3 Finally, perform any addition or subtraction. Write the answer in simplest form.	$=\frac{23}{14}-\frac{6}{14}$ $=\frac{17}{14}=1\frac{3}{14}$

3.7 Mixed fraction problems

Navigator

Answers page 629

Q1 Column 1, Q2 (a), Q3 (a) & (b), Q4, Q5, Q6, Q11, Q14	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q13, Q15	Q1 Columns 2 & 3, Q2, Q3, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15

Fluency

WE19

1 Simplify the following using the correct order of operations.

(a) $2+\frac{3}{4}\times\frac{1}{3}$ (b) $5-\frac{7}{8}\times\frac{1}{7}$ (c) $\frac{5}{11}\div(5+4)$

(d) $\frac{3}{4}\times\left(\frac{1}{3}+\frac{1}{6}\right)\times\frac{1}{2}$ (e) $\frac{7}{8}\times\left(\frac{9}{14}-\frac{2}{7}\right)$ (f) $\frac{5}{6}\times\left(\frac{2}{5}+\frac{1}{3}\right)$

(g) $1\frac{5}{8}+\left(1\frac{3}{4}-1\frac{1}{8}\right)\times 5$ (h) $2\frac{3}{4}+\left(4\frac{1}{2}-3\frac{1}{4}\right)\times\frac{1}{2}$ (i) $1\frac{3}{5}+\left(3\frac{3}{10}-1\frac{1}{5}\right)\div 2$

(j) $1\frac{1}{4}-\left(3\frac{1}{3}-2\frac{5}{6}\right)\div 2$ (k) $3\frac{3}{8}+\left(8\frac{1}{4}-6\frac{3}{8}\right)\div 3$ (l) $\frac{7}{9}\div\left(1\frac{5}{6}-1\frac{3}{5}\right)\times\frac{1}{5}$

2 (a) $\frac{3}{7}\times\frac{1}{3}+5=$

A $5\frac{1}{7}$ B $5\frac{1}{3}$ C $5\frac{5}{21}$ D $5\frac{7}{9}$

(b) $6-\frac{9}{13}\div\frac{3}{4}=$

A $5\frac{1}{13}$ B $5\frac{25}{52}$ C $5\frac{12}{13}$ D $7\frac{1}{13}$

Understanding

3 Write out the following questions using fractions, the operations symbols ($\times$, $\div$, $+$, $-$) and brackets where necessary. Then, evaluate using the correct order of operations.

(a) Add $\frac{7}{8}$ and $\frac{5}{6}$, multiply the total by 2.

(b) Find the product of $\frac{4}{5}$ and $\frac{1}{3}$, then add $1\frac{1}{2}$.

(c) Find the difference between $2\frac{4}{5}$ and $1\frac{2}{3}$, then multiply by 3.

(d) Find out how many $\frac{2}{3}$s in 5, then multiply by 4.

4 A survey was taken of 120 people as they left Wally's Sandwich Bar. Write your answers to the following in simplest form.

(a) If 80 were male, what fraction were male?

(b) What fraction were female?

(c) If 45 were under 21 years of age, what fraction were under 21?

(d) What fraction were 21 or over?

(e) If $\frac{3}{4}$ had bought a drink, how many had bought a drink?

(f) Of those who bought a drink, $\frac{2}{5}$ had bought a salad roll. How many people had a drink and a salad roll?

5 Grant is training in his backyard pool for the 1500 m freestyle at the next Olympics. Unfortunately, the pool is only 20 m in length.

(a) What fraction of the total 1500 m race would he have swum after completing just 1 lap?

(b) How many laps would he have to complete to swim 1500 m?

(c) What fraction of the total 1500 m race would he have swum after completing 6 laps?

(d) How far would he have swum if he had completed $\frac{1}{10}$ of the race distance?

(e) How far would he have swum if he had completed $\frac{3}{4}$ of the race distance?

6 Valerie is running laps of her school oval to train for Rigby High's Mini Marathon. She ran $4\frac{1}{2}$ laps before having to rest. After her rest, she ran another $3\frac{1}{4}$ laps before stopping again. Then, she struggled through another $\frac{1}{3}$ of a lap. How many laps did she complete altogether?

7 One weekend, Louisa and Andrew walk from Ferndale to Greenhill and back, a total of $2\frac{3}{4}$ km. The next weekend they walk from Ferndale through Greenhill to Highvale, which is $5\frac{1}{3}$ km.

How far (in km) is it from:

(a) Ferndale to Greenhill

(b) Greenhill to Highvale

(c) Ferndale to Highvale and back again?

8 Jeremy worked at an amusement park for 4 hours at the normal hourly rate, then another $2\frac{1}{2}$ hours at $1\frac{1}{2}$ times the normal rate. What was the equivalent number of normal hours he was to be paid?

Reasoning

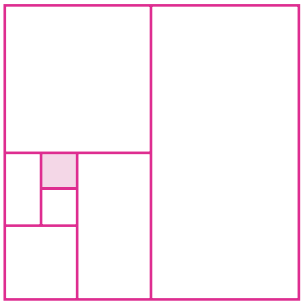
9 Theresa, Isabelle and Bryce worked on a mathematical problem and came up with the answers $8\frac{1}{2}$, $8\frac{2}{3}$ and $8\frac{3}{5}$, respectively. The correct answer was $8\frac{7}{12}$. Which of the three students was closest to the correct answer?

10 Despina's department store advertises a winter sale in which they claim everything is between $\frac{1}{3}$ and $\frac{1}{2}$ off. For which of the following items is this incorrect?

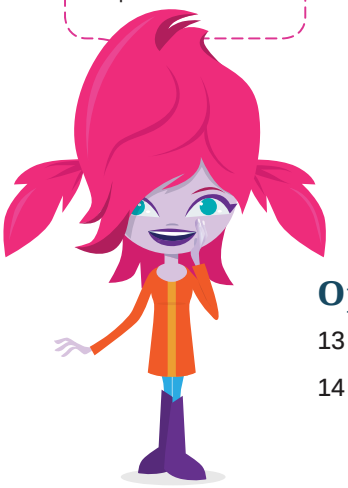
Item	Normal price	Sale price
Bath towels	$22	$12
Camera	$900	$600
Cutlery set	$250	$120
Microwave oven	$540	$350
Jeans	$60	$35

11 The Happy Valley Farming Cooperative has divided its square block of land in the following way. Each section of land is exactly half of the section next to it.

The May family has been given the shaded region to farm. If the total block of land is 1, what fraction of the total *don't* the May family farm?



Hint: Draw a diagram or flowchart to show this process.



12 Prashanth has agreed to sell badges for a charity. He receives a large box and places them, with permission, at the school reception. After several days, Prashanth took an estimate of the number of badges he had left. He emptied out the box, divided the pile of badges roughly into halves, and put half back in the box. He then halved the pile he had left, and returned half to the box. He did this one more time, after which he counted the number of badges in the pile, and found he had 13.

(a) What fraction of the total number of badges does 13 represent?

(b) How many badges did Prashanth have in the box, approximately?

Open-ended

13 Write down three numbers in fraction form that are less than $\frac{7}{8}$ but greater than $\frac{1}{2}$.

14 A and B are two different numbers selected from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where A is always greater than B.

(a) What values can $\frac{A+B}{A-B}$ have? Give three possibilities.

(b) What is the largest value this fraction can have?

15 'Ideal fractions' are pairs of fractions whose sum and product are the same. (They give the same number when added as when multiplied.)

(a) Show that $\frac{7}{3}$ and $\frac{7}{4}$ are ideal fractions.

(b) Show that $\frac{5}{3}$ and $\frac{5}{2}$ are ideal fractions.

(c) Look carefully at the numerators and denominators of the fraction pairs in (a) and (b). Try to find a pattern.

(d) Write down another pair of ideal fractions.

Outside the Square — Problem solving

Mega-equivalents

1 Use four of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 to make a fraction equivalent to $\frac{1}{2}$ that has two digits in the numerator and in the denominator (e.g. $\frac{34}{68}$).

2 Now, use six digits to make another fraction equivalent to $\frac{1}{2}$ that has three digits in the numerator and denominator.

3 See if you can form another fraction equivalent to $\frac{1}{2}$ using eight digits (four in numerator and denominator).

4 Can you make a fraction equivalent to $\frac{1}{2}$ using all 10 digits?

Strategy options

- Guess and check.
- Test all possible combinations.

Challenge 3

1 If $\frac{1}{3}$ of a number is 16, then $\frac{3}{8}$ of this number is:

A 8 B 16 C 18 D 24

2 Kerry bought a new water tank for her garden. The tank is initially empty, but fills to $\frac{2}{3}$ full of capacity after rain. The tank now holds 480 litres of water. How many litres does the tank hold when full?

3 If $\frac{\nabla}{16}$ lies between $\frac{1}{4}$ and $\frac{3}{8}$, and ∇ is a whole number, then ∇ equals:

A 5 B 6 C 7 D 8

4 Two proper fractions (in simplest form) are subtracted, the first with a denominator of 10, and the second with a denominator of 6. If the answer is given in simplest form, the largest possible numerator that it could have is:

A 8 B 9 C 11 D 22

5 The number halfway between $\frac{1}{5}$ and $\frac{1}{20}$ is:

A $\frac{1}{10}$ B $\frac{1}{8}$ C $\frac{7}{40}$ D $\frac{1}{4}$

6 Two frogs live together in a backyard pond. One frog croaks every 4 minutes, the other frog croaks every $2\frac{1}{2}$ minutes. If they both croak together at 11 a.m., what is the first time after 1 p.m. that they will croak together?

7 $2 + \dfrac{6}{1 + \frac{1}{2}}$ equals:

A 2 B 4 C 6 D 8

8 Express this fraction in its simplest form:

$$\cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{3}}}}$$

9 If 60 is added to one-third of a number, the resulting value is double the number. What is the number?

10 Five numbers are put in a row from smallest to largest, the difference between each adjacent number (a number and the number next to it) is the same. If the first number is $\frac{1}{8}$ and the last is $\frac{1}{6}$, what are the three numbers in between (in simplest form)?

Mathspace

FRACTURED RAVINE

While exploring the mountains with your friends you fall down a ravine! Armed with your fraction skills, can you survive the perils that await you?

Equipment: 1 die, 2–4 brains

Rules:

- Each player begins with 100 life points.
- If you lose all your life points you are out (dead).
- Take turns to roll the die and move forward that number of spaces. Follow the directions given in the space that you land on.
- All players must stop at the 'Leap of Faith' regardless of what number they have rolled.
- The goal: to be the first to reach sunlight.

START

You sprain an ankle.

$1\frac{1}{2} + 3\frac{1}{4} + 4\frac{2}{8} =$

Deduct the solution from your life points total.

You call out 'Cooee' and cause a rock fall.

$\frac{3}{4}$ of 16 =

Deduct the answer from your life points total.

Your torch runs out of batteries.

$\frac{1}{3}$ of 60 =

Deduct the answer from your life points total.

Your abseil rope gets caught on a sharp rock.

$\frac{2}{5}$ of 30 =

Deduct the answer from your life points total.

You run out of drinking water.

$12\frac{1}{3} + 3\frac{1}{6} - 4\frac{1}{2} =$

Deduct the solution from your life points total.

The oxygen level decreases rapidly.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction = $\frac{3}{4}$.

Find the fraction of 60 that this would be,

e.g. $\frac{3}{4}$ of 60 = 45.

Deduct this from your life points total.

You begin to feel claustrophobic.

Roll the die and multiply the number you rolled by $\frac{2}{5}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

You eat the last of the food supply.

Roll the die and multiply the number you rolled by $\frac{7}{2}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

A rock falls and breaks all the bones in one of your feet.

The human body contains 206 bones. If approximately $\frac{1}{8}$ of these are in each foot, how many bones did you break (rounded to the nearest whole number)?

Deduct this from your life points total.

LEAP OF FAITH

ALL EXPLORERS STOP HERE

There is a large crevasse ahead, but it's the only way forward—you need to find the courage to jump!

Roll the die and substitute the number you roll into this fraction $\frac{3}{\square}$.

If the fraction created is greater than (or equal to) 1 you've made it across.

If the fraction created is less than 1 you must wait until you can roll again.

You reach a ladder and see sunlight above.

All explorers must stop at each rung of the ladder.

(Players don't need to roll to move.)

Rung 1:

Find what fraction of your first name is made up of vowels. e.g. 'Natalie' $= \frac{4}{7}$. Do the same for your last name . Multiply one of these fractions by 50 and round to the nearest whole number.

Deduct this from your life points total.

Rung 2:

If the fraction of your first name that is made up of vowels is less than or equal to $\frac{1}{2}$, *deduct 10 from your life points total*. If it's greater than $\frac{1}{2}$, *deduct 15.*

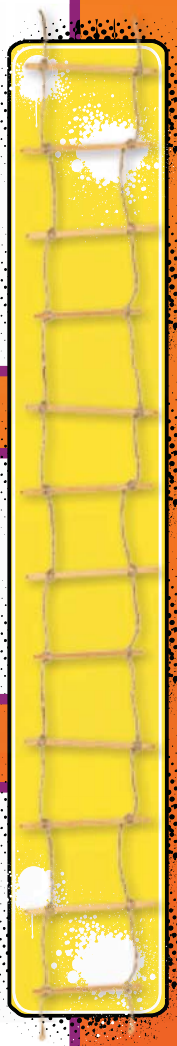
Rung 3:

You are on the final rung.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. roll 1 = 3, roll 2 = 4, fraction = $\frac{3}{4}$. You need to roll a fraction larger than $\frac{2}{3}$ to make the final step out of the ravine.

Every time you fail you must deduct 10 points from your life points total.



SUNLIGHT! YOU SURVIVED. WELL DONE

Chapter review 3

D.I.Y. Summary

Key Words

cancelling	improper fraction	mixed number	simplest form
denominator	inverse	numerator	simplify
equivalent fractions	Lowest Common Denominator (LCD)	proper fraction	unit fraction
fraction			

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 In a fraction, the _____ tells us how many parts we have, and the _____ tells us the size of each part.

2 A _____ has a value between 0 and 1.

3 To _____ a fraction, divide the numerator and denominator by a common factor. This process is known as _____.

4 Dividing by a fraction is the same as multiplying by the _____ of the fraction.

5 An _____ has a numerator that is greater than or equal to the denominator.

6 _____ are located at exactly the same position on a number line.

7 A _____ is a fraction with a numerator of 1.

Fluency

1 Here are 3 dozen (36) eggs. Ex. 3.1, 3.2

(a) If $\frac{4}{9}$ of the eggs are slightly cracked, how many eggs are damaged?

(b) The farmer's 3 best hens laid 15 of the eggs pictured. What fraction of the total is this? Write your answer in simplest form.

2 Write the value of the fractions located by the arrows on this number line: Ex. 3.1

3 Write (a) 6 and (b) 11 as improper fractions with denominators of: Ex. 3.1

(i) 3 (ii) 10 (iii) 1

4 (a) 3 pizzas are shared between 5 students. Write the amount of pizza each student gets as a fraction. Ex. 3.1

(b) 7 packets of lollies are shared between 4 students. Write the number of packets each student receives as a mixed number.

5 Find equivalent fractions by copying and completing the following. Ex 3.2

(a) $\frac{2}{7} = \frac{12}{\square}$ (b) $\frac{12}{18} = \frac{\square}{6}$ (c) $\frac{15}{30} = \frac{45}{\square}$ (d) $\frac{\square}{24} = \frac{5}{4}$

6 Write the following fractions in their simplest form. Ex 3.2

(a) $\frac{12}{15}$ (b) $\frac{48}{20}$ (c) $3\frac{60}{72}$ (d) $1\frac{8}{40}$

7 (a) Write $3\frac{2}{7}$ as an improper fraction. Ex 3.2

(b) Write $\frac{50}{9}$ as a mixed number.

8 Copy and complete the following by inserting either $<$, $>$ or $=$. Ex 3.3

(a) $\frac{5}{13} \_\_\_\_ \frac{1}{2}$ (b) $\frac{6}{16} \_\_\_\_ \frac{30}{80}$ (c) $\frac{5}{12} \_\_\_\_ \frac{3}{8}$ (d) $\frac{8}{9} \_\_\_\_ \frac{9}{10}$

9 Evaluate the following. Estimate first to check that your answers are reasonable. Ex 3.4

(a) $\frac{5}{12} + \frac{7}{8}$ (b) $\frac{7}{18} - \frac{2}{9}$ (c) $\frac{2}{3} - \frac{2}{7}$ (d) $\frac{1}{6} + \frac{7}{10}$

10 Evaluate the following. Write your answers as mixed numbers in simplest form. Ex 3.4

(a) $3\frac{2}{5} - 1\frac{3}{4}$ (b) $2\frac{1}{4} + 5\frac{11}{12}$ (c) $8 - 2\frac{5}{7}$ (d) $4\frac{5}{6} - 4\frac{2}{3}$

11 Calculate the following. Ex 3.5

(a) $\frac{3}{7}$ of \$28 (b) $\frac{2}{9}$ of $\frac{3}{4}$

12 Evaluate the following. Ex 3.5

(a) $\frac{6}{11} \times \frac{55}{18}$ (b) $2\frac{4}{9} \times 5$ (c) $\frac{2}{3} \times \frac{9}{16} \times 1\frac{1}{7}$

13 Evaluate the following. Ex 3.6

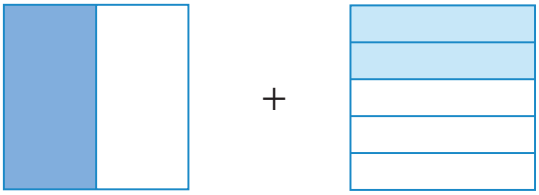
(a) $9 \div \frac{3}{4}$ (b) $21 \div 3\frac{2}{7}$ (c) $\frac{6}{5} \div \frac{32}{15}$ (d) $3\frac{1}{4} \div 3\frac{1}{3}$

14 Simplify by using the correct order of operations. Ex 3.7

(a) $4 + \frac{5}{6} \times 2\frac{1}{3}$ (b) $7\frac{2}{5} - \left(3\frac{1}{10} - \frac{3}{5}\right)$ (c) $\frac{7}{8} \times \left(\frac{3}{4} + 4 \times \frac{3}{4}\right)$

Understanding

15 (a) What fraction sum is represented by the following two grids?



(b) Calculate the fraction sum using the grid method or any other method.

16 Write a fraction or a mixed number in simplest form to show each of these:

(a) eight chocolate biscuits in a packet of 24

(b) two complete '6-packs' of soft drink cans, with one can left over

(c) one packet of twelve colour pencils and four pencils in a second packet

(d) nine hours sleep in one day.

17 There are 12 boys and 15 girls in class 7A. 6 of the students have blonde hair, and 2 have red hair. Write the following as fractions of the whole class, in simplest form.

(a) the number of girls

(b) the number of blonde-haired students

(c) the number of students with neither blonde nor red hair

18 Write each set of numbers in order from smallest to largest, by using strategies for comparing fractions.

(a) $\frac{3}{4}, \frac{1}{2}, 1, \frac{1}{3}, \frac{3}{5}$

(b) $2, \frac{13}{5}, \frac{5}{4}, \frac{30}{50}, 1\frac{2}{5}$

19 Jamal gets 7 out of 9 shots at goal in the netball ring, whereas Kayla gets 9 out of 12 shots in. Determine who is the more accurate shooter by comparing fractions.

20 Elise and Reece each have an identical block of chocolate. Elise eats $\frac{2}{5}$ and Reece eats $\frac{2}{3}$.

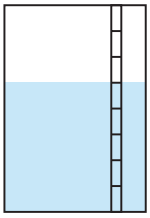
(a) How much more has Reece eaten, as a fraction of a block?

(b) Together, have Elise and Reece eaten more or less than a whole block of chocolate? Estimate, then calculate your answer.

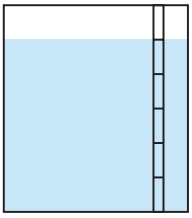
21 Gary had $1\frac{1}{2}$ bags of cement in his shed. He used $\frac{2}{3}$ of a bag to mix up some concrete. What fraction of a full bag does he have left?

22 Jarrod owns a petrol station that has three main fuel tanks. There is a dip stick in each tank that shows the level of fuel remaining in each tank. How many litres are left in each tank?

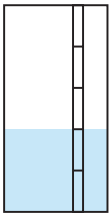
(a) Full tank = 48 000 L



(b) Full tank = 60 000 L



(c) Full tank = 30 000 L



23 Here is a recipe for a choc-orange cake:

180 g butter	$\frac{2}{3}$ cup castor sugar	4 eggs
$1\frac{3}{4}$ cups self-raising flour	2 tablespoons cocoa	$\frac{1}{2}$ cup orange juice

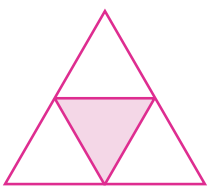
Rewrite the ingredients to make a cake that is one-quarter of the size of this cake:

Reasoning

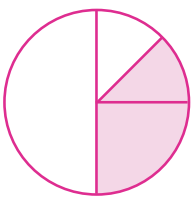
24 State the fraction of the area of these shapes that is shaded.

(a)

(b)



(c)



25 45 minutes into a Fun Run, Katya had completed 4 km of the 10 km course, whereas Mabok had completed 7 km of the 15 km course.

(a) Who had the least distance still to run?

(b) Who had completed a bigger fraction of their particular course?

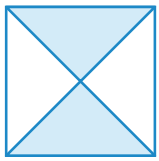
26 Which of the following will give an answer of 1?

A $\frac{1}{4} \div \frac{1}{4}$ B $\frac{1}{2} \div \frac{1}{4}$ C $1 \div \frac{1}{4}$ D $4 \div \frac{1}{4}$

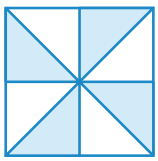
NAPLAN practice 3

Numeracy: Non-calculator

1 Which diagram does not have exactly $\frac{1}{2}$ of the area shaded?

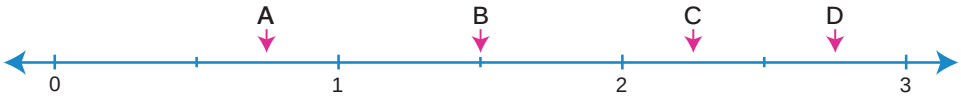
A 

B 

C 

D 

2 Which arrow is pointing closest to the location of $\frac{11}{8}$ on this number line?



3 Kevin is mixing sand, gravel and cement to make concrete. Sand makes up $\frac{1}{2}$ of the mixture, and gravel is $\frac{1}{3}$. What fraction of the mixture is cement?

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$

4 Esther is making vanilla ice-cream using this recipe:

5 eggs
6 cups milk
1 cup sugar
$1\frac{1}{2}$ teaspoons vanilla
2 cans condensed milk

This recipe makes 6 servings. How many teaspoons of vanilla will Esther need to make 30 servings?

A $5\frac{1}{2}$ B $7\frac{1}{2}$ C $30\frac{1}{2}$ D 45

Numeracy: Calculator allowed

5 A school has 200 students. 48 of the students are involved in the school production. The fraction of students who are involved in the school production is closest to:

A one-fifth B one-quarter C one-third D one-half

6 Which of the following fractions is greater than 2 but less than 3?

A $\frac{3}{4}$ B $\frac{12}{9}$ C $\frac{17}{7}$ D $\frac{15}{5}$

7 Imran ran $\frac{1}{6}$ of a cross-country course. He ran 3 km. How long is the course?

A 6 km B 9 km C 12 km D 18 km

8 Which fraction lies exactly halfway between $\frac{3}{8}$ and $\frac{4}{8}$ on the number line?

A $\frac{3}{16}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{7}{16}$