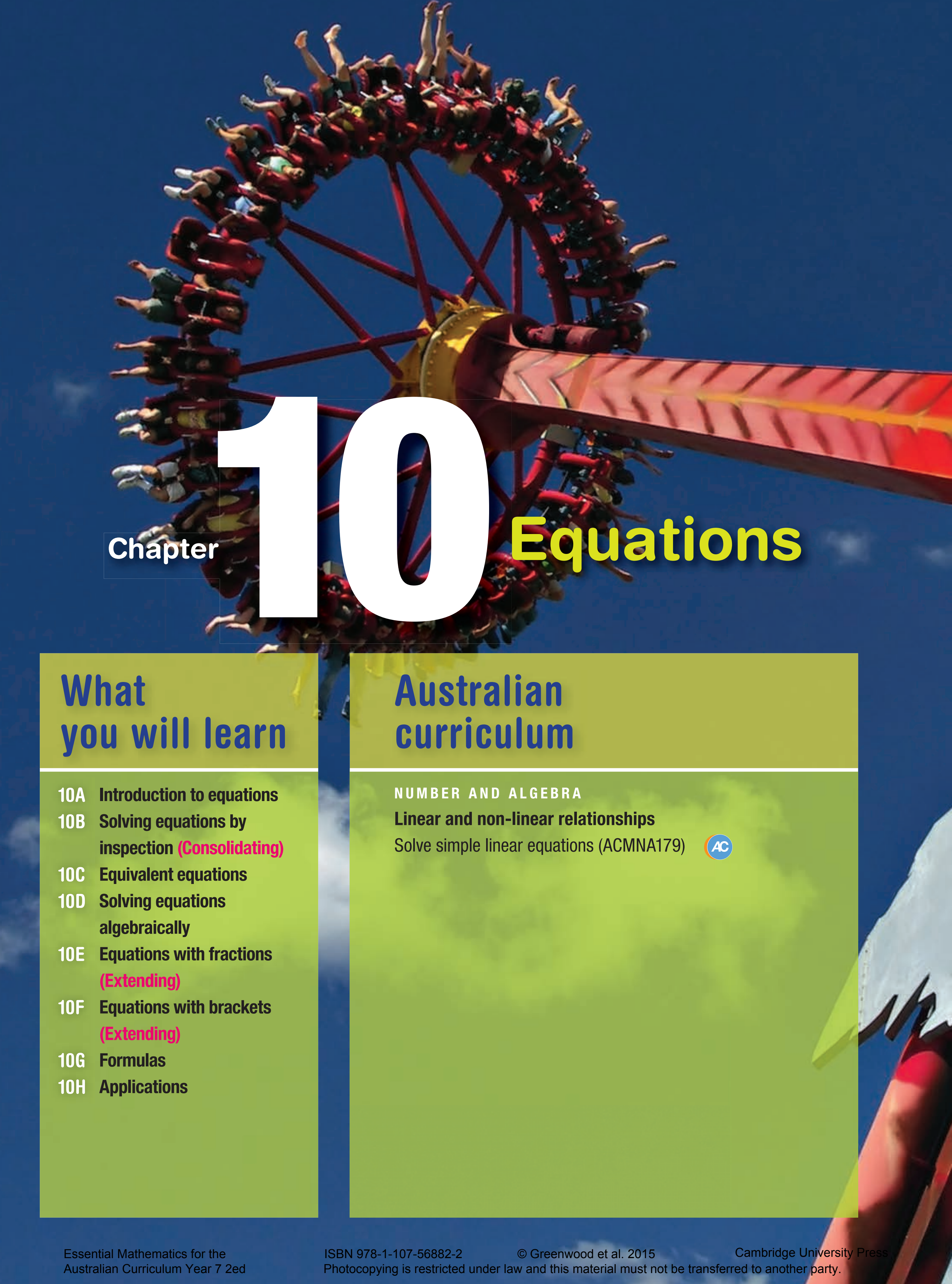


Chapter 10 Equations

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve simple linear equations (ACMNA179)

ISBN 978-1-107-56882-2

Cambridge University Press

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Theme park equations

Equations are used widely in mathematics and in many other fields. Whenever two things are equal, or should be equal, there is the potential to use the study of equations to help deal with such a situation.

Knowledge of mathematics and physics is vitally important when designing theme park rides. Engineers use equations to 'build' model rides on a computer so that safety limits can be determined in a virtual reality in which nobody gets injured.

Algebraic equations are solved to determine the dimensions and strengths of structures required to deal safely with the combined forces of weight, speed and varying movement. Passengers might scream with a mixture of terror and excitement but they must return unharmed to earth!

At Dreamworld on the Gold Coast, Queensland, 'The Claw' swings 32 people upwards at 75 km/h to a maximum height of 27.1 m (9 storeys), simultaneously spinning 360° at 5 r.p.m. (revolutions per minute). 'The Claw' is the most powerful pendulum ride on the planet. It is built to scare!

10A Introduction to equations

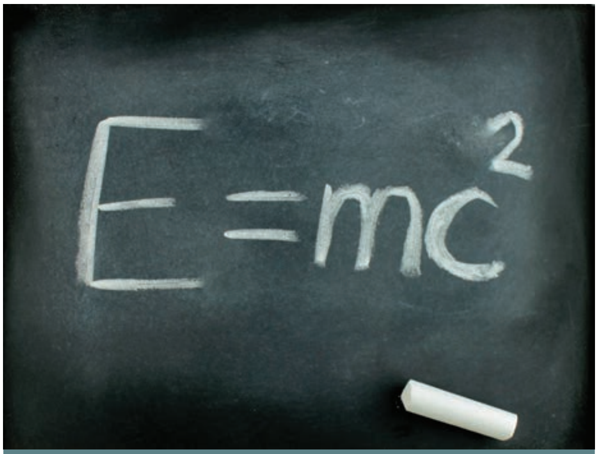
An equation is a mathematical statement used to say that two expressions have the same value. It will always consist of two expressions that are separated by an equals sign (=).

Sample equations include:

$$3 + 3 = 6$$
$$30 = 2 \times 15$$
$$100 - 30 = 60 + 10$$

which are all true equations.

An equation does not have to be true. For instance, $2 + 2 = 17$ and $5 = 3 - 1$ and $10 + 15 = 12 + 3$ are all false equations.



This equation was proposed by the famous scientist Albert Einstein (1879–1955). It explains the special theory of relativity.

If an equation contains pronumerals, one cannot generally tell whether the equation is true or false until values are substituted for the pronumerals. For example, $5 + x = 7$ could be true (if x is 2) or it could be false (if x is 15).

Let's start: Equations – true or false?

Rearrange the following five symbols to make as many different equations as possible.

$$5, 2, 3, +, =$$

- Which of them are true? Which are false?
- Is it always possible to rearrange numbers and operations to make true equations?

Key ideas

- An **expression** is a collection of pronumerals, numbers and operators without an equals sign (e.g. $2x + 3$).
- An **equation** is a mathematical statement stating that two things are equal, (e.g. $2x + 3 = 4y - 2$).
- Equations have a left-hand side (LHS), a right-hand side (RHS) and an equals sign in between.

$$\underbrace{2x + 3}_{\text{LHS}} = \underbrace{4y - 2}_{\text{RHS}}$$

- Equations are mathematical statements that can be true (e.g. $2 + 3 = 5$) or false (e.g. $5 + 7 = 21$).
- If a pronumeral is included in an equation, you need to know the value to substitute before deciding whether the equation is true. For example, $3x = 12$ would be true if 4 is substituted for x, but it would be false if 10 is substituted.

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Example 1 Identifying equations

Which of the following are equations?

a $3 + 5 = 8$ **b** $7 + 7 = 18$ **c** $2 + 12$ **d** $4 = 12 - x$ **e** $3 + u$

SOLUTION	EXPLANATION
a $3 + 5 = 8$ is an equation.	There are two expressions (i.e. $3 + 5$ and 8) separated by an equals sign.
b $7 + 7 = 18$ is an equation.	There are two expressions separated by an equals sign. Although this equation is false, it is still an equation.
c $2 + 12$ is not an equation.	This is just a single expression. There is no equals sign.
d $4 = 12 - x$ is an equation.	There are two expressions separated by an equals sign.
e $3 + u$ is not an equation.	There is no equals sign, so this is not an equation.

Example 2 Classifying equations

For each of the following equations, state whether it is true or false.

a $7 + 5 = 12$
b $5 + 3 = 2 \times 4$
c $12 \times (2 - 1) = 14 + 5$
d $3 + 9x = 60 + 6$, if $x = 7$
e $10 + b = 3b + 1$, if $b = 4$
f $3 + 2x = 21 - y$, if $x = 5$ and $y = 8$

SOLUTION	EXPLANATION
a true	The left-hand side (LHS) and right-hand side (RHS) are both equal to 12, so the equation is true.
b true	LHS $= 5 + 3 = 8$ and RHS $= 2 \times 4 = 8$, so both sides are equal.
c false	LHS $= 12$ and RHS $= 19$, so the equation is false.
d true	If x is 7, then: LHS $= 3 + 9 \times 7 = 66$, RHS $= 60 + 6 = 66$
e false	If b is 4, then: LHS $= 10 + 4 = 14$, RHS $= 3(4) + 1 = 13$
f true	If $x = 5$ and $y = 8$, then: LHS $= 3 + 2(5) = 13$, RHS $= 21 - 8 = 13$

Example 3 Writing equations from a description

Write equations for each of the following scenarios.

a The sum of x and 5 is 22.

b The number of cards in a deck is x. In 7 decks there are 91 cards.

c Priya's age is currently j. In 5 years' time her age will equal 17.

d Corey earns \$$w$ per year. He spends $\frac{1}{12}$ on sport and $\frac{2}{13}$ on food. The total amount Corey spends on sport and food is \$15 000.

SOLUTION	EXPLANATION
a $x + 5 = 22$	The sum of x and 5 is written $x + 5$.
b $7x = 91$	$7x$ means $7 \times x$ and this number must equal the 91 cards.
c $j + 5 = 17$	In 5 years' time Priya's age will be 5 more than her current age, so $j + 5$ must be 17.
d $\frac{1}{12} \times w + \frac{2}{13} \times w = 15000$	$\frac{1}{12}$ of Corey's wage is $\frac{1}{12} \times w$ and $\frac{2}{13}$ of his wage is $\frac{2}{13} \times w$.

Exercise 10A

1–4	3, 4	—

UNDERSTANDING

Example 1 **1** Classify each of the following as an equation (E) or not an equation (N).

a $7 + x = 9$
b $2 + 2$
c $2 \times 5 = t$
d $10 = 5 + x$
e $2 = 2$
f $7 \times u$
g $10 \div 4 = 3p$
h $3 = e + 2$
i $x + 5$

Example 2a–c **2** Classify each of these equations as true or false:

a $2 + 3 = 5$
b $3 + 2 = 6$
c $5 - 1 = 6$

Example 2d **3** If $x = 2$, is $10 + x = 12$ true or false?

Example 2e **4** Consider the equation $4 + 3x = 2x + 9$.

a If $x = 5$, state the value of the left-hand side (LHS).
b If $x = 5$, state the value of the right-hand side (RHS).
c Is the equation $4 + 3x = 2x + 9$ true or false when $x = 5$?

5(½), 6, 7, 9(½)	5(½), 6, 7, 8–9(½)	5–9(½)

FLUENCY

5 For each of the following equations, state whether it is true or false.

a $10 \times 2 = 20$ **b** $12 \times 11 = 144$ **c** $3 \times 2 = 5 + 1$
d $100 - 90 = 2 \times 5$ **e** $30 \times 2 = 32$ **f** $12 - 4 = 4$
g $2(3 - 1) = 4$ **h** $5 - (2 + 1) = 7 - 4$ **i** $3 = 3$
j $2 = 17 - 14 - 1$ **k** $10 + 2 = 12 - 4$ **l** $1 \times 2 \times 3 = 1 + 2 + 3$
m $2 \times 3 \times 4 = 2 + 3 + 4$ **n** $100 - 5 \times 5 = 20 \times 5$ **o** $3 - 1 = 2 + 5 - 5$

6 If $x = 3$, state whether each of these equations is true or false.

a $5 + x = 7$ **b** $x + 1 = 4$ **c** $13 - x = 10 + x$ **d** $6 = 2x$

7 If $b = 4$, state whether each of the following equations is true or false.

a $5b + 2 = 22$ **b** $10 \times (b - 3) = b + b + 2$
c $12 - 3b = 5 - b$ **d** $b \times (b + 1) = 20$

Example 2f

8 If $a = 10$ and $b = 7$, state whether each of these equations is true or false.

a $a + b = 17$ **b** $a \times b = 3$ **c** $a \times (a - b) = 30$
d $b \times b = 59 - a$ **e** $3a = 5b - 5$ **f** $b \times (a - b) = 20$
g $21 - a = b$ **h** $10 - a = 7 - b$ **i** $1 + a - b = 2b - a$

Example 3a

9 Write equations for each of the following.

a The sum of 3 and x is equal to 10. **b** When k is multiplied by 5, the result is 1005.
c The sum of a and b is 22. **d** When d is doubled, the result is 78.
e The product of 8 and x is 56. **f** When p is tripled, the result is 21.
g One-quarter of t is 12.
h The sum of q and p is equal to the product of q and p.

10, 11	10, 12	10, 12, 13

PROBLEM-SOLVING

Example 3b–d

10 Write true equations for each of these problems. You do not need to solve them.

a Chairs cost $c at a store. The cost of 6 chairs is $546.

b Patrick works for x hours each day. In a 5-day working week, he works $37\frac{1}{2}$ hours in total.

c Pens cost $a each and pencils cost $b. Twelve pens and three pencils cost $28 in total.

d Amy is f years old. In 10 years' time her age will be 27.

e Andrew's age is j and Hailey's age is m. In 10 years' time their combined age will be 80.

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PROBLEM-SOLVING

11 Find a value of m that would make this equation true: $10 = m + 7$.

12 Find two possible values of k that would make this equation true: $k \times (8 - k) = 12$.

13 If the equation $x + y = 6$ is true, and x and y are both whole numbers between 1 and 5, what values could they have?

14(½)	14(½)	14

REASONING

14 Equations involving pronumerals can be split into three groups:

A: Always true, no matter what values are substituted.

N: Never true, no matter what values are substituted.

S: Sometimes true but sometimes false, depending on the values substituted.

Categorise each of these equations as either A, N or S.

a $x + 5 = 11$
b $12 - x = x$
c $a = a$
d $5 + b = b + 5$
e $a = a + 7$
f $5 + b = b - 5$
g $0 \times b = 0$
h $a \times a = 100$
i $2x + x = 3x$
j $2x + x = 4x$
k $2x + x = 3x + 1$
l $a \times a + 100 = 0$

Equation permutations

—	—	15, 16

ENRICHMENT

15 For each of the following, rearrange the symbols to make a true equation.

a 6, 2, 3, ×, =
b 1, 4, 5, –, =
c 2, 2, 7, 10, –, ÷, =
d 2, 4, 5, 10, –, ÷, =

16 **a** How many different equations can be produced using the symbols 2, 3, 5, +, =?

b How many of these equations are true?

c Is it possible to change just one of the numbers above and still produce true equations by rearranging the symbols?

d Is it possible to change just the operation above (i.e. +) and still produce true equations?

Many mathematical equations need to be solved to build and launch space stations into orbit.

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10B Solving equations by inspection CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

Solving an equation is the process of finding the values that pronumerals must take in order to make the equation true. Pronumerals are also called ‘unknowns’ when solving equations. For simple equations, it is possible to find a solution by trying a few values for the pronumeral until the equation is true. This method does not guarantee that we have found all the solutions (if there is more than one) and it will not help if there are no solutions, but it can be a useful and quick method for simple equations.

Let’s start: Finding the missing value

- Find the missing values to make the following equations true.

  $10 \times \square - 17 = 13$

  $27 = 15 + 3 \times \square$

  $2 \times \square + 4 = 17$
- Can you always find a value to put in the place of $\square$ in any equation?

Key ideas

- **Solving** an equation means finding the values of any pronumerals that make the equation true. These values are called **solutions** to the equation.
- An **unknown** in an equation is a pronumeral whose value needs to be found in order to make the equation true.
- One method of solving equations is by **inspection** (also called **trial and error**), which involves inspecting (or trying) different values and seeing which ones make the equation true.

Example 4 Finding the missing number

For each of these equations, find the value of the missing number that would make it true.

a $\square \times 7 = 35$ **b** $20 - \square = 14$

SOLUTION	EXPLANATION
a 5	$5 \times 7 = 35$ is a true equation.
b 6	$20 - 6 = 14$ is a true equation.

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Example 5 Solving equations by inspection

Solve each of the following equations by inspection.

a $c + 12 = 30$ **b** $5b = 20$ **c** $2x + 13 = 21$

SOLUTION	EXPLANATION
a $c + 12 = 30$ $c = 18$	The unknown variable here is c. $18 + 12 = 30$ is a true equation.
b $5b = 20$ $b = 4$	The unknown variable here is b. Recall that $5b$ means $5 \times b$, so if $b = 4$, $5b = 5 \times 4 = 20$.
c $2x + 13 = 21$ $x = 4$	The unknown variable here is x. Trying a few values: $x = 10$ makes LHS $= 20 + 13 = 33$, which is too large. $x = 3$ makes LHS $= 6 + 13 = 19$, which is too small. $x = 4$ makes LHS $= 21$.

Exercise 10B

1–4	3	—

UNDERSTANDING

1 If the missing number is 5, classify each of the following equations as true or false.

a $\square + 3 = 8$ **b** $10 \times \square + 2 = 46$

c $10 - \square = 5$ **d** $12 = 6 + \square \times 2$

2 For the equation $\square + 7 = 13$:

a Find the value of the LHS (left-hand side) if $\square = 5$.

b Find the value of the LHS if $\square = 10$.

c Find the value of the LHS if $\square = 6$.

d What value of $\square$ would make the LHS equal to 13?

Example 4

3 Find the value of the missing numbers.

a $4 + \square = 7$ **b** $2 \times \square = 12$ **c** $13 = \square + 3$ **d** $10 = 6 + \square$

e $42 = \square \times 7$ **f** $100 - \square = 30$ **g** $\square \times 4 = 80$ **h** $\square + 12 = 31$

4 State the unknown pronumeral in each of the following equations.

a $4 + x = 12$ **b** $50 - c = 3$

c $4b + 2 = 35$ **d** $5 - 10d = 2$

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5–6(½)	5–6(½), 7	5–7(½)

FLUENCY

Example 5a, b

5 Solve the following equations by inspection.

- **a** $8 \times y = 64$
- **b** $6 \div l = 3$
- **c** $l \times 3 = 18$
- **d** $4 - d = 2$
- **e** $l + 2 = 14$
- **f** $a - 2 = 4$
- **g** $s + 7 = 19$
- **h** $x \div 8 = 1$
- **i** $12 = e + 4$
- **j** $r \div 10 = 1$
- **k** $13 = 5 + s$
- **l** $0 = 3 - z$

Example 5c

6 Solve the following equations by inspection.

- **a** $2p - 1 = 5$
- **b** $3p + 2 = 14$
- **c** $4q - 4 = 8$
- **d** $4v + 4 = 24$
- **e** $2b - 1 = 1$
- **f** $5u + 1 = 21$
- **g** $5g + 5 = 20$
- **h** $4(e - 2) = 4$
- **i** $45 = 5(d + 5)$
- **j** $3d - 5 = 13$
- **k** $8 = 3m - 4$
- **l** $8 = 3o - 1$

7 Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

- **a** $4 \times (x + 1) - 5 = 11$
- **b** $7 + x = 2 \times x$
- **c** $(3x + 1) \div 2 = 8$
- **d** $10 - x = x + 2$
- **e** $2 \times (x + 3) + 4 = 12$
- **f** $15 - 2x = x$

8, 9	8–10	10–12

PROBLEM-SOLVING

8 Find the value of the number in each of these examples.

- **a** A number is doubled and the result is 22.
- **b** 3 less than a number is 9.
- **c** Half of a number is 8.
- **d** 7 more than a number is 40.
- **e** A number is divided by 10, giving a result of 3.
- **f** 10 is divided by a number, giving a result of 5.

9 Justine is paid \$10 an hour for x hours. During a particular week, she earns \$180.

- **a** Write an equation involving x to describe this situation.
- **b** Solve the equation by inspection to find the value of x.

10 Karim's weight is w kg and his brother is twice as heavy, weighing 70 kg.

- **a** Write an equation involving w to describe this situation.
- **b** Solve the equation by inspection to find the value of w.

11 Taylah buys x kg of apples at \$4.50 per kg. She spends a total of \$13.50.

- **a** Write an equation involving x to describe this situation.
- **b** Solve the equation by inspection to find x.

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PROBLEM-SOLVING

12 Yanni's current age is y years. In 12 years' time he will be three times as old.

a Write an equation involving y to describe this situation.

b Solve the equation by inspection to find y.

13	13	13, 14

REASONING

13 **a** Solve the equation $x + (x + 1) = 19$ by inspection.

b The expression $x + (x + 1)$ can be simplified to $2x + 1$. Use this observation to solve $x + (x + 1) = 181$ by inspection.

14 There are three consecutive whole numbers that add to 45.

a Solve the equation $x + (x + 1) + (x + 2) = 45$ by inspection to find the three numbers.

b An equation of the form $x + (x + 1) + (x + 2) = ?$ has a whole number solution only if the right-hand side is a multiple of 3. Explain why this is the case. (Hint: Simplify the LHS.)

Multiple pronumerals

—	—	15

ENRICHMENT

15 When multiple pronumerals are involved, inspection can still be used to find a solution. For each of the following equations find, by inspection, one pair of values for x and y that make them true.

a $x + y = 8$

b $x - y = 2$

c $3 = 2x + y$

d $x \times y = 6$

e $12 = 2 + x + y$

f $x + y = x \times y$

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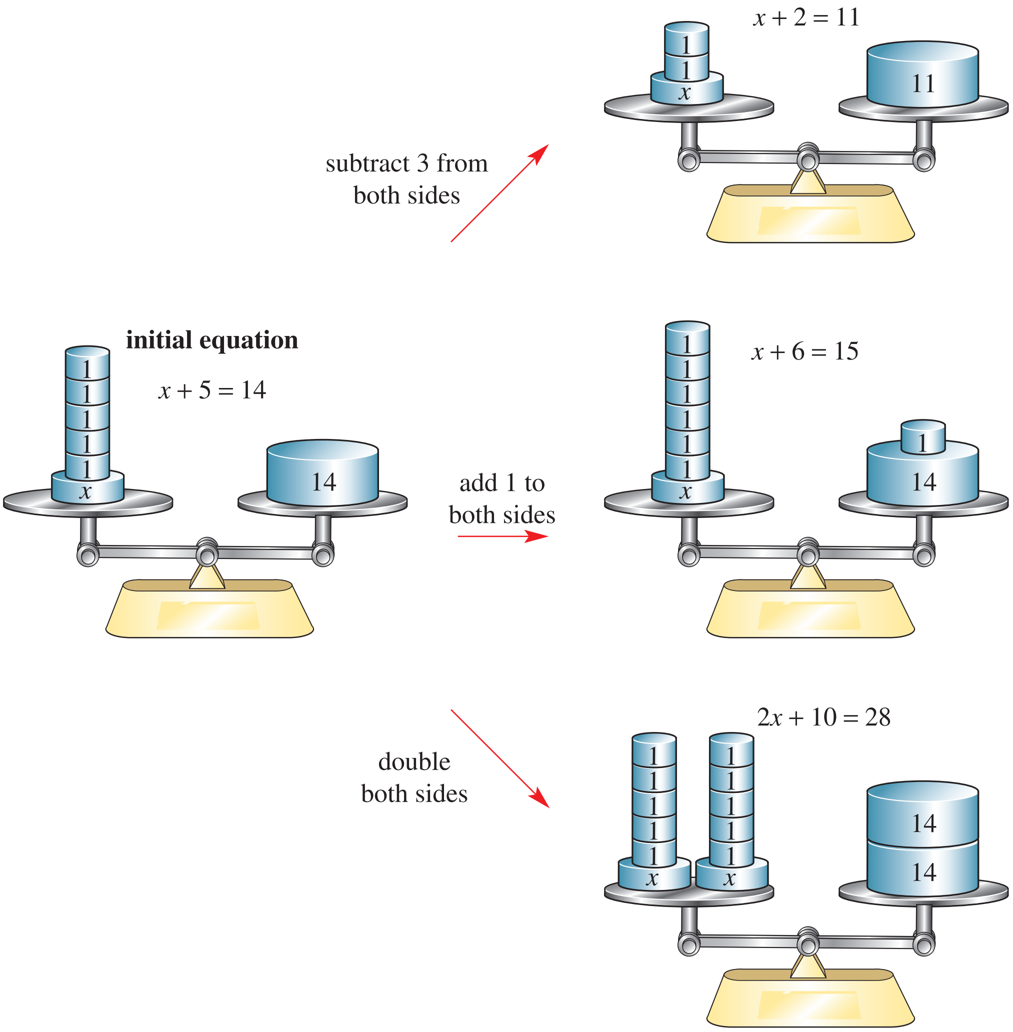
10C Equivalent equations

Interactive
Widgets
HOTsheets
Walkthroughs

Sometimes, two equations essentially express the same thing. For example, the equations $x + 5 = 14$, $x + 6 = 15$ and $x + 7 = 16$ are all made true by the same value of x. Each time, we have added one to both sides of the equation.

We can pretend that true equations are about different objects that have the same weight. For instance, to say that $3 + 5 = 8$ means that a 3 kg block added to a 5 kg block weighs the same as an 8 kg block.



A true equation stays true if we ‘do the same thing to both sides’, such as adding a number or multiplying by a number. The exception to this rule is that multiplying both sides of any equation by zero will always make the equation true, and dividing both sides of any equation by zero is not permitted because nothing can be divided by zero. If we do the same thing to both sides we will have an equivalent equation.

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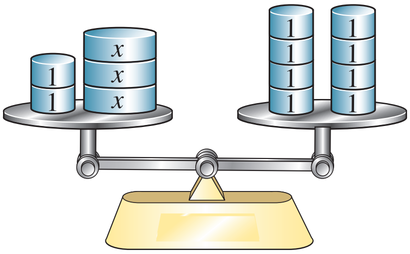
Let's start: Equations as scales

The scales in the diagram show $2 + 3x = 8$.



- What would the scales look like if two '1 kg' blocks were removed from both sides?
- What would the scales look like if the two '1 kg' blocks were removed just from the left-hand side? (Try to show whether they would be level.)
- Use scales to illustrate why $4x + 3 = 4$ and $4x = 1$ are equivalent equations.

Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side with the right-hand side of the equation

Example 6 Applying an operation

For each equation, find the result of applying the given operation to both sides and then simplify.

a $2 + x = 5\ [+4]$

b $7x = 10\ [\times 2]$

c $30 = 20b\ [\div 10]$

d $7q - 4 = 10\ [+4]$

SOLUTION	EXPLANATION
a $2 + x = 5$	
$2 + x + 4 = 5 + 4$	The equation is written out, and 4 is added to both sides.
$x + 6 = 9$	Simplify the expressions on each side.
b $7x = 10$	
$7x \times 2 = 10 \times 2$	The equation is written out, and both sides are multiplied by 2.
$14x = 20$	Simplify the expressions on each side.
c $30 = 20b$	
$\frac{30}{10} = \frac{20b}{10}$	The equation is written out, and both sides are divided by 10.
$3 = 2b$	Simplify the expressions on each side.

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d $7q - 4 = 10$
$7q - 4 + 4 = 10 + 4$
$7q = 14$

The equation is written out, and 4 is added to both sides.

Simplify the expressions on each side.

Example 7 Showing that equations are equivalent

Show that these pairs of equations are equivalent by stating the operation used.

a $2x + 10 = 15$ and $2x = 5$
b $5 = 7 - x$ and $10 = 2(7 - x)$
c $10(b + 3) = 20$ and $b + 3 = 2$

SOLUTION

EXPLANATION

a Both sides have had 10 subtracted.

$$\begin{tikzcd} 2x + 10 = 15 \arrow[d, bend right, "-10"'] \arrow[d, bend left, "-10"] \\ 2x = 5 \end{tikzcd}$$

$2x + 10 - 10$ simplifies to $2x$, so we get the second equation by subtracting 10.

b Both sides have been multiplied by 2.

$$\begin{tikzcd} 5 = 7 - x \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 10 = 2(7 - x) \end{tikzcd}$$

$2(7 - x)$ represents the RHS; i.e. $7 - x$, being multiplied by 2.

c Both sides have been divided by 10.

$$\begin{tikzcd} 10(b + 3) = 20 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ b + 3 = 2 \end{tikzcd}$$

Remember $10(b + 3)$ means $10 \times (b + 3)$. If we have $10(b + 3)$, we get $b + 3$ when dividing by 10.

Exercise 10C

1, 2	1	—

UNDERSTANDING

Example 6a

1 Write an equation that results from adding 10 to both sides of each of these equations.

a $10d + 5 = 20$ **b** $7e = 31$ **c** $2a = 12$ **d** $x = 12$

2 Match up each of these equations (**a** to **e**) with its equivalent equation (i.e. **A** to **E**), where 3 has been added to both sides.

a $10 + x = 14$	**A** $12x + 3 = 123$
b $x + 1 = 13$	**B** $x + 13 = 11x + 3$
c $12 = x + 5$	**C** $13 + x = 17$
d $x + 10 = 11x$	**D** $x + 4 = 16$
e $12x = 120$	**E** $15 = x + 8$

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10C

3–4(½)	3–4(½), 5	3–5(½)

FLUENCY

Example 6b–d

3 For each equation, show the result of applying the listed operations to both sides.

a $5 + x = 10$ [+1] **b** $3x = 7$ [×2] **c** $12 = 8q$ [÷4]
d $9 + a = 13$ [–3] **e** $7 + b = 10$ [+5] **f** $5 = 3b + 7$ [–5]
g $2 = 5 + a$ [+2] **h** $12x - 3 = 3$ [+5] **i** $7p - 2 = 10$ [+2]

Example 7

4 Show that these pairs of equations are equivalent by stating the operation used.

a $4x + 2 = 10$ and $4x = 8$ **b** $7 + 3b = 12$ and $9 + 3b = 14$
c $20a = 10$ and $2a = 1$ **d** $4 = 12 - x$ and $8 = 2(12 - x)$
e $18 = 3x$ and $6 = x$ **f** $12 + x = 3$ and $15 + x = 6$
g $4(10 + b) = 80$ and $10 + b = 20$ **h** $12x = 5$ and $12x + 4 = 9$

5 For each of the following equations, find the equivalent equation that is the result of adding 4 to both sides and then multiplying both sides by 3.

a $x = 5$ **b** $2 = a + 1$ **c** $d - 4 = 2$
d $7 + a = 8$ **e** $3y - 2 = 7$ **f** $2x = 6$

6	6, 7	7, 8

PROBLEM-SOLVING

6 Match up each of these equations (**a** to **e**) with its equivalent equation (i.e. **A** to **E**), stating the operation used.

a $m + 10 = 12$ **A** $7 - m = 6$
b $3 - m = 2$ **B** $5m = 18$
c $12m = 30$ **C** $6m = 10$
d $5m + 2 = 20$ **D** $6m = 15$
e $3m = 5$ **E** $m + 12 = 14$

7 For each of the following pairs of equations, show they are equivalent by listing the two steps required to transform the first equation to the second.

a $x = 5$ and $3x + 2 = 17$ **b** $m = 2$ and $10m - 3 = 17$
c $5(2 + x) = 15$ and $x = 1$ **d** $10 = 3x + 10$ and $0 = x$

8 For each of the following equations, write an equivalent equation that you can get in one operation. Your equation should be simpler (i.e. smaller) than the original.

a $2q + 7 = 9$ **b** $10x + 3 = 10$ **c** $2(3 + x) = 40$ **d** $x \div 12 = 5$

9	9, 10	9–11

REASONING

9 Sometimes two equations that look quite different can be equivalent.

a Show that $3x + 2 = 14$ and $10x + 1 = 41$ are equivalent by copying and completing the following.

$$\begin{tikzcd}
3x + 2 = 14 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
3x = 12 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\
\underline{\quad} = \underline{\quad} \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\
\underline{\quad} = \underline{\quad} \arrow[d, bend right, "+1"'] \arrow[d, bend left, "+1"] \\
10x + 1 = 41
\end{tikzcd}$$

b Show that $5x - 3 = 32$ and $x + 2 = 9$ are equivalent. (Hint: Try to go via the equation $x = 7$.)

c Show that $(x \div 2) + 4 = 9$ and $(x + 8) \div 2 = 9$ are equivalent.

10 As stated in the rules for equivalence listed in the **Key ideas**, multiplying both sides by zero is not permitted.

a Write the result of multiplying both sides of the following equations by zero.

i $3 + x = 5$ **ii** $2 + 2 = 4$ **iii** $2 + 2 = 5$

b Explain in a sentence why multiplying by zero does not give a useful equivalent equation.

11 Substituting pronumerals can be thought of as finding equivalent equations. Show how you can start with the equation $x = 3$ and find an equivalent equation with:

a $7x + 2$ on the LHS **b** $8 + 2x$ on the LHS

Equivalence relations

—	—	12

ENRICHMENT

12 Classify each of the following statements as true or false, justifying your answer.

a Every equation is equivalent to itself.

b If equation 1 and equation 2 are equivalent, then equation 2 and equation 1 are equivalent.

c If equation 1 and equation 2 are equivalent, and equation 2 and equation 3 are equivalent, then equation 1 and equation 3 are equivalent.

d If equation 1 and equation 2 are *not* equivalent, and equation 2 and equation 3 are *not* equivalent, then equation 1 is *not* equivalent to equation 3.

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10D Solving equations algebraically

Interactive
Widgets
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Walkthroughs

A soccer player preparing for a game will put on shin pads, then socks and, finally, soccer boots. When the game is over, these items are removed in reverse order: first the boots, then the socks and, finally, the shin pads. Nobody takes their socks off before their shoes. A similar reversal of procedures occurs with equivalent equations.


The order in which things are done matters in both sports and maths.

Here are three equivalent equations.

$$\begin{tikzcd}
x = 3 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
2x = 6 \arrow[d, bend right, "+4"'] \arrow[d, bend left, "+4"] \\
2x + 4 = 10
\end{tikzcd}$$

We can undo the operations around x by doing the opposite operation in the reverse order.

$$\begin{tikzcd}
2x + 4 = 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
2x = 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x = 3
\end{tikzcd}$$

Because these equations are equivalent, this means that the solution to $2x + 4 = 10$ is $x = 3$. An advantage with this method is that solving equations by inspection can be very difficult if the solution is not just a small whole number.

Let's start: Attempting solutions

Georgia, Kartik and Lucas try to solve the equation $4x+ 8 = 40$. They present their attempted solutions below.

Georgia

$$\begin{tikzcd}
4x + 8 = 40 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
x + 8 = 10 \arrow[d, bend right, "-8"'] \arrow[d, bend left, "-8"] \\
x = 2
\end{tikzcd}$$

Kartik

$$\begin{tikzcd}
4x + 8 = 40 \arrow[d, bend right, "-8"'] \arrow[d, bend left, "+8"] \\
4x = 48 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
x = 12
\end{tikzcd}$$

Lucas

$$\begin{tikzcd}
4x + 8 = 40 \arrow[d, bend right, "-8"'] \arrow[d, bend left, "-8"] \\
4x = 32 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
x = 8
\end{tikzcd}$$

- Which of the students has the correct solution to the equation? Justify your answer by substituting each student's final answer.
- For each of the two students with the incorrect answer, explain the mistake they have made in their attempt to have equivalent equations.
- What operations would you do to both sides if the original equation was $7x – 10 = 11$?

Key ideas

- To solve an equation, find a simpler equation that is equivalent. Repeat this until the solution is found.
- A simpler equation can be found by applying the opposite operations in reverse order.
  e.g. for $5x + 2 = 17$, we have:

$$\begin{tikzcd} x \arrow[r, bend left, "\times 5"] & 5x \arrow[r, bend left, "+2"] & 5x+2 \end{tikzcd}$$

  So we solve the equation by ‘undoing’ them in reverse order.

$$\begin{tikzcd} 5x+2 \arrow[r, bend left, "-2"] & 5x \arrow[r, bend left, "\div 5"] & x \end{tikzcd}$$

  This gives the solution:

$$\begin{tikzcd} 5x+2=17 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 5x=15 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x=3 \end{tikzcd}$$

- A solution can be checked by substituting the value to see if the equation is true.
  e.g. LHS $= 5(3) + 2 = 17$ and RHS $= 17$.

Example 8 Solving one-step equations

Solve each of the following equations algebraically.

a $5x = 30$ **b** $17 = y - 21$ **c** $10 = \frac{q}{3}$

SOLUTION

EXPLANATION

a

$$\begin{tikzcd} 5x=30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x=6 \end{tikzcd}$$

So the solution is $x = 6$.

The opposite of × 5 is ÷ 5.
By dividing both sides by 5, we get an equivalent equation. Recall that $5x \div 5$ simplifies to x.

b

$$\begin{tikzcd} 17=y-21 \arrow[d, bend right, "+21"'] \arrow[d, bend left, "+21"] \\ 38=y \end{tikzcd}$$

So the solution is $y = 38$.

The opposite of -21 is $+21$.

Write the pronumeral on the LHS.

c

$$\begin{tikzcd} 10=\frac{q}{3} \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 30=q \end{tikzcd}$$

So the solution is $q = 30$.

Multiplying both sides by 3 gives an equivalent equation that is simpler. Note that $\frac{q}{3} \times 3 = q$.

Write the pronumeral on the LHS.

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Example 9 Solving two-step equations

Solve each of the following equations algebraically and check the solution.

a $7 + 4a = 23$ **b** $\frac{d}{3} - 2 = 4$ **c** $12 = 2(e + 1)$

SOLUTION

EXPLANATION

a

$$\begin{tikzcd} 7 + 4a = 23 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\ 4a = 16 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ a = 4 \end{tikzcd}$$

Check:

$$\begin{aligned} \text{LHS} &= 7 + 4a & \text{RHS} &= 23 \checkmark \\ &= 7 + 4 \times 4 \\ &= 7 + 16 \\ &= 23 \end{aligned}$$

At each step, try to make the equation simpler by applying an operation to both sides.

Choose the opposite operations based on $7 + 4a$:

$$a \xrightarrow{\times 4} 4a \xrightarrow{+7} 7 + 4a$$

Opposite operations: –7, then ÷ 4.

Check that our equation is true by substituting $a = 4$ back into the LHS and RHS. Both sides are equal, so $a = 4$ is a solution.

b

$$\begin{tikzcd} \frac{d}{3} - 2 = 4 \arrow[d, bend right, "+2"'] \arrow[d, bend left, "+2"] \\ \frac{d}{3} = 6 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ d = 18 \end{tikzcd}$$

Check:

$$\begin{aligned} \text{LHS} &= \frac{d}{3} - 2 & \text{RHS} &= 4 \checkmark \\ &= \frac{18}{3} - 2 \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

At each step, try to make the equation simpler by applying an operation to both sides.

The opposite of –2 is +2 and the opposite of ÷ 3 is × 3.

Check that our equation is true by substituting $d = 18$ back into the LHS and RHS. Both sides are equal, so $d = 18$ is a solution.

c

$$\begin{tikzcd} 12 = 2(e + 1) \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ 6 = e + 1 \arrow[d, bend right, "-1"'] \arrow[d, bend left, "-1"] \\ 5 = e \end{tikzcd}$$

So the solution is $e = 5$.

Check:

$$\begin{aligned} \text{LHS} &= 12 & \text{RHS} &= 2(e + 1) \\ & & &= 2(5 + 1) \\ & & &= 2 \times 6 \\ & & &= 12 \checkmark \end{aligned}$$

At each step, try to make the equation simpler by applying an operation to both sides.
The opposite of × 2 is ÷ 2 and the opposite of + 1 is – 1.

Write the solution on the LHS.

Check that our equation is true by substituting $e = 5$ back into the equation.

Exercise 10D

1–4	2, 4	—

UNDERSTANDING

1 State whether each of the following equations is true or false.

a $x + 4 = 7$, if $x = 3$
b $b - 2 = 7$, if $b = 5$
c $7(d - 6) = d$, if $d = 7$
d $g + 5 = 3g$, if $g = 2$
e $f \times 4 = 20$, if $f = 3$

2 Consider the equation $7x = 42$.

a Copy and complete the diagram opposite.
b What is the solution to the equation $7x = 42$?

$$\begin{tikzcd} 7x = 42 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\ x = \_\_ \end{tikzcd}$$

3 The equations $g = 2$ and $12(g + 3) = 60$ are equivalent. What is the solution to the equation $12(g + 3) = 60$?

4 Copy and complete the following, showing which operation was used. Remember that the same operation must be used for both sides.

a
$$\begin{tikzcd} 5 + a = 30 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ a = 25 \end{tikzcd}$$

b
$$\begin{tikzcd} 10b = 72 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ b = 7.2 \end{tikzcd}$$

c
$$\begin{tikzcd} 12 = \frac{c}{4} \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ 48 = c \end{tikzcd}$$

d
$$\begin{tikzcd} 8 = c - 12 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ 20 = c \end{tikzcd}$$

5(½), 6, 7, 8(½)	5(½), 6, 7, 8(½), 9, 10	6–10(½)

FLUENCY

Example 8

5 Solve the following equations algebraically.

a $6m = 54$
b $g - 9 = 2$
c $s \times 9 = 81$
d $i - 9 = 1$
e $7 + t = 9$
f $8 + q = 11$
g $4y = 48$
h $7 + s = 19$
i $24 = j \times 6$
j $12 = l + 8$
k $1 = v \div 2$
l $19 = 7 + y$
m $k \div 5 = 1$
n $2 = y - 7$
o $8z = 56$
p $13 = 3 + t$
q $b \times 10 = 120$
r $p - 2 = 9$
s $5 + a = 13$
t $n - 2 = 1$

6 Copy and complete the following to solve the given equations algebraically.

a
$$\begin{tikzcd} 7a + 3 = 38 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ 7a = 35 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_ = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} 4b - 10 = 14 \arrow[d, bend right, "+10"'] \arrow[d, bend left, "+10"] \\ \_\_ = \_\_ \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_ = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 2(q + 6) = 20 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ q + 6 = \_\_ \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_ = \_\_ \end{tikzcd}$$

d
$$\begin{tikzcd} 5 = \frac{x}{10} + 3 \arrow[d, bend right, "-3"'] \arrow[d, bend left, "-3"] \\ \_\_ = \frac{x}{10} \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_ = \_\_ \end{tikzcd}$$

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FLUENCY

7 For each of these equations, state the first operation you would apply to both sides to solve it.

a $2x + 3 = 9$
b $4x - 7 = 33$
c $5(a + 3) = 50$
d $22 = 2(b - 17)$

Example 9

8 For each of the following equations:

- **i** Solve the equation algebraically, showing your steps.
- **ii** Check your solution by substituting the value into the LHS and RHS.

a $6f - 2 = 64$
b $\frac{k}{4} + 9 = 10$
c $5x - 4 = 41$
d $3(a - 8) = 3$
e $5k - 9 = 31$
f $\frac{a}{3} + 6 = 8$
g $2n - 8 = 14$
h $\frac{n}{4} + 6 = 8$
i $1 = 2g - 7$
j $30 = 3q - 3$
k $3z - 4 = 26$
l $17 = 9 + 8p$
m $10d + 7 = 47$
n $38 = 6t - 10$
o $9u + 2 = 47$
p $7 = 10c - 3$
q $10 + 8q = 98$
r $80 = 4(y + 8)$
s $4(q + 8) = 40$
t $7 + 6u = 67$

9 Solve the following equations, giving your solution as fractions.

a $4x + 5 = 8$
b $3 + 5k = 27$
c $22 = (3w + 7) \times 2$
d $10 = 3 \times (2 + x)$
e $3 = (8x + 1) \div 2$
f $3(x + 2) = 7$

10 Solve the following equations algebraically. (Note: The solutions for these equations are negative numbers.)

a $4r + 30 = 2$
b $2x + 12 = 6$
c $10 + \frac{t}{2} = 2$
d $\frac{y}{4} + 10 = 4$
e $-3x = 15$
f $4 = 2k + 22$
g $2x = -12$
h $5x + 20 = 0$
i $0 = 2x + 3$

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11, 12 | 12–14 | 13–15

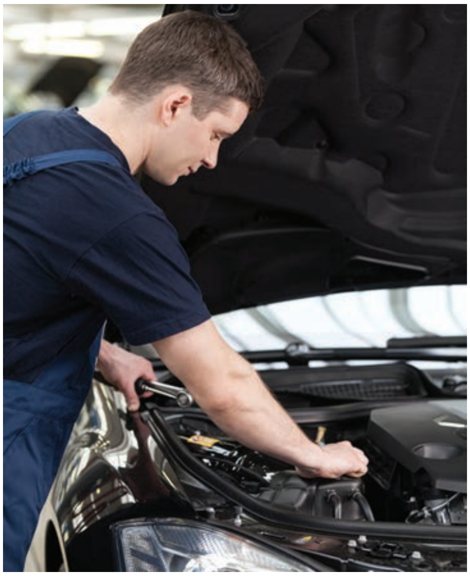
PROBLEM-SOLVING

11 For each of the following, write an equation and solve it algebraically.

- **a** The sum of x and 5 is 12.
- **b** The product of 2 and y is 10.
- **c** When b is doubled and then 6 is added, the result is 44.
- **d** 7 is subtracted from k. This result is tripled, giving 18.
- **e** 3 is added to one-quarter of b, giving a result of 6.
- **f** 10 is subtracted from half of k, giving a result of 1.

12 Danny gets paid \$12 per hour, plus a bonus of \$50 for each week. In one week he earned \$410.

- **a** Write an equation to describe this, using n for the number of hours worked.
- **b** Solve the equation algebraically and state the number of hours worked.



13 Jenny buys 12 pencils and 5 pens for the new school year. The pencils cost \$1.00 each.

- **a** If pens cost \$$x$ each, write an expression for the total cost.
- **b** The total cost was \$14.50. Write an equation to describe this.
- **c** Solve the equation algebraically, to find the total cost of each pen.
- **d** Check your solution by substituting your value of x into $12 + 5x$.

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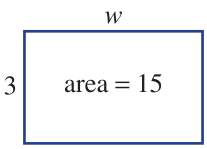
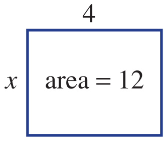
PROBLEM-SOLVING

14 Write equations and solve them algebraically to find the unknown value in each of the following diagrams.

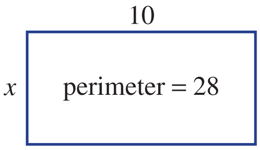
a



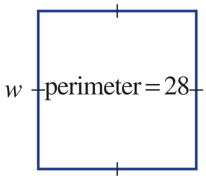
b



c



d



15 Solve the following equations algebraically.

a $7(3 + 5x) - 21 = 210$ **b** $(100x + 13) \div 3 = 271$ **c** $3(12 + 2x) - 4 = 62$

16	16, 17	16, 17, 18(½)

REASONING

16 Write five different equations that give a solution of $x = 2$.

17 **a** Show that $2x + 5 = 13$ and $5x = 20$ are equivalent by filling in the missing steps.

b Show that $10 + 2x = 20$ and $2(x - 3) = 4$ are equivalent.

c If two equations have the same solution, does this guarantee they are equivalent? Justify your answer.

d If two equations have different solutions, does this guarantee they are not equivalent? Justify your answer.

$$\begin{tikzcd}
2x + 5 = 13 \arrow[d, "-5"', "-5"] \\
\_\_ = \_\_ \arrow[d, "?"', "?"] \\
x = 4 \arrow[d, "?"', "?"] \\
5x = 20
\end{tikzcd}$$

18 Nicola has attempted to solve four equations. Describe the mistake she has made in each case.

a

$$\begin{tikzcd}
4x + 2 = 36 \arrow[d, "\div 4"', "\div 4"] \\
x + 2 = 9 \arrow[d, "-2"', "-2"] \\
x = 7
\end{tikzcd}$$

b

$$\begin{tikzcd}
3x + 10 = 43 \arrow[d, "-10"', "-10"] \\
3x = 33 \arrow[d, "-3"', "-3"] \\
x = 30
\end{tikzcd}$$

c

$$\begin{tikzcd}
2a + 5 = 11 \arrow[d, "-5"', "-5"] \\
2a = 16 \arrow[d, "\div 2"', "\div 2"] \\
a = 8
\end{tikzcd}$$

d

$$\begin{tikzcd}
7 + 12a = 43 \arrow[d, "-12"', "-12"] \\
7 + a = 31 \arrow[d, "-7"', "-7"] \\
a = 24
\end{tikzcd}$$

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Pronumerals on both sides	—	—	19

19 If an equation has a pronumeral on both sides, you can subtract it from one side and then use the same method as before. For example:

$$\begin{tikzcd} 5x+4=3x+10 \arrow[d, bend right, "-3x"'] \arrow[d, bend left, "-3x"] \\ 2x+4=10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x=6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x=3 \end{tikzcd}$$

Solve the following equations using this method.

a $5x + 2 = 3x + 10$

b $8x - 1 = 4x + 3$

c $5 + 12l = 20 + 7l$

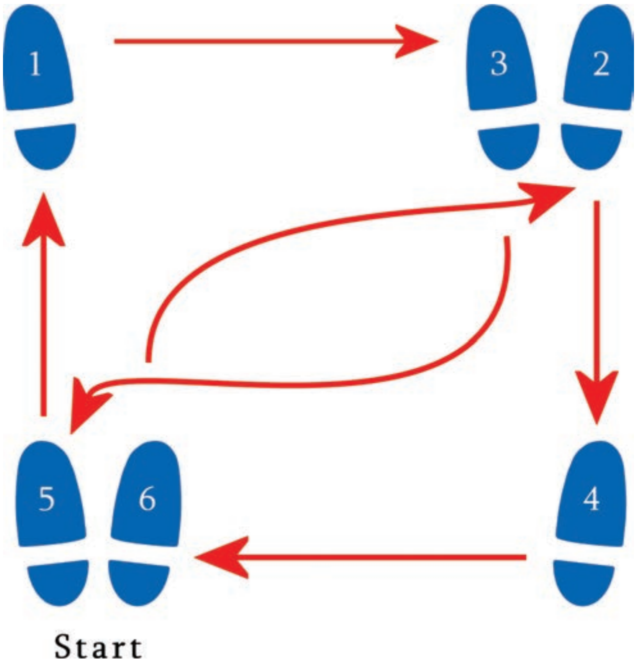
d $2 + 5t = 4t + 3$

e $12s + 4 = 9 + 11s$

f $9b - 10 = 8b + 9$

g $5j + 4 = 10 + 2j$

h $3 + 5d = 6 + 2d$



Just like dance steps, a strict order must be followed when solving equations algebraically.

10E Equations with fractions EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

Solving equations that involve fractions is straightforward once we recall that, in algebra, $\frac{a}{b}$ means $a \div b$. This means that if we have a fraction with b on the denominator, we can multiply both sides by b to get a simpler, equivalent equation.

Let's start: Fractional differences

Consider these three equations.

a $\frac{2x+3}{5} = 7$ **b** $\frac{2x}{5} + 3 = 7$ **c** $2\left(\frac{x}{5}\right) + 3 = 7$

- Solve each of them (by inspection or algebraically).
- Compare your solutions with those of your classmates.
- Why do two of the equations have the same solution?

Key ideas

- Recall that $\frac{a}{b}$ means $a \div b$.
- To solve an equation that has a fraction on one side, multiply *both* sides by the denominator.

$$\begin{tikzcd} \frac{x}{5} = 4 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ x = 20 \end{tikzcd}$$

- If neither side of an equation is a fraction, do not multiply by the denominator.

$$\begin{tikzcd} \frac{x}{3} + 5 = 8 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ \ldots \end{tikzcd}$$ ✗ Do not do this

$$\begin{tikzcd} \frac{x}{3} + 5 = 8 \arrow[d, bend right, "-5"'] \arrow[d, bend left, "-5"] \\ \frac{x}{3} = 3 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = 9 \end{tikzcd}$$ ✓ Do this

- The expressions $\frac{x}{3} + 2$ and $\frac{x+2}{3}$ are different, as demonstrated in these flow charts.

$$x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+2} \frac{x}{3} + 2 \quad \text{vs} \quad x \xrightarrow{+2} x + 2 \xrightarrow{\div 3} \frac{x+2}{3}$$

Example 10 Solving equations with fractions

Solve each of the following equations.

a $\frac{a}{7} = 3$

b $\frac{5y}{3} = 10$

c $\frac{3x}{4} + 7 = 13$

d $\frac{2x - 3}{5} = 3$

SOLUTION	EXPLANATION
a $$\begin{tikzcd} \frac{a}{7} = 3 \arrow[d, bend right, "\times 7"'] \arrow[d, bend left, "\times 7"] \\ a = 21 \end{tikzcd}$$	Multiplying both sides by 7 removes the denominator of 7.
b $$\begin{tikzcd} \frac{5y}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 5y = 30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ y = 6 \end{tikzcd}$$	Multiplying both sides by 3 removes the denominator of 3. The equation $5y = 30$ can be solved normally.
c $$\begin{tikzcd} \frac{3x}{4} + 7 = 13 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\ \frac{3x}{4} = 6 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ 3x = 24 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ x = 8 \end{tikzcd}$$	First, we subtract 7 because we do not have a fraction by itself on the LHS. Once there is a fraction by itself, multiply by its denominator (in this case, 4) and solve the equation $3x = 24$ as you would normally.
d $$\begin{tikzcd} \frac{2x - 3}{5} = 3 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ 2x - 3 = 15 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\ 2x = 18 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x = 9 \end{tikzcd}$$	First, multiply both sides by 5 to remove the denominator. Then solve the equation $2x - 3 = 15$ as you would normally.

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Exercise 10E

1–4 | 4 | —

UNDERSTANDING

1 Classify each of the following as true or false.

a $\frac{a}{5}$ means $a \div 5$.

b $\frac{q}{12}$ means $12 \div q$.

c $\frac{4+a}{3}$ means $(4+a) \div 3$.

d $\frac{4+a}{3}$ means $4 + (a \div 3)$.

e $\frac{12+3q}{4}$ means $(12+3q) \div 4$.

f $2 + \frac{x}{5}$ means $(2+x) \div 5$.

2 **a** If $x = 10$, find the value of $\frac{x+4}{2}$.

b If $x = 10$, find the value of $\frac{x}{2} + 4$.

c State whether the following is true or false: $\frac{x+4}{2}$ and $\frac{x}{2} + 4$ are equivalent expressions.

Example 10a

3 Fill in the missing steps to solve each of these equations.

a

$$\begin{tikzcd}
\frac{b}{4} = 11 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\
b = \_\_
\end{tikzcd}$$

b

$$\begin{tikzcd}
\frac{d}{5} = 3 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\
\_\_ = \_\_
\end{tikzcd}$$

c

$$\begin{tikzcd}
\frac{h}{4} = 7 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\
\_\_ = \_\_
\end{tikzcd}$$

d

$$\begin{tikzcd}
\frac{p}{13} = 2 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\
\_\_ = \_\_
\end{tikzcd}$$

4 For each of the following equations (**a** to **d**), choose the appropriate first step (i.e. **A** to **D**) needed to solve it.

a $\frac{x}{3} = 10$

b $\frac{x}{3} + 2 = 5$

c $\frac{x-3}{2} = 1$

d $\frac{x}{2} - 3 = 5$

A Multiply both sides by 2.

B Multiply both sides by 3.

C Subtract 2 from both sides.

D Add 3 to both sides.

5–6(½) | 5–7(½) | 5–7(½)

FLUENCY

Example 10b

5 Solve the following equations algebraically.

a $\frac{m}{6} = 2$

b $\frac{c}{9} = 2$

c $\frac{s}{8} = 2$

d $\frac{r}{5} = 2$

e $\frac{3u}{5} = 12$

f $\frac{2y}{9} = 4$

g $\frac{5x}{2} = 10$

h $\frac{3a}{8} = 6$

i $\frac{4h}{5} = 8$

j $\frac{3j}{5} = 9$

k $\frac{5v}{9} = 5$

l $\frac{3q}{4} = 6$

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FLUENCY

Example 10c, d

6 Solve the following equations algebraically. Check your solutions using substitution.

a $\frac{h+15}{12}=2$ **b** $\frac{y+5}{11}=1$ **c** $\frac{j+8}{11}=1$ **d** $\frac{b-2}{2}=1$

e $\frac{7u-12}{9}=1$ **f** $14+\frac{4t}{9}=18$ **g** $1=\frac{w+5}{11}$ **h** $1=\frac{4r-13}{3}$

i $\frac{2q}{9}+2=4$ **j** $\frac{s+2}{5}=1$ **k** $\frac{3l}{2}+9=21$ **l** $12=\frac{2z}{7}+10$

m $1=\frac{v-4}{7}$ **n** $\frac{f-2}{7}=1$ **o** $9=4+\frac{5x}{2}$ **p** $3=\frac{7+4d}{9}$

q $\frac{7n}{5}+14=21$ **r** $\frac{7m+7}{4}=21$ **s** $3=\frac{7p}{4}-11$ **t** $\frac{4a-6}{5}=6$

7 Solve the following equations algebraically. (Note: The solutions to these equations are negative numbers.)

a $\frac{y+4}{3}=1$ **b** $\frac{a}{10}+2=1$ **c** $\frac{2x}{5}+10=6$ **d** $\frac{x}{4}+12=0$

e $0=12+\frac{2u}{5}$ **f** $\frac{3y}{5}+8=2$ **g** $1=\frac{-2u-3}{5}$ **h** $-2=\frac{4d}{5}+2$

8	8, 9	9, 10

PROBLEM-SOLVING

8 In each of the following cases, write an equation and solve it to find the number.

a A number, t, is halved and the result is 9.

b One-third of q is 14.

c A number, r, is doubled and then divided by 5. The result is 6.

d 4 is subtracted from q and this is halved, giving a result of 3.

e 3 is added to x and the result is divided by 4, giving a result of 2.

f A number, y, is divided by 4 and then 3 is added, giving a result of 5.

9 A group of five people go out for dinner and then split the bill evenly. They each pay $31.50.

a If b represents the total cost of the bill, in dollars, write an equation to describe this situation.

b Solve this equation algebraically.

c What is the total cost of the bill?

10 Lee and Theo hired a tennis court for a cost of $\$x$, which they split evenly. Out of his own pocket, Lee also bought some tennis balls for $5.

a Write an *expression* for the total amount of money that Lee paid.

b Given that Lee paid $11 in total, write an equation and solve it to find the total cost of hiring the court.

c State how much money Theo paid for his share of hiring the tennis court.

REASONING	11	11, 12	11–13

11 **a** Explain, in one sentence, the difference between $\frac{2x+3}{5}$ and $\frac{2x}{5}+3$.

b What is the first operation you would apply to both sides to solve $\frac{2x+3}{5}=7$?

c What is the first operation you would apply to both sides to solve $\frac{2x}{5}+3=7$?

d Are there any values of x for which $\frac{2x+3}{5}$ and $\frac{2x}{5}+3$ are equal to each other?

12 Sometimes an equation's solution will be a fraction. For example, $2x=1$ has the solution $x=\frac{1}{2}$.

a Give another equation that has $x=\frac{1}{2}$ as its solution.

b Find an equation that has the solution $x=\frac{5}{7}$.

c Could an equation have the solution $x=-\frac{1}{2}$? Justify your answer.

13 Dividing by 2 and multiplying by $\frac{1}{2}$ have the same effect.

For example, $6\div 2=3$ and $6\times\frac{1}{2}=3$.

a Show how each of these equations can be solved algebraically.

i $\frac{x}{2}=5$

ii $\frac{1}{2}\times x=5$

b Solve the two equations $\frac{x+4}{3}=10$ and $\frac{1}{3}(x+4)=10$ algebraically, showing the steps you would use at each stage clearly.

c How does rewriting divisions as multiplications change the first step when solving equations?

Fractional answers

ENRICHMENT	—	—	14

14 Solve each of the following equations, giving your answers as a fraction.

a $\frac{2x+5}{4}=3$

b $\frac{3x-4}{6}=\frac{3}{4}$

c $\left(\frac{7+2x}{4}\right)\times 3=10$

d $\frac{1}{2}=\frac{3x-1}{5}$

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Progress quiz

10A **1** Classify each of the following as an equation (E) or not an equation (N).

a $4 + w = 9$ **b** $x + 12$ **c** $3 + 10 = 17$ **d** $20 + 6$

10A **2** If $x = 4$, state whether each of these equations is true (T) or false (F).

a $3x - 4 = 8$ **b** $5x + 6 = 25$ **c** $9 - x = x + 1$ **d** $2x = 5x - 8$

10A **3** Write an equation for each of the following. (You do not need to solve the equations you write.)

a The sum of 5 and m is equal to 12.

b When d is doubled, the result is 24.

c The product of 9 and x is 72.

d Canaries cost \$$c$ each and budgies cost \$$b$ each. Three canaries and four budgies cost \$190.

10B **4** Solve the following equations by inspection.

a $a - 5 = 4$ **b** $36 = x \times 9$ **c** $m \div 5 = 6$ **d** $3a + 1 = 7$

10C **5** For each equation, find the result of applying the given operation to both sides. You *do not* need to solve the equation.

a $4x = 7$ (multiply by 5) **b** $2a + 5 = 21$ (subtract 5)

10C **6** State the operation applied to get from the first to the second equation in each of the following.

a $? \left(\begin{aligned} 4x - 8 &= 12 \\ 4x &= 20 \end{aligned} \right) ?$ **b** $? \left(\begin{aligned} 18 &= 6x \\ 3 &= x \end{aligned} \right) ?$ **c** $? \left(\begin{aligned} 3a &= 5 \\ 6a &= 10 \end{aligned} \right) ?$

10D **7** Solve each of the following equations algebraically.

a $a + 7 = 19$ **b** $w - 4 = 27$ **c** $k \div 9 = 7$ **d** $5m = 40$

10D **8** Solve each of the following equations algebraically and check your solution.

a $12x = 132$ **b** $8 + 3a = 29$ **c** $42 = 8m - 6$ **d** $100 = 5(y + 12)$

10E **Ext** **9** Solve the following equations algebraically.

a $\frac{c}{4} = 6$ **b** $\frac{3a}{4} = 6$ **c** $\frac{h + 16}{4} = 7$ **d** $15 = \frac{2m}{5} - 7$

10E **Ext** **10** Write an equation for each of these situations and solve it to find the number.

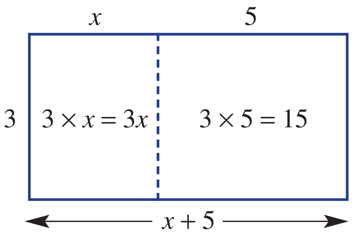
a A number, m, is divided by 2 and then 5 is added, giving a result of 11.

b 7 is added to x and the result is halved, giving a result of 9.

10F Equations with brackets EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

Recall from Chapter 5 that expressions with brackets can be expanded using the observation shown at right about rectangles' areas.



So $3(x + 5)$ is equivalent to $3x + 15$.

When solving $3(x + 5) = 21$, we could first divide both sides by 3 or we could first expand the brackets, giving $3x + 15 = 21$, and then subtract 15. For some equations, the brackets must be expanded first.

Let's start: Removing brackets

a Draw two rectangles, with areas $4(x + 3)$ and $5(x + 2)$.

b Use these to show that $4(x + 3) + 5(x + 2)$ is equivalent to $9x + 22$.

c Can you solve the equation $4(x + 3) + 5(x + 2) = 130$?

Key ideas

- To expand brackets, use the **distributive law**, which states that:

  $a(b + c) = ab + ac$ e.g. $3(x + 4) = 3x + 12$

  $a(b - c) = ab - ac$ e.g. $4(b - 2) = 4b - 8$
- **Like terms** are terms that contain exactly the same pronumerals and can be collected to simplify expressions. For example, $3x + 4 + 2x$ can be simplified to $5x + 4$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

Example 11 Expanding brackets

Expand each of the following.

a $4(x + 3)$ **b** $6(q - 4)$ **c** $5(3a + 4)$

SOLUTION	EXPLANATION
a $4(x + 3) = 4x + 12$	Using the distributive law: $4(x + 3) = 4x + 12$
b $6(q - 4) = 6q - 24$	Using the distributive law: $6(q - 4) = 6q - 24$
c $5(3a + 4) = 15a + 20$	Using the distributive law: $5(3a + 4) = 5 \times 3a + 20$

Example 12 Simplifying expressions with like terms

Simplify each of these expressions.

a $2x + 5 + x$ **b** $3a + 8a + 2 - 2a + 5$

SOLUTION	EXPLANATION
a $2x + 5 + x = 3x + 5$	Like terms are $2x$ and x. These are combined to give $3x$.
b $3a + 8a + 2 - 2a + 5 = 9a + 7$	Like terms are combined: $3a + 8a - 2a = 9a$ and $2 + 5 = 7$

Example 13 Solving equations by expanding brackets

Solve each of these equations by expanding brackets first.

a $3(x + 2) = 18$ **b** $7 = 7(4q - 3)$ **c** $3(b + 5) + 4b = 29$

SOLUTION **EXPLANATION**

a

$$3(x + 2) = 18$$

Use the distributive law to expand the brackets.

$$\begin{tikzcd} 3x + 6 = 18 \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\ 3x = 12 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ x = 4 \end{tikzcd}$$

Solve the equation by performing the same operations to both sides.

b

$$7 = 7(4q - 3)$$

Use the distributive law to expand brackets.

$$\begin{tikzcd} 7 = 28q - 21 \arrow[d, bend right, "+21"'] \arrow[d, bend left, "+21"] \\ 28 = 28q \arrow[d, bend right, "\div 28"'] \arrow[d, bend left, "\div 28"] \\ 1 = q \end{tikzcd}$$

Solve the equation by performing the same operations to both sides.

So $q = 1$ is the solution.

c

$$3(b + 5) + 4b = 29$$
$$3b + 15 + 4b = 29$$

$$\begin{tikzcd} 7b + 15 = 29 \arrow[d, bend right, "-15"'] \arrow[d, bend left, "-15"] \\ 7b = 14 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\ b = 2 \end{tikzcd}$$

Use the distributive law to expand brackets.
Collect like terms to simplify the expression.
Solve the equation by performing the same operations to both sides.

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Exercise 10F

1–3	2, 3	—

UNDERSTANDING

1 Which of the following is the correct expansion of $5(x + 2)$?

A $5 \times x + 2$ **B** $5x + 2$ **C** $5x + 10$ **D** $10x$

2 Fill in the missing numbers.

a $3(x + 2) = 3x + \square$ **b** $4(3a + 1) = 12a + \square$

c $2(b + 1) = \square b + 2$ **d** $6(2c + 3) = \square c + 18$

3 Answer true or false to each of the following.

a $4x + 3x$ can be simplified to $7x$. **b** $2a + 4b$ can be simplified to $6ab$.

c $6p - 4p$ can be simplified to $2p$. **d** $7a + 3 + 2a$ can be simplified to $9a + 3$.

e $2b + 3$ can be simplified to $5b$. **f** $20x - 12x + 3y$ can be simplified to $32x + 3y$.

4–8(½)	4–9(½)	4–9(½)

FLUENCY

Example 11

4 Expand each of the following.

a $2(x + 1)$ **b** $5(2b + 3)$ **c** $2(3a - 4)$ **d** $5(7a + 1)$

e $4(3x + 4)$ **f** $3(8 - 3y)$ **g** $12(4a + 3)$ **h** $2(u - 4)$

Example 12

5 Simplify these expressions by collecting like terms.

a $3a + a + 2$ **b** $5 + 2x + x$ **c** $2b - 4 + b$ **d** $5a + 12 - 2a$

e $5x + 3 + x$ **f** $3k + 6 - 2k$ **g** $7 + 2b - 1$ **h** $6k - k + 1$

Example 13a

6 Solve the following equations by expanding the brackets first. Check your solutions by substituting them in.

a $2(10 + s) = 32$ **b** $2(5 + l) = 12$ **c** $3(p - 7) = 6$ **d** $8(y + 9) = 72$

e $8(4 + q) = 40$ **f** $7(p + 7) = 133$ **g** $8(m + 7) = 96$ **h** $22 = 2(b + 5)$

i $25 = 5(2 + p)$ **j** $63 = 7(p + 2)$ **k** $9(y - 6) = 27$ **l** $2(r + 8) = 32$

Example 13b

7 Solve these equations by expanding the brackets first.

a $6(3 + 2d) = 54$ **b** $8(7x - 7) = 56$ **c** $3(2x - 4) = 18$ **d** $27 = 3(3 + 6e)$

e $44 = 4(3a + 8)$ **f** $30 = 6(5r - 10)$ **g** $10 = 5(9u - 7)$ **h** $3(2q - 9) = 39$

Example 13c

8 Solve the following equations by first expanding the brackets. You will need to simplify the expanded expressions by collecting like terms.

a $5(4s + 4) + 4s = 44$ **b** $5i + 5(2 + 2i) = 25$ **c** $3(4c - 5) + c = 50$

d $3(4 + 3v) - 4v = 52$ **e** $5(4k + 2) + k = 31$ **f** $4q + 6(4q - 4) = 60$

g $40 = 4y + 6(2y - 4)$ **h** $44 = 4f + 4(2f + 2)$ **i** $40 = 5t + 6(4t - 3)$

9 Solve the following equations algebraically. (Note: The answers to these equations are negative numbers.)

a $3(u + 7) = 6$ **b** $2(k + 3) = 0$ **c** $6(p - 2) = -18$

d $16 = 8(q + 4)$ **e** $5(2u + 3) = 5$ **f** $3 = 2(x + 4) + 1$

g $4(p - 3) + p = -32$ **h** $3(r + 4) + 2r + 40 = 2$ **i** $2(5 + x) - 3x = 15$

10(½)	10	10

PROBLEM-SOLVING

10 For each of the following problems:

- **i** Write an equation.
- **ii** Solve your equation by first expanding any brackets.

a 5 is added to x and then this is doubled, giving a result of 14.

b 3 is subtracted from q and the result is tripled, giving a final result of 30.

c A number, x, is doubled and then 3 is added. This number is doubled again to get a result of 46.

d 4 is added to y and this is doubled. Then the original number, y, is subtracted, giving a result of 17.

11	11, 12	12, 13(½)

REASONING

11 For each of the following equations, explain why there are no solutions by first simplifying the LHS.

a $2(x + 5) - 2x = 7$

b $3(2x + 1) + 6(2 - x) = 4$

c $4(2x + 1) - 10x + 2(x + 1) = 12$

12 Consider the equation $2(3x + 4) - 6x + 1 = 9$.

a Show that this equation is true if $x = 0$.

b Show that this equation is true if $x = 3$.

c Explain why this equation is always true.

d Give an example of another equation involving brackets that is always true, where one side contains a variable but the other side is just a number.

13 For equations like $4(3x + 2) = 44$, you have been expanding the brackets first. Since $4(3x + 2) = 44$ is the same as $4 \times (3x + 2) = 44$, you can just start by dividing both sides by 4. Without expanding brackets, solve the equations in Question **6** by dividing first.

Expanding multiple brackets

—	—	14

ENRICHMENT

14 Solve each of the following equations by expanding all sets of brackets.

a $6(2j - 4) + 4(4j - 3) = 20$

b $3(4a + 5) + 5(1 + 3a) = 47$

c $2(5a + 3) + 3(2a + 3) = 63$

d $222 = 3(4a - 3) + 5(3a + 3)$

e $77 = 2(3c - 5) + 3(4c + 5)$

f $240 = 4(3d + 3) + 6(3d - 2)$

g $2(x + 3) + 4(x + 5) = 32$

h $4(x + 5) + 4(x - 5) = 24$

i $2(3x + 4) + 5(6x + 7) + 8(9x + 10) = 123$

10G Formulas

Interactive

Often, two or more pronumerals (or variables) represent related quantities. For example, the speed (s) at which a car travels and the time (t) it takes to arrive at its destination are related variable quantities. A formula is an equation that contains two or more pronumerals and shows how they are related.

Widgets

HOTsheets

Walkthroughs

Let's start: Fahrenheit and Celsius

In Australia, we measure temperature in degrees Celsius, whereas in the USA it is measured in degrees Fahrenheit. A formula to convert between them is $F = \frac{9C}{5} + 32$.

- At what temperature in degrees Fahrenheit does water freeze?
- At what temperature in degrees Fahrenheit does water boil?
- What temperature is 100° Fahrenheit in Celsius? Do you know what is significant about this temperature?

Key ideas

- The **subject** of an equation is a pronumeral that occurs by itself on the left-hand side. For example, T is the subject of $T = 4x + 1$.
- A **formula** or **rule** is an equation that contains two or more pronumerals (or variables), one of which is the subject of the equation.
- To use a formula, first substitute all the known values into the equation and then solve it to find the final value.

Example 14 Applying a formula

Consider the rule $k = 3b + 2$. Find the value of:

a k if $b = 5$ **b** k if $b = 10$ **c** b if $k = 23$

SOLUTION	EXPLANATION
a $k = 3 \times 5 + 2$ $= 17$	Substitute $b = 5$ into the equation.
b $k = 3 \times 10 + 2$ $= 32$	Substitute $b = 10$ into the equation.
c $\begin{array}{rcl} & 23 = 3b + 2 & \\ -2 & & -2 \\ & 21 = 3b & \\ \div 3 & & \div 3 \\ & 7 = b & \end{array}$ Therefore, $b = 7$.	Substitute $k = 23$ into the equation. Now solve the equation to find the value of b.

Example 15 Applying a formula involving three pronumerals

Consider the rule $Q = w(4 + t)$. Find the value of:

a Q if $w = 10$ and $t = 3$ **b** t if $Q = 42$ and $w = 6$

SOLUTION	EXPLANATION
a $Q = 10(4 + 3)$ $= 10 \times 7$ $= 70$	Substitute $w = 10$ and $t = 3$ to evaluate.
b $42 = 6(4 + t)$ $42 = 24 + 6t$ -24 -24 $18 = 6t$ $\div 6$ $\div 6$ $3 = t$ Therefore $t = 3$.	Substitute $Q = 42$ and $w = 6$. Expand the brackets and then solve the equation.

Exercise 10G

1, 2 | 2 | —

UNDERSTANDING

1 State whether each of the following equations is a rule (R) or not a rule (N).

a $2x + 5 = 10$ **b** $y = 3x + 5$ **c** $F = ma$

d $5 - q = 3$ **e** $w = 12 - v$ **f** $P = I + k - 3$

2 Substitute:

a $x = 3$ into the expression $5x$

b $x = 7$ into the expression $4(x + 2)$

c $y = 3$ into the expression $20 - 4y$

d $y = 10$ into the expression $\frac{y + 4}{7}$

3–5 | 3–6 | 3, 5, 6

FLUENCY

Example 14

3 Consider the rule $h = 2m + 1$. Find:

a h if $m = 3$

b h if $m = 4$

c m if $h = 17$. Set up an equation and solve it algebraically.

d Find m if $h = 21$. Set up an equation and solve it algebraically.

4 Consider the formula $y = 5 + 3x$. Find:

a y if $x = 6$ **b** x if $y = 17$ **c** x if $y = 26$

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10G

FLUENCY

Example 15

5 Consider the rule $A = q + t$. Find:

a A if $q = 3$ and $t = 4$ **b** q if $A = 5$ and $t = 1$ **c** t if $A = 3$ and $q = 3$

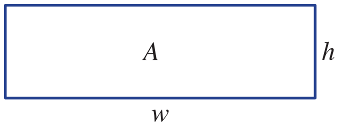
6 Consider the formula $G = 7x + 2y$. Find:

a G if $x = 3$ and $y = 3$ **b** x if $y = 2$ and $G = 11$ **c** y if $G = 31$ and $x = 3$

7	7, 8	8, 9

PROBLEM-SOLVING

7 The formula for the area of a rectangle is $A = w \times h$, where w is the rectangle's width and h is the rectangle's height.



a Set up and solve an equation to find the width of a rectangle with $A = 20$ and $h = 4$.

b A rectangle is drawn for which $A = 25$ and $w = 5$.

- **i** Set up and solve an equation to find h. **ii** What type of rectangle is this?

8 The perimeter for a rectangle is given by $P = 2(w + h)$. Find the:

a perimeter when $w = 3$ and $h = 5$ **b** value of h when $P = 10$ and $w = 2$

c area of a rectangle if its perimeter is 20 and width is 4

9 To convert between temperatures in Celsius and Fahrenheit the rule is $F = \frac{9C}{5} + 32$.

a Find F if $C = 20$. **b** Find the value of C if $F = 50$.

c Find the temperature in Celsius if it is 53.6° Fahrenheit.

d Marieko claims that the temperature in her city varies between 68° Fahrenheit and 95° Fahrenheit. What is the difference, in Celsius, between these two temperatures?

10	10	10, 11

REASONING

10 Rearranging a formula involves finding an equivalent equation that has a different variable on one side by itself. For example, the formula $S = 6g + b$ can be rearranged to make g by itself: Now we have a formula that can be used to find g once S and b are known.

$$\begin{tikzcd} S = 6g + b \arrow[d, bend right, "-b"'] \arrow[d, bend left, "-b"] \\ S - b = 6g \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ \frac{S - b}{6} = g \end{tikzcd}$$

a Rearrange $S = 5d + 3b$ to make a rule where d is by itself.

b Rearrange the formula $F = \frac{9C}{5} + 32$ to make C by itself.

c Rearrange the formula $Q = 3(x + 12) + x$ to make x by itself. (Hint: You will need to expand the brackets first.)

11 A taxi company charges different amounts of money based on how far the taxi travels and how long the passenger is in the car. Although the company has not revealed the formula it uses, some sample costs are shown below.

Distance (D) in km	Time (t) in minutes	Cost (C) in dollars
10	20	30
20	30	50

a Show that the rule $C = D + t$ is consistent with the values above.

b Show that the rule $C = 3D$ is not consistent with the values above.

c Show that the rule $C = 2D + 10$ is consistent with the values above.

d Try to find at least two other formulas that the taxi company could be using, based on the values shown.

AFL equations

—	—	12

12 In Australian Rules Football (AFL), the score, S, is given by $S = 6g + b$, where g is the number of goals scored and b is the number of 'behinds' (i.e. near misses).

a Which team is winning if the Abbotsford Apes have scored 11 goals ($g = 11$) and 9 behinds ($b = 9$), whereas the Brunswick Baboons have scored 12 goals and 2 behinds?

b The Camberwell Chimpanzees have scored 7 behinds and their current score is $S = 55$. Solve an equation algebraically to find how many goals the team has scored.

c In some AFL competitions, a team can score a 'supergoal', which is worth 9 points. If q is the number of supergoals that a team kicks, write a new formula for the team's score.

d For some rare combinations of goals and behinds, the score equals the product of g and b. For example, 4 goals and 8 behinds gives a score of $6 \times 4 + 8 = 32$, and $4 \times 8 = 32$. Find all the other values of g and b that make the equation $6g + b = gb$ true.

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10H Applications

Interactive

Widgets

HOTsheets

Walkthroughs

Our methods for solving equations can be applied to many situations in which equations occur.

Let's start: Stationery shopping

Sylvia bought 10 pencils and 2 erasers for $20.40. Edward bought 5 pencils and 3 erasers for $12.60.

- Use the information above to work out how much Karl will pay for 6 pencils and 5 erasers.
- Describe how you got your answer.
- Is there more than one possible solution?

Key ideas

- To solve a problem, follow these steps.
  - Define pronumerals to stand for unknown numbers. (e.g. let e = cost of an eraser in dollars.)
  - Write an equation to describe the problem.
  - Solve the equation algebraically if possible, or by inspection.
  - Ensure you answer the original question, and include the correct units (e.g. dollars, years, cm).

Example 16 Solving a problem using equations

The sum of Kate's current age and her age next year is 19. How old is Kate?

SOLUTION	EXPLANATION
Let k = Kate's current age in years.	Define a pronumeral to stand for the unknown number.
$k + (k + 1) = 19$	Write an equation to describe the situation. Note that $k + 1$ is Kate's age next year.
$$\begin{tikzcd} 2k + 1 = 19 \arrow[d, bend right, "-1"'] \arrow[d, bend left, "-1"] \\ 2k = 18 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ k = 9 \end{tikzcd}$$	Simplify the LHS and then solve the equation algebraically.
Kate is currently 9 years old.	Answer the original question.

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Exercise 10H

1, 2	1, 2	—

UNDERSTANDING

1 For each of the following problems, choose the best pronumeral definition.

a Problem: Monique's age next year is 12. How old is she now?

- **A** Let m = Monique's current age.
- **B** Let m = Monique.
- **C** Let $m = 12$.
- **D** Let m = Monique's age next year.
- **E** Let m = this year.

b Problem: Callan has 15 boxes, which weigh a total of 300 kg. How much does each box weigh?

- **A** Let $w = 15$.
- **B** Let $w = 300$.
- **C** Let w = the weight of one box.
- **D** Let w = the number of boxes.
- **E** Let w = the total weight.

c Problem: Jared's family has a farm with cows and sheep. The total number of animals is 200 and there are 71 cows. How many sheep are there?

- **A** Let x = the size of a sheep.
- **B** Let x = the total number of animals.
- **C** Let x = the number of sheep.
- **D** Let x = the number of cows.
- **E** Let x = Jared's age.

2 Solve the following equations algebraically.

a $5x = 30$ **b** $7a + 2 = 16$ **c** $2k - 3 = 15$

3–5	3–6	3–6

FLUENCY

Example 16

3 Launz buys a car and a trailer for a combined cost of \$40 000. The trailer costs \$2000.

a Define a pronumeral for the car's cost.

b Write an equation to describe the problem.

c Solve the equation algebraically.

d Hence, state the cost of the car.

4 Meghan buys 12 pens for a total cost of \$15.60.

a Define a pronumeral for the cost of one pen.

b Write an equation to describe the problem.

c Solve the equation algebraically.

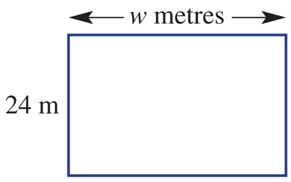
d Hence, state the cost of one pen.

FLUENCY

5 Jonas is paid \$17 per hour and gets paid a bonus of \$65 each week. One particular week he earned \$643.

- **a** Define a pronumeral for the number of hours Jonas worked.
- **b** Write an equation to describe the problem.
- **c** Solve the equation algebraically.
- **d** How many hours did Jonas work in that week?

6 This rectangular paddock has an area of 720 m^2.



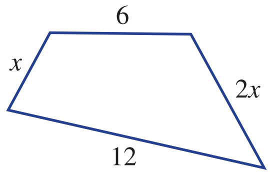
- **a** Write an equation to describe the problem, using w for the paddock's width.
- **b** Solve the equation algebraically.
- **c** How wide is the paddock?
- **d** What is the paddock's perimeter?

7, 8	8–10	11–14

PROBLEM-SOLVING

7 A number is doubled, then 3 is added and the result is doubled again. This gives a final result of 34. Set up and solve an equation to find the original number, showing all the steps clearly.

8 The perimeter of the shape shown is 30. Find the value of x.



9 Alexa watches some television on Monday, then twice as many hours on Tuesday, then twice as many hours again on Wednesday. If she watches a total of $10\frac{1}{2}$ hours from Monday to Wednesday, how much television did Alexa watch on Monday?

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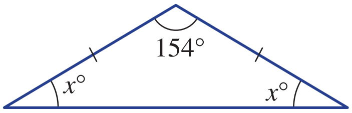
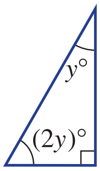
10 Marcus and Sara's combined age is 30. Given that Sara is 2 years older than Marcus, write an equation and find Marcus' age.

11 An isosceles triangle is shown below. Write an equation and solve it to find $x°$, the unknown angle. (Remember: The sum of angles in a triangle is 180°.)

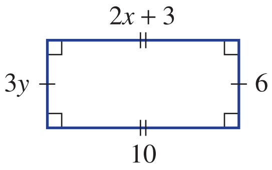


12 Find the value of y in the triangle shown here, by first writing an equation.



13 A rectangle has width w and height h. The perimeter and area of the rectangle are equal. Write an equation and solve it by inspection to find some possible values for w and h. (Note: There are many solutions to this equation. Try to find a few.)

14 Find the values of x and y in the rectangle shown.

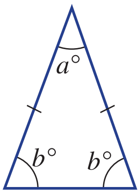


15	15	15, 16

15 If photocopying costs 35 cents a page and p is the number of pages photocopied, which of the following equations have possible solutions? Justify your answers. (Note: Fraction answers are not possible because you must still pay 35 cents even if you photocopy only part of a page.)

a $0.35p = 4.20$ **b** $0.35p = 2.90$ **c** $0.35p = 2.80$

16 Assume that an isosceles triangle is drawn so that each of its three angles is a whole number of degrees. Prove that the angle a must be an even number of degrees.

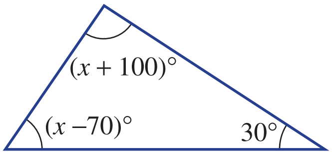


10H Strange triangles

—	—	17

17 Recall that the sum of angles in a triangle is 180°.

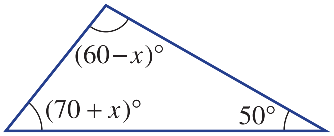
a David proposes the following triangle, which is not drawn to scale.



i Find the value of x.

ii Explain what makes this triangle impossible.

b Helena proposes the following triangle, which is also not drawn to scale.



i Explain why the information in the diagram is not enough to find x.

ii What are the possible values that x could take?

c Design a geometric puzzle, like the one in part **a**, for which the solution is impossible.

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Investigation

Theme parks

There are thousands of theme parks all over the world which offer a vast array of rides that are built to thrill. By surfing the internet, you can discover the longest, tallest, fastest and scariest rides. Although prices are kept competitive, theme parks need to make a profit so that they can maintain safety standards and continue to build new and more exciting rides.

Thrill World and Extreme Park are two theme parks. Both charge different prices for entry and for each ride. Their prices are:

- Thrill World: $20 entry and $5 per ride
- Extreme Park: $60 entry and $3 per ride

a Copy and complete the table below for each theme park. The total cost for the day includes the entry cost and cost of the rides.

Number of rides (n)	1	2	3	4	5	6	7	8	. . .	20	21	22	23	24	25
Thrill World total cost $$T$	$25								. . .						
Extreme Park total cost $$E$	$63								. . .						

b Write a formula for:

i T, the total cost, in dollars, for n rides at Thrill World

ii E, the total cost, in dollars, for n rides at Extreme Park

c For each of these thrill seekers, use an equation to calculate how many rides they went on.

i Amanda, who spent $105 at Thrill World **ii** George, who spent $117 at Extreme Park

d Refer to your completed table to determine the number of rides that will make the total cost for the day the same at each theme park.

e A third theme park, Fun World, decides to charge no entry fee but to charge $10 per ride. Find the minimum number of rides that you could go on at Fun World before it becomes cheaper at:

i Thrill World **ii** Extreme Park

f Fun World changes its pricing policy after it decides that it will be unable to compete by pricing its rides so high, so it decides to charge an entry fee and then make all rides free.

Investigate how much Fun World should charge to attract customers while still making profits that are similar to those of Thrill World and Extreme Park. Provide some mathematical calculations to support your conclusions.

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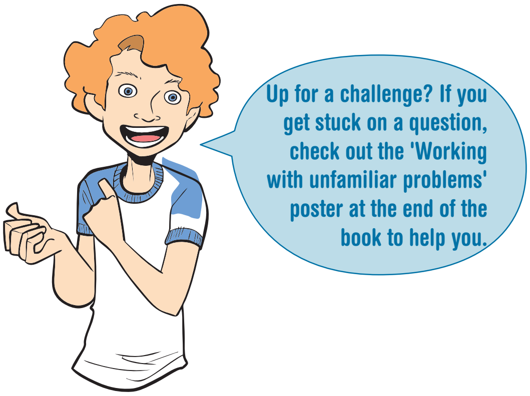
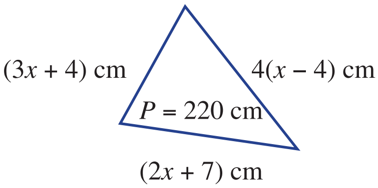
Problems and challenges



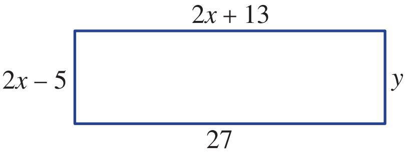
1 Find the unknown number in the following problems.

- **a** A number is added to half of itself and the result is 39.
- **b** A number is doubled, then tripled, then quadrupled. The result is 696.
- **c** One-quarter of a number is subtracted from 100 and the result is 8.
- **d** Half of a number is added to 47, and the result is the same as the original number doubled.
- **e** A number is increased by 4, the result is doubled and then 4 is added again to give an answer of 84.

2 A triangle has sides $2x + 7$, $3x + 4$ and $4(x - 4)$ measured in cm. If the perimeter of the triangle is 220 cm, find the difference between the longest and shortest sides.



3 For this rectangle, find the values of x and y and determine the perimeter. The sides are measured in cm.



4 A rectangular property has a perimeter of 378 m. It is divided up into five identical rectangular house blocks as shown in this diagram. Determine the area of each house block.

5 Find the values of a, b and c, given the clues:

$5(a + 2) + 3 = 38$ and $2(b + 6) - 2 = 14$ and $3a + 2b + c = 31$

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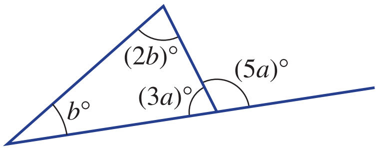
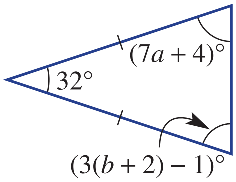
6 Find the values of a and b for each of these geometric figures.

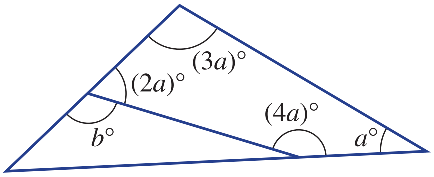
a



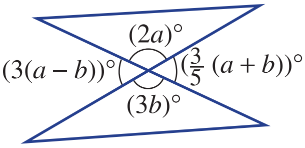
b



c



d



7 James's nanna keeps goats and chickens in her backyard. She says to James, 'There are 41 animals in this yard'. James says to his nanna, 'There are 134 animal legs in this yard'.
How many goats and how many chickens are in the paddock?

8 Five consecutive numbers have the middle number $3x - 1$. If the sum of these five numbers is 670 find the value of x and list the five numbers.

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Chapter summary

Equations

Solving an equation

Finding pronumeral values to make equation true
e.g. $15 + x = 20$
solution: $x = 5$

Formulas or rules are types of equations.
e.g. $F = ma$, $A = l \times w$, $P = 4x$

e.g.
$$\begin{array}{rcl} -5 & 3x+5=38 & -5\\ & 3x=33 & \\ \div 3 & x=11 & \div 3\end{array}$$

e.g.
$$\begin{array}{rcl} \times 4 & \frac{x}{4}=7 & \times 4\\ & x=28 & \end{array}$$

e.g.
$$\begin{array}{rcl} +2 & 5x-2=33 & +2\\ & 5x=35 & \\ \div 5 & x=7 & \div 5\end{array}$$

Equations with fractions are solved by multiplying by the denominator when fraction is by itself. (Ext)

e.g.
$$\begin{array}{rcl} -2 & 2+\frac{x}{7}=4 & -2\\ & \frac{x}{7}=2 & \\ \times 7 & x=14 & \times 7\end{array}$$

e.g.
$$\begin{array}{rcl} \times 4 & \frac{3x+9}{4}=6 & \times 4\\ -9 & 3x+9=24 & -9\\ & 3x=15 & \\ \div 3 & x=5 & \div 3\end{array}$$

Equivalent equations

are obtained by performing the same operation to both sides
e.g.
$$\begin{array}{rcl} +8 & 12+3x=5 & +8\\ & 20+3x=13 & \end{array}$$

To solve algebraically, use equivalent equations and opposite operations.

Main expression	First step
$3x+5$	-5
$12x$	$\div 12$
$5x-2$	$+2$
$\frac{x}{4}$	$\times 4$

Brackets should be expanded and like items collected.
Use the distributive law to expand. (Ext)

e.g. $4(x+2) = 4x + 8$

e.g.
$$\begin{array}{rcl} & 5(x+3)+2x=29 & \\ & 5x+15+2x=29 & \\ -15 & 7x+15=29 & -15\\ & 7x=14 & \\ \div 7 & x=2 & \div 7\end{array}$$

Applications

1 Use a pronumeral for the unknown.
2 Write an equation.
3 Solve the equation.
4 Answer the original question.

e.g. John's age in 3 years will be 15. How old is he now?

1 Let j = John's current age in years
2 $j + 3 = 15$
3 $\begin{array}{rcl} -3 & j+3=15 & -3\\ & j=12 & \end{array}$
4 John is currently 12 years old.

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Multiple-choice questions

10A **1** If $x = 3$, which one of the following equations is true?

A $4x = 21$ **B** $2x + 4 = 12$ **C** $9 - x = 6$

D $2 = x + 1$ **E** $x - 3 = 4$

10A **2** When 11 is added to the product of 3 and x, the result is 53. This can be written as:

A $3x + 11 = 53$ **B** $3(x + 11) = 53$ **C** $\frac{x}{3} + 11 = 53$

D $\frac{x + 11}{3} = 53$ **E** $3x - 11 = 53$

10B **3** Which of the following values of x make the equation $2(x + 4) = 3x$ true?

A 2 **B** 4 **C** 6

D 8 **E** 10

10C **4** The equivalent equation that results from subtracting 3 from both sides of $12x - 3 = 27$ is:

A $12x = 24$ **B** $12x - 6 = 24$ **C** $12x - 6 = 30$

D $9x - 3 = 24$ **E** $12x = 30$

10D **5** To solve $3a + 5 = 17$, the first step to apply to both sides is to:

A add 5 **B** divide by 3 **C** subtract 17

D divide by 5 **E** subtract 5

10D **6** The solution to $2t - 4 = 6$ is:

A $t = 1$ **B** $t = 3$ **C** $t = 5$

D $t = 7$ **E** $t = 9$

10E **Ext** **7** The solution of $\frac{2x}{7} = 10$ is:

A $x = 35$ **B** $x = 70$ **C** $x = 20$

D $x = 30$ **E** $x = 5$

10E **Ext** **8** The solution to the equation $10 = \frac{3p + 5}{2}$ is:

A $p = 5$ **B** $p = 20$ **C** $p = 15$

D $p = 7$ **E** $p = 1$

10F **Ext** **9** The solution of $3(u + 1) = 15$ is:

A $u = 5$ **B** $u = 4$ **C** $u = 11$

D $u = 6$ **E** $u = 3$

10G **10** A formula relating A, p and t is $A = 3p - t$. If $A = 24$ and $t = 6$, then p equals:

A 18 **B** 4 **C** 30

D 2 **E** 10

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Short-answer questions

10A **1** Classify each of the following equations as true or false.

- **a** $4 + 2 = 10 - 2$
- **b** $2(3 + 5) = 4(1 + 3)$
- **c** $5w + 1 = 11$, if $w = 2$
- **d** $2x + 5 = 12$, if $x = 4$
- **e** $y = 3y - 2$, if $y = 1$
- **f** $4 = z + 2$, if $z = 3$

10A **2** Write an equation for each of the following situations. You do not need to solve the equations.

- **a** The sum of 2 and u is 22.
- **b** The product of k and 5 is 41.
- **c** When z is tripled the result is 36.
- **d** The sum of a and b is 15.

10B **3** Solve the following equations by inspection.

- **a** $x + 1 = 4$
- **b** $x + 8 = 14$
- **c** $9 + y = 10$
- **d** $y - 7 = 2$
- **e** $5a = 10$
- **f** $\frac{a}{5} = 2$

10C **4** For each equation, find the result of applying the given operation to both sides and then simplify.

- **a** $2x + 5 = 13\ [-5]$
- **b** $7a + 4 = 32\ [-4]$
- **c** $12 = 3r - 3\ [+3]$
- **d** $15 = 8p - 1\ [+1]$

10D **5** Solve each of the following equations algebraically.

- **a** $5x = 15$
- **b** $r + 25 = 70$
- **c** $12p + 17 = 125$
- **d** $12 = 4b - 12$
- **e** $5 = 2x - 13$
- **f** $13 = 2r + 5$
- **g** $10 = 4q + 2$
- **h** $8u + 2 = 66$

10E **Ext** **6** Solve the following equations algebraically.

- **a** $\frac{3u}{4} = 6$
- **b** $\frac{8p}{3} = 8$
- **c** $3 = \frac{2x + 1}{3}$
- **d** $\frac{5y}{2} + 10 = 30$
- **e** $4 = \frac{2y + 20}{7}$
- **f** $\frac{4x}{3} + 4 = 24$

10F **Ext** **7** Expand the brackets in each of the following expressions.

- **a** $2(3 + 2p)$
- **b** $4(3x + 12)$
- **c** $7(a + 5)$
- **d** $9(2x + 1)$

10F **Ext** **8** Solve each of these equations by expanding the brackets first.

- **a** $2(x - 3) = 10$
- **b** $27 = 3(x + 1)$
- **c** $48 = 8(x - 1)$
- **d** $60 = 3y + 2(y + 5)$
- **e** $7(2z + 1) + 3 = 80$
- **f** $2(5 + 3q) + 4q = 40$

10F **Ext** **9** Consider the equation $4(x + 3) + 7x - 9 = 10$.

- **a** Is $x = 2$ a solution?
- **b** Show that the solution to this equation is *not* a whole number.

10F **Ext** **10**

- **a** Does $3(2x + 2) - 6x + 4 = 15$ have a solution? Justify your answer.
- **b** State whether the following are solutions to $5(x + 3) - 3(x + 2) = 2x + 9$.
  - **i** $x = 2$
  - **ii** $x = 3$

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10G **11** The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$, where h is the height of the trapezium,and a and b represent the parallel sides.

- **a** Set up and solve an equation to find the area of a trapezium with height 20 cm and parallel sides of 15 cm and 30 cm.
- **b** Find the height of a trapezium whose area is 55 cm^2 and has parallel sides 6 cm and 5 cm, respectively.

10G **12** Consider the rule $F = 3a + 2b$. Find:

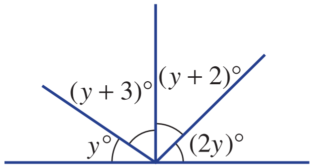
- **a** F if $a = 10$ and $b = 3$
- **b** b if $F = 27$ and $a = 5$
- **c** a if $F = 25$ and $b = 8$

10H **13** For each of the following problems, write an equation and solve it to find the unknown value.

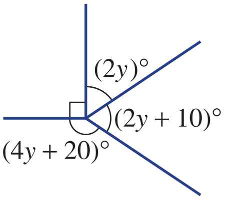
- **a** A number is added to three times itself and the result is 20. What is the number?
- **b** The product of 5 and a number is 30. What is the number?
- **c** Juanita's mother is twice as old as Juanita. The sum of their ages is 60. How old is Juanita?
- **d** A rectangle has a width of 21 cm and a perimeter of 54 cm. What is its length?

10H **14** Find the value of y for each of these figures.

a



b



Extended-response questions

1 Udhav's mobile phone plan charges a 15-cents connection fee and then 2 cents per second for every call.

- **a** How much does a 30-second call cost?
- **b** Write a rule for the total cost, C, in cents, for a call that lasts t seconds.
- **c** Use your rule to find the cost of call that lasts 80 seconds.
- **d** If a call cost 39 cents, how long did it last? Solve an equation to find t.
- **e** If a call cost $1.77, how long did it last?
- **f** On a particular day, Udhav makes two calls – the second one lasting twice as long as the first, with a total cost of $3.30. What was the total amount of time he spent on the phone?

2 Gemma is paid $$x$ per hour from Monday to Friday, but earns an extra $2 per hour during weekends. During a particular week, she worked 30 hours during the week and then 10 hours on the weekend.

- **a** If $x = 12$, calculate the total wages Gemma was paid that week.
- **b** Explain why her weekly wage is given by the rule $W = 30x + 10(x + 2)$.
- **c** Use the rule to find Gemma's weekly wage if $x = 16$.
- **d** If Gemma earns $620 in one week, find the value of x.
- **e** If Gemma earns $860 in one week, how much did she earn from Monday to Friday?

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