Lesson 16: Adding fractions

Lesson type: Diagnostic-exploratory lesson   Time Allocated: 1-1.5 hours

Overview:

Aim: For students to develop a visual model for adding fractions with unrelated denominators.

Concepts targeted:
- Fractions need to be "fair". If the pieces are to be given the same name, then the size of each piece needs to be the same.
- The "whole" needs to be the same when comparing fractions.
- The "whole" does not become bigger when we add in extra fractions. If we have more pieces than needed to make one whole, we have more than one, not a bigger "one".

Main task:
- Draw one third and one fifth in different ways and explore ways of adding these together in a visual model.

Resources:
You will need lots of A4 paper as well as scissors and coloured pens

Process:

Step 1: Diagnostic Question
Ask students to add $\frac{1}{3}$ and $\frac{1}{5}$ together by drawing pictures. Give them paper and coloured pens to use. They can also try simply folding paper to make their thirds and fifths, colouring in the appropriate parts.

Step 2: Confrontational Questions

For Misconception 1: See Lesson 7.

For Misconception 2:
Choose the piece that they nominated as a third and ask, "Is this piece still a third now that you have five pieces?" If they are still confused, ask, "So, are thirds and fifths the same size as each other?" If still confused, see Lesson 7.

For Misconception 3:
If they start with one shape to show one fraction, then add a **second shape** to show the other fraction ask, "If I had a really yummy chocolate cake and I wanted to eat a third of it, I could do it like you have shown. But now, I need to eat a fifth. I know - I'll go and buy a whole different cake and eat a fifth of that one instead. What do you think? Can I use two different cakes before I use up the first cake, or do I need to eat a fifth of the one that I started on?" Once they have realised that you can't use two separate wholes, go to the Leading Questions on the following page.
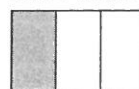
Common Misconceptions:

1. Students who draw unequal sized pieces for either thirds or fifths
   This is not thirds.

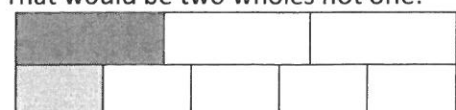2. Students who add extra pieces to their whole:
   Start with:

   But then add two more:

3. Students who make their whole bigger to show the other fraction. That would be two wholes not one:

   This is not $\frac{1}{3} + \frac{1}{5}$! Also, it is not $\frac{2}{8}$.

Step 3: Leading Questions

Give the kids a rectangular piece of paper and get them to use that to make thirds, then draw on the board the different ways that they suggest, making sure to colour in one third (horizontal and vertical ways of dividing the rectangle into thirds are very important – others are not so important at this stage). Then do the exact same process with fifths, but using a second colour. Ask the kids if there is a way that they can put the third and the fifth together on the same cake. See the steps below.
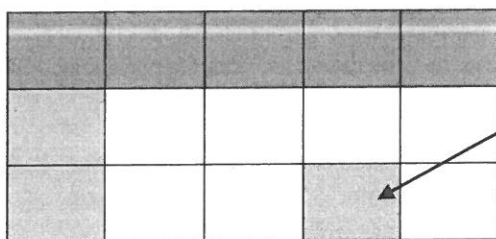
1. Have the students draw different ways to make $1/3$, then put these on the board:
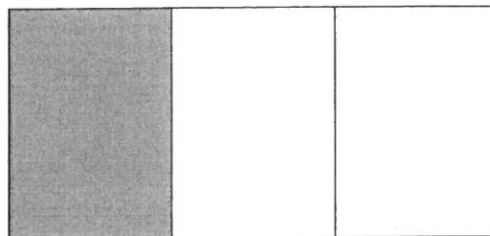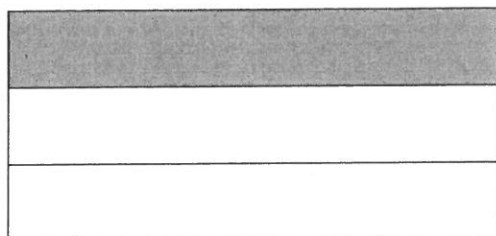
2. Repeat with ways to draw $1/5$:
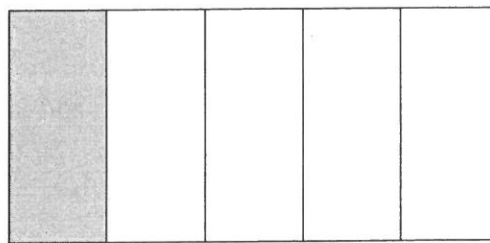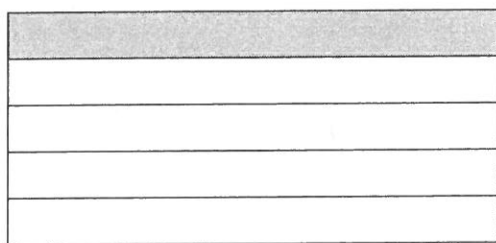
3. Ask students to put both together:

> Watch out for the overlap. Remember to account for it somewhere else.

Step 4: Generalising Questions

- Students who cannot develop an appropriate picture might still be able to solve a deductive pattern. Give them the answer ($8/15$) and see if they can solve the problem deductively instead (work out how to get $8/15$ from $1/3 + 1/5$).
- Use the same process to add other unit fractions, then progress to non-unit fractions (e.g. $2/5 + 1/3$)
- Extend the same process to fractions that would add to give more than one whole (e.g. $2/3 + 1/2$), realising that you will need to start on a new whole once one of the wholes is complete.

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