Technology Exploration GeoGebra

Equipment required: 1 brain, 1 computer with GeoGebra

Versions of this Exploration are available for other technologies in Pearson Reader.

Investigating angles on parallel lines

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are eleven icons called tools. When you hover the mouse over the arrow in the bottom right-hand corner of the tool icon the arrow turns red and the name of the tool appears. By clicking on this arrow a drop-down list of more tools appears. Instructions on how to use the tool will appear in the top right-hand corner of the screen.

1 Click on the View menu. Deselect ‘Axes’, ‘Grid’ and ‘Algebra View’. (Alternatively, you can right click in the space where the drawings appear, called the graphics view, to do the same for the ‘Axes’ and the ‘Grid’.)

2 Click on the Options menu. Select ‘Labelling’, then ‘New Points only’. Click on Options again, then select ‘Rounding’, and ‘0 Decimal Places’.

3 If a larger font is required, click on the options menu and select ‘Font Size’. Choose an appropriate size from the list provided.

Creating a pair of parallel lines and a transversal

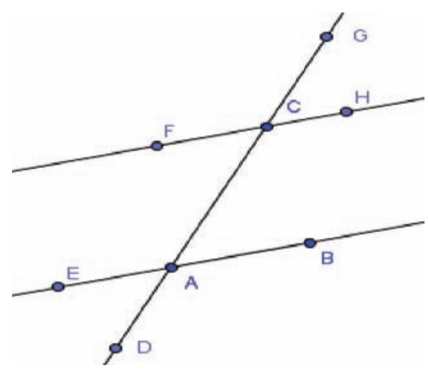
Parallel lines are lines that are equidistant from each other (the distance between the lines is the same all the way along them). A transversal is a line that cuts two or more other lines. (The lines cut by a transversal do not have to be parallel.)

4 Construct a line using the ‘Line Through two Points’ tool , then click on any two points on the page.

5 Click on the small arrow on the fourth tool from the left and select the ‘Parallel Line’ option . Create a parallel line by clicking on the line and then on anywhere above the line. (You know when you are about to select the line as it becomes darker and the cursor turns to an arrow when you hover over it.)

6 Create a transversal by selecting the ‘Line Through two Points’ tool , then clicking on point *A* and then on point *C*.

7 Select the ‘New Point’ tool . Use this to place points on the lines as shown below. Keep the points in the same order as we will refer to them in the following steps. They are placed in a clockwise direction from under point *A*. Points will be placed on the line only if the line gets darker when your mouse hovers over it. It does not matter if your transversal or your parallel lines slope in the opposite direction, just ensure that the points are placed in the same positions.



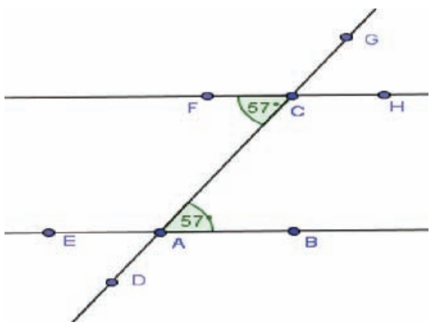
If you make a mistake, you can either:

- right click on the object and select ‘Delete’, or
- click the ‘Edit’ menu and select ‘Undo’, or
- press ‘ctrl’ + ‘z’.

Alternate angles and parallel lines

8 Click on the small arrow on the 8th tool from the left.

Select the 'Angle' tool . Click on point *B*, then *A* and then *C* (∠*BAC*). Notice that the vertex of the angle is the middle letter. Click on ∠*FCA* in the same way. Points must be selected in a clockwise order, so if you get the reflex angle, delete it and try selecting the points in the opposite order. Your diagram should look similar to the one below. You have just marked a pair of alternate angles. Alternate angles sit on opposite sides of a transversal.



9 Right click inside one of the marked angles and select 'Object Properties'. A pop-up box will appear. On the 'Colour' tab click on a different colour. With the pop-up box still open, click inside the other marked angle and select the same colour (the colour will be stored in the 'Recent' box on the right-hand side of the pop-up box). Close the pop-up box.

10 Click on the 'Angle' tool . Mark ∠*CAE* and ∠*ACH* as described in step **8**. You have just marked another pair of alternate angles.

11 Click on the 'Select' tool . (Pressing 'Escape' also takes you to the 'Select' tool.) Click on point *B* and drag it about.

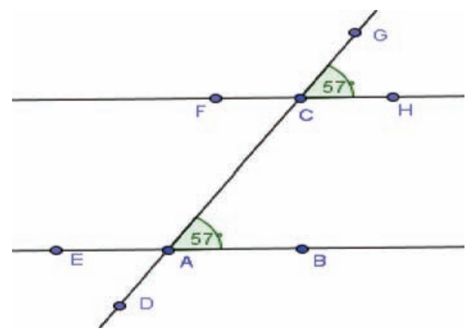
(a) You have just marked two pairs of alternate angles between a pair of parallel lines. What do you notice as you move the lines about?

(b) Copy and complete the statement: 'Alternate angles between parallel lines are ___________'.

12 Right click inside a marked angle and select delete, repeat for the other marked angles. (If some of the points have disappeared, move point *B* until they reappear.)

Corresponding angles and parallel lines

13 Click on the 'Angle' tool , then mark ∠*HCG* and then ∠*BAC* by clicking on the points. Your diagram should look similar to the one below. You have just marked a pair of corresponding angles. Corresponding angles sit on the same side of the transversal, one underneath the other.



14 Follow the process in step **9** to change the colour of both angles.

15 Repeat steps **13** and **14** for ∠*ACH* and ∠*DAB*, for ∠*EAD* and ∠*FCA*, and for ∠*CAE* and ∠*GCF*. Make sure that each pair of angles has a different colour. You have just marked three more pairs of corresponding angles.

16 Click on the 'Select' tool (or press escape). Click on point *B* and drag it about. Take notice of the angle sizes.

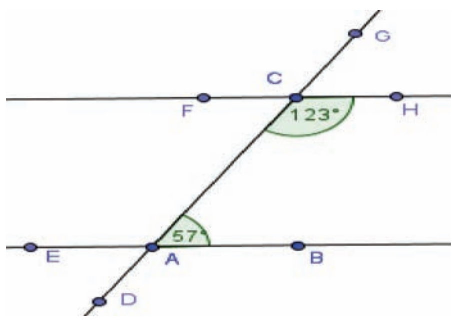
(a) What do you notice about each pair of corresponding angles as you move the lines about?

(b) Copy and complete the statement: 'Corresponding angles on parallel lines are ___________.'

17 Delete the marked angles as described in step **12**.

Co-interior angles and parallel lines

18 Click on the 'Angle' tool, then mark $\angle ACH$ and $\angle BAC$ by selecting the points. Your diagram should look similar to the one below. You have marked a pair of co-interior angles (also known as allied angles). Co-interior angles sit on the same side of the transversal and on the inside of the lines.



19 Follow the process in step **9** to change the colour of both angles.

20 Repeat steps **18** and **19** for $\angle CAE$ and $\angle FCA$ to mark another pair of co-interior angles.

21 Click on the 'Select' tool (or press escape), then click on point B and drag it about. Take note of the angle sizes.

- **(a)** What do you notice about each pair of co-interior angles as you move the line about?
- **(b)** Copy and complete the statement: 'Co-interior angles between parallel lines sum to _____°'.

22 Clear the angles as described in step **12**.

Taking it further

23 Select the 'New Point' tool and place a point on the interval between points A and C. This will be point I.

24 Click on the small arrow on the fourth tool from the left. Select the 'Parallel Line' tool. Create another parallel line by clicking on the line AB and then on the new point I.

25 Select the 'New Point' tool and place a point J on the new line to the left of point I and another point K to the right of point I.

26 Select the 'Angle' tool. Click on $\angle IAE$.

27 You may use the 'Angle' tool to assist you with the following.

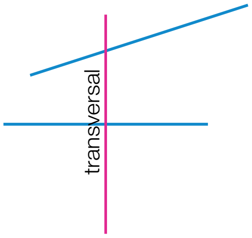
Find two angles that are alternate, two angles that are corresponding, and two angles that are co-interior to $\angle IAE$. Mark each of these angles using different colours to indicate the three different angle types. Use correct terminology to name the angles.

28 Using the 'Angle' tool, mark and colour all of the angles that are the same size.

- **(a)** Can you see any pairs of vertically opposite angles? Name as many pairs as you can.
- **(b)** Can you see any pairs of adjacent supplementary angles? Name as many pairs as you can.

8.4 Angles and parallel lines

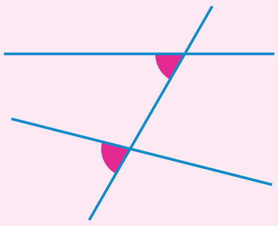
A **transversal** is a line that **intersects** (crosses or transverses) two or more other lines, as shown.



Angles formed when a transversal cuts two lines

When a transversal intersects two other lines, pairs of angles are formed. These angles are given special names.

Corresponding angles

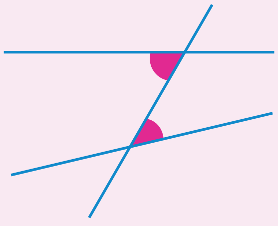


Both of these angles either lie above or below the two lines cut by the transversal and on the same side of it.

'Corresponding' means 'matching' so corresponding angles are in matching positions.

There are 4 pairs of corresponding angles formed when a transversal cuts two lines.

Alternate angles

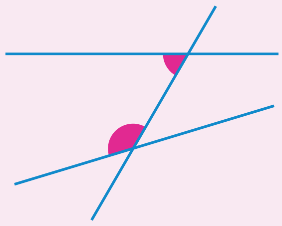


Both of these angles lie between the two lines cut by the transversal but are on opposite sides of it.

'Alternate' means 'swap' so alternate angles swap sides of the transversal.

There are two pairs of alternate angles formed when a transversal cuts two lines

Co-interior (allied) angles



Both of these angles lie between the lines cut by the transversal and on the same side of it.

'Co' means 'with' and 'interior' means 'inside' so co-interior angles are inside and on the same side of the transversal.

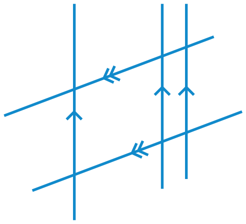
There are two pairs of co-interior angles formed when a transversal cuts two lines.

Co-interior angles are also called 'allied' angles.

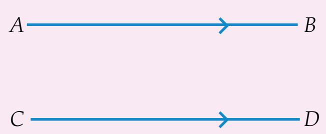
Parallel lines are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. The lines on a page of lined paper are parallel.

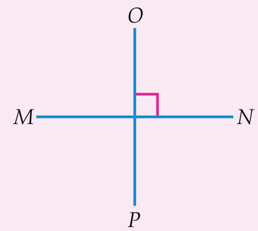
Lines that are parallel are marked with an arrow pointing in the same direction.

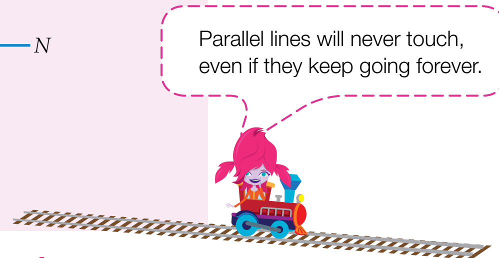
If more than one set of parallel lines appears in a diagram we use more than one arrow.



Perpendicular lines intersect at right angles.

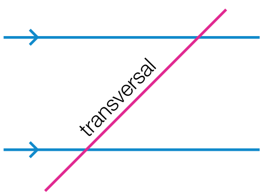


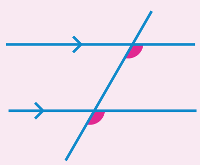
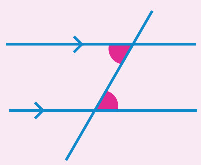
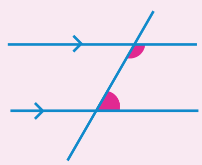




Properties of angles formed when a transversal cuts parallel lines

When parallel lines are crossed by a transversal, the pairs of angles on parallel lines described have special properties.



Corresponding angles	Alternate angles	Co-interior (allied) angles
		
Corresponding angles on parallel lines are *equal*.	Alternate angles on parallel lines are *equal*.	Co-interior (allied) angles on parallel lines are *supplementary*. They add to 180°.

The opposite of the above is also true. If corresponding or alternate angles are equal or if co-interior angles are supplementary when two lines are cut by a transversal, the two lines are parallel.

8.4 Angles and parallel lines

Navigator

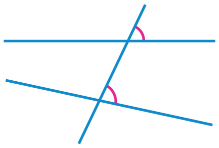
Q1, Q2, Q3, Q4, Q5, Q6 Columns 1 & 2, Q7, Q8 Column 1, Q9, Q10, Q13 Column 1, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 2, Q9, Q11, Q12, Q13 Column 2, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 3, Q11, Q12, Q13 Column 3, Q14, Q15

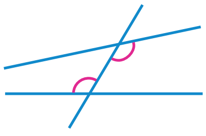
Answers page 669

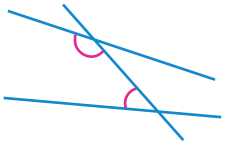
Equipment required: Protractor for Question **6 (a)**

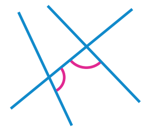
Fluency

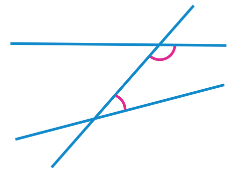
1 Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

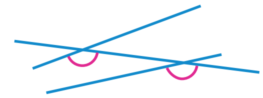
(a) 

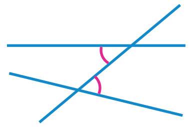
(b) 

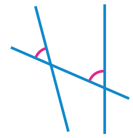
(c) 

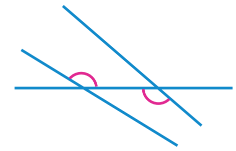
(d)


(e)


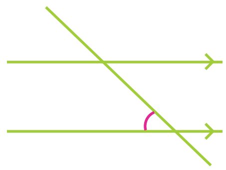
(f)


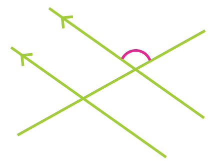
(g)


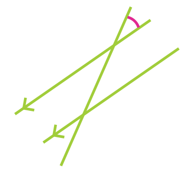
(h)


(i)


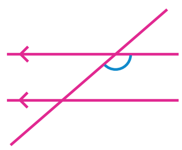
2 Copy each of the following and mark in an angle corresponding to the one shown.

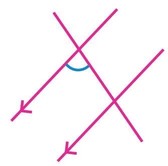
(a)


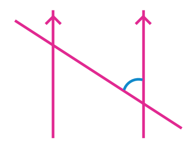
(b)


(c)


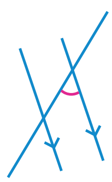
3 Copy each of the following and mark in an angle alternate to the one shown.

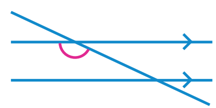
(a)


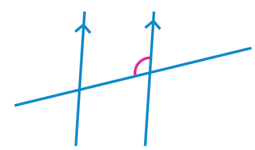
(b)


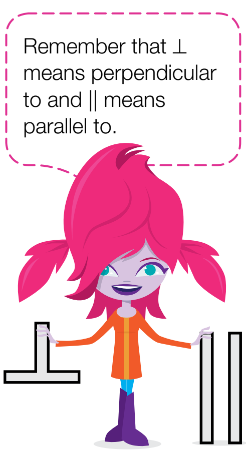
(c)


4 Copy each of the following and mark in an angle that is co-interior with the one shown.

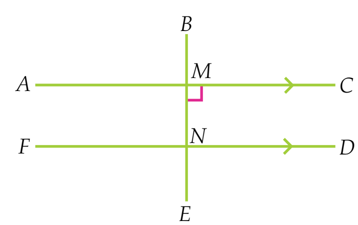
(a)


(b)


(c)




5 Which of the following statements is not true about the diagram shown?



A $\angle AMB$ is corresponding to $\angle FNB$

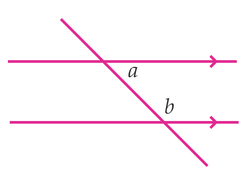
B $\angle BNF$ is alternate to $\angle EMC$

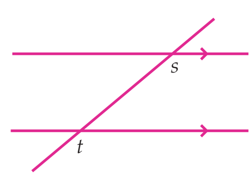
C $\angle AMN$ and $\angle CMN$ are co-interior angles

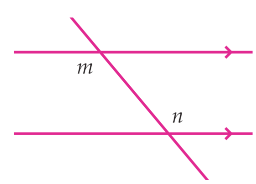
D BE is $\perp$ to FD

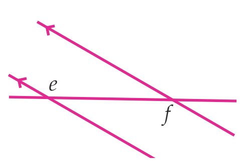
Understanding

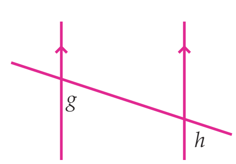
6 **(a)** Use a protractor to measure the labelled angles in each diagram below. State whether each pair of angles is corresponding, alternate or co-interior.

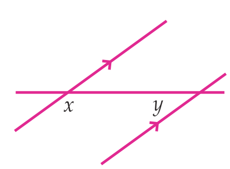
(i)


(ii)


(iii)


(iv)


(v)


(vi)


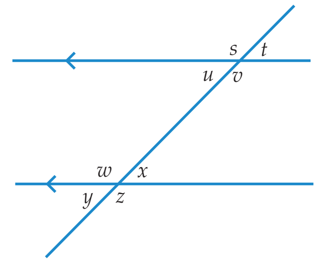
(b) State which pairs of angles in part **(a)** are equal. Which pairs of angles add to 180°?

7 State which angle in the diagram is:

(a) corresponding to u (b) alternate to v

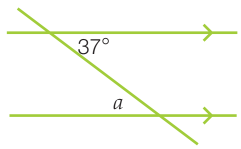
(c) co-interior with w (d) co-interior with x

(e) corresponding to w (f) alternate to u.

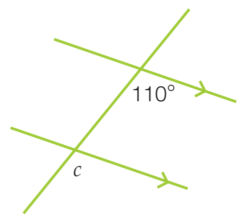


8 Find the value of the pronumerals in each case, and give a reason for your answer (e.g. corresponding angles).

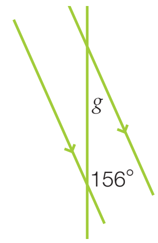
(a)



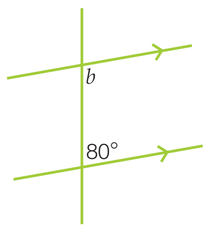
(b)



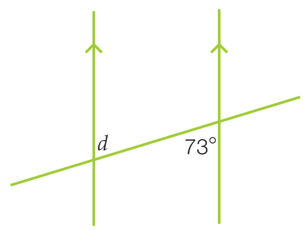
(c)



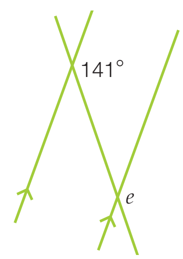
(d)



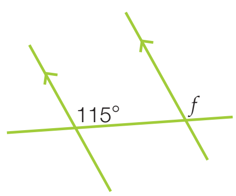
(e)



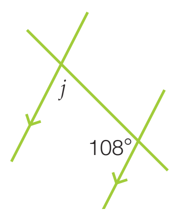
(f)



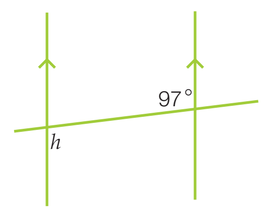
(g)



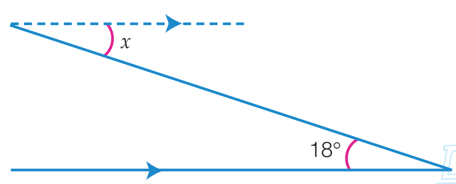
(h)



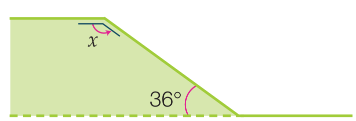
(i)



9 A pilot of a plane flying parallel to the ground can see the runway as it is coming in to land. The angle of approach to the runway is 18°. What angle must the pilot turn the plane down through to start the approach?

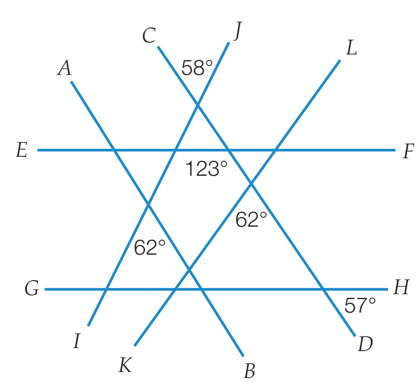


10 In an underground carpark a ramp is being constructed. The ramp will ascend at an angle of 36°. At what angle must the supports be positioned at the top of the ramp?



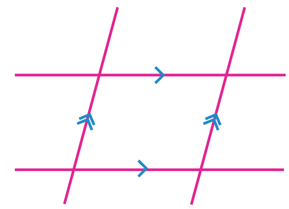
Reasoning

11 Choose the pair of lines that are parallel and explain why you choose them.

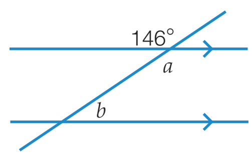


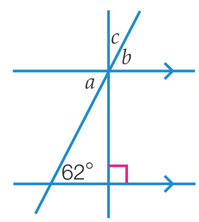
12 When a pair of parallel lines is cut by two parallel transversals, how many pairs of angles are formed that are:

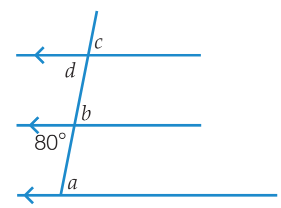
(a) corresponding
(b) alternate
(c) co-interior?

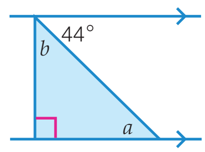


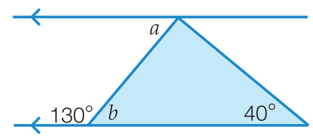
13 Find the value of each pronumeral below, giving a reason for your answer.

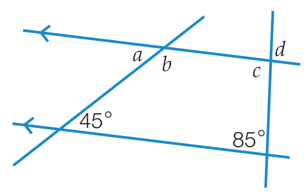
(a)


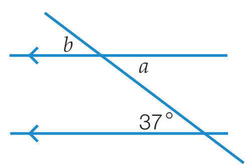
(b)


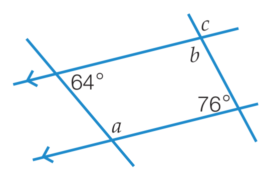
(c)


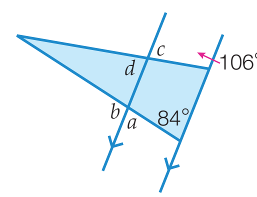
(d)


(e)


(f)


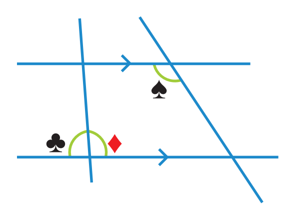
(g)


(h)


(i)


Open-ended

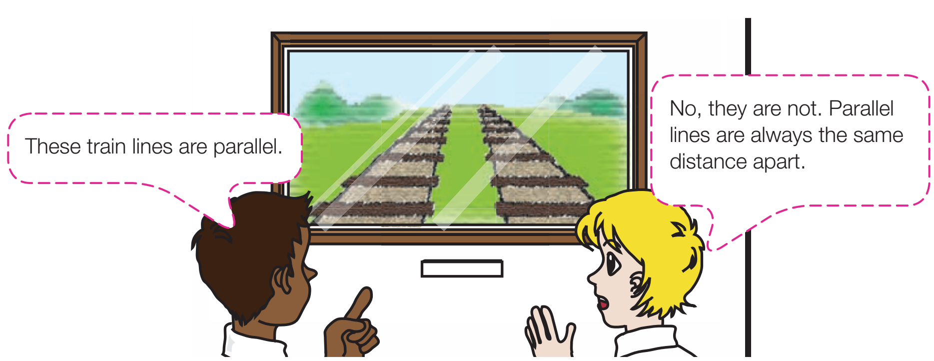
14 Aileen has drawn a pair of parallel lines with two transversals. She has marked three angles using different symbols.



(a) Copy the diagram into your book. Mark another three angles using the same symbols as Aileen so that there is a pair of corresponding angles, alternate angles and co-interior angles.

(b) In how many different ways can the three other angles be arranged, with no pair of angles sharing an angle with another pair? Draw a diagram for each combination.

15



How would you explain to the students whether the train lines are parallel or not?

Half-time 8

Equipment required: Protractor for Questions **3** and **5**

1 Draw the following angles by estimating their size. (Do not use a protractor.) **Ex 8.1**

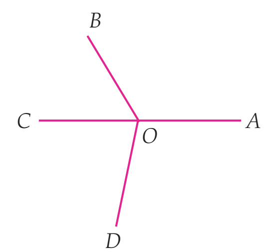
(a) 90° **(b)** 180° **(c)** 360°

(d) 34° **(e)** 75° **(f)** 120°

2 **(a)** Name the two acute angles in the diagram opposite. **Ex 8.2**

(b) Name the three obtuse angles in the diagram opposite.

(c) Give another name for $\angle AOB$.



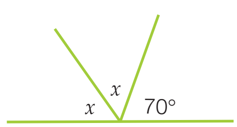
3 Draw the following angles using a protractor. **Ex 8.1**

(a) 90° **(b)** 180° **(c)** 360°

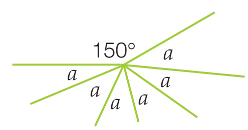
(d) 34° **(e)** 75° **(f)** 120°

4 Find the value of the pronumerals in each case and give reasons for your answer. **Ex 8.3**

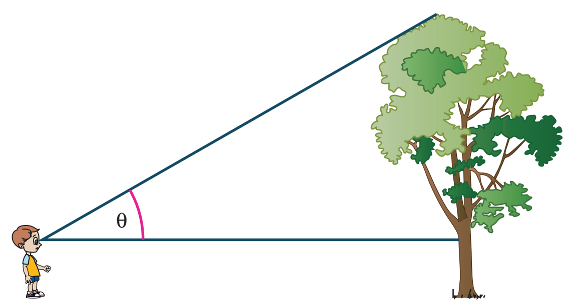
(a)



(b)

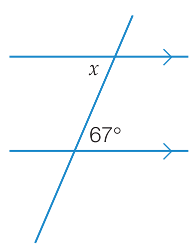


5 Measure the angle marked in the diagram opposite. **Ex 8.1**

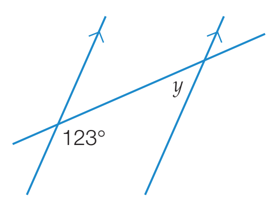


6 Find the value of the pronumerals in each case and give reasons for your answer. **Ex 8.4**

(a)



(b)



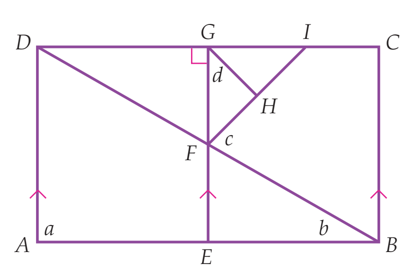
7 Using letter names such as $\angle ABC$: **Ex 8.3, 8.4**

(a) name an angle corresponding to a

(b) name an angle alternate to b

(c) name an angle supplementary to c

(d) name an angle complementary to d.

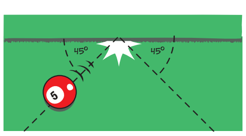


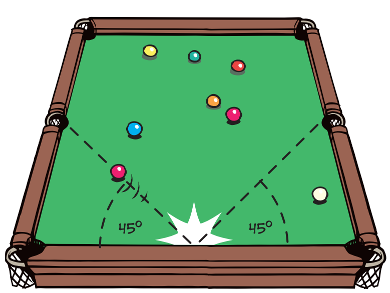
Billiard ball bounces

Equipment required: 1 or 2 brains, 1-centimetre grid paper, protractor, ruler

The game of billiards is played on a large rectangular table that is covered in felt and surrounded by a cushion along the edges. Players use a long stick, called a cue, to hit balls into pockets around the edges of the table. Snooker and pool are variations of the game that have become extremely popular over time. The size of the table can vary from 12 feet (3.7 m) along the larger side to 9 feet (2.7 m), 8 feet (2.4 m) or 7 feet (2.1 m) in length.

When planning their shots, good billiards players use the fact that a ball bounces off the side of a table at the same angle at which it hits, as shown here:





Here, the ball approaches the cushion at an angle of 45° and rebounds at an angle of 45°.

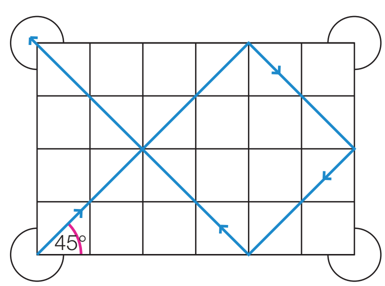
The ball may bounce off the sides several times before it either stops rolling or goes into a pocket. For this investigation, we will assume that the ball keeps rolling until it falls into a pocket.

The Big Question

Can we predict how many times a billiard ball will bounce on tables of different sizes?

Can we predict which pocket the ball will fall into?

Engage



Here, the ball is hit from the bottom left corner at an angle of 45° and bounces off the sides of the table three times before falling into the opposite pocket along the short side.

1 (a) To see what would happen if we made the table bigger, use your grid paper to draw a similar 'table' to the one above, but make it 6 cm × 8 cm. Starting in the same corner (bottom left), trace the path of a ball hit at an angle of 45°, bouncing it off the sides until the path ends in a pocket. You should find the ball bounces five times and falls into the opposite pocket on the long side.

(b) What would happen if the table was square?

Explore

2 (a) Draw up tables of the following dimensions on your grid paper (you might like to share this task with a partner).

1×2, 1×4, 2×3, 2×4, 2×8, 3×4, 3×6, 3×8, 3×12, 4×5, 4×6, 4×7, 4×8, 5×6, 5×7, 5×8, 6×7, 6×8, 6×9, 7×9, 8×10, 9×12

(b) Starting in the same corner each time, trace the path of the ball hit at an angle of 45°. Count the number of bounces, and also note which pocket the ball falls into.

Strategy options

- Make a table
- Look for a pattern.

Explain

3 (a) Collect all of the drawings you have made of the tables and their ball paths. To help you see any patterns or connections between them, group together the tables that have something in common. Some of the groups you could make are:

– tables where the pattern traced by the path of the ball is identical

– tables where the ball passes through every square on the grid.

(b) Once you have made these groups, compare the length, width and 'bounce' numbers for each table in the group. Are they connected in some way? Compare the starting and finishing pockets of the balls in each group. Is there a pattern here?

Elaborate

4 (a) Write a couple of sentences that answer the Big Question about predicting the number of bounces for tables of different sizes.

(b) Write a couple of sentences that answer the Big Question about predicting which pocket the ball will fall into.

Evaluate

5 (a) Consider how you worked on this task and the methods that you used. How did you organise or keep track of your results? Could you have done this better?

(b) Did grouping your results help you to spot patterns and connections between them? Which groups were useful?

Extend

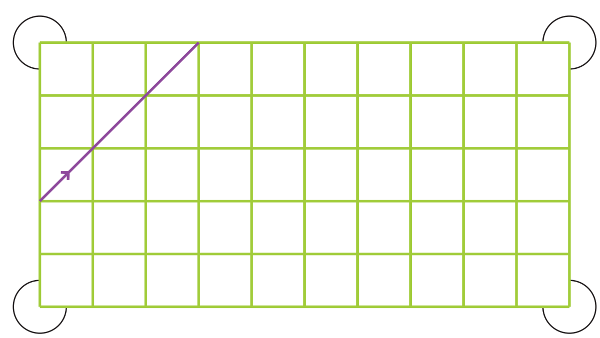
6 (a) Predict the number of bounces and which pocket the ball will fall into for the following table sizes.

(i) 18×27 (ii) 17×19

In which of these tables will the ball path pass through every square on the grid?

Draw these tables and check your prediction.

(b) Draw a 5×10 table on grid paper and trace the path of the ball hit from this position.



What do you notice? Try starting at different positions and comment on what you find.

(c) A billiard ball bouncing at the same angle at which it hits is an example of 'the law of reflection'. Investigate this law and other examples of where it can be seen.

8.5 Polygons

We draw many shapes on a flat surface called a plane. These are called two-dimensional or **plane shapes**.

Plane shapes whose sides are all straight lines are called **polygons**.

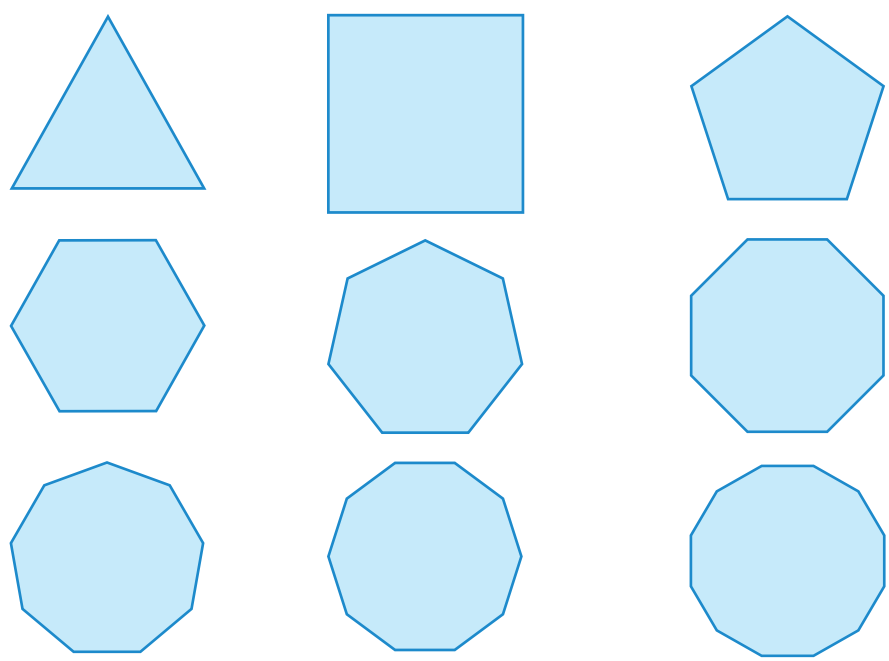
Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

The word polygon is made up of two Greek words—*poly* (meaning many) and *gon* (meaning angle), so a polygon is a many-angled shape. As you can see in the following diagrams of polygons, the number of sides in a polygon is equal to its number of angles.

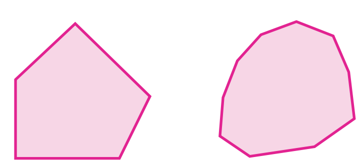
The table opposite shows the names given to the first 10 polygons. Note that an undecagon is rarely used.

A **regular polygon** has all sides of equal length and all angles of equal size. The number of sides gives the name of the polygon.

You are probably familiar with the regular polygons shown below.

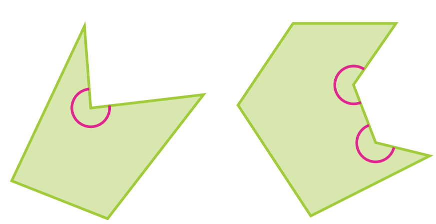


If the sides are not all equal, they are called **irregular** and the polygon is called an irregular polygon.



Concave and convex polygons

All of the polygons shown so far have been **convex** polygons, as they contain no interior angles greater than 180°. Opposite are examples of **concave** polygons which always contain at least one internal angle greater than 180°. (The sides cave in.)



8.5 Polygons

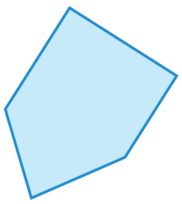
Navigator

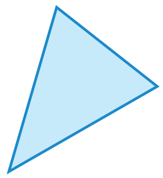
Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7

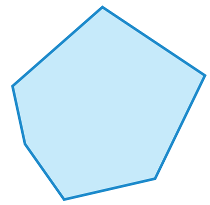
Answers page 670

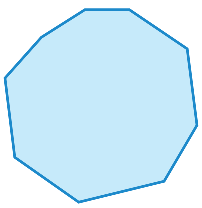
Fluency

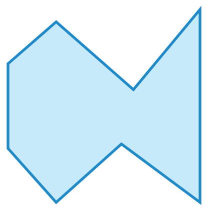
1 Name each of the polygons below. State whether each one is concave or convex.

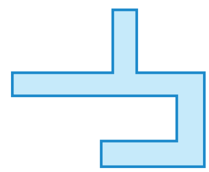
(a)


(b)


(c)


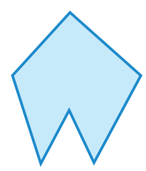
(d)


(e)


(f)


(g)


(h)


(i)


2 Choose the correct answer.

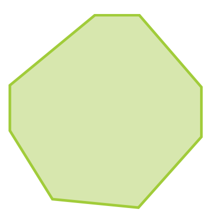
(a) The shape opposite is:

A a hexagon

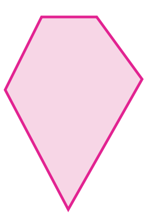
B a heptagon

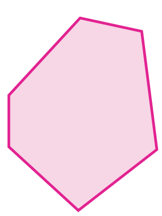
C an octagon

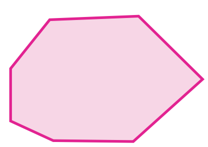
D a nonagon

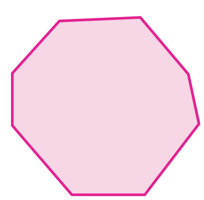


(b) Which of the shapes below is a heptagon?

A


B


C


D


Understanding

3 Name the type of polygon in each photograph.

(a)

(b)

(c)

(d)

Reasoning

4 What is the greatest number of right angles that a pentagon can have? Demonstrate your answer.

5 (a) If a quadrilateral can have two sets of parallel sides, how many sets of parallel sides can a hexagon have?

(b) Based on your results from part **(a)**, how many pairs of parallel sides can an octagon and a decagon have? Explain any pattern you have found.

(c) Can this pattern be applied to a pentagon or to a heptagon? Explain why or why not.

Open-ended

6 Using a ruler, draw any:

(a) concave quadrilateral (b) concave hexagon (c) concave octagon.

7 Name some common objects that contain regular polygons.

Outside the Square Puzzle

The hex is gone

Equipment required: 1 brain, a pair of scissors

This is a regular hexagon.

On loose paper, draw at least six regular hexagons that are each about a quarter of an A4 page. Cut out the hexagons and use them to assist you with the following questions.

(a) How could a regular hexagon be cut into two pieces which, when put together, make a parallelogram?

(b) How could a regular hexagon be cut into three pieces which, when put together, make a rhombus?

(c) How could a regular hexagon be cut into four pieces which, when put together, make two equilateral triangles?