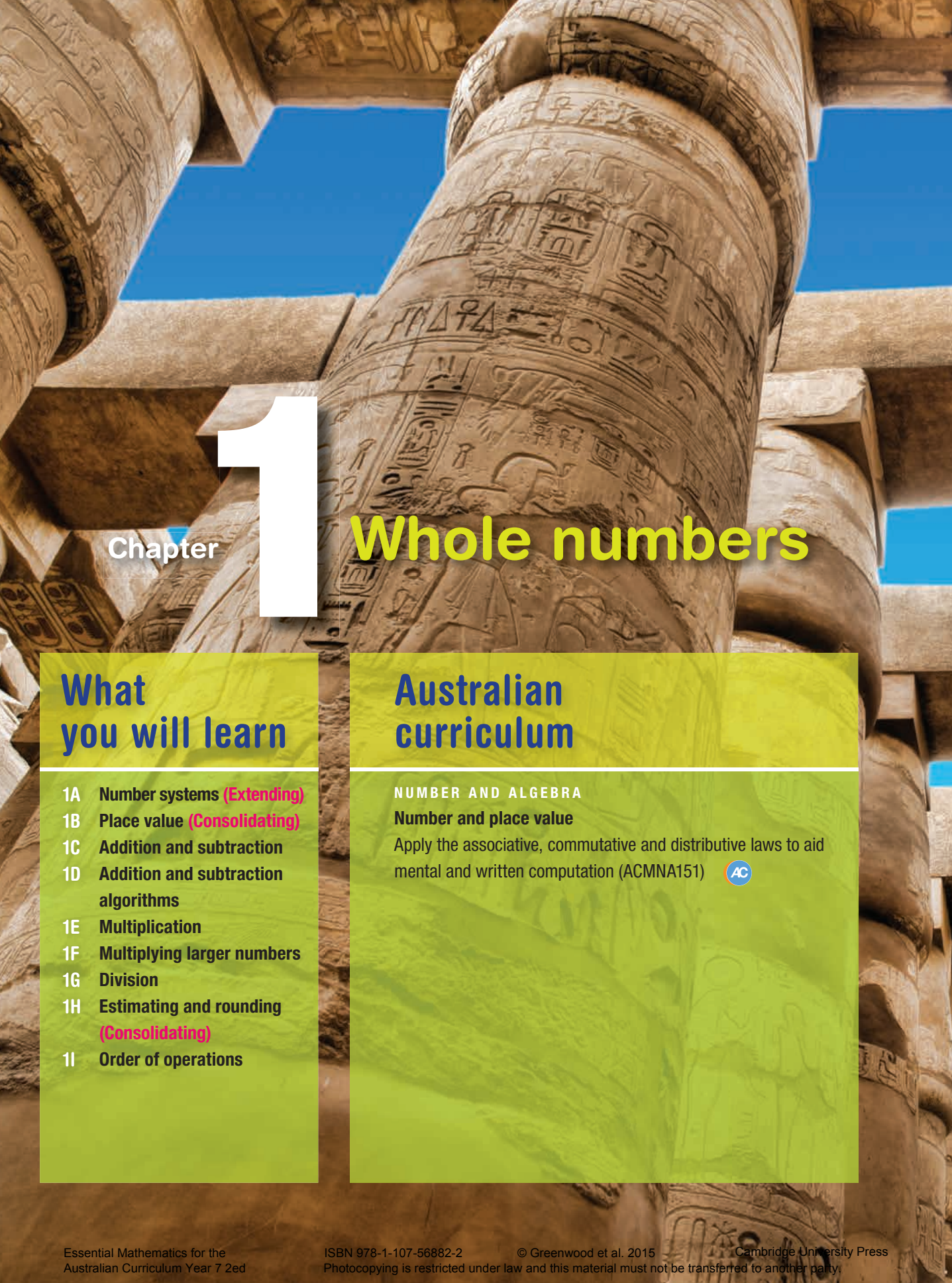

Chapter 1

Whole numbers

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151) AC

Essential Mathematics for the Australian Curriculum Year 7 2ed
ISBN 978-1-107-56882-2

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Whole numbers in the ancient world and now

Whole numbers and number systems have been used for thousands of years to help count objects and record information.

In ancient Egypt in about 3000 BCE if one hundred and twenty-four blocks of stone were needed to build a monument, this number would have been represented as

Today we use whole numbers to help deal with all sorts of situations.

Here are some examples:

- Recording the number of points in a game
- Calculating the number of pavers required for a terrace
- Counting the number of items purchased at a shop
- Tallying the number of votes in an election
- Calculating the approximate distance between two towns.

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1A Number systems EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

Throughout the ages and in different countries, number systems were developed and used to help people count and communicate with numbers. From the ancient Egyptians to the modern day, different systems have used pictures and symbols to represent whole numbers. Some of the well-known number systems are the Egyptian, Babylonian, Roman, modern Chinese and the Hindu-Arabic or decimal system.

The Roman numerals on this stone show the distance to the next village.

Let's start: Count like a Roman

Here are the letters used in the Roman number system for some numbers that you know.

Number	1	2	3	4	5	6	7	8	9	10	50	100
Roman numerals	I	II	III	IV	V	VI	VII	VIII	IX	X	L	C

- What numbers do you think XVII and XIX represent?
- Can you write the numbers 261 and 139 using Roman numerals?

Key ideas

Egyptian number system

- Records show that this number system was used from about 3000 BCE.
- **Hieroglyphics** were used to represent numbers.
- From about 1600 BCE hieroglyphics were used to represent groups of 10, 100, 1000 etc.
- Symbols of the same type were grouped in twos or threes and arranged vertically.

Number	1	10	100	1000	10 000	100 000	1 000 000
Hieroglyphic							
Description	Stick or staff	Arch or heel bone	Coil of rope	Lotus flower	Bent finger or reed	Tadpole or frog	Genie

- Examples:

3 

5 

21 

342 

- Note that the hieroglyphics with the larger value are written in front (i.e. on the left).
- There was no symbol for the number zero.

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Key ideas

Babylonian number system

- From about 1750 BCE the ancient Babylonians used a very sophisticated number system and its origins have been traced to about 3000 BCE.
- Symbols called **cuneiform** (wedge shapes) were used to represent numbers.
- The symbols were written into clay tablets, which were then allowed to dry in the Sun.
- The number system is based on the number 60, but a different wedge shape was used to represent groups of 10.

The Hanging Gardens of Babylon, built for his wife by King Nebuchadnezzar II around 600 BCE, were one of the seven wonders of the ancient world.

- The system is positional in that the position of each wedge shape helps determine its value. So ▼▼ means 2 but ▼ ◀▼ means 62.

Number	1	10	60
Symbol	▼	◀	▼
Description	Upright wedge shape	Sideways wedge	Upright wedge shape

- To represent zero, they used a blank space or sometimes a small slanted wedge shape for zeros inside a number.
- Examples:

5 ▼▼▼ 11 ◀▼ 72 ▼ ◀▼▼ 121 ▼▼ ▼

Roman number system

- Some capital letters are used and are called Roman numerals.
- The Roman number system was developed in about the third century BCE and remained the dominant system in many parts of the world until about the Middle Ages. It is still used today in many situations.
- A smaller letter value to the left of a larger letter value indicates subtraction. For example, IV means 5 – 1 = 4 and XC means 100 – 10 = 90. Only one letter can be placed to the left for subtraction. Is, Xs and Cs are the numerals that can be used to reduce the next two larger numerals. So X, for example, can be used to reduce L and C but not D.

Number	1	5	10	50	100	500	1000
Symbol	I	V	X	L	C	D	M

- Examples:

2	4	21	59	90
II	IV	XXI	LIX	XC

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Example 1 Using ancient number systems

Write each of the numbers 3, 15 and 144 using the given number systems.

a Egyptian **b** Babylonian **c** Roman

SOLUTION

a 3 

15 

144 

b 3 

15 

144 

c 3 III

15 XV

144 CXLIV

EXPLANATION

 means 1

 means 10

 means 100

 means 1

 means 10

 means 60

I means 1

V means 5
X means 10

C means 100
XL means 40
IV means 4

Exercise 1A

1–3 | 3 | —

UNDERSTANDING

1 Which number system uses these symbols?

a cuneiform (wedge shapes), e.g. 

b capital letters, e.g. V and L

c hieroglyphics (pictures), e.g.  and 

2 Draw the symbols used in these number systems for the given numbers.

a Egyptian

i 1 **ii** 10 **iii** 100 **iv** 1000

b Babylonian

i 1 **ii** 10 **iii** 60

c Roman

i 1 **ii** 5 **iii** 10 **iv** 50 **v** 100

3 In the Roman system, IV does not mean 1 + 5 to give 6. What do you think it means?

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1A

4, 5 | 4–6 | 4–6(½)

FLUENCY

Example 1

4 Write these numbers using the given number systems.

a Egyptian

i 3 **ii** 21 **iii** 114 **iv** 352

b Babylonian

i 4 **ii** 32 **iii** 61 **iv** 132

c Roman

i 2 **ii** 9 **iii** 24 **iv** 156

5 What number do these groups of symbols represent?

a Egyptian

i  **ii**  **iii**  **iv** 

b Babylonian

i  **ii**  **iii**  **iv** 

c Roman

i IV **ii** VIII **iii** XVI **iv** XL

6 Work out the answer to each of these problems. Write your answer using the same number system that is given in the question.

a XIV + XXII

b 

c 

d DCLXIX + IX

7, 8 | 8–10 | 8–10

PROBLEM-SOLVING

7 In ancient Babylon, a person adds goats to another group of  .
How many goats are there in total? Write your answer using the Babylonian number system.

8 An ancient Roman counts the number of people in three queues. The first queue has XI, the second has LXII and the third has CXV. How many people are there in total? Write your answer using the Roman number system.

9 One Egyptian house is made from  stones and a second house is made from stones.
How many more stones does the first house have? Write your answer using the Egyptian number system.

10 Which number system (Egyptian, Babylonian or Roman) uses the least number of symbols to represent these numbers?

a 55 **b** 60 **c** 3104

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1A

11(½) | 11(½), 12 | 11(½), 12, 13

REASONING

11 In the Roman system Is, Xs and Cs are used to reduce either of the next two larger numerals. So 9 is IX, not VIIII; and 49 is XLIX, not IL.
Also, only one numeral can be used to reduce another number. So 8 is VIII, not IIX.
Write these numbers using Roman numerals.

a 4	**b** 9	**c** 14	**d** 19
e 29	**f** 41	**g** 49	**h** 89
i 99	**j** 449	**k** 922	**l** 3401

12 The Egyptian system generally uses more symbols than the other systems described here. Can you explain why? How many symbols are used for the number 999?

13 In the Babylonian system ▼ stands for 1, but because they did not use a symbol for zero at the end of a number, it also represents 60. People would know what it meant, depending on the situation it was used. Here is how it worked for large numbers. The dots represent empty spaces.

1	60	3600
▼	▼ ……	▼ …… ……

a Write these numbers using the Babylonian system.

i 12 **ii** 72 **iii** 120 **iv** 191 **v** 3661 **vi** 7224

b Can you explain why ▼ …… …… represents 3600?

c What would ▼ …… …… …… represent?

Other number systems

— | — | 14

ENRICHMENT

14 Other well-known number systems include:

i Mayan
ii modern Chinese
iii ancient Greek.

Look up these number systems on the internet or in other books. Write a brief sentence covering the points below.

a When and where the number systems were used.
b What symbols were used?
c Examples of numbers using these symbols.

An ancient Mayan carving.

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1B Place value CONSOLIDATING

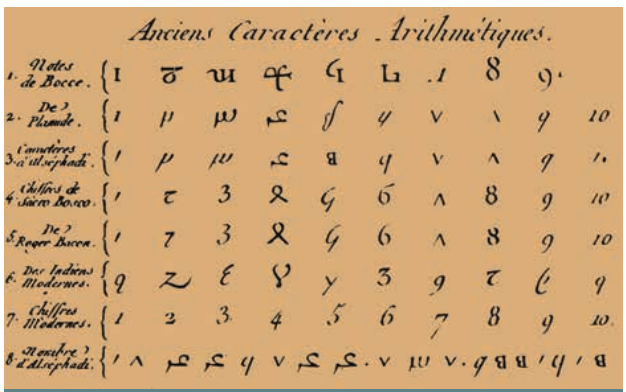
Interactive

Widgets

HOTsheets

Walkthroughs

The commonly used number system today, called the decimal system or base 10, is also called the Hindu-Arabic number system. Like the Babylonian system, the value of the digit depends on its place in the number, but only one digit is used in each position. A digit for zero is also used. The decimal system originated in ancient India about 3000 BCE and spread throughout Europe through Arabic texts over the next 4000 years.



The famous 'Histoire de la Mathematique', a French document showing the history of the Hindu-Arabic number system over thousands of years.

Let's start: Largest and smallest

Without using decimal points, repeated digits or a zero (0) at the start of a number, see if you can use all the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to write down:

- the largest possible number
- the smallest possible number.

Can you explain why your numbers are, in fact, the largest or smallest possible?

Key ideas

- The Hindu-Arabic or **decimal system** uses base 10. This means powers of 10 (1, 10 or 10^1, 100 or 10^2, 1000 or 10^3, ...) are used to determine the place value of a digit in a number.
- The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called **digits**.
- The value of each digit depends on its place in the number. The **place value** of the digit 2 in the number 126, for example, is 20.
- $3 \times 1000 + 2 \times 100 + 5 \times 10 + 4 \times 1$ (or $3 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 4 \times 1$) is said to be the **expanded form** of the **basic numeral** 3254.

thousands hundreds tens ones

$3\ 2\ 5\ 4 = 3 \times 1000 + 2 \times 100 + 5 \times 10 + 4 \times 1$

expanded form

- Symbols used to compare numbers include the following.

= (is equal to)	$1 + 3 = 4$	or	$10 - 7 = 3$
≠ (is not equal to)	$1 + 3 \neq 5$	or	$11 + 38 \neq 50$
> (is greater than)	$5 > 4$	or	$100 > 37$

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Key ideas

≥ (is greater than or equal to)	$5 \geq 4$	or	$4 \geq 4$
< (is less than)	$4 < 5$	or	$13 < 26$
≤ (is less than or equal to)	$4 \leq 5$	or	$4 \leq 4$
≈ or ≒ (is approximately equal to)	$4.02 \approx 4$	or	$8997 \fallingdotseq 9000$

Example 2 Finding place value

Write down the place value of the digit 4 in these numbers.

a 437 **b** 543 910

SOLUTION	EXPLANATION
a $4 \times 100 = 400$	4 is worth 4×100 3 is worth 3×10 7 is worth 7×1
b $4 \times 10\,000 = 40\,000$	5 is worth $5 \times 100\,000$ 4 is worth $4 \times 10\,000$ 3 is worth 3×1000 9 is worth 9×100 1 is worth 1×10

Example 3 Writing in expanded form

Write 517 in expanded form.

SOLUTION	EXPLANATION
$517 = 5 \times 100 + 1 \times 10 + 7 \times 1$	Write each digit separately and multiply by the appropriate power of 10.

Place value is very important, such as in a bank cheque.

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Exercise 1B

1–3 | 3 | —

UNDERSTANDING

1 Choose one of the words 'ones', 'tens', 'hundreds' or 'thousands' to describe each number.

a 100 **b** 1000 **c** 10 **d** 1

2 Write down these numbers using digits.

a two hundred and sixty-three
b seven thousand four hundred and twenty-one
c thirty-six thousand and fifteen
d one hundred thousand and one

3 Which symbol (next to the capital letters) matches the given words?

A = **B** ≠ **C** > **D** ≥
E < **F** ≤ **G** ≈

a is not equal to **b** is less than **c** is greater than or equal to
d is equal to **e** is greater than **f** is less than or equal to
g is approximately equal to

4–7(½) | 4–7(½), 8 | 4–8(½)

FLUENCY

Example 2

4 Write down the place value of the digit 7 in these numbers.

a 37 **b** 71 **c** 379 **d** 704
e 1712 **f** 7001 **g** 45 720 **h** 170 966

5 Write down the place value of the digit 2 in these numbers.

a 126 **b** 2143 **c** 91 214 **d** 1 268 804

6 State whether each of these statements is true or false.

a $5 > 4$ **b** $6 = 10$ **c** $9 \neq 99$ **d** $1 < 12$
e $22 \leq 11$ **f** $126 \leq 126$ **g** $19 \geq 20$ **h** $138 > 137$

Example 3

7 Write these numbers in expanded form.

a 17 **b** 281 **c** 935 **d** 20
e 4491 **f** 2003 **g** 10 001 **h** 55 555

8 Write these numbers, given in expanded form, as a basic numeral.

a $3 \times 100 + 4 \times 10 + 7 \times 1$
b $9 \times 1000 + 4 \times 100 + 1 \times 10 + 6 \times 1$
c $7 \times 1000 + 2 \times 10$
d $6 \times 100\,000 + 3 \times 1$
e $4 \times 1\,000\,000 + 3 \times 10\,000 + 7 \times 100$
f $9 \times 10\,000\,000 + 3 \times 1000 + 2 \times 10$

9 | 9(½), 10 | 10, 11

PROBLEM-SOLVING

9 Arrange these numbers from smallest to largest.

a 55, 45, 54, 44
b 729, 29, 92, 927, 279
c 23, 951, 136, 4
d 435, 453, 534, 345, 543, 354
e 12 345, 54 321, 34 512, 31 254
f 1010, 1001, 10 001, 1100, 10 100

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1B

PROBLEM-SOLVING

10 How many numbers can be made using the given digits? Digits are not allowed to be used more than once and all digits must be used.

a 2, 8 and 9 **b** 1, 6 and 7 **c** 2, 5, 6 and 7

11 You are given three different non-zero digits, for example; 2, 5 and 8. How many three digit numbers can be formed from your three given digits if digits can be used more than once?

REASONING

12 | 12 | 12, 13

12 The letters used here represent the digits of a number. Write each one in expanded form.

a *ab* **b** *abcd* **c** *a*0000*a*

13 By considering some of the other number systems (Egyptian, Babylonian or Roman) explained in the previous section, describe the main advantages of the Hindu-Arabic system.

ENRICHMENT

Large numbers and expanded form — | — | 14

14 It is convenient to write very large numbers in expanded form with index notation.
Here is an example.
$50\,000\,000 = 5 \times 10\,000\,000 = 5 \times 10^7$

a Explain why it is convenient to write large numbers in this type of expanded form.

b 3200 can also be written in the form 32×10^2. All the non-zero digits are written down and then multiplied by a power of 10. Similarly, write each of these numbers in the same way.

i 4100 **ii** 370 000 **iii** 21 770 000

c Write each of these numbers as basic numerals.

i 381×10^2 **ii** 7204×10^3 **iii** 1028×10^6

d Write these numbers in expanded form, just as you did in the examples above. Research them if you do not know what they are.

i 1 million **ii** 1 billion **iii** 1 trillion

iv 1 googol **v** 1 googolplex

In 2008 in Zimbabwe, bank notes were issued in trillions of dollars, but soon became worthless due to inflation.

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1C Addition and subtraction

Interactive

Widgets

HOTsheets

Walkthroughs

The process of finding the total value of two or more numbers is called addition. The words 'plus', 'add' and 'sum' are also used to describe addition.

The process for finding the difference between two numbers is called subtraction. The words 'minus', 'subtract' and 'take away' are also used to describe subtraction.

What's the difference in our heights?

Let's start: Your mental strategy

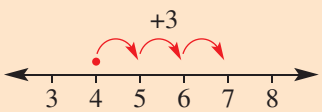
Many problems that involve addition and subtraction can be solved mentally without the use of a calculator or complicated written working.

Consider $98 + 22 - 31 + 29$

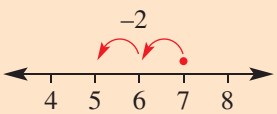
How would you work this out? What are the different ways it could be done mentally? Explain your method.

Key ideas

- The symbol + is used to show addition or find a sum.
  e.g. $4 + 3 = 7$



- $a + b = b + a$ e.g. $4 + 3 = 3 + 4$
  - This is the **commutative law** for addition, meaning that the order does not matter.
- $a + (b + c) = (a + b) + c$ e.g. $4 + (11 + 3) = (4 + 11) + 3$
  - This is called the associative law for addition, meaning that it does not matter which pair is added first.



- The symbol – is used to show subtraction or find a difference.
  e.g. $7 - 2 = 5$
- $a - b \neq b - a$ (in general ...) e.g. $4 - 3 \neq 3 - 4$
- $a - (b - c) \neq (a - b) - c$ e.g. $8 - (4 - 2) \neq (8 - 4) - 2$
- Mental addition and subtraction can be done using different strategies.
  - **Partitioning** (grouping digits in the same position)

$$171 + 23 = 100 + (70 + 20) + (1 + 3)$$
$$= 194$$

  - **Compensating** (making a 10, 100 etc. and then adjusting or compensating by adding or subtracting)

$$46 + 9 = 46 + 10 - 1$$
$$= 55$$

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Key ideas

- **Doubling or halving** (making a double or half and then adjusting with addition or subtraction)

$$75 + 78 = 75 + 75 + 3 = 150 + 3 = 153 \qquad 124 - 61 = 124 - 62 + 1 = 62 + 1 = 63$$

Example 4 Mental addition and subtraction

Use the suggested strategy to mentally work out the answer.

a 132 + 156 (partitioning)
b 25 + 19 (compensating)
c 56 – 18 (compensating)
d 35 + 36 (doubling or halving)

SOLUTION	EXPLANATION
a 132 + 156 = 288	100 + 30 + 2 100 + 50 + 6 200 + 80 + 8
b 25 + 19 = 44	25 + 19 = 25 + 20 – 1 = 45 – 1 = 44
c 56 – 18 = 38	56 – 18 = 56 – 20 + 2 = 36 + 2 = 38
d 35 + 36 = 71	35 + 36 = 35 + 35 + 1 = 70 + 1 = 71

Exercise 1C

1–5 | 5 | — | UNDERSTANDING

1 **a** List three words that mean *addition*.
b List three words that mean *subtraction*.

2 Write the number which is:
a 3 more than 7
b 58 more than 11
c 7 less than 19
d 137 less than 157

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1C

UNDERSTANDING

3 **a** Find the sum of these pairs of numbers.
i 2 and 6 **ii** 19 and 8 **iii** 62 and 70

b Find the difference between these pairs of numbers.
i 11 and 5 **ii** 29 and 13 **iii** 101 and 93

4 State whether each of these statements is true or false.
a $4 + 3 > 6$ **b** $11 + 19 \geq 30$ **c** $13 - 9 < 8$
d $26 - 15 \leq 10$ **e** $1 + 7 - 4 \geq 4$ **f** $50 - 21 + 6 < 35$

5 Give the result for each of the following.
a 7 plus 11 **b** 22 minus 3
c the sum of 11 and 21 **d** 128 add 12
e 36 take away 15 **f** the difference between 13 and 4

6(½), 7–10 6–11(½) 6–11(½)

FLUENCY

Example 4a **6** Mentally find the answers to these sums. Hint: Use the partitioning strategy.
a $23 + 41$ **b** $71 + 26$ **c** $138 + 441$
d $246 + 502$ **e** $937 + 11$ **f** $1304 + 4293$
g $140\,273 + 238\,410$ **h** $390\,447 + 201\,132$ **i** $100\,001 + 101\,010$

7 Mentally find the answers to these differences. Hint: Use the partitioning strategy.
a $29 - 18$ **b** $57 - 21$ **c** $249 - 137$
d $1045 - 1041$ **e** $4396 - 1285$ **f** $10101 - 100$

Example 4b **8** Mentally find the answers to these sums. Hint: Use the compensating strategy.
a $15 + 9$ **b** $64 + 11$ **c** $19 + 76$
d $18 + 115$ **e** $31 + 136$ **f** $245 + 52$

Example 4c **9** Mentally find the answers to these differences. Hint: Use the compensating strategy.
a $35 - 11$ **b** $45 - 19$ **c** $156 - 48$
d $244 - 22$ **e** $376 - 59$ **f** $5216 - 199$

Example 4d **10** Mentally find the answers to these sums and differences. Hint: Use the doubling or halving strategy.
a $25 + 26$ **b** $65 + 63$ **c** $121 + 123$
d $240 - 121$ **e** $482 - 240$ **f** $1006 - 504$

11 Mentally find the answers to these mixed problems.
a $11 + 18 - 17$ **b** $37 - 19 + 9$ **c** $101 - 15 + 21$
d $136 + 12 - 15$ **e** $28 - 10 - 9 + 5$ **f** $39 + 71 - 10 - 10$
g $1010 - 11 + 21 - 1$ **h** $5 - 7 + 2$ **i** $10 - 25 + 18$

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1C

12–14 | 13–15 | 15–17

PROBLEM-SOLVING

12 Gary worked 7 hours on Monday, 5 hours on Tuesday, 13 hours on Wednesday, 11 hours on Thursday and 2 hours on Friday. What is the total number of hours that Gary worked during the week?

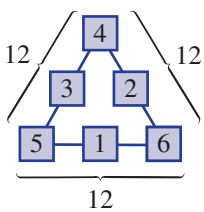
13 In a batting innings, Phil hit 126 runs and Mario hit 19 runs. How many more runs did Phil hit compared to Mario?

14 A farmer reduced his cattle numbers from 86 to 54. How many cows were taken away?

15 Bag A has 18 marbles and bag B has 7 fewer marbles than bag A. What is the total number of marbles?

16 Matt has 36 cards and Andy has 35 more cards than Matt. If they combine their cards, how many do they have in total?

17 Each side on a magic triangle adds up to the same number, as shown in this example with a sum of 12 on each side.

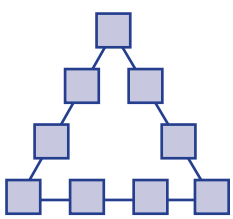


a Place each of the digits from 1 to 6 in a magic triangle with three digits along each side so that each side adds up to the given number.

i 9 **ii** 10

b Place each of the digits from 1 to 9 in a magic triangle with four digits along each side so that each side adds up to the given number.

i 20 **ii** 23



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18 | 18, 19 | 19, 20

1C

REASONING

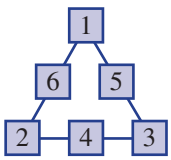
18 **a** The mental strategy of partitioning is easy to apply for 23 + 54 but harder for 23 + 59. Explain why.

b The mental strategy of partitioning is easy to apply for 158 – 46 but harder for 151 – 46. Explain why.

19 Complete these number sentences if the letters a, b and c represent numbers.

a $a + b = c$ so $c - \_\_ = a$ **b** $a + c = b$ so $b - a = \_\_$

20 This magic triangle uses the digits 1 to 6, and has each side adding to the same total. This example shows a side total of 9.



a How many different side totals are possible using the same digits?

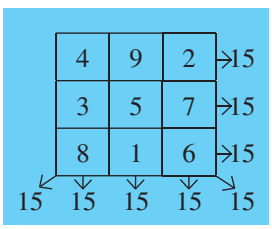
b Explain your method.

Magic squares

— | — | 21, 22

ENRICHMENT

21 A magic square has every row, column and main diagonal adding to the same number, called the magic sum. For example, this magic square has a magic sum of 15.
Find the magic sums for these squares, then fill in the missing numbers.



a

6		
7	5	
2		

b

10		
	11	13
		12

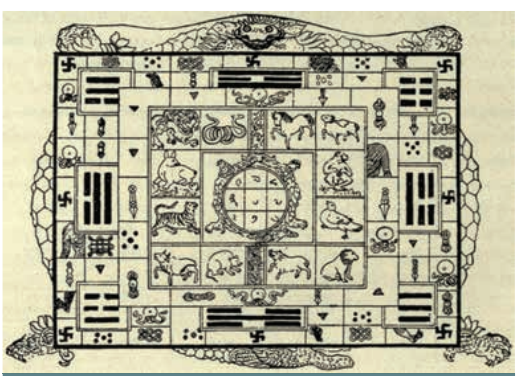
c

15	20	
14		
19		

d

1	15		4
	6		9
		11	
13		2	16

22 The sum of two numbers is 87 and their difference is 29. What are the two numbers?



This magic square was known in ancient China as a 'Lo Shu' square and uses only the numbers 1 to 9. It is shown in the middle of this ancient design as symbols on a turtle shell, surrounded by the animals which represent the traditional Chinese names for the years.

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1D Addition and subtraction algorithms

It is not always practical to solve problems involving addition and subtraction mentally. For more complex problems a procedure involving a number of steps can be used and this helps to give the answer. Such a procedure is called an algorithm.

For the addition algorithm, if two digits add to more than 9, then the higher place value digit in the sum can be carried to the next column.

For the subtraction algorithm, if two digits subtract to be less than 0, then a 1 can be borrowed from the next column to form an extra 10.

Let's start: The missing digits

Discuss what numbers should go in the empty boxes.
Give reasons for your answers.

$$\begin{array}{r} 1\,\square\,4 \\ +\ 9\ 5\ \square \\ \hline 1\,\square\,2\ 5 \end{array} \qquad \begin{array}{r} \square\ 5\ \square \\ -\ 1\,\square\,4 \\ \hline 9\ 4 \end{array}$$

Key ideas

- An **algorithm** is a procedure involving a number of steps that eventually leads to the answer to a problem.
- **Addition algorithm**
  - Arrange the numbers vertically so that the digits with similar place value are in the same column.
  - Add digits in the same column, starting on the right.
  - If the digits add to more than 9, carry the 10 to the next column.

$$\begin{array}{r} {}^{1}234 \\ 192 \\ \hline 426 \end{array} \qquad \begin{array}{l} 4 + 2 = 6 \\ 3 + 9 = 12 \\ 1 + 2 + 1 = 4 \end{array}$$

- **Subtraction algorithm**
  - Arrange the numbers vertically so that the digits with similar place value are in the same column.
  - Subtract digits in the same column top-down and starting on the right.
  - If the digits subtract to less than 0, borrow a 1 from the next column to form an extra 10.

$$\begin{array}{r} {}^{1}2\,{}^{1}59 \\ 1\ 82 \\ \hline 77 \end{array} \qquad \begin{array}{l} 9 - 2 = 7 \\ 15 - 8 = 7 \\ 1 - 1 = 0 \end{array}$$

- Calculators may be used to check your answers.

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Example 5 Using the addition algorithm

Give the result for each of these sums.

a
$$\begin{array}{r} 26 \\ +\,66 \\ \hline \end{array}$$

b
$$\begin{array}{r} 439 \\ +\,172 \\ \hline \end{array}$$

SOLUTION

a
$$\begin{array}{r} {}^{1}26 \\ +\,66 \\ \hline 92 \end{array}$$

b
$$\begin{array}{r} {}^{1}4{}^{1}39 \\ +\;1\;7\;2 \\ \hline 6\;1\;1 \end{array}$$

EXPLANATION

Add the digits vertically.

$6 + 6 = 12$, so carry the 1 to the tens column.

$9 + 2 = 11$, carry a 1 to the tens column.

$1 + 3 + 7 = 11$, so carry a 1 to the hundreds column.

Example 6 Using the subtraction algorithm

Give the result for each of these differences.

a
$$\begin{array}{r} 74 \\ -\,15 \\ \hline \end{array}$$

b
$$\begin{array}{r} 3240 \\ -\,2721 \\ \hline \end{array}$$

SOLUTION

a
$$\begin{array}{r} {}^{6}\not{7}{}^{1}4 \\ -\;1\;5 \\ \hline 5\;9 \end{array}$$

b
$$\begin{array}{r} {}^{2}\not{3}{}^{1}2\,{}^{3}\not{4}{}^{1}0 \\ -\;2\;7\;2\;1 \\ \hline 5\;1\;9 \end{array}$$

EXPLANATION

Borrow 1 from 7 to make $14 - 5 = 9$.

Then subtract 1 from 6 (not 7).

Borrow 1 from 4 to make $10 - 1 = 9$.

Subtract 2 from 3 (not 4).

Borrow 1 from 3 to make $12 - 7 = 5$.

Note that $2 - 2 = 0$ and you do not need to show a 0 before the 5.

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Exercise 1D

1–3(½) | 3(½) | —

UNDERSTANDING

1 Mentally find the results to these simple sums.

a 8 + 9 **b** 87 + 14 **c** 138 + 6 **d** 99 + 11

e 998 + 7 **f** 19 + 124 **g** 102 + 99 **h** 52 + 1053

2 Mentally find the results to these simple differences.

a 13 – 5 **b** 36 – 9 **c** 75 – 8 **d** 100 – 16

e 37 – 22 **f** 104 – 12 **g** 46 – 17 **h** 1001 – 22

3 What is the missing number in these problems?

a
$$\begin{array}{r} 27 \\ +\,31 \\ \hline 5\square \end{array}$$

b
$$\begin{array}{r} 36 \\ +\,15 \\ \hline 5\square \end{array}$$

c
$$\begin{array}{r} 123 \\ +\,\,91 \\ \hline 2\square 4 \end{array}$$

d
$$\begin{array}{r} 46 \\ +\,\square 4 \\ \hline 110 \end{array}$$

e
$$\begin{array}{r} 24 \\ -\,1\square \\ \hline 12 \end{array}$$

f
$$\begin{array}{r} 67 \\ -\,48 \\ \hline \square 9 \end{array}$$

g
$$\begin{array}{r} 162 \\ -\,\square 1 \\ \hline 81 \end{array}$$

h
$$\begin{array}{r} 14\square 2 \\ -\,623 \\ \hline 809 \end{array}$$

4–5(½), 7–8(½) | 4–8(½) | 4–8(½)

FLUENCY

Example 5

4 Give the answer to each of these sums. Check your answer with a calculator.

a
$$\begin{array}{r} 36 \\ +\,51 \\ \hline \end{array}$$

b
$$\begin{array}{r} 74 \\ +\,25 \\ \hline \end{array}$$

c
$$\begin{array}{r} 17 \\ +\,24 \\ \hline \end{array}$$

d
$$\begin{array}{r} 47 \\ +\,39 \\ \hline \end{array}$$

e
$$\begin{array}{r} 129 \\ +\,97 \\ \hline \end{array}$$

f
$$\begin{array}{r} 458 \\ +\,287 \\ \hline \end{array}$$

g
$$\begin{array}{r} 1041 \\ +\,882 \\ \hline \end{array}$$

h
$$\begin{array}{r} 3092 \\ +\,1988 \\ \hline \end{array}$$

5 Show your working to find the result for each of these sums.

a 85 + 76 **b** 131 + 94 **c** 1732 + 497 **d** 988 + 987

6 Give the result for each of these sums.

a
$$\begin{array}{r} 17 \\ 26 \\ +\,34 \\ \hline \end{array}$$

b
$$\begin{array}{r} 126 \\ +\,47 \\ +\,19 \\ \hline \end{array}$$

c
$$\begin{array}{r} 152 \\ 247 \\ +\,19 \\ \hline \end{array}$$

d
$$\begin{array}{r} 2197 \\ 1204 \\ +\,807 \\ \hline \end{array}$$

e 946 + 241 + 27 + 9 **f** 1052 + 839 + 7 + 84

Example 6

7 Find the answers to these differences. Check your answer with a calculator.

a
$$\begin{array}{r} 54 \\ -\,23 \\ \hline \end{array}$$

b
$$\begin{array}{r} 85 \\ -\,65 \\ \hline \end{array}$$

c
$$\begin{array}{r} 46 \\ -\,27 \\ \hline \end{array}$$

d
$$\begin{array}{r} 94 \\ -\,36 \\ \hline \end{array}$$

e
$$\begin{array}{r} 125 \\ -\,89 \\ \hline \end{array}$$

f
$$\begin{array}{r} 241 \\ -\,129 \\ \hline \end{array}$$

g
$$\begin{array}{r} 358 \\ -\,279 \\ \hline \end{array}$$

h
$$\begin{array}{r} 491 \\ -\,419 \\ \hline \end{array}$$

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FLUENCY

8 Show your working to find the answer to each of these differences.

a 32 – 16 **b** 124 – 77 **c** 613 – 128 **d** 1004 – 838

9, 10 | 10–12 | 11–13

PROBLEM-SOLVING

9 Farmer Green owns 287 sheep, farmer Brown owns 526 sheep and farmer Grey owns 1041 sheep. How many sheep are there in total?

10 A car's odometer shows 12 138 kilometres at the start of a journey and 12 714 kilometres at the end of the journey. How far was the journey?

11 Two different schools have 871 and 950 students enrolled.

a How many students are there in total?

b Find the difference in the number of students between the schools.

12 Find the missing numbers in these sums.

a
$$\begin{array}{r} 3\,\square \\ +\,5\;3 \\ \hline \square\;1 \end{array}$$

b
$$\begin{array}{r} 1\,\square\;4 \\ +\quad 7\,\square \\ \hline \square\;9\;1 \end{array}$$

c
$$\begin{array}{r} \square\,\square \\ +\square\;4\;7 \\ \hline 9\;1\;4 \end{array}$$

13 Find the missing numbers in these differences.

a
$$\begin{array}{r} 6\,\square \\ -\,2\;8 \\ \hline \square\;4 \end{array}$$

b
$$\begin{array}{r} 2\,\square\;5 \\ -\square\;8\,\square \\ \hline 8\;1 \end{array}$$

c
$$\begin{array}{r} 3\,\square\,\square\;2 \\ -\quad 9\;2\,\square \\ \hline \square\;1\;6\;5 \end{array}$$

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1D

14 | 14, 15 | 14, 15

REASONING

14 **a** First work out the answer to these simple problems before doing part **b**.

i $28 + 18 - 17$ **ii** $36 - 19 + 20$

b For part **i** above, is it possible to work out $18 - 17$ and then add this total to 28?

c For part **ii** above, is it possible to work out $19 + 20$ and then subtract this total from 36?

d Can you suggest a good mental strategy for part **ii** above that gives the correct answer?

15 **a** What are the missing digits in this sum?

b Explain why there is more than one possible set of missing numbers in the sum given opposite. Give some examples.

$$\begin{array}{r} 2\ \square\ 3 \\ +\ \square\ \square\ \square \\ \hline 4\ 2\ 1 \end{array}$$

More magic squares

— | — | 16–18

ENRICHMENT

16 Complete these magic squares.

a

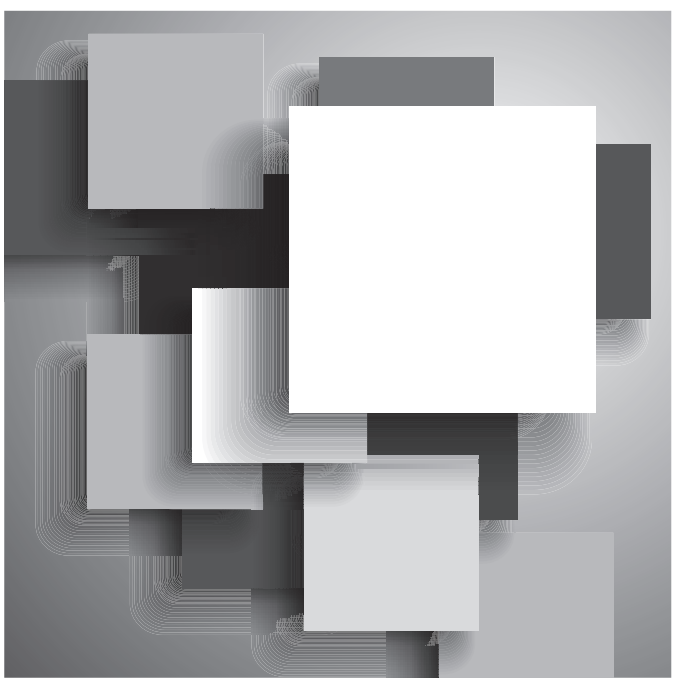
62	67	60
		65

b

101		114	
	106		109
	110		
113	103	102	116

17 The sum of two numbers is 978 and their difference is 74. What are the two numbers?

18 Make up some of your own problems like Question **17** and test them on a friend.



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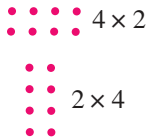
1E Multiplication

Interactive

The multiplication of two numbers represents a repeated addition.

For example, 4×2 could be thought of as 4 groups of 2 or $2 + 2 + 2 + 2$.

Similarly, 4×2 could be thought of as 2 groups of 4 or 2×4 or $4 + 4$.



Widgets

HOTsheets

Walkthroughs

Let's start: Which is the correct mental strategy?

Three students explain their method for finding the answer to 124×8.

- Billy says that you can do 124×10 to get 1240, then subtract 2 to get 1238.
- Lea says that you halve 124 and 8 twice each to give $31 \times 2 = 62$.
- Surai says that you multiply 8 by 4 to give 32, 8 by 2 to give 16 and 8 by 1 to give 8. She says the total is therefore $32 + 16 + 8 = 56$.

Are any of the students correct and can you explain any errors in their thinking?

Key ideas

- Finding the **product** of two numbers involves multiplication. We say 'the product of 2 and 3 is 6'.
- $a \times b = b \times a$ e.g. $2 \times 3 = 3 \times 2$
  - This is the **commutative law** for multiplication, meaning that the order does not matter.
- $(a \times b) \times c = a \times (b \times c)$ e.g. $(3 \times 5) \times 4 = 3 \times (5 \times 4)$
  - This is the **associative law** for multiplication, meaning it does not matter which pair is multiplied first.
- The multiplication algorithm for multiplying by a single digit involves:
  - Multiplying the single digit by each digit in the other number, starting from the right.
  - Carrying and adding any digits with a higher place value to the total in the next column.

$$\begin{array}{r} {}^{1}23 \\ \times \quad 4 \\ \hline 92 \end{array} \qquad \begin{array}{l} 4 \times 3 = 12 \\ 4 \times 2 + 1 = 9 \end{array}$$

- Mental strategies for multiplication include:
  - Knowing your multiplication tables off by heart.
  - Using the commutative law by changing the order. For example, 43×2 might be thought of more easily as 2 groups of 43 or 2×43.
  - Using the commutative and associative law by altering the order if more than one number is being multiplied.

$9 \times 7 = 63$ $\quad 12 \times 3 = 36$

$$\begin{aligned} 5 \times 11 \times 2 &= 5 \times 2 \times 11 \\ &= 10 \times 11 \\ &= 110 \end{aligned}$$

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- Using the **distributive law** by making a 10, 100 etc. and then adjusting by adding or subtracting. The distributive law is $a \times (b + c) = (a \times b) + (a \times c)$ or $a \times (b - c) = (a \times b) - (b \times c)$. This will be used more extensively in the algebra chapters.

$6 \times 21 = (6 \times 20) + (6 \times 1)$
$= 120 + 6$
$= 126$

$7 \times 18 = (7 \times 20) - (7 \times 2)$
$= 140 - 14$
$= 126$

- Using the doubling and halving strategy by doubling one number and halving the other.

$5 \times 7 \times 4 = 10 \times 7 \times 2$
$= 70 \times 2$
$= 140$

- Using factors to split a number.

$11 \times 12 = 11 \times 6 \times 2$
$= 66 \times 2$
$= 132$

Example 7 Using mental strategies for multiplication

Use a mental strategy to find the answer to each of these products.

a 7×6 **b** 3×13 **c** 4×29 **d** 5×24 **e** 7×14

SOLUTION	EXPLANATION
a $7 \times 6 = 42$	7×6 or 6×7 should be memorised (from multiplication tables).
b $3 \times 13 = 39$	$3 \times 13 = (3 \times 10) + (3 \times 3) = 30 + 9 = 39$ (The distributive law is being used.)
c $4 \times 29 = 116$	$4 \times 29 = (4 \times 30) - (4 \times 1) = 120 - 4 = 116$ (The distributive law is being used.)
d $5 \times 24 = 120$	$5 \times 24 = 10 \times 12 = 120$ (The doubling and halving strategy is being used.)
e $7 \times 14 = 98$	$7 \times 14 = 7 \times 7 \times 2 = 49 \times 2 = 98$ (Factors of 14 are used.)

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Example 8 Using the multiplication algorithm

Give the result for each of these products.

a 31×4 **b** 197×7

SOLUTION **EXPLANATION**

a
$$\begin{array}{r} 31 \\ \times\ 4 \\ \hline 124 \end{array}$$

$4 \times 1 = 4$

$4 \times 3 = 12$

b
$$\begin{array}{r} {}^{6}1\ {}^{4}9\ \ 7 \\ \times\ \ \ \ \ 7 \\ \hline 1\ 3\ 7\ 9 \end{array}$$

$7 \times 7 = 49$ (carry the 4)

$7 \times 9 + 4 = 67$ (carry the 6)

$7 \times 1 + 6 = 13$

Exercise 1E

1, 2(½), 3 | 3 | —

UNDERSTANDING

1 Write the next three numbers in these patterns.

a 4, 8, 12, 16, __ **b** 11, 22, 33, __ **c** 17, 34, 51, __

2 Are these statements true or false?

a $4 \times 3 = 3 \times 4$ **b** $2 \times 5 \times 6 = 6 \times 5 \times 2$ **c** $11 \times 5 = 10 \times 5$

d $3 \times 32 = 3 \times 30 + 3 \times 2$ **e** $5 \times 18 = 10 \times 9$ **f** $21 \times 4 = 2 \times 42$

g $19 \times 7 = 20 \times 7 - 19$ **h** $39 \times 4 = 40 \times 4 - 1 \times 4$ **i** $64 \times 4 = 128 \times 8$

3 What is the missing digit in these products?

a
$$\begin{array}{r} 2\ 1 \\ \times\ \ 3 \\ \hline 6\ \square \end{array}$$

b
$$\begin{array}{r} 3\ 6 \\ \times\ \ 5 \\ \hline 18\ \square \end{array}$$

c
$$\begin{array}{r} 7\ 6 \\ \times\ \ 2 \\ \hline 1\ \square\ 2 \end{array}$$

d
$$\begin{array}{r} 4\ 0\ 2 \\ \times\ \ \ \ 3 \\ \hline 1\ \square\ 0\ 6 \end{array}$$

4–7 | 4–5(½), 6, 7(½), 8 | 4–8(½)

FLUENCY

Example 7a

4 Using your knowledge of multiplication tables, give the answer to these products.

a 8×7 **b** 6×9 **c** 12×4 **d** 11×11

e 6×12 **f** 7×5 **g** 12×9 **h** 13×3

Example 7b, c

5 Find the results to these products mentally. Hint: Use the distributive law strategy – subtraction for **a** to **d** and addition for **e** to **h**.

a 3×19 **b** 6×29 **c** 4×28 **d** 38×7

e 5×21 **f** 4×31 **g** 6×42 **h** 53×3

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1E

FLUENCY

Example 7d, e

6 Find the answer to these products mentally. Hint: Use the double and halve strategy or split a number using its factors.

a 4×24 **b** 3×18 **c** 6×16 **d** 24×3

Example 8

7 Give the result of each of these products, using the multiplication algorithm. Check your results using a calculator.

a 33×2 **b** 43×3 **c** 72×6 **d** 55×3

e 129×2 **f** 407×7 **g** 526×5 **h** 3509×9

8 Find the answer to these products, showing your working.

a 47×5 **b** 1391×3 **c** 9×425 **d** 7×4170

9, 10 | 10–12 | 11–13

PROBLEM-SOLVING

9 Eight tickets costing $33 each are purchased for a concert. What is the total cost of the tickets?

10 A circular race track is 240 metres long and Rory runs seven laps. How far does Rory run in total?

11 Reggie and Angelo combine their packs of cards. Reggie has five sets of 13 cards and Angelo has three sets of 17 cards. How many cards are there in total?

12 Sala purchases some goods for a party at an outlet store and has $100 to spend. She selects eight bottles of drink for $2 each, 13 food packs at $6 each and 18 party hats at 50 cents each. Does she have enough money to pay for all the items?

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1E

PROBLEM-SOLVING

13 Find the missing digits in these products.

a
$$\begin{array}{r} 3\ 9 \\ \times\quad 7 \\ \hline 2\square 3 \end{array}$$

b
$$\begin{array}{r} 2\ 5 \\ \times\quad \square \\ \hline 1\,2\ 5 \end{array}$$

c
$$\begin{array}{r} 7\ 9 \\ \times\quad \square \\ \hline \square\,3\ 7 \end{array}$$

d
$$\begin{array}{r} 1\ 3\ 2 \\ \times\quad\ \ \square \\ \hline 10\square\ 6 \end{array}$$

e
$$\begin{array}{r} 2\square \\ \times\quad 7 \\ \hline \square\,8\ 9 \end{array}$$

f
$$\begin{array}{r} \square\square \\ \times\quad 9 \\ \hline 3\ 5\ 1 \end{array}$$

g
$$\begin{array}{r} 2\ 3\square \\ \times\quad\ \ 5 \\ \hline 1\square\,6\ 0 \end{array}$$

h
$$\begin{array}{r} \square\square\ 4 \\ \times\quad\ \ \square \\ \hline \square\,1\ 9\ 8 \end{array}$$

	14	14, 15	15, 16

REASONING

14 The commutative and associative laws for multiplication mean that numbers can be multiplied in any order. So $(a \times b) \times c = (b \times a) \times c = b \times (a \times c) =$ _____, where the brackets show which numbers are multiplied first. In how many ways can $2 \times 3 \times 5$ be calculated?

15 The distributive law can help to work out products mentally.
e.g. $7 \times 31 = (7 \times 30) + (7 \times 1) = 210 + 7 = 217$
Write each of the following as single products. Do not find the answer.

a $3 \times 20 + 3 \times 1$ **b** $9 \times 50 + 9 \times 2$ **c** $7 \times 30 + 7 \times 2$
d $5 \times 100 - 5 \times 3$ **e** $a \times 40 - a \times 2$ **f** $a \times 200 + a \times 3$

16 How many different ways can the two spaces be filled in this problem? Explain why.

$$\begin{array}{r} 2\square 3 \\ \times\quad 4 \\ \hline 8\square 2 \end{array}$$

Missing digits | — | — | 17, 18

ENRICHMENT

17 Find all the missing digits in these products.

a
$$\begin{array}{r} \square\ 1\ \square \\ \times\quad\ \ 7 \\ \hline \square\ 5\ 1\ \square \end{array}$$

b
$$\begin{array}{r} 2\ 9\ \square \\ \times\quad\ \ 3 \\ \hline 8\square\square \end{array}$$

18 The product of two numbers is 132 and their sum is 28.
What are the two numbers?

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1F Multiplying larger numbers

Interactive

Widgets

HOTsheets

Walkthroughs

There are many situations that require the multiplication of large numbers – for example, the total revenue from selling 40 000 tickets at $23 each, or the area of a rectangular park with length and width dimensions of 65 metres by 122 metres. To complete such calculations by hand requires the use of a suitable algorithm.

How much revenue came from selling tickets to this game?

Let's start: Spot the errors

There are three types of errors in the working shown for this problem. Find the errors and describe them.

$$\begin{array}{r} 271 \\ \times\ 13 \\ \hline 613 \\ 271 \\ \hline 1273 \end{array}$$

Key ideas

- When multiplying by 10, 100, 1000, 10 000 etc. each digit moves to the left by the number of zeros, e.g. $45 \times 1000 = 45\,000$.
- A strategy for multiplying by multiples of 10, 100 etc. is to first multiply by the number without the zeros then add the zeros to the answer later.
  For example, $21 \times 3000 = 21 \times 3 \times 1000 = 63 \times 1000 = 63\,000$
- The algorithm for multiplying large numbers involves dividing the problem into smaller products and then adding the totals.

$$\begin{array}{rcl} 143 & & \\ \times\ 14 & & \\ \hline {}^{1}572 & \leftarrow & 143 \times 4 \\ 1430 & \leftarrow & 143 \times 10 \\ \hline 2002 & \leftarrow & 1430 + 572 \end{array}$$

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Example 9 Multiplying large numbers

Give the result for each of these products.

a 37×100 **b** 45×70 **c** 614×14

SOLUTION	EXPLANATION
a $37 \times 100 = 3700$	Move the 3 and the 7 two places to the left and add two zeros.
b $45 \times 70 = 45 \times 7 \times 10$ $= 315 \times 10$ $= 3150$	First multiply by 7, then multiply by 10 later. 45 × 7 315
c 614 × 14 2456 6140 8596	First multiply 614×4. Then multiply 614×10. Add the totals to give the answer.

Exercise 1F

1–3 | 3 | —

UNDERSTANDING

1 What is the missing digit in these products?

a $72 \times 10 = 7\square 0$ **b** $13 \times 100 = 130\square$ **c** $49 \times 100 = 49\square 0$ **d** $924 \times 10 = 92\square 0$

2 What is the missing number in these products?

a $15 \times \_\_ = 1500$ **b** $329 \times \_\_ = 3290$ **c** $92 \times \_\_ = 920\,000$

3 State if the following calculations are correct. If they are incorrect, find the correct answer.

a
26
× 4
84

b
39
× 14
156
39
195

c
92
× 24
368
1840
2208

d
102
× 24
408
240
648

How could you calculate the number of pieces of fruit on this stall without counting them all?

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1F

4–6(½) | 4–7(½) | 4–7(½)

FLUENCY

Example 9a

4 Give the result of each of these products.

a 4×100 **b** 29×10 **c** 183×10 **d** 46×100

e 50×1000 **f** 630×100 **g** 1441×10 **h** $2910 \times 10\,000$

Example 9b

5 Use the suggested strategy in Example 9b to find these products.

a 17×20 **b** 36×40 **c** 92×70 **d** 45×500

e 138×300 **f** 92×5000 **g** 317×200 **h** 1043×9000

Example 9c

6 Use the multiplication algorithm to find these products.

a 37×11 **b** 72×19 **c** 126×15 **d** 428×22

e 396×46 **f** 416×98 **g** 380×49 **h** 1026×33

7 First estimate the answers to these products, then use a calculator to check.

a 19×11 **b** 26×21 **c** 37×15 **d** 121×18

8, 9 | 9–11 | 10–12

PROBLEM-SOLVING

8 A pool area includes 68 square metres of paving at $32 per square metre. What is the total cost of paving?

9 Waldo buys 215 metres of pipe at $28 per metre. What is the total cost of piping?

10 How many seconds are there in one day?

11 Find the missing digits in these products.

a
$$\begin{array}{r} 2\square \\ \times\ 1\ 7 \\ \hline 1\square\ 1 \\ 2\square\ 0 \\ \hline \square\square\ 1 \end{array}$$

b
$$\begin{array}{r} 1\square\ 3 \\ \times\ \ 1\square \\ \hline \ \ \square\ 2\ 9 \\ 1\square\ 3\square \\ \hline \square\square\ 5\square \end{array}$$

c
$$\begin{array}{r} \square\square \\ \times\ \ 3\ 7 \\ \hline \ \ 3\ 4\ 3 \\ \square\ 4\square\square \\ \hline \square\square\square\square \end{array}$$

d
$$\begin{array}{r} \square\ 2\square \\ \times\ \ \ 2\square \\ \hline \ \ 1\ 2\ 6 \\ \square\ 5\ 2\square \\ \hline \square\ 6\square\square \end{array}$$

12 There are 360 degrees in a full turn. How many degrees does the minute hand on a clock turn in one week?

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1F

13 | 13, 14 | 14, 15

REASONING

13 The product of two whole numbers is less than their sum. Neither number is zero or less. What must be true about one of the numbers?

14 If both numbers in a multiplication problem have at least three digits, then the algorithm needs to be expanded. Use the algorithm to find these products.

a $\begin{array}{r} 294 \\ \times\ 136 \\ \hline \end{array}$ **b** $\begin{array}{r} 1013 \\ \times\ 916 \\ \hline \end{array}$ **c** $\begin{array}{r} 3947 \\ \times\ 1204 \\ \hline \end{array}$ **d** $\begin{array}{r} 47126 \\ \times\ 3107 \\ \hline \end{array}$

15 Can you work out these computations using an effective mental strategy? Look to see if you can first simplify each question.

a $98 \times 16 + 2 \times 16$

b $33 \times 26 - 3 \times 26$

c $19 \times 15 + 34 \times 17 - 4 \times 17 + 1 \times 15$

d $22 \times 19 - 3 \times 17 + 51 \times 9 - 1 \times 9 + 13 \times 17 - 2 \times 19$

Multiplication puzzle

— | — | 16, 17

ENRICHMENT

16 **a** What is the largest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

b What is the smallest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

$\square\square\square \times \square\square$

17 The product of two whole numbers is 14 391 and their difference is 6. What are the two numbers?

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Progress quiz

1A **1** Write the number 134 using the given number systems.

Ext

a Egyptian **b** Roman

1B **2** Write the number 50862 in expanded form.

1C **3** Use the suggested strategy to mentally work out the answer.

a 143 + 232 (partitioning) **b** 35 + 29 (compensating)

c 74 – 17 (compensating) **d** 35 + 36 (doubling)

1D **4** Give the result for each of these problems.

a Addition

$$\begin{array}{r} 18 \\ +\ 44 \\ \hline \end{array}$$

b Subtraction

$$\begin{array}{r} 124 \\ -\ 46 \\ \hline \end{array}$$

1E **5** Using your knowledge of multiplication tables, give the answer to these products.

a 7×4 **b** 9×8 **c** 12×9 **d** 5×9

1E **6** Use the distributive law strategy to find the answer to each of these products. Show your working.

a 6×14 **b** 5×39

1E **7** Give the result of each of these products, using the multiplication algorithm. Show your working.

a
$$\begin{array}{r} 84 \\ \times\ 3 \\ \hline \end{array}$$

b
$$\begin{array}{r} 237 \\ \times\ 4 \\ \hline \end{array}$$

c 2146×7

1F **8** Give the result of each of these products. Show your working.

a 38×100 **b** 24×70 **c** 513×200

1F **9** Use the multiplication algorithm to find these products.

a
$$\begin{array}{r} 26 \\ \times\ 12 \\ \hline \end{array}$$

b
$$\begin{array}{r} 317 \\ \times\ 16 \\ \hline \end{array}$$

1D **10** Two different schools have 948 and 1025 students enrolled.

a How many students are there in total?

b Find the difference in the number of students between the schools.

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1G Division

Division involves finding the number of equal groups into which a particular number can be divided. This can be achieved both with and without a remainder or 'left over'. Dividing 20 apples among five people and $10 000 between three bank accounts are examples of when division can be used.

Multiplication and division are reverse operations, and this is shown in this simple example:

$7 \times 3 = 21$ So, $21 \div 3 = 7$ or $21 \div 7 = 3$

Let's start: Arranging counters

A total of 24 counters sit on a table. Using whole numbers, in how many ways can the counters be divided into equal-sized groups with no counters remaining?

- Is it also possible to divide the counters into equal-sized groups but with two counters remaining?
- If five counters are to remain, how many equal-sized groups can be formed and why?

Key ideas

- The number of equal-sized groups formed from the division operation is called the **quotient**.
- The total being divided is called the **dividend** and the size of the equal groups is called the **divisor**.
- Any amount remaining after division into equal-sized groups is called the **remainder**.

  $7 \div 3 = 2$ and 1 remainder $= 2\frac{1}{3}$

  total being divided (dividend) / size of equal groups (divisor) / quotient

  $7 \div 3 = 2$ and 1 remainder means
  $7 = 2 \times 3 + 1$
  $37 \div 5 = 7$ and 2 remainder means
  $37 = 7 \times 5 + 2$
- $a \div b \neq b \div a$ (in general)
  - The commutative law does not hold for division, e.g. $8 \div 2 \neq 2 \div 8$
- $(a \div b) \div c \neq a \div (b \div c)$
  - The associative law does not hold for division, e.g. $(8 \div 4) \div 2 \neq (4 \div 2)$
- The short division algorithm involves first dividing into the digit with the highest place value and then carrying any remainder to the next digit, working from left to right.

  $413 \div 3 = 137$ and 2 remainder
  $= 137\frac{2}{3}$

  $4 \div 3 = 1$ and 1 rem. / $11 \div 3 = 3$ and 2 rem. / $23 \div 3 = 7$ and 2 rem.

  $3\overline{)4^{1}1^{2}3}$ with quotient 137

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Key ideas

- Mental division can be done using different strategies.
  - Knowing your multiplication tables off by heart.
  - $63 \div 9 = ?$ is the same as asking $9 \times ? = 63$.
  - Making a convenient multiple of the divisor and then adjusting by adding or subtracting. Below is an application of the distributive law.

$$\begin{aligned} 84 \div 3 &= (60 + 24) \div 3 \\ &= (60 \div 3) + (24 \div 3) \\ &= 20 + 8 \\ &= 28 \end{aligned} \qquad \begin{aligned} 84 \div 3 &= (90 - 6) \div 3 \\ &= (90 \div 3) - (6 \div 3) \\ &= 30 - 2 \\ &= 28 \end{aligned}$$

  - Halving both numbers. If both numbers in the division are even, then halve both numbers.

$$\begin{aligned} 70 \div 14 &= 35 \div 7 \\ &= 5 \end{aligned}$$

Example 10 Using mental strategies for division

Use a mental strategy to find the quotient.

a $84 \div 7$ **b** $93 \div 3$ **c** $128 \div 8$

SOLUTION	EXPLANATION
a $84 \div 7 = 12$	$7 \times ? = 84$ (Use your knowledge from multiplication tables.)
b $93 \div 3 = 31$	$93 \div 3 = (90 \div 3) + (3 \div 3) = 30 + 1$ (This uses the distributive law.)
c $128 \div 8 = 16$	$128 \div 8 = 64 \div 4 = 32 \div 2 = 16$ (Halve both numbers repeatedly.)

Example 11 Using the short division algorithm

Use the short division algorithm to find the quotient and remainder.

a $3\overline{)37}$ **b** $7\overline{)195}$

SOLUTION	EXPLANATION
a $3\overline{)37}^{\,12}$ $37 \div 3 = 12$ and 1 remainder.	$3 \div 3 = 1$ with no remainder. $7 \div 3 = 2$ with 1 remainder.

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b $7\overline{)19^{5}5}$ $\quad$ $2\ 7$

195 ÷ 7 = 27 and 6 remainder.

7 does not divide into 1.

19 ÷ 7 = 2 with 5 remainder.

55 ÷ 7 = 7 with 6 remainder.

Exercise 1G

1–3 | 3 | —

UNDERSTANDING

1 Write the number that is missing in these statements.

a 8 ÷ 2 = 4 is the same as 4 × ? = 8.

b 36 ÷ 12 = 3 is the same as ? × 12 = 36.

c 42 ÷ ? = 6 is the same as 6 × 7 = 42.

d 72 ÷ 6 = ? is the same as 12 × 6 = 72.

2 What is the remainder when:

a 2 is divided into 7?

b 5 is divided into 37?

c 42 is divided by 8?

d 50 is divided by 9?

3 Write the missing digit in each of these divisions.

a $3\overline{)5\ 1}$ $\quad$ $\square 7$

b $7\overline{)8\ 4}$ $\quad$ $\square 2$

c $5\overline{)1\ 2\ 5}$ $\quad$ $2\square$

d $9\overline{)1\ 3\ 5}$ $\quad$ $1\square$

4–5(½), 6–8 | 4–5(½), 6–9 | 4–9(½)

FLUENCY

Example 10a **4** Use your knowledge of multiplication tables to find the quotient.

a 28 ÷ 7 **b** 36 ÷ 12 **c** 48 ÷ 8 **d** 45 ÷ 9

e 56 ÷ 8 **f** 63 ÷ 7 **g** 96 ÷ 12 **h** 121 ÷ 11

Example 10b **5** Find the answer to these using a mental strategy. Hint: Use the distributive law strategy.

a 63 ÷ 3 **b** 76 ÷ 4 **c** 57 ÷ 3 **d** 205 ÷ 5

e 203 ÷ 7 **f** 189 ÷ 9 **g** 906 ÷ 3 **h** 490 ÷ 5

Example 10c **6** Find the answers to these using a mental strategy. Hint: Use the halving strategy by halving both numbers.

a 88 ÷ 4 **b** 124 ÷ 4 **c** 136 ÷ 8 **d** 112 ÷ 16

7 Write the answers to these divisions, which involve 0s and 1s.

a 26 ÷ 1 **b** 1094 ÷ 1 **c** 0 ÷ 7 **d** 0 ÷ 458

Example 11 **8** Use the short division algorithm to find the quotient and remainder.

a $3\overline{)71}$ **b** $7\overline{)92}$ **c** $2\overline{)139}$ **d** $6\overline{)247}$

e $4\overline{)2173}$ **f** $3\overline{)61\,001}$ **g** $5\overline{)4093}$ **h** $9\overline{)90\,009}$

9 Use the short division algorithm to find the quotient and remainder.

a 526 ÷ 4 **b** 1691 ÷ 7 **c** 2345 ÷ 6 **d** 92 337 ÷ 8

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1G

10–12 | 11–14 | 14–16

PROBLEM-SOLVING

10 If 117 food packs are divided equally among nine families, how many packs does each family receive?

11 Spring Fresh Company sells mineral water in packs of six bottles. How many packs are there in a truck containing 744 bottles?

12 A bricklayer earns $1215 in a week.

a How much does he earn per day if he works Monday to Friday?

b How much does he earn per hour if he works 9 hours per day?

13 A straight fence has two end posts as well as other posts that are divided evenly along the fence 4 metres apart. If the fence is to be 264 metres long, how many posts are needed, including the end posts?

14 Friendly Taxis can take up to four passengers each. How many taxis are required to transport 59 people?

15 A truck can carry up to 7 tonnes of rock. What is the minimum number of trips needed to transport 130 tonnes of rock?

16 All the rows, columns and main diagonals in the magic square multiply to give 216. Can you find the missing numbers?

	9	12
		1

17, 18 | 17–19 | 20–22

REASONING

17 Write down the missing numbers.

a $37 \div 3 = 12$ and $\square$ remainder means $37 = \square \times 3 + 1$

b $96 \div 7 = \square$ and 5 remainder means $96 = 13 \times \square + 5$

c $104 \div 20 = 5$ and $\square$ remainder means $104 = \square \times 20 + 4$

18 Pies are purchased wholesale at 9 for $4. How much will it cost to purchase 153 pies?

19 Give the results to these problems, if a represents any number.

a $a \div 1$

b $0 \div a$

c $a \div a$

20 A number less than 30 leaves a remainder of 3 when divided by 5 and a remainder of 2 when divided by 3. What two numbers meet the given conditions?

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1G

REASONING

21 As you know $a \div b$ is not generally equal to $b \div a$. However, can you find a situation where $a \div b = b \div a$?

22 The short division algorithm can also be used to divide by numbers with more than one digit.

e.g. $215 \div 12 = 17$ and 11 remainder.

$21 \div 12 = 1$ and 9 remainder.

$95 \div 12 = 7$ and 11 remainder.

$$12\overline{)21^{9}5}\quad 1\ 7$$

Use the short division algorithm to find the quotient and remainder.

a $371 \div 11$ **b** $926 \div 17$ **c** $404 \div 13$

d $1621 \div 15$ **e** $2109 \div 23$ **f** $6914 \div 56$

Long, short division — — 23–25

ENRICHMENT

23 Use the short division algorithm to find the quotient and remainder.

a $1247 \div 326$ **b** $1094 \div 99$ **c** $26\,401 \div 1432$

24 The magic product for this square is 6720. Find the missing numbers.

1	6		56
40		2	3
14			
			10

25 Instead of carrying out a complex division algorithm, you could convert the divisor into a smaller pair of factors and complete two simpler division questions to arrive at the correct answer.

For example: $1458 \div 18 = (1458 \div 2) \div 9$

$= 729 \div 9$

$= 81$

Use factors to help you calculate the following.

a $555 \div 15$ **b** $860 \div 20$ **c** $3600 \div 48$

d $1456 \div 16$ **e** $6006 \div 42$ **f** $2024 \div 22$

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1H Estimating and rounding CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

Many theoretical and practical problems do not need precise or exact answers. In such situations reasonable estimations can provide enough information to solve the problem.

The total revenue from the Australian Open tennis tournament depends on crowd numbers. Estimates would be used before the tournament begins to predict these numbers. An estimate for the total revenue might be $8 million.

Let's start: The tennis crowd

Here is a photo of a crowd at a tennis match. Describe how you might estimate the number of people in the photo. What is your answer? How does your answer differ from those of others in your class?

How can you estimate the number of spectators?

Key ideas

- Estimates or approximations to the answers of problems can be found by **rounding** numbers to the nearest 10, 100, 1000 etc.
- If the next digit is 0, 1, 2, 3 or 4, then round down.
- If the next digit is 5, 6, 7, 8 or 9, then round up.
- **Leading digit approximation** rounds the first digit to the nearest 10 or 100 or 1000 etc.
  e.g. For 932 use 900 For 968 use 1000
- The symbol $\approx$ means '**approximately equal to**'. The symbol $\fallingdotseq$ can also be used.

Example 12 Rounding

Round these numbers as indicated.

a 86 (to the nearest 10)

b 4142 (to the nearest 100)

SOLUTION	EXPLANATION
a $86 \approx 90$	The digit after the 8 is greater than or equal to 5, so round up.
b $4142 \approx 4100$	The digit after the 1 is less than or equal to 4, so round down.

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Example 13 Using leading digit approximation

Estimate the answers to these problems by rounding each number to the leading digit.

a 42×7 **b** 95×326

SOLUTION	EXPLANATION
a $42 \times 7 \approx 40 \times 7$ $= 280$	The leading digit in 42 is the 4 in the 'tens' column.
b $95 \times 326 \approx 100 \times 300$ $= 30\,000$	The nearest 'ten' to 95 is 100, and the leading digit in 326 is in the 'hundreds' column.

Example 14 Estimating with operations

Estimate the answers to these problems by rounding both numbers as indicated.

a 115×92 (to the nearest 100) **b** $2266 \div 9$ (to the nearest 10)

SOLUTION	EXPLANATION
a $115 \times 92 \approx 100 \times 100$ $= 10\,000$	115 rounds to 100 and 92 rounds to 100.
b $2266 \div 9 \approx 2270 \div 10$ $= 227$	2266 rounds to 2270 and 9 rounds to 10.

Exercise 1H

1, 2 | 2 | —

UNDERSTANDING

1 State whether these numbers have been rounded up or down.

a $59 \approx 60$ **b** $14 \approx 10$ **c** $137 \approx 140$

d $255 \approx 260$ **e** $924 \approx 900$ **f** $1413 \approx 1000$

2 For the given estimates, decide if the approximate answer is going to give a larger or smaller result compared to the true answer.

a $58 + 97 \approx 60 + 100$ **b** $24 \times 31 \approx 20 \times 30$

c $130 - 79 \approx 130 - 80$ **d** $267 - 110 \approx 270 - 110$

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1H

3–7(½) | 3–7(½) | 3–7(½)

FLUENCY

Example 12

3 Round these numbers as indicated.

a 59 (nearest 10) b 32 (nearest 10) c 124 (nearest 10)
d 185 (nearest 10) e 231 (nearest 100) f 894 (nearest 100)
g 96 (nearest 10) h 584 (nearest 100) i 1512 (nearest 1000)

4 Round these numbers using leading digit approximation; i.e. round to the first digit.

a 21 b 29 c 136 d 857
e 5600 f 92 104 g 9999 h 14

5 Estimate the answers to these problems by first rounding both numbers as indicated.

a 72 + 59 (nearest 10) b 138 – 61 (nearest 10)
c 275 – 134 (nearest 10) d 841 + 99 (nearest 10)
e 203 – 104 (nearest 100) f 815 + 183 (nearest 100)
g 990 + 125 (nearest 100) h 96 + 2473 (nearest 100)
i 1555 – 555 (nearest 1000)

Example 13

6 Use leading digit approximation to estimate the answer.

a 29×4 b $124 + 58$ c $232 - 106$ d $61 \div 5$
e $394 \div 10$ f 97×21 g $1390 + 3244$ h $999 - 888$

Example 14

7 Estimate the answers to these problems by rounding both numbers as indicated.

a 29×41 (nearest 10) b 92×67 (nearest 10)
c 124×173 (nearest 100) d 2402×3817 (nearest 1000)
e $48 \div 11$ (nearest 10) f $159 \div 12$ (nearest 10)
g $104 \div 11$ (nearest 10) h $2493 \div 103$ (nearest 100)

8, 9 | 9–11 | 10–12

PROBLEM-SOLVING

8 Many examples of Aboriginal art include dot paintings. Here is one example. Estimate the number of dots it contains.

9 A digger can dig 29 scoops per hour and work 7 hours per day. Approximately how many scoops can be dug over 10 days?

10 Most of the pens at a stockyard are full of sheep. There are 55 pens and one of the pens has 22 sheep. Give an estimate for the total number of sheep at the stockyard.

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PROBLEM-SOLVING

11 A whole year group of 159 students is roughly divided into 19 groups. Estimate the number in each group.

12 It is sensible sometimes to round one number up if the other number is going to be rounded down. Use leading digit approximation to estimate the answers to these problems.

a 11×19 **b** 129×954 **c** 25×36 **d** 1500×2500

13a, b | 13 | 13

REASONING

13 The letters a and b represent numbers. Which of the words 'smaller' or 'larger' completes these sentences?

a If a and b are both rounded up, then compared to the true answer the approximate answer to:

i $a + b$ will be _____.

ii $a \times b$ will be _____.

b If only a is rounded up, but b is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be _____.

ii $a \div b$ will be _____.

c If only b is rounded up, but a is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be _____.

ii $a \div b$ will be _____.

d If only b is rounded down, but a is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be _____.

ii $a \div b$ will be _____.

Maximum error | — | — | 14

ENRICHMENT

14 When rounding numbers before a calculation is completed, it is most likely that there will be an error. This error can be large or small, depending on the type of rounding involved.
For example, when rounding to the nearest 10, $71 \times 11 \approx 70 \times 10 = 700$.
But $71 \times 11 = 781$, so the error is 81.

a Calculate the error if these numbers are rounded to the nearest 10 before the multiplication is calculated.

i 23×17 **ii** 23×24 **iii** 65×54 **iv** 67×56

b Explain why the error in parts **i** and **iii** is much less than the error in parts **ii** and **iv**.

c Calculate the error if these numbers are rounded to the nearest 10 before the division is calculated.

i $261 \div 9$ **ii** $323 \div 17$ **iii** $99 \div 11$ **iv** $396 \div 22$

d Explain why the approximate answers in parts **i** and **ii** are less than the correct answer, and why the approximate answers in parts **iii** and **iv** are more than the correct answer.

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1I Order of operations

Interactive

When combining the operations of addition, subtraction, multiplication and division, a particular order needs to be followed. Multiplication and division sit higher in the order than addition and subtraction, and this relates to how we might logically interpret simple mathematical problems put into words.

Widgets

HOTsheets

Walkthroughs

Consider these two statements.

- 2 groups of 3 chairs plus 5 chairs.
- 5 chairs plus 2 groups of 3 chairs.

In both cases, there are $2 \times 3 + 5 = 11$ chairs. This means that $2 \times 3 + 5 = 5 + 2 \times 3$.

This also suggests that for $5 + 2 \times 3$ the multiplication should be done first.

Let's start: Minimum brackets

- How might you use brackets to make this statement true?

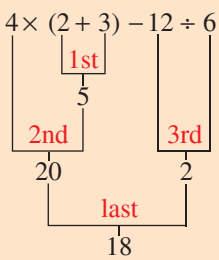
$$2 + 3 \times 5 - 3 \div 6 + 1 = 2$$

- What is the minimum number of pairs of brackets needed to make it true?

Key ideas

- When working with more than one operation:
  - Deal with **brackets** (also known as parentheses) first.
  - Do **multiplication** and **division** next, working from left to right.
  - Do **addition** and **subtraction** last, working from left to right.
- Recall $(a + b) + c = a + (b + c)$ but $(a - b) - c \neq a - (b - c)$

  $(a \times b) \times c = a \times (b \times c)$ but $(a \div b) \div c \neq a \div (b \div c)$
- Brackets can sit inside other brackets.
  - Square brackets can also be used. e.g. $[2 \times (3 + 4) - 1] \times 3$
  - Always deal with the inner brackets first.
- Note that some calculators apply the order of operations and some do not.



Example 15 Using order of operations

Use order of operations to answer the following.

a $5 + 10 \div 2$ b $18 - 2 \times (4 + 6) \div 5$

SOLUTION	EXPLANATION
a $5 + 10 \div 2 = 5 + 5$ $= 10$	Do the division before the addition.

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b $18 - 2 \times (4 + 6) \div 5 = 18 - 2 \times 10 \div 5$
$= 18 - 20 \div 5$
$= 18 - 4$
$= 14$

Deal with brackets first.

Do the multiplication and division next, working from left to right.

Do the subtraction last.

Example 16 Using order of operations in worded problems

Find the difference between 76 and 43, triple this result and, finally, subtract the quotient of 35 and 7.

SOLUTION

a $3 \times (76 - 43) - 35 \div 7 = 3 \times 33 - 5$
$= 99 - 5$
$= 94$

EXPLANATION

First, write the problem using symbols and numbers.
Use brackets for the difference since this operation is to be completed first.

Exercise 1I

1(½), 2 | 2 | —

UNDERSTANDING

1 Which operation (addition, subtraction, multiplication or division) is done first in the following?

a $2 + 5 - 3$ **b** $5 \div 5 \times 2$ **c** $2 \times 3 \div 6$
d $5 \times 2 + 3$ **e** $7 \div 7 - 1$ **f** $(6 + 2) \times 3$
g $(8 \div 4) - 1$ **h** $4 + 7 \times 2$ **i** $8 - 10 \div 5$
j $10 - 2 + 3$ **k** $6 + 2 \times 3 - 1$ **l** $5 \times (2 + 3 \div 3) - 1$

2 Classify these statements as true or false.

a $5 \times 2 + 1 = (5 \times 2) + 1$ **b** $10 \times (3 + 4) = 10 \times 3 + 4$
c $21 - 7 \div 7 = (21 - 7) \div 7$ **d** $9 - 3 \times 2 = 9 - (3 \times 2)$

3–4(½), 5, 6 | 3–4(½), 5, 6 | 3–6(½)

FLUENCY

Example 15a

3 Use order of operations to find the answers to the following. Check your answers using a calculator and see if it applies the order of operations.

a $2 + 3 \times 7$ **b** $5 + 8 \times 2$ **c** $10 - 20 \div 2$
d $22 - 16 \div 4$ **e** $6 \times 3 + 2 \times 7$ **f** $1 \times 8 - 2 \times 3$
g $18 \div 9 + 60 \div 3$ **h** $2 + 3 \times 7 - 1$ **i** $40 - 25 \div 5 + 3$
j $63 \div 3 \times 7 + 2 \times 3$ **k** $78 - 14 \times 4 + 6$ **l** $300 - 100 \times 4 \div 4$

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11

Example 15b

FLUENCY

4 Use order of operations to find the answer to the following problems. Check your answers using a calculator and see if it applies the order of operations.

a $2 \times (3 + 2)$	**b** $18 \div (10 - 4)$	**c** $(19 - 9) \div 5$
d $(100 + 5) \div 5 + 1$	**e** $2 \times (9 - 4) \div 5$	**f** $50 \div (13 - 3) + 4$
g $16 - 2 \times (7 - 5) + 6$	**h** $(7 + 2) \div (53 - 50)$	**i** $14 - (7 \div 7 + 1) \times 2$
j $(20 - 10) \times (5 + 7) + 1$	**k** $3 \times (72 \div 12 + 1) - 1$	**l** $48 \div (4 + 4) \div (3 \times 2)$

5 These computations involve brackets within brackets. Ensure you work with the inner brackets first.

a $2 \times [(2 + 3) \times 5 - 1]$	**b** $[10 \div (2 + 3) + 1] \times 6$	**c** $26 \div [10 - (17 - 9)]$
d $[6 - (5 - 3)] \times 7$	**e** $2 + [103 - (21 + 52)] - (9 + 11) \times 6 \div 12$	

Example 16

6 Find the answer to these worded problems by first writing the sentence using numbers and symbols.

a Triple the sum of 3 and 6.
b Double the quotient of 20 and 4.
c The quotient of 44 and 11 plus 4.
d 5 more than the product of 6 and 12.
e The quotient of 60 and 12 is subtracted from the product of 5 and 7.
f 15 less than the difference of 48 and 12.
g The product of 9 and 12 is subtracted from double the product of 10 and 15.

7, 8 | 8, 9 | 8–10

PROBLEM-SOLVING

7 A delivery of 15 boxes of books arrives, each box containing eight books. The bookstore owner removes three books from each box. How many books still remain in total?

8 In a class, eight students have three TV sets at home, four have two TV sets, 13 have one TV set and two students have no TV sets. How many TV sets are there in total?

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11

PROBLEM-SOLVING

9 Insert brackets into these statements to make them true.

a $4 + 2 \times 3 = 18$ **b** $9 \div 12 - 9 = 3$ **c** $2 \times 3 + 4 - 5 = 9$

d $3 + 2 \times 7 - 3 = 20$ **e** $10 - 7 \div 21 - 18 = 1$ **f** $4 + 10 \div 21 \div 3 = 2$

g $20 - 31 - 19 \times 2 = 16$ **h** $50 \div 2 \times 5 - 4 = 1$ **i** $25 - 19 \times 3 + 7 \div 12 + 1 = 6$

10 The amount of $100 is divided into two first prizes of equal value and three second prizes of equal value. Each prize is a whole number of dollars and first prize is at least four times the value of second prize. If second prize is more than $6, find the amount of each prize.

11 | 11, 12 | 12, 13

REASONING

11 Decide if the brackets given in each statement are actually necessary; that is, do they make any difference to the problem?

a $2 + (3 \times 6) = 20$ **b** $(2 + 3) \times 6 = 30$ **c** $(20 \times 2) \times 3 = 120$

d $10 - (5 + 2) = 3$ **e** $22 - (11 - 7) = 18$ **f** $19 - (10 \div 2) = 14$

g $(40 \div 10) \div 4 = 1$ **h** $100 \div (20 \div 5) = 25$ **i** $2 \times (3 + 2) \div 5 = 2$

12 The letters a, b and c represent numbers. Decide if the brackets are necessary in these expressions.

a $a + (b + c)$ **b** $a - (b - c)$ **c** $a \times (b \times c)$ **d** $a \div (b \div c)$

13 Simplify the following. Assume $b \neq 0$, and $a \neq 0$.

a $a + b - a$ **b** $(a - a) \times b$ **c** $a + b \div b$ **d** $a \times b \div a$

Operation in rules — | — | 14

ENRICHMENT

14 Using whole numbers and any of the four operations (+, –, ×, ÷), describe how you would obtain the 'Finish' number from the 'Start' number in each of these tables. Your rule must work for every pair of numbers in its table.

a

Start	Finish
1	3
2	5
3	7
4	9

b

Start	Finish
1	0
2	3
3	6
4	9

c

Start	Finish
3	10
4	17
5	26
6	37

Make up your own table with a 'secret' rule and test it on a friend.

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Investigation

The abacus

The abacus is a counting device that has been used for thousands of years. Abacuses were used extensively by merchants, traders, tax collectors and clerks before modern-day numerals systems were developed. Counting boards called Abax date back to 500 BCE. These were wood or stone tablets with grooves, which would hold beans or pebbles.

The modern abacus is said to have originated in China in about the thirteenth century and includes beads on wires held in a wooden frame.

A German woodcut from 1508 showing an abacus in use by the gentleman on the right, while a mathematician (at left) writes algorithms.

A modern abacus with 13 wires.

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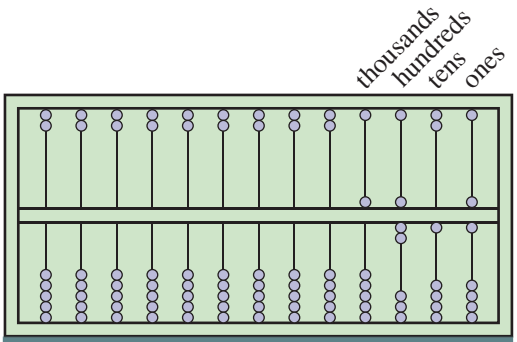
There are 5 beads on one side of a modern abacus worth 1 each and 2 beads on the opposite side worth 5 each.

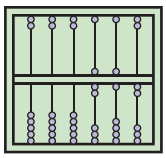
- Each wire represents a different unit, e.g. ones, tens, hundreds etc.
- Beads are counted only when they are pushed towards the centre.



Here is a diagram showing the number 5716.

a What numbers are showing on the abacus diagrams below? Only the first six wires are showing.

i



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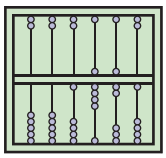

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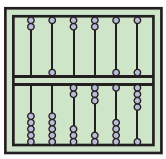

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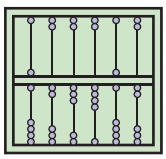

b Draw abacus diagrams showing these numbers.

i 57 **ii** 392 **iii** 6804 **iv** 290 316

c Imagine adding two numbers using an abacus by sliding beads along their wires. Clearly explain the steps taken to add these numbers.

i $11 + 7$ **ii** $2394 + 536$

d Imagine subtracting two numbers using an abacus by sliding beads along their wires. Clearly explain the steps taken to subtract these numbers.

i $23 - 14$ **ii** $329 - 243$

e Multiplication is calculated as a repeated addition.
e.g. $3 \times 21 = 21 + 21 + 21$
Clearly explain the steps involved when using an abacus to multiply these numbers.

i 3×42 **ii** 5×156

f Division is calculated as a repeated subtraction.
e.g. $63 \div 21 = 3$, since $63 - 21 - 21 - 21 = 0$
Clearly explain the steps involved when using an abacus to divide these numbers.

i $28 \div 7$ **ii** $405 \div 135$

g See if you can find a real abacus or computer abacus with which to work. Use the abacus to show how you can do the problems in Questions **c** to **f** above.

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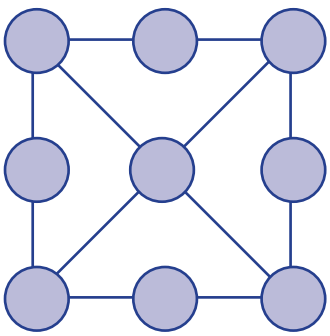
Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 The extra dollar. The cost of dinner for two people is $45 and they both give the waiter $25 each. Of the extra $5 the waiter is allowed to keep $3 as a tip and returns $1 to each person. So the two people paid $24 each, making a total of $48, and the waiter has $3. The total is therefore $48 + $3 = $51. Where did the extra $1 come from?

2 The sum along each line is 15. Can you place each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make this true?



3 Ethan starts at 2637 and counts backwards by eights. He stops counting when he reaches a number less than 10. What is this final number?

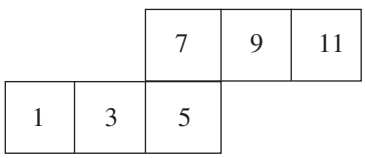
4 Make the total of 100 out of all the numbers 2, 3, 4, 7 and 11, using each number only once. You can use any of the operations (+, –, ×, ÷), as well as brackets.

5 A leaking tap loses 1 drop of water per second. If 40 of these drops of water make a volume of 10 mL, how many litres of water are wasted from this tap in mL:

a in 1 day? (round answer to the nearest unit)

b in 1 year? (round answer to the nearest 100)

6 When this shape is folded to make a cube, three of the sides will meet at every vertex (corner) of the cube. The numbers on these three sides can be multiplied together.
Find the smallest and largest of these products.



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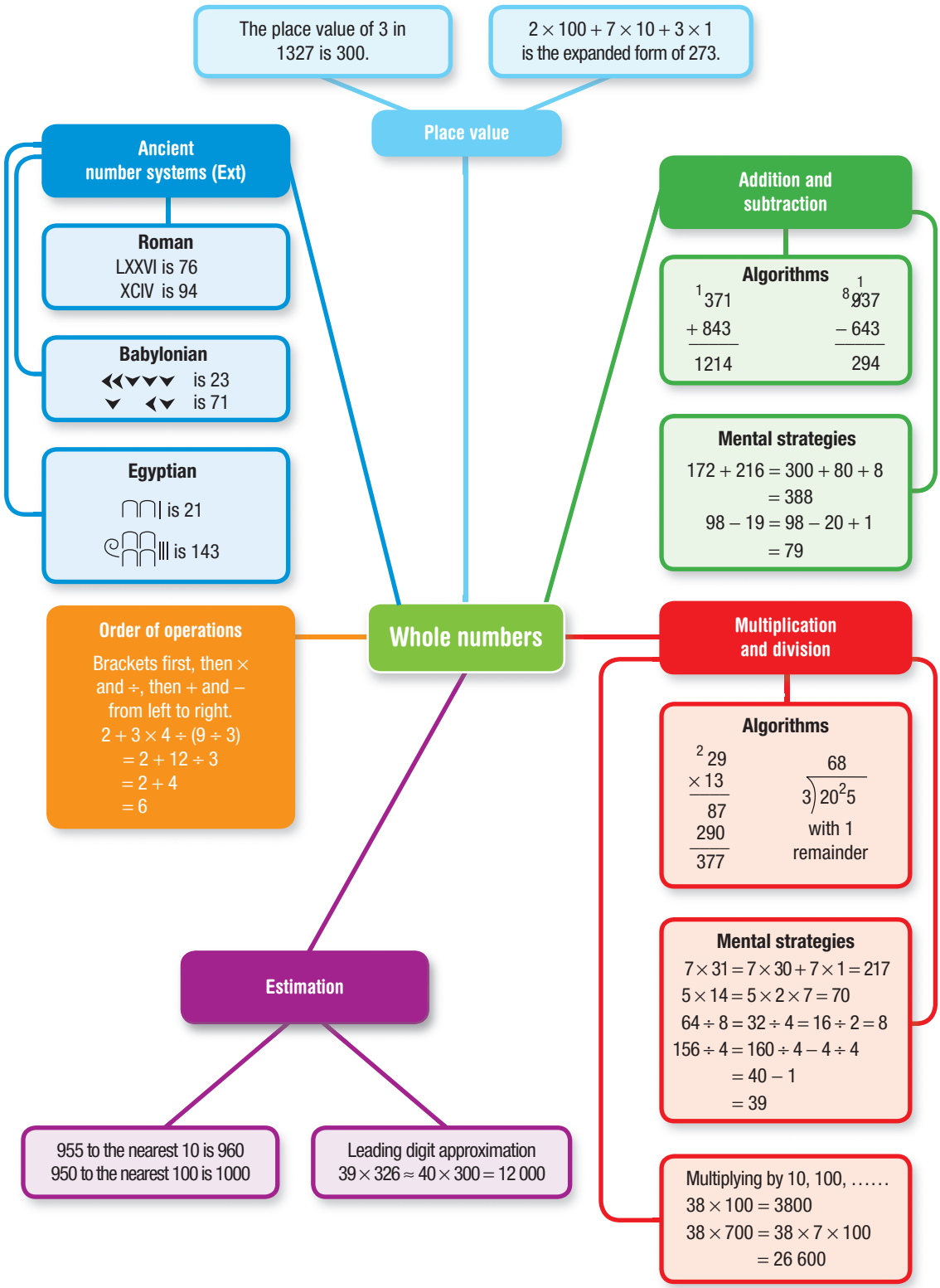
Chapter summary



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Chapter review

Multiple-choice questions

1A Ext **1** The correct Roman numerals for the number 24 are:

A XXIII **B** XXIV **C** XXXLIV **D** IVXX **E** IXXV

1B **2** $3 \times 1000 + 9 \times 10 + 2 \times 1$ is the expanded form of:

A 3920 **B** 392 **C** 3092 **D** 3902 **E** 329

1C/E **3** Which of the following is not true?

A $2 + 3 = 3 + 2$ **B** $2 \times 3 = 3 \times 2$ **C** $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

D $5 \div 2 \neq 2 \div 5$ **E** $7 - 2 = 2 - 7$

1C **4** The sum of 198 and 103 is:

A 301 **B** 304 **C** 299 **D** 199 **E** 95

1C **5** The difference between 378 and 81 is:

A 459 **B** 297 **C** 303 **D** 317 **E** 299

1E **6** The product of 7 and 21 is:

A 147 **B** 141 **C** 21 **D** 140 **E** 207

1G **7** The missing digit in this division

$$\begin{array}{r} 1\ \ 1\ \ 8 \\ 7\overline{)\square\,{}^{1}2\,{}^{5}6} \end{array}$$

is:

A 6 **B** 1 **C** 9 **D** 8 **E** 7

1G **8** The remainder when 317 is divided by 9 is:

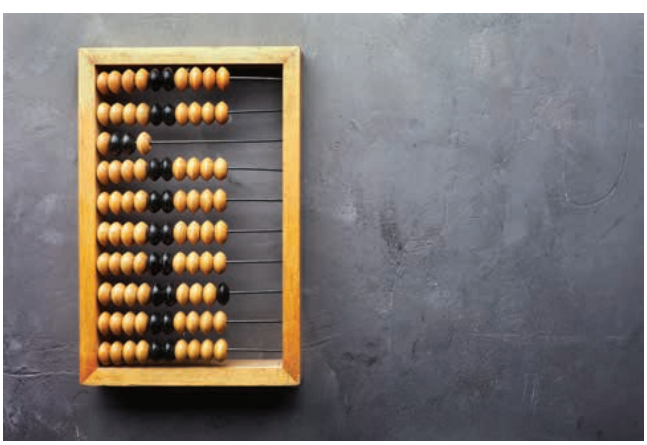
A 7 **B** 5 **C** 2 **D** 1 **E** 0

1G **9** 458 rounded to the nearest 100 is:

A 400 **B** 500 **C** 460 **D** 450 **E** 1000

1I **10** The answer to $[2 + 3 \times (7 - 4)] \div 11$ is:

A 1 **B** 5 **C** 11 **D** 121 **E** 0



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Short-answer questions

1A Ext **1** Write these numbers using the given number systems.

a Egyptian

i 3 **ii** 31 **iii** 326

b Babylonian

i 12 **ii** 60 **iii** 132

c Roman

i 14 **ii** 40 **iii** 146

1B **2** Write down the place value of the digit 5 in these numbers.

a 357 **b** 5249 **c** 356 612

1C **3** Use a mental strategy to find these sums and differences.

a $124 + 335$ **b** $687 - 324$ **c** $59 + 36$ **d** $256 - 39$

1D **4** Use an algorithm and show your working for these sums and differences.

a $76 + 52$ **b** $1528 + 796$ **c** $329 - 138$ **d** $2109 - 1814$

1E/G **5** Use a mental strategy to answer the following.

a 5×19 **b** 22×6 **c** 5×44 **d** $123 \div 3$

e $264 \div 8$ **f** $96 \div 4$ **g** 29×1000 **h** 36×300

i $14\,678 \div 1$

1F/G **6** Use an algorithm and show your working for the following.

a 157×9 **b** 27×13 **c** $7\overline{)327}$ **d** $4\overline{)30162}$

1D/F/G **7** Find the missing digits in the following.

a
$$\begin{array}{r} 2\square 3 \\ +\,7\,3\,\square \\ \hline 9\,6\,1 \end{array}$$

b
$$\begin{array}{r} \square\,2\,\square \\ -\,4\,\square\,3 \\ \hline 2\,5\,6 \end{array}$$

c
$$\begin{array}{r} \quad\quad\square\,3 \\ \times\quad 2\,\square \\ \hline \quad\square\,7\,1 \\ \square\,0\,6\,0 \\ \hline \square\square\,3\,1 \end{array}$$

d $5\overline{)\square\,4\,\square\,1\,\square}$ with no remainder

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Chapter review

1H **8** Round these numbers as indicated.

a 72 (nearest 10) **b** 3268 (nearest 100) **c** 951 (nearest 100)

1H **9** Use leading digit approximation to estimate the answers to the following.

a $289 + 532$ **b** 22×19 **c** 452×11 **d** $99 \div 11$

1I **10** Use order of operations to find the answers to the following.

a $3 \times (2 + 6)$ **b** $6 - 8 \div 4$ **c** $2 \times 8 - 12 \div 6$

d $(5 + 2) \times 3 - (8 - 7)$ **e** $0 \times (988\,234 \div 3)$ **f** $1 \times (3 + 2 \times 5)$

Extended-response questions

1 A city tower construction uses 4520 tonnes of cement trucked from a factory that is 7 kilometres from the construction site. Each cement mixer can carry 7 tonnes of cement, and the cement costs $85 per truck load for the first 30 loads and $55 per load after that.

a How many loads of cement are needed? Add a full load for any remainder.

b Find the total distance travelled by the cement mixers to deliver all loads, assuming they need to return to the factory after each load.

c Find the total cost of cement needed to make concrete for the tower construction.

d A different cement supplier offers a price of $65 per 8-tonne truck, no matter how many loads are needed. Find the difference in the cost of cement for the tower by this supplier compared to the original supplier.

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2 One night Ricky and her brother Micky decide to have some fun at their father's sweet shop. In the shop they find 7 tins of 135 jelly beans each, 9 packets of 121 choc buds, 12 jars of 70 smarties and 32 packets of 5 liquorice sticks.

- **a** Find the total number of sweets that Ricky and Micky find that night.
- **b** Find the difference between the number of choc buds and the number of smarties.
- **c** Ricky and Micky decide to divide each type of sweet into groups of 7 and then eat any remainder. Which type of sweet will they eat the most of and how many?
- **d** After eating the remainders, they round the total of each sweet using leading digit approximation. If they round down they put the spare sweets in their pockets. If they round up they borrow any spare sweets from their pockets. Any leftover in their pockets they can eat. Do Ricky and Micky get to eat any more sweets?

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