
1

1 Whole numbers

It's a Santa stampede! Another Guinness World Record, this time for 'the largest gathering of Santa Clauses' (12 695 of them!).

For nearly 60 years, *Guinness World Records* has been recording an increasing number of bizarre feats, including 'the most cockroaches eaten' (36), and 'the most consecutive skateboard frontside ollies off a half pipe ramp' (34). The book itself holds the record for being the best-selling copyrighted series of all time. We seem to have an obsession with wanting to know the most, the biggest, the fastest and the tallest.

Keeping records requires a number system with which to count, order, measure and calculate. Of course, we have more serious numbers to work with in our personal lives: how much money we spend, save and earn; how far and how often we travel; and how much of the Earth's resources we use. We find totals, work out differences, divide up quantities, and make estimations of size and number on a daily basis. To operate in this world, you need to be able to work with numbers!

Forum

Is it important to keep 'world records'? If so, in which areas of achievement should records be kept?

What kinds of other records do we keep?

Why learn this?

You are heading to the checkout with 2 $18 T-shirts and a $65 pair of jeans. Will $100 be enough to pay for them? If you swim 12 laps of a 50 metre pool 4 times a week, how many metres a week is that? Working confidently with numbers means more than just being able to add, subtract, multiply and divide. It means being able to choose which skill to use, to estimate and round numbers, and to have a range of mental skills to use so you can work things out efficiently and accurately.

After completing this chapter you will be able to:

- choose and use a range of mental strategies for calculations
- understand how the properties of numbers can be used to calculate efficiently
- estimate answers to problems using rounding strategies
- interpret and work with numbers in index form
- apply the order of operations
- solve problems involving whole numbers.

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from the Online Student Book.

1 What is the place value of the red digit?

(a) 45 783

A eight B tens C eighty three D hundreds

(b) 1 264 184

A two B hundreds C ten thousands D hundred thousands

2 (a) Rearrange the following numbers in ascending order (from smallest to largest).

567, 4500, 0, 74, 11 100, 6008, 12, 602

(b) Rearrange the following numbers in descending order (from largest to smallest).

1200, 204, 987, 2196, 240, 95, 2400, 1010

3 Find:

(a) 50 000 + 6000 + 800 + 90 + 5

(b) 7 000 000 + 20 000 + 5000 + 70 + 3

4 (a) Round 1245 off to the nearest:

(i) 10 (ii) 100 (iii) 1000

(b) Round 8983 off to the nearest:

(i) 10 (ii) 100 (iii) 1000

5 Find:

(a) $3 \times 2 \times 3$ (b) $5 \times 3 \times 3 \times 2$ (c) $2 \times 2 \times 2$ (d) $10 \times 10 \times 10 \times 10$

6 Set out these calculations in your preferred way and work out the answers.

(a) 456 + 56 (b) 16 + 2047 (c) 90 + 1267 + 341

7 Set out these calculations in your preferred way and work out the answers.

(a) 298 – 123 (b) 854 – 227 (c) 1406 – 249

8 Set out these calculations in your preferred way and work out the answers.

(a) 45×7 (b) 134×5 (c) 34×95

9 Set out these calculations in your preferred way and work out the answers.

(a) $844 \div 4$ (b) $3708 \div 9$ (c) $897 \div 7$

Key Words

associative law	divisor	mathematical conventions	quotient
base	estimate	order of operations	remainder
commutative law	expanded form	perfect cubes	rounding
cube root	index	perfect squares	square root
distributive law	index form	power	
dividend	indices	product	

Mental strategies

1.1

This section covers some strategies that are useful for doing calculations in your head. Try writing your answers out first, then move on to doing them mentally. You might find some methods easier to use than others, or have different strategies of your own. That's okay—people are different in the way they like to approach things.

Strategy 1 – Make easy numbers

This strategy uses two properties of numbers:

1 The order in which you add or multiply any two numbers does not change the result.

For example: $2 \times 3 = 6$ and $3 \times 2 = 6$
$4 + 5 = 9$ and $5 + 4 = 9$

This is known as the commutative law.

2 The order in which three or more numbers are added or the order in which they are multiplied is not important. In the following examples, brackets are used to show which pair of numbers is added or multiplied first.

Multiplication:

$(2 \times 3) \times 5$ and $2 \times (3 \times 5)$
$= 6 \times 5$ $= 2 \times 15$
$= 30$ $= 30$

Addition:

$(6 + 7) + 8$ and $6 + (7 + 8)$
$= 13 + 8$ $= 6 + 15$
$= 21$ $= 21$

This is known as the associative law.

It is important to realise, however, that the commutative and associative laws do not apply to subtraction and division.

For example: $(9 - 5) - 4$ but $9 - (5 - 4)$
$= 4 - 4$ $= 9 - 1$
$= 0$ $= 8$

$(40 \div 10) \div 2$ but $40 \div (10 \div 2)$
$= 4 \div 2$ $= 40 \div 5$
$= 2$ $= 8$

The order in which you add a group of numbers and the order in which you multiply a group of numbers makes no difference to the result. This is not true for subtraction or division.

Multiples of 10 are easy to add and multiply, so we can use the commutative and associative laws to 'shuffle' the calculation to create multiples of 10.

For example:

$7 + 9 + 3$ is the same as: $3 + 7 + 9$
$= 10 + 9$

$5 \times 37 \times 2$ is the same as: $5 \times 2 \times 37$
$= 10 \times 37$

We can split one number into two parts, then use the associative law to add one part to the other number to create a multiple of 10.

For example: $135 + 46$
$= 135 + 5 + 41$ (splitting 46 into 5 and 41)
$= 140 + 41$ (adding 5 to 135)

Worked Example 1 WE1

Calculate the following using the 'make easy numbers' strategy.

(a) $7 + 32 + 13$ (b) $2 \times 13 \times 5$ (c) $293 + 568$

Thinking		Working
(a) 1	Rearrange the addition to form 'easy' numbers, such as multiples of 10.	(a) $7 + 32 + 13$ $= 7 + 13 + 32$
2	Perform these calculations first.	$= 20 + 32$
3	Complete the question.	$= 52$
(b) 1	Rearrange the multiplication to form 'easy' numbers, such as multiples of 10.	(b) $2 \times 13 \times 5$ $= 2 \times 5 \times 13$ $= 10 \times 13$
2	Perform these calculations first.	$= 130$
3	Complete the question.	
(c) 1	Split one number into two parts, then add one part to the other number to create an 'easy' number. (Here, we have split 7 away from 568 and added it to 293.)	(c) $293 + 568$ $= 293 + 7 + 561$ $= 300 + 561$
2	Perform the calculation to complete the question.	$= 861$

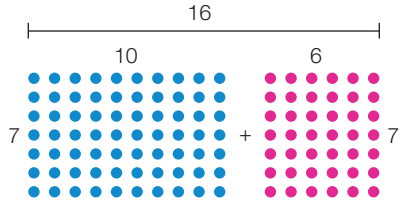
Strategy 2 – Use the distributive law

This strategy uses a property of numbers called the **distributive law**. The distributive law allows us to multiply a large number by splitting it up into 10s and 1s (or 100s, 10s and 1s), multiplying each part separately, then adding or subtracting each of the products. (When two numbers are multiplied together, the result is called the **product**.)

For example, we can break 7×16 down into 7 lots of 10 plus 7 lots of 6.

We can represent the multiplication in an array diagram:

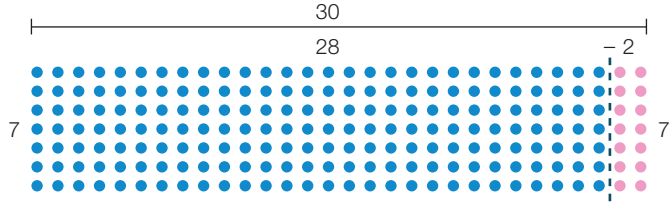
7×16
$= 7 \times (10 + 6)$ (because $16 = 10 + 6$)
$= 7 \times 10 + 7 \times 6$
$= 70 + 42$
$= 112$



In a similar way, we can write 19 as $20 - 1$, or 28 as $30 - 2$. For example, we can break 7×28 down into 7 lots of 30 minus 7 lots of 2.

We can represent this multiplication as an array diagram (the number subtracted is on the right of the dotted line).

7×28
$= 7 \times (30 - 2)$ (because $28 = 30 - 2$)
$= 7 \times 30 - 7 \times 2$
$= 210 - 14$
$= 196$



To multiply by a large number, split it up into 10s and 1s (or 100s, 10s and 1s). Multiply by these separately, then add or subtract each of the products.

Worked Example 2 WE2

Evaluate the following using the distributive law.

(a) 7×22 (b) 15×9

Thinking		Working
(a) 1	Split the number you are multiplying by into 10s and 1s, writing it in brackets.	(a) 7×22 $= 7 \times (20 + 2)$
2	Multiply the tens and the ones by the number in front of the brackets.	$= 7 \times 20 + 7 \times 2$ $= 140 + 14$
3	Add the two products together.	$= 154$
(b) 1	Round one of the numbers off to the nearest multiple of 10, then write it as a subtraction in brackets. (Here, we write 9 as (10 – 1).)	(b) 15×9 $= 15 \times (10 - 1)$
2	Multiply the numbers inside the brackets by the number in front.	$= 15 \times 10 - 15 \times 1$ $= 150 - 15$
3	Complete the subtraction.	$= 135$

1.1 Mental strategies

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Column 1, Q5, Q6, Q7, Q9, Q11, Q15	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Column 2, Q4, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q15	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Answers page 612

Fluency

1 Calculate the following using the 'make easy numbers' strategy. WE1

(a) $8 + 23 + 42$ (b) $15 + 57 + 35$ (c) $64 + 79 + 56$

(d) $5 \times 6 \times 2$ (e) $4 \times 6 \times 5$ (f) $2 \times 42 \times 5$

(g) $5 \times 7 \times 6$ (h) $5 \times 3 \times 8$ (i) $5 \times 14 \times 4$

(j) $47 + 73$ (k) $124 + 56$ (l) $211 + 169$

(m) $37 + 128 + 63$ (n) $77 + 78 + 23$ (o) $89 + 116 + 11$

2 Evaluate the following using the distributive law. WE2

(a) 17×9 (b) 19×8 (c) 49×6

(d) 6×31 (e) 7×52 (f) 5×43

(g) 99×9 (h) 77×3 (i) 57×8

(j) 14×11 (k) 15×13 (l) 16×12

(m) 101×8 (n) 113×5 (o) 124×11

3 Use any appropriate mental strategy to work out the following.

(a) $23 + 41 + 57$ (b) $347 + 156$ (c) $335 - 170$

(d) $8 \times 9 \times 5$ (e) 14×7 (f) 21×9

(g) 103×6 (h) 22×11 (i) 3×194

(j) $147 + 213$ (k) 19×14 (l) $4 \times 7 \times 15$

Understanding

4 Choose the correct answer to the following question.

23×7 could be calculated by:

A multiplying 3 and 7, then adding 20

B multiplying 20 and 7, then adding 3

C multiplying 20 and 7, multiplying 3 and 7, then adding the products together

D multiplying 20, 3 and 7 altogether.

5 Bilal has completed the first 3 stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

(a) the total distance that Bilal has ridden so far

(b) how far Bilal still has to ride, if the total rally distance is 480 km.

6 The Year 7s at Mountain View Secondary College are undertaking a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings 8 classes of 25 students will need.

7 Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

(a) how much Jason has saved after 17 weeks

(b) how much he still has to save if the cricket gear he wants costs $189.

8 Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

(a) Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

(b) List two reasons why the actual number of cars might be less than your answer to (a).

9 Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?

10 Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

(a) 8 notepads at 98 cents each

(b) 3 gluesticks at 77 cents each

(c) 12 pens at 59 cents each

(d) 5 boxes of paperclips at 82 cents each

Reasoning

11 Below are some mistakes made by students on a test, and their explanation of the method they used. Write what each student has done incorrectly and what the answer should be.

(a) $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So, then I added 21.'

(b) $35 \times 3 = 140$. Sam: 'I doubled 35, then doubled my answer to get 140.'

(c) $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'

12 Four students were asked to write down a mental maths strategy for calculating 9×29. Here are their suggestions:

Alex: $10 \times 29 - 29$ Indrah: $10 \times 29 - 9$
Khalid: $9 \times 30 - 9$ Lucy: $9 \times 30 - 30$

(a) Who has written a correct strategy?

(b) Why do two different strategies give the same answer?

(c) Explain what is wrong with the other strategies.

Open-ended

13 Tranh is in the hardware shop buying some supplies. He has 5 picture hooks, which are 28 cents each, 12 curtain rings, which are 15 cents each, and a small hammer costing $6. As he walks to the checkout, Tranh wonders if he has enough money, as he only has $10 in his pocket. Describe the mental calculations Tranh could do to be sure he has enough money to pay for his items. Is $10 enough?

14 Describe how you could use your calculator to work out 11×23 if the '1' button was broken.

15 Brendan has conducted a survey of the number of pets owned by each member of his class. His results are: 2, 2, 2, 1, 1, 2, 3, 3, 2, 2, 1, 4, 1, 3, 5, 2, 4, 1, 2, 1, 3, 2, 1.

Brendan has been trying to use his calculator to add up his list of numbers, but he keeps losing his place in the list. Suggest a method that Brendan could use to organise his numbers and use his calculator more efficiently. Use your method to add up Brendan's list.

Outside the Square Game

Greed

Equipment required: 2 brains, 2 dice

How to win:
The winner is the first player to reach or pass 500.

How to play:
On your turn, roll the dice and make a two-digit number (for example, if you roll a 4 and a 3, you can make 34 or 43). Write this number down.

Roll again, make another two-digit number and add it to the first one.

Keep rolling, and adding up the total as you go – BUT, if you roll a 'double' (two of the same number), all your score for that turn is lost, and your turn ends. (This rule does not apply for a player's very first roll.)

You can choose to stop at any time and 'bank' your score. A banked score is carried over into your next turn and cannot be lost. Circle your banked score after each turn to help you keep track of things.

Maths 4 Real

Numbers around the world

Throughout history, different civilisations and cultures have developed various systems and symbols for numbers. Some of these systems travelled to other parts of the world as people migrated and traded, while others were (and still are) only used by small groups with a certain language or culture.

The Hindu-Arabic system

0 1 2 3 4 5 6 7 8 9

You are very familiar with the above ten symbols that we use to represent numbers. Have you ever thought about where these symbols come from? They have their origins in the 'Brahmi numerals', which were used in India from about 300 BC. They were adopted and modified by the Arabic people, who brought them to Europe during the Middle Ages. The Arabs also developed a symbol for zero, calling it *sifr*. Because of its history, we refer to our number system as the 'Hindu-Arabic system'. The Brahmi numerals are shown in the second row of the table below. They have evolved over time to become the more familiar symbols in the top row.

1	2	3	4	5	6	7	8	9
—	=	≡	𑁤	𑁥	𑁦	𑁧	𑁨	𑁩

The Roman system

The ancient Romans used letters as their symbols for numbers. Roman numerals are still used today on clocks, watches and monuments, or to show the date when something (such as a movie) was made.

I	II	III	IV	V	VI	VII
1	2	3	4	5	6	7
VIII	**IX**	**X**	**L**	**C**	**D**	**M**
8	9	10	50	100	500	1000

In Roman numerals, the letter **I**, **X** and **C** before a higher value letter means we take it away from the higher value. The rule is that you can only subtract these from the next two higher Roman numerals. For example, 4 is **IV** and 9 is **IX** but 49 is not **IL** (it is **XLIX**; made up of **XL** for 40 and **IX** for 9). So, **I** can be subtracted from **V** and **X**, **X** can be subtracted from **L** and **C** and **C** can be subtracted from **D** and **M**.

39 is **XXXIX**

(10 + 10 + 10 + 9)

2644 is **MMDCXLIV**

(1000 + 1000 + 500 + 100 + (50 – 10) + (5 – 1))

1 Write out these numbers using Roman numerals:

(a) 8 **(c)** 439

(b) 84 **(d)** 1975

2 Perform the following calculations. Write your answers using Roman numerals.

(a) XXIX + CLII

(b) MDCCLX – MCDXCIX

3 What is the largest Roman numeral you can make that uses one of each of the different symbols: **C**, **L**, **M**, **X**, **D**, **V**, **I**? Write your answer in Roman and Hindu-Arabic numerals.

The Chinese character system

0	1	2	3	4	5	6
〇	一	二	三	四	五	六

7	8	9	10	100	1000	10000
七	八	九	十	百	千	万

The Chinese character system uses two symbols to show each of the place values represented. The first symbol indicates how many, and the second symbol indicates the place value. However, when there is only one of a particular place value only one symbol is used. If a place value is zero then no symbols are needed. Chinese numbers are written vertically.

For example:

270	19	8005
二百七十 (two (lots of) one hundred and seven (lots of) ten)	十九 (ten and nine)	八千五 (eight (lots of) one thousand and five)

4 Write out these numbers using the Chinese character system:

(a) 37 **(c)** 1053

(b) 823 **(d)** 6400

5 Perform the following calculations. Write your answers using Chinese characters.

(a) 五百六十 + 八百九

(b) 千 − 三百二十七

6 Did you find it difficult to perform calculations with the Roman and Chinese systems? What are the advantages of using the Hindu-Arabic system for calculations?

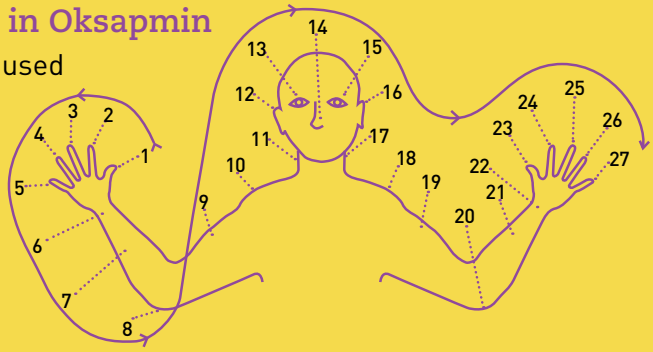
Non-base 10 systems

The Hindu-Arabic system is a 'base 10' or 'decimal' system. The place value of each digit in a number is 10 times bigger than the next position to the right, for example:
367 = 3 'hundreds' + 6 'tens' + 7 'ones', or 300 + 60 + 7.

Many indigenous cultures of Australia, Torres Strait and Papua New Guinea have counting systems that use body parts to indicate numbers. These systems are not base 10 systems. For example, the people of Sandaun province, Papua New Guinea (who speak the Oksapmin language) use the parts of the upper body to count in lots of 27.

Counting in Oksapmin

Body parts used to count in Oksapmin



1 *tipun* "thumb"	*kin tən* "other-side eye"	15
2 *ləwatipun* "index finger"	*nat tən* "other-side ear"	16
3 *bumlip* "middle finger"	*gwel tən* "other-side side of neck"	17
4 *xətlip* "ring finger"	*kat tən* "other-side shoulder"	18
5 *xətxət* "little finger"	*tuwət tən* "other-side upper arm"	19
6 *xadəp* "wrist"	*amun tən* "other-side elbow"	20
7 *bes* "forearm"	*bes tən* "other-side forearm"	21
8 *amun* "elbow"	*xadəp tən* "other-side wrist"	22
9 *tuwət* "upper arm"	*tipun tən* "other-side thumb"	23
10 *kat* "shoulder"	*ləwatipun tən* "other-side index finger"	24
11 *gwel* "side of neck"	*bumlip tən* "other-side middle finger"	25
12 *nat* "ear"	*xətlip tən* "other-side ring finger"	26
13 *kin* "eye"	*xətxət tən* "other-side little finger"	27
	lum "nose"	14

7 **(a)** In Oksapmin what word would you use to say '15'?

(b) If you wanted to say 'I have eight chickens' in Oksapmin, what body part would you point to?

8 The Hindu-Arabic number system is used extensively across the world. List some advantages and disadvantages of having one predominant number system.

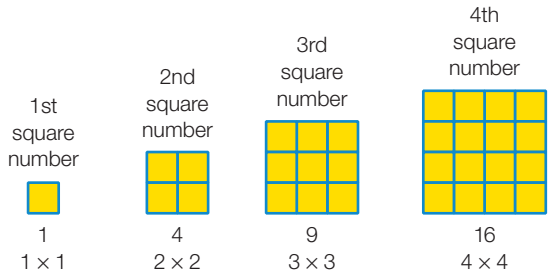
Research:

- Investigate the number systems of other ancient civilisations (such as the Mayans or Babylonians) or other indigenous cultures.
- Find out how to write and say numbers in another language, such as Japanese or Greek.

1.2 Indices

Square numbers

Square numbers (or **perfect squares**) are numbers that can be represented by squares of different side lengths, as shown below. 1, 4, 9, 16 and 25 are square numbers.



The number of small squares in each larger square is equal to the number of smaller squares along one side, multiplied by itself. We can think of it as multiplying the length and width of the larger square.

We have a shorthand way of writing square numbers. 4×4, for example, is written as 4^2. The small '2' means that two 4s are multiplied together. We say 4^2 as 'four squared'.

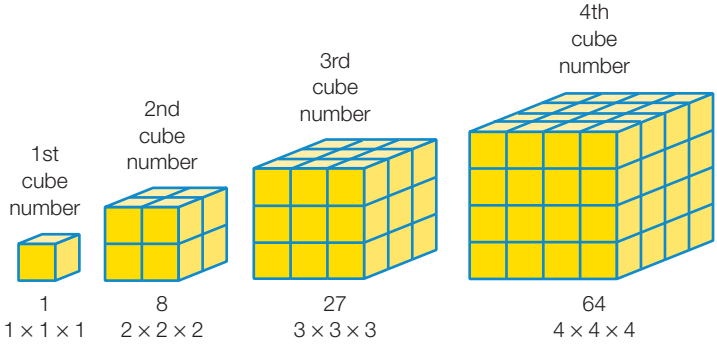
We can write the sequence of perfect squares as:

$1^2, \quad 2^2, \quad 3^2, \quad 4^2, \quad 5^2, \quad 6^2, \quad 7^2 \ldots$

$= 1, \quad 4, \quad 9, \quad 16, \quad 25, \quad 36, \quad 49 \ldots$

Cube numbers

Cube numbers (or **perfect cubes**) are numbers that can be represented by blocks arranged in a cube pattern, as shown below. 1, 8, 27 and 64 are cube numbers.



The number of smaller cubes in each larger cube is equal to the number of smaller cubes along one side, multiplied by itself, and itself again. We can think of it as multiplying the length, width and height of the cube.

Using our shorthand, we can write, for example, $2 \times 2 \times 2$ as 2^3. The small '3' indicates that three 2s are multiplied together. We say 2^3 as 'two cubed'.

We can write the sequence of perfect cubes as:

$1^3, \quad 2^3, \quad 3^3, \quad 4^3, \quad 5^3 \ldots$

$= 1, \quad 8, \quad 27, \quad 64, \quad 125 \ldots$

To 'square' a number, multiply it by itself:

$$6^2 = 6 \times 6$$
$$= 36$$

To 'cube' a number, multiply it by itself and itself again, so that the number is written three times in all:

$$4^3 = 4 \times 4 \times 4$$
$$= 64$$

Do not confuse 4^3 with 4×3.

Square roots and cube roots

Finding the square root or cube root of a number is the reverse of squaring or cubing a number. For example, the square root of 16 is the number that gives 16 when multiplied by itself. In this case, that number is 4, because $4^2 = 16$. We write $\sqrt{16} = 4$, where the symbol $\sqrt{\ }$ means 'the square root of'. The cube root of 8 is 2, because $2^3 = 8$. We write $\sqrt[3]{8} = 2$, where $\sqrt[3]{\ }$ means 'the cube root of'.

Only the roots of perfect squares and perfect cubes are whole numbers. Many roots are decimal numbers and a calculator may be used to find them.

Index notation

We can extend our shorthand for square numbers and cube numbers to write other large numbers produced by repeated multiplication, by continuing a pattern.

If:	$3^2 = 3 \times 3$	$(= 9)$
and	$3^3 = 3 \times 3 \times 3$	$(= 27)$
then,	$3^4 = 3 \times 3 \times 3 \times 3$	$(= 81)$
	$3^5 = 3 \times 3 \times 3 \times 3 \times 3$	$(= 243)$

This shorthand notation is called index form. The large number (the one being multiplied repeatedly) is called the base. The small number, written up high to the right of the base, is called the power, or index. (The plural of index is indices.) The index tells you how many times the base will appear when written in expanded form (that is, written as a series of multiplications).

Index form:

5^4 ← index (power)
5 ← base

Expanded form: $5 \times 5 \times 5 \times 5$

Say: '5 to the power of four', or '5 to the fourth' or 'base 5, index 4'.

The value of 5^4 is 625.

Any number to the power of 1 is itself. $5^1 = 5$.

Worked Example 3 WE3

Write each of the following numbers in expanded form, then find its value.

(a) 4^3 (b) 8^5

Thinking		Working
(a) 1	Identify the base and the index. Multiply the base by itself according to what the index is.	(a) $4^3 = 4 \times 4 \times 4$
2	Perform the multiplication.	$= 64$
(b) 1	Identify the base and the index. Multiply the base by itself according to what the index is.	(b) $8^5 = 8 \times 8 \times 8 \times 8 \times 8$
2	Perform the multiplication.	$= 32\,768$

Indices on the calculator

Many calculators have a key to help you work efficiently with indices. It usually looks like ^ or x^y. For example, to calculate 12^4, you would press: 1 2 ^ 4 = or 1 2 x^y 4 =.

Some calculators also have an x^2 key and/or an x^3 key for calculating square and cube numbers. For example, to find 17^2, press: 1 7 x^2 =. To find 2^3, press: 2 x^3 =.

The ^ or x^y key can also be used to calculate squares and cubes.

Square roots and cube roots on the calculator

Scientific calculators have a square root key, which usually looks like this: $\sqrt{\ }$.

To find the square root of, for example, 256, press: $\sqrt{\ }$ 2 5 6 =. (If you try this, you should find that the answer is 16.) If that doesn't work, try entering the number, then pressing the $\sqrt{\ }$ key.

To find a cube root on the calculator, use the $\sqrt[x]{\ }$ key. To find the cube root of, for example, 512, press: 3 $\sqrt[x]{\ }$ 5 1 2 =. (If you try this, you should find that the answer is 8.)

Powers of 10

The number system that we use for counting is based on powers of ten:

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$ etc.

Here, we use the word 'power' to refer to the actual value of the number in index form. So, the first four powers of 10 are: 10, 100, 1000, 10 000.

Notice that when we write the value of the number in index form, the number of zeroes is equal to the index. For example, 10^3 has an index of 3, and its value, 1000, has 3 zeroes.

1.2 Indices

Navigator

Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q10, Q11, Q12, Q13, Q15, Q17, Q19, Q22, Q24	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7 Columns 2 & 3, Q8, Q9 Columns 1 & 2, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q22, Q23, Q24	Q1 Column 3, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7 Columns 3 & 4, Q9, Q11, Q12 Columns 2 & 3, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24

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Equipment required: Calculator for Questions 3, 10, 12, 17, 19, 21(a)

Fluency

WE3

1 Write each of the following numbers in expanded form, then find its value.

(a) 2^3 (b) 2^4 (c) 2^6

(d) 1^8 (e) 0^7 (f) 10^3

(g) 6^4 (h) 5^5 (i) 8^6

(j) 11^3 (k) 12^4 (l) 14^5

2 Write each of the following in index form.

(a) $8 \times 8 \times 8$ (b) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

(c) $12 \times 12 \times 12 \times 12 \times 12$ (d) $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$

(e) seventeen cubed (f) nineteen squared

(g) eight to the power of 4 (h) thirteen to the power of seven

(i) base 11, index 7 (j) base 9, index 6

3 Find the value of each of the following.

(a) 4^7 (b) 6^6 (c) 9^4

(d) 8^3 (e) 7^5 (f) 15^3

(g) seventeen squared (h) fifty cubed (i) the cube of 4

(j) 5×10^2 (k) 4×10^3 (l) $10^3 \times 2$

(m) 2 to the power of 10 (n) 3 to the power of 9 (o) base 1, index 8

4 Write in words how we could say each of the following.

(a) 5^2 (b) 31^2 (c) 3^3 (d) 27^3

(e) 4^5 (f) 9^6 (g) 1^4 (h) 7^7

5 If 1 is the first square number and 4 is the second, write the 5th, 6th and 7th square numbers.

6 If 1 is the first cube number and 8 is the second, write the 4th, 5th and 6th cube numbers.

7 Evaluate:

(a) $\sqrt{25}$ (b) $\sqrt{49}$ (c) $\sqrt{100}$ (d) $\sqrt{121}$

(e) $\sqrt{1}$ (f) $\sqrt{0}$ (g) $\sqrt{4900}$ (h) $\sqrt{400}$

(i) $\sqrt[3]{8}$ (j) $\sqrt[3]{64}$ (k) $\sqrt[3]{1000}$ (l) $\sqrt[3]{1}$

(m) $\sqrt[3]{0}$ (n) $\sqrt[3]{125}$ (o) $\sqrt[3]{8000}$ (p) $\sqrt[3]{27\,000}$

8 (a) In $2^3 = 8$ the base number is:

A 2 B 3 C 6 D 8

(b) The square of 5 is:

A 2.2 B 10 C 25 D 125

(c) The cube root of 729 is:

A 9 B 27 C 243 D 2187

Understanding

9 Evaluate the following without using a calculator.

(a) $1^4 + 2^2$ (b) $2^3 - 1^6$ (c) $3^4 - 2^4$

(d) $2^3 \times 2^2$ (e) $2^2 \times 2^4$ (f) $3^3 \times 3^2$

(g) $2^5 + 5^2 - 6^2$ (h) $10^2 + 3^2 - 4^3$ (i) $5^2 - 1^6 + 3^3$

10 (a) Arrange these numbers in ascending order:

$4^5, 5^4, 1^{200}, 10^3, 4^6, 5^5$

(b) Arrange these numbers in descending order:

$100^2, 10^5, 1^{1000}, 0^{100}, 3^2, 2^3$

11 (a) Write down two numbers between 5 and 40 that are both even and square.

(b) Write down two numbers between 30 and 90 that are both odd and square.

12 Evaluate these with your calculator, using the most efficient method possible:

(a) $16^3 - 4096$ (b) $21^4 - 4481$ (c) $15^4 - 5625$

(d) $36^3 \times 53$ (e) $14^4 \times 14$ (f) $19^5 \times 21$

(g) $2^{12} + 2^{18}$ (h) $3^{10} + 12^4$ (i) $4^9 + 3^{11}$

13 Only the square roots of perfect squares are whole numbers. Square roots of other numbers lie in between the perfect square roots (they are decimal numbers). For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{20}$ would lie in between 4 and 5. Find which two consecutive whole numbers the following lie between. (Consecutive numbers come one after the other; e.g. 8 and 9.)

(a) $\sqrt{10}$ (b) $\sqrt{5}$ (c) $\sqrt{20}$ (d) $\sqrt{62}$

(e) $\sqrt{99}$ (f) $\sqrt{2}$ (g) $\sqrt{70}$ (h) $\sqrt{108}$

14 Arrange the following in ascending order.

(a) $2^4, \sqrt[3]{64}, 5, \sqrt{49}$ (b) $7^2, \sqrt[3]{125}, 9, \sqrt{60}$

(c) $\sqrt[3]{8}, 10, \sqrt{90}, 4^2$ (d) $\sqrt{80}, 8, \sqrt{105}, 3^3$

15 (a) Complete the following table of the powers of 10.

Index form	Expanded form	Value
10^1	10	10
10^2	10×10	
10^3		1000
10^4		

(b) For each row of the table, compare the index numbers in the first column with the number of zeroes in the third column. Describe the pattern.

16 For each of the following, find the missing power of 10.

(a) $20\,000 = 2 \times 10^{\square}$ (b) $400 = 4 \times 10^{\square}$ (c) $5000 = 5 \times 10^{\square}$

(d) $300\,000 = 3 \times 10^{\square}$ (e) $80 = 8 \times 10^{\square}$ (f) $7\,000\,000 = 7 \times 10^{\square}$

(g) $150\,000 = 15 \times 10^{\square}$ (h) $91\,500 = 915 \times 10^{\square}$ (i) $2\,340\,000 = 234 \times 10^{\square}$

Reasoning

17 (a) Use your calculator to work out $5^2 \times 5^3$ and $5 \times 5 \times 5 \times 5 \times 5$.

(b) Now, try it with $2^3 \times 2^6$ and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

(c) What can you conclude?

18 How many whole numbers have the value of their square root in between:

(a) 2 and 3

(b) 5 and 6

(c) 8 and 9?

19 (a) Use your calculator to answer TRUE or FALSE to each of the following statements.

(i) 4^6 is bigger than 6^4

(ii) 2^{10} is bigger than 10^2

(iii) 3^9 is bigger than 9^3

(iv) 19^2 is bigger than 2^{19}

(b) Look at your answers for part (a) and, *without* using your calculator, answer TRUE or FALSE to each of the following statements.

(i) 9^8 is bigger than 8^9

(ii) 2^{100} is bigger than 100^2

20 (a) The number 10^{100} was called a googol by the mathematician Edward Kasner.

(i) How many 10s are multiplied together to give a googol?

(ii) How many zeroes would follow the 1 in a googol?

(b) If you raise the number ten to the power of a googol, you get a number called a googolplex.

(i) How many 10's are multiplied together to give a googolplex?

(ii) How many zeroes would follow the 1 in a googolplex? How much time do you think you save by writing a googolplex in index form?

21 (a) Complete the following.

$11^2 =$ ____ $111^2 =$ ____ $1111^2 =$ ____

(b) Look at the pattern in part (a) and, *without* using a calculator, copy and complete the following.

$11\,111^2 =$ ____ $111\,111^2 =$ ____ $1\,111\,111^2 =$ ____

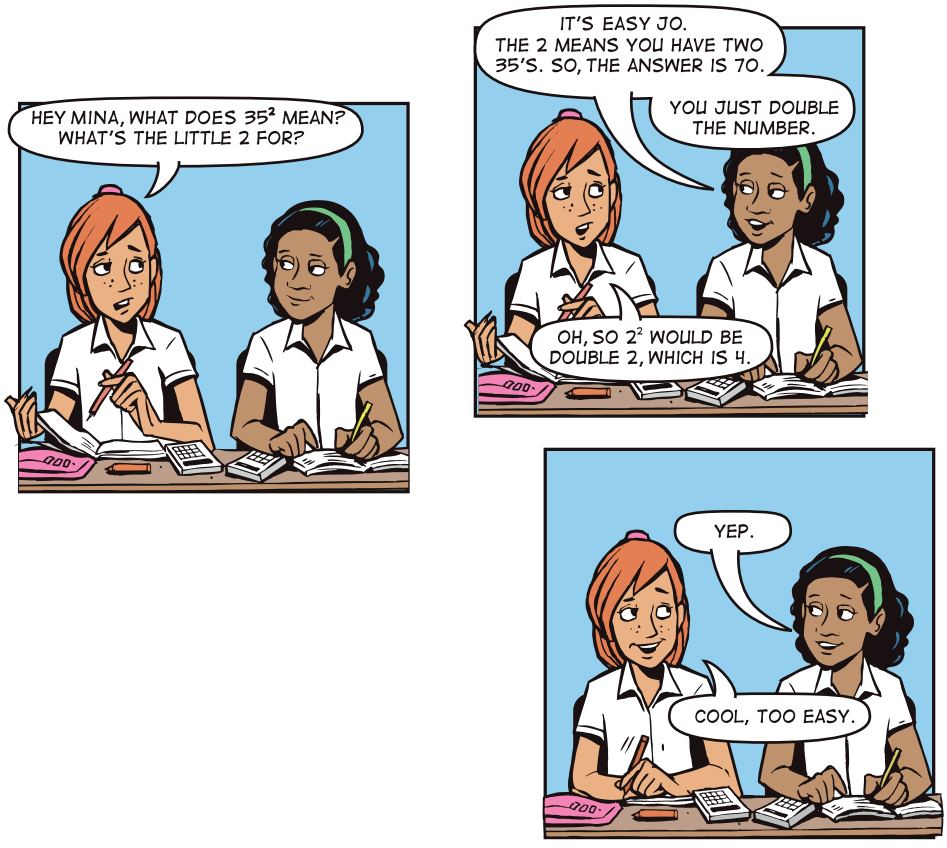
Open-ended

22 Use the digits 1, 2 and 3 to create at least three numbers that are greater than 500.

23 (a) A number has two digits when it is squared and three digits when it is cubed. What might be the number?

(b) A number has three digits when it is squared and four digits when it is cubed. What might be the number?

24



(a) What is wrong with Mina's explanation? Write down a better explanation that Mina could give to Jo.

(b) What question did Jo get right? Why was this the case?

Outside the Square

Problem solving

Teacher's age

Cheryl, who is a teenager, was curious about her maths teacher's age.

"Well", said the teacher, "Right now the second digit of my age is a square number and the product of my digits is the cube of a number. In ten years' time my age will be a square number and 25 years from now my age will be a number that has been cubed."

"I know your age," Cheryl said with confidence. "It's also interesting that the difference between our ages is one more than an odd number which has been squared."

How old is the teacher?

How old is Cheryl?

Strategy options

- Guess and check.
- Make a table.
- Work backwards.

Investigation

How many squares on a chessboard?

Equipment required: 1 brain, graph or grid paper

The Big Question

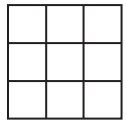
A chessboard is an 8×8 grid of squares. What is the total number of squares on a chessboard?

Engage

1 Shape puzzles, such as the one below, have been appearing in puzzle books for many years.

Write down how many squares there are in this 3×3 grid.

People who are tricked by this puzzle forget there are squares of more than one size. In addition, several squares overlap each other. By drawing out the grid and shading, show that there are 14 squares of three different sizes in the 3×3 grid.



Explore

2 To solve a puzzle such as this one, it helps to have a system. Use your results from Question 1 to fill in the following table. The 1×1 and 2×2 grids have been done for you.

	Number of squares of each type				
Grid size	Side of length 1	Side of length 2	Side of length 3	Side of length 4	Total squares in whole grid
1×1	1				1
2×2 	4	1			5
3×3 					

3 Now, extend the system of counting the number of squares of each size to work out how many squares there are in a 4×4, 5×5 … through to an 8×8 grid (a chessboard). Extend the above table to record the number of each size of square, and the total number of squares in each grid.

Strategy options

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

Explain

4 Describe any patterns you can see in your table of results. (Consider the numbers of each size square in a grid and how they change as the grid gets larger.)

Elaborate

5 State your answer to the Big Question. Present your results so that they clearly show how you arrived at your answer.

6 Use your pattern to predict the total number of squares on: (a) a 10×10 grid (b) any size grid.

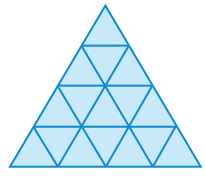
Evaluate

7 (a) Consider how you worked on the investigation and the methods you used. Describe the strategy or method you used to make sure that you counted all the squares. How confident are you that your answer is correct?

(b) Did any of your methods or strategies change as you went? Did you develop any shortcuts that made your working easier? What were they?

Extend

8 Using a similar method, work out how many triangles there are in this pattern.



1.3 More strategies for multiplication and division

The result of a multiplication calculation is called the product. The numbers being multiplied together are sometimes called multipliers.

The result of a division calculation is called the **quotient**. The number being divided is the **dividend** and the number we are dividing by is the **divisor**. Sometimes after a division we are left with a **remainder**.

18 multiplier	×	27 multiplier	=	486 product	
32 dividend	÷	10 divisor	=	3 quotient	rem 2 remainder

You should be very familiar with multiplying any two numbers from 0 to 10 together, such as 4×3, 8×6, or 9×5. If you know a lot of multiplication facts, you also know a lot of division facts; for example, $63 \div 7 = 9$ because $9 \times 7 = 63$. Following on from strategies 1 and 2 covered in section 1.1, here are two more strategies that are useful for multiplying and dividing numbers, including numbers larger than the ones on the multiplication table. You may have other methods that you like to use.

Strategy 3 – Work in stages

Multiplying a number by 4 is the same as multiplying by 2 twice ($4 = 2 \times 2$); that is, doubling and then doubling again. To multiply by 6, we could multiply by 3, and double the answer ($6 = 3 \times 2$). Similarly, we could divide by 4 by dividing by 2 twice. Dividing by 6 is the same as dividing by 2, then dividing by 3 (you won't want to use this strategy if the number is odd).

Multiply or divide in stages by breaking the multiplier or divisor down and doing a series of simpler multiplications or divisions.

Worked Example 4 WE4

Calculate the following using the 'work in stages' strategy.

(a) 34×8 (b) $900 \div 15$

Thinking	Working
(a) 1 Break the multiplier down into a series of simpler multiplications.	(a) 34×8 $= 34 \times 4 \times 2$ $= 34 \times 2 \times 2 \times 2$
2 Perform one of these multiplications.	$= 68 \times 2 \times 2$

	3	Perform the next multiplication (× 2 again).	$= 136 \times 2$
	4	Keep multiplying until the multiplication is complete (× 2 again).	$= 272$
(b)	1	Break the divisor down into a series of simpler divisions.	(b) $900 \div 15$ $= 900 \div 3 \div 5$
	2	Perform one of these divisions (÷ 3).	$= 300 \div 5$
	3	Perform the next division (÷ 5). Keep dividing until the division is complete.	$= 60$

Strategy 4 – Use an array

Multiplying two two-digit or three-digit numbers is a little more challenging than multiplying two one-digit numbers. We can extend our 'use the distributive law' strategy from section 1.1 to both numbers and multiply them in an array, or grid.

Worked Example 5

WE5

Calculate 23×47 using the 'use an array' strategy:

Thinking	Working
1 Split the numbers to be multiplied into 10s and 1s. Draw up a grid. Place one broken-up number on top of the columns and one to the left of the rows.	$23 = 20 + 3$ $\quad 47 = 40 + 7$ 20 | 3 40 | | 7 | |
2 Multiply each of the 10s and 1s with each of the other 10s and 1s, to get a product in each of the four grid spaces.	20 | 3 40 | 800 | 120 7 | 140 | 21
3 Add the two products in each of the rows. (Alternatively, you could add the columns.)	20 | 3 40 | 800 | 120 | 920 7 | 140 | 21 | 161
4 Add these two sums to get your final answer.	$\begin{array}{r} 920 \\ +\ 161 \\ \hline 1081 \end{array}$

Multiplying by multiples of 10

Multiplying a whole number by a power of 10 such as 10, 100 or 1000 makes the value of each digit in the number 10, 100 or 1000 times bigger. We show this by adding zeroes onto the end of the number. The number of zeroes added is the same as the number of zeroes in the power of 10 (and the same as the power number).

For example: $3 \times 10 = 30$ (3×10^1)
$7 \times 1000 = 7000$ (7×10^3)
$15 \times 100 = 1500$ (15×10^2)

To multiply a whole number by a multiple of 10 such as 40, 800 or 12 000, or to multiply two multiples of 10 together, we can use the associative law to rearrange the calculation and multiply in stages.

For example: $7 \times 40 = 7 \times 4 \times 10$
$= 28 \times 10$
$= 280$

30×600
$= 3 \times 10 \times 6 \times 100$
$= 3 \times 6 \times 10 \times 100$
$= 18 \times 1000$
$= 18\,000$

To multiply whole numbers by numbers that are multiples of 10:

1. write each multiple of 10 as the product of a number and a power of 10
2. rewrite the multiplication, grouping the powers of 10 together
3. multiply the other numbers using the total number of zeroes in the powers of 10 to write the correct number of zeroes in the answer.

Dividing by multiples of 10

Consider the following divisions.

$50 \div 10 = 5$
$500 \div 100 = 5$
$500 \div 10 = 50$
$50\,000 \div 100 = 500$
$5\,000\,000 \div 10\,000 = 500$

The number of zeroes in the answer is the difference between the number of zeroes in the numbers being divided.

A quick way to show this type of division is to draw a line through them. This is often called 'cancelling out' zeroes. For example: $50\,0\cancel{00} \div 1\cancel{00}$
$= 500 \div 1$
$= 500$

It is important to remember what 'cancelling out zeroes' means—you are dividing both numbers by a power of 10. For example, crossing out three zeroes means that you are dividing by 1000.

To divide by whole numbers that are multiples of 10:

1. divide both numbers by 10 until one number is no longer a multiple of 10. Show this by cancelling zeroes
2. perform the simplified division.

Worked Example 6 WE6

Calculate the following.

(a) 7×900 (b) 6000×30 (c) $4500 \div 90$

Thinking			Working	
(a)	1	Rewrite the multiple of 10 as the product of a power of 10 and another number.	(a)	7×900 $= 7 \times 9 \times 100$
	2	Multiply the first two numbers.		$= 63 \times 100$
	3	Multiply by the power of ten.		$= 6300$
(b)	1	Rewrite each multiple of 10 as the product of a number and a power of 10.	(b)	6000×30 $= 6 \times 1000 \times 3 \times 10$
	2	Rearrange the calculation, placing the powers of 10 together and the other numbers together.		$= 6 \times 3 \times 1000 \times 10$
	3	Perform the multiplications.		$= 18 \times 10\,000$ $= 180\,000$
(c)	1	Divide both numbers by 10 until one number is no longer a multiple of 10.	(c)	$4500 \div 90$
	2	Write down the simpler division that results.		$= 450 \div 90$
	3	Perform the division.		$= 50$

1.3 More strategies for multiplication and division

Navigator

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Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4 Column 1, Q5, Q7, Q8, Q12, Q14, Q15	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Column 2, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency

WE4

1 Calculate the following using the 'work in stages' strategy.

(a) 27×4 (b) 41×6 (c) 62×8

(d) $600 \div 4$ (e) $153 \div 9$ (f) $864 \div 16$

(g) 35×9 (h) $390 \div 6$ (i) 70×15

(j) 15×12 (k) $450 \div 25$ (l) $360 \div 24$

2 Calculate the following using the 'use an array' strategy.

WE5

(a) 12×41 (b) 17×26 (c) 19×68

(d) 23×61 (e) 31×49 (f) 39×56

(g) 42×47 (h) 49×53 (i) 55×55

(j) 61×82 (k) 72×77 (l) 96×51

3 Calculate the following.

WE6

(a) 700×3 (b) 400×5 (c) 2000×13

(d) $360 \div 20$ (e) $5400 \div 200$ (f) $8000 \div 500$

(g) 1200×400 (h) $400 \times 32\,000$ (i) $7000 \times 20\,000$

(j) $12\,000 \div 20$ (k) $350\,000 \div 700$ (l) $8\,400\,000 \div 4000$

4 Calculate the following using any suitable strategy.

(a) $180 \div 20$ (b) $450 \div 90$ (c) $800 \div 25$

(d) 40×19 (e) 53×20 (f) 102×18

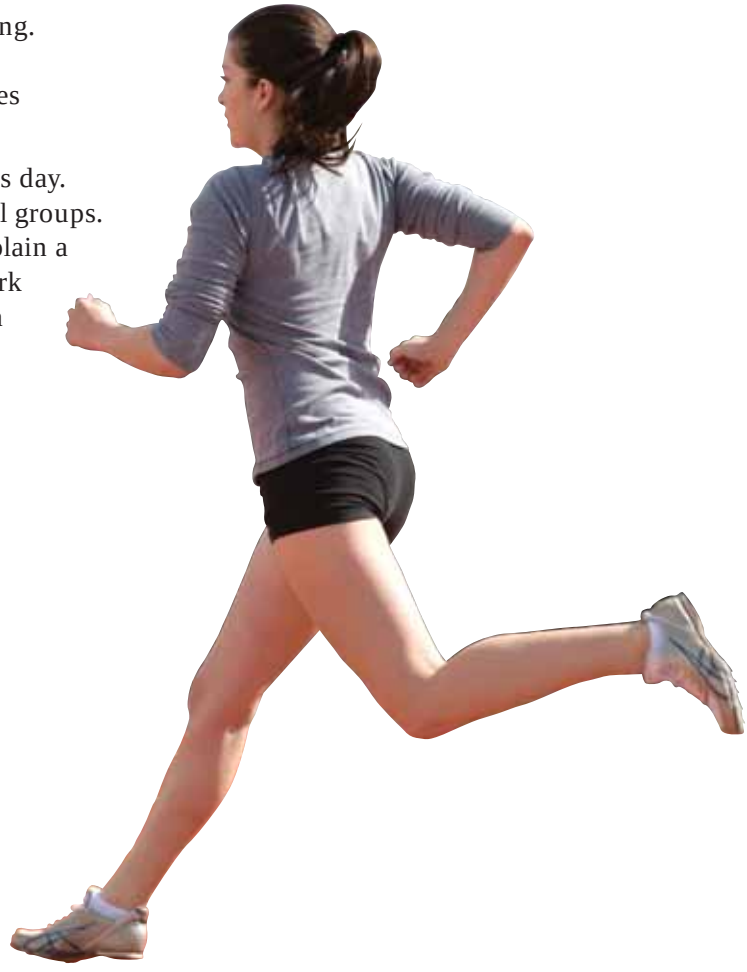
(g) $2700 \div 18$ (h) $1075 \div 25$ (i) $2250 \div 15$

(j) 120×12 (k) 59×72 (l) 98×25

Understanding

5 Charlotte is a distance runner in training. She runs 20 laps of a 400 metre race track 6 times a week. How many metres does Charlotte run in a week?

6 Year 7 students are practising for sports day. They need to be divided into five equal groups. There are 130 students in the year. Explain a quick way the teacher could use to work out how many students will be in each group, and find the answer.



7 Marie's monthly mobile phone bill is $52. If she continued to pay this amount every month, use a mental or written strategy to calculate how much Marie will have paid in phone bills by the end of her 2 year contract.

8 A group of eight friends have won $2000 and want to divide it equally between them. How much will each receive?

9 Rehan is paying back a bank loan that he used to buy a house. His repayments are set at $1800 per month for 30 years. How much money will Rehan have paid back at the end of this period of time?

10 Dharma has printed 3700 brochures advertising his business. He wants to place them in the letterboxes of houses in the local area. Dharma estimates that there are about 40 houses in each street. How many streets will he be able to cover with 3700 brochures?

Reasoning

11 A travelling salesperson has lost his calculator. He needs to work out the individual cost of an item that is packaged into groups of 12 items, and costs $660 for the pack.

(a) He decides to approximate by dividing by 10. Find the answer he obtains.

(b) Calculate the actual cost of each individual item.

(c) He sells five items at his approximated cost. How much did he lose or gain?

12 The 'use an array' method of multiplication can be extended from two-digit to three-digit numbers by adding an extra row and an extra column to the grid. For example, 213×145 can be calculated using the following array.

	200	10	3
100			
40			
5			

Use an extended grid to calculate the following.

(a) 162×246 (b) 356×412 (c) 107×560

13 Which of the following is an incorrect method for calculating $240 \div 4$?

A Halve 240, then halve the answer.

B Calculate $2 \div 4$, $4 \div 4$ and $0 \div 4$, then add the results together.

C Calculate $200 \div 4$ and $40 \div 4$, then add the quotients.

D Calculate $24 \div 4$, then multiply the answer by 10.

Open-ended

14 ◆ × ● = 1620.

(a) What could ◆ and ● be? Find at least seven different combinations of whole numbers.

(b) Find at least three combinations where both ◆ and ● are two-digit numbers.

15 Here is part of Kim's maths homework. She had been learning about splitting up numbers to be multiplied, so she decided to use that strategy to complete the following.

Multiplication

(1) 9×24
$= 9 \times 2 + 9 \times 4$
$= 18 + 36$
$= 54$

(2) 12×43
$= 12 \times 4 + 12 \times 3$
$= 48 + 36$
$= 84$

(a) Kim is a bit worried that her answers don't look big enough. What are the correct answers?

(b) Explain to Kim the mistake she has made in both questions.

(c) Give Kim some advice so that she can avoid similar mistakes in the future.

Outside the Square Puzzle

Wizard maths

Henry and his friends Hilary and Rob are out shopping in Oblique Lane. Oblique Lane is a street of wizard shops in which goods are purchased using wizard money. Wizard money, however, is different to non-wizard money.

Wizard money consists of aureas, denaris and cuprums.

1 aurea is worth 17 denaris.
1 denari is worth 29 cuprums.

1 Henry, Hilary and Rob each have 10 aureas to spend. Here are their shopping lists. Add up the total cost of each list, then work out the change they will get in aureas, denaris and cuprums. Use the above information to help you with your calculations.

Henry

Item	*Price*
1 cloak:	2 aureas, 4 denaris
1 large bag owl pellets:	3 aureas, 15 cuprums
2 bottles elderflower juice:	14 denaris, 27 cuprums

Hilary

Item	*Price*
10 scrolls of parchment:	1 aurea, 14 denaris,
1 box of quills:	11 denaris, 23 cuprums
Reading Ancient Runes book:	5 aureas, 13 denaris, 10 cuprums

Rob

Item	*Price*
Wizard Monthly magazine:	9 denaris, 21 cuprums
New wand:	6 aureas, 16 denaris
1 box of tricky toffee:	1 aurea, 9 denaris, 17 cuprums

2 Invent a wizard shopping list of your own and give it to a friend to add up. (Make sure you can work out the answer too, though!)

Half-time 1

Ex 1.1, 1.3

1 Calculate the following by using an appropriate mental strategy.

(a) $4 \times 17 \times 5$ (b) $15 \times 23 \times 2$ (c) $33 + 45 + 107$

(d) 40×19 (e) $37 + 49 + 153$ (f) $127 + 65 + 69$

Ex 1.2

2 (a) Write the value of the following.

(i) 6^3 (ii) 5^5 (iii) 10^1 (iv) 7^2

(v) $\sqrt{49}$ (vi) $\sqrt[3]{8}$ (vii) $\sqrt{100}$ (viii) $\sqrt[3]{64}$

(b) Which of your answers to (a) is:

(i) a perfect square (ii) a perfect cube?

Ex 1.1, 1.3

3 Calculate the following. Use mental strategies if possible.

(a) 19×9 (b) 41×8 (c) 5×38 (d) 22×17

(e) 147×3 (f) 12×53 (g) 67×71 (h) 92×84

Ex 1.3

4 Calculate the following divisions. Use mental strategies if possible.

(a) $136 \div 4$ (b) $174 \div 6$ (c) $580 \div 5$ (d) $940 \div 20$

(e) $128 \div 8$ (f) $351 \div 9$ (g) $270 \div 15$ (h) $1344 \div 24$

Ex 1.2

5 Write the following in index form.

(a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ (b) 6 to the power of 5 (c) base 4, index 7

(d) fifteen squared (e) twenty-one cubed (f) $41 \times 41 \times 41$

Ex 1.3

6 Calculate the following.

(a) 70×60 (b) 150×20 (c) 400×300

(d) 250×800 (e) $3600 \div 40$ (f) $9000 \div 300$

(g) $7500 \div 1500$ (h) $124000 \div 200$ (i) $60000 \div 400$

Ex 1.1, 1.3

7 13 000 tickets were sold to a music concert at a cost of $52 each. Use a combination of strategies to calculate the amount made by the promoters of the concert from ticket sales.

Estimating and rounding

1.4

Often, we do not need to know the exact answer to a problem. An estimate, or approximate answer, is accurate enough.

We serve about 40 customers every hour. They pay approximately $10 per meal. That's approximately $4800 into the till in a 12 hour day.

It's approximately 750 km from Melbourne to Adelaide. I estimate it will take me 10 hours to drive there, and cost about $450 in fuel.

Estimating by rounding to the first digit

Rounding is an important skill for making good estimations.

Consider the following.

46 is *rounded up* to 50 (because it lies in between 40 and 50, and is closer to 50).

44 is *rounded down* to 40 (because it lies in between 40 and 50, and is closer to 40).

When rounding, we look at the digit to the right of the one being rounded. If it is 0, 1, 2, 3 or 4 we round down. If it is 5, 6, 7, 8 or 9 we round up.

This means that:

173 is rounded *up* to 200

137 is rounded *down* to 100

4500 is rounded *up* to 5000

4499 is rounded *down* to 4000

25 374 is rounded *up* to 30 000

24 985 is rounded *down* to 20 000

This type of rounding is called 'rounding to the first digit'. This means that the rounded number has only zeroes after the first digit.

Rounding to the first digit helps us estimate the answers to multiplication and division questions.

To round a number to the first digit, look at the second digit of the number. If it is 0,1, 2, 3 or 4, keep the first digit the same and replace the other digits with zeroes. If it is 5, 6, 7, 8 or 9, increase the first digit by 1, and replace the other digits with zeroes.

When rounding, we use the symbol ≈ which means 'approximately equal to'.

Worked Example 7 WE7

Round the following numbers to the first digit.

(a) 361 (b) 2050 (c) 8

Thinking	Working
(a) Look at the second digit. Because it is 5 or greater, increase the first digit by one and replace the following digits with zeroes.	(a) $361 \approx 400$
(b) Look at the second digit. Because it is less than 5, keep the first digit the same and replace the following digits with zeroes.	(b) $2050 \approx 2000$
(c) Because there is only one digit, this number is already rounded to the first digit.	(c) 8

Estimating with multiplication and division

Multiplication and division can be estimated using rounding to the first digit. This will not give an exact answer to a problem, but an approximate one.

Worked Example 8 WE8

Calculate an approximate answer to the following by first rounding each number to the first digit.

(a) 368×52 (b) $77\,483 \div 421$

Thinking	Working
(a) 1 Round both numbers to the first digit.	(a) 368×52 $\approx 400 \times 50$
2 Multiply the two rounded numbers together and write the approximate answer.	$\approx 4 \times 100 \times 5 \times 10$ $\approx 20 \times 1000$ $\approx 20\,000$
(b) 1 Round both numbers to the first digit.	(b) $77\,483 \div 421$ $\approx 80\,000 \div 400$
2 Divide the two rounded numbers by an appropriate power of 10 by 'cancelling zeroes'. (Here, we have divided both numbers by 100.)	$\approx 80\,000 \div 400$ $\approx 800 \div 4$
3 Do the simple division and write the approximate answer.	≈ 200

Estimating using other ways of rounding

Instead of rounding off to the first digit, we could round off to a closer multiple of 5 or 10 that is convenient to work with. This would give a more accurate estimate of the actual answer.

Worked Example 9 WE9

Calculate an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

(a) $4587 \div 49$ (b) 197×23

Thinking		Working
(a) 1	Round off the numbers that you are dividing to convenient numbers. (Here, one number is rounded to the nearest 100, the other to the nearest 10.)	(a) $4587 \div 49$ $\approx 4600 \div 50$
2	Perform the division with the rounded numbers, cancelling zeroes first.	$\approx 4600 \div 50$ $\approx 460 \div 5$
3	Change the non-zero digits of the larger number to the nearest multiple of the smaller number.	$\approx 450 \div 5$
4	Do the simple division and write the approximate answer.	≈ 90
(b) 1	Round off the numbers that you are multiplying to convenient numbers. (Here, one number is rounded to the nearest 10, the other to the nearest 5.)	(b) 197×23 $\approx 200 \times 25$
2	Do the simplified multiplication.	$\approx 2 \times 100 \times 25$ $\approx 2 \times 25 \times 100$
3	Write the approximate answer.	≈ 5000

Overestimating and underestimating

You can use an estimate of the answer to a problem to check whether your calculated answer is 'reasonable'. It is useful to be able to judge whether your estimate is greater than (an 'overestimate') or less than (an 'underestimate') the final answer. To do this, we need to consider the way in which the numbers were rounded.

Consider these examples of multiplication by rounding to the first digit.

(a) 157×66
$\approx 200 \times 70$
$\approx 14\,000$

Actual answer: 10 362

(b) 218×46
$\approx 200 \times 50$
$\approx 10\,000$

Actual answer: 10 028

In (a), the actual answer is a lot less than the estimate! This is because both numbers were rounded up by a large amount.

In (b), the actual answer is 10 028—very close. This is because one number was rounded up, and the other down, by comparable amounts.

When dividing, we need to be aware of the effect of rounding the original numbers up or down.

Consider these examples of division by rounding to the first digit.

(a) $2320 \div 9$
$\approx 2000 \div 10$
$= 200$

Actual answer: 257.8

(b) $6432 \div 33$
$\approx 6000 \div 30$
$= 200$

Actual answer: 194.9

In (a), the big difference between the approximate answer and the actual answer is because the rounded calculation resulted in a smaller number being divided by a larger one.

In (b), the approximate answer is very close to the actual answer. This is because both numbers were rounded down—a smaller number divided by a smaller number.

To achieve a reasonable approximate answer:

- when multiplying, round one number up and the other number down
- when dividing, both numbers should be rounded up, or both rounded down.

1.4 Estimating and rounding

Navigator

Answers page 614

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q8, Q10, Q12, Q13, Q14	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14	Q1 Columns 3 & 4, Q2 Column 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q15

Fluency

WE7

1 Round the following numbers to the first digit.

(a) 68	(b) 74	(c) 10	(d) 6
(e) 619	(f) 650	(g) 483	(h) 970
(i) 4846	(j) 3723	(k) 9510	(l) 9643
(m) 75 000	(n) 716 599	(o) 800 050	(p) 1 801 021

WE8

2 Calculate an approximate answer to the following by first rounding each number to the first digit.

(a) 681×41	(b) 547×84	(c) 62×819
(d) $2940 \div 51$	(e) $3199 \div 62$	(f) $1955 \div 78$
(g) 141×837	(h) 104×8946	(i) 7340×250
(j) 9×6511	(k) 250×950	(l) $75\,342 \div 80$
(m) $11\,397 \div 213$	(n) $27\,496 \div 487$	(o) $2547 \times 30\,995$

WE9

3 Calculate an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

(a) 41×26	(b) 122×54	(c) 191×14
(d) $387 \div 43$	(e) $379 \div 22$	(f) $248 \div 52$
(g) 1587×23	(h) $33\,997 \times 21$	(i) 4036×253
(j) $3527 \div 52$	(k) $10\,803 \div 95$	(l) $44\,895 \div 14$
(m) $23\,938 \div 356$	(n) $15\,337 \div 2498$	(o) 5089×2492

4 In each case, which of these estimates do you think is the closest? Don't try to count or measure.

(a) About how many people are there in the photo?

A 700

B 4000

C 30 000

D 100 000

(b) Approximately how far is it:

(i) from Alice Springs to the South Australian border, by road?

A 75 km

B 150 km

C 300 km

D 600 km

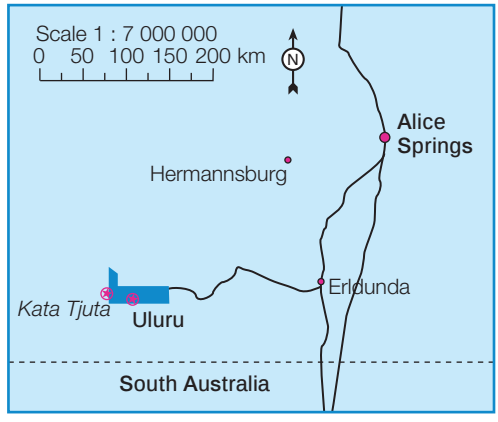
(ii) directly from Uluru to Alice Springs (i.e. in a straight line)?

A 40 km

B 350 km

C 789 km

D 1000 km



Understanding

5 Estimate the following by rounding the smaller number to the first digit and the larger number to that same place value (e.g. if the smaller number is rounded to the nearest 10, round the larger number to the nearest 10 also).

(a) $1247 + 323$ (b) $10\,290 + 178$ (c) $14\,532 + 2788$

(d) $823 - 79$ (e) $1176 - 241$ (f) $453 - 276$

6 For each of the following, find an estimate for the answer by first rounding each number to the first digit. State whether the actual answer will be higher or lower than the estimate.

(a) 364×57 (b) 2398×426 (c) 153×16

(d) $1996 \div 24$ (e) $943 \div 86$ (f) $597 \div 136$

7 In each case, choose the best estimate from the alternatives given. Don't do the actual calculation.

(a) 321×73

A 210 B 2100 C 2163 D 21 000

(b) 405×950

A 360 000 B 400 000 C 450 000 D 3 600 000

(c) 7865 ÷ 24

A 400 B 900 C 1600 D 210 000

(d) 999 160 ÷ 527

A 220 B 330 C 660 D 2000

8 Last weekend, 255 345 people attended the AFL matches. They paid, on average, $22 to attend their game.

(a) Round the attendance to the first digit.

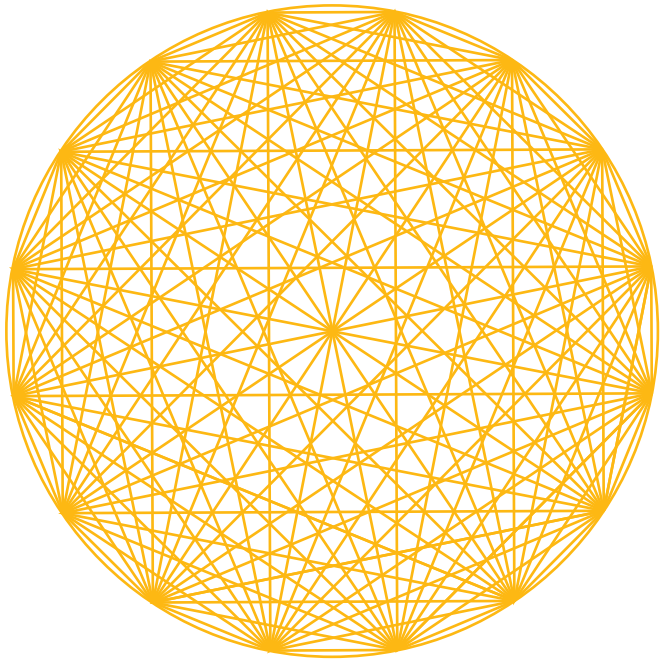
(b) Use this estimate, and a rounded estimate for the ticket price, to find an estimate for the total entry cost.

9 (a) Tahnee earns $38 295 per year. Approximately how much is this per week? Calculate by rounding to convenient numbers first.

(b) Catriona earns $175 per week from her part-time job. Approximately how much is this per year? Calculate by rounding to convenient numbers first.

Reasoning

10 (a) Approximately how many straight lines are there in this picture?



A 100

B 500

C 5000

D 10 000

(b) Explain a way of finding the exact number of lines without counting them all.

11 79×5003 and 76×5488 both give the same approximate answer of 400 000 when each number is rounded to the first digit. Which of the actual answers to the two calculations will be closer to 400 000? Explain how you arrived at your answer.

12 (a) Use rounding to the first digit to find an approximate answer for 1344 ÷ 21.

(b) Find an approximate answer for 1344 ÷ 21 by first rounding to convenient numbers, such as multiples of 10.

(c) Which method do you think has given a closer estimate to the actual answer? Give a reason for your prediction.

(d) Work out the actual answer and see if you were correct.

13 Ms Kelly, the assistant principal at Summerhill Secondary, is organising buses to transport students to the pool for the swimming carnival. Each bus can seat 52 people, and she has 428 staff and students going (although she doesn't know how many might be sick or away on the day). How many buses should Ms Kelly order? Why is estimating by rounding not helpful in this situation?

Open-ended

14 Natalie is out shopping. She is heading towards the checkout when she suddenly wonders if she has enough money. She mentally rounds her purchases off in her head and adds them up.

Here is Natalie's rounded off, mental list:

T-shirt ≈ $20
Earrings ≈ $10
Necklace ≈ $10
Lip gloss ≈ $10
Total ≈ $50

'Terrific!' thinks Natalie. 'I've got exactly enough!'

The actual prices of the items are: T-shirt $17; earrings $14; necklace $13; lip gloss $9.

(a) There is nothing wrong with Natalie's rounding. Why is she in trouble when she gets to the checkout?

(b) Describe to Natalie a 'safer' method for rounding money.

15 Write down three pairs of numbers that when rounded to the first digit, then multiplied together, give:

(a) 3200 (b) 200 (c) 100 000 (d) 42 000

Outside the Square Game

Estimation frustration

Equipment required: 2 brains, 2 dice, 1 calculator

How to win:
The winner is the first person to 40 points.

How to play:

1 Roll the dice for your partner. Make a two-digit number from the numbers on the dice (e.g. rolling a 4 and a 3 means you can make 34 or 43—your choice). Write this number down.

2 Roll again, make another two-digit number, and write this one down also.

3 Show these two numbers to your partner. They now have just 10 seconds to write down an estimate of what the product of these two numbers will be.

4 Use a calculator to work out the answer, and compare it to your partner's estimate.

5 Score according to the table below.

How close were they?	Points given
Spot on	20 points
Less than 100 off	10 points
Between 100 and 200 off	5 points
More than 200 off	0 points

6 Now, get your partner to roll the dice for you.

1.5 Order of operations

Jamie, Josh and Ali all worked out the following problem: $1 + 2 \times 3 + 4$.

- Jamie's answer was 13.
- Josh's answer was 11.
- Ali's answer was 15.

Why were there different answers?

We can't have different people getting different answers for the same calculation. Mathematicians have agreed on some rules about the order in which to do the four operations of $\times$, $+$, $-$ and $\div$. Rules that mathematicians agree to work by are called **mathematical conventions**.

The '**order of operations**' rules:

1. Always do calculations in brackets first.
2. Next, do any calculations with indices (powers).
3. Do the multiplication and division next, in order from left to right, as you come to them.
4. Finally, do the addition and subtraction, in order from left to right, as you come to them.

According to the order of operations rules, who had the correct answer to the problem above: Jamie, Josh or Ali?

Worked Example 10

WE10

Calculate the following, showing all steps of working.

(a) $24 + 6 \div 2 - 1 \times 4$ (b) $14 - 9 + 6^2 \div (7 + 2) \times 3$

Thinking	Working
(a) 1 There are no brackets and no indices, so do multiplications and divisions in the order in which they appear.	(a) $24 + 6 \div 2 - 1 \times 4$ $= 24 + 3 - 1 \times 4$ $= 24 + 3 - 4$
2 Now, do additions and subtractions in the order in which they appear.	$= 27 - 4$ $= 23$

(b) 1	Do the calculation in brackets first.	(b) $14 - 9 + 6^2 \div (7 + 2) \times 3$
		$= 14 - 9 + 6^2 \div 9 \times 3$
2	Evaluate any numbers in index form.	$= 14 - 9 + 36 \div 9 \times 3$
3	Do multiplications and divisions in the order in which they appear.	$= 14 - 9 + 4 \times 3$ $= 14 - 9 + 12$
4	Do additions and subtractions in the order in which they appear.	$= 5 + 12$ $= 17$

1.5 Order of operations

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5 Column 1, Q7, Q9, Q11, Q12, Q13	Q1 Column 2, Q2, Q4, Q5 Column 2, Q6, Q7, Q8 Column 1, Q9, Q10, Q11, Q12, Q13	Q1 Column 2, Q4, Q5 Column 2, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Answers page 614

Fluency

WE10

1 Calculate the following, showing all steps of working.

(a) $1 + 8 \times 3$
(b) $8 - 5 \div 5$
(c) $6 \times 2 - 1$
(d) $8 \div 4 - 2$
(e) $25 - 2 \times 11$
(f) $6 \div 3 + 3 \times 5$
(g) $8 - 24 \div 12 + 3$
(h) $8 \times 3 \div 4 \times 2$
(i) $9 \times (10 - 7) \div 3$
(j) $24 \div (7 + 5) \times 5^2$
(k) $88 \div 8 - 6 \times (5 - 4)$
(l) $12 \times 5 + 4 \times (10 - 4)$
(m) $38 - 7 \times 2^2 + 13 - 4 \div 2$
(n) $9 - 2 + 5 + 3 \times 4 \div 6$
(o) $23 - 5 + (17 - 2) \times 3 + 5^2$
(p) $28 \div 7 \times 3^2 + (5 - 1) \div 2 + 3$

2 State TRUE or FALSE for the following.

(a) For $2 + 6 \times 4$ we would calculate $2 + 6$ first.
(b) For $6 + 12 \div 3$ we would calculate $12 \div 3$ first.
(c) For $8 + 40 \div (3 + 5) \times 10^2$ we would calculate $8 + 40$ first.
(d) For $24 + 6 \div 2 - 1 \times 4$ we would calculate $6 \div 2$ first.
(e) $4 + 12 \div 2$ simplifies to 8.
(f) $20 \div 5 - 1$ simplifies to 3.

3 (a) $8 + 6 - 4 \div 2$ is equal to:

A 5 B 9 C 12 D 16

(b) $12 - 8 \div 4 + 2^2$ is equal to:

A 5 B 6 C 9 D 14

Understanding

4 (a) Which calculation would you do first and which would you do second in each of these questions?

(i) $14 - 2 \times 6 + 2 \times 2$
(ii) $(15 \div 5) + 6 \times 3$

(b) Calculate the value of the expressions in (a).

5 Put brackets into these statements, where necessary, to make them true.

(a) $6 + 6 \times 3 = 36$

(b) $10 - 4 \times 5 = 30$

(c) $9 - 8 \times 6 + 4 = 10$

(d) $12 + 6 \div 7 - 4 = 14$

(e) $6 \div 3 + 3 \times 5 = 5$

(f) $3 \times 6 \div 8 - 4 + 5 = 2$

(g) $3 \times 10 - 7 \div 9 + 12 = 13$

(h) $18 \div 3 \times 5 - 3 + 2 = 14$

(i) $7 + 3 \div 4 + 1 = 2$

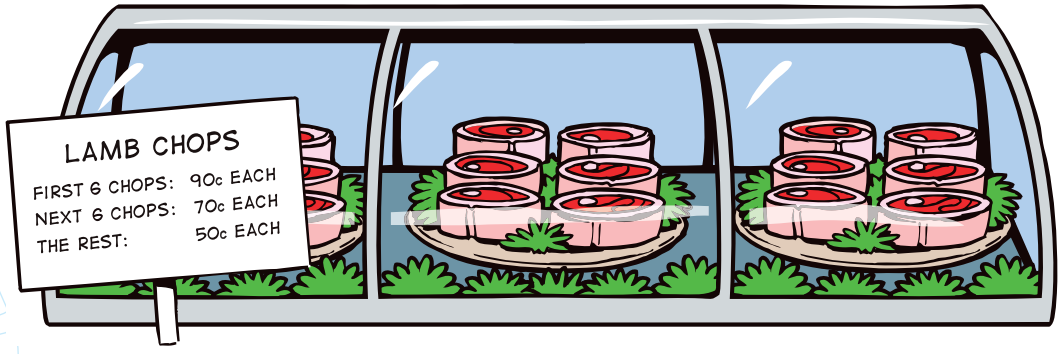
(j) $5 - 3 \times 8 - 6 \div 2 = 2$

6 During 'peak' time, Shania's mobile phone company charges 73c per minute, plus a 29c 'flagfall' (a single amount charged as soon as a call is connected). During 'off-peak' time, there is no flagfall, and a rate 50c per minute is charged per call. Shania has $20 credit on her phone. She makes one 5-minute call during 'peak' time, and one 8-minute call during 'off-peak' time.

(a) Calculate the total cost of the two calls. Show your working clearly.

(b) Using the symbols +, –, ×, and at least one pair of brackets, write down a calculation that Shania could do to work out how much credit she had left on her phone after the two calls were made.

7 A butcher had an oversupply of lamb chops. He decided to tempt people to buy more of them by putting up the following sign.



Stuart bought 8 chops, Mahalia bought 11 chops, and Sofia bought 17 chops.

The amount Stuart paid can be worked out using the following calculation.

$6 \times 90 + 2 \times 70$. This gives an answer of 680 cents, which can then be written as $6.80.

(a) Write down a calculation similar to the one above, to work out how much Mahalia paid, then do the calculation.

(b) Write down a calculation similar to the one above, to work out how much Sofia paid, then do the calculation.

(c) How much more than Stuart did Sofia pay?

8 Replace each $*$ with one of the four symbols (+, –, ×, ÷) to make the statement true.

(a) $2 + 21 * 3 = 9$

(b) $15 - 6 * 2 = 12$

(c) $14 - 8 * 6 = 0$

(d) $7 * 5 * 6 = 29$

(e) $14 * 3 * 2 = 15$

(f) $(24 * 6) * 10 = 3$

(g) $8 * 5 * 2 - 6 = 12$

(h) $12 * 2 + 1 * 9 = 15$

9 Replace each * with either <, > or = to make the statement true.

(a) $6 \times (4 \div 2) \times 3 * (6 \times 4) \div 2 \times 3$

(b) $(1 + 4) \times 20 \div 5 * 1 + (4 \times 20) \div 5$

(c) $100 + 10 \div 10 * (100 + 10) \div 10$

(d) $36 \div 6 \times (3 - 3) * 36 \div 6 \times 3 - 3$

Remember < means 'is less than' and > means 'is greater than' when read from left to right.

Reasoning

10 Remove the unnecessary brackets from these statements.

(a) $(4 \times 3) + (6 - 2) = 16$

(b) $8 + (4 \div 2) + (6 \times 2) = 22$

(c) $4 + (20 \div 5) \times (3 + 7) = 44$

(d) $6 \times (3 + 2) - (12 \div 2) = 24$

11 Sean typed the following into his calculator: $8 + 6 \div 2 + 3 \times 5$. Shazhad typed the same into the 'calculator' application of his mobile phone. Sean's answer was 26. Shazhad's answer was 50.

(a) Do the calculation to decide who's answer was correct.

(b) How did the other calculator come up with a different answer?

Open-ended

12 By placing one pair of brackets in different positions, show all the possible answers for this question: $8 + 4 \times 6 \div 2 - 1$

13 Jing has worked out that the answer to $30 \div 2 \times (2 + 3)$ is 3. Here is his working.

$30 \div 2 \times (2 + 3)$
$= 30 \div 2 \times 5$ Step 1
$= 30 \div 10$ Step 2
$= 3$ Step 3

Jing's teacher has marked it as incorrect. What is the correct answer? Explain to Jing which step of his working is incorrect, what he should have done instead, and why.

Outside the Square

Problem solving

Four 4s = 100?

1 Using the number 4 exactly four times, together with any of the four operations (+, –, ×, ÷) and brackets if you need them, see if you can make the numbers 0 to 9. A way to get 5 has been done for you. There is more than one way in many cases.

0 =
1 =
2 =
3 =
4 =
5 = $(4 \times 4 + 4) \div 4$
6 =
7 =
8 =
9 =

2 See how many numbers up to 100 you can find with just four 4s. You might like to work with a partner or in a small group.

3 Using the digits of the year 2013 exactly once, together with brackets and any of the four operations, see how many of the numbers from 1 to 10 you can make.

e.g. $(3 + 1) \times 2 + 0 = 8$

Strategy options
- Guess and check.
- Test all possible combinations.

Mathspace

The Operation Theatre

There are many weird and interesting facts about the human body. Calculate the following expressions using the correct order of operations to discover some of these facts.

1 The number of muscles it takes to produce human speech.

$36 \div 2 \times 4$

CODE LETTER: T

2 The number of shades of grey a human eye can distinguish

$246 + 352 - (43 + 55)$

CODE LETTER: N

3 The highest recorded speed of a sneeze (in km per hour).

$125 + 200 \div 4 - 2 \times 5$

CODE LETTER: E

4 The number of different scents your nose can detect.

$(6000 + 1500 \div 5) - 43 \times 100$

CODE LETTER: W

5 Babies are born with cartilage knee caps. They don't turn into bone until the child reaches between which two ages?

(a) $3 \times 4 \div 12 \times 2$ CODE LETTER: Y

(b) $30 - 6 \times 8 \div 12 \times 6$ CODE LETTER: B

6 **(a)** The number of bones you are born with.

$200 - 50 + (30 - 1) \times 5 + 5$ CODE LETTER: H

(b) The number of bones an adult has.

$618 \div (56 - 47) \times 3$ CODE LETTER: G

7 The average human will shed approximately how many billion flakes of skin a day?

$30 - 7 \times 3 + 10 - 36 \div 4$

CODE LETTER: I

8 The number of million blood cells destroyed in the human body every second.

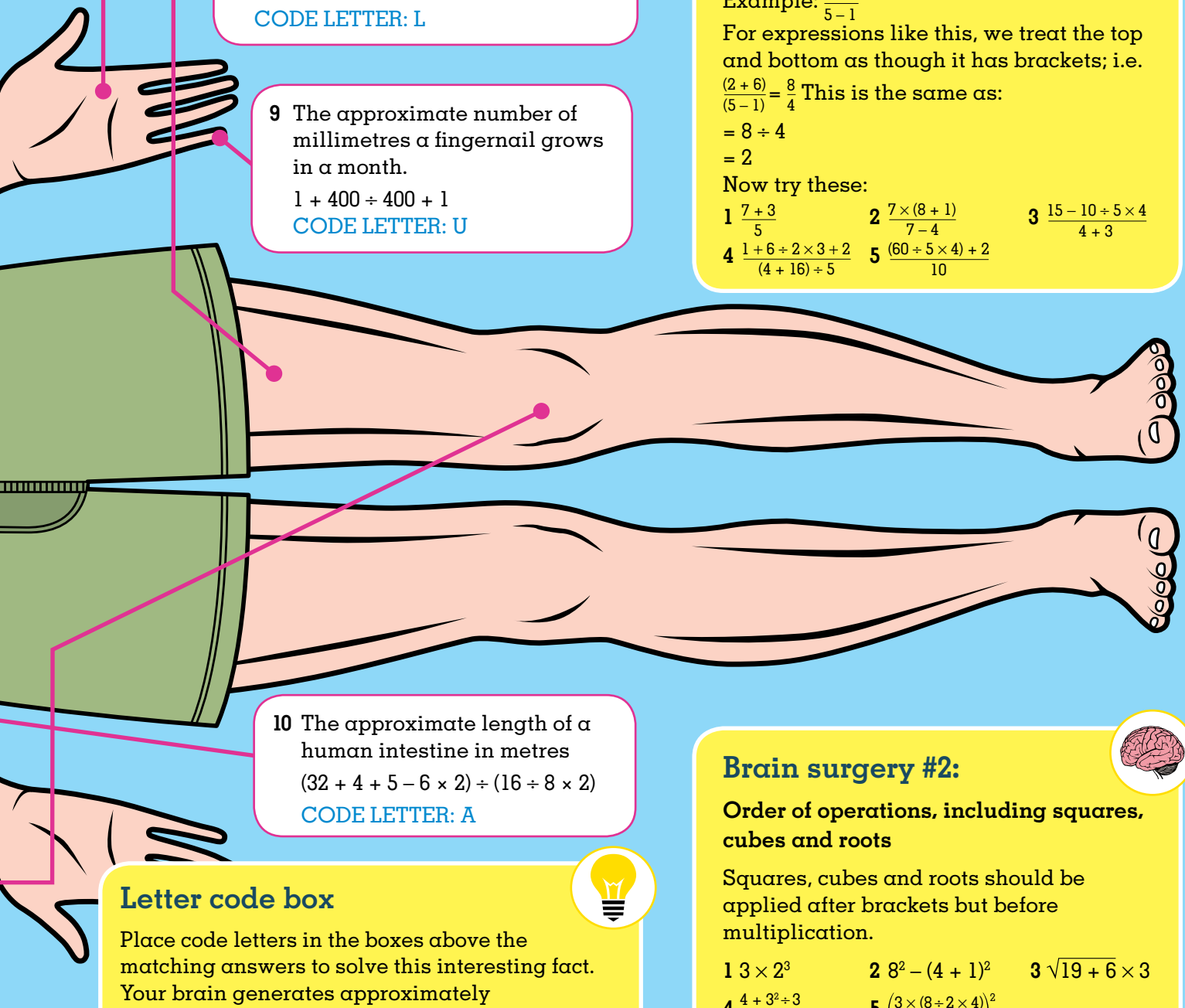
$90 \div (7 + 11) \times 3$

CODE LETTER: L

9 The approximate number of millimetres a fingernail grows in a month.

$1 + 400 \div 400 + 1$

CODE LETTER: U



10 The approximate length of a human intestine in metres

$(32 + 4 + 5 - 6 \times 2) \div (16 \div 8 \times 2)$

CODE LETTER: A

Letter code box

Place code letters in the boxes above the matching answers to solve this interesting fact.
Your brain generates approximately

☐	☐	☐	☐	☐	☐	watts
72	2000	165	500	72	2	

of power while you're awake - enough to illuminate

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
7	15	10	206	300	72	6	3	15	6

WARNING: Advanced order of operations

Brain surgeons need to be very smart, see if you can calculate these trickier expressions.

Brain surgery #1:

Order of operations using a fraction bar

Example: $\frac{2+6}{5-1}$

For expressions like this, we treat the top and bottom as though it has brackets; i.e. $\frac{(2+6)}{(5-1)} = \frac{8}{4}$ This is the same as:

$= 8 \div 4$

$= 2$

Now try these:

1 $\frac{7+3}{5}$ **2** $\frac{7 \times (8+1)}{7-4}$ **3** $\frac{15 - 10 \div 5 \times 4}{4+3}$

4 $\frac{1 + 6 \div 2 \times 3 + 2}{(4+16) \div 5}$ **5** $\frac{(60 \div 5 \times 4) + 2}{10}$

Brain surgery #2:

Order of operations, including squares, cubes and roots

Squares, cubes and roots should be applied after brackets but before multiplication.

1 3×2^3 **2** $8^2 - (4+1)^2$ **3** $\sqrt{19+6} \times 3$

4 $\frac{4 + 3^2 \div 3}{\sqrt{16+3}}$ **5** $\left(\frac{3 \times (8 \div 2 \times 4)}{\sqrt{64}}\right)^2$

1.6 Mixed whole number problems

Maths problems in daily life do not come as 'ready made' calculations all set out for you to do. They come in situations that you need to first understand in order to decide which skills to use.

Some words that are associated with the four operations are listed below. You might be able to think of some others. Identifying these in a problem will help you to apply the right operation.

+ Sum	− Difference	× Product	÷ Quotient
add	subtract	multiply	divide
total	less than	how many times more	goes into
altogether	minus	lots of	shared between

This section contains problems that will require you to read the information carefully and decide which skills to use. Many problems require more than one step or calculation. You should use the skills and strategies covered in this chapter to help you.

1.6 Mixed whole number problems

Navigator

Answers page 615

Q1, Q2, Q3, Q6, Q7, Q8, Q9, Q11, Q12, Q15	Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q15, Q16	Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Fluency

1 (a) The Australian cricket team made 425 in their first innings and 299 in their second. How many runs did they make in total for the match?

(b) A new car cost $28 500. Five years later it was worth $19 275. By how much had the value of the car decreased in that time?

(c) A piano teacher charges $27 for each 1-hour lesson. How much would it cost for 13 lessons?

(d) Sula has a job that pays her $12 per hour. How many hours did Sula work in a week for which her pay was $372?

2 Harvey 'Scoop' Roberts, a journalist with the *Monthly Farm News*, can type 60 words a minute. How long does it take him to type an article of 1800 words?

3 Wendy is training to be an Olympic swimmer. Every morning she swims 3600 m in a 50 m pool. How many laps is that?

Understanding

4 Little Lucy is exactly 5 weeks old. How many minutes old is she?

5 The highest mountain in the world, measured from sea level, is the Himalayan peak of Mount Everest. It is 8848 m above sea level. If we measure mountains that start under the ocean, the tallest mountain in the world from base to tip is Mauna Kea on the island of Hawaii. It measures 10 203 m, of which 4205 m is above sea level.

(a) How much of Mauna Kea is below sea level?

(b) If we don't count the part of Mauna Kea that is under water, how much higher is Mount Everest?

6 Cathy is out shopping for bargains. She bought two pairs of $68 shoes in a store that said 'buy one pair, get the second pair half price'. She bought two $19 T-shirts where the special deal was 'buy one, get $10 off the second'. Cathy also bought a pair of earrings for $25. What was the total cost of her shopping spree?

7 Mick is a baker in a supermarket. He makes hot cross buns and packs them in bags of 6. He stacks 4 shelves with 8 bags on each shelf. At the end of the day, Mike has 3 bags of buns left. How many hot cross buns did he sell?

8 The two longest rivers in the world are the Amazon (6448 km) and the Nile (6670 km). The longest river in Australia is the Darling (2739 km).

(a) How much longer is the Nile than the Amazon?

(b) How much longer is the Nile than the Darling?

(c) How much longer is the Amazon than the Darling?

9 The width of a painting including the frame is 85 cm. If the frame is 6 cm wide all the way around, what is the width of the unframed painting?

10 The Pizza Pit-Stop employs five people. The two cooks work 36 hours each per week for $18 an hour. The three waiters work 30 hours each per week for $15 an hour. What does the Pizza Pit-Stop pay its five employees in total per week?

Reasoning

11 Sam has 28 model planes on a display shelf. Mark has double Sam's number of model planes. Harvey has half as many planes as Sam. What is the difference between the number of model planes that Harvey and Mark have?

12 Hiram is moving house. He has 13 boxes of possessions, but can only fit 4 at a time in his car.

(a) Beginning the day at his old house and ending it at his new house, how many times will Hiram need to travel between the two houses in order to shift all his boxes?

(b) If the distance between the two houses is 7 km, how far will he travel?

13 Jules Verne wrote about travelling around the world in 80 days.

(a) Write 80 days as a number of weeks plus a number of days.

(b) How many months, weeks and days is it, if we assume there are 30 days in a month?

(c) How many months, weeks and days is it, if we assume that there are 4 weeks in a month?

(d) Explain why your answers to (b) and (c) are different.

14 Weighing Jason's pet dog Scruffy at the vet's was a problem, as Scruffy kept jumping off the scales. The vet held Scruffy and stood on the scales. Then, Jason held Scruffy and stood on the scales. Finally, the vet, Jason and Scruffy all stood on the scales.

The scales read as follows:
The vet and Scruffy = 91 kg
Jason and Scruffy = 56 kg
The vet, Jason and Scruffy = 138 kg

Work out how much Scruffy weighs.

Open-ended

15 (a) Write two numbers that have a sum greater than 90 but a product less than 1800.

(b) Write two numbers that have a difference of less than 10 but have a sum greater than one-quarter of their product.

16 Simon owns a company that produces fruit juice. The juice comes in three different-sized bottles—300 mL, 600 mL and 1000 mL (1 L), weighing 300 g, 600 g and 1000 g (1 kg), respectively. Simon packs same-size bottles into boxes of 20, and delivers them by truck to shops. Simon's truck can hold a maximum load of 1000 kg.

(a) For each of the different-sized bottles, write down the mass in kg of a box of 20 bottles. (Remember, 1000 g = 1 kg.)

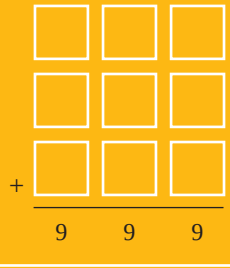
(b) How many boxes of each type of juice can Simon get on the truck to make a load of 1000 kg?

(c) Find at least two different combinations of boxes that would make a load of 1000 kg.

Outside the Square — Problem solving

Nine hundred and ninety-nine

Copy the following boxes into your work book.



Using each of the digits from 1 to 9 once in each of the boxes, create three three-digit numbers whose sum is 999.

(It may help to write the digits 1 to 9 on small pieces of paper, so you can move them around the boxes easily.)

How many different combinations can you find?

Aim to find at least four combinations of different three-digit numbers.

Strategy options
- Test all possible combinations.
- Break problem into manageable parts.

Challenge 1

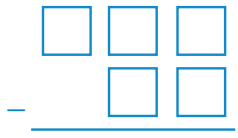
1 The digits 5, 6, 7, 8 and 9 can be arranged to form five-digit even numbers. The tens digit in the smallest of these numbers is:

A 6 B 7 C 8 D 9

2 How many different numbers can you make by multiplying two one-digit even numbers?

A 6 B 8 C 9 D 10

3 Each of the digits 3, 5, 6, 7 and 8 is placed one to a box in the diagram opposite. If the two-digit number is subtracted from the three-digit number, the smallest difference is:



A 261 B 269 C 271 D 278

4 Kim and Brian are given savings accounts by their grandmother, with the same amount of money in each. Kim is a good saver and puts an extra $5 into her account each week. Brian takes $1 out of his account every week. After 10 weeks, Kim has 3 times as much in her account as Brian has in his account. What was the starting amount in each account?

5 The product of the ages of two teenagers and their father is 15 181. How old is the father?

6 Look at the following number arrangement.

1

3 5

7 9 11

13 15 … …

What will be the sum of the seventh row? Look for a pattern in your row totals. What will be the sum of the one-hundredth row?

7 How many square numbers are there between 8 and 8008?

8 Find the next three terms in this number pattern 4, 7, 11, 18, 29, ...

9 The counting numbers are arranged in six columns as shown. In which column will 1001 appear?

A	B	C	D	E	F
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
…	…	…	…	20	19

10 (a) Write down the values of 0^2, 1^2, 2^2, 3^2, … , 9^2.

(b) Consider the last digit of each of the values found in (a). What are the only digits that a square number can end in?

(c) What digits can't a perfect square end in?

(d) Which of the following numbers can't be perfect squares?

(i) 327 (ii) 324 (iii) 343 (iv) 289 (v) 732

(e) Without using a calculator, write 961 as a perfect square by first deciding what the square root could end in and then thinking about the factors of 900.

Chapter review 1

D.I.Y. Summary

Key Words

associative law	divisor	mathematical conventions	quotient
base	estimate	order of operations	remainder
commutative law	expanded form	perfect cubes	rounding
cube root	index	perfect squares	square root
distributive law	index form	power	
dividend	indices	product	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 For the number 7^3, 7 is the _____ and 3 is the _____, or _____.

2 1, 9 and 25 are all examples of _____. 1, 8 and 27 are all examples of _____.

3 The _____ of 8 is 2.

4 In the statement $97 \div 3 = 32$ rem 1, 97 is the _____, 3 is the _____, 32 is the _____, and 1 is the _____.

5 One strategy for making multiplication easier is to use the _____; for example, $6 \times 17 = 6 \times (10 + 7)$.

To find the value of a statement such as $(9 + 5) \times 3 - 2$, we would use the _____.

6 The 'long way' of writing a number in index form is in _____.

7 The _____ of 16 is 4, because $4 \times 4 = 16$.

8 $7 \times 5 = 5 \times 7$ is an example of the _____.

9 When two or more numbers are multiplied together, the answer is called the _____.

10 We can obtain an _____ of the answer to a calculation by _____ to the first digit.

Fluency

1 Use an appropriate mental strategy to help calculate each of the following. Ex 1.1

(a) $89 + 53$ (b) $71 + 67$ (c) $93 + 125 + 7$ (d) $5 \times 12 \times 2$

(e) 21×15 (f) 19×23 (g) $4 \times 9 \times 5$ (h) 32×17

2 Find the value of the following. Ex 1.2

(a) 12^2 (b) 5^3 (c) 10^5

(d) $\sqrt{64}$ (e) $\sqrt{900}$ (f) $\sqrt[3]{27}$

(g) $5^3 + 2^2$ (h) $8^4 - 3^5$ (i) $2^3 \times 3^2$

3 Write each of the following numbers in index form. Ex 1.2

(a) $7 \times 7 \times 7 \times 7 \times 7$ (b) ten cubed

(c) five squared (d) twelve to the power of eight

4 Calculate the following using any suitable strategy. Ex.1.3

(a) 35×40 (b) 850×200 (c) 757×7 (d) 98×31

(e) $4400 \div 200$ (f) 27×59 (g) $612 \div 4$ (h) $756 \div 7$

5 Use rounding to the first digit to find approximate answers to the following. Ex.1.4

(a) 35×241 (b) 1763×27 (c) $3299 \div 64$ (d) $57\,008 \div 192$

(e) 159×286 (f) 3427×963 (g) $4398 \div 487$ (h) $86\,005 \div 334$

6 Find approximate answers to the following by first rounding each number to a convenient multiple of 10 or 100. Ex.1.4

(a) $554 \div 7$ (b) $2136 \div 19$ (c) $15\,624 \div 216$ (d) 28×604

(e) 157×419 (f) 523×1998 (g) $54\,327 \div 915$ (h) $12\,794 \div 183$

7 Calculate: Ex.1.5

(a) $9 \div (2 + 1) - 2$ (b) $(3 \times 8) \div 4 + 7$ (c) $12 - 6 \times 2 + 11$

(d) $7 + 12 \div 4 - 1 \times 2^2$ (e) $(13 - 5 \times 2) + (20 \div 10)$ (f) $[5 \times (9 + 1)] - 3^2$

8 Misha pays $112 for two T-shirts and a pair of jeans. If the T-shirts were $23 each, how much did the jeans cost? Ex.1.6

Understanding

9 Cartons of milk are packed into boxes which are then stacked into larger crates for transport to supermarkets. If there are 20 cartons in a box, and 9 boxes in a crate, use mental strategies to calculate how many cartons of milk are delivered to a supermarket that ordered 4 crates.

10 Use rounding to the first digit to estimate the answers to the following, then state whether the actual answer will be higher, lower, or close to your estimate.

(a) 3741×22 (b) 265×341 (c) $25\,736 \div 49$ (d) $96\,001 \div 17$

11 Sasha runs a small free-range egg farm. On Tuesday, she collected 283 eggs from her hens.

(a) How many cartons will she be able to fill, if each carton can hold 12 eggs?

(b) How many eggs will she have left over?

12 In the calculation of $2 \times [30 \div (4 - 1)] + 6$ the first operation to do is:

A $+$ B $-$ C $\times$ D $\div$

13 Put brackets into these statements, where necessary, to make them true.

(a) $4 \times 2 + 3 \div 5 - 1 = 3$ (b) $5 + 1 \div 6 + 4 + 2 = 7$

14 Melissa's salary is $57 935 per year.

(a) Approximately how much is this per week? Calculate by rounding to the first digit first.

(b) Consider how you rounded the numbers in (a). Will Melissa's actual weekly salary be more or less than the approximate answer you calculated?

15 Which two consecutive numbers do the following lie between?

(Hint: Consider the perfect squares on either side of the number being considered.)

(a) $\sqrt{12}$ (b) $\sqrt{30}$ (c) $\sqrt{88}$ (d) $\sqrt{150}$

16 The local restaurant pays its casual employees $15 per hour. The following table shows the number of hours worked by some of the casual staff. Work out the pay for each of thmese workers and the total of the wages bill.

Employee	Number of hours worked
Jessica	12
Amy	11
Aaron	9
Nikkita	6
Josef	14

17 Replace each * with one of the four operations (+, –, ×, ÷) to make the statement true.

(a) $9 * 7 * 3 = 30$ (b) $16 * 4 \times 2 * 2 = 12$

18 As Jared drove into a carpark, he noticed that the cost of parking was $6 for the first hour, then $3 for every hour after that. Jared's car was parked for 6 hours.

(a) Write down a calculation that Jared could use to work out how much his parking has cost him.

(b) As Jared drives out of the carpark, he gives the attendant $40. Use your answer to part (a) to calculate how much change he will receive.

19 Write these numbers in expanded form and then work out the answer.

(a) $5^3 + 2^2$ (b) $8^4 - 3^5$ (c) $(3^2 - 2^3) \times 6^2$

20 (a) Complete this sequence of the powers of 2:

$2^1 = 2$ $2^4 =$ _____

$2^2 = 4$ $2^5 =$ _____

$2^3 = 8$ $2^6 =$ _____

(b) Will 2^{10} be larger or smaller than 1000? Write the difference.

21 Put these numbers in ascending order: 2^4, $\sqrt{121}$, 10^2, 3^3, 4, $\sqrt{81}$

Reasoning

22 Michelle has $17 in her purse. Michelle's dad empties out the 'loose change' jar and divides the money equally between his 3 children. Michelle now has $24 in her purse. How much loose change was in the jar?

23 The train to Suntown has broken down and 224 passengers are stranded. Carly, the train controller, needs to order some buses for them. Each bus can seat 42 people. How many buses should Carly order? Explain why rounding down to the first digit is not useful in this situation.

24 How many whole numbers have the value of their square root in between 4 and 5?

25 Hamish has a collection of 12 alien figurines. His friend Ben has half this amount. Hamish's brother Sean has three times the amount that Ben has. The difference between the number of aliens in Sean's collection and the number in Hamish's is:

A 6 B 12 C 18 D 24

26 Explain why 3^2 is not equal to 3×2.

27 Will the actual answer to 57×196 be greater or less than an approximate answer calculated by rounding each number to the first digit? Explain how you decided.

NAPLAN practice 1

Numeracy: Non-calculator

1 Another way of writing 7^3 is:

A $7 + 3$ B 7×3

C $7 \times 7 \times 7$ D $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2 What is the best way to estimate the total cost of these items: skateboard, $87; knee pads, $31; helmet $54?

A $80 + $30 + $50

B $80 + $30 + $60

C $90 + $30 + $50

D $90 + $40 + $60

3 The correct answer to $18 + 12 \div (5 + 1)$ is:

A 5 B 6 C 7 D 20

4 The answer to 24×379 would be closest to:

A 1000 B 2000 C 5000 D 10 000

5 Which one of these has the same value as 13×5?

A $10 + 3 + 5$ B $10 \times 3 + 5$ C $10 \times 5 + 3$ D $10 \times 5 + 15$

Numeracy: Calculator allowed

6 'Mini-Munches' chocolate bars can be bought individually for 85c each or in packs of 6 for $5. You need to buy 34 bars for a party. What is the least amount you can pay?

7 Tom is stacking a drinks fridge in the supermarket with cans of cola. The fridge has 3 shelves. On each shelf Tom can place 5 rows of 12 cans. How many cans will fit in the fridge?

8 The area of New Zealand is 268 676 square kilometres. What is this area rounded to the nearest thousand square kilometres?

A 268 700 B 268 000 C 269 000 D 270 000

9 The government of Victoria asked all people in the State to reduce their water usage to less than 155 L per person per day. Which of the following families has not achieved this target?

Family name	Number of people in the family	Total water used by the family in 1 day
Bennett	5	760 L
Shwarma	4	632 L
Nicolai	6	918 L
Zhang	3	459 L