Half-time 5

1 Which rule describes the given information? Ex. 5.3

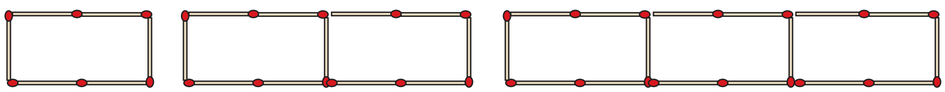
'Subtract 45 from x to get y.'

A $y = x + 45$ B $y = x - 45$ C $y = x \times 45$ D $y = x \div 45$

2 For each of the following rules, evaluate y by substituting the given values of x. Ex. 5.4

(a) $y = x - 4$ when $x = 6$ (b) $y = 2(x - 7)$ when $x = -13$ (c) $y = 54 - 3x$ when $x = -9$

3 If s represents the number of shapes formed and M is the number of matchsticks used, which rule is correct for this pattern? Ex. 5.5



A $M = 5s + 1$ B $M = 6s$ C $M = 5s - 1$ D $M = 4s + 2$

4 Write an equation to describe each of the following. Ex. 5.2

(a) Tim has p footy cards. He collects 7 more. He now has 28 cards.

(b) 20 is obtained when 4 is subtracted from the product of x and 7.

(c) A bird aviary has w parrots and g canaries. When 4 of the canaries are sold, there are 14 birds left in the aviary. How many birds are in the aviary now?

5 If y is equal to 4 plus the quotient of x divided by 2: Ex. 5.3

(a) draw a flowchart for this rule

(b) write the rule using algebra

(c) copy and complete this table of values for the rule.

x	2	8	10	5	13
y					

6 Nicola has 6 identical bags of lollies, plus an extra 11 loose lollies. If n represents the number of lollies in one bag, write an expression for the total number of lollies Nicola has. Ex. 5.1

7 Given the rule $y = 4x - 3$, which statement is not true? Ex. 5.4

A When $x = 6$, $y = 21$. B When $x = 5$, $y = 17$.

C When $x = 4$, $y = 13$. D When $x = 3$, $y = 0$.

8 The cost of an international phone call is 35 cents for every minute, plus a one-off 'flagfall' charge of 50 cents. Ex. 5.4

(a) Write a rule to calculate the total cost of an international phone call. Use the pronumeral C for the cost and m for the length of the call, in minutes.

(b) Use your rule to calculate the cost of a 6-minute phone call. Write your answer in dollars.

(c) Ravi wants to call his parents in Singapore. He has \$8. Will this be enough money for a 20-minute call?

Investigation

Richie's Restaurant

Equipment required: 1 brain

Richie has just bought 30 new rectangular tables for his restaurant. One table can seat six people. He needs to arrange them according to two conditions.

- The tables must be set up so that he can seat at least one group of 6, 8, 10, 12, 14, 16 and 18 people all at the same time.
- He wants to seat the maximum number of people possible.

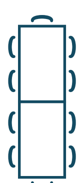
The Big Question

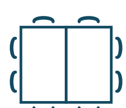
How should Richie arrange his tables to fulfil both of the conditions?

Engage

To seat more than six people, there are two ways in which Richie can join the tables together:

Lengthways or widthways





1 How many people can be seated around two tables if they are joined together:

(a) lengthways **(b)** widthways?

2 For each of the two ways of joining the tables, copy and complete the following table of values. Draw the tables and count the number of people around them if necessary.

Number of joined tables (t)	1	2	3	4	5	6	7
Number of people seated (p)							

Explore

3 If Richie had no restrictions on the way he could set up his tables, what is the maximum number of people he could seat?

4 If Richie now sets up his tables to fulfil the given conditions, draw up a seating plan of Richie's restaurant, showing your arrangement of the 30 tables. How many people can be seated using this arrangement?

Strategy options

- Draw a diagram.
- Guess and check.
- Make a table.
- Look for a pattern.
- Test all possible combinations.

Explain

5 Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient (uses fewer tables)? Is this type of arrangement always more efficient?

6 Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

7 For each of the two arrangements, write the rule that connects the number of tables (t) to the number of people that can be seated (p).

Elaborate

8 Consider the coefficient and the constant that appears in each of your rules. Explain where they come from.

9 How do the two rules show that one type of table arrangement is more efficient than the other?

10 Given what you now know about the efficiency of different table arrangements, modify your plan from Question **4**, if necessary, to seat a greater number of people, while still fulfilling Richie's condition.

11 Draw your improved seating arrangement to answer the Big Question.

Evaluate

12 Consider the way in which you worked on this problem. How did you approach it? Could you have gone about it a different way?

13 Did you use your tables of values (from Question **2**) or the algebra rules to help you solve the problem?

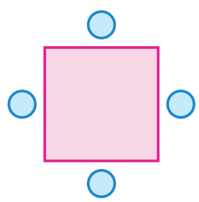
14 How confident are you that your final solution is the best solution?

15 Make a list of all the other factors you would need to consider if you were setting up tables in your own restaurant.

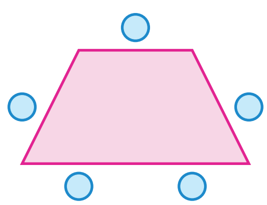
Extend

16 For his next restaurant, Richie would like to use some different-shaped tables. He could use:

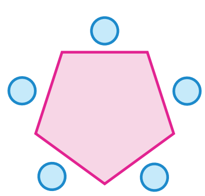
(a) square tables that sit 4 people



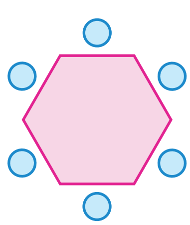
(b) trapezoidal tables that seat 5 people



(c) pentagonal tables that seat 5 people or



(d) hexagonal tables that seat 6 people.



Write a report to explain to Richie why none of these different table shapes would be suitable if he wants to seat the maximum number of people while fulfilling the given conditions and using no more than 30 tables.

5.6 Simplifying expressions with addition and subtraction

Like terms

Like terms:

- have the same pronumeral part
- can have different coefficients; $2x$ and $5x$ are like terms
- can have pronumerals in a different order; ab and ba are like terms
- can have pronumerals raised to exactly the same power; $7x^3$ and $4x^3$ are like terms.

Unlike terms

Unlike terms:

- have different pronumeral parts; $2x$ and $2y$ are unlike terms
- can have the the same pronumerals raised to different powers; x^2 and x are unlike terms ($2^2 \neq 2$).

Because pronumerals represent numbers, we can use number laws in algebra to change the order of pronumerals.

$2 \times 3 = 3 \times 2$, so $ab = ba$ (commutative law)
$(3 \times 2) \times 5 = 3 \times (2 \times 5)$, so $(ab)c = a(bc)$ (associative law)
$abc = acb = bac = bca = cab = cba$

When terms have different pronumeral parts, we usually write the pronumerals in alphabetical order. This helps to identify like terms.

For example, $6abc$, $3bca$, $4dab$, $8cad$, $11adb$, $7cda$ can be written as $6abc$, $3abc$, $4abd$, $8acd$, $11abd$ and $7acd$. We can now see that $6abc$ and $3bca$ are like terms, $4dab$ and $11adb$ are like terms and $8cad$ and $7cda$ are also like terms.

Worked Example 10

WE10

Which of the following are like terms?

(a) $4x$ and $7x$ **(b)** $5ab$ and $9b$ **(c)** x and $12x$

(d) 8 and $8z$ **(e)** h^3 and $6h^3$ **(f)** xy and x^2y

Thinking	Working
(a) Do the terms have exactly the same pronumeral part?	(a) $4x$ and $7x$ are like terms.
(b) Do the terms have exactly the same pronumeral part?	(b) $5ab$ and $9b$ are not like terms.

(c) Do the terms have exactly the same pronumeral part?	(c) x and $12x$ are like terms.
(d) Do the terms have exactly the same pronumeral part?	(d) 8 and $8z$ are not like terms.
(e) Do the terms have exactly the same pronumeral part?	(e) h^3 and $6h^3$ are like terms.
(f) Do the terms have exactly the same pronumeral part?	(f) xy and x^2y are not like terms.

Adding and subtracting like terms

Maddie bought 6 identical packets of party balloons for her birthday party. Each packet contained x balloons, so she bought $6x$ balloons. At home, Maddie found that she had 4 more identical packets, so she added $4x$ balloons to the $6x$ balloons she has just bought. Now, she had 10 packets, all containing x balloons, so she had a total of $10x$ balloons.

$6x$ and $4x$ are like terms. When we add $6x$ and $4x$ we can simplify them to $10x$ by adding their coefficients.

$6x + 4x = 10x$

Maddie didn't need all the balloons, so she took 2 packets back to the store. She returned $2x$ balloons. She now has 8 packets of balloons, so she has $8x$ balloons for her party.

$10x$ and $2x$ are like terms. When we subtract $2x$ from $10x$ we can simplify them to $8x$ by subtracting their coefficients.

$10x - 2x = 8x$

We can simplify an expression by adding or subtracting like terms. This is called 'collecting like terms'. We add or subtract like terms by adding or subtracting their coefficients.

If Maddy buys 3 identical packets of balloons that contain x balloons and 2 different packets that each contain y balloons, we can show that she now has $3x + 2y$ balloons. $3x + 2y$ cannot be simplified any further, as $3x$ and $2y$ are unlike terms.

Examples:

$6c + 9c = 15c$
$12y + 3y - 4y = 11y$
$3x + 4y + 5x = 8x + 4y$
$7ab + 2ba = 9ab$

$8x - 3x = 5x$
$16abc - 13abc = 3abc$
$6x^2 + 4x^2 + 10x^2 = 20x^2$
$2x + 3x^2 - x = x + 3x^2$

$3ab + 4bc$ cannot be simplified, because ab and bc are not like terms.
$2x + 2x^2$ cannot be simplified because x and x^2 are not like terms.

Checking by substitution

We can replace a pronumeral with any number to check that our simplification is correct.

For example: $6x + 4x = 10x$
$10x - 2x = 8x$

for all values of x.

If $x = 20$,
$6 \times 20 + 4 \times 20 = 10 \times 20$
$10 \times 20 - 2 \times 20 = 8 \times 20$

If $x = 7$,
$6 \times 7 + 4 \times 7 = 10 \times 7$
$10 \times 7 - 2 \times 7 = 8 \times 7$

This is an application of the distributive law.

Worked Example 11

WE11

Simplify each expression by collecting like terms.

(a) $4y + 7y$ (b) $2mn - 4mn$ (c) $8a + 4 - 3a$

Thinking	Working
(a) 1 Identify like terms by looking for terms that have exactly the same pronumeral part.	(a) $4y + 7y$
2 Simplify like terms by adding the coefficients of the terms.	$= 11y$
(b) 1 Identify like terms by looking for terms that have exactly the same pronumeral part.	(b) $2mn - 4mn$
2 Simplify like terms by subtracting the coefficients of the terms.	$= -2mn$
(c) 1 Identify the like terms.	(c) $8a + 4 - 3a$
2 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($-3a$).	$= 8a - 3a + 4$
3 Simplify like terms by subtracting the coefficients of the terms.	$= 5a + 4$

Worked Example 12

WE12

Simplify each expression where possible by collecting like terms.

(a) $4x + 3y - 9x + 2y$ (b) $6ab - 4a + 3ab - 7$

Thinking	Working
(a) 1 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($- 9x$).	(a) $4x + 3y - 9x + 2y$ $= 4x - 9x + 3y + 2y$
2 Simplify the like terms by adding or subtracting the coefficients of the terms.	$= -5x + 5y$
(b) 1 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($+ 3ab - 4a$).	(b) $6ab - 4a + 3ab - 7$ $= 6ab + 3ab - 4a - 7$
2 Simplify the like terms by adding the coefficients of the terms.	$= 9ab - 4a - 7$

- Only like terms can be added together or subtracted.
- The operation sign (+ or –) moves with the term that follows it.

5.6 Simplifying expressions with addition and subtraction

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q11	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q11	Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11

Answers page 652

Fluency

1 Which of the following are like terms? WE10

(a) $3k$ and $5k$ **(b)** $4y$ and $5z$ **(c)** $11y$ and $12v$

(d) 17 and 8 **(e)** $6ab$ and $7bc$ **(f)** $2xy$ and $14yx$

(g) $4xyz$ and yzx **(h)** mnp and $3npm$ **(i)** 3, 5 and x

2 Simplify each expression by collecting like terms. WE11

(a) $3x + 4x$ **(b)** $12y + 4y$ **(c)** $11a + 2a$

(d) $13y - 9y$ **(e)** $7m - 3m$ **(f)** $-10x - x$

(g) $3mn + 2nm$ **(h)** $8xy - 3xy$ **(i)** $-2z + 3z + 4$

(j) $x + 4y + 6x$ **(k)** $10w + 3z + 4w$ **(l)** $17a + 2b - 5a$

3 Simplify each expression where possible by collecting like terms. WE12

(a) $3e + 4f + 7e - 6f$ **(b)** $12p + 8q - 4p - 12q$ **(c)** $8t + 15s - 16t - 3s$

(d) $6x - 4y + 3x + y^2$ **(e)** $12w - 2z + 3w - 3z^2$ **(f)** $10m + 9n - n - 5m^2$

(g) $3a + 17 + 4a - 8$ **(h)** $14x - 4 - 6x + 9$ **(i)** $21y - 36 - y - 17$

4 **(a)** A like term for $7x$ is:

A $12x$ **B** $3 + x$ **C** 7 **D** xy

(b) A like term for $6wxy$ is:

A $6w$ **B** $6x$ **C** $6y$ **D** $12xyw$

Understanding

5 Simplify each expression where possible.

(a) $5e + 14f + 8e + 6$ **(b)** $18x + 6y - 4y + 2$ **(c)** $4 + 6x + 5y - 3$

(d) $2x - 3y + 4$ **(e)** $9m - 2n + 3mn$ **(f)** $6a + 11b - 5c - 7$

6 Write each of the following rules using algebra, substitute the numbers 2, 3 and 4 into each rule, then answer TRUE or FALSE for each of these statements.

(a) These two rules are the same:

Rule 1: 'Take any number and multiply it by four. Take the same number and multiply it by six. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

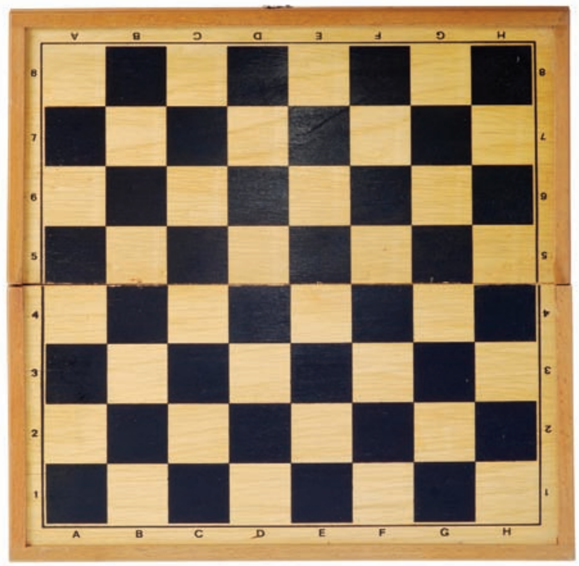
(b) These two rules are the same:

Rule 1: 'Take any number and multiply it by fifteen. Take the same number and multiply it by ten. Subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by five.'

Reasoning

7 A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.



(a) If x is the number of coins on the first square, how many are lying on the:

(i) second square

(ii) third square

(iii) sixth square?

(b) How many coins, in total, are there on the first six squares of the board?

(c) If each coin's value is $\$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins $378.

8 On each of her birthdays, Georgia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write an expression for the amount of money Georgia received on her fourth birthday.

(b) How much money, in total, did Georgia receive for her first four birthdays? Write your answer in terms of d.

(c) How much money did Georgia receive altogether for her first four birthdays if $d = 8$?

(d) If Georgia receives $567 on her fifth birthday, how much did she receive on her first birthday?

9 Kristian opens a bank account by depositing $\$y$. He then deposits $\$x$ every week and makes no withdrawals or other deposits.

The amount in his account after 5 weeks is:

A $x + 5y$ B $y + x + 5$ C $y + 5x$ D $5y + 5x$

Open-ended

10 This algebra pattern consists of the terms $x + y$, $2x + 3y$, $3x + 5y$, $4x + 7y$, … where the coefficient of x increases by 1 each time and the coefficient of y increases by 2 each time.

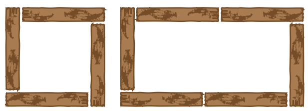
(a) Write an expression for the sixth term.

(b) Write an expression for the sum of the first five terms.

(c) Find two sets of values for x and y so that the sum of the first five terms is a multiple of 10.

(d) Find the sum for your chosen values.

11 A garden bed is made using rectangular timber planks. The planks are arranged to form square and rectangular garden beds as shown in the diagram. Let N be the total number of planks used to make the garden bed, L be the number of planks for the length of the bed and W be the number of planks for the width of the bed.



(a) Write down a rule that will find the number of planks needed for square garden beds of any side length.

(b) Write down a rule that will find the number of planks needed for a rectangular garden bed of length L and width W.

(c) Give examples of each type of garden bed using no more than 30 planks.

(d) If the length must always be double the width, rewrite the rule for the rectangular garden bed using only W as the pronumeral.

(e) Using this new rule, give examples of different types of rectangular garden beds using no more than 30 planks.

Outside the Square Puzzle

Latin squares

a	d	b	c
c	b	d	a
d	a	c	b
b	c	a	d

The sum of each row, column and diagonal is represented by $a + b + c + d$.

If you divide the table into quarters, what is the sum of the four squares in each quarter?

Can you find other 2×2 groups of squares in the table whose sum is $a + b + c + d$?

Draw the two diagonals on the table. What can you say about the remaining rows and columns; that is, the rows and columns that are not on the diagonals?

Draw some 4×4 tables in your exercise book.

Pick four different values for a, b, c and d and make up some Latin squares, using the Latin square above to help you. Make sure that you check that your answers are actually Latin squares.

Leave out some of the values and ask your partner to find the missing numbers to complete your Latin square.

Your complete Latin square might look like:

9	8	14	3
3	14	8	9
8	9	3	14
14	3	9	8

You might then change it to this one before asking your partner to complete the Latin square.

9	8		
3		8	
8		3	
14	3		8

Age matters

Equipment required: 1 brain, 1 computer with an Excel spreadsheet program

Poh, Quentin and Rasheed are three friends who share the same birthday. They are now twelve and a half years old (150 months) and they have decided, after learning some algebra, that they may be able to negotiate a better pocket money agreement with their parents than the amount they get at present. They decide that the agreement they reach with their parents will run for 24 months.

Poh thinks that she will do quite well if she can get her parents to agree to pay her using the following formula.

$P = 2m - 285$ where m is her age in months and P is her monthly payment in dollars.

Quentin thinks he has a better plan. His formula is:

$Q = 3m - 440$ where m is his age in months and Q is his monthly payment in dollars.

Rasheed disagrees with the other two. He believes he has the winning formula.

$R = 4m - 595$ where m is his age in months and R is his monthly payment in dollars.

Versions of this Exploration are available for other technologies in Pearson Reader.

Never use units in a formula. Units (if any) need to be defined in the heading.

1. To see who has the best formula, open an Excel spreadsheet and set up four columns. Use P (\$) to represent Poh's pocket money, Q (\$) to represent Quentin's pocket money and R (\$) to represent Rasheed's pocket money. Enter Age in months (m) in A1, P (\$) in B1, Q (\$) in C1 and R (\$) in D1.
2. Enter 150 in A2 and 151 in A3. Highlight A2 and A3 and drag down to A13 using the small black cross on the bottom right-hand side of A3. Enter formulas in the formula bar for cells B2 (=2*A2 – 285), C2 (=3*A2 – 440) and D2 (=4*A2 – 595) as shown.
3. Total the amount of pocket money Poh receives in the first 12 months by entering the formula shown in cell B14 (=SUM(B2:B13)).

This screen shot shows the formulas you need to enter.

	A	B	C	D
1	Age in months(m)	P($)	Q($)	R($)
2	150	=2*A2-285	=3*A2-440	=4*A2-595
3	151			
4	152			
5	153			
6	154			
7	155			
8	156			
9	157			
10	158			
11	159			
12	160			
13	161			
14		=SUM(B2:B13)		

If you have entered each formula correctly your spreadsheet should look like this.

	A	B	C	D
1	Age in months(m)	P($)	Q($)	R($)
2	150	15	10	5
3	151			
4	152			
5	153			
6	154			
7	155			
8	156			
9	157			
10	158			
11	159			
12	160			
13	161			
14		15		

4 To fill the columns B, C and D, highlight B2 to D2, click on the small black cross on the right bottom corner of D2 and drag the formulas down to D13. Drag the formula shown in B14 across C14 and D14 to total the pocket money each received in 12 months.

Use your spreadsheet to answer the following questions.

5 How much would they each get paid in the first month if their parents agreed to this new arrangement?

6 Is there any time when they would all get the same amount for the month? If so, when does it happen and how much would they receive that month?

7 How much would each of them get on their 13th birthday ($m = 156$)?

8 What is the total amount each of them would receive in the first 12 months of their agreement?

9 Who has received the most pocket money after 12 months?

10 Extend your formulas for another 12 months. Who has received the most pocket money over 24 months? Is your answer the same as in Question **9**?

11 Which student has the best formula? Give reasons for your choice. What do you think each student would say to convince their parents to adopt their formula?

Taking it further

12 Poh's mum suggests a different formula. Her formula reads like this. $M = 2y + 2$ where y is the age in years and M is the pocket money in dollars. She says she will round to the nearest dollar.

(a) Add this formula to your spreadsheet, use the round function to round to the nearest dollar (=round(2*A2/12+2,0) in cell E2) and determine the total amount she would pay in the first 12 months.

	A	B	C	D	E
1	Age in mo	P($)	Q($)	R($)	M($)
2	150	15	10	5	27
3	151				
4	152				
5	153				
6	154				
7	155				
8	156				
9	157				
10	158				
11	159				
12	160				
13	161				
14		15			

(b) Compare the amount of pocket money Poh would earn in the first 12 months using this formula with the other three formulas.

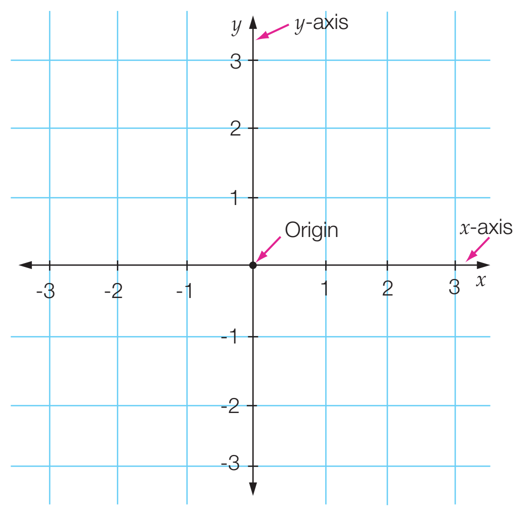
(c) Poh's older sister advises her to be careful before she decides which formula to use. Which formula would you advise her to use over a 2-year period. Justify your recommendation.

5.7 The Cartesian plane

In 1637, the French mathematician René Descartes developed a reference system that allowed any point on a plane to be located accurately. It was called the **Cartesian plane** (a plane is a flat 2-dimensional surface) in his honour, although it is also known as the number plane. This was an exciting idea as now algebra could be used to solve geometrical problems.

A Cartesian plane is constructed by drawing two lines at right angles to each other, one horizontal and the other vertical. The point at which they cross is called the **origin**. The horizontal line is called the x-axis and the vertical line is called the y-axis. Both **axes** (plural of axis) are number lines that extend infinitely in both directions. The integers on the axes are used to form a grid that allows any point to be located in reference to the origin.

The Cartesian plane is more accurate than alphanumeric grid systems used by street directories and spreadsheets, as the Cartesian plane locates points, whereas the alphanumeric system finds an area within a square or cell.



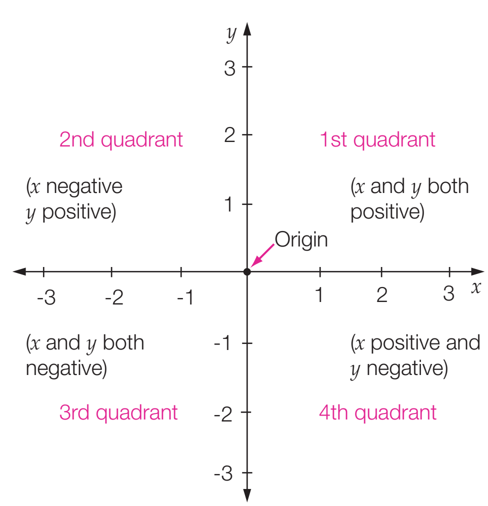
The position of any point on a number plane is described by a pair of numbers called the **coordinates** of the point. Coordinates are always written in brackets as an **ordered pair** (x, y). Locating any point on the plane involves two moves from the origin.

(x, y)

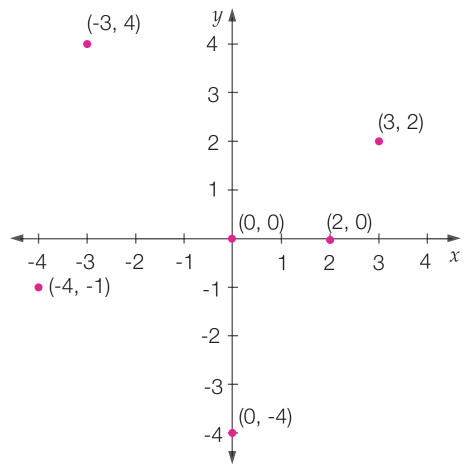
The x-coordinate shows the move to the left (–) or to the right (+) of the origin.

The y-coordinate shows the move up (+) or down (–) from the origin.

The x- and y-axes divide any plane into four **quadrants**. We number the quadrants starting with the quadrant in which both the x- and the y-coordinates have positive values. This is the 1st quadrant. We move in an anticlockwise direction to the 2nd, 3rd and 4th quadrants.



The following points and their Cartesian coordinates are shown on the Cartesian plane at right: (3, 2), (-3, 4), (-4, -1), (0, 0), (2, 0) and (0, -4).



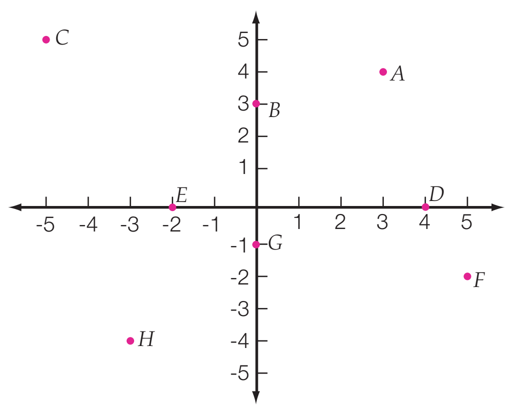
- We always move in the horizontal direction first and then in the vertical direction. This is a mathematical convention.
- The name 'ordered pair' tells you that order is important. The x-coordinate is always written first, and the y-coordinate is written second (x, y).
- The coordinates of the origin are (0, 0).

Worked Example 13

WE13

(a) Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



Thinking	Working
(a) For each point, find the number of units it is to the left or right of the origin. This is the x-coordinate of the point. Then, find the number of units it is up or down from the origin. This is the y-coordinate. The + and - signs indicate direction. (Point A is 3 units to the right of the origin and 4 units up from the origin.) Write the coordinates as an ordered pair.	(a) $A = (3, 4)$ $B = (0, 3)$ $C = (-5, 5)$ $D = (4, 0)$ $E = (-2, 0)$ $F = (5, -2)$ $G = (0, -1)$ $H = (-3, -4)$
(b) Identify the quadrant that each point is in. Points that lie on the x-axis or on the y-axis are not in a quadrant.	(b) A is in the 1st quadrant, C is in the second quadrant, H is in the 3rd quadrant and F is in the 4th quadrant. B, D, E and G lie on an axis, so they are not in any quadrant.

5.7 The Cartesian plane

Navigator

Answers page 653

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

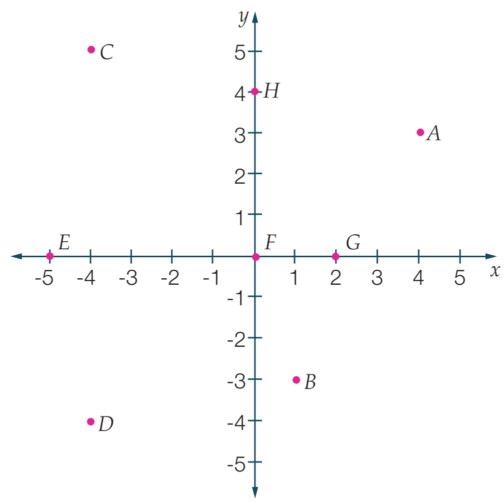
Equipment required: Graph paper for Questions **7**, **8** and **13**

Fluency

WE13

1 **(a)** Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



2 **(a)** State whether each of the following coordinates is in quadrant 1, quadrant 2, quadrant 3 or quadrant 4.

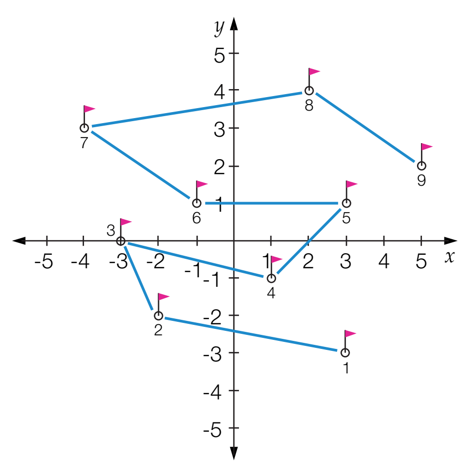
(i) (-2, 6) **(ii)** (5, -1) **(iii)** (-7, -4) **(iv)** (12, 10)

(v) (-32, 12) **(vi)** (51, -1) **(vii)** (-87, -90) **(viii)** (-2, 21)

(b) State whether each coordinate lies on the x-axis, the y-axis, or on both the x-axis and the y-axis.

(i) (0, 5) **(ii)** (0, -8) **(iii)** (0, 0) **(iv)** (-4, 0) **(v)** (15, 0)

3 The layout of a nine-hole golf course is shown on the number plane. Write in order the ordered pairs of the nine holes.



4 **(a)** A point is 1 unit right and 4 units up from the origin of a Cartesian plane. The coordinates of the point are:

A (4, 1) **B** (-4, 1)
C (1, 4) **D** (-1, 4)

(b) A point is 5 units left and 2 units up from the origin of a number plane. The coordinates of the point are:

A (-5, 2) **B** (2, -5) **C** (5, -2) **D** (-2, 5)

(c) A point is 4 units down and 3 units left of the origin of a Cartesian plane. The coordinates of the point are:

A (-4, -3) **B** (-3, -4) **C** (3, -4) **D** (-3, 4)

5 (a) The origin has the coordinates:

A (1, 0) B (0, 1) C (1, 1) D (0, 0)

(b) One coordinate that lies on the x-axis is:

A (0, 4) B (1, 1) C (2, 4) D (1, 0)

(c) One coordinate that lies on the y-axis is:

A (3, 0) B (3, 3) C (0, -3) D (-3, 0)

Understanding

6 (a) Which of the following points will give a vertical line passing through the x-axis when joined in a straight line to (10, -6)?

A (-6, -2) B (4, -6) C (10, 0) D (0, 10)

(b) Which of the following points will give a horizontal line passing through the y-axis when joined in a straight line to (8, 13)?

A (4, 2) B (8, -3) C (-5, 0) D (0, 13)

7 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -9 to 9 on the x-axis and -4 to 4 along the y-axis. Plot the following points and join them in the order given to form a picture. You may like to colour your picture when complete.

(-4, -1) (-5, -1) (-5, 0) (-9, 1) (-6, 3) (-1, 2) (-4, -1) (-4, -2) (-1, -4) (0, -3) (-1, 2) (0, 3) (1, 2) (6, 3) (9, 1) (5, 0) (5, -1) (4, -1) (1, 2) (0, -3) (1, -4) (4, -2) (4, -1) STOP

Now join (3, 4) to (0, 2.5) and join (-3, 4) to (0, 2.5).

8 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -8 to 9 along the x-axis and -11 to 11 along the y-axis. Join each of the following sets of points in the order given. When you reach the word STOP, lift your pencil and start again from the next pair of coordinates.

Join (3, 0) (3, 5) (2, 5) (2, 4) (1, 4) (1, 5) (0, 5) (0, 4) (-1, 4) (-1, 5) (-2, 5) (-2, 4) (-3, 4) (-3, 5) (-4, 5) (-4, -9) (3, -9) (3, 0) (4, 0) (4, -1) (5, -1) (5, 0) (6, 0) (6, -1) (7, -1) (7, 0) (8, 0) (8, -9) (3, -9) STOP

Join (5, 0) (5, 7) ($6\frac{1}{2}$, 10) ($6\frac{1}{2}$, 11) (9, 10) ($6\frac{1}{2}$, 10) (8, 7) (8, 0) STOP

Join (-7, -2) (-7, -9) (-4, -9) (-4, 3) (-5, 3) (-5, 2) (-6, 2) (-6, 3) (-7, 3) (-7, 6) ($-7\frac{1}{2}$, 8) (-8, 6) (-8, -1) (-7, -2) STOP

Join (-2, -6) (-1, -5) (0, -5) (1, -6) (-2, -6) (-2, -9) (-3, -11) (2, -11) (1, -9) (1, -6) STOP

Join (3, 5) (3, 7) ($3\frac{1}{2}$, 9) (4, 7) (4, 4) (3, 1) STOP Join (-4, 5) (-4, 9) (-2, 8) (-4, 8) STOP

Join (6, -5) (7, -5) (7, -2) (6, -2) (6, -5) STOP Join (6, 6) (7, 6) (7, 3) (6, 3) (6, 6) STOP

Join (2, -1) (2, 2) (1, 2) (1, -1) (2, -1) STOP Join (-3, -1) (-2, -1) (-2, -4) (-3, -4) (-3, -1) STOP

Join (-6, -3) (-5, -3) (-5, 1) (-6, 1) (-6, -3) STOP Join (3, 7) to (4, 7) STOP

Join (5, 7) to (8, 7) STOP Join (-8, 6) to (-7, 6) STOP

Join (-2, -6) to (-3, -1) STOP Join (1, -6) to (2, -11) STOP

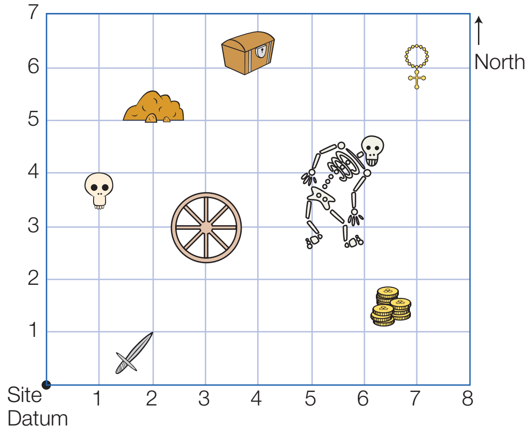
9 (a) Which of the following coordinate pairs lies furthest up on the Cartesian plane?

A (-3, 2) B (2, 3) C (3, -2) D (1, -3)

(b) Which of the following coordinate pairs is the furthest to the left on the Cartesian plane?

A (-3, 2) B (2, 3) C (3, -2) D (1, -3)

10 The grid shown allows an archaeologist to reconstruct the layout of an old tomb. (Archaeologists call the point (0, 0) the datum point.) State the point(s) at which each of the following is found.



(a) the centre of the base of the treasure chest

(b) the middle of the rock face base

(c) the tip of the dagger

(d) the centre of the chariot wheel

(e) the right wrist and the left shoulder of the skeleton

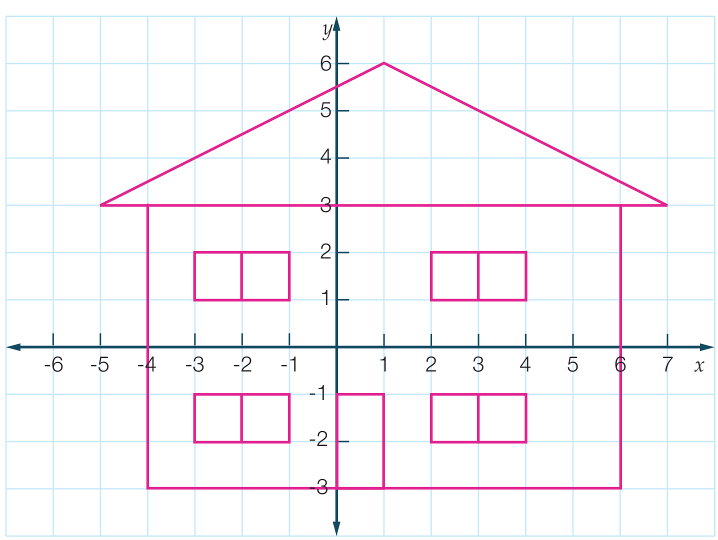
(f) the four grid points that enclose all the coins

(g) the top of the skull lying by itself

(h) the chain at the point where the cross is attached

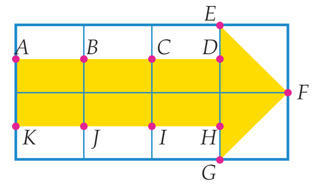
Reasoning

11 Write instructions using coordinates so that this drawing could be reproduced by another student.



Open-ended

12 Draw this arrow shape on a Cartesian grid so that it lies in at least three quadrants, and give the coordinates of each of the points A to K.



13 On a piece of grid or graph paper, rule a set of axes to form a Cartesian plane. Allow for a scale from -5 to +5 on both axes.

(a) Using more than one quadrant on your Cartesian plane, use straight lines to draw a simple design of your own.

(b) Write instructions using coordinates so that your design could be reproduced by another student.

(c) Give your instructions to another student to see if they can draw your design exactly.

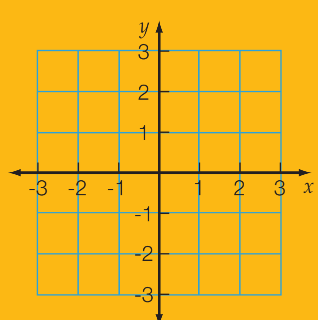
Outside the Square Game

Line up

Equipment required: 2 brains, grid paper, die

How to win:
Five games are played and the player with the greatest total of points is the winner.

How to play:



1. Copy the grid. Take turns to roll the die. The aim is to get three consecutive coordinates in a row, a column or a diagonal.
2. The number on the die is multiplied by 1 and -1 and then split into a sum or difference of two numbers used as the coordinates. For example, if a 3 is rolled, the coordinates might be (0, 3), (3, 0), (0, -3), (-3, 0), (1, 2), (2, -1), (1, -2), (-1, 2), (-1, -2), (-2, -1).

The player can mark any one of these positions.

3. Some combinations cannot be used. For example, if a 6 is rolled, the only combinations that fit on the playing board are (3, 3), (-3, -3), (-3, 3), (3, -3) but not, for example, (-5, 1) or (6, 0).
4. If a player cannot mark a coordinate because it is already marked, they miss a turn.
5. When a player has three in a row, the number of points earned is the total of the numbers, ignoring the minus sign.

For example, for the coordinates (-3, 2), (-2, 2) and (3, 2), the points awarded are
3 + 2 + 2 + 2 + 3 + 2 = 14.

6. Then, clear the board for the next round.
7. Five games are played and the player with the greatest total of points is the winner.