Chapter 4

# Fractions and percentages

## What you will learn



## Australian curriculum

NUMBER AND ALGEBRA

**Real numbers**

Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line (ACMNA152)

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)

Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

Express one quantity as a fraction of another with and without the use of digital technologies (ACMNA155)

Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)

Recognise and solve problems involving simple ratios. (ACMNA173)

**Money and financial mathematics**

Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)


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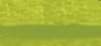
## Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

# Ancient Egyptian fractions

The ancient Egyptians used fractions over 4000 years ago. The Egyptian sky god Horus was a falcon-headed man whose eyes were believed to have magical healing powers. Egyptian jewellery, ornaments or clothing decorated with the Eye of Horus design were regarded as good luck charms for health and magical protection from evil.

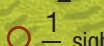
The six parts in the Eye of Horus design represent the six ways that information enters the brain. These six different parts or symbols represented the six fractions used by ancient Egyptian mathematics. For example, instead of writing $\frac{1}{2}$, Egyptians would write  , and instead of writing $\frac{1}{8}$ they would write  .

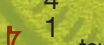
Eye of Horus fraction symbols are found in ancient Egyptian medical prescriptions for mixing 'magical' medicine. Amazingly, modern doctors still use the eye of Horus (  ) symbolism when they write ℞(Rx) at the start of a prescription.

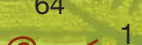
 $\frac{1}{8}$ thought (eyebrow closest to brain)

 $\frac{1}{16}$ hearing (pointing to ear)

 $\frac{1}{2}$ smell (pointing to nose)

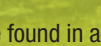
 $\frac{1}{4}$ sight (pupil of the eye)

 $\frac{1}{64}$ touch (leg touching the ground)

 $\frac{1}{32}$ taste (curled top of wheat plant)

A proportion or fraction can be written using a combination of these symbols. For example:

$\frac{3}{4}$ =  and

$\frac{3}{16}$ =  .

Which symbols would represent $\frac{7}{8}$? Can $\frac{1}{3}$ be written using the Eye of Horus symbols?

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# 4A What are fractions? CONSOLIDATING





The word 'fraction' comes from the Latin word 'frangere', which means 'to break into pieces'.

Although the following sentences are not directly related to the mathematical use of fractions, they all contain words that are related to the orginal Latin term 'frangere' and they help us gain an understanding of exactly what a fraction is.

- *The fragile vase smashed into a hundred pieces when it landed on the ground.*
- *After the window was broken, several fragments were found on the floor.*
- *She fractured her leg in two places.*
- *The computer was running slowly and needed to be defragmented.*
- *The elderly gentleman was becoming very frail in his old age.*

Can you think of any other related sentences?


Fragments of a broken object are all fractions of the whole.

Brainstorm specific common uses of fractions in everyday life. The list could include cooking, shopping, sporting, building examples and more.

## Let's start: What strength do you like your cordial?

- Imagine preparing several jugs of different strength cordial. Samples could include $\frac{1}{4}$ strength cordial, $\frac{1}{5}$ strength cordial, $\frac{1}{6}$ strength cordial, $\frac{1}{8}$ strength cordial.
- In each case, describe how much water and how much cordial is needed to make a 1 litre mixture. Note: 1 litre (L) = 1000 millilitres (mL).
- On the label of a Cottee's cordial container, it suggests 'To make up by glass or jug: add five parts water to one part Cottee's Fruit Juice Cordial, according to taste'. What fraction of cordial do Cottee's suggest is the best?

## Key ideas

- A fraction is made up of a numerator (**u**p) and a denominator (**d**own).

  For example: $\frac{3}{5}$ ← numerator, ← denominator
  - The **denominator** tells you how many parts the whole is divided up into.
  - The **numerator** tells you how many of the divided parts you have selected.
  - The horizontal line separating the numerator and the denominator is called the **vinculum**.
- A **proper fraction** or **common fraction** is less than a whole, and therefore the numerator must be smaller than the denominator.

  For example: $\frac{2}{7}$ is a proper fraction.

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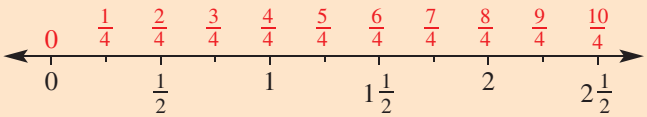
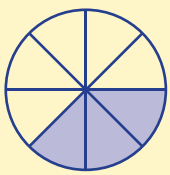
Key ideas

- An **improper fraction** is greater than a whole, and therefore the numerator must be larger than the denominator.
  For example: $\frac{5}{3}$ is an improper fraction.
- We can represent fractions on a number line.
  This number line shows the whole numbers 0, 1 and 2. Each unit has then been divided equally into four segments, therefore creating 'quarters'.




- Whole numbers can be represented as fractions.
  On the number line above we see that 1 is the same as $\frac{4}{4}$ and 2 is the same as $\frac{8}{4}$.
- We can represent fractions using area. If a shape is divided into regions of equal areas, then shading a certain number of these regions will create a fraction of the whole shape.
  For example:  Fraction shaded = $\frac{3}{4}$

## Example 1 Understanding the numerator and the denominator



a Into how many pieces has the whole pizza been divided?
b How many pieces have been selected (shaded)?
c In representing the shaded fraction of the pizza:
  i What must the denominator equal?
  ii What must the numerator equal?
  iii Write the amount of pizza selected (shaded) as a fraction.

| SOLUTION | EXPLANATION |
|---|---|
| a 8 | Pizza cut into 8 equal pieces. |
| b 3 | 3 of the 8 pieces are shaded in blue. |
| c i 8 | Denominator shows the number of parts the whole has been divided into. |
| ii 3 | Numerator tells how many of the divided parts you have selected. |
| iii $\frac{3}{8}$ | Shaded fraction is the numerator over the denominator; i.e. 3 out of 8 divided pieces. |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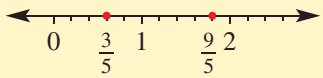
## Example 2 Representing fractions on a number line

Represent the fractions $\frac{3}{5}$ and $\frac{9}{5}$ on a number line.

**SOLUTION**




**EXPLANATION**

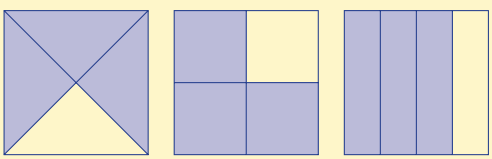
Draw a number line starting at 0 and mark on it the whole numbers 0, 1 and 2.

Divide each whole unit into five segments of equal length. Each of these segments has a length of one-fifth.

## Example 3 Shading areas

Represent the fraction $\frac{3}{4}$ in three different ways, using a square divided into four equal regions.

**SOLUTION**



**EXPLANATION**

Ensure division of square creates four equal areas. Shade in three of the four regions.

## Exercise 4A

1–4 | 4 | —

UNDERSTANDING

**1** **a** State the denominator of this proper fraction: $\frac{2}{9}$.

**b** State the numerator of this improper fraction: $\frac{7}{5}$.

**2** Group the following list of fractions into proper fractions, improper fractions and whole numbers.

**a** $\frac{7}{6}$ **b** $\frac{2}{7}$ **c** $\frac{50}{7}$ **d** $\frac{3}{3}$

**e** $\frac{3}{4}$ **f** $\frac{5}{11}$ **g** $\frac{1}{99}$ **h** $\frac{9}{4}$

**i** $\frac{11}{8}$ **j** $\frac{10}{10}$ **k** $\frac{5}{1}$ **l** $\frac{121}{5}$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4A

UNDERSTANDING

Example 1

**3** Answer the following questions for each of the pizzas (**A** to **D**) drawn below.

**a** Into how many pieces has the whole pizza been divided?

**b** How many pieces have been selected (shaded)?

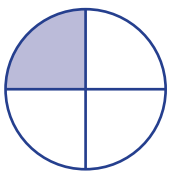
**c** In representing the shaded fraction of the pizza:

**i** What must the denominator equal?

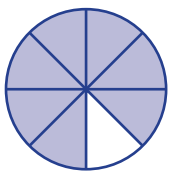
**ii** What must the numerator equal?

**iii** Write the amount of pizza selected (shaded) as a fraction.

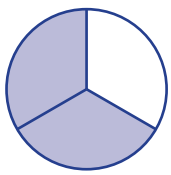
**A**



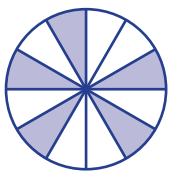
**B**



**C**



**D**



**4** Find the whole numbers amongst the following list of fractions. Hint: There are five whole numbers to find.

**a** $\frac{15}{4}$ **b** $\frac{14}{8}$ **c** $\frac{12}{5}$ **d** $\frac{30}{15}$

**e** $\frac{17}{3}$ **f** $\frac{30}{12}$ **g** $\frac{12}{12}$ **h** $\frac{33}{10}$

**i** $\frac{53}{3}$ **j** $\frac{9}{3}$ **k** $\frac{50}{20}$ **l** $\frac{28}{7}$

**m** $\frac{96}{8}$ **n** $\frac{24}{5}$ **o** $\frac{62}{4}$ **p** $\frac{1031}{2}$

5(½), 6 | 5(½), 6, 7(½) | 5(½), 6, 7(½)

FLUENCY

Example 2

**5** Represent the following fractions on a number line.

**a** $\frac{3}{7}$ and $\frac{6}{7}$ **b** $\frac{2}{3}$ and $\frac{5}{3}$ **c** $\frac{1}{6}$ and $\frac{5}{6}$

**d** $\frac{2}{4}$ and $\frac{11}{4}$ **e** $\frac{11}{5}$ and $\frac{8}{5}$ **f** $\frac{5}{4}$, $\frac{9}{4}$ and $\frac{3}{2}$

Example 3

**6** Represent each of these fractions in three different ways, using a rectangle divided into equal regions.

**a** $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** $\frac{2}{6}$

**7** Write the next three fractions for each of the following fraction sequences.

**a** $\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}$, ___, ___, ___

**b** $\frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$, ___, ___, ___

**c** $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$, ___, ___, ___

**d** $\frac{11}{7}, \frac{10}{7}, \frac{9}{7}, \frac{8}{7}$, ___, ___, ___

**e** $\frac{13}{2}, \frac{11}{2}, \frac{9}{2}, \frac{7}{2}$, ___, ___, ___

**f** $\frac{3}{4}, \frac{8}{4}, \frac{13}{4}, \frac{18}{4}$, ___, ___, ___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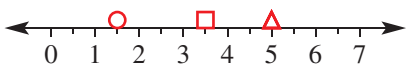
4A

8 | 8, 9 | 9, 10

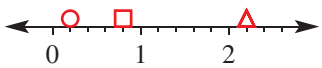
PROBLEM-SOLVING

**8** What fractions correspond to each of the different shapes positioned on these number lines?

**a**

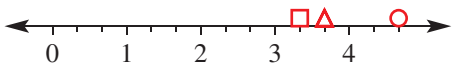



**b**




**c**

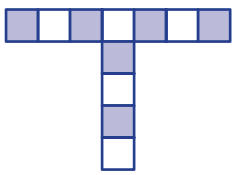

**d**




**9** What operation (i.e. +, –, × or ÷) does the vinculum relate to?

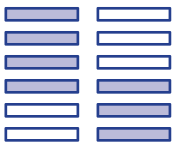
**10** For each of the following, state what fraction of the diagram is shaded.

**a**



**b**

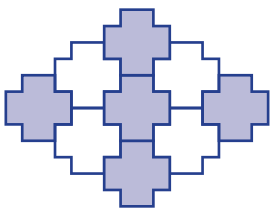
**c**



**d**

**e**

**f**



11 | 11, 12 | 12, 13

REASONING

**11** For each of the following, write the fraction that is describing part of the total.

- **a** After one day of a 43-kilometre hike, they had completed 12 kilometres.
- **b** From 15 starters, 13 went on and finished the race.
- **c** Rainfall for 11 months of the year was below average.
- **d** One egg is broken in a carton that contains a dozen eggs.
- **e** Two players in the soccer team scored a goal.
- **f** The lunch stop was 144 kilometres into the 475-kilometre trip.
- **g** Seven members in the class of 20 have visited Australia Zoo.
- **h** One of the car tyres is worn and needs replacing.
- **i** It rained three days this week.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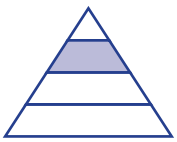
4A

REASONING

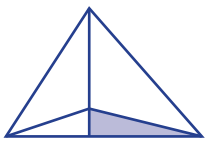
**12** Explain the logic behind the terms 'proper fraction' and 'improper fraction'.

**13** Which diagram has one-quarter shaded?

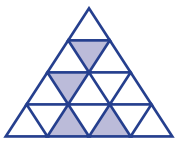
**a**



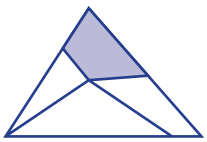
**b**



**c**



**d**



## Adjusting concentration — — 14

ENRICHMENT

**14 a** A 250-millilitre glass of cordial is made by mixing four parts water to one part cordial.

- **i** What fraction of the glass is cordial?
- **ii** What amount of cordial is required?

**b** Fairuz drinks 50 millilitres of the glass and thinks it's 'too strong'. So he fills the glass back up with 50 millilitres of pure water.

- **i** How much cordial is in the glass now?
- **ii** What fraction of the glass is cordial?

**c** Fairuz drinks 50 millilitres of the drink but he still thinks it is 'too strong'. So, once again, he fills the glass back up with 50 millilitres of pure water.

- **i** How much cordial is in the glass now?
- **ii** What fraction of the glass is cordial?

**d** Lynn prefers her cordial much stronger compared with Fairuz. When she is given a glass of the cordial that is mixed at four parts to one, she drinks 50 millilitres and decides it is 'too weak'. So she fills the glass back up with 50 millilitres of straight cordial.

- **i** How much cordial is in Lynn's glass after doing this once?
- **ii** What fraction of the glass is cordial?

**e** Like Fairuz, Lynn needs to repeat the process to make her cordial even stronger. So, once again, she drinks 50 millilitres and then tops the glass back up with 50 millilitres of straight cordial.

- **i** How much cordial is in Lynn's glass now?
- **ii** What fraction of the glass is cordial?

**f** If Fairuz continues diluting his cordial concentration in this manner and Lynn continues strengthening her cordial concentration in this manner, will either of them ever reach pure water or pure cordial? Explain your reasoning.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# 4B Equivalent fractions and simplified fractions

Interactive

Widgets

HOTsheets

Walkthroughs

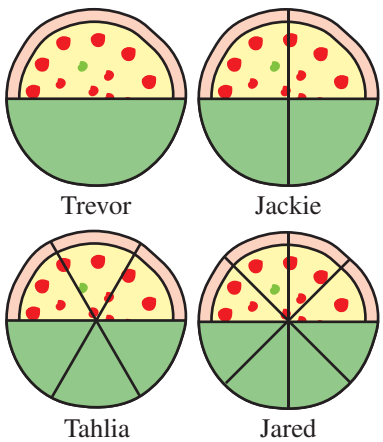
Often fractions may look very different when in fact they have the same value.

For example, in an AFL football match, 'half-time' is the same as 'the end of the second quarter'. We can say that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions. In both situations, the equivalent fraction of the game has been completed.

There are four quarters played out in a game of AFL football.

Consider a group of friends eating pizzas during a sleepover. The pizzas are homemade and each person cuts up their pizza as they like.

Trevor cuts his pizza into only two pieces, Jackie cuts hers into four pieces, Tahlia into six pieces and Jared into eight pieces. The shaded pieces are the amount that they have eaten before it is time to start the second movie.




By looking at the pizzas, it is clear to see that Trevor, Jackie, Tahlia and Jared have all eaten the same amount of pizza. We can therefore conclude that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions.

This is written as $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

## Let's start: Fraction clumps

- Prepare a class set of fraction cards. (Two example sets are provided below.)
- Hand out one fraction card to each student.
- Students then arrange themselves into groups of equivalent fractions.
- Set an appropriate time goal by which this task must be completed.
- Repeat the process with a second set of equivalent fraction cards.

**Sample sets of fraction cards**

*Class set 1*

$\frac{1}{2}, \frac{3}{12}, \frac{3}{24}, \frac{10}{80}, \frac{1}{3}, \frac{8}{40}, \frac{1}{5}, \frac{3}{6}, \frac{1}{8}, \frac{5}{40}, \frac{3}{9}, \frac{1}{4}, \frac{1000}{4000}, \frac{100}{200}, \frac{10}{50}, \frac{2}{16}, \frac{10}{30}, \frac{13}{39}, \frac{5}{10}, \frac{7}{14}, \frac{2}{6}, \frac{7}{28}, \frac{2}{10}, \frac{4}{20}, \frac{2}{8}$

*Class set 2*

$\frac{2}{3}, \frac{6}{14}, \frac{3}{18}, \frac{4}{10}, \frac{2}{12}, \frac{24}{64}, \frac{11}{66}, \frac{4}{6}, \frac{3}{7}, \frac{30}{70}, \frac{12}{32}, \frac{3}{8}, \frac{10}{15}, \frac{5}{30}, \frac{1}{6}, \frac{2000}{5000}, \frac{21}{49}, \frac{300}{800}, \frac{6}{9}, \frac{9}{21}, \frac{2}{5}, \frac{14}{35}, \frac{20}{30}, \frac{6}{16}, \frac{22}{55}$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Key ideas

- **Equivalent fractions** are fractions that mark the same place on a number line.
  For example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.
- Equivalent fractions are produced by multiplying the numerator and denominator by the same number. This number can be any whole number greater than 1.
- Equivalent fractions can also be produced by dividing the numerator and denominator by the same number.
- **Simplifying fractions** involves writing a fraction in its 'simplest form' or 'easiest form' or 'most convenient form'. To do this, the numerator and the denominator must be divided by their **highest common factor (HCF)**.
- It is a mathematical convention to write all answers involving fractions in their simplest form.

## Example 4 Producing equivalent fractions

Write four equivalent fractions for $\frac{2}{3}$.

**SOLUTION**

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

Many other fractions are also possible.

Other common choices include:

$\frac{20}{30}, \frac{200}{300}, \frac{2000}{3000}, \frac{40}{60}$

**EXPLANATION**

$\frac{2}{3} \overset{\times 2}{=} \frac{4}{6} \overset{\times 3}{=} \frac{6}{9} \overset{\times 4}{=} \frac{8}{12} \overset{\times 5}{=} \frac{10}{15}$

## Example 5 Checking for equivalence

By writing either = or ≠ between the fractions, state whether the following pairs of fractions are equivalent or not equivalent.

a $\frac{1}{3} \quad \frac{3}{7}$

b $\frac{4}{5} \quad \frac{20}{25}$

**SOLUTION**

a $\frac{1}{3} \neq \frac{3}{7}$

b $\frac{4}{5} = \frac{20}{25}$

**EXPLANATION**

Convert to a common denominator.

$\frac{1}{3} = \frac{1 \times 7}{3 \times 7} = \frac{7}{21}$ and $\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$, $\frac{7}{21} \neq \frac{9}{21}$

$\frac{4}{5} = \frac{4 \times 5}{5 \times 5} = \frac{20}{25}$


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## Example 6 Converting to simplest form

Write these fractions in simplest form.

**a** $\frac{12}{20}$ **b** $\frac{7}{42}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{12}{20} = \frac{3 \times \cancel{4}}{5 \times \cancel{4}} = \frac{3}{5}$ | The HCF of 12 and 20 is 4.<br>Both the numerator and the denominator are divided by the HCF of 4. |
| **b** $\frac{7}{42} = \frac{\cancel{7} \times 1}{\cancel{7} \times 6} = \frac{1}{6}$ | The HCF of 7 and 42 is 7.<br>The 7 is 'cancelled' from the numerator and the denominator. |

## Exercise 4B

1–5 | 5 | —

UNDERSTANDING

**1** Which of the following fractions are equivalent to $\frac{1}{2}$?

$\frac{3}{5}, \frac{3}{6}, \frac{3}{10}, \frac{2}{4}, \frac{11}{22}, \frac{7}{15}, \frac{8}{12}, \frac{2}{1}, \frac{5}{10}, \frac{6}{10}$

**2** Which of the following fractions are equivalent to $\frac{8}{20}$?

$\frac{4}{10}, \frac{1}{5}, \frac{6}{20}, \frac{8}{10}, \frac{16}{40}, \frac{2}{5}, \frac{4}{12}, \frac{12}{40}, \frac{80}{200}, \frac{1}{4}$

**3** Fill in the missing numbers to complete the following strings of equivalent fractions.

**a** $\frac{1}{3} = \frac{\square}{6} = \frac{4}{\square} = \frac{\square}{30} = \frac{\square}{60} = \frac{100}{\square}$

**b** $\frac{2}{8} = \frac{\square}{4} = \frac{\square}{12} = \frac{6}{\square} = \frac{\square}{80} = \frac{10}{\square}$

**4** In the following lists of equivalent fractions, circle the fraction that is in its simplest form.

**a** $\frac{3}{15}, \frac{10}{50}, \frac{2}{10}, \frac{1}{5}$ **b** $\frac{100}{600}, \frac{3}{18}, \frac{1}{6}, \frac{7}{42}$

**c** $\frac{4}{6}, \frac{2}{3}, \frac{16}{24}, \frac{20}{30}$ **d** $\frac{9}{12}, \frac{15}{20}, \frac{6}{8}, \frac{3}{4}$


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4B

UNDERSTANDING

**5** Fill in the gaps to reduce these fractions to their simplest form.

**a** $\frac{10}{30}$ **i** HCF = ☐ **ii** $\frac{10}{30} = \frac{1 \times \square}{3 \times \square}$. Therefore, simplest form is $\frac{\square}{3}$.

**b** $\frac{4}{18}$ **i** HCF = ☐ **ii** $\frac{4}{18} = \frac{2 \times \square}{9 \times \square}$. Therefore, simplest form is $\frac{\square}{9}$.

**c** $\frac{4}{28}$ **i** HCF = ☐ **ii** $\frac{4}{28} = \frac{1 \times \square}{7 \times \square}$. Therefore, simplest form is $\frac{1}{\square}$.

**d** $\frac{9}{15}$ **i** HCF = ☐ **ii** $\frac{9}{15} = \frac{3 \times \square}{5 \times \square}$. Therefore, simplest form is $\frac{\square}{\square}$.

6–9(½) 6–10(½) 6–10(½)

FLUENCY

Example 4

**6** Write four equivalent fractions for each of the fractions listed.

**a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{2}{5}$ **d** $\frac{3}{5}$

**e** $\frac{2}{9}$ **f** $\frac{3}{7}$ **g** $\frac{5}{12}$ **h** $\frac{3}{11}$

**7** Find the unknown value to make the equation true.

**a** $\frac{3}{4} = \frac{?}{12}$ **b** $\frac{5}{8} = \frac{?}{80}$ **c** $\frac{6}{11} = \frac{18}{?}$ **d** $\frac{2}{7} = \frac{16}{?}$

**e** $\frac{3}{?} = \frac{15}{40}$ **f** $\frac{?}{1} = \frac{14}{7}$ **g** $\frac{?}{10} = \frac{24}{20}$ **h** $\frac{13}{14} = \frac{?}{42}$

**i** $\frac{2}{7} = \frac{10}{?}$ **j** $\frac{19}{20} = \frac{190}{?}$ **k** $\frac{11}{21} = \frac{55}{?}$ **l** $\frac{11}{?} = \frac{44}{8}$

Example 5

**8** Write either = or ≠ between the fractions, to state whether the following pairs of fractions are equivalent or not equivalent.

**a** $\frac{1}{2}$ $\frac{5}{8}$ **b** $\frac{4}{8}$ $\frac{2}{4}$ **c** $\frac{3}{7}$ $\frac{30}{60}$

**d** $\frac{5}{9}$ $\frac{15}{18}$ **e** $\frac{11}{15}$ $\frac{33}{45}$ **f** $\frac{1}{2}$ $\frac{402}{804}$

**g** $\frac{12}{36}$ $\frac{1}{3}$ **h** $\frac{18}{24}$ $\frac{21}{28}$ **i** $\frac{6}{18}$ $\frac{11}{33}$

Example 6

**9** Write the following fractions in simplest form.

**a** $\frac{15}{20}$ **b** $\frac{12}{18}$ **c** $\frac{10}{30}$ **d** $\frac{8}{22}$

**e** $\frac{14}{35}$ **f** $\frac{2}{22}$ **g** $\frac{8}{56}$ **h** $\frac{9}{27}$

**i** $\frac{35}{45}$ **j** $\frac{36}{96}$ **k** $\frac{120}{144}$ **l** $\frac{700}{140}$

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## 4B

FLUENCY

**10** These lists of fractions are meant to contain only fractions in their simplest form; however, there is one mistake in each list. Find the fraction that is not in simplest form and rewrite it in its simplest form.

**a** $\frac{1}{3}, \frac{3}{8}, \frac{5}{9}, \frac{7}{14}$

**b** $\frac{2}{5}, \frac{12}{16}, \frac{16}{9}, \frac{13}{37}$

**c** $\frac{12}{19}, \frac{4}{42}, \frac{5}{24}, \frac{6}{61}$

**d** $\frac{7}{63}, \frac{9}{62}, \frac{11}{81}, \frac{13}{72}$

11 | 11, 12 | 12

PROBLEM-SOLVING

**11** A family block of chocolate consists of 12 rows of 6 individual squares. Tania eats 16 individual squares. What fraction of the block, in simplest terms, has Tania eaten?

**12** Four people win a competition that allows them to receive $\frac{1}{2}$ a tank of free petrol. Find how many litres of petrol the drivers of these cars receive.

**a** Ford Territory with a 70-litre tank

**b** Nissan Patrol with a 90-litre tank

**c** Holden Commodore with a 60-litre tank

**d** Mazda 323 with a 48-litre tank

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4B

13 | 13, 14 | 14, 15

REASONING

**13** Justin, Joanna and Jack are sharing a large pizza for dinner. The pizza has been cut into 12 equal pieces. Justin would like $\frac{1}{3}$ of the pizza, Joanna would like $\frac{1}{4}$ of the pizza and Jack will eat whatever is remaining. By considering equivalent fractions, determine how many slices each person gets served.

**14** J.K. Rowling's first book, *Harry Potter and the Philosopher's Stone*, is 225 pages long. Sam plans to read the book in three days, reading the same number of pages each day.

**a** How many pages should Sam read each day?

**b** The fraction $\frac{75}{225}$ of the book is equivalent to what fraction in simplest form?

By the end of the second day, Sam is on track and has read $\frac{2}{3}$ of the book.

**c** How many pages of the book is $\frac{2}{3}$ equivalent to?

**15** A fraction when simplified is written as $\frac{3}{5}$. What could the fraction have been before it was simplified?

## Equivalent bars of music

— | — | 16

ENRICHMENT

**16** Each piece of music has a time signature. A common time signature is called $\frac{4}{4}$ time, and is actually referred to as Common time!

Common time, or $\frac{4}{4}$ time, means that there are four 'quarter notes' (or crotchets) in each bar.

Listed below are the five most commonly used musical notes.

 – whole note (fills the whole bar) – semibreve

 – half note (fills half the bar) – minim

 – quarter note (four of these to a bar) – crotchet

 – eighth note (eight to a bar) – quaver

 – sixteenth note (sixteen to a bar) – semi-quaver

**a** Write six different 'bars' of music in $\frac{4}{4}$ time.

Carry out some research on other types of musical time signatures.

**b** Do you know what the time signature $\frac{12}{8}$ means?

**c** Write three different bars of music for a $\frac{12}{8}$ time signature.

**d** What are the musical symbols for different length rests?

**e** How does a dot (or dots) written after a note affect the length of the note?


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# 4C Mixed numbers CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

As we have seen in this chapter, a fraction is a common way of representing part of a whole number. For example, a particular car trip may require $\frac{2}{3}$ of a tank of petrol.

On many occasions, you may need whole numbers plus a part of a whole number. For example, a long interstate car trip may require $2\frac{1}{4}$ tanks of petrol.

When you have a combination of a whole number and a fraction this number is known as a **mixed number**.

A long car trip may require a full tank of petrol and another fraction of a tank as well.

## Let's start: Pizza frenzy

With a partner, attempt to solve the following pizza problem. There is more than one answer.

At Pete's pizza shop, small pizzas are cut into four equal slices, medium pizzas are cut into six equal slices and large pizzas are cut into eight equal slices.

For a class party, the teacher ordered 13 pizzas, which the students ate with ease. After the last slice was eaten, a total of 82 slices of pizza had been eaten by the students. How many pizzas of each size did the teacher order?

## Key ideas

- A number is said to be a **mixed number** when it is a mix of a whole number plus a proper fraction.

$2\frac{3}{5}$ is a mixed number

whole number: $2$ — proper fraction: $\frac{3}{5}$

- **Improper fractions** (fractions greater than a whole, where the numerator is greater than the denominator) can be converted to mixed numbers or whole numbers.

$\frac{15}{4} = 3\frac{3}{4}$ (improper fraction → mixed number)

$\frac{16}{4} = 4$ (improper fraction → whole number)


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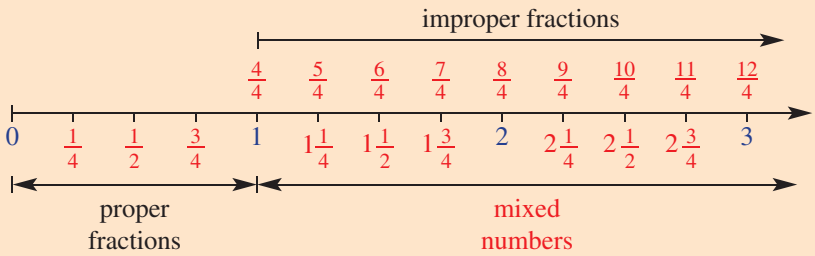
Key ideas

- Mixed numbers can be converted to improper fractions.
- In general, improper fractions should be written as mixed numbers, with the fraction part written in simplest form.
- A number line helps show the different types of fractions.




## Example 7 Converting mixed numbers to improper fractions

Convert $3\frac{1}{5}$ to an improper fraction.

**SOLUTION**

$3\frac{1}{5} = 1 + 1 + 1 + \frac{1}{5}$

$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$

$= \frac{16}{5}$

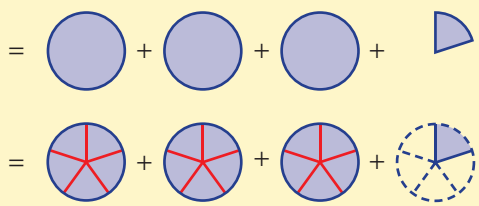
or

$3\frac{1}{5} = \frac{15}{5} + \frac{1}{5}$

$= \frac{16}{5}$

**EXPLANATION**

$3\frac{1}{5}$ = 3 wholes + $\frac{1}{5}$ of a whole



Short-cut method:

Multiply the whole number part by the denominator and then add the numerator.

$3 \times 5 + 1 = 16$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## Example 8 Converting improper fractions to mixed numbers

Convert $\frac{11}{4}$ to a mixed number.

**SOLUTION**

**Method 1**

$$\frac{11}{4} = \frac{8+3}{4} = \frac{8}{4} + \frac{3}{4} = 2 + \frac{3}{4} = 2\frac{3}{4}$$

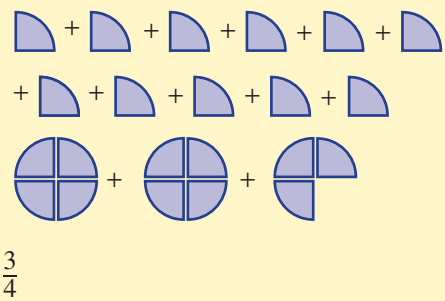
**Method 2**

$$4\overline{)11} \quad 2 \text{ rem } 3$$

So $\frac{11}{4} = 2\frac{3}{4}$

**EXPLANATION**

$\frac{11}{4}$ = 11 quarters



$= 2\frac{3}{4}$

## Example 9 Writing mixed numbers in simplest form

Convert $\frac{20}{6}$ to a mixed number in simplest form.

**SOLUTION**

$$\frac{20}{6} = 3\frac{2}{6} = 3\frac{1\times\cancel{2}}{3\times\cancel{2}} = 3\frac{1}{3}$$

or

$$\frac{20}{6} = \frac{10\times\cancel{2}}{3\times\cancel{2}} = \frac{10}{3} = 3\frac{1}{3}$$

**EXPLANATION**

Method 1: Convert to mixed number and then simplify the fraction part.

Method 2: Simplify the improper fraction first and then convert to a mixed number.

Each pane of glass is $\frac{1}{12}$ of the whole window.

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## Exercise 4C

1–5 | 5 | —

UNDERSTANDING

**1** Between which two whole numbers do the following mixed numbers lie?

**a** $2\frac{1}{2}$ **b** $11\frac{1}{7}$ **c** $36\frac{8}{9}$

**2** Work out the total number of pieces in each of these situations.

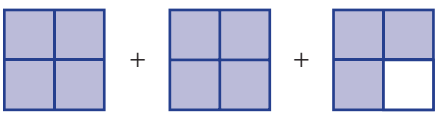
**a** four pizzas cut into six pieces each

**b** 10 Lego trucks, where each truck is made from 36 Lego pieces

**c** five jigsaw puzzles with 12 pieces in each puzzle

**d** three cakes cut into eight pieces each

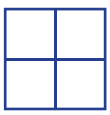
**3** The mixed number $2\frac{3}{4}$ can be represented in 'window shapes' as

$2\frac{3}{4} =$  $+ \quad +$

Represent the following mixed numbers using 'window shapes'.

**a** $1\frac{1}{4}$ **b** $1\frac{3}{4}$ **c** $3\frac{2}{4}$ **d** $5\frac{2}{4}$

**4** A 'window shape' consists of four panes of glass. How many panes of glass are there in the following number of 'window shapes'?

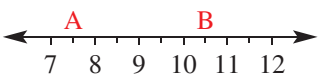


**a** 2 **b** 3 **c** 7 **d** 11

**e** $4\frac{1}{4}$ **f** $1\frac{3}{4}$ **g** $2\frac{2}{4}$ **h** $5\frac{4}{4}$

**5** What mixed numbers correspond to the letters written on each number line?

**a**




**b**


**c**


6–8(½) | 6–8(½) | 6–8(½)

FLUENCY

Example 7

**6** Convert these mixed numbers to improper fractions.

**a** $2\frac{1}{5}$ **b** $1\frac{3}{5}$ **c** $3\frac{1}{3}$ **d** $5\frac{2}{3}$ **e** $4\frac{1}{7}$ **f** $3\frac{3}{7}$

**g** $2\frac{1}{2}$ **h** $6\frac{1}{2}$ **i** $4\frac{2}{3}$ **j** $11\frac{1}{3}$ **k** $8\frac{2}{5}$ **l** $10\frac{3}{10}$

**m** $6\frac{1}{9}$ **n** $2\frac{7}{9}$ **o** $5\frac{2}{8}$ **p** $2\frac{5}{8}$ **q** $1\frac{11}{12}$ **r** $3\frac{5}{11}$

**s** $4\frac{5}{12}$ **t** $9\frac{7}{12}$ **u** $5\frac{15}{20}$ **v** $8\frac{3}{100}$ **w** $64\frac{3}{10}$ **x** $20\frac{4}{5}$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## 4C

FLUENCY

Example 8

**7** Convert these improper fractions to mixed numbers.

**a** $\frac{7}{5}$ **b** $\frac{4}{3}$ **c** $\frac{5}{3}$ **d** $\frac{7}{4}$

**e** $\frac{11}{3}$ **f** $\frac{21}{5}$ **g** $\frac{16}{7}$ **h** $\frac{10}{4}$

**i** $\frac{12}{7}$ **j** $\frac{19}{6}$ **k** $\frac{20}{3}$ **l** $\frac{41}{4}$

**m** $\frac{35}{8}$ **n** $\frac{26}{5}$ **o** $\frac{48}{7}$ **p** $\frac{41}{3}$

**q** $\frac{37}{12}$ **r** $\frac{81}{11}$ **s** $\frac{93}{10}$ **t** $\frac{78}{7}$

**u** $\frac{231}{100}$ **v** $\frac{333}{10}$ **w** $\frac{135}{11}$ **x** $\frac{149}{12}$

Example 9

**8** Convert these improper fractions to mixed numbers in their simplest form.

**a** $\frac{10}{4}$ **b** $\frac{28}{10}$ **c** $\frac{16}{12}$ **d** $\frac{8}{6}$

**e** $\frac{18}{16}$ **f** $\frac{30}{9}$ **g** $\frac{40}{15}$ **h** $\frac{60}{25}$

9 | 9, 10 | 10, 11

PROBLEM-SOLVING

**9** Draw a number line from 0 to 5 and mark on it the following fractions.

**a** $\frac{2}{3}, 2, \frac{5}{3}, 3\frac{1}{3}$ **b** $\frac{3}{4}, \frac{12}{4}, 2\frac{1}{4}, 3\frac{1}{2}$ **c** $\frac{4}{5}, \frac{14}{5}, 3\frac{1}{5}, \frac{10}{5}, \frac{19}{5}$

**10** Fill in the gaps for the following number patterns.

**a** $1\frac{1}{3}, 1\frac{2}{3}, 2, \_\_\_, 2\frac{2}{3}, 3, 3\frac{1}{3}, \_\_\_, \_\_\_, 4\frac{1}{3}, 4\frac{2}{3}, 5$

**b** $\frac{3}{7}, \frac{5}{7}, 1, 1\frac{2}{7}, \_\_\_, 1\frac{6}{7}, \_\_\_, 2\frac{3}{7}, 2\frac{5}{7}, \_\_\_, 3\frac{2}{7}, \_\_\_, \_\_\_$

**c** $\frac{3}{5}, 1\frac{1}{5}, 1\frac{4}{5}, \_\_\_, 3, 3\frac{3}{5}, \_\_\_, \_\_\_, 5\frac{2}{5}, \_\_\_, 6\frac{3}{5}, \_\_\_$

**11** Four friends order three large pizzas for their dinner. Each pizza is cut into eight equal slices. Simone has three slices, Izabella has four slices, Mark has five slices and Alex has three slices.

**a** How many pizza slices do they eat in total?

**b** How many pizzas do they eat in total? Give your answer as a mixed number.

**c** How many pizza slices are left uneaten?

**d** How many pizzas are left uneaten? Give your answer as a mixed number.


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4C

12 12 12

REASONING

**12** **a** Patricia has three sandwiches that are cut into quarters and she eats all but one-quarter. How many quarters does she eat?

**b** Phillip has five sandwiches that are cut into halves and he eats all but one-half. How many halves does he eat?

**c** Crystal has $x$ sandwiches that are cut into quarters and she eats them all but one-quarter. How many quarters does she eat?

**d** Byron has $y$ sandwiches that are cut into thirds and he eats all but one-third. How many thirds does he eat?

**e** Felicity has $m$ sandwiches that are cut into $n$ pieces and she eats them all. How many pieces does she eat?

## Mixed number swap meet

— — 13

ENRICHMENT

**13** **a** Using the digits 1, 2 and 3 only once, three different mixed numbers can be written.

**i** Write down the three possible mixed numbers.

**ii** Find the difference between the smallest and highest mixed numbers.

**b** Repeat part **a** using the digits 2, 3 and 4.

**c** Repeat part **a** using the digits 3, 4 and 5.

**d** Predict the difference between the largest and smallest mixed number when using only the digits 4, 5 and 6. Check to see if your prediction is correct.

**e** Write down a rule for the difference between the largest and smallest mixed numbers when using any three consecutive integers.

**f** Extend your investigation to allow mixed numbers where the fraction part is an improper fraction.

**g** Extend your investigation to produce mixed numbers from four consecutive digits.


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# 4D Ordering fractions





You already know how to order a set of whole numbers.

For example: 3, 7, 15, 6, 2, 10 are a set of six different whole numbers that you could place in ascending or descending order.

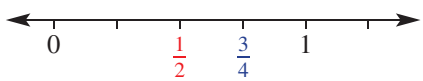
In ascending order, the correct order is: 2, 3, 6, 7, 10, 15.

In descending order, the correct order is: 15, 10, 7, 6, 3, 2.

In this section you will learn how to write different fractions in ascending and descending order. To be able to do this we need to compare different fractions and we do this through our knowledge of equivalent fractions (see Section 4B).

Remember a fraction is greater than another fraction if it lies to the right of that fraction on a number line.

$\frac{3}{4} > \frac{1}{2}$




## Let's start: The order of five

- As a warm-up activity, ask five volunteer students to arrange themselves in alphabetical order, then in height order and, finally, in birthday order.
- Each of the five students receives a large fraction card and displays it to the class.
- The rest of the class must then attempt to order the students in ascending order, according to their fraction card. It is a group decision and none of the five students should move until the class agrees on a decision.
- Repeat the activity with a set of more challenging fraction cards.

## Key ideas

- To **order** (or arrange) fractions we must know how to compare different fractions. This is often done by considering equivalent fractions.
- If the numerators are the same, the smallest fraction is the one with the biggest denominator, as it has been divided up into the most pieces.

  For example: $\frac{1}{7} < \frac{1}{2}$
- If the denominators are the same, the smallest fraction is the one with the smallest numerator.

  For example: $\frac{3}{10} < \frac{7}{10}$

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Key ideas

- To order two fractions with different numerators and denominators, we can use our knowledge of equivalent fractions to produce fractions with a common denominator and then compare the numerators.
- The **lowest common denominator (LCD)** is the lowest common multiple of the different denominators.
- **Ascending** order is when numbers are ordered going *up*, from smallest to largest.
- **Descending** order is when numbers are ordered going *down*, from largest to smallest.

## Example 10 Comparing fractions

Place the correct mathematical symbol <, = or >, in between the following pairs of fractions to make true mathematical statements.

a $\frac{2}{5} \square \frac{4}{5}$

b $\frac{1}{3} \square \frac{1}{5}$

c $\frac{2}{3} \square \frac{3}{5}$

d $2\frac{3}{7} \square \frac{16}{7}$

| SOLUTION | EXPLANATION |
|---|---|
| a $\frac{2}{5} \boxed{<} \frac{4}{5}$ | Denominators are the same, therefore compare numerators. |
| b $\frac{1}{3} \boxed{>} \frac{1}{5}$ | Numerators are the same.<br>Smallest fraction has the biggest denominator. |
| c $\frac{2}{3} \square \frac{3}{5}$ | LCD of 3 and 5 is 15.<br>Produce equivalent fractions. |
| $\frac{10}{15} \boxed{>} \frac{9}{15}$. Hence, $\frac{2}{3} \boxed{>} \frac{3}{5}$. | Denominators now the same, therefore compare numerators. |
| d $2\frac{3}{7} \square \frac{16}{7}$ | Convert mixed number to an improper fraction. |
| $\frac{17}{7} \boxed{>} \frac{16}{7}$. Hence, $2\frac{3}{7} \boxed{>} \frac{16}{7}$. | Denominators are the same, therefore compare numerators. |


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## Example 11 Ordering fractions

Place the following fractions in ascending order.

**a** $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$

**b** $1\frac{3}{5}, \frac{7}{4}, \frac{3}{2}, 2\frac{1}{4}, \frac{11}{5}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{45}{60}, \frac{48}{60}, \frac{40}{60}$ | LCD of 3, 4 and 5 is 60. Produce equivalent fractions with denominator of 60. |
| $\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$ | Order fractions in ascending order. |
| $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ | Rewrite fractions back in original form. |
| **b** $\frac{8}{5}, \frac{7}{4}, \frac{3}{2}, \frac{9}{4}, \frac{11}{5}$ | Express all fractions as improper fractions. |
| $\frac{32}{20}, \frac{35}{20}, \frac{30}{20}, \frac{45}{20}, \frac{44}{20}$ | LCD of 2, 4 and 5 is 20. Produce equivalent fractions with a denominator of 20. |
| $\frac{30}{20}, \frac{32}{20}, \frac{35}{20}, \frac{44}{20}, \frac{45}{20}$ | Order fractions in ascending order. |
| $\frac{3}{2}, 1\frac{3}{5}, \frac{7}{4}, \frac{11}{5}, 2\frac{1}{4}$ | Rewrite fractions back in original form. |

## Exercise 4D

1–4(½) | 3–4(½) | —

UNDERSTANDING

**1** Circle the largest fraction in each of the following lists.

**a** $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$

**b** $\frac{4}{3}, \frac{2}{3}, \frac{7}{3}, \frac{5}{3}$

**c** $\frac{5}{11}, \frac{9}{11}, \frac{3}{11}, \frac{4}{11}$

**d** $\frac{8}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}$

**2** State the lowest common multiple of the following sets of numbers.

**a** 2, 5 **b** 3, 7 **c** 5, 4 **d** 6, 5

**e** 3, 6 **f** 2, 10 **g** 4, 6 **h** 8, 6

**i** 2, 3, 5 **j** 3, 4, 6 **k** 3, 8, 4 **l** 2, 6, 5

**3** State the lowest common denominator of the following sets of fractions.

**a** $\frac{1}{3}, \frac{3}{5}$ **b** $\frac{2}{4}, \frac{3}{5}$ **c** $\frac{4}{7}, \frac{2}{3}$ **d** $\frac{2}{10}, \frac{1}{5}$

**e** $\frac{4}{6}, \frac{3}{8}$ **f** $\frac{5}{12}, \frac{2}{5}$ **g** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ **h** $\frac{4}{3}, \frac{3}{4}$

**4** Fill in the gaps to produce equivalent fractions.

**a** $\frac{2}{5} = \frac{\square}{15}$ **b** $\frac{2}{3} = \frac{\square}{12}$ **c** $\frac{1}{4} = \frac{\square}{16}$

**d** $\frac{3}{7} = \frac{\square}{14}$ **e** $\frac{3}{8} = \frac{\square}{40}$ **f** $\frac{5}{6} = \frac{\square}{18}$


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4D

5–7(½) | 5–7(½) | 5–7(½)

FLUENCY

Example 10

**5** Place the correct mathematical symbol <, = or >, in between the following pairs of fractions to make true mathematical statements.

**a** $\frac{3}{5} \square \frac{1}{5}$ **b** $\frac{7}{9} \square \frac{2}{9}$ **c** $\frac{2}{2} \square \frac{3}{3}$ **d** $\frac{13}{18} \square \frac{17}{18}$

**e** $\frac{1}{4} \square \frac{1}{3}$ **f** $\frac{1}{10} \square \frac{1}{20}$ **g** $\frac{1}{7} \square \frac{1}{5}$ **h** $\frac{3}{5} \square \frac{18}{30}$

**i** $\frac{2}{3} \square \frac{1}{3}$ **j** $\frac{4}{5} \square \frac{3}{4}$ **k** $\frac{5}{6} \square \frac{9}{10}$ **l** $\frac{5}{7} \square \frac{15}{21}$

**m** $\frac{7}{11} \square \frac{3}{5}$ **n** $1\frac{2}{3} \square 1\frac{1}{2}$ **o** $3\frac{3}{7} \square \frac{15}{4}$ **p** $\frac{12}{5} \square \frac{19}{8}$

Example 11

**6** Place the following fractions in ascending order.

**a** $\frac{3}{5}, \frac{8}{5}, 1\frac{2}{5}$ **b** $\frac{5}{9}, \frac{1}{3}, \frac{2}{9}$

**c** $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ **d** $\frac{5}{6}, \frac{3}{5}, \frac{2}{3}$

**e** $2\frac{1}{4}, \frac{11}{4}, \frac{5}{2}, 3\frac{1}{3}$ **f** $\frac{15}{8}, \frac{11}{6}, \frac{7}{4}, \frac{5}{3}$

**g** $2\frac{7}{10}, \frac{9}{4}, \frac{11}{5}, 2\frac{1}{2}, 2\frac{3}{5}$ **h** $4\frac{4}{9}, \frac{15}{3}, 4\frac{10}{27}, 4\frac{2}{3}, 4\frac{1}{6}$

**7** Place the following fractions in descending order, without finding common denominators.

**a** $\frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$ **b** $\frac{3}{5}, \frac{3}{7}, \frac{3}{6}, \frac{3}{8}$

**c** $\frac{7}{2}, \frac{7}{5}, \frac{7}{8}, \frac{7}{7}$ **d** $\frac{1}{15}, \frac{1}{10}, \frac{1}{50}, \frac{1}{100}$

**e** $7\frac{1}{11}, 8\frac{3}{5}, 5\frac{4}{9}, 10\frac{2}{3}$ **f** $2\frac{1}{3}, 2\frac{1}{9}, 2\frac{1}{6}, 2\frac{1}{5}$

8 | 8, 9 | 9, 10

PROBLEM-SOLVING

**8** Place the following cake fractions in decreasing order of size.

**A** sponge cake shared equally by four people = $\frac{1}{4}$ cake

**B** chocolate cake shared equally by eleven people = $\frac{1}{11}$ cake

**C** carrot and walnut cake shared equally by eight people = $\frac{1}{8}$ cake

**9** Four friends, Dean, David, Andrea and Rob, all competed in the Great Ocean Road marathon. Their respective finishing times were $3\frac{1}{3}$ hours, $3\frac{5}{12}$ hours, $3\frac{1}{4}$ hours and $3\frac{4}{15}$ hours. Write down the correct finishing order of the four friends.

**10** Rewrite the fractions in each set with their lowest common denominator and then write the next two fractions that would continue the pattern.

**a** $\frac{2}{9}, \frac{1}{3}, \frac{4}{9}, \_, \_$ **b** $\frac{1}{2}, \frac{5}{4}, 2, \_, \_$

**c** $\frac{11}{6}, \frac{3}{2}, \frac{7}{6}, \_, \_$ **d** $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \_, \_$


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## 4D

11 | 11, 12 | 12, 13

REASONING

**11** Write a fraction that lies between the following pairs of fractions.

**a** $\frac{3}{5}, \frac{3}{4}$ **b** $\frac{1}{4}, \frac{1}{2}$ **c** $\frac{2}{7}, \frac{1}{6}$

**d** $\frac{17}{20}, \frac{7}{10}$ **e** $2\frac{1}{3}, 2\frac{1}{5}$ **f** $8\frac{7}{10}, 8\frac{3}{4}$

**12** Write the whole number values that ? can take so that $\frac{?}{3}$ lies between:

**a** 2 and 3 **b** 5 and $5\frac{1}{2}$

**13** Thomas and Nathan had a doughnut eating race to see who could eat the most doughnuts in 1 minute. Before the race started Thomas cut each of his doughnuts into fifths to make them just the right bite-size. Nathan decided to cut each of his doughnuts into quarters before the race. After 1 minute of frenzied eating, the stop whistle blew. Thomas had devoured 28 fifths of doughnut and Nathan had munched his way through 22 quarters of doughnut.

**a** Who won the doughnut eating race?

**b** What was the winning doughnut margin? Express your answer in simplest form.

### Shady designs

— | — | 14

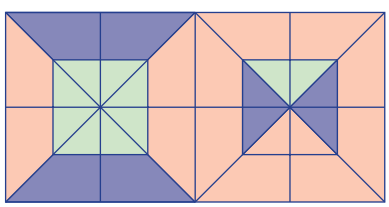
ENRICHMENT

**14 a** For each of the diagrams shown, work out what fraction of the rectangle is coloured purple. Explain how you arrived at each of your answers.

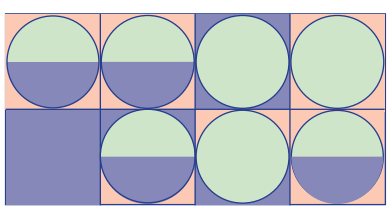
**b** Redraw the shapes in order from the largest amount of purple to the smallest.

**c** Design and shade two more rectangle designs.

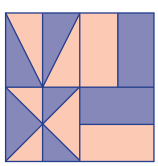
**i**



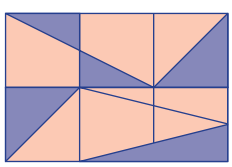
**ii**



**iii**



**iv**



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# 4E Adding fractions

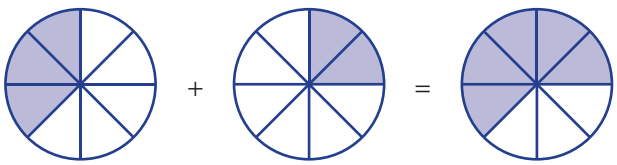
Interactive

Widgets

HOTsheets

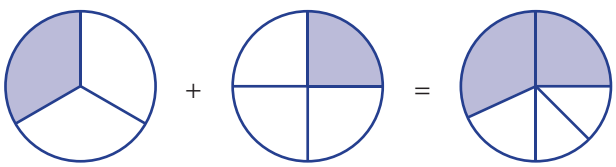
Walkthroughs

Fractions with the same denominator can be easily added together.



$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Fractions with different denominators cannot be added together so easily.



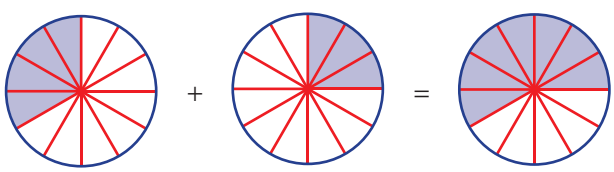
$\frac{1}{3} + \frac{1}{4} = ?$

Note:

$\frac{1}{3} + \frac{1}{4} \neq \frac{1}{7}$

$\frac{1}{3} + \frac{1}{4} \neq \frac{2}{7}$

But with a common denominator it is possible.



$\frac{1}{3} + \frac{1}{4} = ?$

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

## Let's start: 'Like' addition

Pair up with a classmate and discuss the following.

**1** Which of the following pairs of numbers can be simply added together without having to carry out any form of conversion?

| | | |
|---|---|---|
| **a** 6 goals, 2 goals | **b** 11 goals, 5 behinds | **c** 56 runs, 3 wickets |
| **d** 6 hours, 5 minutes | **e** 21 seconds, 15 seconds | **f** 47 minutes, 13 seconds |
| **g** 15 cm, 3 m | **h** 2.2 km, 4.1 km | **i** 5 kg, 1680 g |
| **j** $\frac{2}{7}, \frac{3}{7}$ | **k** $\frac{1}{4}, \frac{1}{2}$ | **l** $2\frac{5}{12}, 1\frac{1}{3}$ |

Does it become clear that we can only add pairs of numbers that have the *same* unit? In terms of fractions, we need to have the same _____?

**2** By choosing your preferred unit (when necessary), work out the answer to each of the problems above.

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## Key ideas

- Fractions can be simplified using addition *only* if they are 'like' fractions; that is, they must have the **same denominator**. This means they have been divided up into the same number of pieces.

**Same denominators**

- If two or more fractions have the same denominator, to add them together simply add the numerators and keep the denominator. This allows you to find the total number of divided pieces.

**Different denominators**

- If the denominators are different, we must use our knowledge of equivalent fractions to convert them to fractions with the same **lowest common denominator (LCD)**.
- To do this, carry out these steps.
  1. Find the LCD (often, but not always, found by multiplying denominators).
  2. Convert fractions to their equivalent fractions with the LCD.
  3. Add the numerators and write this total above the LCD.
- After adding fractions, always look to see if your answer needs to be simplified.

### Example 12 Adding 'like' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{3}{5}$ b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}$

| SOLUTION | EXPLANATION |
|---|---|
| a $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$ | The denominators are the same; i.e. 'like', therefore simply add the numerators. |
| b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}=\frac{14}{11}$ | Denominators are the same, so add numerators. |
| $=1\frac{3}{11}$ | Simplify answer by converting to a mixed number. |

### Example 13 Adding 'unlike' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{1}{2}$ b $\frac{3}{4}+\frac{5}{6}$


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| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}$ | LCD is 10.<br>Write equivalent fractions with the LCD. |
| $=\frac{7}{10}$ | Denominators are the same, so add numerators. |
| **b** $\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}$ | LCD is 12.<br>Write equivalent fractions with the LCD. |
| $=\frac{19}{12}$ | Denominators are the same, so add numerators. |
| $=1\frac{7}{12}$ | Simplify answer to a mixed number. |

## Example 14 Adding mixed numbers

Simplify:

**a** $3\frac{2}{3}+4\frac{2}{3}$ **b** $2\frac{5}{6}+3\frac{3}{4}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** **Method 1** | |
| $3+4+\frac{2}{3}+\frac{2}{3}=7+\frac{4}{3}$ | Add the whole number parts together.<br>Add the fraction parts together. |
| $=8\frac{1}{3}$ | Noting that $\frac{4}{3}=1\frac{1}{3}$, simplify the answer. |
| **Method 2** | |
| $\frac{11}{3}+\frac{14}{3}=\frac{25}{3}$ | Convert mixed numbers to improper fractions. Have the same denominators, so add numerators. |
| $=8\frac{1}{3}$ | Convert improper fraction back to a mixed number. |
| **b** **Method 1** | |
| $2+3+\frac{5}{6}+\frac{3}{4}$ | Add the whole number parts together.<br>LCD of 6 and 4 is 12.<br>Write equivalent fractions with LCD. |
| $=5+\frac{10}{12}+\frac{9}{12}$ | |
| $=5+\frac{19}{12}$ | Add the fraction parts together. |
| $=6\frac{7}{12}$ | Noting that $\frac{19}{12}=1\frac{7}{12}$, simplify the answer. |


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**Method 2**

$\frac{17}{6} + \frac{15}{4} = \frac{34}{12} + \frac{45}{12}$

$= \frac{79}{12}$

$= 6\frac{7}{12}$

Convert mixed numbers to improper fractions.
Write equivalent fractions with LCD.
Add the numerators together.

Simplify answer back to a mixed number.

## Exercise 4E

1–4 | 4 | —

UNDERSTANDING

**1** Copy the following sentences into your workbook and fill in the gaps.

**a** To add two fractions together, they must have the same _____.

**b** When adding fractions together, if they have the same _____, you simply add the _____.

**c** When adding two or more fractions where the _____ are different, you must find the _____ _____ _____.

**d** After carrying out the addition of fractions, you should always _____ your answer to see if it can be _____.

**2** Copy the following sums into your workbook and fill in the empty boxes.

**a** $\frac{3}{8} + \frac{2}{8} = \frac{\square}{8}$

**b** $\frac{4}{7} + \frac{1}{7} = \frac{\square}{7}$

**c** $\frac{1}{3} + \frac{1}{4}$

$= \frac{\square}{12} + \frac{\square}{12}$

$= \frac{\square}{12}$

**d** $\frac{2}{5} + \frac{3}{4}$

$= \frac{\square}{20} + \frac{\square}{20}$

$= \frac{\square}{20} = 1\frac{\square}{20}$

**3** State the LCD for the following pairs of 'incomplete' fractions.

**a** $\frac{}{5} + \frac{}{3}$ **b** $\frac{}{4} + \frac{}{5}$ **c** $\frac{}{2} + \frac{}{3}$ **d** $\frac{}{6} + \frac{}{3}$

**e** $\frac{}{2} + \frac{}{8}$ **f** $\frac{}{5} + \frac{}{10}$ **g** $\frac{}{7} + \frac{}{11}$ **h** $\frac{}{3} + \frac{}{9}$

**i** $\frac{}{12} + \frac{}{8}$ **j** $\frac{}{2} + \frac{}{18}$ **k** $\frac{}{15} + \frac{}{10}$ **l** $\frac{}{12} + \frac{}{16}$

**4** The following sums have been completed, but only six of them are correct. Copy them into your workbook, then place a tick beside the six correct answers and a cross beside the six incorrect answers.

**a** $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ **b** $\frac{1}{3} + \frac{1}{4} = \frac{2}{7}$ **c** $\frac{2}{5} + \frac{4}{5} = \frac{6}{10}$ **d** $\frac{1}{11} + \frac{3}{11} = \frac{4}{11}$

**e** $\frac{3}{5} + \frac{4}{5} = 1\frac{2}{5}$ **f** $\frac{2}{7} + \frac{2}{7} = \frac{2}{7}$ **g** $\frac{7}{12} + \frac{4}{12} = \frac{11}{12}$ **h** $\frac{4}{9} + \frac{4}{5} = \frac{4}{14}$

**i** $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$ **j** $\frac{1}{2} + \frac{2}{5} = \frac{3}{7}$ **k** $2\frac{2}{7} + 3\frac{1}{7} = 5\frac{3}{7}$ **l** $1\frac{2}{3} + 2\frac{1}{5} = 3\frac{3}{8}$

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4E

5–8(½) | 5–8(½) | 6(½), 8(½)

FLUENCY

Example 12a

**5** Add the following fractions.

**a** $\frac{1}{8}+\frac{4}{8}$ **b** $\frac{2}{7}+\frac{3}{7}$ **c** $\frac{1}{5}+\frac{3}{5}$ **d** $\frac{3}{11}+\frac{6}{11}$

**e** $\frac{5}{8}+\frac{2}{8}$ **f** $\frac{1}{12}+\frac{6}{12}$ **g** $\frac{3}{15}+\frac{4}{15}$ **h** $\frac{3}{9}+\frac{2}{9}$

Example 12b

**i** $\frac{6}{7}+\frac{3}{7}$ **j** $\frac{7}{10}+\frac{6}{10}$ **k** $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$ **l** $\frac{12}{19}+\frac{3}{19}+\frac{8}{19}$

Example 13a

**6** Add the following fractions.

**a** $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{3}+\frac{3}{5}$ **c** $\frac{1}{2}+\frac{1}{6}$ **d** $\frac{1}{4}+\frac{1}{3}$

**e** $\frac{2}{5}+\frac{1}{4}$ **f** $\frac{1}{5}+\frac{3}{4}$ **g** $\frac{2}{7}+\frac{1}{3}$ **h** $\frac{3}{8}+\frac{1}{5}$

Example 13b

**i** $\frac{3}{5}+\frac{5}{6}$ **j** $\frac{4}{7}+\frac{3}{4}$ **k** $\frac{8}{11}+\frac{2}{3}$ **l** $\frac{2}{3}+\frac{3}{4}$

Example 14a

**7** Simplify:

**a** $1\frac{1}{5}+2\frac{3}{5}$ **b** $3\frac{2}{7}+4\frac{1}{7}$ **c** $11\frac{1}{4}+1\frac{2}{4}$ **d** $1\frac{3}{9}+4\frac{2}{9}$

**e** $5\frac{2}{3}+4\frac{2}{3}$ **f** $8\frac{3}{6}+12\frac{4}{6}$ **g** $9\frac{7}{11}+9\frac{7}{11}$ **h** $4\frac{3}{5}+7\frac{4}{5}$

Example 14b

**8** Simplify:

**a** $2\frac{2}{3}+1\frac{3}{4}$ **b** $5\frac{2}{5}+1\frac{5}{6}$ **c** $3\frac{1}{2}+8\frac{2}{3}$ **d** $5\frac{4}{7}+7\frac{3}{4}$

**e** $8\frac{1}{2}+6\frac{3}{5}$ **f** $12\frac{2}{3}+6\frac{4}{9}$ **g** $17\frac{8}{11}+7\frac{3}{4}$ **h** $9\frac{7}{12}+5\frac{5}{8}$

9, 10 | 10, 11 | 11, 12

PROBLEM-SOLVING

**9** Myles, Liza and Camillus work at a busy cinema complex. For a particular movie, Myles sells $\frac{3}{5}$ of all the tickets and Liza sells $\frac{1}{3}$.

**a** What fraction of movie tickets are sold by Myles and Liza, together?

**b** If all of the movie's tickets are sold, what is the fraction sold by Camillus?


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## 4E

PROBLEM-SOLVING

**10** Martine loves to run and play. Yesterday, she ran for $2\frac{1}{4}$ kilometres, walked for $5\frac{2}{5}$ kilometres and skipped for $\frac{1}{2}$ a kilometre. What was the total distance that Martine ran, walked and skipped?

**11** Jackson is working on a 1000-piece jigsaw puzzle. After 1 week, he has completed $\frac{1}{10}$ of the puzzle. After 2 weeks he has completed another $\frac{2}{5}$ of the puzzle. In the third week, Jackson completed another $\frac{1}{4}$ of the puzzle.

- **a** By the end of the third week, what fraction of the puzzle has Jackson completed?
- **b** How many pieces of the puzzle does Jackson place in the second week?
- **c** What fraction of the puzzle is still unfinished by the end of the third week? How many pieces is this?

**12** A survey of Year 7 students' favourite sport is carried out. A total of 180 students participate in the survey. One-fifth of students reply that netball is their favourite, one-quarter reply rugby and one-third reply soccer. The remainder of students leave the question unanswered.

- **a** What fraction of the Year 7 students answered the survey question?
- **b** What fraction of the Year 7 students left the question unanswered?
- **c** How many students did not answer the survey question?

13 | 13, 14 | 13, 14

REASONING

**13** Fill in the empty boxes to make the following fraction sums correct.

- **a** $\frac{1}{\square} + \frac{1}{\square} = \frac{7}{10}$
- **b** $\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} = \frac{7}{8}$
- **c** $\frac{3}{\square} + \frac{\square}{4} = \frac{17}{20}$
- **d** $\frac{2}{\square} + \frac{\square}{3} + \frac{4}{\square} = 1$

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4E

REASONING

**14** Four students each read the same English novel over two nights, for homework. The table shows what fraction of the book was read on each of the two nights.

| Student | First night | Second night |
|---|---|---|
| Mikhail | $\frac{2}{5}$ | $\frac{1}{4}$ |
| Jim | $\frac{1}{2}$ | $\frac{1}{10}$ |
| Vesna* | $\frac{1}{4}$ | $\frac{1}{5}$ |
| Juliet | $\frac{7}{12}$ | $\frac{1}{20}$ |

*Vesna woke up early on the third morning and read another $\frac{1}{6}$ of the novel before leaving for school.

Place the students in order, from least to most, according to what fraction of the book they had read by their next English lesson.

ENRICHMENT

## Raise it to the max — — 15

**15 a** Using the numbers 1, 2, 3, 4, 5 and 6 only once, arrange them in the boxes below to, first, produce the maximum possible answer, and then the minimum possible answer. Work out the maximum and minimum possible answers.

$$\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square}$$

**b** Repeat the process for four fractions using the digits 1 to 8 only once each. Again, state the maximum and minimum possible answers.

**c** Investigate maximum and minimum fraction statements for other sets of numbers and explain your findings.

**d** Explain how you would arrange the numbers 1 to 100 for 50 different fractions if you were trying to achieve the maximum or minimum sum.



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# 4F Subtracting fractions

Interactive

Widgets

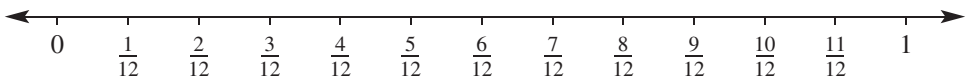
HOTsheets

Walkthroughs

Subtracting fractions is very similar to adding fractions. You must establish the **lowest common denominator** (**LCD**) if one does not exist and this is done through producing equivalent fractions. Then, instead of adding numerators at the final step, you simply carry out the correct subtraction.

Complications can arise when subtracting mixed numbers and **Example 16** shows the available methods that can be used to overcome such problems.

## Let's start: Alphabet subtraction




- Copy into your workbook the number line above.
- Place the following letters in the correct position on the number line.

$A = \frac{2}{3}$ $B = \frac{5}{12}$ $C = \frac{1}{2}$ $D = \frac{11}{12}$ $E = \frac{1}{12}$ $F = \frac{1}{4}$ $G = \frac{0}{12}$

$H = \frac{1}{3}$ $I = \frac{7}{12}$ $J = \frac{5}{6}$ $K = \frac{12}{12}$ $L = \frac{3}{4}$ $M = \frac{1}{6}$

- Complete the following alphabet subtractions, giving your answer as a fraction and also the corresponding alphabet letter.

**a** J – F **b** A – G **c** D – F – M **d** C – B

**e** K – C **f** L – H – E **g** K – J – E **h** L – I – M

- What does A + B + C + D + E + F + G + H + I – J – K – L – M equal?

## Key ideas

- Fractions can be simplified using subtraction *only* if they are 'like' fractions.
- The process for subtracting fractions is the same as adding fractions, until the final step. At the final step you follow the operation and subtract the second numerator from the first numerator.
- When subtracting mixed numbers, you must have a fraction part that is large enough to allow the other proper fraction to be subtracted from it. If this is not the case at the start of the problem, you may choose to borrow a whole.
  For example:

  $7\frac{1}{2} - 2\frac{3}{4}$ $\frac{1}{2}$ is not big enough to have $\frac{3}{4}$ subtracted from it.

  $6\frac{3}{2} - 2\frac{3}{4}$ Therefore, we choose to borrow a whole from the 7.
- A fail-safe method for subtracting mixed numbers is to convert to improper fractions right from the start.
  For example: $7\frac{1}{2} - 2\frac{3}{4} = \frac{15}{2} - \frac{11}{4}$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## Example 15 Subtracting 'like' and 'unlike' fractions

Simplify:

**a** $\frac{7}{9}-\frac{2}{9}$ **b** $\frac{5}{6}-\frac{1}{4}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{7}{9}-\frac{2}{9}=\frac{5}{9}$ | Denominators are the same, therefore we are ready to subtract the second numerator from the first. |
| **b** $\frac{5}{6}-\frac{1}{4}=\frac{10}{12}-\frac{3}{12}$ | Need to find the LCD, which is 12. Write equivalent fractions with the LCD. |
| $=\frac{7}{12}$ | We have the same denominators now, so subtract second numerator from the first. |

## Example 16 Subtracting mixed numbers

Simplify:

**a** $5\frac{2}{3}-3\frac{1}{4}$ **b** $8\frac{1}{5}-4\frac{3}{4}$

| SOLUTION | EXPLANATION |
|---|---|
| **Method 1: Converting to an improper fraction** | |
| **a** $5\frac{2}{3}-3\frac{1}{4}=\frac{17}{3}-\frac{13}{4}$ | Convert mixed numbers to improper fractions. Need to find the LCD, which is 12. Write equivalent fractions with the LCD. |
| $=\frac{68}{12}-\frac{39}{12}$ | |
| $=\frac{29}{12}$ | We have the same denominators now, so subtract second numerator from the first and convert back to improper fraction. |
| $=2\frac{5}{12}$ | |
| **b** $8\frac{1}{5}-4\frac{3}{4}=\frac{41}{5}-\frac{19}{4}$ | Convert mixed numbers to improper fractions. Need to find the LCD, which is 20. Write equivalent fractions with the LCD. |
| $=\frac{164}{20}-\frac{95}{20}$ | |
| $=\frac{69}{20}$ | We have the same denominators now, so subtract second numerator from the first and convert back to improper fraction. |
| $=3\frac{9}{20}$ | |


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**Method 2: Borrowing a whole number**

**a** $5\frac{2}{3} - 3\frac{1}{4} = \left(5 + \frac{2}{3}\right) - \left(3 + \frac{1}{4}\right)$

$= (5 - 3) + \left(\frac{2}{3} - \frac{1}{4}\right)$

$= 2 + \left(\frac{8}{12} - \frac{3}{12}\right)$

$= 2\frac{5}{12}$

Understand that a mixed number is the addition of a whole number and a proper fraction.
Group whole numbers and group proper fractions.

Simplify whole numbers; simplify proper fractions.

Borrowing a whole was not required.

**b** $8\frac{1}{5} - 4\frac{3}{4} = \left(8 + \frac{1}{5}\right) - \left(4 + \frac{3}{4}\right)$

$= \left(7 + \frac{6}{5}\right) - \left(4 + \frac{3}{4}\right)$

$= (7 - 4) + \left(\frac{6}{5} - \frac{3}{4}\right)$

$= 3 + \left(\frac{24}{20} - \frac{15}{20}\right)$

$= 3\frac{9}{20}$

$\frac{3}{4}$ cannot be taken away from $\frac{1}{5}$ easily.

Therefore, we must borrow a whole.

Group whole numbers and group proper fractions.
Simplify whole numbers; simplify proper fractions.
Borrowing a whole was required.

## Exercise 4F

1–4 | 4 | —

UNDERSTANDING

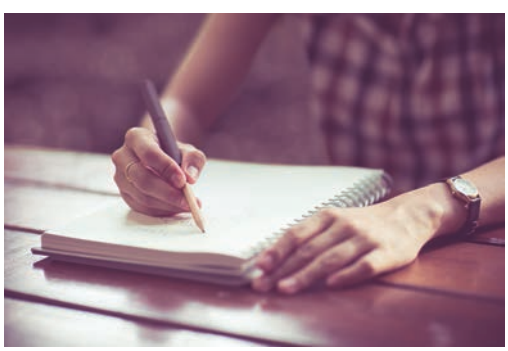
**1** Copy the following sentences into your workbook and fill in the blanks.

**a** To subtract one fraction from another, you must have a common _____.

**b** One fail-safe method of producing a common denominator is to simply _____ the two denominators.

**c** The problem with finding a common denominator that is not the lowest common denominator is that you have to deal with larger numbers and you also need to _____ your answer at the final step.

**d** To find the LCD you can _____ the denominators and then divide by the HCF of the denominators.

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4F

UNDERSTANDING

**2** State the LCD for the following pairs of 'incomplete' fractions.

**a** $\frac{}{4}-\frac{}{6}$ **b** $\frac{}{2}-\frac{}{10}$ **c** $\frac{}{15}-\frac{}{5}$ **d** $\frac{}{6}-\frac{}{9}$

**e** $\frac{}{8}-\frac{}{12}$ **f** $\frac{}{12}-\frac{}{20}$ **g** $\frac{}{14}-\frac{}{8}$ **h** $\frac{}{9}-\frac{}{21}$

**3** Copy these equations into your workbook, and fill in the empty boxes.

**a** $\frac{3}{7}-\frac{2}{7}=\frac{\square}{7}$ **b** $\frac{8}{13}-\frac{5}{13}=\frac{\square}{13}$

**c** $\frac{1}{3}-\frac{1}{4}=\frac{\square}{12}-\frac{\square}{12}$
$=\frac{\square}{12}$

**d** $\frac{4}{5}-\frac{2}{3}=\frac{\square}{15}-\frac{\square}{15}$
$=\frac{\square}{15}$

**4** The following equations have been completed, but only six of them are correct. Copy them into your workbook, then place a tick beside the six correct answers and a cross beside the six incorrect answers.

**a** $\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$ **b** $\frac{3}{5}-\frac{2}{3}=\frac{1}{2}$ **c** $\frac{5}{12}-\frac{5}{10}=\frac{5}{2}$ **d** $\frac{3}{4}-\frac{1}{4}=\frac{2}{4}$

**e** $\frac{8}{11}-\frac{8}{10}=\frac{0}{1}=0$ **f** $\frac{12}{15}-\frac{3}{15}=\frac{9}{15}$ **g** $\frac{2}{3}-\frac{2}{3}=0$ **h** $\frac{5}{7}-\frac{2}{7}=\frac{2}{7}$

**i** $\frac{3}{20}-\frac{2}{20}=\frac{1}{20}$ **j** $2\frac{5}{9}-1\frac{4}{9}=1\frac{1}{9}$ **k** $2\frac{8}{14}-\frac{5}{14}=2\frac{3}{0}$ **l** $\frac{12}{21}-\frac{7}{11}=\frac{5}{10}=\frac{1}{2}$

5–8(½) 5–8(½) 6–8(½)

FLUENCY

Example 15a

**5** Simplify:

**a** $\frac{5}{7}-\frac{3}{7}$ **b** $\frac{4}{11}-\frac{1}{11}$ **c** $\frac{12}{18}-\frac{5}{18}$ **d** $\frac{2}{3}-\frac{1}{3}$

**e** $\frac{3}{5}-\frac{3}{5}$ **f** $\frac{6}{9}-\frac{2}{9}$ **g** $\frac{5}{19}-\frac{2}{19}$ **h** $\frac{17}{23}-\frac{9}{23}$

**i** $\frac{84}{100}-\frac{53}{100}$ **j** $\frac{41}{50}-\frac{17}{50}$ **k** $\frac{23}{25}-\frac{7}{25}$ **l** $\frac{7}{10}-\frac{3}{10}$

Example 15b

**6** Simplify:

**a** $\frac{2}{3}-\frac{1}{4}$ **b** $\frac{3}{5}-\frac{1}{2}$ **c** $\frac{3}{5}-\frac{3}{6}$ **d** $\frac{4}{7}-\frac{1}{4}$

**e** $\frac{1}{2}-\frac{1}{3}$ **f** $\frac{3}{4}-\frac{1}{9}$ **g** $\frac{8}{11}-\frac{1}{3}$ **h** $\frac{4}{5}-\frac{2}{3}$

**i** $\frac{3}{4}-\frac{5}{8}$ **j** $\frac{11}{20}-\frac{2}{5}$ **k** $\frac{5}{12}-\frac{7}{18}$ **l** $\frac{7}{9}-\frac{2}{3}$

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## 4F

FLUENCY

Example 16a

**7** Simplify:

**a** $3\frac{4}{5} - 2\frac{1}{5}$ **b** $23\frac{5}{7} - 15\frac{2}{7}$ **c** $8\frac{11}{14} - 7\frac{9}{14}$ **d** $3\frac{5}{9} - \frac{3}{9}$

**e** $6\frac{2}{3} - 4\frac{1}{4}$ **f** $5\frac{3}{7} - 2\frac{1}{4}$ **g** $9\frac{5}{6} - 5\frac{4}{9}$ **h** $14\frac{3}{4} - 7\frac{7}{10}$

Example 16b

**8** Simplify:

**a** $5\frac{1}{3} - 2\frac{2}{3}$ **b** $8\frac{2}{5} - 3\frac{4}{5}$ **c** $13\frac{1}{2} - 8\frac{5}{6}$ **d** $12\frac{2}{9} - 7\frac{1}{3}$

**e** $8\frac{5}{12} - 3\frac{3}{4}$ **f** $1\frac{3}{5} - \frac{7}{9}$ **g** $11\frac{1}{11} - 1\frac{1}{4}$ **h** $6\frac{3}{20} - 3\frac{2}{3}$

PROBLEM-SOLVING

9, 10 | 10–12 | 11–13

**9** Tiffany poured herself a large glass of cordial. She noticed that the cordial jug has $\frac{3}{4}$ of a litre in it before she poured her glass and only $\frac{1}{5}$ of a litre in it after she filled her glass. How much cordial did Tiffany pour into her glass?

**10** A family block of chocolate is made up of 60 small squares of chocolate. Marcia eats 10 blocks, Jon eats 9 blocks and Holly eats 5 blocks. What fraction of the block of chocolate is left?

**11** Three friends split a restaurant bill. One pays $\frac{1}{2}$ of the bill and one pays $\frac{1}{3}$ of the bill. What fraction of the bill must the third friend pay?

**12** Patty has $23\frac{1}{4}$ dollars, but owes her parents $15\frac{1}{2}$ dollars. How much money does Patty have left after she pays back her parents? Repeat this question using decimals and dollars and cents. Do you get the same answer?

**13** Three cakes were served at a birthday party: an ice-cream cake, a chocolate cake and a sponge cake. $\frac{3}{4}$ of the ice-cream cake was eaten. The chocolate cake was cut into 12 equal pieces, of which 9 were eaten. The sponge cake was divided into 8 equal pieces, with only 1 piece remaining.

**a** What fraction of each cake was eaten?
**b** What fraction of each cake was left over?
**c** What was the total amount of cake eaten during the party?
**d** What was the total amount of cake left over after the party?

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4F

14 | 14, 15 | 15, 16

REASONING

**14** Fill in the empty boxes to make the following fraction sums correct.

**a** $\frac{1}{\square} - \frac{1}{\square} = \frac{1}{12}$

**b** $\frac{\square}{5} - \frac{\square}{2} = \frac{1}{10}$

**c** $2\frac{\square}{3} - 1\frac{\square}{3} = \frac{2}{3}$

**d** $8\frac{1}{\square} - 6\frac{\square}{4} = 1\frac{1}{2}$

**15** Today David's age is one-seventh of Felicity's age. Felicity is a teenager.

**a** In 1 year's time David will be one-fifth of Felicity's age. What fraction of her age will he be in 2 years' time?

**b** How many years must pass until David is one-third of Felicity's age?

**c** How many years must pass until David is half Felicity's age?

**16** Simplify:

**a** **Example 16** shows two possible methods for subtracting mixed numbers: 'Borrowing a whole number' and 'Converting to an improper fraction'. Simplify the following two expressions and discuss which method is the most appropriate for each question.

**i** $2\frac{1}{5} - 1\frac{2}{3}$ **ii** $27\frac{5}{11} - 23\frac{4}{5}$

**b** If you have an appropriate calculator, work out how to enter fractions and check your answers to parts **i** and **ii** above.

## Letter to an absent friend

— | — | 17

ENRICHMENT

**17** Imagine that a friend in your class is absent for this lesson on the subtraction of fractions. They were present yesterday and understood the process involved when adding fractions. Your task is to write a letter to your friend, explaining how to subtract mixed numbers. Include some examples, discuss both possible methods but also justify your favourite method. Finish off with three questions for your friend to attempt and include the answers to these questions on the back of the letter.

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# 4G Multiplying fractions

Interactive

Widgets

HOTsheets

Walkthroughs

What does it mean to multiply two fractions together?

Do you end up with a smaller amount or a larger amount when you multiply two proper fractions together?

What does $\frac{1}{3} \times \frac{2}{3}$ equal?

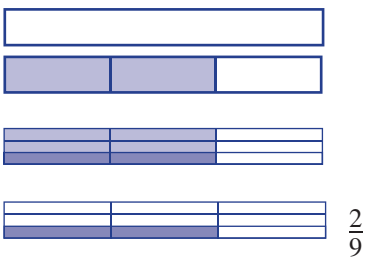
- **'Strip' method**

  Imagine you have a strip of paper.

  You are told to shade $\frac{2}{3}$ of the strip.

  You are now told to shade in a darker colour $\frac{1}{3}$ of your $\frac{2}{3}$ strip.

  The final amount shaded is your answer.




- **'Number line' method**

  Consider the number line from 0 to 1 (shown opposite).

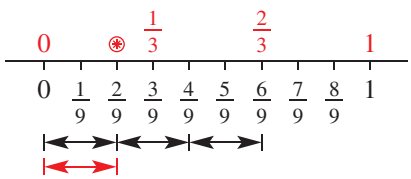
  It is divided into ninths.

  Locate $\frac{2}{3}$.

  Divide this position into three equal pieces (shown as ⟷).

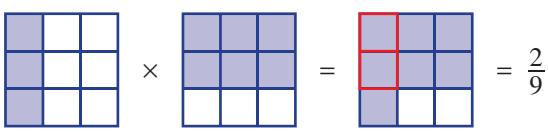
  To locate $\frac{1}{3} \times \frac{2}{3}$ you have only one of the three pieces.

  The final location is your answer (shown as ⟷); i.e. $\frac{2}{9}$.




- **'Shading' method**

  Consider $\frac{1}{3}$ of a square multiplied by $\frac{2}{3}$ of a square.




- **'The rule' method**

  When multiplying fractions, multiply the numerators together and mutiply the denominators together.

$$\frac{1}{3} \times \frac{2}{3} = \frac{1 \times 2}{3 \times 3} = \frac{2}{9}$$

Cutting materials to fit a purpose may involve multiplying a fraction by a fraction.

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## Let's start: 'Clock face' multiplication

Explain and discuss the concept of fractions of an hour on the clock face.
In pairs, students match up the following 10 'clock face' multiplication questions with their correct answer. You may like to place a time limit of 5 minutes on the activity.

Discuss answers at the end of the activity.

| Questions | | Answers | |
|---|---|---|---|
| 1 | $\frac{1}{2}$ of 4 hours | **A** | 25 minutes |
| 2 | $\frac{1}{3}$ of 2 hours | **B** | $1\frac{1}{2}$ hours |
| 3 | $\frac{1}{4}$ of 6 hours | **C** | 5 minutes |
| 4 | $\frac{1}{3}$ of $\frac{1}{4}$ hour | **D** | $\frac{1}{4}$ hour |
| 5 | $\frac{1}{4}$ of $\frac{1}{3}$ hour | **E** | 2 hours |
| 6 | $\frac{1}{3}$ of $\frac{3}{4}$ hour | **F** | 2 hours 40 minutes |
| 7 | $\frac{1}{10}$ of $\frac{1}{2}$ hour | **G** | $\frac{1}{12}$th hour |
| 8 | $\frac{1}{5}$ of $\frac{1}{2}$ hour | **H** | 40 minutes |
| 9 | $\frac{2}{3}$ of 4 hours | **I** | $\frac{1}{10}$th hour |
| 10 | $\frac{5}{6}$ of $\frac{1}{2}$ hour | **J** | 3 minutes |


**Key ideas**

- Fractions do *not* need to have the same denominator to be multiplied together.
- To multiply fractions, multiply the numerators together and multiply the denominators together.
  - In symbols: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

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## Key ideas

- If possible, 'simplify', 'divide' or 'cancel' fractions before multiplying.
  - Cancelling can be done *vertically* or *diagonally*.
  - Cancelling can never be done *horizontally*.

$$\frac{3}{5} \times \frac{\cancel{4}^{1}}{\cancel{8}^{2}} \quad \text{cancelling vertically} \quad \checkmark$$

$$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{6}^{2}} \quad \text{cancelling diagonally} \quad \checkmark$$

Never do this! →

$$\frac{\cancel{3}^{1}}{5} \times \frac{\cancel{6}^{2}}{7} \quad \text{cancelling horizontally} \quad \times$$

- A whole number can be written as a fraction with a denominator of 1.
- 'of', '×', 'times', 'lots of' and 'product' all refer to the same mathematical operation of multiplying.
- Mixed numbers must be changed to improper fractions before multiplying.
- Final answers should be written in simplest form.

### Example 17 Finding a simple fraction of a quantity

Find:

**a** $\frac{2}{3}$ of 15 bananas

**b** $\frac{3}{10}$ of 50 lollies

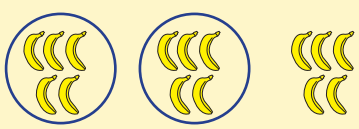
**SOLUTION**

**a** $\frac{2}{3}$ of 15 bananas

$\left(\frac{1}{3} \text{ of } 15\right) \times 2 = 10$

**EXPLANATION**

Divide 15 bananas into 3 equal groups.
Therefore, 5 in each group.
Take 2 of the groups.



Answer is 10 bananas.

**b** $\frac{3}{10}$ of 50 lollies

$\left(\frac{1}{10} \text{ of } 50\right) \times 3 = 15$

Divide 50 into 10 equal groups.
Therefore, 5 in each group.
Take 3 of the groups.
Therefore, answer is 15 lollies.

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## Example 18 Multiplying proper fractions

Find:

**a** $\frac{2}{3} \times \frac{1}{5}$ **b** $\frac{3}{4} \times \frac{8}{9}$ **c** $\frac{4}{8}$ of $\frac{3}{6}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5}$ | Multiply the numerators together. |
| $= \frac{2}{15}$ | Multiply the denominators together.<br>The answer is in simplest form. |
| **b** $\frac{3}{4} \times \frac{8}{9} = \frac{{}^{1}\cancel{3} \times 8^{2}}{{}_{1}\cancel{4} \times 9^{3}}$ | Cancel first. |
| $= \frac{2}{3}$ | Then multiply numerators together and denominators together. |
| **c** $\frac{4}{8}$ of $\frac{3}{6} = \frac{4}{8} \times \frac{3}{6}$ | Change 'of' to multiplication sign. |
| $= \frac{{}^{1}\cancel{4} \times 3^{1}}{{}_{2}\cancel{8} \times 6^{2}}$ | Cancel and then multiply the numerators and the denominators. |
| $= \frac{1}{4}$ | The answer is in simplest form. |

## Example 19 Multiplying proper fractions by whole numbers

Find:

**a** $\frac{1}{3} \times 21$ **b** $\frac{2}{5}$ of 32

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{1}{3} \times 21 = \frac{1}{{}_{1}\cancel{3}} \times \frac{\cancel{21}^{7}}{1}$ | Rewrite 21 as a fraction with a denominator equal to 1.<br>Cancel and then multiply numerators and denominators. |
| $= \frac{7}{1}$ | $7 \div 1 = 7$ |
| $= 7$ | |
| **b** $\frac{2}{5}$ of $32 = \frac{2}{5} \times \frac{32}{1}$ | Rewrite 'of' as a multiplication sign.<br>Write 32 as a fraction. |
| $= \frac{64}{5}$ | Multiply numerators and denominators. |
| $= 12\frac{4}{5}$ | Convert answer to a mixed number. |


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## Example 20 Multiplying improper fractions

Find:

**a** $\frac{5}{3} \times \frac{7}{2}$ **b** $\frac{8}{5} \times \frac{15}{4}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{5}{3} \times \frac{7}{2} = \frac{5 \times 7}{3 \times 2}$ | Multiply the numerators together. |
| $= \frac{35}{6} = 5\frac{5}{6}$ | Multiply the denominators together.<br>Convert the answer to a mixed number. |
| **b** $\frac{8}{5} \times \frac{15}{4} = \frac{{}^{2}\cancel{8} \times \cancel{15}^{3}}{{}_{1}\cancel{5} \times \cancel{4}^{1}}$ | Cancel first. |
| $= \frac{6}{1} = 6$ | Multiply 'cancelled' numerators together and 'cancelled' denominators together.<br>Write the answer in simplest form. |

## Example 21 Multiplying mixed numbers

Find:

**a** $2\frac{1}{3} \times 1\frac{2}{5}$ **b** $6\frac{1}{4} \times 2\frac{2}{5}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5}$ | Convert mixed numbers to improper fractions. |
| $= \frac{49}{15}$ | Multiply numerators together.<br>Multiply denominators together. |
| $= 3\frac{4}{15}$ | Write the answer in simplest form. |
| **b** $6\frac{1}{4} \times 2\frac{2}{5} = \frac{{}^{5}\cancel{25}}{{}_{1}\cancel{4}} \times \frac{\cancel{12}^{3}}{\cancel{5}^{1}}$ | Convert to improper fractions.<br>Simplify fractions by cancelling. |
| $= \frac{15}{1}$ | Multiply numerators and denominators together. |
| $= 15$ | Write the answer in simplest form. |

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## Exercise 4G

1–5 | 5 | —

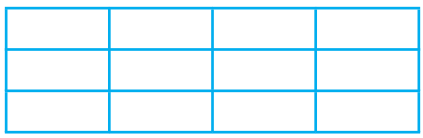
UNDERSTANDING

**1** Copy these sentences into your workbook and fill in the blanks.

- **a** A proper fraction has a value that is between _____ and _____.
- **b** An improper fraction is always greater than _____.
- **c** A mixed number consists of two parts, a _____ _____ part and a _____ _____ part.

**2** When multiplying a whole number by a proper fraction, do you get a smaller or larger answer when compared with the whole number? Explain your answer.

**3** Copy into your workbook the grid shown opposite.



- **a** On your diagram, shade in blue $\frac{1}{3}$ of the grid.
- **b** Now shade in red $\frac{1}{4}$ of the shaded blue.
- **c** You have now shaded $\frac{1}{4}$ of $\frac{1}{3}$. What fraction is this of the original grid?

Example 17

**4** Use drawings to show the answer to these problems.

- **a** $\frac{1}{3}$ of 12 1ollies
- **b** $\frac{1}{5}$ of 10 pencils
- **c** $\frac{2}{3}$ of 18 donuts
- **d** $\frac{3}{4}$ of 16 boxes
- **e** $\frac{3}{8}$ of 32 dots
- **f** $\frac{3}{7}$ of 21 triangles

**5** One of the following four methods is the correct solution to the problem $\frac{1}{2} \times \frac{1}{5}$. Find the correct solution and copy it into your workbook.

**A**
$\frac{1}{2} \times \frac{1}{5}$
$= \frac{1+1}{2+5}$
$= \frac{2}{7}$

**B**
$\frac{1}{2} \times \frac{1}{5}$
$= \frac{1 \times 1}{2 \times 5}$
$= \frac{2}{10}$

**C**
$\frac{1}{2} \times \frac{1}{5}$
$= \frac{5}{10} \times \frac{2}{10}$
$= \frac{7}{20}$

**D**
$\frac{1}{2} \times \frac{1}{5}$
$= \frac{1 \times 1}{2 \times 5}$
$= \frac{1}{10}$

6–9(½) | 6–10(½) | 6–10(½)

FLUENCY

Example 18

**6** Evaluate:

- **a** $\frac{3}{4} \times \frac{1}{5}$
- **b** $\frac{2}{7} \times \frac{1}{3}$
- **c** $\frac{2}{3} \times \frac{5}{7}$
- **d** $\frac{4}{9} \times \frac{2}{5}$
- **e** $\frac{2}{3} \times \frac{3}{5}$
- **f** $\frac{4}{7} \times \frac{1}{4}$
- **g** $\frac{3}{4} \times \frac{1}{3}$
- **h** $\frac{5}{9} \times \frac{9}{11}$
- **i** $\frac{3}{6} \times \frac{5}{11}$
- **j** $\frac{2}{3} \times \frac{4}{8}$
- **k** $\frac{8}{11} \times \frac{3}{4}$
- **l** $\frac{2}{5} \times \frac{10}{11}$
- **m** $\frac{2}{7}$ of $\frac{3}{5}$
- **n** $\frac{3}{4}$ of $\frac{2}{5}$
- **o** $\frac{5}{10}$ of $\frac{4}{7}$
- **p** $\frac{6}{9}$ of $\frac{3}{12}$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## 4G

FLUENCY

Example 19

**7** Find:

**a** $\frac{1}{3}$ of 18 **b** $\frac{1}{5}$ of 45 **c** $\frac{2}{3}$ of 24 **d** $\frac{3}{5}$ of 25

**e** $\frac{2}{7}$ of 42 **f** $\frac{1}{4}$ of 16 **g** $\frac{4}{5}$ of 100 **h** $\frac{3}{7}$ of 77

Example 20

**8** Find:

**a** $\frac{5}{2} \times \frac{7}{3}$ **b** $\frac{6}{5} \times \frac{11}{7}$ **c** $\frac{6}{4} \times \frac{11}{5}$ **d** $\frac{9}{6} \times \frac{13}{4}$

**e** $\frac{8}{5} \times \frac{10}{3}$ **f** $\frac{21}{4} \times \frac{8}{6}$ **g** $\frac{10}{7} \times \frac{21}{5}$ **h** $\frac{14}{9} \times \frac{15}{7}$

Example 21

**9** Find:

**a** $1\frac{3}{5} \times 2\frac{1}{3}$ **b** $1\frac{1}{7} \times 1\frac{2}{9}$ **c** $3\frac{1}{4} \times 2\frac{2}{5}$ **d** $4\frac{2}{3} \times 5\frac{1}{7}$

**10** Find:

**a** $\frac{6}{5} \times \frac{8}{3}$ **b** $\frac{1}{2} \times \frac{3}{8}$ **c** $\frac{3}{4}$ of $5\frac{1}{3}$ **d** $7\frac{1}{2} \times 4\frac{2}{5}$

**e** $\frac{3}{7}$ of $\frac{2}{3}$ **f** $1\frac{1}{2} \times 2\frac{1}{4}$ **g** $\frac{8}{9} \times \frac{6}{20}$ **h** $\frac{15}{4} \times \frac{8}{5}$

PROBLEM-SOLVING

11, 12 | 12, 13 | 12–14

**11** At a particular secondary college, $\frac{2}{5}$ of the Year 7 students are boys.

**a** What fraction of the Year 7 students are girls?

**b** If there are 120 Year 7 students, how many boys and girls are there?

**12** To paint one classroom, $2\frac{1}{3}$ litres of paint are required.
How many litres of paint are required to paint five identical classrooms?

**13** A scone recipe requires $1\frac{3}{4}$ cups of self-raising flour and $\frac{3}{4}$ of a cup of cream. James is catering for a large group and needs to quadruple the recipe. How much self-raising flour and how much cream will he need?

**14** Julie has finished an injury-plagued netball season during which she was able to play only $\frac{2}{3}$ of the matches. The season consisted of 21 matches. How many games did Julie miss as a result of injury?

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4G

15 | 15, 16 | 16, 17

REASONING

**15** Not all of the following fraction equations are correct. Copy them into your workbook, then place a tick beside those that are correct and a cross beside those that are wrong. Provide the correct solution for those you marked as incorrect.

**a** $\frac{1}{3}+\frac{1}{4}=\frac{1}{7}$ **b** $\frac{1}{3}+\frac{1}{4}=\frac{1}{12}$ **c** $\frac{1}{3}\times\frac{1}{4}=\frac{2}{7}$

**d** $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$ **e** $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$ **f** $\frac{1}{3}-\frac{1}{4}=\frac{0}{-1}$

**16** Circle the correct alternative for the following statement and justify your answer. Using an example, explain why the other alternatives are incorrect.

*When multiplying a proper fraction by another proper fraction the answer is:*

**A** a whole number **B** a mixed numeral

**C** an improper fraction **D** a proper fraction

**17** Write two fractions that:

**a** multiply to $\frac{3}{5}$ **b** multiply to $\frac{3}{4}$ **c** multiply to $\frac{1}{7}$

Who are we? | — | — | 18

ENRICHMENT

**18 a** Using the clues provided, work out which two fractions are being discussed.

- We are two proper fractions.
- Altogether we consist of four different digits.
- When added together our answer will still be a proper fraction.
- When multiplied together you could carry out some cancelling.
- The result of our product, when simplified, contains no new digits from our original four.
- Three of our digits are prime numbers and the fourth digit is a cube number.

**b** Design your own similar question and develop a set of appropriate clues. Have a classmate try and solve your question.

**c** Design the ultimate challenging 'Who are we?' question. Make sure there is only one possible answer.


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# 4H Dividing fractions

Interactive

Widgets

HOTsheets

Walkthroughs

Remember that division used to be referred to as 'how many'.

Thinking of division as 'how many' helps us to understand dividing fractions.

For example, to find $\frac{1}{2} \div \frac{1}{4}$, think of $\frac{1}{2}$ how many $\frac{1}{4}$s, or how many $\frac{1}{4}$s are in a $\frac{1}{2}$?

Consider this strip of paper that is divided into four equal sections.

In our example of $\frac{1}{2} \div \frac{1}{4}$, we have only $\frac{1}{2}$ a strip, so we will shade in half the strip.

By thinking of the $\div$ sign as 'how many', the question is asking how many quarters are in half the strip. From our diagram, we can see that the answer is 2. Therefore, $\frac{1}{2} \div \frac{1}{4} = 2$.

In a game of football, when it is half-time, you have played two quarters. This is another way of confirming that $\frac{1}{2} \div \frac{1}{4} = 2$.

## Let's start: 'Divvy up' the lolly bag

To 'divvy up' means to divide up, or divide out, or share equally.

Consider a lolly bag containing 24 lollies.
In pairs, students answer the following questions.

- How many lollies would each person get if you 'divvy up' the lollies between three people?
- If you got $\frac{1}{3}$ of the lollies in the bag, how many did you get?

How many ways can these 24 lollies be divided?

Can you see that 'divvying up' by 3 is the same as getting $\frac{1}{3}$? Therefore, $\div 3$ is the same as $\times \frac{1}{3}$.

- How many lollies would each person get if you 'divvy up' the lollies between eight people?
- If you got $\frac{1}{8}$ of the lollies in the bag, how many did you get?

Can you see that 'divvying up' by 8 is the same as getting $\frac{1}{8}$? Therefore, $\div 8$ is the same as $\times \frac{1}{8}$.

- What do you think is the same as dividing by $n$?
- What do you think is the same as dividing by $\frac{a}{b}$?


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## Key ideas

- To find the **reciprocal** of a fraction, you must **invert** the fraction. This is done by swapping the numerator and the denominator. 'Inverting' is sometimes known as turning the fraction upside down, or flipping the fraction.
  - The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

  For example: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
- Dividing by a number is the same as multiplying by its reciprocal.

  For example: $15 \div 3 = 5$ and $15 \times \frac{1}{3} = 5$.
  - Dividing by 2 is the same as multiplying by $\frac{1}{2}$.
- When asked to divide by a fraction, instead choose to multiply by the fraction's reciprocal.

  Therefore, to divide by $\frac{a}{b}$ we multiply by $\frac{b}{a}$.
- When dividing, mixed numbers must be changed to improper fractions.

### Example 22 Finding reciprocals

State the reciprocal of the following.

**a** $\frac{2}{3}$ **b** 5 **c** $1\frac{3}{7}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. | The numerator and denominator are swapped. |
| **b** Reciprocal of 5 is $\frac{1}{5}$. | Think of 5 as $\frac{5}{1}$ and then invert. |
| **c** Reciprocal of $1\frac{3}{7}$ is $\frac{7}{10}$. | Convert $1\frac{3}{7}$ to an improper fraction; i.e. $\frac{10}{7}$, and then invert. |

### Example 23 Dividing a fraction by a whole number

Find:

**a** $\frac{5}{8} \div 3$ **b** $2\frac{3}{11} \div 5$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{5}{8} \div 3 = \frac{5}{8} \times \frac{1}{3}$ | Change the $\div$ sign to a $\times$ sign and invert the 3 $\left(\text{or } \frac{3}{1}\right)$. |
| $= \frac{5}{24}$ | Multiply the numerators and denominators. |


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**b** $2\frac{3}{11} \div 5 = \frac{25}{11} \div \frac{5}{1}$ — Convert the mixed number to an improper fraction. Write 5 as an improper fraction.

$= \frac{{}^{5}\cancel{25}}{11} \times \frac{1}{\cancel{5}^{1}}$ — Change the ÷ sign to a × sign and invert the divisor. Simplify by cancelling.

$= \frac{5}{11}$ — Multiply numerators and denominators.

## Example 24 Dividing a whole number by a fraction

Find:

**a** $6 \div \frac{1}{3}$ **b** $24 \div \frac{3}{4}$

**SOLUTION** **EXPLANATION**

**a** $6 \div \frac{1}{3} = \frac{6}{1} \times \frac{3}{1}$ — Instead of ÷ $\frac{1}{3}$, change to × $\frac{3}{1}$.

$= \frac{18}{1} = 18$ — Simplify.

**b** $24 \div \frac{3}{4} = \frac{{}^{8}\cancel{24}}{1} \times \frac{4}{\cancel{3}^{1}}$ — Instead of ÷ $\frac{3}{4}$, change to × $\frac{4}{3}$.

$= 32$ — Cancel and simplify.

## Example 25 Dividing fractions by fractions

Find:

**a** $\frac{3}{5} \div \frac{3}{8}$ **b** $2\frac{2}{5} \div 1\frac{3}{5}$

**SOLUTION** **EXPLANATION**

**a** $\frac{3}{5} \div \frac{3}{8} = \frac{3}{5} \times \frac{8}{3}$ — Change the ÷ sign to a × sign and invert the divisor. (Note: The divisor is the second fraction.)

$= \frac{8}{5} = 1\frac{3}{5}$ — Cancel and simplify.

**b** $2\frac{2}{5} \div 1\frac{3}{5} = \frac{12}{5} \div \frac{8}{5}$ — Convert mixed numbers to improper fractions.

$= \frac{{}^{3}\cancel{12}}{\cancel{5}^{1}} \times \frac{{}^{1}\cancel{5}}{\cancel{8}^{2}}$ — Change the ÷ sign to a × sign and invert the divisor.

$= \frac{3}{2} = 1\frac{1}{2}$ — Cancel, multiply and simplify.

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## Exercise 4H

1–4 | 4 | —

UNDERSTANDING

**1** Which of the following is the correct first step for finding $\frac{3}{5} \div \frac{4}{7}$?

**A** $\frac{3}{5} \times \frac{7}{4}$ **B** $\frac{5}{3} \times \frac{4}{7}$ **C** $\frac{5}{3} \times \frac{7}{4}$

**2** Write the correct first step for each of these division questions. (Do not go on and find the final answer.)

**a** $\frac{5}{11} \div \frac{3}{5}$ **b** $\frac{1}{3} \div \frac{1}{5}$ **c** $\frac{7}{10} \div \frac{12}{5}$ **d** $\frac{8}{3} \div 3$

**3** When dividing mixed numbers, the first step is to convert to improper fractions and the second step is to multiply by the reciprocal of the divisor. Write the correct first and second steps for each of the following mixed number division questions. (Do not go on and find the final answer.)

**a** $2\frac{1}{2} \div 1\frac{1}{3}$ **b** $24 \div 3\frac{1}{5}$ **c** $4\frac{3}{11} \div 5\frac{1}{4}$ **d** $\frac{8}{3} \div 11\frac{3}{7}$

**4** Make each sentence correct, by inserting the word *more* or *less* in the gap.

**a** $10 \div 2$ gives an answer that is _____ than 10.

**b** $10 \div \frac{1}{2}$ gives an answer that is _____ than 10.

**c** $\frac{3}{4} \div \frac{2}{3}$ gives an answer that is _____ than $\frac{3}{4}$.

**d** $\frac{3}{4} \times \frac{3}{2}$ gives an answer that is _____ than $\frac{3}{4}$.

**e** $\frac{5}{7} \div \frac{8}{5}$ gives an answer that is _____ than $\frac{5}{7}$.

**f** $\frac{5}{7} \times \frac{5}{8}$ gives an answer that is _____ than $\frac{5}{7}$.

5–8(½) | 5–9(½) | 5–9(½)

FLUENCY

Example 22

**5** State the reciprocal of each of the following.

**a** $\frac{5}{7}$ **b** $\frac{3}{5}$ **c** $\frac{2}{9}$ **d** $\frac{1}{8}$

**e** $2\frac{1}{3}$ **f** $4\frac{3}{5}$ **g** $1\frac{5}{6}$ **h** $8\frac{2}{3}$

**i** 12 **j** 101 **k** $\frac{1}{9}$ **l** 1

Example 23

**6** Find:

**a** $\frac{3}{4} \div 2$ **b** $\frac{5}{11} \div 3$ **c** $\frac{8}{5} \div 4$ **d** $\frac{15}{7} \div 3$

**e** $2\frac{1}{4} \div 3$ **f** $5\frac{1}{3} \div 4$ **g** $12\frac{4}{5} \div 8$ **h** $1\frac{13}{14} \div 9$


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## 4H

FLUENCY

Example 24

**7** Find:

**a** $5 \div \frac{1}{4}$ **b** $7 \div \frac{1}{3}$ **c** $10 \div \frac{1}{10}$ **d** $24 \div \frac{1}{5}$

**e** $12 \div \frac{2}{5}$ **f** $15 \div \frac{3}{8}$ **g** $14 \div \frac{7}{2}$ **h** $10 \div \frac{3}{2}$

Example 25

**8** Find:

**a** $\frac{2}{7} \div \frac{2}{5}$ **b** $\frac{1}{5} \div \frac{1}{4}$ **c** $\frac{3}{7} \div \frac{6}{11}$ **d** $\frac{2}{3} \div \frac{8}{9}$

**e** $2\frac{1}{4} \div 1\frac{1}{3}$ **f** $4\frac{1}{5} \div 3\frac{3}{10}$ **g** $12\frac{1}{2} \div 3\frac{3}{4}$ **h** $9\frac{3}{7} \div 12\frac{4}{7}$

**9** Find:

**a** $\frac{3}{8} \div 5$ **b** $22 \div \frac{11}{15}$ **c** $2\frac{2}{5} \div 1\frac{3}{4}$ **d** $\frac{3}{4} \div \frac{9}{4}$

**e** $7 \div \frac{1}{4}$ **f** $2\frac{6}{15} \div 9$ **g** $7\frac{2}{3} \div 1\frac{1}{6}$ **h** $\frac{3}{5} \div \frac{2}{7}$

PROBLEM-SOLVING

10, 11 | 11–13 | 12–14

**10** If $2\frac{1}{4}$ leftover pizzas are to be shared between three friends, what fraction of pizza will each friend receive?

**11** A property developer plans to subdivide $7\frac{1}{2}$ acres of land into blocks of at least $\frac{3}{5}$ of an acre. Through some of the land runs a creek, where a protected species of frog lives. How many of the blocks can the developer sell if two blocks must be reserved for the creek and its surroundings?

**12** Miriam cuts a 10-millimetre thick sisal rope into four equal pieces. If the rope is $3\frac{3}{5}$ metres long before it is cut, how long is each piece?

**13** A carpenter takes $\frac{3}{4}$ of an hour to make a chair. How many chairs can he make in 6 hours?

**14** Justin is a keen runner and regularly runs at a pace of $3\frac{1}{2}$ minutes per kilometre. Justin finished a Sunday morning run in 77 minutes. How far did he run?


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4H

| | 15 | 15, 16 | 16, 17 |
|---|---|---|---|

REASONING

**15** Pair up the equivalent expressions and state the simplified answer.

$\frac{1}{2}$ of 8 | $12 \div 4$ | $10 \times \frac{1}{2}$ | $10 \div 2$

$3 \div \frac{1}{2}$ | $12 \times \frac{1}{4}$ | $\frac{1}{2} \div \frac{1}{8}$ | $3 \times 2$

**16** Find:

**a** $\frac{3}{8} \times \frac{4}{5} \div \frac{2}{3}$ **b** $\frac{3}{8} \div \frac{4}{5} \div \frac{2}{3}$ **c** $\frac{3}{8} \div \frac{4}{5} \times \frac{2}{3}$ **d** $\frac{3}{8} \times \frac{4}{5} \times \frac{2}{3}$

**17 a** A car travels 180 kilometres in $1\frac{1}{2}$ hours. How far will it travel in 2 hours if it travels at the same speed?

**b** A different car took $2\frac{1}{4}$ hours to travel 180 kilometres. How far did it travel in 2 hours, if it maintained the same speed?

| You provide the question | — | — | 18 |
|---|---|---|---|

ENRICHMENT

**18** Listed below are six different answers.
You are required to make up six questions that will result in the following six answers.
All questions must involve a division sign. Your questions should increase in order of difficulty by adding extra operation signs and extra fractions.

**a** Answer 1: $\frac{3}{5}$ **b** Answer 2: $2\frac{1}{3}$ **c** Answer 3: $\frac{7}{1}$

**d** Answer 4: 0 **e** Answer 5: $\frac{1}{100}$ **f** Answer 6: $4\frac{4}{5}$


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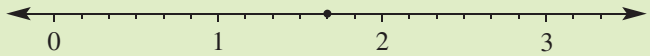
# Progress quiz

**4A** **1** Consider the fraction $\frac{3}{4}$.

- **a** Represent this fraction on a diagram.
- **b** State the denominator of this fraction.
- **c** State the numerator of this fraction.
- **d** Represent this fraction on a number line.
- **e** Is this a proper fraction, an improper fraction or a mixed numeral?

**4A** **2** What fraction is represented on the number line shown?




Write it as an improper fraction and as a mixed number.

**4B** **3** Write three equivalent fractions for $\frac{2}{5}$.

**4B/C** **4** Write these fractions in simplest form.

- **a** $\frac{4}{10}$
- **b** $\frac{15}{30}$
- **c** $\frac{14}{6}$
- **d** $\frac{24}{8}$

**4C** **5** Convert $1\frac{3}{5}$ to an improper fraction.

**4C** **6** Convert $\frac{13}{4}$ to a mixed number.

**4D** **7** Place the correct mathematical symbol $<, =$ or $>$ between the following pairs of fractions to make true mathematical statements.

- **a** $\frac{2}{3} \quad \frac{5}{9}$
- **b** $\frac{4}{5} \quad \frac{24}{30}$
- **c** $1\frac{1}{5} \quad \frac{12}{10}$
- **d** $\frac{5}{9} \quad \frac{18}{20}$

**4D** **8** Write the following fractions in ascending order: $\frac{1}{2}, \frac{2}{3}, \frac{9}{4}, \frac{4}{9}$.

**4E** **9** Add the following fractions together.

- **a** $\frac{4}{7} + \frac{2}{7}$
- **b** $\frac{2}{5} + \frac{3}{10}$
- **c** $\frac{3}{4} + \frac{2}{5}$
- **d** $1\frac{3}{4} + 3\frac{1}{2}$

**4F** **10** Simplify:

- **a** $\frac{5}{12} - \frac{1}{3}$
- **b** $\frac{5}{6} - \frac{1}{4}$
- **c** $5\frac{1}{2} - 3\frac{1}{5}$

**4G** **11** Find:

- **a** $\frac{3}{5}$ of \$560
- **b** $\frac{2}{3} \times \frac{7}{11}$
- **c** $\frac{2}{3} \times \frac{9}{10}$
- **d** $1\frac{1}{3} \times \frac{9}{16}$

**4H** **12** State the reciprocal of:

- **a** $\frac{4}{5}$
- **b** 6
- **c** $2\frac{1}{2}$
- **d** $\frac{1}{12}$

**4H** **13** Find:

- **a** $\frac{7}{8} \div 4$
- **b** $8 \div \frac{3}{4}$
- **c** $\frac{3}{5} \div \frac{7}{15}$

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# 4I Fractions and percentages

Interactive

Widgets

HOTsheets

Walkthroughs

We come across percentages in many everyday situations. Interest rates, discounts, test results and statistics are just some of the common ways in which we deal with percentages. Percentages are closely related to fractions. A percentage is another way of writing a fraction with a denominator of 100. Therefore, 87% means that if something is divided into 100 pieces you would have 87 of them.

A fraction can be interpreted as a percentage of the total.

## Let's start: Student ranking

Five students completed five different Mathematics tests. Each of the tests was out of a different number of marks. The results are shown below. Your task is to rank the five students in descending order, according to their test result.

- Matthew scored 15 out of a possible 20 marks.
- Mengna scored 36 out of a possible 50 marks.
- Maria scored 33 out of a possible 40 marks.
- Marcus scored 7 out of a possible 10 marks.
- Melissa scored 64 out of a possible 80 marks.

Change these test results to equivalent scores out of 100, and therefore state the percentage test score for each student.

## Key ideas

- The symbol, **%**, means 'per cent'. This comes from the Latin words *per centum*, which means out of 100. Therefore, 75% means 75 out of 100.
- We can write percentages as fractions by changing the % sign to a denominator of 100 (meaning out of 100).
  For example: $37\% = \frac{37}{100}$
- We can convert fractions to percentages through our knowledge of equivalent fractions.
  For example: $\frac{1}{4} = \frac{25}{100} = 25\%$
- To convert any fraction to a percentage, multiply by 100.
  For example: $\frac{3}{8} \times 100 = \frac{3}{8} \times \frac{100}{1} = \frac{75}{2} = 37\frac{1}{2}$. Therefore $\frac{3}{8} = 37\frac{1}{2}\%$
- Common percentages and their equivalent fractions are shown in the table below. It is useful to know these.

| Fraction | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{1}{8}$ | $\frac{2}{3}$ | $\frac{3}{4}$ |
|---|---|---|---|---|---|---|---|
| Percentage | 50% | $33\frac{1}{3}\%$ | 25% | 20% | $12\frac{1}{2}\%$ | $66\frac{2}{3}\%$ | 75% |


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## Example 26 Converting percentages to fractions

Express these percentages as fractions or mixed numbers in their simplest form.

**a** 17% **b** 36% **c** 140%

| SOLUTION | EXPLANATION |
|---|---|
| **a** $17\% = \frac{17}{100}$ | Change % sign to a denominator of 100. |
| **b** $36\% = \frac{36}{100}$ | Change % sign to a denominator of 100. |
| $= \frac{9 \times 4}{25 \times 4}$ | Cancel HCF. |
| $= \frac{9}{25}$ | Answer is now in simplest form. |
| **c** $140\% = \frac{140}{100}$ | Change % sign to a denominator of 100. |
| $= \frac{7 \times 20}{5 \times 20}$ | Cancel HCF. |
| $= \frac{7}{5} = 1\frac{2}{5}$ | Convert answer to a mixed number. |

## Example 27 Converting to percentages through equivalent fractions

Convert the following fractions to percentages.

**a** $\frac{5}{100}$ **b** $\frac{11}{25}$

| SOLUTION | EXPLANATION |
|---|---|
| **a** $\frac{5}{100} = 5\%$ | Denominator is already 100, therefore simply write number as a percentage. |
| **b** $\frac{11}{25} \overset{\times 4}{=} \frac{44}{100}$ | Require denominator to be 100. Therefore, multiply numerator and denominator by 4 to get an equivalent fraction. |
| $= 44\%$ | |


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## Example 28 Converting to percentages by multiplying by 100%

Convert the following fractions to percentages.

**a** $\frac{3}{8}$ **b** $3\frac{3}{5}$

**SOLUTION** | **EXPLANATION**

**a** $\frac{3}{8} \times 100 = \frac{3}{\cancel{8}_{2}} \times \frac{\cancel{100}^{25}}{1}$

$= \frac{75}{2} = 37\frac{1}{2}$

$\therefore \frac{3}{8} = 37\frac{1}{2}\%$

Multiply by 100%.
Simplify by cancelling HCF.
Write your answer as a mixed number.

**b** $3\frac{3}{5} \times 100 = \frac{18}{\cancel{15}_{1}} \times \frac{\cancel{100}^{20}}{1}$

$= 360$

$\therefore 3\frac{3}{5} = 360\%$

Convert mixed number to improper fraction.
Cancel and simplify.

## Exercise 4I

1–3 | 3 | —

UNDERSTANDING

**1** Change these test results to equivalent scores out of 100, and therefore state the percentage.

**a** 7 out of 10 = _____ out of 100 = _____%
**b** 24 out of 50 = _____ out of 100 = _____%
**c** 12 out of 20 = _____ out of 100 = _____%
**d** 1 out of 5 = _____ out of 100 = _____%
**e** 80 out of 200 = _____ out of 100 = _____%
**f** 630 out of 1000 = _____ out of 100 = _____%

**2** Write these fraction sequences into your workbook and write beside each fraction the equivalent percentage value.

**a** $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ **b** $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ **c** $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$

**3** **a** If 14% of students in Year 7 are absent due to illness, what percentage of Year 7 students are at school?
**b** If 80% of the Geography project has been completed, what percentage still needs to be finished?

4–7(½) | 4–7(½) | 4–7(½)

FLUENCY

Example 26a,b

**4** Express these percentages as fractions in their simplest form.

**a** 11% **b** 71% **c** 43% **d** 49%
**e** 25% **f** 30% **g** 15% **h** 88%


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## 4I

FLUENCY

Example 26c

**5** Express these percentages as mixed numbers in their simplest form.

| | | | |
|---|---|---|---|
| **a** 120% | **b** 180% | **c** 237% | **d** 401% |
| **e** 175% | **f** 110% | **g** 316% | **h** 840% |

Example 27

**6** Convert these fractions to percentages, using equivalent fractions.

| | | | |
|---|---|---|---|
| **a** $\frac{8}{100}$ | **b** $\frac{15}{100}$ | **c** $\frac{97}{100}$ | **d** $\frac{50}{100}$ |
| **e** $\frac{7}{20}$ | **f** $\frac{8}{25}$ | **g** $\frac{43}{50}$ | **h** $\frac{18}{20}$ |
| **i** $\frac{56}{50}$ | **j** $\frac{27}{20}$ | **k** $\frac{20}{5}$ | **l** $\frac{16}{10}$ |

Example 28

**7** Convert these fractions to percentages by multiplying by 100%.

| | | | |
|---|---|---|---|
| **a** $\frac{1}{8}$ | **b** $\frac{1}{3}$ | **c** $\frac{4}{15}$ | **d** $\frac{10}{12}$ |
| **e** $1\frac{3}{20}$ | **f** $4\frac{1}{5}$ | **g** $2\frac{36}{40}$ | **h** $\frac{13}{40}$ |

8, 9 | 9, 10 | 9–11

PROBLEM-SOLVING

**8** A bottle of lemonade is only 25% full.
- **a** What fraction of the bottle has been consumed?
- **b** What percentage of the bottle has been consumed?
- **c** What fraction of the bottle is left?
- **d** What percentage of the bottle is left?

**9** A lemon tart is cut into eight equal pieces. What percentage of the tart does each piece represent?

**10** Petrina scores 28 out of 40 on her Fractions test. What is her score as a percentage?

**11** The Heathmont Hornets basketball team have won 14 out of 18 games. They still have two games to play. What is the smallest and the largest percentage of games the Hornets could win for the season?


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41

12 | 12, 13 | 13, 14

REASONING

**12** Lee won his tennis match with the score 6–4, 6–2, 6–1.

**a** What fraction of games did he win?

**b** What percentage of games did he win?

**13** Scott and Penny have just taken out a home loan, with an interest rate of $5\frac{1}{2}\%$. Write this interest rate as a fraction.

**14** Write each of the following percentages as fractions.

**a** $2\frac{1}{2}\%$ **b** $8\frac{1}{4}\%$ **c** $12\frac{1}{2}\%$ **d** $33\frac{1}{3}\%$

## Lottery research

— | — | 15

ENRICHMENT

**15** Conduct research on a major lottery competition. If possible:

**a** Find out, on average, how many tickets are sold each week.

**b** Find out, on average, how many tickets win a prize each week.

**c** Determine the percentage chance of winning a prize.

**d** Determine the percentage chance of winning the various divisions.

**e** Work out the average profit the lottery competition makes each week.


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# 4J Percentage of a number

Interactive

Widgets

HOTsheets

Walkthroughs

A common application of percentages is to find a certain percentage of a given number. Throughout life you will come across many examples where you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, personal improvements, salary increases, commission rates and more.

In this exercise we will focus on the mental calculation of percentages.

Retail sales may involve a percentage of the original price being taken away.

## Let's start: Percentages in your head

It is a useful skill to be able to quickly calculate percentages mentally.
Calculating 10% or 1% is often a good starting point. You can then multiply or divide these values to arrive at other percentage values.

- In pairs, using mental arithmetic only, calculate these 12 percentages.

| | | | |
|---|---|---|---|
| **a** 10% of $120 | **b** 10% of $35 | **c** 20% of $160 | **d** 20% of $90 |
| **e** 30% of $300 | **f** 30% of $40 | **g** 5% of $80 | **h** 5% of $420 |
| **i** 2% of $1400 | **j** 2% of $550 | **k** 12% of $200 | **l** 15% of $60 |

- Check your answers with a classmate or your teacher.
- Design a quick set of 12 questions for a classmate.
- Discuss helpful mental arithmetic skills to increase your speed at calculating percentages.

## Key ideas

■ To find the percentage of a number we:
1. Express the required percentage as a fraction.
2. Change the 'of' to a multiplication sign.
3. Express the number as a fraction.
4. Follow the rules for multiplication of fractions.

$$25\% \text{ of } 60 = \frac{25}{100} \times \frac{60}{1}$$
$$= 15$$

■ Percentage of a number $= \frac{\text{percentage}}{100} \times \text{number}$

## Example 29 Finding the percentage of a number

Find:

**a** 30% of 50 **b** 15% of 400


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| SOLUTION | EXPLANATION |
|---|---|
| **a** $30\% \text{ of } 50 = \frac{30}{100} \times \frac{50}{1}$ | Write % as a fraction. |
| $= \frac{30}{2} = 15$ | Cancel and simplify. |
| Mental arithmetic: | |
| 10% of 50 = 5 | |
| Hence, 30% of 50 = 15. | Multiply by 3 to get 30%. |
| **b** $15\% \text{ of } 400 = \frac{15}{100} \times \frac{400}{1}$ | Write % as a fraction. |
| $= \frac{15 \times 4}{1} = 60$ | Cancel and simplify. |
| Mental arithmetic: | |
| 10% of 400 = 40, 5% of 400 = 20 | Halve to get 5%. |
| Hence, 15% of 400 = 60. | Multiply by 3 to get 15%. |

## Example 30 Solving a worded percentage problem

Jacqueline has saved up $50 to purchase a new pair of jeans. She tries on many different pairs but only likes two styles, Evie and Next. The Evie jeans are normally $70 and are on sale with a 25% discount. The Next jeans retail for $80 and have a 40% discount for the next 24 hours. Can Jacqueline afford either pair of jeans?

| SOLUTION | EXPLANATION |
|---|---|
| **Evie jeans** | |
| Discount = 25% of $70 | Calculate the discount on the Evie jeans. |
| $= \frac{25}{100} \times \frac{70}{1} = \$17.50$ | Find 25% of $70. |
| Sale price = $70 – $17.50 | Find the sale price by subtracting the discount. |
| = $52.50 | |
| **Next jeans** | |
| Discount = 40% of $80 | Calculate the discount on the Next jeans. |
| $= \frac{40}{100} \times \frac{80}{1} = \$32$ | Find 40% of $80. |
| Sale price = $80 – $32 | Find the sale price by subtracting the discount. |
| = $48 | |
| Jacqueline can afford the Next jeans. | |


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## Exercise 4J

1–2 | 2 | —

UNDERSTANDING

**1** Copy and complete the following sentences.

**a** Finding 10% of a quantity is the same as dividing the quantity by _____.
**b** Finding 1% of a quantity is the same as dividing the quantity by _____.
**c** Finding 50% of a quantity is the same as dividing the quantity by _____.
**d** Finding 100% of a quantity is the same as dividing the quantity by _____.
**e** Finding 20% of a quantity is the same as dividing the quantity by _____.
**f** Finding 25% of a quantity is the same as dividing the quantity by _____.

**2** Without calculating the exact values, determine which alternative (**i** or **ii**) has the highest value.

**a** **i** 20% of $400 **ii** 25% of $500
**b** **i** 15% of $3335 **ii** 20% of $4345
**c** **i** 3% of $10 000 **ii** 2% of $900
**d** **i** 88% of $45 **ii** 87% of $35

3–4(½), 5 | 3–4(½), 5, 6(½) | 3–4(½), 5, 6(½)

FLUENCY

Example 29

**3** Find:

| | | | |
|---|---|---|---|
| **a** 50% of 140 | **b** 10% of 360 | **c** 20% of 50 | **d** 30% of 90 |
| **e** 25% of 40 | **f** 25% of 28 | **g** 75% of 200 | **h** 80% of 250 |
| **i** 5% of 80 | **j** 4% of 1200 | **k** 5% of 880 | **l** 2% of 9500 |
| **m** 11% of 200 | **n** 21% of 400 | **o** 12% of 300 | **p** 9% of 700 |

**4** Find:

| | | | |
|---|---|---|---|
| **a** 120% of 80 | **b** 150% of 400 | **c** 110% of 60 | **d** 400% of 25 |
| **e** 125% of 12 | **f** 225% of 32 | **g** 146% of 50 | **h** 3000% of 20 |

**5** Match the questions with their correct answer.

| Questions | Answers |
|---|---|
| 10% of $200 | $8 |
| 20% of $120 | $16 |
| 10% of $80 | $20 |
| 50% of $60 | $24 |
| 20% of $200 | $25 |
| 5% of $500 | $30 |
| 30% of $310 | $40 |
| 10% of $160 | $44 |
| 1% of $6000 | $60 |
| 50% of $88 | $93 |

**6** Find:

| | | |
|---|---|---|
| **a** 30% of $140 | **b** 10% of 240 millimetres | **c** 15% of 60 kilograms |
| **d** 2% of 4500 tonnes | **e** 20% of 40 minutes | **f** 80% of 500 centimetres |
| **g** 5% of 30 grams | **h** 25% of 12 hectares | **i** 120% of 120 seconds |

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4J

PROBLEM-SOLVING

7, 8 | 8–10 | 10–12

**7** Harry scored 70% on his Percentages test. If the test is out of 50 marks, how many marks did Harry score?

Example 30

**8** Grace wants to purchase a new top and has $40 to spend. She really likes a red top that was originally priced at $75 and has a 40% discount ticket on it. At another shop, she also likes a striped hoody, which costs $55. There is 20% off all items in the store on this day. Can Grace afford either of the tops?

**9** In a student survey, 80% of students said they received too much homework. If 300 students were surveyed, how many students felt they get too much homework?

**10** 25% of teenagers say their favourite fruit is watermelon. In a survey of 48 teenagers, how many students would you expect to write watermelon as their favourite fruit?

**11** At Gladesbrook College, 10% of students walk to school, 35% of students catch public transport and the remainder of students are driven to school. If there are 1200 students at the school, find how many students:

- **a** walk to school
- **b** catch public transport
- **c** are driven to school

**12** Anthea has just received a 4% salary increase. Her wage before the increase was $2000 per week.

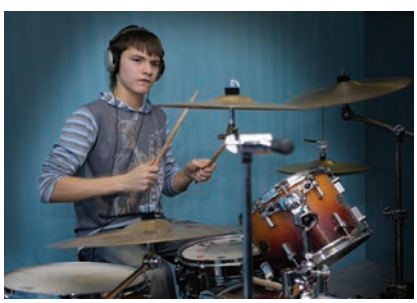
- **a** How much extra money does Anthea receive due to her salary rise?
- **b** What is Anthea's new salary per week?
- **c** How much extra money does Anthea receive per year?

REASONING

13 | 13, 14 | 15–18

**13** Sam has 2 hours of 'free time' before dinner is ready. He spends 25% of that time playing computer games, 20% playing his drums, 40% playing outside and 10% reading a book.

- **a** How long does Sam spend doing each of the four different activities?
- **b** What percentage of time does Sam have remaining at the end of his four activities?
- **c** Sam must set the table for dinner, which takes 5 minutes. Does he still have time to get this done?

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## 4J

REASONING

**14** Gavin mows 60% of the lawn in 48 minutes. How long will it take him to mow the entire lawn if he mows at a constant rate?

**15** Find:

**a** 20% of (50% of 200)
**b** 10% of (30% of 3000)
**c** 5% of (5% of 8000)
**d** 80% of (20% of 400)

**16** Which is larger: 60% of 80 or 80% of 60?

**17** Tom did the following calculation: $120 \div 4 \div 2 \times 3$. What percentage of 120 did Tom find?

**18** **a** If 5% of an amount is \$7, what is 100% of the amount?

**b** If 25% of an amount is \$3, what is $12\frac{1}{2}$% of the amount?

ENRICHMENT

### Waning interest — — 19

**19** When someone loses interest or motivation in a task, they can be described as having a 'waning interest'. Jill and Louise are enthusiastic puzzle makers, but they gradually lose interest when tackling very large puzzles.

**a** Jill is attempting to complete a 5000-piece jigsaw puzzle in 5 weeks. Her interest drops off, completing 100 fewer pieces each week.

**i** How many pieces must Jill complete in the first week to ensure that she finishes the puzzle in the 5-week period?
**ii** What percentage of the puzzle does Jill complete during each of the 5 weeks?
**iii** What is the percentage that Jill's interest wanes each week?

**b** Louise is attempting to complete an 8000-piece jigsaw puzzle in 5 weeks. Her interest drops off at a constant rate of 5% per week.

**i** What percentage of the puzzle must Louise complete in the first week to ensure she finishes the puzzle in the 5-week period?
**ii** Record how many pieces of the puzzle Louise completes each week and the corresponding percentage of the puzzle.
**iii** Produce a table showing the cumulative number of pieces completed and the cumulative percentage of the puzzle completed over the 5-week period.

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# 4K Expressing a quantity as a proportion

Interactive

Widgets

HOTsheets

Walkthroughs

Sometimes we want to know the proportion of a certain quantity compared to a given total or another quantity. This may be done using a fraction, percentage or ratio. The Earth's surface, for example, is about 70% ocean. So, the proportion of land could be written as 30% (as a percentage) or $\frac{3}{10}$ (as a fraction). The ratio of land to ocean could be described as 30 parts of land to 70 parts of ocean. Alternatively, the ratio could be expressed as 3 parts of land to 7 parts of ocean.

The proportion of land to sea in this photo of the Whitsunday Islands, Queensland, could be expressed as a fraction, percentage or ratio.

## Let's start: Tadpole proportion

Scientists Hugh and Jack take separate samples of tadpoles, which include green and brown tadpoles, from their local water channels. Hugh's sample contains 3 green tadpoles and 15 brown tadpoles, whereas Jack's sample contains 27 green tadpoles and 108 brown tadpoles.

- Find the proportion of green tadpoles in each of Hugh and Jack's samples.
- Use both fractions and percentages to compare the proportions.
- Which sample might be used to convince the local council that there are too many brown tadpoles in the water channels?

## Key ideas

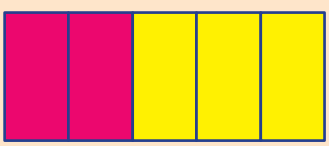
- To express one quantity as a fraction of another:

$$\text{fraction} = \frac{\text{amount}}{\text{total}}$$

- To express one quantity as a percentage of another:

$$\text{percentage} = \frac{\text{amount}}{\text{total}} \times \frac{100}{1}$$

- A ratio compares parts of a total.



$$\text{Red fraction} = \frac{2}{5}$$

$$\text{Red percentage} = \frac{2}{5} \times \frac{100}{1} = 40\%$$

Ratio = 2 parts red to 3 parts yellow

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## Example 31 Expressing as a proportion

Express the following as both a fraction and percentage of the total.

**a** $40 out of a total of $200

**b** 24 green ducks out of a total of 30 ducks

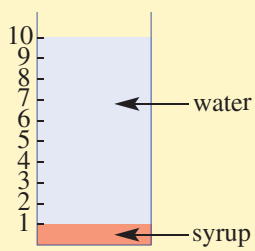
| SOLUTION | EXPLANATION |
|---|---|
| **a** Fraction $= \frac{40}{200}$ | Write the given amount and divide by the total. Then simplify the fraction. |
| $= \frac{1}{5}$ | |
| Percentage $= \frac{40}{200} \times \frac{100}{1}$ | Multiply the fraction by 100 to convert to a percentage. |
| $= 20\%$ | |
| **b** Fraction $= \frac{24}{30}$ | There is a total of 24 brown ducks out of a total of 30. |
| $= \frac{4}{5}$ | |
| Percentage $= \frac{24}{30} \times \frac{100}{1}$ | Use the same fraction and multiply by 100. |
| $= 80\%$ | |

## Example 32 Using ratios

A glass of cordial is 1 part syrup to 9 parts water.

**a** Express the amount of syrup as a fraction of the total.

**b** Express the amount of water as a percentage of the total.




| SOLUTION | EXPLANATION |
|---|---|
| **a** Fraction $= \frac{1}{10}$ | There is a total of 10 parts, including 1 part syrup. |
| **b** Percentage $= \frac{9}{10} \times \frac{100}{1}$ | There is a total 9 parts water in a total of 10 parts. |
| $= 90\%$ | |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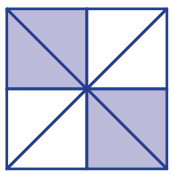
## Exercise 4K

1, 2 | 2 | —

UNDERSTANDING

**1** This square shows some coloured triangles and some white triangles.



- **a** How many triangles are coloured?
- **b** How many triangles are white?
- **c** What fraction of the total is coloured?
- **d** What percentage of the total is coloured?
- **e** What fraction of the total is white?
- **f** What percentage of the total is white?

**2** A farmer's pen has 2 black sheep and 8 white sheep.

- **a** How many sheep are there in total?
- **b** What fraction of the sheep are black?
- **c** What fraction of the sheep are white?
- **d** What percentage of the sheep are black?
- **e** What percentage of the sheep are white?

3(½), 4–6 | 3(½), 4–7 | 3–4(½), 5–7

FLUENCY

Example 31

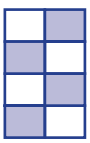
**3** Express the following as both a fraction and a percentage of the total.

- **a** 30 out of a total of 100
- **b** 3 out of a total of 5
- **c** \$10 out of a total of \$50
- **d** \$60 out of a total of \$80
- **e** 2 kg out of a total of 40 kg
- **f** 14 g out of a total of 28 g
- **g** 3 L out of a total of 12 L
- **h** 30 mL out of a total of 200 mL

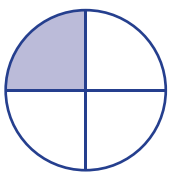
**4** Write each coloured area as both a fraction and percentage of the total area.

**a**

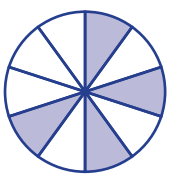
**b**



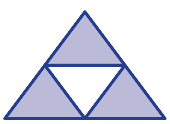
**c**



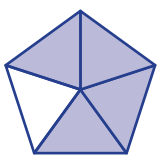
**d**



**e**



**f**



Example 32

**5** A jug of lemonade is made up of 2 parts of lemon juice to 18 parts of water.

- **a** Express the amount of lemon juice as a fraction of the total.
- **b** Express the amount of lemon juice as a percentage of the total.

**6** A mix of concrete is made up of 1 part of cement to 4 parts of sand.

- **a** Express the amount of cement as a fraction of the total.
- **b** Express the amount of cement as a percentage of the total.
- **c** Express the amount of sand as a fraction of the total.
- **d** Express the amount of sand as a percentage of the total.

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## 4K

FLUENCY

**7** A pair of socks is made up of 3 parts of wool to 1 part of nylon.
- **a** Express the amount of wool as a fraction of the total.
- **b** Express the amount of wool as a percentage of the total.
- **c** Express the amount of nylon as a fraction of the total.
- **d** Express the amount of nylon as a percentage of the total.

PROBLEM-SOLVING

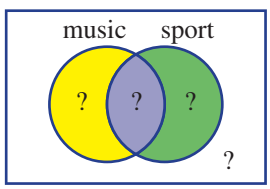
8, 9 | 9–11 | 10–12

**8** Gillian pays $80 tax out of her income of $1600. What percentage of her income does she keep?

**9** Over summer, a dam's water volume reduces from 20 megalitres to 4 megalitres. What fraction of the water in the dam has been lost?

**10** Express the following as a fraction and percentage of the total.
- **a** 20 cents of $5
- **b** 14 days out of 5 weeks
- **c** 15 centimetres removed from a total length of 3 metres
- **d** 3 seconds taken from a world record time of 5 minutes
- **e** 180 grams of a total of 9 kilograms
- **f** 1500 centimetres from a total of 0.6 kilometres

**11** Of 20 students, 10 play sport and 12 play a musical instrument, with some of these students playing both sport and music. Two students do not play any sport or musical instrument.




- **a** What fraction of the students play both sport and a musical instrument?
- **b** What percentage of the students play a musical instrument but not a sport?

**12** An orchard of 80 apple trees is tested for diseases. 20 of the trees have blight disease, 16 have brown rot disease and some trees have both. A total of 48 trees have neither blight nor brown rot.
- **a** What percentage of the trees has both diseases?
- **b** What fraction of the trees has blight but does not have brown rot?

REASONING

13 | 13, 14 | 15–17

**13** For a recent class test, Ross scored 45 out of 60 and Maleisha scored 72 out of 100. Use percentages to show that Ross obtained the higher mark.

**14** The prices of two cars are reduced for sale. A hatch priced at $20 000 is now reduced by $3000 and a 4WD priced at $80 000 is now reduced by $12 800. Determine which car has the largest percentage reduction, giving reasons.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4K

REASONING

**15** A yellow sports drink has 50 g of sugar dissolved in fluid that weighs 250 g, including the weight of the sugar. A blue sports drink has 57 g of sugar dissolved in fluid that weighs 300 g, including the weight of the sugar. Which sports drink has the least percentage of sugar? Give reasons.

**16** A room contains $a$ girls and $b$ boys.

**a** Write an expression using the pronumerals $a$ and $b$ for the fraction of:

**i** boys in the room **ii** girls in the room

**b** Write an expression using the pronumerals $a$ and $b$ for the percentage of:

**i** boys in the room **ii** girls in the room

**17** A mixture of dough has $a$ parts of flour to $b$ parts of water.

**a** Write an expression for the fraction of flour.

**b** Write an expression for the percentage of water.

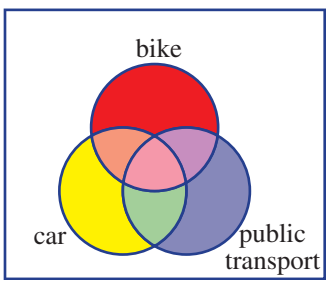
ENRICHMENT

## Transport turmoil — — 18

**18** A class survey of 30 students reveals that the students use three modes of transport to get to school: bike, public transport and car. All of the students used at least one of these three modes of transport in the past week.

Twelve students used a car to get to school and did not use any of the other modes of transport. One student used all three modes of transport and one student used only a bike for the week. There were no students who used both a bike and a car but no public transport. Five students used both a car and public transport but not a bike. Eight students used only public transport.

Use this diagram to help answer the following.




**a** How many students used both a bike and public transport but not a car?

**b** What fraction of the students used all three modes of transport?

**c** What fraction of the students used at least one mode of transport, including a bike?

**d** What fraction of the students used at least one mode of transport, including public transport?

**e** What percentage of students used public transport and a car during the week?

**f** What percentage of students used either public transport or a car or both during the week?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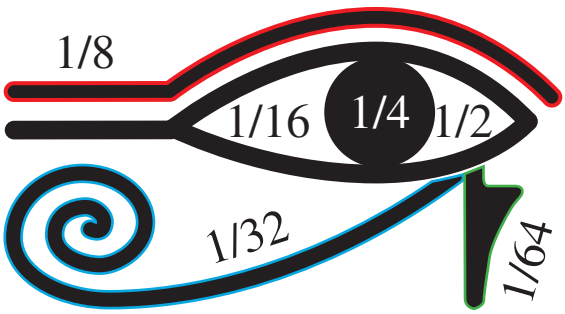
# Investigation

## Egyptian fractions




The fractions in the ancient Egyptian Eye of Horus were used for dividing up food and land, as well as portions of medicine. They are called **unitary** fractions because all the numerators are 1.

Clearly, the ancient Egyptians had no calculators or precise measuring instruments; nevertheless, by repeatedly dividing a quantity in half, the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ or $\frac{1}{32}$ were combined to estimate any other fraction.

Imagine that you are an ancient Egyptian baker and wish to share your last three loaves of bread equally between four people.

First, you cut two loaves in half and give half a loaf to each of your four customers.

You have one loaf remaining and you can cut that into quarters (i.e. half and then half again).

So each of your four customers now receives half a loaf and one-quarter of a loaf, which is $\frac{3}{4}$ of a loaf.

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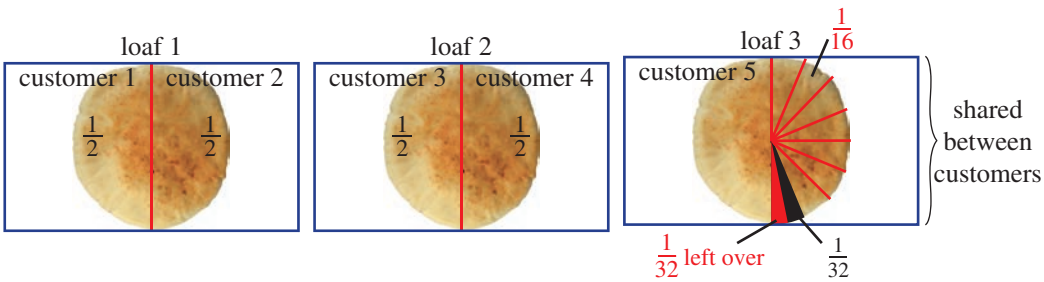
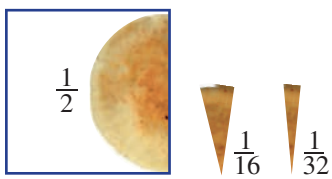
**Using ancient Egyptian fractions, how could three loaves be divided equally between five people?**

First, cut the loaves in half and give each customer $\frac{1}{2}$ ( ) a loaf. The remaining half loaf can be cut into eight parts and each person is given $\frac{1}{8}$ of $\frac{1}{2} = \frac{1}{16}$th ( ) of a loaf. There is a small portion left $\left(3 \text{ portions of } \frac{1}{16}\right)$, so these portions can be divided in half and each customer given $\frac{1}{2}$ of $\frac{1}{16} = \frac{1}{32}$ ( ) of a loaf.




Each customer has an equal share $\frac{1}{2} + \frac{1}{16} + \frac{1}{32}$ ( ) of the loaf and the baker will have the small $\frac{1}{32}$ ( ) of a loaf left over.




If each loaf is divided *exactly* into five parts, the three loaves would have 15 equal parts altogether and each customer could have three parts of the 15; $\frac{3}{15} = \frac{1}{5}$th of the total or $\frac{3}{5}$th of one loaf.

$\frac{3}{5} = 0.6$ and $\frac{1}{2} + \frac{1}{16} + \frac{1}{32} = 0.59375 \approx 0.6$ ($\approx$ means approximately equal).

So even without calculators or sophisticated measuring instruments, the ancient Egyptian method of repeated halving gives quite close approximations to the exact answers.

**Task**

Using diagrams, explain how the following portions can be divided equally using only the ancient Egyptian unitary fractions of $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{32}$

**a** three loaves of bread shared between eight people
**b** one loaf of bread shared between five people
**c** two loaves of bread shared between three people

Include the Egyptian Eye of Horus symbols for each answer, and determine the difference between the exact answer and the approximate answer found using the ancient Egyptian method.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# Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

**1** Find the sum of all fractions in the form $\frac{a}{b}$ where the numerator $a$ is less than the denominator $b$, and $b$ is an integer from 2 to 10 inclusive.

**2** At the end of each practice session, Coach Andy rewards his swim team by distributing 30 pieces of chocolate according to effort. Each swimmer receives a *different* number of whole pieces of chocolate. Suggest possible numbers (all different) of chocolate pieces for each swimmer attending practice when the chocolate is shared between:

**a** 4 swimmers
**b** 5 swimmers
**c** 6 swimmers
**d** 7 swimmers

**3** In this magic square the sum of the fractions in each row, column and diagonal is the same. Find the value of each letter in this magic square.

| | | |
|---|---|---|
| $\frac{2}{5}$ | A | $\frac{4}{5}$ |
| B | C | D |
| E | $\frac{1}{2}$ | 1 |

**4** You are given four fractions: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$. Using any three of these fractions, complete each number sentence below.

**a** $\_\_\_\_ + \_\_\_\_ \times \_\_\_\_ = \frac{7}{30}$

**b** $\_\_\_\_ \div \_\_\_\_ - \_\_\_\_ = \frac{7}{6}$

**c** $\_\_\_\_ + \_\_\_\_ - \_\_\_\_ = \frac{13}{60}$

**5** When a \$50 item is increased by 20%, the final price is \$60. Yet when a \$60 item is reduced by 20%, the final price is not \$50. Explain.


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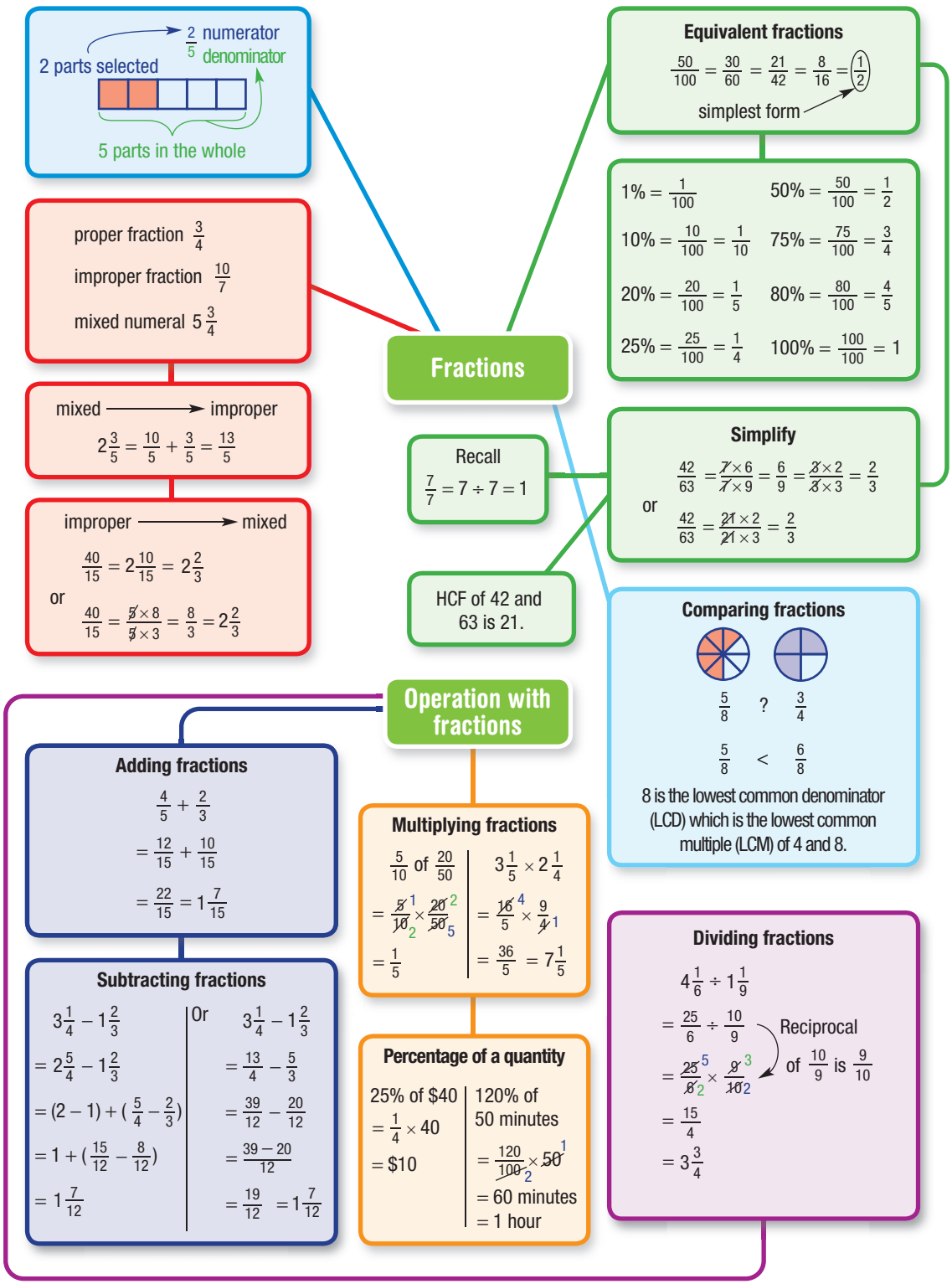
Chapter summary

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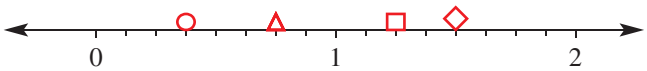
Chapter review

## Multiple-choice questions

4A **1** Which set of fractions corresponds to each of the different shapes positioned on the number line?




**A** $\frac{3}{8}, \frac{6}{8}, 1\frac{3}{8}, \frac{12}{8}$ **B** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{4}, \frac{12}{8}$ **C** $\frac{1}{2}, \frac{3}{4}, \frac{9}{8}, 1\frac{5}{8}$

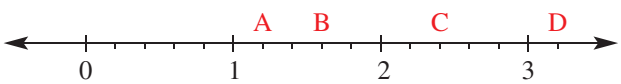
**D** $\frac{2}{8}, \frac{3}{4}, 1\frac{3}{8}, 1\frac{1}{2}$ **E** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{2}, \frac{14}{8}$

4B **2** Which of the following statements is not true?

**A** $\frac{3}{4} = \frac{9}{12}$ **B** $\frac{6}{11} = \frac{18}{33}$ **C** $\frac{3}{10} = \frac{15}{40}$

**D** $\frac{13}{14} = \frac{39}{42}$ **E** $\frac{2}{7} = \frac{16}{56}$

4C **3** Which set of mixed numbers corresponds to the letters written on the number line?




**A** $1\frac{1}{5}, 1\frac{3}{5}, 2\frac{2}{5}, 3\frac{1}{5}$ **B** $1\frac{2}{5}, 1\frac{3}{5}, 2\frac{3}{5}, 3\frac{1}{5}$ **C** $1\frac{1}{5}, 1\frac{2}{5}, 2\frac{2}{5}, 3\frac{2}{5}$

**D** $1\frac{2}{5}, 1\frac{4}{5}, 2\frac{2}{5}, 3\frac{2}{5}$ **E** $1\frac{1}{5}, 1\frac{3}{5}, 2\frac{3}{5}, 3\frac{1}{5}$

4D **4** Which is the lowest common denominator for this set of fractions? $\frac{7}{12}, \frac{11}{15}, \frac{13}{18}$

**A** 60 **B** 120 **C** 180 **D** 3240 **E** 90

4D **5** Which of the following fraction groups is in correct descending order?

**A** $\frac{1}{5}, \frac{1}{3}, \frac{2}{2}$ **B** $\frac{3}{4}, \frac{3}{5}, \frac{3}{8}, \frac{3}{7}$ **C** $\frac{5}{8}, \frac{4}{5}, \frac{3}{8}, \frac{2}{3}$

**D** $\frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{1}{100}$ **E** $2\frac{1}{5}, 2\frac{8}{15}, 2\frac{2}{3}, 2\frac{3}{4}$

4E **6** Which problem has an incorrect answer?

**A** $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ **B** $\frac{3}{4} + \frac{5}{12} = \frac{5}{16}$ **C** $\frac{3}{4} \times \frac{5}{12} = \frac{5}{16}$

**D** $5\frac{2}{3} - 3\frac{1}{4} = 2\frac{5}{12}$ **E** $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

4E **7** Three friends share a pizza. Kate eats $\frac{1}{5}$ of the pizza, Archie eats $\frac{1}{3}$ of the pizza and Luke eats the rest. What fraction of the pizza does Luke eat?

**A** $\frac{4}{12}$ **B** $\frac{2}{3}$ **C** $\frac{14}{15}$ **D** $\frac{7}{15}$ **E** $\frac{8}{15}$

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4D/I **8** Which list is in correct ascending order?

**A** 0.68, $\frac{3}{4}$, 0.76, 77%, $\frac{13}{40}$

**B** $\frac{7}{8}$, 82%, 0.87, $\frac{12}{15}$, 88%

**C** 21%, 0.02, 0.2, 0.22, $\frac{22}{10}$

**D** $\frac{14}{40}$, 0.3666, $0.3\dot{6}$, 37%, $\frac{93}{250}$

**E** 0.76, 72%, $\frac{3}{4}$, 0.68, $\frac{13}{40}$

4C **9** $\frac{60}{14}$ can be written as:

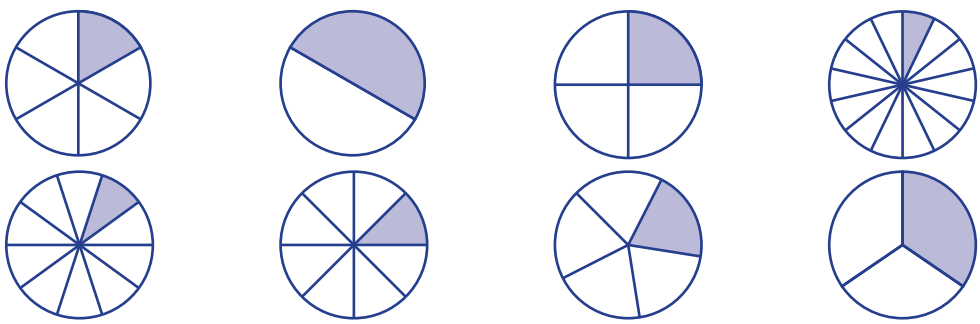
**A** $4\frac{2}{7}$ **B** $2\frac{4}{7}$ **C** $4\frac{2}{14}$ **D** $7\frac{4}{7}$ **E** $5\frac{1}{7}$

4G **10** $\frac{17}{25}$ of a metre of material is needed for a school project. How many centimetres is this?

**A** 65 cm **B** 70 cm **C** 68 cm **D** 60 cm **E** 75 cm

## Short-answer questions

4A **1** List the shaded fractions in correct ascending order.



4B **2** Write four fractions equivalent to $\frac{3}{5}$ and write a sentence to explain why they are equal in value.

4B **3** Write the following fractions in simplest form.

**a** $\frac{18}{30}$ **b** $\frac{8}{28}$ **c** $\frac{35}{49}$

4C **4** Convert each of the following to a mixed number in simplest form.

**a** $\frac{15}{10}$ **b** $\frac{63}{36}$ **c** $\frac{45}{27}$ **d** $\frac{56}{16}$

4D **5** Place the correct mathematical symbol $<, =$ or $>$, in between the following pairs of fractions to make true mathematical statements.

**a** $\frac{2}{7} \square \frac{4}{7}$

**b** $\frac{3}{8} \square \frac{1}{8}$

**c** $1\frac{2}{3} \square 1\frac{3}{5}$

**d** $3\frac{1}{9} \square \frac{29}{9}$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Chapter review

4D **6** State the largest fraction in each list.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ b $\frac{3}{8}, \frac{2}{8}, \frac{5}{8}, \frac{1}{8}$

4D **7** State the lowest common multiple for each pair of numbers.

a 2, 5 b 3, 7 c 8, 12

4D **8** State the lowest common denominator for each set of fractions.

a $\frac{1}{2}, \frac{3}{5}$ b $\frac{2}{3}, \frac{3}{7}$ c $\frac{3}{8}, \frac{5}{12}$

4D **9** Rearrange each set of fractions in descending order.

a $1\frac{3}{5}, \frac{9}{5}, 2\frac{1}{5}$ b $\frac{14}{8}, \frac{11}{6}, \frac{9}{4}, \frac{5}{3}$ c $5\frac{2}{3}, \frac{47}{9}, 5\frac{7}{18}, 5\frac{1}{9}, 5\frac{1}{3}$

4E/F **10** Determine the simplest answer for each of the following.

a $\frac{3}{8}+\frac{1}{8}$ b $\frac{1}{3}+\frac{1}{2}$ c $\frac{3}{8}+\frac{5}{6}$

d $2\frac{7}{15}+3\frac{3}{10}$ e $\frac{7}{8}-\frac{3}{8}$ f $5\frac{1}{4}-2\frac{3}{4}$

g $\frac{3}{4}-\frac{2}{5}+\frac{7}{8}$ h $8\frac{7}{12}-4\frac{7}{9}+2\frac{1}{3}$ i $13\frac{1}{2}+5\frac{7}{10}-6\frac{3}{5}$

4G **11** Find:

a $\frac{1}{3}\times 21$ b $\frac{4}{5}$ of 100 c $\frac{3}{4}$ of 16

d $\frac{8}{10}\times\frac{25}{4}$ e $\frac{2}{3}$ of $\frac{1}{4}$ f $3\frac{1}{8}\times 2\frac{2}{5}$

4H **12** Determine the reciprocal of each of the following.

a $\frac{3}{4}$ b $\frac{7}{12}$ c $2\frac{3}{4}$ d $5\frac{1}{3}$

4H **13** Perform these divisions.

a $\frac{6}{10}\div 3$ b $64\div 3\frac{1}{5}$

c $6\frac{2}{5}\div 1\frac{6}{10}$ d $\frac{3}{8}\div 1\frac{1}{4}\div 1\frac{1}{2}$

4I **14** Copy the table into your workbook and complete.

| Percentage form | 36% | | | 140% | | 18% |
|---|---|---|---|---|---|---|
| Fraction | | $2\frac{1}{5}$ | $\frac{5}{100}$ | | $\frac{11}{25}$ | |

4J **15** Determine which alternative (**i** or **ii**) is the better value discount.

a **i** 25% of \$200 **ii** 20% of \$260

b **i** 5% of \$1200 **ii** 3% of \$1900

4K **16** Express the following as both a fraction and percentage of the total.

a 6 out of 10 b \$4 out of 20

c 50 cents out of \$8 d 600 mL out of 2 L


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## Extended-response questions

**1** Evaluate each of the following.

**a** $3\frac{1}{4}+1\frac{3}{4}\times 2\frac{1}{2}$

**b** $5\div 3\frac{1}{3}+4\frac{3}{8}-\frac{5}{12}$

**c** $7\frac{2}{5}+2\frac{1}{10}\div 2\frac{4}{5}\times 3\frac{3}{4}$

**d** $3\frac{5}{7}+6\frac{1}{4}\div\left(3\frac{3}{8}-\frac{3}{4}\right)$

**2** The length of one side of a triangle is $\frac{5}{12}$ of the perimeter and a second side has length $\frac{5}{28}$ of the perimeter. If these two sides have a total length of 77 centimetres, determine the triangle's perimeter as a mixed number.

**3** **a** A sale on digital cameras offers 20% discount. Determine the sale price of a camera that was originally priced at $220.

**b** The sale price of a DVD is $18. This is 25% less than the original marked price. Determine the original price of this DVD.

**4** Perform the following calculations.

**a** Increase $440 by 25%.

**b** Decrease 300 litres by 12%.

**c** Increase $100 by 10% and then decrease that amount by 10%. Explain the reason for the answer.

**d** When $\$A$ is increased by 20%, the result is $300. Calculate the result if $\$A$ is decreased by 20%.

**5** When a Ripstick is sold for $200 the shop makes 25% profit on the price paid for it.
If this $200 Ripstick is now sold at a discount of 10%, what is the percentage profit of the price at which the shop bought the Ripstick?
At what price should the Ripstick be sold to make 30% profit?

**6** At Sunshine School there are 640 primary school students and 860 secondary students.
For their Christmas family holiday, 70% of primary school students go to the beach and 45% of secondary students go to the beach.
Determine the overall percentage of students in the whole school that has a beach holiday for Christmas. Write this percentage as a mixed number.


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