Investigation

Angle sum of polygons

Equipment required: 1–2 brains, two pieces of paper, ruler and protractor

The Big Question

Is there a relationship between the number of sides of a polygon and the angle sum of that polygon?

Engage

1 **(a)** On a blank sheet of paper, use a ruler to draw a triangle of reasonable size (about a quarter of a page).

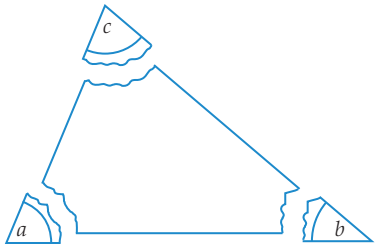
(b) Use a protractor to measure each angle.

(c) Add these angles together. Did you get 180°?

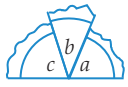
(d) Repeat this with a second triangle.

2 **(a)** Draw a third triangle on a separate piece of paper and cut it out carefully.

(b) Tear off the angles as shown.



(c) Place all the vertices together with arms touching. (Did they form a straight line?)



(d) Paste these angles together on your page.

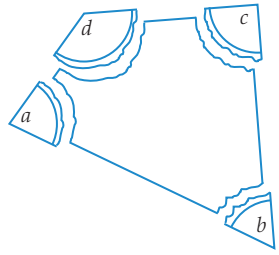
3 **(a)** On your first sheet of paper, use a ruler to draw a quadrilateral.

(b) Measure each angle.

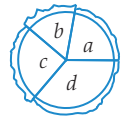
(c) Add these together. Did you get 360°?

(d) Draw another quadrilateral and cut it out carefully.

(e) Tear off the angles as shown.



(f) Place all the vertices together with arms touching. (Did they form a revolution?)



(g) Paste these angles together on your first page.

(h) Draw a second quadrilateral and divide it into two triangles.

(i) Calculate the angle sum of the quadrilateral.

Explore

4 On a blank sheet of paper, use a ruler to draw a pentagon, a hexagon, a heptagon and an octagon. Make sure that each polygon is a reasonable size (about a quarter of a page).

5 Use your protractor to measure the angles of each shape.

6 Find the angle sum for each shape. Did you get a multiple of 180° for each shape?

7 Use straight lines from vertex to vertex to divide each shape into as many triangles as you can, but do not allow any of the lines to cross. Count the number of triangles in each shape.

8 Multiply the number of triangles by 180° for each polygon.

9 Copy the following table and enter your results.

Strategy options

- Make a table.
- Make a model.
- Look for a pattern.

Polygon	Number of angles	Number of triangles	Angle sum (°)	180° × Number of triangles (°)
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				

Explain

10 Explain how you found a relationship between the number of angles (column 1) and the number of triangles (column 2) for each shape.

11 Explain how you found a relationship between the number of angles (or sides) of a polygon and the angle sum of that polygon.

Elaborate

12 Answer the Big Question by writing the rule you have found, describing carefully any of the variables you have used.

13 Use your rule to find the angle sum of a nonagon, a decagon and a dodecagon.

14 Draw each of these and measure each angle.

15 Find the sum of the angles for each shape and compare your answers with your answers in Question **13**.

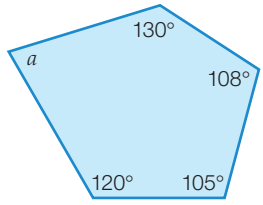
Evaluate

16 Did you get exactly the same answer when you measured your angles and added them as you did when you multiplied the number of triangles by 180°. Why could they be different? Which is more accurate?

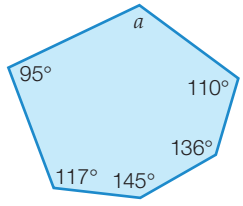
Extend

17 Calculate the size of the angle a in each of the following polygons.

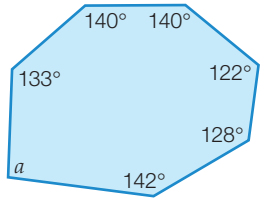
(a)



(b)



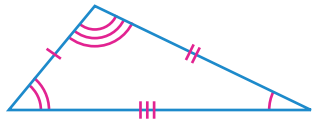
(c)



Triangles

8.6

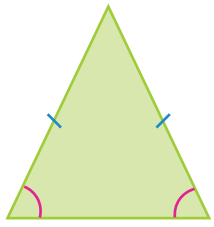
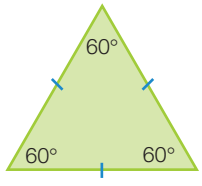
A triangle is the simplest polygon with three straight sides and three angles.



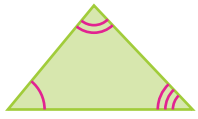
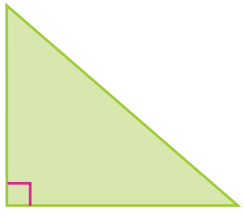
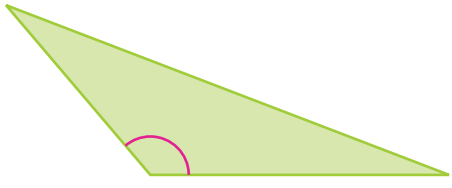
Classifying triangles

Triangles are classified according to their sides or angles.

When using the side lengths to classify a triangle, there are three main types:

Scalene triangle—no equal sides and no equal angles.	
Isosceles triangle—two equal sides and two equal base angles, which are opposite the two equal sides.	
Equilateral triangle—three equal sides and three equal angles of 60° each.	

When using angles to classify a triangle, the three main types are:

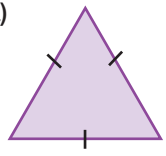
Acute-angled triangle—all three angles are acute (less than 90°).	
Right-angled triangle—one angle is a right angle (equal to 90°).	
Obtuse-angled triangle—one angle is obtuse (greater than 90° but less than 180°).	

When the angles or lines of a shape are marked in a particular way, it means that all other angles or sides with that same marking are equal in size.

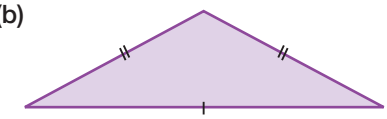
Worked Example 6 WE6

Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

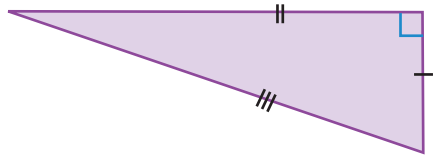
(a)



(b)



(c)



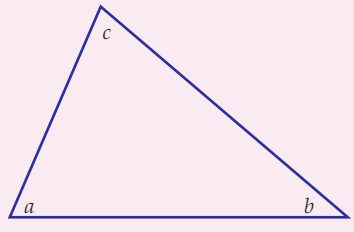
Thinking	Working
(a) (i) How many sides are the same length? (three)	(a) (i) equilateral triangle
(ii) What type of angle is the largest angle? (acute)	(ii) acute-angled triangle
(iii) Combine the side and angle name.	(iii) an acute-angled equilateral triangle
(b) (i) How many sides are the same length? (two equal sides)	(b) (i) isosceles triangle
(ii) What type of angle is the largest angle? (obtuse)	(ii) obtuse-angled triangle
(iii) Combine the side and angle name.	(iii) an obtuse-angled isosceles triangle
(c) (i) How many sides are the same length? (none)	(c) (i) scalene triangle
(ii) What type of angle is the largest angle? (one right angle)	(ii) right-angled triangle
(iii) Combine the side and angle name.	(iii) a right-angled scalene triangle

(Note: In **(a)**, it is unnecessary to describe an equilateral triangle with an angle name, as all angles must be 60° and therefore acute angles.)

Angle sum in a triangle

In any triangle, the three angles have a sum of 180°.

$a + b + c = 180$



Angles in a triangle using technology

We can explore the sum of the angles in a triangle using the GeoGebra geometry software package, or a CAS calculator. Here we will use GeoGebra.

Open a GeoGebra program. Deselect the axes found under the View menu, as they are not needed here, but select the Algebra View.

To construct a triangle, select the polygon tool  . Now, on the drawing pad, left click for the first point, move the cursor and left click again for the second point, repeat for the third point and return to the first point to complete the triangle. Move in an anticlockwise direction so that the interior angles will be measured.

To measure the angles, select the angle tool  and then 'poly' in the Algebra View (the panel on the LHS of the screen). All the interior angles will now be measured.

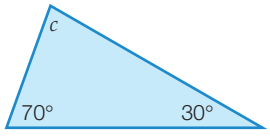
Using the input box at the bottom of the screen, select Sum from the drop-down Command list on the RHS. Use the drop-down variables list (next to Command) to enter the angle variables inside { } brackets; e.g. Sum[{α,β,γ}]. In Algebra View, you will see the sum displayed as $\delta = 180°$.

Use the select tool  to select a vertex of the triangle. Hold down the left click and move the selected point around. You will see the angle change, but the sum remains constant at 180°. Select another vertex and move it around. Repeat this procedure with the third vertex.

Worked Example 7 WE7

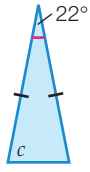
Find the size of the angle labelled c in each of the following triangles.

(a)



(b)

(c)

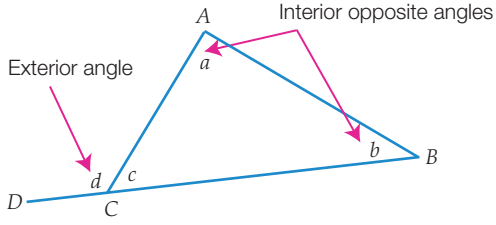


Thinking		Working
(a) **1**	The three angles add to 180°. Write an equation that shows this.	(a) $c + 30 + 70 = 180$
2	Simplify the equation by adding the given angles.	$c + 100 = 180$
3	Solve the equation (subtract 100 from both sides).	$c = 80°$
(b) **1**	The three angles add to 180°. Write an equation that shows this. (Remember that the small square in the triangle indicates a 90° angle.)	(b) $c + 90 + 16 = 180$
2	Simplify the equation by adding the given angles.	$c + 106 = 180$
3	Solve the equation (subtract 106 from both sides).	$c = 74°$

(c) 1	The three angles add to 180°. Write an equation that shows this. (Because this is an isosceles triangle, the two angles opposite the equal sides are equal.)	(c) $c + c + 22 = 180$
2	Simplify the equation by adding like terms.	$2c + 22 = 180$
3	Solve the equation (subtract 22 from both sides, then divide by 2).	$2c = 158$ $c = 79°$

Exterior angle of a triangle

An exterior angle of a triangle is formed on the outside of a triangle when one of the sides is extended.



In the diagram, $\angle d$ is the exterior angle.

There can be six different exterior angles formed on any triangle. Can you find them all? What relationship does the exterior angle have to the interior angles of a triangle?

We know that $\angle a + \angle b + \angle c = 180°$ (angles of a triangle add to 180°)

$\angle a + \angle b + \angle c - \angle c = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle a + \angle b = 180° - \angle c$

$\angle c + \angle d = 180°$ (angles on a straight line add to 180°)

$\angle c - \angle c + \angle d = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle d = 180° - \angle c$

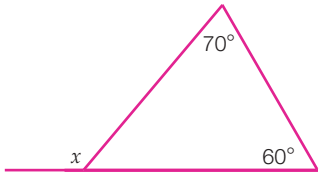
Therefore, $\angle a + \angle b + = \angle d$ (both equal to $180° - \angle c$)

The exterior angle of a triangle is equal to the sum of the two interior opposite angles in the triangle.

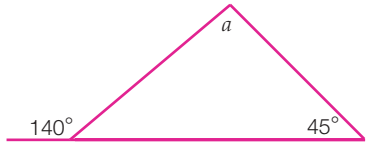
Worked Example 8 WE8

Calculate the value of the pronumeral in each of the following.

(a)



(b)



Thinking	Working
(a) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the exterior angle.	(a) $x = 60 + 70$ $x = 130°$
(b) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the unknown angle.	(b) $140 = a + 45$ $a = 140 - 45$ $a = 95°$

8.6 Triangles

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19

Answers page 671

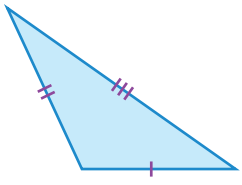
Equipment required: Protractor and ruler for Question **4**

Fluency

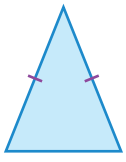
1 Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

WE6

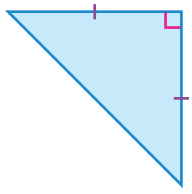
(a)



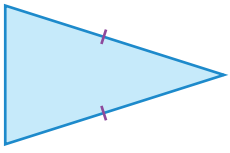
(b)



(c)



(d)



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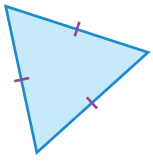

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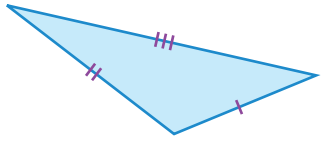

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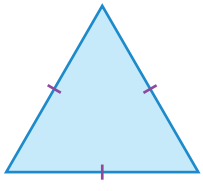

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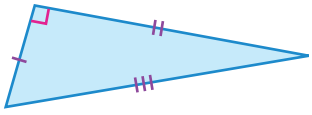
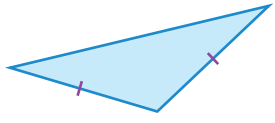
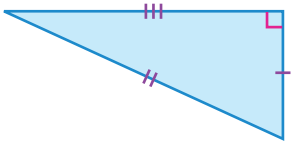


(i)

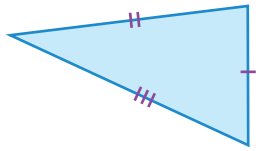


(j)



(k)

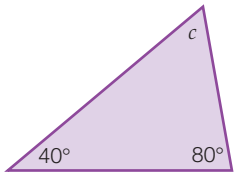
(l)



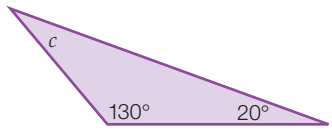
WE7

2 Find the size of the angle labelled c in each of the following triangles.

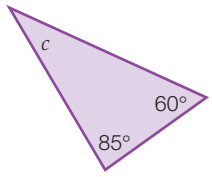
(a)



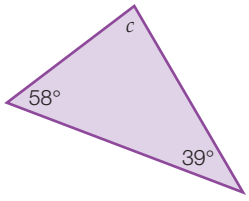
(b)



(c)



(d)



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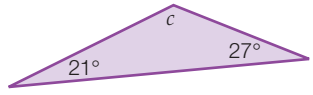

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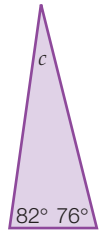

(g)

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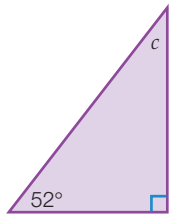
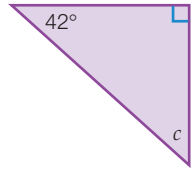


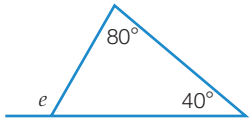
(i)



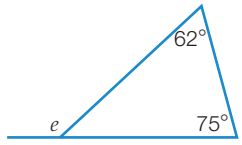
WE8

3 Calculate the value of the pronumeral in each of the following.

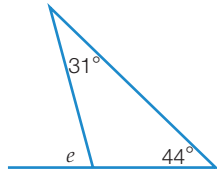
(a) **(i)**



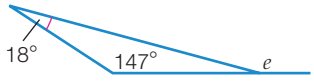
(ii)



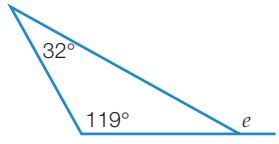
(iii)



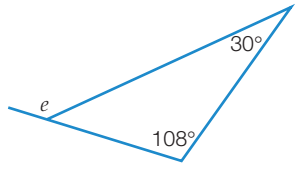
(iv)



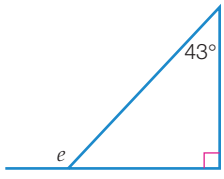
(v)



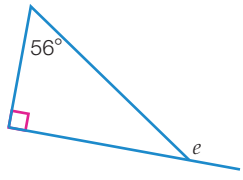
(vi)



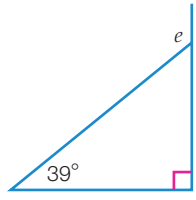
(vii)



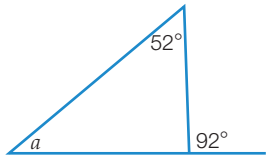
(viii)



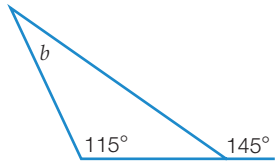
(ix)



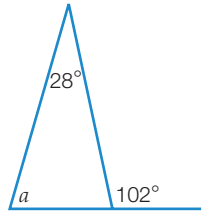
(b) **(i)**



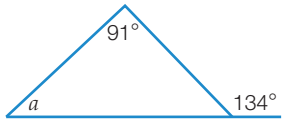
(ii)



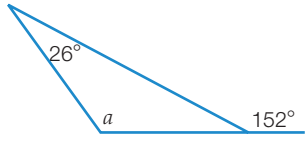
(iii)



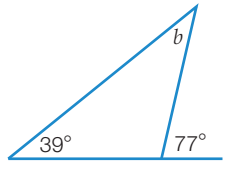
(iv)



(v)



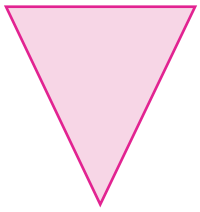
(vi)



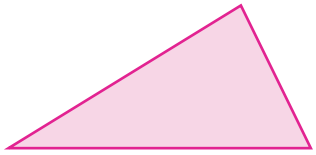
Understanding

4 Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one. Give the angle names as well, using a protractor if necessary to check the angle types.

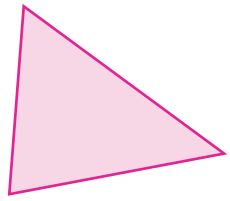
(a)



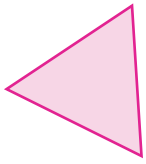
(b)



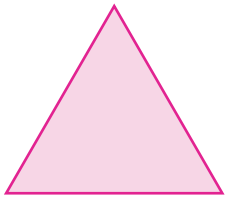
(c)



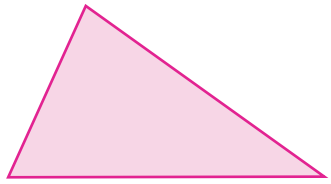
(d)



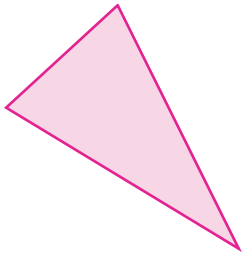
(e)



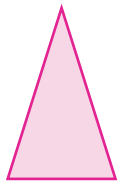
(f)



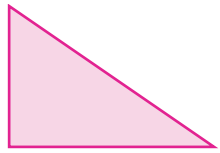
(g)



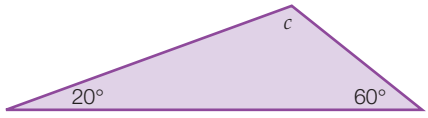
(h)



(i)



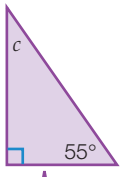
5 (a) The size of angle c is:



A 20° **B** 60°

C 80° **D** 100°

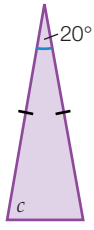
(b) The size of angle c is:



A 35° **B** 45°

C 55° **D** 125°

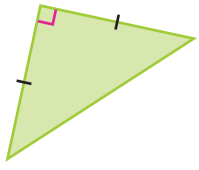
(c) The size of angle c is:



A 20° **B** 80°

C 90° **D** 160°

6 The triangle shown is best described as:



A isosceles, acute-angled

B isosceles, right-angled

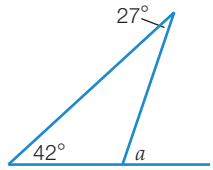
C scalene, acute-angled

D scalene, right-angled

7 **(a)** The value of the pronumeral is:

A 15° **B** 63°

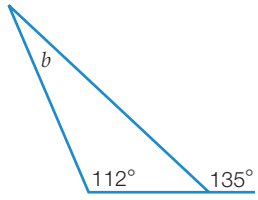
C 69° **D** 111°



(b) The value of the pronumeral is:

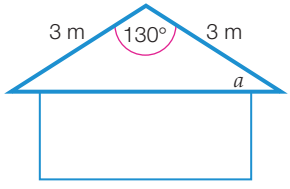
A 23° **B** 36°

C 42° **D** 68°

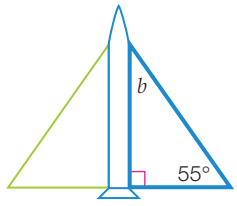


8 Name the type of triangle shown in bold in each diagram below. (Side or angle names may be used.) Find the value of any unknown angles marked with a pronumeral.

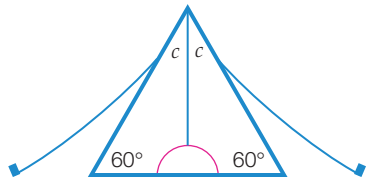
(a)



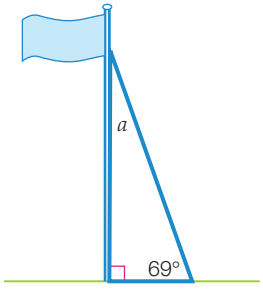
(b)



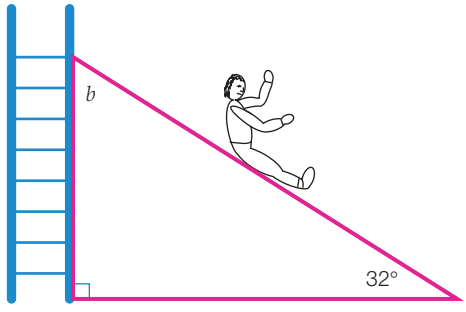
(c)



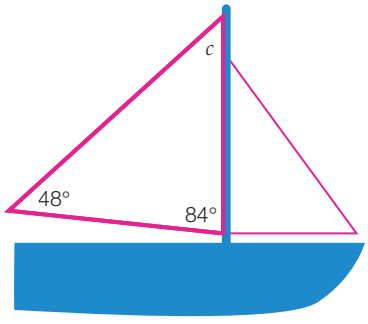
(d)



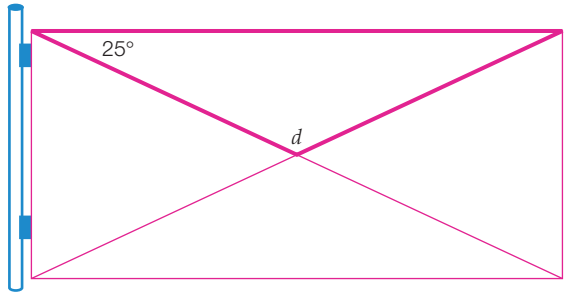
(e)



(f)



(g)



9 The diagonal of a square cuts the square into two triangles. Give the side and angle name of the triangles formed.

10 Two equilateral triangles are placed next to each other so that two sides are joined along their whole length to form a quadrilateral.

(a) What can you say about the side lengths of the quadrilateral (four-sided shape) formed?

(b) What will be the size of each of the angles in the quadrilateral?

Reasoning

11 Is it possible to have a triangle with more than one obtuse angle? Explain by drawing a diagram.

12 What is the minimum number of acute angles a triangle must have?

13 Give the side and angle names of each of the following triangles, with reasons why.

(a) a triangle with one angle measuring 36° and another angle measuring 54°

(b) a triangle with one angle measuring 28° and another angle measuring 124°

(c) a triangle with two angles measuring 60° each

(d) a triangle with one angle measuring 12° and another angle measuring 12°

14 Explain why the following triangles are not possible.

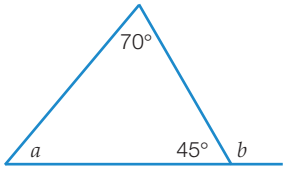
(a) a triangle with angles 55°, 65° and 70°

(b) an equilateral triangle with three 55° angles

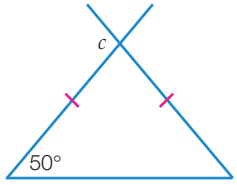
(c) isosceles triangle with 45°, 55° and 80° angles

15 Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

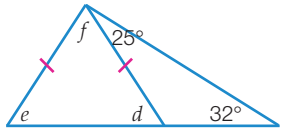
(a)



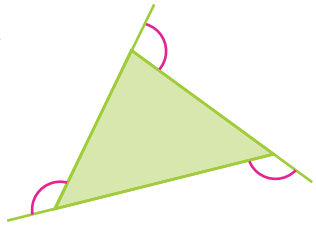
(b)



(c)



16 Find the sum of the marked angles in the diagram on the right without measuring the angles.



Open-ended

17 Draw three different obtuse-angled, scalene triangles.

18 Draw an example of:

(a) an acute-angled scalene triangle

(b) an acute-angled isosceles triangle.

19 Draw three different triangles with an exterior angle between 80° and 100°.

Outside the Square Puzzle

Eggs, spoons and turtles

1 Jane, Scotty, Jessica, Bruce and Michael all took part in their school egg-and-spoon race. Find the order in which they finished the race.

- Bruce finished the race in between the two girls.
- The winner had six letters in their name.
- Jane finished faster than Michael and slower than Jessica.

2 Four students and their turtles took part in the annual School Turtle Championships. Match the students to their turtles, and find the order in which they finished.

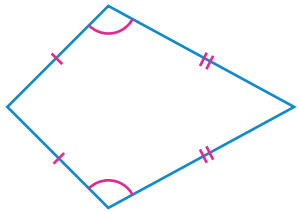
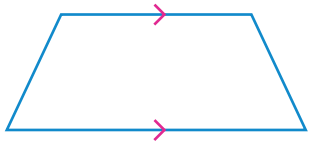
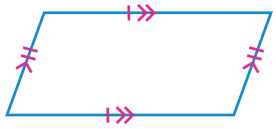
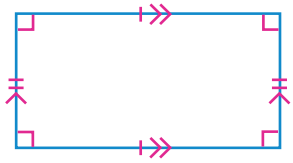
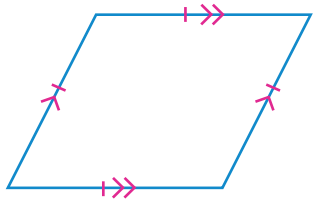
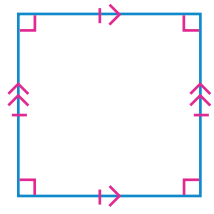
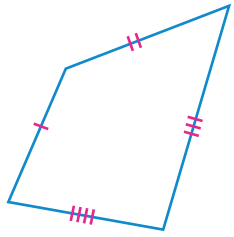
- Chloe owns Sandy, who didn't finish in last place.
- Percy finished immediately in front of Vicky's turtle.
- Myrtle finished behind Ernest, who is owned by Vicky.
- Ethan was celebrating as his turtle finished two places higher than Warren's.

8.7 Quadrilaterals

A plane shape with four straight sides is called a **quadrilateral**. We name quadrilaterals by looking at the properties of their sides and angles.

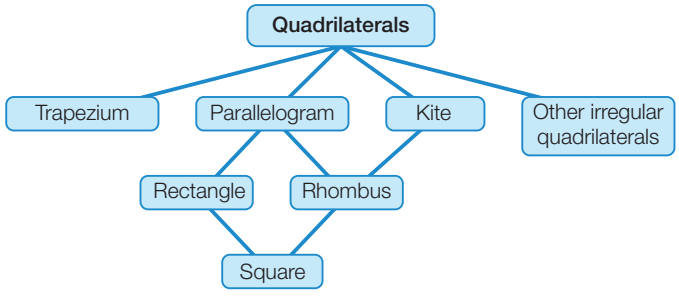
Quadrilaterals

PP Discovery Task

Quadrilateral	Description
Kite 	Two pairs of equal adjacent sides (sides next to each other) and one pair of equal opposite angles.
Trapezium 	Only one pair of parallel sides.
Parallelogram 	Two pairs of parallel sides. Opposite sides and angles are equal.
Rectangle 	Two pairs of opposite sides equal and all internal angles are right angles (equal to 90°).
Rhombus 	All four sides equal; opposite angles are equal. (This can be thought of as a parallelogram with all sides equal or a 'pushed over' square.)
Square 	All sides equal and all internal angles are right angles (equal to 90°).
Irregular quadrilateral 	A four-sided plane shape that has no special qualities.

Parallel side pairs are indicated by marking the side with the same number of arrows in the same direction, as shown.

A quadrilateral may have more than one correct name. For example, a square may also be called a rectangle, because it has all the angle and side properties of a rectangle (two pairs of parallel sides and 90° angles). It could also be called a rhombus, because its pairs of opposite sides are equal and its opposite angles are equal.



Angle sum of a quadrilateral

The sum of the angles in a quadrilateral is 360°.

Angles in a quadrilateral using technology

Use GeoGebra to construct a quadrilateral with the polygon tool as you did to construct a triangle (p. 459). Measure and sum the interior angles. This time, the angle sum will be 360°. Once again, use the select tool to move all four points one by one. The angles will change but the angle sum stays constant at 360°.

Angle properties of special quadrilaterals

The diagonally opposite angles of a parallelogram are equal.

The two interior opposite angles of a kite where unequal sides meet are equal.

8.7 Quadrilaterals

Navigator

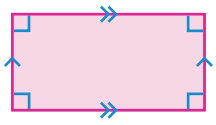
Q1, Q2, Q3, Q4 Column 1, Q5, Q6 Column 1, Q7, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q17, Q20	Q1, Q2, Q3, Q4 Column 2, Q5, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20	Q1, Q2, Q3, Q4 Column 3, Q5, Q6 Column 3, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21

Answers page 672

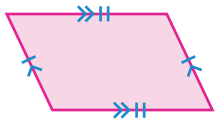
Fluency

1 Give the most accurate name for each shape below.

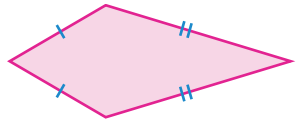
(a)



(b)



(c)



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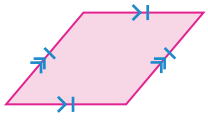
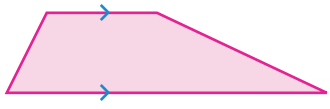
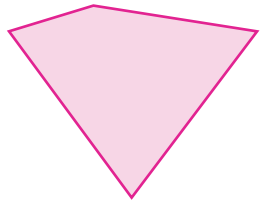


(e)



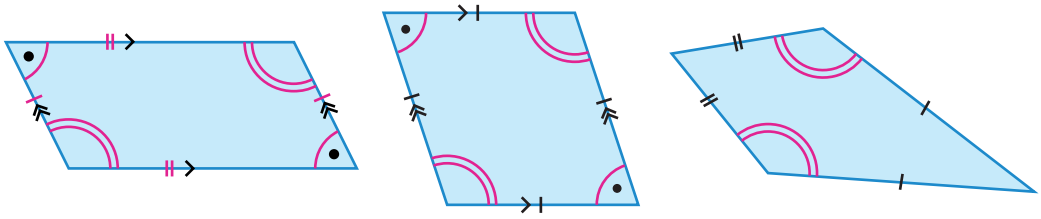
(f)



2 A quadrilateral has both pairs of opposite sides parallel and equal in length. We can be sure this quadrilateral is a:

A parallelogram **B** rhombus **C** rectangle **D** square

3 State TRUE or FALSE for each of the following, referring to the three quadrilaterals below.



(a) The opposite angles in a parallelogram are equal.

(b) All four angles in a parallelogram are equal.

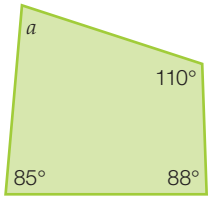
(c) The opposite angles in a rhombus are equal.

(d) All four angles in a rhombus are equal.

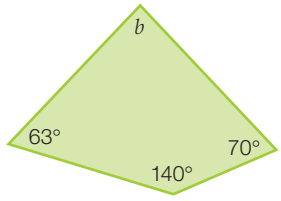
(e) One pair of opposite angles in a kite are equal.

4 Find the size of the unknown angle represented by a letter in each of the following quadrilaterals.

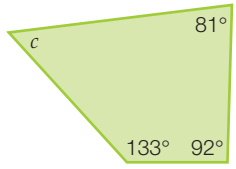
(a)



(b)



(c)



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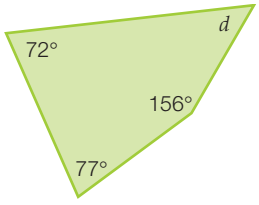
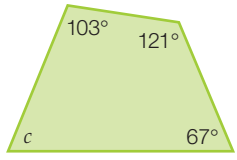


(e)



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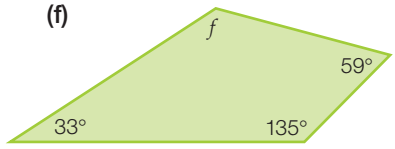
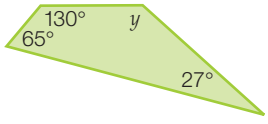
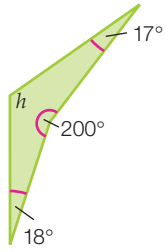


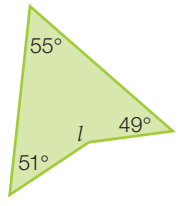
(g)



(h)



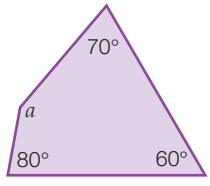
(i)



5 (a) The size of angle a shown is:

A 70° **B** 90°

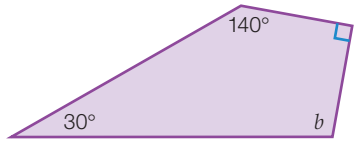
C 100° **D** 150°



(b) The size of angle b shown is:

A 10° **B** 90°

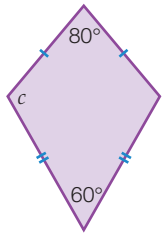
C 100° **D** 170°



(c) The size of angle c shown is:

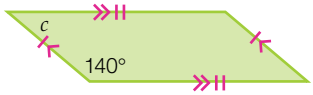
A 55° **B** 80°

C 110° **D** 140°

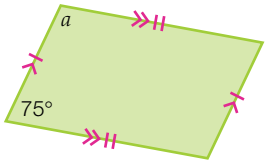


6 Find the size of the angle labelled with a letter in each quadrilateral.

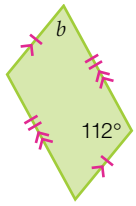
(a)



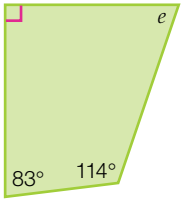
(b)



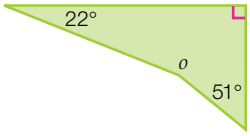
(c)



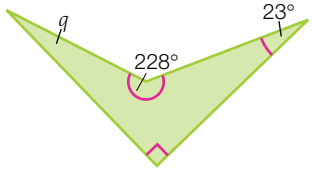
(d)



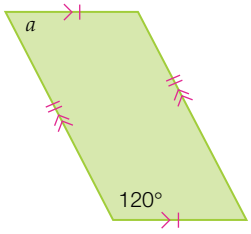
(e)



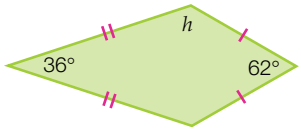
(f)



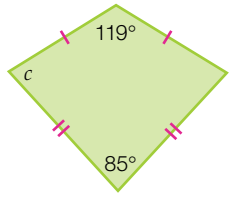
(g)



(h)



(i)

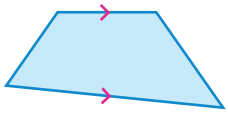


Understanding

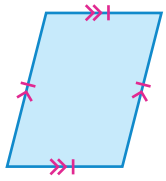
7 The diagrams below were created by a tired illustrator. What type of shape do you think he meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines are correct.

For Question **7**, make sure you do the markings for things such as equal sides, parallel sides and right angles.

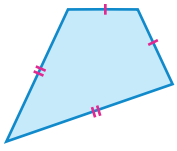
(a)



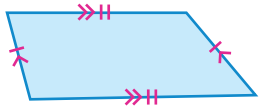
(b)



(c)

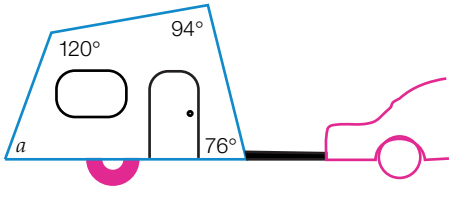


(d)

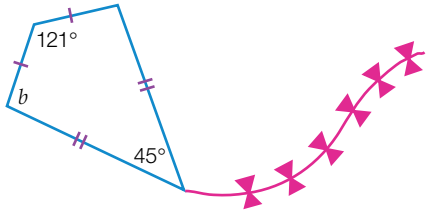


8 Find the angle represented by the pronumeral in each quadrilateral below. Give reasons to justify your answer.

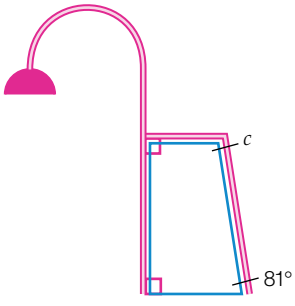
(a)



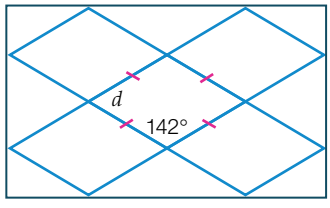
(b)



(c)



(d)



9 An angle in a parallelogram measures 40°. Draw a diagram and mark in the size of each of the other three angles.

10 A quadrilateral has one 45° angle and three unknown angles that are equal to each other. What is the size of each of the three unknown angles?

Reasoning

11 All rhombuses are parallelograms, but not all parallelograms are rhombuses. Explain.

12 Is a parallelogram a type of trapezium? Explain.

13 All rectangles are parallelograms, but not all parallelograms are rectangles. Is this true? Explain why/why not.

14 What is the maximum number of:

(a) obtuse angles a quadrilateral can have? Confirm by drawing a diagram.

(b) reflex angles a quadrilateral can have? Confirm by drawing a diagram.

15 A quadrilateral has two angles of 70° and 34°. The other two angles are equal to each other.

(a) What is the size of the two unknown angles?

(b) What type of quadrilateral could this shape be?

16 Can a quadrilateral have the angles 35°, 85°, 130° and 120°? Explain your answer.

17 What properties of a square make it also classifiable as:

(a) a rectangle **(b)** a rhombus **(c)** a parallelogram?

18 Sam is looking at a kite, which has two given angles of 50° and 120°, but it is unclear which angle is which and where they are located on the kite. He thinks that the best way to solve a problem is to draw a diagram. Help Sam by drawing three different kites that could contain the two known angles of 50° and 120°.

Open-ended

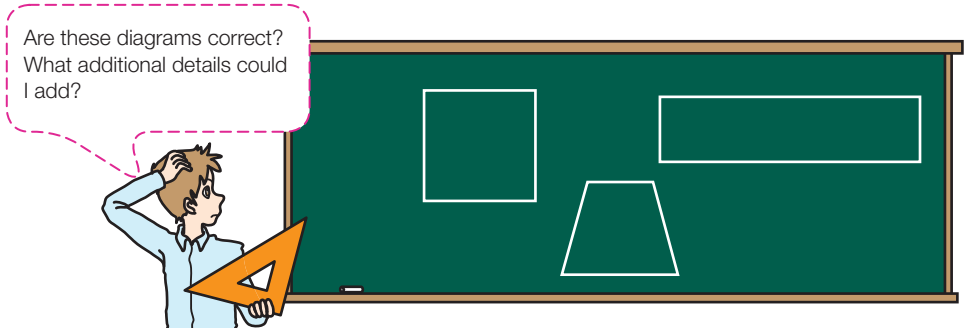
19 A quadrilateral has two angles equal to 80°.

(a) Name one type of quadrilateral that this could be and draw it, marking in the 80° angles.

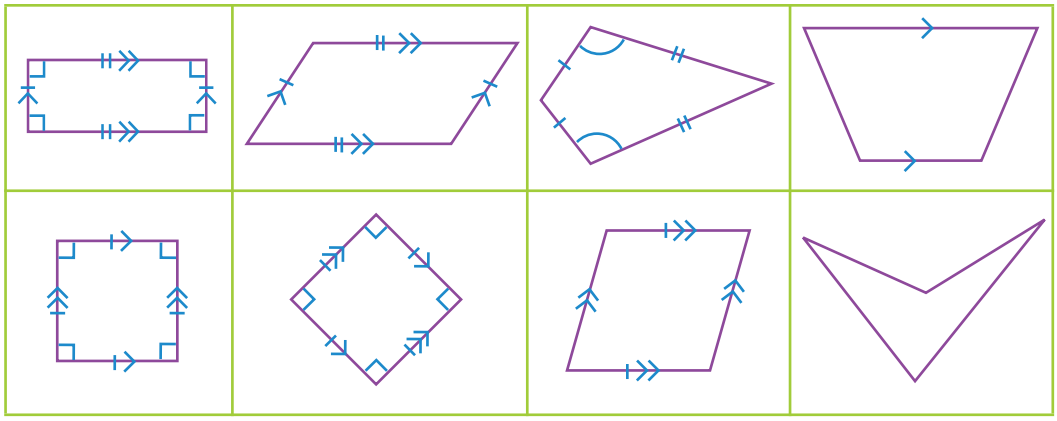
(b) How many different types of quadrilaterals can have two angles equal to 80°? List the different types.

(c) Draw an example of each, marking in the 80° angles.

20 Jack is asked by his teacher to draw a square, a rectangle and a trapezium. Do Jack's diagrams include enough information to clearly depict a square, a rectangle and a trapezium? Redraw his diagrams, adding any missing details.



21 Divide these quadrilaterals into two groups. Explain the reason(s) you used to divide them into each group.

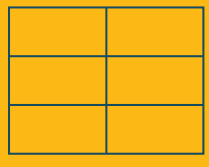


Outside the Square

Problem solving

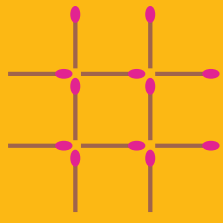
Quadrilateral quandaries

1 How many rectangles can you count here altogether? (Include rectangles tha are the same size even if they are drawn in different positions.)

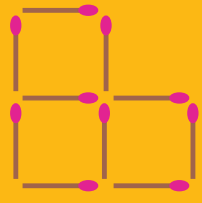


2 Find how many different size squares can be drawn on the grid shown.

3 Move three matchsticks in the arrangement shown to create three squares that touch each other.



4 Remove one match and re-position three others to form one square and two parallelograms.



Strategy options

- Draw a diagram.
- Look for a pattern.