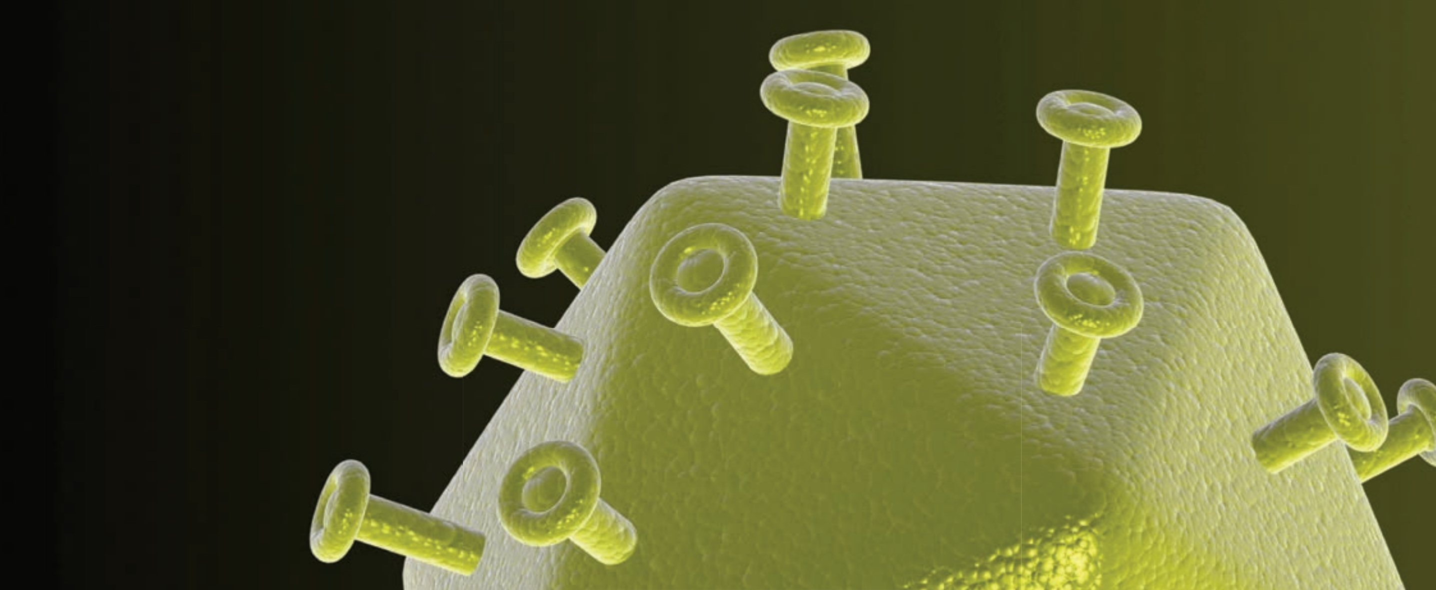

Chapter 9 Polygons, solids and transformations

What you will learn

Australian curriculum

MEASUREMENT AND GEOMETRY

Location and transformation

Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)

Geometric reasoning

Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)

Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)

Shape

Draw different views of prisms and solids formed from combinations of solids (ACMMG161)

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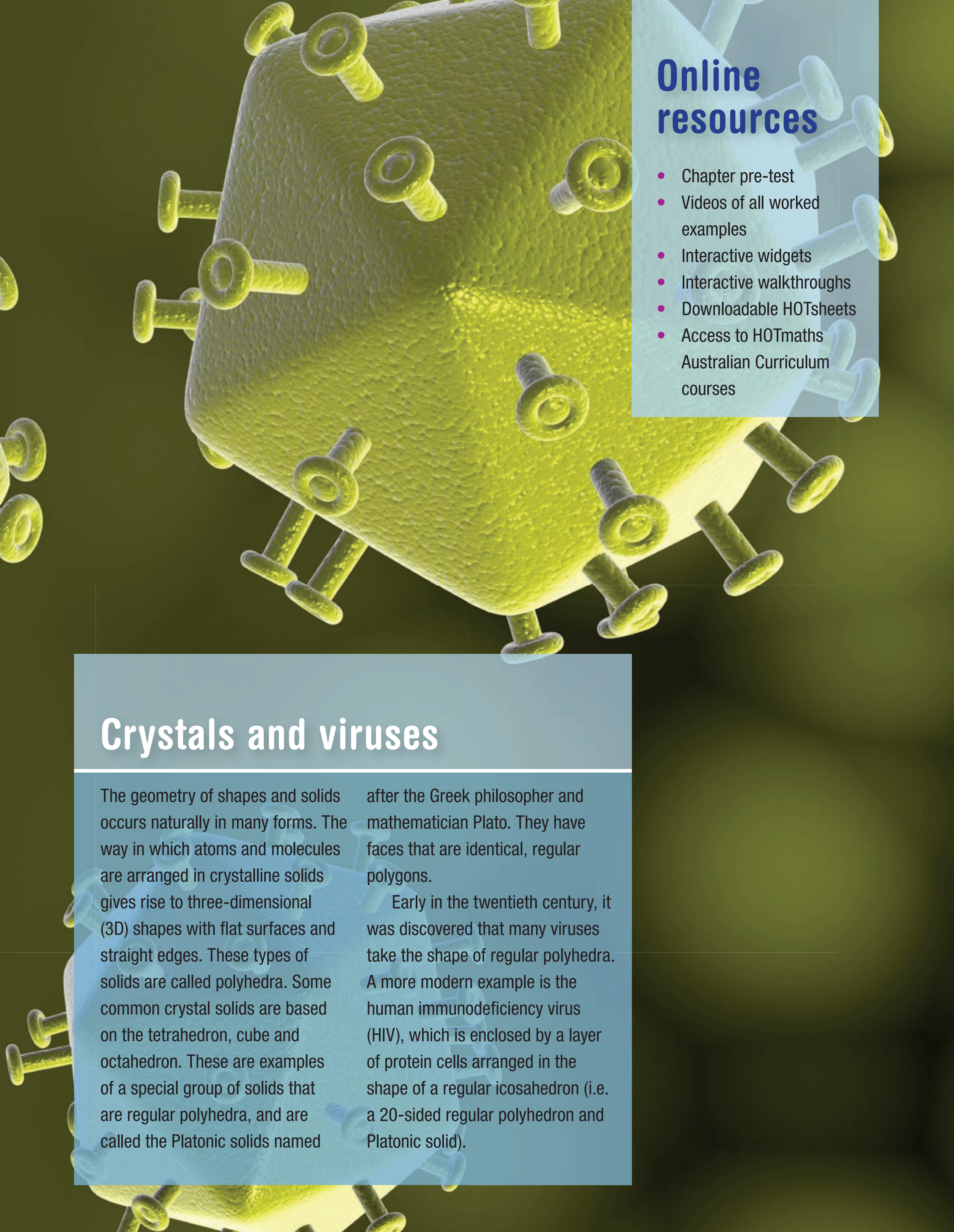

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Crystals and viruses

The geometry of shapes and solids occurs naturally in many forms. The way in which atoms and molecules are arranged in crystalline solids gives rise to three-dimensional (3D) shapes with flat surfaces and straight edges. These types of solids are called polyhedra. Some common crystal solids are based on the tetrahedron, cube and octahedron. These are examples of a special group of solids that are regular polyhedra, and are called the Platonic solids named after the Greek philosopher and mathematician Plato. They have faces that are identical, regular polygons.

Early in the twentieth century, it was discovered that many viruses take the shape of regular polyhedra. A more modern example is the human immunodeficiency virus (HIV), which is enclosed by a layer of protein cells arranged in the shape of a regular icosahedron (i.e. a 20-sided regular polyhedron and Platonic solid).

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9A Polygons

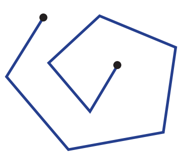
Interactive
Widgets
HOTsheets
Walkthroughs

Polygons are closed plane shapes with straight sides. Each side is a segment and joins with two other sides at points called vertices. The number of sides, angles and vertices are the same for each type of polygon, and this number determines the name of the polygon. The word *polygon* comes from the Greek words *poly* meaning 'many' and *gonia* meaning 'angle'.

This tower has a base that is shaped like a hexagon, or 6-sided polygon.

Let's start: How hard is it to draw an octagon?

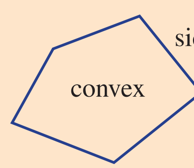
Try to draw an 8-sided shape with no inside angle that is bigger than 180°. This may not be as easy as you think! Remember that you must link the last drawn segment to the point at which you started.



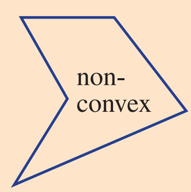
This is not a good example!

Key ideas

- **Polygons** are closed plane figures with straight sides. A side is also called an **edge**.
- A **vertex** is the point at which two sides of a shape meet. (*Vertices* is the plural form of vertex.)



- **Convex** polygons have all vertices pointing outward and all interior (inside) angles smaller than 180°.
- **Non-convex** (or concave) polygons have at least one vertex pointing inward and at least one interior angle bigger than 180°.



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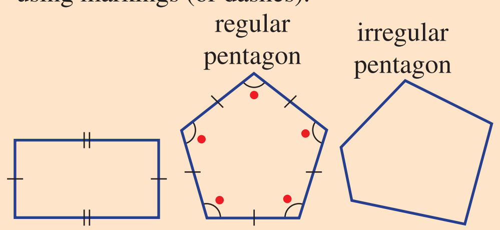
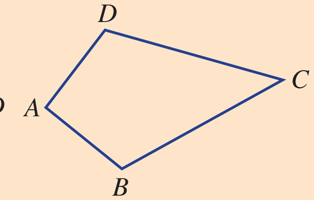
Key ideas

- Polygons are classified by the number of sides they have.
- **Regular** polygons have sides of equal length and angles of equal size.
  - In a diagram, sides of equal length are shown using markings (or dashes).

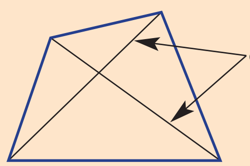


Number of sides	Type
3	Triangle or trigon
4	Quadrilateral or tetragon
5	Pentagon
6	Hexagon
7	Heptagon or septagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- Polygons are usually named with capital letters for each vertex and in succession, clockwise or anticlockwise.

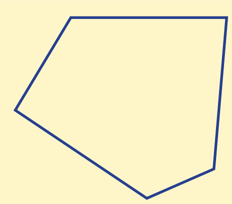


- A **diagonal** is a segment that joins two vertices, dividing a shape into two parts.



Example 1 Classifying polygons

a State the type of this shape and whether it is convex or non-convex.
b Is the shape regular or irregular?



SOLUTION	EXPLANATION
a convex pentagon	The shape has 5 sides and all the vertices are pointing outward.
b irregular	The sides are not of equal length and the angles are not equal.

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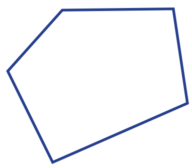
Exercise 9A

UNDERSTANDING		
1, 2	2	—

1 Consider these three polygons.

i


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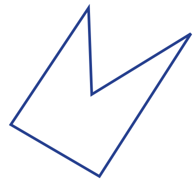

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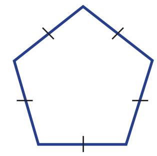


a The three shapes are an example of what type of polygon?
b Which shape(s) are convex and why?
c Which shape(s) are non-convex and why?
d Complete the sentence. The third shape is called a ____________ ____________.

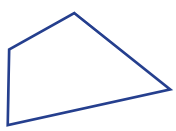
2 Draw an example of each of these shapes.
a convex hexagon **b** non-convex pentagon **c** convex nonagon

FLUENCY		
3–5	3(½), 4, 5, 6(½)	3(½), 4, 5, 6(½)

3 How many sides do each of these shapes have?
a pentagon **b** triangle **c** decagon **d** heptagon
e undecagon **f** quadrilateral **g** nonagon **h** hexagon
i octagon **j** dodecagon

Example 1

4 **a** Which of the given shapes are convex?
b State the type of polygon by considering its number of sides.

i


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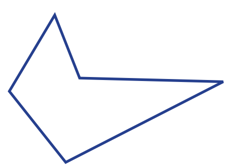

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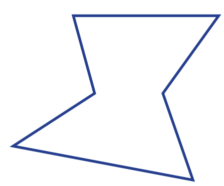

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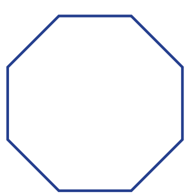
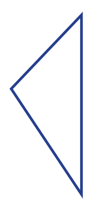
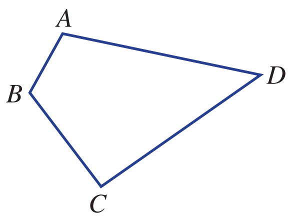


v


vi


5 State the type of polygon and name it, using the vertex labels; e.g. triangle ABC.

a


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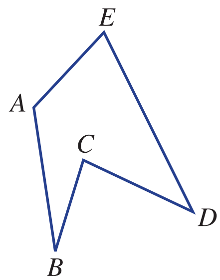
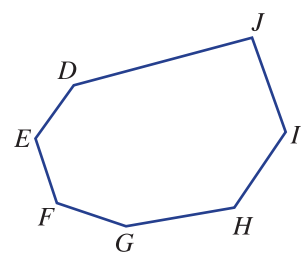


c


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9A

FLUENCY

6 Which of the following are not polygons?

a circle **b** square **c** rectangle **d** oval
e cylinder **f** cube **g** line **h** segment

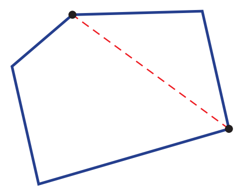
7	7, 8	7, 8

PROBLEM-SOLVING

7 A diagonal between two vertices divides a polygon into two parts.

a What is the maximum (i.e. largest) number of diagonals that can be drawn for the following shapes if the diagonals are *not* allowed to cross?

i convex pentagon **ii** convex decagon



b What is the maximum number of diagonals that can be drawn for the following shapes if the diagonals *are* allowed to cross?

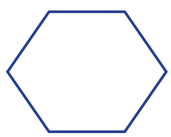
i convex pentagon **ii** convex decagon

8 Draw line segments to show how you would divide the given shapes into the shapes listed below.

a

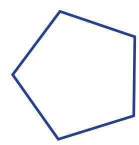
two triangles

b



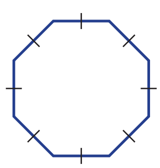
one rectangle and two triangles

c



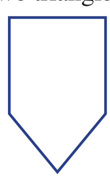
three triangles

d



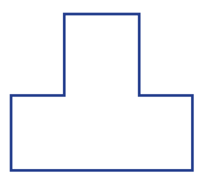
four triangles and one square

e



two quadrilaterals

f



one pentagon and one heptagon

9	9, 10	10–12

REASONING

9 State whether each of the following statements is true or false.

a A regular polygon will have equal interior (i.e. inside) angles.
b The size of the angles inside a pentagon are the same as the angles inside a decagon.
c An irregular polygon must always be non-convex.
d Convex polygons are not always regular.

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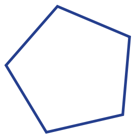
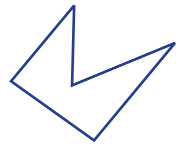
9A

REASONING

10 a For each of these pentagons, draw the five diagonals that join the vertices.

i 

ii 

b What types of polygons have at least one diagonal outside the shape. Why?

11 An equi-angular shape has all of its interior angles of equal size. Are all equi-angular shapes regular polygons? Draw some examples to investigate.

12 An equilateral shape has all of its sides equal. Are all equilateral shapes regular polygons?

Rules for diagonals

—	—	13

ENRICHMENT

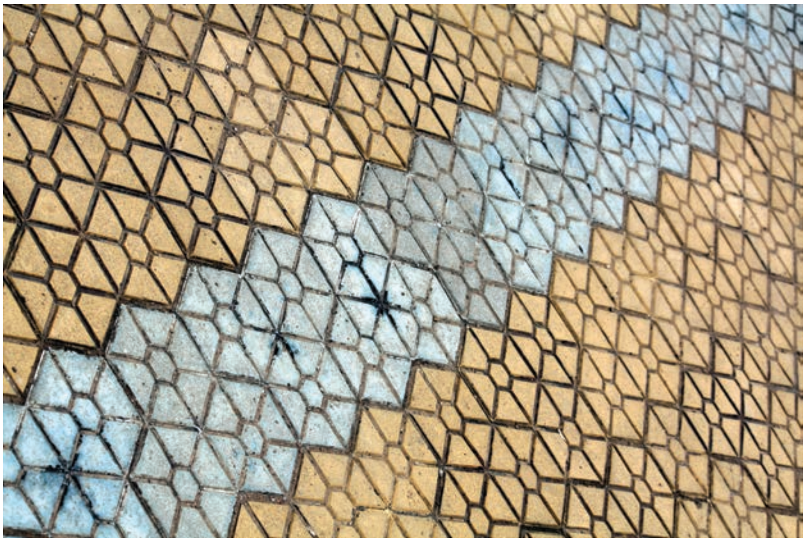
13 a Copy the table below into your workbook and complete it.

Number of sides	3	4	5	6	7
Number of diagonals (not allowed to cross)	0	1			
Number of diagonals (allowed to cross)	0	2			

b If a polygon has n sides, find a rule for:

i the number of diagonals (not allowed to cross)

ii the number of diagonals (allowed to cross)



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9B Triangles

The word 'triangle', meaning 'three angles', describes a shape with three sides. The triangle is an important building block in mathematical geometry. Similarly, it is important in the practical world of building and construction owing to the rigidity of its shape.


Interlocking triangles give this bridge's frame strength and stability.

Interactive

Widgets

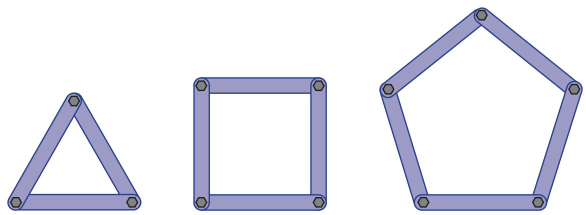
HOTsheets

Walkthroughs

Let's start: Stable shapes

Consider these constructions, which are made from straight pieces of steel and bolts.

Assume that the bolts are not tightened and that there is some looseness at the points where they are joined.

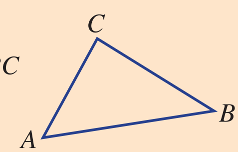


- Which shape(s) do you think could lose their shape if a vertex is pushed?
- Which shape(s) will not lose their shape when pushed? Why?
- For the construction(s) that might lose their shape, what could be done to make them rigid?

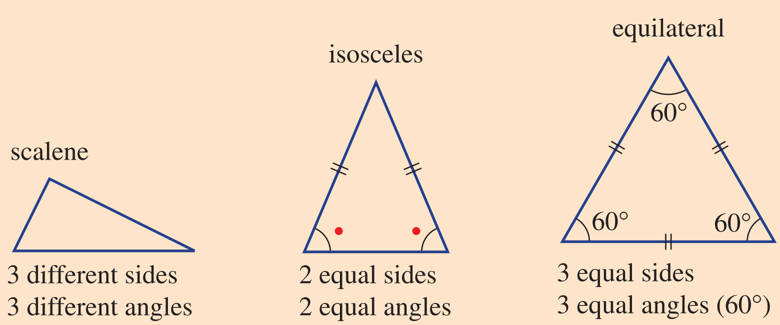
Key ideas

- **Triangles** can be named using the vertex labels.

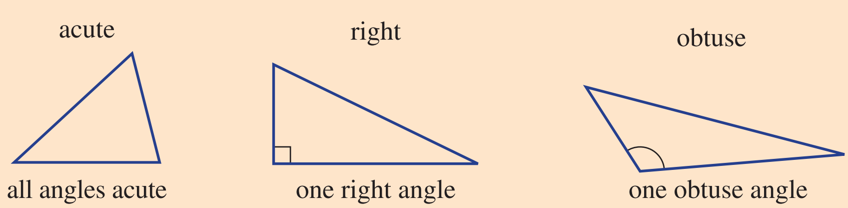
triangle ABC or $\Delta\ ABC$



- Triangles are classified by their **side lengths**.



- Triangles are also classified by the size of their **interior angles**.



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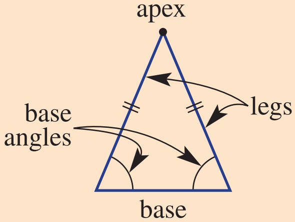
Key ideas

- The parts of an **isosceles triangle** are named as shown opposite. The base angles are equal and two sides (called the legs) are of equal length. The two sides of equal length are opposite the equal angles.

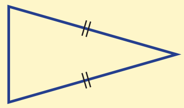


- Sides of equal length are indicated by matching markings.

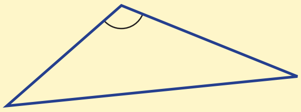
Example 2 Classifying triangles

Classify these triangles by:

a



b



i their side lengths (i.e. scalene, isosceles or equilateral)
ii their angles (i.e. acute, right or obtuse)

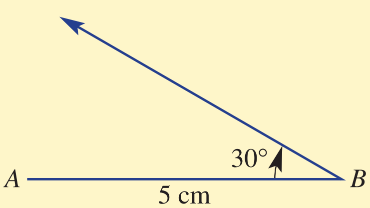
SOLUTION	EXPLANATION
a **i** isosceles	Has 2 sides of equal length.
ii acute	All angles are acute.
b **i** scalene	Has 3 different side lengths.
ii obtuse	Has 1 obtuse angle.

Example 3 Drawing triangles

Draw a triangle ABC with $AB = 5$ cm, $\angle ABC = 30^\circ$ and $\angle BAC = 45^\circ$.

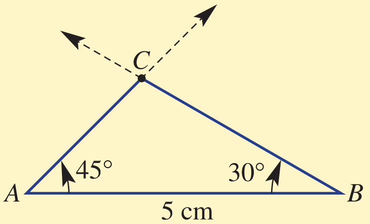
SOLUTION / **EXPLANATION**

a



First, measure and draw segment AB.
Then use a protractor to form the angle 30° at point B.

b



Then use a protractor to form the angle 45° at point A.
Mark point C and join with A and B.

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Exercise 9B

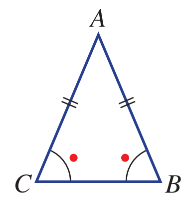
UNDERSTANDING

1, 2	2	—

1 Draw an example of each of the triangles given below. Refer back to the **Key ideas** in this section to check that the features of each triangle are correct.

a scalene **b** isosceles **c** equilateral
d acute **e** right **f** obtuse

2 Answer these questions, using the point labels A, B and C for the given isosceles triangle.



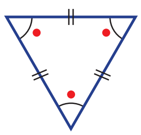
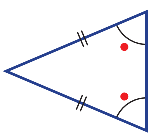
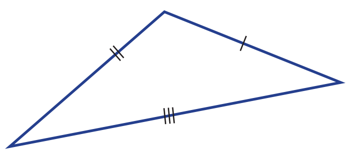
a Which point is the apex?
b Which segment is the base?
c Which two segments are of equal length?
d Which two angles are the base angles?

FLUENCY

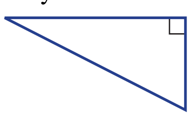
3–5	3–5	3–5

Example 2

3 Classify each of these triangles according to their side lengths (i.e. scalene, isosceles or equilateral).

a

b

c


4 Classify each of these triangles according to their angles (i.e. acute, right or obtuse).

a

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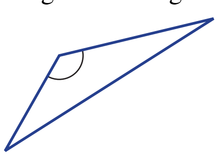
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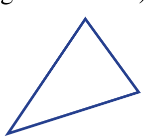

Example 3

5 Use a protractor and ruler to draw the following triangles.

a triangle ABC with $AB = 5$ cm, $\angle ABC = 40^\circ$ and $\angle BAC = 30^\circ$
b triangle DEF with $DE = 6$ cm, $\angle DEF = 50^\circ$ and $\angle EDF = 25^\circ$
c triangle ABC with $AB = 5$ cm, $\angle ABC = 35^\circ$ and $BC = 4$ cm

PROBLEM-SOLVING

6, 7	6, 7	7, 8

6 Is it possible to draw any of the following? If yes, give an example.

a an acute triangle that is also scalene
b a right triangle that is also isosceles
c an equilateral triangle that is also obtuse
d a scalene triangle that is also right angled

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9B

PROBLEM-SOLVING

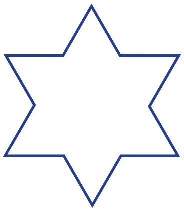
7 What is the smallest number of identical equilateral triangles needed to form each of these shapes?

a a diamond

b an equilateral triangle made up of more than one other equilateral triangle

c a hexagon

d a 6-pointed star (shown opposite) (Note: Overlapping is not allowed.)



8 Draw an example of a triangle that fits the triangle type in both the row and column. Are there any cells in the table for which it is impossible to draw a triangle?

Triangles	Scalene	Isosceles	Equilateral
Acute			
Right			
Obtuse			

9	9, 10	10, 11

REASONING

9 a Is it possible to divide every triangle into two right triangles using one line segment? Explore with diagrams.

b Which type of triangle can always be divided into two *identical* right triangles?

10 Try drawing a triangle with side lengths 4 cm, 5 cm and 10 cm. Explain why this is impossible.

11 a Is the side opposite the largest angle in a triangle always the longest?

b Can you draw a triangle with two obtuse angles? Explain why or why not.

Unique triangles

—	—	12

ENRICHMENT

12 Investigate whether it is possible to draw more than one triangle with the following information. If not, then the single triangle you have drawn is unique.

a $\angle ABC = 20°$, $\angle BCA = 50°$, $\angle BAC = 110°$

b $AB = 5$ cm, $AC = 4$ cm, $\angle BAC = 70°$

c $AB = 6$ cm, $\angle BAC = 70°$

d $AB = 5$ cm, $BC = 4$ cm, $\angle BAC = 50°$

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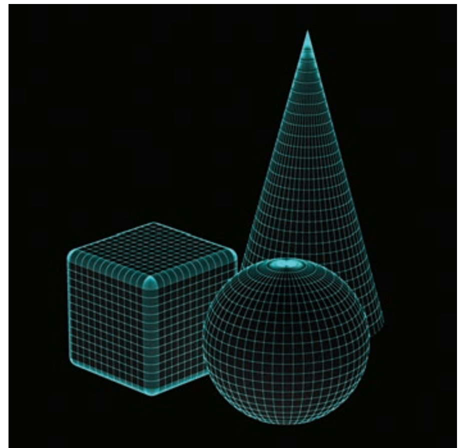
9C Constructing triangles EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

Triangles can be constructed with a high degree of accuracy using a ruler and a pair of compasses. Alternatively, computer geometry can be a useful tool for assisting in the construction process, as well as for exploring general properties of shapes.


Geometry software allows you to draw and manipulate triangles and other shapes.

Let's start: I am not unique

Use a protractor and ruler to draw a triangle ABC with the properties:

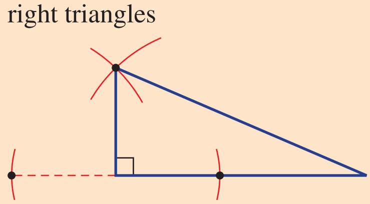
$AB = 6$ cm, $\angle BAC = 20^\circ$ and $BC = 5$ cm

- Compare your triangle with those drawn by other students.
- Is your triangle the same shape? If not, can you explain why?
- How many triangles are possible?

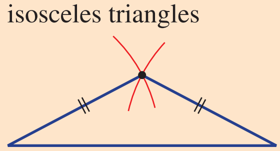
Key ideas

- **Arcs** drawn using a pair of compasses can help to construct triangles accurately.
- Computer geometry or a pair of compasses can be used to accurately construct:

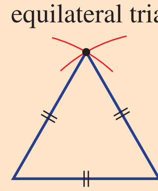
right triangles



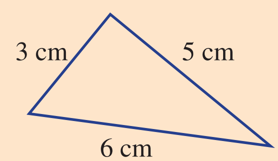
isosceles triangles



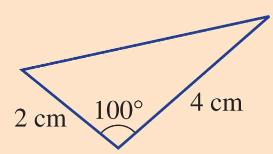
equilateral triangles



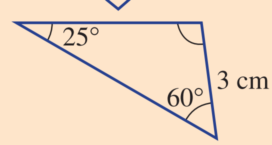
- The following information is sufficient to construct a single **unique triangle**.
  - 3 sides (**SSS**)



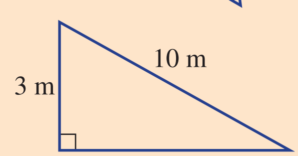
  - 2 sides and the angle between them (**SAS**). The angle is also known as the included angle.



  - 2 angles and 1 side (**AAS**) (in any order)



  - a right angle, hypotenuse length and another side length (**RHS**)



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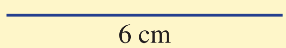
Example 4 Constructing a triangle

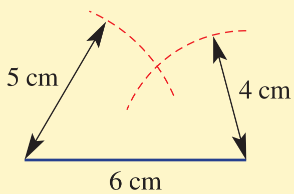
Construct a triangle with side lengths 6 cm, 4 cm and 5 cm.

SOLUTION

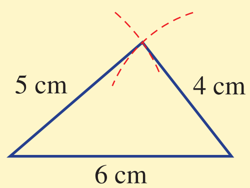
EXPLANATION



Use a ruler to draw a segment 6 cm in length.



Construct two arcs with radius 4 cm and 5 cm, using each end of the segment as the centres.



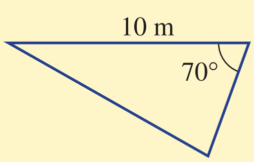
Mark the intersection point of the arcs and draw the two remaining segments.

Example 5 Deciding if triangles are unique

Is there enough information given to accurately construct a unique triangle? If yes, write SSS, SAS, AAS or RHS, whichever one suits best.

a triangle ABC with $AB = 5$ cm, $AC = 6$ cm and $\angle BAC = 60^\circ$

b

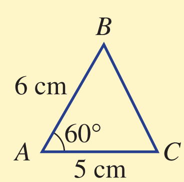


SOLUTION

EXPLANATION

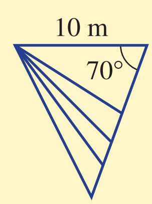
a yes; SAS

2 sides and the angle between them are given.



b no

Not enough information is provided to suit SSS, SAS, AAS or RHS. More than one triangle can be drawn with the information.



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Exercise 9C

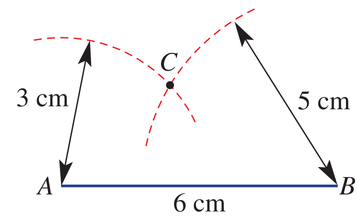
1	1	—

UNDERSTANDING

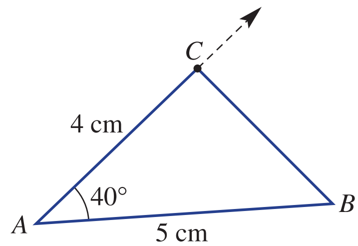
Example 4

1 Use a protractor, pair of compasses and a ruler to construct these triangles.

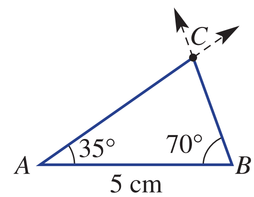
a 3 sides: 3 cm, 5 cm and 6 cm (SSS)
Step 1. Draw a segment, AB, 6 cm long.
Step 2. Draw 2 arcs: one centred at A with radius 3 cm and the other centred at B with radius 5 cm.
Step 3. Mark the point C and join to points A and B.



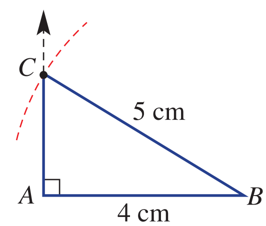
b 2 sides: 5 cm and 4 cm; and the angle between them, 40° (SAS)
Step 1. Draw a segment 5 cm long.
Step 2. Use a protractor to draw the angle 40° at A and then draw the ray AC.
Step 3. Measure the segment AC at 4 cm and mark point C.
Step 4. Join point C with point B.



c 2 angles: 35° and 70°; and a side of length 5 cm between the angles. (This is an example of AAS.)
Step 1. Draw the segment AB 5 cm long.
Step 2. Use a protractor to draw the angles – one at each end.
Step 3. Mark point C and join it with points A and B.



d a right angle, with hypotenuse of length 5 cm and one other side of length 4 cm (RHS)
Step 1. Draw a segment, AB, 4 cm in length.
Step 2. Measure a 90° angle at A and then draw the ray AC.
Step 3. Construct the arc centred at B, using a radius of 5 cm.
Step 4. Mark the point C and join with points A and B.



2–4	2–5	2–5

FLUENCY

Example 5

2 Is there enough information given to accurately construct a unique triangle? If yes, write SSS, SAS, AAS or RHS, whichever one suits best. You may wish to draw a diagram to help display the information.

a triangle ABC with $AB = 4$ cm, $AC = 2$ cm and $BC = 3$ cm
b triangle ABC with $AB = 10$ m and $AC = 5$ cm
c triangle ABC with $AB = 4$ km, $\angle BAC = 30°$ and $AC = 5$ km
d triangle DEF with $\angle DEF = 30°$ and $DE = 9$ cm
e triangle DEF with $\angle DEF = 90°$ and $DF = 3$ m and $DE = 2$ m
f triangle MNO with $\angle MNO = 47°$, $\angle NOM = 70°$ and $NO = 7.2$ m

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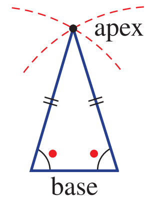
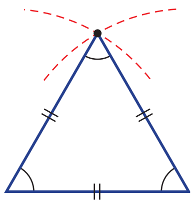
9C

FLUENCY

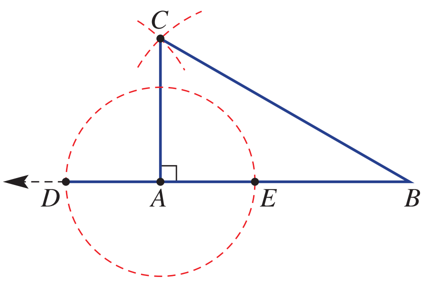
3 Construct an isosceles triangle by following these steps.
  a Draw a base segment of about 4 cm in length.
  b Use a pair of compasses to construct two arcs of equal radius. (Try about 5 cm but there is no need to be exact.)
  c Join the intersection point of the arcs (apex) with each end of the base.
  d Measure the length of the legs to check they are equal.
  e Measure the two base angles to check they are equal.



4 Construct an equilateral triangle by following these steps.
  a Draw a segment of about 4 cm in length.
  b Use a pair of compasses to construct two arcs of equal radius.
  Important: Ensure the arc radius is exactly the same as the length of the segment in part **a**.
  c Join the intersection point of the arcs with the segment at both ends.
  d Measure the length of the three sides to check they are equal.
  e Measure the three angles to check they are all equal and 60°.



5 Construct a right triangle by following these steps.
  a Draw a segment, AB, of about 4 cm in length.
  b Extend the segment AB to form the ray AD. Make AD about 2 cm in length.
  c Construct a circle with centre A and radius AD. Also mark point E.
  d Draw two arcs with centres at D and E, as shown in the diagram. Any radius will do as long as they are equal for both arcs.
  e Mark point C and join with A and B.



6	6, 7	6, 7

PROBLEM-SOLVING

6 Without using a protractor, accurately construct these triangles. Rulers can be used to set the pair of compasses.
  a triangle ABC with AB = 5.5 cm, BC = 4.5 cm and AC = 3.5 cm
  b an isosceles triangle with base length 4 cm and legs 5 cm
  c an equilateral triangle with side length 3.5 cm
  d a right triangle with one side 4 cm and hypotenuse 5 cm

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PROBLEM-SOLVING

7 Use computer geometry to construct these triangles. No measurement is allowed and each triangle does not have to be of a specific size.

a equilateral triangle

b isosceles triangle

c right triangle

8	8	8, 9

REASONING

8 Explain why the following information is not sufficient to draw a single unique triangle. Show that it is not sufficient by drawing at least two different triangles that fit the criteria.

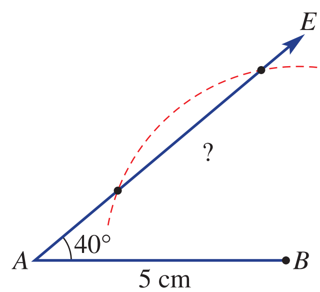
a 3 angles **b** 2 sides

9 **a** Follow these steps to construct this special triangle that has two sides, 5 cm and 4 cm, and one angle of 40° that is not between them.

i Draw AB 5 cm in length.

ii Draw $\angle BAE = 40°$.

iii Draw an arc centred at B with radius 4 cm.



b How many triangles could be formed with the given information? Explain why.

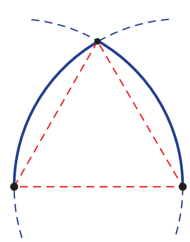
c Can you explain why this situation might be called the 'ambiguous case'?

Gothic arches

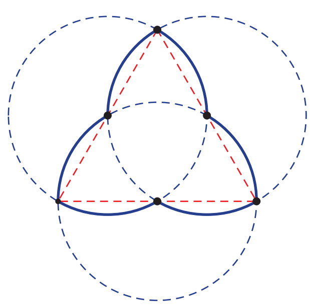
—	—	10

ENRICHMENT

10 **a** The Gothic, or equilateral arch, is based on the equilateral triangle. Try to construct one, using this diagram to help.



b The trefoil uses the midpoints of the sides of an equilateral triangle. Try to construct one, using this diagram to help.



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9D Triangle angle sum

Interactive

Widgets

HOTsheets

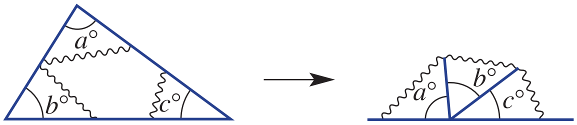
Walkthroughs

The three interior angles of a triangle have a very important property. No matter the shape of the triangle, the three angles always add to the same total.

The different triangles formed by the bridge's cables all have the same angle sum.

Let's start: A visual perspective on the angle sum

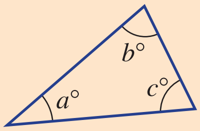
Use a ruler to draw any triangle. Cut out the triangle and tear off the three corners. Then place the three corners together.



What do you notice and what does this tell you about the three angles in the triangle? Compare your results with those of others. Does this work for other triangles?

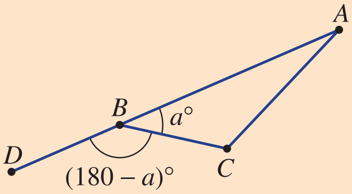
Key ideas

- The **angle sum** of the interior angles of a triangle is 180°.



$a + b + c = 180$

- If one side of a triangle is extended, an **exterior** angle is formed. In the diagram shown opposite, $\angle DBC$ is the exterior angle. The angle $\angle DBC$ is supplementary to $\angle ABC$ (i.e. adds to 180°).

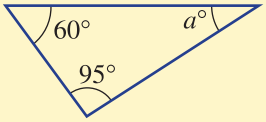


The exterior angle theorem will be looked at more closely in Section 9E.

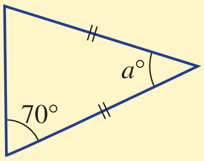
Example 6 Finding an angle in a triangle

Find the value of a in these triangles.

a



b



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SOLUTION | **EXPLANATION**

a $a + 60 + 95 = 180$
$a + 155 = 180$
$a = 25$

The sum of angles in a triangle is 180.
Add the two known angles.
Find the difference between 180 and 155.

b $a + 70 + 70 = 180$

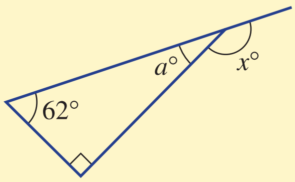
The two angles opposite the sides of equal length (i.e. the base angles) in an isosceles triangle are equal in size.

$a + 140 = 180$
$a = 40$

Add the two equal angles.
Find the difference between 140 and 180.

Example 7 Finding an exterior angle

Find the size of the exterior angle x° in this diagram.



SOLUTION | **EXPLANATION**

$a + 90 + 62 = 180$
$a + 152 = 180$
$a = 28$

The angle sum for a triangle is 180°.
Add the two known angles.
a is the difference between 180 and 152.

$x + 28 = 180$
$x = 152$

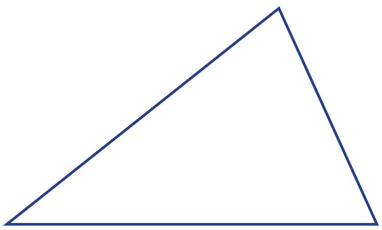
Angles of size x° and a° are supplementary (i.e. they add to 180°).
x is the difference between 180 and 28.

Exercise 9D

1–4	4	—

UNDERSTANDING

1 **a** Use a protractor to measure the three angles in this triangle.
b Add up your three angles. What do you notice?



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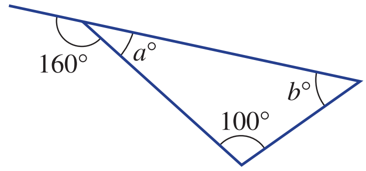
9D

UNDERSTANDING

2 For the triangle opposite, give reasons why:

a a must equal 20

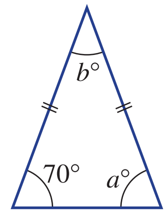
b b must equal 60



3 What is the size of each angle in an equilateral triangle?

4 For the isosceles triangle opposite, give a reason why:

a $a = 70$ **b** $b = 40$



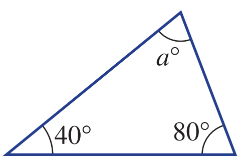
FLUENCY

5–7	5–7	5–7(½)

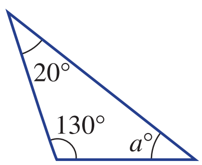
Example 6a

5 Find the value of a in each of these triangles.

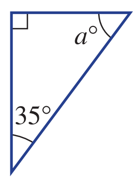
a



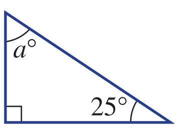
b



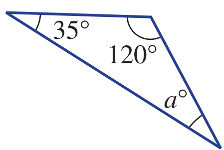
c



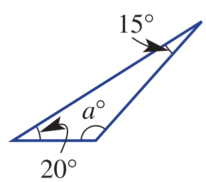
d



e



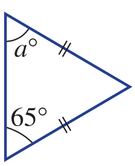
f



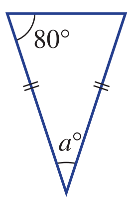
Example 6b

6 Find the value of a in each of these isosceles triangles.

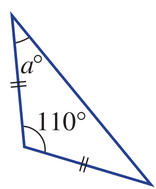
a



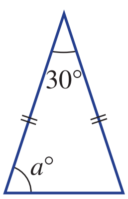
b



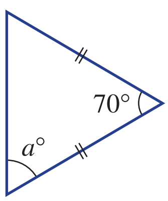
c



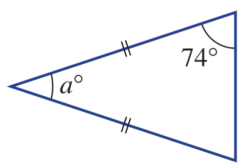
d



e



f



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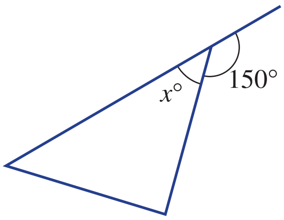
9D

FLUENCY

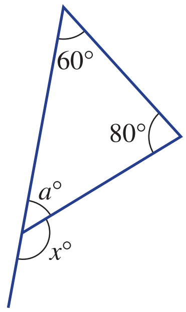
Example 7

7 The triangles below have exterior angles. Find the value of x. For parts **b** to **f**, you will need to first calculate the value of a.

a



b



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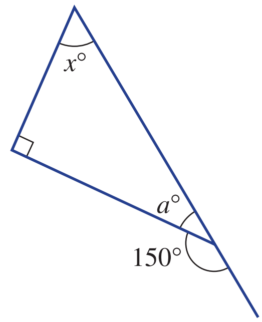
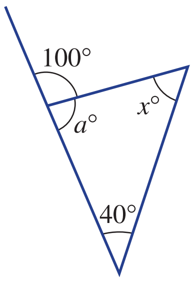


d



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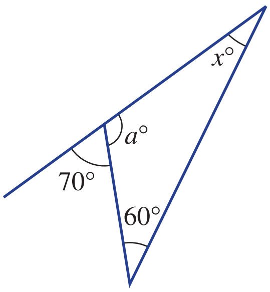

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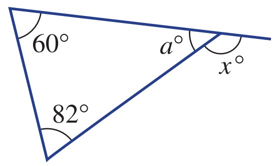
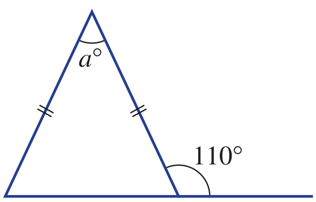


8(½)	8–9(½)	8–9(½), 10

PROBLEM-SOLVING

8 Find the value of a, in each of these triangles.

a



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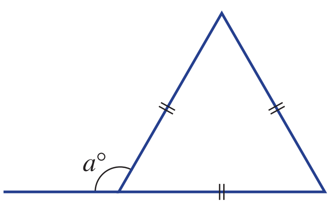

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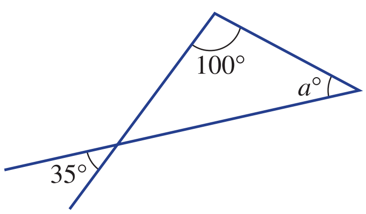

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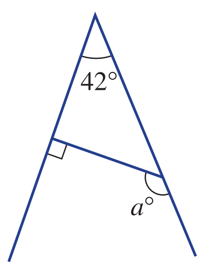

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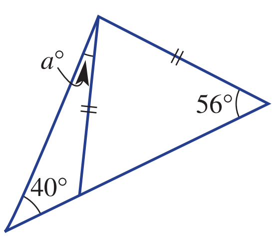

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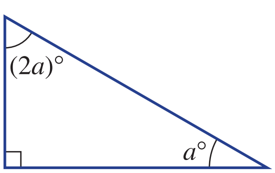

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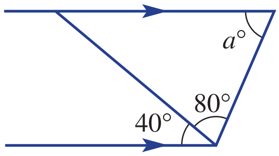
9D

PROBLEM-SOLVING

9 Each of these diagrams has parallel lines. Find the value of a.

a



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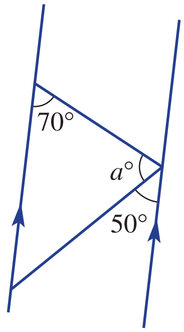

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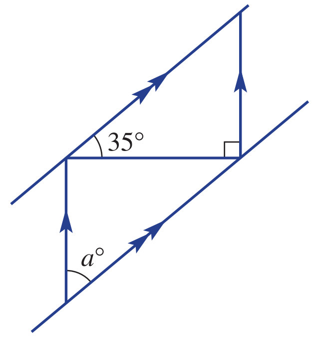

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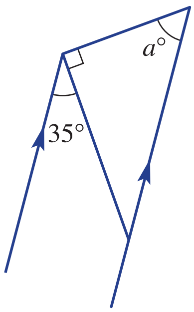

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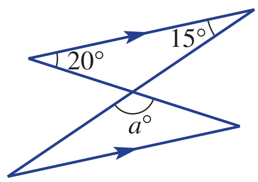

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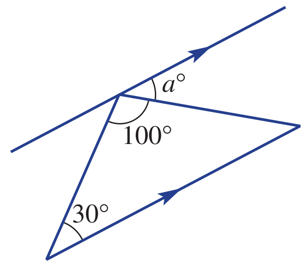
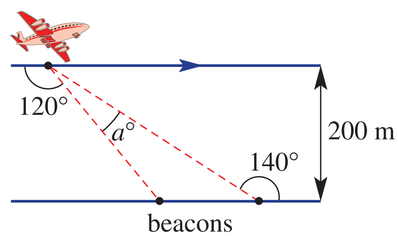


10 A plane flies horizontally 200 m above the ground. It detects two beacons on the ground. Some angles are known, and these are shown in the diagram. Find the angle marked $a°$ between the line of sight to the two beacons.



11	11	11–12

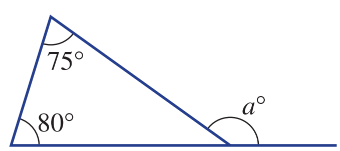
REASONING

11 **a** Find the sum $75° + 80°$.

b Find the value of a in the diagram opposite.

c What do you notice about the answers to parts **a** and **b**?

d Do you think this would be true for other triangles with different angles? Explore.



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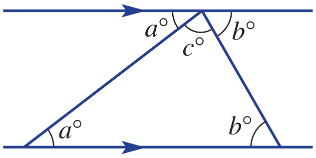
REASONING

12 This diagram includes two parallel lines.



a The angles marked $a°$ are always equal. From the list (corresponding, alternate, cointerior, vertically opposite), give a reason why.

b Give a reason why the angles marked $b°$ are always equal.

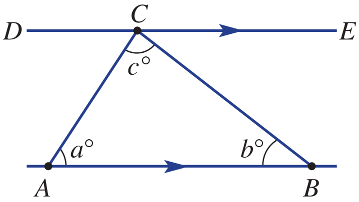
c At the top of the diagram, angles $a°$, $b°$ and $c°$ lie on a straight line. What does this tell you about the three angles $a°$, $b°$ and $c°$ in the triangle?

ENRICHMENT

Proof	—	—	13

13 Complete these proofs. Give reasons for each step where brackets are shown.

a The angle sum in a triangle is 180°.



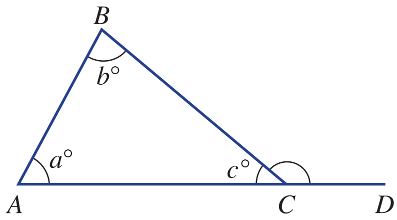
$\angle DCA = a°$ (Alternate to $\angle BAC$ and DE is parallel to AB.)

$\angle ECB =$ ______ (________________________________)

$\angle DCA + \angle ACB + \angle ECB =$ ______ (________________________________)

$\therefore a + b + c =$ ______

b The exterior angle outside a triangle is equal to the sum of the two interior opposite angles.



The Ancient Greek mathematician Euclid of Alexandria is known as the Father of Geometry.

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9E Quadrilaterals

Interactive Widgets HOTsheets Walkthroughs

Quadrilaterals are polygons with four sides. There are special types of quadrilaterals and these are identified by the number of equal side lengths and the number of pairs of parallel lines.


Streets in major cities often form quadrilaterals, and many buildings on them have a similar shape.

Let's start: Quadrilaterals that you know

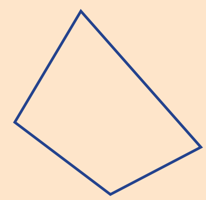
You may already know the names and properties of some of the special quadrilaterals. Which ones do you think have:

- 2 pairs of parallel sides?
- All sides of equal length?
- 2 pairs of sides of equal length?

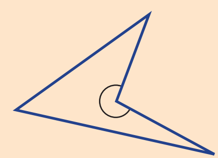
Are there any types of quadrilaterals that you know which you have not yet listed?

Key ideas

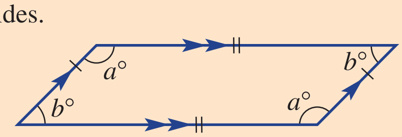
- A **convex quadrilateral** has all four interior angles less than 180°. All vertices point outward.



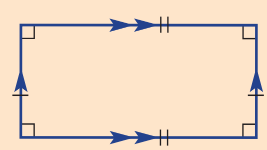
- A **non-convex quadrilateral** has one interior angle greater than 180°.



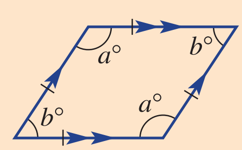
- Parallelograms are quadrilaterals with two pairs of parallel sides.
  - **parallelogram**
    - 2 pairs of parallel sides
    - 2 pairs of sides of equal length
    - opposite angles equal



  - **rectangle** (a parallelogram with all angles 90°)
    - 2 pairs of parallel sides
    - 2 pairs of sides of equal length
    - all angles 90°



  - **rhombus** (or diamond) (a parallelogram with all sides equal)
    - 2 pairs of parallel sides
    - all sides of equal length
    - opposite angles equal



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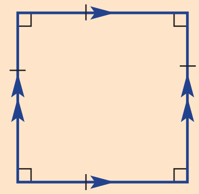
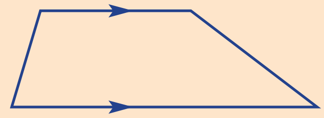
- **square** (a rhombus with all angles 90°)
  - 2 pairs of parallel sides
  - all sides of equal length
  - all angles 90°

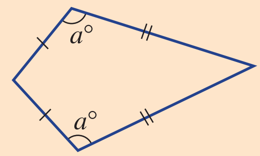


■ Other special quadrilaterals include:

- **trapezium**
  - 1 pair of parallel sides (some like to define a trapezium as a quadrilateral with at least 1 pair of parallel sides; this would make all parallelograms also trapeziums)



- **kite**
  - 2 pairs of sides of equal length
  - 1 pair of opposite angles that are equal in size

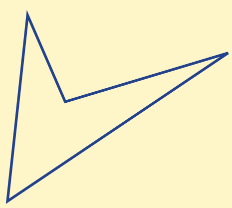


Key ideas

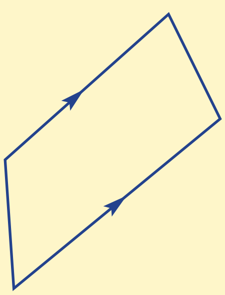
Example 8 Classifying quadrilaterals

State the type of each quadrilateral given below.

a



b



SOLUTION	EXPLANATION
a non-convex quadrilateral	One interior angle is greater than 180°.
b trapezium	There is one pair of parallel sides.

Exercise 9E

1, 2	2	—

UNDERSTANDING

1 Draw an example of each of the quadrilaterals listed. Mark any sides of equal length with single or double dashes, mark parallel lines with single or double arrows and mark equal angles using the letters a and b. (Refer back to the **Key ideas** in this section should you need help.)

a square **b** rectangle **c** rhombus
d parallelogram **e** trapezium **f** kite

2 **a** Draw two examples of a non-convex quadrilateral.
b For each of your drawings, state how many interior angles are greater than 180°.

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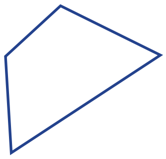
9E

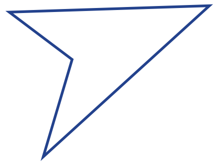
3–5	3–6	3–6

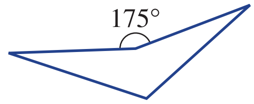
FLUENCY

Example 8

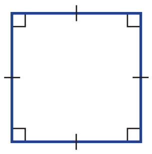
3 Classify each of these quadrilaterals as either convex or non-convex.

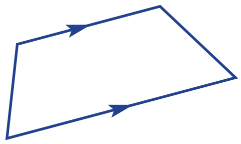
a


b


c


4 State the type of special quadrilateral given below.

a


b


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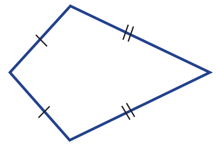

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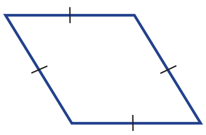
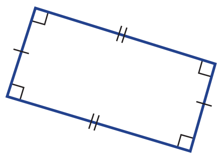


e


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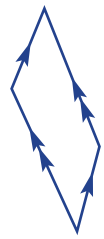
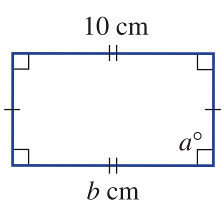
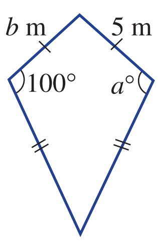


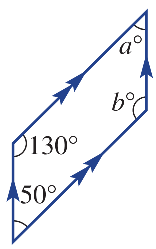
5 Name all the quadrilaterals that have:

- **a** 2 different pairs of sides of equal length
- **b** 2 different pairs of opposite angles that are equal in size
- **c** 2 different pairs of parallel lines
- **d** only 1 pair of parallel lines
- **e** only 1 pair of opposite angles that are equal in size

6 Use your knowledge of the properties of quadrilaterals to find the unknown angles and lengths in each of these diagrams.

a


b


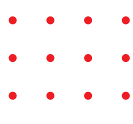
c


7	7, 8	7, 8

PROBLEM-SOLVING

7 Consider this 4×4 grid. Using the dots as vertices, how many different shapes of each kind could be drawn? (Do not count shapes that are of the same size.)

- **a** square
- **b** rectangle (that is not a square)
- **c** rhombus (that is not a square)
- **d** parallelogram (that is not a square, a rectangle or a rhombus)



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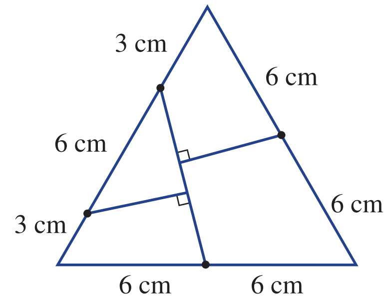
9E

PROBLEM-SOLVING

8 Using the given measurements, accurately draw this equilateral triangle onto a piece of paper and cut it into 4 pieces, as shown. Can you form a square with the four pieces?

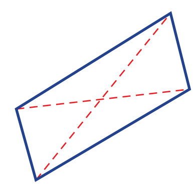


REASONING

9	9	9, 10

9 The diagonals of a quadrilateral are segments that join opposite vertices.

- a List the quadrilaterals that have diagonals of equal length.
- b List the quadrilaterals that have diagonals intersecting at 90°.



10 a Are squares a type of rectangle or are rectangles a type of square? Give an explanation.

- b Are rhombuses a type of parallelogram? Explain.
- c Is it possible to draw a non-convex trapezium?

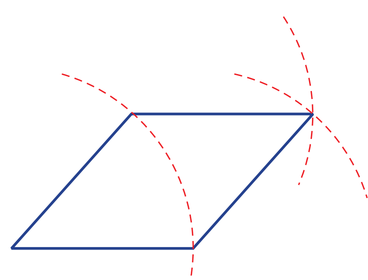
Construction challenge

ENRICHMENT

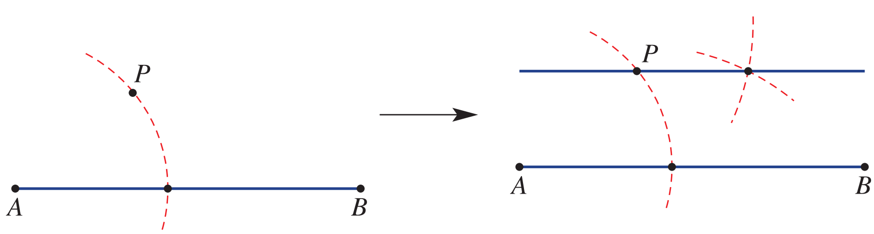
—	—	11

11 Use a pair of compasses and a ruler to construct these figures. Use the diagrams as a guide, then measure to check the properties of your construction.

- a a rhombus with side length 5 cm



- b a line parallel to segment AB and passing through point P

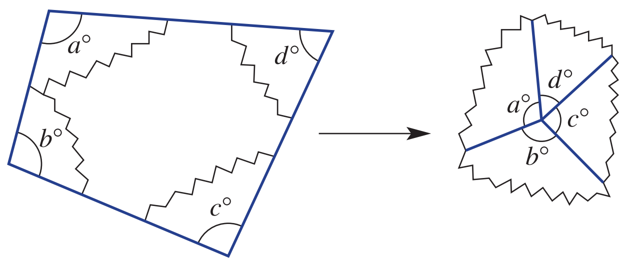


9F Quadrilateral angle sum

Like a triangle, the interior angle sum of a quadrilateral is a fixed number. This is true for both convex and non-convex quadrilaterals.

Let's start: What do a revolution and a quadrilateral have in common?

Use a ruler to draw any quadrilateral. Cut it out and tear off the corners. Arrange them to meet at a point.

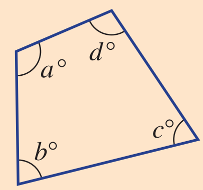


- What does the arrangement tell you about the angles inside a quadrilateral?
- Compare your results with those of others in the class.

Key ideas

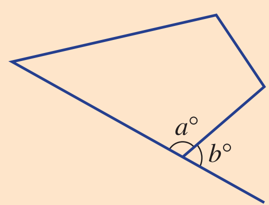
- The **interior angle sum** of a quadrilateral is 360°.

$a + b + c + d = 360$



- **Exterior angles** are formed by extending one side.

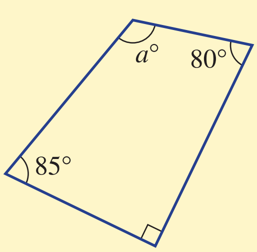
$a + b = 180$



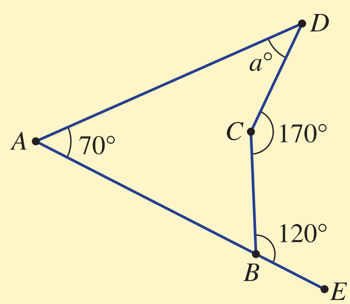
Example 9 Finding unknown angles in quadrilaterals

Find the value of a in each of these quadrilaterals.

a



b



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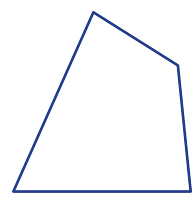
SOLUTION	EXPLANATION
a $a + 85 + 90 + 80 = 360$	The sum of the interior angles is 360°.
$a + 255 = 360$	Add the known angles to simplify.
$a = 105$	a is the difference between 360 and 255.
b Interior $\angle ABC = 180° - 120° = 60°$	$\angle ABC$ and $\angle CBE$ are supplementary.
Interior $\angle BCD = 360° - 170° = 190°$	Angles at a point sum to 360°.
$a + 70 + 60 + 190 = 360$	The sum of the interior angles is 360°.
$a + 320 = 360$	Simplify.
$a = 40$	a is the difference between 360 and 320.

Exercise 9F

UNDERSTANDING	1–3	3	—

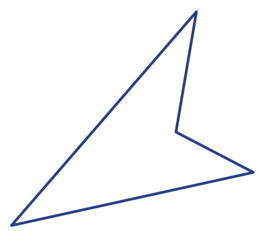
1 a Draw any quadrilateral that is convex (i.e. all interior angles are less than 180°), as shown opposite.

b Measure each interior angle and add them to find the total sum. Check that your answer is close to 360°.



2 a Draw any quadrilateral that is non-convex (i.e. one interior angle is greater than 180°), as shown opposite.

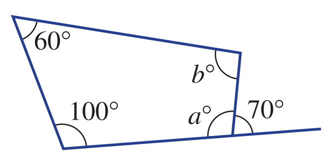
b Measure each interior angle and them add to find the total sum. Check that your answer is close to 360°.



3 In the quadrilateral shown opposite, give a geometrical reason why:

a a must equal 110

b b must equal 90

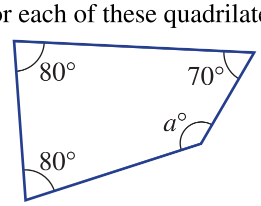


FLUENCY	4, 5	4–6	4, 5(½), 6

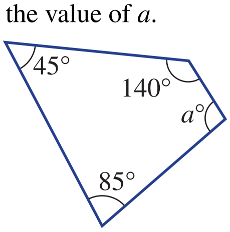
Example 9

4 For each of these quadrilaterals, find the value of a.

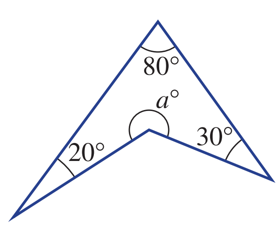
a



b



c



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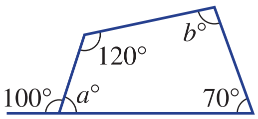
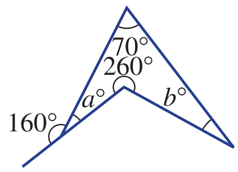
9F

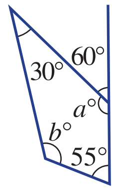
FLUENCY

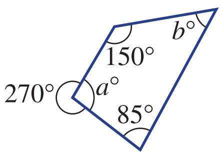
Example 9

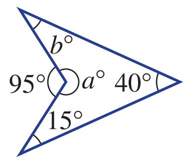
5 Find the values of *a* and *b* in each of the following diagrams.

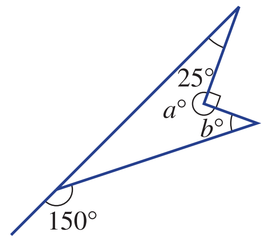
a


b


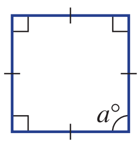
c


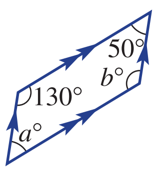
d


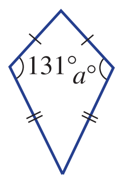
e


f


6 Give the values of *a* (and *b*) in these special quadrilaterals.

a


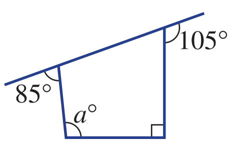
b


c


7	7, 8	8, 9

PROBLEM-SOLVING

7 For each of these diagrams, find the value of *a*. You may need to find some other angles first.

a


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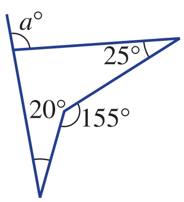
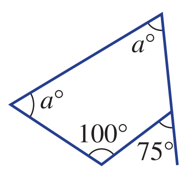
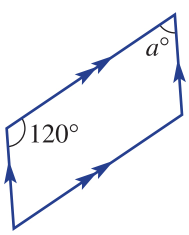
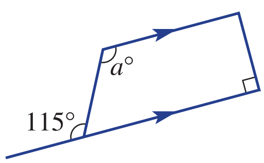
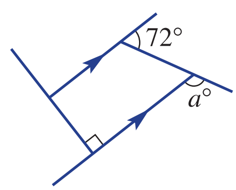


c


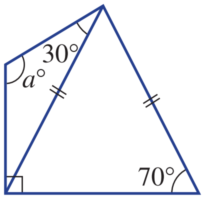
8 The diagrams below include parallel lines. Find the value of *a*.

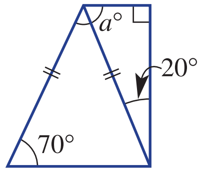
a


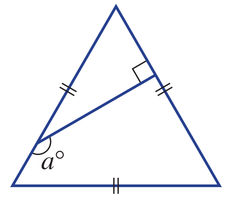
b


c


9 These diagrams include both triangles and quadrilaterals. Find the value of *a*.

a


b


c


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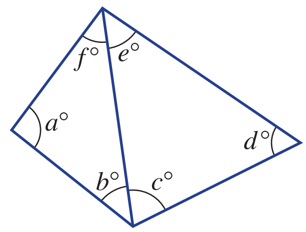
10	10	10, 11

REASONING

10 For the quadrilaterals below, state whether each is possible or impossible. Make drawings to explore.

- **a** all interior angles less than 100°
- **b** all interior angles less than 90°
- **c** more than one interior reflex angle
- **d** more than one interior obtuse angle
- **e** more than three interior acute angles

11 Given is a quadrilateral divided into two triangles. Complete the proof to show that the sum of angles in a quadrilateral is 360°.



$a + b + f =$ __________ (sum of angles in a triangle)

$c + d + e =$ __________ (__________________)

Total sum $= a + b + f + c + d + e$

$=$ __________ $+$ __________

$=$ __________

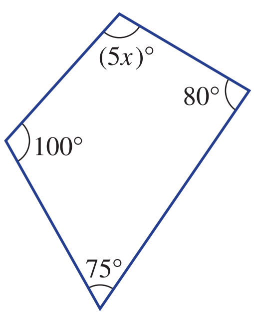
Geometry with algebra

—	—	12

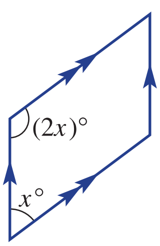
ENRICHMENT

12 Find the value of x for each of these quadrilaterals.

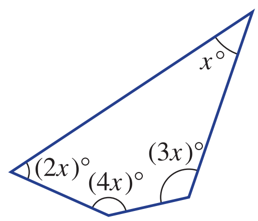
a



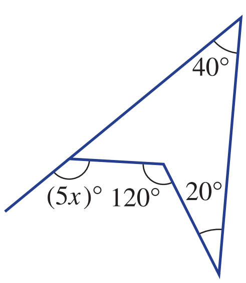
b



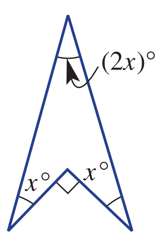
c



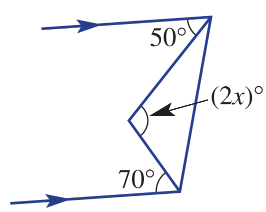
d



e



f



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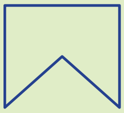
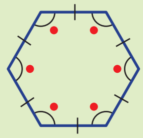
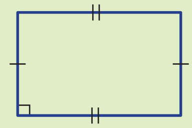
Progress quiz

9A **1** Name the type of shape stating whether it is concave or convex, regular or irregular.

a 

b 

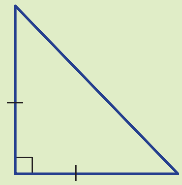
c 

d 

9B **2** Classify this triangle by:

a its side length

b its angles

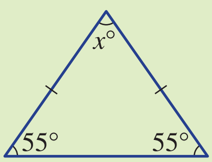


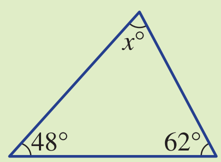
9C **3** Construct an equilateral triangle with sides each 6 cm.

Ext

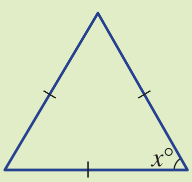
9D **4** What is the angle sum of any triangle?

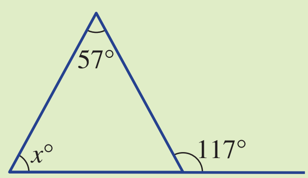
9D **5** Find the value of x in each triangle below.

a 

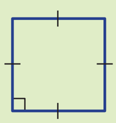
b 

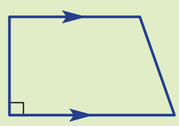
9D **6** Find the size of x in each of these triangles.

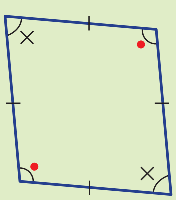
a 

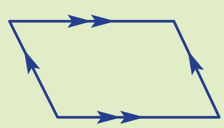
b 

9E **7** State the special type of quadrilateral given in each diagram below.

a 

b 

c 

d 

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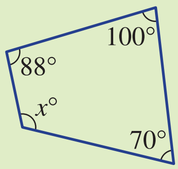
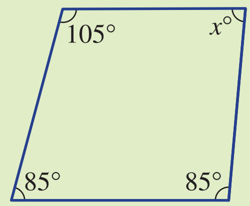
9F **8** Find the value of each pronumeral below.

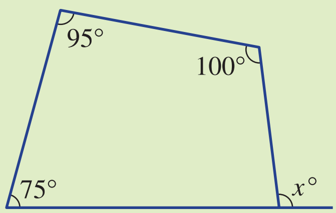
a



b

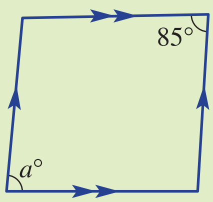


c

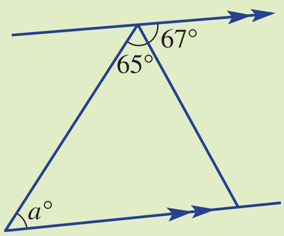


9D/F **9** The diagrams below include parallel lines. Find the value of a.

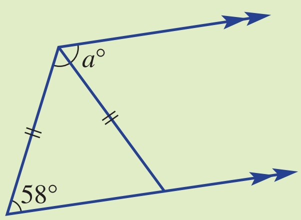
a



b



c



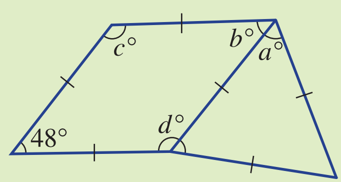
9F **10** Use your combined knowledge of triangles, quadrilaterals, polygons and parallel lines to find:

a the value of a

b the value of b

c the value of c

d the value of d



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9G Symmetry

Interactive
Widgets
HOTsheets
Walkthroughs

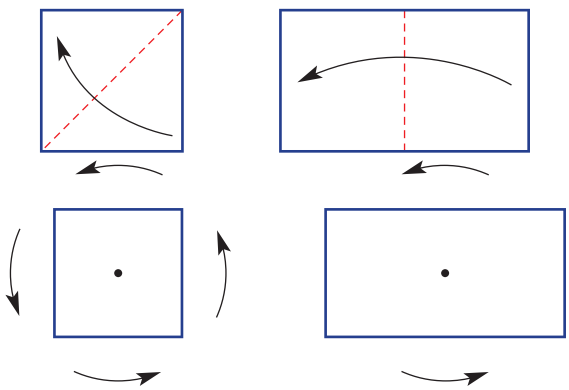
You see many symmetrical geometrical shapes in nature. The starfish and sunflower are two examples. Shapes such as these may have two types of symmetry: line and rotational.

Starfish and sunflowers are both symmetrical, but in different ways.

Let's start: Working with symmetry

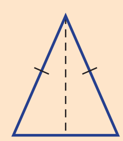
On a piece of paper draw a square (with side lengths of about 10 cm) and a rectangle (with length of about 15 cm and width of about 10 cm), then cut them out.

- How many ways can you fold each shape in half so that the two halves match exactly? The number of creases formed will be the number of lines of symmetry.
- Now locate the centre of each shape and place a sharp pencil on this point. Rotate the shape 360°.
  How many times does the shape make an exact copy of itself in its original position? This number describes the rotational symmetry.

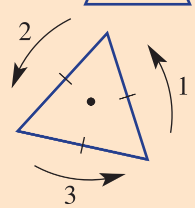


Key ideas

- An axis or **line of symmetry** divides a shape into two equal parts. It acts as a mirror line, with each half of the shape being a reflection of the other.
  - The **order of line symmetry** is the number of axes of symmetry.



- The **order of rotation** is the number of times a shape makes an exact copy of itself (in its original position) after rotating 360°.
  - We say that there is no rotational symmetry if the order of **rotational symmetry** is equal to 1.



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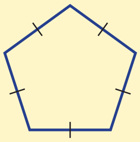
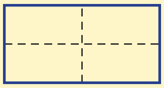
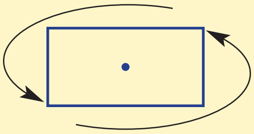
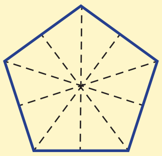
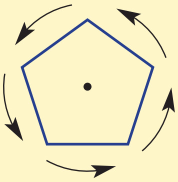
Example 10 Finding the symmetry of shapes

Give the order of line symmetry and of rotational symmetry for each of these shapes.

a rectangle

b regular pentagon



SOLUTION	EXPLANATION
a line symmetry: order 2	
rotational symmetry: order 2	
b line symmetry: order 5	
rotational symmetry: order 5	

Exercise 9G

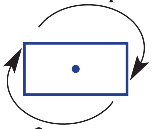
1, 2	2	—

UNDERSTANDING

1 How many ways could you fold each of these shapes in half so that the two halves match exactly? (To help you solve this problem, try cutting out the shapes and folding them.)

a square **b** rectangle **c** equilateral triangle
d isosceles triangle **e** rhombus **f** parallelogram

2 For the shapes listed in Question **1**, imagine rotating them 360° about their centre. How many times do you make an exact copy of the shape in its original position?



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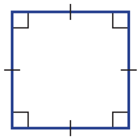
9G

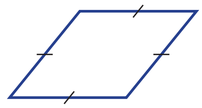
3–7	3–8	3(½), 4–8

FLUENCY

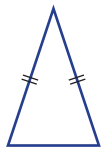
Example 10

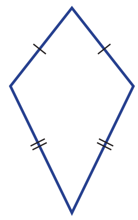
3 Give the order of line symmetry and of rotational symmetry for each shape.

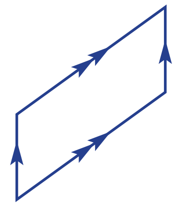
a


b


c


d


e


f


4 Name a type of triangle that has the following properties.

a order of line symmetry 3 and order of rotational symmetry 3

b order of line symmetry 1 and no rotational symmetry

c no line or rotational symmetry

5 List the special quadrilaterals that have these properties.

a line symmetry of order:

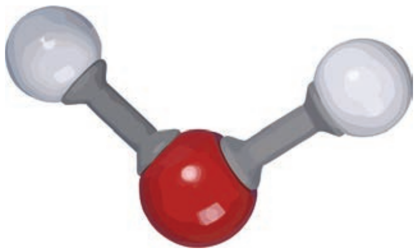
i 1 ii 2 iii 3 iv 4

b rotational symmetry of order:

i 1 ii 2 iii 3 iv 4

6 State the order of line symmetry and rotational symmetry for each of the following.

a


b


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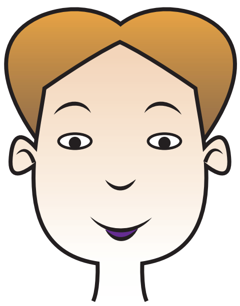
FLUENCY

c



d

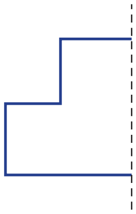
7 Of the capital letters of the alphabet shown here, state which have:

a 1 line of symmetry
b 2 lines of symmetry
c rotational symmetry of order 2

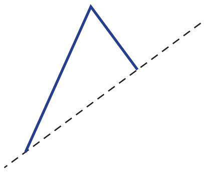
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

8 Complete the other half of these shapes for the given axis of symmetry.

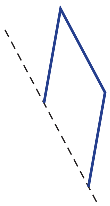
a



b



c



9	9, 10	9, 10

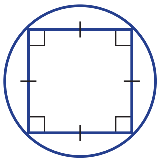
PROBLEM-SOLVING

9 Draw the following shapes, if possible.

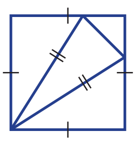
a a quadrilateral with no lines of symmetry
b a hexagon with one line of symmetry
c a shape with line symmetry of order 7 and rotational symmetry of order 7
d a diagram with no line of symmetry but rotational symmetry of order 3
e a diagram with line of symmetry of order 1 and no rotational symmetry

10 These diagrams are made up of more than one shape. State the order of line and of rotational symmetry.

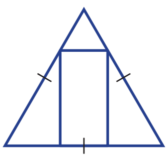
a



b



c



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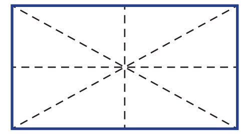
9G

11	11	11, 12

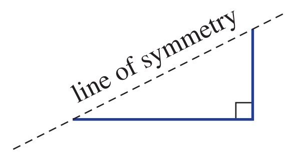
REASONING

11 Many people think a rectangle has four lines of symmetry, including the diagonals.

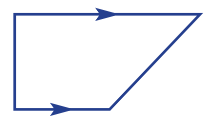


a Complete the other half of this diagram to show that this is not true.

b Using the same method as that used in part **a**, show that the diagonals of a parallelogram are not lines of symmetry.



12 A trapezium has one pair of parallel lines.



a State whether trapeziums always have:

i line symmetry ii rotational symmetry

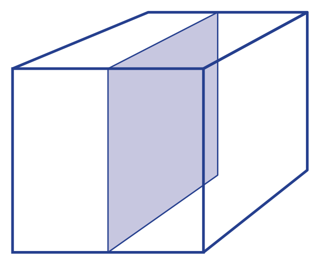
b What type of trapezium will have one line of symmetry?

Symmetry in 3D

—	—	13

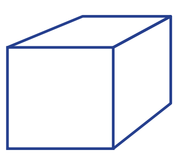
ENRICHMENT

13 Some solid objects also have symmetry. Rather than line symmetry, they have **plane symmetry**. This cube shows one plane of symmetry, but there are more that could be drawn.

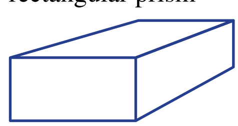


State the number of planes of symmetry for each of these solids.

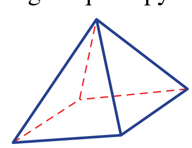
a cube



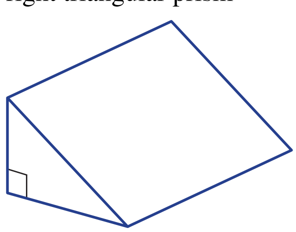
b rectangular prism



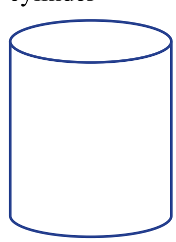
c right square pyramid



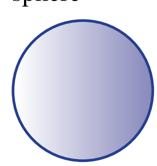
d right triangular prism



e cylinder



f sphere



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9H Reflection and rotation

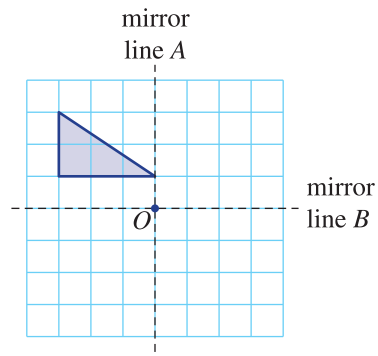
Interactive
Widgets
HOTsheets
Walkthroughs

Reflection and rotation are two types of transformations that involve a change in position of the points on an object. If a shape is reflected in a mirror line or rotated about a point, the size of the shape is unchanged. Hence, the transformations reflection and rotation are said to be isometric.

Let's start: Draw the image

Here is a shape on a grid.

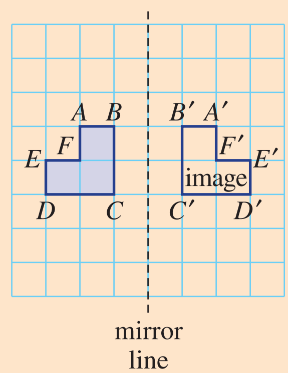
- Draw the image (result) after reflecting the shape in the mirror line A.
- Draw the image (result) after reflecting the shape in the mirror line B.
- Draw the image after rotating the shape about point O by 180°.
- Draw the image after rotating the shape about point O by 90° clockwise.
- Draw the image after rotating the shape about point O by 90° anticlockwise.



Discuss what method you used to draw each image and the relationship between the position of the shape and its image after each transformation.

Key ideas

- **Reflection** and **rotation** are **isometric transformations** that give an **image** of an object or shape without changing its shape and size.
- The **image** of point A is denoted A'.
- A reflection involves a mirror line, as shown in the diagram opposite.



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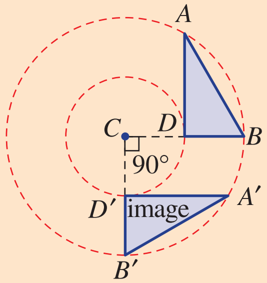
Key ideas

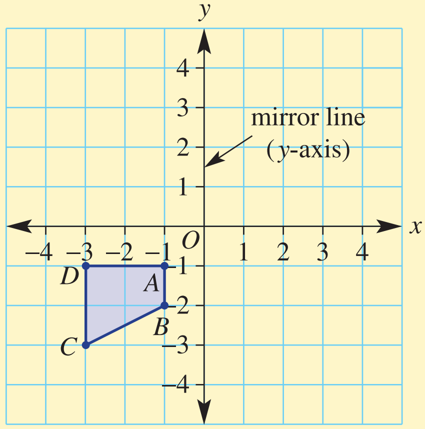
- A rotation involves a centre point of rotation (C) and an angle of rotation, as shown.
  - A pair of compasses can be used to draw each circle, to help find the position of image points.



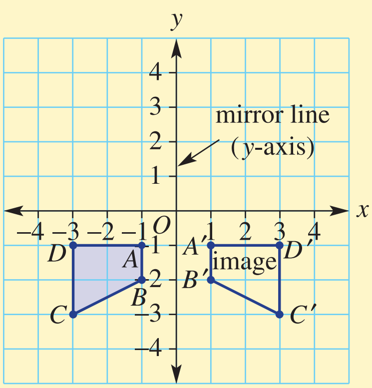
rotation 90° clockwise about C

Example 11 Drawing reflections

Draw the reflected image of this shape and give the coordinates of A', B', C' and D'. The y-axis is the mirror line.



SOLUTION



$A' = (1, -1), B' = (1, -2),\ C' = (3, -3), D' = (3, -1)$

EXPLANATION

Reflect each vertex A, B, C and D about the mirror line. The line segment from each point to its image should be at 90° to the mirror line.

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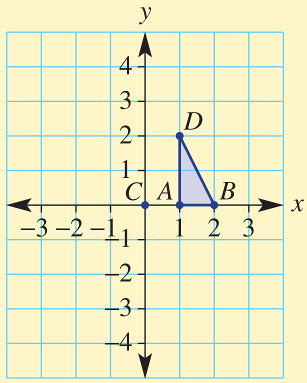
Example 12 Drawing rotations

Draw the image of this shape and give the coordinates of A', B' and D' after carrying out the following rotations.

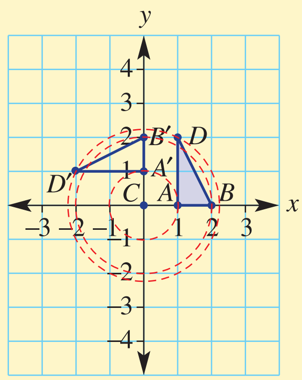
a 90° anticlockwise about C

b 180° about C



SOLUTION | **EXPLANATION**

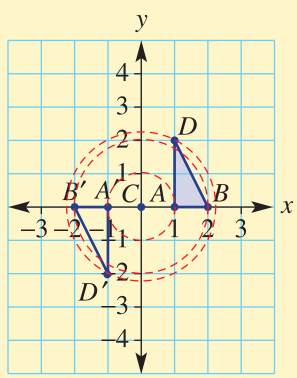
a



$A' = (0, 1), B' = (0, 2), D' = (-2, 1)$

Rotate each point on a circular arc around point C by 90° anticlockwise.

b



$A' = (-1, 0), B' = (-2, 0), D' = (-1, -2)$

Rotate each point on a circular arc around point C by 180° in either direction.

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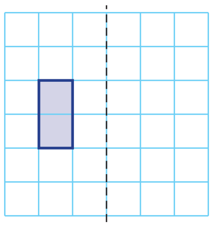
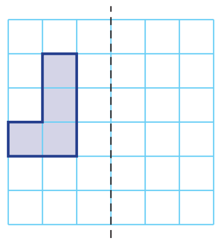
Exercise 9H

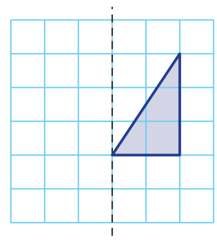
1–3	3	—

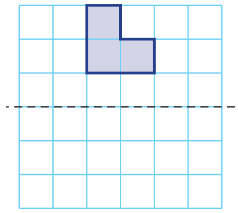
UNDERSTANDING

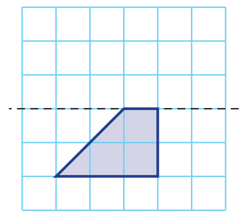
1 Use the grid to reflect each shape in the given mirror line.

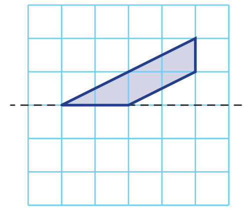
a


b


c


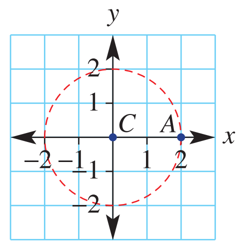
d


e


f


2 Give the coordinates of the image point A' after the point $A(2, 0)$ is rotated about point $C(0, 0)$ by the following angles.

a 180° clockwise b 180° anticlockwise
c 90° clockwise d 90° anticlockwise



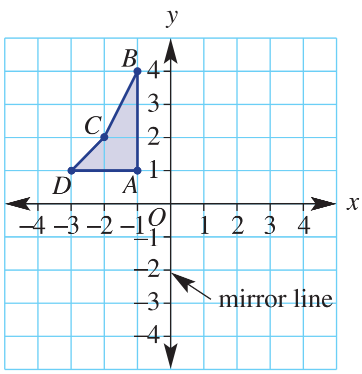
3 a Is the size and shape of an object changed after a reflection?
b Is the size and shape of an object changed after a rotation?

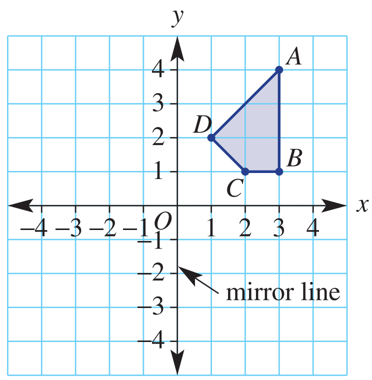
4–6	4–5(½), 6, 7	4–5(½), 6, 7

FLUENCY

Example 11

4 Draw the image of this shape and give the coordinates of A', B', C' and D'. Note that the y-axis is the mirror line for parts **a** to **c**, whereas the x-axis is the mirror line for parts **d** to **f**.

a


b


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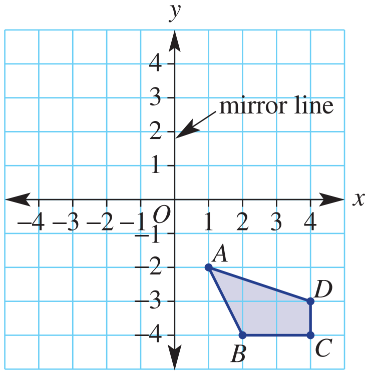
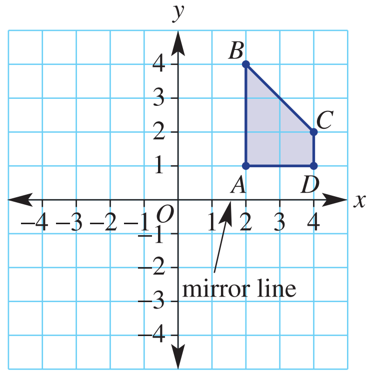
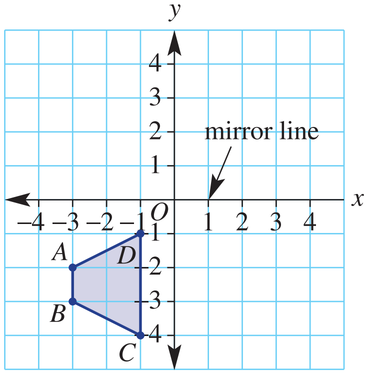
c



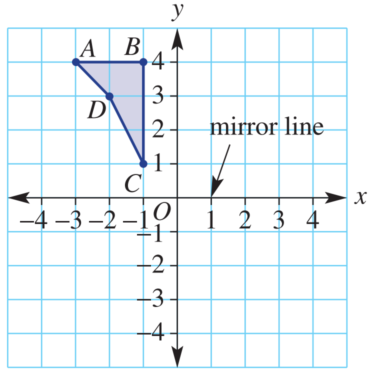
d



e



f



5 Give the new coordinates of the image point A' after point A has been rotated around point $C(0, 0)$ by:

a 180° clockwise

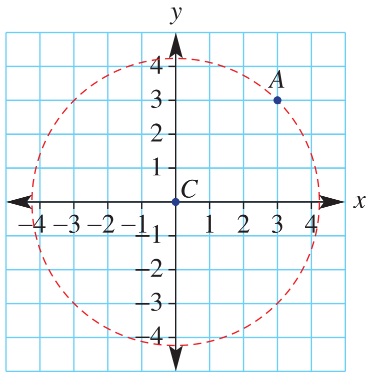
b 90° clockwise

c 90° anticlockwise

d 270° clockwise

e 360° anticlockwise

f 180° anticlockwise



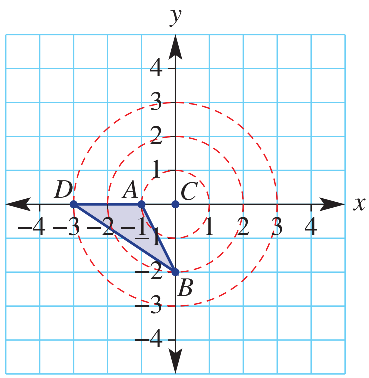
Example 12

6 Draw the image of this shape and give the coordinates of A', B' and D' after the following rotations.

a 90° anticlockwise about C

b 180° about C

c 90° clockwise about C



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9H

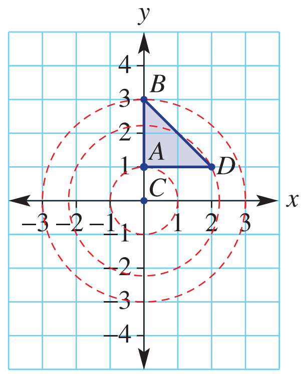
FLUENCY

7 Draw the image of this shape and give the coordinates of A', B' and D' after the following rotations.

a 90° anticlockwise about C

b 180° about C

c 90° clockwise about C

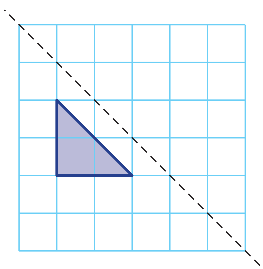


8, 9	8–10	9–11

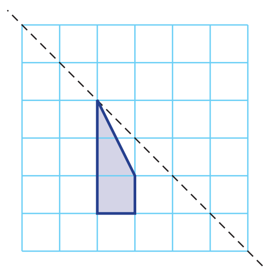
PROBLEM-SOLVING

8 The mirror lines on these grids are at a 45° angle. Draw the reflected image.

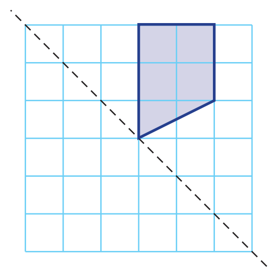
a



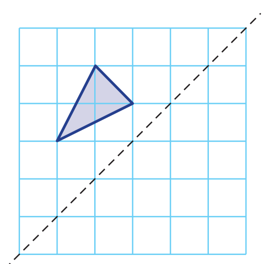
b



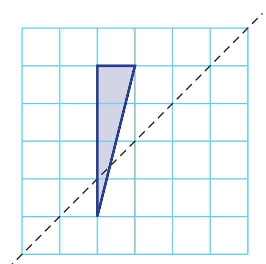
c



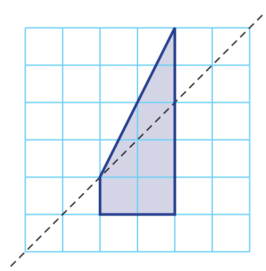
d



e



f



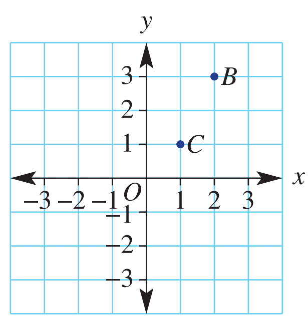
9 On the Cartesian plane, the point $A(-2, 5)$ is reflected in the x-axis and this image point is then reflected in the y-axis. What are the coordinates of the final image?

10 A point, $B(2, 3)$, is rotated about the point $C(1, 1)$. State the coordinates of the image point B' for the following rotations.

a 180°

b 90° clockwise

c 90° anticlockwise



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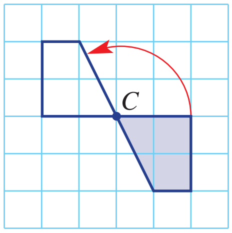
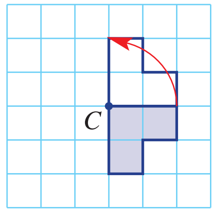
PROBLEM-SOLVING

11 For each shape given, how many degrees has it been rotated?

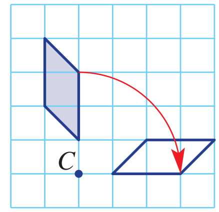
a



b



c



12	12, 13	13, 14

REASONING

12 Write the missing number in these sentences.

a Rotating a point 90° clockwise is the same as rotating a point _____ anticlockwise.

b Rotating a point 38° anticlockwise is the same as rotating a point _____ clockwise.

c A point is rotated 370° clockwise. This is the same as rotating the point _____ clockwise.

13 A point S has coordinates $(-2, 5)$.

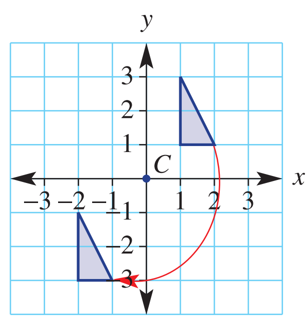
a Find the coordinates of the image point S' after a rotation 180° about $C(0, 0)$.

b Find the coordinates of the image point S' after a reflection in the x-axis followed by a reflection in the y-axis.

c What do you notice about the image points in parts **a** and **b**?

d Test your observation on the point $T(-4, -1)$ by repeating parts **a** and **b**.

14 Explain what is wrong with this attempt at a 180° rotation about $C(0, 0)$.



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9H

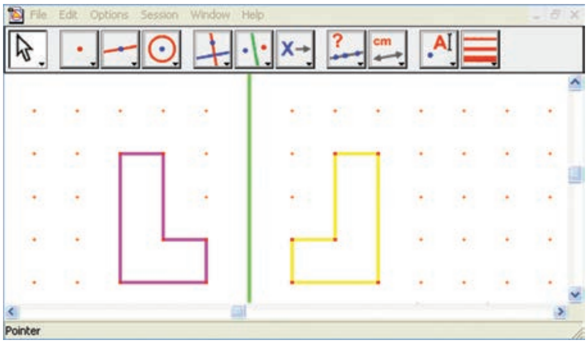
Dynamic geometry exploration

—	—	15, 16

ENRICHMENT

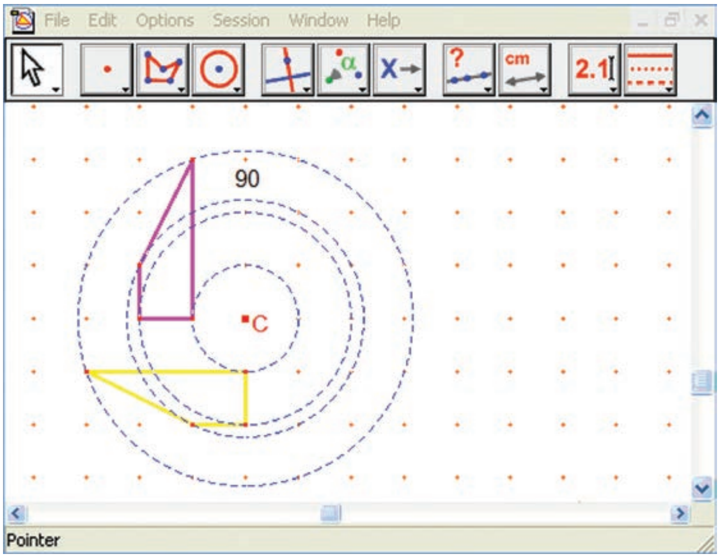
15 Explore reflecting shapes dynamically, using computer geometry.

- **a** On a grid, create any shape using the polygon tool.
- **b** Construct a mirror line.
- **c** Use the reflection tool to create the reflected image about your mirror line.
- **d** Drag the vertices of your original shape and observe the changes in the image. Also try dragging the mirror line.



16 Explore rotating shapes dynamically, using computer geometry.

- **a** On a grid, create any shape using the polygon tool.
- **b** Construct a centre of rotation point and a rotating angle (or number). In Cabri, use the Numerical Edit tool to create a dynamic number.
- **c** Use the rotation tool to create the rotated image that has your nominated centre of rotation and angle. You will need to click on the shape, the centre of rotation and your angle.
- **d** Drag the vertices of your original shape and observe the changes in the image. Also try changing the angle of rotation.



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9I Translation

Interactive

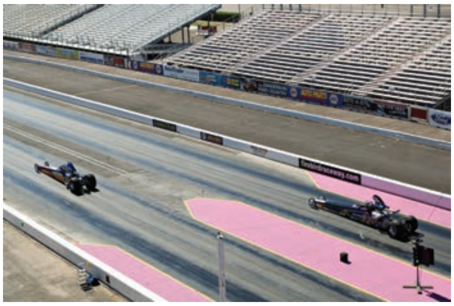
Widgets

HOTsheets

Walkthroughs

The transformation called translation is another isometric transformation because the size and shape of the image is unchanged. Translation involves a shift in an object left, right, up or down. The orientation of a shape is also unchanged.

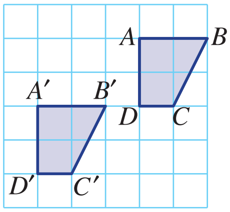
In a dragster race along 300 m of straight track, the main body of the car is translated down the track in a single direction.



Let's start: Describing a translation

Consider this shape $ABCD$ and its image $A'B'C'D'$.

- Use the words left, right, up or down to describe how the shape $ABCD$, shown opposite, could be translated (shifted) to its image.
- Can you think of a second combination of translations that give the same image?
- How would you describe the reverse translation?

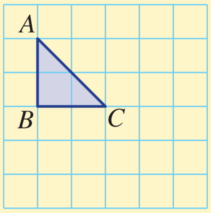


Key ideas

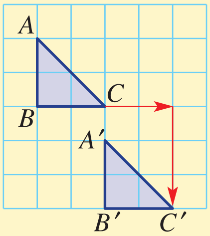
- **Translation** is an isometric transformation involving a shift left, right, up or down.
- Describing a translation involves saying how many units a shape is shifted left, right, up or down.

Example 13 Translating shapes

Draw the image of the triangle ABC after a translation 2 units to the right and 3 units down.



SOLUTION



EXPLANATION

Shift every vertex 2 units to the right and 3 units down. Then join the vertices to form the image.

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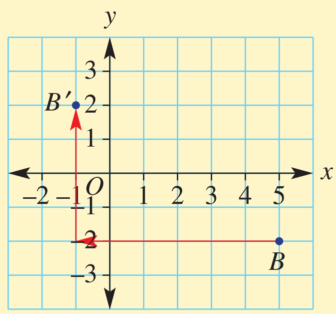
Example 14 Describing translations

A point $B(5, -2)$ is translated to $B'(-1, 2)$. Describe the translation.

SOLUTION

Translation is 6 units left and 4 units up.

EXPLANATION

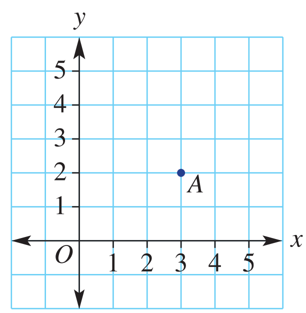


Exercise 9I

1, 2(½), 3	3	—

UNDERSTANDING

1 Point A has coordinates (3, 2). Write the coordinates of the image point A' when point A is translated in each of the following ways.



- **a** 1 unit right
- **b** 2 units left
- **c** 3 units up
- **d** 1 unit down
- **e** 1 unit left and 2 units up
- **f** 3 units left and 1 unit down
- **g** 2 units right and 1 unit down
- **h** 0 units left and 2 units down

2 A point is translated to its image. Write the missing word (i.e. left, right, up or down) for each sentence.

- **a** (1, 1) is translated _____ to the point (1, 3).
- **b** (5, 4) is translated _____ to the point (1, 4).
- **c** (7, 2) is translated _____ to the point (7, 0).
- **d** (3, 0) is translated _____ to the point (3, 1).
- **e** (5, 1) is translated _____ to the point (4, 1).
- **f** (2, 3) is translated _____ to the point (1, 3).
- **g** (0, 2) is translated _____ to the point (5, 2).
- **h** (7, 6) is translated _____ to the point (11, 6).

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9I

UNDERSTANDING

3 The point (7, 4) is translated to the point (0, 1).

- **a** How far left has the point been translated?
- **b** How far down has the point been translated?
- **c** If the point (0, 1) is translated to (7, 4):
  - **i** How far right has the point been translated?
  - **ii** How far up has the point been translated?

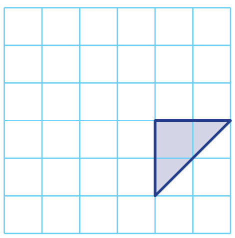
FLUENCY

4, 5–6(½)	4, 5–6(½)	4–6(½)

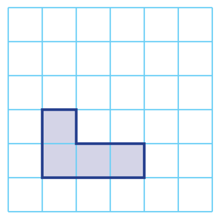
Example 13

4 Draw the image of these shapes after each translation.

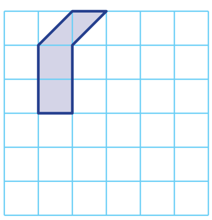
a 3 units left and 1 unit up



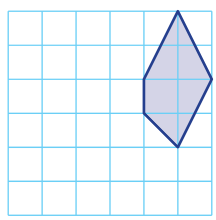
b 1 unit right and 2 units up



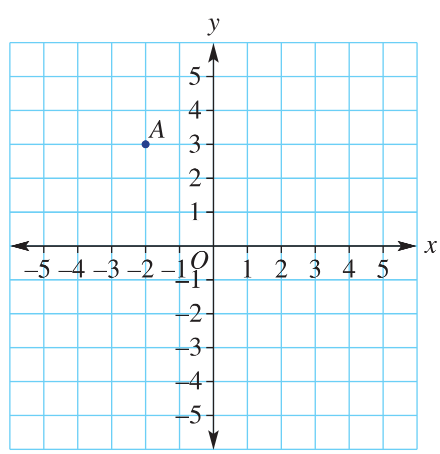
c 3 units right and 2 units down



d 4 units left and 2 units down



5 Point A has coordinates (–2, 3). Write the coordinates of the image point A' when point A is translated in each of the following ways.



- **a** 3 units right
- **b** 2 units left
- **c** 2 units down
- **d** 5 units down
- **e** 2 units up
- **f** 10 units right
- **g** 3 units right and 1 unit up
- **h** 4 units right and 2 units down
- **i** 5 units right and 6 units down
- **j** 1 unit left and 2 units down
- **k** 3 units left and 1 unit up
- **l** 2 units left and 5 units down

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9I

FLUENCY

Example 14

6 Describe the translation when each point is translated to its image. Give your answer similar to these examples: '4 units right' or '2 units left and 3 units up'.

- a $A(1, 3)$ is translated to $A'(1, 6)$.
- b $B(4, 7)$ is translated to $B'(4, 0)$.
- c $C(-1, 3)$ is translated to $C'(-1, -1)$.
- d $D(-2, 8)$ is translated to $D'(-2, 10)$.
- e $E(4, 3)$ is translated to $E'(-1, 3)$.
- f $F(2, -4)$ is translated to $F'(4, -4)$.
- g $G(0, 0)$ is translated to $G'(-1, 4)$.
- h $H(-1, -1)$ is translated to $H'(2, 5)$.
- i $I(-3, 8)$ is translated to $I'(0, 4)$.
- j $J(2, -5)$ is translated to $J'(-1, 6)$.
- k $K(-10, 2)$ is translated to $K'(2, -1)$.
- l $L(6, 10)$ is translated to $L'(-4, -3)$.

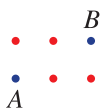
7	7, 8	8, 9

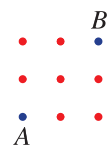
PROBLEM-SOLVING

7 A point, A, is translated to its image, A'. Describe the translation that takes A' to A (i.e. the reverse translation).

- a $A(2, 3)$ and $A'(4, 1)$
- b $B(0, 4)$ and $B'(4, 0)$
- c $C(0, -3)$ and $C'(-1, 2)$
- d $D(4, 6)$ and $D'(-2, 8)$

8 If only horizontal or vertical translations of distance 1 are allowed, how many different paths are there from points A to B on each grid below? No point can be visited more than once.

a


b


9 Starting at $(0, 0)$ on the Cartesian plane, how many different points can you move to if a maximum of 3 units in total can be translated in any of the four directions of left, right, up or down? Do not count the point $(0, 0)$.

10	10	10, 11

REASONING

10 A shape is translated to its image. Explain why the shape's size and orientation is unchanged.

11 A combination of translations can be replaced with one single translation. For example, if $(1, 1)$ is translated 3 units right and 2 units down, followed by a translation of 6 units left and 5 units up, then the final image point $(-2, 4)$ could be obtained with the single translation 3 units left and 3 units up.
Describe the single translation that replaces these combinations of translations.

- a $(1, 1)$ is translated 2 units left and 1 unit up, followed by a translation of 5 units right and 2 units down.
- b $(6, -2)$ is translated 3 units right and 3 units up, followed by a translation of 2 units left and 1 unit down.
- c $(-1, 4)$ is translated 4 units right and 6 units down, followed by a translation of 6 units left and 2 units up.
- d $(-3, 4)$ is translated 4 units left and 4 units down, followed by a translation of 10 units right and 11 units up.

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Combined transformations | — | — | 12

9I

ENRICHMENT

12 Write the coordinates of the image point after each sequence of transformations. For each part, apply the next transformation to the image of the previous transformation.

a $(2, 3)$
- reflection in the x-axis
- reflection in the y-axis
- translation 2 units left and 2 units up

b $(-1, 6)$
- translation 5 units right and 3 units down
- reflection in the y-axis
- reflection in the x-axis

c $(-4, 2)$
- rotation 180° about $(0, 0)$
- reflection in the y-axis
- translation 3 units left and 4 units up

d $(-3, -7)$
- rotation 90° clockwise about $(0, 0)$
- reflection in the x-axis
- translation 6 units left and 2 units down

e $(-4, 8)$
- rotation 90° anticlockwise about $(0, 0)$
- translation 4 units right and 6 units up
- reflection in the x and the y axes


The rotation of the windmills can be analysed as a transformation.

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9J Drawing solids

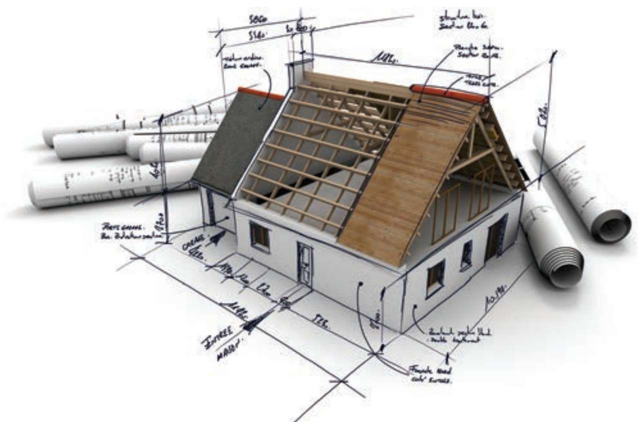
Interactive

Widgets

HOTsheets

Walkthroughs

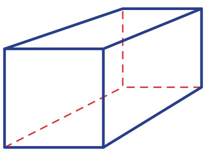
Three-dimensional solids can be represented as a drawing on a two-dimensional surface (e.g. paper or computer screen), provided some basic rules are followed.

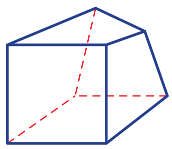


Architects create 3D models of building plans by hand or with computer software.

Let's start: Can you draw a cube?

Try to draw a cube. Here are two bad examples.





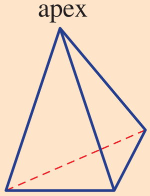
- What is wrong with these drawings?
- What basic rules do you need to follow when drawing a cube?

Key ideas

- Draw cubes and rectangular prisms by keeping:
  - parallel edges pointing in the same direction
  - parallel edges the same length.
- Draw **pyramids** by joining the apex with the vertices on the base.

triangular pyramid (tetrahedron)

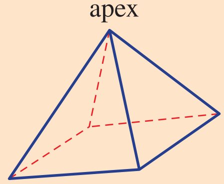
apex



triangular base

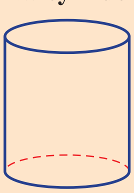
square pyramid

apex

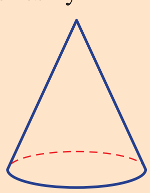


square base

- Draw **cylinders** and **cones** by starting with an oval shape.



cylinder



cone

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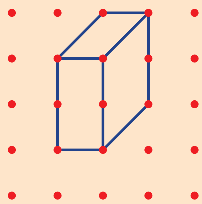
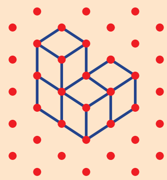
Key ideas

- **Square** and **isometric dot paper** can help to accurately draw solids. Drawings made on isometric dot paper clearly show the cubes that make up the solid.

square dot paper



isometric dot paper

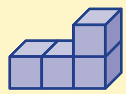


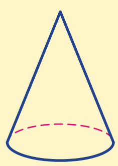
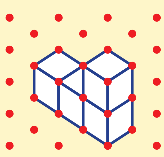
Example 15 Drawing solids

Draw these solids.

a A cone on plain paper

b This solid on isometric dot paper



SOLUTION	EXPLANATION
a 	Draw an oval shape for the base and the apex point. Dot any line or curve which may be invisible on the solid. Join the apex to the sides of the base.
b 	Rotate the solid slightly and draw each cube starting at the front and working back.

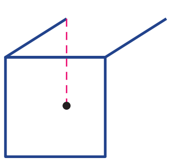
Exercise 9J

1, 2	2	—

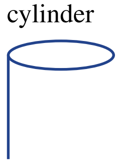
UNDERSTANDING

1 Copy these diagrams and add lines to complete the solid. Use dashed line for invisible edges.

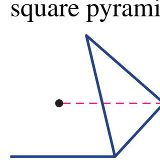
a cube



b cylinder



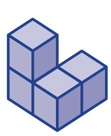
c square pyramid



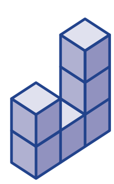
2 Cubes are stacked to form these solids. How many cubes are there in each solid?

a

b



c



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9J

3–6	3–6, 7(½)	3(½), 4–6, 7(½)

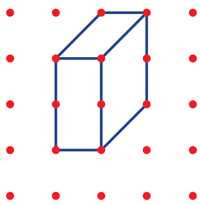
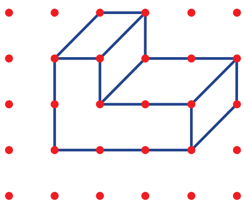
FLUENCY

Example 15a

3 On plain paper draw an example of these common solids.

a cube **b** tetrahedron **c** cylinder
d cone **e** square based pyramid **f** rectangular prism

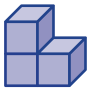
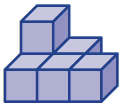
4 Copy these solids onto square dot paper.

a  **b** 

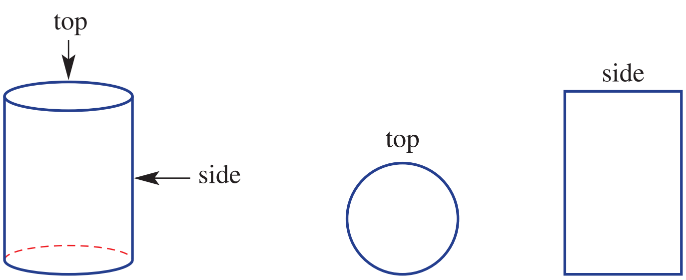
Example 15b

5 Make a copy of the solids in Question **2** on isometric dot paper.

6 Draw these solids onto isometric dot paper.

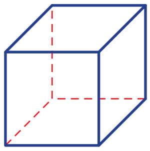
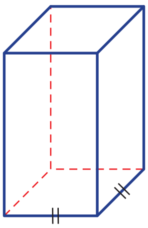
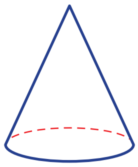
a  **b** 

7 Here is a cylinder with its top view (circle) and side view (rectangle):

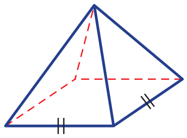
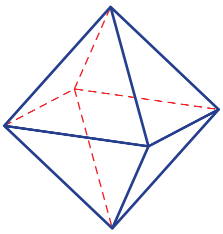
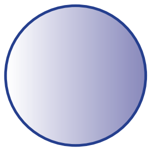


Draw the shapes which are the top view and side view of these solids.

a cube **b** square prism **c** cone

d square pyramid **e** octahedron **f** sphere

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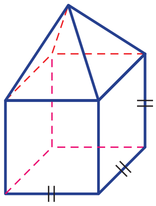
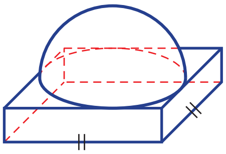
9J

FLUENCY

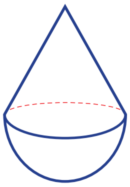
g square pyramid on cube



h hemisphere $\left(\frac{1}{2}\text{ sphere}\right)$ on square prism



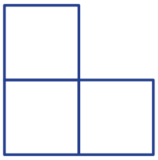
i cone on hemisphere



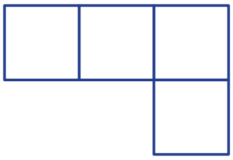
8	8, 9	9, 10

PROBLEM-SOLVING

8 Here is the top (plan or bird's eye) view of a stack of 5 cubes. How many different stacks of 5 cubes could this represent?



9 Here is the top (plan) view of a stack of 7 cubes. How many different stacks of 7 cubes could this represent?



10 Draw these solids, making sure that:

i each vertex can be seen clearly

ii dashed lines are used for invisible edges.

a tetrahedron (solid with 4 faces)

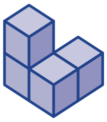
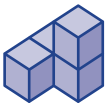
b octahedron (solid with 8 faces)

c pentagonal pyramid (pyramid with pentagonal base)

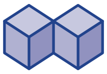
11	11	11, 12

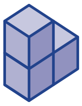
REASONING

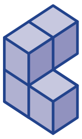
11 Andrea draws two solids as shown. Aiden says that they are drawings of exactly the same solid. Is Aiden correct? Give reasons.

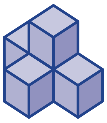
 and 

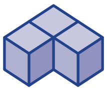
12 Match the solids **a**, **b**, **c** and **d** with an identical solid chosen from **A**, **B**, **C** and **D**.

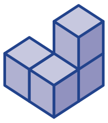
a 

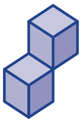
b 

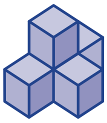
c 

d 

A 

B 

C 

D 

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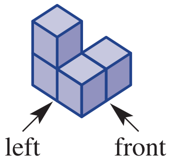
9J

3 viewpoints

—	—	13

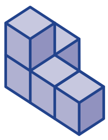
ENRICHMENT

13 This diagram shows the front and left sides of a solid.

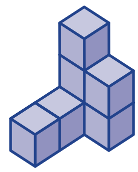


a Draw the front, left and top views of these solids.

i

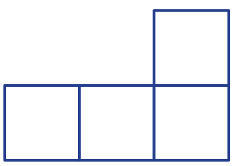


ii

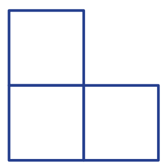


b Draw the solid that has these views.

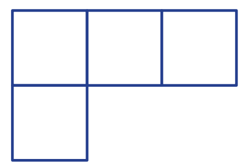
i front



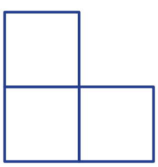
left



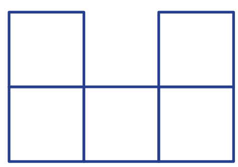
top



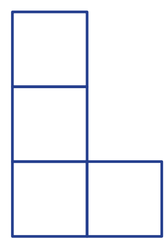
ii front



left



top



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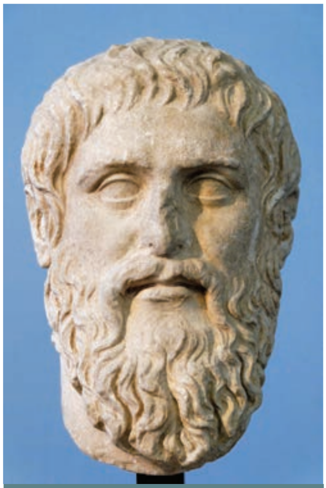
Cambridge University Press

9K Nets and the Platonic solids EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

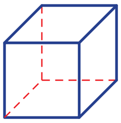
The ancient Greek philosophers studied the properties of polyhedra and how these could be used to explain the natural world. Plato (427–347 BCE) reasoned that the building blocks of all three-dimensional objects were regular polyhedra which have faces that are identical in size and shape. There are 5 regular polyhedra, called the Platonic solids after Plato, which were thought to represent fire, earth, air, water and the universe or cosmos.

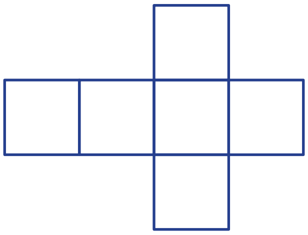


Plato was one of the greatest early philosophers and mathematicians.

Let's start: Net of a cube

Here is one Platonic solid, the regular hexahedron or cube, and its net.



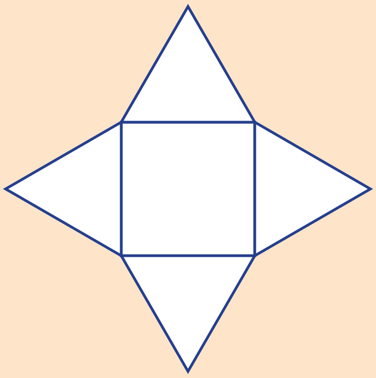


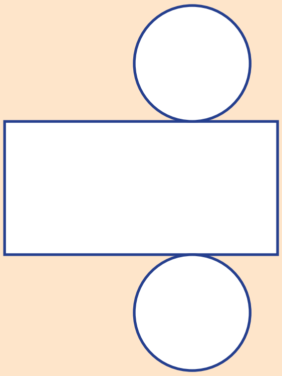
If the faces of the solid are unfolded to form a net, you can clearly see the 6 faces.

Can you draw a different net of a cube? How do you know it will fold to form a cube? Compare this with other nets in your class.

Key ideas

- A **net** of a solid is an unfolded two-dimensional representation of all the faces. Here are two examples.





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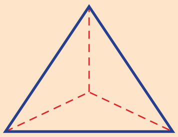
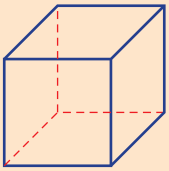
Key ideas

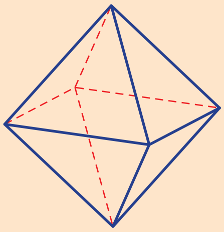
- A **polyhedron** (plural: polyhedra) is a solid with flat faces.
  - They can be named by their number of faces, e.g. tetrahedron (4 faces), hexahedron (6 faces).
- The 5 **Platonic solids** are **regular polyhedra** each with identical regular faces and the same number of faces meeting at each vertex.
  - regular tetrahedron (4 triangular faces)



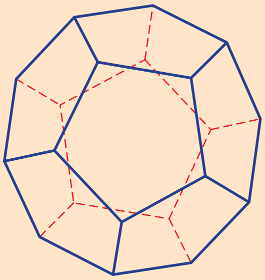
  - regular hexahedron or cube (6 square faces)



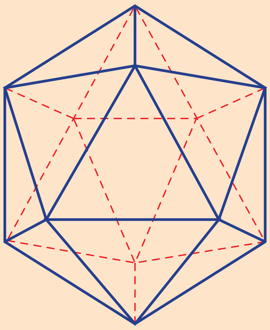
  - regular octahedron (8 triangular faces)



  - regular dodecahedron (12 pentagonal faces)



  - regular icosahedron (20 triangular faces)



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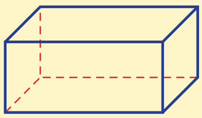
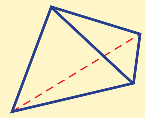
Example 16 Drawing nets

Draw a net for these solids.

a rectangular prism

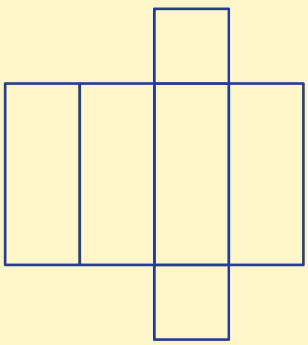


b regular tetrahedron

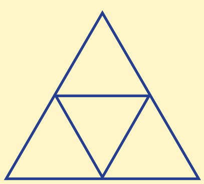


SOLUTION	EXPLANATION
a	This is one possible net for the rectangular prism, but others are also possible.
b	Each triangle is equilateral. Each outer triangle folds up to meet centrally above the centre triangle.

a



b



Exercise 9K

1–4	3, 4	—

UNDERSTANDING

1 Complete these sentences.

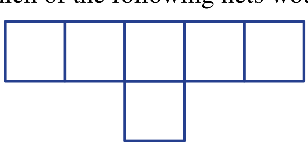
a A regular polygon will have _________ length sides.

b All the faces on regular polyhedra are _________ polygons.

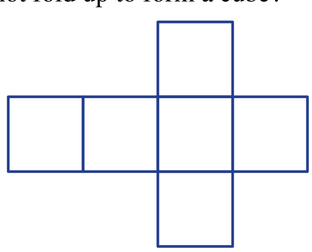
c The _________ solids is the name given to the 5 regular polyhedra.

2 Which of the following nets would not fold up to form a cube?

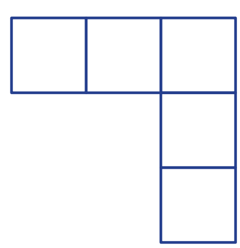
A



B



C



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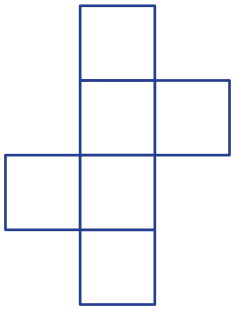
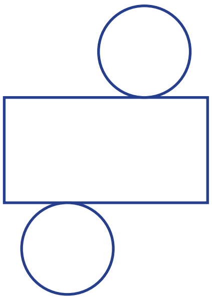
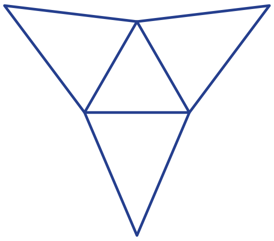
UNDERSTANDING

3 Name the type of shapes that form the faces of these Platonic solids.
a tetrahedron b hexahedron c octahedron
d dodecahedron e icosahedron

4 Name the solids that have the following nets.

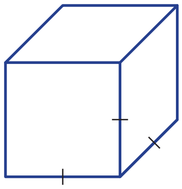
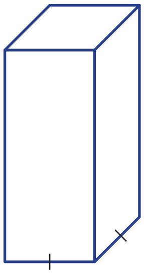
a

b

c


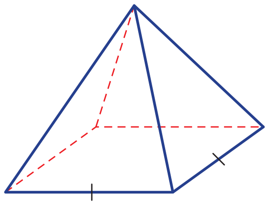
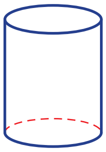
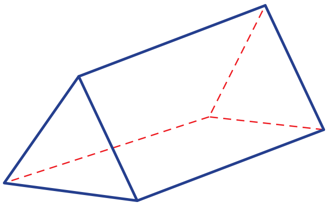
5–7	5–8	5(½), 6–8

FLUENCY

Example 16

5 Draw one possible net for these solids.

a

b

c

d

e

f


6 Which Platonic solid(s) fit these descriptions? There may be more than one.
a Its faces are equilateral triangles.
b It has 20 faces.
c It has 6 vertices.
d It is a pyramid.
e It has 12 edges.
f It has edges which meet at right angles (not necessarily all edges).

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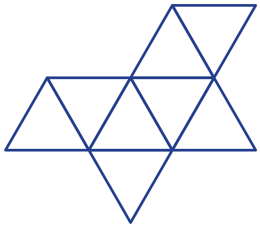
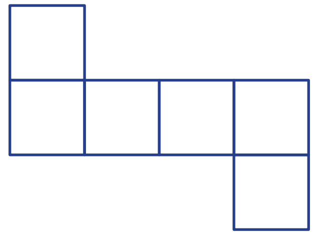
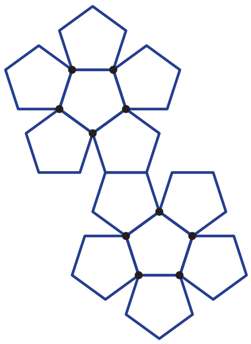
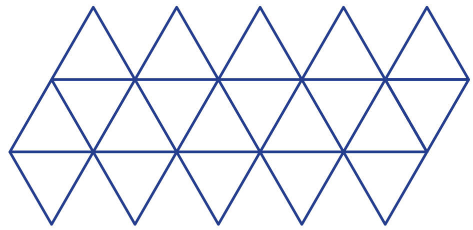
FLUENCY

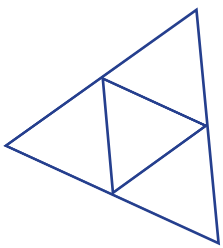
7 Here are nets for the 5 Platonic solids. Name the Platonic solid that matches each one.

a


b


c


d


e


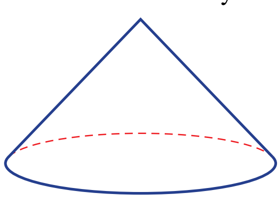
8 How many faces meet at each vertex for these Platonic solids?

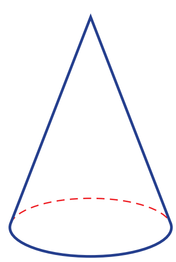
a tetrahedron
b hexahedron
c octahedron
d dodecahdron
e icosahedron

9	9	10

PROBLEM-SOLVING

9 Try drawing a net for a cone. Check by drawing a net and cutting it out to see if it works. Here are two cones to try.

a


b


10 How many different nets are there for these solids? Do not count rotations or reflections of the same net.

a regular tetrahedron
b cube

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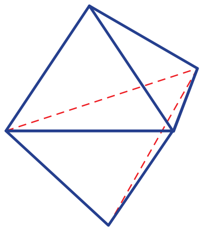
11	11	11

REASONING

11 Imagine gluing two tetrahedrons together by joining two faces as shown, to form a new solid.



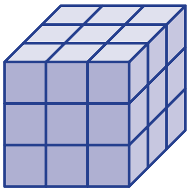
a How many faces are there on this new solid?
b Are all the faces identical?
c Why do you think this new solid is not a Platonic solid.
Hint: Look at the number of faces meeting at each vertex.

Number of cubes

—	—	12

ENRICHMENT

12 Consider a number of 1 cm cubes stacked together to form a larger cube. This one, for example, contains $3 \times 3 \times 3 = 27$ cubes.



a For the solid shown:
i how many 1 cm cubes are completely inside the solid with no faces on the outside?
ii how many 1 cm cubes have at least one face on the outside?

b Complete this table.

n (side length)	1	2	3	4	5
n^3 (number of 1 cm cubes)	1	8			
Number of inside cubes	0				
Number of outside cubes	1				

c For a cube stack of side length n cm, $n \geq 2$ find the rule for:
i the number of cubes in total
ii the number of inside cubes
iii the number of outside cubes

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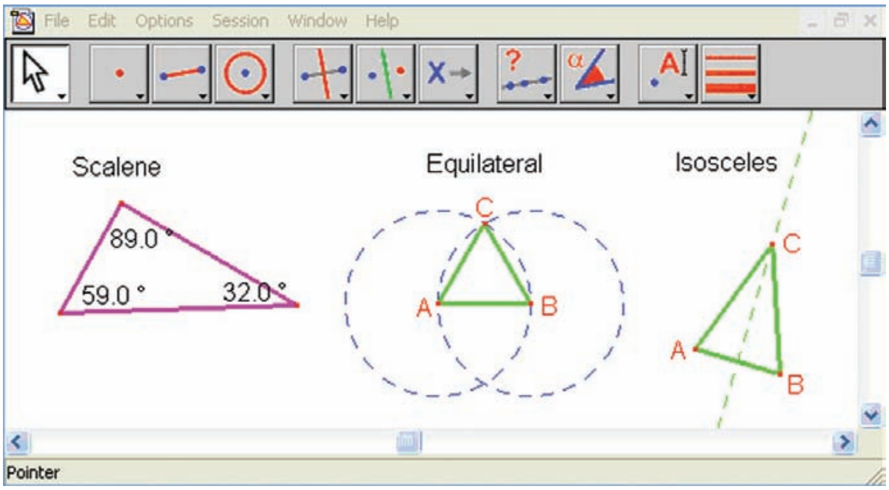
Investigation

Exploring triangles with computer geometry

Scalene

a Use the triangle tool to construct a triangle of any size.
b Measure the three interior angles.
c Use the computer geometry Calculate tool to add up the three interior angles. What do you notice?
d Drag one of the triangle vertices. Does the angle sum change?



Equilateral

a Construct a segment, AB, of any length.
b Construct two circles with centres at A and B, both with exactly the same radius. For the first circle, click at A for the centre and then at B for the radius. For the second circle, do the reverse.
c Place a point where the circles meet and label this C. Then join point C with points A and B.
d Measure the three interior angles. What do you notice?
e Drag the point A or B. What do you notice?

Isosceles

a Construct a segment, AB.
b Use the perpendicular bisector tool to construct the perpendicular bisector of segment AB.
c Place a new point, C, anywhere on the perpendicular line.
d Join point C with points A and B.
e Measure $\angle CAB$ and $\angle CBA$. What do you notice?
f Drag point A, B or C. What do you notice?

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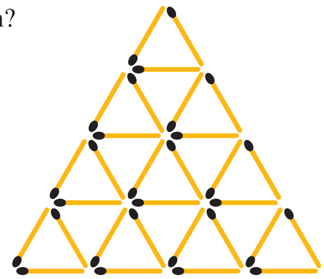
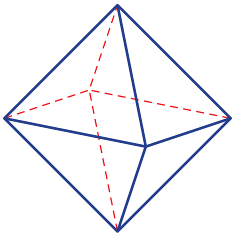
Problems and challenges

1 Use six matches to draw four equilateral triangles.

2 How many equilateral triangles of any size are in this diagram?

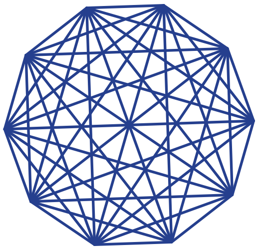


3 A regular octahedron has its corners cut off. How many edges are there in the new shape?



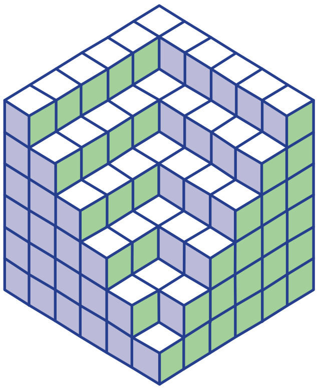
4 A polygon's vertices are joined by diagonals. Without counting the sides, how many diagonals can be drawn in each of these polygons?

a decagon (10 sides)



b 50-gon

5 This solid is made from stacking 1 cm cubes. How many cubes are used?



6 How many pairs of parallel lines are there in any cube?

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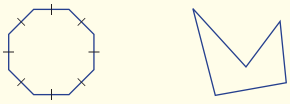
Chapter summary

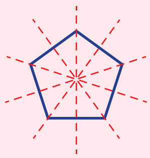
Polygons and transformations

Polygons

regular convex octogon | irregular non-convex pentagon



Symmetry

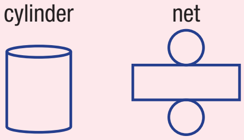


5 lines of symmetry
rotational symmetry of order 5

regular pentagon

Solids

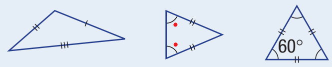
cylinder | net



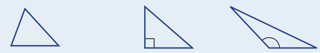
Triangles

Type

scalene | isosceles | equilateral



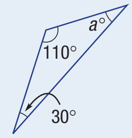
acute | right | obtuse



Constructing triangles

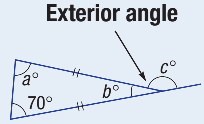
- SSS
- SAS
- AAS
- RHS

Angle sum



$a + 30 + 110 = 180$
$a + 140 = 180$
$a = 40$

Exterior angle



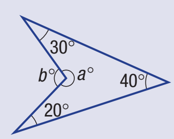
$a = 70$
$b + 140 = 180$
$b = 40$
$c + 40 = 180$
$c = 140$

Quadrilaterals

Special quadrilaterals

- parallelogram
  - rectangle
  - rhombus
  - square
- trapezium
- kite

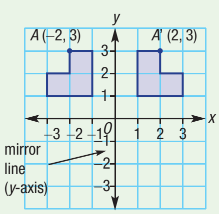
Angle sum



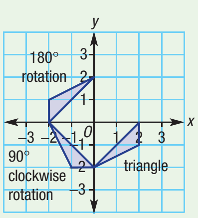
$a + 20 + 40 + 30 = 360$
$a + 90 = 360$
$a = 270$
$b + 270 = 360$
$b = 90$

Transformations

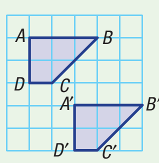
Reflection



Rotation



Translation



2 units right and 3 units down

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Chapter review

Multiple-choice questions

9A
1 A non-convex polygon has:
- **A** all interior angles of 90°
- **B** six sides
- **C** all interior angles less than 180°
- **D** all interior angles greater than 180°
- **E** at least one interior angle greater than 180°

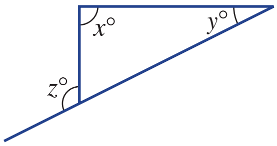
9B
2 The three types of triangles all classified by their interior angles are:
- **A** acute, isosceles and scalene
- **B** acute, right and obtuse
- **C** scalene, isosceles and equilateral
- **D** right, obtuse and scalene
- **E** acute, equilateral and right

9C Ext
3 Which of the following is *not* sufficient to construct a single triangle?
- **A** three sides (SSS)
- **B** two sides and the angle between them (SAS)
- **C** three angles (AAA)
- **D** two angles and one side (AAS)
- **E** a right angle, hypotenuse and one other side (RHS)

9D
4 The angle sum of a triangle is always:

A 120° **B** 360° **C** 270° **D** 180° **E** 90°

9D
5 For the triangle shown opposite, the exterior angle theorem for a triangle says:



A $y = x + z$ **B** $z = x - y$ **C** $z = x + y$
D $x = z + y$ **E** $z = y - x$

9E
6 The quadrilateral that has 2 pairs of sides of equal length and 1 pair of angles of equal size is called a:

A kite **B** trapezium **C** rhombus **D** triangle **E** square

9F
7 Three angles inside a quadrilateral add to 275°. The fourth angle is:

A 750° **B** 95° **C** 285° **D** 125° **E** 85°

9G
8 A rhombus has line symmetry of order:

A 0 **B** 1 **C** 2 **D** 3 **E** 4

9H
9 The point $T(-3, 4)$ is reflected in the x-axis; hence, the image point T' has coordinates:

A $(3, 4)$ **B** $(-3, 4)$ **C** $(0, 4)$ **D** $(3, -4)$ **E** $(-3, -4)$

9I
10 The translation that takes $A(2, -3)$ to $A'(-1, 1)$ could be described as:
- **A** 3 units left
- **B** 4 units up
- **C** 3 units left and 4 units up
- **D** 1 unit right and 2 units down
- **E** 1 unit left and 2 units down

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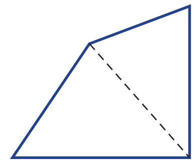
Short-answer questions

9A **1** How many sides do these polygons have?

a pentagon **b** heptagon **c** undecagon

9A **2** A diagonal inside a polygon joins two vertices. Find how many diagonals can be drawn inside a quadrilateral if the shape is:

a convex **b** non-convex



9C Ext **3** Use a protractor and ruler to draw these triangles.

a triangle ABC with $AB = 4$ cm, $\angle CAB = 25^\circ$ and $\angle ABC = 45^\circ$

b triangle ABC with $AB = 5$ cm, $\angle BAC = 50^\circ$ and $AC = 5$ cm

9C Ext **4** Use a protractor, pair of compasses and a ruler to construct these triangles.

a triangle ABC with $AB = 5$ cm, $BC = 6$ cm and $AC = 3$ cm

b triangle ABC with $AB = 6$ cm, $\angle BAC = 35^\circ$ and $AC = 5$ cm

9C Ext **5** Is there sufficient information to construct a triangle in each of following cases? If yes, write SSS, SAS, AAS or RHS.

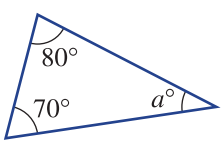
a triangle ABC with $\angle ABC = 20^\circ$, $\angle BAC = 40^\circ$ and $AB = 6$ cm

b triangle DEF with $DE = 9$ cm and $\angle DEF = 72^\circ$

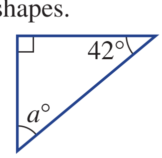
c triangle STU with $\angle STU = 90^\circ$, $SU = 10$ cm and $ST = 6$ cm

9D **6** Find the value of a in each of these shapes.

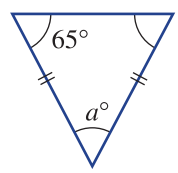
a



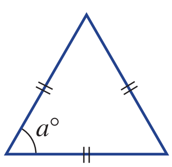
b



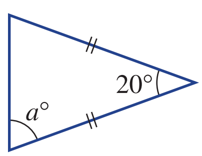
c



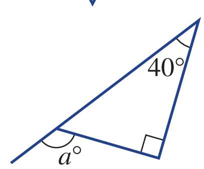
d



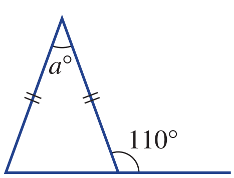
e



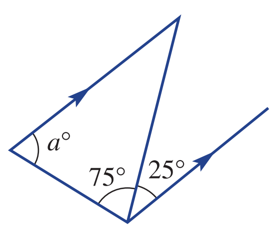
f



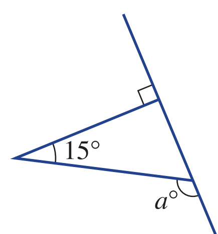
g



h

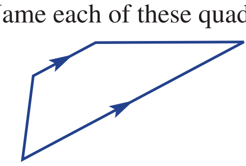


i

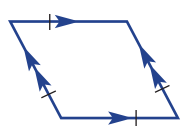


9E **7** Name each of these quadrilaterals.

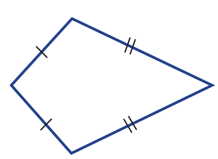
a



b



c



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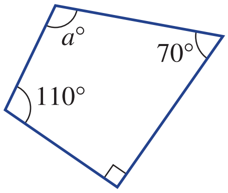
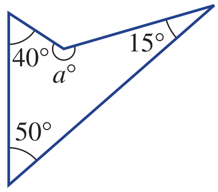
Chapter review

9F **8** Find the value of a, marked in these quadrilaterals.

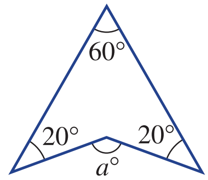
a



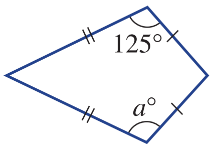
b



c



d



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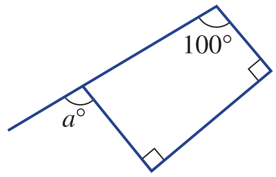
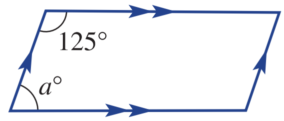


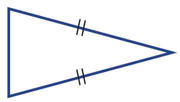
f



9G **9** Name the order of line and rotational symmetry for each of these diagrams.

a

b



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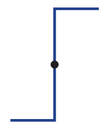
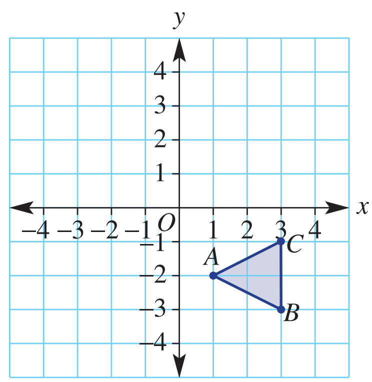


9H **10** Write the coordinates of A', B' and C' when this shape is reflected in the following mirror lines.

a the y-axis **b** the x-axis

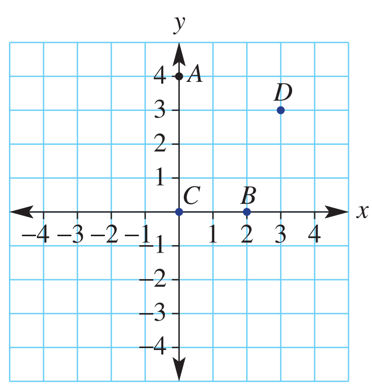


9H **11** Points $A(0, 4)$, $B(2, 0)$ and $D(3, 3)$ are shown here. Write down the coordinates of the image points A', B' and D' after each of the following rotations.

a $180°$ about $C(0, 0)$

b $90°$ clockwise about $C(0, 0)$

c $90°$ anticlockwise about $C(0, 0)$



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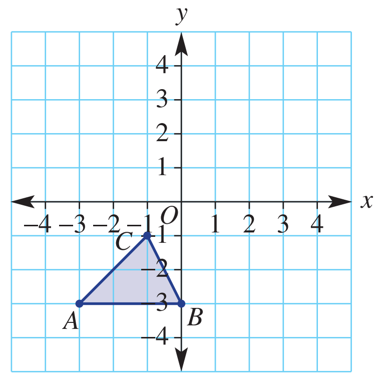
9I

12 Write the coordinates of the vertices A', B' and C' after each of these translations.



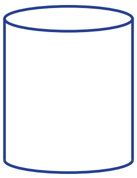
a 4 units right and 2 units up

b 1 unit left and 4 units up

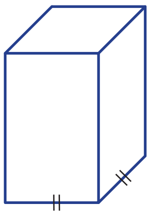
9J

13 Draw a side view, top view and net for each of these solids.

a

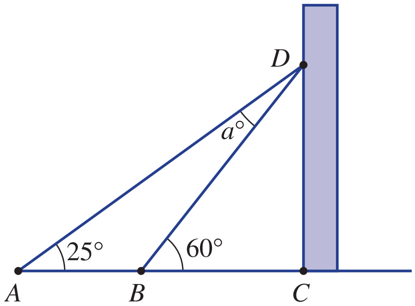


b



Extended-response questions

1 Two cables support a vertical tower, as shown in the diagram opposite, and the angle marked $a°$ is the angle between the two cables.



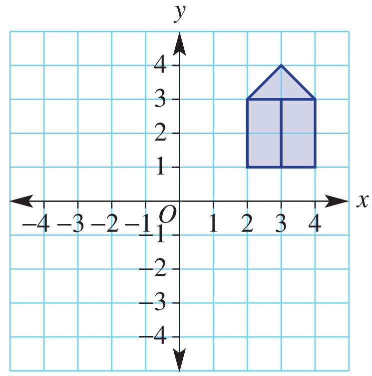
a Find $\angle BDC$.

b Find $\angle ADC$.

c Find the value of a.

d If $\angle DAB$ is changed to 30° and $\angle DBC$ is changed to 65°, will the value of a stay the same? If not, what will be the new value of a?

2 Shown is a drawing of a simple house on a Cartesian plane. Draw the image of the house after these transformations.



a translation 5 units left and 4 units down

b reflection in the x-axis

c rotation 90° anticlockwise about $C(0, 0)$

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