
10

Transformation and visualisation

10

Why is The Pentagon a pentagon?

In 1941, a radical new design for a government building impressed the President of the USA.

Located near Washington, DC, the building known as 'The Pentagon' was built during World War II to house the War Department, now the United States Department of Defense. It is one of the largest buildings in the world, despite being only five storeys high (the steel required for extra storeys was needed for warships and weapons). The five-sided shape was chosen because it was the best fit to the shape of the land originally chosen for the location of the building. President Roosevelt changed the location, but kept the shape because, 'nothing like it has ever been done that way before'. The Pentagon is a regular pentagon. Each of the five sides and the five angles formed where the sides meet are equal. Because it is a regular shape, the Pentagon has a high degree of reflectional and rotational symmetry.

Forum

How do you think the builders of the Pentagon ensured that each of the five sides and angles were equal?

Looking at the Pentagon from above:

How many shapes can you identify within the main pentagon shape?

How many straight lines can you draw that divide the shape into two identical halves?

Many buildings and structures that are considered 'beautiful' are symmetrical. Do you think we are naturally 'drawn' towards symmetry?

Why learn this?

Everyday we see and work with geometrical figures. We get food and drink from cylindrical cans, pack things into prism-shaped boxes, and play sport on rectangular courts with spherical balls. Each of these objects has been designed so that its shape suits its purpose. Many careers involve design, from architecture (designing buildings), to industrial design (designing products for a mass market) to graphic design (designing the images used in advertising, books, magazines and websites). Knowledge of geometry and appreciation of symmetry is important in these and many other careers.

After completing this chapter you will be able to:

- translate, reflect and rotate shapes
- describe the process and compare the results of different transformations
- identify lines of symmetry
- identify the order of reflectional symmetry and the order of rotational symmetry
- recognise symmetry in design and nature
- visualise and draw 3D shapes
- draw elevations and plan views of 3D shapes.

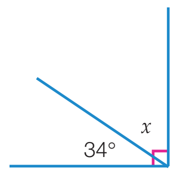
Recall 10

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

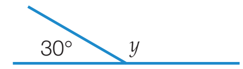
Worksheet R10.1

1 Find the size of the unknown angle in each of the following.

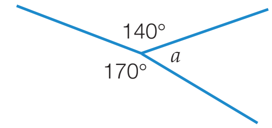
(a)



(b)



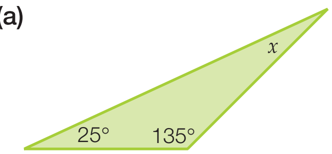
(c)



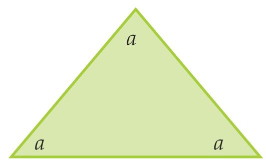
Worksheet R10.2

2 Find the size of the unknown angle in each of the following.

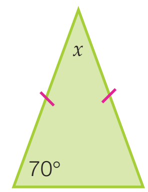
(a)



(b)



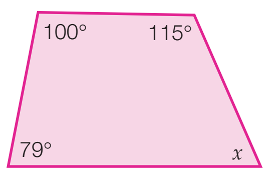
(c)



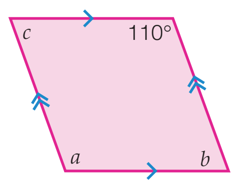
Worksheet R10.3

3 Find the size of the unknown angle in each of the following.

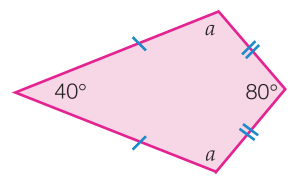
(a)



(b)



(c)



Worksheet R10.4

4 Draw an example of:

(a) a rhombus
(b) a trapezium
(c) an equilateral triangle
(d) a kite
(e) an isosceles triangle
(f) a parallelogram.

Mark all sides that are equal and/or parallel. Use letter names to name your figures.

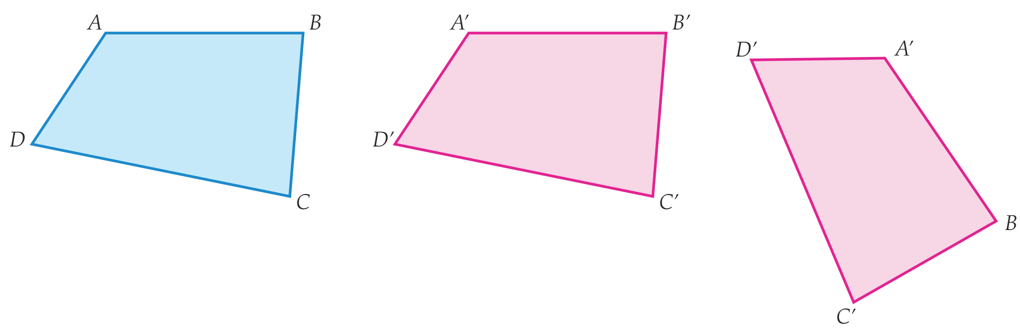
Key Words

asymmetrical	isometric	perpendicular distance	rotation
axis of symmetry	line of reflection	plan views	rotational symmetry
centre of rotation	order of reflectional symmetry	reflection	transformation
elevations	order of rotational symmetry	reflectional symmetry	translation
image	original	rotate	

Translations

10.1

A **transformation** changes a particular shape (or figure) or set of points according to a defined rule. A transformation may move, turn, flip or change the shape of a figure. The transformations we will deal with here are those that move a shape but retain its original shape and size. The first figure is called the **original** and the transformed shape is called the **image** of the transformation. We give the original shape letter names such as A, B, C, D and the image letter names such as A', B', C', D'. Transformations may change the position only, or they may involve a change in orientation such as a turn or a reflection.



Original

Image 1
Change in position only

Image 2
Change in position and orientation

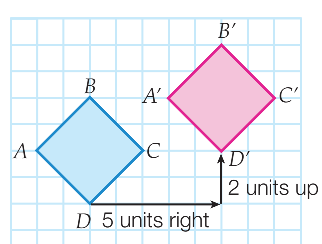
There are three types of transformations in which the shape does not change:

- translation
- reflection
- rotation.

Each type of transformation moves a figure in a particular way. Transformations can be combined to make a complex move and can be reversed to move the image back to its original position and orientation.

Translations

A **translation** occurs when a figure is shifted or slid across the page to change its position. When a figure undergoes translation, the image is moved without turning. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both. The example at right shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as [5, 2].

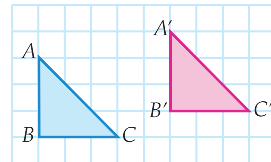


- Vertices of an image are denoted by a dash; e.g. A' or B'. We say 'A dash' or 'A prime'.
- To distinguish between the original figure and the translated image, you can use a different colour or broken lines.
- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

Worked Example 1

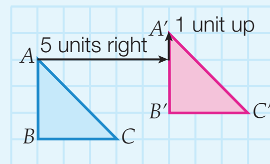
WE1

Describe the translation shown in the diagram.



Thinking / Working

1 Start by selecting one of the vertices and count the units in the horizontal and vertical directions to get to the corresponding translated vertex.



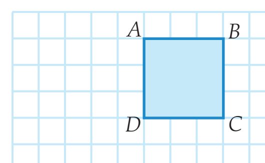
2 Write the translation in words or as an ordered pair.

5 units right and 1 unit up or [5, 1].

Worked Example 2

WE2

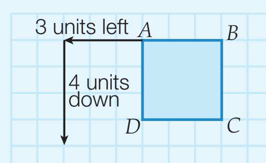
Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.



Thinking / Working

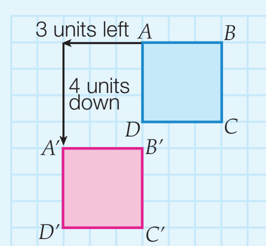
1 Start by selecting one of the vertices (*A*) and move according to the given translation.

2 Label this vertex with a letter using image notation (*A*′).



3 Repeat with the remaining vertices and then draw the resulting image in a different colour.

4 Label all vertices of the image using image notation.



If our shape is drawn on the Cartesian plane, we can identify the coordinates of the vertices and then find the coordinates of the transformed shape after a given transformation. In this section, we will identify the coordinates of a shape after a given translation.

Worked Example 3

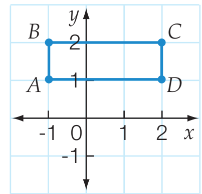
WE3

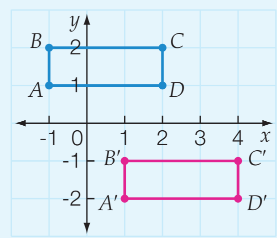
(a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (-1, 1)$, $B = (-1, 2)$, $C = (2, 2)$, $D = (2, 1)$
(b) Translate each vertex by the given translation, labelling each vertex with image notation. (Here, we move each point 2 units right and 3 units down. This is a translation of [2, -3].)	(b) 
(c) Identify the coordinates of each vertex on the resulting image.	(c) $A' = (1, -2)$, $B' = (1, -1)$, $C' = (4, -1)$, $D' = (4, -2)$
(d) Look for the connection between the original coordinates, the transformation and the image coordinates.	(d) Adding 2 to the x-coordinate of the original and subtracting 3 from the y-coordinate of the original will give the image coordinates.

10.1 Translations

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18

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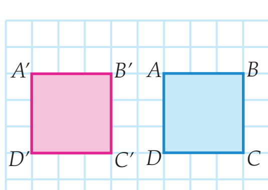
Equipment required: Grid paper for Questions **2**, **3**, **9**, **13(b)**, **15(c)**, **16** and **17**

Fluency

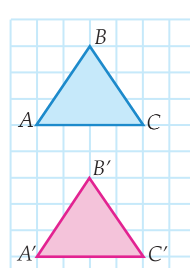
1 Describe the translation shown in each of the following diagrams.

WE1

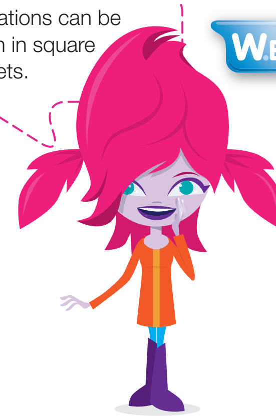
(a)



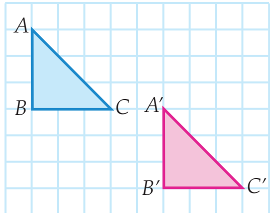
(b)



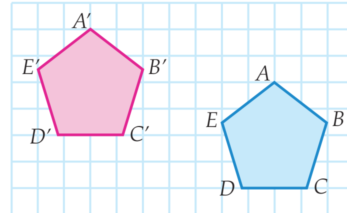
Translations can be written in square brackets.



(c)



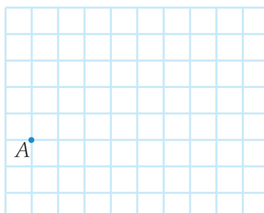
(d)



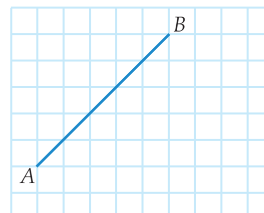
WE2

2 Copy each of the following onto grid paper and draw the resulting image after the translation.

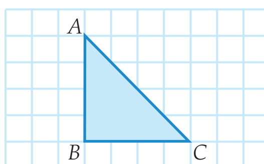
(a) 6 units right and 2 units up



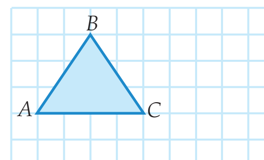
(b) 6 units left and 6 units down



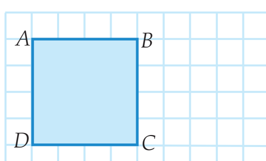
(c) 6 units left and 1 unit down



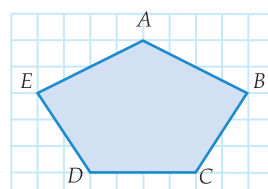
(d) 1 unit right and 6 units down



(e) [7, -5]



(f) [-6, 4]



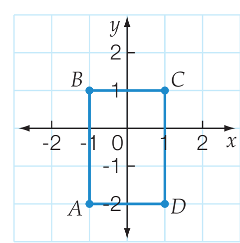
WE3

3 (a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 1 unit to the left and 2 units up.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



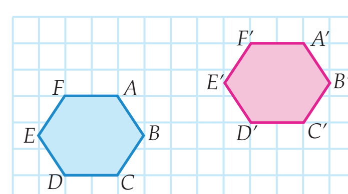
4 How has the hexagon in the diagram been translated?

A 7 units up and 2 units right

B 7 units right and 2 units up

C 7 units left and 2 units down

D 6 units right and 2 units up



5 What would be the reverse translation of 4 units left and 8 units down?

A 4 units up and 8 units right

B 4 units left and 8 units up

C 4 units left and 8 units down

D 4 units right and 8 units up

6 Write the translation [-2, 6] using direction instructions (i.e. right, left, up, down).

7 Write the translation of 3 units left and 4 units down using square brackets.

8 Write the reverse translation of 10 units right and 4 units up using square brackets.

Understanding

9 (a) Plot the points (-1, 4), (2, 3) and (0, 2) on a Cartesian plane and label them A, B and C.

(b) Join the points to form triangle ABC, then translate the shape 2 units right and 1 unit down, labelling the image with image notation.

(c) Write the coordinates of the translated points.

10 A figure is translated 4 units left, 3 units down, 6 units left and 7 units up. What would be the final position of the image compared to that of the original figure?

11 What single translation would be equivalent to 12 units down, 5 units right, 7 units left, 8 units up and 2 units right?

12 What single translation would be equivalent to a translation of [2, 3] followed by a translation of [-2, 6]?

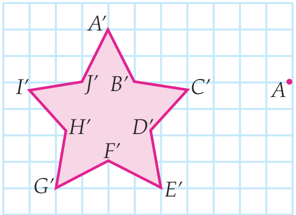
Reasoning

13 (a) Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(6, -1)$ is translated 2 units left and 2 units up.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

14 If a particular translation was 2 units right, what would be the single translation that would be equivalent to repeating 2 units right a hundred times?

15 The following diagram is of a translated image and a single point of the original figure.



(a) What translation has occurred to the original figure?

(b) What is the reverse translation?

(c) Use this to reverse the translation and draw the original figure.

Open-ended

16 Plot three points on a Cartesian plane and write their coordinates. Decide upon a particular translation and use this translation to translate your shape. Draw the image after the translation and write down the coordinates of the translated points using image notation.

17 On grid paper, draw a figure that has 4 vertices. Then, translate that figure in any direction and identify the reverse translation required to get the image back to the original figure.

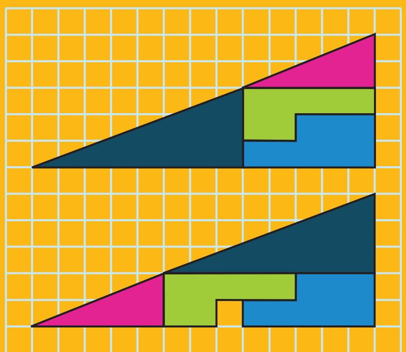
18 Provide an explanation either for or against the following statement:

'If a figure undergoes more than one translation, the order in which you translate a figure does not affect the final position of the image.'

Outside the Square Puzzle

Jigsaw paradox

This Jigsaw Paradox orginates from the famous Curry's Paradox and was modified by Martin Gardner in 1956. This paradox or puzzle shows two arrangements of four identical shapes to create two right-angled triangles, but one has a 1×1 hole in it. Both triangles have 13 units for the base and 5 units for the height.



How can this be true? Can you explain what happened to the missing 1×1 square?

10.2 Reflections

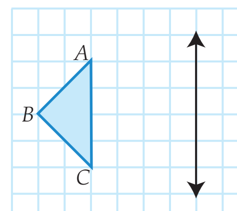
A **reflection** is a transformation that creates an image of the original figure in the same way that a mirror creates an image when you look into it. A reflection is therefore often called a mirror image. It is a reversed or 'flipped over' version of the original about a **line of reflection**. A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed.

Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side. The distance used is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection. To reflect a figure, reflect each vertex on the figure and then join the points in order.

Worked Example 4

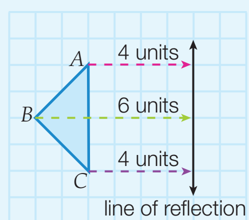
WE4

Copy the following figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



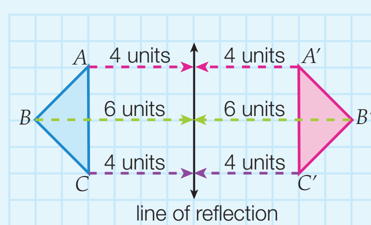
Thinking / Working

1 Measure the perpendicular distance between each vertex and the line of reflection.



2 Use the distance between the vertices on the line of reflection to plot the points on the opposite side of the line of reflection and reproduce the original figure.

3 Label the image vertices with image notation.



- A reflection always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same; for example, the size of the side lengths or angles will stay the same.

We can find the image of a shape drawn on a Cartesian plane after it is reflected in either the x- or y-axis.

Worked Example 5

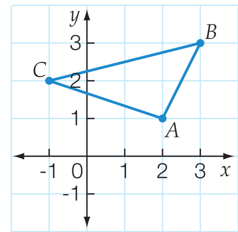
WE5

(a) **(i)** Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

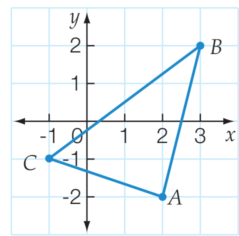


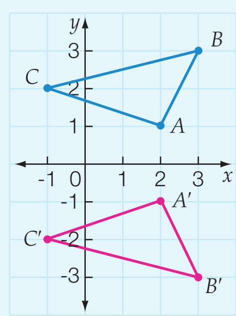
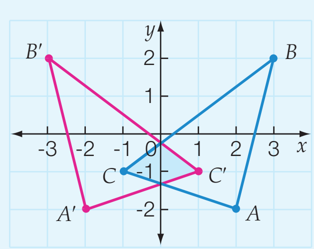
(b) **(i)** Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.



Thinking	Working
(a) **(i)** Use the x- and x-axes to identify the Cartesian coordinates of the points.	(a) (i) $A = (2, 1)$, $B = (3, 3)$, $C = (-1, 2)$
(ii) Reflect each point in the x-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	(ii) 
(iii) Identify the coordinates of each vertex on the resulting image.	(iii) $A' = (2, -1)$, $B' = (3, -3)$, $C' = (-1, -2)$
(iv) Look for the connection between the coordinates of the original and the image.	(iv) The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original.
(b) **(i)** Use the x- and y-axes to identify the Cartesian coordinates of the points.	(b) (i) $A = (2, -2)$, $B = (3, 2)$, $C = (-1, -1)$
(ii) Reflect each point in the y-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	(ii) 

(iii) Identify the coordinates of each vertex on the resulting image.	(iii) $A' = (-2, -2)$, $B' = (-3, 2)$, $C' = (1, -1)$
(iv) Look for the connection between the coordinates of the original and the image.	(iv) *The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.*

10.2 Reflections

Navigator

Answers page 687

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13

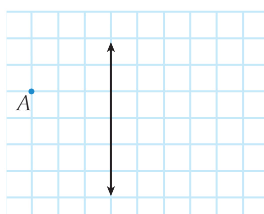
Equipment required: Grid paper for Questions **1–3**, **5–8** and **10–13**

Fluency

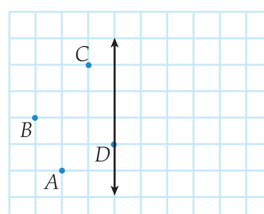
WE4

1 Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection shown.

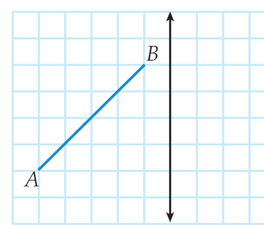
(a)



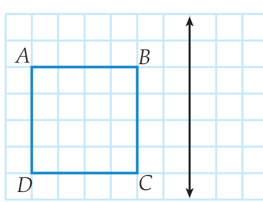
(b)



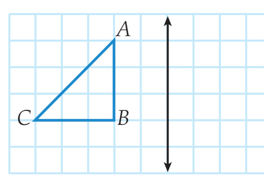
(c)



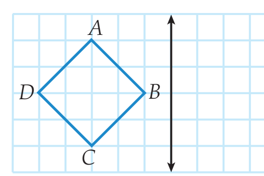
(d)



(e)



(f)



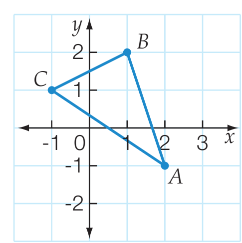
WE5

2 (a) (i) Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

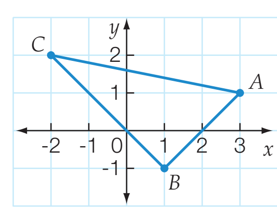


(b) (i) Write down the coordinates of each vertex on the given shape.

(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

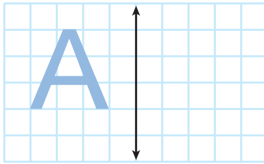
(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

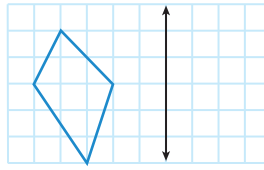


3 Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection shown.

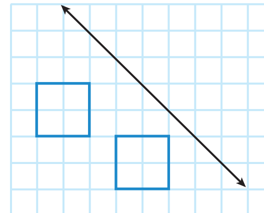
(a)



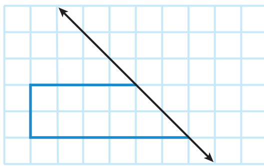
(b)



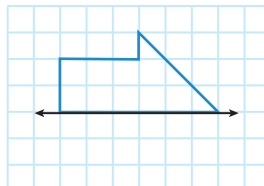
(c)



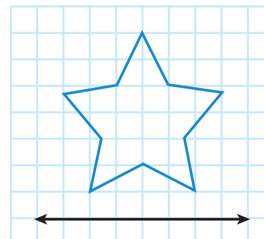
(d)



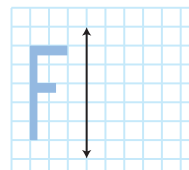
(e)



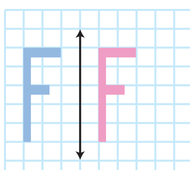
(f)



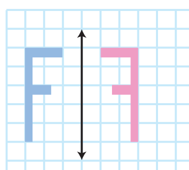
4 Which of the following options is the correct reflection of the capital F in the vertical line of reflection?



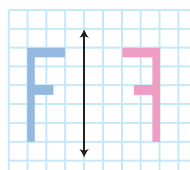
A



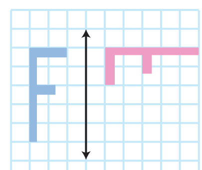
B



C



D



Understanding

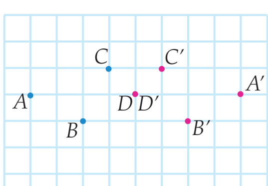
5 (a) Plot the points (-1, 5), (3, 1) and (2, -1) on a Cartesian plane and label them A, B and C.

(b) Join the points to form triangle ABC, then reflect the shape first in the x-axis and then in the y-axis.

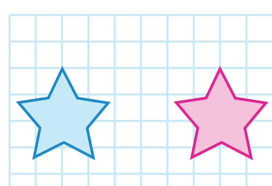
(c) Write the coordinates of the points after the second reflection.

6 Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

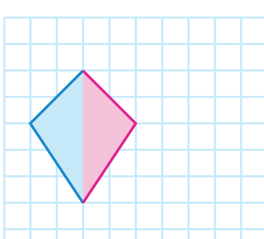
(a)



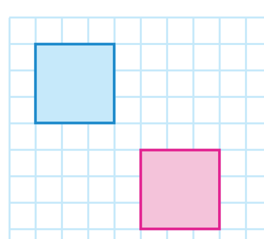
(b)



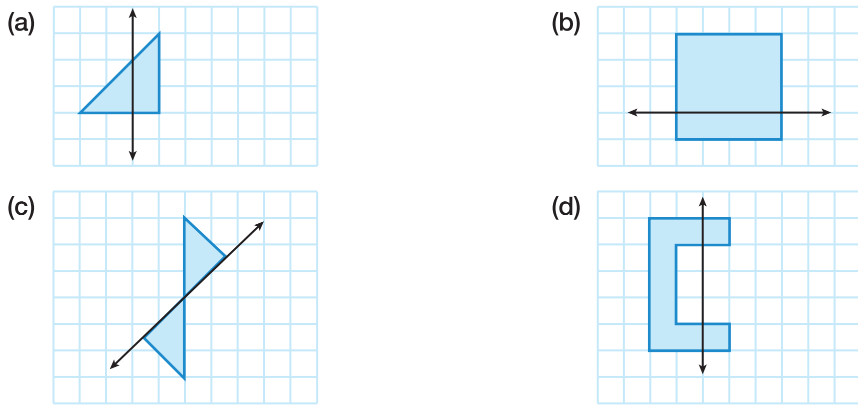
(c)



(d)



7 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.



Reasoning

8 (a) Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the x-axis.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

(c) Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the y-axis.

(d) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(c)**.

9 Bruno has a digital clock on his glass bedside table. When he looked at the time, he noticed that at that particular instance, the time 8.30 was reflected perfectly on the table. Thinking this to be a little odd, he decided to find out the following.

(a) List the number of minutes past the hour when the minute part of the time reflects the correct time (include 00).

(b) In a 12-hour mode, list the hours when the hour part of the time reflects the correct time.

(c) In a 12-hour period, use your answers to parts **(a)** and **(b)** to find how many times the reflection on the table gives the correct time in a 12-hour mode.

(d) In a 24-hour period, how many times does the reflection in the table give the correct time in a 24-hour mode?

(e) Is your answer to part **(d)** double your answer to part **(c)**? Explain your answer.

Open-ended

10 Plot three points on a Cartesian plane and write their coordinates. Reflect this shape in either the x- or y-axis. Draw the image after the reflection and write down the coordinates of the reflected points using image notation.

11 On grid paper, draw a figure with four vertices. Reflect this figure in:

(a) a horizontal line of reflection

(b) a vertical line of reflection.

12 Repeat Question **11** with a figure with five, six, or seven vertices.

13 On grid paper, draw a figure with four vertices.

(a) Reflect this figure in a horizontal line of reflection and then in a vertical line of reflection.

(b) Reflect the same figure in the vertical line of reflection and then in the horizontal line of reflection.

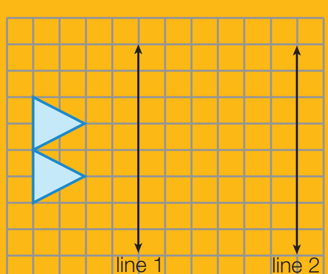
(c) Comment on the images found in parts **(a)** and **(b)**.

Outside the Square — Problem solving

Mirror, mirror on the wall, does it matter where I stand at all?

Equipment required: 1 brain, grid paper

The diagram below consists of two blue triangles and two vertical lines of reflection.



To begin, reflect the two triangles in the line of reflection labelled 'line 1' and then in the line of reflection labelled 'line 2' and note the final location of the image. Next, switch the order in which the two triangles are reflected so that they are reflected in line 2 first, and then in line 1, and note the final location of the image.

Does the order in which these triangles are reflected in either line affect the final location and orientation of the final image? What did you notice about the final image? Can this transformation be described another way?

To justify your decision, experiment with different situations, such as drawing the two triangles originally on the right-hand-side of line 2 or in-between the lines. You can also alter the position of the lines of reflection. For example, line 1 and line 2 could be two horizontal lines or one could be horizontal and the other vertical.

Strategy options

- Draw a diagram.
- Test all possible combinations.

10.3 Rotations

We also transform a figure when we **rotate** it around a fixed point. This is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter O.

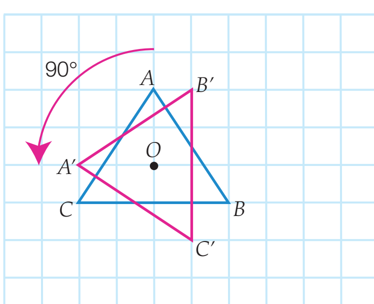
When rotating any figure you need to know:

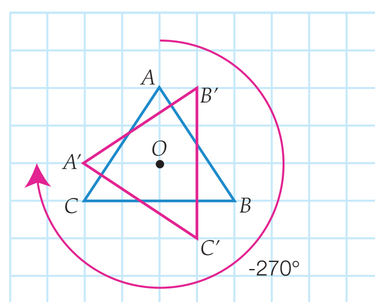
- the location of the centre of rotation
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The centre of rotation can be located inside or outside the figure and is the only point that does not rotate. The size of the angle of rotation is generally in multiples of 30° or 45°. Common angles of rotation are 30°, 45°, 60°, 90°, 180° and 270°. A rotation of 360° will rotate the image of the figure through a full revolution about the centre of rotation and will return the figure to its original position.

- The vertices of a rotated image are labelled in the same order as the original figures.
- Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.
- Two rotations that result in an image in the same position will always add to 360°. For example, 90° clockwise is equivalent to 270° anticlockwise as 90° + 270° = 360°.
- A rotation in an anticlockwise direction is a positive rotation.
- A rotation in a clockwise direction is a negative rotation.

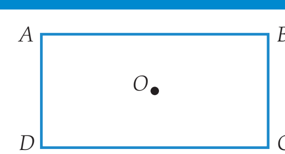
A clockwise rotation of 180° is the same as an anticlockwise rotation of 180°.

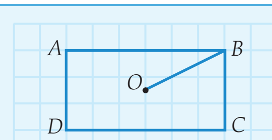




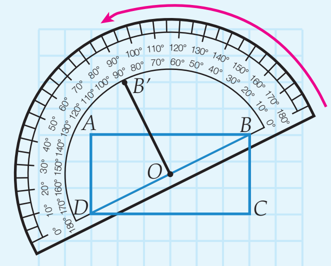
Worked Example 6

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about O.

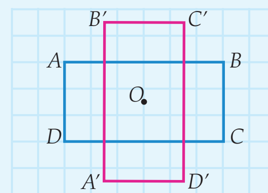


Thinking	Working
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.	

2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex. Plot the resulting point. The new point must be the same distance from *O* as the original point.



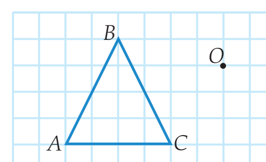
3 Continue the process with the other vertices and connect the resulting points to produce the original figure in its rotated position.



The same steps are used when the centre of rotation is outside the shape. The distance between the vertices and the centre of rotation is the same before and after the rotation, similar to reflection of an object in a line of reflection. Measuring this distance and checking that it is the same before and after the rotation is an easy way to check that the rotation is correct.

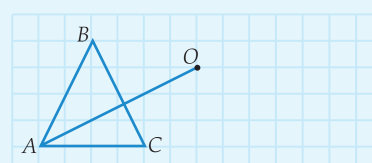
Worked Example 7

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (-180°) about *O*.

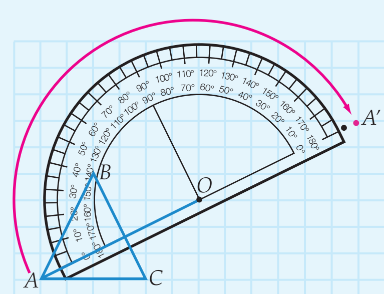


Thinking	Working

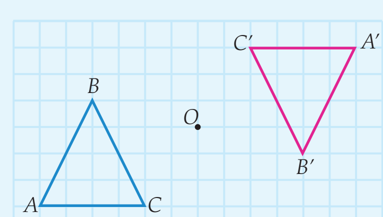
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.



2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex and plot a point. Extend *AO* through this point and mark the point *A*′ so that *OA*′ is the same length as *OA*.



3 Continue the process with the other vertices and connect the resulting points to reproduce the original figure in its rotated position.

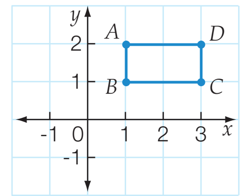


We can find the coordinates of an image after a rotation of 90°, 180° or 270° in a clockwise or anticlockwise direction about the origin.

Worked Example 8

WE8

(a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) **(i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) **(i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

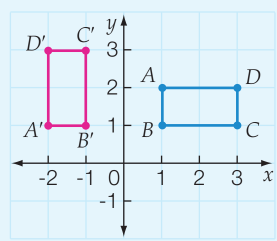
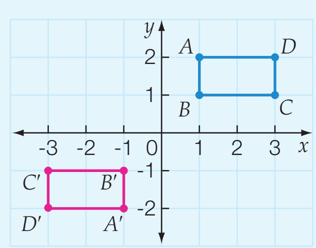
(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) **(i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

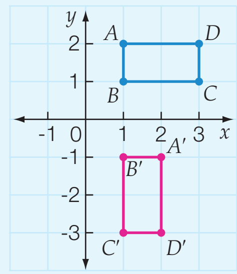
Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (1, 2)$, $B = (1, 1)$, $C = (3, 1)$, $D = (3, 2)$
(b) **(i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 90° in an anticlockwise direction about the origin.)	(b) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-2, 1)$, $B' = (-1, 1)$, $C' = (-1, 3)$, $D' = (-2, 3)$
(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.	(iii) The x-coordinates of the image vertices are the negative of the y-coordinates of the original vertices and the y-coordinates of the image vertices are the x-coordinates of the original vertices.
(c) **(i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 180° in an anticlockwise direction about the origin.)	(c) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-1, -2)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, -2)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

(d) (i) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 270° or -90°.)

(d) (i)



(ii) Identify the coordinates of each vertex on the resulting image.

(ii) $A' = (2, -1)$, $B' = (1, -1)$, $C' = (1, -3)$, $D' = (2, -3)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.

10.3 Rotations

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 (a), Q11, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12 Column 2, Q13, Q14, Q15, Q16

Answers page 689

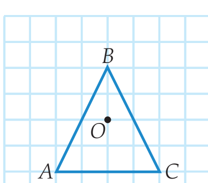
Equipment required: Protractor and grid paper for Questions **1**, **2**, **3**, **6**, **9 (b)**, **10**, **14** and **16**

Fluency

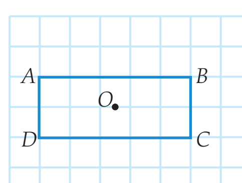
1 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

WE6

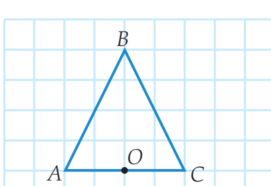
(a) Rotate 180° in an anticlockwise direction (180°) about *O*.



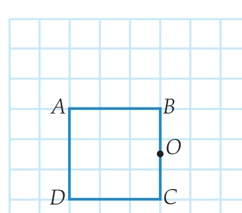
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.



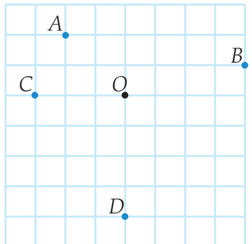
(d) Rotate 90° in a clockwise direction (-90°) about *O*.



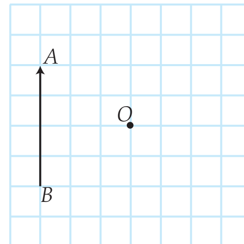
WE7

2 Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

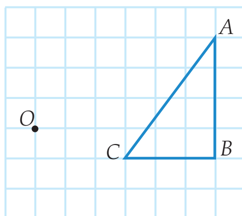
(a) Rotate each of the points 90° in a clockwise direction (-90°) about O.



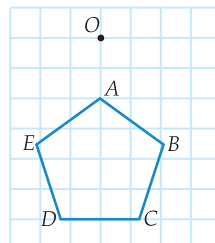
(b) Rotate 270° in an anticlockwise direction (270°) about O.



(c) Rotate 180° in a clockwise direction (-180°) about O.

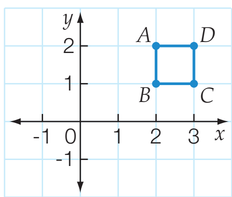


(d) Rotate 90° in an anticlockwise direction (90°) about O.



WE8

3 (a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) (i) Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) (i) Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) (i) Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

4 Which of the following statements about rotation is incorrect?

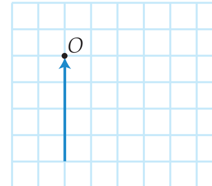
A Rotation can be in either a clockwise or an anticlockwise direction.

B A rotation of 360° will result in the image located in the same position as the original figure.

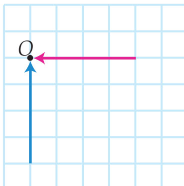
C A rotation will always produce an image that is a perfect replica and in the same orientation as the original figure.

D A rotation of 90° in a clockwise direction is equivalent to a rotation of 270° in an anticlockwise direction about the same centre of rotation.

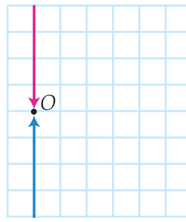
5 This arrow turns about the centre of rotation O. What does the shape look like after a turn of 180° anticlockwise?



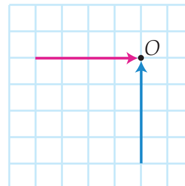
A



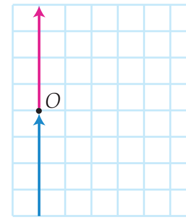
B



C



D



Understanding

6 **(a)** Plot the points (1, 4), (2, 4), (2, 2) and (1, 2) on a Cartesian plane and label them A, B, C and D.

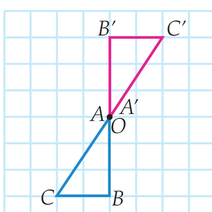
(b) Join the points to form a figure $ABCD$, then rotate the figure 90° in a clockwise direction about point A.

(c) Write the coordinates of the rotated points using image notation.

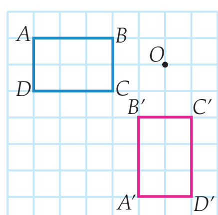
7 A clockwise rotation of 120° is the same as an anticlockwise rotation of how many degrees?

8 Find the rotation (the size of the rotation and the direction) that has taken place to produce the following images. Identify more than one rotation that will achieve the same result.

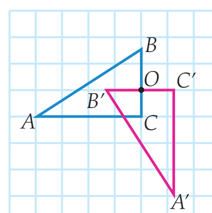
(a)



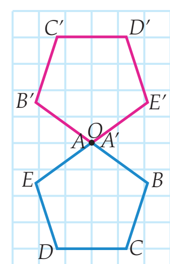
(b)



(c)



(d)



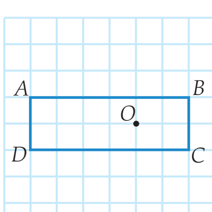
Reasoning

9 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(3, -2)$ is rotated 180° clockwise about the origin.

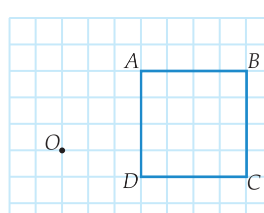
(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

10 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

(a) Rotate 90° in an anticlockwise direction about O.



(b) Rotate 90° in a clockwise direction about O.



11 The following are the capital letters of the alphabet.

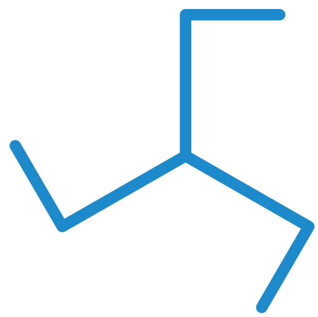
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which letters of the alphabet are able to be rotated about a centre of rotation in either direction and at an angle of less than 360° to produce an identical image in the same orientation as the original letter?

(b) Four of these letters have something else in common with each other. Can you identify these four letters? Describe the similarity you have noticed.

12 Estimate the smallest angle through which the following figures need to be rotated so the resulting image will match the original figure.

(a)



(b)

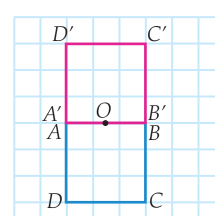
(c)

(d)

(e)

(f)

13 Is the following transformation a rotation of 180° around point *O*, a reflection along the line *AB* or both? Provide an explanation.



Open-ended

14 Plot four points on a Cartesian plane and write their coordinates. Decide upon a particular rotation and use this rotation to rotate your shape. Draw the image after the rotation and write down the coordinates of the rotated points using image notation.

15 Explain whether you agree or disagree with the following statement.

'A rotation of 180° in either direction and about any point of rotation can be equivalent to reflecting the same figure in a line of reflection through that point.'

16 On grid paper, draw two figures, a regular and an irregular polygon with the same number of sides. Decide upon a centre of rotation for each polygon that is on a vertex of that polygon.

(a) Rotate each of the figures by 90° about the centre of rotation in a clockwise direction and draw the image.

(b) Repeat this four times.

(c) Rotate the figures 45° about the centre of rotation.

(d) Repeat this eight times.

(e) Comment on the final images in parts **(b)** and **(d)**.

Outside the Square — Puzzle

Tangram magic

Equipment required: 1–2 brains, 2 pieces of grid paper, blank paper, ruler, pens and scissors

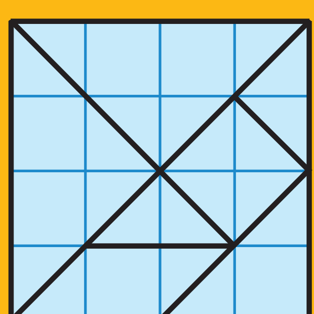
Tangram is an ancient Chinese puzzle. The object of the puzzle is to rearrange the pieces of a square (the puzzle pieces) to form as many different shapes using all seven pieces.

Before you start this puzzle you will need to make the seven pieces required to form the shapes.

Step 1 On a piece of grid paper, rule a square. The larger the square, the larger the pieces will be to work with. An 8 cm × 8 cm square is recommended.

Step 2 Use the tangram template below and divide your square into seven pieces as shown.

Step 3 Cut out your pieces carefully.



The tangram pieces will be:

- one square
- two small congruent (identical) isosceles triangles
- two large congruent isosceles triangles
- one medium isosceles triangle
- one parallelogram.

Once the pieces are cut out, it is time to begin. Form as many of the 12 shapes given below with your tangram pieces. You can translate, reflect and rotate the pieces, but remember to use all seven pieces.

When you have formed a shape, draw it carefully on blank paper, showing how the pieces fit together. Keep a list of the shapes you have made.