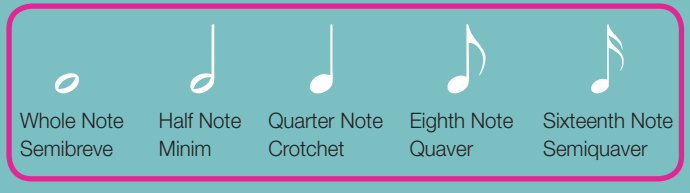
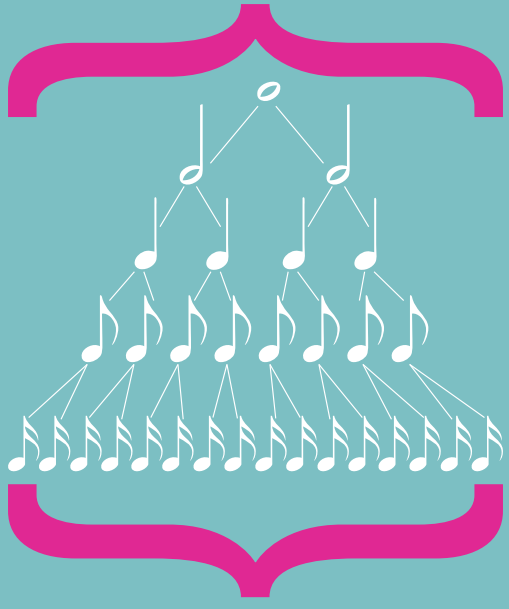
Maths 4 Real

MUSICAL FRACTIONS

In western cultures, music is written using symbols called 'notes'. Music notes have different shapes that indicate sounds of different durations, that is, they tell the musician how long the sound should be played for. The note symbols and their names are:



The note names are based on fractions, because music notes 'add' together in the same way fractions do. This tree diagram shows us how shorter notes add together to give longer notes.



From the diagram, we can see that:

2 half notes are equal in length to 1 whole note:	4 quarter notes are equal in length to 1 whole note:	2 quarter notes are equal in length to 1 half note:
$\frac{1}{2}+\frac{1}{2}=1$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$	$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Note that:

Eighth notes and sixteenth notes are often joined in pairs, like this:

Or groups of four, like this:

1 Using the tree diagram and examples above, copy and complete the questions below by writing one equivalent note after the equals sign. Write the fraction sum underneath.

(a) **(b)** **(c)** **(d)**

We can also add different types of notes together, e.g.:

$\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$ $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$

2 Copy and complete the questions below by writing one note in the space. Write the fraction sums underneath.

(a)  **(b)** 

(c)  **(d)** 

3 Copy and complete the questions below by writing 2 or more different notes in the spaces (there are several ways this can be done).

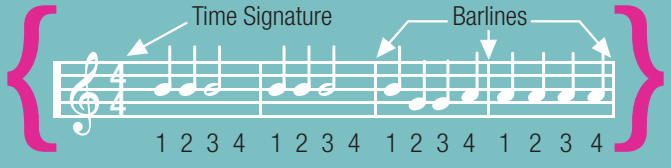
(a)  **(b)** 

(c)  **(d)** 

Time Signatures

Music is written on 5 parallel lines called a 'stave' and in sections called 'bars'. Each bar contains the same number of 'beats', or counts. 'Barlines' are drawn across the lines of the stave. You can see 4 bars and barlines in the piece of music below.

The 'time signature' is placed at the beginning of the first bar of a piece of music, and looks like a fraction. The top number tells us the number of beats in each bar, while the bottom number tells us what type of note counts as one beat.



A common time signature is $\frac{4}{4}$. There are 4 beats in each bar, and each beat is worth 1 quarter note. In $\frac{4}{4}$ the note values in each bar must add up to 4 quarter notes (1 whole note).

4 (a) Copy these bars, then use quarter notes or half notes to fill in the missing beats. Check that the fraction sum for each bar adds up to $\frac{4}{4}$.

(b) Copy these bars, then use any combination of half, quarter, eighth or sixteenth notes to fill in the missing beats. Make sure you check the fraction sum of each bar.

5 Write your own piece of music in the four bars below. Use any number or combination of notes, just make sure that every bar 'adds up' to 4 quarter notes.

Research

Find out about the music notation of other cultures, such as traditional Chinese or Indian music. Write some simple questions or exercises like the ones in this task to teach others what you have learnt.

Investigation

Tangram teaser

Equipment required: 1 brain, 3E Tangram template, scissors

The tangram is an ancient Chinese puzzle. It has seven pieces that can be arranged to make hundreds of shapes and patterns, including the large square you can see here.

The Big Question

How many different squares can be made by using various combinations of the seven tangram pieces? If the largest square is one whole, what are the fraction values of the smaller squares?

Engage

The tangram has seven individual pieces of five different shapes: two large triangles, two small triangles, one medium triangle, one small square and one parallelogram.

1 If the large square that is made up of all the pieces represents one whole, estimate the fraction that is represented by:

   (a) a large triangle

   (b) the medium triangle

   (c) a small triangle

   (d) the small square

   (e) the parallelogram.

2 Check your estimates by cutting out the parts on your tangram template and using them to visually show what fraction of the large square they cover.

3 Not all seven pieces must be used to make a square. A smaller square can be made using four pieces, for example:

By adding up the fractions represented by each piece, work out what fraction of the large seven-piece square is taken up by this four-piece square. Write your answer in simplest form.

Explore

4 Experiment with different numbers and combinations of pieces to make as many different squares as possible. Make sure you keep a visual record of each different square that you make. (Hint: There is more than one way of making the big, seven-piece square.)

Strategy options

- Draw a diagram.
- Make a table.
- Test all possible combinations.

Explain

5 Present accurate drawings of all the squares you have made, clearly showing the parts in each. Underneath your drawings, write a fraction sum that shows how each of the pieces add to give a fraction of the big, seven-piece square. How many different squares have you found?

Elaborate

6 Sometimes, the same four or five pieces can be used to make more than one square. How did you decide whether two squares were 'different' to each other?

7 Using your definition of 'different,' summarise your findings by answering the Big Question.

Evaluate

8 Consider how you worked on the investigation and the methods you used. Do you think you have the complete set of solutions to this problem? Explain your answer.

9 **(a)** Did you find this task challenging, or straightforward?

(b) What was the most difficult part of the task?

(c) Would you tackle puzzles like the tangram in your own time?

Extend

10 Try any or all of the following.

(a) Can you form a triangle using:

(i) two tangram pieces

(ii) three tangram pieces

(iii) four tangram pieces

(iv) five tangram pieces

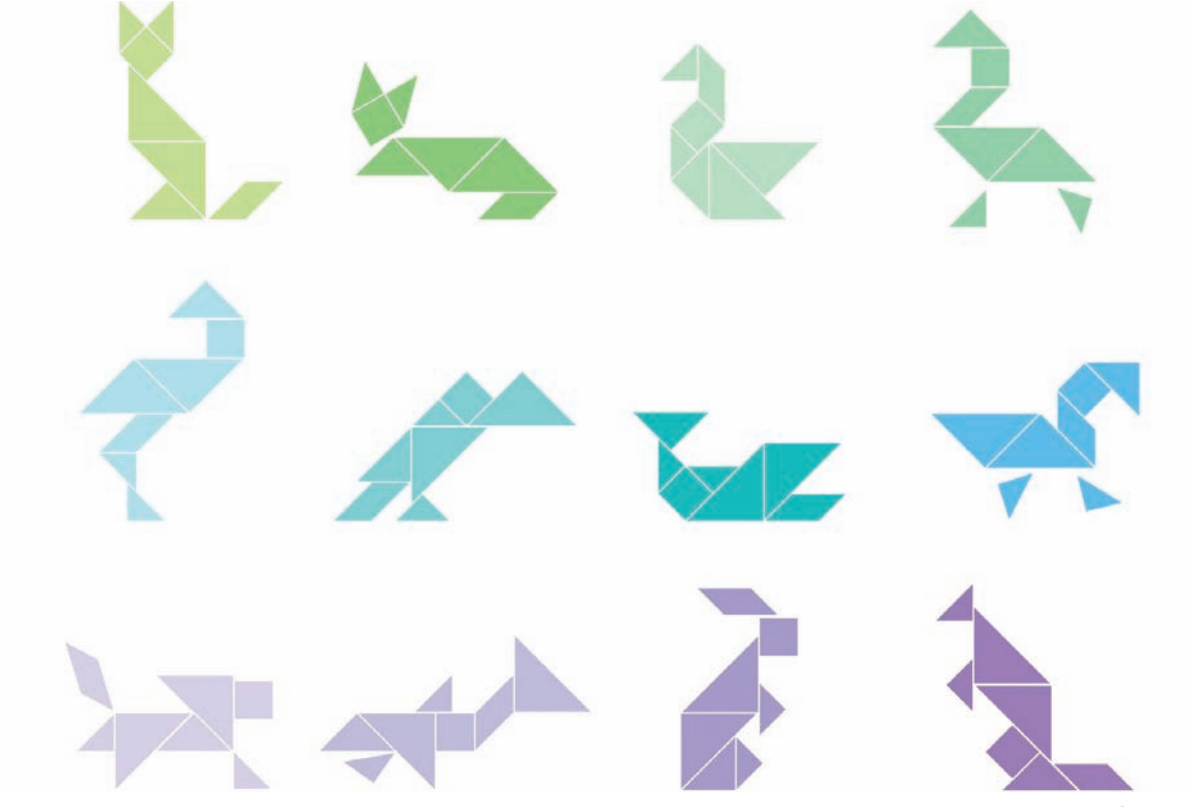
(v) six tangram pieces

(vi) all seven tangram pieces?

Draw a diagram of each triangle formed and write a fraction sum to show what fraction of the large square each triangle represents.

(b) Use the tangram pieces to make one or both of your initials.

(c) Use the tangram designs below as inspiration to make your own tangram designs.



3.5 Multiplying fractions

The multiplication sign, ×, means '*lots of*', or simply '*of*'.
For example, 8 lots of \$5 = 8 × \$5 = \$40.

Finding a fraction of a whole amount

We know that three-quarters of an hour is 45 minutes: $\frac{3}{4}$ of 60 = 45.

We can show how to calculate this in two ways:

1 We could find one-quarter of an hour, then multiply by three.

$\frac{1}{4}$ of 60 = 60 ÷ 4 = 15

$\frac{3}{4}$ of 60 = 3 × 15 = 45

This method is called the 'Unit fraction method'.
A **unit fraction** is a fraction with a numerator of 1.
In the above example, the unit fraction is $\frac{1}{4}$.

(This method works well if the denominator of the fraction is a factor of the whole number. This makes the division step straightforward.)

2 We could replace the word 'of' with the multiplication sign, ×, and multiply:

$$\frac{3}{4} \text{ of } 60 = \frac{3 \times 60}{4 \times 1} \quad \text{(writing 60 as an improper fraction with a denominator of 1)}$$

$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{60}^{15}}{1} \quad \text{(multiplying the numerators together and the denominators together, cancelling common factors first)}$$

$$= \frac{45}{1}$$

$$= 45$$

To find a fraction of a whole number, either:

- find the unit fraction (by dividing by the denominator), then multiply by the numerator

or

- write the whole number as an improper fraction and multiply the numerators together and the denominators together. Cancel any common factors between numerators and denominators before multiplying (this avoids having to work with large numbers).

Worked Example 14 WE14

Find $\frac{2}{5}$ of 40.

Method 1: Find the unit fraction, then multiply

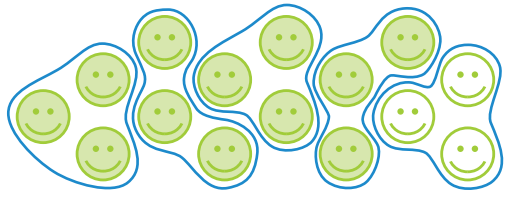
Thinking	Working
1 Find the unit fraction (in this case, $\frac{1}{5}$) by dividing the whole number by the denominator.	$40 \div 5 = 8$
2 Multiply the unit fraction amount by the numerator.	$2 \times 8 = 16$
3 Write the answer.	$\frac{2}{5}$ of $40 = 16$

Method 2: Multiply two fractions

Thinking	Working
1 Replace 'of' with '×' and write the whole number as an improper fraction with a denominator of 1.	$\frac{2}{5}$ of 40
2 Multiply the numerators and denominators together, cancelling common factors first (here, we cancel a common factor of 5).	$= \frac{2}{\cancel{5}_{1}} \times \frac{\cancel{40}^{8}}{1}$ $= \frac{16}{1}$
3 Simplify if possible.	$= 16$

Finding a fraction of a fraction

Our 'smiley collection' from page 107 can help us understand what it means to find a fraction of a fraction.



In Worked Example 1, we found $\frac{4}{5}$ of 15 by dividing the collection into five equal groups and counting how many smileys were in four of those groups.

$\frac{4}{5}$ of 15 = 12

If we wanted to find $\frac{3}{4}$ of $\frac{4}{5}$, we would take the 12 smileys *as our new whole*, ignoring the fifth group. Finding $\frac{3}{4}$ of this new whole means taking three of the four groups, or nine smileys.

$\frac{3}{4}$ of 12 = 9

These nine of the 15 original smileys represent $\frac{3}{4}$ of $\frac{4}{5}$.

$\frac{3}{4}$ of $\frac{4}{5}$ of 15 = 9

They also represent $\frac{3}{5}$ of the original collection: $\frac{3}{5}$ of 15 = 9.

The following Worked Example shows how multiplying $\frac{3}{4}$ and $\frac{4}{5}$ gives $\frac{3}{5}$.

Worked Example 15 WE15

Find $\frac{3}{4}$ of $\frac{4}{5}$.

Thinking	Working
1 Change the 'of' to ×.	$\frac{3}{4}$ of $\frac{4}{5}$
2 Multiply the numerators and denominators together, cancelling common factors before multiplying. (Here, the common factor is 4.)	$= \frac{3}{\cancel{4}_1} \times \frac{\cancel{4}^1}{5}$ $= \frac{3}{5}$

To find a fraction of a fraction:

- write mixed numbers as improper fractions before multiplying
- replace 'of' with a '×' symbol
- cancel any common factors
- multiply the numerators together and the denominators together
- simplify the answer further if necessary.

Worked Example 16 WE16

Find $4 \times 2\frac{3}{20}$. Write your answer as a mixed number, if appropriate.

Thinking	Working
1 Write both numbers as improper fractions. (Whole numbers can be written as fractions with a denominator of 1.)	$4 \times 2\frac{3}{20}$ $= \frac{4}{1} \times \frac{43}{20}$
2 Cancel any common factors between numerators and denominators. (Here, the common factor is 4.)	$= \frac{\cancel{4}^1}{1} \times \frac{43}{\cancel{20}_5}$
3 Multiply the simplified numerator and the denominator.	$= \frac{1 \times 43}{1 \times 5} = \frac{43}{5}$
4 Write the answer as a mixed number.	$= 8\frac{3}{5}$

3.5 Multiplying fractions

Navigator

Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Column 1, Q5, Q6, Q8, Q10, Q11, Q13, Q14, Q16, Q19	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q4 Column 2, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q19	Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 3, Q5, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19

Answers page 638

Fluency

1 Find the following.

WE14

(a) $\frac{1}{3}$ of 18 (b) $\frac{1}{4}$ of 12 (c) $\frac{1}{6}$ of 18 (d) $\frac{1}{9}$ of 36

(e) $\frac{3}{4}$ of 28 (f) $\frac{4}{5}$ of 30 (g) $\frac{6}{7}$ of 56 (h) $\frac{3}{8}$ of 72

(i) $\frac{3}{2}$ of 20 (j) $\frac{6}{5}$ of 100 (k) $\frac{11}{10}$ of 50 (l) $\frac{5}{4}$ of 60

2 Find the following.

WE15

(a) $\frac{1}{2}$ of $\frac{2}{13}$ (b) $\frac{2}{9} \times \frac{1}{2}$ (c) $\frac{4}{5} \times \frac{5}{9}$ (d) $\frac{3}{7} \times \frac{7}{13}$

(e) $\frac{5}{7} \times \frac{1}{10}$ (f) $\frac{3}{5}$ of $\frac{10}{11}$ (g) $\frac{5}{12}$ of $\frac{6}{7}$ (h) $\frac{5}{6}$ of $\frac{1}{11}$

(i) $\frac{6}{7} \times \frac{3}{8}$ (j) $\frac{9}{10} \times \frac{5}{6}$ (k) $\frac{4}{7}$ of $\frac{2}{3}$ (l) $\frac{3}{10} \times \frac{9}{25}$

3 Find the following. Write your answers as mixed numbers, if appropriate.

WE16

(a) $3 \times \frac{2}{9}$ (b) $\frac{3}{4} \times 8$ (c) $\frac{5}{8} \times 2$ (d) $3 \times \frac{5}{12}$

(e) $5 \times 2\frac{3}{10}$ (f) $8 \times 1\frac{3}{4}$ (g) $1\frac{4}{5} \times 15$ (h) $5\frac{1}{12} \times 8$

(i) $5\frac{1}{3} \times \frac{3}{8}$ (j) $3\frac{1}{2} \times 3\frac{1}{5}$ (k) $4\frac{1}{5} \times 4\frac{2}{7}$ (l) $1\frac{1}{8} \times 3\frac{5}{9}$

When cancelling common factors, don't cancel out two things on the same line—cancel something on the bottom with something on the top.

4 Calculate the following, giving your answers in simplest form.

(a) $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ (b) $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ (c) $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$

(d) $1\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ (e) $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ (f) $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$

5 (a) $\frac{2}{5}$ of $2\frac{1}{2}$ cups of sugar is:

A $\frac{1}{5}$ cup B $\frac{4}{10}$ cup C 1 cup D $2\frac{1}{5}$ cups

(b) $\frac{1}{5}$ of $\frac{1}{3}$ of 15 is:

A 1 B 2 C 3 D 5

Understanding

6 A group of 7 workers shared third prize in a local raffle. The prize was worth $2170.

(a) What fraction of the prize money does each worker receive?

(b) How much money does each worker get?

7 The proceeds of a $48 000 inheritance were to be shared between four cousins. Each cousin was to receive a specific fraction of the inheritance. Michael was to receive $\frac{5}{24}$, Marcus $\frac{1}{12}$, Lily $\frac{3}{8}$ and Julie $\frac{1}{3}$. How much money did each cousin receive?

8 A chocolate bar was made up of 45 equal pieces. Ling had to share it equally with her brother and sister, Gao and Chen.

(a) How many pieces did each person receive? Copy this chocolate block into your book and divide it up to show each person's share.

(b) Ling then decided to divide her share of chocolate equally between herself and four of her friends.

(i) How many pieces did each person receive?

(ii) What fraction of the original block is this?

(iii) On your drawing of the chocolate bar show the fraction that Ling and her friends receive.

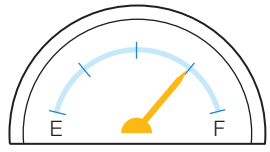
9 Annika cut one-quarter of an apple pie in halves to share with her brother Arjun. What fraction of the original whole pie do they each now have?

10 Draw a rectangle that is 5 cm long and 4 cm wide. Divide it lengthways into fifths. Shade in $\frac{1}{5}$. Now, take the shaded section and divide it into quarters. Shade in $\frac{3}{4}$ of this section.

(a) What fraction of the original rectangle have you shaded twice?

(b) Write the fraction multiplication that is shown by your diagram.

11 A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.



(a) How much petrol has been used?

(b) How much petrol is still in the tank?

12 Melissa surveyed 75 Year 7 students. $\frac{2}{3}$ said they enjoyed studying mathematics. $\frac{4}{5}$ of those who enjoyed mathematics also said they enjoyed studying science.

(a) How many students said they enjoyed mathematics?

(b) How many students said they enjoyed mathematics and science?

13 Angelina is making Custard Cream cookies. Below are the ingredients that make 24 cookies.

180 g butter $\frac{1}{2}$ cup icing sugar $1\frac{1}{4}$ cups self-raising flour $\frac{1}{3}$ cup custard powder

Rewrite the ingredients to make 60 cookies. (Hint: What do you need to multiply 24 by to get 60?)

Reasoning

14 Without doing the calculation, which is larger: $1\frac{1}{2} \times 12$ or $\frac{1}{3} \times 9$?

15 Without doing any calculations, say which of the following would give the biggest answer.

A $\frac{1}{2} \times \frac{3}{4}$ **B** $\frac{2}{5} \times \frac{2}{3}$ **C** $\frac{3}{4} \times \frac{6}{5}$ **D** $1\frac{1}{2} \times 1\frac{2}{5}$

16 The missing number in the statement $\frac{5}{12} \times \frac{\square}{10} = \frac{5}{8}$ is:

A 4 **B** 5 **C** 10 **D** 15

Open-ended

17 Write two fractions that multiply to give $\frac{1}{6}$ as the simplified answer.

18 $\frac{1}{\square} \times 3\square = 1\square$

Fill each of the boxes with a single digit (0–9) to make the statement correct. There are six possible combinations. Try to find at least three of them.

19 Asher was asked to find $\frac{3}{5}$ of 60. His answer was 4. Here is his working:

$$\frac{\cancel{3}^{1}}{5} \times \frac{\cancel{60}^{20}}{1} = \frac{20}{5} = 4$$

Asher knows his answer is not correct, because he knows that $\frac{3}{5}$ is bigger than one half, and 4 is nowhere near half of 60! Explain where Asher has gone wrong. Include the correct working and answer to the problem.

Outside the Square Game

Fringo!

Equipment required: 2 brains, 2 dice

How to win:
The winner is the first person to have four fractions in a row (horizontal, vertical or diagonal) crossed off their grid. When they do, they should call out 'Fringo!'

How to play:
Fringo is fraction bingo! Draw a 4×4 grid into your book. Fill in your grid with any 16 of the numbers, fractions and mixed numbers on this list:

$1, 2, 3, 4, 5, 6, \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, 1\frac{1}{3}, 1\frac{2}{3}, \frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{1}{6}, \frac{5}{6}$

Take turns in rolling the dice one at a time. Make the number you roll first into the denominator of a unit fraction and write it down. For example, if you roll a 4, write down $\frac{1}{4}$. Multiply the number you roll second by this unit fraction, and write the answer in simplest form.

For example, Roll 1 = 4, Roll 2 = 2. Write down $\frac{1}{4} \times 2$ and calculate it (in this case, $\frac{2}{4}$, which simplifies to $\frac{1}{2}$).

If you have the answer to the fraction multiplication in your grid, cross it off.

(HINT: You may realise after playing a few rounds that some answers come up more than others. Thinking about where the best places are to put these on the grid will increase your chance of winning. You could also investigate just how many ways the fractions on the list can be made.)

Mathspace

FRACTURED RAVINE

While exploring the mountains with your friends you fall down a ravine! Armed with your fraction skills, can you survive the perils that await you?

Equipment: 1 die, 2–4 brains

Rules:

- Each player begins with 100 life points.
- If you lose all your life points you are out (dead).
- Take turns to roll the die and move forward that number of spaces. Follow the directions given in the space that you land on.
- All players must stop at the 'Leap of Faith' regardless of what number they have rolled.
- The goal: to be the first to reach sunlight.

START

You sprain an ankle.

$1\frac{1}{2} + 3\frac{1}{4} + 4\frac{2}{8} =$

Deduct the solution from your life points total.

You call out 'Cooee' and cause a rock fall.

$\frac{3}{4}$ of 16 =

Deduct the answer from your life points total.

Your torch runs out of batteries.

$\frac{1}{3}$ of 60 =

Deduct the answer from your life points total.

Your abseil rope gets caught on a sharp rock.

$\frac{2}{5}$ of 30 =

Deduct the answer from your life points total.

You run out of drinking water.

$12\frac{1}{3} + 3\frac{1}{6} - 4\frac{1}{2} =$

Deduct the solution from your life points total.

The oxygen level decreases rapidly.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction = $\frac{3}{4}$.

Find the fraction of 60 that this would be,

e.g. $\frac{3}{4}$ of 60 = 45.

Deduct this from your life points total.

You begin to feel claustrophobic.

Roll the die and multiply the number you rolled by $\frac{2}{5}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

You eat the last of the food supply.

Roll the die and multiply the number you rolled by $\frac{7}{2}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

A rock falls and breaks all the bones in one of your feet.

The human body contains 206 bones. If approximately $\frac{1}{8}$ of these are in each foot, how many bones did you break (rounded to the nearest whole number)?

Deduct this from your life points total.

LEAP OF FAITH

ALL EXPLORERS STOP HERE

There is a large crevasse ahead, but it's the only way forward—you need to find the courage to jump!

Roll the die and substitute the number you roll into this fraction $\frac{3}{\square}$.

If the fraction created is greater than (or equal to) 1 you've made it across.

If the fraction created is less than 1 you must wait until you can roll again.

You reach a ladder and see sunlight above.

All explorers must stop at each rung of the ladder.

(Players don't need to roll to move.)

Rung 1:

Find what fraction of your first name is made up of vowels. e.g. 'Natalie' $= \frac{4}{7}$. Do the same for your last name . Multiply one of these fractions by 50 and round to the nearest whole number.

Deduct this from your life points total.

Rung 2:

If the fraction of your first name that is made up of vowels is less than or equal to $\frac{1}{2}$, *deduct 10 from your life points total.* If it's greater than $\frac{1}{2}$, *deduct 15.*

Rung 3:

You are on the final rung.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction $= \frac{3}{4}$. You need to roll a fraction larger than $\frac{2}{3}$ to make the final step out of the ravine.

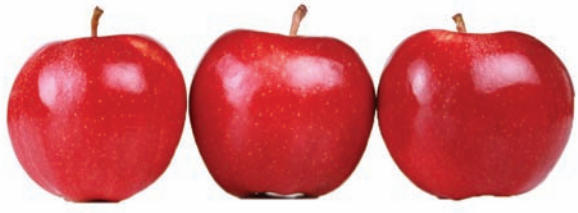
Every time you fail you must deduct 10 points from your life points total.

SUNLIGHT! YOU SURVIVED. WELL DONE

3.6 Dividing fractions

Whole numbers divided by fractions

Here are three whole apples.



If we cut each apple into quarters, how many apple quarters will there be? We can think of this as 'how many quarters in 3' and write it as $3 \div \frac{1}{4} = ?$

We have 4 quarters in each apple, so there are 12 quarters in 3 apples: $3 \div \frac{1}{4} = 12$.

If we cut the apples into fifths, we would have fifteen fifths: $3 \div \frac{1}{5} = 15$.

If we cut the apples into sixths, we would have eighteen sixths: $3 \div \frac{1}{6} = 18$.

We can get our answer by multiplying the whole number by the denominator of the fraction.

i.e. $3 \div \frac{1}{4}$	$3 \div \frac{1}{5}$	$3 \div \frac{1}{6}$
$= 3 \times \frac{4}{1}$	$= 3 \times \frac{5}{1}$	$= 3 \times \frac{6}{1}$
$= 12$	$= 15$	$= 18$

The fractions have been turned upside down, or 'inverted', to show they are being multiplied like whole numbers.

Inverting a fraction (turning it upside down) is also known as 'finding the **inverse**'.

The inverse of $\frac{1}{4}$ is $\frac{4}{1}$ or simply 4.

What about dividing by fractions where the numerator is not 1, such as $3 \div \frac{3}{4}$?

If we put our 12 apple quarters on the previous page into groups of 3 quarters, there will be 4 groups. Because $\frac{3}{4}$ is three times as big as $\frac{1}{4}$, the answer to $3 \div \frac{3}{4}$ is one-third as big as $3 \div \frac{1}{4}$.

We can achieve the same answer by multiplying 3 by $\frac{4}{3}$, the inverse of $\frac{3}{4}$.

$$3 \div \frac{3}{4}$$
$$= \frac{{}^{1}\cancel{3}}{1} \times \frac{4}{{}_{1}\cancel{3}}$$
$$= \frac{4}{1}$$
$$= 4$$

To divide a whole number by a fraction, multiply by the inverse of the fraction.

The result of a division calculation is called the 'quotient'.

Another way of dividing a fraction (or a whole number) by a fraction is to convert them both to the same type of fraction; that is, fractions with the same denominator (the LCD). A common denominator means we have fractions of the same type, like having two measurements in the same units. Once we have a common denominator, we can perform the division on just the numerators. This is demonstrated in Method 2 of the Worked Example below.

Worked Example 17 WE17

Calculate $9 \div \frac{3}{8}$.

Method 1: Multiply by the inverse

Thinking	Working
1 Write the whole number as an improper fraction with a denominator of 1.	$\frac{9}{1} \div \frac{3}{8}$
2 Find the inverse of the second fraction (turn it upside down) and change the ÷ to ×.	$= \frac{9}{1} \times \frac{8}{3}$
3 Cancel any common factors between numerators and denominators. (Here, the common factor is 3.)	$= \frac{{}^{3}\cancel{9}}{1} \times \frac{8}{{}_{1}\cancel{3}}$
4 Multiply the simplified numerators and denominators.	$= \frac{3 \times 8}{1 \times 1} = \frac{24}{1}$
5 Write the answer.	$= 24$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Write the whole number as an improper fraction with the same denominator as the fraction (in this case, 8).	LCD = 8 $9 = \frac{72}{8}$
2 Rewrite the division using the two fractions.	$9 \div \frac{3}{8}$ $= \frac{72}{8} \div \frac{3}{8}$
3 Now that the denominators are the same, we can ignore them and perform the division with just the numerators.	$= 72 \div 3$ $= 24$

Fractions divided by fractions

The 'invert the fraction you are dividing by and multiply' method also applies when dividing one fraction by another.

The 'use equivalent fractions with the LCD' method can also be used.

Convert mixed numbers to improper fractions before dividing.

Worked Example 18

WE18

Calculate $1\frac{1}{6} \div \frac{2}{3}$.

Method 1: Multiply by the inverse

Thinking	Working
1 If there are any mixed numbers, convert them to improper fractions.	$1\frac{1}{6} = \frac{7}{6}$ $\frac{7}{6} \div \frac{2}{3}$
2 Find the inverse of the second fraction (turn it upside down) and change the $\div$ to $\times$.	$= \frac{7}{6} \times \frac{3}{2}$
3 Cancel common factors and perform the simplified multiplication.	$= \frac{7}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2}$ $= \frac{7 \times 1}{2 \times 2}$ $= \frac{7}{4}$
4 Write the answer as a mixed number.	$= 1\frac{3}{4}$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Find the LCD. Write the mixed number as an improper fraction with the LCD. Write the fraction as an equivalent fraction with the LCD.	LCD = 6 $1\frac{1}{6} = \frac{7}{6}$ $\frac{2}{3} = \frac{4}{6}$
2 Rewrite the division using the fractions with the LCD.	$1\frac{1}{6} \div \frac{2}{3}$ $= \frac{7}{6} \div \frac{4}{6}$
3 Perform the division with just the numerators, and write the answer.	$= 7 \div 4$ $= 1\frac{3}{4}$

3.6 Dividing fractions

Navigator

Q1 Columns 1–3, Q2 Columns 1 & 2, Q4, Q5, Q6, Q8, Q9, Q12, Q15	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q15	Q1 Column 4, Q2 Column 4, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

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Fluency

1 Calculate the following.

WE17

(a) $6 \div \frac{1}{4}$ **(b)** $2 \div \frac{1}{6}$ **(c)** $3 \div \frac{1}{7}$ **(d)** $4 \div \frac{1}{9}$

(e) $4 \div \frac{2}{9}$ **(f)** $4 \div \frac{4}{5}$ **(g)** $5 \div \frac{3}{7}$ **(h)** $7 \div \frac{4}{9}$

(i) $6 \div 1\frac{1}{8}$ **(j)** $8 \div 2\frac{2}{3}$ **(k)** $6 \div 3\frac{1}{5}$ **(l)** $3 \div 2\frac{1}{4}$

2 Calculate the following.

WE18

(a) $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{1}{4} \div \frac{1}{6}$ **(c)** $\frac{3}{4} \div \frac{2}{3}$ **(d)** $\frac{7}{10} \div \frac{1}{2}$

(e) $\frac{1}{3} \div 2$ **(f)** $\frac{1}{5} \div 4$ **(g)** $\frac{3}{4} \div 3$ **(h)** $\frac{9}{10} \div 6$

(i) $\frac{9}{10} \div 1\frac{3}{5}$ **(j)** $\frac{7}{6} \div 2\frac{1}{12}$ **(k)** $\frac{4}{5} \div 1\frac{8}{9}$ **(l)** $\frac{9}{10} \div 2\frac{2}{3}$

(m) $2\frac{2}{3} \div 1\frac{1}{3}$ **(n)** $5\frac{1}{2} \div 5\frac{3}{8}$ **(o)** $4\frac{1}{2} \div 1\frac{1}{5}$ **(p)** $4\frac{5}{7} \div 2\frac{3}{4}$

3 **(a)** $5 \div \frac{15}{8} =$

A $\frac{8}{75}$ **B** $\frac{3}{8}$ **C** $2\frac{2}{3}$ **D** $\frac{75}{8}$

(b) $\frac{16}{7} \div \frac{20}{49} =$

A $\frac{5}{28}$ **B** $\frac{320}{343}$ **C** $\frac{343}{320}$ **D** $5\frac{3}{5}$

4 Copy and complete the following. Do all working in your head.

(a) There are _____ halves in one, so $1 \div \frac{1}{2} =$ _____.

(b) There are _____ halves in three, so $3 \div \frac{1}{2} =$ _____.

(c) There are _____ quarters in one, so $1 \div \frac{1}{4} =$ _____.

(d) There are _____ quarters in two, so $2 \div \frac{1}{4} =$ _____.

5 The inverse of $\frac{1}{3}$ is:

A $\frac{1}{6}$ **B** $\frac{2}{3}$ **C** 1 **D** 3

Understanding

6 There is $\frac{5}{8}$ of a metre of ribbon in the textiles class cupboard. Students require $\frac{1}{10}$ of a metre each for their projects. How many students will be able to use the ribbon?

7 Scientists in a medical laboratory work in shifts of $3\frac{1}{2}$ hours each. How many whole shifts are to be worked during a period of 42 hours?

8 Hannah is a baker whose specialty is scones. She has a sack containing 80 cups of flour. Her scone recipe uses $2\frac{2}{3}$ of a cup of flour for each batch of scones. How many batches of scones will Hannah get from her sack of flour?

9 On a 15 km Fun Run course, drinks stations are placed every $2\frac{1}{2}$ km from the start. How many drinks stations will there be on the course? (There are no drinks stations at the start or finish lines.)

10 The hit movie 'Exterminator 5' runs for $1\frac{3}{4}$ hours. A local cinema complex plans to screen it back to back (alternating the screenings between 2 cinemas). The first screening will start at 1:30 p.m. The last screening must finish by 11 p.m. How many times can the movie be screened within this time period?

Reasoning

11 The missing number from the statement $\frac{3}{4} \div \frac{\square}{8} = 2$ is:

A 1 **B** 2 **C** 3 **D** 4

12 Without doing a calculation, state which will give the larger number as a result:
$\frac{4}{5} \div \frac{1}{3}$, or $\frac{4}{5} \div \frac{2}{3}$?

13 Consider this sequence: $5 \div \frac{3}{4} = 6\frac{2}{3}$, $5 \div \frac{3}{5} = 8\frac{1}{3}$, $5 \div \frac{3}{6} = 10$, $5 \div \frac{3}{7} = 11\frac{2}{3}$.

(a) Why does the quotient (the answer to the division) increase as the denominator of the fractions in red is increased?

(b) Predict the answer to the next term in the sequence; i.e. $5 \div \frac{3}{8}$ will be ...

Open-ended

14 $4 \div \frac{2}{\square} = 1\square$

Use digits from 0 to 9 to fill in the boxes to make a correct statement. Find at least two different combinations.

15 Here is Gabi's working on a fraction division problem:

$$3 \div \frac{1}{4} = 3 \div 4 = \frac{3}{4}$$

Her friend Natalie has worked it out differently. This is what Natalie wrote:

$$3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$$

'That can't be right', said Gabi—12 is way too big to be the answer!

Which of the girls' working is correct? Explain what the other has done wrong.

Outside the Square — Problem solving

Fill in the blanks

1 Fill in the blanks with positive whole numbers to make mathematically true statements. Do not use the same number twice within a statement. See if you can come up with at least two different combinations.

(a) $\frac{\blacklozenge}{4} + \frac{1}{\blacklozenge} = \frac{\blacklozenge}{20}$ **(b)** $\frac{\bigstar}{\bigstar} - \frac{\bigstar}{6} = \frac{\bigstar}{12}$

2 Use four of the digits 1, 3, 4, 5, 6, 7 to make two fractions whose sum is close to but less than 1.

$\frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} < 1$

Try to come up with at least three combinations. Which combination is the closest to 1?

Strategy options

- Guess and check.
- Test all possible combinations