2

Integers

2

Texting aliens

Mathematics is said to be the language that we could use to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same. It is believed that the best way to send a message would be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system. In 1974, the Arecibo telescope in Puerto Rico broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; however, this is not surprising given the distance it would have to travel. Later in this chapter you can learn about another way prime numbers are used to send information .

Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10; however, sometimes we count by 2, 7, 60, 360 and 365. What do we count using these numbers?

Why learn this?

Understanding relationships between numbers allows us to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for our study of many other areas of mathematics. Negative numbers are an important set of numbers that we will also consider in this chapter. Temperatures, elevations, goal differences and money owed are a few examples of the uses of negative numbers.

After completing this chapter you will be able to:

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative integers to represent quantities
- compare and order integers
- add and subract integers.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from the Online Student Book.

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

2 (a) List all the digits with which an even number can end.

(b) List all the digits with which an odd number can end.

3 Copy and complete each of the following by writing a < (less than) or > (greater than) sign between the given values.

(a) 10 _____ 7 (b) 3 _____ 6 (c) 2 _____ 0 (d) 0 _____ 5

4 Calculate:

(a) $3 + 8 + 12$ (b) $22 + 19 - 7$ (c) $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ (e) $72 - 39 + 14$ (f) $51 + 43 - 11 - 7$

5 Write the following temperatures in order from coldest to warmest.

(a) 15°C, 7°C, 0°C, -4°C, 21°C, -11°C

(b) 5°C, -3°C, 10°C, -25°C, 32°C, -14°C

6 Write the following in expanded form, then evaluate.

(a) 7^2 (b) 3^4 (c) 2^6 (d) 1^9

7 Calculate the following.

(a) $3^2 \times 5^2$ (b) $4^3 \div 2^3$ (c) $8^2 + 6^2$ (d) $9^2 - 7^2$

Key Words

common factor	factor	positive
common multiple	Highest Common Factor (HCF)	prime factor
composite number	integers	prime number
coprime	loss	profit
deposit	Lowest Common Multiple (LCM)	withdrawal
divisibility	multiple	
divisible	negative	

Multiples, factors and divisibility

2.1

Multiples and factors

The numbers 1, 2, 3, 4, 5, ... are called the whole numbers, or the counting numbers. (Any time we use '...' in mathematics, we are saying the pattern is infinite, or goes on forever.)

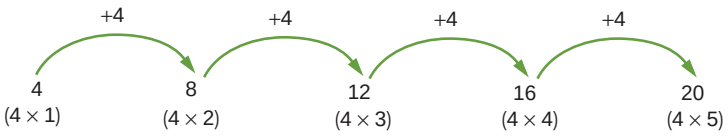
We find the multiples of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

	1×7	2×7	3×7	4×7	5×7	...
Multiples of 7	7	14	21	28	35	...

Another way to create a list of multiples of a number is to start at the number and add it repeatedly.

For example, the multiples of 4 are:



The first in the sequence of multiples of a number is always the number itself. We can see from the above table and sequence that the first multiple of 7 is 7 (1×7), and the first multiple of 4 is 4 (1×4).

A factor is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, we can see some factors:

	7	14	21	28	35	...
Some factors	1, 7	2, 7	3, 7	4, 7	5,7	...

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a positive integer has. We can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$
and $28 = 2 \times 14$
and $28 = 1 \times 28$

So, 28 has a total of six factors: 1, 2, 4, 7, 14 and 28.

Worked Example 1 WE1

Find all the factors of each of the following numbers.

(a) 12 (b) 110

Thinking			Working
(a)	1	Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(a) $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
	2	List the factors from smallest to largest.	Factors of 12: 1, 2, 3, 4, 6, 12.
(b)	1	Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(b) $1 \times 110 = 110$ $2 \times 55 = 110$ $5 \times 22 = 110$ $10 \times 11 = 110$
	2	List the factors from smallest to largest.	Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. We include 7 only once in the list of factors for 49. If we reach such a pair, this also tells us we have finished finding the pairs of numbers.

Divisibility

Another way of considering factors and multiples is to talk about **divisibility**. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all refer to the same idea:

Two factors of 35 are 5 and 7.
35 is divisible by 5 and 7.
Both 5 and 7 go into 35 exactly, without any remainder.
5 multiplied by 7 gives 35.
35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help us determine which numbers are divisible by others. For larger numbers, we can use some tests that enable us to determine whether one number is divisible by another. These tests are summarised in the following table.

A number is divisible by ...	If it passes this divisibility test
2	The last digit is an even number (0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked Example 2

WE2

Determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 (b) 4 (c) 5 (d) 6

Thinking		Working	
(a) 1	Add up the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3.	(a) 75: $7 + 5 = 12$ 98: $9 + 8 = 17$ 110: $1 + 1 + 0 = 2$ 132: $1 + 3 + 2 = 6$	✓ ✗ ✗ ✓
2	State the answer for each number considered.	75 and 132 are divisible by 3. 98 and 110 are not divisible by 3.	
(b) 1	Look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4.	(b) 75 98 110 132	✗ ✗ ✗ ✓
2	State the answer for each number considered.	132 is divisible by 4. 75, 98 and 110 are not divisible by 4.	
(c) 1	Is the last digit 5 or 0?	(c) 75 98 110 132	✓ ✗ ✓ ✗
2	State the answer for each number considered.	75 and 110 are divisible by 5. 98 and 132 are not divisible by 5.	
(d) 1	Write down the even numbers (these are divisible by 2). Add up the digits in each of these numbers and see whether the number is divisible by 3.	(d) Using the working from (a): 98: 17 110: 2 132: 6	 ✗ ✗ ✓
2	State the answer for each number considered.	132 is divisible by 6. 75, 98 and 110 are not divisible by 6.	

Multiples of a whole number are found by multiplying it by another whole number

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

Common multiples

A common multiple of two numbers is a number that both of them divide into exactly. Changing the multiple table from the start of the section slightly, we get:

	1 and 7	2 and 7	3 and 7	4 and 7	5 and 7	...
A common multiple	7	14	21	28	35	...

This table only gives one common multiple for each pair of numbers. There are an infinite number of others. The Lowest Common Multiple (LCM) of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56... . The LCM of 2 and 7 is 14. There is no highest common multiple.

Worked Example 3

WE3

Find the Lowest Common Multiple (LCM) of the following set of numbers, by first listing the multiples of each: 4 and 6.

Thinking	Working
1 List the first few multiples of the first number.	4: 4, 8, 12, 16, 20, 24, ...
2 List the first few multiples of the second number.	6: 6, 12, 18, 24, 30, 36, ...
3 Circle the first number that appears in both lists. This is the Lowest Common Multiple (LCM).	LCM of 4 and 6 is 12.

Common factors

A common factor of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

	7 and 14	4 and 20	9 and 15	8 and 40	12 and 18
Common factors	1, 7	1, 2, 4	1, 3	1, 2, 4, 8	1, 2, 3, 6

1 will always be a common factor of any set of numbers. Sometimes it's important for us to find the Highest Common Factor (HCF) of two numbers. From the above table, we can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, the smaller number is the HCF. For example, the HCF of 4 and 20 is 4 and the HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

Worked Example 4 WE4

Find the Highest Common Factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Thinking	Working
1 List all factors of the first number.	12: (1) (2) (3) 4, (6) 12
List all factors of the second number.	18: (1) (2) (3) (6) 9, 18
2 Circle the factors appearing in both lists. These are the common factors.	
3 Select the largest number that appears in both lists. This is the Highest Common Factor (HCF).	HCF of 12 and 18 is 6.

The Lowest Common Multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The Highest Common Factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The Highest Common Factor is also known as the Greatest Common Divisor (GCD).

2.1 Multiples, factors and divisibility

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1–3, Q4 Columns 1–3, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15, Q18, Q23	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2–4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20 (a), Q23, Q24	Q1 Columns 3 & 4, Q2, Q3 Column 3, Q4 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q24, Q25

Answers page 617

Fluency

1 Find all the factors of each of the following numbers. WE1

(a) 18	(b) 16	(c) 23	(d) 24
(e) 20	(f) 35	(g) 36	(h) 42
(i) 53	(j) 60	(k) 77	(l) 84

2 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. WE2

(a) 3	(b) 4	(c) 5	(d) 8	(e) 9

WE3

3 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5 (b) 3 and 9 (c) 5 and 25
(d) 5 and 6 (e) 4 and 7 (f) 8 and 12
(g) 7 and 9 (h) 10 and 12 (i) 6 and 11
(j) 9 and 12 (k) 20 and 50 (l) 8 and 14
(m) 3, 4 and 5 (n) 2, 25 and 50 (o) 20, 50 and 60

WE4

4 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15 (b) 8 and 24 (c) 5 and 12 (d) 26 and 36
(e) 11 and 33 (f) 28 and 70 (g) 44 and 22 (h) 10 and 30
(i) 40 and 70 (j) 32 and 60 (k) 35 and 70 (l) 42 and 48

5 (a) The lowest common multiple of 8 and 1 is:

A 8 B 16 C 24 D 80

(b) Which of the following is a factor of 34?

A 4 B 12 C 17 D 68

6 (a) A number divisible by 2, 3 and 10 is:

A 6 B 15 C 60 D 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 B 38 and 52 C 38 and 60 D 52 and 60

Understanding

7 (a) Which one of the following numbers is not a multiple of 8?

A 4 B 24 C 72 D 88

(b) Which of the following is not a factor of 42?

A 1 B 6 C 21 D 84

8 State TRUE or FALSE for the following.

(a) 346 is a multiple of 3. (b) 872 is divisible by 6.
(c) 2 is a factor of 348. (d) 52 is a multiple of 4.
(e) 854 is divisible by 9. (f) 3 is a factor of 56 902.

9 For each group of numbers, find (i) the LCM and (ii) the HCF.

(a) 4, 6 and 10 (b) 6, 8 and 12
(c) 8, 12 and 16 (d) 10, 25 and 40

10 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is _____ by 8.
(b) 6 is a _____ of 54, so 54 is a multiple of 6.
(c) 72 is divisible by 9, so that makes it a _____ of 9.
(d) 4 is a factor of 60, so 60 is _____ by 4.

11 (a) If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

12 Mrs Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Mrs Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

13 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

14 The smallest number divisible by 3, 4 and 5 is:

A 12 B 24 C 30 D 60

15 If two events occur at different time intervals, the lowest common multiple (LCM) of the two time intervals is the point when the two events coincide, or occur together. Use this information to answer the following question.

In a city lighting display, one set of lights flashes every 25 seconds and the other set flashes every minute. If they are turned on at the same time, write down the next three times when the two sets of lights flash together.

16 (a) Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning

17 Peter power-walked around an oval while Mei-ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei-ling jogged 24 laps in the hour.

(a) How many times did Mei-ling pass Peter?

(b) How many times did Mei-ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei-ling pass Peter exactly on the spot where they started?

18 (a) Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

100 000	(2)	3	(4)	(5)	6	(8)	9	(10)
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4 it will also be divisible by _____.

(ii) If a number is divisible by 9 it will also be divisible by _____.

(c) Explain your answers to (b).

19 The test to determine whether a number is divisible by 6 is to test whether it is divisible by 2 *and* 3. Explain why the test works.

20 A *perfect* number is a number for which the sum of its factors (excluding itself) equals the number. The first perfect number is 6, as $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

21 An *abundant* number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next 2 abundant numbers. (Both are less than 40.)

22 (a) How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

Open-ended

23 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Draw three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is most practical for a chocolate box? Explain your answer.

24 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

25 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites

2.2

A number that has exactly two factors, itself and 1, is a **prime number.**

A number that has more than two factors is called a **composite number.**

The number 1 is a special number. It is neither prime nor composite.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **coprime** if their highest common factor is 1.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BCE to 195 BCE. He was the first person to calculate a value for the circumference of the Earth. Another thing he was famous for was his 'sieve'.

See if you can reproduce what he did.

Copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating until there is no 'next number'.

Step 6 Write the factors of each of the circled numbers. What types of numbers are these?

Step 7 Write the factors of any five of the crossed out numbers, except for 1.

Step 8 Which type of number—circled or crossed out—has more factors? Explain why.

2.2 Primes and composites

Navigator

Answers page 618

Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14, Q17	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18	Q1, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18

Fluency

Use 'The sieve of Eratosthenes' on the previous page to help you answer questions 1–6.

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the primes between 20 and 60.

4 (a) The first prime number after 50 is:

A 51 B 53 C 55 D 57

(b) A number coprime with 18 is:

A 9 B 21 C 24 D 25

Understanding

To show that a number is composite, you only need to show that one of the divisibility tests works.

5 (a) What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write TRUE or FALSE for each of the following statements.

(a) 21 is prime (b) 38 is composite

(c) 59 is prime (d) 49 is prime

(e) 5 and 7 are coprime (f) 5 and 6 are coprime

(g) All even numbers greater than 2 are composite

(h) All odd numbers are primes

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 (b) 621 (c) 9909

(d) 4 516 803 (e) 87 912 404 (f) 2 871 025

8 Which of the following pairs of numbers are coprime? Give reasons for your choice(s).

(a) 9 and 17 (b) 8 and 11 (c) 13 and 52 (d) 27, 63

Reasoning

9 Explain why any pair of prime numbers is coprime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 (a) Find the numbers closest to 100 that are coprime with 100.

(b) Find the numbers closest to 36 that are coprime with 36.

15 Will a prime number always be coprime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be coprime.

Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

Outside the Square Puzzle

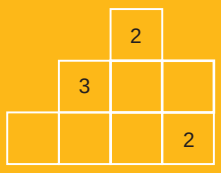
Gold digger 1

It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. She has marked on two separate maps for you the places next to where gold lies. If a number 3 is in a box, then it means that there are 3 pieces of gold in adjacent squares, either horizontally, vertically or diagonally. (Adjacent squares share an edge or a corner.) None of the already numbered squares contains a piece of gold, and no square contains more than one piece of gold.

Your task is to find exactly where the gold lies, so your team can get to it first and win the competition. There is only one possible solution for each map.

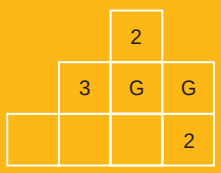
Basic techniques

Look for an easy opening:

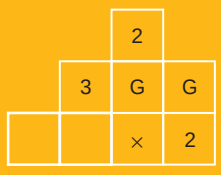


The top square (numbered 2) is only touching 2 other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.

Go back to other squares:



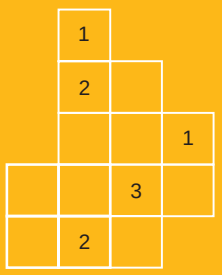
Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it is touching is empty. Mark this square with a cross.



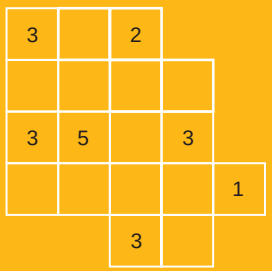
Now, look at the 3, and you can see that it is already next to one piece of gold, and is only touching two other empty squares, so both of these must also contain gold.

Now, copy the following maps and find the gold.

(a)



(b)



Mathspace

FACTOR PILLAR

FACTORPILLAR

Equipment required: 1 counter per player, 1 die, 2–4 brains, calculator (optional)

How to play:

- Roll to decide who goes first.
- Players take turns to roll the die and move forward the number of spaces shown.
- The aim: to move from start to finish and get as many points as possible by finding factors of numbers.

When a player lands on a number, they must list all the factors of that number. The number of factors is their score. For example, a player lands on 6: factors are 1, 2, 3 and 6; score = 4.

But beware! If another player notices you have missed one or more factors they gain five points. If they suggest a factor that isn't correct they lose five points.

LARGE CIRCLE: If a player correctly finds all the factors of the number in a large circle, they earn double the points for that number.

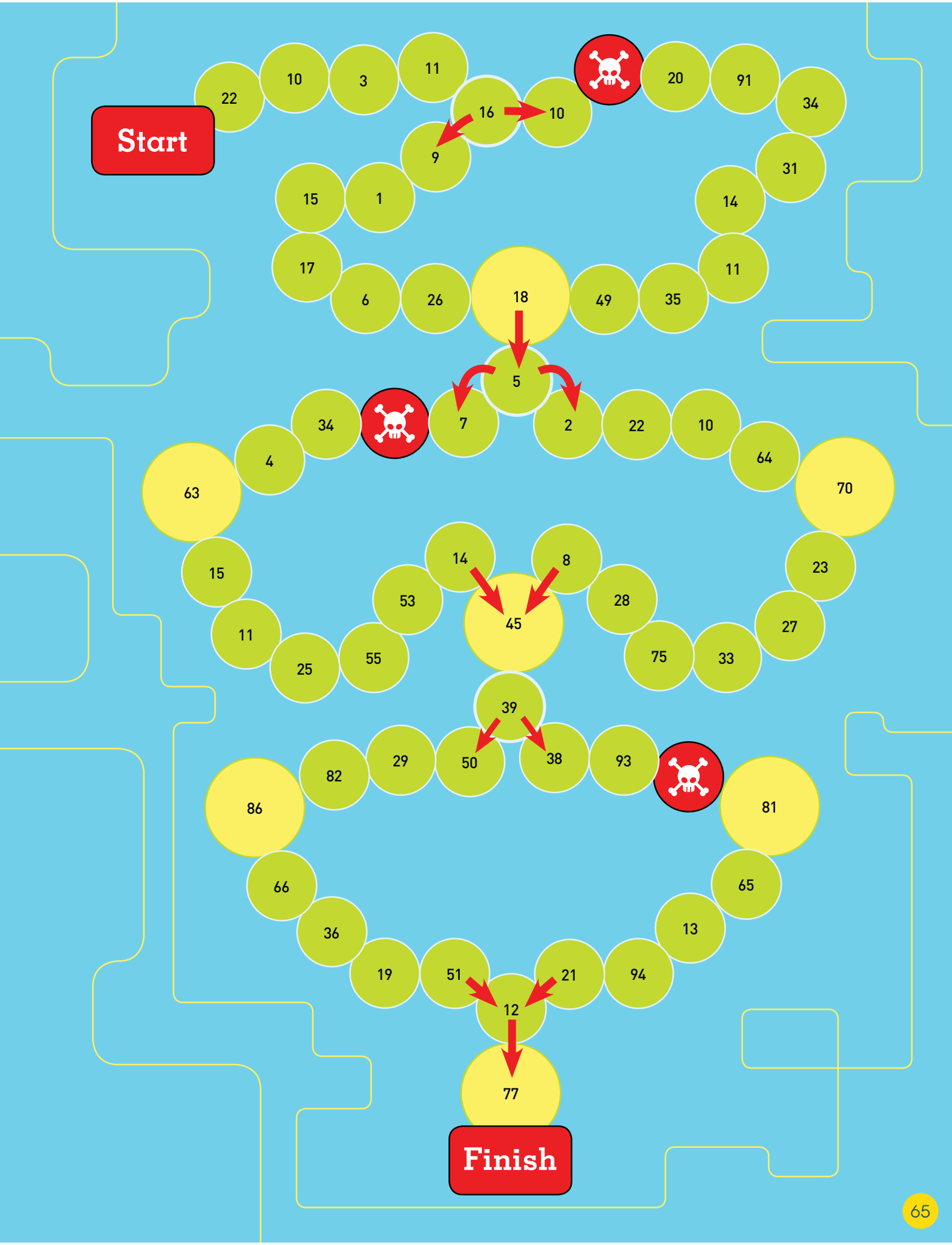
CROSSBONES: If a player lands on a circle containing a skull and crossbones, they miss a turn.

SQUARE NUMBER: If a player lands on a square number and calls 'Square Power' within two seconds, they have an extra turn.

PRIME NUMBER: If a player lands on a prime number and calls 'Prime Power' within two seconds, they can double their points for that number; if they don't call 'Prime Power' they only get two points for the two factors.

Players may not move backwards. When a player reaches a circle with two arrows leading from it they can decide which path to take.

The game ends when one player reaches or crosses the finish line. The player with the highest number of points at that moment is declared the winner (even if they haven't reached the finish line).


Start
22
10
3
11
16
10
9
20
91
34
31
15
1
14
17
6
26
18
49
35
11
5
7
2
22
10
64
34
4
63
70
15
14
8
23
53
45
28
11
27
25
55
75
33
39
50
38
93
29
82
86
81
66
65
36
13
19
51
21
94
12
77
Finish

2.3 Prime factors

A prime factor of a number is a factor that is also a prime number.

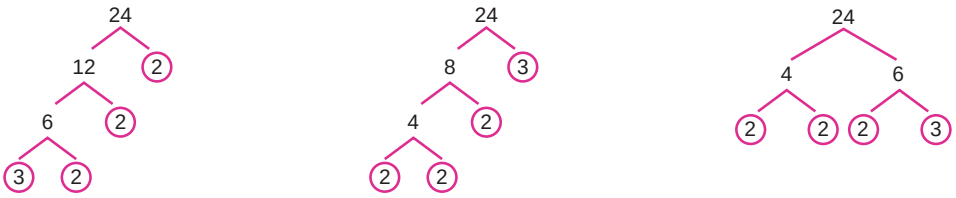
Every whole number can be written as the unique product of its prime factors.

Using a factor tree to find prime factors

A factor tree is a useful way of finding the prime factors of a number. To construct a factor tree:

1. Draw two lines ('branches') extending out from the number, in an upside-down 'V' shape.
2. Write two factors of the number at the end of the branches.
3. If one of the factors is prime, circle it. If it is composite, draw another two branches and split the factor into two smaller factors.
4. Repeat step 3 until all branches end in a circled, prime number.

There is often more than one way for a factor tree to be constructed. Three versions of the factor tree for 24 are shown here. They look different; however, the prime factors that appear on the ends of the branches are the same: 2, 2, 2, 3.



Once all of the branches end in prime factors, we have found all of the prime factors, and we write the number as the product of those factors. We write the factors in ascending order (smallest to largest) and, if any are repeated, we write them in index form.

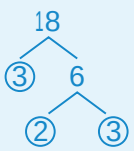
For example: $24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$

Worked Example 5

WE5

Draw a factor tree for the number 18, then express the number as a product of its prime factors in index form.

Thinking	Working
1 Write the number as the product of two factors.	$18 = 3 \times 6$
2 Show each factor at the end of a branch and circle it if it is prime.	
3 Split any composite factor into two smaller factors, continuing until the factors at the end of each branch are prime.	
4 Write the number as a product of the primes in index form and in ascending order (smallest to largest).	$18 = 2 \times 3 \times 3$ $= 2 \times 3^2$

Using repeated division to find prime factors

We can also find the prime factors of a number by doing a series of divisions by small prime factors, such as 2, 3 or 5. We can use the divisibility rules to identify these smaller prime factors; for example, if the number is even, we can divide by 2. The divisibility rules are listed in section 2.1 on page 55.

We keep dividing by the same, or a different, prime factor until the final answer is 1. The list of all the divisors will be the list of the prime factors.

Worked Example 6 WE6

Use repeated division to find the prime factors of 84. Express the number as a product of its prime factors in index form.

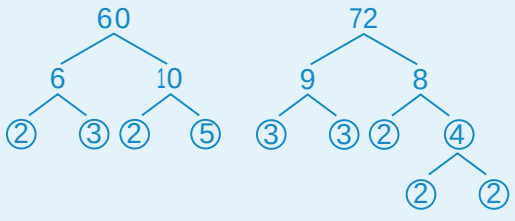
Thinking	Working
1 Identify a prime factor of the number and divide it into the number. Continue dividing by the same or a different prime factor until the last answer is 1.	$2\overline{)84}$ $2\overline{)42}$ $3\overline{)21}$ $7\overline{)7}$ 1
2 The prime factors are listed down the left side of the division calculation. Write the number as a product of these.	$84 = 2 \times 2 \times 3 \times 7$
3 Write the number as a product of the prime factors in index form and in ascending order.	$84 = 2^2 \times 3 \times 7$

Using prime factors to find the HCF (highest common factor)

If we have two numbers written as a product of their prime factors, the HCF is the product of all the common prime factors.

Worked Example 7 WE7

Use prime factors to find the HCF of the following pair of numbers: 60 and 72.

Thinking	Working
1 Use factor trees to find the prime factors.	
2 Write each number as a product of its prime factors. Do not use index form.	$60 = (2 \times 2 \times 3) \times 5$ $72 = 2 \times (2 \times 2 \times 3) \times 3$
3 Circle the group of factors that is common to both numbers (there must be the same number of factors in each group).	
4 Find the product of the common prime factors. This is the HCF.	$HCF = 2 \times 2 \times 3$ $= 12$

2.3 Prime factors

Navigator

Answers page 618

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q9, Q10, Q11, Q14	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q9, Q10, Q11, Q12, Q13, Q14	Q1 Column 4, Q2 Columns 2 & 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15

Fluency

WE5

1 Draw a factor tree for each of the following numbers, then express the number as a product of its prime factors in index form.

(a) 8	(b) 12	(c) 20	(d) 48
(e) 14	(f) 26	(g) 68	(h) 44
(i) 64	(j) 72	(k) 108	(l) 144
(m) 200	(n) 750	(o) 1000	(p) 1236

WE6

2 Use repeated division to find the prime factors of the following numbers. Express each number as a product of its prime factors in index form.

(a) 8	(b) 12	(c) 28	(d) 36
(e) 39	(f) 77	(g) 51	(h) 38
(i) 30	(j) 63	(k) 96	(l) 132
(m) 168	(n) 198	(o) 288	(p) 212

WE7

3 Use prime factors to find the HCF of the following pairs of numbers.

(a) 8 and 20	(b) 12 and 18	(c) 12 and 36
(d) 36 and 48	(e) 24 and 56	(f) 28 and 84
(g) 64 and 96	(h) 70 and 98	(i) 60 and 105
(j) 66 and 110	(k) 80 and 128	(l) 130 and 156

4 (a) 120 written as the product of its prime factors is:

A $2 \times 4 \times 15$ B $3 \times 8 \times 5$ C $2 \times 2 \times 5 \times 6$ D $2 \times 2 \times 2 \times 3 \times 5$

(b) The prime factors of 550 are:

A 2, 5, 11 B 2, 10, 55 C 5, 10, 11 D 2, 11, 25

5 Which numbers are the product of these prime factorisations?

(a) $2^3 \times 3^2$	(b) $2^2 \times 3 \times 5^2$	(c) $2^4 \times 3^3 \times 5^3$
(d) $3^2 \times 5 \times 7^2$	(e) $2^3 \times 5^2 \times 7 \times 11$	(f) $2^6 \times 11^2$

6 $6 = 2 \times 3$ and $8 = 2^3$. Use these facts to write the following numbers as products of prime numbers.

(a) 48	(b) 96 (48×2)	(c) 144 (48×3)

7 $33 = 3 \times 11$ and $10 = 2 \times 5$. Use these facts to write the following numbers as products of prime numbers.

(a) 330	(b) 660	(c) 990

Understanding

8 The prime factors of 192 are 2 and 3 and the prime factors of 42 are 2, 3 and 7. List the prime factors of 192×42.

9 (a) The HCF of a pair of numbers can also be used to find the lowest common multiple (LCM). Copy and complete the following method.

Prime factors of 36: $2 \times \_\_ \times \_\_ \times \_\_$

Prime factors of 100: $2 \times \_\_ \times \_\_ \times \_\_$

Highest common factor (HCF) of 36 and 100: $2 \times \_\_ = \_\_$

Factors of 36 not used to find the HCF: $\_\_ \times 3 = \_\_$

Factors of 100 not used to find the HCF: $5 \times \_\_ = \_\_$

HCF × all unused factors of 36 × all unused factors of 100: $\_\_ \times 3 \times \_\_ \times 5 \times \_\_ = 900$

The result is the lowest common multiple of 36 and 100.

(b) Use this method to find the LCM for the following pairs of numbers.

(i) 8 and 18	(ii) 12 and 18	(iii) 8 and 20
(iv) 8 and 72	(v) 20 and 26	(vi) 20 and 800
(vii) 20 and 1000	(viii) 400 and 800	(ix) 800 and 1000

10 Marcus is making identical balloon arrangements for a party. He has 84 red balloons and 54 blue balloons. He would like to sort his balloons into groups, each group containing the same number of red and blue balloons, with no balloons left over.

(a) Use prime factors to find the highest common factor of 84 and 54.

(b) If Marcus split his balloons into this number of groups, how many red and blue balloons would be in each group?

Reasoning

11 A number less than 550 is the product of four prime factors: 2, 3, 5 and a fourth prime factor between 15 and 20. What is the fourth factor, and what is the number?

12 (a) Find the smallest number that can be written in prime factor form with:

(i) exactly two different prime factors

(ii) exactly three different prime factors

(iii) exactly four different prime factors.

(b) Repeat (a), but remove the condition that the factors are different.

13 140 tickets were sold to the school concert. Each ticket sold was numbered from 1 to 140 and a lucky door prize was awarded to each person who had a ticket with a number that was a multiple of 15, but not a multiple of 9, 10 or 25. Five friends had the ticket numbers 45, 80, 90, 105 and 135.

(a) Do any of them have a winning ticket? If so, what is this winning number?

(b) If 250 tickets were sold, how many winning tickets would there be? Find the winning numbers.

Open-ended

14 A number between 100 and 200 is the product of four primes and has three prime factors (one factor is repeated). Explain how you might go about finding this number and write down two such numbers.

15 To find the prime factors of a number, Ray uses the divisibility tests. He then writes down the number from each divisibility test that worked.

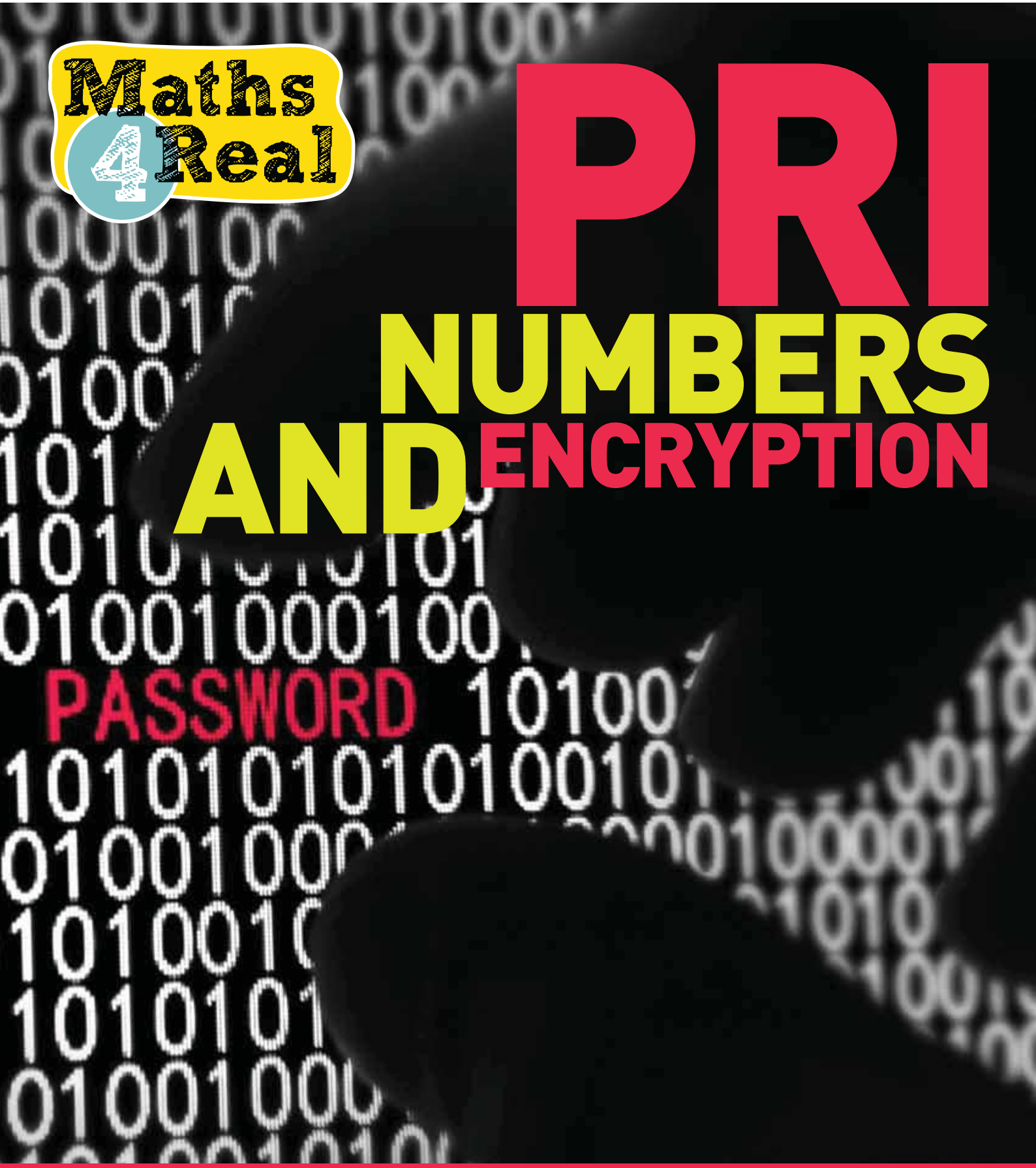
(a) Using this method, what would Ray say are the prime factors of 20?

(b) What is wrong with Ray's method?

(c) Suggest how Ray's method can be improved so that it is accurate.

Maths 4 Real

PRIME NUMBERS AND ENCRYPTION



{A B C D E F G H I J K L M

ME

How secure is the PIN that people use to access an ATM? What about the passwords and credit card numbers used for online shopping and banking? Some people are wary of placing personal details on the internet, fearing that they may be accessed by unauthorised people. Cryptography is the study of hiding information by turning it into code that cannot be understood by others. A piece of information (such as a PIN) is 'encrypted', or coded, and sent to the intended recipient (the bank), who is able to 'decrypt' it. One method used to code data is RSA encryption. It uses two large prime numbers multiplied together to obtain the 'public key', which is used to encrypt the message. To decrypt the message, you first need to find the 2 original primes (the 'private' key).

How easily can this be done? It depends on the size of the numbers. The larger the two prime numbers that are multiplied, the harder it is to find them, and the more secure the encryption.

1 Which two prime numbers were multiplied together to give each of these encryption keys?

(a) 77 **(b)** 38 **(c)** 202 **(d)** 143

2 Multiply the following prime numbers together to get encryption keys.

(a) 3 and 17

(b) 131 and 727

(c) 313 and 93139

DIY Decryption

3 **(a)** Write out the first five prime numbers.

(b) Find the six smallest keys that can be created by multiplying two primes together.

(c) How many factors does each key in part **(b)** have?

(d) Will every key created by multiplying two primes together have this number of factors? Why?

4 Here's a simplified version of how decryption using prime numbers works. Suppose the encrypted message is W L R G Y S B E M R K C R E B J C V and your key is 713.

Find the two prime numbers whose product is 713 (23 and 31). Now write the digits of these primes under the code as follows:

W	L	R	G	Y	S	B	E	M	R	K	C	R	E	B	J	C	V
2	3	3	1	2	3	3	1	2	3	3	1	2	3	3	1	2	3

The digits tell you how many places to shift each letter along the alphabet. The 2 under the W means that you shift two letters forward through the alphabet from W to Y so that the decoded message has Y as its first letter. To get the second letter, you move 3 places forward from L in the alphabet, arriving at the letter O.

(a) Continue decrypting the code to get the full message.

(b) Describe what you did with the coded letter Y.

5 Decode this message using 943 as the key.

K X P G Q F O N D M N H K B E L N L N S Y K Y D

Note: You will need to decide which order to write the two primes. Only one will work.

6 Encrypt your own message using this method. Use prime numbers under 100 and keep the message under 25 letters. Give your coded message and key to someone to decrypt.

7 Why must prime numbers be used as encryption keys? Why can't we use any number, such as 120, as a key?

Research

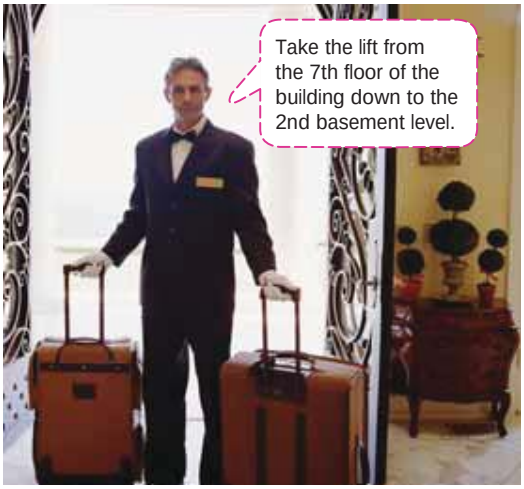
Find out about GIMPS (Great Internet Mersenne Prime Search) and how prime numbers are used in email encryption.

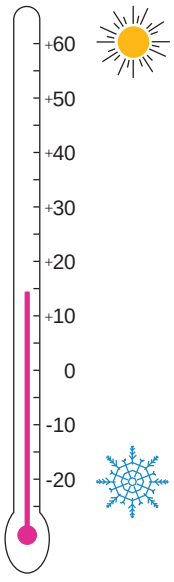
Consider whether governments should ban encryption so that intelligence agencies such as Australian Security Intelligence Organisation (ASIO) can read everyone's emails.

Make a list of the potential problems and dangers that can arise from placing personal information on the internet.

N O P Q R S T U V W X Y Z }

2.4 Introduction to integers





All of the above statements describe more than just the size of a number—they also give a direction. We can use a + (positive) sign or a – (negative) sign to show this direction. Below is a list of words usually associated with + or –.

+ (positive)	- (negative)
up	down
increase	decrease
gain	lose
deposit	withdrawal
profit	loss
above	below

The words 'deposit' and 'withdrawal' are banking words, used to describe the movement of money into and out of a bank account. 'Profit' and 'loss' are words used to describe whether money has been made or lost by a business.

Integers

The integers are all of the whole numbers that you are familiar with (1, 2, 3 …), the negative equivalents of these (-1, -2, -3 …), and zero, which is neither positive nor negative.

We indicate whether a number is positive or negative by placing a '+' or '-' in front of the number. For example, we write '+4' and say 'positive four'. We write '-9', and say 'negative nine'.

Worked Example 8 WE8

Write down an integer suggested by each of the following.

(a) depositing $660 into your bank account

(b) diving 5 m below the surface of the ocean

Thinking		Working
(a) 1	Does the statement suggest an increase or a decrease? (In this case, 'deposit' suggests an increase, so use a positive sign.)	(a)
2	Write the number with the appropriate sign.	+660
(b) 1	Does the statement suggest an increase or a decrease? (In this case, 'below' suggests a decrease, so use a negative sign.)	(b)
2	Write the number with the appropriate sign.	-5

Comparing integers

The number line is extremely useful when working with integers.

Up until now, we have used only the positive section of the number line, which extended to the right of zero. Now, we will extend the number line to the left of 0 to include negative numbers. Notice that the negative numbers form a mirror image of the positive numbers on the other side of zero.

As we move along the number line, the numbers get larger as we move to the right, and smaller if we move to the left. Look at the positions of +5 and -10 on the number line shown. +5 is further to the right than -10. We say that +5 *is greater than* -10, and write +5 > -10. We could also say that -10 *is less than* +5, and write -10 < +5.

Note: Often, the + sign is left off positive numbers; for example, 6 has the same value as +6. From now on, we will leave the + signs off the positive numbers on the number line, because there is no need for them.

If a number has no sign in front of it, we assume it is positive. 7 = +7

'>' is the symbol for 'is greater than'.

'<' is the symbol for 'is less than'.

Worked Example 9 WE9

Write a > or < symbol between each pair of integers to make a correct statement.

(a) -9 _____ -4 (b) 0 _____ -10 (c) 7 _____ -48

Thinking	Working
(a) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, -9 is to the left of -4.) If it is to the right, insert >. If it is to the left, insert <.	(a) -9 < -4
(b) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 0 is to the right of -10.) If it is to the right, insert >. If it is to the left, insert <.	(b) 0 > -10
(c) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 7 is to the right of -48.) If it is to the right, insert >. If it is to the left, insert <.	(c) 7 > -48

Worked Example 10 WE10

Arrange the following integers in ascending order (from smallest to largest).

-2, 3, -7, 0, 6

Thinking	Working
1 Mark the integers on a number line as shown.	
2 Now, list the marked integers in order, as they appear on the number line, moving from left to right.	-7, -2, 0, 3, 6

The further to the right a number is on a horizontal number line, the greater its value.

The further to the left a number is on a horizontal number line, the smaller its value.

Vertical number lines can also represent directed numbers, where the higher a number is on the line, the greater its value. A thermometer is an example of a vertical number line.

2.4 Introduction to integers

Navigator

Q1 Column 1, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q8, Q11, Q12, Q13, Q14, Q16, Q18	Q1 Column 1, Q2 Columns 2–4, Q3 Columns 2–3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9 Column 1, Q11, Q12, Q14, Q15, Q16, Q17, Q18	Q1 Column 2, Q2 Columns 3 & 4, Q3 Column 3, Q4 Column 3, Q5 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18

Answers page 619

Fluency

WE8

1 Write down an integer suggested by each of the following.

(a) a profit of $300
(b) a loss of $50 000
(c) 8 seconds before blast-off
(d) 4 hours after take-off
(e) 36 m above sea level
(f) a deposit of $45
(g) a withdrawal of $20
(h) a win by 12 points
(i) a 2 point loss
(j) down 7 floors
(k) up 28 floors
(l) I owe the bank $40 000.
(m) I have $225 in the bank.
(n) The water level dropped by 9 m.

WE9

2 Write a > or < symbol between each pair of integers to make a correct statement.

(a) +6 _____ -3
(b) +1 _____ -3
(c) -7 _____ +5
(d) -2 _____ +6
(e) -12 _____ -4
(f) -7 _____ -1
(g) -2 _____ -11
(h) -9 _____ 0
(i) 0 _____ -2
(j) -35 _____ +7
(k) -4 _____ +67
(l) -11 _____ +75

WE10

3 Arrange the following integers in ascending order (from smallest to largest).

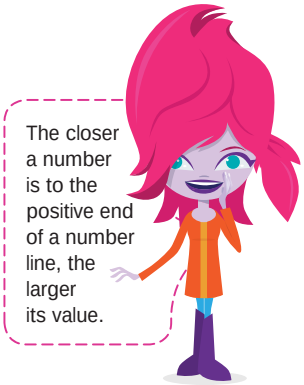
(a) -3, -12, 6, -4, -1
(b) -11, 10, 9, 2, -10
(c) 1, -6, -4, 9, 2
(d) -8, -5, 7, 1, -6
(e) -1, -4, -7, -8, -12
(f) -56, 23, -79, 0, 6

4 Arrange the following integers in descending order (from largest to smallest).

(a) -5, -6, 3, 7, 8
(b) -8, 7, 10, 1, -12
(c) 1, -8, -4, -9, -6
(d) -12, 0, -5, 7, -11
(e) -11, -5, -2, -12, -9
(f) 0, 79, -54, 4, -9

5 State the opposite of:

(a) west 300 km
(b) 3 days late
(c) 7°C below zero
(d) add 5
(e) subtract 22
(f) go left 3 m
(g) -2
(h) +16
(i) -350



6 (a) The directed number +2 is suggested by:

A going up two flights of stairs

B going down two flights of stairs

C losing 2 kg in weight

D the temperature going from 19°C to 17°C.

(b) The directed number -4 is suggested by:

A going up four flights of stairs

B the rain gauge filling with 4 mm of rain

C gaining 4 kg in weight

D the temperature going from 22°C to 18°C.

7 Write TRUE or FALSE for each of the following.

(a) $58 > -60$ (b) $-44 < -50$ (c) $-29 > 30$ (d) $-31 < -29$

(e) $-75 > 70$ (f) $-92 > -11$ (g) $19 > -100$ (h) $82 > -99$

Understanding

8 Write down the integers that lie between:

(a) -4 and 3 (b) -2 and 2 (c) -9 and -4

(d) -5 and 0 (e) -37 and -42 (f) -120 and -115

9 Find the next three numbers in each pattern.

(a) -18, -16, -14, ____, ____, ____ (b) -20, -15, -10, ____, ____, ____

(c) 9, 6, 3, ____, ____, ____ (d) -6, -12, -18, ____, ____, ____

(e) 30, 20, 10, ____, ____, ____ (f) 12, 7, 2, ____, ____, ____

10 The balances of four traders' bank accounts show the following figures.

Ben	$428	Sue	-$23
Damien	-$260	Carol	$325

(a) Who has the largest account?

(b) Who owes their bank the most?

11 The minimum and maximum temperatures recorded at Uluru during one week were:

Day	Maximum (°C)	Minimum (°C)
Monday	19	-3
Tuesday	16	-7
Wednesday	22	0
Thursday	20	-1
Friday	23	2
Saturday	20	-2
Sunday	18	-4

(a) On which day was the lowest minimum recorded?

(b) When was the highest minimum recorded?

(c) Which day had the greatest difference between the minimum and the maximum?

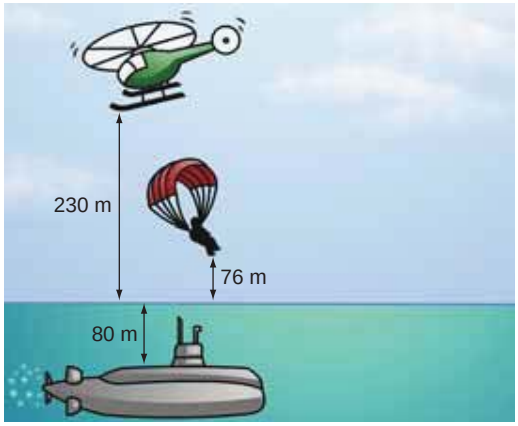
12 In a combined operation, the airforce and navy have a helicopter and a submarine at the levels shown.

A parachutist has 76 m to fall before hitting the ocean.

(a) How far is the parachutist from the submarine?

(b) What distance from the helicopter is the parachutist?

(c) How far apart are the helicopter and the submarine?



Reasoning

13 (a) If you cycled 5 km west, then 3 km east, then 2 km west, what single journey could you have taken to get to the finishing point?

(b) What is the opposite of this single journey?

14 (a) If you got in a lift on the ground floor, then travelled 7 floors up, then 2 floors up, then 6 floors down and finally 1 floor up, what single journey could you have taken to get to your final floor?

(b) What is the opposite of this single journey?

15 On Monday, Sam withdrew $80 from his bank account at an ATM. On Tuesday, his pay of $350 was deposited into the account, and he also deposited $40 that he had received for his birthday. On Thursday, Sam used the account to pay a $120 bill online, and on Friday he withdrew $50 from an ATM.

(a) Did Sam have more or less money at the end of the week than at the start?

(b) How much more or less did he have?

Open-ended

16 Write any three negative integers greater than -8.

17 The game of indoor cricket is similar to regular cricket in that batsmen can score 1, 2, 3, 4 or 6 runs off each ball that is bowled to them. It is different in that pairs of batsmen stay in for 4 overs (an 'over' is a group of 6 balls). If they lose their wicket, or 'get out', in these 4 overs, they do not leave the pitch. Instead, 3 runs are subtracted from their score, which is shown as the combined total of the runs scored by both batsmen.

(a) In their first over, Angelo and Murray scored as follows: 2, -3 (wicket), 1, 2, 4, -3 (wicket). What was their overall score at the end of the over?

(b) At the end of their first over, Saul and Jeremy's score was 4. On each ball of the over, they had either scored runs or lost their wicket. List at least three ways they could have reached 4 in six balls.

18 Sharif is a computer technician. He works in an office building with 25 floors, helping fix computer problems for other employees. Write a short story using this sequence of directed numbers, describing a day at work for Sharif.

0, +21, -13, +7, 0, +4, 0, -2, +5, 0, -1, -2, 0

Outside the Square Puzzle

Temperature tangle

Equipment: access to the internet or a library may be necessary

Australia is known for its warm summers, but it can get cold here too!

Solve this puzzle to discover the place that holds the record for the coldest recorded temperature in Australia.

1 Write the numbers 1–13 down your page.

2 For the 13 situations listed below, find the matching temperature.

3 For each of the 13 statements in order, write the temperature and the letter next to it in your book.

4 If the temperatures and their corresponding letters are written in the correct order, you will spell out the name of Australia's coldest place.

1	Boiling point of water	70°C	P
2	Oven temperature required to bake a cake	37°C	E
3	Temperature inside your fridge at home	0°C	O
4	Coldest air temperature recorded on Earth (Vostok Station, Antarctica)	180°C	H
5	Temperature of the surface of the Sun	51°C	A
6	Freezing point of water	-15°C	T
7	Hottest air temperature recorded on Earth (Al 'Aziziyah, Libya)	4°C	A
8	Temperature inside your freezer at home	100°C	C
9	Body temperature of a healthy human	-196°C	S
10	Drinking temperature of a cup of tea	-40°C	S
11	Hottest temperature recorded in Australia (Oodnadatta, SA)	58°C	T
12	Temperature of a freezer inside an ice-cream factory	5700°C	L
13	Temperature of liquid nitrogen	-89°C	R

Half-time 2

1 (a) Find the lowest common multiple (LCM) of 9 and 15. Ex 2.1

(b) Find the highest common factor (HCF) of 48 and 72.

2 The common factors of 18 and 30 are: Ex 2.1

A 1, 18, 23 B 1, 2, 3, 6 C 1, 9, 18, 30 D 2, 3, 9, 18

3 Write each of the following as either a positive or a negative integer. Ex 2.4

(a) a deposit of $85 into a bank account (b) a weight loss of 7 kg

(c) a win by 3 goals (d) a profit of $28 000

(e) 40 m below sea level (f) going up 11 floors in a building

4 List all the prime numbers between 30 and 50. Ex 2.2

5 Write a < (less than) or > (greater than) symbol between each of the following to make a true statement. Ex 2.4

(a) 6 _____ 10 (b) -4 _____ 7 (c) 3 _____ -12 (d) -8 _____ -15

6 Draw a factor tree for each of the following, then express each number as a product of its prime factors, in index form. Ex 2.3

(a) 54 (b) 60 (c) 98 (d) 104

7 (a) A number is divisible by 15 if it is divisible by both 3 and 5. Use divisibility tests to decide if 76 905 is divisible by 15. Ex 2.1

(b) Show, using two divisibility tests, that 4734 is divisible by 6.

8 Write the following sets of integers in ascending order (smallest to largest). Ex 2.4

(a) -37, 7, -30, 0, -3, 3 (b) 54, -20, -1, -5, -40

9 Use prime factors to find the HCF of the following pair of numbers: 48 and 120. Ex 2.3

10 The sum of two prime numbers is 30. The numbers cannot be: Ex 2.2

A 7 and 23 B 11 and 19 C 13 and 17 D 14 and 16

11 Three racing cars, Honda, Holden and Ford, take 25 seconds, 30 seconds and 50 seconds, respectively, to complete one lap of a circuit. They start the race at the same time and from the same position. Ex 2.1

(a) How many seconds will pass before the cars are at the same position on the track?

(b) How many laps had each car completed by this time?

(c) How many laps did the Honda complete in the time taken by the Ford to complete 6 laps?

2.5 Adding and subtracting positive integers

In the following sections you will learn how to perform calculations with integers. A number line can be very useful for this.

Adding positive integers

Adding two positive integers is the straightforward addition you have been doing for years.

For example, $^{+}1 + {}^{+}2 = {}^{+}3$, which can simply be written as $1 + 2 = 3$.

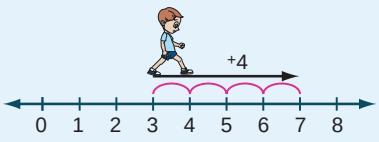
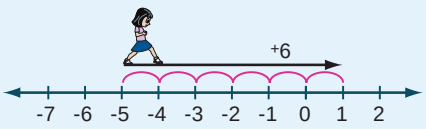
To add a positive number, move that many spaces along the number line to the right (the positive direction) from the starting position.

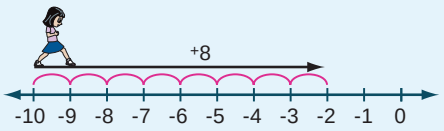
The steps are the same, regardless of whether our starting position is a positive or a negative integer, as we can see in the Worked Example below.

Worked Example 11 WE11

Use a number line to perform each of the following additions.

(a) $3 + 4$ (b) $-5 + 6$ (c) $-10 + 8$

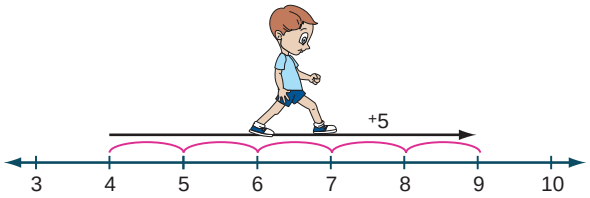
Thinking		Working
(a) 1	Draw or imagine a number line. Start at the first number (3) and walk the number of steps indicated by the second number (4) in the positive direction (to the right).	(a) 
2	Write the addition as a number sentence.	$3 + 4 = 7$
(b) 1	Draw or imagine a number line. Start at the first number (-5) and walk the number of steps indicated by the second number (6) in the positive direction (to the right).	(b) 
2	Write the addition as a number sentence.	$-5 + 6 = 1$

(c)	1	Draw or imagine a number line. Start at the first number (-10) and walk the number of steps indicated by the second number (8) in the positive direction (to the right).	(c) 
	2	Write the addition as a number sentence.	-10 + 8 = -2

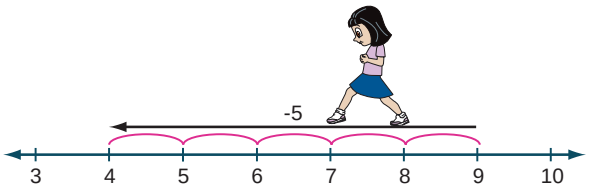
Subtracting positive integers

Subtraction and addition are opposite operations. We have shown addition as starting on a point on the number line and walking along a number of spaces to the right. Because subtraction is the opposite of addition, we subtract by moving to the left on the number line.

Addition:
Move to the right
4 + 5 = 9



Subtraction:
Move to the left
9 – 5 = 4



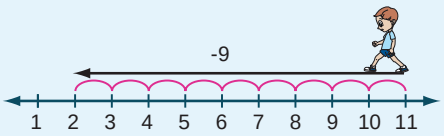
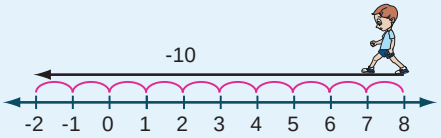
To subtract a positive number, move that many spaces along the number line to the left (the negative direction).

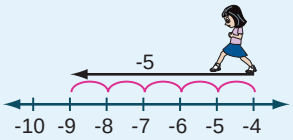
Worked Example 12

WE12

Use a number line to perform each of the following subtractions.

(a) 11 – 9 (b) 8 – 10 (c) -4 – 5

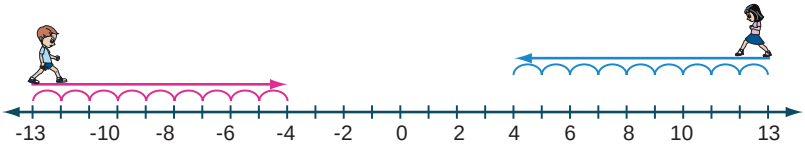
Thinking			Working
(a)	1	Draw or imagine a number line. Start at the first number (11) and walk the number of steps indicated by the second number (9) in the negative direction (to the left).	(a) 
	2	Write the subtraction as a number sentence.	11 – 9 = 2
(b)	1	Draw or imagine a number line. Start at the first number (8) and walk the number of steps indicated by the second number (10) in the negative direction (to the left).	(b) 
	2	Write the subtraction as a number sentence.	8 – 10 = -2

(c) 1	Draw or imagine a number line. Start at the first number (-4) and walk the number of steps indicated by the second number (5) in the negative direction (to the left).	(c) 
2	Write the subtraction as a number sentence.	-4 – 5 = -9

Part (b) of the Worked Example above shows that if we subtract a larger number (such as 10) from a smaller one (such as 8), we will walk past the zero into the negative side of the number line.

Using number line symmetry

The number line can be thought of as being a reflection, or mirror image, around 0. This symmetry can be useful when working with integers. For example, the journey to calculate -13 + 9 (moving to the right) is the mirror image of the journey to calculate +13 – 9 (moving to the left), with answers of -4 and +4, respectively.



So, to calculate -85 + 23, we could simply do 85 – 23, then place a negative sign in front of the answer.

85 – 23 = 62 -85 + 23 = -62

2.5 Adding and subtracting positive integers

Navigator

Answers page 620

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q5, Q6, Q8, Q12, Q13	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency

WE11

1 Use a number line to perform each of the following additions.

(a) 8 + 3	(b) 7 + 1	(c) 3 + 8	(d) 1 + 7
(e) -9 + 7	(f) -11 + 6	(g) -10 + 5	(h) -12 + 6
(i) -8 + 14	(j) -7 + 19	(k) -5 + 30	(l) -20 + 40
(m) -30 + 20	(n) -17 + 13	(o) -75 + 55	(p) -87 + 56

WE12

2 Use a number line to perform each of the following subtractions.

(a) 5 – 3	(b) 8 – 2	(c) 12 – 7	(d) 11 – 11
(e) 8 – 14	(f) 6 – 17	(g) 10 – 18	(h) 11 – 15
(i) 30 – 40	(j) -3 – 5	(k) 2 – 22	(l) -5 – 7
(m) -9 – 3	(n) -24 – 6	(o) -35 – 34	(p) -66 – 58

3 (a) $-5 + 3$ may be calculated by following which of these instructions?

A start at $+5$ and walk 3 steps to the right, arriving at 8

B start at -5 and walk 3 steps to the right, arriving at -2

C start at $+5$ and walk 3 steps to the left, arriving at 2

D start at -5 and walk 3 steps to the left, arriving at -8

(b) $-15 - 10$ may be described by which number line journey?

A start at -15 and walk 10 steps to the right, arriving at -5

B start at $+10$ and walk 15 steps to the left, arriving at -5

C start at $+15$ and walk 10 steps to the left, arriving at $+5$

D start at -15 and walk 10 steps to the left, arriving at -25

4 Calculate the following.

(a) $3 + 4 + 5$ (b) $5 + 2 + 4$ (c) $7 + 1 + 4$

(d) $-8 + 6 + 9$ (e) $-29 + 5 + 6$ (f) $-37 + 7 + 8$

(g) $6 - 7 - 4$ (h) $3 - 5 - 8$ (i) $12 - 16 - 4$

(j) $14 + 9 - 10$ (k) $23 - 30 + 6$ (l) $44 - 51 + 8$

Understanding

5 The temperature in the desert is $-12°C$, but rises by 20 degrees. What is the new temperature?

6 A lift is at the third basement floor (3 floors below ground level). It moves up 14 levels, then stops. On which floor does it stop?

7 A submarine 30 m below the surface of the ocean rises 16 m, then dives a further 20 m. What depth is it at now?

8 A diver jumps from a platform 12 m high and dives 17 m to the bottom of a pool. How deep is the pool?

9 You have \$5 in your pocket, but you owe a friend \$12 and someone else owes you \$20. If everyone pays up, how much will you have at the end?

10 Write the following number sentences in words.

(a) $+5 - 7 = -2$

(b) $-8 + 9 = +1$

(b) $-4 - 3 = -7$

Reasoning

11 (a) Show the following pairs of calculations on separate number lines.

(i) $-1 + 2$ and $+2 - 1$ (ii) $-7 + 4$ and $+4 - 7$ (iii) $-13 + 6$ and $+6 - 13$

(b) What do you notice? Comment on your observation.

(c) Use your observation from (b) to write the answers to the following without using a number line.

(i) $-37 + 86$ (ii) $-72 + 95$ (iii) $-104 + 119$

12 (a) Draw a number line from -12 to 12. Draw a number line journey showing $5 - 3 = 2$ and the 'mirror image' journey showing $-5 + 3 = -2$.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences:

(i) $8 - 6 = 2$ (ii) $5 - 4 = 1$ (iii) $-10 + 6 = -4$ (iv) $-12 + 9 = -3$

(c) Calculate these additions on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) $-23 + 17$ (ii) $-48 + 27$ (iii) $-59 + 34$ (iv) $-62 + 43$

Open-ended

13 The thermometer at a weather station in the desert near Alice Springs read -4°C at 6 a.m. The temperature then rose until it reached a maximum of 25°C at 12 p.m. What might the thermometer readings have been at 7, 8, 9, 10 and 11 a.m.?

14 Write down three different positive integers that when added to -13 give a negative answer.

Outside the Square — Problem solving

The slippery snail

A snail is at the bottom of a low garden wall 40 cm high. If the snail manages to move 4 cm up the wall each day, but slips down 3 cm each night, how long does it take to reach the top of the wall?

Strategy options

- Draw a diagram.
- Act it out.
- Look for a pattern.

2.6 Adding and subtracting negative integers

The '+' and '–' signs can be used to show an operation (addition or subtraction) or a direction (positive or negative). Where necessary, to avoid confusion, brackets are placed around the number and the direction sign in front of it, to separate it from an operation symbol.

For example, $^{+}2 + (-3)$ is said as 'positive 2 add negative 3'.

$-4 - (^{+}7)$ is said as 'negative 4 subtract positive 7'.

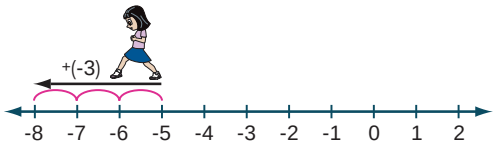
Adding negative integers

Imagine that you owe a friend \$5. We could write this as -5. Now, imagine that you owe a second friend \$3, which we can write as -3. How much do you owe altogether? The answer is \$8, which we can write as -8.

We 'add up' your total debt by calculating the sum: $-5 + (-3) = -8$. We can see that adding a negative number gives a 'more negative' result; that is, a number that is further towards the negative end of the number line.

We can show this on the number line in the following way.

1. Start at -5.
2. Walk along the number line 3 steps in the negative direction (to the left).
3. Arrive at the answer of -8.



$-5 + (-3)$ on the number line shows the same journey as the subtraction $-5 - 3$, or $-5 - (^{+}3)$.

The movement along the number line is the same, and the end result of -8 is the same.

This means that adding -3 and subtracting $^{+}3$ are exactly the same process.

To add a negative number, subtract its opposite.

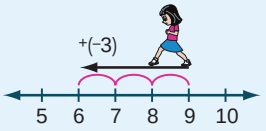
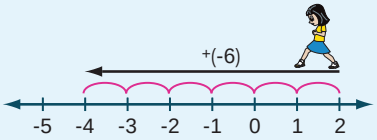
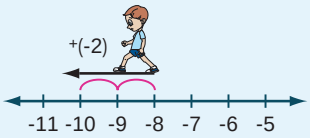
$-4 + (-7)$
$= -4 - 7$
$= -11$

$^{+}2 + (-3)$
$= ^{+}2 - 3$
$= -1$

Worked Example 13 WE13

Use a number line to perform each of the following additions.

(a) 9 + (-3) (b) 2 + (-6) (c) -8 + (-2)

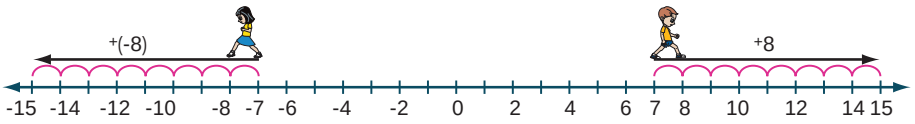
Thinking		Working
(a) 1	Draw or imagine a number line. Start at the first number (9) and walk the number of steps indicated by the second number (-3) in the negative direction (to the left).	(a) 
2	Write the addition as a number sentence.	9 + (-3) = 6
(b) 1	Draw or imagine a number line. Start at the first number (2) and walk the number of steps indicated by the second number (-6) in the negative direction (to the left).	(b) 
2	Write the addition as a number sentence.	2 + (-6) = -4
(c) 1	Draw or imagine a number line. Start at the first number (-8) and walk the number of steps indicated by the second number (-2) in the negative direction (to the left).	(c) 
2	Write the addition as a number sentence.	-8 + (-2) = -10

Using number line symmetry

The symmetry of the number line can be useful when adding negative numbers.

To calculate 7 + 8, we start at 7 and move to the right on the number line 8 steps to 15.

-7 + -8 is the mirror image of this. We start at -7, facing to the left and move to the left on the number line 8 steps to -15.

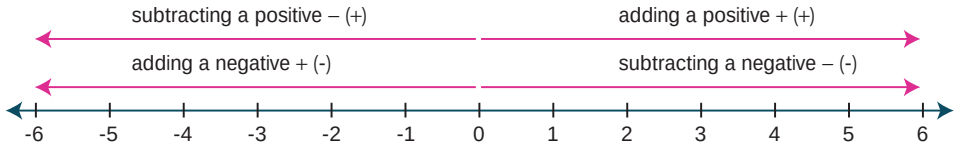


So, to calculate -47 + (-59), we can perform the 'mirror image' calculation of 47 + 59, then place a negative sign in front of the answer.

Subtracting negative integers

It can be difficult to imagine what it means to subtract a negative number. We have seen, however, that adding a negative number means moving along the number line to the left; that is, in the negative direction. Subtraction is the opposite of addition. So, subtracting a negative number means moving along the number line to the right; that is, in the positive direction.

We can think of subtracting a negative number as making the other number 'less negative', which, in turn, makes it 'more positive'.



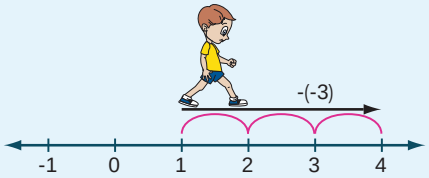
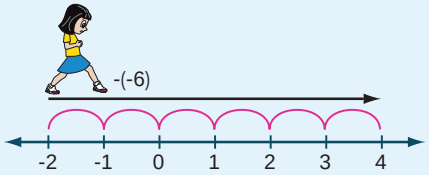
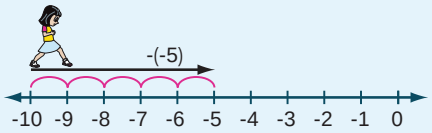
To subtract a negative number, add its opposite.

3 – (-1)	-7 – (-5)
= 3 + 1	= -7 + 5
= 4	= -2

Worked Example 14 WE14

Use a number line to perform each of the following subtractions.

(a) 1 – (-3) (b) -2 – (-6) (c) -10 – (-5)

Thinking	Working
(a) 1 Draw or imagine a number line. Start at the first number (1) and walk the number of steps indicated by the second number (-3) in the positive direction (to the right).	(a) 
2 Write the subtraction as a number sentence.	1 – (-3) = 4
(b) 1 Draw or imagine a number line. Start at the first number (-2) and walk the number of steps indicated by the second number (-6) in the positive direction (to the right).	(b) 
2 Write the subtraction as a number sentence.	-2 – (-6) = 4
(c) 1 Draw or imagine a number line. Start at the first number (-10) and walk the number of steps indicated by the second number (-5) in the positive direction (to the right).	(c) 
2 Write the subtraction as a number sentence.	-10 – (-5) = -5

Note that we say the above Worked Examples in the following way.

- +1 – (-3) is 'positive one subtract negative three'
- -2 – (-6) is 'negative two subtract negative six'
- -10 – (-5) is 'negative ten subtract negative five'

The number line journeys above show that subtracting a negative number results in a journey in the positive direction of the number line. This is the same journey as adding the positive equivalent of the number.

2.6 Adding and subtracting negative integers

Navigator

Answers page 621

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q5, Q6, Q9, Q10, Q11, Q12, Q15	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q15	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15

Fluency

WE13

1 Use a number line to perform each of the following additions.

(a) $5 + (-4)$	(b) $3 + (-2)$	(c) $12 + (-3)$	(d) $8 + (-2)$
(e) $2 + (-6)$	(f) $4 + (-9)$	(g) $10 + (-17)$	(h) $9 + (-20)$
(i) $-2 + (-7)$	(j) $-3 + (-8)$	(k) $8 + (-22)$	(l) $10 + (-24)$
(m) $-5 + (-5)$	(n) $-30 + (-60)$	(o) $-17 + (-12)$	(p) $-62 + (-71)$

WE14

2 Use a number line to perform each of the following subtractions.

(a) $10 - (-1)$	(b) $4 - (-2)$	(c) $5 - (-5)$	(d) $6 - (-6)$
(e) $-3 - (-11)$	(f) $-9 - (-12)$	(g) $-4 - (-20)$	(h) $-17 - (-11)$
(i) $-3 - (-15)$	(j) $-9 - (-8)$	(k) $-5 - (-13)$	(l) $-19 - (-13)$
(m) $-30 - (-20)$	(n) $-55 - (-45)$	(o) $-67 - (-41)$	(p) $-81 - (-53)$

3 (a) The finishing point for the number line journey $+6 + (-12)$ may be found by following which of these instructions?

- A start at $+6$ and walk 12 steps to the left, finishing at -6
- B start at $+6$ and walk 12 steps to the right, finishing at $+18$
- C start at -12 and walk 6 steps to the left, finishing at -18
- D start at -6 and walk 12 steps to the right, finishing at $+6$

(b) The answer to $+3 - (-20)$ may be found by following which of these instructions?

- A start at $+3$ and walk 20 steps to the left, arriving at -17
- B start at $+3$ and walk 20 steps to the right, arriving at 23
- C start at -20 and walk 3 steps to the right, arriving at -17
- D start at -3 and walk 20 steps to the left, arriving at -23

4 Calculate the following.

(a) $9 + (-7) + (-2)$	(b) $6 + (-5) + (-7)$	(c) $9 + (-12) + (-4)$
(d) $-1 + (-4) + (-5)$	(e) $10 + (-8) + (-7)$	(f) $24 + (-12) + (-6)$
(g) $2 - (-1) - (-2)$	(h) $1 - (-6) - (-2)$	(i) $-4 - (-7) + (-6)$
(j) $-7 - (-10) + (-5)$	(k) $12 + (-16) + (-4)$	(l) $48 + (-32) - (-6)$

Understanding

5 The temperature at sunset was 12°C. During the night it fell by 16°C to the minimum temperature. What was the minimum temperature?

6 A worker steps into a lift on the 7th floor of an office building and travels down 10 levels. Which level does he finish at? Write your answer as an integer.

7 Your company is $2000 in debt. You then receive notice that you owe the bank $1500. What is your financial state now?

8 A farmer lowers a bucket from the top of a 1 m high well using a 10 m rope attached to the handle. If she lowers the whole length of rope, how far below ground does the bucket reach?

9 A bird flying 6 m above sea level suddenly spots a fish and dives 9 m straight down before catching the fish. How far below the surface was the fish?

10 A miner is sitting on a ledge 8 m below the top of a vertical mine shaft when he drops his torch. If the torch falls 23 m before hitting the bottom of the shaft, how deep is the shaft?

11 Write the following number sentences in words.

(a) $-5 + (-6) = -11$ (b) $+7 - (-8) = +15$ (c) $-9 - (-14) = +5$

Reasoning

12 (a) Draw a number line from -12 to +12. Draw number line journeys showing $2 + 3 = 5$ and $-2 + (-3) = -5$.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences.

(i) $1 + 6 = 7$ (ii) $4 + 7 = 11$ (iii) $-3 + (-8) = -11$ (iv) $-5 + (-4) = -9$

(c) Work out these calculations on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) $-21 + (-15)$ (ii) $-28 + (-27)$ (iii) $-36 + (-41)$ (iv) $-65 + (-29)$

13 Evaluate the following.

(a) $12 - (+6)$ (b) $12 - (-6)$ (c) $-3 - (+6)$ (d) $-3 - (-6)$

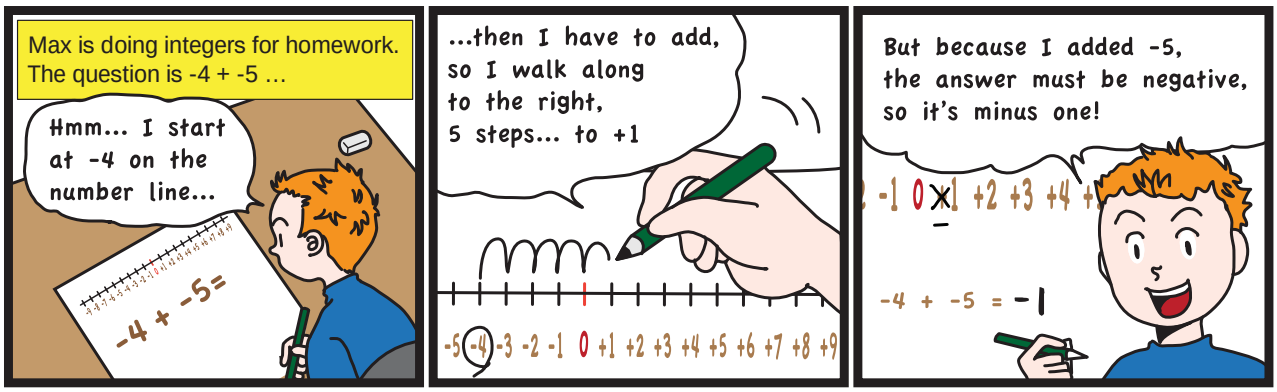
(e) Copy and complete the following sentence.

Whether you start with a positive or a negative integer, if you subtract a positive number, then the result is _____ and if you subtract a negative number, then the result is _____.

Open-ended

14 Write down three integers that when subtracted from -21 give a positive answer.

15



(a) What is the correct answer?

(b) Explain to Max where he went wrong, and how to do these types of questions correctly in the future.

Investigation

Heat beads and ice blocks

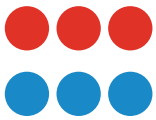
Equipment required: 1 brain, 10 red and 10 blue counters (or any other pair of colours)

The Big Question

Positive and negative integers can be represented by different coloured counters. How can we use them to show integer addition and subtraction?

Engage

We will use the red counters to represent positive integers and the blue counters to represent negative integers.

For example:  $= {}^{+}3$

 $= -5$

Because they are opposites, 1 blue counter will cancel out, or 'annihilate', 1 red counter. In other words, 1 blue + 1 red = 0. We can think of this as 1 'heat bead' (red) cancelling out 1 'ice block' (blue).

We can write this as a number sentence: $1 + -1 = 0$.

We can use combinations of red and blue counters to represent integers, by cancelling red and blue pairs.

For example:

 -7

+

 -4

= -3

(We have shown here that 4 red and 4 blue pairs cancelled each other.)

1 Place 3 red counters on the table to represent $^{+}3$. Now, place 2 blue counters underneath them.

What number is represented now (remembering that 2 blues will cancel 2 reds)?

Complete the number sentence: $^{+}3 + -2 =$ _____ .

2 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) $^{+}3 + -4$

(b) $^{+}2 + -5$

(c) $^{+}4 + -6$

3 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) $-4 + {}^{+}1$

(b) $-7 + {}^{+}3$

(c) $-9 + {}^{+}2$

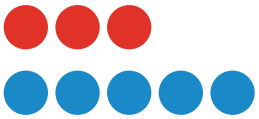
Explore

4 So far, we have used the counters to model addition. Modelling subtraction can be a little more challenging.

Strategy options

- Draw a diagram.
- Act it out.

(a) What number is shown by this collection of counters?



(b) If you removed the red counters, what number would be shown now?

Complete the number sentence:
$-2 - ({}^{+}3) =$ _____

5 Notice that in order to perform this subtraction, we had to have 3 red counters present, so that we could show the subtraction by removing them. They were balanced or 'annihilated' by 3 of the blue counters, so they had no effect on the starting number.

Use your counters to do the following subtractions. You may need to add pairs of red and blue counters in order to have enough to remove.

(a) $-4 - {}^{+}2$

(b) $-1 - {}^{+}3$

(c) $-2 - {}^{+}5$

6 Use your counters to do the following subtractions. Again, you may need to add pairs of counters at the beginning.

(a) $+3 - +4$

(b) $+2 - +5$

(c) $+4 - +6$

7 Looking at the counters diagram in question 4, what number would be shown if you removed all of the blue counters? Write this as a subtraction.

8 Use your counters to do the following subtractions.

(a) $-4 - -1$

(b) $-7 - -3$

(c) $-9 - -2$

9 Use your counters to do the following subtractions.

(a) $-2 - -4$

(b) $+1 - -5$

(c) $-3 - -7$

10 Use your counters to do the following subtractions.

(a) $+2 - -4$

(b) $+5 - -1$

(c) $+3 - -7$

Explain

11 (a) What do you notice about the answers to question 2 and question 6? What conclusion can you draw from this?

(b) What do you notice about the answers to question 3 and question 8? What conclusion can you draw from this?

(c) What do you notice about the answers to question 9 and question 10? Why is this the case?

12 Why was it necessary to use counters of both colours to represent your starting number in questions 5 and 6, and 9 and 10?

13 Helen wants to use her red and blue counters to show $+3 - -5$. She has lined up 3 red counters to show $+3$.

She has no blue counters to remove. Explain how Helen can include 5 blue counters in her representation of $+3$.

Elaborate

14 For each of the following, choose an integer between -10 and $+10$, and use counters of one or both colours to represent it. (For example, you might choose -3, which you could show with 3 blue counters, or 2 red and 5 blue counters.) Then, add or subtract a second number of counters to show the operation described. Write a number sentence for the operation you have modelled, and draw a diagram of counters.

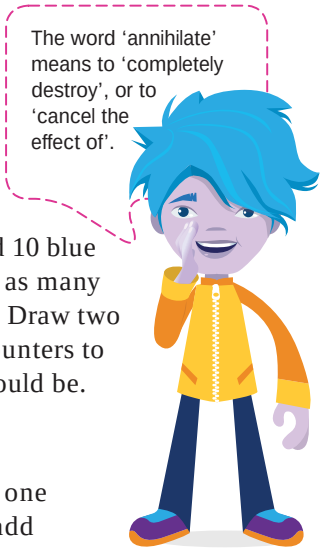
(a) adding a positive integer to a negative integer

(b) adding a negative integer to a positive integer

(c) subtracting a positive integer from a negative integer

(d) subtracting a negative integer from a positive integer

15 Explain why this method of using counters to show integer addition and subtraction is called the 'Annihilation method'.



16 Sam has made a number using some of his 10 red and 10 blue counters. He has used twice as many of one colour than the other. Draw two different arrangements of counters to show what Sam's number could be.

Evaluate

17 Using counters in this way is one method of learning how to add and subtract negative numbers. Another method is to use a number line, which is presented elsewhere in this chapter.

Do you find one method easier to work with than the other? Which one? Why do you think this is?

Extend

18 Can you use your counters to show multiplication?

(a) Start by showing 2×-4 and 3×-3. It may be useful to remember that multiplication means 'groups of'.

(b) Can you show -4×-2 or -3×-3? How is this method of using counters limited here?

2.7 Simplifying addition and subtraction

You have seen that we can simplify operations with directed numbers by omitting the '+' sign for positive numbers. For example, +7 can be written simply as 7. We can use this to write additions and subtractions more simply.

+4 + (+5) = +9 can be written as 4 + 5 = 9

+4 – (+5) = -1 can be written as 4 – 5 = -1

We can simplify other expressions by replacing the two signs between the numbers with a single sign.

We have seen that adding a negative number and subtracting a positive number are the same. Both result in a movement to the left on the number line; that is, subtraction.

7 + (-3) is the same as 7 – (+3). Both can be written as 7 – 3.

+ (-) and – (+) can be replaced by –.

We have also seen that adding a positive number and subtracting a negative number are the same. Both result in a movement to the right on the number line; that is, addition.

6 + (+2) is the same as 6 – (-2). Both are equal to 6 + 2.

+ (+) and – (-) can be replaced by +.

We can summarise these observations with the following rules for adding and subtracting integers:

If the two signs are the same, the result is addition.
+ (+) = +
– (-) = +

If the two signs are opposite, the result is subtraction.
– (+) = –
+ (-) = –

To apply these rules the two signs must be *next* to each other when the calculation is written down. The rules do not apply otherwise.

A physical method

If you find the above rules a little bit difficult to remember, here is another, physical method of adding and subtracting directed numbers:

1. Draw or imagine a number line. Place your pen, or imagine you are standing, on the first number.
2. Look at the operations symbol:

   If it is +, turn and face right (the positive end of the number line).

   If it is –, turn and face left (the negative end of the number line).
3. Look at the sign of the second number:

   If it is positive, walk forward that many paces.

   If it is negative, walk backwards that many paces.

This method can help you become familiar with knowing which combinations of symbols result in addition (movement to the right of the number line), and which result in subtraction (movement to the left of the number line).

Worked Example 15 WE15

Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

(a) $4 + (+9)$ (b) $-1 + (-3)$ (c) $10 - (+1)$ (d) $-6 - (-5)$

Thinking			Working
(a)	1	Adding a positive number is straightforward addition. Replace + (+) with +.	(a) $4 + (+9)$ $= 4 + 9$
	2	Calculate.	$= 13$
(b)	1	Adding a negative number is equivalent to subtraction. Replace + (-) with –.	(b) $-1 + (-3)$ $= -1 - 3$
	2	Calculate.	$= -4$
(c)	1	Subtracting a positive number is straightforward subtraction. Replace – (+) with –.	(c) $10 - (+1)$ $= 10 - 1$
	2	Calculate.	$= 9$
(d)	1	Subtracting a negative number is equivalent to addition. Replace – (-) with +.	(d) $-6 - (-5)$ $= -6 + 5$
	2	Calculate.	$= -1$

2.7 Simplifying addition and subtraction

Navigator

Answers page 621

Q1 Columns 1 & 2, Q2, Q3 Columns 1–3, Q4, Q5 Columns 1 & 2, Q6, Q7, Q10, Q13	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q5 Columns 2 & 3, Q6, Q8, Q9, Q10, Q12, Q13	Q1 Column 4, Q2, Q3 Column 4, Q4, Q5 Columns 2–4, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Fluency

WE15

1 Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

(a) $+5 + (+2)$ (b) $-1 + (+4)$ (c) $+5 + (+6)$ (d) $-9 + (+7)$

(e) $+5 - (+3)$ (f) $-8 - (+4)$ (g) $+1 - (+12)$ (h) $-3 - (+7)$

(i) $+12 + (-3)$ (j) $+10 + (-4)$ (k) $+7 + (-9)$ (l) $+4 + (-8)$

(m) $-7 - (-6)$ (n) $-3 - (-9)$ (o) $+2 - (-10)$ (p) $+1 - (-1)$

(q) $-7 + (-3)$ (r) $-9 + (-3)$ (s) $-4 - (-9)$ (t) $-2 - (-6)$

2 State TRUE or FALSE for the following.

(a) $25 + (+5) = 25 - 5$ (b) $30 + (-10) = 30 - 10$

(c) $15 - (+20) = 15 - 20$ (d) $40 - (-50) = 40 - 50$

3 Calculate:

(a) $6 - 18$ (b) $9 - 20$ (c) $-5 + 9$ (d) $-3 + 6$

(e) $-12 + 7$ (f) $-10 + 2$ (g) $-9 - 4$ (h) $-7 - 5$

(i) $-60 + 30$ (j) $-52 + 21$ (k) $-27 + 84$ (l) $-28 + 93$

4 (a) $-9 - (-6) - (+4)$ simplifies to:

A $-9 - 6 - 4$ B $-9 + 6 + 4$ C $-9 + 6 - 4$ D $9 + 6 + 4$

(b) $14 - (+4) + (-3)$ is the same as:

A $-14 - 4 - 3$ B $14 - 4 - 3$ C $14 - 4 + 3$ D $14 + 4 + 3$

5 Calculate:

(a) $-6 + 7 + 5$ (b) $-4 + 6 + 5$ (c) $-12 + 3 + 2$ (d) $-11 + 3 + 4$

(e) $-8 + 5 - 3$ (f) $-6 + 2 - 4$ (g) $-4 + 12 - 4$ (h) $-3 + 10 - 6$

(i) $-5 - 1 - 6$ (j) $-2 - 5 - 4$ (k) $36 - 40 - 5$ (l) $43 - 50 - 2$

Understanding

6 Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions for the month listed below.

Credit	$30
Debit	-$80
Credit	$200
Debit	-$60
Debit	-$500

What is your account balance at the end of the month?

7 A maintenance worker in an office block starts work on the ground floor, and then travels as follows.

UP 10 floors
DOWN 7 floors
DOWN 5 floors
UP 4 floors
DOWN 6 floors
UP 12 floors

(a) What was the highest floor the worker reached?

(b) What was the lowest floor he travelled to?

(c) Which floor did he finish on?

8 Diana makes and sells soft toys. She keeps records of her profits and losses over a ten-week period. These records are shown in the table. Find Diana's overall profit or loss during this time.

Week	Result	
1	Profit	$120
2	Profit	$25
3	Loss	-$70
4	Profit	$210
5	Loss	-$150
6	Loss	-$180
7	Loss	-$90
8	Profit	$40
9	Loss	-$160
10	Loss	-$95

Reasoning

9 Complete the following addition and subtraction grids. Begin with the number at the top of each column, and either add or subtract the numbers at the left of each row.

(a)

+	-6	+20		31
+4				
		+13		
	+2			
			-23	22

(b)

–	8	-11		
	5			
-7			26	
+5				-30
		-1		

10 A 'magic square' is a square grid of numbers where the sum of each of the rows, columns and diagonals is the same number.

	6	−1
	2	
5		

(a) What is the magic sum for this 3×3 magic square?

(b) Complete the magic square in part (a).

11 You can make new magic squares by adding or subtracting the same number from every number in an existing magic square.

(a) Make a new 3×3 magic square by subtracting 4 from each number in the magic square in question 10 (a).

(b) What is the new magic sum? Explain why the new magic sum is not 4 less than the old magic sum.

Open-ended

12 In the sport of golf, each 'hole' has a particular number of 'shots' it should take you to get the ball from the tee to the hole. This is known as the 'par' for the hole. If you take this number of shots to get the ball in the hole, your score for the hole is 0. If your number of shots for the hole is *less than par*, this is shown on the bottom row of the scorecard by a *negative number*. If your number of shots is *greater than par*, this is shown with a *positive number*.

(a) Carla played nine holes of golf. She scored under par on four holes, par on three holes and over par for the rest. If Carla took a total of 35 shots to complete her round, fill in what her scorecard may have looked like.

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken										35
Score compared to par										

(b) Carla played another round of nine holes. Her overall score was -1 ('1 under par'). What could her scorecard have looked like, if she did not score 'par' (0) on any hole?

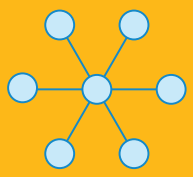
13 Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

Outside the Square Puzzle

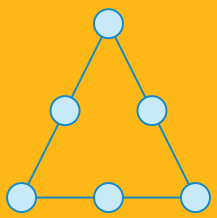
Pattern sums

Copy and complete each of the following patterns. Arrange the numbers given, one in each circle, so that the sum of each line is equal to the stated value.

1 Use $^{-}4, ^{-}3, ^{-}2, ^{-}1, 0, 1, 2$; sum = $^{-}3$



2 Use $^{-}2, ^{-}1, 0, 1, 2, 3$; sum = 0



Challenge 2

1 What are the next three numbers in each pattern?

(a) 17, 13, 9, 5, …

(b) -31, -25, -19, -13, …

2 The product of three brothers' ages is 72 and their sum is 14. The youngest of the brothers are twins. What are the ages of the brothers?

3 Find three consecutive numbers (numbers that come one after another, such as 5, 6, 7) so that the sum of the first and third numbers is 172.

4 There are 150 students in a school's Year 7. For a performance in the school hall, chairs are arranged in rows, with the same number of chairs in each row, so that all 150 students are seated with no spare seats. If 10 more chairs were added to each row, everyone could be seated in 4 fewer rows, allowing the people in the back row to be closer to the stage. How many chairs were in each row in the original seating arrangement?

5 What is the sum of all the digits in the numbers from 1 to 100?

6 Shannon needs to know the total amount of money her friends have raised for the 'Save the Koala' fund. All they will tell her is this:

Patricia and Tim have collected $130 between them.
If Tim and Jenni pooled their money, they would have $150.
Jenni and Robyn have $100 altogether.
Robyn and Hilary have just $70 in total.
If Hilary and Patricia combined their money, they would have $90.
Tell Shannon what she needs to know.

7 A number less than 100 gives a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6. Find the number.

8 41 is a prime number. If the order of its digits is reversed, it becomes 14 which is NOT a prime number. The number of two-digit prime numbers that DO give a prime number when their digits are reversed is:

A 8 B 9 C 10 D 11

9 (a) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 19$?

A 1 B 2 C 3 D 4

(b) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 29$?

A 3 B 4 C 5 D 6

(c) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 59$?

A 9 B 10 C 11 D 12

(d) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 99$?

A 18 B 19 C 20 D 21

Chapter review 2

D.I.Y. Summary

Key Words

common factor	factor	positive
common multiple	Highest Common Factor (HCF)	prime factor
composite number	integers	prime number
coprime	loss	profit
deposit	Lowest Common Multiple (LCM)	withdrawal
divisibility	multiple	
divisible	negative	

1 A _____ of 6 is 18. The _____ of 6 and 4 is 12.

2 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the _____s of 18. The _____ of 18 and 27 is 9.

4 A number that is not _____ by any numbers other than 1 and itself is called a _____.

5 A number with more than two factors is called a _____.

6 When you put money into a bank account, you are making a _____.

7 The addition of two negative numbers will always give a _____ answer.

8 If you sell something for less money than you bought it for, you have made a _____.

9 Every whole number greater than 1 can be written as the unique product of its _____s.

10 You make a _____ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a _____.

12 Two numbers are _____ if their highest common factor is 1.

Fluency

1 Find the LCM of: Ex 2.1

(a) 9 and 6 (b) 9 and 12 (c) 10 and 15

2 List all the factors of: Ex 2.1

(a) 36 (b) 48 (c) 51 (d) 100

3 Find the HCF of: Ex 2.1

(a) 24 and 56 (b) 18 and 72 (c) 45 and 80

4 Copy the following table and do the divisibility tests. Circle the number if the original number is divisible by it. Ex 2.1

5301	2	3	4	5	6	9	10
10 000	2	3	4	5	6	9	10
333 333	2	3	4	5	6	9	10
31 700	2	3	4	5	6	9	10
43 521 820	2	3	4	5	6	9	10

5 State whether each of the following numbers is a prime number or a composite number, and explain why. Ex 2.2

(a) 5 (b) 16 (c) 77 (d) 276 350

6 By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors. Ex 2.3

(a) 24 (b) 30 (c) 88 (d) 200

7 Use prime factors to find the HCF of: Ex 2.3

(a) 27 and 36 (b) 72 and 96 (c) 108 and 240

8 Write a directed number suggested by the following. Ex 2.4

(a) 14 degrees below zero (b) an altitude of 200 metres

9 State the opposite of: Ex 2.4

(a) north 5 km (b) adding 27

10 Write < or > between the following pairs of numbers to make a true statement. Ex 2.4

(a) -52 _____ 25 (b) 19 _____ -20

11 Arrange the following numbers in ascending order. Ex 2.4

(a) -7, 12, 0, -9, 7 (b) 4, -4000, 40, 400

12 Calculate: Ex 2.5

(a) +16 + 2 (b) -3 + 18 (c) -15 + 5 (d) +9 – 3

(e) +1 – 5 (f) +16 – 8 (g) +7 – 12 (h) -14 – 18

13 Calculate: Ex 2.6

(a) +7 + (-10) (b) +9 + (-6) (c) -11 – (-4) (d) -4 – (-4)

(e) -12 – (-5) (f) +5 + (-3) (g) -8 – (-5) (h) -5 + (-7)

14 Rewrite the following with a single sign between the integers, then evaluate. Ex 2.7

(a) 9 – (+11) (b) -3 + (+10) (c) -10 – (-21) (d) 8 + (-12)

(e) -4 – (-41) (f) -14 + (+28) (g) -5 – (+8) – (-2) (h) 4 + (-9) + (+2)

Understanding

15 Use the words 'multiple', factor' and 'divisible' to complete the following sentences.

(a) 45 is _____ by 9, so that makes it a _____ of 9.

(b) 8 is a _____ of 56, so 56 is a _____ of 8.

(c) 27 is _____ by 3, so that makes it a _____ of 3.

16 If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations.

17 Use the symmetry of the number line to help you calculate the following.

(a) -31 + 19 (b) -54 – 27 (c) -22 + (-38) (d) -9 – (61)

18 Describe the number line journey you could follow to find the value of:

(a) +3 + (-8) (b) -6 – (+5)

19 For each pair of numbers, state whether or not they are coprime. If not, explain why.

(a) 11 and 27 (b) 51 and 63 (c) 14 and 35 (d) 24 and 55

20 In the game of indoor cricket, 3 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game.

Ball 1: 2 Runs
Ball 2: Wicket
Ball 3: 1 Run
Ball 4: Wicket
Ball 5: 1 Run
Ball 6: 4 Runs

What was the score at the end of the over?

21 There are 84 male and 108 female guests at an official dinner. The dinner organiser wants to have an equal number of male and female guests at each table.

(a) Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

(b) Use your answer from (a) to find the number of male and female guests at each table.

22 Find the first common multiple of 2, 7 and 9 that is greater than 500.

23 (a) What is the first prime number after 70?

(b) What is the first composite number after 70?

24 Michelle made deposits of $210, $25, $45 and $66 into her bank account during one month, and withdrawals of $35, $56, $214 and $102 during the same period.

(a) At the end of the period, had her balance increased or decreased?

(b) By how much had it increased or decreased?

25 Joanna and Petra are on two different ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level?.

26 Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers.

Reasoning

27 Copy the following and write <, = or > to make true statements.

(a) $4 - (-7) \_\_\_\_ 4 + 7$

(b) $-3 + 2 \_\_\_\_ 3 - 2$

(c) $5 + (-3) \_\_\_\_ 5 - (+3)$

(d) $-8 - 9 \_\_\_\_ -8 + 9$

28 If you know that a number is divisible by 8, what other numbers do you also know it is divisible by?

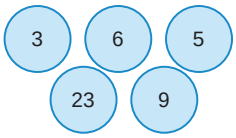
29 A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor?

NAPLAN practice 2

Numeracy: Non-calculator

1 Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

A -15 B -7 C 7 D 15

2 

The sum of the composite numbers shown above is:

A 8 B 15 C 18 D 31

3 A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does he get out on?

4 The number 42 written as a product of prime factors is:

A 21×2 B $2 \times 3 \times 7$ C 1×42 D 6×7

5 On a sunny winter's day in Moscow, the temperature at midday was 3ºC. By midnight it had dropped to -9ºC. The integer that represents this change is:

A -12 B -3 C 3 D 12

Numeracy: Calculator allowed

6 A train passes a town every week and every 10 days an aeroplane flies overhead. If the train and the plane were observed to be at the town on a certain day, after how many more days will both appear again at the same time?

A 10 B 11 C 50 D 70

7 Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit: $55
Withdrawal: -$75
Withdrawal: -$33
Deposit: $85
Withdrawal: -$27

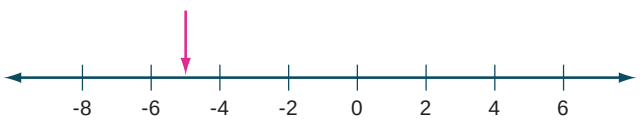
How much does Vin have in his account at the end of the month?

8 The ages in years of three people are 65, 39 and 52. The highest common factor of the three ages is:

A 1 B 13 C 39 D 165

9 The arrow is pointing to an integer on the number line.

What number is at this position? ____



Mixed review A

Fluency

1 Write these integers in ascending order. Ex 2.4

8, 17, -10, 0, -25, 32, -48

2 Write the following in index form. Ex 1.2

(a) 9 squared (b) $7 \times 7 \times 7 \times 7$ (c) 4 cubed

3 List all numbers divisible by both 8 and 6 that are less than 100. Ex 2.1

4 Write an integer to represent the following. Ex 2.4

(a) a bank withdrawal of $570 (b) a win by 5 points

5 Write < or > between the following pairs of numbers to make a true statement. Ex 2.4

(a) -27 ____ 14 (b) 0 ____ -35

6 Calculate: Ex 1.3

(a) $8000 \div 200$ (b) $1200 \div 4$ (c) $45\,000 \div 90$

(d) 30×120 (e) 400×1500 (f) 2000×5000

7 Use a mental strategy to calculate the following. Ex 1.1

(a) $4 \times 17 \times 5$ (b) $183 + 220$ (c) 42×19

(d) 36×11 (e) $169 + 71$ (f) $5 \times 24 \times 8$

8 Evaluate: Ex 1.5

(a) $6 \times 4 \div 2 \times 6$ (b) $5 + 6 \times 7$ (c) $18 + 12 - 7 + 6$

(d) $2 + 5 \times 9$ (e) $18 \div 6 - 3$ (f) $8 \times (15 - 5)$

9 Calculate the following. Ex 2.5, 2.6

(a) $-9 + 7$ (b) $5 + (-8)$ (c) $-3 - (-7)$ (d) $-6 - 11$

10 Estimate the answers to the following by rounding to the first digit. Ex 1.4

(a) 17×93 (b) 46×281 (c) 337×240

(d) $953 \div 11$ (e) $8195 \div 237$ (f) $12\,495 \div 5400$

11 Arrange the following numbers in ascending order (from smallest to largest). Ex 2.4

(a) 5, 0, -15, 10, -5 (b) -300, 3, 0, -30, 3000

12 Find the lowest common multiple of: Ex 2.1

(a) 8 and 12 (b) 12 and 16

13 Find the highest common factor of: Ex 2.1

(a) 36 and 27 (b) 64 and 72

14 List the factors of each of these numbers and state whether each number is prime or composite. Ex 2.2

(a) 18 (b) 23 (c) 44 (d) 79

15 Write each number as the product of its prime factors in index form. Ex 2.3

(a) 63 (b) 48 (c) 72 (d) 120

Ex 2.7

16 Simplify the following by writing a single sign between the values, then calculate the answer.

(a) $-6 + (-7)$ (b) $4 - (-11)$ (c) $-2 + (^{+}7)$

(d) $22 + (-9)$ (e) $-5 - (-10)$ (f) $18 + (^{+}3)$

Understanding

17 A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now?

18 Which of the following numbers are:

(a) prime

(b) perfect squares

(c) powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

19 What is the first perfect cube that is divisible by both 3 and 4?

20 Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower.

(a) 256×37 (b) 1379×24 (c) $5498 \div 46$

21 An Easter show runs for 7 days. The total attendance at this year's show was 62 982.

(a) Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

(b) If each person paid an average ticket price of $12, use your answer from (a) to calculate how much money the show organisers made from ticket sales? Use some mental or written strategies to calculate your answer.

Reasoning

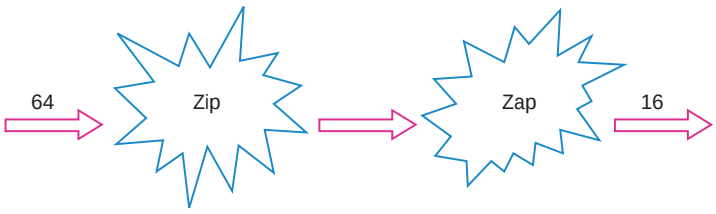
22 On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account.

(a) By the end of the week, did Kiran have more or less money in his bank account?

(b) How much more or less?

23 The number 64 passes through two 'magic clouds ' Zip and Zap and emerges as the number 16.

Which of the following can describe what Zip and Zap did to the number passing through?



A Zip: square root, Zap: square

B Zip: cube root, Zap: square

C Zip: square root, Zap: nothing

D Zip: nothing, Zap: cube root