

Mathspace

Fwoooosh!

I CAN'T CONTROL THE SHIP!
WE'LL CRASH INTO THE VOLCANO!!
LET'S GET OUT OF HERE!

Zoom

CRASH!

PLANETPOLYGON

While exploring the universe, you and your co-pilot Violet enter the atmosphere of the strange planet Polygon. Your ship malfunctions, and to avoid certain death you both eject out of the ship. On landing you are knocked unconscious. When you wake, you look around—where is Violet and why is your spacecraft crumpled?

You find a note on your PCD (Personal Communication Device).

Message #1: 2.15 p.m. Alien code

I couldn't find you when I woke. I've met a group of aliens who seem to call themselves Polygons. They don't have a verbal language but communicate instead with shapes. Through our conversation I think I have managed to decode some of their language.

I asked them what happened to our ship, they answered, and I have described the shapes. Here is approximately what they said.

- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only
- quadrilateral with 1 pair of parallel sides only
- 3-sided shape with 2 equal angles
- special rectangle with equal side lengths
- 3-sided shape with 3 different side lengths
- parallelogram that has all equal sides, but is not a square
- 2 pairs of parallel sides, with 90 degree angles (but is not a square)
- hexagonal-looking shape
- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only

1 Decode what the aliens said happened to the ship.

Shape	Possible meaning
square	catapulted
kite	flying thing
isosceles triangle	volcano
rhombus	large moogal
rectangle	paw
trapezium	into
polygon	squashed
scalene triangle	hit



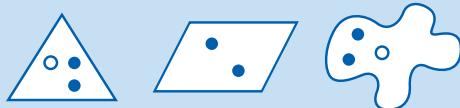
Message #2: 3.15 p.m. Alien intelligence test

The Polygons are willing to give us the necessary materials to fix our spaceship, but we both have to pass their test. Good luck, from Violet.

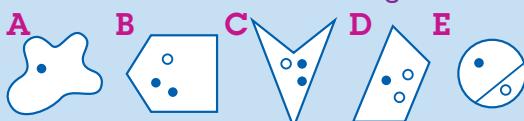
2 (a) These are Kwoogals:



These are not Kwoogals:

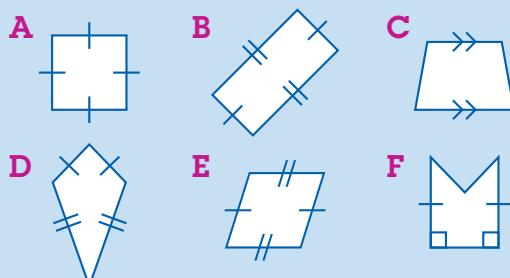


(i) Which one of these is a Kwoagal?



(ii) Write the definition of a Kwoagal.

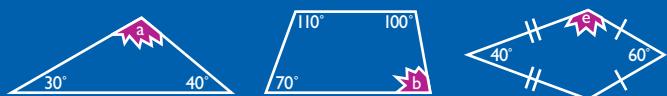
(b) Which of these are parallelograms?
(More than one answer is okay.)



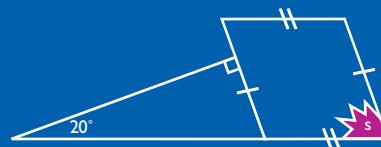
Message #3: 4.18 p.m. Spaceship blueprints

I've found the blueprints for our spaceship but some of the angles have been burnt in the crash. Can you calculate what the missing angles should be?

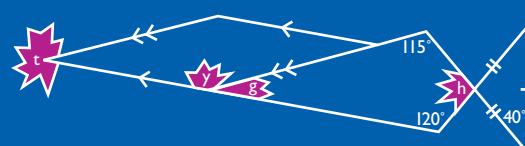
Fuel cells



Neutron absorber



Black hole combustion unit



What is a moogal?

Use your answers for the spaceship blueprint message to solve the code below.

110° 70° 100° 110° 25° 25° 155°

80° 130° 110° 70° 335°

8.8

Compass constructions

Using a ruler, a pencil and a compass, we can construct many angles and shapes without measuring any angles with a protractor. Constructions may also be made using a CAS calculator or computer geometry software.

Most constructions are based on two basic techniques: bisecting a line and bisecting an angle. (Bisect means to divide into halves.) A **perpendicular bisector** bisects a line at right angles. We use a sharp pencil for drawing lines and for use in our compass, and we leave all construction lines on our diagrams.

Unless otherwise instructed, lines and compass openings should be between 5 and 8 cm long and angles between 40° and 50° .

Worked Example 9

WE 9

Using only a compass, a ruler and a pencil:

(a) bisect a line

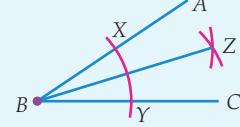
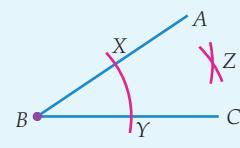
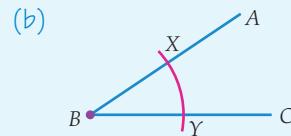
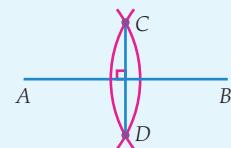
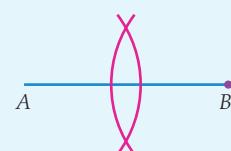
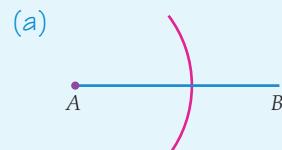
(b) bisect an angle.

Thinking

- (a) 1 Use your ruler to draw a line \overline{AB} . Place the point of the compass on point A so that the compass opening is more than half the length of the line and draw an arc above and below the line.
- 2 Keeping the compass at the same opening, place your compass point on point B and draw an arc to cut the first arc above and below the line.
- 3 With your ruler join the points CD where the arcs cross. As this line crosses AB at right angles, it is the perpendicular bisector of the line.

- (b) 1 Draw an acute angle. Place the point of the compass on the vertex of $\angle ABC$ and draw an arc to cut both arms of your angle at X and Y .
- 2 Keeping the compass at the same opening, place your compass point on point X and draw an arc. Repeat this using point Y so that the arcs cross.
- 3 Using a ruler, draw a line connecting the vertex of $\angle ABC$ with Z , the point of intersection of the arcs. This line BZ bisects the angle (cuts the angle in half).

Working



An equilateral triangle not only has three equal sides but also has three equal angles of 60° . A square has four equal sides and all angles are right angles, so we can use our previous constructions to help us construct a square.

Worked Example 10

WE 10

Using only a compass, a ruler and a pencil:

- (a) construct an equilateral triangle (b) construct a square.

Thinking

- (a) 1 Use your ruler to draw a line \overline{AB} . Open your compass to the length of line AB . Place the point of the compass on point A and draw an arc above the line. Place the point of the compass on point B and draw another arc above the line so that the arcs cross at point C .
- 2 Using a ruler, join points A and B to point C . $\triangle ABC$ is an equilateral triangle. Mark all sides as equal and all angles as 60° .

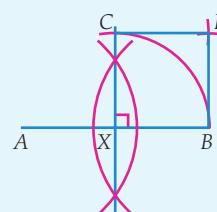
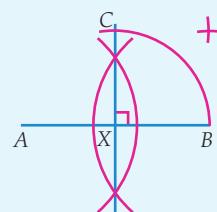
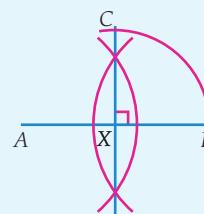
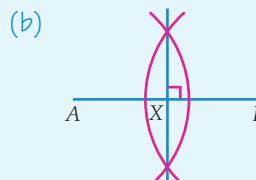
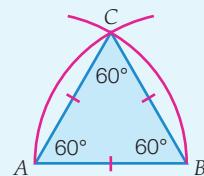
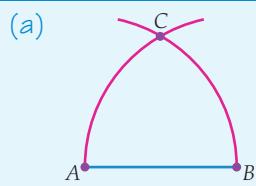
- (b) 1 Construct a right angle on a horizontal line AB by constructing the perpendicular bisector of the line.

- 2 Place your compass point on point X , the point of intersection of AB and the perpendicular bisector. With the compass opened to length BX , cut the perpendicular bisector at point C .

- 3 Keeping the compass opening the same, place the compass point on point B and make an arc above the line. Place the compass point on point C and make an arc to cut the last arc at D .

- 4 Join B to D and the point C to point D . $BXCD$ is a square.

Working



Worked Example 11

WE 11

Using only a compass, a ruler and a pencil:

- (a) construct an angle equal to another angle (b) construct a pair of parallel lines.

Thinking

- (a) 1 Draw an acute $\angle ABC$.

- 2 Use a compass and, with the compass point on point B , draw an arc through point C .

- 3 Draw a line segment and label one end point Y . Without changing the opening on your compass and, with the compass point on point Y , draw an arc to cut the line segment at Z .

- 4 Place the compass point on point C and alter the compass opening so that it measures the length \overline{CA} . Place the compass point on point Z and draw an arc above \overline{YZ} .

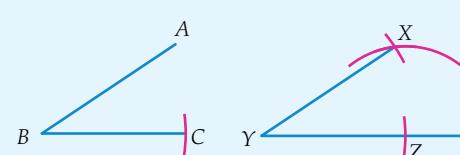
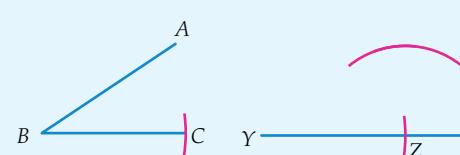
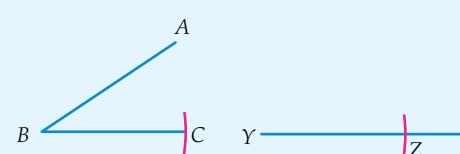
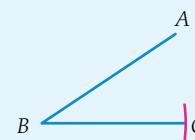
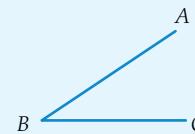
- 5 Place the compass point on point B and alter the compass opening so that it measures the length \overline{BA} . Place the compass point on point Y and draw an arc to cut the previous arc at X . Join \overline{XY} .

- (b) 1 Draw a line segment \overline{AB} . Draw another line \overline{EH} to intersect with \overline{AB} . Label the point of intersection G . Choose a point F between E and G . Using the instructions in part (a), construct acute $\angle YFG$ equal to acute $\angle BGH$.

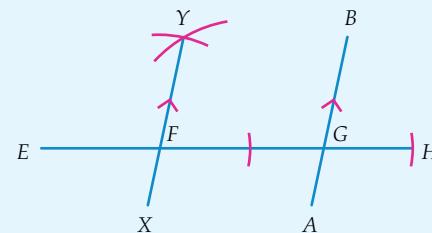
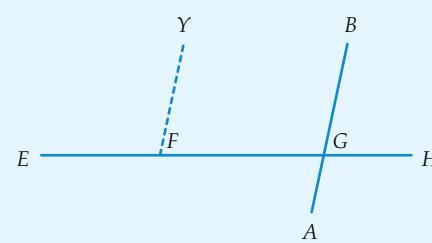
- 2 Draw a line segment through \overline{YF} and extend to a point X .
 \overline{XY} will be parallel to \overline{AB} , as you have constructed equal corresponding angles.

Working

(a)



(b)



8.8 Compass constructions

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q8,
Q10, Q13

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9,
Q10, Q11, Q13

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8,
Q9, Q10, Q11, Q12, Q13

**Answers
page 673**

Equipment required: Compass, ruler, pencil

Fluency

Understanding

- 4 Using only a compass, a ruler and a pencil:

 - (a) construct a 90° angle
 - (b) bisect this angle to make a 45° angle.

5 Using only a compass, a ruler and a pencil:

 - (a) construct a 60° angle
 - (b) bisect the 60° angle constructed in part (a) to make a 30° angle
 - (c) bisect a line 10 cm long.

6 Using only a compass, a ruler and a pencil construct a parallelogram.

Reasoning

- 7 Construct a triangle with one angle equal to 30° and another angle equal to 45° and the arm between these two angles equal to 8 cm.
 - 8 (a) Construct an angle of 75° .
(b) Construct an angle of 15° .
(c) Construct an angle of 135° .
 - 9 Construct an isosceles right-angled triangle where the two equal sides are 5 cm long.
 - 10 Construct an equilateral triangle on each side of a square.
 - 11 Use a compass to draw a circle. Select a point on the circle. Using the same compass opening, divide the circle perimeter into 6 equal pieces. Join the points of intersection in order to make a hexagon. Explain why this construction works.
 - 12 Use a compass to draw a circle. Construct an octagon by dividing the perimeter of the circle into 8 equal pieces. Explain how your knowledge of constructing a 90° angle and bisecting an angle helped you.

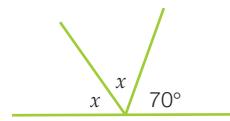
Open-ended

- 13** Choose any two polygon shapes and use your construction knowledge to make a design.

Challenge 8

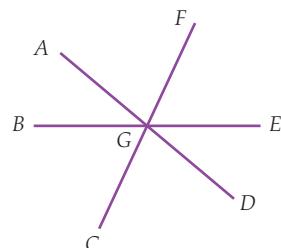


- 1 The smaller angle between the hands of a clock at half past two is:
A 90° **B** 105° **C** 120° **D** 135°
- 2 Two angles are complementary and one angle is 46° more than the other. What is the size of the larger angle?
- 3 Find the value of the pronumerals in the following.



- 4 In the diagram, $\angle BGF = 115^\circ$ and $\angle AGE = 140^\circ$.

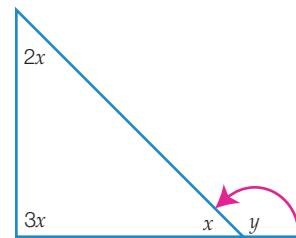
What is the size of $\angle CGD$?



- 5 Two angles are supplementary and one angle is 54° more than the other. What is the size of the smaller angle?

- 6 In the diagram, the value of y is:

- A** 30°
- B** 60°
- C** 90°
- D** 150°



- 7 In the diagram, the value of x is:

- A** 110°
- B** 70°
- C** 40°
- D** 30°

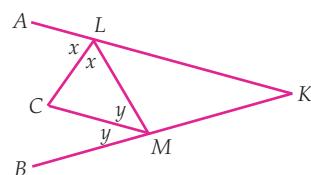


- 8 (a) Shortly after 12 o'clock, the hour and minute hands of a clock form an angle of 55° . What is the time now?

- (b) Noor observes this and leaves, returning after 40 minutes. What is the acute angle between the hands now?

- 9 In the diagram, ALK and BMK are straight lines. The bisector of $\angle ALM$ meets the bisector of $\angle BML$ at C . If $\angle LCM$ is 70° , then $\angle LKM$, in degrees, is:

- A** 10°
- B** 20°
- C** 40°
- D** 110°



Chapter review

8

D.I.Y. Summary

Key Words

acute angle	equilateral triangle	plane	right angle
acute-angled triangle	intersect	plane shapes	right-angled triangle
alternate angles	irregular	polygon	scalene triangle
angle	isosceles triangle	protractor	square
arms	kite	quadrilateral	straight angle
co-interior (allied) angles	line segment	ray	supplementary angles
complementary angles	obtuse angle	rectangle	transversal
concave	obtuse-angled triangle	reflex angle	trapezium
convex	parallel lines	regular polygon	vertex
corresponding angles	perpendicular bisector	revolution	vertically opposite angles
degree	perpendicular lines	rhombus	

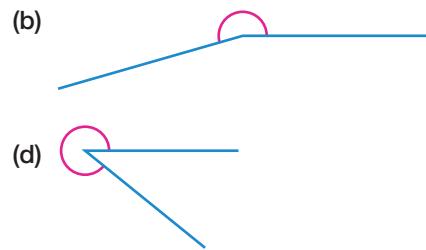
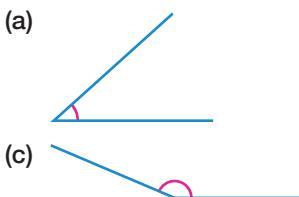
Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

- 1 To measure an angle, the centre point of the _____ must match with the _____ of the angle. The lines must match with the _____ of the angle.
- 2 40° and 50° are _____.
- 3 _____ add to 180° s.
- 4 Lines drawn at right angles to each other are called _____.
- 5 The angle names listed in order from largest to smallest are _____, _____, _____, _____, _____, and _____.
- 6 The most general name for a four-sided figure is a _____.
- 7 Lines in the same plane that never meet are called _____.
- 8 An _____ has two sides equal in length and one unequal side.
- 9 A closed shape with only straight line sides is called a _____.
- 10 A square is a _____ with right angles.
- 11 A _____ has three sides all of different lengths.
- 12 A line crossing other lines is called a _____.
- 13 A _____ is a quadrilateral with one pair of parallel sides.
- 14 Angles that are between parallel lines and on the same side of the transversal are _____ and are _____.

Equipment required: Protractor for Questions 1, 3 and 14, and a compass for Question 13

Fluency

- 1 Use a protractor to measure each of the following angles.



Ex. 8.1

- 2 Match the following angle sizes with the drawn angles. Do not use a protractor.

A 20°

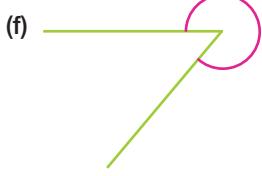
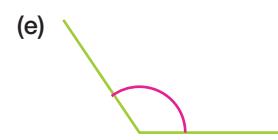
B 55°

C 120°

D 210°

E 270°

F 310°



Ex. 8.1

- 3 Use a protractor and a ruler to draw an angle of each size stated.

(a) 56°

(b) 162°

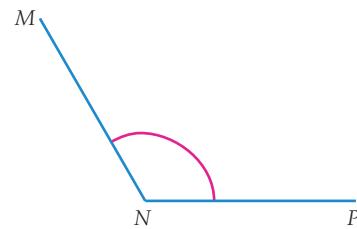
(c) 257°

(d) 304°

Ex. 8.1

- 4 Which of the following is not true of the angle shown?

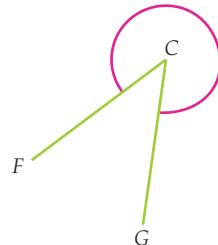
- A It is named $\angle NPM$.
- B It is named $\angle MNP$.
- C It is named $\angle PNM$.
- D It is an obtuse angle.



Ex. 8.2

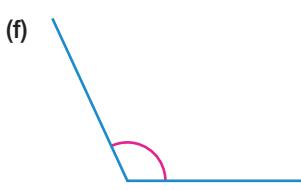
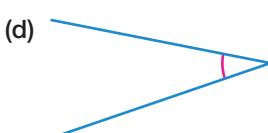
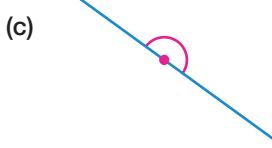
- 5 Which of the following is true of the angle shown?

- A It is obtuse.
- B It is reflex.
- C It is less than 180° .
- D It is called $\angle CFG$.



Ex. 8.2

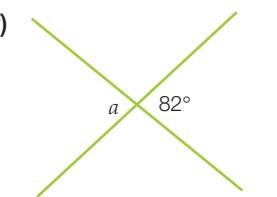
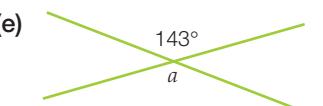
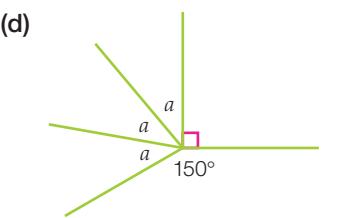
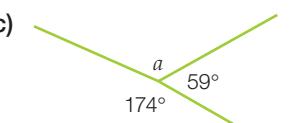
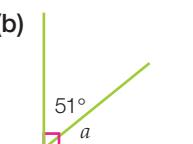
- 6 State the type of each angle below (i.e. acute etc.).



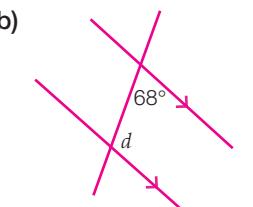
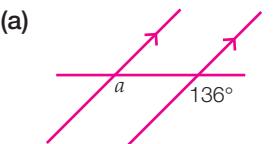
Ex. 8.2

Ex. 8.3

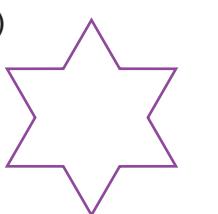
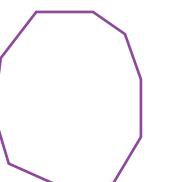
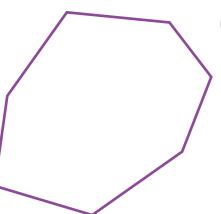
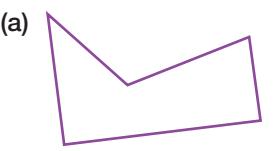
7 Find the value of a in each of the following diagrams.



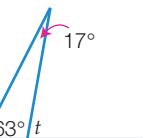
8 Find the value of the pronumeral in the following diagrams.

Ex. 8.4

9 Name the following polygons, also stating whether they are regular or irregular, convex or concave.

Ex. 8.5

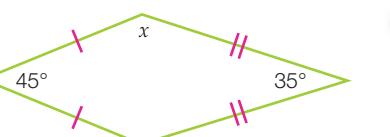
10 Find the value of the pronumeral in the diagram.

**Ex. 8.6**

11 Find the unknown angle in each of the following quadrilaterals. Make sure you support your answer with reason(s).

Ex. 8.7

12 Find the value of x in the following quadrilateral. Make sure you support your answer with reason(s).

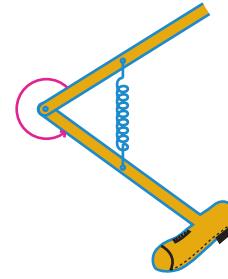
**Ex. 8.7**

13 Use a compass to construct a square of side length 4 cm.

Ex. 8.8

Understanding

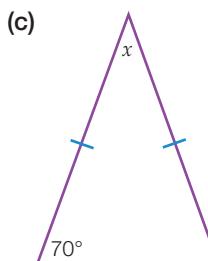
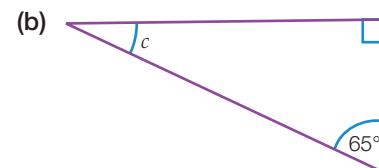
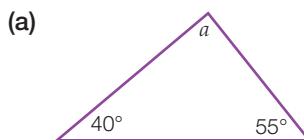
- 14 Eloise has constructed a mechanical leg to kick a football as part of her physical education investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



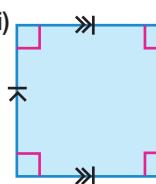
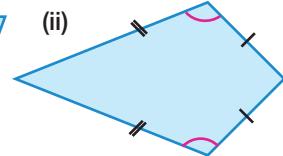
- 15 One angle in a right-angled triangle is 45° .

- (a) What is the value of the other two angles?
(b) The triangle described in (a) was named a right-angled triangle. Give another name for this triangle.

- 16 Find the missing angle in each of the following triangles. Make sure you support your answer with reason(s).



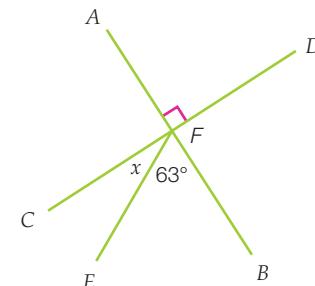
- 17 Using the shapes below, answer TRUE or FALSE to the following statements.



- (a) All four shapes are quadrilaterals.
(b) (i) and (ii) are kites.
(c) (i), (iii) and (iv) could be classified as parallelograms.
(d) (i) is a rhombus.
(e) (iii) is a rectangle.

Reasoning

- 18 Find the value of the angle x , giving reasons for each step.



- 19 What type of angle would result from adding an acute angle and a right angle?

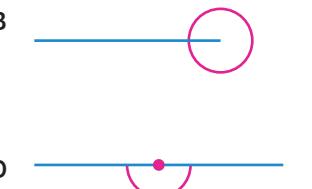
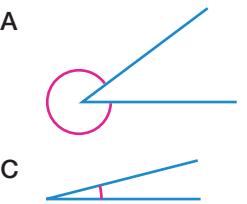
- 20 What type of angle would result from adding a straight angle and an obtuse angle?

- 21 What types of angle could result from subtracting a reflex angle from a revolution?

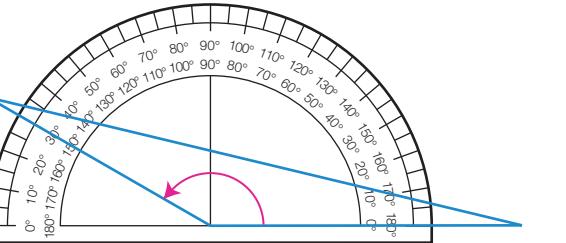
NAPLAN practice 8

Numeracy: Non-calculator

1 Which is the largest angle?

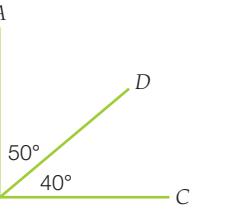


2 What is the size of the angle in the triangle marked by the arrow?



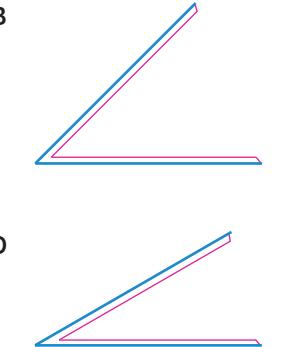
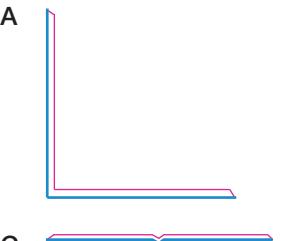
3 $\angle ABD$ and $\angle DBC$ are best described as:

- A vertically opposite angles
- B supplementary angles
- C angles at a point
- D complementary angles.

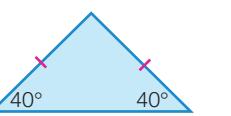
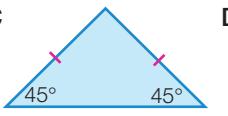
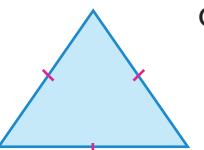
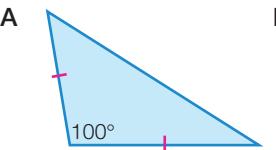


Numeracy: Calculator allowed

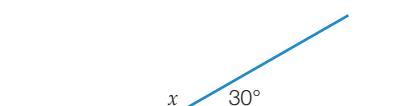
4 Which diagram shows a book opened to about 30° ?



5 Which one of these is a right-angled isosceles triangle?



6 What is the value of x in this diagram?



A 30°

B 60°

C 120°

D 150°

Mixed review D

Equipment required: Protractor for Question 15(c)

Fluency

- 1 Simplify:

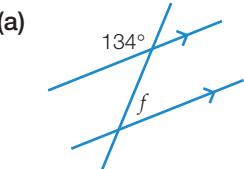
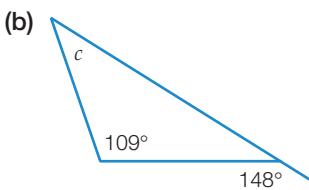
(a) $2a + 3b + 4a - 7b$
(b) $6a + 3b - 2c - b + 5c$
Ex. 5.6
- 2 Write out factor trees for each of the following numbers, then express each number as the product of its prime factors in index form.

(a) 45
(b) 24
(c) 630
Ex. 2.3
- 3 Write the following percentages as fractions in simplest form.

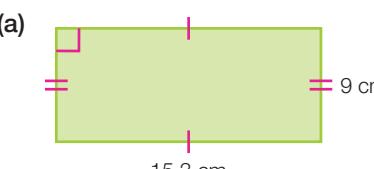
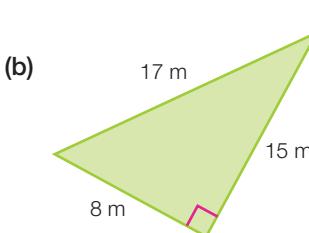
(a) 75%
(b) 50%
(c) 30%
(d) 4%
Ex. 4.7
- 4 Evaluate without using a calculator:

(a) 1.1×0.23
(b) $0.24 + 3.7 - 1.503$
Ex. 4.4, 4.5
- 5 Solve the following equations using algebra.

(a) $3x - 4 = -7$
(b) $\frac{x+5}{3} = -12$
Ex. 7.4
- 6 Find the value of the pronumeral in the following diagrams.

(a) 
(b) 
Ex. 8.4, 8.6
- 7 Evaluate without a calculator:

(a) $3\frac{1}{5} + 2\frac{1}{2} - 1\frac{3}{4}$
(b) $2\frac{1}{4} \times 1\frac{1}{3} \div 3\frac{1}{5}$
Ex. 3.4–3.6
- 8 Find the area of the following shapes.

(a) 
(b) 
Ex. 6.4, 6.5

Understanding

- 9 The table on the right shows the minimum and maximum temperatures at Frosty Hollow for 1 week in winter.
 - (a) On which day was the lowest maximum recorded?
 - (b) On which day was the highest minimum recorded?
 - (c) Which day had the greatest difference between minimum and maximum?

Day	Minimum (°C)	Maximum (°C)
Monday	-4	5
Tuesday	-8	0
Wednesday	-2	1
Thursday	-5	-1
Friday	-6	6
Saturday	-4	4
Sunday	-9	2

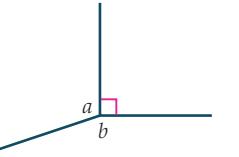
- 10 Which of the following is the correct answer to $(4 + 3) \times 10 - 4 \div 2$?

A 68 B 35 C 33 D 21

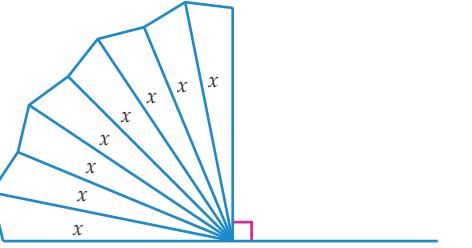
Reasoning

- 11 The angles in this diagram can be called:

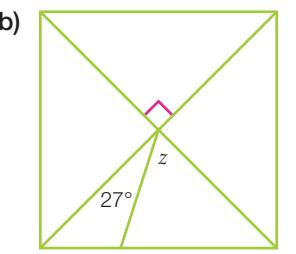
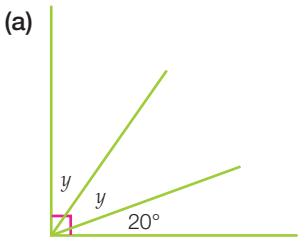
A vertically opposite B complementary
C supplementary D angles at a point.



- 12 A fan, when opened out, looks like this. What is the size of the angle x between the folds of the fan?



- 13 Find the value of the pronumeral.

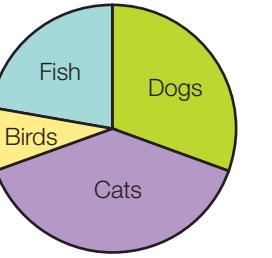


- 14 What two angles are formed by the two hands on a clock face at 5.00 p.m.?

- 15 John has drawn this pie chart, showing the popularity of pets.

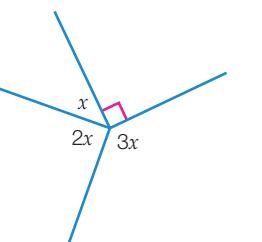
- (a) Estimate the angle at the centre of each wedge.
(b) Without using a protractor, how could you check to see how reasonable the estimates you made in (a) are?
(c) Using a protractor, measure the size of each wedge.

Popularity of pets



- 16 You enter a lift on the fourth floor. Once inside, you are trapped and the lift takes you on an adventure! First, you go up five floors, then fall six floors, then fall another two floors before finally going up one floor and coming to rest. The doors open, much to your relief. What floor are you on?

- 17 Find the size of each angle in the diagram by first finding the value of x .



- 18 (a) Draw and label an acute angle $\angle ABC$.

- (b) There is another angle in your diagram, apart from the acute angle. Classify it.
(c) Name the angle in (b).

- 19 Name as many pairs of complementary angles, supplementary angles, vertically opposite angles and angles in a revolution as you can see in the figure.

