

Investigation



The great escape

Equipment required: 1 brain

Farmer Sue has a paddock to which she brings her cows to wait for milking and a large yard next to it for her free-range chickens. Every now and then, some of her chickens escape into the cow paddock.

One day, the gate connecting the chicken yard and the cow paddock was left wide open and all of Farmer Sue's chickens escaped into the cow paddock where all of her cows were waiting to be milked. Farmer Sue raced over and counted 40 heads and 112 legs.

The Big Question

How many chickens and cows did Farmer Sue own?

Engage

- 1 Last week, Farmer Sue found 4 chickens and 8 cows in the paddock and had to race around to catch the chickens and return them to their own yard.
- (a) How many heads would she have counted?
 - (b) How many legs would she have counted?
 - (c) If there had been 8 chickens and 4 cows:
 - (i) how many heads would she have counted?
 - (ii) how many legs would she have counted?

Explore

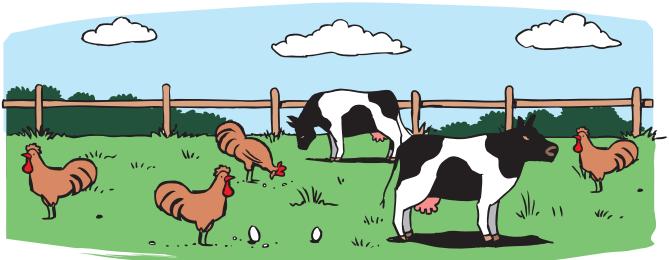
- 2 If n represents the number of cows in the paddock and c represents the number of chickens that have escaped into the paddock:
- (a) write an equation to show 12 heads altogether
 - (b) what are all the possible combinations of cows and chickens that would give a total of 12 heads? Present your answers in a list or table.
 - (c) Write an equation to show 40 legs altogether.
 - (d) Substitute $n = 8$ and $c = 4$ into your equation to make a true statement.



Strategy options

- Guess and check.
- Make a table.
- Make a model.
- Look for a pattern.

- 3 (a) If Farmer Sue counted 40 heads, choose two possible values for the number of cows and chickens she might own.
- (b) For each choice, using an appropriate equation, calculate the number of legs she would have counted.
- (c) Can you now find the values for n and c that would give 40 heads and 112 legs?



Explain

- 4 If the number of heads stays the same but the number of legs changes, explain how the numbers of each animal in the paddock change.

Elaborate

- 5 (a) State your answer to the Big Question. Explain how it was obtained.
- (b) What would be the solution for n and c if the number of heads was still 12 but the number of legs was now only 32? Show how you worked this out.

Evaluate

- 6 (a) What methods did you use to solve this problem?
- (b) Did you use a particular problem-solving strategy or method?
- (c) Why did you choose that particular method?
- (d) Do you think this was the best method?
- (e) Can you think of any other ways to solve this problem?

Extend

- 7 Deep in the ocean, a scuba diver discovers a collection of 21 crabs and octopuses together in the same small area. He counts 198 legs altogether. (A crab has 10 legs and an octopus has 8.) How many crabs and octopuses were there in this Octopus' Garden?

The case of the confusing clues

Cara Loft is on a secret mission. The golden calculator has been stolen by the mysterious Dr Equation. Where has he hidden it? All Cara has is a series of clues spread throughout the rather creepy House of Calculus. You need to help Cara by finding your way through the maze. As you move through the maze, you will enter some tricky 'equation chambers'.



When you enter the chamber:

- make sure that you enter through a coloured number or symbol and write it down (they form an equation you will need to solve at the end).
- To exit the equation chamber, solve the equation and move out through the exit showing the correct solution.

When you finally exit the maze, you need to complete the practice session to help you get ready for the even harder next section. Good luck!

START

Practice session 1

Solve these equations:

1 $3x + 1 = 10$

6 $\frac{2x}{3} + 1 = 5$

2 $4x - 4 = 12$

7 $\frac{5x}{2} + 1 = 11$

3 $8x - 1\frac{1}{2} = 6\frac{1}{2}$

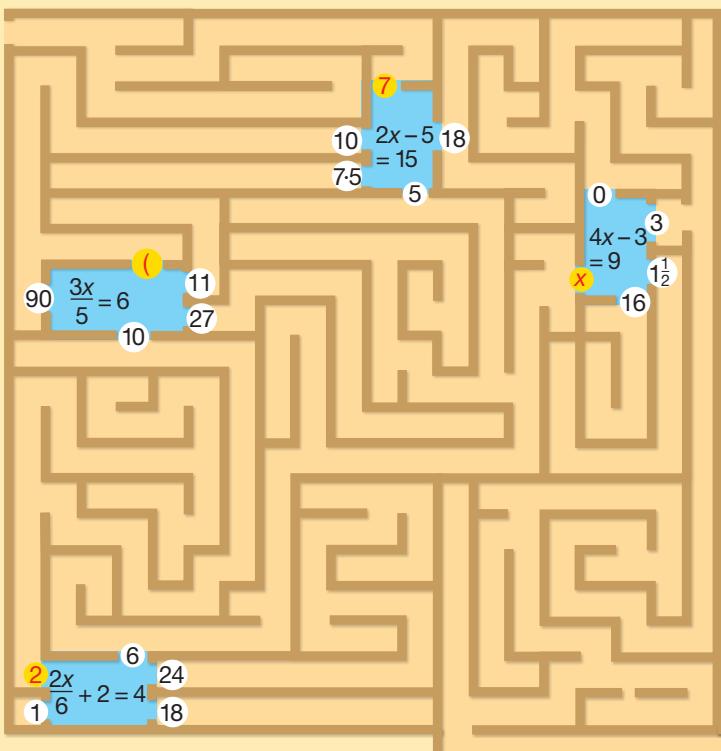
8 $\frac{7x}{5} - 3 = 11$

4 $7x - 3.5 = 31.5$

9 $\frac{8x}{6} - 4 = 0$

5 $5x + \frac{1}{3} = 15\frac{1}{3}$

10 $\frac{9x}{10} + 1 = 8.2$

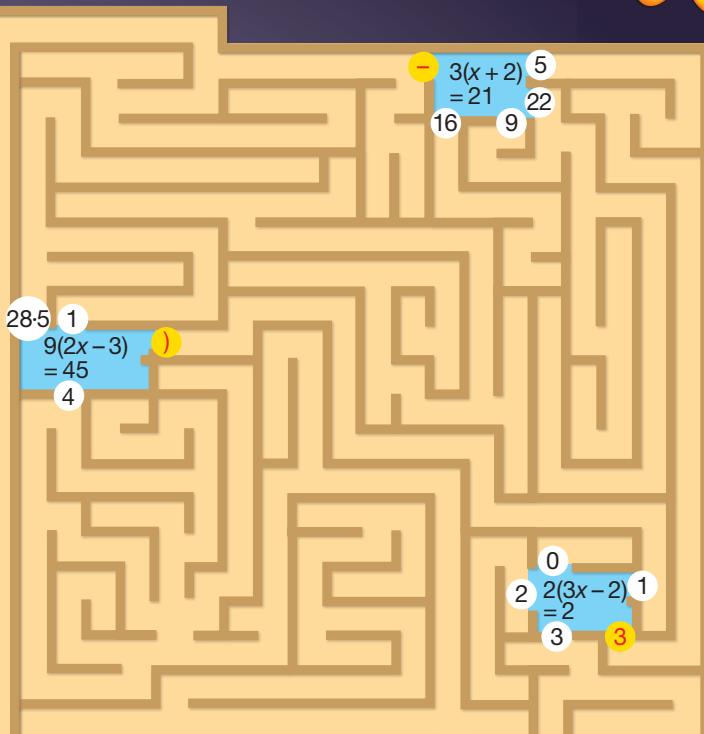




Practice session 2

Solve these equations:

- | | | | |
|----------|-----------------|-----------|------------------------|
| 1 | $3(x + 2) = 9$ | 6 | $\frac{x + 8}{2} = 7$ |
| 2 | $4(x - 5) = 12$ | 7 | $\frac{x + 3}{4} = 3$ |
| 3 | $2(x + 1) = 16$ | 8 | $\frac{x - 8}{3} = 2$ |
| 4 | $5(x - 3) = 35$ | 9 | $\frac{x + 1}{2} = 5$ |
| 5 | $6(x - 2) = 24$ | 10 | $\frac{x - 8}{11} = 2$ |



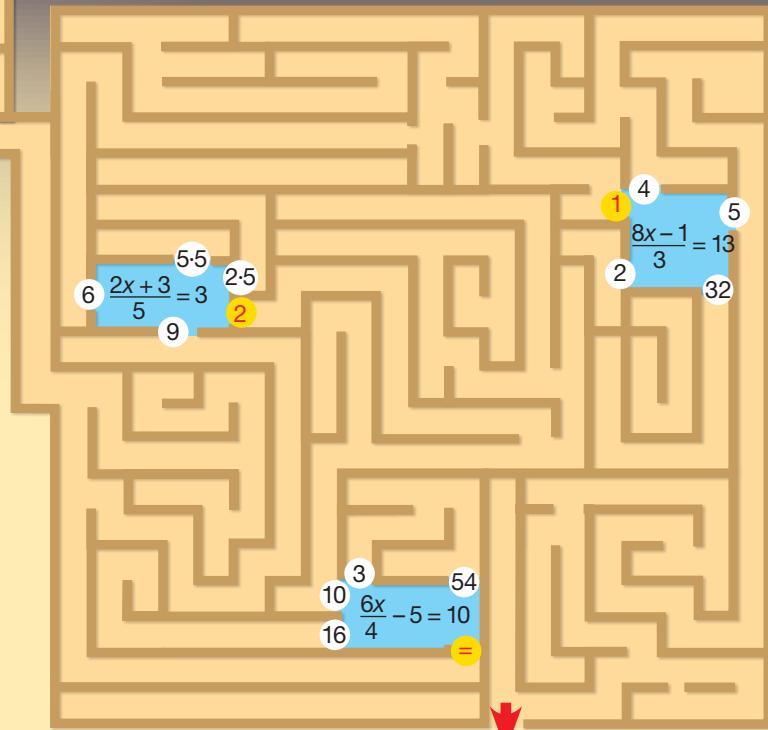
Practice session 3

Solve these equations:

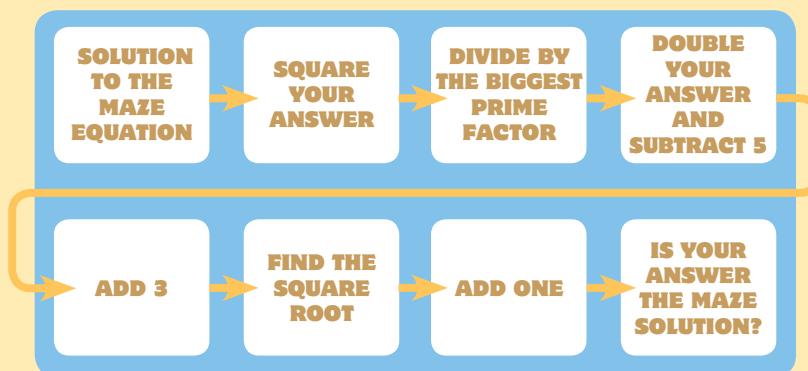
- | | | | |
|----------|-----------------------|----------|-------------------------|
| 1 | $5x + 4 = 9$ | 5 | $\frac{7x}{5} + 1 = 15$ |
| 2 | $4(x - 3) = 8$ | 6 | $\frac{5x}{3} + 3 = 13$ |
| 3 | $\frac{x + 1}{8} = 3$ | 7 | $7x - 8 = 20$ |
| 4 | $9(x - 5) = 90$ | 8 | $\frac{x + 2}{6} = 4$ |

Congratulations!

You have successfully mastered the House of Calculus. Or have you? Where is the golden calculator? Dr Equation has left you a final challenge. Piece together the clues you have collected in order and then solve the equation that they form. You then need to use your answer to attempt the biggest challenge of all—the sinister ‘Steps of Arithmetica’. Successfully meet the challenge and you will find the golden calculator. Fail and you will have to go back to the start!



FINISH



Challenge 7



- 1 When half of a number is increased by 15 the result is 49. The original number is:

A 68

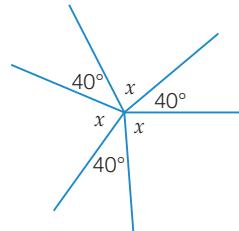
B 34

C 32

D 17

- 2 Write an equation and solve it to find the value of x in the diagram.

3 Solve the equation: $\frac{3x - 5}{4} = -6$.



- 4 If $59 + x = 73 + y$, then:

A $x = 14 - y$

B $x = 14 + y$

C $x = y - 14$

D $x = 14y$

- 5 Using only the digits 1, 2, 3 and 7 and the two mathematical symbols + and = only once, we can create a true number sentence $2^3 = 1 + 7$.

- (a) Using the digits 2, 3, 4, 5 and the two mathematical symbols + and = only once and a number as a power, create a true number sentence.

- (b) Using any different four digits from 0 to 9 and the two mathematical symbols + and = only once and one of the numbers as a power, create five different true number sentences. Do not use $2^3 = 1 + 7$ or your answer to (a). Use 0 and 2 as a power only once.

- (c) Use once only, any different four digits from 0 to 9 and the two mathematical symbols – and = only once and one of the numbers as a power to create five true number sentences. Use 0 and 1 as a power only once.

- 6 The inverse (reciprocal) of $\frac{2}{3}$ is $\frac{3}{2}$. If the inverse of $\frac{4x}{5}$ is $\frac{1}{20}$, then the value of x is:

A $\frac{1}{25}$

B $\frac{1}{16}$

C 16

D 25

- 7 Solve for y :

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{y} = 1$$

- 8 Vladmir buys an equal number of 55-cent and \$1.10 stamps and spends \$80.85. How many stamps did he buy altogether? Form an equation and solve it.

- 9 A Lotto winner won $\frac{9}{10}$ of the total prize pool. Shortly after, she spent $\frac{3}{4}$ of her winnings, but still had \$2700 left. What was the value of the total prize pool? Form an equation and solve it.

- 10 If $\frac{1}{3} > \frac{1}{4} > \frac{1}{m}$ and the difference between the first two fractions equals the difference between the last two fractions, find the value of m . (Hint: Write $\frac{1}{m}$ on one side of the equation before you take the inverse.)

- 11 Two integers, a and b , do not end in zero. If the product ab is a multiple of 10 and $a > b$, the last digit of $a - b$ cannot be:

A 1

B 3

C 5

D 7

Chapter review

7

D.I.Y. Summary

Key Words

backtracking	false number sentence	solve
balance method	guess, check and improve	solving by inspection
checking by substitution	inverse operations	true number sentence
equivalent equations	solution	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

- 1 An equation can be solved by performing the _____ on both sides of the equation.
- 2 To _____ an equation is to find the value of the unknown that will make a true number sentence.
- 3 $4 + 5 = 10$ is a _____.
- 4 One way of solving an equation is by looking at the equation to see what number might be the answer. This is called _____.
- 5 Equations that have the same solution are said to be _____.
- 6 _____ is a method we can use to solve equations. It can be shown by moving backwards along a flowchart.
- 7 _____ is another method used to solve equations.

Fluency

- 1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

Ex. 7.1

(a) $5 \times 4 = 30 - 2$ (b) $2 \times 7 + 6 = 9 + 5$

- 2 Find the unknown number in each of the following sentences.

Ex. 7.1

- (a) Six subtracted from a number is equal to nine.
(b) A number divided by four is equal to seven.

- 3 Find the solution to each of the following equations by inspection.

Ex. 7.2

(a) $2x = 18$ (b) $x + 7 = 10$ (c) $\frac{x}{5} = 3$
(d) $x - 4 = 11$ (e) $x + 3 = 15$ (f) $x - 4 = 24$

- 4 Solve the following equations using guess, check and improve.

Ex. 7.2

(a) $3x + 25 = 16$ (b) $\frac{x}{7} + 51 = 63$ (c) $\frac{3x + 5}{4} = 8$

- 5 List the inverse (or opposite) operation to each of the following.

Ex. 7.3

(a) $+ 5$ (b) $\div 3$ (c) $\times 8$ (d) $- 4$

- 6 Solve each of the following equations using a flowchart and backtracking.

(a) $3x - 7 = 2$

(b) $\frac{x+6}{5} = 4$

(c) $4(x+1) = 20$

(d) $\frac{x}{8} - 2 = 3$

Ex. 7.3

- 7 Use the balance method to solve the following equations.

(a) $4x + 7 = 19$

(b) $5(x-2) = 20$

(c) $\frac{x}{3} + 5 = 10$

(d) $\frac{x-8}{6} = 3$

(e) $\frac{x+4}{5} = 5$

(f) $\frac{x-7}{3} = 10$

Ex. 7.4

- 8 The equation $5x - 6 = 4$ has the solution:

A $x = 1$

B $x = 2$

C $x = 3$

D $x = 4$

Ex. 7.4

- 9 Danni bought 5 tickets to a show. She was given \$27.50 change from \$200. Use an equation to find how much each ticket cost. Let t be the cost of a ticket.

Ex. 7.5

Understanding

- 10 Which equation describes this situation?

A restaurant seats a maximum of 64 patrons. On Saturday afternoon, it is fully booked for the evening meal service. At 5.30 p.m., a group booking for 16 is cancelled. If p represents the number of people who are still booked, the equation that can be formed is:

A $p - 16 = 64$

B $p = 64 + 16$

C $16p = 64$

D $p + 16 = 64$

- 11 Ross buys three dim sims and a bottle of juice.

(a) If each dim sim costs x cents and the bottle of juice costs \$1.40, write an expression for the total cost (in cents) of the food and juice.

(b) Use an equation to find the cost of one dim sim if Ross spends a total of \$2.75.

- 12 The air temperature at 4 p.m. is 28°C . Over the past hour, the temperature has risen by 5°C . What was the temperature at 3 p.m.? (Form an equation to solve by letting t represent the air temperature at 3 p.m.)

Reasoning

- 13 A triangle  made from matchsticks has three sides of equal length. If more matchsticks are added to make another triangle, we obtain this shape .

(a) How many matchsticks of equal length does this shape have?

(b) A third triangle is added to the shape. How many matchsticks of equal length does this shape have?

(c) If n represents the number of triangles joined together in a row, write, using the pronumeral n , the number of matchsticks of equal length the shape has in total.

(d) (i) Use your answer to (c) to write down the equation for n triangles joined together that have 97 matchsticks of equal length.

(ii) Solve the equation obtained in part (d) (i).

NAPLAN practice 7

Numeracy: Non-calculator

- 1 Jan is baking biscuits. She puts her biscuits in 3 equal rows. When they are baked, she eats one. She has 44 biscuits left. How many biscuits did she have in each row?

A 3

B 10

C 15

D 20

- 2 A rectangular garden bed has a perimeter of 30 metres and a length of 10 metres. If the width of the garden can be written as w , which equation best represents this situation?

A $w + 10 = 30$

B $w + 30 = 10$

C $2w + 10 = 30$

D $2w + 20 = 30$

10 metres



- 3 What is the missing number that makes this number sentence true?

$$3 \times \square = 12 + 3 \times 6$$

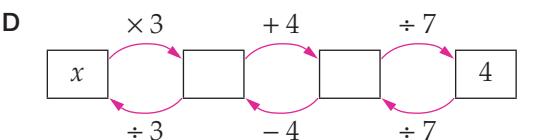
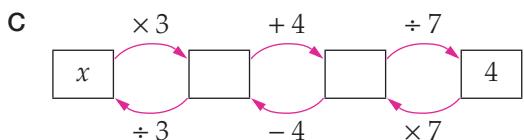
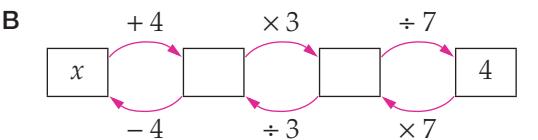
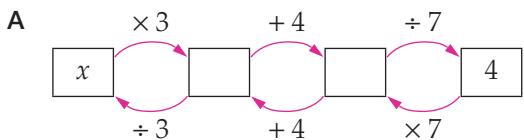
A 1

B 7

C 9

D 10

- 4 Voula thinks of a number and asks Renza to guess what it is. Voula states that after the number is tripled, four added and the result divided by seven, the answer is four. Which flowchart best represents how Renza might find Voula's mystery number? Let the mystery number = x .



Numeracy: Calculator allowed

- 5 A pack of 4 donuts costs \$3.60.

A pack of 6 donuts costs \$4.00.

You need to buy 34 donuts.

What is the least amount you can pay?

A \$23.60

B \$26.40

C \$26.50

D \$29.90

- 6 The cooking time for a turkey is 30 minutes plus 20 minutes per kilogram. If M is the mass of the turkey in kilograms, the total cooking time in minutes is:

A $20 + 30M$

B $20M + 30$

C $20 \times 30M$

D $30 - 20M$

- 7 Julie, Nikki and Romina earned a total of \$3400 in the last week. Julie earned twice as much as Nikki whereas Romina earned \$200 more than Nikki. How much did Nikki earn in the last week?

A \$600

B \$800

C \$1000

D \$1600

- 8 The solution to $\frac{3(x-4)}{2} = 15$ is:

A $x = 6$

B $x = 10$

C $x = 14$

D $x = 58$