Chapter 5 Algebra

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)

Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

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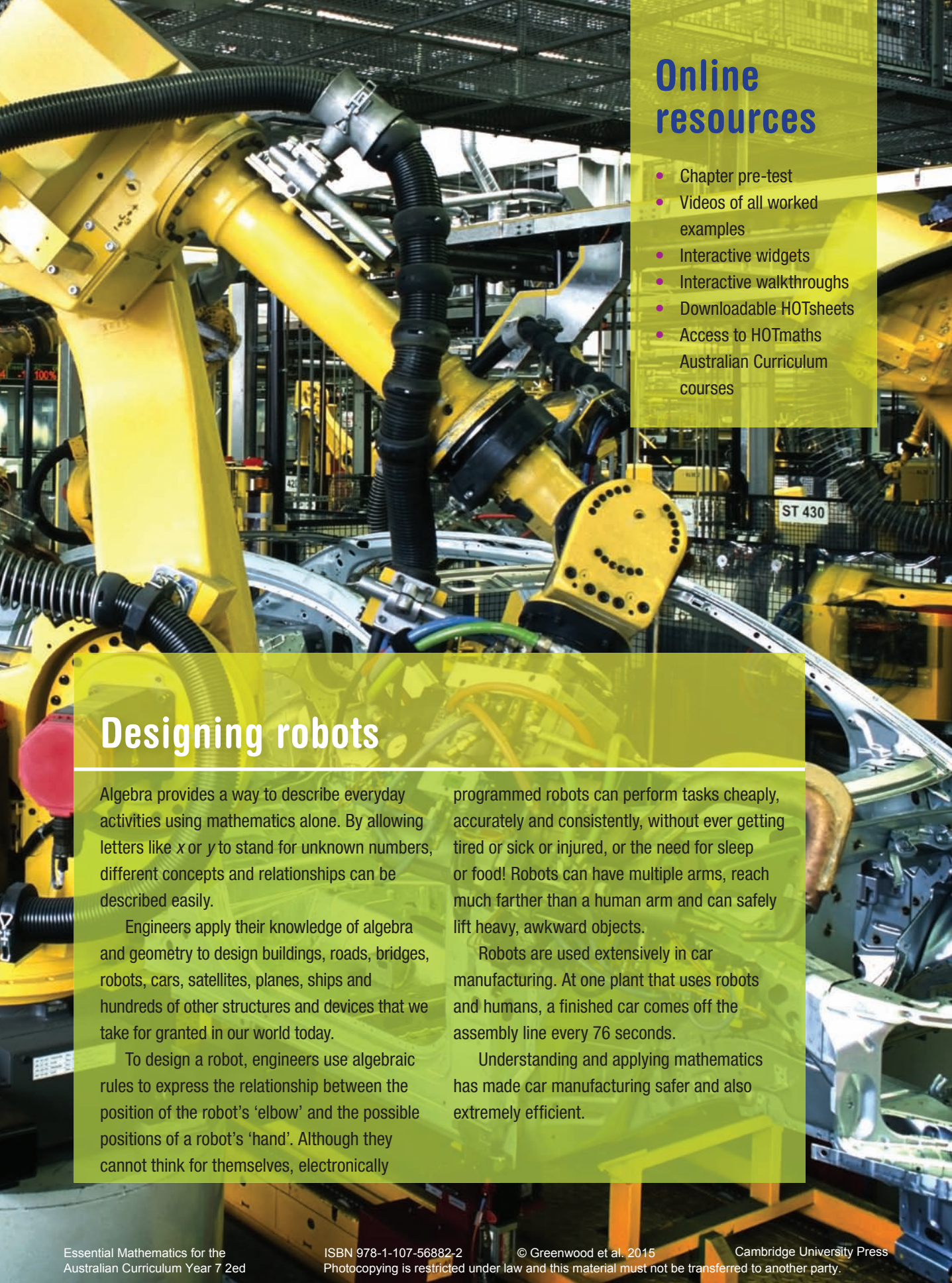

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Designing robots

Algebra provides a way to describe everyday activities using mathematics alone. By allowing letters like x or y to stand for unknown numbers, different concepts and relationships can be described easily.

Engineers apply their knowledge of algebra and geometry to design buildings, roads, bridges, robots, cars, satellites, planes, ships and hundreds of other structures and devices that we take for granted in our world today.

To design a robot, engineers use algebraic rules to express the relationship between the position of the robot's 'elbow' and the possible positions of a robot's 'hand'. Although they cannot think for themselves, electronically programmed robots can perform tasks cheaply, accurately and consistently, without ever getting tired or sick or injured, or the need for sleep or food! Robots can have multiple arms, reach much farther than a human arm and can safely lift heavy, awkward objects.

Robots are used extensively in car manufacturing. At one plant that uses robots and humans, a finished car comes off the assembly line every 76 seconds.

Understanding and applying mathematics has made car manufacturing safer and also extremely efficient.

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5A Introduction to algebra

Interactive

Widgets

HOTsheets

Walkthroughs

A pronumeral is a letter that can represent a number. The choice of letter used is not significant mathematically, but can be used as an aid to memory. For instance, h might stand for someone's height and w might stand for someone's weight.

The table shows the salary Petra earns for various hours of work if she is paid \$12 an hour.

Numbers of hours	Salary earned (\$)
1	$12 \times 1 = 12$
2	$12 \times 2 = 24$
3	$12 \times 3 = 36$
n	$12 \times n = 12n$

Rather than writing $12 \times n$, we write $12n$ because multiplying a pronumeral by a number is common and this notation saves space. We can also write $18 \div n$ as $\frac{18}{n}$.

Using pronumerals we can work out a total salary for any number of hours of work.

Let's start: Pronumeral stories

Ahmed has a jar with b biscuits. He eats 3 biscuits and then shares the rest equally among 8 friends. Each friend receives $\frac{b-3}{8}$ biscuits. This is a short story for the expression $\frac{b-3}{8}$.

- Try to create another story for $\frac{b-3}{8}$, and share it with others in the class.
- Can you construct a story for $2t + 12$? What about $4(k + 6)$?

Key ideas

- In algebra, letters can be used to stand for numbers. A **pronumeral** is a letter that stands for a number. If a pronumeral could represent *any* number rather than just one, it is also called a **variable**.
- $a \times b$ is written as ab and $a \div b$ is written as $\frac{a}{b}$.
- A **term** consists of numbers and pronumerals combined with multiplication or division. For example, 5 is a term, x is a term, $9a$ is a term, abc is a term, $\frac{4xyz}{3}$ is a term.
- A term that does not contain any pronumerals is called a **constant term**. All numbers by themselves are constant terms.
- An (**algebraic**) **expression** consists of numbers and pronumerals combined with any mathematical operations. For example, $3x + 2yz$ is an expression and $8 \div (3a - 2b) + 41$ is also an expression. Any term is also an expression.

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Key ideas

- A **coefficient** is the number in front of a pronumeral. For example, the coefficient of y in the expression $8x + 2y + z$ is 2. If there is no number in front, then the coefficient is 1, since $1z$ and z are equal.

Example 1 The terminology of algebra

a List the individual terms in the expression $3a + b + 13c$.

b State the coefficient of each pronumeral in the expression $3a + b + 13c$.

c Give an example of an expression with exactly two terms, one of which is a constant term.

SOLUTION	EXPLANATION
a There are three terms: $3a$, b and $13c$.	Each part of an expression is a term. Terms get added (or subtracted) to make an expression.
b The coefficient of a is 3, the coefficient of b is 1 and the coefficient of c is 13.	The coefficient is the number in front of a pronumeral. For b the coefficient is 1 because b is the same as $1 \times b$.
c $27a + 19$ (There are many other possible expressions.)	This expression has two terms, $27a$ and 19, and 19 is a constant term because it is a number without any pronumerals.

Example 2 Writing expressions from word descriptions

Write an expression for each of the following.

a 5 more than k **b** 3 less than m **c** the sum of a and b

d double the value of x **e** the product of c and d

SOLUTION	EXPLANATION
a $k + 5$	5 must be added to k to get 5 more than k.
b $m - 3$	3 is subtracted from m.
c $a + b$	a and b are added to obtain their sum.
d $2 \times x$ or $2x$	x is multiplied by 2. The multiplication sign is optional.
e $c \times d$ or cd	c and d are multiplied to obtain their product.

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Example 3 Expressions involving more than one operation

Write an expression for each of the following without using the × or ÷ symbols.

a p is halved, then 4 is added

b the sum of x and y is taken and then divided by 7

c the sum of x and one-seventh of y

d 5 is subtracted from k and the result is tripled

SOLUTION	EXPLANATION
a $\frac{p}{2} + 4$	p is divided by 2, then 4 is added.
b $(x + y) \div 7 = \frac{x + y}{7}$	x and y are added. This whole expression is divided by 7. By writing the result as a fraction, the brackets are no longer needed.
c $x + \frac{y}{7}$ or $x + \frac{1}{7}y$	x is added to one-seventh of y, which is $\frac{y}{7}$.
d $(k - 5) \times 3 = 3(k - 5)$	5 subtracted from k gives the expression $k - 5$. Brackets must be used to multiply the whole expression by 3.

Exercise 5A

1, 2 | 1, 2 | —

UNDERSTANDING

Example 1

1 The expression $4x + 3y + 24z + 7$ has four terms.

a List the terms.

b What is the constant term?

c What is the coefficient of x?

d Which letter has a coefficient of 24?

Example 2

2 Match each of the word descriptions on the left with the correct mathematical expression on the right.

a the sum of x and 4

b 4 less than x

c the product of 4 and x

d one-quarter of x

e the result from subtracting x from 4

f 4 divided by x

A $x - 4$

B $\frac{x}{4}$

C $4 - x$

D $4x$

E $\frac{4}{x}$

F $x + 4$

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5A

FLUENCY

3, 4–5(½) | 3, 4–5(½), 6 | 3–6(½)

3 For each of the following expressions, state:

i the number of terms; and

ii the coefficient of n.

a $17n + 24$

b $31 - 27a + 15n$

c $15nw + 21n + 15$

d $15a - 32b + 2n + \frac{4}{3}xy$

e $n + 51$

f $5bn - 12 + \frac{d}{5} + 12n$

4 Write an expression for each of the following without using the × or ÷ symbols.

a 1 more than x

b the sum of k and 5

c double the value of u

d 4 lots of y

e half of p

f one-third of q

g 12 less than r

h the product of n and 9

i t is subtracted from 10

j y is divided by 8

Example 3

5 Write an expression for each of the following without using the × or ÷ symbols.

a 5 is added to x, then the result is doubled.

b a is tripled, then 4 is added.

c k is multiplied by 8, then 3 is subtracted.

d 3 is subtracted from k, then the result is multiplied by 8.

e The sum of x and y is multiplied by 6.

f x is multiplied by 7 and the result is halved.

g p is halved and then 2 is added.

h The product of x and y is subtracted from 12.

6 Describe each of these expressions in words.

a $7x$

b $a + b$

c $(x + 4) \times 2$

d $5 - 3a$

PROBLEM-SOLVING

7, 8 | 8–10 | 8–11

7 Nicholas buys 10 lolly bags from a supermarket.

a If there are 7 lollies in each bag, how many lollies does he buy in total?

b If there are n lollies in each bag, how many lollies does he buy in total?
Hint: Write an expression involving n.

8 Mikayla is paid $\$x$ per hour at her job. Write an expression for each of the following amounts (in $).

a How much does Mikayla earn if she works 8 hours?

b If Mikayla gets a pay rise of $3 per hour, what is her new hourly wage?

c If Mikayla works for 8 hours at the increased hourly rate, how much does she earn?

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5A

PROBLEM-SOLVING

9 Recall that there are 100 centimetres in 1 metre and 1000 metres in 1 kilometre. Write expressions for each of the following.

- **a** How many metres are there in x km?
- **b** How many centimetres are there in x metres?
- **c** How many centimetres are there in x km?

10 A group of people go out to a restaurant, and the total amount they must pay is $\$A$. They decide to split the bill equally. Write expressions to answer the following questions.

- **a** If there are 4 people in the group, how much do they each pay?
- **b** If there are n people in the group, how much do they each pay?
- **c** One of the n people has a voucher that reduces the total bill by \$20. How much does each person pay now?

11 There are many different ways of describing the expression $\frac{a+b}{4}$ in words. One way is: 'The sum of a and b is divided by 4.' What is another way?

REASONING

12 | 12, 14 | 13, 14

12 If x is a whole number between 10 and 99, classify each of these statements as true or false.

- **a** x must be smaller than $2 \times x$.
- **b** x must be smaller than $x + 2$.
- **c** $x - 3$ must be greater than 10.
- **d** $4 \times x$ must be an even number.
- **e** $3 \times x$ must be an odd number.

13 If b is an even number greater than 3, classify each of these statements as true or false.

- **a** $b + 1$ must be even.
- **b** $b + 2$ could be odd.
- **c** $5 + b$ could be greater than 10.
- **d** $5b$ must be greater than b.

14 If c is a number between 10 and 99, sort the following in ascending order (i.e. smallest to largest): $3c$, $2c$, $c - 4$, $c \div 2$, $3c + 5$, $4c - 2$, $c + 1$, $c \times c$.

ENRICHMENT

Many words compressed — | — | 15

15 One advantage of writing expressions in symbols rather than words is that it takes up less space. For instance, 'twice the value of the sum of x and 5' uses eight words and can be written as $2(x + 5)$. Give an example of a worded expression that uses more than 10 words and then write it as a mathematical expression.

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5B Substituting and evaluating

Interactive

Widgets

HOTsheets

Walkthroughs

Evaluation of expressions involves replacing pronumerals (like x and y) with numbers and obtaining a single number as a result. For example, we can evaluate $4 + x$ when x is 11, to get 15.

Let's start: Sum to 10

The pronumerals x and y could stand for any number.

- What numbers could x and y stand for if you know that $x + y$ must equal 10? Try to list as many pairs as possible.
- If $x + y$ must equal 10, what values could $3x + y$ equal? Find the largest and smallest values.

Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value. For example, if $x = 3$ and $y = 8$, then $x + 2y$ evaluated gives $3 + 2 \times 8 = 19$.
- A term like $4a$ means $4 \times a$. When substituting a number we must include the multiplication sign, since two numbers written as 42 is very different from the product 4×2.
- Once an expression contains no pronumerals, evaluate using the normal order of operations seen in Chapter 1:
  - brackets
  - multiplication and division from left to right
  - addition and subtraction from left to right.

  For example: $(4 + 3) \times 2 - 20 \div 4 + 2 = 7 \times 2 - 20 \div 4 + 2$
  $= 14 - 5 + 2$
  $= 9 + 2$
  $= 11$

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Example 4 Substituting a pronumeral

Given that $t = 5$, evaluate:

a $t + 7$ **b** $8t$ **c** $\frac{10}{t} + 4 - t$

SOLUTION	EXPLANATION
a $t + 7 = 5 + 7$ $= 12$	Replace t with 5 and then evaluate the expression, which now contains no pronumerals.
b $8t = 8 \times t$ $= 8 \times 5$ $= 40$	Insert $\times$ where it was previously implied, then substitute in 5. If the multiplication sign is not included, we might get a completely incorrect answer of 85.
c $\frac{10}{t} + 4 - t = \frac{10}{5} + 4 - 5$ $= 2 + 4 - 5$ $= 1$	Replace all occurrences of t with 5 before evaluating. Note that the division $(10 \div 5)$ is calculated before the addition and subtraction.

Example 5 Substituting multiple pronumerals

Substitute $x = 4$ and $y = 7$ to evaluate these expressions.

a $5x + y + 8$ **b** $80 - (2xy + y)$

SOLUTION	EXPLANATION
a $5x + y + 8 = 5 \times x + y + 8$ $= 5 \times 4 + 7 + 8$ $= 20 + 7 + 8$ $= 35$	Insert the implied multiplication sign between 5 and x before substituting the values for x and y.
b $80 - (2xy + y) = 80 - (2 \times x \times y + y)$ $= 80 - (2 \times 4 \times 7 + 7)$ $= 80 - (56 + 7)$ $= 80 - 63$ $= 17$	Insert the multiplication signs, and remember the order in which to evaluate. Note that both occurrences of y are replaced with 7.

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Example 6 Substituting with powers and roots

If $p = 4$ and $t = 5$, find the value of:

a $3p^2$ **b** $t^2 + p^3$ **c** $\sqrt{p^2 + 3^2}$

SOLUTION	EXPLANATION
a $3p^2 = 3 \times p \times p$ $= 3 \times 4 \times 4$ $= 48$	Note that $3p^2$ means $3 \times p \times p$ not $(3 \times p)^2$.
b $t^2 + p^3 = 5^2 + 4^3$ $= 5 \times 5 + 4 \times 4 \times 4$ $= 25 + 64$ $= 89$	t is replaced with 5, and p is replaced with 4. Remember that 4^3 means $4 \times 4 \times 4$.
c $\sqrt{p^2 + 3^2} = \sqrt{4^2 + 3^2}$ $= \sqrt{25}$ $= 5$	Recall that the square root of 25 must be 5 because $5 \times 5 = 25$.

Exercise 5B

1–5 | 2, 3 | —

UNDERSTANDING

1 Use the correct order of operations to evaluate the following.

a $4 + 2 \times 5$ **b** $7 - 3 \times 2$

c $3 \times 6 - 2 \times 4$ **d** $(7 - 3) \times 2$

Example 4a

2 What number would you get if you replaced b with 5 in the expression $8 + b$?

3 What number is obtained when $x = 3$ is substituted into the expression $5 \times x$?

4 What is the result of evaluating $10 - u$ if u is 7?

5 Calculate the value of $12 + b$ if:

a $b = 5$ **b** $b = 8$

c $b = 60$ **d** $b = 0$

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5B

6–8(½), 9 | 6–7(½), 8, 9, 10(½) | 6–10(½)

FLUENCY

Example 4b,c

6 If $x = 5$, evaluate each of the following. Set out your solution in a manner similar to that shown in **Example 4**.

a $x + 3$
b $x \times 2$
c $14 - x$
d $2x + 4$
e $3x + 2 - x$
f $13 - 2x$
g $2(x + 2) + x$
h $30 - (4x + 1)$
i $\frac{20}{x} + 3$
j $(x + 5) \times \frac{10}{x}$
k $\frac{x + 7}{4}$
l $\frac{10 - x}{x}$
m $7x + 3(x - 1)$
n $40 - 3x - x$
o $x + x(x + 1)$
p $\frac{30}{x} + 2x(x + 3)$
q $100 - 4(3 + 4x)$
r $\frac{6(3x - 8)}{x + 2}$

Example 5

7 Substitute $a = 2$ and $b = 3$ into each of these expressions and evaluate.

a $2a + 4$
b $3a - 2$
c $a + b$
d $3a + b$
e $5a - 2b$
f $7ab + b$
g $ab - 4 + b$
h $2 \times (3a + 2b)$
i $100 - (10a + 10b)$
j $\frac{12}{a} + \frac{6}{b}$
k $\frac{ab}{3} + b$
l $\frac{100}{a + b}$

8 Evaluate the expression $5x + 2y$ when:

a $x = 3$ and $y = 6$
b $x = 4$ and $y = 1$
c $x = 7$ and $y = 3$
d $x = 0$ and $y = 4$
e $x = 2$ and $y = 0$
f $x = 10$ and $y = 10$

9 Copy and complete each of these tables.

a

n	1	2	3	4	5	6
$n + 4$	5			8		

b

x	1	2	3	4	5	6
$12 - x$			9			

c

b	1	2	3	4	5	6
$2(b - 1)$						

d

q	1	2	3	4	5	6
$10q - q$						

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5B

FLUENCY

Example 6

10 Evaluate each of the following, given that $a = 9, b = 3$ and $c = 5$.

a $3c^2$ **b** $5b^2$ **c** $a^2 - 3^3$ **d** $2b^2 + \frac{a}{3} - 2c$

e $\sqrt{a} + \sqrt{3ab}$ **f** $\sqrt{b^2 + 4^2}$ **g** $24 + \frac{2b^3}{6}$ **h** $(2c)^2 - a^2$

PROBLEM-SOLVING

11 | 11, 13 | 12, 13

11 A number is substituted for b in the expression $7 + b$ and gives the result 12. What is the value of b?

12 A number is substituted for x in the expression $3x - 1$. If the result is a two-digit number, what value might x have? Try to describe all the possible answers.

13 Copy and complete the table.

x	5	9	12			
$x + 6$	11			7		
$4x$	20				24	28

REASONING

14 | 14 | 14, 15

14 Assume x and y are two numbers, where $xy = 24$.

a What values could x and y equal if they are whole numbers? Try to list as many as possible.

b What values could x and y equal if they can be decimals, fractions or whole numbers?

15 Dugald substitutes different whole numbers into the expression $5 \times (a + a)$. He notices that the result always ends in the digit 0. Try a few values and explain why this pattern occurs.

ENRICHMENT

Missing numbers — | — | 16

16 a Copy and complete the following table. Note: x and y are whole numbers $(0, 1, 2, 3, \ldots)$ for this table.

x	5	10	7			
y	3	4		5		
$x + y$			9	14	7	
$x - y$	2				3	8
xy		40			10	0

b If x and y are two numbers where $x + y$ and $x \times y$ are equal, what values might x and y have? Try to find at least three (they do not have to be whole numbers).

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5C Equivalent expressions

Interactive

Widgets

HOTsheets

Walkthroughs

In algebra, as when using words, there are often many ways to express the same thing. For example, we can write 'the sum of x and 4' as $x + 4$ or $4 + x$, or even $x + 1 + 1 + 1 + 1$.

No matter what number x is, $x + 4$ and $4 + x$ will always be equal. We say that the expressions $x + 4$ and $4 + x$ are equivalent because of this.

By substituting different numbers for the pronumerals it is possible to see whether two expressions are equivalent. Consider the four expressions in this table.

	$3a + 5$	$2a + 6$	$7a + 5 - 4a$	$a + a + 6$
$a = 0$	5	6	5	6
$a = 1$	8	8	8	8
$a = 2$	11	10	11	10
$a = 3$	14	12	14	12
$a = 4$	17	14	17	14

From this table it becomes apparent that $3a + 5$ and $7a + 5 - 4a$ are equivalent, and that $2a + 6$ and $a + a + 6$ are equivalent.

Let's start: Equivalent expressions

Consider the expression $2a + 4$.

- Write as many different expressions as possible that are equivalent to $2a + 4$.
- How many equivalent expressions are there?
- Try to give a logical explanation for why $2a + 4$ is equivalent to $4 + a \times 2$.

This collection of pronumerals and numbers can be arranged into many different equivalent expressions.

Key ideas

- Two expressions are called **equivalent** when they are always equal, regardless of what numbers are substituted for the pronumerals.
  For example:
  - $x + 12$ is equivalent to $12 + x$, because the order in which numbers are added is not important.
  - $3k$ is equivalent to $k + k + k$, because multiplying by a whole number is the same as adding repeatedly.
- The rules of algebra are used to prove that two expressions are equivalent, but a table of values can be helpful to test whether expressions are likely to be equivalent.

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Example 7 Equivalent expressions

Which two of these expressions are equivalent: $3x + 4$, $8 - x$, $2x + 4 + x$?

SOLUTION

$3x + 4$ and $2x + 4 + x$ are equivalent.

EXPLANATION

By drawing a table of values, we can see straight away that $3x + 4$ and $8 - x$ are not equivalent, since they differ for $x = 2$.

	$x = 1$	$x = 2$	$x = 3$
$3x + 4$	7	10	13
$8 - x$	7	6	5
$2x + 4 + x$	7	10	13

$3x + 4$ and $2x + 4 + x$ are equal for all values, so they are equivalent.

Exercise 5C

1, 2 | 2 | —

UNDERSTANDING

1 a Copy the following table into your workbook and complete.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$2x + 2$				
$(x + 1) \times 2$				

b Fill in the gap: $2x + 2$ and $(x + 1) \times 2$ are _____ expressions.

2 a Copy the following table into your workbook and complete.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$5x + 3$				
$6x + 3$				

b Are $5x + 3$ and $6x + 3$ equivalent expressions?

3, 4 | 3–5 | 3–5

FLUENCY

3 Demonstrate that $6x + 5$ and $4x + 5 + 2x$ are equivalent by completing the table.

	$6x + 5$	$4x + 5 + 2x$
$x = 1$		
$x = 2$		
$x = 3$		
$x = 4$		

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5C

Example 7

FLUENCY

4 For each of the following, choose a pair of equivalent expressions.

a $4x, 2x + 4, x + 4 + x$

b $5a, 4a + a, 3 + a$

c $2k + 2, 3 + 2k, 2(k + 1)$

d $b + b, 3b, 4b - 2b$

5 Match up the equivalent expressions below.

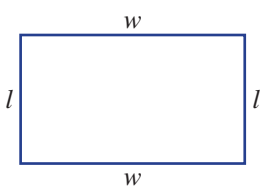
a $3x + 2x$	**A** $6 - 3x$
b $4 - 3x + 2$	**B** $2x + 4x + x$
c $2x + 5 + x$	**C** $5x$
d $x + x - 5 + x$	**D** $4 - x$
e $7x$	**E** $3x + 5$
f $4 - 3x + 2x$	**F** $3x - 5$

6 | 6, 7 | 7, 8

PROBLEM-SOLVING

6 Write two different expressions that are equivalent to $4x + 2$.

7 The rectangle shown below has a perimeter given by $w + l + w + l$. Write an equivalent expression for the perimeter.



8 There are many expressions that are equivalent to $3a + 5b + 2a - b + 4a$.
Write an equivalent expression with as few terms as possible.

9 | 10, 11 | 10–12

REASONING

9 The expressions $a + b$ and $b + a$ are equivalent and only contain two terms. How many expressions are equivalent to $a + b + c$ and contain only three terms?
Hint: Rearrange the pronumerals.

10 Prove that no two of these three expressions are equivalent: $4 + x, 4x, x - 4, x \div 4$.

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5C

REASONING

11 Generalise each of the following patterns in numbers to give two equivalent expressions. The first one has been done for you.

- **a** Observation: $3 + 5 = 5 + 3$ and $2 + 7 = 7 + 2$ and $4 + 11 = 11 + 4$.
  Generalised: The two expressions $x + y$ and $y + x$ are equivalent.
- **b** Observation: $2 \times 5 = 5 \times 2$ and $11 \times 5 = 5 \times 11$ and $3 \times 12 = 12 \times 3$.
- **c** Observation: $4 \times (10 + 3) = 4 \times 10 + 4 \times 3$ and $8 \times (100 + 5) = 8 \times 100 + 8 \times 5$.
- **d** Observation: $100 - (4 + 6) = 100 - 4 - 6$ and $70 - (10 + 5) = 70 - 10 - 5$.
- **e** Observation: $20 - (4 - 2) = 20 - 4 + 2$ and $15 - (10 - 3) = 15 - 10 + 3$.
- **f** Observation: $100 \div 5 \div 10 = 100 \div (5 \times 10)$ and $30 \div 2 \div 3 = 30 \div (2 \times 3)$.

12 **a** Show that the expression $4 \times (a + 2)$ is equivalent to $8 + 4a$ using a table of values for a between 1 and 4.

- **b** Write an expression using brackets that is equivalent to $10 + 5a$.
- **c** Write an expression without brackets that is equivalent to $6 \times (4 + a)$.

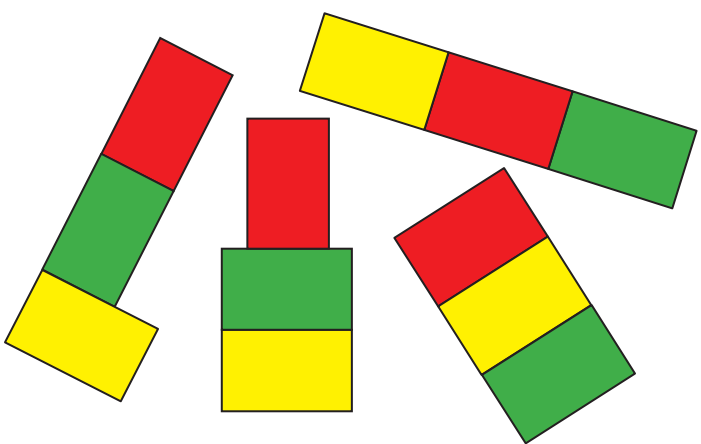
Thinking about equivalence — — 13–14

ENRICHMENT

13 $3a + 5b$ is an expression containing two terms. List two expressions containing three terms that are equivalent to $3a + 5b$.

14 Three expressions are given: expression A, expression B and expression C.

- **a** If expressions A and B are equivalent, and expressions B and C are equivalent, does this mean that expressions A and C are equivalent? Try to prove your answer.
- **b** If expressions A and B are not equivalent, and expressions B and C are not equivalent, does this mean that expressions A and C are not equivalent? Try to prove your answer.



Each shape above is made from three identically-sized tiles of width w and length l. Which of the shapes have the same perimeter?

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5D Like terms

Whenever we have terms with exactly the same pronumerals, they are called 'like terms' and can be collected and combined. For example, $3x + 5x$ can be simplified to $8x$. If the two terms do not have exactly the same pronumerals, they must be kept separate; for example, $3x + 5y$ cannot be simplified – it must be left as it is.

Recall from arithmetic that numbers can be multiplied in any order (e.g. $5 \times 3 = 3 \times 5$). This means pronumerals can appear in a different order within a term and give equivalent expressions (e.g. ab and ba are equivalent).

Let's start: Simplifying expressions

- Try to find a simpler expression that is equivalent to

$$1a + 2b + 3a + 4b + 5a + 6b + \ldots + 19a + 20b$$

- What is the longest possible expression that is equivalent to $10a + 20b + 30c$? Assume that all coefficients must be whole numbers greater than zero.
- Compare your expressions to see who has the longest one.

Key ideas

- **Like terms** are terms containing exactly the same pronumerals, although not necessarily in the same order.
  - $5ab$ and $3ab$ are like terms.
  - $4a$ and $7b$ are not like terms.
  - $2acb$ and $4bac$ are like terms.
- Like terms can be combined within an expression to create a simpler expression that is equivalent. For example, $5ab + 3ab$ can be simplified to $8ab$.
- If two terms are not like terms (such as $4x$ and $5y$), they can still be added to get an expression like $4x + 5y$, but this expression cannot be simplified further.

Example 8 Identifying like terms

Which of the following pairs are like terms?

a $3x$ and $2x$ **b** $3a$ and $3b$ **c** $2ab$ and $5ba$

d $4k$ and k **e** $2a$ and $4ab$ **f** $7ab$ and $9aba$

SOLUTION	EXPLANATION
a $3x$ and $2x$ are like terms.	The pronumerals are the same.
b $3a$ and $3b$ are not like terms.	The pronumerals are different.

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c	$2ab$ and $5ba$ are like terms.	The pronumerals are the same, even though they are written in a different order (one a and one b).
d	$4k$ and k are like terms.	The pronumerals are the same.
e	$2a$ and $4ab$ are not like terms.	The pronumerals are not exactly the same (the first term contains only a and the second term has a and b).
f	$7ab$ and $9aba$ are not like terms.	The pronumerals are not exactly the same (the first term contains one a and one b, but the second term contains two copies of a and one b).

Example 9 Simplifying using like terms

Simplify the following by collecting like terms.

a $7b + 2 + 3b$

b $12d - 4d + d$

c $5 + 12a + 4b - 2 - 3a$

d $13a + 8b + 2a - 5b - 4a$

e $12uv + 7v - 3vu + 3v$

SOLUTION	EXPLANATION
a $7b + 2 + 3b = 10b + 2$	$7b$ and $3b$ are like terms, so they are added. They cannot be combined with the term 2 because it is not 'like' $7b$ or $3b$.
b $12d - 4d + d = 9d$	All the terms here are like terms. Remember that d means $1d$ when combining them.
c $5 + 12a + 4b - 2 - 3a$ $= 12a - 3a + 4b + 5 - 2$ $= 9a + 4b + 3$	$12a$ and $3a$ are like terms. We subtract $3a$ because it has a minus sign in front of it. We can also combine the 5 and the 2 because they are like terms.
d $13a + 8b + 2a - 5b - 4a$ $= 13a + 2a - 4a + 8b - 5b$ $= 11a + 3b$	Combine like terms, remembering to subtract any term that has a minus sign in front of it.
e $12uv + 7v - 3vu + 3v$ $= 12uv - 3vu + 7v + 3v$ $= 9uv + 10v$	Combine like terms. Remember that $12uv$ and $3vu$ are like terms (i.e. they have the same pronumerals), so $12uv - 3uv = 9uv$.

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Exercise 5D

1, 2 | 2 | —

UNDERSTANDING

1 For each of the following terms, state all the pronumerals that occur in it.

a $4xy$ **b** $3abc$ **c** $2k$ **d** pq

2 Copy the following sentences into your workbook and fill in the gaps to make the sentences true. More than one answer might be possible.

a $3x$ and $5x$ are _____ terms.
b $4x$ and $3y$ are not _____ _____.
c $4xy$ and $4yx$ are like _____.
d $4a$ and _____ are like terms.
e $x + x + 7$ and $2x + 7$ are _____ expressions.
f $3x + 2x + 4$ can be written in an equivalent way as _____.

3–5(½) | 3–5(½) | 3–5(½)

FLUENCY

Example 8

3 Classify the following pairs as like terms (L) or not like terms (N).

a $7a$ and $4b$ **b** $3a$ and $10a$ **c** $18x$ and $32x$ **d** $4a$ and $4b$
e 7 and $10b$ **f** x and $4x$ **g** $5x$ and 5 **h** $12ab$ and $4ab$
i $7cd$ and $12cd$ **j** $3abc$ and $12abc$ **k** $3ab$ and $2ba$ **l** $4cd$ and $3dce$

4 Simplify the following by collecting like terms.

a $a + a$ **b** $3x + 2x$ **c** $4b + 3b$ **d** $12d - 4d$
e $15u - 3u$ **f** $14ab - 2ab$ **g** $8ab + 3ab$ **h** $4xy - 3xy$

Example 9

5 Simplify the following by collecting like terms.

a $2a + a + 4b + b$ **b** $5a + 2a + b + 8b$ **c** $3x - 2x + 2y + 4y$
d $4a + 2 + 3a$ **e** $7 + 2b + 5b$ **f** $3k - 2 + 3k$
g $7f + 4 - 2f + 8$ **h** $4a - 4 + 5b + b$ **i** $3x + 7x + 3y - 4x + y$
j $10a + 3 + 4b - 2a$ **k** $4 + 10h - 3h$ **l** $10x + 4x + 31y - y$
m $10 + 7y - 3x + 5x + 2y$ **n** $11a + 4 - 3a + 9$ **o** $3b + 4b + c + 5b - c$
p $7ab + 4 + 2ab$ **q** $9xy + 2x - 3xy + 3x$ **r** $2cd + 5dc - 3d + 2c$
s $5uv + 12v + 4uv - 5v$ **t** $7pq + 2p + 4qp - q$ **u** $7ab + 32 - ab + 4$

6, 7 | 7, 8 | 7, 8, 9(½)

PROBLEM-SOLVING

6 Ravi and Marissa each work for n hours per week. Ravi earns \$27 per hour and Marissa earns \$31 per hour.

a Write an expression for the amount Ravi earns in one week (in dollars).
b Write an expression for the amount Marissa earns in one week (in dollars).
c Write a simplified expression for the total amount Ravi and Marissa earn in one week (in dollars).

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5D

PROBLEM-SOLVING

7 The length of the line segment shown could be expressed as $a + a + 3 + a + 1$.

a Write the length in the simplest form.
b What is the length of the segment if a is equal to 5?

8 Let x represent the number of marbles in a standard-sized bag. Xavier bought 4 bags and Cameron bought 7 bags. Write simplified expressions for:

a the number of marbles Xavier has
b the number of marbles Cameron has
c the total number of marbles that Xavier and Cameron have
d the number of *extra* marbles that Cameron has compared to Xavier

9 Simplify the following by collecting like terms.

a $3xy + 4xy + 5xy$	**b** $4ab + 5 + 2ab$	**c** $5ab + 3ba + 2ab$
d $10xy - 4yx + 3$	**e** $10 - 3xy + 8xy + 4$	**f** $3cde + 5ecd + 2ced$
g $4 + x + 4xy + 2xy + 5x$	**h** $12ab + 7 - 3ab + 2$	**i** $3xy - 2y + 4yx$

	10	10	11

REASONING

10 **a** Test, using a table of values, that $3x + 2x$ is equivalent to $5x$.
b Prove that $3x + 2y$ is not equivalent to $5xy$.

11 **a** Test that $5x + 4 - 2x$ is equivalent to $3x + 4$.
b Prove that $5x + 4 - 2x$ is not equivalent to $7x + 4$.
c Prove that $5x + 4 - 2x$ is not equivalent to $7x - 4$.

How many rearrangements?	—	—	12

ENRICHMENT

12 The expression $a + 3b + 2a$ is equivalent to $3a + 3b$.

a List two other expressions with three terms that are equivalent to $3a + 3b$.
b How many expressions, consisting of exactly three terms added together, are equivalent to $3a + 3b$? All coefficients must be whole numbers greater than 0.

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Progress quiz

5A **1** For the expression $7a + 4b + c + 9$, answer the following.

- **a** State the number of terms.
- **b** List the individual terms.
- **c** State the coefficient of b.
- **d** What is the constant term?

5A **2** Write an expression for each of the following.

- **a** the product of m and p
- **b** the sum of a and k
- **c** 8 more than t
- **d** 4 less than w

5A **3** Write an expression for each of the following without using the $\div$ or $\times$ symbols.

- **a** m is halved, then 7 is added.
- **b** 7 is added to m and then the result is halved.
- **c** The sum of a and k is taken and then divided by 3.
- **d** The sum of a and one-third of k.
- **e** 12 is subtracted from d and the result is tripled.
- **f** d is tripled and 12 is subtracted from the result.

5B **4** If $x = 4$, evaluate each of the following.

- **a** $3x + 7$
- **b** $\frac{20}{x} + 2 - x$
- **c** $18 - (2x + 1)$

5B **5** Substitute $a = 5$ and $b = 2$ into each of these expressions and evaluate.

- **a** $3a + b + 7$
- **b** $20 - (a + 2b)$
- **c** $\frac{35}{a + b}$
- **d** $5 + a^2 - 2b^2$
- **e** $\sqrt{a^2 - 16}$

5C **6** Which two of the following expressions are equivalent?
$3a + 4$, $4a + 3$, $4 + 3a$, $7a$

5D **7** Classify the following pairs as like terms (L) or not like terms (N).

- **a** $3a$ and $8a$
- **b** $3x$ and $3xy$
- **c** 6 and $6a$
- **d** $4mp$ and $5pm$

5D **8** Simplify the following by collecting like terms.

- **a** $7a + 2b + 5 + a + 3b$
- **b** $2cd + 4c + 8d + 5dc - c + 4$

5C **9** Archie has two part-time jobs each paying \$8 per hour. He works x hours at one job and y hours at the other. Write two equivalent expressions for the total amount of money, in dollars, that he earns.

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5E Multiplying and dividing expressions

To multiply a number by a pronumeral, we have already seen we can write them next to each other. For example, $7a$ means $7 \times a$, and $5abc$ means $5 \times a \times b \times c$. The order in which numbers or pronumerals are multiplied is unimportant, so $5 \times a \times b \times c = a \times 5 \times c \times b = c \times a \times 5 \times b$. When writing a product without $\times$ signs, the numbers are written first.

We write $\frac{7xy}{3xz}$ as shorthand for $(7xy) \div (3xz)$.

We can simplify fractions like $\frac{10}{15}$ by dividing by common factors such as $\frac{10}{15} = \frac{\cancel{5} \times 2}{\cancel{5} \times 3} = \frac{2}{3}$.

Similarly, common pronumerals can be cancelled in a division like $\frac{7xy}{3xz}$, giving $\frac{7\cancel{x}y}{3\cancel{x}\cancel{z}} = \frac{7y}{3z}$.

Let's start: Rearranging terms

$5abc$ is equivalent to $5bac$ because the order of multiplication does not matter. In what other ways could $5abc$ be written?

$5 \times a \times b \times c = ?$

- $a \times b$ is written as ab.
- $a \div b$ is written as $\frac{a}{b}$.
- $a \times a$ is written as a^2.
- Because of the commutative property of multiplication (e.g. $2 \times 7 = 7 \times 2$), the order in which values are multiplied is not important. So $3 \times a$ and $a \times 3$ are equivalent.
- Because of the associative property of multiplication (e.g. $3 \times (5 \times 2)$ and $(3 \times 5) \times 2$ are equal), brackets are not required when only multiplication is used. So $3 \times (a \times b)$ and $(3 \times a) \times b$ are both written as $3ab$.
- Numbers should be written first in a term and pronumerals are generally written in alphabetical order. For example, $b \times 2 \times a$ is written as $2ab$.
- When dividing, any common factor in the numerator and denominator can be cancelled.
  For example: $\frac{{}^{2}\cancel{4}a^{1}\cancel{b}}{{}_{1}\cancel{2}^{1}\cancel{b}c} = \frac{2a}{c}$

Example 10 Simplifying expressions with multiplication

a Write $4 \times a \times b \times c$ without multiplication signs.

b Simplify $4a \times 2b \times 3c$, giving your final answer without multiplication signs.

c Simplify $3w \times 4w$.

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SOLUTION	EXPLANATION
a $4 \times a \times b \times c = 4abc$	When pronumerals are written next to each other they are being multiplied.
b $4a \times 2b \times 3c = 4 \times a \times 2 \times b \times 3 \times c$ $= 4 \times 2 \times 3 \times a \times b \times c$ $= 24abc$	First insert the missing multiplication signs. Rearrange to bring the numbers to the front. $4 \times 2 \times 3 = 24$ and $a \times b \times c = abc$, giving the final answer.
c $3w \times 4w = 3 \times w \times 4 \times w$ $= 3 \times 4 \times w \times w$ $= 12w^2$	First insert the missing multiplication signs. Rearrange to bring the numbers to the front. $3 \times 4 = 12$ and $w \times w$ is written as w^2.

Example 11 Simplifying expressions with division

a Write $(3x + 1) \div 5$ without a division sign.

b Simplify the expression $\frac{8ab}{12b}$.

SOLUTION	EXPLANATION
a $(3x + 1) \div 5 = \frac{3x + 1}{5}$	The brackets are no longer required as it becomes clear that all of $3x + 1$ is being divided by 5.
b $\frac{8ab}{12b} = \frac{8 \times a \times b}{12 \times b}$ $= \frac{2 \times \cancel{4} \times a \times \cancel{b}}{3 \times \cancel{4} \times \cancel{b}}$ $= \frac{2a}{3}$	Insert multiplication signs to help spot common factors. 8 and 12 have a common factor of 4. Cancel out the common factors of 4 and b.

Exercise 5E

1–4 | 3, 4 | —

UNDERSTANDING

1 Chen claims that $7 \times d$ is equivalent to $d \times 7$.

a If $d = 3$, find the values of $7 \times d$ and $d \times 7$.

b If $d = 5$, find the values of $7 \times d$ and $d \times 7$.

c If $d = 8$, find the values of $7 \times d$ and $d \times 7$.

d Is Chen correct in his claim?

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5E

UNDERSTANDING

2 Classify each of the following statements as true or false.

a $4 \times n$ can be written as $4n$.
b $n \times 3$ can be written as $3n$.
c $4 \times b$ can be written as $b + 4$.
d $a \times b$ can be written as ab.
e $a \times 5$ can be written as $50a$.
f $a \times a$ can be written as $2a$.

3 **a** Simplify the fraction $\frac{12}{18}$. (Note: This is the same as $\frac{2 \times 6}{3 \times 6}$.)

b Simplify the fraction $\frac{2000}{3000}$. (Note: This is the same as $\frac{2 \times 1000}{3 \times 1000}$.)

c Simplify $\frac{2a}{3a}$. (Note: This is the same as $\frac{2 \times a}{3 \times a}$.)

4 Match up these expressions with the correct way to write them.

a $2 \times u$	**A** $3u$
b $7 \times u$	**B** $\frac{5}{u}$
c $5 \div u$	**C** $2u$
d $u \times 3$	**D** $\frac{u}{5}$
e $u \div 5$	**E** $7u$

5–8(½) 5–8(½) 5–8(½)

FLUENCY

Example 10a

5 Write each of these expressions without any multiplication signs.

a $2 \times x$	**b** $5 \times p$	**c** $8 \times a \times b$
d $3 \times 2 \times a$	**e** $7 \times 4 \times f$	**f** $5 \times 2 \times a \times b$
g $2 \times 8 \times x \times y$	**h** $2 \times b \times 5$	**i** $x \times 7 \times z \times 4$

Example 10b,c

6 Simplify these expressions.

a $3a \times 12$	**b** $7d \times 9$	**c** $2 \times 4e$
d $3 \times 5a$	**e** $4a \times 3b$	**f** $7e \times 9g$
g $8a \times bc$	**h** $4d \times 7af$	**i** $a \times 3b \times 4c$
j $2a \times 4b \times c$	**k** $4d \times 3e \times 5fg$	**l** $2cb \times 3a \times 4d$
m $w \times w$	**n** $a \times a$	**o** $3d \times d$
p $2k \times k$	**q** $p \times 7p$	**r** $q \times 3q$
s $6x \times 2x$	**t** $3z \times 5z$	**u** $9r \times 4r$

Example 11a

7 Write each expression without a division sign.

a $x \div 5$	**b** $z \div 2$	**c** $a \div 12$
d $b \div 5$	**e** $2 \div x$	**f** $5 \div d$
g $x \div y$	**h** $a \div b$	**i** $(4x + 1) \div 5$
j $(2x + y) \div 5$	**k** $(2 + x) \div (1 + y)$	**l** $(x - 5) \div (3 + b)$

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5E

Example 11b

FLUENCY

8 Simplify the following expressions by dividing by any common factors. Remember that $\frac{a}{1} = a$.

a $\frac{2x}{5x}$ **b** $\frac{5a}{9a}$ **c** $\frac{9ab}{4b}$ **d** $\frac{2ab}{5a}$

e $\frac{2x}{4}$ **f** $\frac{9x}{12}$ **g** $\frac{10a}{15a}$ **h** $\frac{30y}{40y}$

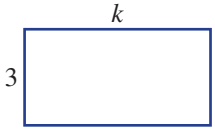
i $\frac{4a}{2}$ **j** $\frac{21x}{7x}$ **k** $\frac{4xy}{2x}$ **l** $\frac{9x}{3xy}$

9, 10 | 10, 12 | 10–12

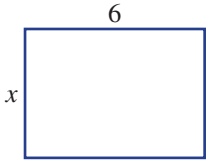
PROBLEM-SOLVING

9 Write a simplified expression for the area of the following rectangles. Recall that for rectangles, *Area* = *width* × *length*.

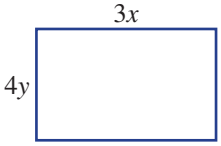
a



b



c



10 The weight of a single muesli bar is x grams.

a What is the weight of 4 bars? Write an expression.

b If Jamila buys n bars, what is the total weight of her purchase?

c Jamila's cousin Roland buys twice as many bars as Jamila. What is the total weight of Roland's purchase?

11 Five friends go to a restaurant. They split the bill evenly, so each spends the same amount.

a If the total cost is \$100, how much do they each spend?

b If the total cost is \$$C$, how much do they each spend? Write an expression.

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5E

PROBLEM-SOLVING

12 Replace the question marks with algebraic terms to make these equivalence statements true.

a $4c \times ? \times b = 12abc$

b $2a \times 2b \times ? = 28abc$

c $\frac{14ab}{?} = 2a$

d $\frac{12xy}{?} = x$

e $\frac{50x}{?} \times y = 5y$

13 | 13, 14 | 13, 14

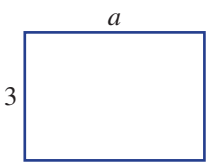
REASONING

13 The expression $3 \times 2p$ is the same as the expression $\underset{(1)}{2p} + \underset{(2)}{2p} + \underset{(3)}{2p}$.

a What is a simpler expression for $2p + 2p + 2p$? Hint: Combine like terms.

b $3 \times 2p$ is shorthand for $3 \times 2 \times p$. How does this relate to your answer in part **a**?

14 The area of the rectangle shown is $3a$. The length and width of this rectangle are now doubled.



a Draw the new rectangle, showing its dimensions.

b Write a simplified expression for the area of the new rectangle.

c Divide the area of the new rectangle by the area of the old rectangle. What do you notice?

d What happens to the area of the original rectangle if you triple both the length and the width?

Managing powers | — | — | 15

ENRICHMENT

15 The expression $a \times a$ can be written as a^2 and the expression $a \times a \times a$ can be written as a^3.

a What is $3a^2b^2$ when written in full with multiplication signs?

b Write $7 \times x \times x \times y \times y \times y$ without any multiplication signs.

c Simplify $2a \times 3b \times 4c \times 5a \times b \times 10c \times a$.

d Simplify $4a^2 \times 3ab^2 \times 2c^2$.

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5F Expanding brackets EXTENDING

Interactive

We have already seen that there are different ways of writing two equivalent expressions. For example, $4a + 2a$ is equivalent to $2 \times 3a$, even though they look different.

Note that $3(7 + a) = 3 \times (7 + a)$, which is equivalent to 3 lots of $7 + a$.

Widgets

So, $3(7 + a) = 7 + a + 7 + a + 7 + a$

$= 21 + 3a$

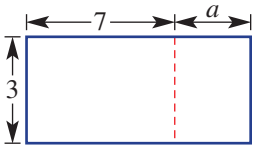
HOTsheets

It is sometimes useful to have an expression that is written with brackets, like $3 \times (7 + a)$, and sometimes it is useful to have an expression that is written without brackets, like $21 + 3a$.

Walkthroughs

Let's start: Total area

What is the total area of the rectangle shown at right? Try to write two expressions, only one of which includes brackets.



Key ideas

- **Expanding** (or **eliminating**) brackets involves writing an equivalent expression without brackets. This can be done by writing the bracketed portion a number of times or by multiplying each term.

  $2(a + b) = a + b + a + b$ or $2(a + b) = 2 \times a + 2 \times b$

  $= 2a + 2b$ $= 2a + 2b$
- To eliminate brackets, you can use the **distributive law**, which states that:

  $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
- The distributive law is used in arithmetic. For example:

$$\begin{aligned} 5 \times 27 &= 5(20 + 7) \\ &= 5 \times 20 + 5 \times 7 \\ &= 100 + 35 \\ &= 135 \end{aligned}$$

- The process of removing brackets using the distributive law is called **expansion**.
- When expanding, every term inside the brackets must be multiplied by the term outside the brackets.

Many of the simpler expressions in algebra can be thought of in terms of the areas of rectangles.

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Example 12 Expanding brackets by simplifying repeated terms

Repeat the expression that is inside the brackets and then collect like terms. The number outside the brackets is the number of copies required.

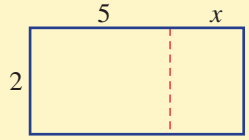
a $2(a + k)$ **b** $3(2m + 5)$

SOLUTION	EXPLANATION
a $2(a + k) = a + k + a + k$ $= 2a + 2k$	Two copies of the expression $a + k$. Simplify by collecting the like terms.
b $3(2m + 5) = 2m + 5 + 2m + 5 + 2m + 5$ $= 6m + 15$	Three copies of the expression $2m + 5$. Simplify by collecting the like terms.

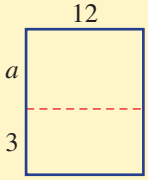
Example 13 Expanding brackets using rectangle areas

Write two equivalent expressions for the area of each rectangle shown, only one of which includes brackets.

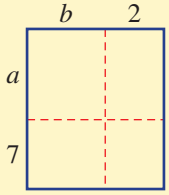
a



b



c



SOLUTION	EXPLANATION
a Using brackets: $2(5 + x)$	The whole rectangle has height 2 and width $5 + x$.
Without brackets: $10 + 2x$	The smaller rectangles have area $2 \times 5 = 10$ and $2 \times x = 2x$, so they are added.
b Using brackets: $12(a + 3)$	The dimensions of the whole rectangle are 12 and $a + 3$. Note that, by convention, we do not write $(a + 3)12$.
Without brackets: $12a + 36$	The smaller rectangles have area $12 \times a = 12a$ and $12 \times 3 = 36$.
c Using brackets: $(a + 7)(b + 2)$	The whole rectangle has height $a + 7$ and width $b + 2$. Note that brackets are used to ensure we are multiplying the entire height by the entire width.
Without brackets: $ab + 2a + 7b + 14$	The diagram can be split into four rectangles, with areas ab, $2a$, $7b$ and 14.

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Example 14 Expanding using the distributive law

Expand the following expressions.

a $5(x + 3)$ **b** $8(a - 4)$

c $3(a + 2b)$ **d** $5a(3p - 7q)$

SOLUTION	EXPLANATION
a $5(x + 3) = 5 \times x + 5 \times 3$	Use the distributive law: $5(x + 3) = 5x + 5 \times 3$
$= 5x + 15$	Simplify the result.
b $8(a - 4) = 8 \times a - 8 \times 4$	Use the distributive law with subtraction:
$= 8a - 32$	$8(a - 4) = 8a - 8 \times 4$
	Simplify the result.
c $3(a + 2b) = 3 \times a + 3 \times 2b$	Use the distributive law:
$= 3a + 6b$	$3(a + 2b) = 3a + 3 \times 2b$
	Simplify the result, remembering that $3 \times 2b = 6b$.
d $5a(3p - 7q) = 5a \times 3p - 5a \times 7q$	Expanding: $5a(3p - 7q) = 5a \times 3p - 5a \times 7q$
$= 15ap - 35aq$	Simplify the result, remembering that $5a \times 3p = 15ap$ and $5a \times 7q = 35aq$.

Exercise 5F

1–4 | 2, 3 | —

UNDERSTANDING

Example 12

1 The expression $3(a + 2)$ can be written as $(a + 2) + (a + 2) + (a + 2)$.

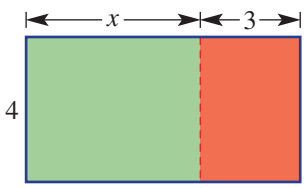
a Simplify this expression by collecting like terms.

b Write $2(x + y)$ in full without brackets and simplify the result.

c Write $4(p + 1)$ in full without brackets and simplify the result.

d Write $3(4a + 2b)$ in full without brackets and simplify the result.

2 The area of the rectangle shown can be written as $4(x + 3)$.



a What is the area of the green rectangle?

b What is the area of the red rectangle?

c Write the total area as an expression, without using brackets.

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5F

UNDERSTANDING

3 Copy and complete the following calculation using the distributive law.

a $3 \times 21 = 3 \times (20 + 1)$
$= 3 \times 20 + 3 \times 1$
$=$ ____ + ____
$=$ ____

b $7 \times 34 = 7 \times (30 + 4)$
$= 7 \times$ ___ $+ 7 \times$ ___
$=$ ____ + ____
$=$ ____

c $5 \times 19 = 5 \times (20 - 1)$
$= 5 \times$ ___ $- 5 \times$ ___
$=$ ____ $-$ ____
$=$ ____

4 **a** Copy and complete the following table. Remember to follow the rules for correct order of operations.

	$4(x + 3)$	$4x + 12$
$x = 1$	$= 4(1 + 3)$ $= 4(4)$ $= 16$	$= 4(1) + 12$ $= 4 + 12$ $= 16$
$x = 2$		
$x = 3$		
$x = 4$		

b Fill in the gap: The expressions $4(x + 3)$ and $4x + 12$ are _____.

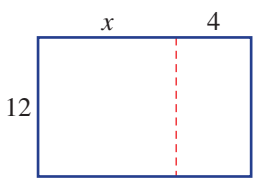
5, 6–8(½) 5, 6–8(½) 5, 6–8(½)

FLUENCY

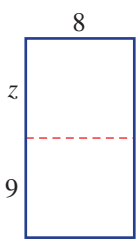
Example 13

5 For the following rectangles, write two equivalent expressions for the area.

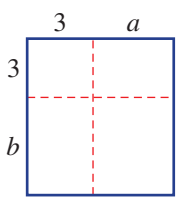
a



b



c



Example 14a,b

6 Use the distributive law to expand the following.

a $6(y + 8)$	**b** $7(l + 4)$	**c** $8(s + 7)$	**d** $4(2 + a)$
e $7(x + 5)$	**f** $3(6 + a)$	**g** $9(9 - x)$	**h** $5(j - 4)$
i $8(y - 8)$	**j** $8(e - 7)$	**k** $6(e - 3)$	**l** $10(8 - y)$

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5F

FLUENCY

Example 14c

7 Use the distributive law to expand the following.

a $10(6g - 7)$	**b** $5(3e + 8)$	**c** $5(7w + 10)$	**d** $5(2u + 5)$
e $7(8x - 2)$	**f** $3(9v - 4)$	**g** $7(q - 7)$	**h** $4(5c - v)$
i $2(2u + 6)$	**j** $6(8l + 8)$	**k** $5(k - 10)$	**l** $9(o + 7)$

Example 14d

8 Use the distributive law to expand the following.

a $6i(t - v)$	**b** $2d(v + m)$	**c** $5c(2w - t)$	**d** $6e(s + p)$
e $d(x + 9s)$	**f** $5a(2x + 3v)$	**g** $5j(r + 7p)$	**h** $i(n + 4w)$
i $8d(s - 3t)$	**j** $f(2u + v)$	**k** $7k(2v + 5y)$	**l** $4e(m + 10y)$

9, 10 | 10, 11 | 10–12

PROBLEM-SOLVING

9 Write an expression for each of the following and then expand it.

a A number, x, has 3 added to it and the result is multiplied by 5.

b A number, b, has 6 added to it and the result is doubled.

c A number, z, has 4 subtracted from it and the result is multiplied by 3.

d A number, y, is subtracted from 10 and the result is multiplied by 7.

10 In a school classroom there is one teacher as well as an unknown number of boys and girls.

a If the number of boys is b and the number of girls is g, write an expression for the total number of people in the classroom, including the teacher.

b The teacher and all the students are each wearing two socks. Write two different expressions for the total number of socks being worn, one with brackets and one without.

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5F

PROBLEM-SOLVING

11 When expanded, $4(3x + 6y)$ gives $12x + 24y$. Find two other expressions that expand to $12x + 24y$.

12 The distance around a rectangle is given by the expression $2(l + w)$, where l is the length and w is the width. What is an equivalent expression for this distance?

	13	13, 14	14, 15

REASONING

13 Use a diagram of a rectangle like that in Question **2** to demonstrate that $5(x + 3) = 5x + 15$.

14 Use a diagram of a rectangle to prove that $(a + 2)(b + 3) = ab + 2b + 3a + 6$.

15 When expanded, $5(2x + 4y)$ gives $10x + 20y$.

a How many different ways can the missing numbers be filled with whole numbers for the equivalence $\square(\square x + \square y) = 10x + 20y$?

b How many different expressions expand to give $10x + 20y$ if fractions or decimals are included?

Expanding sentences	—	—	16

ENRICHMENT

16 Using words, people do a form of expansion. Consider these two statements:

Statement A: 'John likes tennis and football.'

Statement B: 'John likes tennis and John likes football.'

Statement B is an 'expanded form' of statement A, which is equivalent in its meaning but more clearly shows that two facts are being communicated. Write an 'expanded form' of the following sentences.

a Rosemary likes Maths and English.

b Priscilla eats fruit and vegetables.

c Bailey and Lucia like the opera.

d Frank and Igor play video games.

e Pyodir and Astrid like fruit and vegetables.

(Note: There are four facts being communicated here.)

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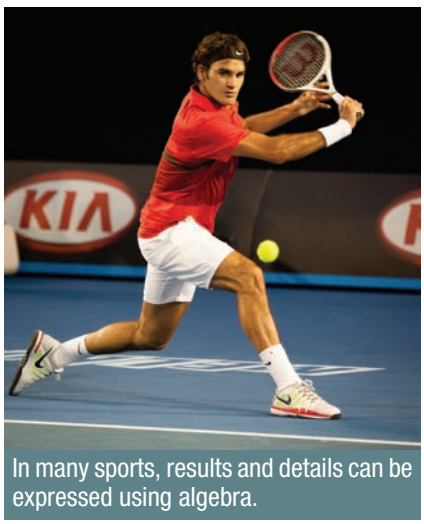
5G Algebraic modelling EXTENDING

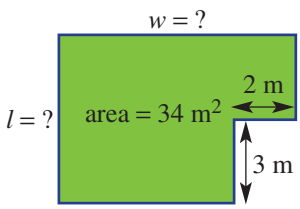
An algebraic expression can be used to describe problems relating to many different areas, including costs, speeds and sporting results. Much of modern science relies on the application of algebraic rules and formulas. It is important to be able to convert word descriptions of problems to mathematical expressions in order to solve these problems mathematically.



In many sports, results and details can be expressed using algebra.

Let's start: Garden bed area

The garden shown at right has an area of 34 m^2, but the width and length are unknown.



- What are some possible values that w and l could equal?
- Try to find the dimensions of the garden that make the fencing around the outside as small as possible.

Key ideas

- Many different situations can be modelled with algebraic expressions.
- To apply an expression, the pronumerals should be defined clearly. Then known values should be substituted for the pronumerals.

Example 15 Applying an expression

The perimeter of a rectangle is given by the expression $2w + 2l$, where w is the width and l is the length.

a Find the perimeter of a rectangle if $w = 5$ and $l = 7$.

b Find the perimeter of a rectangle with width 8 cm and length 3 cm.

SOLUTION	EXPLANATION
a $2w + 2l = 2(5) + 2(7)$ $= 10 + 14$ $= 24$	To apply the rule, we substitute $w = 5$ and $l = 7$ into the expression. Evaluate using the normal rules of arithmetic (i.e. multiplication before addition).
b $2w + 2l = 2(8) + 2(3)$ $= 16 + 6$ $= 22$ cm	Substitute $w = 8$ and $l = 3$ into the expression. Evaluate using the normal rules of arithmetic, remembering to include appropriate units (cm) in the answer.

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Example 16 Constructing expressions from problem descriptions

Write expressions for each of the following.

a The total cost, in dollars, of 10 bottles, if each bottle costs $\$x$.

b The total cost, in dollars, of hiring a plumber for n hours. The plumber charges a \$30 call-out fee plus \$60 per hour.

c A plumber charges a \$60 call-out fee plus \$50 per hour. Use an expression to find how much an 8-hour job would cost.

SOLUTION	EXPLANATION
a $10x$	Each of the 10 bottles costs $\$x$, so the total cost is $10 \times x = 10x$.
b $30 + 60n$	For each hour, the plumber charges \$60, so must pay $60 \times n = 60n$. The \$30 call-out fee is added to the total bill.
c Expression for cost: $60 + 50n$ If $n = 8$, then cost is $60 + 50 \times 8 = \$460$	Substitute $n = 8$ to find the cost for an 8-hour job. Cost will be \$460.

Exercise 5G

1–3 | 3 | —

UNDERSTANDING

1 The area of a rectangle is given by the expression $w \times l$, where w is its width and l is its length.

a Find the area if $w = 5$ and $l = 7$.

b Find the area if $w = 2$ and $l = 10$.

2 The perimeter of a square with width w is given by the expression $4w$.

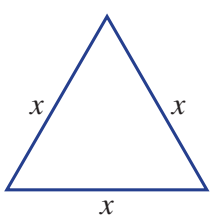
a Find the perimeter of a square with width 6 cm (i.e. $w = 6$).

b Find the perimeter of a square with width 10 m (i.e. $w = 10$).

3 Consider the equilateral triangle shown.

a Write an expression that gives the perimeter of this triangle.

b Use your expression to find the perimeter if $x = 12$.



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5G

4–7 4–8 5–8

FLUENCY

Example 16a

4 If pens cost \$2 each, write an expression for the cost, in dollars, of n pens.

5 If pencils cost \$$x$ each, write an expression for the cost, in dollars, of:

a 10 pencils

b 3 packets of pencils, if each packet contains 5 pencils

c k pencils

6 A car travels at 60 km/h, so in n hours it has travelled $60n$ kilometres.

a How far does the car travel in 3 hours (i.e. $n = 3$)?

b How far does the car travel in 30 minutes?

c Write an expression for the total distance (in km) travelled in n hours for a motorbike with speed 70 km/h.

7 A carpenter charges a \$40 call-out fee and then \$80 per hour. This means the total cost, in dollars, for x hours of work is $40 + 80x$.

a How much would it cost for a 2-hour job (i.e. $x = 2$)?

b How much would it cost for a job that takes 8 hours?

c The call-out fee is increased to \$50. What is the new expression for the total cost, in dollars, of x hours?

Example 16b

8 Match up the word problems with the expressions (**A** to **E**) below.

a The area of a rectangle with height 5 and width x.

b The perimeter of a rectangle with height 5 and width x.

c The total cost, in dollars, of hiring a DVD for x days if the price is \$1 per day.

d The total cost, in dollars, of hiring a builder for 5 hours if the builder charges a \$10 call-out fee and then \$$x$ per hour.

e The total cost, in dollars, of buying a \$5 magazine and a book that costs \$$x$.

A $10 + 2x$

B $5x$

C $5 + x$

D x

E $10 + 5x$

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9, 10 | 10–12 | 11–13

5G

PROBLEM-SOLVING

9 A plumber charges a $50 call-out fee and $100 per hour.

a Copy and complete the table below.

Number of hours	1	2	3	4	5
Total costs ($)					

b Find the total cost if the plumber works for t hours. Give an expression.

c Substitute $t = 30$ into your expression to find how much it will cost for the plumber to work 30 hours.

10 To hire a tennis court, you must pay a $5 booking fee plus $10 per hour.

a What is the cost of booking a court for 2 hours?

b What is the cost of booking a court for x hours? Write an expression.

c A tennis coach hires a court for 7 hours. Substitute $x = 7$ into your expression to find the total cost.

11 Adrian's mobile phone costs 30 cents to make a connection, plus 60 cents per minute of talking. This means that a t-minute call costs $30 + 60t$ cents.

a What is the cost of a 1-minute call?

b What is the cost of a 10-minute call? Give your answer in dollars.

c Write an expression for the cost of a t-minute call in dollars.

12 In Australian Rules football a goal is worth 6 points and a 'behind' is worth 1 point. This means the total score for a team is $6g + b$, if g goals and b behinds are scored.

a What is the score for a team that has scored 5 goals and 3 behinds?

b What are the values of g and b for a team that has scored 8 goals and 5 behinds?

c If a team has a score of 20, this could be because $g = 2$ and $b = 8$. What are the other possible values of g and b?

13 In a closing-down sale, a shop sells all CDs for $\$c$ each, books cost $\$b$ each and DVDs cost $\$d$ each. Claudia buys 5 books, 2 CDs and 6 DVDs.

a What is the cost of Claudia's order? Give your answer as an expression involving b, c and d.

b Write an expression for the cost of Claudia's order if CDs doubled in price and DVDs halved in price.

c As it happens, the total price Claudia ends up paying is the same in both situations. Given that CDs cost $12 and books cost $20 (so $c = 12$ and $b = 20$), how much do DVDs cost?

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5G

14 | 14 | 14, 15

REASONING

14 A shop charges $\$c$ for a box of tissues.

- **a** Write an expression for the total cost, in dollars, of buying n boxes of tissues.
- **b** If the original price is tripled, write an expression for the total cost, in dollars, of buying n boxes of tissues.
- **c** If the original price is tripled and twice as many boxes are bought, write an expression for the total cost in dollars.

15 Hiring a basketball court costs $10 for a booking fee, plus $30 per hour.

- **a** Write an expression for the total cost in dollars to hire the court for x hours.
- **b** For the cost of $40, you could hire the court for 1 hour. How long could you hire the court for the cost of $80?
- **c** Explain why it is *not* the case that hiring the court for twice as long costs twice as much.
- **d** Find the average cost per hour if the court is hired for a 5 hour basketball tournament.
- **e** Describe what would happen to the *average* cost per hour if the court is hired for many hours (e.g. more than 50 hours).

Mobile phone mayhem

— | — | 16

ENRICHMENT

16 Rochelle and Emma are on different mobile phone plans, as shown below.

	Connection	Cost of minute
Rochelle	20 cents	60 cents
Emma	80 cents	40 cents

- **a** Write an expression for the cost of making a t-minute call using Rochelle's phone.
- **b** Write an expression for the cost of making a t-minute call using Emma's phone.
- **c** Whose phone plan would be cheaper for a 7-minute call?
- **d** What is the length of call for which it would cost exactly the same for both phones?
- **e** Investigate current mobile phone plans and describe how they compare to those of Rochelle's and Emma's plans.

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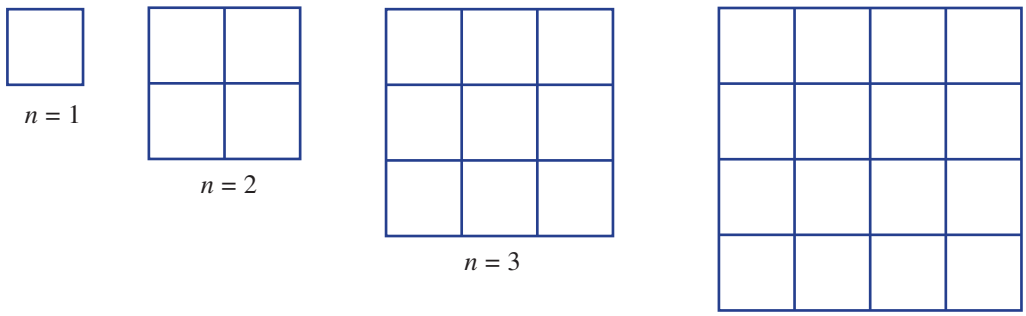
Investigation

Fencing paddocks

A farmer is interested in fencing off a large number of 1 m × 1 m foraging regions for the chickens. Consider the pattern below.



a For $n = 2$, the outside perimeter is 8 m, the area is 4 m^2 and the total length of fencing required is 12 m. Copy and complete the following table.

n	1	2	3	4	5	6
Outside perimeter (m)		8				
Area (m^2)		4				
Fencing required		12				

b Write an expression for:

i the total outside perimeter of the fenced section

ii the total area of the fenced section.

c The farmer knows that the expression for the total amount of fencing is one of the following. Which one is correct? Prove to the farmer that the others are incorrect.

i $6n$ **ii** $(n+1)^2$ **iii** $n \times 2 \times (n+1)$

d Use the correct formula to work out the total amount of fencing required if the farmer wants to have a total area of 100 m^2 fenced off.

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In a spreadsheet application these calculations can be made automatically. Set up a spreadsheet as follows.

	A	B	C	D
1	n	Perimeter	Area	Total fencing
2	0	0	0	0
3	=A2+1	=A3*4	=A3*A3	=A3*2*(A3+1)

Drag down the cells until you have all the rows from $n = 0$ to $n = 30$.

e Find the amount of fencing needed if the farmer wants the total area to be at least:

i 25 m^2 **ii** 121 m^2

iii 400 m^2 **iv** 500 m^2

f If the farmer has 144 m of fencing, what is the maximum area his grid could have?

g For each of the following lengths of fencing, give the maximum area, in m^2, that the farmer could contain in the grid.

i 50 m **ii** 200 m **iii** 1 km **iv** 40 km

h In the end, the farmer decides that the overall grid does not need to be a square, but could be any rectangular shape. Design rectangular paddocks with the following properties.

a perimeter = 20 m and area = 21 m^2

b perimeter = 16 m and fencing required = 38 m^2

c area = 1200 m^2 and fencing required = 148 m

d perimeter = 1 km and fencing required is less than 1.5 km

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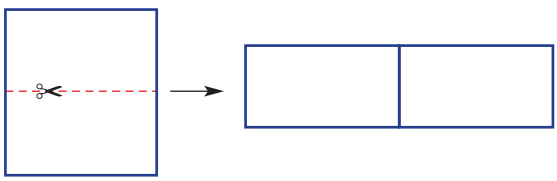
Problems and challenges

1 If $x + y = 8$ and $y + m = 17$ find the value of $x + 2y + m$.

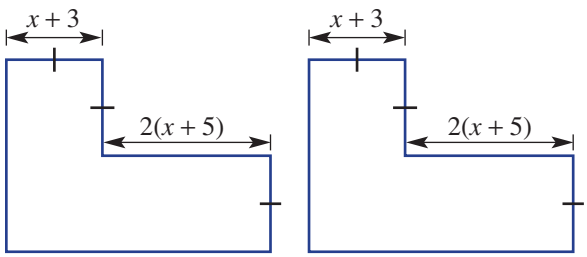
2 A square is cut in half and the two identical rectangles are joined to form a rectangle as shown in this diagram.



Find an expression for the perimeter of the rectangle if the square has a side length of:

a $2m$ **b** $4(x + 3)$ **c** $w + y$

3 These two identical 'L' shapes are to be joined along identical (matching) sides without any overlap. Find a simplified algebraic expression for the largest and smallest possible perimeters of the joined shapes and also for the difference between these two perimeters. Calculate this difference when $x = 10$ cm. The diagrams are not drawn to scale.



4 In a list of five consecutive integers, the middle integer is $3a + 2$. Find two equivalent expressions for the sum of these five integers: one expanded and simplified, and one factorised.

5 Find the values of the pronumerals below in the following sum/product tables.

a

			Sum
	a	b	c
	d	24	32
Sum	12	e	48

b

			Product
	a	b	18
	2	c	d
Product	12	e	180

6 What is the coefficient of x once the expression $x + 2(x + 1) + 3(x + 2) + 4(x + 3) + \ldots + 100(x + 99)$ is simplified completely?

7 Think of any number and then perform the following operations. Add 5, then double the result, then subtract 12, then subtract the original number, then add 2. Use algebra to explain why you now have the original number again. Then design a puzzle like this yourself and try it on friends.

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Chapter summary

Algebra

Creating expressions

6 more than k: $k + 6$
Product of 4 and x: $4x$
10 less than b: $b - 10$
Half of q: $\frac{q}{2}$
The sum of a and b is tripled: $3(a + b)$

Pronumerals

are letters used to represent numbers
g: number of grapes in a bunch
d: distance travelled (in metres)

Terms are pronumerals and numbers combined with $\times$, $\div$
e.g. $4x$, $10y$, $\frac{a}{3}$, 12
3a means $3 \times a$
$\frac{b}{10}$ means $b \div 10$

Like terms have exactly the same pronumerals.
$5a$ and $3a$ ✓
$2ab$ and $12ba$ ✓
$7ab$ and $2a$ ×

Algebraic expressions

Combination of numbers, pronumerals and operations, e.g. $2xy + 3yz$, $\frac{12}{x} - 3$

Equivalent expressions

Always equal when pronumerals are substituted.
e.g. $2x + 3$ and $3 + 2x$ are equivalent.
$4(3x)$ and $12x$ are equivalent.

Substitution

Replacing pronumerals with values.
e.g. $5x + 2y$ when $x = 10$ & $y = 3$ becomes $5(10) + 2(3) = 50 + 6 = 56$
e.g. q^2 when $q = 7$ becomes $7^2 = 49$

To **simplify** an expression, find a simpler expression that is equivalent.

Combining like terms gives a way to simplify.
e.g. $4a + 2 + 3a = 7a + 2$
$3b + 5c + 2b - c = 5b + 4c$
$12xy + 3x - 5yx = 7xy + 3x$

Applications (Ext)

Commonly used expressions
$A = l \times w$
$P = 2l + 2w$

Cost is $50 + 90x$
call-out fee
hourly rate

Expanding brackets (Ext)

$3(a + 4) = 3a + 12$
$5k(10 - 2j) = 50k - 10kj$
Using the distributive law gives an equivalent expression.

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Multiple-choice questions

5A **1** In the expression $3x + 2y + 4xy + 7yz$ the coefficient of y is:
A 3 **B** 2 **C** 4 **D** 7 **E** 16

5B **2** If $t = 5$ and $u = 7$, then $2t + u$ is equal to:
A 17 **B** 32 **C** 24 **D** 257 **E** 70

5B **3** If $x = 2$, then $3x^2$ is equal to:
A 32 **B** 34 **C** 12 **D** 25 **E** 36

5D **4** Which of the following pairs does *not* consist of two like terms?
A $3x$ and $5x$ **B** $3y$ and $12y$ **C** $3ab$ and $2ab$
D $3cd$ and $5c$ **E** $3xy$ and yx

5D **5** A fully simplified expression equivalent to $2a + 4 + 3b + 5a$ is:
A 4 **B** $5a + 5b + 4$ **C** $10ab + 4$
D $7a + 3b + 4$ **E** $11ab$

5E **6** The simplified form of $4x \times 3yz$ is:
A $43xyz$ **B** $12xy$ **C** $12xyz$ **D** $12yz$ **E** $4x3yz$

5E **7** The simplified form of $\frac{21ab}{3ac}$ is:
A $\frac{7b}{c}$ **B** $\frac{7ab}{ac}$ **C** $\frac{21b}{3c}$ **D** 7 **E** $\frac{b}{7c}$

5F Ext **8** When brackets are expanded, $4(2x + 3y)$ becomes:
A $8x + 3y$ **B** $2x + 12y$ **C** $8x + 8y$ **D** $24x$ **E** $8x + 12y$

5F Ext **9** The fully simplified form of $2(a + 7b) - 4b$ is:
A $2a + 10b$ **B** $2a + 3b$ **C** $a + 3b$
D $2a + 14b - 4b$ **E** $2a + 18b$

5A **10** A number is doubled and then 5 is added. The result is then tripled. If the number is represented by k, then an expression for this description is:
A $3(2k + 5)$ **B** $6(k + 5)$ **C** $2k + 5$
D $2k + 15$ **E** $30k$

Short-answer questions

5A **1** **a** List the four individual terms in the expression $5a + 3b + 7c + 12$.
b What is the constant term in the expression above?

5A **2** Write an expression for each of the following.
a 7 is added to u **b** k is tripled **c** 7 is added to half of r
d 10 is subtracted from h **e** the product of x and y **f** x is subtracted from 12

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Chapter review

5B **3** If $u = 12$, find the value of:

a $u + 3$ **b** $2u$ **c** $\frac{24}{u}$ **d** $3u - 4$

5B **4** If $p = 3$ and $q = 5$, find the value of:

a pq **b** $p + q$ **c** $2(q - p)$ **d** $4p + 3q$

5B **5** If $t = 4$ and $u = 10$, find the value of:

a t^2 **b** $2u^2$ **c** $3 + \sqrt{t}$ **d** $\sqrt{10tu}$

5C **6** For each of the following pairs of expressions, state whether they are equivalent (E) or not equivalent (N).

a $5x$ and $2x + 3x$ **b** $7a + 2b$ and $9ab$

c $3c - c$ and $2c$ **d** $3(x + 2y)$ and $3x + 2y$

5D **7** Classify the following pairs as like terms (L) or not like terms (N).

a $2x$ and $5x$ **b** $7ab$ and $2a$ **c** $3p$ and p

d $9xy$ and $2yx$ **e** $4ab$ and $4aba$ **f** $8t$ and $2t$

g $3p$ and 3 **h** $12k$ and $120k$

5D **8** Simplify the following by collecting like terms.

a $2x + 3 + 5x$ **b** $12p - 3p + 2p$ **c** $12b + 4a + 2b + 3a + 4$

d $12mn + 3m + 2n + 5nm$ **e** $1 + 2c + 4h - 3o + 5c$ **f** $7u + 3v + 2uv - 3u$

5E **9** Simplify the following expressions involving products.

a $3a \times 4b$ **b** $2xy \times 3z$ **c** $12f \times g \times 3h$ **d** $8k \times 2 \times 4lm$

5E **10** Simplify the following expressions involving quotients.

a $\frac{3u}{2u}$ **b** $\frac{12y}{20y}$ **c** $\frac{2ab}{6b}$ **d** $\frac{12xy}{9yz}$

5F Ext **11** Expand the following expressions using the distributive law.

a $3(x + 2)$ **b** $4(p - 3)$ **c** $7(2a + 3)$ **d** $12(2k + 3l)$

5F Ext **12** Give two examples of expressions that expand to give $12b + 18c$.

5G Ext **13** If tins of paints weigh 9 kg, write an expression for the weight in kg of t tins of paint.

5G Ext **14** If there are g girls and b boys in a room, write an expression for the total number of children in the room.

5G Ext **15** Write an expression for the total number of books that Analena owns if she has x fiction books and twice as many non-fiction books.

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Extended-response questions

1 A taxi driver charges $3.50 to pick up passengers and then $2.10 per kilometre travelled.

a State the total cost if the trip length is:

i 10 km

ii 20 km

iii 100 km

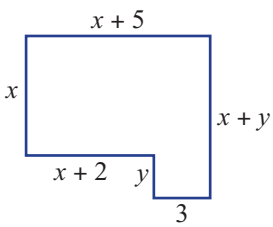
b Write an expression for the total cost, in dollars, of travelling a distance of d kilometres.

c Use your expression to find the total cost of travelling 40 km.

d Prove that your expression is not equivalent to $2.1 + 3.5d$ by substituting in a value for d.

e Another taxi driver charges $6 to pick up passengers and then $1.20 per kilometre. Write an expression for the total cost (in dollars) of travelling d kilometres in this taxi.

2 An architect has designed a room, shown opposite, for which x and y are unknown. (All measurements are in metres.)



a Find the perimeter of this room if $x = 3$ and $y = 2$.

b It costs $3 per metre to install skirting boards around the perimeter of the room. Find the total cost of installing skirting boards if the room's perimeter is $x = 3$ and $y = 2$.

c Write an expression for the perimeter (in metres) of the room and simplify it completely.

d Write an expanded expression for the total cost, in dollars, of installing skirting boards along the room's perimeter.

e Write an expression for the total floor area in m^2.

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