Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Tasks
## Set 1

## TASK LIST




**Produced by a Department of Education - MAWA partnership project**

**Written for the Year 7 Australian Curriculum**

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# The Implexus-5000

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 200: THE IMPLEXUS-5000

## Overview

This task revises the concept of indices. A basic knowledge of multiplication is assumed. A good follow-on for this task would be Task 201 Investigating Indices.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

## Students can demonstrate

- *fluency* when they
  - write numbers in index notation
- *understanding* when they
  - connect the relationship between expanded notation, index notation and standard form
- *reasoning* when they
  - explain why a number raised to the power of 0 is always 1

This is the IMPLEXUS-5000. It looks like a microwave, but it isn't. It takes the number 1 and makes it bigger.

All you do is put in the number 1 into the IMPLEXUS-5000 and choose the "growth factor" you would like from the control panel.

| | | |
|---|---|---|
| x2 | x3 | x4 |
| x5 | x6 | x7 |
| x8 | x9 | x10 |

So, if you chose x3, after one second the IMPLEXUS-5000 would beep and you would have a shiny number 3 come out (instead of the 1 that you put in there)

And if you chose x5, after one second the IMPLEXUS-5000 would beep and you would have shiny number 5 (because the 1 you put in multiplied by five equals 5)

Of course, like any microwave you can also choose how long the IMPLEXUS-5000 grows your number for.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

So if you put in the number 1, and choose a "growth factor" of x3 and a "growth time" of 2 seconds, what number would the IMPLEXUS-5000 give you?

It would give you a 9, because 1 x 3 x 3 = 9

## Activity 1

1. List 5 different numbers using the IMPLEXUS-5000. Write down the “growth factor” and the “growth time” you would need to make each number.
   Various Answers

2. What numbers could the IMPLEXUS-5000 grow in 2 seconds? List as many as you can find.
   4, 9, 16, 25, 36, 49, 64, 81, 100

3. What numbers could the IMPLEXUS-5000 grow in 3 seconds? List as many as you can find.
   8, 27, 64, 125, 216, 343, 512, 729, 1000

4. What is the biggest number you could grow using the IMPLEXUS-5000?
   Growth factor: x10; Growth time: 9 seconds
   1,000,000,000

5. DISCUSSION QUESTION: Why do you think there isn’t a growth factor of x1 on the IMPLEXUS-5000?
   A growth factor of one keeps our number the same, so it isn’t needed.

6. DISCUSSION QUESTION: What would you get if you chose a growth factor of x3 for 0 seconds? Do you get the same answer for a different growth factor? Why or Why not?
   You would get 1 – although you have chosen a growth factor, you haven’t allowed any time for the number to grow.

## Activity 2

Lets say that you grew the number 8 by choosing a “growth factor” of x2 and a “growth time” of 3 seconds.

How could you tell someone who doesn’t speak English to grow your number?

You could do it like this:

$$1 \times 2 \times 2 \times 2$$

This form is called expanded notation because we write out each step individually.

1. Have a go at writing out the following numbers in expanded notation.

   a. Growth Factor: x2, Growth time: 4 Seconds
      1 x2 x2 x2 x2

   b. Growth Factor: x3, Growth time: 2 Seconds
      1 x3 x3

   c. Growth Factor: x5, Growth time: 3 Seconds
      1 x5 x5 x5

d. Growth Factor: x2, Growth time: 6 Seconds

1 x2 x2 x2 x2 x2 x2

e. Growth Factor: x21, Growth time: 3 Seconds

1 x21 x21 x21

2. For each of the situations in Question 1, write down the number that you would grow in standard form.

16  9  125  64  9261

3. DISCUSSION QUESTION: Do we need to include the number 1 in our expanded notation? Why or Why not?

No. Multiplying any number by 1 does not change its value, so it is not really necessary to include it in our expanded notation.

Activity 3

Now imagine that you chose growth factor of x10 and that you left your number in the IMPLEXUS-5000 for 180 seconds.

You wouldn't want to write that out in expanded form! It would take too long.

So here is another way we can write these numbers:

Index ("growth time")

$$10^{180}$$

Base ("growth factor")

The 10 is your growth factor which Mathematicians call the **base** and 180 is the growth time which mathematicians call an **index.**

Write down the following in expanded form, index notation and standard form.

| Instructions | Expanded Form | Index Notation | Standard form |
|---|---|---|---|
| Growth Factor: x10<br>Growth Time: 5 seconds | 10 x10 x10 x10 x10 | $10^5$ | 100 000 |
| Growth Factor: x7<br>Growth Time: 3 seconds | 7 x7 x7 | $7^3$ | 343 |
| Growth Factor: x12<br>Growth Time: 2 seconds | 12 x12 | $12^2$ | 144 |
| Growth Factor: x2<br>Growth Time: 4 seconds | 2 x2 x2 x2 | $2^4$ | 16 |
| Growth Factor: x3<br>Growth Time: 6 seconds | 3 x3 x3 x3 x3 x3 | $3^6$ | 729 |
| Growth Factor: x6<br>Growth Time: 1 seconds | 6 | $6^1$ | 6 |
| Growth Factor: x9<br>Growth Time: 7 seconds | 9 x9 x9 x9 x9 x9 x9 | $9^7$ | 4 782 969 |
| Growth Factor: x3<br>Growth Time: 8 seconds | 3 x3 x3 x3 x3 x3 x3 x3 | $3^8$ | 6561 |
| Growth Factor: x1<br>Growth Time: 10 seconds | 1 x1 x1 x1 x1 x1 x1 x1 x1 x1 | $1^{10}$ | 1 |
| Growth Factor: x8<br>Growth Time: 0 seconds | 1 | $8^0$ | 1 |

This is the IMPLEXUS-5000. It looks like a microwave, but it isn't. It takes the number 1 and makes it bigger.

All you do is put in the number 1 into the IMPLEXUS-5000 and choose the "growth factor" you would like from the control panel.

| | | |
|---|---|---|
| x2 | x3 | x4 |
| x5 | x6 | x7 |
| x8 | x9 | x10 |

So, if you chose x3, after one second the IMPLEXUS-5000 would beep and you would have a shiny number 3 come out (instead of the 1 that you put in there).

And if you chose x5, after one second the IMPLEXUS-5000 would beep and you would have shiny number 5 (because the 1 you put in multiplied by five equals 5).

Of course, like any microwave you can also choose how long the IMPLEXUS-5000 grows your number for.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

So if you put in the number 1, and choose a "growth factor" of x3 and a "growth time" of 2 seconds, what number would the IMPLEXUS-5000 give you?

It would give you a 9, because 1 x 3 x 3 = 9

Activity 1

1. List 5 different numbers using the IMPLEXUS-5000. Write down the “growth factor” and the “growth time” you would need to make each number.

2. What numbers could the IMPLEXUS-5000 grow in 2 seconds? List as many as you can find.

3. What numbers could the IMPLEXUS-5000 grow in 3 seconds? List as many as you can find.

4. What is the biggest number you could grow using the IMPLEXUS-5000?

5. DISCUSSION QUESTION: Why do you think there isn’t a growth factor of x1 on the IMPLEXUS-5000?

6. DISCUSSION QUESTION: What would you get if you chose a growth factor of x3 for 0 seconds? Do you get the same answer for a different growth factor? Why or Why not?

Activity 2

Let's say that you grew the number 8 by choosing a "growth factor" of x2 and a "growth time" of 3 seconds.

How could you tell someone who doesn't speak English to grow your number?

You could do it like this:

$$1 \times 2 \times 2 \times 2$$

This form is called expanded notation because we write out each step individually.

1. Have a go at writing out the following numbers in expanded notation.

   f. Growth Factor: x2, Growth time: 4 Seconds

   g. Growth Factor: x3, Growth time: 2 Seconds

   h. Growth Factor: x5, Growth time: 3 Seconds

   i. Growth Factor: x2, Growth time: 6 Seconds

   j. Growth Factor: x21, Growth time: 3 Seconds

2. For each of the situations in Question 1, write down the number that you would grow in standard form.

3. DISCUSSION QUESTION: Do we need to include the number 1 in our expanded notation? Why or Why not?

## Activity 3

Now imagine that you chose growth factor of x10 and that you left your number in the IMPLEXUS-5000 for 180 seconds.

You wouldn't want to write that out in expanded form! It would take too long.

So here is another way we can write these numbers:

Index ("growth time")

$$10^{180}$$

Base ("growth factor")

The 10 is your growth factor which Mathematicians call the **base** and 180 is the growth time which mathematicians call an **index.**

Write down the following in expanded form, index notation and standard form.

| Instructions | Expanded Form | Index Notation | Standard form |
|---|---|---|---|
| Growth Factor: x10<br>Growth Time: 5 seconds | | | |
| Growth Factor: x7<br>Growth Time: 3 seconds | | | |
| Growth Factor: x12<br>Growth Time: 2 seconds | | | |
| Growth Factor: x2<br>Growth Time: 4 seconds | | | |
| Growth Factor: x3<br>Growth Time: 6 seconds | | | |
| Growth Factor: x6<br>Growth Time: 1 seconds | | | |
| Growth Factor: x9<br>Growth Time: 7 seconds | | | |
| Growth Factor: x3<br>Growth Time: 8 seconds | | | |
| Growth Factor: x1<br>Growth Time: 10 seconds | | | |
| Growth Factor: x8<br>Growth Time: 0 seconds | | | |

Department of Education

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# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Investigating Indices

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 201: INVESTIGATING INDICES

## Overview

This task investigates the patterns generated in different sets of index numbers. It is assumed that students are familiar with index notation (See Task 200).

## Students will need

- calculators (optional)

## Relevant content descriptions from the Western Australian Curriculum

- Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

## Students can demonstrate

- *fluency* when they
  - write numbers with index notation
  - calculate accurately with indexed numbers
- *understanding* when they
  - see the relationship between expanded notation, index notation and standard form
- *reasoning* when they
  - can describe and explain patterns in base 10, base 3, and base 2 numbers
- *problem solving* when they
  - identify and describe patterns in indexed numbers

Activity 1

1. The table below follows a simple pattern. Identify the pattern and use it to complete the following table.

| | | |
|---|---|---|
| $7 \times 10^0$ | 7 x 1 | 7 |
| $7 \times 10^1$ | 7 x 10 | 70 |
| $7 \times 10^2$ | 7 x 10 x 10 | 700 |
| $7 \times 10^3$ | 7 x 10 x 10 x 10 | 7000 |
| $7 \times 10^4$ | 7 x 10 x 10 x 10 x 10 | 70 000 |
| $7 \times 10^5$ | 7 x 10 x 10 x 10 x 10 x 10 | 700 000 |
| $7 \times 10^6$ | 7 x 10 x 10 x 10 x 10 x 10 x 10 | 7 000 000 |
| $7 \times 10^7$ | 7 x 10 x 10 x 10 x 10 x 10 x 10 x 10 | 70 000 000 |
| $7 \times 10^8$ | 7 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 | 700 000 000 |
| $7 \times 10^9$ | 7 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 | 7 000 000 000 |
| $7 \times 10^{10}$ | 7 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 | 70 000 000 000 |

2. Describe the patterns you noticed in the table.
   As the power of 10 increases by 1, the standard number has 1 more zero written on the end of it.

3. How do these patterns help you to multiply numbers of 10s?
   If a whole number is multiplied by powers of 10, you take the index number and write that many zeroes after the number you are multiplying.

## Activity 2

1. Complete the following table

| | | |
|---|---|---|
| $3^0$ | 1 | 1 |
| $3^1$ | 3 | 3 |
| $3^2$ | 3 x 3 | 9 |
| $3^3$ | 3 x 3 x 3 | 27 |
| $3^4$ | 3 x 3 x 3 x 3 | 81 |
| $3^5$ | 3 x 3 x 3 x 3 x 3 | 243 |
| $3^6$ | 3 x 3 x 3 x 3 x 3 x 3 | 729 |
| $3^7$ | 3 x 3 x 3 x 3 x 3 x 3 x 3 | 2187 |
| $3^8$ | 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 | 6561 |
| $3^9$ | 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 | 19 683 |
| $3^{10}$ | 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 | 59 049 |
| $3^{11}$ | 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 | 177 147 |

2. Have a look at the last digit of each of the numbers in the last column. What do you notice?
   It follows a pattern, 3, 9, 7, 1, 3, 9, 7, 1, . . .

3. What do you think the last digit of $3^{12}$ would be? How do you know?
   The index 12 is a multiple of 4 so the last digit will be 1 since it is the last digit in the group of four (3, 9, 7, 1).

4. What do you think the last digit of $3^{24}$ would be?
   It would also be 1.

5. Can you write a general rule to describe this pattern?
   Divide the index by 4.
   - If it is divisible by 4, the last digit of the standard number will be 1.
   - If there is a remainder of 1, the last digit will be 3,
   - If there is a remainder of 2, the last digit will be 9
   - If there is a remainder of 3, the last digit will be 7

6. Create your own table using a different base number. What patterns do you notice?
   Powers of 2 end in 2, 4, 8 or 6
   Powers of 4 end in 4 or 6
   Powers of 5 always end in 5
   Powers of 6 always end in 6
   Powers of 7 end in 7, 9, 3 or 1
   Powers of 8 end in 8, 4, 2 or 6
   Powers of 9 end in 9 or 1
   etc.

Activity 3

Your friend Ruby has the following five cards:

$\boxed{2^0}$ $\boxed{2^1}$ $\boxed{2^2}$ $\boxed{2^3}$ $\boxed{2^4}$ $\boxed{2^5}$

1. Write down the value of each of the cards in standard form.
   1, 2, 4, 8, 16, 32

2. Ruby selects the two cards below and adds their values together. What is the total of her two cards?

   $\boxed{2^1}$ $\boxed{2^4}$

   17

3. What is the smallest value Ruby could make using her cards?
   1

4. What is the largest value Ruby could make using her cards, if she can only use each card once?
   63

5. Ruby selects three cards and adds their values together. She gets a total of 13. Which three cards did she select?
   $2^0 + 2^2 + 2^3$

6. How could Ruby make a total of 25?
   $2^0 + 2^3 + 2^4$

7. Ruby wants to make every number between 1 and 63 using her cards and thinks she can do it without having duplicates of any of her cards.

   See if you can prove Ruby right. Use the space below to show how each total can be made.

   (Hint: Try to be systematic in your approach. Can you use a list, table or diagram to help you?)

$1 = 2^0$
$2 = 2^1$
$3 = 2^0 + 2^1$
$4 = 2^2$
$5 = 2^0 + 2^2$
$6 = 2^1 + 2^2$
$7 = 2^0 + 2^1 + 2^2$
$8 = 2^3$
$9 = 2^0 + 2^3$
$10 = 2^1 + 2^3$
$11 = 2^0 + 2^1 + 2^3$
$12 = 2^2 + 2^3$
$13 = 2^0 + 2^2 + 2^3$
$14 = 2^1 + 2^2 + 2^3$
$15 = 2^0 + 2^1 + 2^2 + 2^3$
$16 = 2^4$
$17 = 2^0 + 2^4$
$18 = 2^1 + 2^4$
$19 = 2^0 + 2^1 + 2^4$
$20 = 2^2 + 2^4$
$21 = 2^0 + 2^2 + 2^4$
$22 = 2^1 + 2^2 + 2^4$
$23 = 2^0 + 2^1 + 2^2 + 2^4$
$24 = 2^3 + 2^4$
$25 = 2^0 + 2^3 + 2^4$
$26 = 2^1 + 2^3 + 2^4$
$27 = 2^0 + 2^1 + 2^3 + 2^4$
$28 = 2^2 + 2^3 + 2^4$
$29 = 2^0 + 2^2 + 2^3 + 2^4$
$30 = 2^1 + 2^2 + 2^3 + 2^4$
$31 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4$
$32 = 2^5$
$33 = 2^0 + 2^5$
$34 = 2^1 + 2^5$
$35 = 2^0 + 2^1 + 2^5$
$36 = 2^2 + 2^5$
$37 = 2^0 + 2^2 + 2^5$
$38 = 2^1 + 2^2 + 2^5$
$39 = 2^0 + 2^1 + 2^2 + 2^5$
$40 = 2^3 + 2^5$
$41 = 2^0 + 2^3 + 2^5$
$42 = 2^1 + 2^3 + 2^5$
$43 = 2^0 + 2^1 + 2^3 + 2^5$
$44 = 2^2 + 2^3 + 2^5$
$45 = 2^0 + 2^2 + 2^3 + 2^5$
$46 = 2^1 + 2^2 + 2^3 + 2^5$
$47 = 2^0 + 2^1 + 2^2 + 2^3 + 2^5$
$48 = 2^4 + 2^5$
$49 = 2^0 + 2^4 + 2^5$
$50 = 2^1 + 2^4 + 2^5$
$51 = 2^0 + 2^1 + 2^4 + 2^5$
$52 = 2^2 + 2^4 + 2^5$
$53 = 2^0 + 2^2 + 2^4 + 2^5$
$54 = 2^1 + 2^2 + 2^4 + 2^5$
$55 = 2^0 + 2^1 + 2^2 + 2^4 + 2^5$
$56 = 2^3 + 2^4 + 2^5$
$57 = 2^0 + 2^3 + 2^4 + 2^5$
$58 = 2^1 + 2^3 + 2^4 + 2^5$
$59 = 2^0 + 2^1 + 2^3 + 2^4 + 2^5$
$60 = 2^2 + 2^3 + 2^4 + 2^5$
$61 = 2^0 + 2^2 + 2^3 + 2^4 + 2^5$
$62 = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$
$63 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$

Activity 1

1. The table below follows a simple pattern. Identify the pattern and use it to complete the following table.

| | | |
|---|---|---|
| | 7 x 1 | 7 |
| | 7 x 10 | |
| | 7 x 10 x 10 | 700 |
| $7 \times 10^3$ | 7 x 10 x 10 x 10 | |
| $7 \times 10^4$ | | 70 000 |
| | | |
| | 7 x 10 x 10 x 10 x 10 x 10 x 10 | |
| $7 \times 10^7$ | | |
| | | |
| | | |
| | | 70 000 000 000 |

2. Describe the patterns you noticed in the table.

3. How do these patterns help to you multiply numbers of 10s?

Activity 2

1. Complete the following table.

| | | |
|---|---|---|
| | 1 | 1 |
| | 3 | 3 |
| | 3 x 3 | 9 |
| $3^3$ | 3 x 3 x 3 | |
| $3^4$ | | 81 |
| | | |
| | 3 x 3 x 3 x 3 x 3 x 3 | |
| $3^7$ | | |
| | | |
| | | |
| | | |
| | | |

2. Look at the last digit of each of the numbers in the last column. What do you notice?

3. What do you think the last digit of $3^{12}$ would be? How do you know?

4. What do you think the last digit of $3^{24}$ would be?

5. Can you write a general rule to describe this pattern?

6. Create your own table using a different base number. What patterns do you notice?

Activity 3

Your friend Ruby has the following five cards:

| $2^0$ | $2^1$ | $2^2$ | $2^3$ | $2^4$ | $2^5$ |
|---|---|---|---|---|---|

1. Write down the value of each of the cards in standard form.

2. Ruby selects the two cards below and adds their values together. What is the total of her two cards?

| $2^1$ | $2^4$ |
|---|---|

3. What is the smallest value Ruby could make using her cards?

4. What is the largest value Ruby could make using her cards, if she can only use each card once?

5. Ruby selects three cards and adds their values together. She gets a total of 13. Which three cards did she select?

6. How could Ruby make a total of 25?

7. Ruby wants to make every number between 1 and 63 using her cards and thinks she can do it without having duplicates of any of her cards.

   See if you can prove Ruby right. Use the space below to show how each total can be made.

   (Hint: Try to be systematic in your approach. Can you use a list, table or diagram to help you?)

Department of Education

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Prime Factorisation

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 202: PRIME FACTORISATION

## Overview

This task revises the concept of prime and composite numbers and introduces the concept of prime factorisation. It may be useful to complete Task 200 The IMPLEXUS-5000 before starting this task.

## Students will need

- Calculators (optional)

## Relevant content descriptions from the Western Australian Curriculum

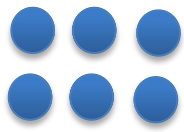
- Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

## Students can demonstrate

- *fluency* when they
    - can describe and identify prime and composite numbers
    - represent the common unit fractions as decimals as in Activity 1
- *understanding* when they
    - can identify the prime factors of any number
- *reasoning* when they
    - describe patterns they notice in the first 100 primes

Activity 1

1. Using the Internet or a mathematical dictionary, find and write definitions for the following terms then provide an illustration or example for each.

| Word | Definition | Illustration/Example |
|---|---|---|
| Array | An arrangement of items in rows and columns |  |
| Composite number | A counting number that has more than two factors | 1, 2, 3, 4, 6 and 12 are all factors of 12.<br><br>Therefore, 12 is a composite number |
| Factor | Counting numbers that you multiply together to get another number | 2 x 3 = 6<br>2 and 3 are factors of 6 |
| Multiple | The result of multiplying a whole number by an integer | 3 x 1 = 3<br>3 x 2 = 6<br>3 x 3 = 9<br>3, 6 and 9 are multiples of 3 |
| Prime number | A prime number has exactly two factors, 1 and itself<br><br>Note that 1 has only one factor and is therefore NOT a prime | 3, 5, 7 are all primes |

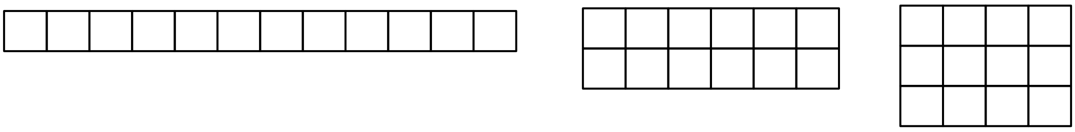
Activity 2

1. There are many ways to work out the factors of a number.

   a. You can use a T-chart to systematically list all the pairs of numbers that multiply to give you the answer you want. Below is an example of a T-chart for the factors of 12.

| Factors of 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| 4 | 3 |
| 6 | 2 |
| 12 | 1 |

   b. You can use grid paper to build rectangular arrays.
   For example, if we wanted to find all the factors of 12, we could draw as many different types of rectangles of 12 units as possible. The length and width of each rectangle will give you a different set of factors.



   The factors of twelve are 1 and 12 (from the first rectangle), 2 and 6 (from the second rectangle) and 3 and 4 from the last rectangle.

2. Using any method you like, find the factors of the following numbers.

   a. 6 1, 2, 3, 6
   b. 18 1, 2, 3, 6, 9
   c. 27 1, 3, 9, 27
   d. 35 1, 5, 7, 35
   e. 36 1, 2, 3, 4, 6, 9, 12, 18, 36
   f. 9 1, 3, 9
   g. 22 1, 2, 11, 22

3. Using any method you like, find the factors of the following numbers.
   a. 29 1, 29
   b. 17 1, 17
   c. 23 1, 23
   d. 5 1, 5
   e. 3 1, 3
   f. 11 1, 11
   g. 19 1, 19

4. Did you notice a difference between the answers in Question 2 and the answers in Question 3? What did you notice?
   The answers in Question 2 above show more than two factors, while the answers in Question 3 all show exactly 2 factors.

Activity 3 – Finding Prime Numbers

Here is a way of finding all the prime numbers between 1 and 100. By the end of this activity you should have all the prime numbers circled and all the composite numbers highlighted

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

***The number 1 has been shaded in to remind you that it is not considered to be a prime or a composite number.

1. Circle the number 2. (This has already been done for you)
2. Highlight every multiple of 2.
3. Circle the number 3.
4. Highlight every multiple of 3.
5. Skip over the number 4 because it has already been highlighted.
6. Circle the number 5.
7. Highlight every multiple of 5.
8. Skip over the number 6 and circle the number 7.
9. Highlight every multiple of 7.
10. Repeat the process until every number (except 1) is either circled or multiplied.

11. How many prime numbers are there between 1 and 100?
    25

12. How many primes are less than 10? Less than 20? Less than 30? Keeping going and describe any patterns you notice.
    Less than 10 – 4
    Less than 20 – 8
    Less than 30 – 10
    Less than 40 – 12
    Less than 50 – 15
    Less than 60 – 17
    Less than 70 – 19
    Less than 80 – 22
    Less than 90 – 24
    Less than 100 - 25

    The number of primes seems to decrease as we get into higher values.

13. Twin primes are two primes that have a difference of 2.

    For example 5 and 7 are twin primes.

    Using the Sieve of Eratosthenes that you completed above, list all the twin primes you can find.
    3 and 5, 5 and 7, 11 and 13, 17 and 19, 41 and 43, 71 and 73

14. Symmetrical primes are numbers where the digits have been reversed.

    For example 17 and 71 are symmetrical primes.

    a. 23 and 32 are NOT symmetrical primes. Can you explain why not?
       32 is not a prime number (it is a composite number) therefore it can't be a symmetrical prime.

    b. Using the Sieve above, list all the pairs of symmetrical primes you can find.
       13 and 31, 17 and 71, 37 and 73, 79 and 97

## Activity 4

1. Using the Sieve of Eratosthenes that you completed in Activity 3, list all the prime numbers between 1 and 100.

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

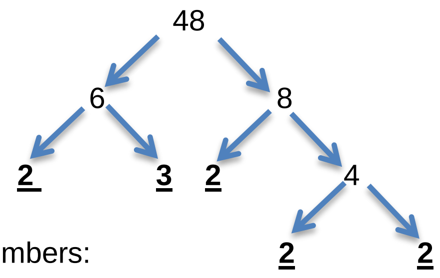
2. Using only the prime numbers listed in Question 1 and the multiplication operation you can make every number between 2 and 50. See if you can complete the table below.

| Number | Expanded Form | Index Notation | | Number | Expanded Form | Index Notation |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | | 27 | 3 x 3 x 3 | $3^3$ |
| 3 | 3 | 3 | | 28 | 2 x 2 x 7 | $2^2$ x 7 |
| 4 | 2 x 2 | $2^2$ | | 29 | 29 | 29 |
| 5 | 5 | 5 | | 30 | 2 x 3 x 5 | 2 x 3 x 5 |
| 6 | 2 x 3 | 2 x 3 | | 31 | 31 | 31 |
| 7 | 7 | 7 | | 32 | 2 x 2 x 2 x 2 x 2 | $2^5$ |
| 8 | 2 x 2 x 2 | $2^3$ | | 33 | 3 x 11 | 3 x 11 |
| 9 | 3 x 3 | $3^2$ | | 34 | 2 x 17 | 2 x 17 |
| 10 | 2 x 5 | 2 x 5 | | 35 | 5 x 7 | 5 x 7 |
| 11 | 11 | 11 | | 36 | 2 x 2 x 3 x 3 | $2^2$ x $3^2$ |
| 12 | 2 x 2 x 3 | $2^2$ x 3 | | 37 | 37 | 37 |
| 13 | 13 | 13 | | 38 | 2 x 19 | 2 x 19 |
| 14 | 2 x 7 | 2 x 7 | | 39 | 3 x 13 | 3 x 13 |
| 15 | 3 x 5 | 3 x 5 | | 40 | 2 x 2 x 2 x 5 | $2^3$ x 5 |
| 16 | 2 x 2 x 2 x 2 | $2^4$ | | 41 | 41 | 41 |
| 17 | 17 | 17 | | 42 | 2 x 3 x 7 | 2 x 3 x 7 |
| 18 | 2 x 3 x 3 | 2 x $3^2$ | | 43 | 43 | 43 |
| 19 | 19 | 19 | | 44 | 2 x 2 x 11 | $2^2$ x 11 |
| 20 | 2 x 2 x 5 | $2^2$ x 5 | | 45 | 3 x 3 x 5 | $3^2$ x 5 |
| 21 | 3 x 7 | 3 x 7 | | 46 | 2 x 23 | 2 x 23 |
| 22 | 2 x 11 | 2 x 11 | | 47 | 47 | 47 |
| 23 | 23 | 23 | | 48 | 2 x 2 x 2 x 2 x 3 | $2^4$ x 3 |
| 24 | 2 x 2 x 2 x 3 | $2^3$ x 3 | | 49 | 7 x 7 | $7^2$ |
| 25 | 5 x 5 | $5^2$ | | 50 | 2 x 5 x 5 | 2 x $5^2$ |
| 26 | 2 x 13 | 2 x 13 | | | | |

3. Discussion Question: What method did you use to complete the table? If you had to the task again with a different set of numbers, would you do it the same way, or would you do something different?

   Various answers

4. In Question 2 the composite numbers have been **factorised**; i.e., expressed as the product of primes. Another method of factorising composites is to use factor trees:




   Write the number you want to factorise:

   Choose any two factors of the original number:

   Find factors of the factors:

   Repeat the process until you are left with only prime numbers:

   Take each number at the end of the branches and multiply them together as 2 x 3 x 2 x 2 x 2, which can be re-ordered as 2 x 2 x 2 x 2 x 3.

   Simplify by using index notation where possible $2^4$ x 3.

   So the prime factorisation for 48 is $2^4$ x 3.

5. Use the factor tree method to find the prime factors of the following numbers
   a. 81 = $3^4$
   b. 64 = $2^5$
   c. 88 = $2^3$ x 11
   d. 71 = 71
   e. 100 = $2^2$ x $5^2$

Activity 1

1. Using the Internet or a mathematical dictionary, find and write definitions for the following terms then provide an illustration or example for each.

| Word | Definition | Illustration/Example |
|---|---|---|
| Array | | |
| Composite number | | |
| Factor | | |
| Multiple | | |
| Prime number | | |

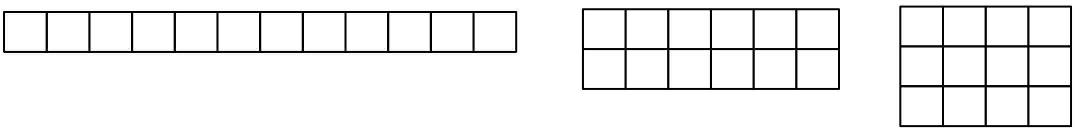
Activity 2

1. There are many ways to work out the factors of a number.

   a. You can use a T-chart to systematically list all the pairs of numbers that multiply to give you the answer you want. Below is an example of a T-chart for the factors of 12.

| Factors of 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| 4 | 3 |
| 6 | 2 |
| 12 | 1 |

   b. You can use grid paper to build rectangular arrays.
   For example, if we wanted to find all the factors of 12, we could draw as many different types of rectangles of 12 units as possible. The length and width of each rectangle will give you a different set of factors.



   The factors of twelve are 1 and 12 (from the first rectangle), 2 and 6 (from the second rectangle) and 3 and 4 from the last rectangle.

2. Using any method you like, find the factors of the following numbers.
   a. 6
   b. 18
   c. 27
   d. 35
   e. 36
   f. 9
   g. 22

3. Using any method you like, find the factors of the following numbers.
   a. 29
   b. 17
   c. 23
   d. 5
   e. 3
   f. 11
   g. 19

4. Did you notice a difference between the answers in Question 2 and the answers in Question 3? What did you notice?

Activity 3

Here is a way of finding all the prime numbers between 1 and 100. By the end of this activity you should have all the prime numbers circled and all the composite numbers highlighted

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

***The number 1 has been shaded in to remind you that it is not considered to be a prime or a composite number.

1. Circle the number 2. (This has already been done for you)
2. Highlight every multiple of 2.
3. Circle the number 3.
4. Highlight every multiple of 3.
5. Skip over the number 4 because it has already been highlighted.
6. Circle the number 5.
7. Highlight every multiple of 5.
8. Skip over the number 6 and circle the number 7.
9. Highlight every multiple of 7.
10. Repeat the process until every number (except 1) is either circled or multiplied.

11. How many prime numbers are there between 1 and 100?

12. How many primes are less than 10? Less than 20? Less than 30? Keeping going and describe any patterns you notice.

13. Twin primes are two primes that have a difference of 2.

    For example 5 and 7 are twin primes.

    Using the Sieve of Eratosthenes that you completed above, list all the twin primes you can find.

14. Symmetrical primes are numbers where the digits have been reversed.

    For example 17 and 71 are symmetrical primes.

    a. 23 and 32 are NOT symmetrical primes. Can you explain why not?

    b. Using the Sieve above, list all the pairs of symmetrical primes you can find.

Activity 4

1. Using the Sieve of Eratosthenes that you completed in Activity 3, list all the prime numbers between 1 and 100.

2. Using only the prime numbers listed in Question 1 and the multiplication operation, you can make every number between 2 and 50. See if you can complete the table below.

| Number | Expanded Form | Index Notation | | Number | Expanded Form | Index Notation |
|---|---|---|---|---|---|---|
| 2 | 2 | $2^1$ | | 27 | | |
| 3 | | | | 28 | | |
| 4 | 2 x 2 | $2^2$ | | 29 | | |
| 5 | | | | 30 | | |
| 6 | | | | 31 | | |
| 7 | | | | 32 | | |
| 8 | | | | 33 | | |
| 9 | | | | 34 | | |
| 10 | | | | 35 | | |
| 11 | | | | 36 | 2 x 2 x 3 x 3 | $2^2 \times 3^2$ |
| 12 | | | | 37 | | |
| 13 | | | | 38 | | |
| 14 | | | | 39 | | |
| 15 | | | | 40 | | |
| 16 | | | | 41 | | |
| 17 | | | | 42 | 2 x 3 x 7 | 2 x 3 x 7 |
| 18 | | | | 43 | 43 | 43 |
| 19 | | | | 44 | | |
| 20 | | | | 45 | | |
| 21 | | | | 46 | | |
| 22 | | | | 47 | | |
| 23 | | | | 48 | | |
| 24 | | | | 49 | | |
| 25 | | | | 50 | | |
| 26 | | | | | | |

3. Discussion Question: What method did you use to complete the table? If you had to perform the task again with a different set of numbers, would you do it the same way, or would you do something different?

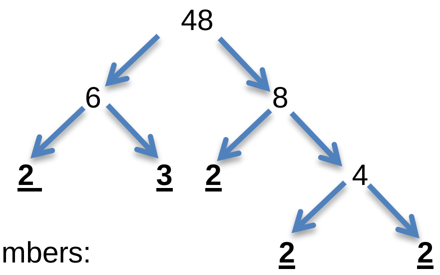
4. In Question 2 the composite numbers have been **factorised**; i.e., expressed as the product of primes. Another method of factorising composites is to use factor trees:

   Write the number you want to factorise:

   Choose any pair of factors of the original number:

   Find factors of the factors:

   Repeat the process until you are left with only prime numbers:




   Take each number at the end of the branches and multiply them together as 2 x 3 x 2 x 2 x 2, which can be re-ordered as 2 x 2 x 2 x 2 x 3.

   Simplify by using index notation where possible $2^4 \times 3$.

   So the prime factorisation for 48 is $2^4 \times 3$.

5. Use the factor tree method to find the prime factorisation of the following numbers.

   a. 81
   b. 64
   c. 88
   d. 71
   e. 100

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Introducing Variables

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 209: INTRODUCING VARIABLES

## Overview

This task is a series of activities designed to informally introduce algebra and the idea of variables to students.

## Students will need

- calculators
- For each pair of students:
  - One paper cup with a mixture of counters and paperclips (instead of paperclips you could use pebbles, bottle caps, pegs, or any other small objects)
  - Paper cup with only counters in it.

## Relevant content descriptions from the Western Australian Curriculum

- Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)
- Create algebraic expression and evaluate them by substituting a given value for each variable (ACMNA176)

## Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
  - substitute a given value into a pre-determined algorithm
- *understanding* when they
  - can explain the concept of a variable in their own words
- *reasoning* when they
  - use manipulatives to solve equations
- *problem solving* when they
  - devise strategies for adding a set of numbers quickly

<u>Activity 1 – Teacher-led activity</u>

1. Put the Hundreds Chart (provided, next page) up on the white board.

2. Challenge a student to do battle with you. Ask them to come up to the front of the room.

3. Explain the rules to the class:
   - You will choose another student to select a number from inside the red square.
   - You and the battle student will try to be the first person to find the total of the square that has been chosen and all the squares touching it.
   - The loser sits down and the winner gets to pick the next person to do battle with.

Example: If the number 12 is chosen then the sum is 1+2+3+11+12+13+21+ 22+23 = 108.

| | | |
|---|---|---|
| *1* | *2* | *3* |
| *11* | *12* | *13* |
| *21* | *22* | *23* |

(The trick to winning: Take the number chosen and multiply it by 9.)

4. Play the game for several rounds. Boast about your brilliance.

5. Play a different version of the game where the student tells you the total they get for their number, and you and the student doing battle have to work out what that number is (Trick: Divide by 9).

6. Boast about your brilliance some more.

7. After a few rounds, students will start to come up with their own strategies for winning. Ask students how they do it.

8. Ideas to discuss
   - The number each student chose was not known in advance, and is therefore a variable.
     Ask students how they could represent this idea on paper.
   - Before each student chose a number, there was no way you could work out an answer, but you could tell someone what you were going to do with the number.
     Ask students how they could write down these instructions for someone else. Is there are shorter way of writing down these instructions?
   - Once the students had chosen a number, then we could work out the answer.
     Ask students how they could represent this information on paper.
   - Variables change. Sometimes -
     - there is nothing more you can do with a variable,
     - you can replace a variable with a number and then perform a calculation
     - you can be given answer and you can work out the variable.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## Activity 2 Teacher-led Activity

1. For this activity, work in pairs, and decide who is "A" and who is "B".

2. A will make up a **secret** algorithm and write it down somewhere safe (without B seeing what it is!).

   Example: Take a number, double it and add 5

3. B will choose a number and tell A what the number is.

   Example: B chooses the number 6

4. A will then use B's number and, using the secret algorithm, come up with a number that they give back to B.

   Example: A gives B the number 17 (because 6 x 2 + 5 = 17)

5. B tries to guess the algorithm. If they are not ready to guess, or they make an incorrect guess, they choose another number and repeat the process.

6. When B has guessed the algorithm, the students should switch roles.

Encourage A to write their algorithm down somewhere safe, so that they remember it.

Encourage B to write down both the number they give to A and the number they get back so that it is easier for them to identify patterns.

Prompt cards have been included on the next page for students who need help creating an algorithm. Use of these is optional.

| | |
|---|---|
| Take a Number | Double it |
| Add 3 | Subtract 6 |
| Multiply by 4 | Divide by 2 |
| Add the number you started with | Multiply by 5 |
| Subtract 1 | Divide by 3 |

Activity 3  Teacher-led Activity

Before the class, organise pairs of cups with different numbers of counters and paperclips in them (See the next page for suggestions about the number of paperclips and the number of counters in each cup).

You will need one pair of cups for each pair of students.

Place each pair of cups at various intervals around the room.

When the students arrive, proceed as follows;

1. Hold up one pair of cups and tell students that both the cups weigh the same.

2. Tip the first cup out on the desk and show students that it contains both paperclips and counters.
Example: 4 paperclips and 3 counters

3. Tip out the second cup and show students that it only contains counters.
Example: 11 counters

4. Write an algebraic expression to represent the situation on the board.
Example: $4x + 3 = 11$

5. Ask students how much one paperclip weighs in terms of counters.

6. Allow students to explain how you could work out the mass of one paperclip. After each step, write down what has happened in algebraic terms
Example:
Student: You could take 3 counters off each side.
Teacher: Takes 3 counters off each side, and writes on board: $4x + 3 - 3 = 11 – 3$.
Then simplifies equation to: $4x = 8$
Student: Now divide both sides by four groups.
Teacher: Creates four groups on each side, and writes on board: $4x \div 4 = 8 \div 4$

7. Have each pair of students move to one pair of cups. Ask students to find the mass of the paperclip in terms of counters. Insist that they write down each step algebraically as they go.

8. Once students have solved their equation they can move to a different set of cups.

9. Repeat the process, but have paperclips in both cups.

10. Complete the tables below, where $x$ is the number of paper clips.

**Variable on one side only**

| Cup 1 | Cup 2 | Answers |
|---|---|---|
| 3 paperclip, 4 counters | 34 counters | $3x + 4 = 34$, so $x$ (1 paperclip) = 10 counters |
| 2 paperclips, 1 counters | 19 counters | $2x + 1 = 19$, so $x$ (1 paperclip) = 9 counters |
| 2 paperclips, 1 counter | 17 counters | $2x + 1 = 17$, so $x$ (1 paperclip) = 8 counters |
| 3 paperclips, 8 counters | 14 counters | $3x + 8 = 14$, so $x$ (1 paperclip) = 2 counters |
| 3 paperclips, 2 counters | 5 counters | $3x + 2 = 29$, so $x$ (1 paperclip) = 9 counters |
| 2 paperclips, 4 counters | 14 counters | $2x + 4 = 20$, so $x$ (1 paperclip) = 8 counters |
| 2 paperclips, 6 counters | 18 counters | $2x + 6 = 22$, so $x$ (1 paperclip) = 8 counters |
| 2 paperclips, 8 counters | 22 counters | $2x + 8 = 26$, so $x$ (1 paperclip) = 9 counters |
| 3 paperclips, 5 counters | 8 counters | $3x + 5 = 32$, so $x$ (1 paperclip) = 9 counters |
| 2 paperclips, 4 counters | 22 counters | $2x + 4 = 20$, so $x$ (1 paperclip) = 8 counters |
| 2 paperclips, 5 counters | 23 counters | $2x + 5 = 21$, so $x$ (1 paperclip) = 8 counters |
| 2 paperclips, 3 counters | 23 counters | $2x + 3 = 21$, so $x$ (1 paperclip) = 9 counters |
| 2 paperclips, 3 counters | 13 counters | $2x + 3 = 13$, so $x$ (1 paperclip) = 5 counters |
| 4 paperclips, 3 counters | 27 counters | $4x + 3 = 27$, so $x$ (1 paperclip) = 6 counters |

**Variable on both sides**

| Cup 1 | Cup 2 | Answers |
|---|---|---|
| 2 paperclips, 2 counters | 1 paperclip, 8 counters | $2x + 2 = x + 8$, so $x = 6$ |
| 4 paperclips, 3 counters | 2 paperclips, 13 counters | $4x + 3 = 2x + 13$, so $x = 5$ |
| 5 paperclips, 1 counter | 2 paperclips, 10 counters | $5x + 1 = 2x + 10$, so $x = 3$ |
| 5 paperclips, 2 counters | 2 paperclips, 14 counters | $5x + 2 = 2x + 14$, so $x = 4$ |
| 2 paperclips, 1 counter | 1 paperclip, 8 counters | $2x + 1 = x + 8$, so $x = 7$ |
| 5 paperclips, 1 counter | 1 paperclip, 17 counters | $5x + 1 = x + 17$, so $x = 4$ |
| 12 paperclips, 2 counters | 2 paperclips, 22 counters | $12x + 2 = 2x + 22$, so $x = 2$ |
| 2 paperclips, 11 counters | 7 paperclips, 1 counter | $2x + 11 = 7x + 1$, so $x = 2$ |
| 1 paperclip, 25 counters | 12 paperclips, 3 counters | $x + 25 = 12x + 3$, so $x = 2$ |
| 4 paperclips, 10 counters | 2 paperclips, 28 counters | $4x + 10 = 2x + 28$, so $x = 9$ |
| 2 paperclips, 37 counters | 8 paperclips, 7 counters | $2x + 37 = 8x + 7$, so $x = 5$ |
| 4 paperclips, 64 counters | 7 paperclips, 13 counters | $4x + 64 = 7x + 13$, so $x = 17$ |
| 4 paperclips, 17 counters | 7 paperclips, 2 counters | $4x + 17 = 7x + 2$, so $x = 5$ |
| 5 paperclips, 8 counters | 3 paperclips, 12 counters | $5x + 8 = 3x + 12$, so $x = 2$ |

Activity 1 – Teacher-led activity

The teacher will put this Hundreds Chart on the white board.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

You are to do battle with battle the teacher to see who can add a set of numbers the quickest.

The rules of the contest are as follows:

- A student selects a number from inside the red square;
- If the student picks say 12, as in the diagram on the next page;
- You and the teacher battle to see who can first add the numbers on all the squares touching the square with 12; and
- The winner picks another student to repeat the battle with the teacher.

Example:
If the number 12 is chosen on the Hundred Chart, as in the diagram on the next page, then the sum of the numbers is 1+2+3+11+12+13+21+22+23 = 108

| | | |
|---|---|---|
| *1* | *2* | *3* |
| *11* | *12* | *13* |
| *21* | *22* | *23* |

Can you come up with any useful strategies for adding all the numbers quickly?

<u>Activity 2 (Teacher-led Activity)</u>

1. For this activity, work in pairs, and decide who is “A” and who is “B”.

2. A will make up a **<u>secret</u>** algorithm and write it down somewhere safe (without B seeing what it is!).

3. B will choose a number and tell A what the number is.

4. A will then use B’s number and, using the secret algorithm, come up with a number that they give back to B.

5. B tries to guess the algorithm. If they are not ready to guess, or they make an incorrect guess, they choose another number and repeat the process.

6. When B has guessed the algorithm, the students should switch roles.

Prompt cards have been included on the next page for students who need help creating an algorithm. Use of these is optional.

| | |
|---|---|
| Take a Number | Double it |
| Add 3 | Subtract 6 |
| Multiply by 4 | Divide by 2 |
| Add the number you started with | Multiply by 5 |
| Subtract 1 | Divide by 3 |

Activity 3  Teacher-led Activity

Before the class, the teacher will organise pairs of cups with different numbers of counters and paperclips in them. Teacher will give out one pair of cups for each pair of students.

The teacher will hold up one pair of cups and tell you that both the cups weigh the same.

Tip the first cup out on the desk to show that it contains both paperclips and counters.

Tip out the second cup and show that it only contains counters.

1. Write an algebraic expression to represent the situation that both contents weigh the same.

2. How much does one paperclip weigh in terms of counters?

3. Explain how you could work out the mass of one paperclip. After each step, write down what has happened in algebraic terms.

4. Move with your partner to another pair of cups to find the mass of the paperclip in terms of counters. Write down each step algebraically as you go.

5. Once you have solved their equation then move to a different set of cups.

6. Repeat the process, but have paperclips in both cups.

7. Complete the tables below, where $x$ is the number of paper clips.

**Variable on one side**

| Cup 1 | Cup 2 | Answers |
|---|---|---|
| 3 paperclip, 4 counters | 34 counters | $3x + 4 = 34$, so $x$ (1 paperclip) = 10 counters |
| 2 paperclips, 1 counters | 19 counters | $2x + 1 = 19$, so $x$ (1 paperclip) = 9 counters |
| 2 paperclips, 1 counter | 17 counters | |
| 3 paperclips, 8 counters | 14 counters | |
| 3 paperclips, 2 counters | 5 counters | |
| 2 paperclips, 4 counters | 14 counters | |
| 2 paperclips, 6 counters | 18 counters | |
| 2 paperclips, 8 counters | 22 counters | |
| 3 paperclips, 5 counters | 8 counters | |
| 2 paperclips, 4 counters | 22 counters | |
| 2 paperclips, 5 counters | 23 counters | |
| 2 paperclips, 3 counters | 23 counters | |
| 2 paperclips, 3 counters | 13 counters | |
| 4 paperclips, 3 counters | 27 counters | |

**Variable on both sides**

| Cup 1 | Cup 2 | Answers |
|---|---|---|
| 2 paperclips, 2 counters | 1 paperclip, 8 counters | $2x + 2 = x + 8$, so $x = 6$ |
| 4 paperclips, 3 counters | 2 paperclips, 13 counters | $4x + 3 = 2x + 13$, so $x = 5$ |
| 5 paperclips, 1 counter | 2 paperclips, 10 counters | |
| 5 paperclips, 2 counters | 2 paperclips, 14 counters | |
| 2 paperclips, 1 counter | 1 paperclip, 8 counters | |
| 5 paperclips, 1 counter | 1 paperclip, 17 counters | |
| 12 paperclips, 2 counters | 2 paperclips, 22 counters | |
| 2 paperclips, 11 counters | 7 paperclips, 1 counter | |
| 1 paperclip, 25 counters | 12 paperclips, 3 counters | |
| 4 paperclips, 10 counters | 2 paperclips, 28 counters | |
| 2 paperclips, 37 counters | 8 paperclips, 7 counters | |
| 4 paperclips, 64 counters | 7 paperclips, 13 counters | |
| 4 paperclips, 17 counters | 7 paperclips, 2 counters | |
| 5 paperclips, 8 counters | 3 paperclips, 12 counters | |

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Graphing Stories

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 210: GRAPHING STORIES

## Overview

This task is designed to introduce students to travel graphs. In the first activity they try to graph motion contained in 15-second video clips. The second activity allows students to be creative by describing a scenario and providing the graph for it. The third activity asks students to read and interpret a travel graph.

## Students will need

- access to the internet (Activity 1 only)
- grid paper

## Relevant content descriptions from the Western Australian Curriculum

- Investigate, interpret and analyse graphs from authentic data (ACMNA180)

## Students can demonstrate

- *fluency* when they
  - construct a simple travel graph
- *understanding* when they
  - can explain what horizontal sections on a travel graph mean
  - can explain what the different slopes on a travel graph mean
- *reasoning* when they
  - interpret the information contained in a travel graph
- *problem solving* when they
  - calculate distances and speeds from a travel graph.

Activity 1 – Teacher-led Activity

1. The teacher will go to this website: http://graphingstories.com/ and print out the worksheet provided, making enough copies for the class.

2. After playing several videos, such as the following, you are to graph each story.
   Show these and/or other videos so that have a framework to proceed.

   - Height of Waist off Ground by Adam Poetzal
   - Bum Height off Ground by Kathy Lehner
   - Distance from Bench by Kenneth Lawler
   - Elevation of Plane by Jose Luis Ibarra

3. Compare your graphs to each other and to the answer provided at the end of the video.

4. Discuss some of the features of each graph:

   a. What does a flat line in the graph mean?
      There is no change.

   b. Why is the line steeper in some places than others?
      The rate of change is faster.

   c. What would the graph look like if the person went faster? Slower? Changed their speed in the middle?
      The slope would be steeper if the person went faster, and less steep if slower. A change of speed would show as a change in the slope of the graph.

Activity 2

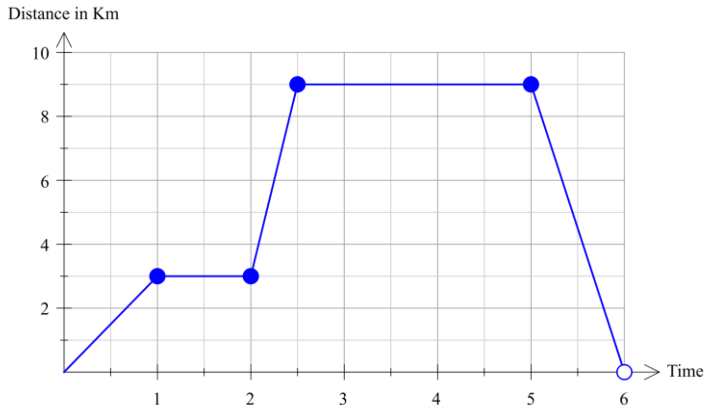
1. Write your own story describing a 15-second event similar to the ones you have just seen. It could be something that has happened to you recently, or it can be completely made up.
   Answers will vary.

2. On a separate piece of grid paper, draw a graph that represents your story.
   Answers will vary.

3. Swap graphs with someone else. Let them tell you what they think your story is about. Try to guess what their story is about.
   Answers will vary.

4. How did your guess compare to the story that your friend wrote?
   Answers will vary.

## Activity 3




John walked to Shane's house for a visit.

After an hour, John and Shane decided to go to the movies.

Shane's mum dropped the two boys at the cinemas.

When the movie was over, John caught the bus home.

The travel graph above shows John's journey. Use it to answer the following questions.

1. How far was Shane's house from John's?
   3 km
2. How long did John take to walk to Shane's house?
   1 hour
3. How fast was John travelling?
   3 km/hour
4. The graph is horizontal in two places. Why is this?
   Because John stopped at these two places (Shane's house and the cinema).
5. How far from John's house is the cinema?
   9 km
6. How far from Shane's house is the cinema?
   6 km
7. How long did it take to travel to the cinema?
   Half an hour
8. How fast were Shane and John travelling on their way to the cinema?
   12 km/hour
9. When was John travelling faster; walking to Shane's house, or being driven to the cinema? How does the graph show that John was travelling faster for this part of the journey?
   John was travelling faster on the way to the cinema. The graph is steeper for this part of the journey
10. How long was the movie?
    Two and half an hours
11. How fast was the bus that took John home travelling?
    9 km/hour

Activity 1 – Teacher-led Activity

1. The teacher will go to this website: http://graphingstories.com/ and print out the worksheet provided, making enough copies for the class.

2. After playing several videos, such as the following, you are to graph each story.

   - Height of Waist off Ground by Adam Poetzal
   - Bum Height off Ground by Kathy Lehner
   - Distance from Bench by Kenneth Lawler
   - Elevation of Plane by Jose Luis Ibarra

3. Compare your graphs to each other and to the answer provided at the end of the video.

4. Discuss some of the features of each graph:

   a. What does a flat line in the graph mean?

   b. Why is the line steeper in some places than others?

   c. What would the graph look like if the person went faster? Slower? Changed their speed in the middle?

Activity 2

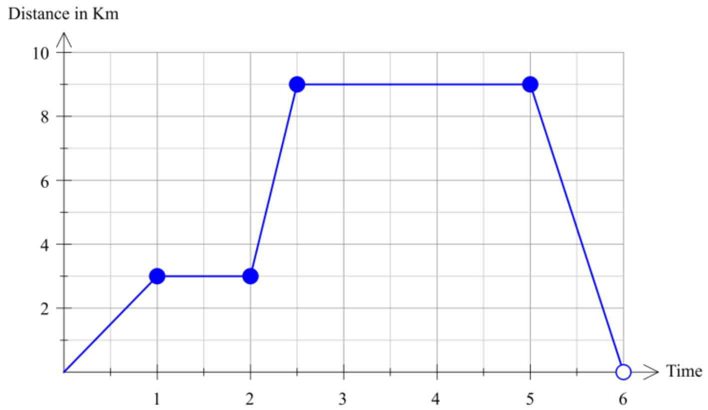
1. Write your own story describing a 15-second event similar to the ones you have just seen. It could be something that has happened to you recently, or it can be completely made up.

2. On a separate piece of grid paper, draw a graph that represents your story.

3. Swap graphs with someone else. Let them tell you what they think your story is about. Try to guess what their story is about.

4. How did your guess compare to the story that your friend wrote?

Activity 3




John walked to Shane's house for a visit.

After an hour, John and Shane decided to go to the movies.

Shane's mum dropped the two boys at the cinemas

When the movie was over, John caught the bus home.

The travel graph above shows John's journey. Use it to answer the following questions.

1. How far was Shane's house from John's?

2. How long did John take to walk to Shane's house?

3. How fast was John travelling?

4. The graph is horizontal in two places. Why is this?

5. How far from John's house is the cinema?

6. How far from Shane's house is the cinema?

7. How long did it take to travel to the cinema?

8. How fast were Shane and John travelling on their way to the cinema?

9. When was John travelling the fastest; walking to Shane's house, or being driven to the cinema? How does the graph show that John was travelling faster for this part of the journey?

10. How long was the movie?

11. How fast was the bus that took John home travelling?

Department of Education

# Year 7 Mathematics

## Number & Algebra Activity

# Three Laws

Produced by a Department of Education - MAWA partnership project
written for the Year 7 Australian Curriculum

# TASK 212: THREE LAWS

## Overview

This task is designed to introduce students to the Associative, Commutative and Distributive laws. Students work individually to research one of the given laws, they check their research with their peers who have researched the same law, and then teach other students the given topic. Students then work in groups to create questions and worked solutions on each of the topics.

## Students will need

- access to the internet

## Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (AMNA151)

## Students can demonstrate

- *fluency* when they
  - explain the associative, commutative and distributive laws
- *understanding* when they
  - create key-point summaries of each law
- *reasoning* when they
  - provide worked solutions to each of the questions that they have created
- *problem solving* when they
  - write their own practice questions on each of the laws.

Activity

1. The teacher will ‘label’ each student A, B or C.

2. You are to investigate these laws based on your letter.
   A’s – Associative Law
   B’s – Commutative Law
   C’s – Distributive Law

3. You can use the Internet, or any other resources available in the room to research the topic, so that you can teach other students about your topic later.
   Have Internet and appropriate textbook sources available.

4. At the end of the research session, all the A’s should have a group meeting to make sure your findings all agree; while the B’s and C’s are do the same thing.
   It may be more conducive to split each larger group into smaller ones.

5. Once your group is satisfied that everyone is an expert on the given topic, you are to prepare a short presentation to explain the concept to the rest of the class.
   Remind students to include visual aids and examples.
   Ensure students understand that they will each be presenting this on their own.

6. Organise yourselves into groups of three. Each group should have an A, B and C student in it.

7. Take turns teaching your own topic while the other two students write a key-point summary.

8. Once your group have all completed the key-point summaries, the group is to write at least 5 questions, as follows.

- There must be at least one question for each of the three laws.
- There must be at least one worded question.
- There must be worked solutions for each of the questions.

These questions can be collected and used as practice for students in a following lesson.

<u>Activity</u>

1. The teacher will 'label' each student A, B or C.

2. You are to investigate these laws based on your letter.
   A's – Associative Law
   B's – Commutative Law
   C's – Distributive Law

3. You can use the Internet, or any other resources available in the room to research the topic, so that you can teach other students about your topic later.

4. At the end of the research session, all the A's should have a group meeting to make sure your findings all agree; while the B's and C's are do the same thing.

5. Once your group is satisfied that everyone is an expert on the given topic, you are to prepare a short presentation to explain the concept to the rest of the class.

6. Organise yourselves into groups of three. Each group should have an A, B and C student in it.

7. Take turns teaching your own topic while the other two students write a key-point summary.

8. Once your group have all completed the key-point summaries, the group is to write at least 5 questions, as follows.

   - There must be at least one question for each of the three laws.
   - There must be at least one worded question.
   - There must be worked solutions for each of the questions.

9. These questions can be collected and used as practice for students in a following lesson.

Department of Education

The Mathematical Association of Western Australia Inc.

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Fascinating Fractions

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 213: FASCINATING FRACTIONS

## Overview

This activity is an extension activity designed to give students lots of practice adding fractions with different denominators. It is assumed that students are already familiar with adding and subtracting fractions with different denominators and comparing fractions, and have completed Task 218.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)
- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)

## Students can demonstrate

- *fluency* when they
  - add fractions with different denominators
  - compare fractions using equivalence
- *reasoning* when they
  - identify the pattern when adding two fractions
- *problem solving* when they
  - use a range of strategies to solve problems involving adding fractions
  - find unit fractions that sum to one
  - use unit fractions and addition to create new fractions.

Activity 1

1. Using only the numbers 0, 1, 2 and 3 write all the possible fractions representing one or less.
   $\frac{0}{1}$ $\frac{0}{2}$ $\frac{0}{3}$ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{3}$

2. Simplify any fractions that you can. Re-write your fraction list below.
   Hint: $\frac{0}{2}$ and $\frac{0}{3}$ both simplify to $\frac{0}{1}$.
   $\frac{0}{1}$ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

3. Write the fractions in order from smallest to largest.
   $\frac{0}{1}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{1}$

A sequence of fractions listed in order of size is called a *Farey sequence* (after the mathematician who investigated them). The Farey sequence you have found in the question above is referred to as $F_3$ because the largest digit in the sequence is 3.

4. Find the $F_4$ sequence. (Use the numbers 0,1 2, 3, and 4 and repeat the steps outlined in Questions 1–3).
   $\frac{0}{1}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{1}$

5. Which fractions are in the $F_4$ sequence that weren't in the $F_3$ sequence?
   $\frac{1}{4}$ $\frac{3}{4}$

6. Which new fractions do you think will be in the $F_5$ sequence? Make a prediction.
   Various answers

7. Find the $F_5$ sequence.
   $\frac{0}{1}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{1}{1}$

8. Which fractions are in the $F_5$ sequence that weren't in the $F_4$ sequence? How did this compare to your prediction in question 6?
   Various answers

9. Choose any three consecutive fractions from your answer to Question 7. Write your choices below.
   Various answers

10. Add the smallest fraction to the biggest fraction in your set and simplify the result. What answer do you get? How does this relate to the three numbers you chose in Question 9?
    The middle number is half way between the first and third number.

11. Does this work for any three consecutive numbers in a Farey sequence? Show your working below.

    Yes it does

<u>Activity 2</u>

Consider the following sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots = \underline{\qquad\qquad}$$

1. What patterns do you notice?

   Various answers; e.g., the denominator doubles at each step; the denominators are all multiples of two; the denominators are all powers of two.

2. What do the 3 dots just before the equals sign mean?

   The pattern keeps going forever; i.e., 'and so on'.

3. Without performing any calculations, what do you think the total of adding all these fractions would be? Write down your best guess.

   Various answers

4. Take a blank sheet of paper
   a. Divide it in half.
   b. Shade one half and label it $\frac{1}{2}$.
   c. Divide the unshaded side in half again.
   d. Shade half of the half and label $\frac{1}{4}$ .
   e. Divide the unshaded part in half again.
   f. Shade in and label $\frac{1}{8}$ .
   g. Repeat the process for as long as you can.

5. Will you ever completely colour in the whole page? Why/Why not?

   No. You keep shading half of the space that's left, so you will get closer and closer to filling the page, but never actually do it.

6. Using what you have discovered, what do you think the answer to the fraction sum at the top of the page would be now? Justify your answer.

   It will get very close to one, but never actually reach it, just like you get very close to having the whole page coloured in with out ever getting it all done.

## Activity 3

Fractions aren't that new. In fact, the Egyptians were using fractions over 3000 years ago!

However, they wrote their fractions a little differently to us. All their fractions had 1 as the numerator (also known as "unit fractions").

For example the Egyptians used $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$ but they would never use $\frac{2}{3}$ or $\frac{3}{4}$ .

If the ancient Egyptians wanted to write a fraction with a different numerator, such as $\frac{3}{4}$, they would write it as a sum of different unit fractions; i.e., $\frac{1}{2} + \frac{1}{4}$ , as this equals $\frac{3}{4}$.

If they wanted to write a fraction such as $\frac{2}{3}$, then they would not write $\frac{1}{3} + \frac{1}{3}$ as this used the same fraction twice. Instead they would have written $\frac{1}{2} + \frac{1}{6}$!

1. How many fractions that are smaller than 1 can you create using the list of fractions below. Remember you can add two or more fractions together, but you can't use a fraction twice!

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{1}{8} \quad \frac{1}{9} \quad \frac{1}{10}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 19/90 | 13/42 | 13/30 | 5/8 | 53/72 | 4/5 | 13/15 | 19/20 |
| 9/40 | 14/45 | 4/9 | 9/14 | 26/35 | 17/21 | 13/15 | 19/20 |
| 17/72 | 13/40 | 9/20 | 2/3 | 3/4 | 17/21 | 243/280 | 29/30 |
| 17/70 | 12/35 | 11/24 | 41/60 | 3/4 | 73/90 | 7/8 | 44/45 |
| 16/63 | 7/20 | 10/21 | 25/36 | 95/126 | 33/40 | 443/504 | 119/120 |
| 4/15 | 13/36 | 1/2 | 7/10 | 23/30 | 5/6 | 25/28 | |
| 15/56 | 11/30 | 8/15 | 17/24 | 43/56 | 301/360 | 191/210 | |
| 5/18 | 3/8 | 7/12 | 32/45 | 7/9 | 59/70 | 11/12 | |
| 7/24 | 11/28 | 3/5 | 29/40 | 47/60 | 17/20 | 58/63 | |
| 3/10 | 5/12 | 11/18 | 61/84 | 19/24 | 31/36 | 157/168 | |

2. Draw a number line and place all the fractions you found in Question 1 on it.
   Various answers

3. The Egyptians could also write the number 1 as a sum of different fractions. For example, the sum $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$ uses 3 different unit fractions and adds to one.

   Using exactly 4 different unit fractions there are different ways to write the number one. See if you can find them. You can use any unit fractions you wish.

   $^1/_2 + {}^1/_3 + {}^1/_9 + {}^1/_{18}$    $^1/_2 + {}^1/_4 + {}^1/_5 + {}^1/_{20}$    $^1/_2 + {}^1/_3 + {}^1/_8 + {}^1/_{24}$    $^1/_2 + {}^1/_4 + {}^1/_6 + {}^1/_{12}$

Activity 1

1. Using only the numbers 0, 1, 2 and 3 write all possible fractions representing one or less.

2. Simplify any fractions that you can. Re-write your fraction list below.
   Hint: $\frac{0}{2}$ and $\frac{0}{3}$ both simplify to $\frac{0}{1}$.

3. Write the fractions in order from smallest to largest.

A sequence of fractions listed in order of size is called a *Farey sequence* (after the mathematician who investigated them). The Farey sequence you have found in the question above is referred to as $F_3$ because the largest digit in the sequence is 3.

4. Find the $F_4$ sequence. (Use the numbers 0,1 2, 3, and 4 and repeat the steps outlined in Questions 1–3).

5. Which fractions are in the $F_4$ sequence that weren't in the $F_3$ sequence?

6. Which new fractions do you think will be in the $F_5$ sequence? Make a prediction.

7. Find the $F_5$ sequence.

8. Which fractions are in the $F_5$ sequence that weren't in the $F_4$ sequence? How did this compare to your prediction in question 6?

9. Choose any three consecutive fractions from your answer to Question 7. Write your choices below.

10. Add the smallest fraction to the biggest fraction in your set and simplify the result. What answer do you get? How does this relate to the three numbers you chose in Question 9?

11. Does this work for any three consecutive numbers in a Farey sequence? Show your working below.

Activity 2

Consider the following sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots = \_\_\_\_\_\_$$

1. What patterns do you notice?

2. What do the 3 dots just before the equals sign mean?

3. Without performing any calculations, what do you think the total of adding all these fractions would be? Write down your best guess.

4. Take a blank sheet of paper
   a. Divide it in half.
   b. Shade one half and label it $\frac{1}{2}$.
   c. Divide the unshaded side in half again.
   d. Shade half of the half and label $\frac{1}{4}$.
   e. Divide the unshaded part in half again.
   f. Shade in and label $\frac{1}{8}$ .
   g. Repeat the process for as long as you can.

5. Will you ever completely colour in the whole page? Why/Why not?

6. Using what you have discovered, what do you think the answer to the fraction sum at the top of the page would be now? Justify your answer.

Activity 3

Fractions aren't that new. In fact, the Egyptians were using fractions over 3000 years ago!

However, they wrote their fractions a little differently to us. All their fractions had 1 as the numerator (also known as "unit fractions").

For example the Egyptians used $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$ but they would never use $\frac{2}{3}$ or $\frac{3}{4}$.
If the ancient Egyptians wanted to write a fraction with a different numerator, such as $\frac{3}{4}$, they would write it as a sum of different unit fractions; i.e., $\frac{1}{2} + \frac{1}{4}$ as this equals $\frac{3}{4}$.

If they wanted to write a fraction such as $\frac{2}{3}$, then they would not write $\frac{1}{3} + \frac{1}{3}$ as this used the same fraction twice. Instead they would have written $\frac{1}{2} + \frac{1}{6}$!

1. How many fractions that are smaller than 1 can you create using the list of fractions below. Remember you can add two or more fractions together, but you can't use a fraction twice!

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{1}{8} \quad \frac{1}{9} \quad \frac{1}{10}$$

2. Draw a number line and place all the fractions you found in Question 1 on it.

3. The Egyptians could also write the number 1 as a sum of different fractions. For example the sum $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$ uses 3 different unit fractions and adds up to one.

   Using exactly 4 different unit fractions there are different ways to write the number one. See if you can find them. You can use any unit fractions you wish.

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Activity

# Divide and Conquer Fractions

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 214: DIVIDE AND CONQUER FRACTIONS

## Overview

This task is designed to introduce the division of fractions. It is assumed that students are already familiar with multiplication of whole numbers. The task aims to show students why multiplying by a reciprocal works by drawing links to their previous knowledge of multiplication of whole numbers. It is preferable that students attempt this activity without the use of a calculator.

## Students will need

- none required

## Relevant content descriptions from the Western Australian Curriculum

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

## Students can demonstrate

- *fluency* when they
  - use inverse operations to restate equations to make them easier to solve
- *understanding* when they
  - can make different number statements using a composite number, a pair of its factors and the multiplication or division operations
- *reasoning* when they
  - apply their previous knowledge of multiplication and division to the multiplication and division of fractions
  - explain the division of fractions to a friend
- *problem solving* when they
  - use different strategies to solve problems.

Introduction

This activity is based on the multiplication and division of whole numbers. Hopefully you will notice some patterns you haven't seen before!

Activity 1

Below is a set of number statements.

3 x 4 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3

1. Using only whole numbers, create your own set of number statements for 5 more sets of numbers. The easiest way to create a set of number statements is to -
   a. Choose a composite number,
   b. Choose a pair of factors for your number,
   c. Using multiplication and division, write three true statements for your set of numbers.

| *Composite Number* | *Factors* | *Statements* |
|---|---|---|
| 24 | 1, 2, 3, 4, 6, 8, 12, 24 | 3 x 8 = 24, 24 ÷ 8 = 3, 24 ÷ 3 = 8 |
| | Only 1 pair required | Various answers |

2. Can you write a general rule of this pattern? (Hint: Use "a" to represent the composite number and "b" and "c" to represent to the two factors you chose.
   $a \times b = c$
   $c \div b = a$
   $c \div a = b$

3. Solve the following problems.

   a. 48 ÷ 12 = 4
   b. 15 ÷ 3 = 5
   c. 5 x 7 = 35
   d. 4 x 5 = 20
   e. 4 x 7 = 28
   f. 36 ÷ 12 = 3

4. How did you solve the last two? What strategies did you use?
   Various answers.

5. DO NOT solve the problems below. Instead, see if you can rewrite each problem so that it is easier solve. (Hint: Use the pattern you found in Question 2 to help you.)

a. 18 ÷ ______ = 6   18 ÷ 6 = ______ or 6 x ____ = 18; Answer = 3
b. 14 ÷ ______ = 2   14 ÷ 2 = ______ or 2 x ____ = 14; Answer = 7

c. 48 ÷ ______ = 6   48 ÷ 6 = ______ or 6 x ____ = 48; Answer = 8

d. 11 x ______ = 121   121 ÷ 11 = ______ or 11 x ____ = 121; Answer = 11

e. 14 x ______ = 42   42 ÷ 14 = ______ or 14 x ____ = 42; Answer = 3

6. Now find the answers to the questions above.
   Answers included above.

Activity 2

The strategy you used in Activity 1 also works with fractions. For example;

$30 \div \frac{3}{4} = 40$

$30 \div 40 = \frac{3}{4}$

$\frac{3}{4} \times 40 = 30$

1. Create three correct number statements for each of set of numbers below.

| *Numbers* | *Statements* |
|---|---|
| 15, 60, $\frac{1}{4}$ | $60 \times \frac{1}{4} = 15$,<br>$15 \div 60 = \frac{1}{4}$<br>$15 \div \frac{1}{4} = 60$ |
| 20, 30, $\frac{2}{3}$ | $30 \times \frac{2}{3} = 20$<br>$20 \div 30 = \frac{2}{3}$<br>$20 \div \frac{2}{3} = 30$ |
| 20, 100, $\frac{1}{5}$ | $100 \times \frac{1}{5} = 20$<br>$20 \div 100 = \frac{1}{5}$<br>$20 \div \frac{1}{5} = 100$ |
| 16, 40, $\frac{2}{5}$ | $40 \times \frac{2}{5} = 16$<br>$16 \div 40 = \frac{2}{5}$<br>$16 \div \frac{2}{5} = 40$ |
| 14, 2, $^1/_7$ | $14 \times \frac{1}{7} = 2$<br>$2 \div 14 = \frac{1}{7}$<br>$2 \div \frac{1}{7} = 14$ |

2. Can you write a general rule for this pattern? Does it follow the same rule you found in Activity 1?
   $a \times b = c$
   $c \div b = a$
   $c \div a = b$    Yes, it follows the same rule as before.

3. Dividing fractions can be a bit trickier than dividing by whole numbers.

Here is a way to solve the problems.

| | |
|---|---|
| Here is the question: | 18 ÷ ¾ = ? . |
| I prefer multiplication so I can re-write the equation: | ¾ x ? = 18 . |
| I prefer to work with whole numbers so I will multiply both sides by the denominator (in this case, the number 4) | 4 x ¾ x ? = 18 x 4 |
| Then simplify both sides | 3 x ? = 72 . |
| Divide both sides by 3 | (3 x ?) ÷ 3 = 72÷ 3. |
| So the answer is | ? = 24 . |

Use this method, showing each line of working, to solve the following problems.

a. $4 \div \frac{2}{5} = 10$
b. $72 \div \frac{2}{3} = 108$
c. $48 \div \frac{3}{4} = 64$
d. $28 \div \frac{2}{3} = 42$

4. DISCUSSION QUESTION: There is a shorter way to divide by fractions. Can you see what it is?

Various answers; e.g., multiply by the denominator, divide by the numerator

5. Test your theory on the Question 3 items. Does your theory work?

Yes

6. Use this new method to calculate the following:

a. $45 \div \frac{5}{6} = 54$
b. $36 \div \frac{6}{7} = 42$
c. $52 \div \frac{2}{3} = 78$
d. $28 \div \frac{4}{7} = 49$

<u>Activity 3.</u>

Consider the problem the problem $\frac{2}{4} \div \frac{1}{4} = ?$

We know from our previous activities that this can be rearranged to $\frac{1}{4}$ x ? = $\frac{2}{4}$. Now the answer is obvious – it is 2.

1. See if you can complete the following divisions:

a. $\frac{3}{5} \div \frac{1}{5}$ = 3
b. $\frac{4}{7} \div \frac{2}{7}$ = 2
c. $\frac{6}{9} \div \frac{2}{9}$ = 3

2. Consider the problem $\frac{4}{7} \div \frac{5}{7}$

a. We know that this problem can be written as $\frac{5}{7}$ x ? = $\frac{4}{7}$. Will our answer be larger or smaller than 1? How do you know?
It must be smaller than 1, because $\frac{4}{7}$ (the answer) is smaller than $\frac{5}{7}$ (the multiplicand).

b. Find the answer to the above problem.
$\frac{4}{5}$

c. DISCUSSION QUESTION: How did you solve this question? What strategies did you use?
Various answers.

3. Here is a harder problem. Working with a friend, see if you can find the answer to $\frac{4}{9} \div \frac{1}{5}$
$\frac{20}{9} = 2\frac{2}{9}$

4. DISCUSSION QUESTION: How did you solve this question? What strategies did you use?
Various answers.

5. Here is one way of solving the previous question $\frac{4}{9} \div \frac{2}{5}$ = ?

- It is easier to work with multiplication than division — $\frac{2}{5}$ x ? = $\frac{4}{9}$
- I prefer to work with whole numbers, so I will multiply by 5 — 5 x $\frac{2}{5}$ x ? = $\frac{4}{9}$ x 5
- Simplify — 2 x ? = $\frac{20}{9}$ .
- Divide by 2 to get the answer — (2 x ?) ÷ 2 = $\frac{20}{9}$ ÷ 2
- Find the answer — ? = $\frac{20}{18}$ = $\frac{10}{9}$ = $1\frac{1}{9}$

Use the method above to find the solutions to the following divisions.

a. $\frac{3}{5} \div \frac{3}{4}$ = $\frac{4}{5}$

b. $\frac{4}{7} \div \frac{2}{3}$ = $\frac{6}{7}$

c. $\frac{6}{9} \div \frac{7}{5}$ = $\frac{10}{21}$

6. DISCUSSION QUESTION: There is a shorter way to divide by fractions. Can you see what it is?
   Various answers; e.g., multiply by the denominator, divide by the numerator.

7. Test your theory on the divisions from Question 5. Does your theory work?

Multiply by the reciprocal. Emphasise to students that dividing by a number is equivalent to multiplying by its reciprocal.

8. Use this new method to complete the following divisions.

   a. $\frac{1}{6} \div \frac{2}{5} = \frac{1}{6} \times \frac{5}{2} = \frac{5}{12}$

   b. $\frac{3}{4} \div \frac{1}{6} = \frac{3}{4} \times \frac{6}{1} = \frac{18}{4}$

   c. $\frac{2}{7} \div \frac{3}{4} = \frac{2}{7}\, x\, \frac{4}{3} = \frac{8}{21}$

   d. $\frac{1}{6} \div \frac{10}{11} = \frac{1}{6} \times \frac{11}{10} = \frac{11}{60}$

9. Your friend is really struggling with dividing fractions. Write them a short note explaining the easiest way to divide fractions.
   Various answers

Introduction
This activity is based on the multiplication and division of whole numbers. Hopefully you will notice some patterns you haven't seen before!

Activity 1

Below is a set of number statements.
3 x 4 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

1. Using only whole numbers, create your own set of number statements for 5 more sets of numbers. The easiest way to create a set of number statements is to -
   d. Choose a composite number.
   e. Choose a pair of factors for your number.
   f. Using multiplication and division, write three true statements for your set of numbers.

| Composite Number | Factors | Statements |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

2. Can you write a general rule of this pattern? (Hint: use "a" to represent the composite number and "b" and "c" to represent to the two factors you chose)

3. Solve the following problems.

   a. 48 ÷ 12 = _____

   b. 15 ÷ 3 = _____

   c. 5 x 7 = _____

   d. 4 x 5 = _____

   e. 4 x _____ = 28

   f. 36 ÷ _____ = 3

4. How did you solve the last two? What strategies did you use?

5. DO NOT solve the problems below. Instead, see if you can rewrite each problem so that it is easier solve. (Hint: Use the pattern you found in Question 2 to help you.)

   a. 18 ÷ _____ = 6

   b. 14 ÷ _____ = 2

   c. 48 ÷ _____ = 6

   d. 11 x _____ = 121

   e. 14 x _____ = 42

6. Now find the answers to the questions above.

Activity 2

The strategy you used in Activity 1 also works with fractions. For example,

$30 \div \frac{3}{4} = 40$
$30 \div 40 = \frac{3}{4}$
$\frac{3}{4} \times 40 = 30$

1. Create three correct number statements for each set of numbers below.

| Numbers | Statements |
|---|---|
| 15, 60, $\frac{1}{4}$ | |
| 20, 30, $\frac{2}{3}$ | |
| 20, 100, $\frac{1}{5}$ | |
| 16, 40, $\frac{2}{5}$ | |
| 14, 2, $^{1}/_{7}$ | |

2. Can you write a general rule for this pattern? Does it follow the same rule you found in Activity 1?

3. Dividing fractions can be a bit trickier than dividing by whole numbers.

Here is a way to solve the problems.

- Here is the question: $18 \div \frac{3}{4} = ?$ .
- I prefer multiplication so I can re-write the equation: $\frac{3}{4} \times ? = 18$ .
- I prefer to work with whole numbers so I will multiply both sides by the denominator (in this case, the number 4) $4 \times \frac{3}{4} \times ? = 18 \times 4$
- Then simplify both sides $3 \times ? = 72$ .
- Divide both sides by 3 $(3 \times ?) \div 3 = 72 \div 3$
- And the answer is $? = 24$ .

Use this method, showing each line of working, to solve the following problems.

a. $4 \div \frac{2}{5}$

b. $72 \div \frac{2}{3}$

c. $48 \div \frac{3}{4}$

d. $28 \div \frac{2}{3}$

4. DISCUSSION QUESTION: There is a shorter way to divide by fractions. Can you see what it is?

5. Test your theory on the items from Question 3. Does your theory work?

6. Use this new method to calculate the following:

d. $45 \div \frac{5}{2}$

e. $36 \div \frac{6}{5}$

f.$52 \div \frac{2}{3}$

g. $28 \div \frac{4}{7}$

Activity 3.

Consider the problem the problem $\frac{2}{4} \div \frac{1}{4}$ = ?

We know from our previous activities that this can be rearranged to $\frac{1}{4}$ x ? = $\frac{2}{4}$. Now the answer is obvious – it is 2.

1. See if you can complete the following divisions:

   a. $\frac{3}{5} \div \frac{1}{5}$

   b. $\frac{4}{7} \div \frac{2}{7}$

   c. $\frac{6}{9} \div \frac{2}{9}$

2. Consider the problem $\frac{4}{7} \div \frac{5}{7}$

   a. We know that this problem can be written as $\frac{5}{7}$ x ? = $\frac{4}{7}$. Will our answer be larger or smaller than 1? How do you know?

   b. Find the answer to the above problem.

   c. DISCUSSION QUESTION: How did you solve this question? What strategies did you use?

3. Here is a harder problem. Working with a friend, see if you can find the answer to $\frac{4}{9} \div \frac{1}{5}$

4. DISCUSSION QUESTION: How did you solve this question? What strategies did you use?

5. Here is one way of solving the previous question $\frac{4}{9} \div \frac{2}{5} = ?$

- It is easier to work with multiplication than division $\frac{2}{5} \text{ x } ? = \frac{4}{9}$
- I prefer to work with whole numbers, so I will multiply by 5 $5 \text{ x } \frac{2}{5} \text{ x } ? = \frac{4}{9} \text{ x } 5$
- Simplify $2 \text{ x } ? = \frac{20}{9}$ .
- Divide by 2 to get the answer $(2 \text{ x } ?) \div 2 = \frac{20}{9} \div 2$
- Find the answer $? = \frac{20}{18} = \frac{10}{9} = 1\frac{1}{9}$

Use the method above to find the solutions to the following divisions.

a. $\frac{3}{5} \div \frac{3}{4}$

b. $\frac{4}{7} \div \frac{2}{3}$

c. $\frac{6}{9} \div \frac{7}{5}$

6. DISCUSSION QUESTION: There is a shorter way to divide by fractions. Can you see what it is?

7. Test your theory on the divisions from Question 5. Does your theory work?

8 Use this new method to answer the following questions.

a. $\frac{1}{6} \div \frac{2}{5}$

b. $\frac{3}{4} \div \frac{1}{6}$

c. $\frac{2}{7} \div \frac{3}{4}$

d. $\frac{1}{6} \div \frac{10}{11}$

9 Your friend is really struggling with dividing fractions. Write them a short note explaining the easiest way to divide fractions.

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Equivalent Fractions

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 217: EQUIVALENT FRACTIONS

## Overview

This task is designed to revise the idea of equivalent fractions and use equivalence to compare and order fractions on the number line.

## Students will need

- pieces of blank paper
- deck of cards (one between three or four students)
- fraction cards (one between two or three students)
- blu-tack or sticky tape

## Relevant content descriptions from the Western Australian Curriculum

- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)

## Students can demonstrate

- *fluency* when they
    - compare and order fractions on the number line
- *understanding* when they
    - recognise equivalent fractions
- *reasoning* when they
    - explain how to compare and order numbers on the fraction line.

Activity 1

Ensure students have a square or rectangular sheet and give directions as appropriate.

1. Fold a sheet of paper in half. How many parts have been created?
   2

2. Colour in one part of their paper. What fraction of the page is this?
   **½**

3. Now fold the paper in half again as you did before. Then fold the paper in half again. Open the page out flat.
   - How many parts are there in total?
     4
   - How many parts are coloured?
     2
   - What fraction is this?
     **$^2/_4$**
   - Has the amount that's been coloured changed?
     No
   - What can you say about ½ and $^2/_4$?
     They represent the same number; they are equivalent fractions.

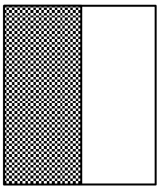
4. By further folding, find at least 3 more fractions that are equivalent to ½.
   Answers will vary.

5. Repeat the process again, using a new piece of paper and starting with a different fraction; e.g., thirds, fifths, etc. Write the equivalences that you found.
   Answers will vary.

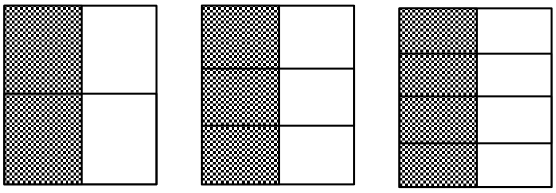
Activity 2

To find equivalent fractions, we could -

- Draw the fraction using columns; e.g., ½ is shown below.

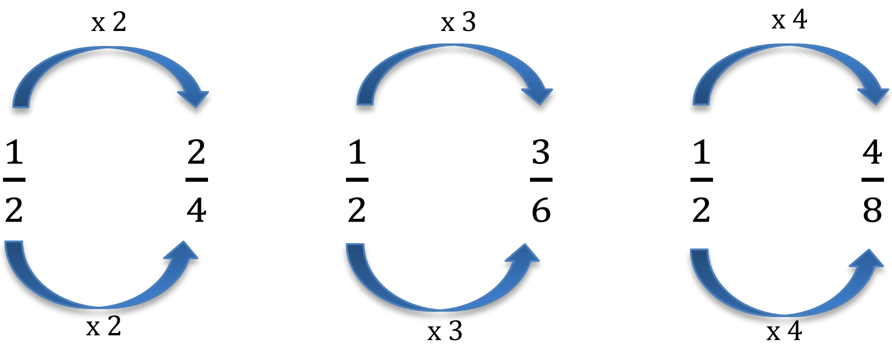


- Break the shape up with more and more horizontal lines



$^{2}/_{4}$ $^{3}/_{6}$ $^{4}/_{8}$

1. Another way to find equivalent fractions is to multiply both the numerator and denominator by the same number. Why does this work?
This means you are multiplying by 1, so the number will not change and only the fraction will look different. For example, $^{1}/_{3} \times {}^{2}/_{2} = {}^{2}/_{6}$




2. For each of the fractions listed below, find at least 3 equivalent fractions and draw diagrams to show how they are equivalent. See below.

   a. ¼

   b. ¾

   c. $^{1}/_{5}$

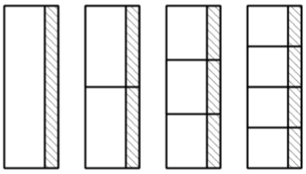
   d. $^{2}/_{5}$

   e. $^{3}/_{7}$

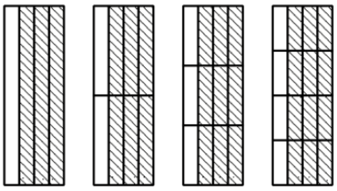
2. For each of your answers to question 2, draw a diagram to show that the fractions you have are equivalent.

Answers may vary, and can be in different forms, with one example for each below:

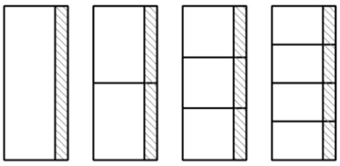
a. ¼ $^{2}/_{8}$ $^{3}/_{12}$ $^{4}/_{16}$



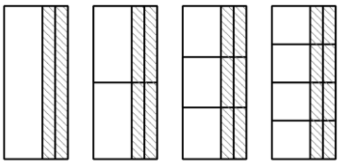
b. ¾ $^{6}/_{8}$ $^{9}/_{12}$ $^{12}/_{16}$



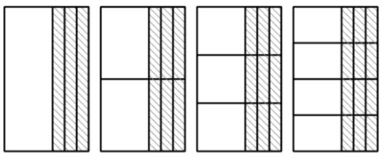
c. $^{1}/_{5}$ $^{2}/_{10}$ $^{3}/_{15}$ $^{4}/_{20}$



d. $^{2}/_{5}$ $^{4}/_{10}$ $^{6}/_{15}$ $^{8}/_{20}$



e. $^{3}/_{7}$ $^{6}/_{14}$ $^{9}/_{21}$ $^{12}/_{28}$



***Or any other acceptable fractions

Activity 3 Teacher-led

1. In this activity you are to work with another student and discuss the solution to this problem:
At a recent party, John ate 2/3 of a pizza; Jane ate 5/6 of a pizza. Who ate more pizza? How do you know?

Have students discuss and explain their ideas. Prompt students to the use of equivalent fractions if necessary. Emphasise the fact that it is easier to compare fractions when they have the same denominator (name).

2. Now try this problem with your partner:
Ella was also at the same party. She had 3/7 of a pizza. Did she eat more or less than John? How do you know?

Again, allow students to discuss and explain their ideas. Prompt students to compare each fraction to another fraction; e.g., ½, or make equivalent fractions so as to compare fractions with the same denominator as each other.

3. Play the “Compare that Fraction” game. Your teacher will direct you.

Have students play the “Compare that Fraction” game. Instructions here:
http://www.education.com/activity/article/capture-that-fraction/

Activity 4

1. Cut out the ‘fraction cards’ from the following pages and draw a number line on the white board. Make it as long as you can and have the numbers run from -3 to 3.

2. Organise students into pairs or small groups. Give each group a card with a positive number on it.

3. Give students a few moments to work out where their card should go on the number line before asking students to come up to the board to stick their card to the number line. If fractions are equivalent, students can stick their card underneath one another.

4. As each student comes up to the board, question their thinking such as follows:
   - How did you decide that number goes there?
     Watch out for misconceptions such as believing the bigger the denominator, the bigger the number, or treating the two numbers in a fraction as separate numbers.
   - How do you know that number is closer to one than to two?
     Answers as appropriate.
   - How can you decide if a number is closer to 0, ½ or a whole?
     If the numerator is closer to 0, than the fraction is closer to zero. If the numerator is close to the denominator, than the fraction is close to 1. If the numerator is approximately half the denominator, then the fraction is close to ½.
   - How do you decide where improper fractions go?
     Students may use different appropriate strategies here.

5. As more cards get placed on the line, encourage other students to question the placement of each card and move any cards, if necessary.

6. When students have agreed on the placement of all the fraction cards, have students consider the negative part of the number line. Remind students that as you travel left along the number line, the numbers are getting smaller and that the size of the number (without the negative sign) tells you how far from zero you are; e.g., -3 is smaller than -2 because it is further away from zero.

7. Give each group a card with a negative fraction on it and repeat the above process.

8. Some questions to ask:
   - How did you decide that number goes there?
   - How do you know that number is closer to negative one than to negative two?
   - How do you know that number is more negative than this one?

   Various answers for these questions.

9. Have students write a short paragraph describing how to compare and order fractions.
   Various answers. Check for misconceptions.

| | | | | |
|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{1}{3}$ | $1\frac{2}{3}$ | $2\frac{1}{4}$ | $\frac{2}{4}$ |
| $2\frac{3}{4}$ | $\frac{1}{5}$ | $1\frac{2}{5}$ | $2\frac{3}{5}$ | $\frac{4}{5}$ |
| $1\frac{2}{6}$ | $\frac{3}{6}$ | $1\frac{2}{7}$ | $2\frac{5}{8}$ | $1\frac{3}{8}$ |
| $\frac{9}{6}$ | $\frac{7}{6}$ | $\frac{9}{7}$ | $\frac{21}{8}$ | $\frac{11}{8}$ |

| | | | | |
|---|---|---|---|---|
| $-\frac{1}{2}$ | $-\frac{1}{3}$ | $-1\frac{2}{3}$ | $-2\frac{1}{4}$ | $-\frac{2}{4}$ |
| $-2\frac{3}{4}$ | $-\frac{1}{5}$ | $-1\frac{2}{5}$ | $-2\frac{3}{5}$ | $-\frac{4}{5}$ |
| $-1\frac{2}{6}$ | $-\frac{3}{6}$ | $-1\frac{2}{7}$ | $-2\frac{5}{8}$ | $-1\frac{3}{8}$ |
| $-\frac{9}{6}$ | $-\frac{7}{6}$ | $-\frac{9}{7}$ | $-\frac{21}{8}$ | $-\frac{11}{8}$ |

<u>Activity 1</u>

1. Fold a sheet of paper in half. How many parts have been created?

2. Colour in one part of their paper. What fraction of the page is this?

3. Now fold the paper in half again as you did before. Then fold the paper in half again. Open the page out flat.
   - How many parts are there in total?
   - How many parts are coloured?
   - What fraction is this?
   - Has the amount that's been coloured changed?
   - What can you say about ½ and ¼?

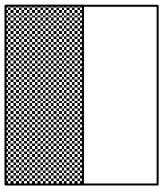
4. By further folding, find at least 3 more fractions that are equivalent to ½.

5. Repeat the process again, using a new piece of paper and starting with a different fraction; e.g., thirds, fifths, etc. Write the equivalences that you found.

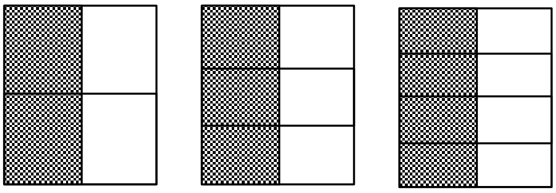
Activity 2

To find equivalent fractions, we could -

- Draw the fraction using columns; e.g., ½ is shown below.

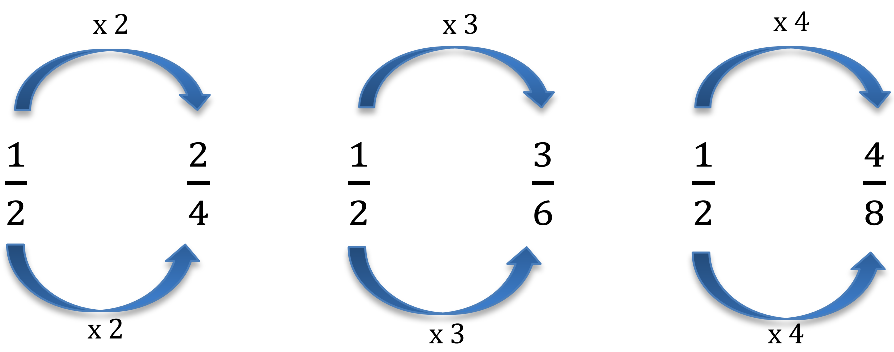


- Break the shape up with more and more horizontal lines



$^{2}/_{4}$ $^{3}/_{6}$ $^{4}/_{8}$

1. Another way to find equivalent fractions is to multiply both the numerator and denominator by the same number. Why does this work?




2. For each of the fractions listed below, find at least 3 equivalent fractions and draw diagrams to show how they are equivalent.

   a. ¼

   b. ¾

   c. $^{1}/_{5}$

   d. $^{2}/_{5}$

   e. $^{3}/_{7}$

3. For each of your answers to Question 2, draw a diagram to show that the fractions you have are equivalent.

Activity 3

1. In this activity you are to work with another student and discuss the solution to this problem:
At a recent party, John ate 2/3 of a pizza; Jane ate 5/6 of a pizza. Who ate more pizza? How do you know?

2. Now try this problem with your partner:
Ella was also at the same party. She had 3/7 of a pizza. Did she eat more or less than John? How do you know?

3. Play the "Compare that Fraction" game. Your teacher will direct you.

Instructions are at: http://www.education.com/activity/article/capture-that-fraction/

Activity 4

In this activity the teacher will have a number line on the board, ranging from -3 to 3.

In pairs or small groups you will receive a card with a positive fraction or mixed numeral on it, and asked to locate its position on the number line, then to justify your choice.
This activity will then be repeated with negative numbers.

| | | | | |
|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{1}{3}$ | $1\frac{2}{3}$ | $2\frac{1}{4}$ | $\frac{2}{4}$ |
| $2\frac{3}{4}$ | $\frac{1}{5}$ | $1\frac{2}{5}$ | $2\frac{3}{5}$ | $\frac{4}{5}$ |
| $1\frac{2}{6}$ | $\frac{3}{6}$ | $1\frac{2}{7}$ | $2\frac{5}{8}$ | $1\frac{3}{8}$ |
| $\frac{9}{6}$ | $\frac{7}{6}$ | $\frac{9}{7}$ | $\frac{21}{8}$ | $\frac{11}{8}$ |

| | | | | |
|---|---|---|---|---|
| $-\frac{1}{2}$ | $-\frac{1}{3}$ | $-1\frac{2}{3}$ | $-2\frac{1}{4}$ | $-\frac{2}{4}$ |
| $-2\frac{3}{4}$ | $-\frac{1}{5}$ | $-1\frac{2}{5}$ | $-2\frac{3}{5}$ | $-\frac{4}{5}$ |
| $-1\frac{2}{6}$ | $-\frac{3}{6}$ | $-1\frac{2}{7}$ | $-2\frac{5}{8}$ | $-1\frac{3}{8}$ |
| $-\frac{9}{6}$ | $-\frac{7}{6}$ | $-\frac{9}{7}$ | $-\frac{21}{8}$ | $-\frac{11}{8}$ |

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Adding Fractions

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 218: ADDING FRACTIONS

### Overview

This task is designed to be an introduction to adding fractions with different denominators. The first task is a simple revision task for adding fractions with the same denominator that helps students to visualise the fractions as they are adding.
The second task introduces different fractions and is designed to allow students to struggle with how to add different fractions.

### Students will need

- scissors
- glue

### Relevant content descriptions from the Western Australian Curriculum

- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)

### Students can demonstrate

- *fluency* when they
  - calculate accurately with fractions
- *understanding* when they
  - recognise equivalence of fractions
  - can explain why fractions with different denominators can't be added as they are
- *reasoning* when they
  - use fraction equivalences to find the fraction sums

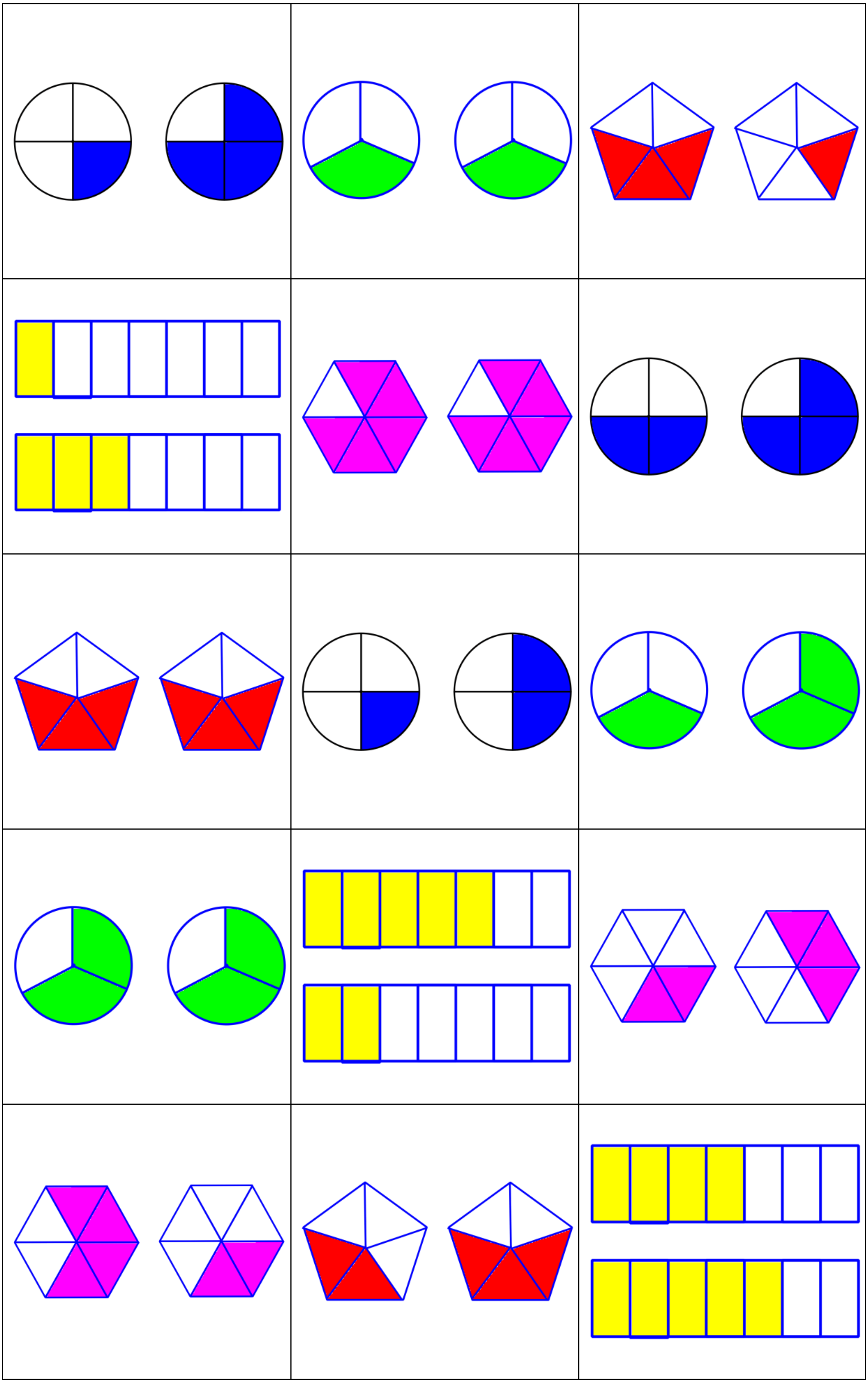
Activity 1

Adding fractions with the same denominator.

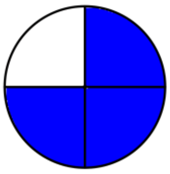
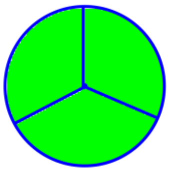
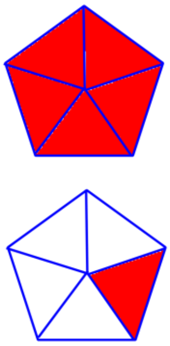
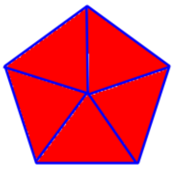
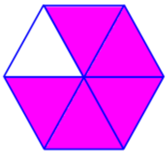
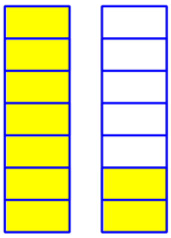
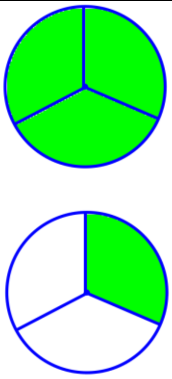
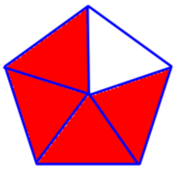
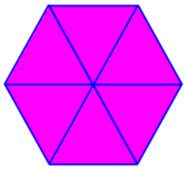
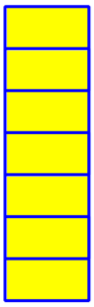
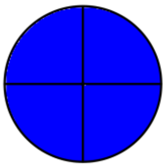
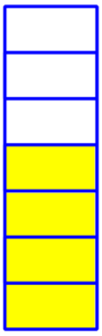
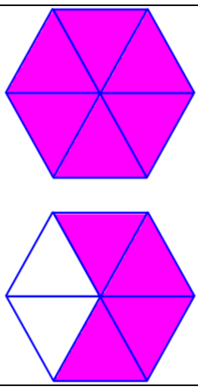
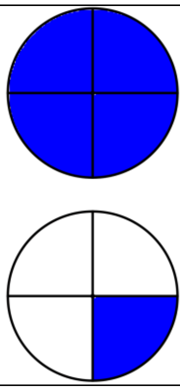
1. Cut out the fraction cards provided on the following pages, and match each picture card with its fractions addition and sum of the addends.

2. Check your work in each case before continuing.

Have students cut out all the cards before continuing. Then ensure they are clear on what is required. Answers as appropriate.



| | | |
|---|---|---|
| $\frac{2}{3}+\frac{2}{3}$ | $\frac{2}{5}+\frac{3}{5}$ | $\frac{4}{6}+\frac{2}{6}$ |
| $\frac{3}{5}+\frac{1}{5}$ | $\frac{1}{4}+\frac{2}{4}$ | $\frac{1}{3}+\frac{1}{3}$ |
| $\frac{1}{4}+\frac{3}{4}$ | $\frac{1}{3}+\frac{2}{3}$ | $\frac{3}{5}+\frac{3}{5}$ |
| $\frac{1}{7}+\frac{3}{7}$ | $\frac{5}{6}+\frac{5}{6}$ | $\frac{2}{4}+\frac{3}{4}$ |
| $\frac{2}{6}+\frac{3}{6}$ | $\frac{5}{7}+\frac{2}{7}$ | $\frac{4}{7}+\frac{5}{7}$ |

| | | |
|---|---|---|
| $\frac{3}{4}$  | $1$  | $1\frac{1}{5}$  |
| $1$  | $\frac{5}{6}$  | $1\frac{2}{7}$  |
| $1\frac{1}{3}$  | $\frac{4}{5}$  | $1$  |
| $1$  | $1$  | $\frac{4}{7}$  |
| $1\frac{4}{6}$  | $\frac{2}{3}$  | $1\frac{1}{4}$  |

## Activity 2 – Teacher Notes

1. Hand out the fraction sum picture cards, fraction sum equation cards and fraction sum answer cards.

2. Ask students to work in pairs or small groups to cut out each card and match them to make a correct equation.

3. All students to attempt the task and struggle with it for a little while.

4. Discuss the following questions as a class:
   a. What is the problem with adding fractions like this?
   b. Can you *accurately* work out the answer to sums like $\frac{2}{3} + \frac{4}{5}$ by looking at them? Why/Why not?
   c. What do we need to do to the fractions so that we can add them?

5. Give students the equivalent fraction sum picture cards and the equivalent fraction sum equation cards.

6. Students should be able to compete the sums now.

7. Have students finish the activity by writing a brief explanation of -
   - why fractions with different denominators cannot be added in their current form;
   - what you must do with fractions of different denominators if you do want to add them.

   Answers should be shared with the class.

Activity 2

Adding fractions with different denominators.

1. Working with a partner, cut out the fraction sum picture cards, the fraction sum equation cards and the fraction answer cards.

2. Match the picture cards with their written sums and their answers.

   Hint: If you are struggling, try grouping the sums into into 3 piles:
   - Sums that you think equal less than 1
   - Sums that you think equal 1
   - Sums that you think equal more than 1.

3. DISCUSSION QUESTIONS:
   a. What is the problem with adding fractions like this?
   We cannot add fractions with unlike denominators without first converting one or both.

   b. Can you *accurately* work out the answer to sums like $\frac{2}{3} + \frac{4}{5}$ by looking at them? Why/Why not?
   No. They need to have the same denominator (name); just as we cannot add metres and centimetres without first converting them to the same name (unit).

   c. What do we need to do to the fractions so that we can add them?
   Convert one or both so they have the same denominator.

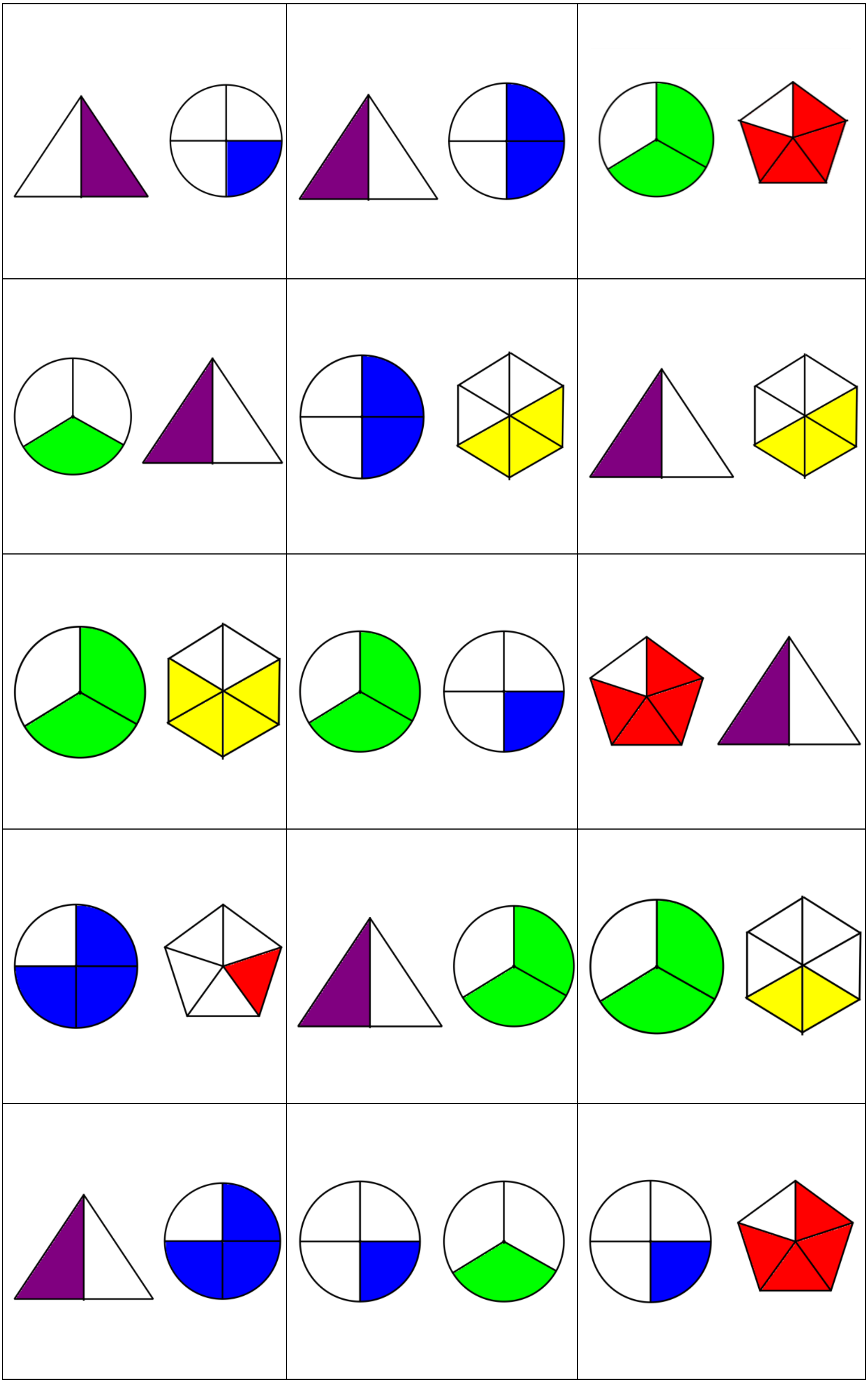
4. Match the equivalent fraction picture cards and the equivalent fraction sum cards you have just been given with the cards you already had. Check your work with the teacher before continuing.
   See answers below.

5. Write a brief explanation of -
   a. Why you cannot add (or subtract!) fractions with different denominators.
   It is impossible as they are different units.

   b. What you must do to fractions with different denominators so they can be added.
   Convert one or more so they all have the same denominator.

   c. Share your answers with the class.

Fraction Sum Picture Cards



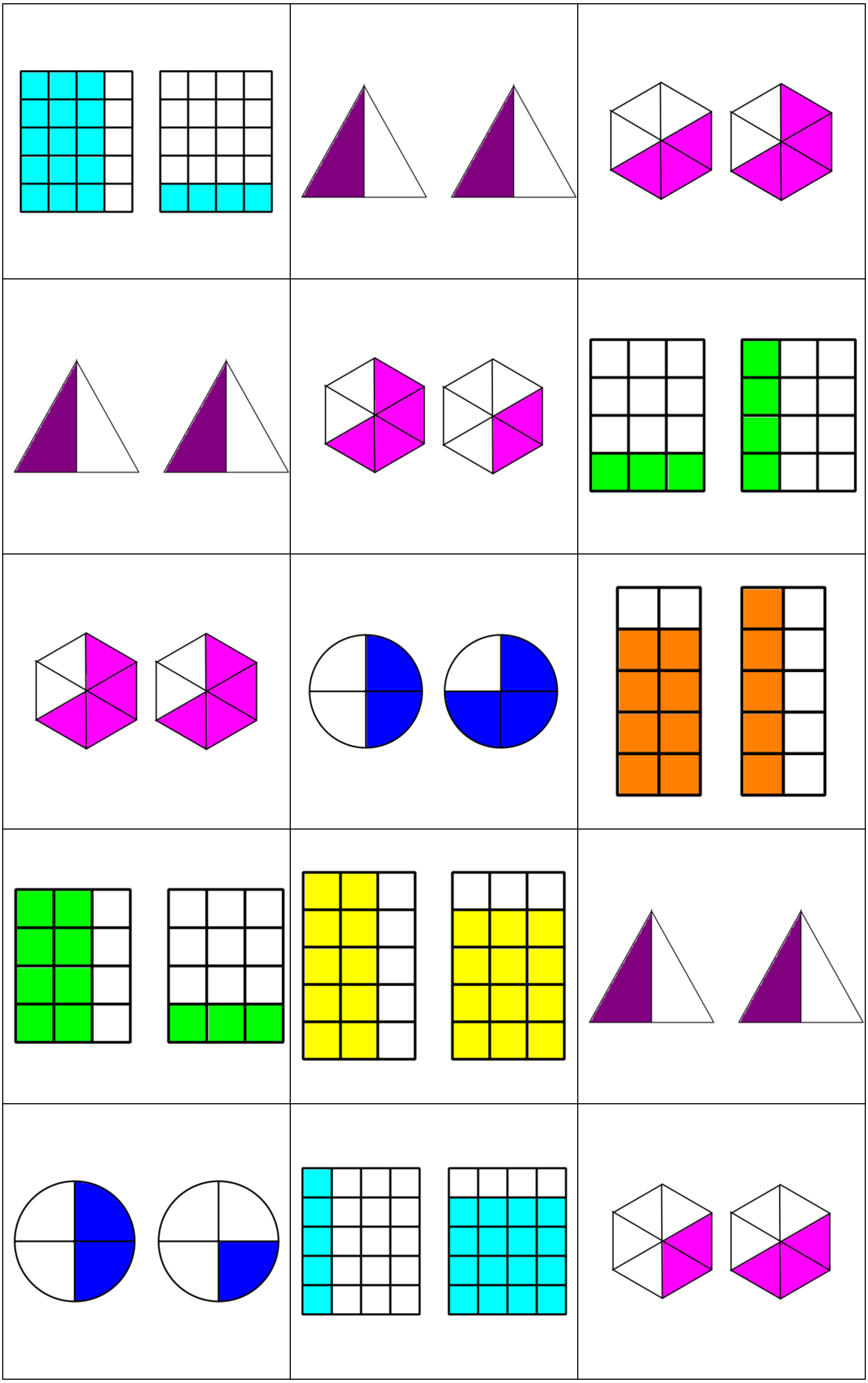
Fraction Sum Equation Cards

| | | |
|---|---|---|
| $\frac{1}{4} + \frac{1}{3}$ | $\frac{4}{5} + \frac{1}{2}$ | $\frac{2}{3} + \frac{4}{6}$ |
| $\frac{1}{4} + \frac{4}{5}$ | $\frac{1}{2} + \frac{3}{6}$ | $\frac{2}{3} + \frac{1}{4}$ |
| $\frac{1}{2} + \frac{3}{4}$ | $\frac{1}{3} + \frac{1}{2}$ | $\frac{2}{4} + \frac{3}{6}$ |
| $\frac{1}{2} + \frac{1}{4}$ | $\frac{2}{3} + \frac{4}{5}$ | $\frac{1}{2} + \frac{2}{4}$ |
| $\frac{2}{3} + \frac{2}{6}$ | $\frac{1}{2} + \frac{2}{3}$ | $\frac{3}{4} + \frac{1}{5}$ |

Answer Cards

| | | |
|---|---|---|
| $\frac{7}{12}$ | $1$ | $1\frac{1}{4}$ |
| $1\frac{1}{20}$ | $\frac{19}{20}$ | $1$ |
| $1$ | $\frac{5}{6}$ | $1\frac{2}{6}$ |
| $1$ | $1\frac{3}{10}$ | $1\frac{1}{6}$ |
| $\frac{11}{12}$ | $1\frac{7}{15}$ | $\frac{3}{4}$ |

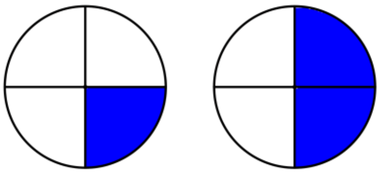
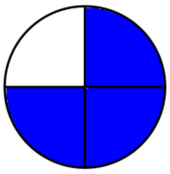
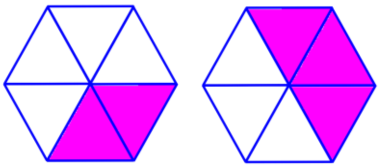
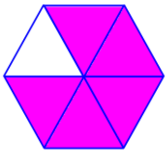
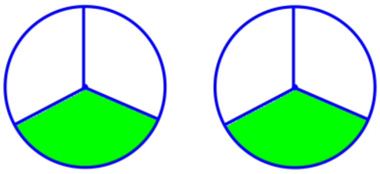
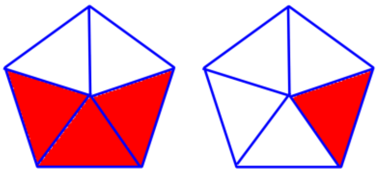
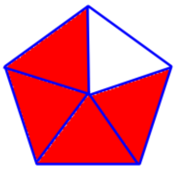
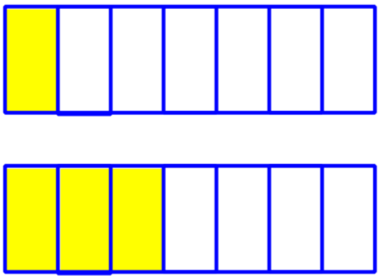
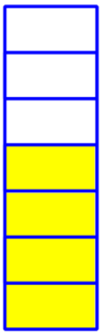
Equivalent Fraction Picture Cards



Equivalent Fraction Sums Cards

| | | |
|---|---|---|
| $\frac{3}{12} + \frac{4}{12}$ | $\frac{8}{10} + \frac{5}{10}$ | $\frac{4}{6} + \frac{4}{6}$ |
| $\frac{5}{20} + \frac{16}{20}$ | $\frac{1}{2} + \frac{1}{2}$ | $\frac{8}{12} + \frac{3}{12}$ |
| $\frac{2}{4} + \frac{3}{4}$ | $\frac{2}{6} + \frac{3}{6}$ | $\frac{1}{2} + \frac{1}{2}$ |
| $\frac{2}{4} + \frac{1}{4}$ | $\frac{10}{15} + \frac{12}{15}$ | $\frac{1}{2} + \frac{1}{2}$ |
| $\frac{4}{6} + \frac{2}{6}$ | $\frac{3}{6} + \frac{4}{6}$ | $\frac{15}{20} + \frac{4}{20}$ |

Activity 1 - Answers

| | | |
|---|---|---|
|  | $\frac{1}{4} + \frac{2}{4}$ | $\frac{3}{4}$  |
|  | $\frac{2}{6} + \frac{3}{6}$ | $\frac{5}{6}$  |
|  | $\frac{1}{3} + \frac{1}{3}$ | $\frac{2}{3}$  |
|  | $\frac{3}{5} + \frac{1}{5}$ | $\frac{4}{5}$  |
|  | $\frac{1}{7} + \frac{3}{7}$ | $\frac{4}{7}$  |

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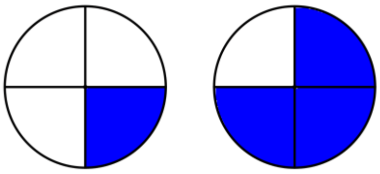
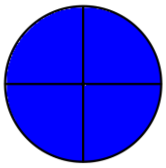
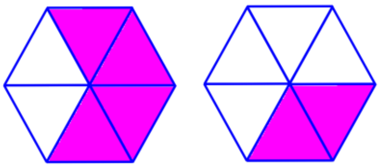
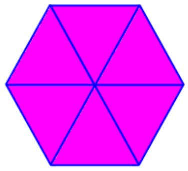
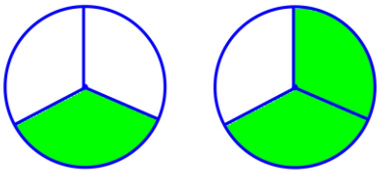
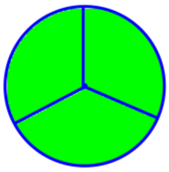
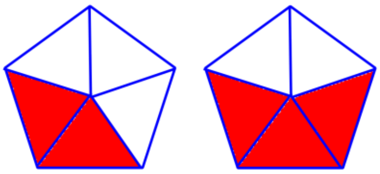
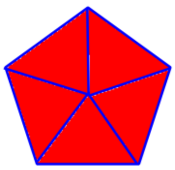
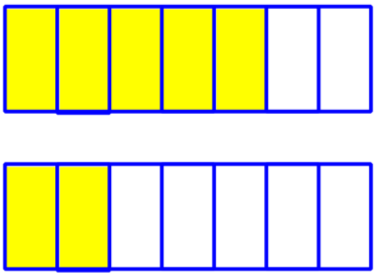
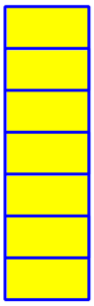
|  | $\frac{1}{4}+\frac{3}{4}$ | 1  |
|  | $\frac{4}{6}+\frac{2}{6}$ | 1  |
|  | $\frac{1}{3}+\frac{2}{3}$ | 1  |
|  | $\frac{2}{5}+\frac{3}{5}$ | 1  |
|  | $\frac{5}{7}+\frac{2}{7}$ | 1  |

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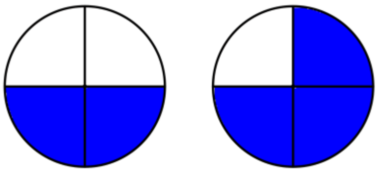
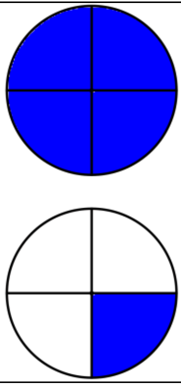
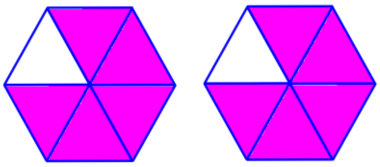
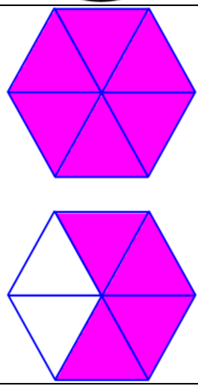
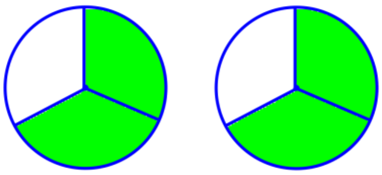
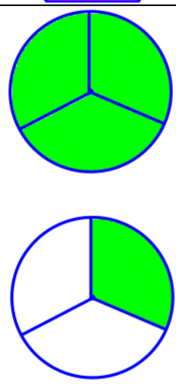
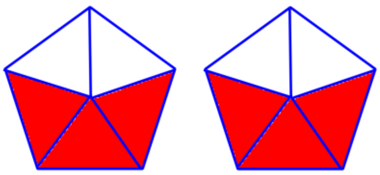
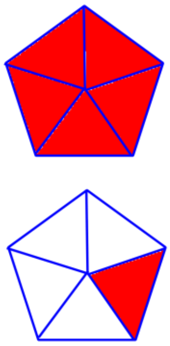
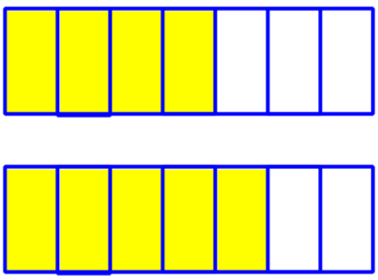
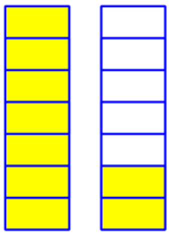
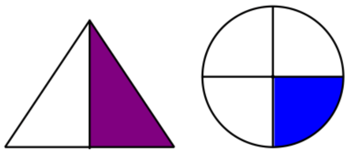
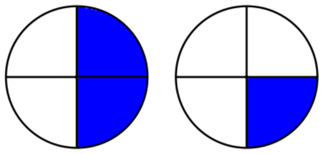
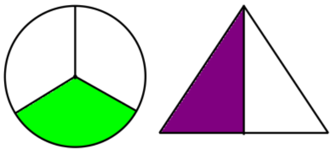
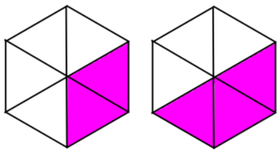
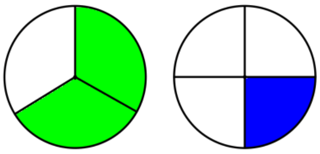
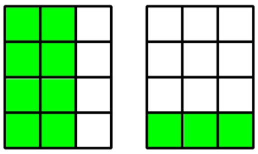
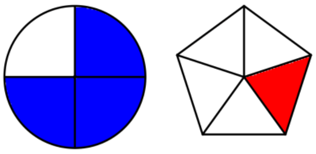
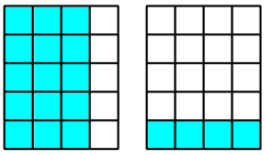
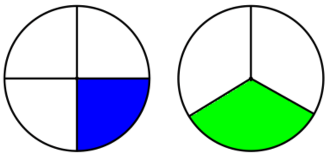
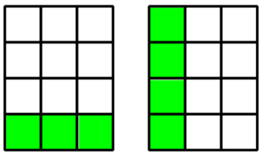
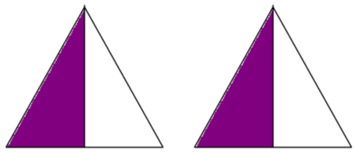
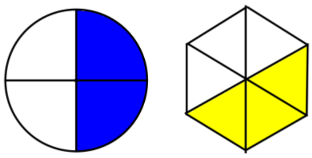
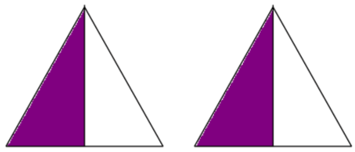
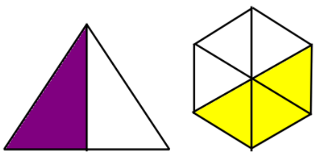
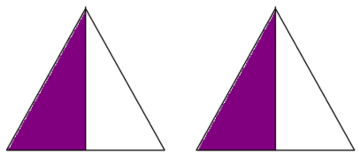
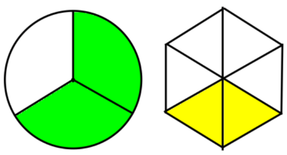
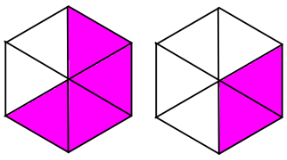
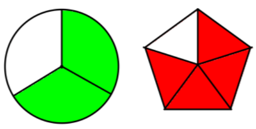
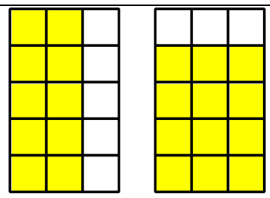
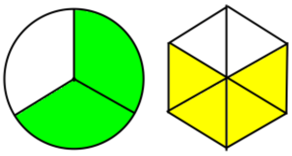
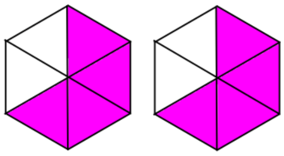
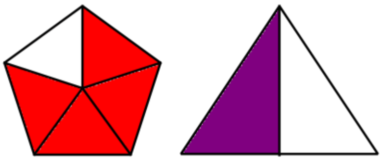
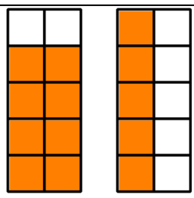
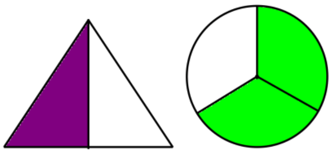
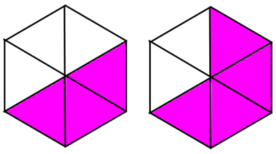
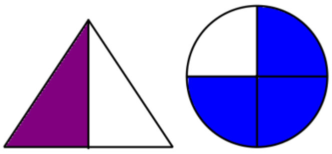
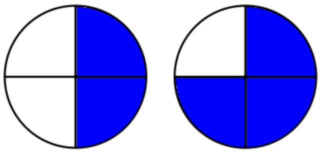
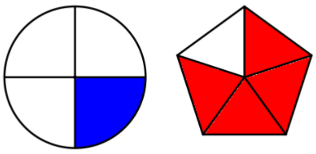
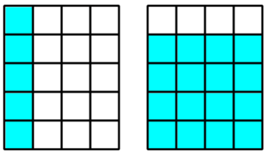
|  | $\frac{2}{4}+\frac{3}{4}$ | $1\frac{1}{4}$  |
|  | $\frac{5}{6}+\frac{5}{6}$ | $1\frac{4}{6}$  |
|  | $\frac{2}{3}+\frac{2}{3}$ | $1\frac{1}{3}$  |
|  | $\frac{3}{5}+\frac{3}{5}$ | $1\frac{1}{5}$  |
|  | $\frac{4}{7}+\frac{5}{7}$ | $1\frac{2}{7}$  |

Activity 2 Answers

| | | |
|---|---|---|
|  $\frac{1}{2}+\frac{1}{4}$ |  $\frac{2}{4}+\frac{1}{4}$ | $\frac{3}{4}$ |
|  $\frac{1}{3}+\frac{1}{2}$ |  $\frac{2}{6}+\frac{3}{6}$ | $\frac{5}{6}$ |
|  $\frac{2}{3}+\frac{1}{4}$ |  $\frac{8}{12}+\frac{3}{12}$ | $\frac{11}{12}$ |
|  $\frac{3}{4}+\frac{1}{5}$ |  $\frac{15}{20}+\frac{4}{20}$ | $\frac{19}{20}$ |
|  $\frac{1}{4}+\frac{1}{3}$ |  $\frac{3}{12}+\frac{4}{12}$ | $\frac{7}{12}$ |

| | | |
|---|---|---|
|  $\frac{1}{2} + \frac{2}{4}$ |  $\frac{1}{2} + \frac{1}{2}$ | $1$ |
|  $\frac{2}{4} + \frac{3}{6}$ |  $\frac{1}{2} + \frac{1}{2}$ | $1$ |
|  $\frac{1}{2} + \frac{3}{6}$ |  $\frac{1}{2} + \frac{1}{2}$ | $1$ |
|  $\frac{2}{3} + \frac{2}{6}$ |  $\frac{4}{6} + \frac{2}{6}$ | $1$ |
|  $\frac{2}{3} + \frac{4}{5}$ |  $\frac{10}{15} + \frac{12}{15}$ | $1\frac{7}{15}$ |

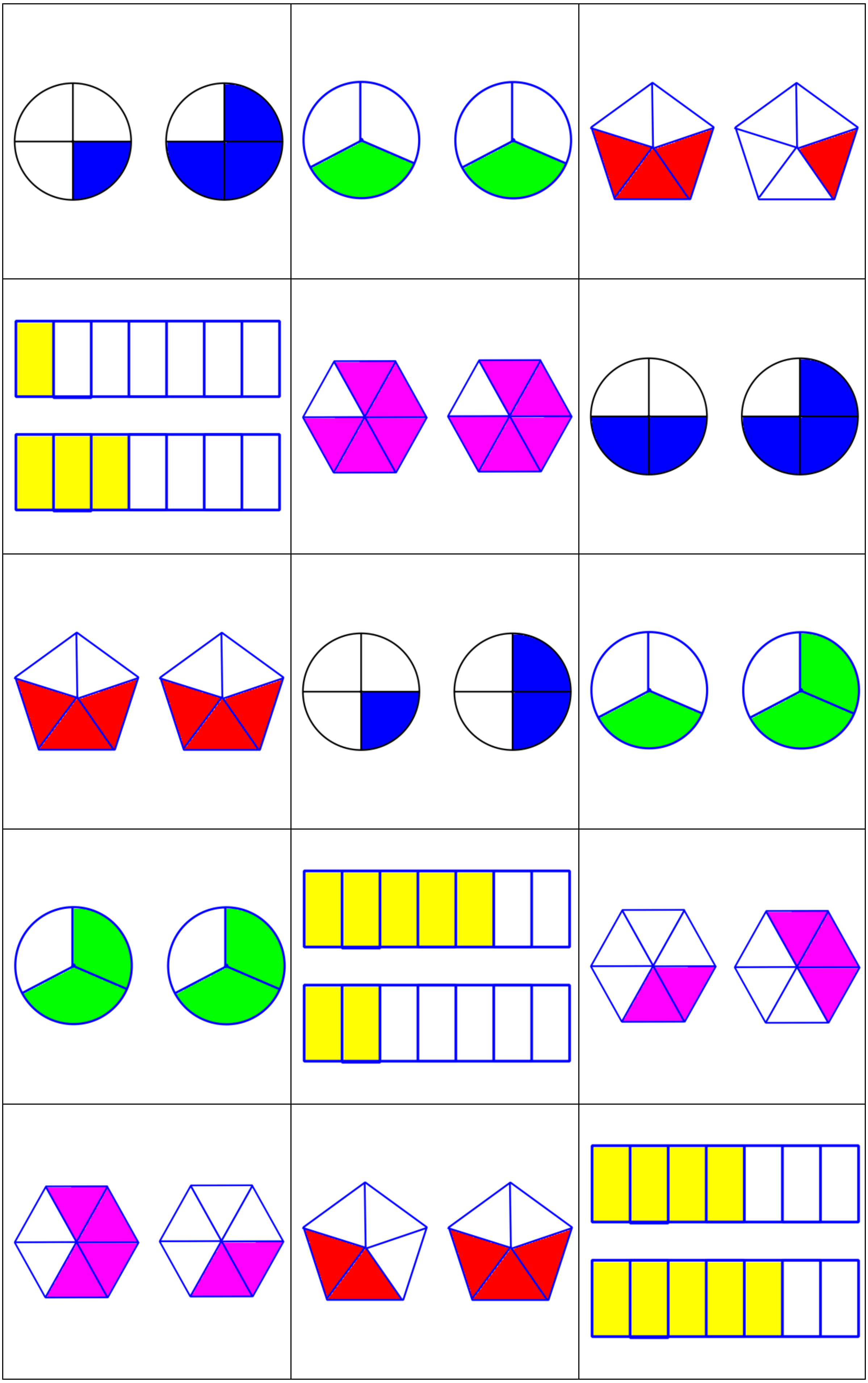
| | | |
|---|---|---|
|  $\frac{2}{3} + \frac{4}{6}$ |  $\frac{4}{6} + \frac{4}{6}$ | $1\frac{2}{6}$ |
|  $\frac{4}{5} + \frac{1}{2}$ |  $\frac{8}{10} + \frac{5}{10}$ | $1\frac{3}{10}$ |
|  $\frac{1}{2} + \frac{2}{3}$ |  $\frac{3}{6} + \frac{4}{6}$ | $1\frac{1}{6}$ |
|  $\frac{1}{2} + \frac{3}{4}$ |  $\frac{2}{4} + \frac{3}{4}$ | $1\frac{1}{4}$ |
|  $\frac{1}{4} + \frac{4}{5}$ |  $\frac{5}{20} + \frac{16}{20}$ | $1\frac{1}{20}$ |

Activity 1

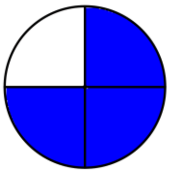
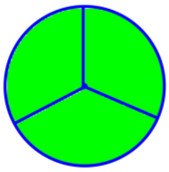
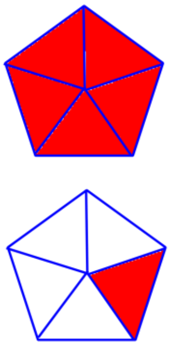
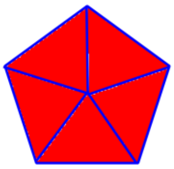
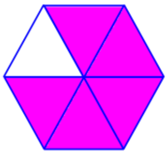
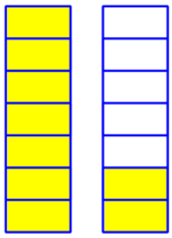
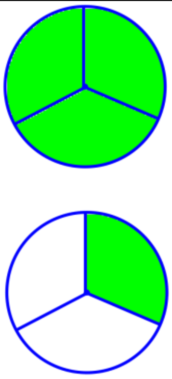
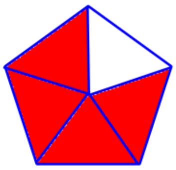
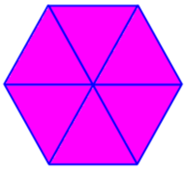
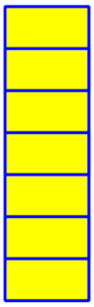
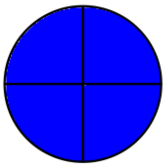
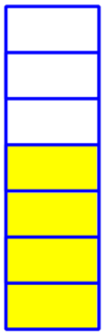
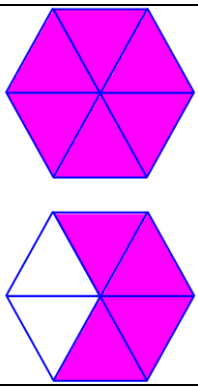
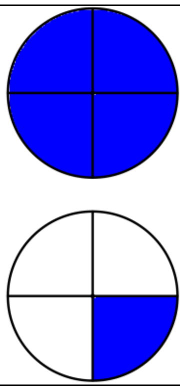
Adding fractions with the same denominator.

1. Cut out the fraction cards provided on the following pages, and match each picture card with its fractions addition and sum of the addends.

2. Check your work in each case before continuing.



| | | |
|---|---|---|
| $\frac{2}{3}+\frac{2}{3}$ | $\frac{2}{5}+\frac{3}{5}$ | $\frac{4}{6}+\frac{2}{6}$ |
| $\frac{3}{5}+\frac{1}{5}$ | $\frac{1}{4}+\frac{2}{4}$ | $\frac{1}{3}+\frac{1}{3}$ |
| $\frac{1}{4}+\frac{3}{4}$ | $\frac{1}{3}+\frac{2}{3}$ | $\frac{3}{5}+\frac{3}{5}$ |
| $\frac{1}{7}+\frac{3}{7}$ | $\frac{5}{6}+\frac{5}{6}$ | $\frac{2}{4}+\frac{3}{4}$ |
| $\frac{2}{6}+\frac{3}{6}$ | $\frac{5}{7}+\frac{2}{7}$ | $\frac{4}{7}+\frac{5}{7}$ |

| | | |
|---|---|---|
| $\frac{3}{4}$  | $1$  | $1\frac{1}{5}$  |
| $1$  | $\frac{5}{6}$  | $1\frac{2}{7}$  |
| $1\frac{1}{3}$  | $\frac{4}{5}$  | $1$  |
| $1$  | $1$  | $\frac{4}{7}$  |
| $1\frac{4}{6}$  | $\frac{2}{3}$  | $1\frac{1}{4}$  |

Activity 2

Adding fractions with different denominators.

1. Working with a partner, cut out the fraction sum picture cards, the fraction sum equation cards and the fraction answer cards.

2. Match the picture cards with their written sums and their answers.

   Hint: If you are struggling, try grouping the sums into 3 piles:
   - Sums that you think equal less than 1
   - Sums that you think equal 1
   - Sums that you think equal more than 1.

3. DISCUSSION QUESTIONS:
   a. What is the problem with adding fractions like this?

   b. Can you *accurately* work out the answer to sums like $\frac{2}{3} + \frac{4}{5}$ by looking at them? Why/Why not?

   c. What do we need to do to the fractions so that we can add them?

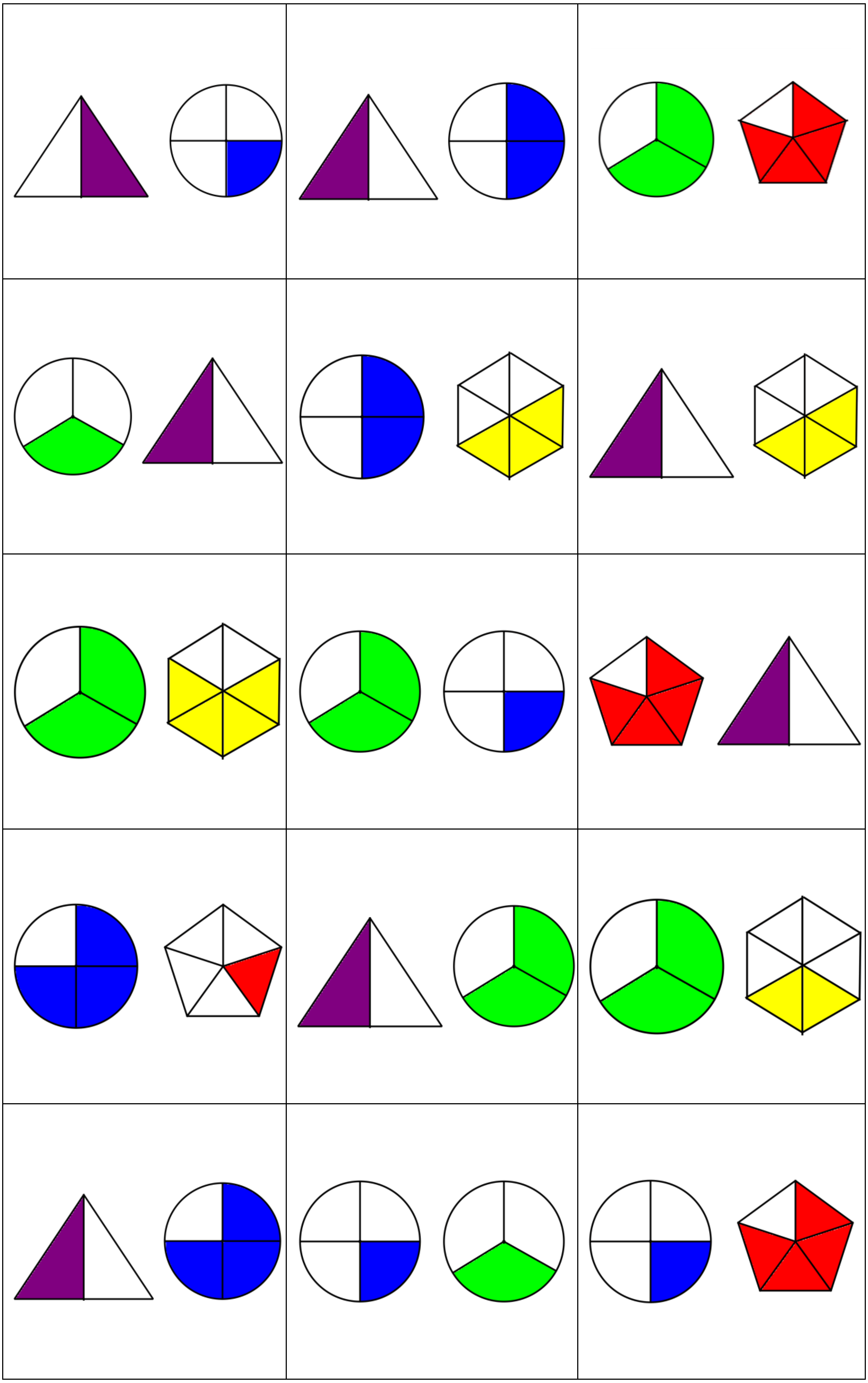
4. Match the equivalent fraction picture cards and the equivalent fraction sum cards you have just been given with the cards you already had. Check your work with the teacher before continuing.

5. Write a brief explanation of;
   a. Why you cannot add (or subtract!) fractions with different denominators.

   b. What you must do to fractions with different denominators so they can be added.

   c. Share your answers with the class.

Fraction Sum Picture Cards



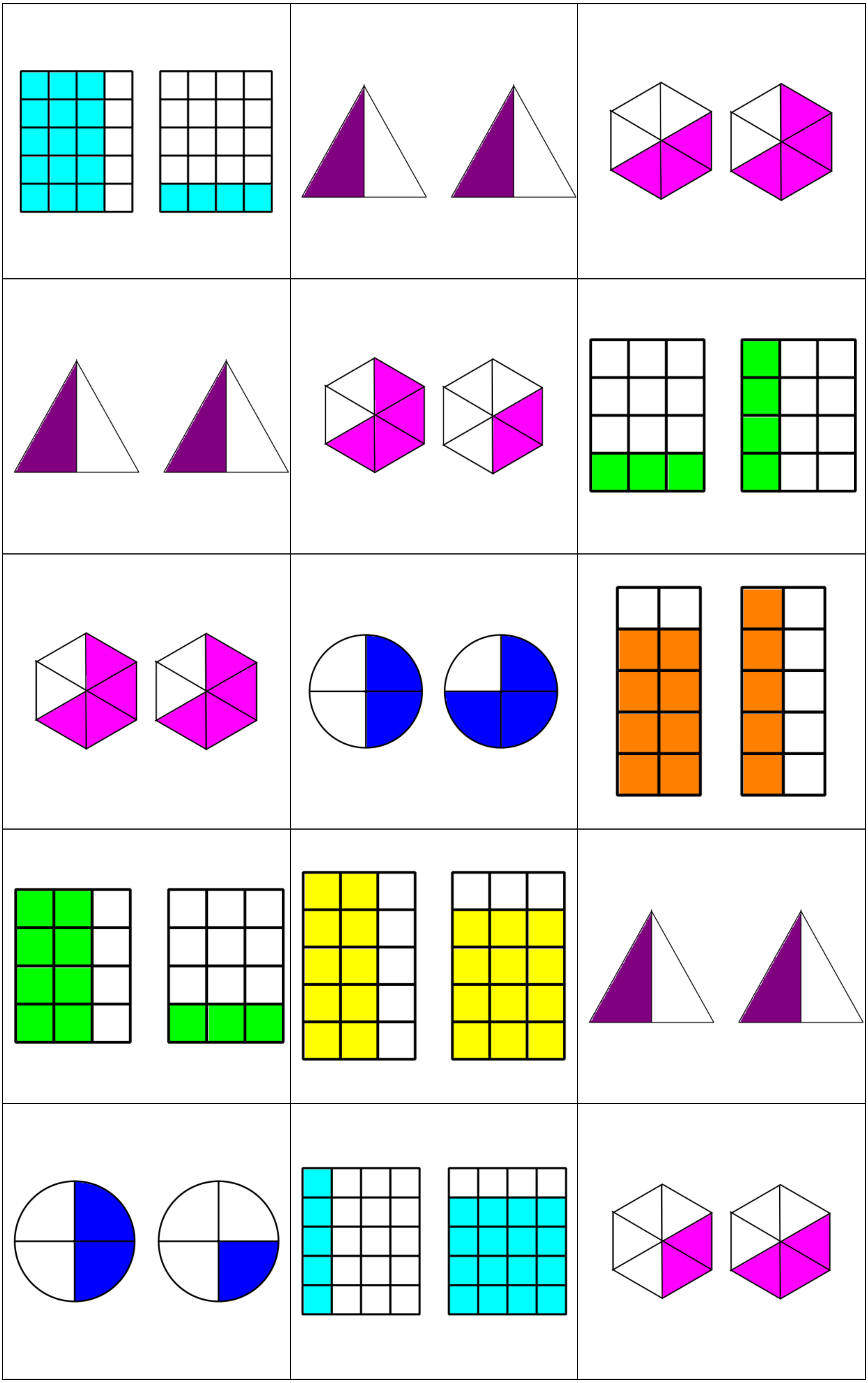
Fraction Sum Equation Cards

| | | |
|---|---|---|
| $\frac{1}{4}+\frac{1}{3}$ | $\frac{4}{5}+\frac{1}{2}$ | $\frac{2}{3}+\frac{4}{6}$ |
| $\frac{1}{4}+\frac{4}{5}$ | $\frac{1}{2}+\frac{3}{6}$ | $\frac{2}{3}+\frac{1}{4}$ |
| $\frac{1}{2}+\frac{3}{4}$ | $\frac{1}{3}+\frac{1}{2}$ | $\frac{2}{4}+\frac{3}{6}$ |
| $\frac{1}{2}+\frac{1}{4}$ | $\frac{2}{3}+\frac{4}{5}$ | $\frac{1}{2}+\frac{2}{4}$ |
| $\frac{2}{3}+\frac{2}{6}$ | $\frac{1}{2}+\frac{2}{3}$ | $\frac{3}{4}+\frac{1}{5}$ |

Answer Cards

| | | |
|---|---|---|
| $\frac{7}{12}$ | $1$ | $1\frac{1}{4}$ |
| $1\frac{1}{20}$ | $\frac{19}{20}$ | $1$ |
| $1$ | $\frac{5}{6}$ | $1\frac{2}{6}$ |
| $1$ | $1\frac{3}{10}$ | $1\frac{1}{6}$ |
| $\frac{11}{12}$ | $1\frac{7}{15}$ | $\frac{3}{4}$ |

Equivalent Fraction Picture Cards



Equivalent Fraction Sums Cards

| | | |
|---|---|---|
| $\frac{3}{12} + \frac{4}{12}$ | $\frac{8}{10} + \frac{5}{10}$ | $\frac{4}{6} + \frac{4}{6}$ |
| $\frac{5}{20} + \frac{16}{20}$ | $\frac{1}{2} + \frac{1}{2}$ | $\frac{8}{12} + \frac{3}{12}$ |
| $\frac{2}{4} + \frac{3}{4}$ | $\frac{2}{6} + \frac{3}{6}$ | $\frac{1}{2} + \frac{1}{2}$ |
| $\frac{2}{4} + \frac{1}{4}$ | $\frac{10}{15} + \frac{12}{15}$ | $\frac{1}{2} + \frac{1}{2}$ |
| $\frac{4}{6} + \frac{2}{6}$ | $\frac{3}{6} + \frac{4}{6}$ | $\frac{15}{20} + \frac{4}{20}$ |

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Fraction Multiplication

Produced by a Department of Education - MAWA partnership project
written for the Year 7 Australian Curriculum

# TASK 219: FRACTION MULTIPLICATION

## Overview

This task is designed to introduce the multiplication of fractions. The task uses the area method to explain how to multiply fractions starting with whole number/fraction questions and moving to fraction/fraction questions. Students are encouraged to look for patterns and discover the "shortcuts" involved with multiplying fractions.
Task 220: Further Fraction Multiplication is a suitable follow on activity for this task.

## Students will need

- No special equipment required

## Relevant content descriptions from the Western Australian Curriculum

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

## Students can demonstrate

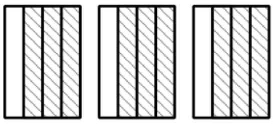
- *fluency* when they
  - can accurately calculate fraction multiplication questions without visual aids
- *understanding* when they
  - can multiply whole numbers by fractions using the area model
  - can multiply fractions by fractions using the area model
- *reasoning* when they
  - identify patterns in fraction multiplication questions that allow them to perform calculations more quickly.
- *problem solving* when they
  - use fraction multiplication to solve word problems.

Activity 1

For each of the questions below, draw the situation and use your diagram to find the answer. For this activity DO NOT simplify your answers.

An example is shown below.

***Find 3 x $\frac{3}{4}$***



*Draw the situation.*
*In this case draw three groups of $\frac{3}{4}$.*

$\frac{9}{4}$

*The number of shaded parts is the numerator.*
*The number of parts in each whole is the denominator.*

---

Answers have not been simplified for this activity to assist students to see the pattern.

1. $2 \times \frac{1}{3} = \frac{2}{3}$
2. $4 \times \frac{3}{4} = \frac{12}{4}$
3. $5 \times \frac{2}{4} = \frac{10}{4}$
4. $4 \times \frac{2}{5} = \frac{8}{5}$
5. $\frac{1}{2} \times 2 = \frac{2}{2}$
6. $\frac{4}{5} \times 3 = \frac{12}{5}$

With appropriate diagrams

7. Cindy bought 2 metres of material, but only used $\frac{1}{4}$ of the material she bought.
   How much material did Cindy use? $\frac{2}{4}$ m

8. Ronan is making porridge for three people. He needs $\frac{2}{3}$ of a cup of porridge for each person. How much porridge does he need in total? $\frac{6}{3}$ cups

9. Ashley bought 30 sparklers and gave $\frac{1}{5}$ to Alex. How many sparklers did Alex get?
   $\frac{30}{5}$ sparklers

Activity 2

While drawing the situations above is easy, it is also very time-consuming. There is a faster way to calculate the answer to these types of problems.

1. Have a look at each of the questions, and their answers, from Activity 1. What do you notice?
   Various answers.

2. Can you see the shortcut? Describe what you've noticed below.
   Multiply the whole number with the numerator of the fraction.

3. Share your thoughts with a partner.

4. If you and your partner disagree with each other, take turns in trying to convince your partner that you are correct.

5. When you and your partner agree, test your theory on the following problems.
   a. $3 \times \frac{4}{6} = \frac{12}{6}$
   b. $5 \times \frac{2}{11} = \frac{10}{11}$
   c. $7 \times \frac{4}{9} = \frac{28}{9}$

6. Check your answer using the area model. Did your theory work?

7. Share your ideas with the class.

Activity 3

Give students the following scenario;
"Imagine that you have $\frac{2}{3}$ of a pizza and you give one quarter of your pizza to a friend. How much do you have now?"

Prompt students with the following questions:

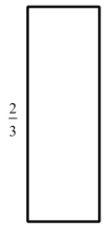
"Do you have more or less pizza than you used to have?" Less

"How would we write down the problem that we are trying to solve?"
Discuss with students that "of" often means multiply. They should come up with $\frac{2}{3} \times \frac{1}{4}$.

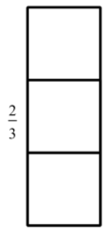
"How can we work out the answer to this?" Allow students to discuss their ideas, then show them the area method for multiplying fractions (as explained below).

*1. Draw a rectangle*

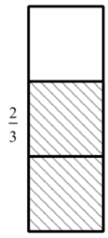
*2. Write the multiplicand along the left side of the rectangle*



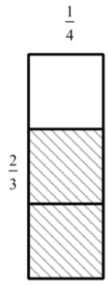

*3. Split the rectangle into parts using horizontal lines*



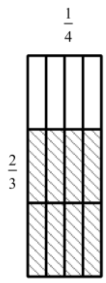

*4. Shade in the fraction given by the multiplicand*



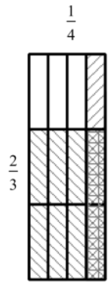
*5. Write the multiplier along the top of the rectangle.*




*6. Break the rectangle into parts using vertical lines.*




*7. Shade in the fraction given by the multiplier*




*8. Find the fraction of the shape that has been cross-hatched.*

*Simplify.*

*This is your answer:*

$^{2}/_{12} = {}^{1}/_{6}$

DISCUSSION QUESTION: Why does this model work?

Encourage students discuss their ideas to arrive at the essential reason.

## Activity 4

Using the area method you have been shown, calculate the answer to the following problems. For this activity. DO NOT simplify your answers.

1. $\frac{3}{8} \times \frac{1}{3} = \frac{3}{24}$
2. $\frac{4}{5} \times \frac{3}{4} = \frac{12}{20}$
3. $\frac{5}{6} \times \frac{2}{4} = \frac{10}{24}$
4. $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$
5. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
6. $\frac{4}{5} \times \frac{7}{10} = \frac{28}{50}$

7. A recipe calls for $\frac{3}{4}$ of a cup of flour, but you only want to make half the amount. How much flour do you need?

   $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ cup

8. At a local school, $\frac{4}{5}$ of students play sport. Of those that do play sport, $\frac{1}{3}$ play basketball. What fraction of the school plays basketball?

   $\frac{1}{3} x \frac{4}{5} = \frac{4}{15}$ of the school

9. Joel collects video games. $\frac{1}{3}$ of Joel's games are racing games and, of those, $\frac{2}{3}$ are car-racing games. What fraction of Joel's collection are car-racing games?

   $\frac{1}{3} x \frac{2}{3} = \frac{2}{9}$ of Joel's collection

Activity 5

While drawing the situations above is easy, it is also very time-consuming. There is a faster way to calculate the answer to these types of problems.

1. Have a close look at each of the questions, and their answers, from Activity 4. Can you see the shortcut? Describe what you've noticed.

   Multiply the numerators to get the numerator of the answer.
   Multiply the denominators to get the denominator of the answer.

2. Share your thoughts with a partner.

3. If you and your partner disagree with each other, take turns in trying to convince your partner that you are correct.

4. When you and your partner agree, test your theory on the following problems.

   a. $\frac{4}{6} \times \frac{4}{6} = \frac{16}{36}$

   b. $\frac{2}{3} \times \frac{2}{11} = \frac{4}{33}$

   c. $\frac{4}{5} \times \frac{4}{9} = \frac{16}{45}$

   d. $\frac{13}{29} \times \frac{7}{13} = \frac{91}{377}$

5. Check several of your answers using the area model. Did your theory work?

   Various examples

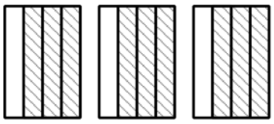
6. Share your ideas with the class.

Activity 1

For each of the questions below, draw the situation and use your diagram to find the answer. For this activity DO NOT simplify your answers.

An example is shown below;

**Find 3 x $\frac{3}{4}$**



Draw the situation.
In this case draw three groups of $\frac{3}{4}$.

$\frac{9}{4}$

The number of shaded parts is the numerator
The number of parts in each whole is the denominator

---

1. 2 x $\frac{1}{3}$
2. 4 x $\frac{3}{4}$
3. 5 x $\frac{2}{4}$
4. 4 x $\frac{2}{5}$
5. $\frac{1}{2}$ x 2
6. $\frac{4}{5}$ x 3

10. Cindy bought 2 metres of material, but only used $\frac{1}{4}$ of the material she bought. How much material did Cindy use?

11. Ronan is making porridge for three people. He needs $\frac{2}{3}$ of a cup of porridge for each person. How much porridge does he need in total?

12. Ashley bought 30 sparklers and gave $\frac{1}{5}$ to Alex. How many sparklers did Alex get?

Activity 2

While drawing the situations above is easy, it is also very time-consuming. There is a faster way to calculate the answer to these types of problems.

8. Have a look at each of the questions, and their answers, from Activity 1. What do you notice?

9. Can you see the shortcut? Describe what you noticed.

10. Share your thoughts with a partner.

11. If you and your partner disagree with each other, take turns in trying to convince your partner that you are correct.

12. When you and your partner agree, test your theory on the following problems.
    a. $3 \times \frac{4}{6}$

    b. $5 \times \frac{2}{11}$

    c. $7 \times \frac{4}{9}$

13. Check your answer using the area model. Did your theory work?

14. Share your ideas with the class.

Activity 3

1. Imagine that you have $\frac{2}{3}$ of a pizza and you give one quarter of your pizza to a friend. How much do you have now?”

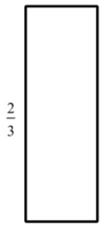
2. Do you have more or less pizza than you used to?

3. How would we write down the problem that we are trying to solve?”

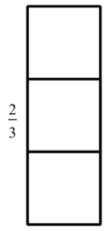
4. How can we work out the answer to this?

1. Draw a rectangle

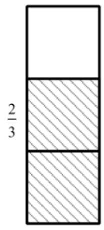
2. Write the multiplicand along the left side of the rectangle



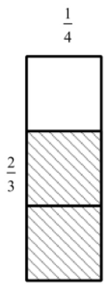

3. Split the rectangle into parts using horizontal lines.



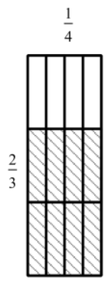

4. Shade in the fraction given by the multiplicand



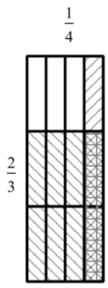

5. Write the multiplier along the top of the rectangle.




6. Split the rectangle into parts using vertical lines.




7. Shade in the fraction given by the multiplier




8. Find the fraction of the shape that has been cross-hatched.

Simplify.

This is your answer:

$^{2}/_{12}$
=
$^{1}/_{6}$

DISCUSSION QUESTION: Why does this model work?

Activity 4

Using the area method you have been shown, calculate the answer to the following problems. For this activity. DO NOT simplify your answers.

1. $\frac{3}{8} \times \frac{1}{3}$

2. $\frac{4}{5} \times \frac{3}{4}$

3. $\frac{5}{6} \times \frac{2}{4}$

4. $\frac{1}{3} \times \frac{2}{5}$

5. $\frac{1}{2} \times \frac{1}{3}$

6. $\frac{4}{5} \times \frac{7}{10}$

10. A recipe calls for $\frac{3}{4}$ of a cup of flour, but you only want to make half the amount. How much flour do you need?

11. At a local school, $\frac{4}{5}$ of students play sport. Of those that do play sport, $\frac{1}{3}$ play basketball. What fraction of the school plays basketball?

12. Joel collects video games. $\frac{1}{3}$ of Joel's games are racing games, and of those $\frac{2}{3}$ are car-racing games. What fraction of Joel's collection are car-racing games?

Activity 5

While drawing the situations above is easy, it is also very time-consuming. There is a faster way to calculate the answer to these types of problems.

7. Have a close look at each of the questions, and their answers, from Activity 4. Can you see the shortcut? Describe what you've noticed.

8. Share your thoughts with a partner.

9. If you and your partner disagree with each other, take turns in trying to convince your partner that you are correct.

10. When you and your partner agree, test your theory on the following problems.

a. $\frac{4}{6} \times \frac{4}{6}$

b. $\frac{2}{3} \times \frac{2}{11}$

c. $\frac{4}{5} \times \frac{4}{9}$

d. $\frac{13}{29} \times \frac{7}{13}$

11. Check several of your answers using the area model. Did your theory work?

12. Share your ideas with the class.

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Further Fraction Multiplication

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 220: FURTHER FRACTION MULTIPLICATION

**Overview**

This task is designed to introduce cross cancelling when multiplying fractions. The task attempts to explain *why* cross cancelling works and gives students the opportunity to practise the skill.

**Students will need**

- no special requirements

**Relevant content descriptions from the Western Australian Curriculum**

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

**Students can demonstrate**

- *fluency* when they
  - can accurately calculate fraction multiplication questions
- *understanding* when they
  - can multiply fractions
- *reasoning* when they
  - use cross-cancelling to make the problems easier
- *problem solving* when they
  - use fraction multiplication and cross cancelling to solve word problems.

Activity 1 – Teacher-led Activity

This activity is designed to show students the cross-cancelling method before multiplying fractions. The explanation is designed to give students an understanding of *why* this method works.

In a previous activity, we learnt to multiply fractions by multiplying the numerators, multiplying the denominators and then simplifying the fraction. Example:

$\frac{4}{9}$ x $\frac{3}{5}$ = $\frac{4\,x\,3}{9\,x\,5}$ = $\frac{12}{45}$ = $\frac{9}{15}$

And for question like this one, I can simplify before I start:

$\frac{3}{9}$ x $\frac{4}{7}$ = $\frac{1}{3}$ x $\frac{4}{7}$ $\frac{1\,x\,4}{3\,x\,7}$ = $\frac{4}{21}$

However, fractions like the last one in the previous activity don't look like they can be simplified and aren't so easy to do without a calculator:
$\frac{13}{29}$ x $\frac{7}{13}$

So what can we do? Well, we already know that we should multiply the numerators and the denominators together:
$\frac{13\,x\,7}{29\,x\,13}$

We also know that we can perform multiplication in any order and get the same result, so we could write the problem like this:
$\frac{13\,x\,7}{13\,x\,29}$

or like this:
$\frac{13}{13}$ x $\frac{7}{29}$

We also know that $\frac{13}{13}$ simplifies to 1, so our answer is just
$\frac{7}{29}$

And we haven't had to do any difficult multiplications in our head.

So if we go back to the original question:
$\frac{13}{29}$ x $\frac{7}{13}$

What we've really done is taken out a factor from the numerator over here:
$\frac{1\ \cancel{13}}{29}$ x $\frac{7}{13}$

And taken out the same factor from the denominator over here:
$\frac{1\ \cancel{13}}{29}$ x $\frac{7}{\cancel{13}\ 1}$

Then proceed with the multiplication in the usual way:
$\frac{1}{29}$ x $\frac{7}{1}$ = $\frac{7}{29}$

Here's another example:
$\frac{7}{12}$ x $\frac{6}{11}$

We start by multiplying the numerators and the denominators together:
$\frac{7 x 6}{12 x 11}$

We can perform multiplication in any order, so I can rewrite the product like this,
$\frac{7 x 6}{11 x 12}$

or like this:
$\frac{7}{11}$ x $\frac{6}{12}$

We can simplify this to:
$\frac{7}{11}$ x $\frac{1}{2}$

And then proceed with the calculation:
$\frac{7 x 1}{11 x 2}$ = $\frac{7}{22}$

If we go back to the original question;
$\frac{7}{12}$ x $\frac{6}{11}$

What we've really done is take a factor of 6 out of the numerator here,
$\frac{7}{12}$ x $\frac{\cancel{6}\ 1}{11}$

and a factor of 6 out of the denominator here:
$\frac{7}{2\ \cancel{12}}$ x $\frac{\cancel{6}\ 1}{11}$

Then proceed with the calculation:
$\frac{7 x 1}{11 x 2}$ = $\frac{7}{22}$

Simplify your answer. However, if we have cross-cancelled properly, we shouldn't have to simplify the answer at all!

At this point, it may be useful to have students write an explanation for why this method works.

Activity 2

1. $\frac{7}{18} \times \frac{9}{22} = \frac{7}{44}$

2. $\frac{7}{8} \times \frac{5}{14} = \frac{5}{16}$

3. $\frac{5}{19} \times \frac{9}{15} = \frac{9}{57}$

4. $\frac{11}{12} \times \frac{5}{22} = \frac{5}{24}$

5. $\frac{5}{8} \times \frac{16}{9} = \frac{10}{9} = 1\frac{1}{9}$

6. Jack ate $\frac{10}{11}$ of a jar of Nutella. Julie at $\frac{1}{12}$ of what Jack ate. What fraction of the jar did Julie eat? $\frac{5}{66}$

7. $\frac{5}{7}$ of Kevin's video games are racing games. If $\frac{14}{23}$ of his racing games are motorcycle racing games, what fraction of his collection are motorcycle racing games. $\frac{10}{23}$

8. Ruby wants to make $\frac{5}{12}$ of a recipe that uses $\frac{3}{7}$ of a cup of flour. How much flour does Ruby need? $\frac{5}{28}$

9. Myles was creating a piece of art using different coloured tiles. $\frac{5}{8}$ of his tiles were black, and he drew sheep on $\frac{7}{10}$ of the black tiles. What fraction of tiles had sheep on them? $\frac{7}{16}$

Activity 3

Creating Sierpinski's Carpet

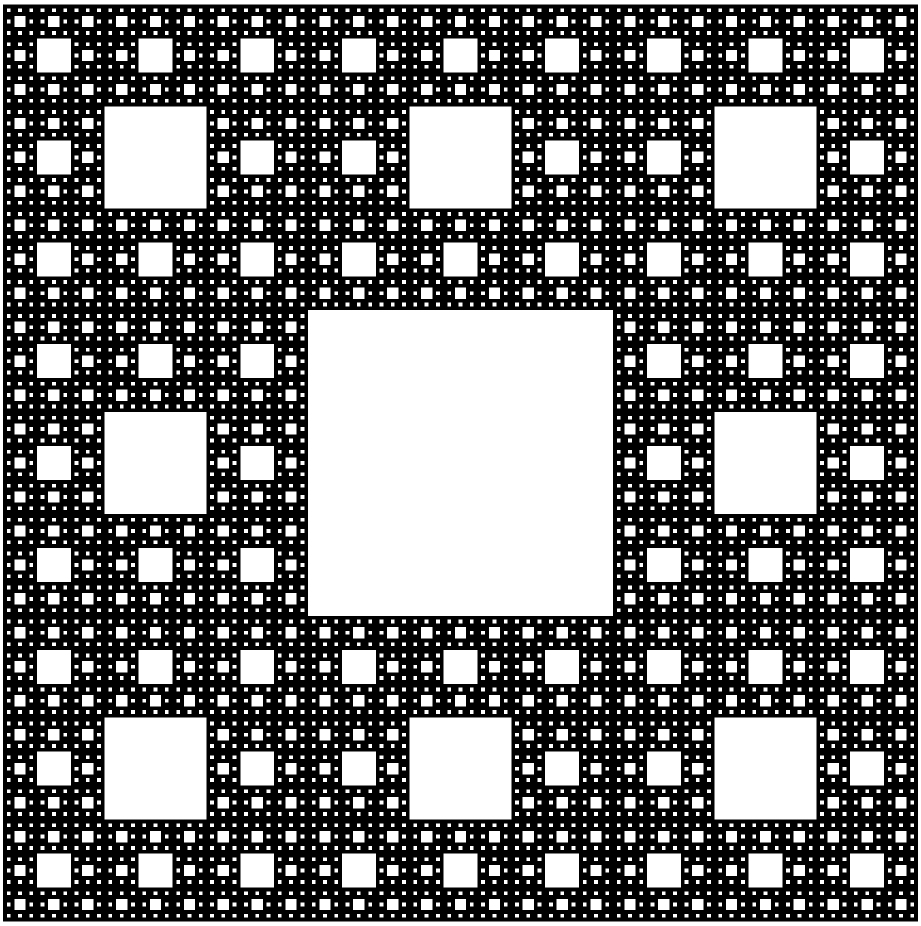
1. Draw a large square.

2. Divide each side into thirds, creating 9 smaller squares.

3. Colour in the middle square.

4. Repeat the steps 1 – 3 for all the squares that are not shaded.

5. When you have reached the stage where you can't go any further, erase the construction lines.



6. Have a look at the shaded squares. How many different sizes of shaded squares do you have? Various answers

For each of the squares you identified in Question 6, find their area as a fraction of the original square you drew. $\frac{1}{9}$ $\frac{1}{9^2}$ $\frac{1}{9^3}$ etc.

7. EXTENSION QUESTION: The pattern you have just created is a type of **fractal.** Research Fractals and see if you can create a different fractal pattern from the one above.

Activity 1

In a previous activity, we learnt to multiply fractions by multiplying the numerators, multiplying the denominators and then simplifying the fraction. Example:

$\frac{4}{9}$ x $\frac{3}{5}$ = $\frac{4 \, x \, 3}{9 \, x \, 5}$ = $\frac{12}{45}$ = $\frac{9}{15}$

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However, fractions like the last one in the previous activity don't look like they can be simplified and aren't so easy to do without a calculator:

$\frac{13}{29}$ x $\frac{7}{13}$

So what can we do? Well, we already know that we should multiply the numerators and the denominators together:

$\frac{13 \, x \, 7}{29 \, x \, 13}$

We also know that we can perform multiplication in any order and get the same result, so we could write the problem like this:

$\frac{13 \, x \, 7}{13 \, x \, 29}$

or like this:

$\frac{13}{13}$ x $\frac{7}{29}$

We also know that $\frac{13}{13}$ simplifies to 1, so our answer is just

$\frac{7}{29}$

And we haven't had to do any difficult multiplications in our head.

So if we go back to the original question:

$\frac{13}{29}$ x $\frac{7}{13}$

What we've really done is taken out a factor from the numerator over here:

$\frac{1 \, \cancel{13}}{29}$ x $\frac{7}{13}$

And taken out the same factor from the denominator over here:

$\frac{1 \, \cancel{13}}{29}$ x $\frac{7}{\cancel{13} \, 1}$

Then proceed with the multiplication in the usual way:

$\frac{1}{29}$ x $\frac{7}{1}$ = $\frac{7}{29}$

Here's another example:

$\frac{7}{12}$ X $\frac{6}{11}$

We start by multiplying the numerators and the denominators together:

$\frac{7\,x\,6}{12\,x\,11}$

We can perform multiplication in any order, so I can rewrite the product like this,

$\frac{7\,x\,6}{11\,x\,12}$

or like this:

$\frac{7}{11}$ X $\frac{6}{12}$

We can simplify this to:

$\frac{7}{11}$ X $\frac{1}{2}$

And then proceed with the calculation:

$\frac{7\,x\,1}{11\,x\,2} = \frac{7}{22}$

If we go back to the original question;

$\frac{7}{12}$ X $\frac{6}{11}$

What we've really done is take a factor of 6 out of the numerator here,

$\frac{7}{12}$ X $\frac{\cancel{6}\,1}{11}$

and a factor of 6 out of the denominator here:

$\frac{7}{2\,\cancel{12}}$ X $\frac{\cancel{6}\,1}{11}$

Then proceed with the calculation:

$\frac{7\,x\,1}{11\,x\,2} = \frac{7}{22}$

If we go back to my original question;

$\frac{7}{12}$ X $\frac{6}{11}$

What we've really done is take a factor of 6 out of the numerator here,

$\frac{7}{12}$ X $\frac{\cancel{6}\,1}{11}$

and a factor of 6 out of the denominator here:

$\frac{7}{2\,\cancel{12}}$ X $\frac{\cancel{6}\,1}{11}$

Then proceed with the calculation:

$\frac{7\,x\,1}{11\,x\,2} = \frac{7}{22}$

Simplify your answer (however, If we have cross-cancelled properly, we shouldn't have to simplify the answer at all!)

Activity 2

1. $\frac{7}{18} \times \frac{9}{22}$

2. $\frac{7}{8} \times \frac{5}{14}$

3. $\frac{5}{19} \times \frac{9}{15}$

4. $\frac{11}{12} \times \frac{5}{22}$

5. $\frac{5}{8} \times \frac{16}{9}$

6. Jack ate $\frac{10}{11}$ of a jar of Nutella. Julie at $\frac{1}{12}$ of what Jack ate. What fraction of the jar did Julie eat?

7. $\frac{5}{7}$ of Kevin's video games are racing games. If $\frac{14}{23}$ of his racing games are motorcycle racing games, what fraction of his collection are motorcycle racing games.

8. Ruby wants to make $\frac{5}{12}$ of a recipe that uses $\frac{3}{7}$ of a cup of flour. How much flour does Ruby need?

9. Myles was creating a piece of art using different coloured tiles. $\frac{5}{8}$ of his tiles were black, and he drew sheep on $\frac{7}{10}$ of the black tiles. What fraction of tiles had sheep on them?

Activity 3

Creating Sierpinski's Carpet

1. Draw a large square.

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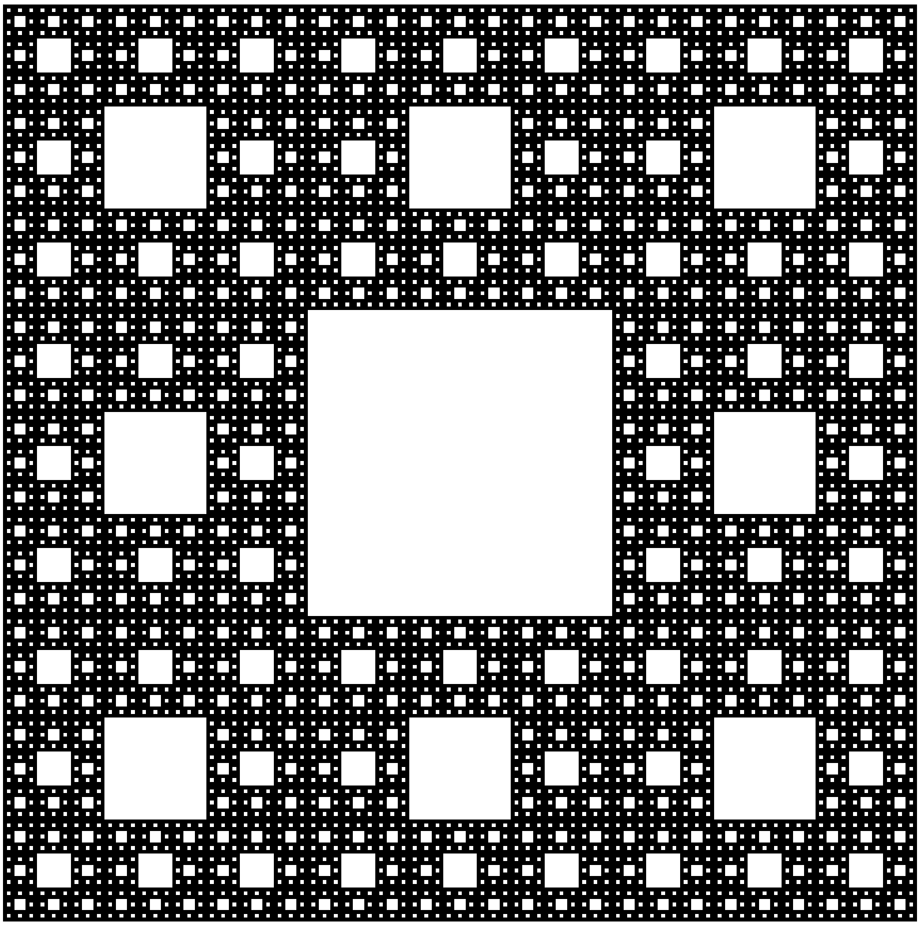


5. When you have reached the stage where you can't go any further, erase the construction lines.

6. Have a look at the shaded squares. How many different sizes of shaded squares do you have?

7. For each of the squares you identified in Question 6, find their area as a fraction of the original square you drew.

8. EXTENSION QUESTION: The pattern you have just created is a type of **FRACTAL.** Research Fractals and see if you can create a different fractal pattern from the one above.

# Year 7 Mathematics

## Number & Algebra Activity

# Comparing, Adding and Subtracting Integers

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 221: COMPARING, ADDING AND SUBTRACTING INTEGERS

## Overview

This task is designed to help students compare, add and subtract integers. Students perform a vocabulary task, followed by comparing integers. Students then play a game that explains the rules of adding and subtracting integers, followed by the practising of this new skill.

## Students will need

- A coin
- deck of cards

## Relevant content descriptions from the Western Australian Curriculum

- Compare, order, add and subtract integers (ACMNA 280)

## Students can demonstrate

- *fluency* when they
  - compare positive and negative numbers correctly
- *understanding* when they
  - calculate with integers correctly
- reasoning when they
  - explain that subtracting a negative makes an addition

## Activity 1

1. Match each word to the correct definition.

| Word | Definition |
|---|---|
| Negative | An amount greater than zero |
| Positive | A mathematical symbol (+ or -) used to indicate whether a quantity is positive or negative |
| Integers | A straight line that has markings on it to represent all real numbers |
| Sign | An amount less than zero |
| Number line | Whole numbers including negatives, positives and zero |

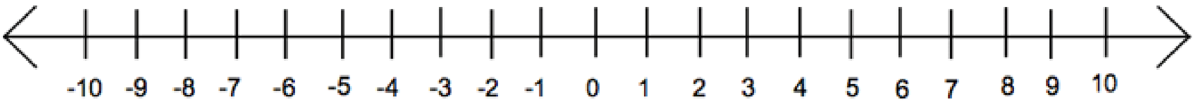
2. Find examples and non-examples for each of the words given above.

| Word | Examples | Non-Examples |
|---|---|---|
| Negative | -1, -2.3, -3 etc. | 0, 1.1, 2, 3, etc. |
| Positive | 0.1, 1, 2.1, 3 etc. | 0, -1.5, -2, -3, |
| Integers | -1, 0, 1, etc. | 3.4, -3.5 etc. |

3. Using the internet, or any other resources available in your room, write a definition for each of these symbols.

| Symbol | Definition |
|---|---|
| + | Positive, plus, addition operation |
| - | Negative, minus, subtraction operation |
| > | Greater than |
| < | Less than |

Activity 2




1. The daily minimum temperatures for Canberra for one week of winter are recorded below. Record these numbers on the number line above.

   5, -3, -1. 0, 4, 3, -4

2. What was the minimum temperature Canberra experienced during that week?
   -4

3. What was the highest?
   5

4. We can use the symbols to compare numbers.
   - = means that two numbers are the same
   - < or “is less than” tells you the first number is further to the left on the number line than the second number. For example -5 < 3
   - > or “is greater than” means that the first number is further to the right on the number line than the second number. For example 6 > -3

   Compare the lowest and highest temperatures using the one of the symbols listed above.
   5 > -4

   There is another way to compare these two numbers. How could you do it? Write your answer below.
   -4 < 5

5. Compare the following numbers using <, > or =. You can use the number line to help you.

   a. 10 > 8

   b. -4 < -2

   c. -7 < 0

   d. -6 < 6

   e. 3 < 4

   f. 4 > -10

   g. 10 > 0

Activity 3

You will need to have a large number line somewhere in the classroom. The number line should go from -15 to 15. If possible, have a picture of a bear at the negative end of the number line and a picture of a pot of gold at the positive end.
You will also need a coin and a deck of cards (A=1, J=11, Q=12, K=13, All other cards are face value).
Give the students the following scenario (you could change the scenario to fit the class' interest; e.g.. star wars theme, minion theme, etc.)

Haha! You've just caught a leprechaun. Now for that pot of gold!
Unfortunately the leprechaun has other ideas and is threatening to feed you to the large bear that has suddenly turned up. You agree to play the following game.
The leprechaun will flip a coin, if it is heads you get to face the pot of gold (a positive), but if its tails you have to turn and face the large and growly bear (definitely a negative).
After that the leprechaun will draw a card from a deck of cards. If it's a black card it will be treated as a positive number and you will walk forward the number of steps given by the card. If it's a red card it will be treated as a negative number and you will walk backwards.
Your starting position will be halfway between the pot of gold and the bear, 15 steps away from each.
If you get to the gold first, you get to keep it. If you get to the bear first, well….
The teacher will display a number line ranging from -15 to 15, and the steps as below are followed, with few examples shown.

| Action | Outcome | Resulting number |
|---|---|---|
| One student will stand in the middle of the number line (i.e.; at zero) | | 0 |
| Another student flips the coin, and the student at the number line faces the direction determined by the coin toss | Head (+) | Student faces to the right |
| A different student chooses a card from a deck of cards. Without turning around, the student at the board should move the number of steps indicated by the card | Black 5 (+5) | Student takes 5 steps forwards<br>Lands on 5 |
| Flip the coin again | Head (+) | Student remains facing to the right |
| Draw a card | Red 7 (-7) | Student takes 7 steps backwards<br>Lands on -2 |
| Flip the coin | Tails (-) | Student faces the left |
| Draw a card | Red 2 (-2) | Student takes 2 steps backwards<br>Lands on 0 |

As you are playing, write the equation that is generated in order to reinforce what the actions mean; e.g., for the above steps 0 + (5) + (-7) – (-2) = 0
Keep playing until the student either gets the gold or gets eaten, and record the process in an equation as was done for the above.
Various answers

Repeat the process with other students and record the actions in a number sentence in each case.
Various answers

To review the process, write a few similar integer addition and subtraction equations on the board and solve them as a class.

Activity 4

1. Use the number lines provided to calculate the answer to the following sums and differences.

a. 7 + 3 = 10
b. 6 + (-4) = 2
c. 8 – 9 = -1
d. -9 + 2 = -7
e. -1 + (-3) = -4
f. -5 – (-8) = 3
g. -1 – 2 = -3

2 Jenny has a negative number and subtracts a number from it. Could her result be a positive number?
Yes

Why/Why not?
If Jenny subtracts a negative number, she will move to the right on the number line. If the number she subtracts is greater than the number she started with, her result will be negative.

3. If Jenny started with a positive number, and added another number, could she get a negative result?
Yes
Why/Why not?
If Jenny adds a negative number she will move to the left of the number line. If the negative number is greater than her starting number, her result will be negative.

4. Consider the following two equations: 7 – (-2) = 9 and 7 + (2) = 9.
Explain, in your own words, why subtracting a negative and adding a positive give the same result.
When you are adding you are facing to the right, and if you adding a positive number you move forward (to the right).
When you are subtracting you face to the left, and if you are subtracting a negative number you move backwards (to the right).
So the overall motion is the same, even if the instructions on how to get there are different.

*Activity 1*

1. Match each word to the correct definition.

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|---|---|
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| Positive | A mathematical symbol (+ or -) used to indicate whether a quantity is positive or negative |
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| Number line | Whole numbers including negatives, positives and zero |

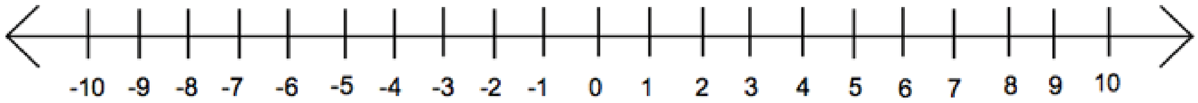
2. Find examples and non-examples for each of the words given above.

| Word | Examples | Non-Examples |
|---|---|---|
| Negative | | |
| Positive | | |
| Integers | | |

3. Using the internet, or any other resources available in your room, write a definition for each of these symbols.

| Symbol | Definition |
|---|---|
| + | |
| - | |
| > | |
| < | |

Activity 2




1. The daily minimum temperatures for Canberra for one week of winter are recorded below. Order these numbers on the number line above.

   5, -3, -1. 0, 4, 3, -4

2. What was the minimum temperature Canberra experienced during that week?

3. What was the highest?

4. We can use the symbols to compare numbers.
   - = means that two numbers are the same
   - < or “less than” tells you the first number is further to the left on the number line than the second number. For example -5 < 3
   - > or “greater than” means that the first number is further to the right on the number line than the second number. For example 6 > -3

   Compare the lowest and highest temperatures using the one of the symbols listed above.

   There is another way to compare these two numbers. How could you do it? Write your answer below.

5. Compare the following numbers using <, > or =. You can use the number line to help you.
   a. 10 _____ 8

   b. -4 _____ -2

   c. -7 _____ 0

   d. -6 _____ 6

   e. 3 _____ 4

   f. 4 _____-10

   g. 10 _____ 0

Activity 3 –Teacher Led Activity

Haha! You've just caught a leprechaun. Now for that pot of gold!
Unfortunately the leprechaun has other ideas and is threatening to feed you to the large bear that has suddenly turned up.
You agree to play the following game.
The leprechaun will flip a coin, if it is heads you get to face the pot of gold (a positive), but if its tails you have to turn and face the large and growly bear (definitely a negative).
After that the leprechaun will draw a card from a deck of cards. If it's a black card it will be treated as a positive number and you will walk forward the number of steps given by the card. If it's a red card it will be treated as a negative number and you will walk backwards.
Your starting position will be halfway between the pot of gold and the bear, 15 steps away from each.
If you get to the gold first, you get to keep it. If you get to the bear first, well….

The teacher will display a number line ranging from -15 to 15, and the steps as below are followed, with few examples shown.

| Action | Outcome | Resulting Number |
|---|---|---|
| One student will stand in the middle of the number line (i.e.; at zero) | | 0 |
| Another student flips the coin, and the student at the number line faces the direction determined by the coin toss | Head (+) | Student faces to the right |
| A different student chooses a card from a deck of cards. Without turning around, the student at the board should move the number of steps indicated by the card | Black 5 (+5) | Student takes 5 steps forwards<br>Lands on 5 |
| Flip the coin again | Head (+) | Student remains facing to the right |
| Draw a card | Red 7 (-7) | Student takes<br>7 steps backwards<br>Lands on -2 |
| Flip the coin | Tails (-) | Student faces the left |
| Draw a card | Red 2 (-2) | Student takes 2 steps backwards<br>Lands on 0 |

As you are playing, write the equation that is generated in order to reinforce what the actions mean; e.g., for the above steps 0 + (5) + (-7) – (-2) = 0
Keep playing until the student either gets the gold or gets eaten, and record the process in an equation as was done for the above.

Repeat the process with other students and record the actions in a number sentence in each case.

Activity 4

1. Use the number lines provided to calculate the answer to the following sums and differences.

    a. 7 + 3

    b. 6 + (-4)

    c. 8 - 9

    d. -9 + 2

    e. -1 + (-3)

    f. -5 – (-8)

    g. -1 – 2

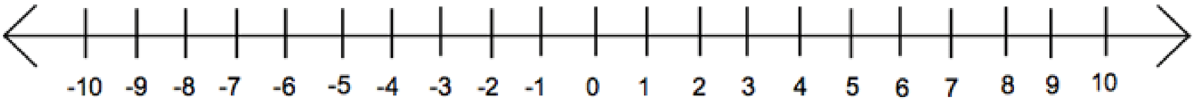
2. Jenny has a negative number and subtracts a number from it. Could her result be a positive number?

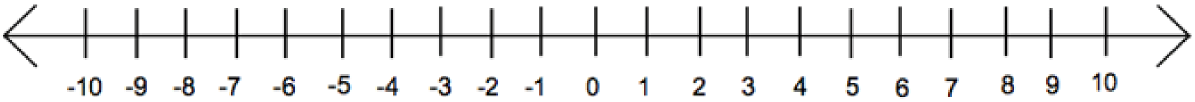
    Why/Why not?

3. If Jenny started with a positive number, and added another number, could she get a negative result?

    Why/Why not?

4. Consider the following two equations: 7 – (-2) = 9 and 7 + (2) = 9.
   Explain, in your own words, why subtracting a negative and adding a positive give the same result.


-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10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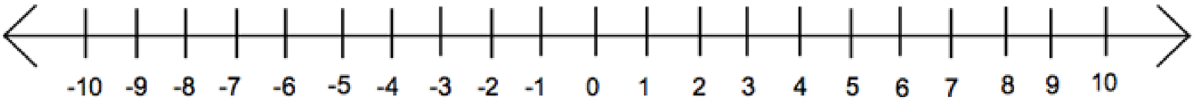
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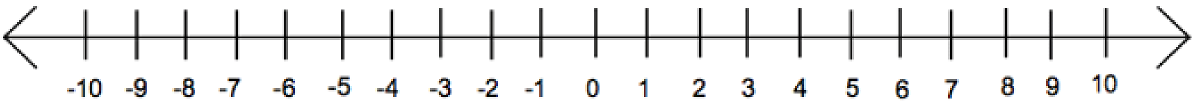
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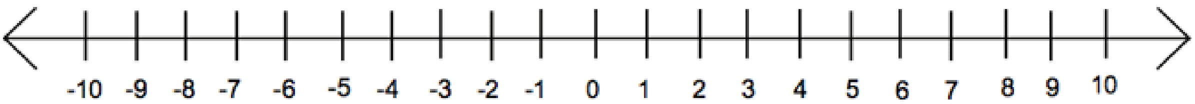
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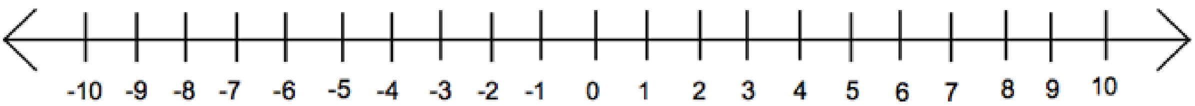
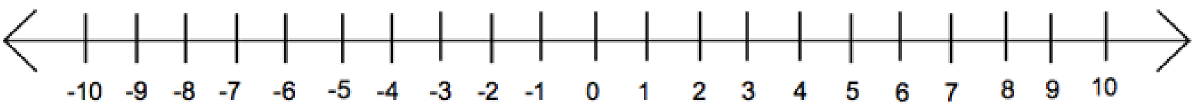

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10


-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Department of Education

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# 'Fracdecipent' Conversions

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 224: 'FRACDECIPENT' CONVERSIONS

## Overview

This task is designed to get students comparing fractions, decimals and percentages. In the first task students use a 100 grid to create a pattern or design of their choice and then the fraction/decimal/per cent of each colour they used.
The second task has students working in pairs to compare and order fractions decimals and percentages.
The final task requires students to find a path through a maze by calculating with various fractions decimals and percentages.

## Students will need

- coloured pencils
- scissors
- glue
- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

## Students can demonstrate

- *fluency* when they
  - represent fractions and decimals in various ways
- *understanding* when they
  - recognise equivalences between fractions decimals and percentages
- *reasoning* when they
  - can explain how to convert between fractions, decimals and percentages
- *problem solving* when they
  - find the maximum- and minimum-value path through a maze

Activity 1

1. DISCUSSION QUESTIONS:
    a. How many squares are there in the grid below? 100
    b. What fraction of the whole grid would one square represent? $\frac{1}{100}$
    c. What percentage of the whole grid would one square represent? 1%
    d. What part of the whole grid would one square represent in decimal notation? 0.01

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| | | | | | | | | | |

2. You are going to colour in each square in the grid above, however there are some rules you must follow;
    a. Every square needs to be coloured in.
    b. You can only use one colour in each square.
    c. There must be 3 to 5 colours in your grid.
    d. Be creative (make a picture, pattern or something abstract!)

Various answers

3. For each of the colours you have used in your design, fill in the table below.

| Colour | Fraction | | Percentage | Decimal |
|---|---|---|---|---|
| | $\frac{\quad}{100}$ | Simplified Fraction | | |
| Various answers | | | | |

4. Working with a friend, and using the information you have both collected in Question 3, work out how to perform the following conversions:

    a. fraction to a percentage
    If the denominator is 100, then write the numerator as a percentage. For example if you have $\frac{25}{100}$ it is 25%.

    b. fraction to a decimal
    Divide the numerator by the denominator; e.g., $\frac{1}{4} = 1 \div 4 = 0.25$.

    c. percentage to a fraction
    Per cent = hundredths, so 25% = $\frac{25}{100}$

    d. percentage to a decimal
    Per cent = hundredths, so simply enter the number of hundredths in decimal form, as in 25% = 0.25.

    e. decimal to a fraction
    If it only has 1 decimal place, write it as a fraction with a denominator of 10
    If it has 2 decimal places, write it as a fraction with a denominator of 100, etc.

    f. convert from a decimal to a percentage
    Hundredths = per cent, so check the number of hundredths for the decimal.
    Example: 0.25 = 25 hundredths = 25%

5. Share your ideas with the class.

    Direct this activity.

6. Create a poster that shows how to make each of the 6 conversions listed in Question 4.

Activity 2

1. Cut out the number cards on the next page and order them from smallest to greatest number.

2. When you have finished, compare your results with another group, and then the class.

3. Discuss the following questions with the class;

   a. What strategies did you use to order your numbers?
   b. How did you decide whether one number was bigger or smaller than another?
   c. What strategies can we use to check our work?
   d. How do we know we are right?

Answers: 0.6%, 6% and 0.06, $\frac{30}{100}$, 0.3004, 0.305, 0.31, $\frac{1}{3}$, 60%, 0.61, $\frac{5}{6}$, 1.3
Direct the discussion re the comparison questions above.

Activity 3

Maze Rules;

- You can move through maze along any line that takes you down or across
- You cannot move along upwards along any line.
- You start the maze with $100.

1. Find the route through the maze below that leaves you with the most money.

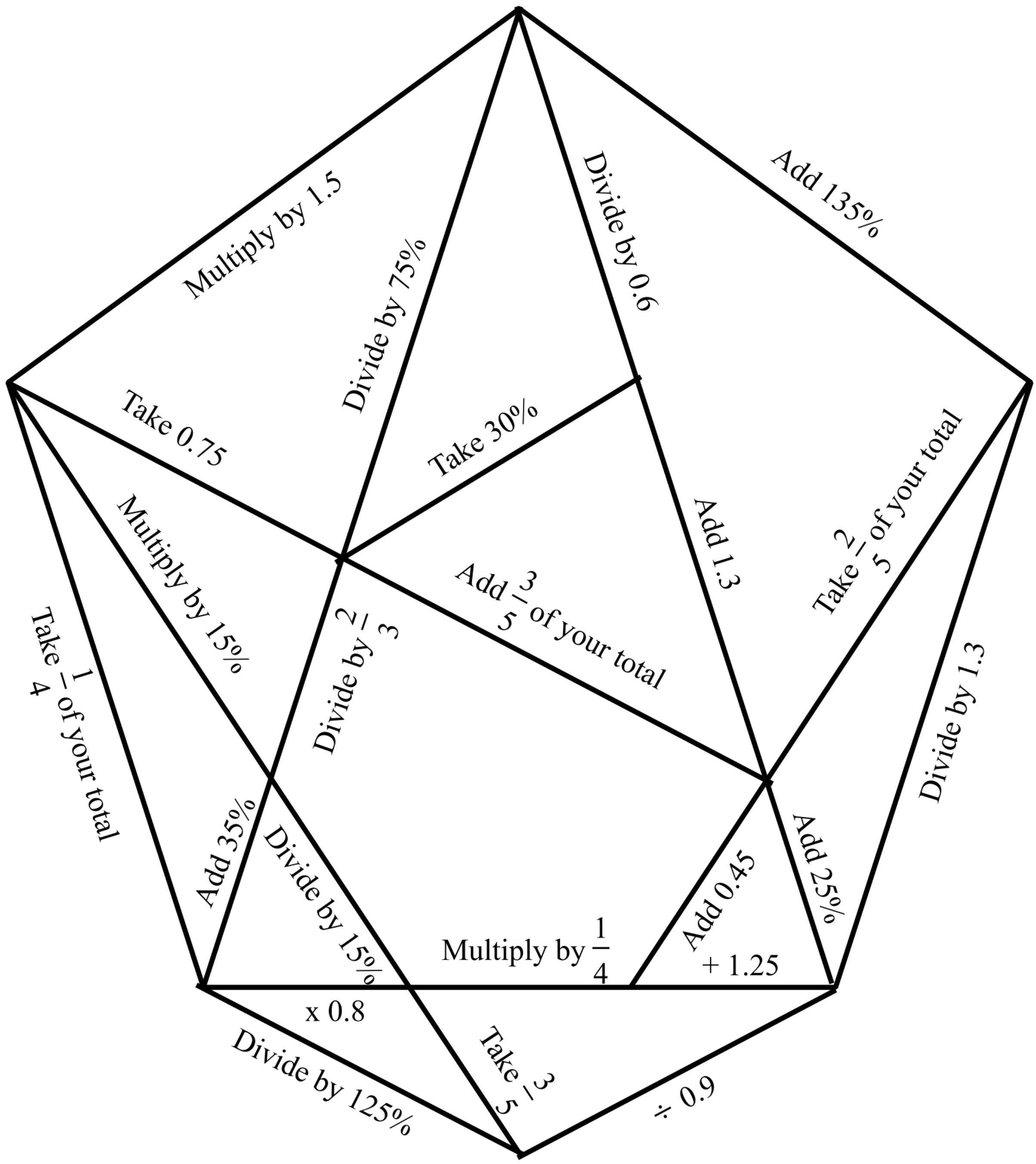
   Divide by 75% → Divide by $\frac{2}{3}$ → Divide by 15% → Take $\frac{3}{5}$ gives $1332.73

2. Find the route through the maze that leaves you with the least money.

   Multiply by 1.5 → Multiply by 15% → Add 35% → x 0.8 → Multiply by ¼ → + 1.25 → ÷0.9 gives $6.47

| | | |
|---|---|---|
| 60% | $\frac{30}{100}$ | 0.305 |
| 1.3 | 0.61 | $\frac{1}{3}$ |
| $\frac{5}{6}$ | 0.6% | 0.06 |
| 0.3004 | 6% | 0.31 |

START




FINISH

Activity 1

1. DISCUSSION QUESTIONS:
    a. How many squares are there in the grid below?
    b. What fraction of the whole grid would one square represent?
    c. What percentage of the whole grid would one square represent?
    d. What part of the whole grid would one square represent in decimal notation?

| | | | | | | | | | |
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2. You are going to colour in each square in the grid above, however there are some rules you must follow;
    a. Every square needs to be coloured in.
    b. You can only use one colour in each square.
    c. There must be 3 to 5 colours in your grid.
    d. Be creative (make a picture, pattern or something abstract!).

3. For each of the colours you have used in your design, fill in the table below.

| Colour | Fraction | | Percentage | Decimal |
|---|---|---|---|---|
| | $\frac{}{100}$ | Simplified Fraction | | |
| | | | | |
| | | | | |
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| | | | | |

4. Working with a friend, and using the information you have both collected in Question 3, work out how to perform the following conversions:

   a. fraction to a percentage

   b. fraction to a decimal

   c. percentage to a fraction

   d. percentage to a decimal

   e. decimal to a fraction

   f. convert from a decimal to a percentage

5. Share your ideas with the class.

6. Create a poster that shows how to make each of the 6 conversions listed in Question 4.

Activity 2

1. Cut out the number cards on the next page and order them from smallest to greatest number.

2. When you have finished, compare your results with another group, and then the class.

3. Discuss the following questions with the class:

   a. What strategies did you use to order your numbers?
   b. How did you decide whether one number was bigger or smaller than another?
   c. What strategies can we use to check our work?
   d. How do we know we are right?

| | | |
|---|---|---|
| 60% | $\frac{30}{100}$ | 0.305 |
| 1.3 | 0.61 | $\frac{1}{3}$ |
| $\frac{5}{6}$ | 0.6% | 0.06 |
| 0.3004 | 6% | 0.31 |

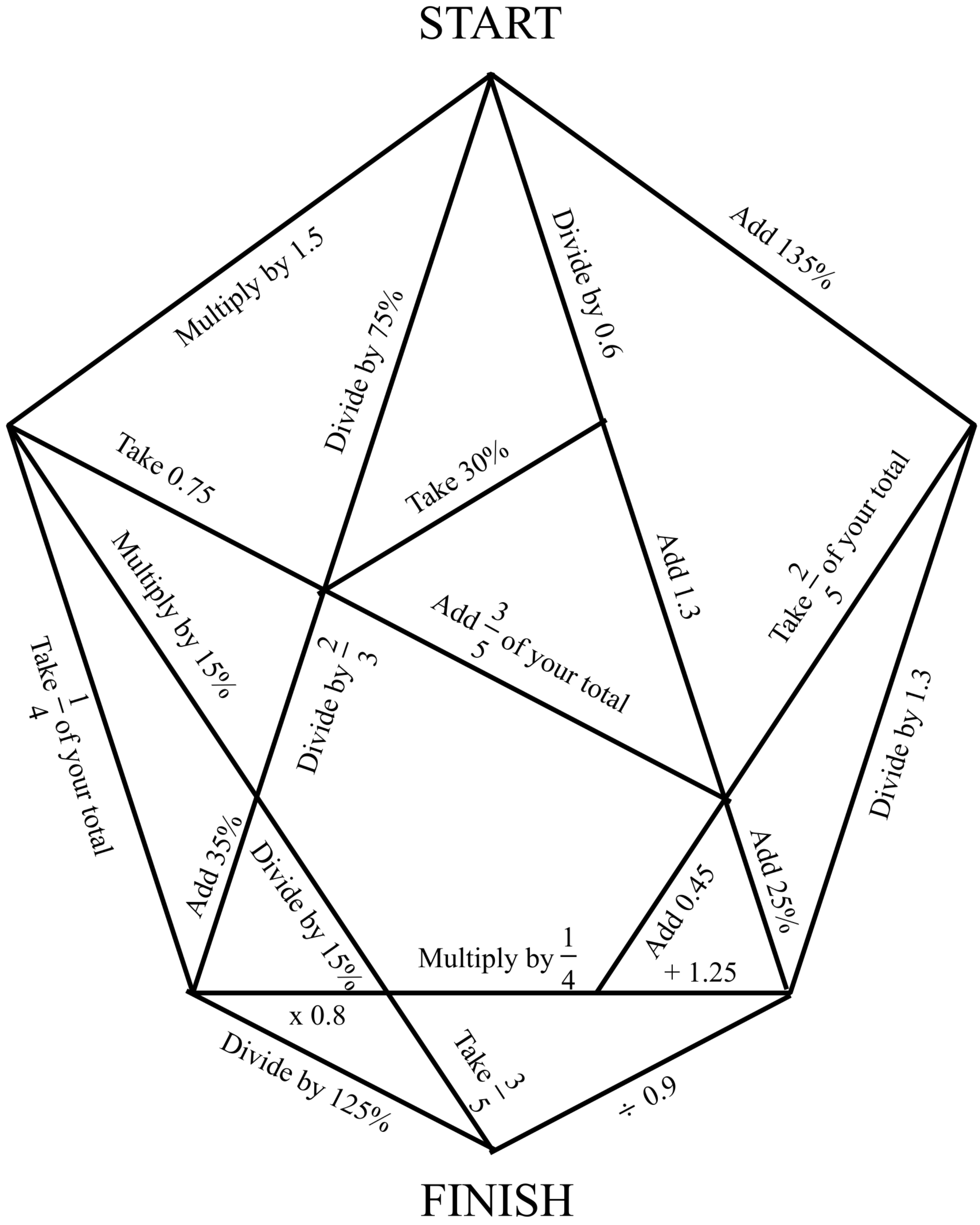
Activity 3

Maze Rules:

- You can move through maze along any line that takes you down or across
- You cannot move along upwards along any line.
- You start the maze with $100.

1. Find the route through the maze below that leaves you with the most money.

2. Find the route through the maze that leaves you with the least money.



Department of Education

# Year 7 Mathematics

## Number & Algebra Activity

# Squares and Square Roots

Produced by a Department of Education - MAWA partnership project
written for the Year 7 Australian Curriculum

# TASK 226: SQUARES AND SQUARE ROOTS

### Overview

This task is designed to introduce students to the concept of square numbers and square roots. Students start by building squares from blocks and progress to investigating the different factors of square numbers. The final activity is a problem solving activity designed to get students using square numbers.

### Students will need

- centicubes
- grid paper
- calculators

### Relevant content descriptions from the Western Australian Curriculum

- Investigate and use square roots of perfect square numbers (ACMNA150)

### Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
- *understanding* when they
  - connect the relationship between squares and square roots
  - estimate square roots of non-square numbers
- *reasoning* when they
  - explain why square numbers have an odd number of factors
- *problem solving* when they
  - identify “happy” numbers and “strange squares”

Activity 1

1. Using centicubes, make a square of side length 2 cm.
   Model as appropriate

2. In the square above the 2 is called the "square root" because it is the length we used to build the square; i.e., it is the starting point or root of the square in the same way that a tree's roots are the starting point of the tree!

   Build a square with a square root of -
   a. 1
   b. 3
   c. 4
   d. 5
   e. 6
   f. 7
   g. 8
   h. 9
   i. 10

   Appropriate models of squares for the above roots.

3. For each of the squares you have built in Questions 1 and 2, complete the table below.

| Square Root (AKA Side Length of Square) | Area of Square |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |
| 10 | 100 |

4. The numbers you wrote in the table above are all called Square Numbers. Why do you think this is?

   They form a perfect square (or various other appropriate answers).

5. How can you use the square root to find the square number?

   Multiply the square root by itself.

6. You should have noticed that you can take the square root and multiply it by itself to get the square number.

   If we were finding the area of a square with a side length of 11 (or *squaring* 11), we could write in like this:
   - In expanded notation as 11 x 11, or
   - in index notation as $11^2$.

   Square the following numbers. Write them in index notation, then find the answers.

   a. 12; $12^2 = 144$

   b. 13; $13^2 = 169$

   c. 14; $14^2 = 196$

   d. 15; $15^2 = 225$

7. Andrew drew a square that had an area of 70 $cm^2$.
   a. Was the square root of Andrew's square a whole number?
   No

   b. Was the square root of Andrew's square longer than 8 cm? How do you know?

   Yes, because 8 x 8 = 64, and Andrew's square is 70 $cm^2$

   c. Was it longer than 9 cm? How do you know?

   No, because 9 x 9 = 81, which is more than the area of Andrew's square.

8. You should have estimated that the square root of Andrew's square is between 8 and 9 cm. See if you can estimate the square roots of the following numbers; i.e., give a number that is too small and too large.

   a. 44 between 6 and 7

   b. 20 between 4 and 5

   c. 56 between 7 and 8

9. Another way to find the square root is to use this symbol '√' on your calculator, which is based on the letter 'r' for 'root'.
   So √9 means "the square root of 9", which is 3.
   Find the square roots of the following numbers.
   a. $\sqrt{16} = 4$
   b. $\sqrt{36} = 6$
   c. $\sqrt{25} = 5$

10. Estimate the answers to the following (without using a calculator).

a. √10 between 3 and 4

b. √31 between 5 and 6

c. √75 between 8 and 9

11. Use a calculator to check your answers to Questions 9 and 10.

Activity 2

1. Below is a list of the first 10 composite numbers that are also square numbers. Find all the factors of these numbers and complete the table below. The first two are done for you.

| Number | List of Factors | Total Number of Factors |
|---|---|---|
| 1 | 1 | 1 |
| 4 | 1, 2, 4 | 3 |
| 9 | 1, 3, 9 | 3 |
| 16 | 1, 2, 4, 8, 16 | 5 |
| 25 | 1, 5, 25 | 3 |
| 36 | 1, 2, 3, 4, 6, 9, 12, 18, 36 | 9 |
| 49 | 1, 7, 49 | 3 |
| 64 | 1, 2, 4, 8, 16, 32, 64 | 7 |
| 81 | 1, 3, 9, 27, 81 | 5 |
| 100 | 1, 2, 4, 5, 10, 20, 25, 50, 100 | 9 |

2. Below is a list of ten composite numbers that are not square numbers. Find all the factors of these numbers and complete the table below.

| Number | List of Factors | Total Number of Factors |
|---|---|---|
| 6 | 1, 2, 3, 6 | 4 |
| 8 | 1, 2, 4, 8 | 4 |
| 10 | 1, 2, 5, 10 | 4 |
| 12 | 1, 2, 3, 4, 6, 12 | 6 |
| 14 | 1, 2, 7, 14 | 4 |
| 22 | 1, 2, 11, 22 | 4 |
| 35 | 1, 5, 7, 35 | 4 |
| 48 | 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 | 10 |
| 70 | 1, 2, 5, 7, 10, 14, 35, 70 | 8 |
| 90 | 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45 90 | 12 |

2. Have a look at the "total number of factors" for each group of numbers. What do you notice?

   The square numbers all have an odd number of factors, while the composite numbers have an even number of factors.

3. Explain why this is the case.

   i. Usually factors come in pairs, as can be represented by rectangles, such as in a 2 x 8 rectangle with an area of 16 units.
   ii. When the rectangle is also a square, such as in the 4 x 4 case, the area is still 16 but one pair of factors is identical (4 and 4).
   iii. This means there is always an odd number of factors for squares.

Activity 3

1. Meta Product Squares
   - Choose a number
   - Multiply the number you have chosen by the number of letters in the word for that number
   - Is your answer a square number?
   - If so, this can be called a Meta Product Square

   How many Meta Product Squares can you find? List them.

   There are many answers, the square meta products less than 50 are listed below.

   four → 4 x 4 letters = 16
   nine → 9 x 4 letters = 36
   eighteen → 18 x 8 = 144
   thirty-six→ 36 x 9 = 329
   forty-nine →49 x 9 = 441

2. Happy Numbers
   To find out whether or not a number is happy, follow the process below.

| | |
|---|---|
| • take each of its digits and square them | 13 →$1^2 + 3^2 = 1 + 9$ |
| • add the answers | 1 + 9 = 10 |
| • repeat as many times as necessary | $1^2 + 0^2 = 1 + 0$ |
| • If you end up with 1 the number is happy. | 1 → therefore 13 is ☺ |

   How many happy numbers can you find? List them.

   There are many answers, and the happy numbers below 50 are listed below;

   1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49

Activity 1

1. Using centicubes, make a square of side length 2 cm.

2. In the square above the 2 is called the “square root” because it is the length we used to build the square; i.e., it is the starting point or root of the square in the same way that a tree’s roots are the starting point of the tree!

   Build a square with a square root of -
      a. 1
      b. 3
      c. 4
      d. 5
      e. 6
      f. 7
      g. 5
      h. 9
      i. 10

3. For each of the squares you have built in Questions 1 and 2, complete the table below.

| Square Root (aka Side Length of Square) | Area of Square |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |

4. The numbers you wrote in the table above are all called “Square Numbers”. Why do you think this is?

5. How can you use the square root to find the square number?

6. You should have noticed that you can take the square root and multiply it by itself to get the square number.

   If we were finding the area of a square with a side length of 11 (or *squaring* 11), we could write in like this;
   - In expanded notation as 11 x 11, or;
   - in index notation as $11^2$

   Square the following numbers. Write them in index notation, then find the answers.

   a. 12

   b. 13

   c. 14

   d. 15

7. Andrew drew a square that had an area of 70 $cm^2$.

   a. Was the square root of Andrew's square a whole number? (Hint: use the table in question 3 to help you!)

   b. Was the square root of Andrew's square longer than 8 cm? How do you know?

   c. Was it longer than 9cm? How do you know?

8. You should have estimated that the square root of Andrew's square is between 8 and 9 cm. See if you can estimate the square roots of the following numbers; i.e., give a number that is too small and too large.

   a. 44

   b. 20

   c. 56

9. Another way to find the square root is to use this symbol '√' on your calculator, which is based on the letter 'r' for 'root'.
   So √9 means "the square root of 9", which is 3.

Find the square root of the following numbers

a. $\sqrt{16}$

b. $\sqrt{36}$

c. $\sqrt{25}$

10. Estimate the answers to the following (without using a calculator).

a. $\sqrt{10}$

b. $\sqrt{31}$

c. $\sqrt{75}$

11. Use a calculator to check your answers to Questions 9 and 10.

<u>Activity 2</u>

1. Below is a list of the first 10 composite numbers that are also square numbers. Find all the factors of these numbers and complete the table below. The first two are done for you.

| Number | List of Factors | Total Number of Factors |
|---|---|---|
| 1 | 1 | 1 |
| 4 | 1, 2, 4 | 3 |
| 9 | | |
| 16 | | |
| 25 | | |
| 36 | | |
| 49 | | |
| 64 | | |
| 81 | | |
| 100 | | |

2. Below is a list of ten composite numbers that are not square numbers. Find all the factors of these numbers and complete the table below.

| Number | List of Factors | Total Number of Factors |
|---|---|---|
| 6 | | |
| 8 | | |
| 10 | | |
| 12 | | |
| 14 | | |
| 22 | | |
| 35 | | |
| 48 | | |
| 70 | | |
| 90 | | |

3. Look at the total number of factors for each of the two groups of numbers. What do you notice?

4. Explain why this is the case.

Activity 3

1. Meta Product Squares
   - Choose a number
   - Multiply the number you have chosen by the number of letters in the word for that number
   - Is your answer a square number?
   - If so, this can be called a Meta Product Square

   How many Meta Product Squares can you find? List them.

2. Happy Numbers.

To find out whether or not a number is happy, follow the process below.

| | |
|---|---|
| • take each of its digits and square them | 13 → $1^2 + 3^2 = 1 + 9$ |
| • add the answers | 1 + 9 = 10 |
| • repeat as many times as necessary | $1^2 + 0^2 = 1 + 0$ |
| • If you end up with 1 the number is happy. | 1 → therefore 13 is ☺ |

How many happy numbers can you find? List them.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Investigating Square Numbers

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 227: INVESTIGATING SQUARE NUMBERS

## Overview

This task is designed to follow on from Task 226. Its aim is to help students find patterns with square numbers. It is assumed that they have already been introduced to square numbers and are comfortable working with them.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- investigate and use square roots of perfect square numbers (ACMNA150)

## Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
- *understanding* when they
  - write prime numbers as the sums of two squares.
- *reasoning* when they
  - identify and explain patterns in sets of square numbers
- *problem solving* when they
  - find the shortest route to visit all the square numbers in a hundred chart

Activity 1

1. Pick any two consecutive counting numbers and write them below.

   Various answers; e.g., 6, 7

2. Add the two numbers and write the total below.

   Various answers; e.g., 6 + 7 = 13

3. Square each number you chose in Question 1, then find the difference between the two squares.

   Various answers; e.g., using the numbers above, $7^2 - 6^2 = 49 – 36 = 13$

4. What do you notice about your answers to Question 2 and Question 3?

   They are the same.

5. Does this work for any two consecutive numbers? Test at least 10 more pairs of numbers below to test your theory.

   Various answers, but the sum of two consecutive numbers and the difference between their squares will always be the same.

6. For the following pairs of numbers, and without using a calculator, predict the difference between their squares.
   a. 34 and 35
      69

   b. 43 and 44
      87

   c. 21 and 22
      43

   d. 1001 and 1002
      2003

7. What rule did you use to make these predictions?

   Add the two numbers together.

8. Check your predictions with a calculator. Were you correct?

   Yes.

Activity 2

Below is a list of all the primes under 50.

| | | | | |
|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 |
| 13 | 17 | 19 | 23 | 29 |
| 31 | 37 | 41 | 43 | 47 |

Ella thinks that she can write every prime number as a sum of two different square numbers, for example $2^2 + 3^2 = 4 + 9 = 13$

1. Is Ella correct? Can you write every prime number as a sum of two squares?

   No, some (e.g., 3) cannot be written as the sum of two squares.

2. Which numbers from the table above can be written as the sum of two squares? Write the list below.

   $1^2 + 1^2 = 1 + 1 = 2$
   $1^2 + 2^2 = 1 + 4 = 5$
   $2^2 + 3^2 = 4 + 9 = 13$
   $1^2 + 4^2 = 1 + 16 = 17$
   $2^2 + 5^2 = 4 + 25 = 29$
   $1^2 + 6^2 = 1 + 36 = 37$
   $4^2 + 5^2 = 16 + 25 = 41$

3. Which numbers from the table above cannot be written as the sum of two squares? Write the list below.

   3, 7, 11, 19, 23, 31, 43, 47

4. For each of the primes you listed in Question 2, do the following:
   a. Take the prime number and add 1
   b. Divide the total by 2
   c. Record your results below.

   All answers should be odd numbers, except for the prime 2, as it is the only even prime.

5. For each of the primes you listed in Question 3, do the following;
   d. Take the prime number and add 1
   e. Divide the total by 2
   f. Record your results below.

   All answers should be even numbers.

6. What is different between the answers you found in Questions 4 and 5?

   Primes that can be written as sum of two squares (except for 2) gave an odd number when we added 1 and divided by 2. Primes that cannot be written as sum of two squares gave an even number when we added 1 and divided by 2.

7. Below is the list of all the primes between 50 and 100. Use your theory from Question 4 to predict which numbers can be written as a sum of two squares.

| | | | | |
|---|---|---|---|---|
| 53 | 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 | 91 |

   53, 61, 73, 89

8. Show the sum of two squares for each number you identified in Question 7.
   $4^2 + 7^2 = 16 + 49 = 53$
   $5^2 + 6^2 = 25 + 36 = 61$
   $3^2 + 8^2 = 9 + 64 = 73$
   $5^2 + 8^2 = 25 + 64 = 89$

9. Did your theory work? If not, come up with a new theory and repeat Questions 7 and 8.
   Various answers

Activity 3

1. On the grid below, highlight every square number.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

2. The "Square Knight" wants to visit every square number on the board.

   However, he has a very peculiar way of moving.
   - He can only move up, down left or right (not diagonally)
   - From any position, he must move 2 squares in whichever direction he chooses, then he must make a 90° turn and move one more square in this new direction.
   - For example, from square 54, the knight has 8 possible moves: He could land on 33, 35, 42, 46, 62, 66, 73, or 75 as shown below.

| | | | | |
|---|---|---|---|---|
| 32 | 33 | 34 | 35 | 36 |
| 42 | 43 | 44 | 45 | 46 |
| 52 | 53 | 54 | 55 | 56 |
| 62 | 63 | 64 | 65 | 66 |
| 72 | 73 | 74 | 75 | 76 |

   Starting at square 1, plan the Knight's journey so that he can visit every square number. Write your route below.
   Various appropriate answers

3. Can this journey be made any shorter? See if you can find a route that takes the least possible moves to visit each square number.

   It is possible to do this in 23 moves as follows:
   1, 13, 25, 4, 16, 28, 9, 30, 49, 57, 36, 44, 56, 64, 83, 62, 81, 73, 85, 77, 98, 79, 100
   Other answers are also possible.

Activity 1

1. Pick any two consecutive counting numbers and write them below.

2. Add the two numbers and write the total below.

3. Square each number you chose in Question 1, then find the difference between the two squares.

4. What do you notice about your answers to Question 2 and Question 3?

5. Does this work for any two consecutive numbers? Test at least 10 more pairs of numbers below to test your theory.

6. For the following pairs of numbers, and without using a calculator, predict the difference between their squares.
    a. 34 and 35
    b. 43 and 44
    c. 21 and 22
    d. 1001 and 1002

7. What rule did you use to make these predictions?

8. Check your predictions with a calculator. Were you correct?

Activity 2

Below is a list of all the primes under 50.

| | | | | |
|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 |
| 13 | 17 | 19 | 23 | 29 |
| 31 | 37 | 41 | 43 | 47 |

Ella thinks that she can write every prime number as a sum of two different square numbers, for example $2^2 + 3^2 = 4 + 9 = 13$

1. Is Ella correct? Can you write every prime number as a sum of two squares?

2. Which numbers from the table above can be written as the sum of two squares? Write the list below

3. Which numbers from the table above cannot be written as the sum of two squares? Write the list below.

4. For each of the primes you listed in Question 2, do the following;
    g. Take the prime number and add 1
    h. Divide the total by 2
    i. Record your results below.

5. For each of the primes you listed in Question 3, do the following;
    j. Take the prime number and add 1
    k. Divide the total by 2
    l. Record your results below.

6. What is different between the answers you found in Questions 4 and 5?

7. Below is the list of all the primes between 50 and 100. Use your theory from Question 4 to predict which numbers can be written as a sum of two squares.

| | | | | |
|---|---|---|---|---|
| 53 | 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 | 91 |

8. Show the sum of two squares for each number you identified in Question 7.

9. Did your theory work? If not, come up with a new theory and repeat Questions 7 and 8.

Activity 3

1. On the grid below, highlight every square number.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

2. The “Square Knight” wants to visit every square number on the board.

   However, he has a very peculiar way of moving.
   - He can only move up, down left or right (not diagonally)
   - From any position, he must move 2 squares in whichever direction he chooses, then he must make a 90° turn and move one more square in this new direction.
   - For example, from square 54, the knight has 8 possible moves: He could land on 33, 35, 42, 46, 62, 66, 73, or 75 as shown below.

| | | | | |
|---|---|---|---|---|
| 32 | 33 | 34 | 35 | 36 |
| 42 | 43 | 44 | 45 | 46 |
| 52 | 53 | 54 | 55 | 56 |
| 62 | 63 | 64 | 65 | 66 |
| 72 | 73 | 74 | 75 | 76 |

Starting at square 1, plan the Knight's journey so that he can visit every square number. Write your route below.

3. Can this journey be made any shorter? See if you can find a route that takes the least possible moves to visit each square number.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Matchstick Patterns

Produced by a Department of Education - MAWA partnership project
Written for the Year 7 Australian Curriculum

# TASK 230: MATCHSTICK PATTERNS

## Overview

This task is designed to help students to -

- identify patterns
- use tables, rules and graphs to record patterns
- use rules and/or graphs to predict patterns
- create their own patterns following given rules.

## Students will need

- matchsticks
- connector blocks or centicubes
- counters
- calculator (optional)

## Relevant content descriptions from the Western Australian Curriculum

- Introduce variables as a way of representing numbers using letters (ACMNA175)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

## Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
- *understanding* when they
  - describe patterns
  - plot points on the Cartesian plane
- *reasoning* when they
  - use a rule, table or graph to predict patterns
- *problem solving* when they
  - use matchsticks or other manipulatives to create a pattern for a given rule.

Activity 1

1. Sharon made the following pattern;

Stage 1 Stage 2 Stage 3

Using matchsticks, build Stages 4, 5 and 6 of the pattern. Draw each stage below.

Stage 4 Stage 5 Stage 6

2. Use your drawings to help you complete the table

| Stage ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | 3 | 5 | 7 | 9 | 11 | 13 |

3. Re-write the numbers from the table above as coordinates. The first two are done for you.
(1, 3) (2, 5) (3, 7) (4, 9) (5, 11) (6, 13)

4. Plot each of the points above on the grid provided.
Join your points with a straight line. Extend your line to the edges of the grid.
Answers as appropriate.

5. Use your graph to predict how many matches would be needed for a -

   a. Stage 8 pattern 17

   b. Stage 10 pattern 21

   c. Stage 14 pattern 29

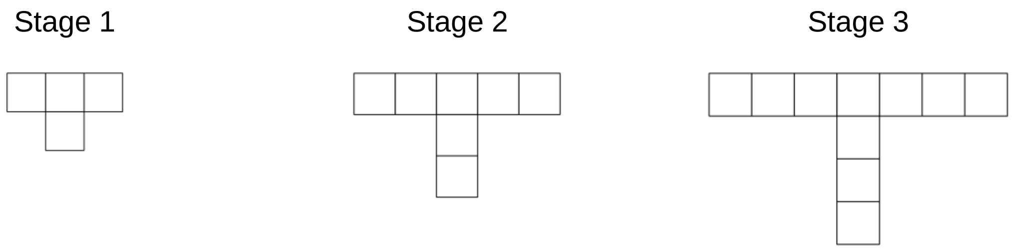
6. How many matches do you think would be needed for a Stage 99 pattern? Answer the questions below to find out

   a. Complete the rule: To find the number of matchsticks you need, take the stage number, multiply it by 2 then add 1.

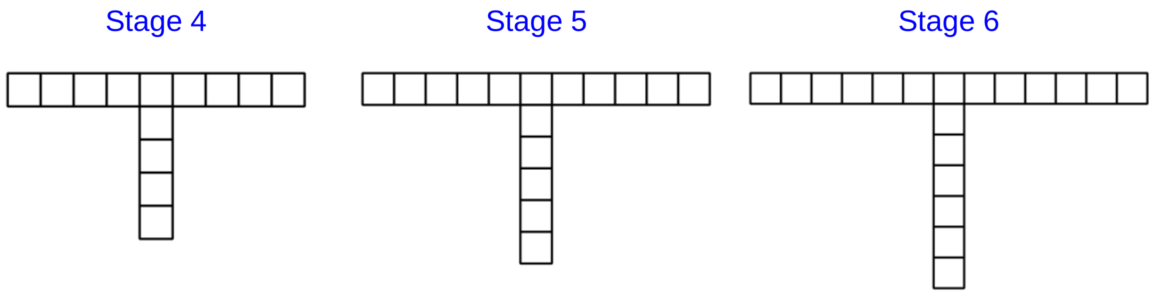
   b. Use the rule above to find out how many matches are needed for a stage 99 pattern 199

Activity 2

1. Jason made the following pattern:



Using connector blocks, build Stages 4, 5 and 6 of the pattern. Draw each stage below.



2. Use your drawings to help you complete the table

| Stage ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | 4 | 7 | 10 | 13 | 16 | 19 |

3. Re-write the numbers from the table above as coordinates.
(1, 4) (2, 7) (3, 10) (4, 13) (5, 16) (6, 19)

4. Plot each of the points above on the grid provided.
Join your points with a straight line. Extend your line to the edges of the grid.

5. Use your graph to predict how many blocks would be needed for a -

   a. Stage 8 pattern
   25

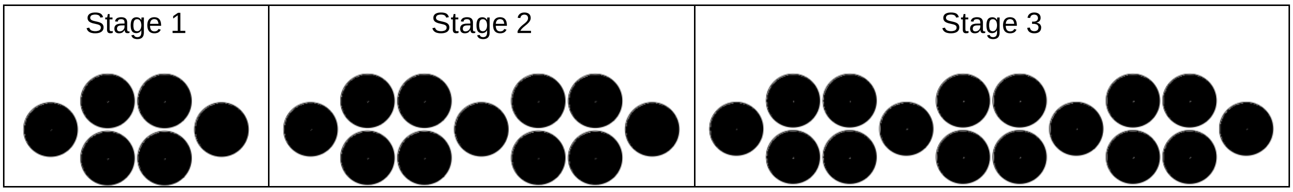
   b. Stage 10 pattern
   31

   c. Stage 14 pattern
   43

6. How many blocks would be needed for a Stage 99 pattern? Find a rule to connect the stage number and the number of blocks.
298.
Multiply the stage number by 3, then add 1

## Activity 3

1. Rachel made the following pattern:




Complete the table.

| Stage ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | 6 | 11 | 16 | 21 | 26 | 31 |

2. Plot each of the points above on the grid provided.
   Join your points with a straight line. Extend your line to the edges of the grid.

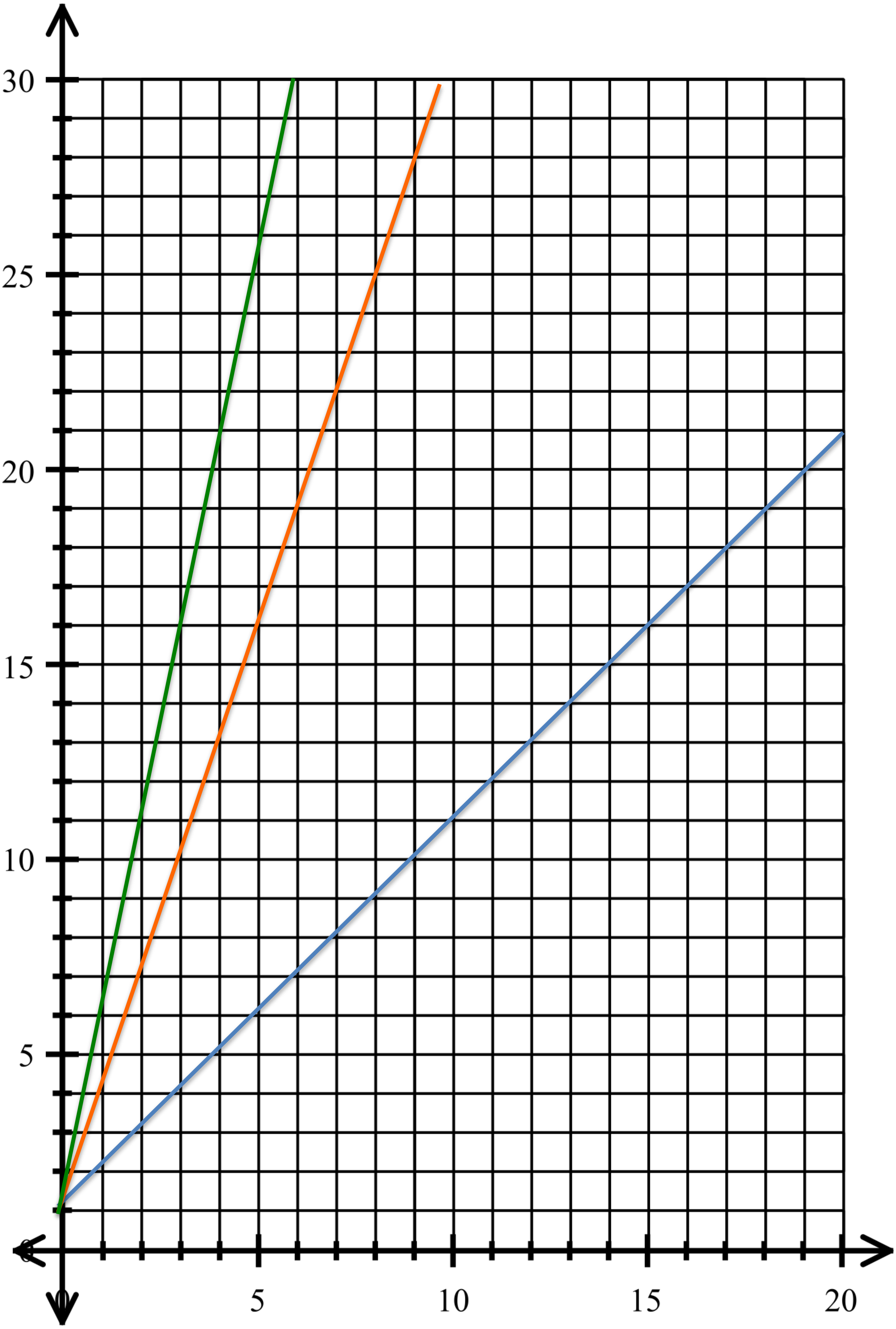
3. Use your graph to predict how many counters would be needed for a -

   a. Stage 8 pattern
      41

   b. Stage 10 pattern
      51

   c. Stage 14 pattern
      71

4. How many counters would be needed for a stage 99 pattern? See if you can find a rule to connect the stage number and the number of counters.
   496
   $y = 5x + 1$


30
25
20
15
10
5
0
0
5
10
15
20

## Activity 4

1. Create your own pattern using matchsticks (or blocks or counters!). Draw the first three stages of your pattern below.
   Various answers

2. Use your pattern to fill out the table below.

| Stage ($x$) | 1 | 2 | 3 | | | |
|---|---|---|---|---|---|---|
| Number of matchsticks ($y$) | | | | | | |

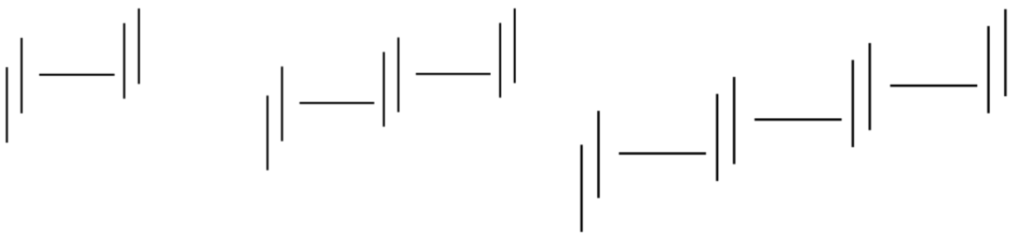
3. Describe in words the rule that your pattern follows.
   Various answers

4. Describe in symbols the rule that your pattern follows.
   Various answers

5. See if you can create a matchstick pattern that follows the following rules. Draw the first 3 stages for each pattern that you create.

   Various answers will be acceptable. Some suggestions are shown below.

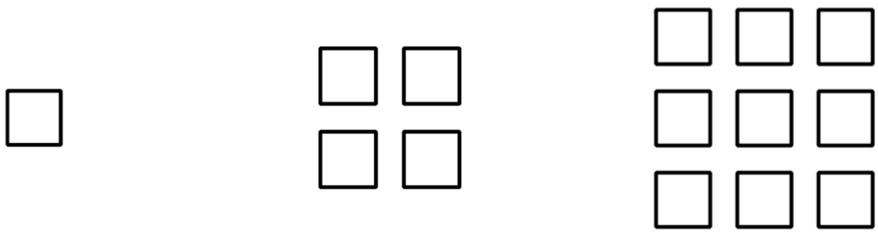
   a. $y = 3x + 2$



   b. $y = 5x + 1$

   c. $y = 3x + 1$

   d. Challenge: $y = x^2$



Activity 1

1. Sharon made the following pattern;


   Using matchsticks, build Stages 4, 5 and 6 of the pattern. Draw each stage below.

2. Use your drawings to help you complete the table

| Stage ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | 3 | 5 | | | | |

3. Re-write the numbers from the table above as coordinates. The first two are done for you.

   (1, 3) (2, 5)

4. Plot each of the points above on the grid provided.
   Join your points with a straight line. Extend your line to the edges of the grid.

5. Use your graph to predict how many matches would be needed for a -

   a. Stage 8 pattern

   b. Stage 10 pattern

   c. Stage 14 pattern

6. How many matches would be needed for a stage 99 pattern? Answer the questions below to find out

   a. Complete the rule: To find the number of matchsticks you need, take the stage

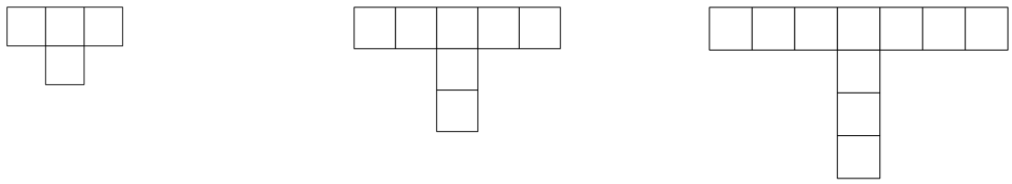
      number, multiply it by _____ then add _____

   b. Use the rule above to find out how many matches are needed for a stage 99 pattern

## Activity 2

1. Jason made the following pattern:

Stage 1 Stage 2 Stage 3



   Using connector blocks, build Stages 4, 5 and 6 of the pattern. Draw each stage below.

2. Use your drawings to help you complete the table

| Stage ($x$) | 1 | 2 | 3 | | | |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | | | | | | |

3. Re-write the numbers from the table above as coordinates.

4. Plot each of the points above on the grid provided.
   Join your points with a straight line. Extend your line to the edges of the grid.

5. Use your graph to predict how many blocks would be needed for a -

   a. Stage 8 pattern

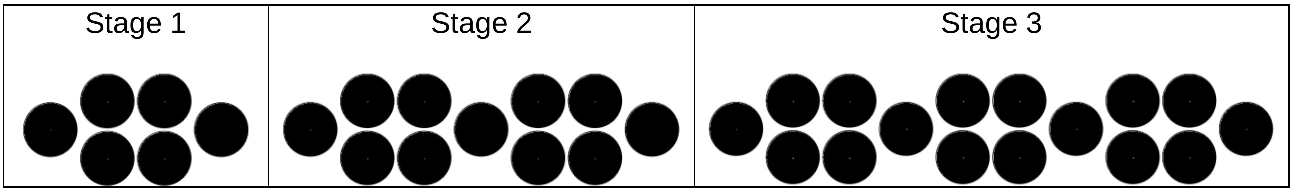
   b. Stage 10 pattern

   c. Stage 14 pattern

6. How many blocks would be needed for a Stage 99 pattern? Find a rule to connect the stage number and the number of blocks.

Activity 3

1. Rachel made the following pattern:




Complete the table

| Stage ($x$) | 1 | 2 | 3 | | | |
|---|---|---|---|---|---|---|
| Number of Matchsticks ($y$) | | | | | | |

2. Plot each of the points above on the grid provided.
Join your points with a straight line. Extend your line to the edges of the grid.

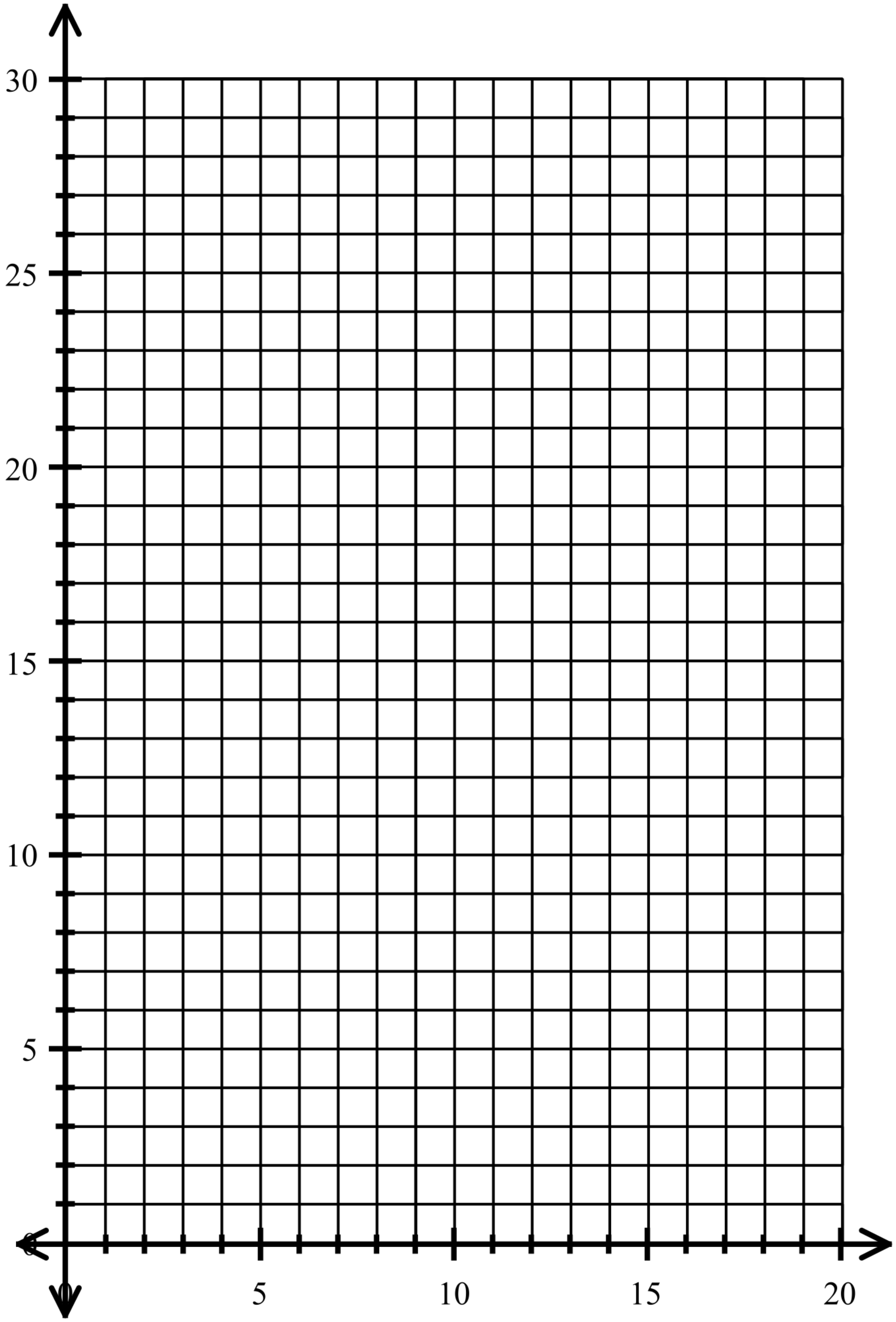
3. Use your graph to predict how many counters would be needed for a -

    a. Stage 8 pattern

    b. Stage 10 pattern

    c. Stage 14 pattern

4. How many counters would be needed for a Stage 99 pattern? Find a rule to connect the stage number and the number of counters.


30
25
20
15
10
5
0
0
5
10
15
20

Activity 4

1. Create your own pattern using matchsticks (or blocks or counters!). Draw the first three stages of your pattern below.

2. Use your pattern to fill out the table below.

| Stage ($x$) | 1 | 2 | 3 | | | |
|---|---|---|---|---|---|---|
| Number of matchsticks ($y$) | | | | | | |

3. Describe in words the rule that your pattern follows.

4. Describe in symbols the rule that your pattern follows.

5. See if you can create a matchstick pattern that follows the rules below. Draw the first 3 stages for each pattern that you create.
   a. $y = 3x + 2$

   b. $y = 5x + 1$

   c. $y = 3x + 1$

   d. Challenge: $y = x^2$

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Best Buys

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 232: BEST BUYS

## Overview

This task is an introduction to the topic of best buys.

## Students will need

- calculators
- access to the internet (Activity 3 only)

## Relevant content descriptions from the Western Australian Curriculum

- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

## Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
- *understanding* when they
  - recognise different ways of determining the answer
- *reasoning* when they
  - select the best buy in a given circumstance
- *problem solving* when they
  - work out the total of a grocery bill if they could buy only want they needed.

Introduction

You great Aunt has promised to give you $5 to spend at the shops. You can buy as many (or as few!) items as you like, but if you try to spend more than $5 you will not get anything at all!

Activity 1

1. Use the catalogues you have been given to help you decide what you will buy.

2. Cut out what you have decided to buy and paste it below (remember to include the price with the picture!)
   Various answers

3. Compare what you have bought with a partner. Write down 3 reasons why what you have bought is a better buy.
   Various answers

4. DISCUSSION QUESTION: Discuss the idea of a better buy with the class.
   Various points as appropriate.

Activity 2 – Teacher-led Activity

1. Tell students you are going to show them six different pairs of goods. The students' job is to decide which of the pair is the better buy and record it on the activity sheet. Do not give students more than a few seconds thinking time at this stage.

2. Discuss each pair with the class, and have students explain why they chose one item over another.

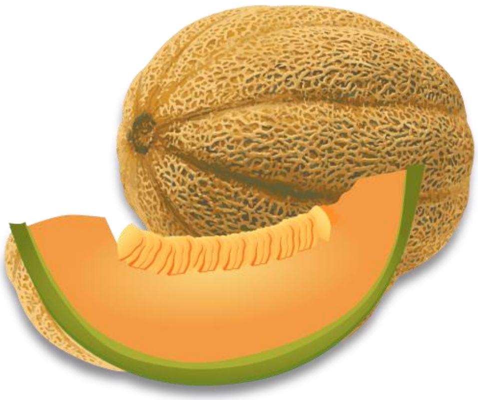
3. Ask students what information we need in order to decide which item is actually the better buy. When they suggest they need to know the price of each item, give them the information and ask them to record it in the given table.

4. Ask students if they agree that the caramel egg is a better buy than the caramel bar. Some students will recognise that the caramel chocolate bar is bigger so that it may be a better buy. At this stage you can give students the mass of each item and students can record the information in their table next to the name of the item.

5. Ask students what they need to do to be able to figure out which item is a better buy. Allow them to discuss strategies for comparing the cost of items with different masses, such as working out the cost per 100 g. When the class has agreed on an approach work through the first few items together as a class.

6. Ask students to find the better buy for each of the other pairs, showing their working out.


OR

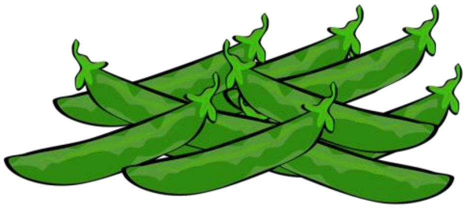
OR


OR


350ml
Strawberry
MILK
350ml
Strawberry
MILK

OR


1 LITRE
FULL
CREAM
MILK
1 LITRE
FULL
CREAM
MILK



OR


ANZAC Biscuits
1kg

OR


45g potato
chips

| Item | Price |
| --- | --- |
| Caramel egg | $1.14 |
| Caramel chocolate bar | $4.34 |
| Rockmelon | $4.98 |
| Watermelon | $2.50 |
| Rice | $3.90 |
| Pasta | $2.15 |
| Flavoured milk | $2.80 |
| Full cream milk | $2.15 |
| Sweet peas | $3.96 |
| Turnips | $1.08 |
| ANZAC biscuits | $11.00 |
| Potato crisps | $1.60 |

| Item | Mass |
| --- | --- |
| Caramel egg | 35 g |
| Caramel chocolate bar | 100 g |
| Rockmelon | 1 kg |
| Watermelon | 1 kg |
| Rice | 2 kg |
| Pasta | 500 g |
| Flavoured milk | 350 mL |
| Full cream milk | 1 L |
| Sweet peas | 400 g |
| Turnips | 180 g |
| ANZAC biscuits | 1 kg |
| Potato crisps | 45 g |

1. Look at the lists of items and their prices.
For each pair listed below, circle the item that you think is the better buy;

a. Caramel egg OR Caramel chocolate bar

b. Rockmelon OR Watermelon

c. Rice OR Pasta

d. Flavoured milk OR Full cream milk

e. Sweet peas OR Turnips

f. ANZAC biscuits OR Potato crisps

Various Answers

2. Complete the following table for each of the items listed in Question 1.

| Item | Price | Item | Price |
|---|---|---|---|
| Caramel egg | $1.14 | Caramel chocolate bar | $4.34 |
| Rockmelon | $4.98 | Watermelon) | $2.50 |
| Rice | $3.90 | Pasta | $2.15 |
| Flavoured milk | $2.80 | Full cream milk | $2.15 |
| Sweet peas | $3.96 | Turnips | $1.08 |
| ANZAC biscuits | $11.00 | Potato crisps | $1.60 |

3. Work out price per 100 grams or 100mL for each item listed above. Show your working below.

Caramel egg - $3.26/100 g
Carmel chocolate bar - $4.34/100 g
Rockmelon - $0.50/100g
Watermelon - $0.25/100 g
Rice - $0.20/100 g
Pasta - $0.43/100 g
Flavoured milk - $0.80/100 mL
Milk - $0.22/100 mL
Sweet peas - $0.99/100 g
Turnips - $0.60/100 g
ANZAC biscuits - $1.10/100 g
Potato crisps - $3.56/100 g

4. Based on your answers to Question 3, list your items from most expensive to least expensive.

   Carmel chocolate bar - $4.34/100g
   Potato crisps - $3.56/100g
   Caramel egg - $3.26/100g
   ANZAC biscuits - $1.10/100g
   Sweet peas - $0.99/100g
   Flavoured milk - $0.80/100ml
   Turnips - $0.60/100g
   Pasta - $0.43/100g
   Watermelon - $0.25/100g
   Milk - $0.22/100ml
   Rockmelon - $0.20/100g
   Rice - $0.20/100g

5. Write a brief explanation of how to determine if one item is a better buy than another.
   Find the unit price (or price per 100g or 100 mL) of each item before comparing.

Activity 3.

You are helping your mum make a special afternoon tea for your family. She has given you $40 and sent you to the shops to buy the ingredients listed below.

You must buy *at least* the amount of each item that your mother has requested, but you may buy more if necessary.

Your mum has also told you that if you come back with everything she needs, she will let you keep the change, so it is in your best interests to buy the cheapest goods.

Here is the list:
5 eggs.
180 g of butter
800 g of plain flour
2½ L of milk
60 g of brown sugar
400 g of milk arrowroot OR marie biscuits
1 loaf of white sliced bread
2 large (350 g) cucumbers
100 g of instant coffee
10 English breakfast tea bags

1. Why might you need to buy more than the amounts noted on your list?

   Various answers. Example:
   Because most goods come pre-packaged, you have to buy a certain amount. For instance, cartons of eggs come in 6, 12 and 18 packs, so even though we only need 5, we would have to buy at least 6 to get the number needed.

2. Using any Australian online grocery shopping site; e.g., coles.com.au, woolworths.com.au, etc., find the best buy for each item and fill in the table below.

| Item | Price of Item | Mass per Volume of Item | Price per 100 g (100 mL) | Price per kg (1 L) | Items per Packet Needed | Total Cost |
|---|---|---|---|---|---|---|
| Eggs | Various answers | | | | | |
| Butter | | | | | | |
| Flour | | | | | | |
| Milk | | | | | | |
| Sugar | | | | | | |
| Biscuits | | | | | | |
| Bread | | | | | | |
| Cucumbers | | | | | | |
| Coffee | | | | | | |
| Tea | | | | | | |
| Total Shopping Bill | | | | | | |
| Change from $40 | | | | | | |

3. If you could buy only the ingredients you needed; i.e., you could buy five eggs and not the whole carton; how much would the groceries have cost you then? Show your working below.

Various answers

Activity 4

1. Determine the best buy in each of the following situations:

   a. Box A contains 420 g of cereal and costs $2.28. Box B contains 700 g of cereal and costs $3.99.
      Box A - $0.54/100 g
      Box B - $0.57/100 g
      Box A is the better buy

   b. Bag C contains 50 g of potato chips and costs $0.99. Bag D contains 110 g of potato chips and costs $1.79.
      Bag C - $1.98/100g
      Bag D - $1.63/100g
      Box D is the better buy

   c. Jar E contains 180 g of coffee and costs $6.39. Jar F contains 120 g of coffee and costs $4.86.
      Jar E - $3.55/100g

Jar F - $4.05/100g
Jar E is the better buy

d. Bottle G contains 220 mL of cleaner and costs $2.64. Bottle H contains 320 mL of cleaner and costs $3.84.
Bottle G - $1.20/100g
Bottle H - $1.20/100g
They are both the same.

2. For each of your answers above, what is the maximum price increase the item could have before the other option becomes the better buy?
   a. Box A’s price would have to increase by 14c for Box B to become the better buy
   b. Bag D’s price would have to increase by 40c for Bag C to become the better buy
   c. Jar E’s price would have to increase by 91c for Jar F to become the better buy
   d. If either bottle increases in price, even by 1 cent, then the remaining product will become the better buy.

Introduction

You great Aunt has promised to give you $5 to spend at the shops. You can buy as many (or as few!) items as you like, but if you try to spend more than $5 you will not get anything at all!

<u>Activity 1</u>

1. Use the catalogues you have been given to help you decide what you will buy.

2. Cut out what you have decided to buy and paste it below (remember to include the price with the picture!)

3. Compare what you have bought with a partner. Write down 3 reasons why what you have bought is a better buy.

4. DISCUSSION QUESTION: Discuss the idea of a better buy with the class.

Activity 2

1. Look at the lists of items and their prices.
For each pair listed below, circle the item that you think is the better buy;

| | | | |
|---|---|---|---|
| a. | Caramel egg | OR | Caramel chocolate bar |
| b. | Rockmelon | OR | Watermelon |
| c. | Rice | OR | Pasta |
| d. | Flavoured milk | OR | Full cream milk |
| e. | Sweet peas | OR | Turnips |
| f. | ANZAC biscuits | OR | Potato crisps |

2. Complete the following table for each of the items listed in Question 1.

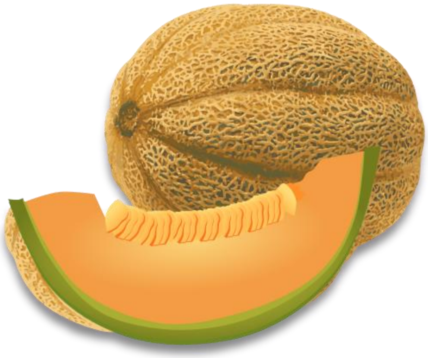
| Item | Price | Item | Price |
|---|---|---|---|
| Caramel egg | | Caramel chocolate bar | |
| Rockmelon | | Watermelon | |
| Rice | | Pasta | |
| Flavoured milk | | Full cream milk | |
| Sweet peas | | Turnips | |
| ANZAC biscuits | | Salt and vinegar chips | |

3. Work out price per 100 grams or 100mL for each item listed above. Show your working out below.

4. Based on your answers to question 3, list your items from most expensive to least expensive.

5. Write a brief explanation of how determine if one item is a better buy than another.


**OR**

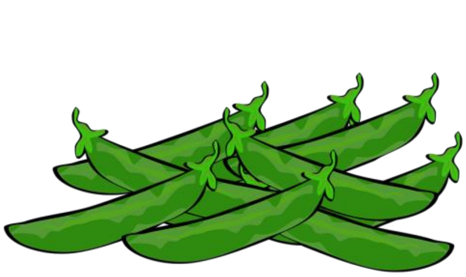
**OR**


**OR**


350 ml
Strawberry MILK
350 ml
Strawberry MILK

**OR**


1 LITRE
FULL CREAM
MILK
1 LITRE
FULL CREAM
MILK



**OR**


ANZAC Biscuits
1kg

**OR**


45g potato chips

| Item | Price |
|---|---|
| Caramel egg | $1.14 |
| Caramel chocolate bar | $4.34 |
| Rockmelon | $4.98 |
| Watermelon | $2.50 |
| Rice | $3.90 |
| Pasta | $2.15 |
| Flavoured milk | $2.80 |
| Full cream milk | $2.15 |
| Sweet peas | $3.96 |
| Turnips | $1.08 |
| ANZAC biscuits | $11.00 |
| Potato crisps | $1.60 |

| Item | Mass |
|---|---|
| Caramel egg | 35 g |
| Caramel chocolate bar | 100 g |
| Rockmelon | 1 kg |
| Watermelon | 1 kg |
| Rice | 2 kg |
| Pasta | 500 g |
| Flavoured milk | 350 mL |
| Full cream milk | 1 L |
| Sweet peas | 400 g |
| Turnips | 180 g |
| ANZAC biscuits | 1 kg |
| Potato crisps | 45 g |

Activity 3.

You are helping your mum make a special afternoon tea for your family. She has given you $40 and sent you to the shops to buy the ingredients listed below.

You must buy *at least* the amount of each item that your mother has requested, but you may buy more if necessary.

Your mum has also told you that if you come back with everything she needs, she will let you keep the change, so it is in your best interests to buy the cheapest goods.

Here is the list:

5 eggs
180 g of butter
800 g of plain flour
2½ L of milk
60 g of brown sugar
400 g of milk arrowroot OR plain biscuits
1 loaf of white sliced bread
2 large (350 g) cucumbers
100 g of instant coffee
10 English breakfast tea bags

1. Why might you need to buy more than the amounts noted on your list?

2. Using any Australian online grocery shopping site; e.g., coles.com.au, woolworths.com.au, etc., find the best buy for each item and fill in the table below.

| Item | Price of Item | Mass per Volume of Item | Price per 100 g (100 mL) | Price per kg (1 L) | Items per Packet Needed | Total Cost |
|---|---|---|---|---|---|---|
| Eggs | | | | | | |
| Butter | | | | | | |
| Flour | | | | | | |
| Milk | | | | | | |
| Sugar | | | | | | |
| Biscuits | | | | | | |
| Bread | | | | | | |
| Cucumbers | | | | | | |
| Coffee | | | | | | |
| Tea | | | | | | |
| Total Shopping Bill | | | | | | |
| Change from $40 | | | | | | |

3. If you could buy only the ingredients you needed – i.e. you could buy five eggs and not the whole carton – how much would the groceries have cost you then? Show your working out below.

<u>Activity 4</u>

3. Determine the best buy in each of the following situations:

   Box A contains 420 g of cereal and costs $2.28. Box B contains 700 g of cereal and costs $3.99.

   a. Bag C contains 50 g of potato chips and costs $0.99. Bag D contains 110 g of potato chips and costs $1.79.

   b. Jar E contains 180 g of coffee and costs $6.39. Jar F contains 120 g of coffee and costs $4.86.

   c. Bottle G contains 220 mL of cleaner and costs $2.64. Bottle H contains 320 mL of cleaner and costs $3.84.

4. For each of your answers above, what is the maximum price increase the item could have before the other option becomes the better buy?

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Activity

# Coin Conundrum

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 234: COIN CONUNDRUM

## Overview

This task is designed to get students using ratios. In Activity 1, students research masses of coins and work out their mass to value ratios. Activity 2 requires students to compare the ratio of the different metals that make up a coin and the final task requires students to determine the best prize using the mass, height or diameter of different coins.

## Students will need

- calculators
- access to the internet
- scales (optional – Activity 3)
- different coins (optional – Activity 3)

## Relevant content descriptions from the Western Australian Curriculum

- Recognise and solve problems using simple ratios (ACMNA173)

## Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
- *reasoning* when they
  - apply known geometric facts to draw conclusions about shapes
  - apply an understanding of ratio
- *problem* solving when they
  - formulate and solve authentic problems using measurements.

Activity 1

You have just helped stop a bank robbery. The bank manager is so grateful for your help he has given you a large bag and told you that you can take as many silver coins as you can carry. Which coin should you pick in order to maximize your reward?

1. How much do you think you can carry? If you are not sure, use 20% of your body mass as a guide and write your estimate below.
   Various answers

2. You can fill your bag with either 5c, 10c, 20c or 50c coins. Which coin will you choose?
   Various answers

3. Go online and find the mass of your coin. Write the mass of the coin below.
   Various answers

4. Using your answers from Question 1 and Question 3, work out how many coins you could carry.
   Various answers

5. Using your answer from Question 4, what are your coins worth?
   Various answers

6. Compare your answer to others in the class. Who got the best reward? How much did they get?
   Various answers

7. DISCUSSION QUESTION: Did you pick the right coin, or should you have chosen something else? How can we decide which coin is the best one to pick?
   Various answers

8. Use the internet to help you fill in the table below:

| Coin | Mass (grams) | Value : Mass Ratio | Value : Mass Ratio ($1 : ?) |
|---|---|---|---|
| 5c | 2.83 | 5 : 2.83 | 1 : 56.6 |
| 10c | 5.65 | 10 : 5.65 | 1 : 56.5 |
| 20c | 11.3 | 20 : 11.3 | 1 : 56.5 |
| 50c | 15.5 | 50 : 15.5 | 1 : 31 |

9. Using the table from Question 8, did you pick the right coin? Justify your answer.
   Various answers

10. If someone could carry exactly 10 kg, how much would their reward be worth if they chose -
    a. 5-cent coins?
       $176.7
    b. 10-cent coins?
       $177
    c. 20-cent coins?
       $177
    d. 50-cent coins?
       $322.5

Activity 2

Below is the list of each of the Australian coins, their mass, and the mass of each of their metal components.

| Coin | Mass (g) | Mass of Copper (g) | Mass of Nickel (g) | Mass of Aluminium (g) |
|---|---|---|---|---|
| $0.05 | 2.83 | 2.12 | 0.71 | - |
| $0.10 | 5.65 | 4.25 | 1.41 | - |
| $0.20 | 11.33 | 8.50 | 2.83 | - |
| $0.50 | 15.55 | 11.66 | 3.89 | - |
| $1.00 | 9.00 | 8.28 | 0.54 | 0.18 |
| $2.00 | 6.60 | 6.07 | 0.40 | 0.13 |

1. Find the ratio of total mass to the mass of copper for each coin, and enter results in the table below.

| Coin | Question 2 Total Mass : Copper Mass | Question 3 Copper Mass : Nickel Mass | Question 7 Value of Copper (to nearest cent) | Question 8 Value of Coin : Value of Copper |
|---|---|---|---|---|
| $0.05 | 283 : 212 | 212 : 71 | $0.01 | 5 : 1 |
| $0.10 | 565 : 425 (113 : 85) | 425 : 141 | $0.03 | 10 : 3 |
| $0.20 | 1133 : 850 | 850 : 283 | $0.05 | 20 : 5 (4 : 1) |
| $0.50 | 1555 : 1166 | 1166 : 389 | $0.07 | 50 : 7 |
| $1.00 | 900 : 828 (25 : 23) | 828 : 54 (46 : 3) | $0.05 | 100 : 5 (20 : 1) |
| $2.00 | 660 : 607 | 607 : 40 | $0.04 | 200 : 4 (50 : 1) |

2. Find the ratio of mass of copper to the mass of nickel in each coin, and enter in the above table.
   See table above

3. If we only had your answers to Question 2 (and not the table given above) could we work out the total mass of each coin? How?
   Yes. You could add the two numbers together.

4. Would this method work for the $1 and $2 coins as well? Why/why not?
   No. There is also Aluminium in these coins, which isn't reflected in the ratio.

5. How could we change our ratio for the $1 and $2 coins so that we could work out the total mass of the coin from its ratio?
   We could have a ratio of copper to nickel to aluminium, so the ratio includes all the parts of the coin.

6. If copper is worth $6000/tonne, how much is the copper in each coin worth? Use the table above.
   See table above

7. Using your answers from Question 4, write a ratio for the value of each coin to the value of the copper in each coin. Use the table above.
   See table above

8. How much would copper need to cost, for the value of the copper in the 5-cent coin to be worth more than the coin itself?

   For this to happen, 2.12 grams of copper would need to cost ***more than*** 5 cents.

   Using 2.12 g/5c, copper would need to cost ***more than*** $23 584.91/tonne.

<u>Activity 3</u>

You have won first prize in a raffle. You can choose one of the four prizes listed below.

- A one-metre line of $2 coins. The coins are lying flat and touching sides.
- One square metre of five-cent pieces. The coins are lying flat, and are arranged in rows with their sides touching
- 1 kg of $1 coins
- A single stack of 50c coins that is 1 metre high.

1. With out doing any calculations, which prize would you pick? Why?
   Various answers

2. See if you can calculate the value of each of the prizes listed above. Show your working below.

For this activity, students can either go on-line and research the dimensions of each coin, or they can measure the coins directly themselves. The Royal Australian Mint and Wikipedia websites were both used to determine the diameters and masses of the coins.

$2-coin – diameter 20.5 mm
1 metre = 100cm = 1000 mm
1000 mm ÷ 20.5 mm = 48.7 coins
(Need to round down, because there isn't enough room to have the 49th coin in the line.)
<u>48 coins x $2 each = $96</u>

5-cent coin – diameter 19.41 mm
1 metre = 100 cm = 1000 mm
1000 mm ÷ 19.41 mm = 51.52 coins (Need to round down, for the same reason as above.)
51 x 51 coins = 2601 coins
<u>2601 coins x $0.05 = $130.05</u>

$1-coin – 9 grams
1 kg = 1000 grams
1000 grams ÷ 9 grams = 111.11 coins (Need to round down, for the same reason as above.)
<u>111 coins x $1 = $111</u>

Height of 50c coin – 2.80 mm
1 metre = 100 cm = 1000 mm
1000 mm ÷ 2.80 mm = 357.14 coins (we will round down, for the same reason as above)
<u>357 coins x $0.50 = $178.50</u>

3. Based on your calculations above, which prize was the best choice?

The best choice was the tower of 50c coins, as it gave you the most money.

Activity 1

You have just helped stop a bank robbery. The bank manager is so grateful for your help he has given you a large bag and told you that you can take as many silver coins as you can carry. Which coin should you pick in order to maximize your reward?

1. How much do you think you can carry? If you are not sure, use 20% of your body mass as a guide and write your estimate below.

2. You can fill your bag with either 5c, 10c, 20c or 50c coins. Which coin will you choose?

3. Go online and find the mass of your coin. Write the mass of the coin below.

4. Using your answers from Question 1 and Question 3, work out how many coins you could carry.

5. Using your answer from Question 4, what are your coins worth?

6. Compare your answer to others in the class. Who got the best reward? How much did they get?

7. DISCUSSION QUESTION: Did you pick the right coin, or should you have chosen something else? How can we decide which coin is the best one to pick?

8. Use the internet to help you fill in the table below:

| Coin | Mass (grams) | Value : Mass Ratio | Value : Mass Ratio ($1 : ?) |
|---|---|---|---|
| 5c | | | |
| 10c | | | |
| 20c | | | |
| 50c | | | |

9. Using the table from Question 8, did you pick the right coin? Justify your answer.

10. If someone could carry exactly 10 kg, how much would their reward be worth if they chose -
    a. 5-cent coins?
    b. 10-cent coins?
    c. 20-cent coins?
    d. 50-cent coins?

<u>Activity 2</u>

Below is the list of each of the Australian coins, their mass, and the mass of each of their metal components.

| Coin | Mass (g) | Mass of Copper (g) | Mass of Nickel (g) | Mass of Aluminium (g) |
|---|---|---|---|---|
| $0.05 | 2.83 | 2.12 | 0.71 | - |
| $0.10 | 5.65 | 4.25 | 1.41 | - |
| $0.20 | 11.33 | 8.50 | 2.83 | - |
| $0.50 | 15.55 | 11.66 | 3.89 | - |
| $1.00 | 9.00 | 8.28 | 0.54 | 0.18 |
| $2.00 | 6.60 | 6.07 | 0.40 | 0.13 |

1. Find the ratio of total mass to the mass of copper for each coin, and enter results in the table below.

| Coin | Question 2<br>Total Mass :<br>Copper Mass | Question 3<br>Copper Mass :<br>Nickel Mass | Question 7<br>Value of Copper<br>(to nearest cent) | Question 8<br>Value of Coin :<br>Value of Copper |
|---|---|---|---|---|
| $0.05 | | | | |
| $0.10 | | | | |
| $0.20 | | | | |
| $0.50 | | | | |
| $1.00 | | | | |
| $2.00 | | | | |

2. Find the ratio of mass of copper to the mass of nickel in each coin, and enter in the table above.

3. If we only had your answers to Question 2 (and not the table given above) could we work out the total mass of each coin? How?

4. Would this method work for the $1 and $2 coins as well? Why/why not?

5. How could we change our ratio for the $1 and $2 coins so that we could work out the total mass of the coin from its ratio?

6. If copper is worth $6000/tonne, how much is the copper in each coin worth? Use the table above.

7. Using your answers from Question 4, write a ratio for the value of each coin to the value of the copper in each coin. Use the table above.

8. How much would copper need to cost, for the value of the copper in the 5-cent coin to be worth more than the coin itself?

Activity 3

You have won first prize in a raffle. You can choose one of the four prizes listed below.

- A one-metre line of $2 coins. The coins are lying flat and touching sides.
- One square metre of five-cent pieces. The coins are lying flat, and are arranged in rows with their sides touching
- 1 kg of $1 coins
- A single stack of 50c coins that is 1 metre high.

1. With out doing any calculations, which prize would you pick? Why?

2. See if you can calculate the value of each of the prizes listed above. Show your working below.

3. Based on your calculations above, which prize was the best choice?

Department of Education

The Mathematical Association of Western Australia Inc.

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Value for Money

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 1: VALUE FOR MONEY

**Overview**

In this task, students will investigate pricing on goods in supermarkets in terms of mass, total cost and cost per 100 grams. One of the activities provides an opportunity for students to see how pricing is made available online for shoppers. It may be necessary to advise students to narrow their searches for appropriate items.

**Students will need**

- calculators
- access to the internet

**Relevant content descriptors from the Western Australian Curriculum**

- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

**Students can demonstrate**

- *fluency* when they
    - calculate cost per 100 g when given mass and total cost as in Activity 4.
    - round monetary amounts to the nearest cent
- *understanding* when they
    - can independently reverse the operations as in Activities 4 and 5; e.g., to determine total cost when given mass and cost per 100 g
- *reasoning* when they
    - explain the difference in the two lists in Activity 1
- *problem solving* when they
    - can determine the correct price for the product as in Activity 4

Activity 1

The table shows the prices and masses of 7 packets of soap. The packets contain different numbers of bars of soap, for example there is 1 bar in packet G and there are 6 bars in packets L and X.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| O | $3.74 | 450 g | $0.83 |
| G | $2.19 | 100 g | $2.19 |
| S | $4.18 | 200 g | $2.09 |
| L | $4.62 | 600 g | $0.77 |
| C | $2.86 | 450 g | $0.64 |
| X | $3.69 | 600 g | $0.62 |
| F | $1.90 | 400 g | $0.48 |

1. Order the seven packets from cheapest packet to the most expensive packet.

   F G C X O S L

2. Order the seven packets from the lightest packet to the heaviest packet.

   G S F O C L X or G S F O C X L or G S F C O L X

   or G S F C O X L

3. Compare these two lists. Is it true to say that "The heavier the packet is, the more expensive it will be"? Give two reasons for your decision.

   The statement is not true.

   L and X have the same mass, but are different prices.

   S is the second lowest in mass but the second highest in price.

4. Considering the cost per 100 g -

   (a) Which packet is the most expensive? G

   (b) Which packet is the cheapest? F

   (c) How can you use the price and the mass to determine this cost per 100 g?

   One way is to divide the price by the number of 100 g.

   Another way is to divide the price by the mass (this gives the cost per gram) and then multiply this answer by 100.

5. Use your answer from 4(c) to check the values in the last column.

6. Sometimes the cheapest soap (least cost per 100 g) is not the best soap for a shopper to buy. Give two reasons to support this statement.

   It might be more soap than the shopper needs.
   It could be the wrong type of soap (industrial).
   They might be allergic to that type of soap.

Activity 2

Use the internet to search for similar data on another product attractive to shoppers.
Possible items to consider: dry dog food, fish food for cats, coffee beans, pizzas
Create a table similar to the one in Activity 1.
Determine the best value for money and suggest circumstances in which it might not be the best option for the shopper who is in need of that product.
Various results

Activity 3

The table shows the prices and masses of 7 packets of wild bird seed.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | $5.93 | 750 g | $0.79 |
| 2 | $5.50 | 2 kg | $0.28 |
| 3 | $2.96 | 300 g | $0.99 |
| 4 | $5.50 | 2 kg | $0.28 |
| 5 | $16.78 | 8 kg | $0.21 |
| 6 | $9.79 | 5 kg | $0.20 |
| 7 | $9.74 | 500 g | $1.95 |

Assuming all packets of bird seed would be suitable for the customer, determine which packet represents the best value for money. Justify your decision.

Packet 6 represents the best value for money because it costs the least amount for 100 g.

Activity 4

A company which supplies meat pies to a local supermarket wants to ensure that its product is the cheapest of all brands sold in that supermarket. At the same time the product has to be as profitable as possible. The company has collected the data below.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | $7.69 | 680 g | $1.13 |
| 2 | $5.78 | 440 g | $1.31 |
| 3 | $11.43 | 480 g | $2.38 |
| 4 | $7.69 | 900 g | $0.85 |
| 5 | $8.65 | 800 g | $1.08 |
| 6 | $4.40 | 200 g | $2.2 |
| 7 | $0.83 | 150 g | $0.55 |

1. Determine the maximum cost per 100 g for the company's meat pies so that the price per 100 g is lower than all the other pies in the list.

   Minimum cost in table is $0.5533333 so, to let the customers see it is cheaper, the price would need to be set at $0.54 (could also be $0.5444444).

2. Use the maximum cost per 100 g determined in the previous questions to calculate the supermarket price for an 880 g packet of the company's pies.

   $0.5444444 x 880 = $4.79

Activity 5

A manufacturer of dairy products supplies the same type of yoghurt in 5 different sizes. She wants to price the items so *that the more the purchaser buys, the better the value for money*. She nominates the cost per 100 g for each size of product.

| Size | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | $1.28 | 80 g | $1.60 |
| 2 | $1.40 | 100 g | $1.40 |
| 3 | $1.80 | 200 g | $0.90 |
| 4 | $2.55 | 300 g | $0.85 |
| 5 | $4.00 | 500 g | $0.80 |
| 6 | $7.50 | 1 kg | $0.75 |

1. Determine the price of each of the different sizes and record your results in the table.

2. Describe your method for determining the price when given the mass and the cost per 100 g.

   Multiply the cost per 100 g by the number of 100 g lots in the mass.

   You could also divide the cost per 100 g by 100 (to get cost per gram) and then multiply this answer by the mass in grams.

3. Use your method from Question 2 to confirm that the prices in the table for Activity 1 are correct.

4. The prices of the two smallest sizes are much higher than for the other products. What other features of these items might be contributing to these higher costs?

   Packaging and handling might be more expensive for the smaller containers.

   These might just happen to be soaps which contain more expensive perfumes or other ingredients.

Activity 6: Further investigation

1. Are all goods marked as price per 100 g? What about liquid products?

   Some goods are exempt, e.g., flowers, books, toys, garden tools.
   Liquid products: Unit pricing is per 100 millilitres or per litre.

2. When was pricing per 100 g introduced in Australia?

   Unit pricing was introduced in Australia in 2009.

3. What are the benefits of having prices marked per 100 g?

   This allows the customer to compare brands and sizes, to select value for money and to save money.

4. Would it be better to mark the goods with prices per kilogram rather than per 100 grams? Explain your answer.

   This could be good for large items as the amounts might look reasonable for people who struggle to understand money and decimals.

   For some items; e.g., chocolate, the comparison might cause some people to change their shopping habits. Easter eggs might be about $40-$50 per kilogram but much larger and heavier blocks of chocolate could be  only $15-$20 per kilogram.

For further information see Australian Competition and Consumer Commission.

<u>Activity 1</u>

The table below shows the prices and masses of 7 packets of soap. The packets contain different numbers of bars of soap. For example, there is one bar in packet G and there are 6 bars in packets L and X.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| O | $3.74 | 450 g | $0.83 |
| G | $2.19 | 100 g | $2.19 |
| S | $4.18 | 200 g | $2.09 |
| L | $4.62 | 600 g | $0.77 |
| C | $2.86 | 450 g | $0.64 |
| X | $3.69 | 600 g | $0.62 |
| F | $1.90 | 400 g | $0.48 |

1. Order the seven packets from the cheapest packet to the most expensive packet.

2. Order the seven packets from the lightest packet to the heaviest packet.

3. Compare these two lists. Is it true to say that “The heavier the packet is, the more expensive it will be”? Give two reasons for your decision.

4. Considering the cost per 100 g

   (a) Which packet is the most expensive?

   (b) Which packet is the cheapest?

   (c) How can you use the price and the mass to determine this cost per 100 g?

5. Use your answer from 4(c) to check the values in the last column.

6. Sometimes the cheapest soap (least cost per 100 g) is not the best soap for a shopper to buy. Give two reasons to support this statement.

<u>Activity 2</u>

Use the internet to search for similar data on another product attractive to shoppers.
Possible items to consider: dry dog food, fish food for cats, coffee beans, pizzas
Create a table similar to the one in Activity 1.
Determine the best value for money and suggest circumstances in which it might not be the best option for the shopper who is in need of that product.

Activity 3

The table below shows the prices and masses of 7 packets of bird-seed.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | $5.93 | 750 g | |
| 2 | $5.50 | 2 kg | |
| 3 | $2.96 | 300 g | |
| 4 | $5.50 | 2 kg | |
| 5 | $16.78 | 8 kg | |
| 6 | $9.79 | 5 kg | |
| 7 | $9.74 | 500 g | |

Assuming all packets of bird seed would be suitable for the customer, determine which packet represents the best value for money. Justify your decision.

Activity 4

A company which supplies meat pies to a local supermarket wants to ensure that its product is the cheapest of all brands sold in that supermarket. At the same time the product has to be as profitable as possible. The company has collected the data below.

| Packet | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | $7.69 | 680 g | |
| 2 | $5.78 | 440 g | |
| 3 | $11.43 | 480 g | |
| 4 | $7.69 | 900 g | |
| 5 | $8.65 | 800 g | |
| 6 | $4.40 | 200 g | |
| 7 | $0.83 | 150 g | |

1. Determine the maximum cost per 100 g for the company's meat pies so that the price per 100 g is lower than all the other pies in the list.

2. Use the maximum cost per 100 g determined in the previous questions to calculate the supermarket price for an 880 g packet of the company's pies.

Activity 5

A manufacturer of dairy products supplies the same type of yoghurt in 5 different sizes. She wants to price the items so *that the more the purchaser buys, the better the value for money*. She nominates the cost per 100 g for each size of product.

| Size | Price | Mass | Cost per 100 g |
|---|---|---|---|
| 1 | | 80 g | \$1.60 |
| 2 | | 100 g | \$1.40 |
| 3 | | 200 g | \$0.90 |
| 4 | | 300 g | \$0.85 |
| 5 | | 500 g | \$0.80 |
| 6 | | 1 kg | \$0.75 |

1. Determine the price of each of the different sizes and record your results in the table.

2. Describe your method for determining the price when given the mass and the cost per 100 g.

3. Use your method from Question 2 to confirm that the prices in the table for Activity 1 are correct.

4. The prices of the two smallest sizes are much higher than for the other products. What other features of these items might be contributing to these higher costs?

Activity 6: Further investigation

1. Are all goods marked as price per 100 g? What about liquid products?

2. When was pricing per 100 g introduced in Australia?

3. What are the benefits of having prices marked per 100 g?

4. Would it be better to mark the goods with prices per kilogram rather than per 100 grams? Explain your answer.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Just Add Zero

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 5: JUST ADD ZERO

### Overview

The purpose of this task is twofold. Firstly it is to challenge students to think carefully about how mathematics is expressed, and secondly it provides an opportunity for students to review multiplication and division of decimals by powers of 10. Students will probably have been taught this skill in Year 6 but this task promotes relevant revision.

### Students will need

- calculators

### Relevant content descriptions from the Western Australian Curriculum

- Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)
- Multiply and divide decimals by powers of 10 (ACMNA130)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

### Students can demonstrate

- *fluency* when they
  - accurately multiply decimals by powers of 10
- *understanding* when they
  - describe patterns with respect to the position of the decimal point
  - relate the number of zeros in the product with the number in the multiplicand
- *reasoning* when they
  - give reasons for mathematical decisions as in Activity 2, Question 8

Introduction

*Just Add Zero* is often heard in mathematics classes but when is it used and what does it mean?

Activity 1

1. Complete the table by adding zero to each of the numbers in the top row.

| Number | 4 | 19 | 401 | 56 | 7092 | 656 |
|---|---|---|---|---|---|---|
| Number + 0 | 4 | 19 | 401 | 56 | 7092 | 656 |

2. What do you get when you add 0 to a number? Explain.

   You get the number itself. You are adding nothing to it.

3. How much greater is a number when you add zero to it?

   It remains the same.

4. Complete the table by multiplying each of the numbers in the top row by 10.
   Use your calculator if necessary to check your answers.

| Number | 4 | 19 | 401 | 56 | 7092 | 656 |
|---|---|---|---|---|---|---|
| Number x 10 | 40 | 190 | 4010 | 560 | 70 920 | 6560 |

5. How many times greater is a number when you multiply the number by 10?

   10 times greater

6. Look at the numbers on the second row. How do they different from the starting numbers?

   They are larger. Compared to the numbers on the top row, there is a zero at the end.

7. Why do you think people say “just add zero” when multiplying by 10?

   They are taught that. It is a quick way of describing the answer.

8. In the true sense of the meaning is *multiplying by 10* the same as *adding zero*?

   No

The following questions can be used to reinforce the idea that *multiplying by 10* is not the same as *adding zero*

9. Complete the table by multiplying each of the numbers in the top row by 10.
   Use your calculator if necessary to check your answers.

| Number | 6.7 | 8.03 | 0.02 | 0.184 | 24.3 | 101.101 |
|---|---|---|---|---|---|---|
| Number x 10 | 67 | 80.3 | 0.2 | 1.84 | 243 | 1011.01 |

10. For these numbers is *multiplying by 10* the same as *adding zero*?
    No

11. Look for a pattern in the answers in the table when you multiply a number with decimals by 10. Describe this pattern.

    The decimal point is now on the right of the digit when it was previously on the left of that digit.
    The decimal point has moved one place to the right.
    When the starting number had one digit after the decimal point, the decimal point has "disappeared". It is assumed to be at the end of a whole number and is not written in.

12. Use your patterns from the previous questions to multiply the following numbers by 10.
    Check your answers with a calculator if necessary.

(a) 501 5010 (b) 34.5 345 (c) 0.98 9.8

(d) 9.01 90.1 (e) 0.0002 0.002 (f) 98 980

(g) 1087 10 870 (h) 1.4378 14.378 (i) 10.07 100.7

<u>Activity 2</u>

1. Is multiplying a number by 100 the same as adding 2 zeros?
   No

2. When multiplying a whole number by 100, can the answer be determined by writing 2 zeros on the end of the starting number?
   Yes

3. When multiplying any decimal number by 100, can the answer be determined by writing 2 zeros on the end of the starting number?
   No

4. When multiplying a whole number by 1000, can the answer be determined by writing 3 zeros on the end of the starting number?
   Yes

5. When multiplying any decimal number by 1000, can the answer be determined by writing 3 zeros on the end of the starting number?
   No

6. Complete the following table. Check that your answers are correct

| Number | 3.2 | 5.08 | 0.06 | 2.438 | 192.921 | 1.6 |
|---|---|---|---|---|---|---|
| Number x 100 | 320 | 508 | 6 | 243.8 | 19 292.1 | 160 |

7. Examine the pattern in the answers in the table. Consider the position of the decimal point. How can you quickly multiply a decimal number by 100?

   Move the decimal point two places to the right
   If there are not two places, add zeros until there are two places.

   Note: If the number is a whole number and the decimal is "hidden" at the end of the number then writing two zeros at the end gives the same result as moving the decimal two places to the right.

8. Do you think it is a good idea to say "When multiplying by 100 just add two zeros? Give a reason for your answer.
   No – because it does not work for all numbers. Also, it is confusing for people who do not realise that it is not addition. It is just writing two zeros at the end.

<u>Activity 3</u>

1. Complete the following table where the numbers in the top row are multiplied by 1000. Check your answers.

| Number | 7.3 | 2.09 | 0.17 | 0.008 | 3.21 | 83.456 |
|---|---|---|---|---|---|---|
| Number x 1000 | 7300 | 2090 | 170 | 8 | 3210 | 83 456 |

2. Examine the pattern in the answers in the table. Consider the position of the decimal point. How can you quickly multiply a decimal number by 1000?

   Move the decimal point three places to the right.
   If there are not three places, add zeros until there are three places.

3. Describe a quick way to multiply a number by -

   (a) 10 000

   Move the decimal point four places to the right.
   If there are not four places, add zeros until there are four places.

   (b) 1 000 000

   Move the decimal point six places to the right.
   If there are not six places, add zeros until there are six places.

4. In the following table each starting number has been multiplied by a power of 10 (i.e.,10, 100, 1000, 10 000 etc.). Complete the table to show what number was used to multiply the starting number.

| Starting number | 3 | 21 | 45 | 16 | 9 |
|---|---|---|---|---|---|
| Multiplied by | 100 | 10 | 10 000 | 1000 | 100 000 |
| Answer | 300 | 210 | 450 000 | 16 000 | 900 000 |
| | | | | | |
| Starting number | 6.6 | 0.42 | 0.03 | 0.7604 | 23.514 |
| Multiplied by | 10 | 1000 | 10 | 100 | 100 |
| Answer | 66 | 420 | 0.3 | 76.04 | 2351.4 |

Activity 4

1. When multiplying a fraction by 10, can you get the answer by writing 0 on the end of the digits?

Consider the following fraction: $\frac{3}{10}$

Are any of the numbers listed below equal to $\frac{3}{10}$ x 10? Circle the ones which are.

$\frac{3}{10}0$ $\frac{30}{10}$ $\frac{3}{100}$ $\frac{30}{100}$

2. For each of the “answers”, describe where the zero has been written.

$\frac{3}{10}0$ on the right of the fraction

$\frac{30}{10}$ on the right of the numerator

$\frac{3}{100}$ on the right of the denominator

$\frac{30}{100}$ on the right of both the denominator and the numerator

3. Of the four possibilities for writing the zero, use the one that works on four other fractions. Record your results below. Examples shown.

| | | | | |
|---|---|---|---|---|
| Fraction | $\frac{1}{10}$ | $\frac{2}{3}$ | $\frac{3}{5}$ | $\frac{7}{10}$ |
| Fraction x 10 | $\frac{10}{10}$ | $\frac{20}{3}$ | $\frac{30}{5}$ | $\frac{70}{10}$ |

4. Did your method work on all four fractions?

Yes

5. Describe how you know that your answer to multiplying the fraction by 10 was correct?

- If you have 10 lots of one tenth, then you have ten tenths.
- You could make a model; e.g., $\frac{2}{3}$, then get 10 of them and you would have twenty thirds.
- The denominator or name of the fraction tells how many parts one whole is divided into, while the numerator tells you how many of these parts you have. Multiplying this number of parts by 10 is the same as multiplying the numerator.

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4. Complete the table by multiplying each of the numbers in the top row by 10.
   Use your calculator if necessary to check your answers.

| Number | 4 | 19 | 401 | 56 | 7092 | 656 |
|---|---|---|---|---|---|---|
| Number x 10 | | | | | | |

5. How many times greater is a number when you multiply the number by 10?

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    Check your answers with a calculator if necessary.

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(d) 9.01 (e) 0.0002 (f) 98

(g) 1087 (h) 1.4378 (i) 10.07

Activity 2

1. Is multiplying a number by 100 the same as adding 2 zeros?

2. When multiplying a whole number by 100, can the answer be determined by writing 2 zeros on the end of the starting number?

3. When multiplying any decimal number by 100, can the answer be determined by writing 2 zeros on the end of the starting number?

4. When multiplying a whole number by 1000, can the answer be determined by writing 3 zeros on the end of the starting number?

5. When multiplying any decimal number by 1000, can the answer be determined by writing 3 zeros on the end of the starting number?

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|---|---|---|---|---|---|---|
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2. For each of the “answers”, describe where the zero has been written.

$\frac{3}{10}0$

$\frac{30}{10}$

$\frac{3}{100}$

$\frac{30}{100}$

3. Of the four possibilities for writing the zero, use the one that you found to work in the previous question, on four other fractions. Record your results below.

| | | | | |
|---|---|---|---|---|
| Fraction | | | | |
| Fraction x 10 | | | | |

4. Did your method work on all four fractions?

5. Describe how you know that your answer to multiplying the fraction by 10 was correct?