
2

Integers

2

Texting aliens.

Mathematics is said to be the language that we could use to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same. It is believed that the best way to send a message would be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system. In 1974, the Arecibo telescope in Puerto Rico (pictured here) broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; this is not surprising given the distance it will have to travel. Later in this chapter you can learn about another way prime numbers are used to send information.

Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10; however, sometimes we count by 2, 7, 60, 360 and 365. What do we count using these numbers?

Why learn this?

Understanding relationships between numbers allows us to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for our study of many other areas of mathematics. Negative numbers are an important set of numbers that we will also consider in this chapter. Temperatures, elevations, goal differences and money owed are a few examples of the uses of negative numbers.

After completing this chapter you will be able to:

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative integers to represent quantities
- compare and order integers
- add and subtract integers.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R2.1

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

PP Worksheet R2.2

2 **(a)** List all the digits with which an even number can end.

(b) List all the digits with which an odd number can end.

PP Worksheet R2.3

3 Copy and complete each of the following by writing a $<$ (less than) or $>$ (greater than) sign between the given values.

(a) 10 ______ 7 **(b)** 3 ______ 6 **(c)** 2 ______ 0 **(d)** 0 ______ 5

PP Worksheet R2.4

4 Calculate:

(a) $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

PP Worksheet R2.5

5 Write the following temperatures in order from coldest to warmest.

(a) 15°C, 7°C, 0°C, -4°C, 21°C, -11°C

(b) 5°C, -3°C, 10°C, -25°C, 32°C, -14°C

PP Worksheet R2.6

6 Write the following in expanded form, then evaluate.

(a) 7^2 **(b)** 3^4 **(c)** 2^6 **(d)** 1^9

PP Worksheet R2.7

7 Calculate the following.

(a) $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

Key Words

common factor	factor	positive
common multiple	Highest Common Factor (HCF)	prime factor
composite number	integers	prime number
coprime	loss	profit
deposit	Lowest Common Multiple (LCM)	withdrawal
divisibility	multiple	
divisible	negative	

2.1 Multiples, factors and divisibility

Multiples and factors

The numbers 1, 2, 3, 4, 5, ... are called the whole numbers, or the counting numbers. (Any time we use '...' in mathematics, we are saying the pattern is infinite, or goes on forever.)

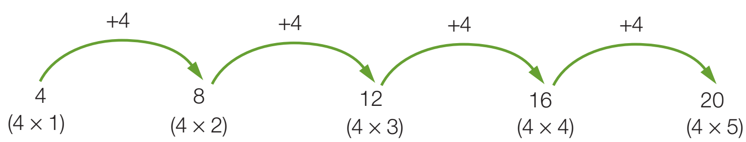
We find the **multiples** of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

	1×7	2×7	3×7	4×7	5×7	...
Multiples of 7	7	14	21	28	35	...

Another way to create a list of multiples of a number is to start at the number and add it repeatedly.

For example, the multiples of 4 are:



The first in the sequence of multiples of a number is always the number itself. We can see from the above table and sequence that the first multiple of 7 is 7 (1×7), and the first multiple of 4 is 4 (1×4).

A **factor** is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, we can see some factors:

	7	14	21	28	35	...
Some factors	1, 7	2, 7	3, 7	4, 7	5,7	...

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a number has. We can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$
and $28 = 2 \times 14$
and $28 = 1 \times 28$

So, 28 has a total of six factors: 1, 2, 4, 7, 14 and 28.

Worked Example 1

WE1

Find all the factors of each of the following numbers.

(a) 12 **(b)** 110

Thinking	Working
(a) 1 Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(a) $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
2 List the factors from smallest to largest.	Factors of 12: 1, 2, 3, 4, 6, 12.
(b) 1 Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(b) $1 \times 110 = 110$ $2 \times 55 = 110$ $5 \times 22 = 110$ $10 \times 11 = 110$
2 List the factors from smallest to largest.	Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. We include 7 only once in the list of factors for 49. If we reach such a pair, this also tells us we have finished finding the pairs of numbers.

Divisibility

Another way of considering factors and multiples is to talk about **divisibility**. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all refer to the same idea.

Two factors of 35 are 5 and 7.
35 is divisible by 5 and 7.
Both 5 and 7 go into 35 exactly, without any remainder.
5 multiplied by 7 gives 35.
35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help us determine which numbers are divisible by others. For larger numbers, we can use some tests that enable us to determine whether one number is divisible by another. These tests are summarised in the following table.

A number is divisible by ...	If it passes this divisibility test
2	The last digit is an even number (0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked Example 2

WE2

Determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Thinking	Working
(a) 1 Add up the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3.	(a) $75: 7 + 5 = 12$ ✓ $98: 9 + 8 = 17$ ✗ $110: 1 + 1 + 0 = 2$ ✗ $132: 1 + 3 + 2 = 6$ ✓
2 State the answer for each number considered.	75 and 132 are divisible by 3. 98 and 110 are not divisible by 3.
(b) 1 Look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4.	(b) 75 ✗ 98 ✗ 110 ✗ 132 ✓
2 State the answer for each number considered.	132 is divisible by 4. 75, 98 and 110 are not divisible by 4.
(c) 1 Is the last digit 5 or 0?	(c) 75 ✓ 98 ✗ 110 ✓ 132 ✗
2 State the answer for each number considered.	75 and 110 are divisible by 5. 98 and 132 are not divisible by 5.
(d) 1 Write down the even numbers (these are divisible by 2). Add up the digits in each of these numbers and see whether the number is divisible by 3.	(d) Using the working from (a): 98: 17 ✗ 110: 2 ✗ 132: 6 ✓
2 State the answer for each number considered.	132 is divisible by 6. 75, 98 and 110 are not divisible by 6.

Multiples of a whole number are found by multiplying it by another whole number.

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

Common multiples

A **common multiple** of two numbers is a number that both of them divide into exactly. Changing the multiple table from the start of the section slightly, we get:

	1 and 7	2 and 7	3 and 7	4 and 7	5 and 7	…
A common multiple	7	14	21	28	35	…

This table only gives one common multiple for each pair of numbers. There is an infinite number of others. The **Lowest Common Multiple (LCM)** of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56,… The LCM of 2 and 7 is 14. There is no highest common multiple.

Worked Example 3

WE3

Find the lowest common multiple (LCM) of the following set of numbers, by first listing the multiples of each: 4 and 6.

Thinking	Working
1 List the first few multiples of the first number.	4: 4, 8, 12, 16, 20, 24, …
2 List the first few multiples of the second number.	6: 6, 12, 18, 24, 30, 36, …
3 Circle the first number that appears in both lists. This is the LCM.	LCM of 4 and 6 is 12.

Common factors

A **common factor** of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

	7 and 14	4 and 20	9 and 15	8 and 40	12 and 18
Common factors	1, 7	1, 2, 4	1, 3	1, 2, 4, 8	1, 2, 3, 6

1 will always be a common factor of any set of numbers. Sometimes it's important for us to find the **Highest Common Factor (HCF)** of two numbers. From the above table, we can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, the smaller number is the HCF. For example, the HCF of 4 and 20 is 4 and the HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

Worked Example 4

WE4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Thinking	Working
1 List all factors of the first number.	12: 1, 2, 3, 4, 6, 12
List all factors of the second number.	18: 1, 2, 3, 6, 9, 18
2 Circle the factors appearing in both lists. These are the common factors.	
3 Select the largest number that appears in both lists. This is the HCF.	HCF of 12 and 18 is 6.

The lowest common multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The highest common factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is also known as the Greatest Common Divisor (GCD).

2.1 Multiples, factors and divisibility

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q4 Columns 1–3, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15, Q18, Q23	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2–4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20(a), Q23, Q24	Q1 Columns 3 & 4, Q2, Q3 Column 3, Q4 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q24, Q25

Answers page 627

Fluency

1 Find all the factors of each of the following numbers. WE1

(a) 18 **(b)** 16 **(c)** 23 **(d)** 24

(e) 20 **(f)** 35 **(g)** 36 **(h)** 42

(i) 53 **(j)** 60 **(k)** 77 **(l)** 84

2 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. WE2

(a) 3 **(b)** 4 **(c)** 5 **(d)** 8 **(e)** 9

WE3

3 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5	**(b)** 3 and 9	**(c)** 5 and 25
(d) 5 and 6	**(e)** 4 and 7	**(f)** 8 and 12
(g) 7 and 9	**(h)** 10 and 12	**(i)** 6 and 11
(j) 9 and 12	**(k)** 20 and 50	**(l)** 8 and 14
(m) 3, 4 and 5	**(n)** 2, 25 and 50	**(o)** 20, 50 and 60

WE4

4 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15	**(b)** 8 and 24	**(c)** 5 and 12	**(d)** 26 and 36
(e) 11 and 33	**(f)** 28 and 70	**(g)** 44 and 22	**(h)** 10 and 30
(i) 40 and 70	**(j)** 32 and 60	**(k)** 35 and 70	**(l)** 42 and 48

A factor of a number can't be larger than the number itself.

5 **(a)** The lowest common multiple of 8 and 1 is:

A 8 **B** 16 **C** 24 **D** 80

(b) Which of the following is a factor of 34?

A 4 **B** 12 **C** 17 **D** 68

6 **(a)** A number divisible by 2, 3 and 5 is:

A 6 **B** 15 **C** 60 **D** 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 **B** 38 and 52 **C** 38 and 60 **D** 52 and 60

Understanding

7 **(a)** Which one of the following numbers is not a multiple of 8?

A 4 **B** 24 **C** 72 **D** 88

(b) Which of the following is not a factor of 42?

A 1 **B** 6 **C** 21 **D** 84

8 State TRUE or FALSE for the following.

(a) 346 is a multiple of 3.
(b) 872 is divisible by 6.
(c) 2 is a factor of 348.
(d) 52 is a multiple of 4.
(e) 854 is divisible by 9.
(f) 3 is a factor of 56 902.

9 For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

(a) 4, 6 and 10
(b) 6, 8 and 12
(c) 8, 12 and 16
(d) 10, 25 and 40

10 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is ____________ by 8.
(b) 6 is a ____________ of 54, so 54 is a multiple of 6.
(c) 72 is divisible by 9, so that makes it a ____________ of 9.
(d) 4 is a factor of 60, so 60 is ____________ by 4.

11 (a) If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

12 Mrs Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Mrs Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

13 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

14 The smallest number divisible by 3, 4 and 5 is:

A 12 B 24 C 30 D 60

15 If two events occur at different time intervals, the lowest common multiple (LCM) of the two time intervals is the point when the two events coincide, or occur together. Use this information to answer the following question.

In a city lighting display, one set of lights flashes every 25 seconds and the other set flashes every minute. If they are turned on at the same time, write down the next three times when the two sets of lights flash together.

16 (a) Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning

17 Peter power-walked around an oval while Mei-ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei-ling jogged 24 laps in the hour.

(a) How many times did Mei-ling pass Peter?

(b) How many times did Mei-ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei-ling pass Peter exactly on the spot where they started?

18 **(a)** Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

100 000	2	3	4	5	6	8	9	10
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4 it will also be divisible by ______.

(ii) If a number is divisible by 9 it will also be divisible by ______.

(c) Explain your answers to **(b)**.

19 The test to determine whether a number is divisible by 6 is to test whether it is divisible by 2 *and* 3. Explain why the test works.

20 A *perfect* number is a number for which the sum of its factors (excluding itself) equals the number. The first perfect number is 6, as $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

21 An *abundant* number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next 2 abundant numbers. (Both are less than 40.)

22 **(a)** How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

Open-ended

23 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Draw three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

24 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

25 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites

2.2

A number that has exactly two factors, itself and 1, is a **prime number**.

A number that has more than two factors is called a **composite number**.

The number 1 is a special number. It is neither prime nor composite.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **coprime** if their highest common factor is 1.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BCE to 195 BCE. He was the first person to calculate a value for the circumference of the Earth. Another thing he was famous for was his 'sieve'.

See if you can reproduce what he did.

Copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating until there is no 'next number'.

Step 6 Write the factors of each of the circled numbers. What types of numbers are these?

Step 7 Write the factors of any five of the crossed out numbers, except for 1.

Step 8 Which type of number—circled or crossed out—has more factors? Explain why.

2.2 Primes and composites

Navigator

Answers page 628

Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14, Q17	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18	Q1, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

Fluency

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the primes between 20 and 60.

4 **(a)** The first prime number after 50 is:

A 51 **B** 53 **C** 55 **D** 57

(b) A number coprime with 18 is:

A 9 **B** 21 **C** 24 **D** 25

Understanding

5 **(a)** What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write TRUE or FALSE for each of the following statements.

(a) 21 is prime. **(b)** 38 is composite.

(c) 59 is prime. **(d)** 49 is prime.

(e) 5 and 7 are coprime. **(f)** 5 and 6 are coprime.

(g) All even numbers greater than 2 are composite.

(h) All odd numbers are primes.

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 **(b)** 621 **(c)** 9909

(d) 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

8 Are the following pairs of numbers coprime? Give reasons for your answer.

(a) 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27, 63

Reasoning

9 Explain why any pair of prime numbers is coprime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 **(a)** Find the numbers closest to 100 that are coprime with 100.

(b) Find the numbers closest to 36 that are coprime with 36.

15 Will a prime number always be coprime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be coprime.

Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

Outside the Square Puzzle

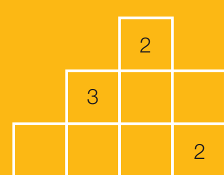
Gold digger 1

It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. She has marked on two separate maps for you the places next to where the gold lies. If a number 3 is in a box, then it means that there are 3 pieces of gold in adjacent squares, either horizontally, vertically or diagonally. (Adjacent squares share an edge or a corner.) None of the already numbered squares contains a piece of gold, and no square contains more than one piece of gold.

Your task is to find exactly where the gold lies, so your team can get to it first and win the competition. There is only one possible solution for each map.

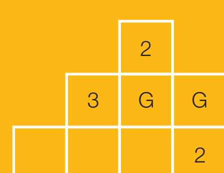
Basic techniques

Look for an easy opening:

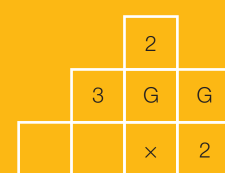


The top square (numbered 2) is only touching 2 other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.

Go back to other squares:



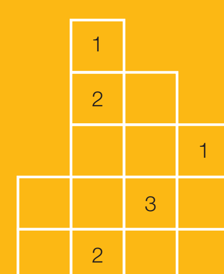
Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it is touching is empty. Mark this square with a cross.



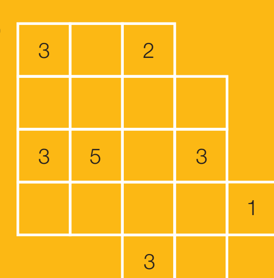
Now, look at the 3, and you can see that it is already next to one piece of gold, and is only touching two other empty squares, so both of these must also contain gold.

Now, copy the following maps and find the gold.

(a)



(b)



Mathspace

FACTOR-PILLAR

FACTORPILLAR

Equipment required: 1 counter per player, 1 die, 2–4 brains, calculator (optional)

How to play:

- Roll to decide who goes first.
- Players take turns to roll the die and move forward the number of spaces shown.
- The aim is to move from start to finish and get as many points as possible by finding factors of numbers.

When a player lands on a number, they must list all the factors of that number. The number of factors is their score. For example, a player lands on 6: factors are 1, 2, 3 and 6; score = 4.

But beware! If another player notices you have missed one or more factors they gain five points. If they suggest a factor that isn't correct they lose five points.

LARGE CIRCLE: If a player correctly finds all the factors of the number in a large circle, they earn double the points for that number.

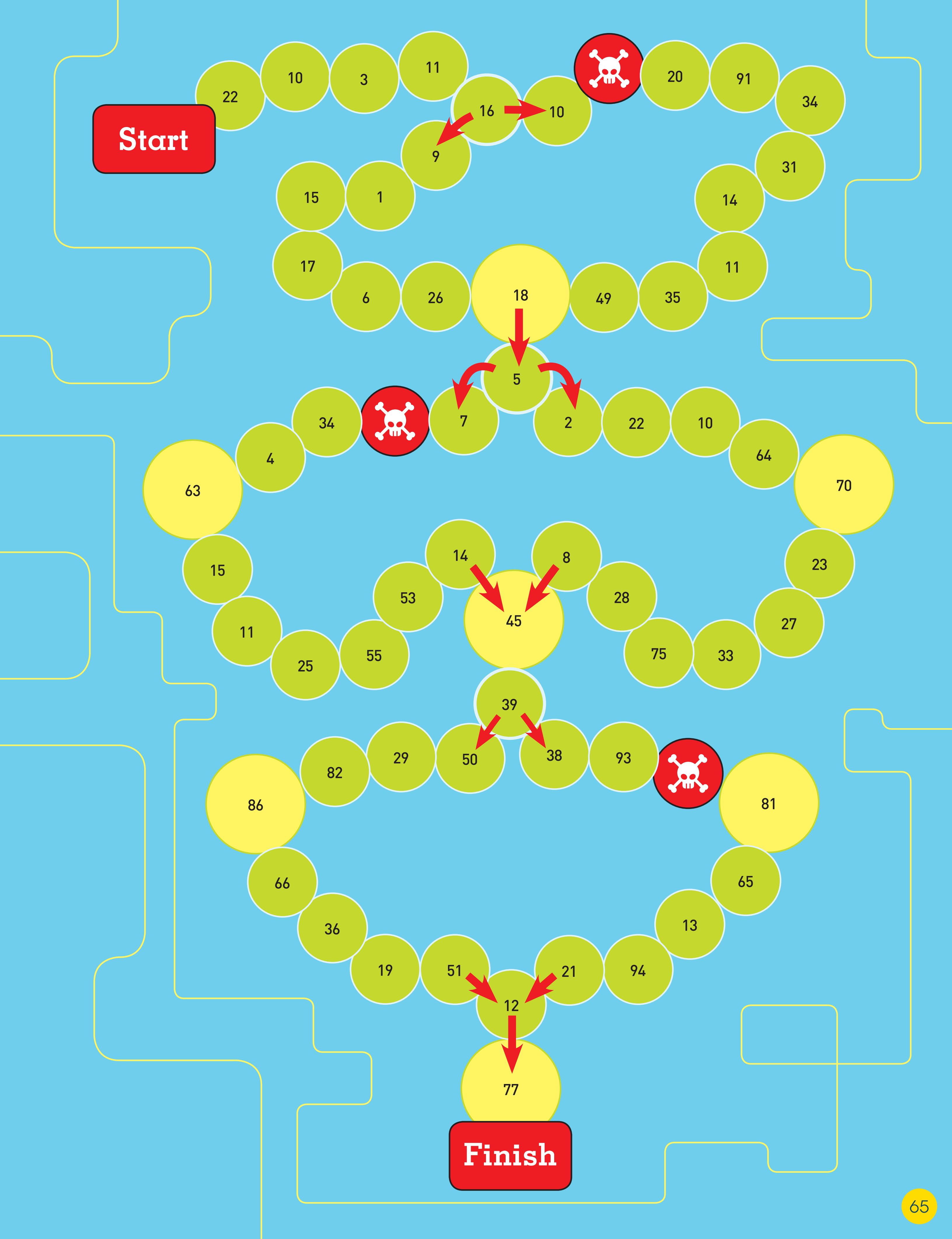
CROSSBONES: If a player lands on a circle containing a skull and crossbones, they miss a turn.

SQUARE NUMBER: If a player lands on a square number and calls 'Square Power' within two seconds, they have an extra turn.

PRIME NUMBER: If a player lands on a prime number and calls 'Prime Power' within two seconds, they can double their points for that number; if they don't call 'Prime Power' they only get two points for the two factors.

Players may not move backwards. When a player reaches a circle with two arrows leading from it they can decide which path to take.

The game ends when one player reaches or crosses the finish line. The player with the highest number of points at that moment is declared the winner (even if they haven't reached the finish line).


Start
22
10
3
11
16
10
20
91
34
31
14
11
35
49
9
1
15
17
6
26
18
5
7
34
4
63
15
11
25
55
53
14
45
8
28
75
33
27
23
70
64
10
22
2
39
50
29
82
86
66
36
19
51
12
21
94
13
65
81
93
38
77
Finish

2.3 Prime factors

A **prime factor** of a number is a factor that is also a prime number.

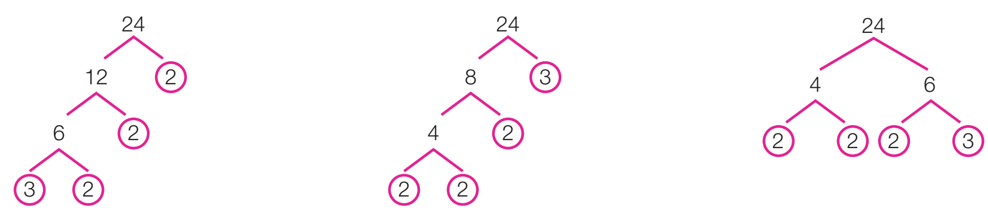
Every whole number can be written as the unique product of its prime factors.

Using a factor tree to find prime factors

A factor tree is a useful way of finding the prime factors of a number. To construct a factor tree:

1. Draw two lines ('branches') extending out from the number, in an upside-down 'V' shape.
2. Write two factors of the number at the end of the branches.
3. If one of the factors is prime, circle it. If it is composite, draw another two branches and split the factor into two smaller factors.
4. Repeat step **3** until all branches end in a circled prime number.

There is often more than one way for a factor tree to be constructed. Three versions of the factor tree for 24 are shown here. They look different; however, the prime factors that appear on the ends of the branches are the same: 2, 2, 2, 3.



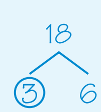
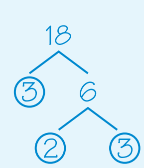
Once all of the branches end in prime factors, we have found all of the prime factors, and we write the number as the product of those factors. We write the factors in ascending order (smallest to largest) and, if any are repeated, we write them in index form.

For example: $24 = 2 \times 2 \times 2 \times 3$
$= 2^3 \times 3$

Worked Example 5

WE 5

Draw a factor tree for the number 18, then express the number as a product of its prime factors in index form.

Thinking	Working
1 Write the number as the product of two factors.	$18 = 3 \times 6$
2 Show each factor at the end of a branch and circle it if it is prime.	
3 Split any composite factor into two smaller factors, continuing until the factors at the end of each branch are prime.	
4 Write the number as a product of the primes in index form and in ascending order (smallest to largest).	$18 = 2 \times 3 \times 3$ $= 2 \times 3^2$

Using repeated division to find prime factors

We can also find the prime factors of a number by doing a series of divisions by small prime numbers, such as 2, 3 or 5. We can use the divisibility rules to identify these smaller prime factors; for example, if the number is even, we can divide by 2. The divisibility rules are listed in section 2.1 on page 55.

We keep dividing by the same, or a different, prime factor until the final answer is 1. The list of all the divisors will be the list of the prime factors.

Worked Example 6

WE6

Use repeated division to find the prime factors of 84. Express the number as a product of its prime factors in index form.

Thinking	Working
1 Identify a prime factor of the number and divide it into the number. Continue dividing by the same or a different prime factor until the last answer is 1.	2) 84 2) 42 3) 21 7) 7 1
2 The prime factors are listed down the left side of the division calculation. Write the number as a product of these.	$84 = 2 \times 2 \times 3 \times 7$
3 Write the number as a product of the prime factors in index form and in ascending order.	$84 = 2^2 \times 3 \times 7$

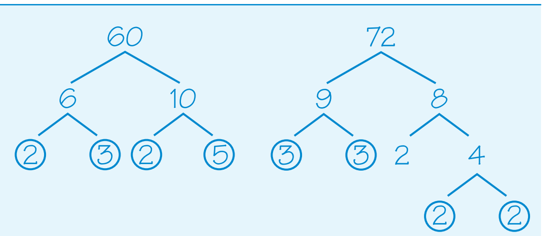
Using prime factors to find the HCF (highest common factor)

If we have two numbers written as a product of their prime factors, the HCF is the product of all the common prime factors.

Worked Example 7

WE7

Use prime factors to find the HCF of the following pair of numbers: 60 and 72.

Thinking	Working
1 Use factor trees to find the prime factors.	
2 Write each number as a product of its prime factors. Do not use index form.	$60 = \boxed{2 \times 2 \times 3} \times 5$ $72 = 2 \times \boxed{2 \times 2 \times 3} \times 3$
3 Circle the group of factors that is common to both numbers (there must be the same number of factors in each group).	
4 Find the product of the common prime factors. This is the HCF.	$\text{HCF} = 2 \times 2 \times 3$ $= 12$

2.3 Prime factors

Navigator

Answers page 628

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q9, Q10, Q11, Q14	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q9, Q10, Q11, Q12, Q13, Q14	Q1 Column 4, Q2 Columns 3 & 4, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15

Fluency

WE5

1 Draw a factor tree for each of the following numbers, then express the number as a product of its prime factors in index form.

(a) 8 **(b)** 12 **(c)** 20 **(d)** 48
(e) 14 **(f)** 26 **(g)** 68 **(h)** 44
(i) 64 **(j)** 72 **(k)** 108 **(l)** 144
(m) 200 **(n)** 750 **(o)** 1000 **(p)** 1236

WE6

2 Use repeated division to find the prime factors of the following numbers. Express each number as a product of its prime factors in index form.

(a) 8 **(b)** 12 **(c)** 28 **(d)** 36
(e) 39 **(f)** 77 **(g)** 51 **(h)** 38
(i) 30 **(j)** 63 **(k)** 96 **(l)** 132
(m) 168 **(n)** 198 **(o)** 288 **(p)** 212

WE7

3 Use prime factors to find the HCF of the following pairs of numbers.

(a) 8 and 20 **(b)** 12 and 18 **(c)** 12 and 36
(d) 36 and 48 **(e)** 24 and 56 **(f)** 28 and 84
(g) 64 and 96 **(h)** 70 and 98 **(i)** 60 and 105
(j) 66 and 110 **(k)** 80 and 128 **(l)** 130 and 156

4 **(a)** 120 written as the product of its prime factors is:

A $2 \times 4 \times 15$ **B** $3 \times 8 \times 5$ **C** $2 \times 2 \times 5 \times 6$ **D** $2 \times 2 \times 2 \times 3 \times 5$

(b) The prime factors of 550 are:

A 2, 5, 11 **B** 2, 10, 55 **C** 5, 10, 11 **D** 2, 11, 25

5 Which numbers are the product of these prime factorisations?

(a) $2^3 \times 3^2$ **(b)** $2^2 \times 3 \times 5^2$ **(c)** $2^4 \times 3^3 \times 5^3$
(d) $3^2 \times 5 \times 7^2$ **(e)** $2^3 \times 5^2 \times 7 \times 11$ **(f)** $2^6 \times 11^2$

6 $6 = 2 \times 3$ and $8 = 2^3$. Use these facts to write the following numbers as products of prime numbers.

(a) 48 **(b)** 96 (48×2) **(c)** 144 (48×3)

7 $33 = 3 \times 11$ and $10 = 2 \times 5$. Use these facts to write the following numbers as products of prime numbers.

(a) 330 **(b)** 660 **(c)** 990

Understanding

8 The prime factors of 192 are 2 and 3 and the prime factors of 42 are 2, 3 and 7. List the prime factors of 192×42.

9 **(a)** The HCF of a pair of numbers can also be used to find the lowest common multiple (LCM). Copy and complete the following method.

Prime factors of 36: 2 × ___ × ___ × ___

Prime factors of 100: 2 × ___ × ___ × ___

Highest common factor (HCF) of 36 and 100: 2 × ___ = ___

Factors of 36 not used to find the HCF: ___ × 3 = ___

Factors of 100 not used to find the HCF: 5 × ___ = ___

HCF × all unused factors of 36 × all unused factors of 100: ___ × 3 × ___ × 5 × ___ = 900

The result is the lowest common multiple of 36 and 100.

(b) Use this method to find the LCM for the following pairs of numbers.

(i) 8 and 18 **(ii)** 12 and 18 **(iii)** 8 and 20

(iv) 8 and 72 **(v)** 20 and 26 **(vi)** 20 and 800

(vii) 20 and 1000 **(viii)** 400 and 800 **(ix)** 800 and 1000

10 Marcus is making identical balloon arrangements for a party. He has 84 red balloons and 54 blue balloons. He would like to sort his balloons into groups, each group containing the same number of red and blue balloons, with no balloons left over.

(a) Use prime factors to find the highest common factor of 84 and 54.

(b) If Marcus split his balloons into this number of groups, how many red and blue balloons would be in each group?

Reasoning

11 A number less than 550 is the product of four prime factors: 2, 3, 5 and a fourth prime factor between 15 and 20. What is the fourth factor, and what is the number?

12 **(a)** Find the smallest number that can be written in prime factor form with:

(i) exactly two different prime factors

(ii) exactly three different prime factors

(iii) exactly four different prime factors.

(b) Repeat **(a)**, but remove the condition that the factors are different.

13 140 tickets were sold to the school concert. Each ticket sold was numbered from 1 to 140 and a lucky door prize was awarded to each person who had a ticket with a number that was a multiple of 15, but not a multiple of 9, 10 or 25. Five friends had the ticket numbers 45, 80, 90, 105 and 135.

(a) Do any of them have a winning ticket? If so, what is their winning number?

(b) If 250 tickets were sold, how many winning tickets would there be? Find the winning numbers.

Open-ended

14 A number between 100 and 200 is the product of four primes and has three prime factors (one factor is repeated). Explain how you might go about finding this number and write down two such numbers.

15 To find the prime factors of a number, Ray uses the divisibility tests. He then writes down the number from each divisibility test that worked.

(a) Using this method, what would Ray say are the prime factors of 20?

(b) What is wrong with Ray's method?

(c) Suggest how Ray's method can be improved so that it is accurate.