7

7 Linear equations

Wiis work wonders. Why do people learn physical tasks at different rates?

Why do some people seem to have 'natural talent' when others struggle to master the basics? University researchers are studying how people learn various skills. They are measuring the motions involved in tasks as simple as playing racquet sports through to more complex activities, such as flying a fighter jet, in the hope that this research will help people to learn these skills faster and more effectively.

One university research project is making use of Nintendo's popular 'Wii' video game technology to create equations that represent skills involved in many human activities from sport to surgery. To do this, a motion-capture device, such as the Wiimote, is used. Data from the Wiimote is used to measure a range of movements that is then used to create mathematical equations. The aim is then to program robots to teach people the most efficient way to learn new tasks. By bringing robotics and virtual reality together, learning will no longer be a matter of trial and error. For example, if you were learning to play tennis, a robotic device, such as a sleeve that prompted you to move your arm the correct way, would make learning easier.

Wiis are already being used in the classroom. How cool is that?

Forum

Researchers are investigating ways that different movements can be recorded by using a Wiimote. Why might the universities be interested in this? How do you think a Wii could be used in classrooms?

Why learn this?

Whether you are deciding how much material to buy to make curtains, working to a family budget, or calculating whether a business has made a profit, equations can be used to represent a variety of situations. Solving them can help us to make informed and accurate decisions.

After completing this chapter you will be able to:

- write an equation from information given in words
- check solutions to equations by substitution
- solve worded problems
- solve everyday problems using equations
- solve equations using a variety of methods.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R7.1

1 Answer TRUE or FALSE to each of the following number sentences.

(a) $3 + 7 = 5 + 6$ (b) $8 \times 3 = 20 + 4$ (c) $\frac{24}{8} = 3 \times 5 - 14$

PP Worksheet R7.2

2 (a) The term $5x$ means:

A $5 + x$ B $5 - x$ C $5 \times x$ D $x \div 5$

(b) The term $\frac{x}{7}$ means:

A $x - 7$ B $x + 7$ C $x \times 7$ D $x \div 7$

(c) $4(x + 3)$ means:

A $4 + (x + 3)$ B $4 \times (x + 3)$ C $4 \times x + 3$ D $4 - (x + 3)$

PP Worksheet R7.3

3 Rewrite each of the following sentences using mathematical symbols and numbers only.

(a) Five added to seven is equal to twelve.

(b) Subtracting two from the product of four and six is equal to twenty-two.

(c) When the sum of eight and six is divided by seven, the answer is equal to two.

PP Worksheet R7.4

4 What is the final result after applying each of the following operations to the number in the box?

(a) $\boxed{8}$ $\times 2$ then -4 then $\div 3$ then $+5$

(b) $\boxed{10}$ $\div 2$ then $\div 5$ then $+6$

PP Worksheet R7.5

5 (a) If $x = 2$, find the value of $4x - 1$.

(b) If $x = 7$, find the value of $\frac{x - 3}{2}$.

Key Words

backtracking	false number sentence	solve
balance method	guess, check and improve	solving by inspection
checking by substitution	inverse operations	true number sentence
equivalent equations	solution	

Number sentences 7.1

Vlado wants change for a \$5 note. His friend Perry gives him two \$2 coins and a \$1 coin in exchange for the note. This can be written as $5 = 2 + 2 + 1$ and is an example of a **true number sentence**.

$2 \times 4 = 5 + 3$ is another example of a true number sentence, as both the left-hand side (LHS) and the right-hand side (RHS) have the same value of 8.

left-hand side (LHS) equals sign right-hand side (RHS)

LHS = RHS

$2 \times 4 = 5 + 4$

LHS ≠ RHS

is an example of a **false number sentence**, as the LHS has a value of 8 and the RHS has a value of 9.

≠ means 'not equal to', < means less than, > means greater than

Worked Example 1

WE1

The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

(a) $5 + 8 = 15$ **(b)** $2 \times 5 - 1 = 7$

Thinking	Working
(a) 1 Find the value of the side that has the coloured number.	(a) $5 + 8 = 13$
2 Compare the answer to the other side of the number sentence.	$13 < 15$
3 Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$5 + 10 = 15$

(b) 1 Find the value of the side that has the coloured number.	(b) $2 \times 5 - 1 = 9$
2 Compare the answer to the other side of the number sentence.	$9 > 7$
3 Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$2 \times 4 - 1 = 7$

Worked Example 2

WE2

Decide which of the following are true number sentences.

(a) $8 \times 3 + 4 = 28$

(b) $\frac{6 \times 5}{3} = 7 + 2$

Thinking	Working
(a) 1 Evaluate the LHS of the number sentence.	(a) LHS $= 8 \times 3 + 4$ $= 28$
2 Compare the LHS and RHS to check whether they are equal.	LHS = RHS
3 State your answer.	The sentence is true.
(b) 1 Evaluate the LHS of the number sentence.	(b) LHS $= \frac{6 \times 5}{3}$ $= 10$
2 Evaluate the RHS of the number sentence.	RHS $= 7 + 2$ $= 9$
3 Compare the LHS and RHS to check whether they are equal.	LHS $\neq$ RHS
4 State your answer.	The sentence is false.

7.1 Number sentences

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q10(a), Q11 (a–d), Q12, Q14, Q16	Q1, Q2 Column 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 (a–f), Q12, Q14, Q16	Q1, Q2 Column 3, Q3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

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Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

WE1

(a) $3 + 4 = 10$ (b) $5 \times 6 = 25$ (c) $\frac{24}{4} = 4$

(d) $22 - 7 = 8 + 4$ (e) $3 \times 7 + 2 = 8 + 9$ (f) $1 + \frac{15}{5} = 6$

2 Decide which of the following are true number sentences.

WE2

(a) $\frac{32}{4} + 7 = 16$ (b) $4 + 3 \times 2 = 14$ (c) $6 + 4 \times 5 = 25$

(d) $30 \div 6 = 4 + 1$ (e) $23 - 7 = 12 + 4$ (f) $8 + 4 = 6 \times 2$

(g) $7 - 5 = 9 \div 3$ (h) $10 \times 8 = 90 - 10$ (i) $\frac{6 + 10 + 12}{7} = \frac{20 + 4}{6}$

3 (a) The number missing from the true number sentence $3 \times \_\_\_\_ = 19 + 5$ is:

A 3 B 6 C 7 D 8

(b) The number missing from the true number sentence $4 \times 2 = \_\_\_\_ + 4$ is:

A 0 B 2 C 4 D 8

(c) The number missing from the true number sentence $\frac{24}{8} + \_\_\_\_ = 10 - 2$ is:

A 4 B 5 C 6 D 8

Understanding

4 Rewrite each true number sentence using numbers and mathematical symbols only.

(a) Forty-three added to five is equal to forty-eight.

(b) Three multiplied by seven is equal to nineteen plus two.

(c) Fifty divided by five is equal to the product of five and two.

(d) When the sum of the numbers six and eight is subtracted from twenty, the answer is equal to twelve divided by two.

5 Write each of the following as a number sentence, then state whether it is true or false.

(a) Nineteen subtracted from thirteen is equal to six.

(b) Four divided by two is equal to eight.

(c) Eleven subtracted from eight is equal to six minus three.

(d) Twelve divided by four is equal to fifteen divided by five.

6 Jane is 5 years older than Todd. If Todd is 7 years old, then write a true number sentence to show Jane's age.

7 Six pens at 60c each cost the same as buying 4 exercise books at 90 cents each. Write this as a true number sentence.

8 The cost of a new shirt is $25 after a discount of $7 is given. Write a true number sentence to show the original price of the shirt.

9 A bus can carry 48 passengers. The bus is full, then 11 people leave while another 5 get on at the next stop. Write a true number sentence to show how many passengers are left on the bus.

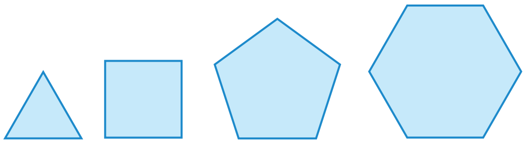
10 Numbers are consecutive if one follows the other. The numbers 8 and 9 are consecutive numbers that add to 17.

(a) Write a true number sentence that shows two consecutive numbers that add to 33.

(b) Write a true number sentence that shows three consecutive numbers whose sum is 54.

Reasoning

11 A piece of wire 60 cm long is used to make a triangle shape that has all three sides the same length. Write a true number sentence to answer each of the following.



(a) How long is each side of the triangle?

(b) The wire is now used to make a square. How long is each side?

(c) The wire is bent to make a regular pentagon. How long is each side?

(d) The wire is then used to make a regular hexagon. How long is each side?

(e) Write a number sentence you could use to determine the length of the side of a shape that has 20 sides of equal length made from the same piece of wire.

(f) Write a number sentence you could use to determine the length of the side of a shape that has 50 sides of equal length made from a wire 200 cm long.

(g) Use a 100 cm length of wire to make a shape whose sides are of equal length. Find three possible values for the number of sides the shape can have if the length of each side is to be a whole number and shorter than 25 cm.

Open-ended

12 Use the numbers 1, 2, 4, 8, the equals sign, =, and as many of the symbols +, –, ×, ÷ as you like to write three different true number sentences whose right-hand side is a number from 1 to 10.

13 A length of wire is used to make a shape that has eight sides of equal length. Find five different whole number values for the length of wire if the length of each side is to be greater than the number of sides.

14 $\square + \square \times \square - \square = 20$

What might the missing numbers be? Give three different answers.

15 Sam and Ruben have written the following number sentences. Sam wrote $3 + 4 \times 21 = 147$. Ruben wrote $3 + 4 \times 21 = 87$.

Who is correct? Why? Write a sentence so that the person who was incorrect can improve their skills.

16 Two different students were given the following number sentence.

$38 - 16 = \square - 12$

They presented the following solutions.

$38 - 16 = \square - 12$	$38 - 16 = \square - 12$
$38 - 12 - 4 = \square - 12$	$38 - (12 + 4) = \square - 12$
$38 - 4 - 12 = \square - 12$	$38 + 4 - 12 = \square - 12$
$34 - 12 = \square - 12$	$42 - 12 = \square - 12$
$\square = 34$	$\square = 42$

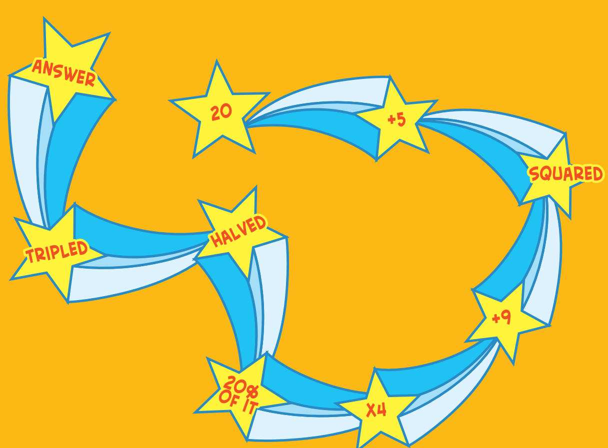
Which of the solutions is correct? Why? What is wrong with the incorrect solution?

Outside the Square Puzzle

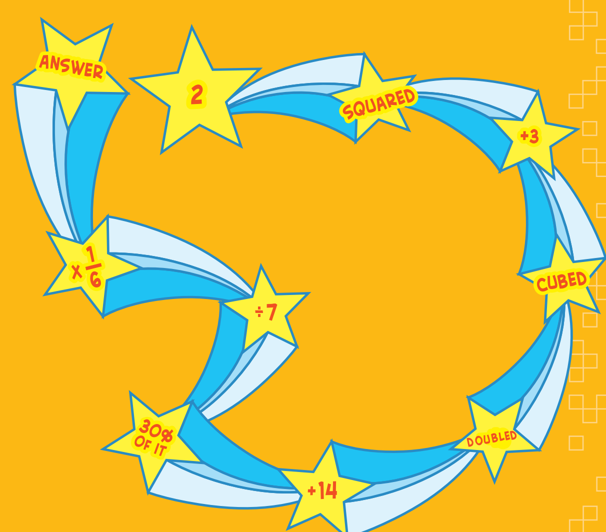
Star quest

Equipment required: 1 brain, 1 watch

(a) Start at the first star and follow the shooting stars. Time yourself to see how long it takes to complete all of the operations.



(b) What about trying something more challenging? Time yourself completing this star quest.



7.2 Introduction to equations

Often, in mathematics, we want to write information in a number sentence but some information is missing. Where a value is unknown, we can use a pronumeral to represent the unknown amount. This number sentence is now called an equation.

$2 \times 4 = 5 + 3$ is a true number sentence. $2 \times 4 = 5 + 4$ is a false number sentence.

$2 \times 4 = 5 + x$ is an equation. Equations always contain an equals sign.

We need to be able to understand and interpret the information given to us in an equation.

The equals sign was invented in 1557 by Welshman Robert Recorde.

Worked Example 3

WE3

Write each of the following equations in words.

(a) $5x - 3 = 1$

(b) $6 = \frac{x}{5} + 2$

Thinking	Working
(a) 1 Call the variable 'a number' and write in words the operation that has been performed on it first, using the correct order of operations.	(a) When a number is multiplied by five
2 Decide what happens next to the LHS and add this on to what you have already written.	and then three is subtracted
3 State the final result. This is the value shown on the RHS.	the result is equal to one.
4 Write the complete equation in words.	When a number is multiplied by five, and then three is subtracted, the result is equal to one.
(b) 1 Call the variable 'a number' and write in words the operation that has been performed on it first using the correct order of operations.	(b) When a number is divided by five
2 Decide what happens next to the RHS and add this on to what you have already written.	and then two is added
3 State the final result. This is the value shown on the LHS.	the result is equal to six.
4 Write the complete equation in words.	When a number is divided by five, and then two is added, the result is equal to six.

We **solve** an equation by finding the unknown value that makes an equation a true number sentence. By substituting $x = 3$ into $2 \times 4 = 5 + x$ we get a true number sentence, so the **solution** to $2 \times 4 = 5 + x$ is $x = 3$.

If x is any other value, we would get a false number sentence.

Equations are useful in solving everyday problems by providing a shorthand way of writing information. There are many different methods used to solve equations.

Solving by inspection

Many equations involve only one operation (+, –, ×, ÷) and are sometimes called one-step equations. For example, $3x = 9$, $\frac{x}{2} = 7$, $x + 3 = 11$, $4 - x = 1$ are one-step equations.

These equations can be solved by simply looking at the equation and guessing the value of x. When we do this, we need to check that our solution will make a true number sentence. This method is called '**solving by inspection**'.

Here are some more one-step equations and their solutions:

For $k - 4 = 5$, the solution is $k = 9$. Check: $9 - 4 = 5$ is a true number sentence.

For $\frac{a}{3} = 5$, the solution is $a = 15$. Check: $\frac{15}{3} = 5$ is a true number sentence.

Worked Example 4

WE4

For the following equation, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

$3x - 4 = 11 \quad (x = 5)$

Thinking	Working
1 Identify the side of the equation that contains the variable.	LHS $= 3x - 4$
2 Substitute the given value for the variable and simplify.	$= 3 \times 5 - 4$ $= 11$
3 Check to see whether this answer is the same as the RHS of the equation.	RHS $= 11$ Yes, $x = 5$ is the solution.

Solving equations using guess, check and improve

We can use '**guess, check and improve**' to solve equations with more than one step.

Worked Example 5

WE5

Find the solution to the following equation by using the guess, check and improve method.

$$\frac{6x - 1}{5} = 7$$

Thinking	Working
1 Guess a value for x and substitute it into the LHS of the equation to see if it makes a true number sentence.	Try $x = 5$ $\frac{6 \times 5 - 1}{5} = \frac{30 - 1}{5}$ $= \frac{29}{5}$ $\neq$ RHS

2 Try a larger value for x and substitute it into the LHS of the equation.	Try $x = 6$ $\frac{6 \times 6 - 1}{5}$ $= \frac{36 - 1}{5}$ $= \frac{35}{5}$ $= 7$ $=$ RHS
3 As this value makes a true number sentence, we have a solution. If the LHS still does not equal the RHS, try another value.	$x = 6$ is the solution to the equation.

7.2 Introduction to equations

Navigator

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Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE3

1 Write each of the following equations in words.

(a) $n + 3 = 10$ (b) $4 + p = 7$ (c) $a + 5 = 6$

(d) $x - 2 = 4$ (e) $k - 7 = 2$ (f) $a - 8 = 6$

(g) $5k = 20$ (h) $7p = 14$ (i) $7a = 21$

(j) $\frac{x}{3} = 4$ (k) $\frac{f}{2} = 6$ (l) $\frac{y}{6} = 12$

(m) $2 + 3x = 7$ (n) $2 + 6x = 10$ (o) $1 + 3x = 8$

WE4

2 For the following equations, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

(a) $x + 6 = 11$ $(x = 5)$ (b) $y + 4 = 10$ $(y = 5)$ (c) $m + 2 = 11$ $(m = 9)$

(d) $d - 6 = 15$ $(d = 21)$ (e) $k - 3 = 8$ $(k = 5)$ (f) $a - 6 = 9$ $(a = 15)$

(g) $3x = 12$ $(x = 3)$ (h) $5a = 20$ $(a = 4)$ (i) $7m = 14$ $(m = 2)$

(j) $\frac{f}{2} = 16$ $(f = 8)$ (k) $\frac{m}{3} = 5$ $(m = 15)$ (l) $\frac{x}{7} = 14$ $(x = 2)$

(m) $5m + 1 = 9$ $(m = 2)$ (n) $2a - 2 = 8$ $(a = 3)$ (o) $5s - 4 = 6$ $(s = 2)$

(p) $\frac{x}{3} + 2 = 5$ $(x = 9)$ (q) $5 + \frac{b}{2} = 9$ $(b = 8)$ (r) $\frac{a}{15} + 2 = 5$ $(a = 30)$

WE5

3 Find the solution to each of the following equations by using the guess, check and improve method.

(a) $2x + 1 = 9$ (b) $6 + 5x = 41$ (c) $7x + 3 = 52$

(d) $\frac{x - 8}{3} = 7$ (e) $\frac{x - 5}{2} = 6$ (f) $\frac{x - 4}{5} = 12$

(g) $3(x - 11) = 21$ (h) $8(x - 2) = 32$ (i) $6(x + 13) = 54$

(j) $\frac{x}{2} - 5 = 33$ (k) $\frac{x}{6} - 8 = 14$ (l) $\frac{x}{3} + 7 = 61$

4 In words, the equation $2 = 5 - \frac{x}{6}$ can be written as:

A two is equal to five added to a number that is divided by six

B five subtracted from six times a number is equal to two

C five added to two is equal to a number that is divided by six

D a number is divided by six and the result is subtracted from five to give an answer of two.

5 (a) Which equation describes this sentence?

Four added to a number is equal to ten.

A $n + 10 = 4$ B $n + 4 = 10n$ C $4 + 10 = n$ D $n + 4 = 10$

(b) Which equation describes this sentence?

A number multiplied by three is equal to six.

A $x + 3 = 6$ B $6x = 3$ C $3x = 6$ D $\frac{x}{3} = 6$

(c) Which equation describes this sentence?

A number divided by four is equal to three.

A $4x = 3$ B $x + 4 = 3$ C $4 - x = 3$ D $\frac{x}{4} = 3$

6 (a) The solution to the equation $x + 3 = 18$ is:

A $x = 6$ B $x = 15$ C $x = 16$ D $x = 21$

(b) The solution to the equation $\frac{x}{2} = 4$ is:

A $x = 1$ B $x = 2$ C $x = 6$ D $x = 8$

(c) The solution to the equation $4 = x - 11$ is:

A -15 B 7 C 11 D 15

Understanding

7 (a) Which equation describes this situation?

The cost, c, in dollars, of some grocery items plus a \$3 delivery charge totals \$55.

A $3c = 55$ B $c - 3 = 55$ C $c + 3 = 55$ D $55 + 3 = c$

(b) Which equation describes this situation?

Each of 4 people receive the same number of lollies, n, when a bag containing 36 lollies is shared between them.

A $4n = 36$ B $4 + n = 36$ C $\frac{n}{4} = 36$ D $n - 4 = 36$

(c) Which equation describes this situation?

A school minibus seats a total of 12 passengers when full. The number of passengers, p, on the bus decreases by 3 at the next stop so that the bus is now half full.

A $p - 3 = 12$ B $3p = 6$ C $p + 3 = 6$ D $p - 3 = 6$

8 Use guess, check and improve to find the solution to the following equations.

(a) $\frac{x}{2} = x - 5$ (b) $3x = 5x - 8$ (c) $\frac{3x - 1}{4} = \frac{2(x + 9)}{5}$

9 Daniel is three years older than his brother Christopher.

(a) Define a variable to represent Daniel's age.

(b) Use this variable to write an expression for Christopher's age.

(c) Write an expression to show how old Christopher will be in 15 years time.

(d) If Christopher will be 32 in 15 years time, write an equation using your answer to part **(c)** as the LHS of the equation and solve it to find Daniel's age now.

Reasoning

10 Cameron has some money in his wallet. Libby has $5.

(a) Define a variable to represent the amount of money Cameron has in his wallet.

(b) If Cameron spends $14, write an expression to show how much money he has left.

(c) If Libby and Cameron now have $23 between them, write an equation and solve it to find how much money Cameron had to start with.

Open-ended

11 Pete is twice as old as Chee.

(a) Choose a value that is a multiple of 3 as the sum of their ages and write the equation that you would use to find Chee's age.

(b) Choose another value that is also a multiple of 3 as the sum of their ages and write a new equation you would use to find Chee's age.

(c) What age might Chee be if the sum of their ages is less than 30?

12 Write a short story that could be represented by the following equations.

(a) $14 - x = 11$ **(b)** $\frac{x}{5} = 12$ **(c)** $6x = 108$

Outside the Square

Puzzle

The great number mystery

What number am I?

- I am a prime number.
- If you triple me, I am made up of two digits.
- If I am multiplied by four, and then divided by five, I am one smaller than my original value.
- If I am squared, I am five less than six times my value.

Solving equations using backtracking

7.3

Some equations are difficult to solve by inspection or guess, check and improve because the solution is not a whole number, or more than one operation is involved.

For example: $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3.75 = 21.8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations but their solutions are not easy to see.

$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x + 6}{8} = 2$ are all two-step equations.

We need to use other methods to solve these equations.

Using flowcharts

In Chapter 5, flowcharts were used to build expressions. Flowcharts can also be used to help us solve equations. To use a flowchart correctly, we need to apply the order of operations that were used to build the expression.

For the expression $2x + 5$, x is first multiplied by 2 to give $2x$, then 5 is added to give us $2x + 5$, so our flowchart looks like this:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x + 5}$$

The order in which we apply the operation is important in building expressions.

If we wanted to add 5 first before multiplying by 2, the flowchart would look like this:

$$\boxed{x} \xrightarrow{+5} \boxed{x + 5} \xrightarrow{\times 2} \boxed{2(x + 5)}$$

This expression is $2(x + 5)$, which is quite different from $2x + 5$.

Applying the same operations in a different order produces different expressions.

Inverse operations

To solve equations using flowcharts, we undo operations used to build the equation by applying **inverse operations** (opposite operations) in the reverse order. We move backwards along the flowchart.

Worked Example 6

WE6

Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a)
$$\boxed{x} \underset{\div 4}{\overset{\times 4}{\rightleftarrows}} \boxed{4x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{4x + 5}$$

(b)
$$\boxed{x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{6x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{6x - 3}$$

Thinking	Working
(a) **1** Above the top arrow write the missing operation needed to build the expression.	$x \xrightarrow{\times 4} 4x \xrightarrow{+5} 4x+5$ $x \xleftarrow{\div 4} 4x \xleftarrow{?} 4x+5$
2 Below the bottom arrow write the missing operation that will undo this expression.	$x \xrightarrow{\times 4} 4x \xrightarrow{+5} 4x+5$ $x \xleftarrow{\div 4} 4x \xleftarrow{-5} 4x+5$
(b) **1** Above the top arrows write the missing operation needed to build each expression.	$x \xrightarrow{\times 6} 6x \xrightarrow{-3} 6x-3$ $x \xleftarrow{?} 6x \xleftarrow{?} 6x-3$
2 Below the bottom arrows write the missing operation needed to undo this expression.	$x \xrightarrow{\times 6} 6x \xrightarrow{-3} 6x-3$ $x \xleftarrow{\div 6} 6x \xleftarrow{+3} 6x-3$

Backtracking

To solve $2x + 5 = 11$, we set up the following flowchart.

$$x \xrightarrow{\times 2} 2x \xrightarrow{+5} 2x+5$$
$$3 \xleftarrow{\div 2} 6 \xleftarrow{-5} 11$$

Step 1 Draw a flowchart for the expression on the LHS of the equation.

Step 2 Add boxes underneath the flowchart boxes and arrow signs in the opposite direction.

Step 3 Write the inverse operations on the arrow signs.

Step 4 Write the number on the RHS of the equation in the box on the right-hand side of the flowchart underneath the expression.

Step 5 Complete the flowchart using the inverse operations.

Step 6 The number under x is the solution.

This method of solving equations is called **backtracking**. Backtracking along this flowchart gives us the solution $x = 3$ to the equation $2x + 5 = 11$.

> When backtracking, we undo an operation by using the inverse (opposite) operation.
>
> $+$ and $-$ are inverse operations
>
> $\times$ and $\div$ are inverse operations

Worked Example 7

WE7

Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

(a)

	$\times 2$		-4	
x		$2x$		$2x-4$
5		10		6
	$\div 2$		$+4$	

(b)

	-4		$\div 2$	
x		$x-4$		
		6		3
	$+4$		$\times 2$	

Thinking / Working

(a) (i) The expression in the last box on the right and the number in the box underneath it form the equation.

(a) (i) Equation is $2x - 4 = 6$

(ii) The solution is the number under x in the first box on the left of the flowchart.

(ii) Solution is $x = 5$

(b) (i) 1 Complete the missing parts of the flowchart.

(b) (i)

	-4		$\div 2$	
x		$x-4$		$\frac{x-4}{2}$
10		6		3
	$+4$		$\times 2$	

2 The expression in the last box on the right and the number in the box underneath it form the equation.

$$\frac{x-4}{2} = 3$$

(ii) The solution is the number under x in the first box on the left of the flowchart.

(ii) $x = 10$

Worked Example 8

WE8

Draw a flowchart and use backtracking to solve the equation: $3x + 4 = 13$.

Thinking / Working

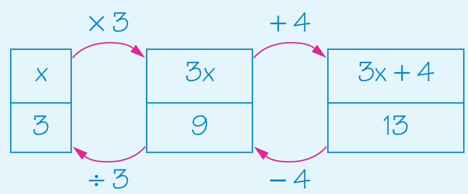
1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. ($\times 3$, $+ 4$)

	$\times 3$		$+4$	
x		$3x$		$3x+4$

2 Write the value from the RHS of the equation under the last box on the right of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. ($\div 3$, $- 4$)

	$\times 3$		$+4$	
x		$3x$		$3x+4$
				13
	$\div 3$		-4	

3 Perform the required operations to obtain a value for x.



4 State the solution. $x = 3$

Checking by substitution

We can check that the answer obtained is the solution to the equation by substituting it into the equation. We evaluate both sides of the equation. If the LHS = RHS, the answer is the solution. This process is called **checking by substitution**.

In Worked Example 8, we would check the solution by substituting $x = 3$ into the left-hand side of the equation.

Check: $\quad \text{LHS} = 3x + 4$
$$\begin{aligned} &= 3 \times 3 + 4 \\ &= 9 + 4 \\ &= 13 \\ &= \text{RHS} \end{aligned}$$

So, $x = 3$ is the solution.

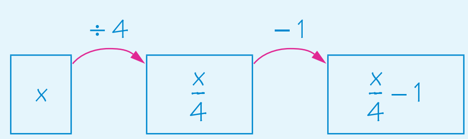
Worked Example 9

WE9

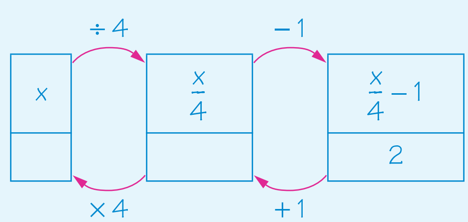
Draw a flowchart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

Thinking	Working

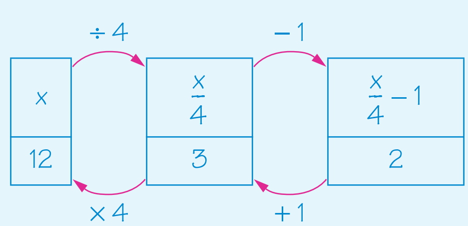
1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. (÷ 4, − 1)



2 Write the value from the RHS of the equation under the right-hand box of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (× 4, + 1)



3 Perform the required operations to obtain a value for x.



4 State the solution. $x = 12$

5 Check the solution by substituting your answer into the left-hand side of the equation. If the left-hand side equals the right-hand side of the equation, you have found the solution.	Check: LHS $= \frac{x}{4} - 1$ $= \frac{12}{4} - 1$ $= 3 - 1$ $= 2$ $=$ RHS

7.3 Solving equations using backtracking

Navigator

Q1 Column 1, Q2 Column 1, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Column 1, Q6 Column 1, Q7 Column 1, Q8 Column 1, Q9, Q10 (a) & (b), Q13	Q1 Column 1, Q2 Column 1, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6 Column 2, Q7 Column 2, Q8 Column 2, Q9, Q10 (a)–(c), Q11, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7 Column 3, Q8 Column 3, Q9, Q10, Q11, Q12, Q13, Q14

Answers page 663

Fluency

WE6

1 Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a)
$$x \xrightarrow{\times 4} \boxed{4x} \xrightarrow{?} \boxed{4x+1}$$
$$x \xleftarrow{\div 4} \boxed{4x} \xleftarrow{?} \boxed{4x+1}$$

(b)
$$x \xrightarrow{\times 5} \boxed{5x} \xrightarrow{?} \boxed{5x-7}$$
$$x \xleftarrow{\div 5} \boxed{5x} \xleftarrow{?} \boxed{5x-7}$$

(c)
$$x \xrightarrow{?} \square \xrightarrow{?} \boxed{2x+4}$$
$$x \xleftarrow{?} \square \xleftarrow{?} \boxed{2x+4}$$

(d)
$$x \xrightarrow{?} \square \xrightarrow{?} \boxed{7x-3}$$
$$x \xleftarrow{?} \square \xleftarrow{?} \boxed{7x-3}$$

(e)
$$x \xrightarrow{?} \square \xrightarrow{?} \boxed{3x-2}$$
$$x \xleftarrow{?} \square \xleftarrow{?} \boxed{3x-2}$$

(f)
$$x \xrightarrow{?} \square \xrightarrow{?} \boxed{8x+1}$$
$$x \xleftarrow{?} \square \xleftarrow{?} \boxed{8x+1}$$

WE7

2 Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

(a)
$$x \xrightarrow{\times 3} 3x \xrightarrow{-1} 3x-1$$
$$4 \xleftarrow{\div 3} 12 \xleftarrow{+1} 11$$

(b)
$$x \xrightarrow{\times 2} 2x \xrightarrow{-3} 2x-3$$
$$5 \xleftarrow{\div 2} 10 \xleftarrow{+3} 7$$

(c)

	$+5$		$\div 4$	
x		$x+5$		$\frac{x+5}{4}$
11		16		4
	-5		$\times 4$	

(d)

	$+7$		$\div 2$	
x		$x+7$		$\frac{x+7}{2}$
9		16		8
	-7		$\times 2$	

(e)

	$\div 6$		$+7$	
x		$\frac{x}{6}$		
		2		9
	$\times 6$		-7	

(f)

	$\div 3$		$+8$	
x		$\frac{x}{3}$		
		5		13
	$\times 3$		-8	

(g)

	-3		$\div 5$	
x				$\frac{x-3}{5}$
				1
	$+3$		$\times 5$	

(h)

	-2		$\div 7$	
x				$\frac{x-2}{7}$
				2
	$+2$		$\times 7$	

(i)

	$+6$		$\times 2$	
x				
				20
	-6		$\div 2$	

(j)

	$\times 5$		-9	
x				
				16
	$\div 5$		$+9$	

(k)

	?		?	
x				$5(x+2)$
				30
	?		?	

(l)

	?		?	
x				$7(x+1)$
				21
	?		?	

WE8

3 Draw a flowchart and use backtracking to solve each of the following equations.

(a) $5x + 3 = 8$ **(b)** $7x + 3 = 24$ **(c)** $2x + 3 = 11$

(d) $2x - 7 = 3$ **(e)** $3x - 2 = 10$ **(f)** $8x - 11 = 13$

WE9

4 Draw a flowchart and use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x}{4} + 5 = 7$ **(b)** $\frac{x}{2} + 6 = 11$ **(c)** $\frac{x}{7} + 3 = 5$

(d) $\frac{x}{2} - 2 = 1$ **(e)** $\frac{x}{5} - 1 = 2$ **(f)** $\frac{x}{2} - 4 = 2$

5 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $3(x + 1) = 12$ **(b)** $2(x + 3) = 16$ **(c)** $2(x + 5) = 14$

(d) $5(x - 6) = 10$ **(e)** $6(x - 4) = 18$ **(f)** $3(x - 5) = 3$

6 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x+8}{3} = 3$ **(b)** $\frac{x+5}{2} = 4$ **(c)** $\frac{x+10}{7} = 2$

(d) $\frac{x-2}{5} = 1$ **(e)** $\frac{x-3}{4} = 2$ **(f)** $\frac{x-5}{2} = 4$

7 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $2x + 3 = 13$ **(b)** $3x + 2 = 11$ **(c)** $5x + 6 = 11$

(d) $5x - 3 = 17$ **(e)** $4x - 1 = 7$ **(f)** $8x - 9 = 7$

(g) $2(x + 7) = 20$ **(h)** $4(x - 3) = 8$ **(i)** $6(x - 1) = 12$

(j) $\frac{x}{2} + 3 = 8$ **(k)** $\frac{x}{7} + 10 = 12$ **(l)** $\frac{x}{3} - 2 = 3$

(m) $\frac{x+2}{4} = 2$ **(n)** $\frac{x-4}{5} = 2$ **(o)** $\frac{x-3}{2} = 2$

8 Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

(a) $\frac{2x+1}{3} = 3$ **(b)** $\frac{3x+7}{2} = 11$ **(c)** $\frac{5x-2}{4} = 7$

(d) $\frac{3x}{5} + 6 = 9$ **(e)** $\frac{2x}{7} + 8 = 12$ **(f)** $\frac{7x}{2} - 10 = 4$

(g) $\frac{2(x+4)}{5} = 6$ **(h)** $\frac{3(x+2)}{4} = 3$ **(i)** $\frac{2(x-3)}{7} = 6$

9 The solution to $\frac{4x}{5} + 6 = 14$ is:

A $x = 1$ **B** $x = 10$ **C** $x = 16$ **D** $x = 25$

Understanding

10 Write an equation using the following information, then solve each equation using backtracking. Use n to represent the unknown number.

(a) A number is doubled, then one is added to the result to give an answer of forty-three.

(b) When four is added to a number and the result is divided by five, the answer is four.

(c) A number is multiplied by seven and the result is divided by three to give an answer of twenty-one.

(d) A number is doubled, then six is added. The result is divided by ten to give an answer of two.

11 Belinda buys 3 packets of pencils. She gives five pencils to her friends. She sells the rest for 10 cents each and makes \$6.70. How many pencils were in each packet? Using n as the number of pencils in the packet, write an equation and use backtracking to answer this question.

Reasoning

12 Ying repairs televisions. The expression for the amount of money ($) she charges is $30x + 20$, where x is the number of hours she takes to repair a TV. Aldo also repairs televisions. The expression for the amount of money ($) he charges is $20x + 50$ where x is the number of hours he takes to repair a TV.

(a) Show the flowchart for the expression $30x + 20$.

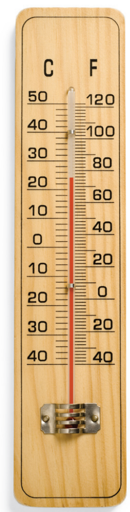
(b) Show the flowchart for the expression $20x + 50$.

(c) Substitute the number $x = 1$ in both flowcharts and write down how much money Ying and Aldo charge for a repair that takes one hour.

(d) Ying charges a customer $95. How long did it take her to repair this TV?

(e) If Aldo charged $95, how long would he have taken?

(f) By trying different values for x, find the number of hours for which the amount charged by Ying and Aldo is the same.



Open-ended

13 If x is the temperature in degrees Celsius, the expression $\frac{9x}{5} + 32$ gives the value of x in degrees Fahrenheit.

(a) Build and complete the flowchart for the expression.

(b) Use your flowchart to find two values for the temperature in degrees Celsius that give a whole number for degrees Fahrenheit.

14 After Ross performs four different operations on a number, he has a result of 10. Write an equation that fits this situation.

Outside the Square Puzzle

Gold digger 2

Your aim is to find the gold. The basic rule on how to solve this kind of puzzle can be found on page 63.

Here are the two maps to solve:

(a)

	2			
	4		1	
4			3	1
				1
	3			2
				1

(b)

3				1	
		4		3	
				3	
0		3		2	

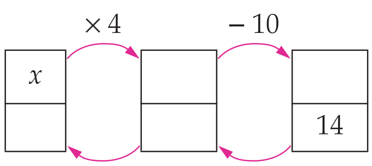
Half-time 7

1 Solve the equation $\frac{x}{12} = 7$ by inspection. Ex.7.2

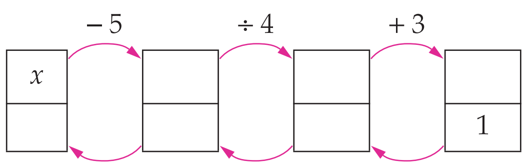
2 The perimeter of a rectangular room is 18 m. If the length of the room is 5 m, what is the width? Form an equation and solve it with backtracking. Ex.7.3

3 Copy and complete the following flowcharts. Write the equation and use the flowchart to find the solution for x. Ex.7.3

(a)



(b)



4 Write a true number sentence that shows two consecutive numbers whose sum is 43. Ex.7.1

5 (a) James is 5 years older than his brother Tom who is 7 years old. If James is j years old: Ex.7.2

(i) write an equation to represent this situation with Tom's age on the RHS

(ii) solve the equation by inspection to find James' age.

(b) Jude is 4 years younger than his sister Amanda who is 16 years old. If Jude is d years old:

(i) write an equation to represent this situation with Amanda's age on the RHS

(ii) solve the equation by inspection to find Jude's age.

6 Use backtracking to solve each of the following equations. Ex.7.3

(a) $3x + 4 = 11$ (b) $\frac{x}{2} + 5 = -3$ (c) $3(x - 8) = 15$

7 Write the equation $5x + 3 = 23$ in words. Ex.7.2

8 The following number sentences are not true. Rewrite them by changing the coloured number so that you have a true number sentence. Ex.7.1

(a) $3 + 6 = 11$ (b) $4 \times 3 = 26 - 6$ (c) $4 + \frac{28}{4} = 18$

9 Kate has a piece of cardboard that is x centimetres wide. She joins a 50-centimetre piece of cardboard of the same length to it and then divides the width into 7 strips so she can paint each strip a different colour of the rainbow. Each strip is 10 cm wide. Use backtracking to find the width of the original piece of cardboard. Ex.7.3

10 State whether the following number sentences are true or false. Ex.7.1

(a) $6 - 4 = 4 - 6$ (b) $2(3 + 5) = 2 \times 3 + 2 \times 5$ (c) $(8 \times 9) \times 2 = 8 \times (9 \times 2)$

11 Pete is five years older than his brother Sam. The sum of their ages is 23. If Pete is p years old, Ex.7.3

(a) write an expression to represent Sam's age

(b) write an equation to represent this situation with 23 on the RHS of the equation

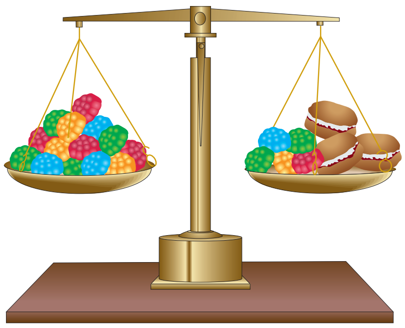
(c) solve the equation using backtracking to find the ages of Pete and Sam.

7.4 Solving equations using the balance method

Scales can be used to measure the mass of an object if other masses are available for comparison. To balance the scales, the two sides of the scales must hold equal masses.

Worked Example 10 WE10

This set of scales is balanced. The left-hand side has 14 lollies; and the right-hand side has three biscuits and five lollies.



(a) If one lolly is taken from the right-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the lolly back?

(c) Starting again, if all of the lollies are taken from the right-hand side, what should be done to the left-hand side to balance the scales?

(d) How many lollies are equivalent to the three biscuits?

(e) How many lollies are equivalent to one biscuit?

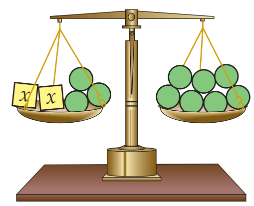
Thinking	Working
(a) Look at the diagram and decide which side is heavier after removing the lolly.	(a) The left-hand side is now heavier.
(b) Balance the scales by doing the same to the other side.	(b) Take a lolly from the left-hand side.
(c) Identify how many lollies were on the right-hand side at the start (5). Take the same amount from the left-hand side.	(c) Take five lollies from the left-hand side.
(d) The beam is horizontal. Look at how many lollies are on the left-hand side (9).	(d) Nine lollies = three biscuits.
(e) The beam is balanced, so the biscuits on the right-hand side equal the lollies on the left-hand side.	(e) Three biscuits = nine lollies. One biscuit = three lollies.

Equivalent equations

Equivalent equations are those that have the same solution. For example, $2x = 4$ is equivalent to $2x + 3 = 7$ as they both have a solution of $x = 2$. This can be checked by substituting $x = 2$ into both equations to show they are both true number sentences.

$2x = 4$	$2x + 3 = 7$
$2 \times 2 = 4$	$2 \times 2 + 3 = 7$

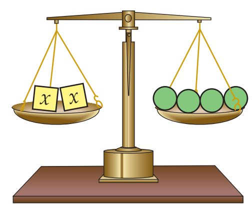
This situation can also be represented with balance scales. The 'equals' sign can be thought of as sitting in the middle of the two balanced sides. These scales represent $2x + 3 = 7$.



$2x + 3 = 7$

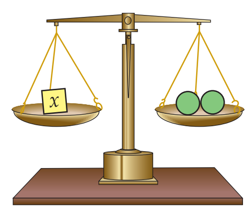
When 3 is taken off both sides, the scales are still balanced and now represent $2x = 4$.

Because the scales are still balanced we say that $2x + 3 = 7$ and $2x = 4$ are equivalent equations.



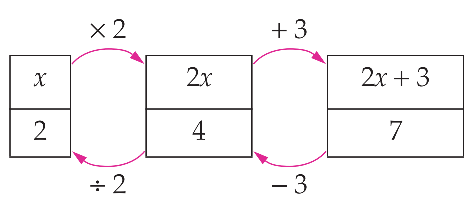
$2x = 4$

As two identical unknown masses equal 4, then one mass equals 2. The scales represent $x = 2$.



$x = 2$

These steps can be shown on a flowchart.



All these equations are equivalent. The simplest equivalent equation is $x = 2$, which is also the solution.

$x = 2$ $\quad 2x = 4$ $\quad 2x + 3 = 7$

> We create equivalent equations by performing the same operation on both sides of the equation.

Worked Example 11

WE11

Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

(a) $x - 5 = 7 \ (+ 6)$ **(b)** $5x + 3 = 8 \ (- 2)$ **(c)** $\frac{x}{2} = 4 \ (\times 2)$ **(d)** $8x = 24 \ (\div 4)$

Thinking			Working
(a)	1	Write the equation.	(a) $x - 5 = 7$
	2	Perform the same operation on both sides (add 6).	$x - 5 + 6 = 7 + 6$
	3	Simplify.	$x + 1 = 13$
(b)	1	Write the equation.	(b) $5x + 3 = 8$
	2	Perform the same operation on both sides (subtract 2).	$5x + 3 - 2 = 8 - 2$
	3	Simplify.	$5x + 1 = 6$
(c)	1	Write the equation.	(c) $\frac{x}{2} = 4$
	2	Perform the same operation on both sides (multiply by 2).	$\frac{x}{2} \times 2 = 4 \times 2$
	3	Simplify.	$x = 8$
(d)	1	Write the equation.	(d) $8x = 24$
	2	Perform the same operation on both sides (divide by 4).	$\frac{8x}{4} = \frac{24}{4}$
	3	Simplify.	$2x = 6$

Solving equations using the balance method

By using a set of scales to represent our equation, we can now solve equations using the **balance method**. Up until now, we have used flowcharts to show us both the order and the operations we need to use to solve equations. With the balance method, we still need to use the same information, but now we show it in a different way.

Consider $2x + 3 = 7$.

	$\times 2$		$+ 3$	
x		$2x$		$2x + 3$
2		4		7
	$\div 2$		$- 3$	

$x = 2$ $\quad$ $2x = 4$ $\quad$ $2x + 3 = 7$

Notice that the three boxes contain three equivalent equations. We have moved backwards through the flowchart and used inverse (opposite) operations (-3, $\div 2$) to solve the equation $2x + 3 = 7$. Let's now show the same steps using the balance method. The order of operations in the flowchart tells us how our equation was built. By moving backwards and performing the inverse operation, we will undo our equation to find the solution.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3 \qquad \text{(subtracting 3 from both sides)}$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2} \qquad \text{(dividing both sides by 2)}$$
$$x = 2$$

Note that the three equivalent equations formed here $2x + 3 = 7$, $2x = 4$, $x = 2$ are the same as the three equations formed from the boxes in the flowchart.

Worked Example 12

WE12

Solve the equation $3x - 7 = 11$ using the balance method. Check your solution by substitution.

Thinking	Working
1 Write the equation. Identify the last operation to be performed on the LHS of it. This is the first operation to be undone. (– 7)	$3x - 7 = 11$
2 Use the inverse operation (+ 7) on both sides of the equals sign and simplify your equation.	$3x - 7 + 7 = 11 + 7$ $3x = 18$
3 Identify the next operation to be undone and apply the inverse operation (÷ 3). If one side of the equation is now the variable by itself, you have found the solution. Otherwise, continue the process until you do have the variable by itself.	$\frac{3x}{3} = \frac{18}{3}$ $x = 6$
4 Check the solution by substitution.	Check: LHS $= 3x - 7$ $= 3 \times 6 - 7$ $= 18 - 7$ $= 11$ $=$ RHS

7.4 Solving equations using the balance method

Navigator

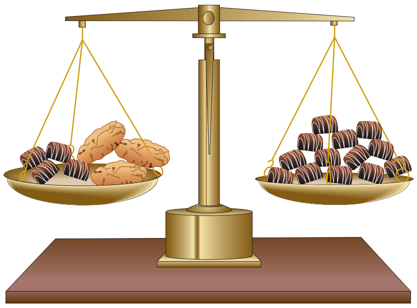
Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q10	Q1, Q2 Column 2, Q3 Column 2, Q4, Q6, Q7, Q8, Q9, Q10	Q1, Q2 Column 2, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Answers page 665

Fluency

WE10

1 This set of scales is balanced. The left-hand side has 4 chocolates and 3 biscuits; and the right-hand side has 13 chocolates.



(a) If one chocolate is taken from the left-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the chocolate back?

(c) Starting again, if all of the chocolates are taken from the left-hand side, what should be done to the right-hand side to balance the scales?

(d) How many chocolates are equivalent to the three biscuits?

(e) How many chocolates are equivalent to one biscuit?

WE11

2 Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

(a) $x + 5 = 8$ $(+ 3)$

(b) $x + 4 = 12$ $(+ 5)$

(c) $x + 1 = 7$ $(+ 6)$

(d) $2x + 2 = 10$ $(- 1)$

(e) $3x + 5 = 10$ $(- 5)$

(f) $6x + 8 = 14$ $(- 5)$

WE12

3 Solve each of the following equations using the balance method. Check your solutions by subsitution.

(a) $3x + 5 = 8$

(b) $5x + 1 = 21$

(c) $7x + 3 = 17$

(d) $2x + 1 = 11$

(e) $2x + 5 = 19$

(f) $2x + 10 = 8$

(g) $3x - 4 = 20$

(h) $5x - 1 = 4$

(i) $6x - 2 = 16$

(j) $4x - 3 = 17$

(k) $2x - 3 = 7$

(l) $4x - 1 = 31$

(m) $\frac{2x}{3} = 8$

(n) $\frac{2x}{7} = 2$

(o) $\frac{5x}{2} = 20$

(p) $\frac{x + 5}{3} = 5$

(q) $\frac{x + 1}{3} = 7$

(r) $\frac{x + 4}{5} = 3$

(s) $2(x - 5) = 14$

(t) $3(x + 2) = 24$

(u) $7(x - 2) = 35$

Inverse operations need to be applied in the correct order to find a solution.

Understanding

4 This set of scales is balanced with 3 apples and 2 bananas on the left-hand side and 7 apples on the right-hand side.

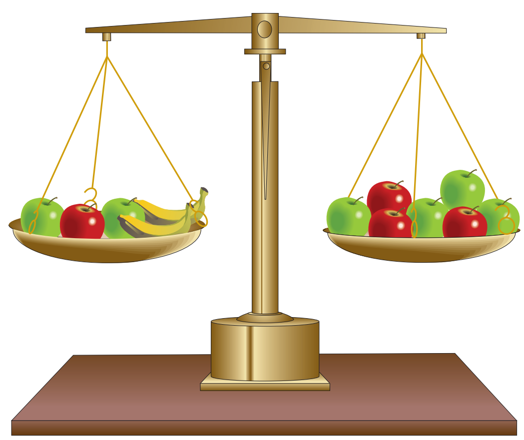
If 4 apples are taken from the right-hand side, the scale can be balanced by:

A taking 4 apples from the left-hand side

B taking 3 apples and 1 banana from the left-hand side

C taking 2 apples and 2 bananas from the left-hand side

D taking 2 bananas from the left-hand side.



5 Trixie's father weighs twice as much as Trixie plus 12 kilograms. If her father weighs 104 kg:

(a) write an equation representing the situation using t to represent Trixie's weight

(b) solve the equation to find out Trixie's weight.

6 Naram has shot a number of 3-point goals as well as 18 other points in a game of basketball. If he shot a total of 30 points in the game:

(a) write this as an equation with p representing the number of 3-point goals shot by Naram

(b) solve your equation to find the number of 3-point goals that he shot.

Reasoning

7 Fatima drives to and from work each day for the five-day working week, and drives an additional 85 kilometres on the weekend. She drives a total of 425 kilometres every week in her car. How far is it from her home to her workplace?

8 George has to take 5 litres of water with him on a boat trip. He has one 1.25 L container and three other containers of equal capacity. Together, the four containers will hold 5 litres. What is the capacity of the other containers?

Open-ended

9 Write down three different equations that are equivalent to $5x + 4 = 14$.

10 Vejay was given the equation $\frac{2x+3}{7} = 3$ to solve and presented the following solution.

$$\frac{2x+3}{7} = 3$$

$$\frac{2x+3}{7} - 3 = 3 - 3$$

$$\frac{2x}{7} = 0$$

$$\frac{2x}{7} \times 7 = 0 \times 7$$

$$2x = 0$$

$$\frac{2x}{2} = \frac{0}{2}$$

$$x = 0$$

(a) Explain to Vejay why his working out is incorrect.

(b) Show Vejay the process he should have used.

(c) How could Vejay avoid making the same mistake next time he's solving equations?

Outside the Square — Problem solving

Paper mountain

Imagine a very, very large sheet of paper, 0.1 mm thick.

If you fold it in half, it is 0.2 mm thick.

How thick will it be if you fold it in half again?

Imagine the paper can be folded an unlimited number of times.

How many folds would be needed to create a paper stack that is taller than Mt Everest? (Mt Everest is 8848 m high.)

Strategy options

- Act it out.
- Look for a pattern.

Maths 4 Real

Dance Party dilemma

Have you ever wondered what is involved in organising a fund-raising event such as a dance party? There would need to be a venue booked, tickets printed, advertising material created and drinks provided, as well as a range of other things.

Believe it or not, a lot of the costs to be considered can be expressed as an equation. Many costs are variables that can change as the number of people attending the party changes. There are also fixed costs to consider, which remain the same no matter how many people attend. Once you have created an equation that represents all of the costs, it becomes possible to calculate how much you should charge each person to enter the event. This would then determine whether you make a profit or a loss.

1 The table below represents the total costs associated with running a dance party. It shows that it would cost $50 to organise the party even if no people attend. Why might this be? Explain.

Number of people n	0	10	20	30	40	50
Total cost ($) C	50	150	250	350	450	550

2 **(a)** What happens to the cost every time you increase the number of people attending the dance party by 10?

(b) Use your answer to **(a)** to determine the increase in cost for one extra person.

3 Using the table in Question **1**, what equation could you use to calculate the costs of running the dance party? Use C to represent cost and n to represent the number of people attending the dance party.

$C = \_\_ \times n + \_\_$

4 Use the equation that you found in Question **3** to calculate the costs of the dance party for the following number of people attending.

(a) 25 **(b)** 36 **(c)** 83

5 Suppose the following table represents the money charged to attend the dance party. As the organiser of the party, this is your income. What equation could you use to represent this? Use I for income and n for the number of people attending.

Number of people, n	0	10	20	30	40	50
Income ($), I	0	120	240	360	480	600

6 Use the formula that you found in Question **5** to calculate the income from the dance party for the following number of people attending.

(a) 32

(b) 53

(c) 95

If your income is bigger than your costs you will make a profit. However, if your costs are greater than your income, you will make a loss.

7 Use your equations from Questions **3** and **5** to complete the following table showing all the costs and income associated with this dance party. Make sure that you also calculate the profit or loss (the difference between the cost and the incomes). Place a negative sign in front of a number to show a loss.

Number of people, *n*	0	15	35	45	63	100
Cost ($), *C*	50	200	400	500		
Income ($), *I*	0	180	420			
Profit/loss ($), *P*	-50	-20	20			

8 The profit or loss made at the party is found by subtracting the cost from the income. Using ***P*** to represent the profit or loss, write a formula that shows this.

9 What is the smallest number of people you would need to attend the dance party in order to make a profit? Show how you worked this out.

Research

Investigate the costs of organising a dance party for 40 of your friends. You need to find the following.

- cost of a suitable venue
- music costs
- catering costs
- cleaning costs

Present a report that outlines a number of different options for each of the above costs. Which is your preferred option and why? What are the benefits of hiring a venue compared to having a party at home?

Technology Exploration Excel

Equipment required: 1 brain, 1 computer with an Excel spreadsheet

Versions of this Exploration are available for other technologies in Pearson Reader.

Algebra using a spreadsheet

Did you know that your computer or calculator can do algebra? A spreadsheet program, such as Excel, uses formulas to do a lot of repetitive substitution. Spreadsheets on a CAS calculator work exactly the same way.

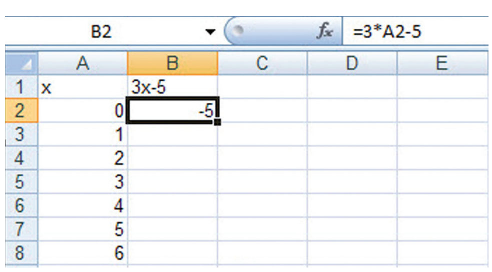
There are many ways to solve algebraic equations. You have been learning some algebraic methods. In this exploration, we are going to use a spreadsheet to find the solution to algebraic equations through a numerical process.

1 Select an Excel spreadsheet or a spreadsheet from a CAS menu and enter x as a heading in cell A1.

(a) To enter x-values from 0 to 6 in column A start at cell A2. Enter 0 and 1, and highlight these two cells. With the cursor on the little black cross on the bottom right-hand corner of cell A3, drag down to A8 or enter =A2+1 into cell A3 and drag this formula down to cell A8.

(b) Enter the expression 3x–5 as a heading in cell B1.

(c) Enter the formula =3×A2–5 into cell B2. This will calculate the value for $3x - 5$ when $x = 0$. The x-value in cell A2 is 0.


B2 | fx =3*A2-5

	A	B	C	D	E
1	x	3x-5			
2	0	-5			
3	1				
4	2				
5	3				
6	4				
7	5				
8	6				

The formula must be changed in each row to use the x-value in column A in that row. The formula will now calculate a new value for $3x - 5$.

(d) Copy this formula down to cell B8 by dragging the little black cross on the bottom right-hand corner of cell B2. This will change the formula to refer to the correct x-value.

2 Use your spreadsheet to find the solution to:

(a) $3x - 5 = 4$

(b) $3x - 5 = 13$

(c) $3x - 5 = -2$

Highlight these solutions using any fill colour you want. Select the cell that contains your solution. Then, use the fill colour icon to select a colour from the font group on the Home tab.

In Excel, a formula always starts with an = sign and * represents a multiplication sign.

3 **(a)** In column C, enter the expression -2x+5 as a heading in C1.

(b) Enter the formula required to calculate a value for the expression $-2x + 5$ when $x = 0$ into cell C2.

(c) Copy this formula down to C8.

You will need to use a single quotation before the negative sign so that the formula is read as text.

4 Use your spreadsheet to find the solution to:

(a) $-2x + 5 = 5$

(b) $-2x + 5 = -3$

(c) $-2x + 5 = -5$

Highlight these solutions with a different fill colour.

5 Use your spreadsheet to find the solution to $3x - 5 = -2x + 5$.

Highlight this solution using another fill colour.

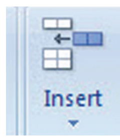
6 We now want to solve $4x - 6 = 30$, so enter 4x–6 as a heading in D1 and =4*A2–6 as a formula in D2.

(a) Fill down to D8. You can see that none of the values = 30, so the solution is not in the 0 to 6 values for x.

(b) We need to extend our values for x, so extend your x-values to $x = 10$.

(c) Now, we can find a solution to $4x - 6 = 30$. Write down the solution and, once again, highlight the solution.

7 Sometimes, our solution might be less than 0, so we will need to insert values smaller than 0 in column A. Insert as many rows as you may want above row 2 and copy the formulas up instead of down.



Left click on row 2, then use 'insert sheet rows' in the 'insert' drop-down menu under the cells group on the Home tab.

Now, find the solution to:

(a) $3x - 5 = -11$ (b) $-2x + 5 = 11$

Highlight with the same fill colours you chose for these expressions previously.

8 Solutions to equations are not always integers. Suppose we are trying to find the solution to $4x - 6 = 25$. We can see that when $x = 7$, $4x - 6 = 22$ and when $x = 8$, $4x - 6 = 26$, but there is no solution for $4x - 6 = 25$. Because 25 is between 22 and 26, the solution will be between 7 and 8. Insert a row between $x = 7$ and $x = 8$ and enter 7.5. This gives us $4x - 6 = 24$. We now know that the solution is between 7.5 and 8.

(a) We can overwrite the 7.5 with 7.7. This gives us $4x - 6 = 24.8$.

(b) Overwriting the 7.7 with 7.75, we now get $4x - 6 = 25$.

Highlight this solution.

	A	B
1	x	4x-6
2	-5	-26
3	-4	-22
4	-3	-18
5	-2	-14
6	-1	-10
7	0	-6
8	1	-2
9	1	-1
10	2	2
11	3	6
12	4	10
13	5	14
14	6	18
15	7	22
16	7.75	25
17	8	26
18	9	30
19	10	34

9 Insert new formulas in your spreadsheet to find the solutions to:

(a) $5x + 6 = 22$

(b) $8x + 1 = 4$

(c) $10x + 3.3 = 140$

Highlight each of these solutions.

Taking it further

10 Try to find the solution to $3x - 4 = 7$. It may look like the solution to this equation is 3.6, but $3 \times 3.6 - 4 = 6.8$, not 7. So, what has gone wrong?

Use the 'increase decimal places' icon found in the number groups on the Home tab to show four decimal places in column A and the column you have used for the expression $3x - 4$ (do not include the heading row). This gets us closer to a solution, but it is still not an exact solution. $x = 3.6666$ gives 6.9998. Do you think you would ever find an exact solution to this question with a spreadsheet? Explain your answer.

11 Give an example of another equation that will have the same problem.

12 'Spreadsheets are very useful for solving equations, but are not as good as algebraic solutions found by hand.' Do you agree with this statement? Give reasons to support your argument.

7.5 Solving problems with equations

Equations can be very useful in solving everyday problems. So far, you have practised solving equations presented to you. Now you will be forming your own equation from information supplied in a worded question. Solving this equation will help you find the unknown value asked for in the question. There may be more than one way to write the equation and solve it.

Worked Example 13

WE13

Phil has 70 cents. He buys a pear and has 25 cents left. If x represents the cost of a pear (in cents), form an equation and solve it to find the cost of the pear.

Thinking	Working
1 Define a variable to represent the unknown quantity.	Let x be the cost of a pear.
2 Form an equation using the information given.	$x + 25 = 70$
3 Identify the first operation to undo and then use the inverse operation on both sides of the equals sign.	$x + 25 - 25 = 70 - 25$ $x = 45$
4 Check the solution.	$\text{LHS} = 45 + 25$ $= 70$ $= \text{RHS}$
5 State the solution in words.	The pear costs 45 cents.

Worked Example 14

WE14

Erica has bought two movie tickets and one bucket of popcorn for a total of $21. If the popcorn costs $5, how much does one movie ticket cost? Using t as the cost of a movie ticket, form an equation and solve it to find the value of t. (Assume both tickets are the same price.)

Thinking	Working
1 Define a variable to represent the unknown quantity.	Let t represent the cost in dollars of one movie ticket.
2 Form an equation using the information given.	$2t + 5 = 21$
3 Identify the first operation to undo and then use the inverse operation on both sides of the equals sign.	$2t + 5 - 5 = 21 - 5$ $2t = 16$
4 Identify the second operation to undo and then use the inverse operation on both sides of the equals sign.	$\frac{2t}{2} = \frac{16}{2}$ $t = 8$
5 Check the solution.	$\text{LHS} = 2t + 5$ $= 2 \times 8 + 5$ $= 16 + 5$ $= 21$ $= \text{RHS}$
6 State the solution in words.	One movie ticket costs \$8.

7.5 Solving problems with equations

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14

Answers page 665

Fluency

1 **(a)** Carmen has 95 cents. She buys an ice-cream and has 20 cents left. If x represents the cost of the ice-cream (in cents), form an equation and solve it to find the cost of the ice-cream. WE13

(b) Matthew can run 100 m in 12.8 seconds. In the school sports, Tony ran 1.5 seconds faster. If t represents Tony's time, form an equation and solve it to find Tony's time.

2 **(a)** Tran bought two sushi rolls and a juice for a total of \$8. If the juice costs \$1.60, how much does one sushi roll cost? Using d as the cost of a sushi roll, form an equation and solve it to find the value of d. (Assume both sushi rolls are the same price.) WE14

(b) Asif bought three DVDs and a CD for \$59. If the CD costs \$21.50, how much does one DVD cost? Using v as the cost of a DVD, form an equation and solve it to find the value of v. (Assume all DVDs are the same price.)

3 A bus has the capacity to seat 45 passengers. Eighteen passengers have boarded the bus and taken seats.

(a) If n represents the number of seats still available, the equation which can be formed is:

A $n - 18 = 45$ **B** $n + 45 = 18$ **C** $18n = 45$ **D** $n + 18 = 45$

(b) Solve the equation formed in part **(a)** to find the number of seats still available on the bus.

4 A roll of fabric is divided evenly between six people. Each person receives 2.5 m of fabric.

(a) If t represents the total length of fabric in the roll (in m), the equation that can be formed is:

A $t = 6 + 2.5$

B $t + 2.5 = 6$

C $\frac{t}{6} = 2.5$

D $2.5t = 6$

(b) Solve the equation formed in part **(a)** to find the total length of the roll of fabric.

Understanding

5 The length of a rectangular vegetable garden is 4 m. The perimeter of wire fencing needed to enclose this garden is 13 m.

(a) If w represents the width (in m) of the vegetable garden, the equation that can be formed is:

A $w + 4 = 13$ B $w + 8 = 13$ C $2w - 8 = 13$ D $2w + 8 = 13$

(b) Solve the equation formed in part **(a)** to find the width of the vegetable garden.

6 Lucas bought a salad sandwich and a container of milk for his lunch.

(a) If the salad sandwich cost $\$x$ and the milk cost \$1.50, write an expression for the total cost of his lunch.

(b) Form an equation and solve it to find the cost of the salad sandwich if Lucas spent \$5.70 on his lunch.

7 (a) If x represents the length (in cm) of a square floor tile, write an expression for its perimeter.

(b) Form an equation and solve it to find the length of the floor tile if its perimeter is 84 cm.

8 A restaurant has several tables that seat four people, but only three tables that seat two people.

(a) If x represents the number of four-person tables available, write an expression for the total number of people that can be seated at the restaurant.

(b) A Saturday night booking for 62 people has been made. Six of these people are to be seated at the three two-person tables. Form an equation and solve it to find how many four-person tables will be needed.

9 At a particular rollerskating rink, it costs \$5 to hire skates and \$3 for each hour spent on the rink. If it costs Natalie \$14, form an equation and solve it to find how many hours she spent rollerskating. Let h represent the number of hours spent rollerskating.

10 The tallest man in recorded history was Robert Wadlow (1918–1940). At age 22, his height was 54 cm less than twice his height at age 5. If his height at age 22 was 272 cm, form an equation and solve it to find his height at age 5.

11 A company called Tool Time hires out electrical tools, charging a non-refundable deposit of \$15 for each item plus an amount per day according to the type of tool.

Tool	Hire charge per day
Electric drill	\$8
Electric sander	\$13
Electric circular saw	\$24
Electric jackhammer	\$35
Concrete mixer	\$48

Let d = the number of days the tools are hired, then form an equation and solve it to answer each of the following.

(a) If Allie hires an electric drill from Tool Time and pays \$63, for how many days did she hire it?

(b) If Serena hires a concrete mixer from Tool Time and pays \$111, for how many days did she hire it?

(c) Kosta hires an electric sander and an electric saw from Tool Time. For how many days did he hire the tools if he paid \$178?

Reasoning

12 The local council has hired a concreting firm to pave some footpaths. Each footpath is to be one metre wide. A wooden frame is needed before the concrete is poured. This frame is made up of one-metre lengths of wood.

The frames for some different lengths of footpath are shown in the diagrams below.

(a) How many pieces of wood are needed to construct the frame of a footpath of length:

(i) 2 metres **(ii)** 3 metres **(iii)** 4 metres?

(b) Write an expression for the number of pieces of wood needed to construct the frame for a footpath of length x metres.

(c) Form an equation and solve it to find how many pieces of wood are needed to construct the frame for a footpath of length:

(i) 6 metres **(ii)** 10 metres **(iii)** 13 metres.

(d) The concreting workers have only 36 pieces of wood. By using your equation and solving it, find the length of the longest footpath that could be poured at the one time.

Open-ended

13 Gary the gardener is keen to do some landscaping on a Saturday and has allowed for 7 hours work. It costs $30 per hour for him to hire a jackhammer and $22 per hour to hire a mulcher. He has budgeted to spend no more than $180. Keeping in mind that he can only use one piece of equipment at a time, what are three options for using the equipment and what would each option cost him?

14 A man is 5 times the age of his son. Given that the sum of their ages is no greater than 70, what are their possible ages? Assume that the answers are whole numbers.

Outside the Square — Problem solving

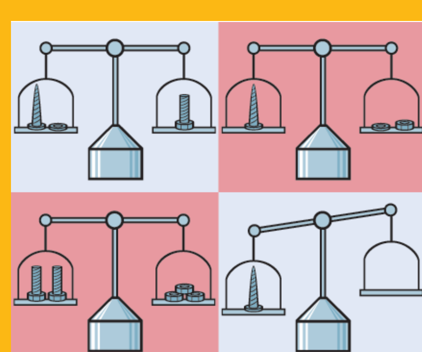
Algebraic puzzles

1 Four numbers are added together and the result is 255. One of the numbers is the square of one of the others. The largest number is double the square number, and the fourth number is 55. Find the numbers.

2 If a screw and a washer balance with a bolt, a screw balances with a washer and a nut, and two bolts balance with three nuts, find out how many washers will balance with a screw.

Strategy options

- Guess and check.
- Break problem into manageable parts.



Investigation

The great escape

Equipment required: 1 brain

Farmer Sue has a paddock to which she brings her cows to wait for milking and a large yard next to it for her free-range chickens. Every now and then, some of her chickens escape into the cow paddock.

One day, the gate connecting the chicken yard and the cow paddock was left wide open and all of Farmer Sue's chickens escaped into the cow paddock where all of her cows were waiting to be milked. Farmer Sue raced over and counted 40 heads and 112 legs.

The Big Question

How many chickens and cows did Farmer Sue own?

Engage

1 Last week, Farmer Sue found 4 chickens and 8 cows in the paddock and had to race around to catch the chickens and return them to their own yard.

(a) How many heads would she have counted?

(b) How many legs would she have counted?

(c) If there had been 8 chickens and 4 cows:

(i) how many heads would she have counted?

(ii) how many legs would she have counted?

Explore

2 If n represents the number of cows in the paddock and c represents the number of chickens that have escaped into the paddock:

(a) write an equation to show 12 heads altogether

(b) what are all the possible combinations of cows and chickens that would give a total of 12 heads? Present your answers in a list or table.

(c) Write an equation to show 40 legs altogether.

(d) Substitute $n = 8$ and $c = 4$ into your equation to make a true statement.

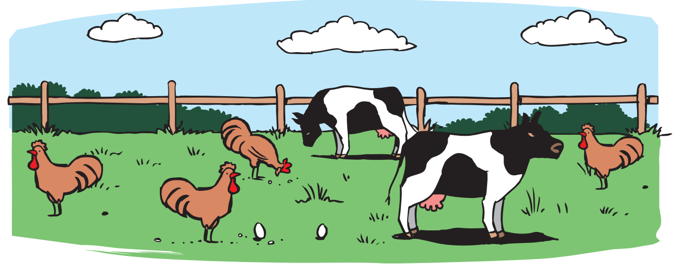
Strategy options

- Guess and check.
- Make a table.
- Make a model.
- Look for a pattern.

3 (a) If Farmer Sue counted 40 heads, choose two possible values for the number of cows and chickens she might own.

(b) For each choice, using an appropriate equation, calculate the number of legs she would have counted.

(c) Can you now find the values for n and c that would give 40 heads and 112 legs?



Explain

4 If the number of heads stays the same but the number of legs changes, explain how the numbers of each animal in the paddock change.

Elaborate

5 (a) State your answer to the Big Question. Explain how it was obtained.

(b) What would be the solution for n and c if the number of heads was still 12 but the number of legs was now only 32? Show how you worked this out.

Evaluate

6 (a) What methods did you use to solve this problem?

(b) Did you use a particular problem-solving strategy or method?

(c) Why did you choose that particular method?

(d) Do you think this was the best method?

(e) Can you think of any other ways to solve this problem?

Extend

7 Deep in the ocean, a scuba diver discovers a collection of 21 crabs and octopuses together in the same small area. He counts 198 legs altogether. (A crab has 10 legs and an octopus has 8.) How many crabs and octopuses were there in this Octopus' Garden?

Mathspace

The case of the confusing clues

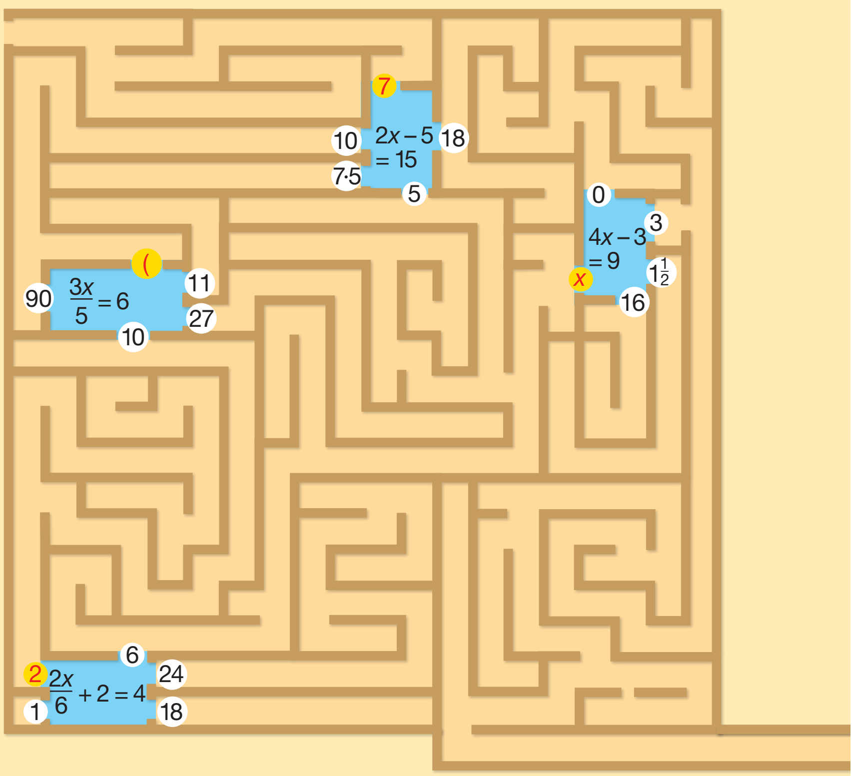
Cara Loft is on a secret mission. The golden calculator has been stolen by the mysterious Dr Equation. Where has he hidden it? All Cara has is a series of clues spread throughout the rather creepy House of Calculus. You need to help Cara by finding your way through the maze. As you move through the maze, you will enter some tricky 'equation chambers'.

When you enter the chamber:

- make sure that you enter through a coloured number or symbol and write it down (they form an equation you will need to solve at the end).
- To exit the equation chamber, solve the equation and move out through the exit showing the correct solution.

When you finally exit the maze, you need to complete the practice session to help you get ready for the even harder next section. Good luck!

START



Practice session 1

Solve these equations:

1 $3x + 1 = 10$

2 $4x - 4 = 12$

3 $8x - 1\frac{1}{2} = 6\frac{1}{2}$

4 $7x - 3.5 = 31.5$

5 $5x + \frac{1}{3} = 15\frac{1}{3}$

6 $\frac{2x}{3} + 1 = 5$

7 $\frac{5x}{2} + 1 = 11$

8 $\frac{7x}{5} - 3 = 11$

9 $\frac{8x}{6} - 4 = 0$

10 $\frac{9x}{10} + 1 = 8.2$

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Practice session 2

Solve these equations:

1 $3(x + 2) = 9$

2 $4(x - 5) = 12$

3 $2(x + 1) = 16$

4 $5(x - 3) = 35$

5 $6(x - 2) = 24$

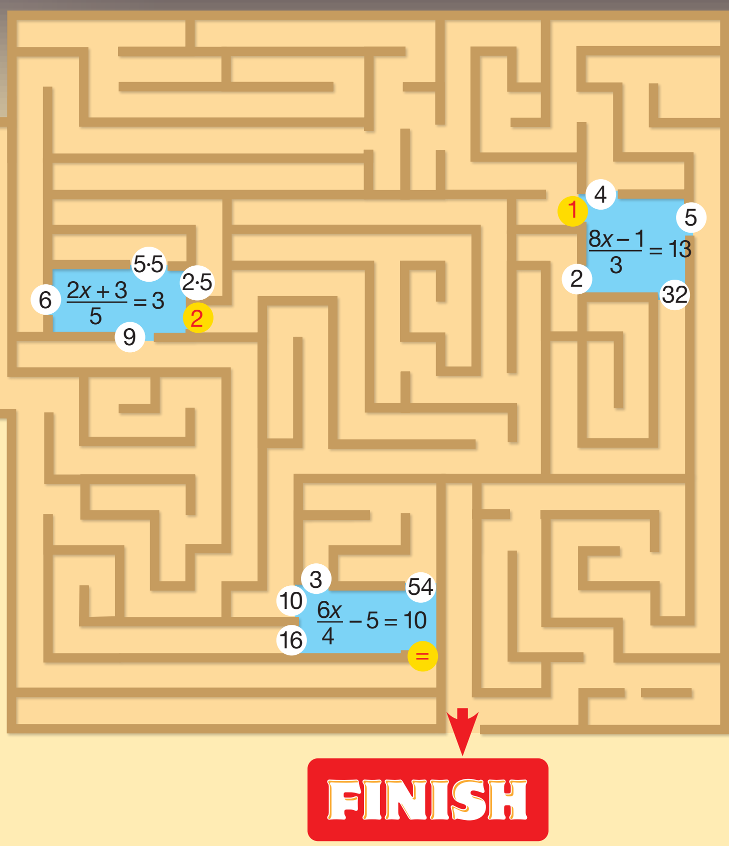
6 $\frac{x + 8}{2} = 7$

7 $\frac{x + 3}{4} = 3$

8 $\frac{x - 8}{3} = 2$

9 $\frac{x + 1}{2} = 5$

10 $\frac{x - 8}{11} = 2$



Practice session 3

Solve these equations:

1 $5x + 4 = 9$

2 $4(x - 3) = 8$

3 $\frac{x + 1}{8} = 3$

4 $9(x - 5) = 90$

5 $\frac{7x}{5} + 1 = 15$

6 $\frac{5x}{3} + 3 = 13$

7 $7x - 8 = 20$

8 $\frac{x + 2}{6} = 4$

Congratulations!

You have successfully mastered the House of Calculus. Or have you? Where is the golden calculator? Dr Equation has left you a final challenge. Piece together the clues you have collected in order and then solve the equation that they form. You then need to use your answer to attempt the biggest challenge of all—the sinister ‘Steps of Arithmetica’. Successfully meet the challenge and you will find the golden calculator. Fail and you will have to go back to the start!

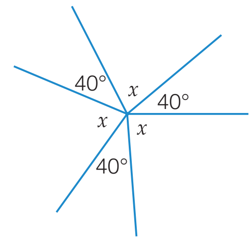
SOLUTION TO THE MAZE EQUATION → SQUARE YOUR ANSWER → DIVIDE BY THE BIGGEST PRIME FACTOR → DOUBLE YOUR ANSWER AND SUBTRACT 5 → ADD 3 → FIND THE SQUARE ROOT → ADD ONE → IS YOUR ANSWER THE MAZE SOLUTION?

Challenge 7

1 When half of a number is increased by 15 the result is 49. The original number is:

A 68 B 34 C 32 D 17

2 Write an equation and solve it to find the value of x in the diagram.



3 Solve the equation: $\frac{3x-5}{4} = -6$.

4 If $59 + x = 73 + y$, then:

A $x = 14 - y$ B $x = 14 + y$

C $x = y - 14$ D $x = 14y$

5 Using only the digits 1, 2, 3 and 7 and the two mathematical symbols + and = only once, we can create a true number sentence $2^3 = 1 + 7$.

(a) Using the digits 2, 3, 4, 5 and the two mathematical symbols + and = only once and a number as a power, create a true number sentence.

(b) Using any different four digits from 0 to 9 and the two mathematical symbols + and = only once and one of the numbers as a power, create five different true number sentences. Do not use $2^3 = 1 + 7$ or your answer to **(a)**. Use 0 and 2 as a power only once.

(c) Use once only, any different four digits from 0 to 9 and the two mathematical symbols − and = only once and one of the numbers as a power to create five true number sentences. Use 0 and 1 as a power only once.

6 The inverse (reciprocal) of $\frac{2}{3}$ is $\frac{3}{2}$. If the inverse of $\frac{4x}{5}$ is $\frac{1}{20}$, then the value of x is:

A $\frac{1}{25}$ B $\frac{1}{16}$ C 16 D 25

7 Solve for y:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{y} = 1$$

8 Vladmir buys an equal number of 55-cent and \$1.10 stamps and spends \$80.85. How many stamps did he buy altogether? Form an equation and solve it.

9 A Lotto winner won $\frac{9}{10}$ of the total prize pool. Shortly after, she spent $\frac{3}{4}$ of her winnings, but still had \$2700 left. What was the value of the total prize pool? Form an equation and solve it.

10 If $\frac{1}{3} > \frac{1}{4} > \frac{1}{m}$ and the difference between the first two fractions equals the difference between the last two fractions, find the value of m. (*Hint*: Write $\frac{1}{m}$ on one side of the equation before you take the inverse.)

11 Two integers, a and b, do not end in zero. If the product ab is a multiple of 10 and $a > b$, the last digit of $a - b$ cannot be:

A 1 B 3 C 5 D 7

Chapter review 7

D.I.Y. Summary

Key Words

backtracking	false number sentence	solve
balance method	guess, check and improve	solving by inspection
checking by substitution	inverse operations	true number sentence
equivalent equations	solution	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 An equation can be solved by performing the ___________ on both sides of the equation.

2 To ___________ an equation is to find the value of the unknown that will make a true number sentence.

3 $4 + 5 = 10$ is a ___________.

4 One way of solving an equation is by looking at the equation to see what number might be the answer. This is called ___________.

5 Equations that have the same solution are said to be ___________.

6 ___________ is a method we can use to solve equations. It can be shown by moving backwards along a flowchart.

7 ___________ is another method used to solve equations.

Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. Ex. 7.1

(a) $5 \times 4 = 30 - 2$ (b) $2 \times 7 + 6 = 9 + 5$

2 Find the unknown number in each of the following sentences. Ex. 7.1

(a) Six subtracted from a number is equal to nine.

(b) A number divided by four is equal to seven.

3 Find the solution to each of the following equations by inspection. Ex. 7.2

(a) $2x = 18$ (b) $x + 7 = 10$ (c) $\frac{x}{5} = 3$

(d) $x - 4 = 11$ (e) $x + 3 = 15$ (f) $x - 4 = 24$

4 Solve the following equations using guess, check and improve. Ex. 7.2

(a) $3x + 25 = 16$ (b) $\frac{x}{7} + 51 = 63$ (c) $\frac{3x + 5}{4} = 8$

5 List the inverse (or opposite) operation to each of the following. Ex. 7.3

(a) $+5$ (b) $\div 3$ (c) $\times 8$ (d) -4

6 Solve each of the following equations using a flowchart and backtracking. Ex 7.3

(a) $3x - 7 = 2$ (b) $\frac{x+6}{5} = 4$ (c) $4(x + 1) = 20$ (d) $\frac{x}{8} - 2 = 3$

7 Use the balance method to solve the following equations. Ex 7.4

(a) $4x + 7 = 19$ (b) $5(x - 2) = 20$ (c) $\frac{x}{3} + 5 = 10$

(d) $\frac{x-8}{6} = 3$ (e) $\frac{x+4}{5} = 5$ (f) $\frac{x-7}{3} = 10$

8 The equation $5x - 6 = 4$ has the solution: Ex 7.4

A $x = 1$ B $x = 2$ C $x = 3$ D $x = 4$

9 Danni bought 5 tickets to a show. She was given \$27.50 change from \$200. Use an equation to find how much each ticket cost. Let t be the cost of a ticket. Ex 7.5

Understanding

10 Which equation describes this situation?

A restaurant seats a maximum of 64 patrons. On Saturday afternoon, it is fully booked for the evening meal service. At 5.30 p.m., a group booking for 16 is cancelled. If p represents the number of people who are still booked, the equation that can be formed is:

A $p - 16 = 64$ B $p = 64 + 16$ C $16p = 64$ D $p + 16 = 64$

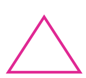
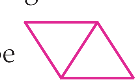
11 Ross buys three dim sims and a bottle of juice.

(a) If each dim sim costs x cents and the bottle of juice costs \$1.40, write an expression for the total cost (in cents) of the food and juice.

(b) Use an equation to find the cost of one dim sim if Ross spends a total of \$2.75.

12 The air temperature at 4 p.m. is 28°C. Over the past hour, the temperature has risen by 5°C. What was the temperature at 3 p.m.? (Form an equation to solve by letting t represent the air temperature at 3 p.m.)

Reasoning

13 A triangle  made from matchsticks has three sides of equal length. If more matchsticks are added to make another triangle, we obtain this shape .

(a) How many matchsticks of equal length does this shape have?

(b) A third triangle is added to the shape. How many matchsticks of equal length does this shape have?

(c) If n represents the number of triangles joined together in a row, write, using the pronumeral n, the number of matchsticks of equal length the shape has in total.

(d) (i) Use your answer to **(c)** to write down the equation for n triangles joined together that have 97 matchsticks of equal length.

(ii) Solve the equation obtained in part **(d) (i)**.

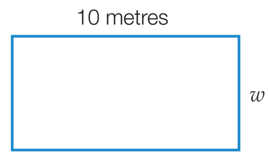
NAPLAN practice 7

Numeracy: Non-calculator

1 Jan is baking biscuits. She puts her biscuits in 3 equal rows. When they are baked, she eats one. She has 44 biscuits left. How many biscuits did she have in each row?

A 3 **B** 10 **C** 15 **D** 20

2 A rectangular garden bed has a perimeter of 30 metres and a length of 10 metres. If the width of the garden can be written as w, which equation best represents this situation?



A $w + 10 = 30$ **B** $w + 30 = 10$

C $2w + 10 = 30$ **D** $2w + 20 = 30$

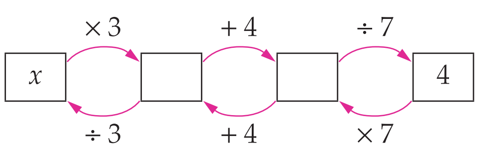
3 What is the missing number that makes this number sentence true?

$3 \times \square = 12 + 3 \times 6$

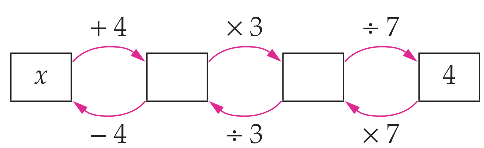
A 1 **B** 7 **C** 9 **D** 10

4 Voula thinks of a number and asks Renza to guess what it is. Voula states that after the number is tripled, four added and the result divided by seven, the answer is four. Which flowchart best represents how Renza might find Voula's mystery number? Let the mystery number = x.

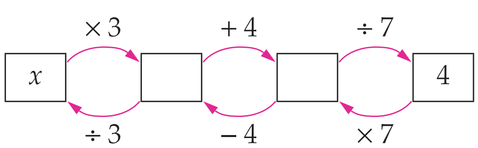
A



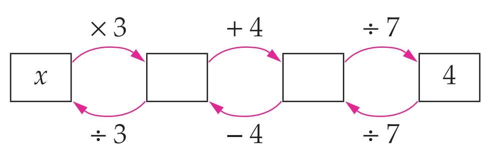
B



C



D



Numeracy: Calculator allowed

5 A pack of 4 donuts costs \$3.60.
A pack of 6 donuts costs \$4.00.
You need to buy 34 donuts.
What is the least amount you can pay?

A \$23.60 **B** \$26.40 **C** \$26.50 **D** \$29.90

6 The cooking time for a turkey is 30 minutes plus 20 minutes per kilogram. If M is the mass of the turkey in kilograms, the total cooking time in minutes is:

A $20 + 30M$ **B** $20M + 30$ **C** $20 \times 30M$ **D** $30 - 20M$

7 Julie, Nikki and Romina earned a total of \$3400 in the last week. Julie earned twice as much as Nikki whereas Romina earned \$200 more than Nikki. How much did Nikki earn in the last week?

A \$600 **B** \$800 **C** \$1000 **D** \$1600

8 The solution to $\frac{3(x - 4)}{2} = 15$ is:

A $x = 6$ **B** $x = 10$ **C** $x = 14$ **D** $x = 58$