



4



# Decimals, percentage and ratio

## 'When is a decimal point not a decimal point?'

A number may contain a point, but this does not make it a decimal point.

Usually, in mathematics, we use numbers to represent quantities, so we can calculate with them. These numbers are called the 'cardinal numbers'. We also use numbers in a less mathematical way—as labels or 'tags' that help to identify things. These numbers are called 'nominal numbers'. Room numbers, telephone numbers, and the footy score in the photo are all examples of nominal numbers. Some nominal numbers include a point that looks like a decimal point, but is actually a 'separator', placed between numbers that are labels for different things. For example, a score of 3.6 in an AFL game means 3 goals and 6 points, not 3 goals and 6 tenths of a fourth goal. A large building such as a hotel might have a room number written as 2.13,

meaning the thirteenth room on the second floor, not 2 rooms and 13 hundredths of a third room.

### Forum

We read the time of 7.30 as 'seven thirty' or 'half past seven'. If we wanted to write this time as a proper decimal number, what would we write? How many minutes would 0.30 hours be? Can you think of other examples where numbers may have a point and look like decimal numbers, but are, in fact, nominal numbers?

Amounts of money, such as \$4.35, are written with a point. Is it a decimal point?

### Why learn this?

An understanding of decimal numbers helps us to measure the wood we need to build a new shelf, work out who won the 100-m freestyle or calculate a household budget. Percentages have many everyday uses, such as measuring performance, advertising discounts or presenting survey results. Ratios and rates help us compare and calculate quantities of the same and different types, such as determining which product size is the best value for money.

#### After completing this chapter you will be able to:

- compare, order and round decimal numbers
- add, subtract, multiply and divide decimal numbers
- convert between decimals, fractions and percentages
- use estimations to check that answers are reasonable
- use percentages to solve problems
- use ratios and rates to compare and calculate amounts
- understand the relationship between ratios, fractions and percentages
- calculate unit prices and determine 'best buys'.

# Recall

# 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

- 1** Copy and complete each of the following by writing < (less than), > (greater than), or = (equal to) between the given numbers.



(a)  $0.1 \underline{\hspace{1cm}}$  0.01      (b)  $2 \underline{\hspace{1cm}}$  2.2      (c)  $0.3 \underline{\hspace{1cm}}$  0.1  
 (d)  $3.2 \underline{\hspace{1cm}}$  4.1      (e)  $0.008 \underline{\hspace{1cm}}$  0.09      (f)  $0.7 \underline{\hspace{1cm}}$  0.07



- 2** The number 43 can be written in expanded form as 'four tens and three ones'. Write the following in expanded form.

(a) 72      (b) 603      (c) 9251      (d) 11 080



- 3** Write each of the following (i) in words and (ii) as decimals.

(a)  $\frac{7}{10}$       (b)  $\frac{8}{1000}$       (c)  $\frac{3}{100}$       (d)  $1\frac{9}{10}$



- 4** Calculate:

(a)	$\begin{array}{r} 34 \\ + 76 \\ \hline \end{array}$	$\begin{array}{r} 925 \\ + 610 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ 409 \\ + 3 \\ \hline \end{array}$
(d)			$\begin{array}{r} 459 \\ 6013 \\ + 27 \\ \hline \end{array}$



- 5** Calculate:

(a)	$\begin{array}{r} 74 \\ - 25 \\ \hline \end{array}$	$\begin{array}{r} 823 \\ - 376 \\ \hline \end{array}$	$\begin{array}{r} 8289 \\ - 384 \\ \hline \end{array}$
(d)			$\begin{array}{r} 2000 \\ - 352 \\ \hline \end{array}$



- 6** Calculate:

(a)  $2 \times 17$       (b)  $25 \times 96$       (c)  $3 \times 48$       (d)  $51 \times 800$



- 7** Perform the following divisions.

(a)  $362 \div 2$       (b)  $9459 \div 9$       (c)  $5600 \div 200$       (d)  $4500 \div 30$



- 8** Calculate:

(a) $70 \times 100$	(b) $12 \times 10\ 000$	(c) $9200 \times 1000$
(d) $80 \div 10$	(e) $3600 \div 100$	(f) $250\ 000 \div 1000$
(g) $1.2 \times 10$	(h) $6.3 \times 100$	(i) $0.0427 \times 1000$
(j) $58 \div 10$	(k) $901 \div 100$	(l) $76.2 \div 1000$



- 9** Write these percentages as fractions in simplest form.

(a) 50%      (b) 10%      (c) 25%      (d) 20%      (e) 1%      (f) 60%

## Key Words

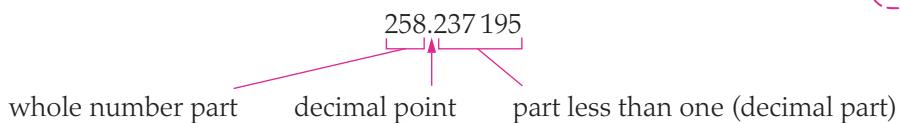
decimal places	part:whole ratio	recurring decimal
decimal point	per cent	round
digit	place value	terminating decimal
equivalent ratios	rate	unit price
part:part ratio	ratio	unitary method

# Place value and comparing decimals

## 4.1

### Decimal place value

Like fractions, decimals are used to write numbers that have a part that is less than one. The **decimal point** is used to separate the whole number part from the part less than one.



The word decimal comes from the Latin word, *dec* or *decem*, meaning 'tenth part'. December used to be the tenth month on the Roman calendar.



The number 258.237195 is made up of nine individual **digits**. Because six of the digits occur after the decimal point, we say that the number has six **decimal places**.

More examples: 3.25 has three digits and two decimal places

45.079 has five digits and three decimal places.

We can write each digit of the number into a 'place value table' to show this:

Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)	Thousands $\frac{1}{1000}$ (0.001)	Ten-thousandths $\frac{1}{10\,000}$ (0.0001)	Hundred-thousandths $\frac{1}{100\,000}$ (0.00001)	Millionths $\frac{1}{1\,000\,000}$ (0.000001)
2	5	8	•	2	3	7	1	9	5

The **place value** of each column (hundreds, tens, ones, tenths, hundredths etc.) is one-tenth of the column to the left of it. As we move across the columns from left to right, we divide by 10 each time. This pattern is continued past the decimal point to get the values of the decimal places. You are probably familiar with the first three decimal places of tenths, hundredths and thousandths; however, we can keep dividing by 10 to get ten-thousandths, hundred-thousandths and millionths.

If there is no digit for a particular place value in a decimal number, we use a zero to show this. For example, five and two tenths and three thousandths is written as 5.203. It is important to include the zero between the 2 and the 3. Leaving it out would mean 5.203 becomes 5.23, which is a different number.

### Expressing decimals in written form

Decimal numbers can be written in several ways:

- Decimal form:

258.237195

- Expanded fractional form:

$$258 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10\,000} + \frac{9}{100\,000} + \frac{5}{1\,000\,000}$$

- Expanded word form:

two hundreds, five tens, eight ones, two tenths, three hundredths, seven thousandths, one ten-thousandth, nine hundred-thousandths and five millionths.

## Expressing decimals in verbal form

- Say each of the digits after the decimal point individually. Do not use whole number language to describe the decimal part of the number. For example, 14.625 is said as 'fourteen point six two five', not 'fourteen point six hundred and twenty five'.
- Fraction equivalents can also be used to describe decimals verbally. For example, 0.63 can be said as 'sixty three hundredths'. This is an equivalent way of saying six tenths and three hundredths.

$$\frac{63}{100} = \frac{6}{10} + \frac{3}{100}$$

### Worked Example 1

WE 1

(a) Write  $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10000}$  as a decimal.

(b) Write 6.2807 in expanded fractional form.

#### Thinking

(a) Imagine the numerators of each fraction placed into their place value columns. You don't actually have to draw them.

(As there are no  $\frac{1}{100}$ s, we write a zero in the 'hundredths' column.)

Ones	Tenths	Hundredths	Thousands	Ten-thousandths
1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
4	•	3	0	6

#### Working

$$(a) 4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10000} \\ = 4.3065$$

(b) 1 Imagine the decimal digits placed into their place value columns.

Ones	Tenths	Hundredths	Thousands	Ten-thousandths
1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
6	•	2	8	0

(b) 6.2807

2 Write a series of fractions using the headings of the place value columns. There is no need to include columns with 0 (e.g.  $\frac{0}{1000}$ ).

$$= 6 + \frac{2}{10} + \frac{8}{100} + \frac{7}{10000}$$

## Worked Example 2

**WE2**

- (a) Write seven units, three tenths, six hundredths, seven thousandths and four hundred-thousandths as a decimal.
- (b) Write 28.0045 in expanded word form.

**Thinking**

**Working**

- (a) Think of, or look at, a place value table.  
Place the digits described in the corresponding place value columns.

(a) 7.367 04

Ones	.	Tenths	Hundredths	Thousands	Ten-thousandths	Hundred-thousandths
1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$	$\frac{1}{100\,000}$
7	•	3	6	7	0	4

- (b) 1 Think of, or look at, a place value table.  
Place the digits in their place value columns, including zero.

(b)

Tens	Ones	.	Tenths	Hundredths	Thousands	Ten-thousandths
10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$
2	8	•	0	0	4	5

- 2 Write the non-zero digits in words, with the place value column heading after each one.

28.0045

= Two tens, eight ones, four thousandths and five ten-thousandths

## Worked Example 3

**WE3**

- (a) Write the value of the 6 in 4.368 as a fraction.
- (b) Write the value of the 9 in 0.0109 in words.

**Thinking**

**Working**

- (a) Identify the place value of the digit, then write the digit as the numerator and the place value as the denominator of the fraction.

(a)  $\frac{6}{100}$

- (b) Identify the place value of the digit, then write the digit, followed by the place value, in words.

(b) nine ten-thousandths

## Comparing decimals

We often need to compare two decimal numbers to decide which one is greater. We might be comparing prices of goods or working out who won a race. Look at the decimal numbers in this photo of the finishing board of the men's 100 m final at the 2008 Olympics. Usain Bolt's time is placed first because it is the smallest decimal number. How much did Bolt win by?

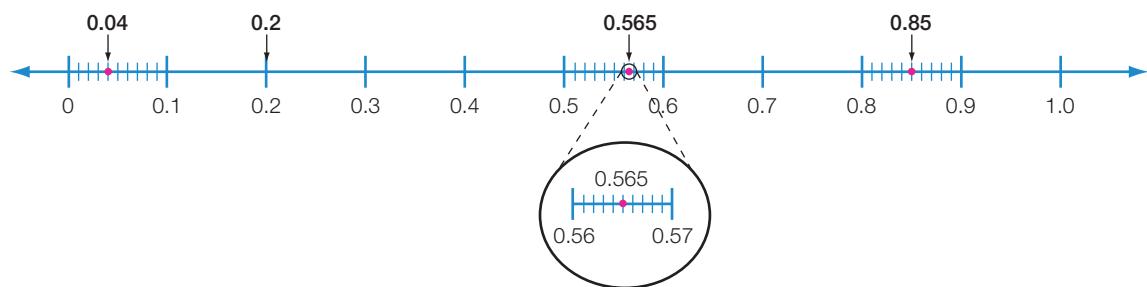
We use the relation symbols of less than ( $<$ ), greater than ( $>$ ) and equal to ( $=$ ) when comparing decimals.

<b>MEN'S 100M FINAL</b>			<b>WR 9.72</b>
1	BOLT Usain	JAM	OR 9.84
2	THOMPSON R.	TRI	9.69 WR
3	DIX Walter	USA	9.89 PB
4	MARTINA C.	AHO	9.91 PB
5	POWELL Asafa	JAM	9.93 NR
6	FRATER M.	JAM	9.97 PB
<b>WIND 0.0</b>			

## Comparing decimals using a number line

We can use a number line to compare decimals. Numbers get larger as we move up the number line from left to right. The number line below is marked off in tenths. The space between 0 and 0.1 and between 0.5 and 0.6 shows 1 tenth divided up into 10 parts, each equal to 1 hundredth.  $\frac{1}{10} = \frac{10}{100}$

The positions of the decimal numbers 0.04, 0.2, 0.565 and 0.85 are shown with arrows.



The interval between 0.56 and 0.57 has been magnified to show the position of 0.565 more clearly. It shows 1 hundredth divided up into 10 parts, each equal to 1 thousandth.  $\frac{1}{100} = \frac{10}{1000}$

From the decimal numbers shown on the number line, we can see that:

The decimal number with the most digits is not necessarily the largest; e.g.  $0.565 < 0.85$ .  
A number's size depends on the value and position of the digits; e.g.  $0.2 > 0.04$ .

## Comparing decimals by comparing digits

The higher its place value, the greater the worth of a digit. For example,  $0.2 > 0.04$ , because  $\frac{2}{10} > \frac{4}{100}$ , so 0.2 lies further to the right on the number line than 0.04. To determine which of two numbers is bigger, we compare the digits in each place value column, beginning with the highest place values.

## Worked Example 4

**We4**

Write < or > between each of the following pairs of decimals to make a true statement.

(a) 5.173 \_\_\_\_\_ 5.0731      (b) 0.472 39 \_\_\_\_\_ 0.4731      (c) 3.5 \_\_\_\_\_ 3.51

**Thinking**

**Working**

- |   |  |
|---|--|
| <p>(a) 1 Compare the whole number parts of each decimal. (Here, they are the same number, 5.)</p> <p>2 If the whole number parts are the same, compare the tenths digits to see which is greater. (Here, 1 is greater than 0, so 5.173 is the larger number.)</p> <p>3 Write a relation symbol (&lt; or &gt;) between the two numbers.</p>  | <p>(a)</p> <p><math>5.173 &gt; 5.0731</math></p>     |
| <p>(b) 1 Compare the whole number parts of each decimal. (Here, they are the same, 0.)</p> <p>2 If the whole number parts are the same, compare the tenths digits. (Here, they are the same, 4.)</p> <p>3 If the tenths digits are the same, compare the hundredths digits. (Here, they are the same, 7.)</p> <p>4 Repeat until you get different digits. (The thousandths digits are 2 and 3, respectively. So, 0.472 39 is less than 0.4731.)</p> <p>5 Write a relation symbol between the two numbers.</p> | <p>(b)</p> <p><math>0.472\ 39 &lt; 0.4731</math></p> |
| <p>(c) 1 Add zeroes to the end of one of the decimals to help you compare. This doesn't change its value (3.5 is the same as 3.50).</p> <p>2 Compare the two decimals as in previous examples. (3.50 has the smaller hundredths digit, so it is the smaller decimal.)</p> <p>3 Write &lt; or &gt; between the two numbers.</p>  | <p>(c)</p> <p><math>3.5 &lt; 3.51</math></p>         |

# 4.1 Place value and comparing decimals

## Navigator

**Answers  
page 641**

Q1, Q2, Q3, Q4 Column 1, Q5  
Column 1, Q7, Q8 (a) & (b), Q9  
Column 1, Q10, Q11, Q13, Q14,  
Q15, Q17, Q19

Q1, Q2, Q3, Q4 Column 2, Q5  
Column 2, Q6, Q8 (c) & (d), Q9  
Column 2, Q10, Q11, Q12, Q14,  
Q15, Q16, Q18, Q19

Q1 (b), Q2 (b), Q3, Q4 Column 2,  
Q5 Column 2, Q6, Q7, Q8 (c) &  
(d), Q9 Column 2, Q11, Q12,  
Q13, Q14, Q15, Q16, Q17, Q18,  
Q19

## Fluency

**WE1**

- 1 (a) Write each of the following as a decimal.

$$(i) 45 + \frac{4}{10} + \frac{6}{100} + \frac{2}{1000}$$

$$(ii) 12 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000} + \frac{3}{10\,000} + \frac{7}{100\,000} + \frac{2}{1\,000\,000}$$

$$(iii) 3 + \frac{7}{10} + \frac{9}{100} + \frac{8}{1000} + \frac{5}{10\,000}$$

$$(iv) 1 + \frac{8}{1000} + \frac{3}{10\,000} + \frac{3}{100\,000} + \frac{2}{1\,000\,000}$$

$$(v) \frac{7}{10} + \frac{8}{100} + \frac{6}{1000} + \frac{6}{100\,000}$$

$$(vi) 7 + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100\,000} + \frac{7}{1\,000\,000}$$

Don't forget to write a zero if you have no digits to go in a place value column.



- (b) Write each of the following in expanded fractional form.

$$(i) 6.63$$

$$(ii) 0.921$$

$$(iii) 0.7345$$

$$(iv) 7.826$$

$$(v) 23.913\,04$$

$$(vi) 45.004\,589$$

- 2 (a) Write each of the following as a decimal.

(i) six ones and five tenths

(ii) nine tenths and seven hundredths

(iii) two tenths, seven hundredths and three thousandths

(iv) three tens, seven ones, four tenths, two hundredths and one ten-thousandth

(v) one ten, four ones, nine tenths, five hundredths, seven thousandths, six ten-thousandths, two hundred-thousandths and three millionths

(vi) seven hundreds, four thousandths, five ten-thousandths and nine hundred-thousandths

- (b) Write each of the following in expanded word form.

$$(i) 5.2$$

$$(ii) 4.9$$

$$(iii) 34.17$$

$$(iv) 0.61$$

$$(v) 2.794$$

$$(vi) 7.5092$$

$$(vii) 35.865\,43$$

$$(viii) 0.820\,027$$

**WE2**

- 3 (a) Write the value of the 2 in each of the following as a fraction.

(i) 6.012      (ii) 0.00452      (iii) 3.287      (iv) 2.034

- (b) Write the value of the 7 in each of the following in words.

(i) 5.734      (ii) 0.0076      (iii) 1.2037      (iv) 8.130037

- 4 Write  $<$  or  $>$  between each of the following pairs of decimals to make a true statement.

- |                         |                         |
|-------------------------|-------------------------|
| (a) 2.4 _____ 0.42      | (b) 2.32 _____ 1.955    |
| (c) 0.65 _____ 0.57     | (d) 0.3003 _____ 0.333  |
| (e) 4.7038 _____ 4.7312 | (f) 8.251 _____ 8.2501  |
| (g) 7.02 _____ 7.002    | (h) 4.7367 _____ 4.7376 |
| (i) 0.927 _____ 0.92734 | (j) 6.013 _____ 6.01    |
| (k) 3.406 _____ 3.4063  | (l) 0.9995 _____ 0.9986 |

- 5 Write TRUE or FALSE for each of the following.

- |                         |                         |
|-------------------------|-------------------------|
| (a) $7.5 < 5.77$        | (b) $4.1 > 4.12$        |
| (c) $6.08 > 6.8$        | (d) $67.54 < 67.504$    |
| (e) $3.023 < 3.203$     | (f) $0.547 > 0.54708$   |
| (g) $2.000012 < 2.0001$ | (h) $4.14529 > 4.20001$ |

- 6 53.017 expressed in expanded word form is:

- A five tens, three ones, one tenth and seven hundredths
- B five tens, three ones, one hundredth and seven thousandths
- C five tenths, three hundredths, one ten-thousandth and seven hundred-thousandths
- D five tens, three ones, one tenth and seven thousandths

- 7 Nine hundredths, four thousandths and three ten-thousandths is equal to:

- A 0.09043      B 0.0943      C 0.943      D 9.43

- 8 For each of the following sets of numbers, copy the number line shown, then mark the positions of the numbers with a labelled arrow. (You may have to estimate the position of some.)



- |                                 |                                |
|---------------------------------|--------------------------------|
| (a) 2.05, 2.09, 2.6, 2.12, 2.59 | (b) 2.0, 2.8, 2.88, 2.9, 2.805 |
| (c) 2.7, 2.4, 2.07, 2.04, 2.407 | (d) 2.2, 2.4, 2.85, 2.35, 2.05 |

- 9 Write each set of decimals in order from smallest to largest.

- |                            |                            |
|----------------------------|----------------------------|
| (a) 2.3, 2.03, 2.13        | (b) 8.7, 8.007, 8.67       |
| (c) 6.646, 6.6403, 6.64    | (d) 0.0095, 0.0905, 0.0509 |
| (e) 5.3281, 5.38, 5.003821 | (f) 3.616, 3.116, 3.661    |
| (g) 0.92, 0.29, 0.092      | (h) 0.85, 0.815, 0.086     |

## Understanding

- 10 Joel wrote down the five-digit number as he saw it on his electronic stopwatch: 37901. However, he forgot to put in the decimal point. Anna knew that the race was timed to a thousandth of a second and was able to put the decimal point in the correct place. Write the number with the decimal point in the correct place.

WE3

WE4

- 11 The difference in times between two skiers in a downhill race was 0.437 of a second.

Write this: (a) in expanded fraction form

(b) as a single fraction.

- 12 For each of the following sets of decimals, draw a section of the number line, marked in either tenths or hundredths, and indicate the position of each number in the set.

(a) 0.4, 0.72, 1.01

(b) 1.7, 1.25, 0.95

(c) 0.06, 0.045, 0.038

(d) 2.9, 3.2, 2.75, 3.06

- 13 Jane records her three best practice times for the 100 m sprint. They are as follows.

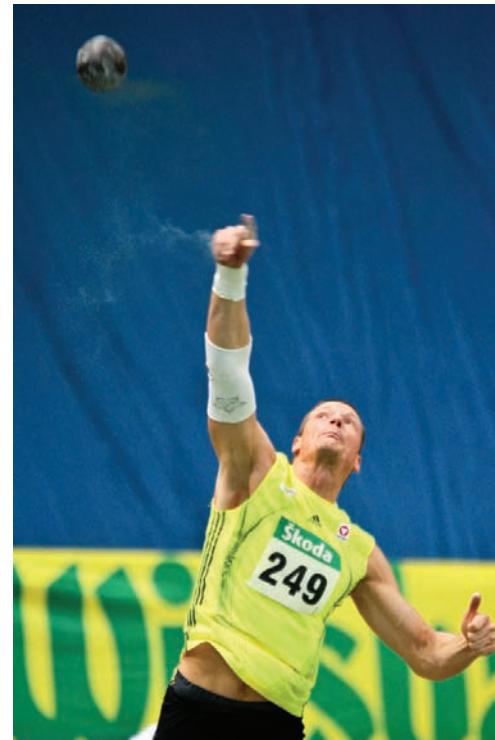
13.95 seconds, 13.08 seconds and

13.69 seconds

(a) Which was Jane's fastest time?

(b) Which was Jane's slowest time?

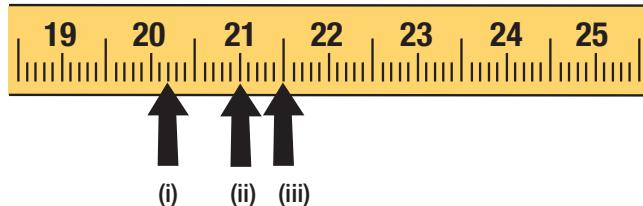
- 14 Egor throws a shot-put the following distances during a competition: 25.6 m, 25.56 m and 25.081 m. Which distance was Egor's longest throw?



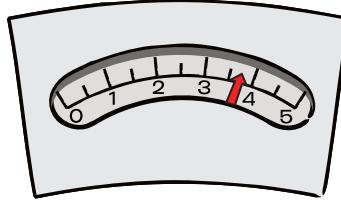
## Reasoning

- 15 Write the value of the decimals indicated by the arrows on the following scales.

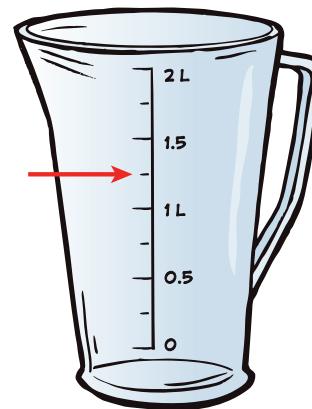
(a)



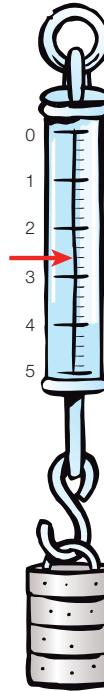
(b)



(c)



(d)



- 16 State the mistakes that the following students have made.

(a) Minh states that the decimal 34.162 has a 2 in the hundredths column.

(b) Al writes four hundreds, nine ones, six tenths and seven hundredths as 409.067.

(c) Max writes seven ones, eight hundredths and nine ten-thousandths as 0.7809.

(d) Polly writes 0.9056 as  $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$ .

## Open-ended

- 17 (a) Write down at least six numbers between 4.5 and 4.7.
- (b) Put your numbers from (a) in ascending order.
- 18 (a) Use any digit from 0 to 9 in each box to make a correct statement. Digits can be repeated. Find at least three different answers.  
 $\square.\square\square < 5.\square$
- (b) Explain any limitations on the other numbers that can be used when certain numbers are used in some boxes.
- 19 Donna has cut out the digits 2, 5 and 7, as well as a decimal point, from coloured paper to stick on a poster to show a decimal number. If she uses all three digits:
- (a) how many different decimal numbers with two decimal places could she make  
(b) how many decimal numbers greater than 5.4 could she make?

# Outside the Square Game

### Duelling decimals

**Equipment required:** 2 brains, 1 die

#### How to win:

The aim is to be the first player to 5 points.

#### How to play:

- One player rolls the die to determine the number of digits in the duel. Each player then puts the appropriate number of boxes on their paper after the decimal point.

e.g. if a 3 is rolled, each would draw:

0 .         0 .

- Each player then takes turns to roll the die and write the number from the roll in a box of their choice.

After all of the boxes have been filled, the player who has created the bigger number wins a point for that round.

For example, the players may have filled in their boxes like this:

0 .  2  5  3      0 .  6  3  1

Here, the second player would win a point.

- This process is repeated with players taking turns to roll the determining number of digits per round.



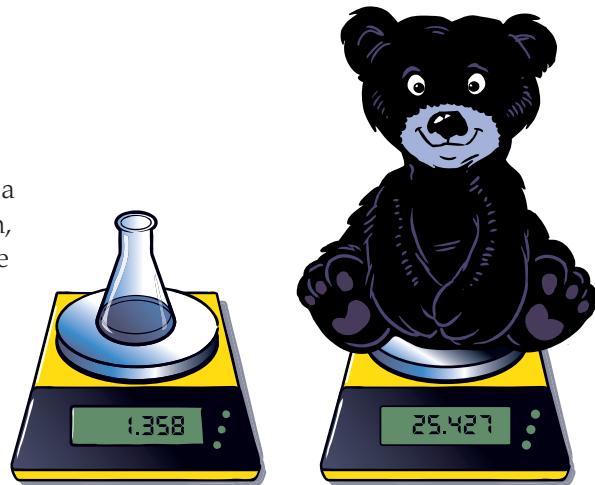
# 4.2

# Rounding decimals

Decimal numbers sometimes contain more decimal places than we require. It depends on the situation.

A scientist might need to know the mass of a chemical accurate to a thousandth of a gram, or three decimal places. They could place the chemical on an electronic scale and get a reading such as 1.358 g.

A zookeeper wanting to find the mass of a bear cub could place the cub on a similar electronic scale and get a reading such as 25.427 kg. However, 2 hundredths or 7 thousandths of a kilogram is an insignificant amount compared to the whole mass of the cub. The zookeeper would **round** the bear cub's mass to 25.4 kg, or just 25 kg.



The number line below shows how the number 1.364 is rounded:

- To the nearest tenth (1 decimal place): 1.364 lies between the tenths values of 1.3 and 1.4. The 6 in the hundredths place value column places it closer to 1.4. 1.364 rounded to the nearest tenth is *rounded up* to 1.4.
- To the nearest hundredth (2 decimal places): 1.364 lies between the hundredths values of 1.36 and 1.37. The 4 in the thousandths column places it closer to 1.36. 1.364 rounded to the nearest hundredth is *rounded down* to 1.36.



To round decimal numbers:

- Step 1** Determine the number of decimal places that you are required to round to. The digit in this place will either remain as it is, or will increase by one.
- Step 2** Look at the next digit to the right of the one that you are rounding to. If this digit is:
- 0, 1, 2, 3 or 4, then leave the digit considered in step 1 as is and delete all digits after it
  - 5, 6, 7, 8 or 9, then increase the digit considered in step 1 by one and delete all digits after it.

If the number in the place value you are rounding to is 9 and you need to increase it, make the 9 a 0, and add one to the digit in front of it. For example, 1.499 rounded to the nearest hundredth is 1.50. We write a 0 in the hundredths column to show this (even though it makes no difference to the value of the number).

## Worked Example 5

**We5**

Round the following decimals to the number of decimal places shown in brackets.

(a) 3.785 (2)

(b) 0.958 34 (3)

(c) 6.214 96 (4)

### Thinking

### Working

(a) 1 Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 3.78 or 3.79.)

(a) 3.785

2 Look at the digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 5 tells us to round up, so the 8 becomes a 9.)

3.785

3 Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.

3.79

(b) 1 Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 0.958 or 0.959.)

(b) 0.958 34

2 Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 3 tells us that we round down, so we leave the 8 as it is.)

0.95834

3 Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.

0.958

(c) 1 Decide which digit you will be rounding. (Here, it is the 9, so the answer will either be 6.2149 or 6.2150.)

(c) 6.214 96

2 Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 6 tells us to round up, but rounding the 9 up will give 10 ten-thousandths, which is equal to 1 thousandth. This means that the thousandths digit is increased by 1.)

6.21496

3 Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. (Here, we leave the zero in the rounded decimal place.)

6.2150

## Rounding money

We write money using decimal notation, usually with 2 decimal places. Because \$1 = 100c, or 10 lots of 10c, the tenths place value column shows how many lots of 10c make up the amount, and the hundredths column shows how many lots of 1c make up the amount.

In the 1990s, Australia stopped using 1 cent and 2 cent pieces, making the 5 cent coin the smallest coin in use. This means that all money amounts paid in cash must be rounded to the nearest 5 cents (5c).

If money is being paid in cash, the following rules apply.

<i>If the amount ends in...</i>	<i>then it is rounded...</i>	<i>the last digit becomes...</i>
1 or 2 cents	down	0
3 or 4 cents	up	5
6 or 7 cents	down	5
8 or 9 cents	up	0

If the amount ends in 0 or 5 cents, then the exact amount is paid.

### Worked Example 6

WE6

Round the following amounts of money to the nearest 5 cents.

(a) \$8.53

(b) \$7.02

(c) \$4.99

#### Thinking

#### Working

- (a) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 3 up to the nearest 5 cents.)

(a) \$8.55

- (b) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 2 down to the nearest multiple if 10 cents, which is 0.)

(b) \$7.00

- (c) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 9 up to the nearest multiple of 10 cents, which is 100.)

(c) \$5.00

# 4.2 Rounding decimals

## Navigator

Q1 Columns 1 & 2, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q10, Q12, Q14, Q16

Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q16

Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16

**Answers**  
page 642

## Fluency

- 1 Round the following decimals to the number of decimal places shown in brackets.

- (a) 4.88 (1)      (b) 6.72 (1)      (c) 7.635 (2)  
 (d) 4.552 (2)      (e) 0.6416 (3)      (f) 3.2772 (2)  
 (g) 0.314 42 (4)      (h) 11.828 55 (2)      (i) 2.917 96 (2)  
 (j) 18.499 95 (3)      (k) 90.899 99 (4)      (l) 18.999 999 (3)  
 (m) 23.4097 (2)      (n) 57.0804 (3)      (o) 259.899 99 (4)

- 2 Round the following amounts of money to the nearest 5 cents.

- (a) \$4.52      (b) \$2.76      (c) \$11.03      (d) \$23.88  
 (e) \$176.35      (f) \$542.06      (g) \$789.54      (h) \$56.75  
 (i) \$6774.99      (j) \$1149.97      (k) \$8989.99      (l) \$9999.99

- 3 0.837 256 rounded to the nearest ten-thousandth is:

- A 0.837      B 0.8372      C 0.83726      D 0.8373



**WE 5**

**WE 6**

## Understanding

- 4 At the 2009 World Athletics Championships, Usain Bolt set a new 100 m record time of 9.58 seconds. Round this time to the nearest tenth of a second.

- 5 The total shown on the receipt for Russell's grocery shopping is \$53.67. If Russell is paying in cash, how much money will he have to hand over?

- 6 Sir Donald Bradman's cricket batting average was 99.943 (rounded to the nearest thousandth). What would be his average rounded to the nearest:

- (a) tenth      (b) hundredth?

- 7 The top temperature recorded one day in Mildura was 41.6°C. What figure should the weather presenter read out in his weather report as the day's maximum if he rounds the temperature reading to the nearest whole number (i.e. no decimal places)?

- 8 The price of a box of chocolates is \$11.97. If Beau buys the chocolates and pays with cash, how much will he pay?

- 9 A physics student calculates the speed of sound to be 352.153 metres per second. In case her equipment is not accurate enough for this result to be reliable, she decides to round it to the nearest tenth. What value does she write in her report, assuming she rounds off correctly?



## Reasoning

- 10** Ruth calculates the length of shelf required for a wall unit to be 1.266 66 m. As her tape measure is only accurate enough to measure in centimetres (cm), she needs to round off this figure to the nearest cm. If 1 cm = 0.01 m, what length should she measure?



- 11** In a cycling sprint race at a velodrome, the following results were recorded: 12.149 s, 12.143 s, 12.138 s, 12.152 s and 12.157 s.

- (a) Write these in order from fastest to slowest.
- (b) It was found that there was a small error in the timing equipment and the results were to be recorded to the nearest hundredth. What were the new times?
- (c) Does this affect the race result?

- 12** (a) The total cost of Rani's shopping was \$20.34. Round this amount to the nearest 5 cents.

The individual costs of the items Rani bought were:

Cereal	\$6.93
Yoghurt	\$4.78
Bananas	\$2.51
Juice	\$3.63
Pasta	\$2.49

- (b) How much more would Rani have paid if the individual prices of the items were each rounded to the nearest 5 cents?

## Open-ended

- 13** Write at least three different decimal numbers that, when rounded, would give the number 3.79.

- 14** Write down two prices that would give \$100 dollars when rounded to:

- (a) the nearest 5 cents
- (b) the nearest dollar.

- 15** Taking 2.548 46 and rounding it to four decimal places gives 2.5485, then rounding this to three decimal places gives 2.549.

What answer do you get when rounding 2.548 46 to three decimal places?

Explain why this is different, and what is wrong with doing it the first way.

- 16** Why do you think 1 and 2 cent coins were taken out of circulation? List two or three possible reasons why a government might make this decision.

# Decimals and fractions

# 4.3

## Writing a decimal as a fraction

As we have seen, a decimal can be written in expanded fractional form. This expanded form can be simplified to describe a decimal as a single fraction or as a mixed number.

$$\begin{aligned}\text{For example: } 3.72 &= 3 + \frac{7}{10} + \frac{2}{100} \\ &= 3 + \frac{70}{100} + \frac{2}{100} \\ &= 3 + \frac{72}{100} \\ &= 3\frac{72}{100} \text{ or } \frac{372}{100} \\ &= 3\frac{18}{25} \text{ or } \frac{93}{25}\end{aligned}$$

The place value of the last digit (hundredths) gives the denominator of the single, unsimplified fraction (100), and the digits themselves form the numerator (72). Remember that fractions should always be given in simplest form.

### Worked Example 7

WE7

Convert each of the following decimals to fractions, giving your answers in simplest form.

(a) 6.28

(b) 0.3125

#### Thinking

- (a) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.
- 2 Simplify the fraction if possible. (Here, a common factor of 4 has been cancelled.)

- (b) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.

- 2 Simplify the fraction if possible. (Here, we cancel common factors of 5, then 25, then 5 again.)

#### Working

$$\begin{aligned}(a) \quad 6.28 &= 6\frac{28}{100} \text{ or } \frac{628}{100} \\ &= 6\frac{7}{25} \text{ or } \frac{157}{25}\end{aligned}$$

$$\begin{aligned}(b) \quad 0.3125 &= \frac{3125}{10\,000} \\ &= \frac{625}{2000} \\ &= \frac{25}{80} \\ &= \frac{5}{16}\end{aligned}$$

The decimal point was first used in the 1600s by mathematician Bartholo Maeus Pitiscus. Thanks, Bart!



## Writing a fraction as a decimal

There are two methods we can use to convert a number in fraction form to decimal form:

**Method 1:** Write the fraction as an equivalent fraction (or mixed number) with a denominator of 10, 100, 1000 etc. Then, write the digits of the numerator of this new fraction as the decimal place values.

$$\text{For example: } \frac{4}{5} = \frac{8}{10} = 0.8$$

$$\frac{13}{20} = \frac{65}{100} = 0.65$$

This method is only convenient for fractions with denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

**Method 2:** The line between the numerator and the denominator is equivalent to the division symbol,  $\div$ . To convert a fraction to a decimal, we simply perform the division. This method can be used for all fractions.

$$\text{For example: } \frac{7}{8} = 7 \div 8 = 0.875$$

$$\frac{37}{40} = 37 \div 40 = 0.925$$

### Worked Example 8

WE8

Convert each of the following fractions to decimals.

(a)  $\frac{7}{5}$

(b)  $\frac{3}{8}$

(c)  $\frac{4}{3}$

#### Thinking

#### Working

(a) 1 Is the denominator a factor of 10, 100 or 1000? (Yes)

(a)  $\frac{7}{5}$

2 Write the fraction as an equivalent fraction with a denominator of 10, 100, or 1000 (10 in this case).

$$= \frac{14}{10}$$

3 If the fraction is improper, write it as a mixed number.

$$= 1\frac{4}{10}$$

4 Write any whole numbers on the left of a decimal point. (If there is no whole number part, write a zero.) Write the numerator of the fraction on the right of the decimal point.

$$= 1.4$$

$$\frac{7}{5} = 1.4$$

- (b) 1 Is the denominator a factor of 10, 100 or 1000? (No)

$$(b) \frac{3}{8}$$

- 2 Write the fraction as a division calculation.

$$3 \div 8$$

$$8) \overline{)3}$$

- 3 Perform the division. If you are unable to begin because the divisor is a greater number (here,  $8 > 3$ ), add a decimal point and a zero to the dividend. Place a decimal point in the quotient and a zero in front of it.

$$3.0 \div 8$$

$$8) \overline{)3.0}$$

- 4 Continue to divide as though the decimal point was not there, adding extra zeroes where required.

$$3.000 \div 8 \quad 8) \overline{)3.000}$$

$$8) \overline{)3.000}$$

- 5 State the answer.

$$\frac{3}{8} = 0.375$$

- (c) 1 Is the denominator a factor of 10, 100 or 1000? (No)

$$(c) \frac{4}{3}$$

- 2 Write the fraction as a division calculation.

$$4 \div 3$$

$$3) \overline{)4}$$

- 3 Perform the division. Add a decimal point and zeroes to the dividend to continue the division, placing a corresponding decimal point in the quotient.

$$4.000 \div 3$$

$$3) \overline{)4.000}$$

- 4 If the division gives a repeating pattern, stop. Write the answer with a dot above the repeating digit.

$$\frac{4}{3} = 1.\dot{3}$$

## Recurring decimals

Dividing the numerator by the denominator sometimes results in a **recurring decimal**—one that has a repeating pattern of digits, such as in part (c) in the previous Worked Example. To write a recurring decimal, place dots or a line above the digits that form the repeating pattern. If it is a single repeating digit, place a dot or a line above that digit. If more than one digit forms the repeating pattern, place a line above the repeating digits, or a dot above the first and last digit in the repeating pattern.

For example:  $\frac{2}{3} = 0.\dot{6}$ ,  $\frac{1}{12} = 0.08\dot{3}$ ,  $\frac{4}{7} = 0.571\dot{4}\dot{2}\dot{8}$  or  $0.\dot{5}71\dot{4}\dot{2}\dot{8}$

A recurring decimal can also be rounded to give an approximate value; for example,  $0.\dot{6}$  rounded to three decimal places is 0.667.

A decimal that is not recurring, but has a finite number of digits (such as 1.4 or 0.375 in the previous Worked Example) is called a **terminating decimal**.

## Fractions and decimals on a calculator

On a scientific calculator, the  $a^{\frac{b}{c}}$  or  $\text{S}\leftrightarrow\text{D}$  key can be used to convert between the fraction and the decimal forms of a number. Pressing the key again will convert it from decimal form back to the fraction.

# 4.3 Decimals and fractions

## Navigator

**Answers**  
page 642

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q12 (a)

Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12 (b), Q13

**Equipment required:** A calculator may be used for Question 2 (i)–(p)

## Fluency

**W.E7**

- 1 Convert each of the following decimals to fractions, giving your answers in simplest form.

- (a) 4.1      (b) 6.5      (c) 8.6      (d) 9.4  
 (e) 4.71      (f) 2.37      (g) 0.35      (h) 2.48  
 (i) 5.009      (j) 0.884      (k) 6.128      (l) 3.172  
 (m) 7.045      (n) 0.088      (o) 6.0015      (p) 1.0075

Make sure you put the correct number of zeroes in the denominator.

**W.E8**

- 2 Convert each of the following fractions to decimals.

- (a)  $\frac{7}{10}$       (b)  $\frac{9}{100}$       (c)  $\frac{17}{100}$       (d)  $\frac{123}{1000}$   
 (e)  $\frac{1}{5}$       (f)  $\frac{3}{25}$       (g)  $\frac{13}{50}$       (h)  $\frac{7}{4}$   
 (i)  $\frac{7}{8}$       (j)  $\frac{11}{16}$       (k)  $\frac{23}{40}$       (l)  $\frac{47}{60}$   
 (m)  $\frac{1}{9}$       (n)  $\frac{7}{12}$       (o)  $\frac{17}{6}$       (p)  $\frac{16}{15}$



- 3 0.55 expressed as a fraction in simplest form is:

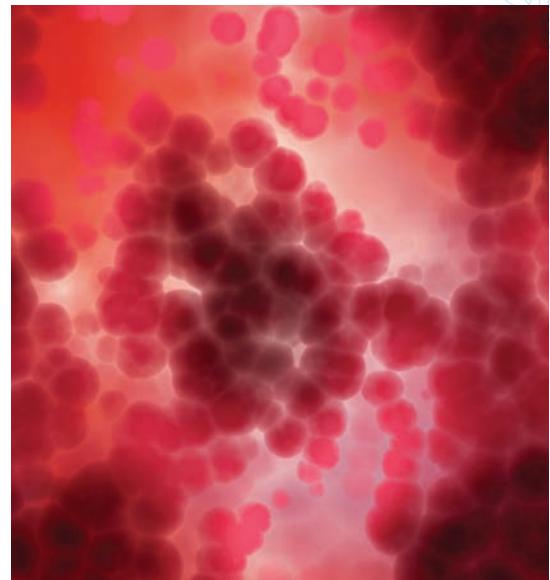
A  $\frac{55}{1000}$       B  $\frac{11}{200}$       C  $\frac{11}{25}$       D  $\frac{11}{20}$

- 4 2.000 47 is equal to:

A  $2\frac{47}{100}$       B  $2\frac{47}{1000}$       C  $2\frac{47}{10\,000}$       D  $2\frac{47}{100\,000}$

## Understanding

- 5 As part of a science experiment, Galina's reaction time was measured to be 0.48 seconds. What fraction of a second is this?
- 6 One inch is approximately equal to 2.54 centimetres. What is this when expressed as a fraction in simplest form?
- 7 Hair grows at an average rate of 1.35 cm per month. Write this as a mixed number in simplest form.
- 8 A virus particle has a diameter of 0.000024 mm. Write this as a fraction in simplest form.
- 9 A red blood cell's diameter is  $\frac{1}{200\,000}$  of a metre. Write this measurement in decimal form.



## Reasoning

- 10 A bank offers its customers an interest rate of 5.85 per cent. Write this rate in single fraction form. Is this rate higher than the  $5\frac{3}{4}$  per cent offered by another bank?
- 11 Jeremy has five different containers and is trying to place them in ascending order of size. The volumes of all five containers are measured in litres but some are given using decimals and others using fractions. The tubs have the following volumes:  $4\frac{1}{5}$ , 4.375, 4.865,  $4\frac{9}{10}$ ,  $4\frac{1}{2}$ . Help Jeremy put them in order from the smallest to the largest.

## Open-ended

- 12 (a) Write three different fractions that have decimal values between 3.4 and 3.5 and have a denominator of 100.
- (b) Write three different fractions that have decimal values between 3.4 and 3.5 where each fraction has a different denominator that is not a power of 10 (such as 100 or 1000).
- 13 Two numbers are rounded to one decimal place to give 5.6. What could those two numbers be? Give your answers in fraction form.

## Outside the Square Puzzle

### What are the neighbours having for dinner?

Four children from the same class in school live on the same street. Find out in which house each child lives, and what they had for dinner yesterday evening.

- Steve didn't have pizza for dinner, which was the meal eaten at house number 15.

- The fish and chips were eaten at house number 1, whereas Joseph lives at house number 11.
- The salad was eaten in a lower house number than the lasagne.

- One of Rowena's classmates lives in house number 5.
- Harriet lives in the highest number house of the four classmates.



# 4.4

# Decimal addition and subtraction

Decimal numbers are added and subtracted in the same way as whole numbers. As with whole numbers, we must be sure that we add or subtract digits of the same place value. The best way to ensure this is to line up the decimal points underneath each other.

To add or subtract decimals, line up the decimal points so that digits with the same place value are underneath each other.

If the numbers have different numbers of decimal places, zeroes can be written in the 'empty' place value columns to help them line up correctly.

## Worked Example 9

WE9

Calculate:  $12.45 + 6 + 0.3678$

### Thinking

- 1 Place the numbers underneath each other with the decimal points lined up. Fill empty place value columns with zeroes if necessary.
- 2 Add as though the values are whole numbers.
- 3 Place a decimal point in the answer so it lines up with the other decimal points.

### Working

$$\begin{array}{r} 12.4500 \\ 6.0000 \\ + 0.3678 \\ \hline 18.8178 \end{array}$$

## Worked Example 10

WE10

Calculate:  $15 - 2.147$

### Thinking

- 1 Write the second number underneath the first with the decimal points lined up. (Write 15 as 15.000)
- 2 Fill empty place value columns with zeroes.
- 3 Subtract, lining up the decimal point in the answer with those in the question.

### Working

$$\begin{array}{r} 15.000 \\ - 02.147 \\ \hline 12.853 \end{array}$$

# 4.4 Decimal addition and subtraction

## Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q13, Q14

Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q15, Q17

Q1 Column 3, Q2 Column 3, Q4, Q5, Q6, Q8, Q11, Q12, Q13, Q15, Q16, Q17

**Answers  
page 642**

## Fluency

1 Calculate:

- |                           |                          |                          |
|---------------------------|--------------------------|--------------------------|
| (a) $8.3 + 7.9$           | (b) $4.85 + 8.09$        | (c) $0.237 + 0.677$      |
| (d) $25.61 + 0.038$       | (e) $5.098 + 21.32$      | (f) $0.025 + 39.786$     |
| (g) $9.703 + 5.624 + 7.5$ | (h) $7.35 + 0.609 + 2.3$ | (i) $0.648 + 7.31 + 0.9$ |
| (j) $6 + 5.017 + 12.9$    | (k) $2.59 + 15 + 0.005$  | (l) $0.0004 + 3.583 + 8$ |

**WE9**

2 Calculate:

- |                   |                   |                       |
|-------------------|-------------------|-----------------------|
| (a) $5.7 - 3.8$   | (b) $7.05 - 2.93$ | (c) $23.982 - 11.735$ |
| (d) $9.663 - 2.7$ | (e) $4.984 - 1.8$ | (f) $7.238 - 3.4$     |
| (g) $9.5 - 2.24$  | (h) $16.2 - 8.75$ | (i) $3.7 - 0.931$     |
| (j) $8 - 7.44$    | (k) $3 - 2.92$    | (l) $93 - 0.698$      |

**WE10**

3 Which of the following shows the correct way to set out the addition of 0.56, 15.092 and 2.7?

A  $\begin{array}{r} 0.56 \\ 15.092 \\ + 2.7 \\ \hline \end{array}$

B  $\begin{array}{r} 0.56 \\ 15.092 \\ + 2.7 \\ \hline \end{array}$

C  $\begin{array}{r} 0.56 \\ 15.092 \\ + 2.7 \\ \hline \end{array}$

D  $\begin{array}{r} 0.56 \\ 15.092 \\ + 2.7 \\ \hline \end{array}$

## Understanding

- 4 Daily rainfall totals for three days over a long weekend were 3.78, 2.5 and 6.42 millimetres. What was the total rainfall over this three-day period?  
 5 John loads his shopping trolley with several items priced as shown.

1 frozen lasagna	\$5.71
2 L orange juice	\$4.25
1 tub of yoghurt	\$3.77
1 packet of cereal	\$6.14

- (a) Find the total cost of John's purchases.  
 (b) Round the total to the nearest 5 cents.

- 6 Cristina's bank account balance was \$335.96 just before she withdrew \$40.45 to pay a bill. How much did she have left in her account after the withdrawal?



- 7 In a gymnastics competition, Adelia scores the following from the five judges: 7.5, 8.5, 7.9, 8 and 8.6. Find her total score.

8 A family drives to a holiday resort and records the car's odometer readings as shown.

Departure: 234.8 km      Arrival: 502.7 km

How far did they travel to get to the resort?

9 During a car tour of Tasmania, Helena travelled distances of 25.64 km, 165.35 km and 5.97 km all in one day. How many kilometres did she travel altogether that day?

10 Georgina pays for \$36.35 worth of groceries with a \$100 note. How much change should she receive?

11 Nico pours 1.625 litres of milk from a full 2-litre container. How much milk is left in the container?



## Reasoning



## Open-ended

- 14 (a) Write two numbers that add up to 2.871.  
(b) Write three numbers, each with three decimal places, that add up to 3.86.  
(c) Explain how you chose the last digit of the numbers you used in (b).

$$\begin{array}{r} 4.0 \\ + 0.80 \\ \hline 7.05 \end{array}$$

- (a) Use one digit from 0 to 9 to fill each box and make a correct sum. Find at least three different combinations.
- (b) Was your choice of digits restricted? Explain how.
- 16 (a) Write two decimal numbers, each with three decimal places, that have a difference of 8.712.
- (b) Write two decimal numbers, each with three decimal places, that have a difference of 8.71.
- (c) Explain how you chose the last digit of the numbers you used in (b).
- 17 Two Australian swimmers finish first and second in the women's 100 m freestyle at the Olympics. The difference in their times is 0.04 seconds. The Olympic record was 53.52 seconds. Assuming that they both break this record by less than  $\frac{2}{10}$  of a second, what could their times be?

# Outside the Square Game

## 1 Closer than you

**Equipment required:** 2–4 brains, 2 dice

### How to win:

The aim is to be the person to get a score as close to 10 as possible.

### How to play:

- 1 Each player draws up a table like the one on the right.
- 2 Take turns to roll both dice, and enter the numbers, one each side of the decimal point, in whatever order you like. Add as you go. You can choose to stop at any point in the table.
- 3 Play the best of three.

	.	.
+	.	.
=	.	.
+	.	.
=	.	.
+	.	.
=	.	.
+	.	.
=	.	.
	.	.

## 2 Decimal zilch

**Equipment required:** 2–4 brains, 3 dice

### How to win:

The aim is to be the person to have the closest score to zero (but not less than zero).

### How to play:

- 1 Each player draws up a table like the one on the right.
- 2 Take turns to roll the three dice. Enter the numbers in the boxes in the next empty line in any order you choose. Subtract from the above total. You can choose to stop at any point in the table.
- 3 Play the best of three.

10	0	0
-	.	.
=	.	.
-	.	.
=	.	.
-	.	.
=	.	.
	.	.