
3

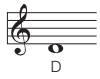
Fractions

3

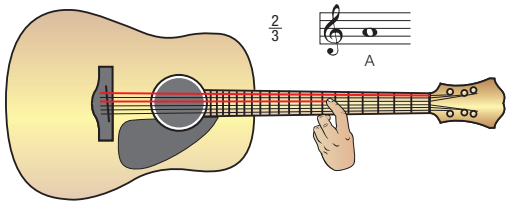
Frets make fractions.

Learning the guitar or violin could help your understanding of fractions.

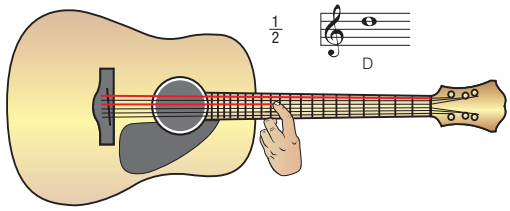
Guitarists and violinists know that shortening a string on their instrument produces a higher sounding note. What they may not realise is that for each note of a scale, the shortened string length is an exact fraction of the full length, or 'open' string. Plucking the third string on a guitar gives the note 'D':



Shortening the string to $\frac{2}{3}$ of its length gives the note 'A'.



Shortening the string to $\frac{1}{2}$ of its length gives the note 'D' an octave higher.



Instrument makers use this information to work out where to place the frets on the necks of guitars. Violins do not have frets, so the player must learn exactly where to place his or her fingers to produce different notes. You will find out other ways in which fractions are involved in music later in this chapter.

Forum

Some studies claim that there is a link between musical and mathematical ability—that people who are good at maths are also good at music. What do you think? What could you do to investigate this theory?

Why learn this?

Whether it be money, measurement or mixing a cake, we do not always work with whole numbers. Fractions are numbers that enable us to work with parts of wholes, and we need to be able to estimate and calculate with them.

After completing this chapter you will be able to:

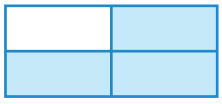
- identify and calculate fractions of a whole
- write one amount as a fraction of another
- simplify fractions and find equivalent fractions
- convert between fractions, whole numbers and mixed numbers
- use strategies to visualise, estimate and order fractions
- add, subtract, multiply and divide fractions
- solve problems involving fractions.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

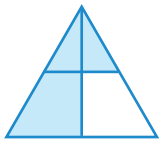
Worksheet R3.1

1 Which of these have three-quarters of the whole shape shaded in?

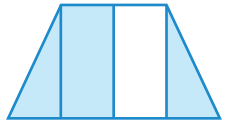
(a) 

(b) 

(c) 

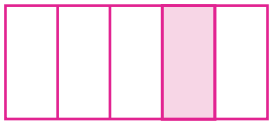
(d) 

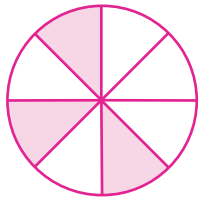
(e) 

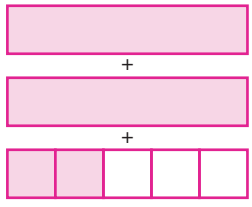
(f) 

Worksheet R3.2

2 Write down a fraction to represent each of the following diagrams or descriptions.

(a) 

(b) 

(c) 

(d) One person's share of a birthday cake divided equally between 10 people.

(e) Joseph eats five out of eight equal-sized slices of a pizza.

(f) two-thirds **(g)** twelve-fifths **(h)** seven-sixteenths

Worksheet R3.3

3 (a) Write these in order from largest to smallest: $\frac{3}{4}$, 1, 0, 2, $1\frac{1}{4}$

(b) Write these in order from smallest to largest: $\frac{3}{8}$, $\frac{1}{8}$, 0, $\frac{7}{8}$, $\frac{11}{8}$, 1, $\frac{9}{8}$

Worksheet R3.4

4 Calculate:

(a) $\frac{4}{7} + \frac{1}{7}$ **(b)** $\frac{8}{11} - \frac{2}{11}$ **(c)** $\frac{2}{3} + \frac{2}{3}$

Worksheet R3.5

5 (a) Write the first five multiples of 8. **(b)** Write the first five multiples of 12.

(c) Write the lowest common multiple (LCM) of 8 and 12.

(d) Write all the factors of 24. **(e)** Write all the factors of 36.

(f) Write the highest common factor (HCF) of 24 and 36.

Key Words

cancelling	improper fraction	mixed number	simplest form
denominator	inverse	numerator	simplify
equivalent fractions	Lowest Common Denominator (LCD)	proper fraction	unit fraction
fraction			

Understanding fractions

3.1

What is a fraction?

A **fraction** is a number that is used to show parts of a whole.

A whole might be an object, a collection of objects or a section of a number line.

If we are dividing a whole into parts to show a fraction, each of the parts must be equal in size.

If we are dividing up a collection of objects into groups to show a fraction, there must be the same number of objects in each group.

To write a fraction, two numbers are required.

$\frac{3}{5}$ ← The **numerator** tells us how many equal parts we have.

← The **denominator** tells us how many equal parts one whole has been divided into.

Worked Example 1

WE1

How many smileys are there in $\frac{4}{5}$ of this collection?

Thinking	Working
1 The denominator shows us how many equal parts to divide the collection into. (Dividing 15 into 5 groups gives 3 smileys in each group.)	$15 \div 5 = 3$ $\frac{1}{5}$ is 3 smileys 
2 The numerator tells us how many of these 'equal parts' we should count for our total. ($\frac{4}{5}$ means take 4 of the 5 groups.)	$4 \times 3 = 12$ 
3 Write the answer.	$\frac{4}{5}$ is 12 smileys

Types of fractions

Proper fractions have a numerator that is less than the denominator. They have values less than 1. Examples: $\frac{1}{2}$ and $\frac{3}{5}$.

Improper fractions have a numerator that is greater than or equal to the denominator. They have values greater than or equal to 1. Examples: $\frac{3}{2}$, $\frac{9}{9}$ and $\frac{17}{6}$.

Mixed numbers have whole number parts and fraction parts written separately. Examples: $1\frac{1}{2}$, $2\frac{5}{6}$ and $34\frac{2}{7}$.

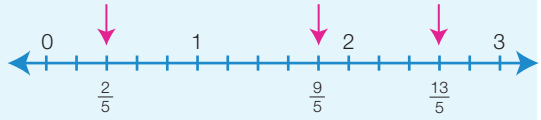
Using a number line to represent fractions

Placing fractions on a number line helps us to understand their size when compared to other numbers.

Drawing up a number line is easier if you can make the distance between whole numbers the same as the denominator. For example, if dividing into thirds, make the distance 3 cm. Then, you can mark off a third for every cm.

Worked Example 2 WE2

Copy this number line and show the positions of these fractions: $\frac{2}{5}$, $\frac{9}{5}$ and $\frac{13}{5}$.

Thinking	Working
1 Copy the number line.	
2 The denominator tells us how to divide up the spaces between the whole numbers (in this case, counting up in fifths). Make sure the distance between each marked division is the same.	
3 For each fraction, look at the numerator and count that many parts along from zero. Indicate the location of the fraction with an arrow.	

Fractions and division

There is an improper fraction that corresponds to each whole number on the number line.

If we fill in the rest of the fractions on the answer line in the Worked Example above, we can see that: $\frac{5}{5} = 1$, $\frac{10}{5} = 2$, $\frac{15}{5} = 3$ and so on.

We also know that: $5 \div 5 = 1$, $10 \div 5 = 2$, $15 \div 5 = 3$.

Writing $\frac{5}{5}$ is the same as writing $5 \div 5$.

We can write any whole number as an improper fraction by writing the numerator as a multiple of the denominator. For example, $3 = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$, because $6 \div 2 = 3$, $9 \div 3 = 3$ and $12 \div 4 = 3$.

The simplest way of writing a whole number as an improper fraction is to write it with a denominator of 1. For example, $3 = \frac{3}{1}$ $(3 \div 1 = 3)$.

Drawing the line between the numerator and the denominator is equivalent to writing the division sign, ÷.

Worked Example 3

WE3

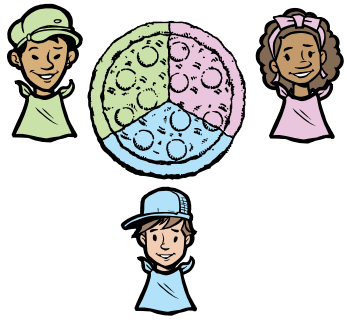
Write the whole number 9 as an improper fraction with a denominator of 4.

Thinking	Working
1 Which number divided by the denominator gives the required whole number? (Which number divided by 4 gives 9?)	$9 = \frac{\square}{4}$
2 Multiply the whole number you want by the denominator to find this number.	$9 \times 4 = 36$
3 Write the answer.	$9 = \frac{36}{4}$

We can consider all fractions in terms of division. Consider the following situation: 1 pizza is shared equally between 3 students. How much pizza does each student get?

We can show the answer is $\frac{1}{3}$ by drawing lines to divide the pizza into 3 equal pieces.

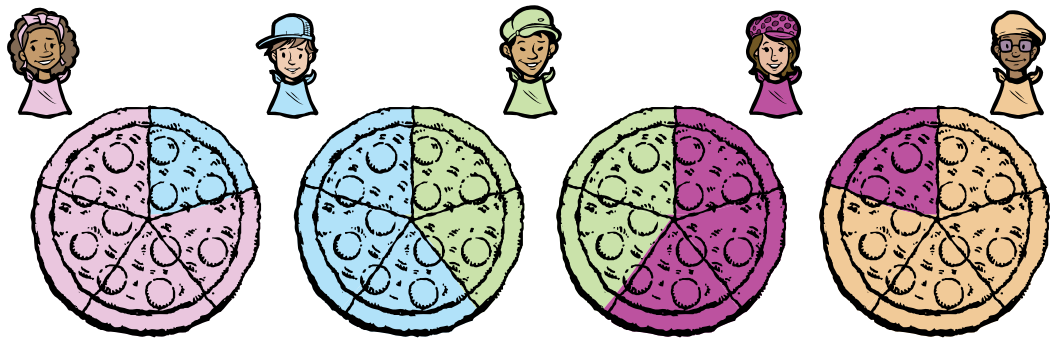
The different colours show which piece goes to which student.



We can write the division of 1 whole pizza among 3 students as $1 \div 3 = \frac{1}{3}$.

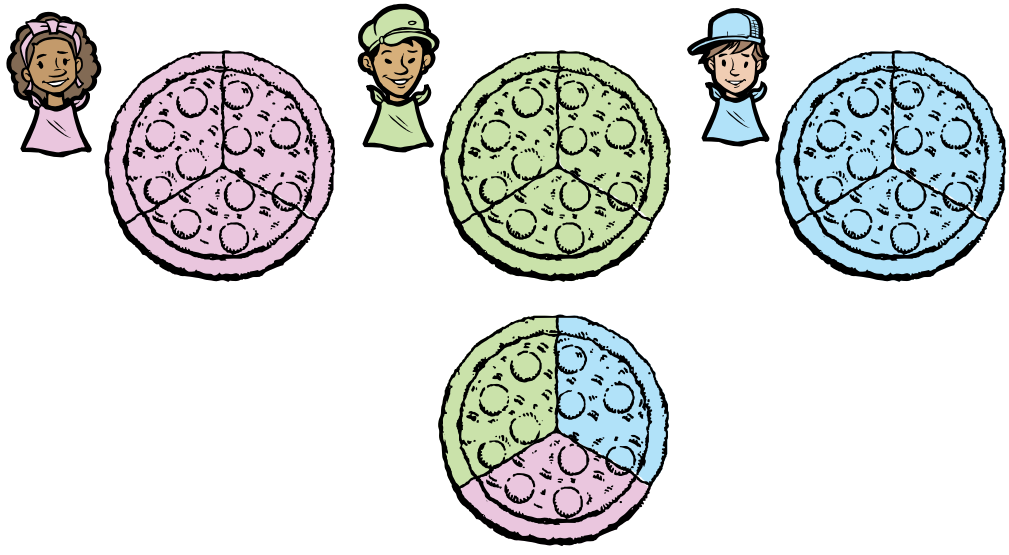
Similarly, if 4 pizzas are shared between 5 students, each student receives $\frac{4}{5}$ of a pizza.

$4 \div 5 = \frac{4}{5}$



If we have a greater number of pizzas than students, each student receives more than a whole pizza. For example, 4 pizzas shared between 3 students gives $4 \div 3 = \frac{4}{3}$ of a pizza each.

From the diagram, we can see that this is equal to 1 whole and $\frac{1}{3}$ of a pizza each, or $1\frac{1}{3}$.



Worked Example 4

WE4

For the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

5 blocks of chocolate are shared equally between 8 students.

Thinking	Working
1 Identify the whole and how many it is being shared between. (The whole is 5 and it is being shared between 8.) Write a division that shows this.	$5 \div 8$
2 Write this division as a fraction.	$= \frac{5}{8}$

3.1 Understanding fractions

Navigator

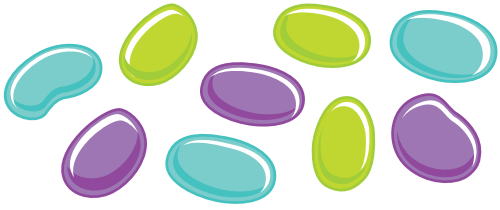
Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q11 (a), Q13, Q16, Q17, Q19 (a), Q20 (b), Q22	Q1, Q2, Q3 (b), Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 (d)–(f), Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19 (b), Q20 (b), Q22	Q2 (b), Q3 (b), Q4, Q5, Q7 Column 3, Q8 (d)–(f), Q9, Q10, Q11 (b), Q12, Q13, Q14, Q15, Q17, Q18, Q19 (c), Q20, Q21, Q22

Answers page 634

Fluency

1 **(a)** How many smileys are there in $\frac{3}{4}$ of this collection?

(b) How many jelly beans are there in $\frac{2}{3}$ of this collection?



WE1

2 **(a)** Copy this number line and show the positions of these fractions: $\frac{3}{4}$, $\frac{8}{4}$, $\frac{5}{4}$ and $-\frac{1}{4}$.

WE2

(b) Copy this number line and show the positions of these fractions: $\frac{5}{6}$, $\frac{8}{6}$, $\frac{1}{6}$ and $-\frac{3}{6}$.

3 **(a)** Write the whole number 5 as an improper fraction with a denominator of:

WE3

(i) 2 **(ii)** 7 **(iii)** 11 **(iv)** 5 **(v)** 1

(b) Write the whole number 13 as an improper fraction with a denominator of:

(i) 2 **(ii)** 5 **(iii)** 8 **(iv)** 13 **(v)** 1

4 **(a)** For each of the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

WE4

(i) 1 pizza is shared equally between 2 students.

(ii) 2 apples are shared equally between 3 students.

(iii) 6 packets of lollies are shared equally between 5 students.

(iv) 10 packets of biscuits are shared equally between 7 students.

(b) In which of the above situations does a student receive more than one whole?

Understanding

5 Here are 16 lollies. How many will you eat if you eat these fractions of the total?

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{8}$

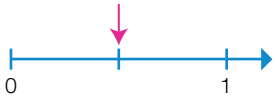
(d) $\frac{5}{8}$

6 **(a)** What fraction of the bananas in this bunch are rotten?

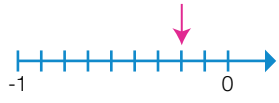
(b) What fraction of the flowers in this vase are red?

7 Write the value of the fraction indicated by the arrow on each of the number lines below.

(a)

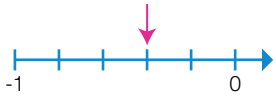


(b)



(c)

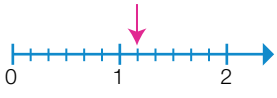
(d)



(e)

(f)

(g)



(h)

(i)

8 Write a fraction or a mixed number to show each of these:

(a) a numerator of 8 and a denominator of 17

(b) denominator of 4 and a numerator of 15

(c) nine chocolate biscuits in a packet of 20

(d) two wholes and two thirds

(e) 4 complete pairs of socks and one odd sock

(f) 3 whole 24-piece blocks of chocolate, with 7 extra pieces.

9 Write a fraction to show each of these:

(a) the weekend days as a fraction of a whole week

(b) 1 hour out of a whole day

(c) 1 second out of a whole minute

(d) 17 minutes out of a whole hour

(e) 157 mL of cola drunk from a 375 mL can

(f) 421L of water in a 500 L rainwater tank.

10 What fraction of this collection of shapes are:

(a) stars

(b) stars or hearts

(c) not hearts?

11 **(a)** Draw a diagram to show that if 5 pizzas are shared equally between 6 students, then each student receives $\frac{5}{6}$ of a pizza.

(b) Draw a diagram to show that if 5 blocks of chocolate are shared equally between 4 students, then each student receives 1 full block and $\frac{1}{4}$ of a second block.

12 Tim is 149 cm tall. His dad is 185 cm tall.

(a) Write Tim's height as a fraction of his dad's height.

(b) Write his dad's height as a fraction of Tim's height.

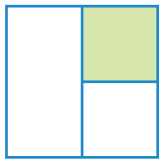
13 **(a)** After driving 8 km, the Johnson family are $\frac{1}{4}$ of the way to the zoo. How far away from the zoo do the Johnsons live?

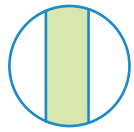
(b) Toby and his Mum shared a bag of lollies: $\frac{2}{3}$ for Toby and $\frac{1}{3}$ for Mum. Mum had 6 lollies. How many did Toby have? How many lollies were in the bag?

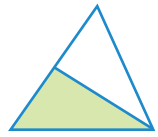
Reasoning

14 4 girls share 5 pizzas evenly between them. 5 boys share 4 of the same size pizzas evenly between them. Write the fraction of pizza that each boy and each girl receives. Who gets more pizza, a boy or a girl?

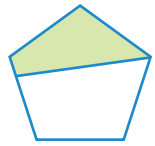
15 To show fractions, a whole must be divided into equal parts. For each of the shapes below, consider whether the shaded section represents one or more equal parts. State the fraction shown. If you don't think it is possible to state a definite fraction, explain why.

(a) 

(b) 

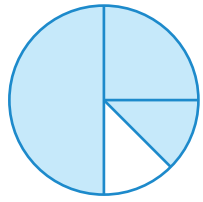
(c) 

(d) 

(e) 

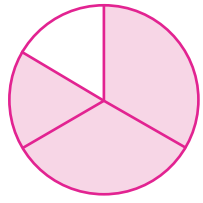
(f) 

16 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

17 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

18 How many whole numbers would appear on the section of the number line between $\frac{1}{2}$ and $\frac{11}{2}$?

Open-ended

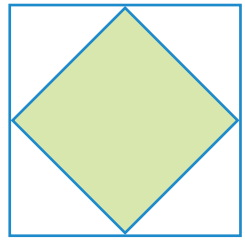
19 Draw a diagram that shows:

(a) three-eighths of an object **(b)** $\frac{5}{6}$ of a collection of objects **(c)** $\frac{9}{3}$

20 (a) This shape  represents $\frac{1}{5}$ of a whole. Draw two examples of what one whole could look like.

(b) This shape  represents $\frac{1}{4}$ of a whole. Draw two examples of what two wholes could look like.

21 Draw a diagram to help you explain why the fraction of this tile that is shaded is $\frac{1}{2}$.



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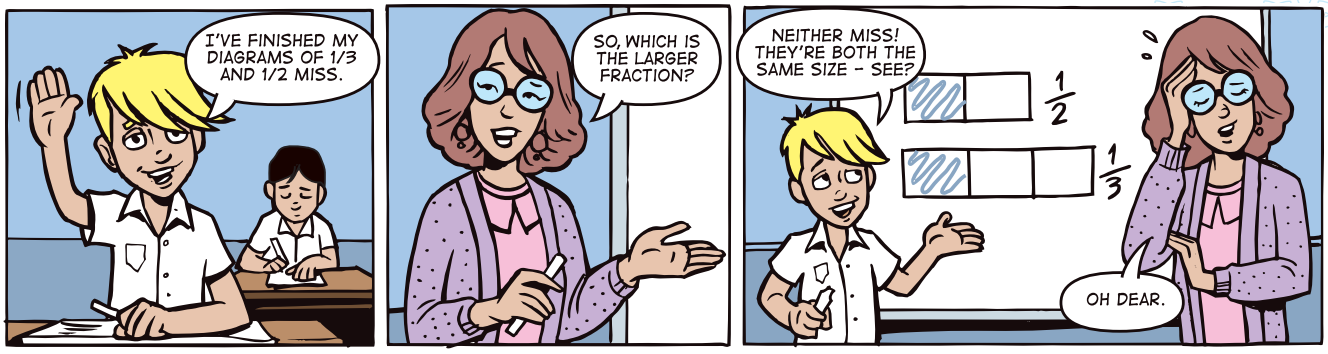


Oliver's diagram is supposed to show which of the fractions $\frac{1}{2}$ or $\frac{1}{3}$ is the larger fraction.

(a) Explain to Oliver what is wrong with his diagram.

(b) Draw a more accurate diagram to help you explain where he has gone wrong.

Outside the Square Puzzle

Fractionally funny fiction

Q: Which English ruler invented fractions?

To find out, write down the letters that correspond to the fractions of each of the following words.

For example, 'the first $\frac{1}{4}$ of 'TERRIFIC' would be the letters 'TE' ('TE-RR-IF-IC')

The letters in each of the 4 boxes form a word.

When you have put together four words, rearrange them to find the answer.

The second $\frac{1}{5}$ of ENTHUSIASM
The middle $\frac{1}{5}$ of WHEEL

The first $\frac{1}{5}$ of KNIFE
The middle $\frac{1}{3}$ of FRINGE
The last $\frac{1}{4}$ of SONG

The last $\frac{1}{6}$ of ENDURE
The third $\frac{1}{6}$ of REFRIGERATOR
The last $\frac{1}{3}$ of HEIGHT
The middle $\frac{1}{7}$ of WASHING

The first $\frac{1}{6}$ of HEAVEN
The middle $\frac{1}{5}$ of CONVENTION
The last $\frac{1}{3}$ of PANTRY

Figured it out?

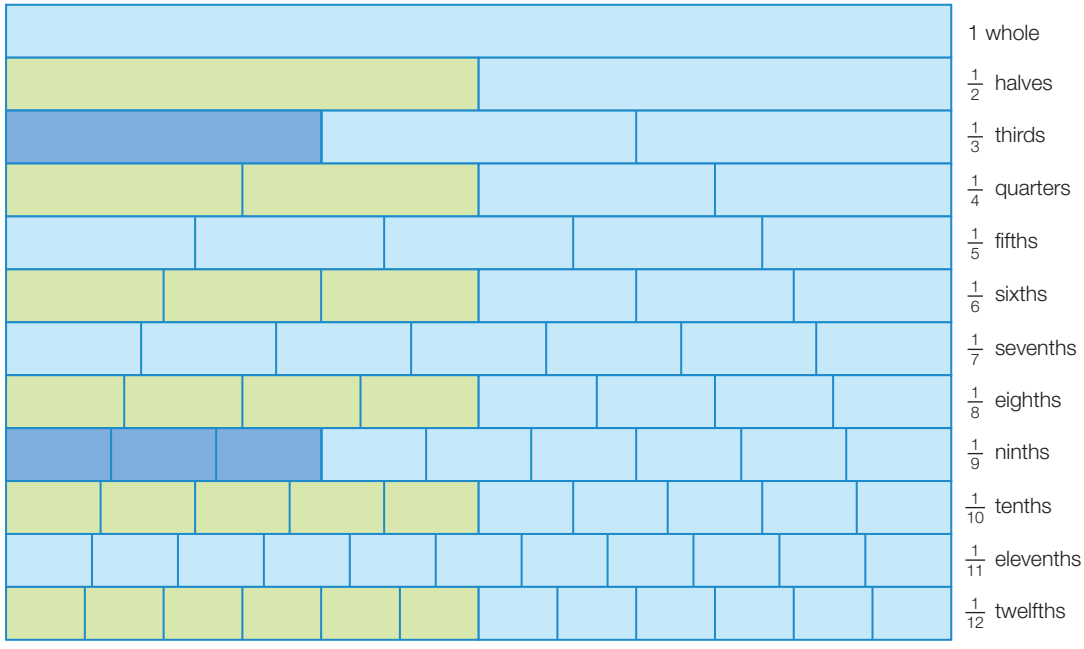
Make up your own puzzle that uses fractions of words to make the answer to a joke or fact.

3.2 Working with fractions

The fraction wall

Discovery Task

This fraction wall is made up of 12 layers of identical rectangles. One layer is left whole at the top, then each layer is divided into halves ($\frac{1}{2}$), thirds ($\frac{1}{3}$), quarters ($\frac{1}{4}$), fifths ($\frac{1}{5}$) … all the way down to twelfths ($\frac{1}{12}$). We can see that the size of the individual fractions (the 'bricks' in the wall) get smaller as we divide the whole into a greater number of pieces.



Equivalent fractions

If we examine the small vertical lines on the fraction wall, we can see that some lie exactly underneath each other (holding a ruler against the lines can help us see this more clearly). These lines in identical positions across the wall indicate **equivalent fractions**—fractions that represent the same amount, but with different numerators and denominators.

For example, if we consider $\frac{1}{2}$, we can see that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (these fractions are all shaded in green on the wall).

If we consider $\frac{1}{3}$, we can see that $\frac{1}{3} = \frac{3}{9}$ ($\frac{1}{3}$ is also equivalent to $\frac{2}{6}$ and $\frac{4}{12}$).

Can you see the pattern that links the numerators and the denominators of the fractions that are equivalent to $\frac{1}{2}$ and $\frac{1}{3}$?

Multiplying the numerator and the denominator of $\frac{1}{2}$ by 2 gives $\frac{2}{4}$: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$.

Similarly, multiplying the numerator and the denominator of $\frac{1}{2}$ by 3, 4, 5 and 6 gives $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and $\frac{6}{12}$.

Dividing the numerator and the denominator by a common factor also produces equivalent fractions: $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$.

(As $\frac{2}{2}$ and $\frac{3}{3}$ are equivalent to 1 whole, multiplying or dividing a fraction by them is the same as multiplying or dividing by 1, which does not change the value of the fraction.)

Equivalent fractions are found either by multiplying both the numerator and the denominator by the same number, or by dividing both the numerator and the denominator by the same number.

Worked Example 5 WE5

Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{7}{10} = \frac{\square}{40}$ **(b)** $\frac{27}{36} = \frac{3}{\square}$

Thinking	Working
(a) 1 Compare the two denominators to determine what you need to multiply the first denominator by to get the second. (To get 40, we multiply 10 by 4.)	(a) $\frac{7}{10} \times \frac{\square}{4} = \frac{\square}{40}$
2 Multiply the numerator by the same number to complete the equivalent fraction.	$\frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$
(b) 1 Compare the two numerators to determine what you need to divide the first numerator by to get the second. (To get 3, we divide 27 by 9.)	(b) $\frac{27}{36} \div \frac{9}{\square} = \frac{3}{\square}$
2 We divide the denominator by the same number to complete the equivalent fraction.	$\frac{27}{36} \div \frac{9}{9} = \frac{3}{4}$

Simplifying fractions

$\frac{1}{4}$ and $\frac{5}{20}$ are equivalent fractions: $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$.

Although they both represent the same amount, $\frac{1}{4}$ is the **simplest form** of this pair of fractions.

A fraction in simplest form is often the easiest to understand and visualise.

To write a fraction in its simplest form, or to **simplify** it, we divide the numerator and the denominator by their highest common factor (HCF).

Answers to fraction questions should always be written in simplest form.

For example, the HCF of 5 and 20 is 5, so: $\frac{5}{20} \div \frac{5}{5} = \frac{1}{4}$ gives us the simplest form of $\frac{5}{20}$.

The process of simplifying fractions is often called '**cancelling**'. To cancel a fraction, we show the division in the following way. Draw a line through the numerator. Divide the numerator by the HCF and write the result of the division next to it. Repeat this for the denominator.

For example: $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} = \frac{1}{4}$

Sometimes, if the HCF is not obvious, we can do several divisions by smaller factors, such as 2 or 3. This is shown in Method 2 of the Worked Example below.

Worked Example 6 WE6

Write the following fraction and mixed number in their simplest form.

(a) $\frac{24}{60}$ (b) $3\frac{12}{28}$

Method 1: Find the HCF

Thinking	Working
(a) 1 Find the HCF of the numerator and the denominator.	(a) HCF of 24 and 60 = 12
2 Divide both the numerator and the denominator by the HCF. (This is shown here using cancelling notation.)	$\overset{2}{\frac{\cancel{24}}{\cancel{60}}}{}_{5}$
3 Write the answer.	$= \frac{2}{5}$
(b) 1 Find the HCF of the numerator and the denominator of the fraction part.	(b) HCF of 12 and 28 = 4
2 Divide both the numerator and the denominator by the HCF.	$3\overset{3}{\frac{\cancel{12}}{\cancel{28}}}{}_{7}$
3 Write the answer.	$= 3\frac{3}{7}$

Method 2: Do several small divisions

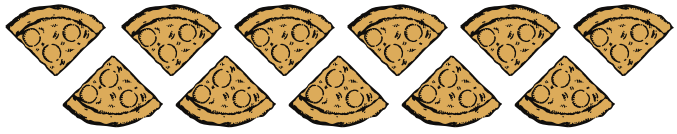
Thinking	Working
(a) 1 Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so we divide by 2.)	(a) $\overset{12}{\frac{\cancel{24}}{\cancel{60}}}{}_{30}$
2 Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again, then by 3.)	$= \overset{6}{\frac{\cancel{12}}{\cancel{30}}}{}_{15}$ $= \overset{2}{\frac{\cancel{6}}{\cancel{15}}}{}_{5}$
3 Write the answer.	$= \frac{2}{5}$

(b) 1	Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so we divide by 2.)	(b) $3\frac{\cancel{12}^{6}}{\cancel{28}_{14}}$
2	Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again.)	$= 3\frac{\cancel{6}^{3}}{\cancel{14}_{7}}$
3	Write the answer.	$= 3\frac{3}{7}$

Mixed numbers and improper fractions

Improper fractions that have a value greater than 1 can be written as mixed numbers. To write an improper fraction as a mixed number, we need to see how many 'wholes' we can make, and what fraction we have left over.

For example, here are 11 slices of pizza. Each slice represents $\frac{1}{4}$, so we have $\frac{11}{4}$.



With 11 quarter slices, we can make 2 whole pizzas with 3 slices left over, or $2\frac{3}{4}$.

This is the same as doing $11 \div 4 = 2$ remainder 3. The 3 becomes the numerator of the fraction part of the mixed number with the same denominator, 4.

To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part. We then write the remainder as the numerator of the fraction part.

Worked Example 7 WE7

Write the improper fraction $\frac{34}{7}$ as a mixed number.

Thinking	Working
1 Divide the numerator by the denominator. Include the remainder with your answer.	$34 \div 7 = 4$ rem 6
2 Write down the whole number part. Write the remainder as the numerator of the fraction part.	$\frac{34}{7} = 4\frac{6}{7}$

Writing a mixed number as an improper fraction is the opposite of the above process. Instead of forming wholes from fraction parts, we 'cut up' our whole numbers into fraction parts, and count the total number of parts that we have.

To write a mixed number as an improper fraction, write the whole number part as an improper fraction with the same denominator. Then, add the proper fraction part.

Worked Example 8 WE8

Write the mixed number $7\frac{3}{5}$ as an improper fraction.

Thinking	Working
1 Write the whole number part as an improper fraction. (Here, 7×5 tells us how many fifths there are in 7.)	$7 \times 5 = 35$ $7 = \frac{35}{5}$
2 Add the proper fraction part.	$\frac{35}{5} + \frac{3}{5} = \frac{38}{5}$
3 Write the mixed number with its equivalent proper fraction.	$7\frac{3}{5} = \frac{38}{5}$

You can probably spot a shortcut for these types of questions—multiply the whole number by the denominator, add the numerator, and place the answer over the denominator: $7\frac{3}{5} = \frac{38}{5}$.

3.2 Working with fractions

Navigator

Answers page 635

Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1–3, Q4 Columns 1–3, Q5 (a) & (b), Q6, Q7, Q8, Q9, Q10, Q11, Q15, Q16, Q17, Q18 (a)–(c), Q19, Q22	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q21, Q22	Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q17, Q20, Q21, Q22

Fluency

WE5

1 Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{3}{4} = \frac{\square}{20}$ (b) $\frac{1}{3} = \frac{7}{\square}$ (c) $\frac{1}{2} = \frac{\square}{52}$ (d) $\frac{2}{5} = \frac{16}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{7}{11} = \frac{\square}{99}$

(i) $\frac{4}{3} = \frac{\square}{27}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{24}{16} = \frac{\square}{4}$ (l) $\frac{72}{36} = \frac{\square}{9}$

(m) $\frac{8}{\square} = \frac{88}{55}$ (n) $\frac{\square}{5} = \frac{42}{35}$ (o) $\frac{54}{\square} = \frac{9}{6}$ (p) $\frac{\square}{36} = \frac{4}{9}$

2 Write the following fractions and mixed numbers in their simplest form.

WE6

If you need to revise how to find the HCF, go to pages 56 and 57.

(a) $\frac{5}{10}$ (b) $\frac{3}{9}$ (c) $\frac{3}{12}$ (d) $\frac{7}{21}$

(e) $\frac{4}{10}$ (f) $\frac{8}{22}$ (g) $\frac{10}{16}$ (h) $\frac{36}{24}$

(i) $2\frac{14}{21}$ (j) $3\frac{5}{30}$ (k) $1\frac{6}{20}$ (l) $12\frac{24}{30}$

(m) $6\frac{20}{45}$ (n) $3\frac{25}{100}$ (o) $2\frac{35}{42}$ (p) $100\frac{12}{84}$

3 Write these improper fractions as mixed numbers.

WE7

(a) $\frac{7}{5}$ (b) $\frac{13}{6}$ (c) $\frac{23}{4}$ (d) $\frac{15}{7}$

(e) $\frac{37}{10}$ (f) $\frac{48}{5}$ (g) $\frac{44}{7}$ (h) $\frac{59}{10}$

(i) $\frac{77}{9}$ (j) $\frac{91}{12}$ (k) $\frac{107}{100}$ (l) $\frac{97}{40}$

4 Write these mixed numbers as improper fractions.

WE8

(a) $1\frac{3}{4}$ (b) $3\frac{1}{5}$ (c) $6\frac{2}{3}$ (d) $5\frac{9}{10}$

(e) $4\frac{3}{8}$ (f) $4\frac{7}{11}$ (g) $10\frac{2}{7}$ (h) $6\frac{8}{11}$

(i) $6\frac{5}{12}$ (j) $3\frac{9}{100}$ (k) $6\frac{8}{9}$ (l) $8\frac{7}{9}$

5 **(a)** Which one of the following is the simplest form of $3\frac{35}{45}$?

A $3\frac{5}{9}$ **B** $3\frac{3}{4}$ **C** $3\frac{7}{9}$ **D** $3\frac{70}{90}$

(b) Which of the following fractions is not equivalent to $\frac{10}{15}$?

A $\frac{2}{3}$ **B** $\frac{30}{45}$ **C** $\frac{40}{60}$ **D** $\frac{20}{25}$

(c) As a mixed number, $\frac{33}{4}$ equals:

A $3\frac{3}{4}$ **B** $7\frac{4}{5}$ **C** $8\frac{1}{4}$ **D** $8\frac{1}{3}$

(d) As an improper fraction, $2\frac{5}{8}$ equals:

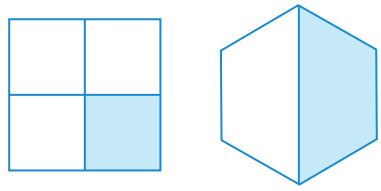
A $\frac{7}{8}$ **B** $\frac{21}{8}$ **C** $\frac{25}{8}$ **D** 21

Understanding

6 Use the fraction wall on page 116 to identify the fraction or fractions that are equivalent to:

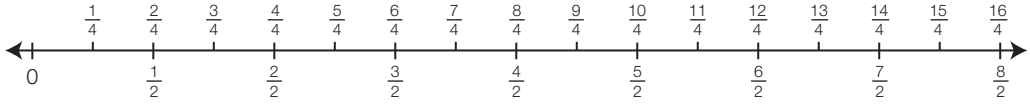
(a) $\frac{3}{5}$ **(b)** $\frac{2}{8}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{9}{12}$

7 **(a)** Copy each of these figures. Write down the fraction that is shaded.



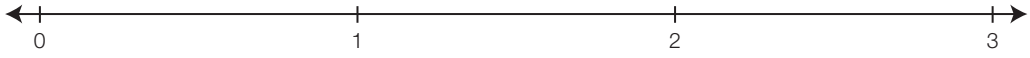
(b) In each figure, rule two diagonal lines joining the opposite vertices (corners). Name the equivalent fraction of the shaded part that you have created.

8 **(a)** The top side of this number line is divided into quarters, and the bottom side into halves.



What do you observe about the positions of the equivalent pairs of fractions $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{2}$ and $\frac{4}{4}$, $\frac{3}{2}$ and $\frac{6}{4}$, and $\frac{4}{2}$ and $\frac{8}{4}$?

(b) Copy the number line below. Divide the top side into thirds and the bottom side into ninths. Use your observation from **(a)** to identify pairs of equivalent fractions.



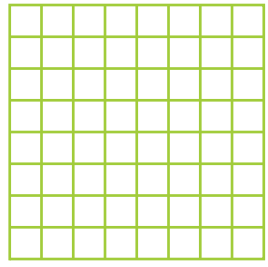
9 Monica, Michelle and Travis are sharing an apple pie that has been cut into 12 equal slices. Monica and Michelle decide to have $\frac{1}{4}$ of the apple pie each and give Travis the remainder. How many slices of pie did each person receive? Draw a diagram that shows this information.

10 Six pizzas were each sliced into 8 equal pieces. If $4\frac{3}{8}$ of the pizzas were eaten, how many slices is that?

11 A block of chocolate has 24 individual squares. How many squares of the chocolate will be gone if $\frac{7}{12}$ of the chocolate is eaten?

12 Talia shaded a fraction of this long rectangle.

How many squares on this grid should Talia shade to show a fraction equivalent to the fraction she shaded in the rectangle above?



13 Write your answers in simplest form.

(a) What fraction of an hour has passed from 3.49 p.m. to 3.51 p.m.?

(b) What fraction of an hour has passed from 11.03 a.m. to 11.23 a.m.?

(c) What fraction of an hour has passed from 8.13 p.m. to 8.25 p.m.?

(d) What fraction of an hour has passed from 2.36 a.m. to 2.45 a.m.?

14 Imagine that the whole rectangle on the top of the fraction wall on page 116 represents 1 hour. How many minutes would each of the individual fractions on the wall represent? Write any answers that are not whole numbers as mixed numbers.

Reasoning

15 How many whole numbers would appear on the section of the number line between $\frac{4}{3}$ and $\frac{51}{9}$? (Hint: Convert these improper fractions to mixed numbers.)

16 Which of the following fractions could not be the simplified form of an original fraction that had a denominator of 40?

A $\frac{3}{8}$ **B** $\frac{2}{5}$ **C** $\frac{3}{7}$ **D** $\frac{9}{10}$

17 Which rows of fractions in the fraction wall on page 116 have no equivalent fractions on the wall? Explain why.

18 For each of the following problems, it may be useful to draw and label a number line with the known fractions. Then, locate the position of the unknown fraction and identify it. Equivalent fractions can help you do this.

Which fraction lies exactly in-between the following fractions on the number line?

(a) $\frac{2}{5}$ and $\frac{3}{5}$ **(b)** $\frac{3}{7}$ and $\frac{4}{7}$ **(c)** $\frac{1}{4}$ and $\frac{2}{4}$

(d) $\frac{6}{10}$ and $\frac{7}{10}$ **(e)** $\frac{1}{9}$ and $\frac{6}{9}$ **(f)** $\frac{2}{11}$ and $\frac{7}{11}$

Open-ended

19 Write three fractions that are equivalent to $\frac{4}{5}$.

20 Write three improper fractions that simplify to $1\frac{2}{3}$.

21 For each of these pairs of fractions, draw a diagram that shows they are equivalent.

(a) $\frac{1}{3}$, $\frac{5}{15}$ **(b)** $\frac{3}{4}$, $\frac{12}{16}$

22 From the fraction wall on page 116 choose three fractions with three different denominators that have no equivalent fractions in the wall.

Outside the Square Puzzle

What fractions are we?

1 I am greater than $\frac{1}{2}$.

My numerator and denominator are both square numbers.

My numerator is an odd number.

The difference between my numerator and denominator is seven.

What fraction am I?

2 My numerator is a single-digit odd number.

My denominator is a two-digit number with both digits the same.

If you add 1 to both my numerator and denominator, you get a fraction equivalent to $\frac{1}{2}$.

What fraction am I?

3 My numerator is a multiple of 5.

My denominator is the LCM of 6 and 9.

I am less than 1.

I am in simplest form.

What fraction am I?