

Chapter

7

Negative numbers

What you will learn

- 7A Integers (**Consolidating**)
- 7B Adding and subtracting positive integers
- 7C Adding and subtracting negative integers
- 7D Multiplication and division of integers (**Extending**)
- 7E Order of operations (**Extending**)
- 7F Substituting integers (**Extending**)
- 7G The number plane

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Compare, order, add and subtract integers (ACMNA280)

Linear and non-linear relationships

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178) 



Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

The coldest place on Earth

The coldest place on Earth is Antarctica, where maximum daily temperatures below 0° Celsius are commonplace. The coldest temperature ever recorded on Earth was about -89°C in 1983 at the Russian Vostok Station in Antarctica. Temperatures on the coast of Antarctica can reach 15°C in summer but average temperatures are generally below 0°C . A layer of ice surrounds and covers almost all of the continent.

The surrounding ice sits both above and below sea level, making it very difficult for ships to reach their destinations. Like the temperature scale, heights above and below sea level can be recorded using both positive and negative numbers.

7A

Integers

CONSOLIDATING



Interactive



HOTsheets



Walkthroughs

The numbers $1, 2, 3, \dots$ are considered to be positive because they are greater than zero (0). Negative numbers extend the number system to include numbers less than zero. All the whole numbers less than zero, zero itself and the whole numbers greater than zero are called integers.

The use of negative numbers dates back to 100 BCE when the Chinese used black rods for positive numbers and red rods for negative numbers in their rod number system. These coloured rods were used for commercial and tax calculations. Later, a great Indian mathematician named Brahmagupta (598–670) set out the rules for the use of negative numbers, using the word *fortune* for positive and *debt* for negative. Negative numbers were used to represent loss in a financial situation.



John Wallis invented the number line.

An English mathematician named John Wallis (1616–1703) invented the number line and the idea that numbers have a direction. This helped define our number system as an infinite set of numbers extending in both the positive and negative directions. Today negative numbers are used in all sorts of mathematical calculations and are considered to be an essential element of our number system.

Let's start: Simple applications of negative numbers

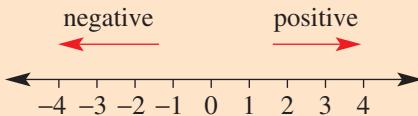
- Try to name as many situations as possible in which negative numbers are used.
- Give examples of the numbers in each case.



- Negative** numbers are numbers less than zero.
- Integers** are whole numbers that can be negative, zero or positive.

$$\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots$$

- A **number line** shows:
 - positive numbers to the right of zero
 - negative numbers to the left of zero.



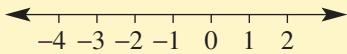
- Each negative number has a positive **opposite**.
3 and -3 are examples of opposite numbers.



Example 1 Drawing a number line

Draw a number line, showing all integers from -4 to 2 .

SOLUTION



EXPLANATION

Use equally spaced markings and put -4 on the left and 2 on the right.



Example 2 Less than or greater than

Insert the symbol $<$ (less than) or $>$ (greater than) into these statements to make them true.

a $-2 \square 3$

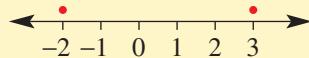
b $-1 \square -6$

SOLUTION

a $-2 < 3$

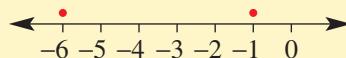
EXPLANATION

-2 is to the left of 3 on a number line.



b $-1 > -6$

-1 is to the right of -6 on a number line.



Exercise 7A

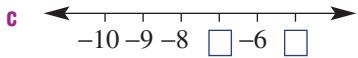
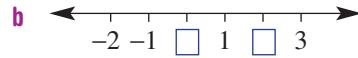
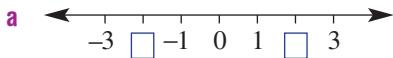
1–3

3

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UNDERSTANDING

- 1 What are the missing numbers on these number lines?



- 2 -5 is the opposite number of 5 , and 5 is the opposite number of -5 . Write down the opposite to these numbers.

a 2

b 6

c -3

d -7

e -15

f 21

g 132

h -1071

- 3 Fill in the blanks using the words *greater* and *less*.

a 5 is _____ than 0

b -3 is _____ than 0

c 0 is _____ than -6

d 0 is _____ than 1

4–5, 6(½), 7

4–5, 6(½), 7

4–7(½)

Example 1

- 4 Draw a number line for each description, showing all the given integers.

a from -2 to 2

b from -5 to 1

c from -10 to -6

d from -32 to -25

FLUENCY

7A

FLUENCY

- 5 List all the integers that fit the given description.

a from -2 up to 4
 c greater than -3 and less than 2
 e less than 4 and greater than -4

b from -7 up to 0
 d greater than -5 and less than 1
 f less than -3 and greater than -10

Example 2

- 6 Insert the symbol $<$ (less than) or $>$ (greater than) into these statements to make them true.

a $7 \square 9$ b $3 \square 2$

c $0 \square -2$ d $-4 \square 0$

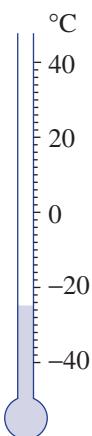
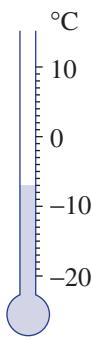
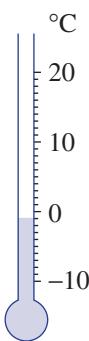
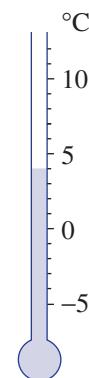
e $-1 \square -5$ f $-7 \square -6$

g $-11 \square -2$ h $-9 \square -13$

i $-3 \square 3$ j $3 \square -3$

k $-130 \square 1$ l $-2 \square -147$

- 7 Give the temperature shown on these thermometers.



8, 9

9, 10

9, 10

PROBLEM-SOLVING

- 8 Arrange these numbers in *ascending* order.

a $-3, -6, 0, 2, -10, 4, -1$

b $-304, 126, -142, -2, 1, 71, 0$

- 9 Write the next three numbers in these simple patterns.

a $3, 2, 1, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

b $-8, -6, -4, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

c $10, 5, 0, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

d $-38, -40, -42, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

e $-91, -87, -83, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

f $199, 99, -1, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

- 10 These lists of numbers show deposits (positive numbers) and withdrawals (negative numbers) for a month of bank transactions. Find the balance at the end of the month.

a Start balance \$200

$-\$10$

$-\$130$

\$25

$-\$100$

\$20

Final balance

b Start balance \$0

\$50

$-\$60$

$-\$100$

\$200

$-\$100$

Final balance

11

11

11, 12

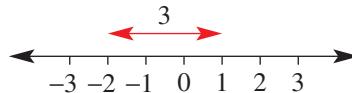
7A

REASONING

- 11 If the height above sea level for a plane is a positive number, then the height for a submarine could be written as a negative number. What is the height relative to sea level for a submarine at these depths?

a 50 m**b** 212.5 m**c** 0 m

- 12 The difference between two numbers could be thought of as the distance between the numbers on a number line. For example, the difference between -2 and 1 is 3 .



Find the difference between these pairs of numbers.

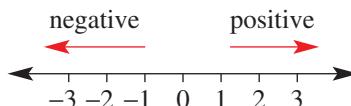
a -1 and 1 **b** -2 and 2 **c** -3 and 1 **d** -4 and 3 **e** -3 and 0 **f** -4 and -1 **g** -10 and -4 **h** -30 and 14

The final position

13

ENRICHMENT

- 13 For these sets of numbers, a positive number means to move right and a negative number means to move left. Start at zero each time and find the final position.

**a** $-1, 4, -5$ **b** $3, -5, -1, 4$ **c** $-5, -1, 3, 1, -2, -1, 4$ **d** $-10, 20, -7, -14, 8, -4$ **e** $-250, 300, -49, -7, 36, -81$ **f** $-7001, 6214, -132, 1493, -217$

7B

Adding and subtracting positive integers



Interactive



Widgets



HOTsheets



Walkthroughs

Adding and subtracting a positive integer can give both positive and negative answers. For example, when the temperature inside a newly installed fridge is 20°C but, after being switched on, the temperature then falls by 25°C , the result is -5°C ; i.e. $20 - 25 = -5$. If a temperature of -10°C rises by 5°C , the result is -5°C ; i.e. $-10 + 5 = -5$.

Let's start: Positive and negative possibilities

Decide if it is possible to find an example of the following.

If so, give a specific example.

- A positive number added to a positive number gives a positive number.
- A positive number added to a positive number gives a negative number.
- A positive number added to a negative number gives a positive number.
- A positive number added to a negative number gives a negative number.
- A positive number subtracted from a positive number gives a positive number.
- A positive number subtracted from a positive number gives a negative number.
- A positive number subtracted from a negative number gives a positive number.
- A positive number subtracted from a negative number gives a negative number.

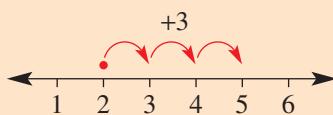


When food goes into a fridge its temperature begins to change.

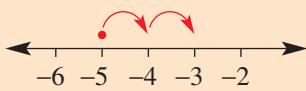
Key ideas

- If a positive number is added to an integer, you move right on a number line.

$$2 + 3 = 5 \quad \text{Start at } 2 \text{ and move right by 3.}$$

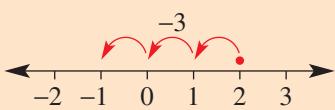


$$-5 + 2 = -3 \quad \text{Start at } -5 \text{ and move right by 2.}$$

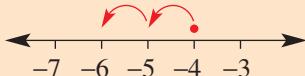


- If a positive number is subtracted from an integer, you move left on a number line.

$$2 - 3 = -1 \quad \text{Start at } 2 \text{ and move left by 3.}$$



$$-4 - 2 = -6 \quad \text{Start at } -4 \text{ and move left by 2.}$$





Example 3 Adding and subtracting positive integers

Calculate the answer to these additions and subtractions.

a $-2 + 3$

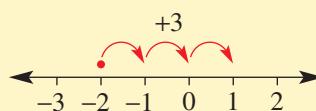
b $-8 + 1$

c $5 - 7$

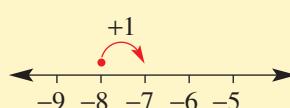
d $-3 - 3$

SOLUTION

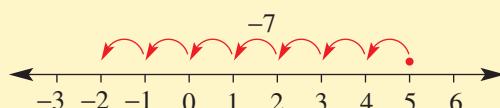
a $-2 + 3 = 1$



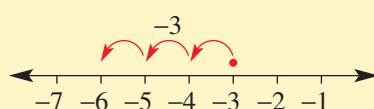
b $-8 + 1 = -7$



c $5 - 7 = -2$



d $-3 - 3 = -6$



1, 2

2

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Exercise 7B

UNDERSTANDING

1 In which direction (i.e. right or left) on a number line do you move for the following calculations?

a 2 is added to -5

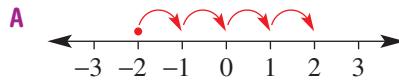
b 6 is added to -4

c 4 is subtracted from 2

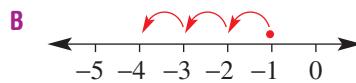
d 3 is subtracted from -4

2 Match up the problems a to d with the number lines A to D.

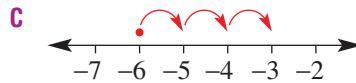
a $5 - 6 = -1$



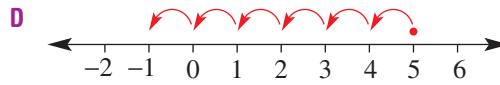
b $-2 + 4 = 2$



c $-1 - 3 = -4$



d $-6 + 3 = -3$



7B

Example 3a,b



- 3 Calculate the answer to these additions. Check your answers using a calculator.

a $-1 + 2$	b $-1 + 4$	c $-3 + 5$	d $-10 + 11$
e $-4 + 3$	f $-5 + 2$	g $-11 + 9$	h $-20 + 18$
i $-4 + 0$	j $-8 + 0$	k $-30 + 29$	l $-39 + 41$
m $-130 + 132$	n $-181 + 172$	o $-57 + 63$	p $-99 + 68$

Example 3c,d



- 4 Calculate the answer to these subtractions. Check your answers using a calculator.

a $4 - 6$	b $7 - 8$	c $3 - 11$	d $1 - 20$
e $-3 - 1$	f $-5 - 5$	g $-2 - 13$	h $-7 - 0$
i $-37 - 4$	j $39 - 51$	k $62 - 84$	l $-21 - 26$
m $-100 - 200$	n $100 - 200$	o $328 - 421$	p $-496 - 138$

- 5 Find the missing number.

a $2 + \square = 7$	b $-2 + \square = 7$	c $-2 + \square = 3$	d $-4 + \square = -2$
e $5 - \square = 0$	f $3 - \square = -4$	g $-9 - \square = -12$	h $-20 - \square = -30$
i $-6 + \square = -1$	j $-8 - \square = -24$	k $\square + 1 = -3$	l $\square + 7 = 2$
m $\square - 4 = -10$	n $\square - 7 = -20$	o $\square + 6 = -24$	p $\square - 100 = -213$

- 6 Evaluate the following. Remember to work from left to right.

a $3 - 4 + 6$	b $2 - 7 - 4$	c $-1 - 4 + 6$	d $-5 - 7 - 1$
e $-3 + 2 - 7 + 9$	f $-6 + 1 - 20 + 3$	g $0 - 9 + 7 - 30$	h $-15 - 20 + 32 - 1$

7, 8

8, 9

8–10

- 7 Determine how much debt remains in these financial situations.

a owes \$300 and pays back \$155	b owes \$20 and borrows another \$35
c owes \$21 500 and pays back \$16 250	

- 8 a The reading on a thermometer measuring temperature rises 18°C from -15°C . What is the final temperature?
 b The reading on a thermometer measuring temperature falls 7°C from 4°C . What is the final temperature?
 c The reading on a thermometer measuring temperature falls 32°C from -14°C . What is the final temperature?

- 9 For an experiment, a chemical solution starts at a temperature of 25°C , falls to -3°C , rises to 15°C and then falls again to -8°C . What is the total change in temperature? Add all the changes together for each rise and fall.

- 10 An ocean sensor is raised and lowered to different depths in the sea. Note that -100 m means 100 m below sea level.

- a If the sensor is initially at -100 m and then raised to -41 m , how far does the sensor rise?
 b If the sensor is initially at -37 m and then lowered to -93 m , how far is the sensor lowered?



PROBLEM-SOLVING

FLUENCY

11 Give an example that suits the description.

- a A positive number subtract a positive number equals a negative number.
 - b A negative number subtract a positive number equals a negative number.
 - c A negative number add a positive number equals a positive number.
 - d A negative number add a positive number equals a negative number.
- 12 a a is a positive integer, b is a positive integer and $a > b$. For each of the following, decide if the result will be positive, negative or zero.
- i $a + b$ ii $a - b$ iii $b - a$ iv $a - a$
- b a is a negative integer and b is a positive integer. Decide if each of the following is *always* true.
- i $a + b$ is positive ii $a - b$ is negative

+ or – combinations

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13

13 Insert + or – signs into these statements to make them true.

- | | |
|--|---|
| a $3 \square 4 \square 5 = 4$ | b $1 \square 7 \square 9 \square 4 = -5$ |
| c $-4 \square 2 \square 1 \square 3 \square 4 = 0$ | d $-20 \square 10 \square 7 \square 36 \square 1 \square 18 = -4$ |
| e $-a \square b \square a \square b = 0$ | f $-a \square a \square 3a \square b \square b = a - 2b$ |

Market Movers Snapshot

Top 20 of ASX 100 Stocks

Rises: 85 Falls: 9 Steadies: 6

ASX Code		Price	Movement by price	Movement by %	
1	IPL INCITEC PIVOT LIMITED	2.992	0.160 ↑	5.65	Chart
2	AQP AQUARIUS PLATINUM LIMITED	5.481	0.260 ↑	4.98	Chart
3	DOW DOWNER EDI LIMITED	4.140	0.190 ↑	4.81	Chart
4	ALL ARISTOCRAT LEISURE LIMITED	3.542	0.130 ↑	3.81	Chart
5	AMC AMCOR LIMITED	6.715	0.200 ↑	3.07	Chart
6	TSE TRANSFIELD SERVICES LIMITED	3.357	0.100 ↑	3.07	Chart
7	FMG FORTESCUE METALS GROUP LTD	4.375	0.120 ↑	2.82	Chart
8	AWE AWE LIMITED	1.675	0.045 ↑	2.76	Chart
9	BLY BOART LONGYEAR LIMITED	2.979	0.080 ↑	2.76	Chart
10	ITO INTOLL GROUP	1.121	0.030 ↑	2.75	Chart
11	MGR MIRVAC GROUP	1.308	0.035 ↑	2.75	Chart
12	CBA COMMONWEALTH BANK OF AUSTRALIA.	51.370	1.370 ↑	2.74	Chart
13	NWS NEWS CORPORATION	16.955	0.420 ↑	2.54	Chart
14	ASX ASX LIMITED	29.636	0.720 ↑	2.49	Chart
15	FXJ FAIRFAX MEDIA LIMITED	1.441	0.035 ↑	2.49	Chart
16	DXS DEXUS PROPERTY GROUP	0.840	0.020 ↑	2.44	Chart
17	GMG GOODMAN GROUP	0.635	0.015 ↑	2.42	Chart
18	JHX JAMES HARDIE INDUSTRIES SE	6.453	0.150 ↑	2.38	Chart
19	MQG MACQUARIE GROUP LIMITED	39.903	0.920 ↑	2.36	Chart
20	ERA ENERGY RESOURCES OF AUSTRALIA LIMITED	13.713	-0.330 ↓	-2.35	Chart

Positive and negative numbers are used to show stock price changes.

7C

Adding and subtracting negative integers



By observing patterns in number calculations, we can see the effect of adding and subtracting negative integers.



Widgets



HOTsheets



Walkthroughs

Addition	Subtraction
$2 + 3 = 5$	$2 - 3 = -1$
$2 + 2 = 4$	$2 - 2 = 0$
$2 + 1 = 3$	$2 - 1 = 1$
$2 + 0 = 2$	$2 - 0 = 2$
$2 + (-1) = 1$	$2 - (-1) = 3$
$2 + (-2) = 0$	$2 - (-2) = 4$
$2 + (-3) = -1$	$2 - (-3) = 5$

So adding -3 is equivalent to subtracting 3, and subtracting -3 is equivalent to adding 3.



If I have \$100 of debt and subtract \$50 of debt, how do I calculate my final balance?

Let's start: Dealing with debt

Let $-\$10$ represent \$10 of debt. Can you write a statement (e.g. $5 + (-10) = -5$) to represent the following financial situations?

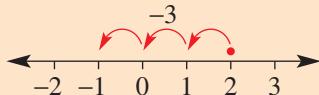
- \$10 of debt is added to a balance of \$5.
- \$10 of debt is added to a balance of $-\$5$.
- \$10 of debt is removed from a balance of $-\$15$.



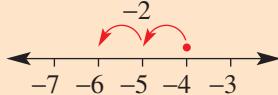
- Adding a negative number is equivalent to subtracting its opposite.

$$a + (-b) = a - b$$

$$2 + (-3) = 2 - 3 = -1$$



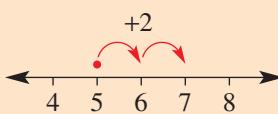
$$-4 + (-2) = -4 - 2 = -6$$



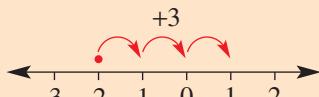
- Subtracting a negative number is equivalent to adding its opposite.

$$a - (-b) = a + b$$

$$5 - (-2) = 5 + 2 = 7$$



$$-2 - (-3) = -2 + 3 = 1$$



- On a number line, the effect of adding or subtracting a negative number is to reverse the direction of the operation.



Example 4 Adding and subtracting negative integers

Calculate the answer to these additions and subtractions.

a $7 + (-2)$

b $-2 + (-3)$

c $1 - (-3)$

d $-6 - (-2)$

SOLUTION

a $7 + (-2) = 7 - 2$

$$= 5$$

b $-2 + (-3) = -2 - 3$

$$= -5$$

c $1 - (-3) = 1 + 3$

$$= 4$$

d $-6 - (-2) = -6 + 2$

$$= -4$$

EXPLANATION

Adding -2 is equivalent to subtracting 2 .

Adding -3 is equivalent to subtracting 3 .

Subtracting -3 is equivalent to adding 3 .

Subtracting -2 is equivalent to adding 2 .

Exercise 7C

1–3(½)

3(½)

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UNDERSTANDING

1 Write down the missing numbers in these sentences. The first one has been done for you.

a $2 + 5$ means that 5 is added to 2 .

b $-3 + 6$ means that \square is added to \square .

c $1 + (-3)$ means that \square is added to \square .

d $-7 + (-11)$ means that \square is added to \square .

e $5 - 3$ means that \square is subtracted from \square .

f $-2 - 6$ means that \square is subtracted from \square .

g $7 - (-3)$ means that \square is subtracted from \square .

h $-7 - (-11)$ means that \square is subtracted from \square .

2 Complete these sentences.

a Adding -4 is equivalent to subtracting \square .

b Adding -6 is equivalent to _____ 6 .

c Adding 5 is equivalent to subtracting \square .

d Adding -11 is equivalent to _____ 11 .

e Subtracting -2 is equivalent to adding \square .

f Subtracting -7 is equivalent to _____ 7 .

7C

UNDERSTANDING

- 3 State whether each of the following is true or false.

a $2 + (-3) = 5$

b $10 + (-1) = 9$

c $-5 + (-3) = -8$

d $-6 + (-2) = -4$

e $5 - (-1) = 4$

f $3 - (-9) = 12$

g $2 - (-3) = 1$

h $-11 - (-12) = -1$

Example 4a,b



- 4 Calculate the answer to these additions. Check your answer using a calculator.

a $3 + (-2)$

b $8 + (-3)$

c $12 + (-6)$

d $9 + (-7)$

e $1 + (-4)$

f $6 + (-11)$

g $20 + (-22)$

h $0 + (-4)$

i $-2 + (-1)$

j $-7 + (-15)$

k $-5 + (-30)$

l $-28 + (-52)$

m $-7 + (-3)$

n $-20 + (-9)$

o $-31 + (-19)$

p $-103 + (-9)$

Example 4c,d



- 5 Calculate the answer to these subtractions. Check your answer using a calculator.

a $2 - (-3)$

b $5 - (-6)$

c $20 - (-30)$

d $29 - (-61)$

e $-5 - (-1)$

f $-7 - (-4)$

g $-11 - (-6)$

h $-41 - (-7)$

i $-4 - (-6)$

j $-9 - (-10)$

k $-20 - (-20)$

l $-96 - (-104)$

m $5 - (-23)$

n $28 - (-6)$

o $-31 - (-19)$

p $-104 - (-28)$

- 6 Find the missing number.

a $2 + \square = -1$

b $3 + \square = -7$

c $-2 + \square = -6$

d $\square + (-3) = 1$

e $\square + (-10) = -11$

f $\square + (-4) = 0$

g $5 - \square = 6$

h $2 - \square = 7$

i $-1 - \square = 3$

j $\square - (-3) = 7$

k $\square - (-10) = 12$

l $\square - (-4) = -20$

m $5 - \square = 11$

n $\square - (-2) = -3$

o $-2 - \square = -4$

p $\square + (-5) = -1$

- 7 Calculate the answer, working from left to right.

a $3 + (-2) + (-1)$

b $2 + (-1) + (-6)$

c $3 - (-1) - (-4)$

d $10 - (-6) + (-4)$

e $-7 - (-1) + (-3)$

f $-20 - (-10) - (-15)$

g $-9 - (-19) + (-16)$

h $-15 - (-20) + (-96)$

i $-13 - (-19) + (-21)$

j $-2 - (-3) - (-5)$

k $-18 - (-16) - (-19)$

l $5 + (-20) - (-26)$

8, 9

9–11

10–12

PROBLEM-SOLVING

- 8 A diver is at a height of -90 m from the surface of the sea. During a diving exercise, the diver rises 50 m, falls 138 m and then rises once again by 35 m. What is the diver's final height from sea level?



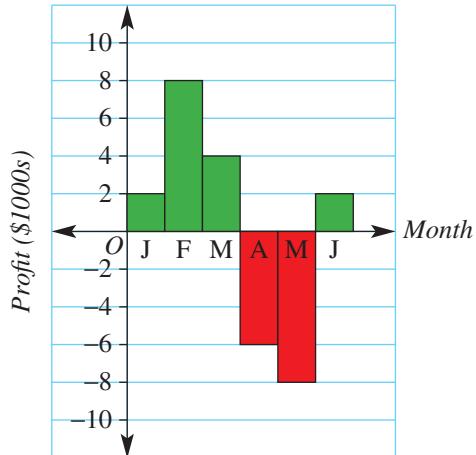
- 9 A small business has a bank balance of $-\$50\,000$. An amount of $\$20\,000$ of extra debt is added to the balance and, later, $\$35\,000$ is paid back. What is the final balance?



- 10 $\$100$ of debt is added to an existing balance of $\$50$ of debt. Later, $\$120$ of debt is removed from the balance. What is the final balance?

- 11 Here is a profit graph showing the profit for each month of the first half of the year for a bakery shop.

- a What is the profit for:
 i February?
 ii April?
 b What is the overall profit for the 6 months?



- 12 Complete these magic squares, using addition. The sum of each row, column and diagonal should be the same.

a

-2		5
	1	
		4

b

		-6
-3		-17
		-7

13 Write these sentences as mathematical statements, e.g. $2 + (-3)$.

- a The sum of 3 and 4.
- b The sum of -2 and -9 .
- c The difference between 5 and -2 .
- d The difference between -2 and 1.
- e The sum of a and the opposite of b .
- f The difference between a and the opposite of b .

14 Simplify these numbers. Hint: In part a, $-(-4)$ is the same as $0 - (-4)$.

a $-(-4)$ b $-(-(-1))$ c $-(-(-(-3)))$

15 a If a is a positive number and b is a negative number, decide if each of the following statements is *always* true.

i $a + b$ is negative ii $a - b$ is positive

b If a is a negative number and b is a negative number, decide if each of the following statements is *always* true.

i $a + b$ is negative ii $a - b$ is positive

c If a and b are both negative numbers and $b < a$, is $a - b$ always positive? Give reasons.

Negative fractions

—

—

16

16 Negative decimals and fractions can be added and subtracted using the same rules as those for integers. Calculate the answer to these sums and differences of fractions.

- | | | | |
|--|--|---|--|
| a $2 + \left(-\frac{1}{2} \right)$ | b $5 + \left(-\frac{4}{3} \right)$ | c $-\frac{1}{2} + \left(-\frac{3}{2} \right)$ | d $-\frac{2}{3} + \left(-\frac{10}{3} \right)$ |
| e $5 - \left(-\frac{1}{3} \right)$ | f $10 - \left(-\frac{3}{2} \right)$ | g $-\frac{5}{4} - \left(-\frac{3}{4} \right)$ | h $-\frac{4}{7} - \left(-\frac{1}{2} \right)$ |
| i $\frac{9}{2} + \left(-\frac{9}{3} \right)$ | j $\frac{9}{2} - \left(-\frac{9}{3} \right)$ | k $4\frac{2}{3} + \left(-1\frac{1}{2} \right)$ | l $5\frac{5}{7} + \left(-4\frac{4}{5} \right)$ |
| m $-\frac{3}{2} + \left(-\frac{1}{3} \right)$ | n $-\frac{7}{4} - \left(-\frac{2}{5} \right)$ | o $3\frac{2}{7} - \left(-1\frac{1}{2} \right)$ | p $-5\frac{1}{6} - \left(-3\frac{2}{5} \right)$ |



In the Celsius scale, temperatures below the freezing point of water (zero degrees) are indicated by negative numbers.

7D

Multiplication and division of integers

EXTENDING



Interactive



Widgets



HOTsheets



Walkthroughs

The rules for multiplication and division of integers can be developed by considering repeated addition.

For example: 4 groups of -3 is $-3 + (-3) + (-3) + (-3) = -12$. So, $4 \times (-3) = -12$.

Also, $-3 \times 4 = -12$ since $a \times b = b \times a$.

We also know that if $5 \times 7 = 35$, then $35 \div 7 = 5$, so if $4 \times (-3) = -12$ then $-12 \div (-3) = 4$. This is saying there are 4 groups of -3 in -12 , which we know from the repeated addition above.

Also, $-12 \div 4 = -3$.

These examples give rise to the rules governing the multiplication and division of negative numbers.

Let's start: Patterns in tables

Complete this table of values for multiplication by noticing the patterns. What does the table of values tell you about the rules for multiplying negative integers?

\times	-3	-2	-1	0	1	2	3
-3				0			
-2				0			
-1				0			
0	0	0	0	0	0	0	0
1				0	1		
2				0	2	4	
3				0			

- The product or quotient of two numbers of the same sign (i.e. positive or negative) is a positive number.

So $a \times b = ab$ and $-a \times (-b) = ab$

e.g. $3 \times 4 = 12$ or $-3 \times (-4) = 12$

and $a \div b = \frac{a}{b}$ and $-a \div (-b) = \frac{a}{b}$

e.g. $12 \div 4 = 3$ or $-12 \div (-4) = 3$

- The product or quotient of two numbers of the opposite sign (i.e. positive and negative) is a negative number.

So $-a \times b = -ab$ and $a \times (-b) = -ab$

e.g. $-3 \times 4 = -12$ or $3 \times (-4) = -12$

and $-a \div b = -\frac{a}{b}$ and $a \div (-b) = -\frac{a}{b}$

e.g. $-12 \div 3 = -4$ or $12 \div (-3) = -4$

Key ideas



Example 5 Multiplying and dividing integers

Calculate these products and quotients.

a $5 \times (-6)$

b $-3 \times (-7)$

c $-36 \div (-4)$

d $-18 \div 9$

SOLUTION

a $5 \times (-6) = -30$

b $-3 \times (-7) = 21$

c $-36 \div (-4) = 9$

d $-18 \div 9 = -2$

EXPLANATION

The two numbers are of opposite sign, so the answer is negative.

The two numbers are of the same sign, so the answer is positive.

Both numbers are negative, so the answer is positive.

The two numbers are of opposite sign, so the answer is negative.



Example 6 Working with multiple operations

Work from left to right to find the answer to $-7 \times 4 \div (-2)$.

SOLUTION

$$\begin{aligned}-7 \times 4 \div (-2) &= -28 \div (-2) \\ &= 14\end{aligned}$$

EXPLANATION

First, calculate -7×4 .

Then calculate $-28 \div (-2)$.

Exercise 7D

1–3

3

—

- 1 Complete these product tables.

a	\times	-2	-1	0	1	2
	-2		0			
	-1		0			
	0	0	0	0	0	
	1		0	1	2	
	2		0			

b	\times	-4	-2	0	2	4
	-4	16				
	-2					
	0					0
	2					
	4					8

- 2 Write down the missing number.

a $2 \times (-3) = -6$, so $-6 \div (-3) = \square$

c $-16 \div 4 = -4$, so $\square \times 4 = -16$

b $2 \times (-3) = -6$, so $-6 \div 2 = \square$

d $16 \div (-4) = -4$, so $\square \times (-4) = 16$

UNDERSTANDING

- 3** Complete each sentence by inserting the missing word *positive* or *negative*.
- The product (\times) of two positive numbers is _____.
 - The product (\times) of two negative numbers is _____.
 - The product (\times) of two numbers with opposite signs is _____.
 - The quotient (\div) of two positive numbers is _____.
 - The quotient (\div) of two negative numbers is _____.
 - The quotient (\div) of two numbers with opposite signs is _____.

4–8(½)

4–9(½)

4–9(½)

Example 5a,b

- 4** Calculate the answer to these products.

a $3 \times (-5)$	b $1 \times (-10)$	c -3×2	d -9×6
e $-8 \times (-4)$	f $-2 \times (-14)$	g $-12 \times (-12)$	h -11×9
i -13×3	j $7 \times (-12)$	k $-19 \times (-2)$	l -36×3
m $-6 \times (-11)$	n $5 \times (-9)$	o $-21 \times (-3)$	p $-36 \times (-2)$

Example 5c,d

- 5** Calculate the answer to these quotients.

a $14 \div (-7)$	b $36 \div (-3)$	c $-40 \div 20$	d $-100 \div 25$
e $-9 \div (-3)$	f $-19 \div (-19)$	g $-25 \div 5$	h $38 \div (-2)$
i $84 \div (-12)$	j $-108 \div 9$	k $-136 \div 2$	l $-1000 \div (-125)$
m $-132 \div (-11)$	n $-39 \div (-3)$	o $78 \div (-6)$	p $-156 \div (-12)$

Example 6

- 6** Work from left to right to find the answer. Check your answer using a calculator.



a $2 \times (-3) \times (-4)$	b $-1 \times 5 \times (-3)$	c $-10 \div 5 \times 2$
d $-15 \div (-3) \times 1$	e $-2 \times 7 \div (-14)$	f $100 \div (-20) \times 2$
g $48 \div (-2) \times (-3)$	h $-36 \times 2 \div (-4)$	i $-125 \div 25 \div (-5)$
j $-8 \div (-8) \div (-1)$	k $46 \div (-2) \times (-3) \times (-1)$	l $-108 \div (-12) \div (-3)$

- 7** Write down the missing number in these calculations.

a $5 \times \square = -35$	b $\square \times (-2) = -8$	c $16 \div \square = -4$	d $-32 \div \square = -4$
e $\square \div (-3) = -9$	f $\square \div 7 = -20$	g $-5000 \times \square = -10\,000$	h $-87 \times \square = 261$
i $243 \div \square = -81$	j $50 \div \square = -50$	k $-92 \times \square = 184$	l $-800 \div \square = -20$

- 8** Remember that $\frac{9}{3}$ means $9 \div 3$. Use this knowledge to simplify each of the following.

a $\frac{-12}{4}$	b $\frac{21}{-7}$	c $\frac{-40}{-5}$	d $\frac{-124}{-4}$
e $\frac{-15}{-5}$	f $\frac{-100}{-20}$	g $\frac{-900}{30}$	h $\frac{20\,000}{-200}$

- 9** Remember that $3^2 = 3 \times 3 = 9$, and $(-3)^2 = -3 \times (-3) = 9$. Use this knowledge to simplify each of the following.

a $(-2)^2$	b $(-1)^2$	c $(-9)^2$	d $(-10)^2$
e $(-6)^2$	f $(-8)^2$	g $(-3)^2$	h $(-1.5)^2$

- 10 List the different pairs of integers that multiply to give these numbers.

a 6**b** 16**c** -5**d** -24

- 11 Insert a multiplication or division sign between the numbers to make a true statement.

a $2 \square - 3 \square - 6 = 1$

b $-25 \square - 5 \square 3 = 15$

c $-36 \square 2 \square - 3 = 216$

d $-19 \square - 19 \square 15 = 15$

- 12 **a** There are two distinct pairs of numbers whose product is -8 and difference is 6. What are the two numbers?

- b** The quotient of two numbers is -11 and their difference is 36. What are the two numbers?
There are two distinct pairs to find.

- 13 2^4 means $2 \times 2 \times 2 \times 2$, and $(-2)^4 = -2 \times (-2) \times (-2) \times (-2)$.

- a** Calculate:

i $(-2)^3$

ii $(-2)^6$

iii $(-3)^3$

iv $(-3)^4$

- b** Which questions from part **a** give positive answers and why?

- c** Which questions from part **a** give negative answers and why?

- 14 $a \times b$ is equivalent to ab , and $2 \times (-3)$ is equivalent to $-(2 \times 3)$. Use this information to simplify these expressions.

a $a \times (-b)$

b $-a \times b$

c $-a \times (-b)$

× and ÷ with negative fractions

- 15 Calculate the answer to these problems containing fractions. Simplify where possible.

a $\frac{1}{2} \times \left(-\frac{1}{2}\right)$

b $\frac{3}{4} \times \left(-\frac{2}{3}\right)$

c $-\frac{5}{7} \times \frac{3}{5}$

d $-\frac{3}{4} \times \left(-\frac{4}{3}\right)$

e $\frac{1}{4} \div \left(-\frac{1}{4}\right)$

f $-\frac{5}{8} \div \frac{1}{2}$

g $-\frac{6}{11} \div \left(-\frac{12}{11}\right)$

h $-\frac{3}{2} \div \left(-\frac{1}{4}\right)$

i $\frac{a}{b} \times \left(-\frac{b}{a}\right)$

j $-\frac{b}{a} \times \left(-\frac{a}{b}\right)$

k $-\frac{a}{b} \div \frac{a}{b}$

l $-\frac{b}{a} \div \left(-\frac{b}{a}\right)$



Progress quiz

7A 1 Draw a number line from -2 to $+3$, showing all the given integers.

7A 2 Copy and insert the symbol $<$ (less than) or $>$ (greater than) into these statements to make them true.

a $-2 \square 1$

b $-9 \square -12$

c $4 \square -5$

7A 3 Arrange these numbers in *ascending* order: $-6, 8, -4, 0, 7$.

7B 4 Calculate the answer to these additions and subtractions.

a $-10 + 12$

b $-4 - 5$

c $26 - 34$

d $-5 - 8 + 9 - 22$

7C 5 Calculate the answer to these additions and subtractions.

a $9 + (-4)$

b $-8 + (-7)$

c $0 + (-3)$

d $12 - (-8)$

7C 6 Calculate the answer, working from left to right.

a $-20 - (-10) - (-15)$

b $10 - (-6) + (-4)$

7D 7 Calculate these products and quotients.

a $4 \times (-3)$

b $-5 \times (-12)$

c $-56 \div 8$

d $-20 \div (-5)$

7D 8 Work from left to right to find the answer.

a $5 \times (-2) \times (-4)$

b $25 \div (-5) \times 6$

c $64 \div (-8) \times (-2)$

d $-40 \div (-4) \div (-5)$

7D 9 Simplify each of the following.

Ext a $(-5)^2$

b $(-2)^3$

c $\frac{-72}{-6}$

d $\frac{-1260}{4}$

7C 10 Ethan has a debt of \$120 on his credit card. He buys another item using his credit card, which adds an extra debt of \$90. At the end of the month \$140 is paid back. What is the final balance on Ethan's credit card?



7E

Order of operations

EXTENDING



We have learnt from our study of positive whole numbers that there is a particular order to follow when dealing with mixed operations and brackets. This order also applies when dealing with negative numbers.

For example: $-2 + 3 \times (-4)$ is different from $(-2 + 3) \times (-4)$.



Widgets



HOTsheets



Walkthroughs

Let's start: Brackets or not?

During a classroom debate about the statement $3 \times (-4) - 8 \div (-2) = -8$:

- Lil says that the statement needs to have more brackets to make it true.
- Max says that even with brackets it is impossible to make it true.
- Riley says that it is correct as it is and there is no need for more brackets.

Who is correct and why?



- When working with more than one operation and with positive and/or negative numbers:

- Deal with brackets first.
- Do multiplication and division next, working from left to right.
- Do addition and subtraction last, working from left to right.

$$\begin{array}{c} -2 \times 3 - (10 + (-2)) \div 4 \\ |2nd| \quad |1st| \\ -6 \qquad \qquad 8 \\ \qquad \qquad \boxed{2} \\ \text{last} \end{array}$$

Example 7 Using order of operations

Use order of operations to evaluate the following.

a $5 + 2 \times (-3)$

b $-6 \times 2 - 10 \div (-5)$

SOLUTION

a $5 + 2 \times (-3) = 5 + (-6)$
 $= -1$

EXPLANATION

Do the multiplication before the addition.

b $-6 \times 2 - 10 \div (-5) = -12 - (-2)$
 $= -12 + 2$
 $= -10$

Do the multiplication and division first.
When subtracting -2 , add its opposite.



Example 8 Order of operations with brackets

Use order of operations to evaluate the following.

a $(-2 - 1) \times 8$

b $5 \div (-10 + 5) + 5$

SOLUTION

$$\begin{aligned} \text{a } (-2 - 1) \times 8 &= -3 \times 8 \\ &= -24 \end{aligned}$$

$$\begin{aligned} \text{b } 5 \div (-10 + 5) + 5 &= 5 \div (-5) + 5 \\ &= -1 + 5 \\ &= 4 \end{aligned}$$

EXPLANATION

Deal with brackets first.

Deal with brackets first. Then do the division before the subtraction.

Exercise 7E

1, 2

2

—

UNDERSTANDING

- 1 Which operation (i.e. addition, subtraction, multiplication or division) is done first in each of the following problems?

a $-2 \div 2 + 1$

b $8 \times (-6) - 4$

c $-3 + 2 \times (-6)$

d $7 - (-8) \div 4$

e $(-2 + 3) \div 5$

f $-6 \div (4 - (-2))$

g $-4 \times 3 \div (-6)$

h $(2 + 3 \times (-2)) + 1$

i $-11 \div (7 - 2 \times (-2))$

- 2 Classify each of the following statements as true or false.

a $-4 + 2 \times 3 = -4 + (2 \times 3)$

b $-4 + 2 \times 3 = (-4 + 2) \times 3$

c $8 \times (2 - (-2)) = 8 \times 4$

d $8 \times (2 - (-2)) = 8 \times 0$

e $-40 - 20 \div (-5) = (-40 - 20) \div (-5)$

f $-40 - 20 \div (-5) = -40 - (20 \div (-5))$

FLUENCY

Example 7

3, 4(½)

3–4(½)

3–4(½)

- 3 Use order of operations to evaluate the following. Check your answer using a calculator.

a $2 + 3 \times (-3)$

b $9 + 10 \div (-5)$

c $20 + (-4) \div 4$

d $18 + (-9) \times 1$

e $10 - 2 \times (-3)$

f $10 - 1 \times (-4)$

g $-8 - (-7) \times 2$

h $-2 \times 4 + 8 \times (-3)$

i $-3 \times (-1) + 4 \times (-2)$

j $12 \div (-6) + 4 \div (-2)$

k $-30 \div 5 - 6 \times 2$

l $-2 \times 3 - 4 \div (-2)$

m $8 \times (-2) - (-3) \times 2$

n $-1 \times 0 - (-4) \times 1$

o $0 \times (-3) - (-4) \times 0 + 0$

Example 8

- 4 Use order of operations to evaluate the following. Check your answer using a calculator.

a $(3 + 2) \times (-2)$

b $(8 - 4) \div (-2)$

c $-3 \times (-2 + 4)$

d $-1 \times (7 - 8)$

e $10 \div (4 - (-1))$

f $(2 + (-3)) \times (-9)$

g $(24 - 12) \div (16 + (-4))$

h $(3 - 7) \div (-1 + 0)$

i $-2 \times (8 - 4) + (-6)$

j $-2 - 3 \times (-1 + 7)$

k $0 + (-2) \div (1 - 2)$

l $1 - 2 \times (-3) \div (-3 - (-2))$

m $(-3 + (-5)) \times (-2 - (-1))$

n $-3 \div (-1 + 4) \times 6$

o $-5 - (8 + (-2)) + 9 \div (-9)$

- 5** A shop owner had bought socks at \$5 a pair but, during an economic downturn, sold them for \$3 a pair. In a particular week, 124 pairs are sold and there are other costs of \$280. What is the shop owner's overall loss for the week?



- 6** A debt of \$550 is doubled and then \$350 of debt is removed each month for 3 months. What is the final balance?

- 7** Insert brackets to make each statement true.

- | | | |
|---|--------------------------------------|---|
| a $-2 + 3 \times 8 = 8$ | b $-10 \div 4 + 1 = -2$ | c $-1 + 7 \times 2 - 15 = -3$ |
| d $-5 - 1 \div (-6) = 1$ | e $3 - 8 \div 5 + 1 = 0$ | f $50 \times 7 - 8 \times (-1) = 50$ |
| g $-2 \times 3 - (-7) - 1 = -21$ | h $-3 + 9 \div (-7) + 5 = -3$ | i $32 - (-8) \div (-3) + 7 = 10$ |
- 8** By inserting only *one* pair of brackets, how many different answers are possible for this calculation? Also include the answers for which brackets are not used.
 $-2 + 8 \times (-4) - (-3)$

- 9** If brackets are removed from these problems, does the answer change?

- | | | |
|--------------------------------|------------------------------------|--|
| a $(2 \times 3) - (-4)$ | b $(8 \div (-2)) - 1$ | c $(-2 + 3) \times 4$ |
| d $9 \div (-4 + 1)$ | e $(9 - (-3) \times 2) + 1$ | f $(-1 + 8 \div (-2)) \times 2$ |

- 10** State if each of the following is generally true or false.

- | | |
|--|--|
| a $(-3 + 1) + (-7) = -3 + (1 + (-7))$ | b $(-3 + 1) - (-7) = -3 + (1 - (-7))$ |
| c $(a + b) + c = a + (b + c)$ | d $(a + b) - c = a + (b - c)$ |
| e $(a - b) + c = a - (b + c)$ | f $(a - b) - c = a - (b - c)$ |

- 11 a** Is the answer to each of the following positive or negative?

- | | |
|--|---|
| i $-6 \times (-4) \times (-8) \times (-108) \times (-96)$ | ii $-100 \div (-2) \div 2 \div (-5)$ |
| iii $(-3)^3$ | iv $-1 \times (-2)^3$ |
| v $\frac{-6 \times (-3) \times 4 \times 7 \times (-3)}{(-2)^2}$ | vi $\frac{(-1)^2 \times (-1)}{(-1)^3 \times (-1)}$ |

- b** Explain the strategy you used to answer the questions in part **a**.

Powers and negative numbers

12,13

7E

ENRICHMENT

12 First, note that:

- $2^4 = 2 \times 2 \times 2 \times 2 = 16$
- $(-2)^4 = -2 \times (-2) \times (-2) \times (-2) = 16$
- $-2^4 = -(2 \times 2 \times 2 \times 2) = -16$

When evaluating expressions with powers, the power is dealt with first in the order of operations.

For example: $((-2)^3 - 1) \div (-3) = (-8 - 1) \div (-3) = -9 \div (-3) = 3$

Evaluate each of the following.

a 2^2

b $(-2)^2$

c -2^2

d $(-2)^5$

e -2^5

f $(3^2 - 1) \times 4$

g $((-3)^3 - 1) \div (-14)$

h $30 \div (1 - 4^2)$

i $-10\ 000 \div (-10)^4$

13 Kevin wants to raise -3 to the power of 4 . He types -3^4 into a calculator and gets -81 .

Explain what Kevin has done wrong.



7F

Substituting integers

EXTENDING



Interactive



Widgets



HOTsheets



Walkthroughs

The process known as substitution involves replacing a pronumeral or letter (sometimes called a variable) with a number. As a car accelerates, for example, its speed could be modelled by the rule $10 + 4t$. So, after 8 seconds we can calculate the car's speed by substituting $t = 8$ into $10 + 4t$.

So $10 + 4t = 10 + 4 \times 8 = 42$ metres per second.

We can also look at the car's speed before time $t = 0$. So at 2 seconds before $t = 0$ (i.e. $t = -2$), the speed would be $10 + 4t = 10 + 4 \times (-2) = 2$ metres per second.



We can use substitution to work out this car's speed at a given time.

Let's start: Order matters

Two students substitute the values $a = -2$, $b = 5$ and $c = -7$ into the expression $ac - bc$. Some of the different answers received are 21, -49, -21 and 49.

- Which answer is correct and what errors were made in the calculation of the other three incorrect answers?

■ Substitute into an expression by replacing pronumerals (letters representing numbers) with numbers.

If $a = -3$ then

$$\begin{aligned} 3 - 7a &= 3 - 7 \times (-3) \\ &= 3 - (-21) \\ &= 3 + 21 \\ &= 24 \end{aligned}$$

■ Brackets can be used around negative numbers to avoid confusion with other symbols.



Example 9 Substituting integers



Evaluate the following expressions using $a = 3$ and $b = -5$.

a $2 + 4a$

b $7 - 4b$

c $b \div 5 - a$

SOLUTION

$$\begin{aligned} a \quad 2 + 4a &= 2 + 4 \times 3 \\ &= 2 + 12 \\ &= 14 \end{aligned}$$

EXPLANATION

Replace a with 3 and evaluate the multiplication first.

b $7 - 4b = 7 - 4 \times (-5)$
 $= 7 - (-20)$
 $= 7 + 20$
 $= 27$

Replace the b with -5 and evaluate the multiplication before the subtraction.

c $b \div 5 - a = -5 \div 5 - 3$
 $= -1 - 3$
 $= -4$

Replace b with -5 and a with 3 , and then evaluate.

Exercise 7F

1–3

3

—

UNDERSTANDING

- 1** Which of the following shows the correct substitution of $a = -2$ into the expression $a - 5$?
A $2 - 5$ **B** $-2 + 5$ **C** $-2 - 5$ **D** $2 + 5$
- 2** Which of the following shows the correct substitution of $x = -3$ into the expression $2 - x$?
A $-2 - (-3)$ **B** $2 - (-3)$ **C** $-2 + 3$ **D** $-3 + 2$
- 3** Rafe substitutes $c = -10$ into $10 - c$ and gets 0. Is he correct? If not, what is the correct answer?

4–6

4–5(½), 6–7

4–7(½)

FLUENCY

Example 9a,b

- 4** Evaluate the following expressions using $a = 6$ and $b = -2$.
a $5 + 2a$ **b** $-7 + 5a$ **c** $b - 6$ **d** $b + 10$
e $4 - b$ **f** $7 - 2b$ **g** $3b - 1$ **h** $-2b + 2$
i $5 - 12 \div a$ **j** $1 - 60 \div a$ **k** $10 \div b - 4$ **l** $3 - 6 \div b$
- 5** Evaluate the following expressions using $a = -5$ and $b = -3$.
a $a + b$ **b** $a - b$ **c** $b - a$ **d** $2a + b$
e $5b + 2a$ **f** $6b - 7a$ **g** $-7a + b + 4$ **h** $-3b - 2a - 1$
- 6** Evaluate these expressions for the given pronumeral values.
a $26 - 4x$ ($x = -3$) **b** $-2 - 7k$ ($k = -1$)
c $10 \div n + 6$ ($n = -5$) **d** $-3x + 2y$ ($x = 3, y = -2$)
e $18 \div y - x$ ($x = -2, y = -3$) **f** $-36 \div a - ab$ ($a = -18, b = -1$)
- 7** These expressions contain brackets. Evaluate them for the given pronumeral values.
(Remember that ab means $a \times b$).
a $2 \times (a + b)$ ($a = -1, b = 6$) **b** $10 \div (a - b) + 1$ ($a = -6, b = -1$)
c $ab \times (b - 1)$ ($a = -4, b = 3$) **d** $(a - b) \times bc$ ($a = 1, b = -1, c = 3$)

7F

8, 9

8, 9

9, 10

PROBLEM-SOLVING

11

11 12

12 13

- 11** Write two different expressions involving x that give an answer of -10 if $x = -5$.

12 Write an expression involving the pronumeral a combined with other integers, so if $a = -4$ the expression would equal these answers.

a -3 **b** 0 **c** 10

13 If a and b are any non-zero integer, explain why these expressions will always give the result of zero.

a $a - b + b - a$ **b** $\frac{a}{a} - 1$ **c** $\frac{(a-a)}{b}$ **d** $\frac{ab}{b} - a$

REASONING

Celsius/Fahrenheit

—

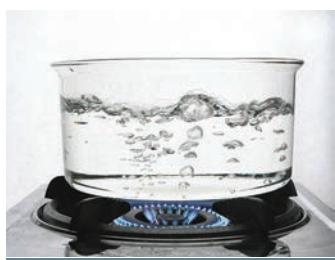
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14

- 14 The Fahrenheit temperature scale ($^{\circ}\text{F}$) is still used today in some countries, but most countries use the Celsius scale ($^{\circ}\text{C}$). 32°F is the freezing point for water (0°C). 212°F is the boiling point for water (100°C).

The formula for converting $^{\circ}\text{F}$ to $^{\circ}\text{C}$ is $^{\circ}\text{C} = \frac{5}{9} \times (^{\circ}\text{F} - 32)$.

- a Convert these temperatures from °F to °C.
 - i 41°F
 - ii 5°F
 - iii -13°F
 - b Can you work out the formula that converts from °C to °F?
 - c Use your rule from part b to check your answers to part a.



The water temperature is 100°C and 212°F.

ENRICHMENT

7G

The number plane



Interactive



Widgets



HOTsheets



Walkthroughs

During the seventeenth century, two well-known mathematicians, René Descartes and Pierre de Fermat, independently developed the idea of a number plane. The precise positions of points are illustrated using coordinates, and these points can be plotted using the axes as measuring guides. This invention revolutionised the study of mathematics and provided a vital link between geometry and algebra. The number plane or coordinate plane, is also called the Cartesian plane (named after Descartes). It uses two axes at right angles that extend in both the positive and negative directions.

Let's start: North, south, east and west

The units for this grid are in metres.

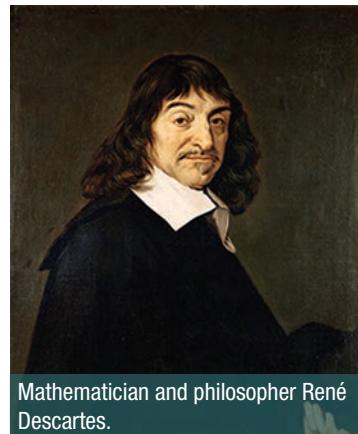
René starts at position O and moves:

- 3 m east
- 2 m south
- 4 m west
- 5 m north.

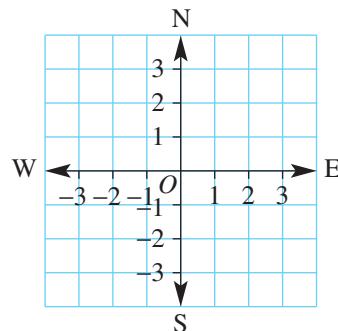
Pierre starts at position O and moves:

- 1 m west
- 3 m south
- 4 m east
- 5 m north.

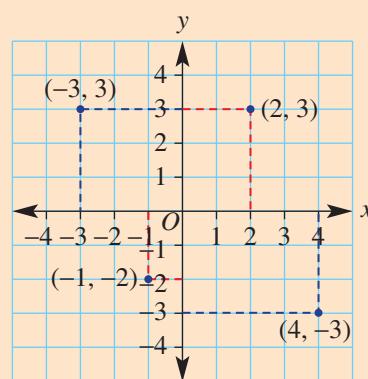
Using the number plane, how would you describe René and Pierre's final positions?



Mathematician and philosopher René Descartes.



- The number plane (or Cartesian plane) uses two axes (x -axis and y -axis) at right angles. Each axis uses a scale that includes both positive and negative numbers.
- A point plotted on the plane has an x - and y -coordinate, which is written as (x, y) . The x -coordinate is written before the y coordinate, as in the alphabet.
- The point $(0, 0)$ is called the origin or O .

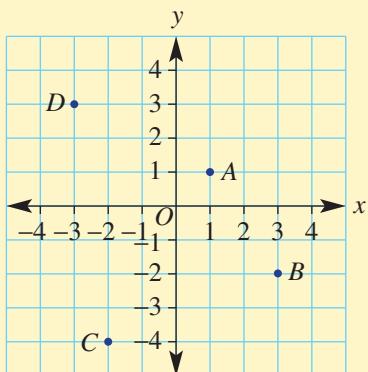


Key ideas



Example 10 Finding coordinates

For the number plane shown, write down the coordinates of the points labelled A, B, C and D.



SOLUTION

$$\begin{aligned}A &= (1, 1) \\B &= (3, -2) \\C &= (-2, -4) \\D &= (-3, 3)\end{aligned}$$

EXPLANATION

For each point, write the x -coordinate first (from the horizontal axis) followed by the y -coordinate (from the vertical axis).

Exercise 7G

1, 2

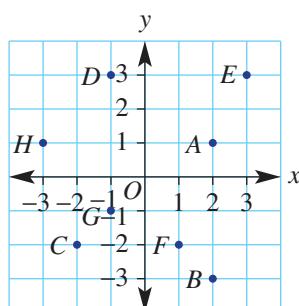
2

—

UNDERSTANDING

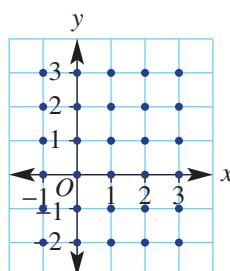
- 1 Match the points A, B, C, D, E, F, G and H with the given coordinates.

- | | | |
|---------------------|---------------------|--------------------|
| a $(-1, 3)$ | b $(2, -3)$ | c $(2, 1)$ |
| d $(-2, -2)$ | e $(3, 3)$ | f $(-3, 1)$ |
| g $(1, -2)$ | h $(-1, -1)$ | |



- 2 Count the number of points, shown as dots, on this plane that have:

- a** both x and y coordinates as positive numbers
- b** an x -coordinate as a positive number
- c** a y -coordinate as a positive number
- d** an x -coordinate as a negative number
- e** a y -coordinate as a negative number
- f** both x and y coordinates as negative numbers
- g** neither x nor y as positive or negative numbers



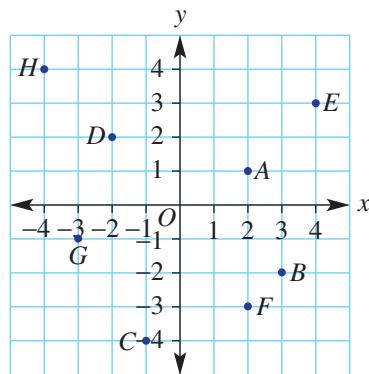
3–6

3–7

3–7

Example 10

- 3** For the Cartesian plane given, write down the coordinates of the points labelled A , B , C , D , E , F , G and H .



- 4 a** Draw a set of axes using 1 cm spacings. Use -4 to 4 on both axes.

- b** Now plot these points.

i $(-3, 2)$

ii $(1, 4)$

iii $(2, -1)$

iv $(-2, -4)$

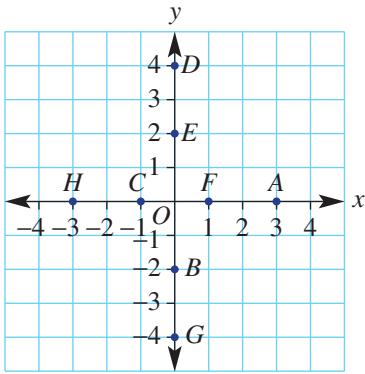
v $(2, 2)$

vi $(-1, 4)$

vii $(-3, -1)$

viii $(1, -2)$

- 5** For the number plane given, write down the coordinates of the points labelled A , B , C , D , E , F , G and H .



- 6** Seven points have the following x and y coordinates.

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

- a** Plot the seven points on a Cartesian plane. Use -3 to 3 on the x -axis and -2 to 4 on the y -axis.

- b** What do you notice about these seven points on the Cartesian plane?

- 7** Seven points have the following x and y coordinates.

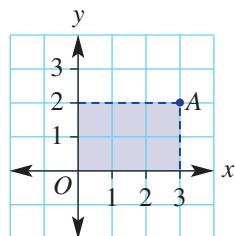
x	-3	-2	-1	0	1	2	3
y	5	3	1	-1	-3	-5	-7

- a** Plot the seven points on a number plane. Use -3 to 3 on the x -axis and -7 to 5 on the y -axis.

- b** What do you notice about these seven points on the number plane?

FLUENCY

- 8** When plotted on the Cartesian plane, what shape does each set of points form?
- a $A(-2, 0), B(0, 3), C(2, 0)$ b $A(-3, -1), B(-3, 2), C(1, 2), D(1, -1)$
 c $A(-4, -2), B(3, -2), C(1, 2), D(-1, 2)$ d $A(-3, 1), B(-1, 3), C(4, 1), D(-1, -1)$
- 9** Using the origin as one corner, the point $A(3, 2)$ as the opposite corner and the axes as two of the sides, a rectangle can be positioned on a set of axes, as shown opposite. Its area is 6 square units. Find the area of the rectangle if the point A is:
- a $(2, 2)$ b $(-3, 2)$
 c $(-1, -4)$ d $(3, -5)$
- 10** Karen's bushwalk starts at a point $(2, 2)$ on a grid map. Each square on the map represents 1 km. If Karen walks to the points $(2, -7)$, then $(-4, -7)$, then $(-4, 0)$ and then $(2, 0)$, how far has she walked in total?
- 11** The points $A(-2, 0), B(-1, ?)$ and $C(0, 4)$ all lie on a straight line. Find the y -coordinate of point B .
- 12** The points $A(-4, 8), B(-1, ?)$ and $C(2, -2)$ all lie on a straight line. Find the y -coordinate of point B .



13

13, 14

13, 14

7G

REASONING

ENRICHMENT

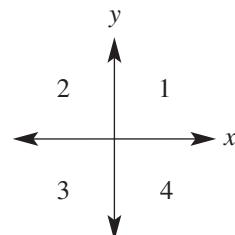
- 13 Consider the points $A(-2, 2)$, $B(0, 2)$ and $C(3, -2)$.

- Which point is closest to $(0, 0)$?
- Which point is farthest from $(0, 0)$?
- List the given points in order from closest to farthest from the origin, O .

- 14 A point (a, b) sits on the number plane in one of the four regions 1, 2, 3 or 4, as shown. These regions are called **quadrants**.

- Name the quadrant or quadrants that include the points where:

i	$a > 0$	ii	$a > 0$ and $b < 0$
iii	$b < 0$	iv	$a < 0$ and $b < 0$
- Shade the region that includes all points for which $b > a$.



Rules and graphs

15

- 15 Consider the rule $y = 2x - 1$.

- Substitute each given x -coordinate into the rule to find the y -coordinate. Then complete this table.

x	-3	-2	-1	0	1	2	3
y							

- Draw a Cartesian plane, using -3 to 3 on the x -axis and -7 to 5 on the y -axis.
- Plot each pair of coordinates (x, y) onto your Cartesian plane.
- What do you notice about the set of seven points?



The Cartesian plane was the starting point for the development of computer-generated graphics and design.



Investigation

Account balance with spreadsheets

If you have money saved in a bank account, your account balance should be positive. If you take out or spend too much money, your account balance may become negative.

- a Set up a spreadsheet to record and calculate a bank balance. Enter the given information describing one week of deposits and withdrawals, as shown.

Book1.xlsx - Microsoft Excel					
	A	B	C	D	
1	Bank account		Opening balance \$320		
2					
3	Date	Detail	Deposits	Withdrawals	Balance
4					\$320
5	May-01	Dinner			\$40
6	May-02	Sailing course fees			\$230
7	May-03	Camp costs			\$70
8	May-04	Deposit	\$100		
9	May-05	2 shirts			\$60
10	May-06	Party food			\$80
11	May-07	Deposit	\$50		
12					

- b i For the given spreadsheet, what is the balance at the end of May 1st?
ii On which day does the balance become negative?
- c Enter this formula into cell E5: =E4+C5-D5
Fill down to reveal the balance after each day.
- d Enter another week of deposits and withdrawals so that the balance shows both positive and negative amounts.
- e Now alter your opening balance. What opening balance is needed so that the balance never becomes negative? Is there more than one value? What is the least amount?
- f Investigate how positive and negative numbers are used on credit card accounts. Give a brief explanation.



Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

- 1** Find the next three numbers in these patterns.
 - a** $3, -9, 27, \underline{\quad}, \underline{\quad}, \underline{\quad}$
 - b** $-32, 16, -8, \underline{\quad}, \underline{\quad}, \underline{\quad}$
 - c** $0, -1, -3, -6, \underline{\quad}, \underline{\quad}, \underline{\quad}$
 - d** $-1, -1, -2, -3, -5, \underline{\quad}, \underline{\quad}, \underline{\quad}$

- 2** Evaluate the following.
 - a** $-100 + (-98) + (-96) + \dots + 98 + 100$
 - b** $(50 - 53) + (49 - 52) + (48 - 51) + \dots + (0 - 3)$
 - c** $2 - 3 + 4 - 5 + 6 - 7 + \dots - 199 + 200$

- 3** Insert brackets and symbols (+, -, \times , \div) into these number sentences to make them true.
 - a** $-3 \underline{\quad} 4 \underline{\quad} -2 = -6$
 - b** $-2 \underline{\quad} 5 \underline{\quad} -1 \underline{\quad} 11 = 21$
 - c** $1 \underline{\quad} 30 \underline{\quad} -6 \underline{\quad} -2 = -3$

- 4** **a** The difference between two numbers is 14 and their sum is 8. What are the two numbers?
b The difference between two numbers is 31 and their sum is 11. What are the two numbers?

- 5** If x and y are integers less than 10 and greater than -10 , how many different integer pairs (x, y) make the equation $x + 2y = 10$ true?

- 6** In the sequence of numbers $\dots, e, d, c, b, a, 0, 1, 1, 2, 3, 5, 8, 13, \dots$ each number is the sum of its two preceding numbers, e.g. $13 = 5 + 8$. What are the values of a, b, c, d and e ?

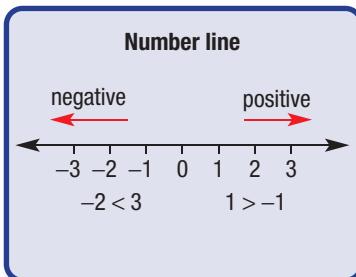
- 7** Given the rule $x^{-m} = \frac{1}{x^m}$, evaluate $-(-5)^{-2}$.

- 8** If $p > q > 0$ and $t < 0$, insert $>$ or $<$ to make each of these a true statement for all values of p, q and t .

a $p + t \underline{\quad} q + t$	b $t - p \underline{\quad} t - q$	c $pt \underline{\quad} qt$
--	--	------------------------------------

- 9** Describe the set of all possible numbers for which the square of the number is greater than the cube of the number.

Chapter summary

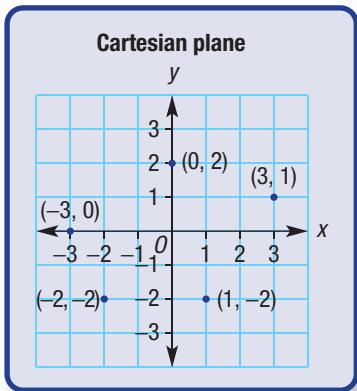


Adding and subtracting positive integers

$$\begin{aligned} -3 + 5 &= 2 \\ -4 + 3 &= -1 \\ 5 - 7 &= -2 \\ -1 - 10 &= -11 \end{aligned}$$

Adding and subtracting negative integers

$$\begin{aligned} 2 + (-3) &= 2 - 3 = -1 \\ -5 + (-4) &= -5 - 4 = -9 \\ 4 - (-3) &= 4 + 3 = 7 \\ -10 - (-6) &= -10 + 6 = -4 \end{aligned}$$



Integers
..., -3, -2, -1,
0, 1, 2, 3, ...

Substitution (Ext)
If $a = -2$ and $b = 4$, then:
 $b - a = 4 - (-2) = 6$
 $ab + 2a = -2 \times 4 + 2 \times (-2)$
 $= -8 + (-4)$
 $= -12$

Order of operations (Ext)

First brackets, then \times or \div
then $+$ or $-$, from left to right.

$$\begin{aligned} 3 \times (5 - (-2)) + 8 \div (-4) \\ = 3 \times 7 + (-2) \\ = 21 + (-2) \\ = 19 \end{aligned}$$

Multiplication (Ext)

$$\begin{aligned} 2 \times 3 &= 6 \\ 2 \times (-3) &= -6 \\ -2 \times 3 &= -6 \\ -2 \times (-3) &= 6 \end{aligned}$$

Division (Ext)

$$\begin{aligned} 10 \div 5 &= 2 \\ 10 \div (-5) &= -2 \\ -10 \div 5 &= -2 \\ -10 \div (-5) &= 2 \end{aligned}$$

Multiple-choice questions

- 7A** 1 When the numbers $-4, 0, -1, 7$ and -6 are arranged from lowest to highest, the correct sequence is:
A $0, -1, -4, -6, 7$ **B** $0, -4, -6, -1, 7$ **C** $-6, -4, -1, 0, 7$
D $-1, -4, -6, 0, 7$ **E** $-6, -1, 0, -4, 7$
- 7B** 2 The difference between -19 and 8 is:
A 152 **B** -11 **C** -27 **D** 11 **E** 27
- 7C** 3 The missing number in $2 - \square = 3$ is:
A 1 **B** -1 **C** 5 **D** -5 **E** 2
- 7C** 4 $5 - (-2) + (-7)$ is equal to:
A -4 **B** 10 **C** 7 **D** 0 **E** 14
- 7A** 5 The temperature inside a mountain hut is initially -5°C . After burning a fire for 2 hours the temperature rises to 17°C . What is the rise in temperature?
A -12°C **B** 12°C **C** 22°C **D** -85°C **E** -22°C
- 7D** 6 The product or quotient of two negative numbers is:
Ext **A** positive **B** negative **C** zero **D** added **E** different
- 7D** 7 $-2 \times (-5) \div (-10)$ is equal to:
Ext **A** -5 **B** 10 **C** -20 **D** 1 **E** -1
- 7E** 8 Which operation (i.e. addition, subtraction, multiplication or division) is completed second in the calculation of $(-2 + 5) \times (-2) + 1$?
Ext **A** addition **B** subtraction **C** multiplication
D division **E** brackets
- 7F** 9 If $a = -2$ and $b = 5$, then $ab - a$ is equal to:
Ext **A** -12 **B** -8 **C** 8 **D** 12 **E** 9
- 7G** 10 The points $A(-2, 3)$, $B(-3, -1)$, $C(1, -1)$ and $D(0, 3)$ are joined on a number plane. What shape do they make?
A triangle **B** square **C** trapezium
D kite **E** parallelogram

Short-answer questions

- 7A** 1 Insert the symbol $<$ (less than) or $>$ (greater than) into each of these statements to make it true.

a $0 \square 7$

b $-1 \square 4$

c $3 \square -7$

d $-11 \square -6$

- 7B/C** 2 Evaluate:

a $2 - 7$

b $-4 + 2$

c $0 - 15$

d $-36 + 37$

e $5 + (-7)$

f $-1 + (-4)$

g $10 - (-2)$

h $-21 - (-3)$

i $1 - 5 + (-2)$

j $-3 + 7 - (-1)$

k $0 + (-1) - 10$

l $-2 - (-3) - (-4)$

- 7C** 3 Find the missing number for each of the following.

a $-2 + \square = -3$

b $-1 + \square = -10$

c $5 - \square = 6$

d $-2 - \square = -4$

e $-1 - \square = 20$

f $-15 - \square = -13$

g $7 + \square = -80$

h $-15 + \square = 15$

- 7D** 4 Evaluate:

Ext

a $5 \times (-2)$

b -3×7

c $-2 \times (-15)$

d $10 \div (-2)$

e $-36 \div 12$

f $-100 \div (-25)$

g $-3 \times 2 \div (-6)$

h $-38 \div (-19) \times (-4)$

- 7D** 5 Find the missing number.

Ext

a $4 \times \square = -8$

b $\square \div -5 = 10$

c $\square \div 9 = -4$

d $-1 \times \square = 1$

- 7E** 6 Use order of operations to find the answers to these expressions.

Ext

a $-2 + 5 \times (-7)$

b $-1 - 18 \div (-2)$

c $-15 \div (1 + 4)$

d $5 - 4 \times (-3) \div (-3)$

e $(-2 - 5) \times (8 \div (-1))$

f $-7 \times ((-4) - 7) + 3$

- 7F** 7 Evaluate the following expressions if $a = 7$, $b = -3$ and $c = -1$.

Ext

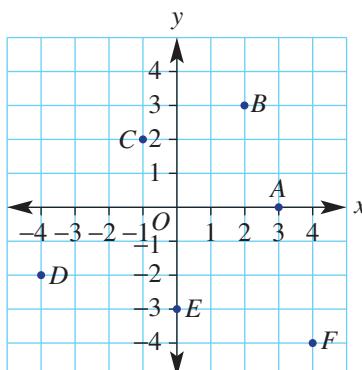
a $a - b$

b $2b - 5a$

c $ab + c$

d $bc - 2a$

- 7G** 8 For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C, D, E and F.



Extended-response questions

- 1 A scientist, who is camped on the ice in Greenland, records the following details in her notepad regarding the temperature over five days. Note that ‘min’ stands for minimum and ‘max’ stands for maximum.
- Monday: min = -18°C , max = -2°C .
 - Decreased 29°C from Monday’s max to give Tuesday’s min.
 - Wednesday’s min was -23°C .
 - Max was only -8°C on Thursday.
 - Friday’s min is 19°C colder than Thursday’s max.
- a What is the overall temperature increase on Monday?
- b What is Tuesday’s minimum temperature?
- c What is the difference between the minimum temperatures for Tuesday and Wednesday?
- d What is the overall temperature drop from Thursday’s maximum to Friday’s minimum?
- e By how much will the temperature need to rise on Friday if its maximum is 0°C ?



- 2 When joined, these points form a picture on the number plane. What is the picture?
 $A(0, 5), B(1, 3), C(1, 1), D(2, 0), E(1, 0), F(1, -2), G(3, -5),$
 $H(-3, -5), I(-1, -2), J(-1, 0), K(-2, 0), L(-1, 1), M(-1, 3), N(0, 5)$