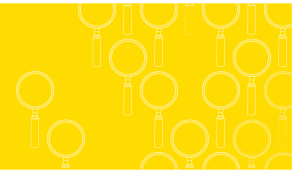## Ten dollars and eighty-nine cents

**Equipment required:** 2 brains may be used for the 'Explore' section

### The Big Question

Why is the answer to a seemingly random collection of calculations always the same?

> Looks like magic, but can it be explained by using maths?

### Engage

**1 (a)** Choose an amount of money less than $10.00 (e.g. $7.25) and write it down.

**(b)** Reverse the digits and write down the amount formed, keeping the decimal point after the first digit (e.g. $5.27).

**(c)** Subtract the smaller number from the larger one (e.g. $7.25 − $5.27 = $1.98).

**(d)** Reverse the digits in your answer to part **(c)** and add the resulting number to your answer to part **(c)** (e.g. $1.98 + $8.91 = $10.89).

**(e)** Repeat parts **(a)** to **(d)** with a different starting amount. Do you get the same final answer?

### Explore

**2** To investigate further, write down at least 10 different amounts (all less than $10.00) using different combinations of digits. Perform the same steps from Question **1** for each of them. Keep a record of which amounts produced the same final answer and which didn't.

**3** Can you see a pattern in the calculations that can explain why the same answer keeps appearing? If you can, try to describe it. If you can't, write down some new amounts and test them. Remember that the three digits you use do not have to all be different —try making some the same and see what happens to the result.

#### Strategy options

- Look for a pattern.
- Seek an exception.
- Break problem into manageable parts.

### Explain

**4** For your calculations that gave $10.89 as the final answer, explain the following.

**(a)** Look at the subtractions of the digits in the hundredths column. What do they all have in common? Explain why this is the case, if you can.

**(b)** Look at the middle digit that results after the subtraction step. What do you notice? Again, explain why this is the case, if you can.

**(c)** Did you find amounts for which the answer was not $10.89? What did the digits of these numbers have in common?

### Elaborate

**5 (a)** For amounts that add to $10.89, what is the sum of the first and last digits of the answer to the subtraction? Explain why.

**(b)** Explain why the final addition step produces $10.89. Why does the addition of the tenths digits always result in an 8?

**6** Summarise what you have found in this investigation by answering the Big Question.

### Evaluate

**7 (a)** Did you find the results to your first few calculations surprising? Do you have more of an understanding now of how the 'magic' works?

**(b)** How many calculations did it take for you to see patterns emerging? What did you first start to notice?

**(c)** Did you feel frustrated at any time during this investigation? If so, how did you deal with the frustration?

### Extend

**8** Try this.

**(a)** Select any three single-digit decimal numbers (e.g. 0.4, 0.6, 0.1).

**(b)** Use these numbers to make six two-digit numbers with one number the units and the other number the tenths (e.g. 4.6, 4.1, 6.1, 6.4, 1.4, 1.6).

**(c)** Add these numbers (e.g. 24.2).

**(d)** Add your original numbers (e.g. 1.1).

**(e)** Divide this into the sum you obtained in part **(c)** (e.g. 24.2 ÷ 1.1).

Did you get 22?

# Decimal multiplication

## Multiplying a decimal by a whole number

To multiply a decimal by a whole number:

Multiply as though both values are whole numbers. Then, place a decimal point in the answer so that it has the same number of decimal places as the decimal number in the question.

To check that your decimal point is in the correct place, it is a good idea to estimate your answer before calculating it.

### Worked Example 11                                                                 W.E 11

Calculate:

**(a)** $4.172 \times 3$          **(b)** $52.63 \times 14$          **(c)** $0.0003 \times 2$

| Thinking | Working |
|---|---|
| **(a) 1** Use rounding to estimate the answer. | (a) $4.172 \times 3 \approx 4 \times 3$ <br> $\approx 12$ |
| **2** Multiply as though both values are whole numbers. | $\begin{array}{r} 4.172 \\ \times \quad ^{2}3 \\ \hline 12.516 \end{array}$ |
| **3** Make sure that there are the same number of decimal places in the answer as in the question (in this case, 3). | $4.172 \times 3 = 12.516$ |
| **4** Check your answer against your estimate to see whether it is reasonable. | *Reasonable* |
| **(b) 1** Use rounding to estimate the answer. | (b) $52.63 \times 14 \approx 50 \times 14$ <br> $\approx 14 \times 5 \times 10$ <br> $\approx 700$ |
| **2** Multiply as though both values are whole numbers. | $\begin{array}{r} 52.63 \\ \times \quad ^{1\,2\,1}14 \\ \hline 21\,052 \\ 52\,630 \\ \hline 73\,6.82 \end{array}$ |
| **3** Make sure that there are the same number of decimal places in the answer as in the question (in this case, 2). | $52.63 \times 14 = 736.82$ |

| | 4 | Check your answer against your estimate to see whether it is reasonable. | | *Reasonable* |

(c) 1 An estimate is not needed here, as we can see that the only non-zero digit will be a 6 (3 × 2).

*(c)*

2 Multiply as though both values are whole numbers. Place a decimal point in the answer so that there are the same number of decimal places as in the decimal number in the question (in this case, 4).

$$\begin{array}{r} 0.0003 \\ \times \qquad 2 \\ \hline 0.0006 \end{array}$$

*0.0003 × 2 = 0.0006*

## Multiplying and dividing decimals by powers of 10

When we multiply a number by 10, we are making each digit in the number 10 times its previous value.

In the place value table, each digit is moved up one place value column to the left to show this. Hundredths become tenths, tenths become ones, ones become tens etc.

For example, $43.25 \times 10 = 432.5$.

| Hundreds 100 | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ 0.01 |
| --- | --- | --- | --- | --- | --- |
| | 4 | 3 | • | 2 | 5 |

× 10   × 10   × 10      × 10

$\times 10 =$

| Hundreds 100 | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) |
| --- | --- | --- | --- | --- |
| 4 | 3 | 2 | • | 5 |

When we divide by 10, we are making each digit one-tenth of its previous value. In the place value table, each digit is moved down one place value column to the right to show this. Tenths become hundredths, ones become tenths, tens become ones etc.

For example, $65.1 \div 10 = 6.51$ as shown below.

| Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ (0.01) |
| --- | --- | --- | --- | --- |
| 6 | 5 | • | 1 | |

÷ 10      ÷ 10      ÷ 10

$\div 10 =$

| Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ (0.01) |
| --- | --- | --- | --- | --- |
| | 6 | • | 5 | 1 |

It is convenient to think of this as moving the decimal point one place to the right (for multiplication) or left (for division). This produces the same answer; however, we should remember it is the digits themselves that actually move as their values increase or decrease.

Multiplying or dividing by 100 is like multiplying or dividing by 10 twice (as $100 = 10 \times 10$). Each digit moves two place value columns. Again, we can think of this as moving the decimal point two places to the right or left.

To *multiply* a decimal by a power of 10:

Move the decimal point to the *right* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$$0.45 \times 10 \ (10^1) = 4.5$$
$$0.45 \times 100 \ (10^2) = 45$$
$$0.45 \times 1000 \ (10^3) = 450$$

To *divide* a decimal by a power of 10:

Move the decimal point to the *left* the same number of place values as zeroes in the power of 10. Fill in any empty place values with zeroes.

$$78.9 \div 10 \ (10^1) = 7.89$$
$$78.9 \div 100 \ (10^2) = 0.789$$
$$78.9 \div 1000 \ (10^3) = 0.0789$$

## Decimal 'shorthand'

Each of the powers of 10 has its own 'name', as shown below with the first nine powers:

$10^1$ = ten
$10^2$ = hundred
$10^3$ = thousand
$10^4$ = ten thousand
$10^5$ = hundred thousand
$10^6$ = million
$10^7$ = ten million
$10^8$ = hundred million
$10^9$ = billion

To write very large numbers, we sometimes write a smaller decimal number and the name of the power. For example, instead of writing 2 300 000, we could write 2.3 million, or $2.3 \times 10^6$. This is equivalent to writing $2.3 \times 1\,000\,000$, or 2 300 000.

## Multiplying decimals by other multiples of 10

Multiples of 10, such as 20, 190, 1700, 8000 etc., can be written as a number multiplied by a power of 10: $20 = 2 \times 10$, $190 = 19 \times 10$, $1700 = 17 \times 100$, $8000 = 8 \times 1000$.

Writing them in this way means we can multiply by the number first, then the power of 10.

### Worked Example 12

WE 12

Calculate:

(a) $0.6295 \times 7000$

(b) $9.81 \times 340\,000$

| Thinking | Working |
|---|---|
| (a) 1 Rewrite the whole number as a product of a number and a power of 10. | (a) $7000 = 7 \times 1000$ |
| 2 Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied. | $0.6295$ $\times 7$ $4.4065$ |
| 3 Multiply the answer by the power of 10 by moving the decimal point to the right. (In this case, 3 places.) | $4.4065 \times 1000$ $= 4406.5$ |

**(b) 1** Rewrite the whole number as a product of a number and a power of 10.

(b) $340\,000 = 34 \times 10\,000$

**2** Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied.

$$\begin{array}{r} 9.81 \\ \times\ ^{2}{}^{3}\ 34 \\ \hline 3924 \\ {}_{1}2{,}9430 \\ \hline 333.54 \end{array}$$

**3** Multiply by the power of 10 by moving the decimal point to the right. (In this case, 4 places.) Write zeroes into the empty place value columns.

$333.54 \times 10\,000$

$= 3\,335\,400$

## Multiplying a decimal by another decimal

To develop a method for multiplying one decimal by another, it is useful to look at some number patterns. We have seen that when we multiply by powers of 10, each digit in the number moves up the same number of place values as the power number. We show this by moving the decimal point. For example:

$0.23 \times 10 = 2.3$          $0.23 \times 100 = 23$          $0.23 \times 1000 = 230$

We can reverse the above calculations by dividing by the powers of 10:

$2.3 \div 10 = 0.23$          $23 \div 100 = 0.23$          $230 \div 1000 = 0.23$

We can see that dividing by powers of 10 corresponds to moving the decimal point to the left the same number of places as zeroes in the power of 10. This is used in the Worked Example below.

## Worked Example 13

**W.E 13**

Calculate:

**(a)** $0.63 \times 0.4$          **(b)** $0.014 \times 0.002$

### Thinking

### Working

**(a) 1** Write both numbers as the division of a whole number by a power of 10.

(a) $0.63 = 63 \div 100$
$0.4 = 4 \div 10$

**2** Multiply as though both numbers are whole numbers. (We do not need to multiply by the zero in 0.4, as it does not affect the outcome.) Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide by 100 and 10, which is the same as dividing by 1000.) If necessary, place a zero in the units place to clearly show the position of the decimal point.

$$\begin{array}{r} 0.63 \\ {}^{2}{}^{1} \\ \times\ 0.4 \\ \hline 0.252 \end{array}$$

$0.63 \times 0.4 = 0.252$

**(b)  1**  Write both numbers as the division of a whole number by a power of 10.

**2**  Multiply as though both numbers are whole numbers (ignoring the zeroes). Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide twice by 1000, which is the same as dividing by 1 000 000.) Place zeroes in the empty place values and a zero in the units place.

(b)  $0.014 = 14 \div 1000$
$0.002 = 2 \div 1000$

$$\begin{array}{r} 0.014 \\ \times \quad 0.002 \\ \hline 0.000\,028 \end{array}$$

$0.014 \times 0.002 = 0.\,000\,028$

To multiply one decimal by another:

**Step 1**  Write both numbers as the division of a whole number and a power of 10.

**Step 2**  Multiply as though both numbers are whole numbers.

**Step 3**  Divide the product by the powers of 10 identified in **Step 1**. Show this by moving the decimal place to the left.

You may be able to spot a useful shortcut from the Worked Example above: The number of decimal places in the answer to the multiplication is equal to the total number of decimal places in the numbers being multiplied.

# **4.5** Decimal multiplication

## Navigator

| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q7, Q9, Q11, Q12, Q13, Q14, Q17, Q21, Q23, Q24 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q17, Q19, Q21, Q22, Q23, Q24, Q25 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q18, Q19, Q20, Q22, Q23, Q24, Q25 |
|---|---|---|

Answers
page 643

## Fluency

**1**  Calculate:

(a)  $7.6 \times 4$

(b)  $5.3 \times 7$

(c)  $6.5 \times 9$

(d)  $5 \times 4.62$

(e)  $0.54 \times 4$

(f)  $6 \times 12.33$

(g)  $8.047 \times 8$

(h)  $0.619 \times 7$

(i)  $3 \times 18.309$

(j)  $22.08 \times 13$

(k)  $48.76 \times 25$

(l)  $37.09 \times 32$

(m)  $0.0004 \times 2$

(n)  $0.0005 \times 7$

(o)  $1.0006 \times 3$

> Remember, the number of decimal places in your answer should be the same as the number in the decimal being multiplied.

WE 11

**2**  Calculate:

(a)  $2.4 \times 60$

(b)  $3.27 \times 70$

(c)  $2.991 \times 40$

(d)  $0.57 \times 200$

(e)  $4.34 \times 500$

(f)  $6.625 \times 400$
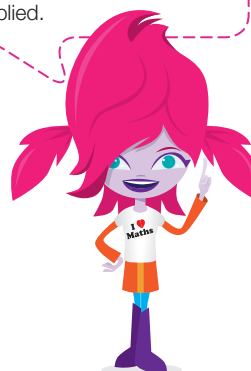
(g)  $4.591 \times 6000$

(h)  $6.38 \times 4300$

(i)  $3.14 \times 9300$

(j)  $0.571 \times 68\,000$

(k)  $0.9465 \times 71\,000$

(l)  $7.4832 \times 55\,000$

WE 12

**WE13**

3 Calculate:

(a) $0.6 \times 0.8$  (b) $0.4 \times 0.6$  (c) $0.9 \times 0.4$

(d) $9.32 \times 0.7$  (e) $4.12 \times 0.3$  (f) $5.26 \times 0.8$

(g) $7.74 \times 0.31$  (h) $9.46 \times 0.22$  (i) $3.76 \times 0.18$

(j) $0.004 \times 0.6$  (k) $0.0308 \times 0.3$  (l) $0.072 \times 0.05$

4 A reasonable estimate for the answer to $1.8 \times 15$ is:

A 3  B 16.8  C 30  D 300

5 How many decimal places will there be in the answer to $12.789 \times 41$?

A 1  B 2  C 3  D 4

6 How many decimal places will there be in the answer to $0.36 \times 1.04$?

A 2  B 3  C 4  D 5

## Understanding

7 Renée buys 52 components from an electronics shop for $0.35 each. What is the total cost of the components?

8 A pill bottle contains 37 pills, each of mass 0.287 grams. What is the total mass of the bottle's contents?

9 Write the following as whole numbers.

(a) 3.5 million  (b) 4.26 billion  (c) 1.3 trillion (1 trillion = 1000 billion)

(d) 57.08 million  (e) 10.2 trillion  (f) 20.045 billion

10 A snail crawls 0.041 metres in an hour. If it kept up this pace, how far could it move in a day?

11 Amy orders 27.5 metres of timber priced at $4.29 per metre from her local hardware store. How much will the timber cost her in total? (Round your answer to the nearest cent.)



12 Gavin sells 300 punnets of berries to a supermarket. If he is paid $1.72 for each punnet, how much money does he receive?

13 In 2010, Mexican businessman Carlos Slim was declared the world's richest man, with a fortune worth 53.5 billion US dollars. Write this amount as a whole number.

14 (a) Marita fills her car's fuel tank with petrol at a cost of $1.35 per litre. If she adds 70 litres, how much must she pay?

(b) Ali pumps 9.5 litres of petrol into his car's tank. How much did the fuel cost him if he paid the same price per litre as Marita? Round your answer to the nearest cent.

15 The instructions on a bag of fertiliser say to spread 0.75 kilograms per square metre of area to be fertilised. Raisa wants to fertilise 2600 square metres of property using this product. How many kilograms of fertiliser should she use?

16 Bryan has calculated that he needs 3.6 square metres of tiles for his bathroom. What will he have to pay if the tiles he wants cost $22.39 per square metre? (Round your answer to the nearest cent.)

## Reasoning

17  Kim works in an ice-cream parlour, and one hot day sells 500 scoops of vanilla. If there is 0.185 litres of ice-cream in each scoop and the day started with 130 litres of vanilla in the freezer, how many litres of vanilla ice-cream are left at the end of the day?



18  Yoghurt contains 167 kilojoules per 100 grams. How many kilojoules would a 175 gram tub contain?

19  In April 2010, the Australian Bureau of Statistics calculated that the population of Australia was 22 296 411.

(a)  Write this number as a decimal number, with the number of millions as the whole number part, and the rest of the number rounded to one decimal place.

(b)  Why would your answer to (a) be the number you would expect to see reported in the news, instead of the 'exact' value?

20  (a)  A woodwork teacher needs 50 pieces of pine, each 0.135 metres long, to give to students to make a particular model. What total length (in metres) of timber does the teacher need? (Ignore any lost length due to saw cuts.)

(b)  If he can buy the pine in 3.5 m lengths, how many pieces does he need, and how much does he have left over?

21  A multi-bag of mini chocolate bars costs $3.95 and contains 12 bars. Each mini chocolate bar can also be purchased individually for 40 cents.

(a)  How much would 12 mini chocolate bars cost if they were purchased individually?

(b)  How much do you save by purchasing the multi-bag?

(c)  Charlotte wants to buy 30 mini bars for a party. Should she buy them individually, or in multi-bags? Which way is cheaper?

22  Wayne is following his grandfather's recipe for tomato sauce, which requires 3.5 pounds of tomatoes. (A pound is an old imperial unit of mass.) Because he only has scales in kilograms, Wayne decides to convert the mass to kilograms. A website tells him that 1 pound equals 0.454 kilograms. What answer should he get when he converts the mass of the tomatoes to kilograms? Round your answer to one decimal place.

23  Francesca read her electricity meter one Saturday, and again at the same time the next Saturday. She recorded the following readings.

First Saturday:       7562.3 kilowatt-hours
Second Saturday:  7638.6 kilowatt-hours

(a)  How many kilowatt-hours of electricity did Francesca's household use in the week?

(b)  If electricity costs $0.13 per kilowatt-hour, find the total cost of the electricity supplied in the week.

## Open-ended

**24** Jayapal has been asked to work out the length if every student in Year 7 lay end to end. He has been told that there are 89 students in Year 7 and that their average height is 1.63 m.

This is his working out:

$89 \times 1 = 89$
$89 \times 6 = 534$
$89 \times 3 = 267$

$$
\begin{array}{r}
89 \\
534 \\
+\ 267 \\
\hline
= 880 \\
\hline
\end{array}
$$

Answer: 8.80 m.

**(a)** Where has Jayapal made his error? Explain how he could use estimation to tell that he was incorrect.

**(b)** Explain how he could modify his method to get the correct answer.

**(c)** Show how you would complete this problem.

**25** Two numbers are multiplied together to give 68.4.

**(a)** Find at least two possible pairs of numbers with one number being a multiple of 10.

**(b)** Find at least two possible pairs of numbers with neither being a multiple of 10.

# Outside the Square | Game

## Multiplicity

**Equipment required:** 2 brains, 1 die, 1 calculator

**How to win:**
The aim of the game is to get the target number as the whole number part of a decimal.

**How to play:**

**1** The first player rolls the die twice to determine the two-digit target number (the first number followed by the second number).

**2** The second player then rolls the die twice to determine the starting number.

**3** The first player puts the starting number into a calculator and can multiply this by any number of their choice, with the aim of reaching the target number.

**4** If they reach a number starting with the two-digit target number, then they're the winner; if not, the other person takes their answer (leave it in the calculator display) and then chooses a number to multiply this by to try to reach the target number.

For example, the first player rolls a 4 and a 3, so the target number is 43.

The second player then rolls two 1s, so the starting number is 11.

The first player chooses to multiply the 11 by 4, which gives an answer of 44—too big.

The second player chooses to multiply the 44 by 0.9, which gives an answer of 39.6—too small.

This continues until a player reaches an answer of 43 point something.

The game can be made more difficult by making the target number a three-digit number, or a two-digit number with one decimal point.

# Half-time 4
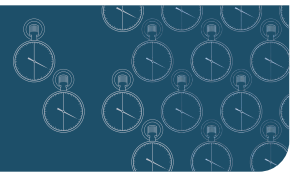
1 Convert the following decimals into fractions (or mixed numbers where appropriate) in simplest form.

   (a) 0.25          (b) 0.3          (c) 3.54          (d) 9.345

2 Put the following decimals in order from smallest to largest.

   9.01, 19.12, 0.03, 0.321, 9.003 24

3 Calculate:

   (a) $12.5 + 7.9$          (b) $124.54 + 8.379$          (c) $0.32 + 2.5$

   (d) $15 - 8.98$          (e) $127.061 - 43.2$          (f) $6 - 4.9$

4 Round the following numbers to three decimal places.

   (a) 4.8765          (b) 13.0037          (c) 2.3498          (d) 23.032 49

5 Calculate the following.

   (a) $7.6 \times 4$          (b) $28.3 \times 5$          (c) $1.37 \times 9$

   (d) $45.2 \times 30$          (e) $0.26 \times 2000$          (f) $6.78 \times 500$

   (g) $4.3 \times 1.7$          (h) $39.82 \times 15.4$          (i) $1.2 \times 0.0005$

6 Franji is saving up to purchase a games console. She has saved the following amounts over the last four weeks: $25, $47.50, $18.70, $9.35.

   (a) How much has she saved in total?

   (b) If the console is $149, how much more does Franji have to save?

7 Write the following in decimal form.

   (a) $23 + \dfrac{4}{10} + \dfrac{9}{100} + \dfrac{7}{10000}$          (b) $14 + \dfrac{235}{1000}$

   (c) 2 hundreds, 9 tens, 3 ones, 4 tenths, 1 hundredth and 5 thousandths

   (d) 9 tens, 0 ones, 6 tenths, 2 hundredths and 7 ten-thousandths

8 Round the following amounts of money to the nearest 5 cents.

   (a) $1.93          (b) $35.08          (c) $57.44          (d) $189.98

9 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary.

   (a) $\dfrac{4}{5}$          (b) $\dfrac{17}{20}$          (c) $\dfrac{2}{3}$          (d) $\dfrac{7}{12}$

10 The times recorded by the finalists in a track cycling sprint race were: 59.023 s, 59.105 s, 59.23 s, 59.125 s. Write the times in order from fastest to slowest.

11 Melinda is out buying some ingredients for a cooking class. She buys:

   12 packets of pasta at $2.49 each
   18 tins of tomatoes at $1.35 each
   6 kg of mince at $8.99 per kg
   3 packets of cheese at $4.78 per packet

   Find the total cost of Melinda's shopping.

# 4.6 Decimal division

When studying division, the following mathematical terms are useful.

- The dividend is the number that is being divided.
- The divisor is the number we are dividing by.
- The quotient is the result of the division (the answer).

These are shown in the example below.

$$\text{quotient} \longrightarrow 8.14$$
$$\text{divisor} \longrightarrow 7\,)\overline{56.98} \longleftarrow \text{dividend}$$

---

When using long or short division to divide a decimal by a whole number:

Line up the decimal point in the quotient with the decimal point in the dividend.

---

## Worked Example 14

**WE 14**

Calculate the following. Round your answers to three decimal places if necessary.

**(a)** $5.28 \div 3$

**(b)** $0.41 \div 7$

| Thinking | Working |
|---|---|
| **(a) 1** Write the question with the setting out shown. | (a) $3\,)\overline{5.28}$ |
| **2** Perform the division, and place a decimal point in the quotient so it lines up with the one in the dividend. | $\begin{array}{r} 1.\,7\,6 \\ 3\,)\overline{5.{}^2 2 {}^1 8} \end{array}$ |
| **(b) 1** Write the question with the setting out shown. | (b) $7\,)\overline{0.41}$ |
| **2** Perform the division, remembering to place a zero in the quotient if the divisor goes zero times into the dividend. Add zeroes to the divisor to continue the division, until there are four decimal places in the answer. | $\begin{array}{r} 0.0\,5\ 8\ 5... \\ 7\,)\overline{0.4\,{}^41\,{}^60\,{}^40} \end{array}$ |
| **3** Round off the answer to three decimal places. | $0.059$ (to 3 d.p.) |

## Dividing decimals by multiples of 10

As we did for multiplying decimals, we can write other multiples of 10 as the product of a number and a power of 10, and do two separate division steps. Remember that we can show division by a power of 10 by moving the decimal point to the left the same number of places as the zeroes in the power of 10.

## Worked Example 15

Calculate the following. Round your answers to three decimal places where necessary.

**(a)** $9.76 \div 400$                    **(b)** $14.7 \div 80$

| Thinking | Working |
|---|---|
| **(a) 1** Break the divisor down into a whole number and a power of 10. | (a)   $9.76 \div 400$<br>$= 9.76 \div 4 \div 100$ |
| **2** Divide by the whole number first (in this case, by the 4). | $\dfrac{2.44}{4 \overline{) 9.^{1}7^{1}6}}$ |
| **3** Now, divide the quotient of the first division by the power of 10. Show this by moving the decimal point. (Here, it is moved 2 places left.) | $2.44 \div 100 = 0.0244$ |
| **4** Round your answer to the specified number of decimal places. | $= 0.024$ (to 3 d.p.) |
| **(b) 1** Break the divisor down into a whole number and a power of 10. | (b)   $14.7 \div 80$<br>$= 14.7 \div 8 \div 10$ |
| **2** Divide by the whole number first (in this case, by the 8). Add zeroes to the divisor to continue the division until there are more decimal places in the answer than the rounded answer requires. | $\dfrac{1.8\ 3\ 7\ 5}{8 \overline{) 14.^{6}7^{3}0^{6}0^{4}0}}$ |
| **3** Now, divide the answer by the power of 10. Show this by moving the decimal point. (Here, it is moved 1 place left.) | $1.8375 \div 10 = 0.18375$ |
| **4** Round your answer to the specified number of decimal places. | $= 0.184$ (to 3 d.p.) |

## Dividing a decimal by another decimal

In order to divide a decimal by another decimal we need to convert it to a problem where we can divide a decimal by a whole number.

Recall that when working with fractions we can multiply or divide the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

For example, $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{10}{20}$. We can apply the same principle to dividends and divisors.

For example, $1 \div 2$ gives the same answer as $2 \div 4$ and $10 \div 20$.

$13.52 \div 1.2$
$= 135.2 \div 12$ (multiplying both numbers by 10)
$= 1352 \div 120$ (multiplying by 10 again)

$7.5 \div 2.5$
$= 15 \div 5$ (multiplying by 2)

> To divide one decimal by another, multiply both numbers so that the divisor is no longer a decimal. Then, divide the new dividend by the new, whole number divisor.

Multiplying both numbers by a power of 10 is often the most convenient. Multiply by the smallest power of 10 that gives a whole number divisor.

## Worked Example 16 — W.E 16

Calculate the following. Round your answer to three decimal places if necessary.

(a) $3.35 \div 0.005$

(b) $10.52 \div 0.9$

| Thinking | Working |
|---|---|
| (a) **1** What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 1000.) | (a) $3.35 \div 0.005$ <br> $= (3.35 \times 1000) \div (0.005 \times 1000)$ |
| **2** Rewrite the question using the new divisor and new dividend. | $= 3350 \div 5$ |
| **3** Calculate the quotient. | $\begin{array}{r} 6\ 70 \\ 5\overline{)33^350} \end{array}$ |
| **4** State the answer. | $3.35 \div 0.005 = 670$ |
| (b) **1** What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 10.) | (b) $10.52 \div 0.9$ <br> $= (10.52 \times 10) \div (0.9 \times 10)$ |
| **2** Rewrite the question using the new divisor and new dividend. | $= 105.2 \div 9$ |
| **3** Calculate the quotient to one more decimal place than the rounded number required. | $\begin{array}{r} 1\,1.\ 6\ 8\ 8\ 8 \\ 9\overline{)10^15.^62^80^80^80} \end{array}$ |
| **4** Round off the quotient to the required number of decimal places. | $= 11.689$ (to 3 d.p.) |

# 4.6 Decimal division

## Navigator

**Answers page 643**

## Fluency

**1** Calculate the following. Round your answers to three decimal places if necessary.

WE 14

Don't forget to write a zero in the quotient if at any stage the divisor 'won't go'.

**TAKE CARE**

(a) $8.48 \div 4$   (b) $12.96 \div 3$   (c) $14.22 \div 9$

(d) $3.3 \div 6$   (e) $2.4 \div 7$   (f) $5.1 \div 9$

(g) $39.42 \div 6$   (h) $10.6 \div 7$   (i) $0.47 \div 9$

(j) $0.42 \div 6$   (k) $1.22 \div 3$   (l) $18.46 \div 7$

(m) $154.33 \div 5$   (n) $84.365 \div 8$   (o) $347.507 \div 7$

**2** Calculate the following. Round your answers to three decimal places if necessary.

WE 15

(a) $3.6 \div 40$   (b) $8.1 \div 90$   (c) $36.216 \div 60$

(d) $96.58 \div 200$   (e) $2.296 \div 400$   (f) $23.4 \div 600$

(g) $233.4 \div 3000$   (h) $40.6 \div 8000$   (i) $18.9 \div 7000$

(j) $68.4 \div 120$   (k) $3.19 \div 110$   (l) $8.97 \div 1300$

**3** Calculate the following. Round your answers to three decimal places if necessary.

WE 16

(a) $5.14 \div 0.2$   (b) $4.12 \div 0.4$   (c) $5.1 \div 0.6$

(d) $15.48 \div 0.04$   (e) $9.018 \div 0.09$   (f) $1.736 \div 0.07$

(g) $2.382 \div 0.006$   (h) $0.3996 \div 0.009$   (i) $0.196 \div 0.008$

(j) $7.7 \div 0.011$   (k) $3.78 \div 0.012$   (l) $1.16 \div 0.008$

(m) $1.74 \div 0.0002$   (n) $8.05 \div 0.0007$   (o) $13.36 \div 0.0008$

**4** How many decimal places will the answer to $298.12 \div 100$ have?

A  1   B  2   C  3   D  4

**5** Evaluating which of the following will give the answer to $3.2 \div 0.005$?

A  $0.0032 \div 5$   B  $320.0 \div 5$   C  $3200.0 \div 5$   D  $32\,000.0 \div 5$

## Understanding

**6** A block of metal of mass 2.312 kg is to be divided equally into 8 pieces. What will the mass of each piece be?

**7** A company makes a profit of 7.86 million dollars. If the profit is to be divided equally among the four company owners, how much would each owner get? (Answer in millions of dollars.)

**8** The total bill for a wedding reception for 100 guests is $9398.55. How much does this work out to per guest? (Round your answer appropriately.)



**9** Anthony needs to mix 0.07 litres of environmentally friendly pest repellent with water each time he fills the tank of his garden sprayer. If he has 0.84 litres of repellent left, how many sprayer tankfuls can he mix?

**10** A jeweller calculated that a popular style of necklace contains 0.009 g of gold. How many necklaces could she make using the 2.332 g of gold in her workshop?

**11** Eleanor's car travels 37.29 km on three litres of fuel. How far would it travel on one litre of fuel?

**12** A 4000 millilitre container of orange juice costs $8.90. Determine the cost per 100 mL. (Round your answer appropriately.)

**13** Simon buys 7 kg of spicy sausages for $32.95. How much is this per kilogram?

**14** Regina has a tub containing 5.8 kilograms of worms which she uses for fishing. If the average worm has a mass of 0.003 kilograms, how many are in her tub?

**15** Hai is making a long-distance call on his mobile. The call costs $0.90 per minute. For how many minutes can he continue his call if he has $5.85 credit left on his phone?

**16** A 1.2 kilometre taxi trip costs Joan $5.70. How much is this per kilometre?

## Reasoning

**17** The captains of two netball teams were discussing which of their teams was taller. The heights of Angie's team were 1.49 m, 1.54 m, 1.65 m, 1.67 m, 1.58 m, 1.73 m and 1.56 m. The heights of Jessica's team were 1.73 m, 1.58 m, 1.59 m, 1.62 m, 1.45 m, 1.51 m and 1.62 m. Add the seven heights and divide the result by seven to find the average (mean) height of each team. Which of the two teams was the taller, on average? Answer correct to two decimal places.

**18** Monica and Celeste were trying to determine the best bargain of three different jars of coffee. The 200 g jar of BestCafe cost $5.95, the 500 g jar of Maxine House was $14.85 and the 300 g tin of Brazilian Roast was $6.75. Which is the best buy in terms of price per 100 grams?

**19** Karen was counting the coins in her cash register drawer and found that she had $1.65 in 5-cent coins, $2.30 in 10-cent coins, $5.20 in 20-cent coins and $7.50 in 50-cent coins. How many of each type of coin did she have?

**20** Jacob received a bill from a rent-a-car firm that gave the total cost as $310. This included the daily cost of $24.80 and $1.20 per kilometre travelled. If Jacob had hired the car for 2 days, how many kilometres did he travel?

## Open-ended

**21** Write three numbers, each with three decimal places, that when divided by 4 give an answer between 0.5 and 1.

**22** For each of the following, write two equivalent divisions by multiplying the divisor by a number that is not a power of 10.

**(a)** $10.05 \div 0.5$

**(b)** $36.04 \div 0.02$

**23** The area of a rectangle is calculated by multiplying the length and the width. If the area of a rectangle is 8.45 m$^2$, what might be the length and width of the sides? Give two possible values for the side dimensions. For each of the side dimensions, work out the related perimeter (distance along the boundary of the rectangle).

**24** Tran divided 5.25 by 5 and wrote down an answer of 1.5.

**(a)** What error did Tran make?

**(b)** Explain to Tran what he needs to do to calculate decimal divisions accurately.

## Outside the Square Puzzle

### Magic square

Copy this 6 × 6 table into your book.

Fill in the blank squares. All the rows and columns must add up to 11.1.

| | | | | | |
|---|---|---|---|---|---|
| 2.1 | | 2.5 | 2.7 | | |
| | 2.5 | 2.8 | | 0.5 | |
| 0.1 | 0.3 | | 1.9 | 3.3 | |
| 0.4 | 0.2 | 1.7 | 1.8 | 3.6 | |
| 2.9 | 3.1 | 0.9 | 1.1 | 1.6 | |
| 3.2 | 3.0 | 1.2 | | 1.3 | 0.8 |