Investigation

Experimental probability

Equipment required: 2 brains, 1 calculator, 1 drawing pin, 1 coin

The Big Question

Can we use probability to decide whether a game of chance is fair?

Engage

1 A drawing pin can land two possible ways. Try dropping the drawing pin a few times and see if it lands point up or point down. Do you think you need to be careful in the way you drop the pin? Should you give it a bit of a 'flip' to help it on its way? Why do you think we might be using the pin instead of a coin or a die?

point up

point down

Explore

Don and Jacob play a game that involves dropping a drawing pin.

If the drawing pin falls point up, Don scores a point. If the drawing pin falls point down, Jacob scores a point. The first one to get to 50 wins.

2 (a) Do you think this is a fair game? Who do you think will win?

(b) Predict the final score. Is it harder to make this sort of prediction than the predictions in the previous exercise?

(c) Play the game with a partner. Decide who gets a score for point up and who gets a score for point down. Tally your results in a table like this:

Point up	𝍸 𝍸 |||
Point down	𝍸 𝍸 𝍸 |||

How close were your predictions?

(d) Compare your results with other pairs of students by constructing a table like this:

	Score	
	Point up	**Point down**
Pair 1	32	50
Pair 2	50	44
Pair 3		

(e) Do you notice any pattern in the results of the games?

(f) **(i)** Find the point up total by adding all the numbers in the point up column.

(ii) Find the point down total by adding up all the numbers in the point down column.

(iii) Then, find the total number of throws by adding the point up and point down totals.

(g) Find the probability that a drawing pin will fall point up by calculating $\frac{\text{point up total}}{\text{total number of throws}}$:

(i) as a fraction

(ii) as a decimal

(iii) as a percentage.

(iv) Could the game still be fair if the percentage is not exactly 50%?

(h) Find the probability that a drawing pin will fall point down by calculating $\frac{\text{point down total}}{\text{total number of throws}}$:

(i) as a fraction

(ii) as a decimal

(iii) as a percentage.

(iv) Could the game still be fair if the percentage is not exactly 50%?

(v) Look at the two probabilities you have found in parts **(g) (ii)** and **(h) (ii)**. Add them together. What do you notice?

(vi) If the game is fair, what do you think each of the two probabilities should be equal to? Is the game fair? Is it close to being a fair game?

Strategy options

- Make a table.
- Act it out.
- Look for a pattern.

Explain

3 How could you change the rules for the drawing pin game to make it fair?

Elaborate

4 Omar and Michelle take it in turns to toss a coin four times. If four heads come up, or if two heads and two tails come up, Omar scores a point. If four tails come up, or if three tails and one head come up, Michelle scores a point. If three heads and one tail come up then they both score a point. The first one to 50 points is the winner.

(a) Do you think this is a fair game? Who do you think will win?

(b) Make a prediction about what you think the final score will be.

(c) Play the game with a partner. Decide who is Omar and who is Michelle.

Tally your results in a table. How close were your predictions?

(d) Compare your results with other pairs of students by constructing a table.

(e) Do you notice any pattern in the results of the games?

(f) **(i)** Find Omar's total by adding all the numbers in the Omar column.

(ii) Find Michelle's total by adding all the numbers in the Michelle column.

(iii) Then, find the total number of throws by adding Omar's and Michelle's totals.

(g) **(i)** Find the fraction of points that Omar gets by calculating:

$$\frac{\text{Omar's total}}{\text{total number of throws}}$$

(ii) Change this to a decimal. Write the probability that Omar will win as Pr(Omar wins) =

(iii) Write the percentage of times Omar will win.

(h) **(i)** Find the fraction of points that Michelle gets by calculating:

$$\frac{\text{Michelle's total}}{\text{total number of throws}}$$

(ii) Change this to a decimal. Write the probability that Michelle will win as Pr(Michelle wins) =

(iii) Write the percentage of times Michelle will win.

(iv) Find Pr(Omar wins) + Pr(Michelle wins).

(v) By looking at your values for Pr(Omar wins) and Pr(Michelle wins), would you say that the game is fair? Is it close to being fair?

Evaluate

5 How could you change the rules for the coin game to make it fair?

Extend

6 What other sorts of probability games do you think you could play? Try to come up with some that do not involve coins or dice.

7 **(a)** Did you affect the result by the way you dropped the pin or tossed the coin?

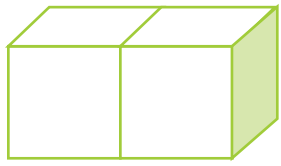
(b) Do you think the tasks in this investigation have helped you understand probability? Explain why or why not?

(c) Do you think it is important for you to know whether a game is fair? Why?

Challenge 9

1 The dots on opposite faces of a die total 7.

Two normal dice are placed next to each other on a table as shown.



Kenny walks around the table.

The greatest number of dots that Kenny can see is:

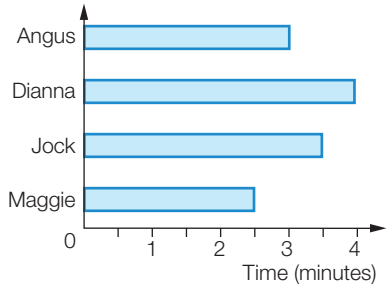
A 30 **B** 34 **C** 36 **D** 42

2 You meet three men at a meeting: Mr Black, Mr Green and Mr White. They are wearing black, green and white bow ties. Mr Black says, "Did you notice that the colour of our ties are different from our names?" The person in the green bow tie says, "Yes, you are correct!" What colour tie is each man wearing?

3 A die is constructed by labelling the faces of a wooden cube with the numbers 1, 1, 2, 3, 3 and 4. When it is tossed, each face has an equal chance of occurring. If this die is rolled once, the probability of rolling an odd number is:

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

4 Four Jack Russell terriers had a race swimming across a pond. The graph shows the time that each dog took to complete the race. Which dog won the race?



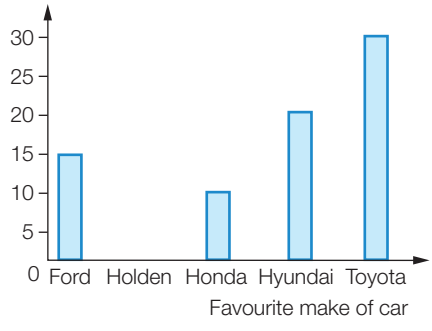
A Angus **B** Dianna

C Jock **D** Maggie

5 Vijay has a bag that contains 3 blue marbles, 6 green marbles, 2 red marbles and 6 yellow marbles. Vijay adds a number of white marbles to the bag and tells Samantha that if he now draws a marble from the bag, the probability of it being blue or green is $\frac{3}{7}$. The number of white marbles that Vijay added to the bag is:

A 3 **B** 4 **C** 5 **D** 6

6 In a survey, 1000 people were asked 'What is your favourite make of car?' Their responses were recorded and then graphed as percentages in a column graph. The column representing Holden has been omitted. What percentage of the people selected a Holden as their favourite make of car?



A 25 **B** 20

C 15 **D** 5

7 Jade has played 20 games of tennis and has won 95% of the games. If she does not lose any more games, how many more games in a row must she win in order to have then won 96% of the games?

A 1 **B** 4 **C** 5 **D** 10

Chapter review 9

D.I.Y. Summary

Key Words

average	continuous data	measures of centre	sample space
bar graph	data	median	stem-and-leaf plot
bimodal	discrete data	mode	tally
bivariate data	dot plot	multimodal	theoretical probability
categorical data	frequency	outcomes	trend
chance	frequency table	outlier	univariate data
class interval	histogram	probability	
column graph	mean	range	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The graphs in this chapter are all ways of presenting _____.

2 The listing of all the outcomes possible in a probability experiment is called the _____.

3 Of the three measures of central tendency considered, the one that represents the value occurring most frequently is called the _____.

4 The probability of an event occurring is another way of saying what _____ there is of the event happening.

5 In a frequency table for grouped data, each group is called a _____.

6 The _____ is the sum of all the values in the data set divided by the number of values in that data set.

Equipment required: Calculator for Question **2**

Fluency

1 A Year 7 class was surveyed about how many television sets their families had at home. The results were:

Ex 9.1

1, 1, 1, 1, 2, 1, 1, 3, 2, 5, 2, 1, 1, 0, 1, 2, 0, 4, 1, 2, 0, 3, 1

Construct a frequency table for this information.

2 Find the mean, median, mode and range for the following sets of data. Use a calculator when necessary, and give your answers correct to two decimal places.

Ex 9.2

(a) 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1

(b) 40, 20, 60, 30, 50, 10

(c) 3.3, 4.1, 3.9, 2.9, 4.4, 4.1, 3.8, 3.3, 4.4, 3.2, 3.9, 4.1, 2.9

3 Draw a bar graph to show the following data that represents the favourite style of TV program for a sample of 100 people. Ex.9.3

Style	Number of people
Comedy	35
Drama	21
Reality	17
Documentary	8
Game	19

4 **(a)** Draw a dot plot to represent the number of customers in a cafe recorded in 15-minute intervals over the course of 8 hours. Ex.9.3

4	4	7	3	6	4	5	9	4	8	8	5	7	9	10	12
11	8	9	8	6	2	4	4	5	5	2	5	3	0	1	1

(b) Calculate the median of the data set.

(c) Do there appear to be any outliers?

5 **(a)** Draw an ordered stem-and-leaf plot to represent the number of runs scored by each of the players at a local cricket club in one round of the season. Ex.9.3

46	64	31	0	50	42	11	7	4	0	5
21	17	115	0	42	61	15	18	5	3	1
40	0	52	56	77	50	41	20	7	2	2
88	17	58	63	14	0	18	19	17	6	2

(b) Calculate the median of the data set.

(c) Do there appear to be any outliers? If so, which values are they?

6 The sector graph at right represents information about the money spent by overseas visitors to Australia. Ex.9.4

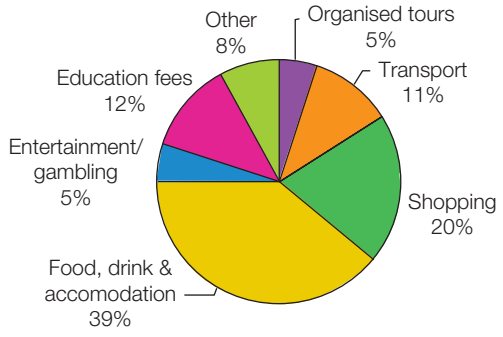
(a) One of the sectors uses an angle of 28.8°. Which one is it?

A Organised tours

B Transport

C Entertainment/gambling

D Other



(b) The Food, drink & accommodation sector uses an angle closest to:

A 95° **B** 110°

C 130° **D** 140°

7 The following are the sales figures for Beeper computer games from 2006 to 2011. Ex.9.5

Year	2006	2007	2008	2009	2010	2011
Units sold	10 000	25 000	35 000	30 000	40 000	45 000

Draw a line graph for this information.

8 The following table gives the mean rainfall, measured in mm, for Cairns (Aero) and Brisbane (Aero). Ex 9.6

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Cairns	396.8	452.5	426.1	198.2	91.4	46.1	29.3	27.0	33.4	39.7	93.4	178.5
Brisbane	109.3	121.6	76.1	63.5	122.8	70.6	27.1	35.2	32.7	65.2	102.0	120.9

Compare the mean rainfall for the two cities using as many statistics as you can.

9 The following table shows the handspan, measured in cm, of twenty 16-year-old students, divided into boys and girls. Ex 9.6

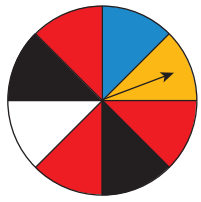
Boys	19.5	21.5	21	22.5	19	19.5	21	21.5	20.5	20
Girls	18	18.5	17.5	17	17	17.5	18	18.5	17	19.5

Draw a parallel dot plot and use it to help you describe the data set.

10 **(a)** When a normal die is rolled, the probability of getting a number less than 5 is equal to: Ex 9.7

A $\frac{1}{5}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** 1

(b) If the spinner shown on the right is spun once, the probability of the pointer landing on red is:



A $\frac{1}{4}$ **B** $\frac{3}{8}$

C $\frac{1}{2}$ **D** $\frac{3}{5}$

Understanding

11 A number of Year 7 students were surveyed about their shoe size. The results were:

$3, 5, 5\frac{1}{2}, 5, 4\frac{1}{2}, 4, 2\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 6, 2, 5\frac{1}{2}, 1\frac{1}{2}, 1, 3\frac{1}{2}, 4\frac{1}{2}, 3, 4, 5, 5\frac{1}{2}, 2\frac{1}{2}, 2, 3\frac{1}{2}, 4\frac{1}{2}, 3, 1, 3\frac{1}{2}, 1\frac{1}{2}, 4, 3\frac{1}{2}$

(a) Construct a frequency table to show this information.

(b) Which shoe size occurred most frequently?

(c) Which shoe size occurred least frequently?

12 Look at the following information.

How much Australians spend on pets each year ($ million)				
Expenditure type	**Dogs**	**Cats**	**Other**	**Total**
Food	560	431	90	1081
Vet charges and prescriptions	230	148	26	404
Pet care products/equipment	153	85	12	250
Pet services	116	35	–	151
Other expenses	109	20	12	141
Total	1168	719	140	2027

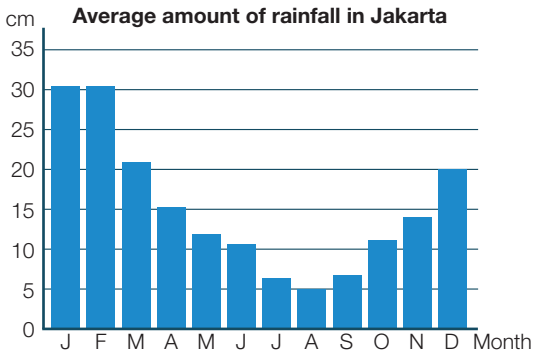
(a) Draw a bar chart showing the various expenditure types for dogs.

(b) Draw a divided bar chart to show the total expenditure for the three categories of pets.

(c) Draw a sector graph to show the various expenditure types for cats.

(d) Explain why a line graph could not be used for any of these sets of data.

13 Look at the following graph, which shows the average amount of rainfall (cm) in the city of Jakarta over 1 year.



(a) What type of graph is this?

(b) What is Jakarta's second driest month?

(c) What is the highest average monthly rainfall?

(d) Which 4 months form the rainy season?

14 Shelley and Douglas play a game in which they toss two coins. Shelley wins if two heads turn up, and Douglas wins if two tails turn up. If both a head and a tail turn up, they throw *one* of the coins again. If this comes up heads, Shelley wins. If it comes up tails, Douglas wins. Do you think this is a fair game?

15 **(a)** What value would the probability that you will be struck by lightning be close to?

(b) What value would the probability that it will rain at some time in the next six months be close to?

(c) If something has a probability of $\frac{1}{2}$ and something else has a probability of $\frac{3}{4}$, which one of the two is more likely to occur?

(d) If someone tells you something has a probability of 1.2 of happening, how do you know that they have made a mistake?

Reasoning

16 Twenty-five Year 7 boys and 25 Year 7 girls had their height measured to the nearest cm. The results are as follows.

Boys:	140	143	144	142	148	148	152	140	146	144
	154	151	145	140	141	147	153	150	149	140
	147	152	148	151	149					
Girls:	142	151	157	138	145	148	143	141	146	154
	156	154	150	143	150	144	148	144	157	156
	146	157	154	145	152					

(a) Construct an ungrouped frequency table for the boys.

(b) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the boys.

(c) Construct an ungrouped frequency table for the girls.

(d) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the girls.

(e) Describe the similarities and differences between the heights of the boys and the girls.

(f) Draw a combined ungrouped frequency table for boys and girls.

(g) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the students.

(h) Describe the heights of the students.

17 In this game you have 12 counters and three dice. You place as many counters as you like on any number of squares, although you must place all 12. You throw the dice and add the numbers shown. If you have any counters on that number you remove them. You continue until the board is clear.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

(a) On which numbers should you never place any counters? Explain why.

(b) On which numbers should you place most counters? Explain why.

(c) Once you have decided on a spread of counters, will every game take the same number of turns to clear the board? Explain your answer.

NAPLAN practice 9

Numeracy: Non-calculator

1 Some students were surveyed about the number of hours spent using a computer over the last weekend. The following results were recorded: 4, 7, 3, 2, 4, 6, 8, 5, 9.

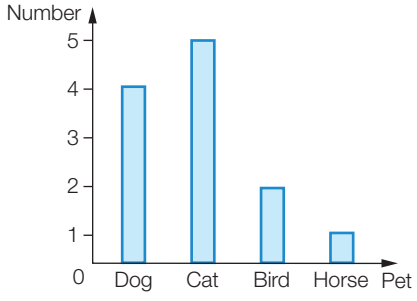
The median result is:

A 4 **B** 5 **C** 6 **D** 9

2 A jar contains 16 red, 12 black, 9 yellow and 8 green lollies.
Without looking, Jen takes one lolly from the jar.
What is the chance that the lolly is yellow?

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

3 Ricky surveyed the members of his cricket team to find out what pets they had. The results are shown in the graph on the right.

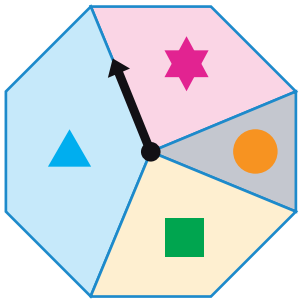


How many pets does the team have altogether?

A 4 **B** 5

C 12 **D** 15

4 Kenny spins the arrow 80 times.



Which table is *most* likely to show his results?

A

Shape section	Number of spins
★	10
●	30
■	20
▲	20

B

Shape section	Number of spins
★	20
●	30
■	10
▲	20

C

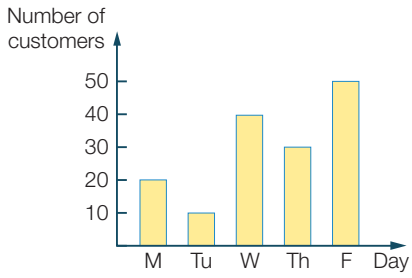
Shape section	Number of spins
★	20
●	10
■	20
▲	30

D

Shape section	Number of spins
★	20
●	20
■	10
▲	30

Numeracy: Calculator allowed

5 A business recorded the number of customers each day for a working week.



(a) Which day was the busiest?

(b) What was the mean number of customers per day?

(c) On which day was the number of customers closest to the mean number of customers per day?

6 The goal attack for the under 13 Panthers netball team scored the following number of goals in the first six games of the season: 14, 19, 23, 20, 28, 19.

The mean number of goals scored is closest to:

A 6 **B** 19.5 **C** 20.5 **D** 126