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Decimals, percentage and ratio

4

'When is a decimal point not a decimal point?'

A number may contain a point, but this does not make it a decimal point.

Usually, in mathematics, we use numbers to represent quantities, so we can calculate with them. These numbers are called the 'cardinal numbers'. We also use numbers in a less mathematical way—as labels or 'tags' that help to identify things. These numbers are called 'nominal numbers'. Room numbers, telephone numbers, and the footy score in the photo are all examples of nominal numbers. Some nominal numbers include a point that looks like a decimal point, but is actually a 'separator', placed between numbers that are labels for different things. For example, a score of 3.6 in an AFL game means 3 goals and 6 points, not 3 goals and 6 tenths of a fourth goal. A large building such as a hotel might have a room number written as 2.13, meaning the thirteenth room on the second floor, not 2 rooms and 13 hundredths of a third room.

Forum

We read the time of 7.30 as 'seven thirty' or 'half past seven'. If we wanted to write this time as a proper decimal number, what would we write? How many minutes would 0.30 hours be? Can you think of other examples where numbers may have a point and look like decimal numbers, but are, in fact, nominal numbers?

Amounts of money, such as $4.35, are written with a point. Is it a decimal point?

Why learn this?

An understanding of decimal numbers helps us to measure the wood we need to build a new shelf, work out who won the 100-m freestyle or calculate a household budget. Percentages have many everyday uses, such as measuring performance, advertising discounts or presenting survey results. Ratios and rates help us compare and calculate quantities of the same and different types, such as determining which product size is the best value for money.

After completing this chapter you will be able to:

- compare, order and round decimal numbers
- add, subtract, multiply and divide decimal numbers
- convert between decimals, fractions and percentages
- use estimations to check that answers are reasonable
- use percentages to solve problems
- use ratios and rates to compare and calculate amounts
- understand the relationship between ratios, fractions and percentages
- calculate unit prices and determine 'best buys'.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

1 Copy and complete each of the following by writing < (less than), > (greater than), or = (equal to) between the given numbers.

Worksheet R4.1

(a) 0.1 _____ 0.01 **(b)** 2 _____ 2.2 **(c)** 0.3 _____ 0.1

(d) 3.2 _____ 4.1 **(e)** 0.008 _____ 0.09 **(f)** 0.7 _____ 0.07

2 The number 43 can be written in expanded form as 'four tens and three ones'. Write the following in expanded form.

Worksheet R4.2

(a) 72 **(b)** 603 **(c)** 9251 **(d)** 11 080

3 Write each of the following **(i)** in words and **(ii)** as decimals.

Worksheet R4.3

(a) $\frac{7}{10}$ **(b)** $\frac{8}{1000}$ **(c)** $\frac{3}{100}$ **(d)** $1\frac{9}{10}$

4 Calculate:

Worksheet R4.4

(a) $\begin{array}{r} 34 \\ +\ 76 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 925 \\ +\ 610 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 67 \\ 409 \\ +\ 3 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 459 \\ 6013 \\ +\ 27 \\ \hline \end{array}$

5 Calculate:

Worksheet R4.5

(a) $\begin{array}{r} 74 \\ -\ 25 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 823 \\ -\ 376 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 8289 \\ -\ 384 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 2000 \\ -\ 352 \\ \hline \end{array}$

6 Calculate:

Worksheet R4.6

(a) 2×17 **(b)** 25×96 **(c)** 3×48 **(d)** 51×800

7 Perform the following divisions.

Worksheet R4.7

(a) $362 \div 2$ **(b)** $9459 \div 9$ **(c)** $5600 \div 200$ **(d)** $4500 \div 30$

8 Calculate:

Worksheet R4.8

(a) 70×100 **(b)** $12 \times 10\,000$ **(c)** 9200×1000

(d) $80 \div 10$ **(e)** $3600 \div 100$ **(f)** $250\,000 \div 1000$

(g) 1.2×10 **(h)** 6.3×100 **(i)** 0.0427×1000

(j) $58 \div 10$ **(k)** $901 \div 100$ **(l)** $76.2 \div 1000$

9 Write these percentages as fractions in simplest form.

Worksheet R4.9

(a) 50% **(b)** 10% **(c)** 25% **(d)** 20% **(e)** 1% **(f)** 60%

Key Words

decimal places	part : whole ratio	recurring decimal
decimal point	per cent	round
digit	place value	terminating decimal
equivalent ratios	rate	unit price
part : part ratio	ratio	unitary method

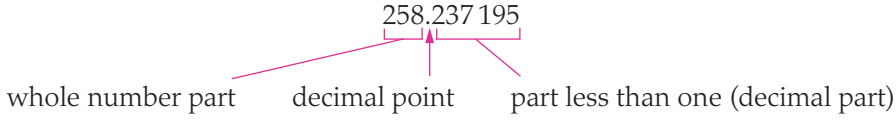
Place value and comparing decimals

4.1

Decimal place value

Like fractions, decimals are used to write numbers that have a part that is less than one. The **decimal point** is used to separate the whole number part from the part less than one.

The word decimal comes from the Latin word, *dec* or *decem*, meaning 'tenth part'. December used to be the tenth month on the Roman calendar.



The number 258.237195 is made up of nine individual **digits**. Because six of the digits occur after the decimal point, we say that the number has six **decimal places**.

More examples: 3.25 has three digits and two decimal places
45.079 has five digits and three decimal places.

We can write each digit of the number into a 'place value table' to show this:

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)	Thousandths $\frac{1}{1000}$ (0.001)	Ten-thousandths $\frac{1}{10\,000}$ (0.0001)	Hundred-thousandths $\frac{1}{100\,000}$ (0.000 01)	Millionths $\frac{1}{1\,000\,000}$ (0.000 001)
2	5	8	•	2	3	7	1	9	5

The **place value** of each column (hundreds, tens, ones, tenths, hundredths etc.) is one-tenth of the column to the left of it. As we move across the columns from left to right, we divide by 10 each time. This pattern is continued past the decimal point to get the values of the decimal places. You are probably familiar with the first three decimal places of tenths, hundredths and thousandths; however, we can keep dividing by 10 to get ten-thousandths, hundred-thousandths and millionths.

If there is no digit for a particular place value in a decimal number, we use a zero to show this. For example, five and two tenths and three thousandths is written as 5.203. It is important to include the zero between the 2 and the 3. Leaving it out would mean 5.203 becomes 5.23, which is a different number.

Expressing decimals in written form

Decimal numbers can be written in several ways:

- Decimal form: 258.237 195
- Expanded fractional form: $258 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10\,000} + \frac{9}{100\,000} + \frac{5}{1\,000\,000}$
- Expanded word form: *two* hundreds, *five* tens, *eight* ones, *two* tenths, *three* hundredths, *seven* thousandths, *one* ten-thousandth, *nine* hundred-thousandths and *five* millionths.

Expressing decimals in verbal form

- Say each of the digits after the decimal point individually. Do not use whole number language to describe the decimal part of the number. For example, 14.625 is said as 'fourteen point six two five', not 'fourteen point six hundred and twenty five'.
- Fraction equivalents can also be used to describe decimals verbally. For example, 0.63 can be said as 'sixty three hundredths'. This is an equivalent way of saying six tenths and three hundredths.

$$\frac{63}{100} = \frac{6}{10} + \frac{3}{100}$$

Worked Example 1 WE1

(a) Write $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$ as a decimal.

(b) Write 6.2807 in expanded fractional form.

Thinking

(a) Imagine the numerators of each fraction placed into their place value columns. You don't actually have to draw them.

(As there are no $\frac{1}{100}$s, we write a zero in the 'hundredths' column.)

Ones $\frac{1}{1}$	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
4	•	3	0	6	5

(b) 1 Imagine the decimal digits placed into their place value columns.

Ones $\frac{1}{1}$	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
6	•	2	8	0	7

2 Write a series of fractions using the headings of the place value columns. There is no need to include columns with 0 (e.g. $\frac{0}{1000}$).

Working

(a) $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$

$= 4.3065$

(b) 6.2807

$= 6 + \frac{2}{10} + \frac{8}{100} + \frac{7}{10\,000}$

Worked Example 2 WE2

(a) Write seven units, three tenths, six hundredths, seven thousandths and four hundred-thousandths as a decimal.

(b) Write 28.0045 in expanded word form.

Thinking

(a) Think of, or look at, a place value table. Place the digits described in the corresponding place value columns.

Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$	Hundred-thousandths $\frac{1}{100\,000}$
7	•	3	6	7	0	4

(b) **1** Think of, or look at, a place value table. Place the digits in their place value columns, including zero.

Tens 10	Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
2	8	•	0	0	4	5

2 Write the non-zero digits in words, with the place value column heading after each one.

Working

(a) 7.367 04

(b)

28.0045
= Two tens, eight ones, four thousandths and five ten-thousandths

Worked Example 3 WE3

(a) Write the value of the 6 in 4.368 as a fraction.

(b) Write the value of the 9 in 0.0109 in words.

Thinking

(a) Identify the place value of the digit, then write the digit as the numerator and the place value as the denominator of the fraction.

(b) Identify the place value of the digit, then write the digit, followed by the place value, in words.

Working

(a) $\frac{6}{100}$

(b) nine ten-thousandths

Comparing decimals

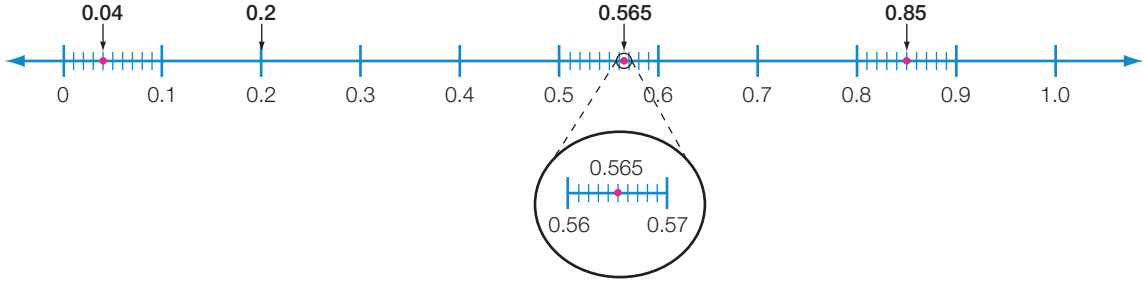
We often need to compare two decimal numbers to decide which one is greater. We might be comparing prices of goods or working out who won a race. Look at the decimal numbers in this photo of the finishing board of the men's 100 m final at the 2008 Olympics. Usain Bolt's time is placed first because it is the smallest decimal number. How much did Bolt win by?

We use the relation symbols of less than (<), greater than (>) and equal to (=) when comparing decimals.

Comparing decimals using a number line

We can use a number line to compare decimals. Numbers get larger as we move up the number line from left to right. The number line below is marked off in tenths. The space between 0 and 0.1 and between 0.5 and 0.6 shows 1 tenth divided up into 10 parts, each equal to 1 hundredth. $\frac{1}{10} = \frac{10}{100}$

The positions of the decimal numbers 0.04, 0.2, 0.565 and 0.85 are shown with arrows.



The interval between 0.56 and 0.57 has been magnified to show the position of 0.565 more clearly. It shows 1 hundredth divided up into 10 parts, each equal to 1 thousandth. $\frac{1}{100} = \frac{10}{1000}$

From the decimal numbers shown on the number line, we can see that:

The decimal number with the most digits is not necessarily the largest; e.g. $0.565 < 0.85$.

A number's size depends on the value and position of the digits; e.g. $0.2 > 0.04$.

Comparing decimals by comparing digits

The higher its place value, the greater the worth of a digit. For example, $0.2 > 0.04$, because $\frac{2}{10} > \frac{4}{100}$, so 0.2 lies further to the right on the number line than 0.04. To determine which of two numbers is bigger, we compare the digits in each place value column, beginning with the highest place values.

Worked Example 4 WE4

Write < or > between each of the following pairs of decimals to make a true statement.

(a) 5.173 _____ 5.0731 **(b)** 0.472 39 _____ 0.4731 **(c)** 3.5 _____ 3.51

Thinking			Working
(a)	**1**	Compare the whole number parts of each decimal. (Here, they are the same number, 5.)	*(a)*
	2	If the whole number parts are the same, compare the tenths digits to see which is greater. (Here, 1 is greater than 0, so 5.173 is the larger number.)	
	3	Write a relation symbol (< or >) between the two numbers.	*5.173 > 5.0731*
(b)	**1**	Compare the whole number parts of each decimal. (Here, they are the same, 0.)	*(b)*
	2	If the whole number parts are the same, compare the tenths digits. (Here, they are the same, 4.)	
	3	If the tenths digits are the same, compare the hundredths digits. (Here, they are the same, 7.)	
	4	Repeat until you get different digits. (The thousandths digits are 2 and 3, respectively. So, 0.472 39 is less than 0.4731.)	
	5	Write a relation symbol beween the two numbers.	*0.472 39 < 0.4731*
(c)	**1**	Add zeroes to the end of one of the decimals to help you compare. This doesn't change its value (3.5 is the same as 3.50).	*(c)*
	2	Compare the two decimals as in previous examples. (3.50 has the smaller hundredths digit, so it is the smaller decimal.)	
	3	Write < or > between the two numbers.	*3.5 < 3.51*

4.1 Place value and comparing decimals

Navigator

Answers page 641

Q1, Q2, Q3, Q4 Column 1, Q5 Column 1, Q7, Q8 (a) & (b), Q9 Column 1, Q10, Q11, Q13, Q14, Q15, Q17, Q19	Q1, Q2, Q3, Q4 Column 2, Q5 Column 2, Q6, Q8 (c) & (d), Q9 Column 2, Q10, Q11, Q12, Q14, Q15, Q16, Q18, Q19	Q1 (b), Q2 (b), Q3, Q4 Column 2, Q5 Column 2, Q6, Q7, Q8 (c) & (d), Q9 Column 2, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19

Fluency

WE1

1 (a) Write each of the following as a decimal.

(i) $45 + \frac{4}{10} + \frac{6}{100} + \frac{2}{1000}$

(ii) $12 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000} + \frac{3}{10\,000} + \frac{7}{100\,000} + \frac{2}{1\,000\,000}$

(iii) $3 + \frac{7}{10} + \frac{9}{100} + \frac{8}{1000} + \frac{5}{10\,000}$

(iv) $1 + \frac{8}{1000} + \frac{3}{10\,000} + \frac{3}{100\,000} + \frac{2}{1\,000\,000}$

(v) $\frac{7}{10} + \frac{8}{100} + \frac{6}{1000} + \frac{6}{100\,000}$

(vi) $7 + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100\,000} + \frac{7}{1\,000\,000}$

(b) Write each of the following in expanded fractional form.

(i) 6.63 **(ii)** 0.921 **(iii)** 0.7345

(iv) 7.826 **(v)** 23.913 04 **(vi)** 45.004 589

WE2

2 (a) Write each of the following as a decimal.

(i) six ones and five tenths

(ii) nine tenths and seven hundredths

(iii) two tenths, seven hundredths and three thousandths

(iv) three tens, seven ones, four tenths, two hundredths and one ten-thousandth

(v) one ten, four ones, nine tenths, five hundredths, seven thousandths, six ten-thousandths, two hundred-thousandths and three millionths

(vi) seven hundreds, four thousandths, five ten-thousandths and nine hundred-thousandths

(b) Write each of the following in expanded word form.

(i) 5.2 **(ii)** 4.9 **(iii)** 34.17 **(iv)** 0.61

(v) 2.794 **(vi)** 7.5092 **(vii)** 35.865 43 **(viii)** 0.820 027

3 **(a)** Write the value of the 2 in each of the following as a fraction.

WE3

(i) 6.012 **(ii)** 0.004 52 **(iii)** 3.287 **(iv)** 2.034

(b) Write the value of the 7 in each of the following in words.

(i) 5.734 **(ii)** 0.0076 **(iii)** 1.2037 **(iv)** 8.130 037

4 Write < or > between each of the following pairs of decimals to make a true statement.

WE4

(a) 2.4 _____ 0.42 **(b)** 2.32 _____ 1.955

(c) 0.65 _____ 0.57 **(d)** 0.3003 _____ 0.333

(e) 4.7038 _____ 4.7312 **(f)** 8.251 _____ 8.2501

(g) 7.02 _____ 7.002 **(h)** 4.7367 _____ 4.7376

(i) 0.927 _____ 0.927 34 **(j)** 6.013 _____ 6.01

(k) 3.406 _____ 3.4063 **(l)** 0.9995 _____ 0.9986

5 Write TRUE or FALSE for each of the following.

(a) $7.5 < 5.77$ **(b)** $4.1 > 4.12$

(c) $6.08 > 6.8$ **(d)** $67.54 < 67.504$

(e) $3.023 < 3.203$ **(f)** $0.547 > 0.547\,08$

(g) $2.000\,012 < 2.0001$ **(h)** $4.145\,29 > 4.200\,01$

6 53.017 expressed in expanded word form is:

A five tens, three ones, one tenth and seven hundredths

B five tens, three ones, one hundredth and seven thousandths

C five tenths, three hundredths, one ten-thousandth and seven hundred-thousandths

D five tens, three ones, one tenth and seven thousandths

7 Nine hundredths, four thousandths and three ten-thousandths is equal to:

A 0.090 43 **B** 0.0943 **C** 0.943 **D** 9.43

8 For each of the following sets of numbers, copy the number line shown, then mark the positions of the numbers with a labelled arrow. (You may have to estimate the position of some.)

(a) 2.05, 2.09, 2.6, 2.12, 2.59 **(b)** 2.0, 2.8, 2.88, 2.9, 2.805

(c) 2.7, 2.4, 2.07, 2.04, 2.407 **(d)** 2.2, 2.4, 2.85, 2.35, 2.05

9 Write each set of decimals in order from smallest to largest.

(a) 2.3, 2.03, 2.13 **(b)** 8.7, 8.007, 8.67

(c) 6.646, 6.6403, 6.64 **(d)** 0.0095, 0.0905, 0.0509

(e) 5.3281, 5.38, 5.003 821 **(f)** 3.616, 3.116, 3.661

(g) 0.92, 0.29, 0.092 **(h)** 0.85, 0.815, 0.086

Understanding

10 Joel wrote down the five-digit number as he saw it on his electronic stopwatch: 37901. However, he forgot to put in the decimal point. Anna knew that the race was timed to a thousandth of a second and was able to put the decimal point in the correct place. Write the number with the decimal point in the correct place.

11 The difference in times between two skiers in a downhill race was 0.437 of a second. Write this: **(a)** in expanded fraction form

(b) as a single fraction.

12 For each of the following sets of decimals, draw a section of the number line, marked in either tenths or hundredths, and indicate the position of each number in the set.

(a) 0.4, 0.72, 1.01

(b) 1.7, 1.25, 0.95

(c) 0.06, 0.045, 0.038

(d) 2.9, 3.2, 2.75, 3.06

13 Jane records her three best practice times for the 100 m sprint. They are as follows. 13.95 seconds, 13.08 seconds and 13.69 seconds

(a) Which was Jane's fastest time?

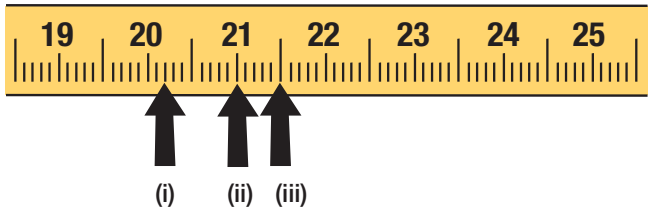
(b) Which was Jane's slowest time?

14 Egor throws a shot-put the following distances during a competition: 25.6 m, 25.56 m and 25.081 m. Which distance was Egor's longest throw?

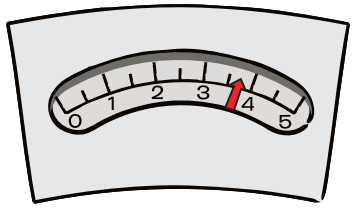
Reasoning

15 Write the value of the decimals indicated by the arrows on the following scales.

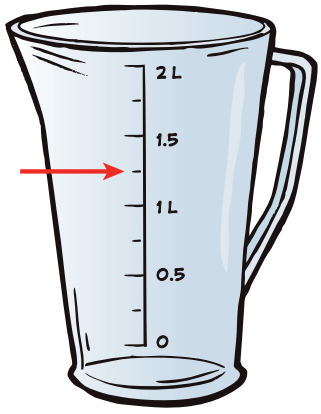
(a)



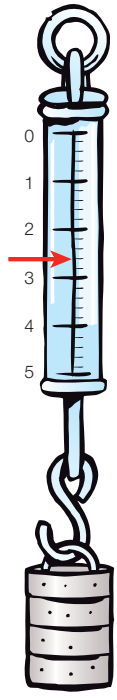
(b)



(c)



(d)



16 State the mistakes that the following students have made.

(a) Minh states that the decimal 34.162 has a 2 in the hundredths column.

(b) Al writes four hundreds, nine ones, six tenths and seven hundredths as 409.067.

(c) Max writes seven ones, eight hundredths and nine ten-thousandths as 0.7809.

(d) Polly writes 0.9056 as $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$.

Open-ended

17 **(a)** Write down at least six numbers between 4.5 and 4.7.

(b) Put your numbers from **(a)** in ascending order.

18 **(a)** Use any digit from 0 to 9 in each box to make a correct statement. Digits can be repeated. Find at least three different answers.

□.□□ < 5.□

(b) Explain any limitations on the other numbers that can be used when certain numbers are used in some boxes.

19 Donna has cut out the digits 2, 5 and 7, as well as a decimal point, from coloured paper to stick on a poster to show a decimal number. If she uses all three digits:

(a) how many different decimal numbers with two decimal places could she make

(b) how many decimal numbers greater than 5.4 could she make?

Outside the Square Game

Duelling decimals

Equipment required: 2 brains, 1 die

How to win:
The aim is to be the first player to 5 points.

How to play:

1 One player rolls the die to determine the number of digits in the duel. Each player then puts the appropriate number of boxes on their paper after the decimal point.

e.g. if a 3 is rolled, each would draw:

0 . □□□ 0 . □□□

2 Each player then takes turns to roll the die and write the number from the roll in a box of their choice.

After all of the boxes have been filled, the player who has created the bigger number wins a point for that round.

For example, the players may have filled in their boxes like this:

0 . 2 5 3 0 . 6 3 1

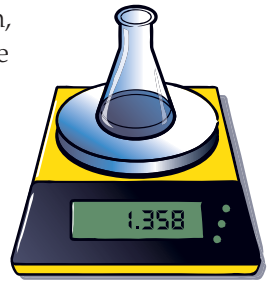
Here, the second player would win a point.

3 This process is repeated with players taking turns to roll the determining number of digits per round.

4.2 Rounding decimals

Decimal numbers sometimes contain more decimal places than we require. It depends on the situation.

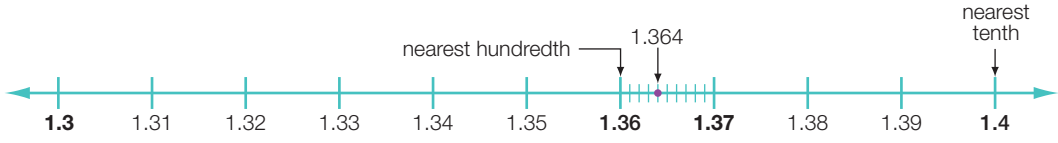
A scientist might need to know the mass of a chemical accurate to a thousandth of a gram, or three decimal places. They could place the chemical on an electronic scale and get a reading such as 1.358 g.



A zookeeper wanting to find the mass of a bear cub could place the cub on a similar electronic scale and get a reading such as 25.427 kg. However, 2 hundredths or 7 thousandths of a kilogram is an insignificant amount compared to the whole mass of the cub. The zookeeper would **round** the bear cub's mass to 25.4 kg, or just 25 kg.

The number line below shows how the number 1.364 is rounded:

- To the nearest tenth (1 decimal place): 1.364 lies between the tenths values of 1.3 and 1.4. The 6 in the hundredths place value column places it closer to 1.4.
  1.364 rounded to the nearest tenth is *rounded up* to 1.4.
- To the nearest hundredth (2 decimal places): 1.364 lies between the hundredths values of 1.36 and 1.37. The 4 in the thousandths column places it closer to 1.36.
  1.364 rounded to the nearest hundredth is *rounded down* to 1.36.



To round decimal numbers:

Step 1 Determine the number of decimal places that you are required to round to. The digit in this place will either remain as it is, or will increase by one.

Step 2 Look at the next digit to the right of the one that you are rounding to. If this digit is:

- 0, 1, 2, 3 or 4, then leave the digit considered in step 1 as is and delete all digits after it
- 5, 6, 7, 8 or 9, then increase the digit considered in step 1 by one and delete all digits after it.

If the number in the place value you are rounding to is 9 and you need to increase it, make the 9 a 0, and add one to the digit in front of it. For example, 1.499 rounded to the nearest hundredth is 1.50. We write a 0 in the hundredths column to show this (even though it makes no difference to the value of the number).

Worked Example 5 WE5

Round the following decimals to the number of decimal places shown in brackets.

(a) 3.785 (2) **(b)** 0.958 34 (3) **(c)** 6.214 96 (4)

Thinking			Working
(a)	**1**	Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 3.78 or 3.79.)	(a) 3.785
	2	Look at the digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 5 tells us to round up, so the 8 becomes a 9.)	3.785
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.	3.79
(b)	**1**	Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 0.958 or 0.959.)	(b) 0.958 34
	2	Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 3 tells us that we round down, so we leave the 8 as it is.)	0.958 34
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.	0.958
(c)	**1**	Decide which digit you will be rounding. (Here, it is the 9, so the answer will either be 6.2149 or 6.2150.)	(c) 6.214 96
	2	Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 6 tells us to round up, but rounding the 9 up will give 10 ten-thousandths, which is equal to 1 thousandth. This means that the thousandths digit is increased by 1.)	6.214 96
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. (Here, we leave the zero in the rounded decimal place.)	6.2150

Rounding money

We write money using decimal notation, usually with 2 decimal places. Because $1 = 100c, or 10 lots of 10c, the tenths place value column shows how many lots of 10c make up the amount, and the hundredths column shows how many lots of 1c make up the amount.

In the 1990s, Australia stopped using 1 cent and 2 cent pieces, making the 5 cent coin the smallest coin in use. This means that all money amounts paid in cash must be rounded to the nearest 5 cents (5c).

If money is being paid in cash, the following rules apply.

If the amount ends in…	*then it is rounded…*	*the last digit becomes…*
1 or 2 cents	down	0
3 or 4 cents	up	5
6 or 7 cents	down	5
8 or 9 cents	up	0

If the amount ends in 0 or 5 cents, then the exact amount is paid.

Worked Example 6 WE6

Round the following amounts of money to the nearest 5 cents.

(a) $8.53 **(b)** $7.02 **(c)** $4.99

Thinking	Working
(a) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 3 up to the nearest 5 cents.)	*(a) $8.55*
(b) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 2 down to the nearest multiple if 10 cents, which is 0.)	*(b) $7.00*
(c) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 9 up to the nearest multiple of 10 cents, which is 100.)	*(c) $5.00*

4.2 Rounding decimals

Navigator

Q1 Columns 1 & 2, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q10, Q12, Q14, Q16	Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q16	Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16

Answers page 642

Fluency

WE5

1 Round the following decimals to the number of decimal places shown in brackets.

(a) 4.88 (1) **(b)** 6.72 (1) **(c)** 7.635 (2)

(d) 4.552 (2) **(e)** 0.6416 (3) **(f)** 3.2772 (2)

(g) 0.314 42 (4) **(h)** 11.828 55 (2) **(i)** 2.917 96 (2)

(j) 18.499 95 (3) **(k)** 90.899 99 (4) **(l)** 18.999 999 (3)

(m) 23.4097 (2) **(n)** 57.0804 (3) **(o)** 259.899 99 (4)

WE6

2 Round the following amounts of money to the nearest 5 cents.

(a) $4.52 **(b)** $2.76 **(c)** $11.03 **(d)** $23.88

(e) $176.35 **(f)** $542.06 **(g)** $789.54 **(h)** $56.75

(i) $6774.99 **(j)** $1149.97 **(k)** $8989.99 **(l)** $9999.99

3 0.837 256 rounded to the nearest ten-thousandth is:

A 0.837 **B** 0.8372 **C** 0.837 26 **D** 0.8373

Understanding

4 At the 2009 World Athletics Championships, Usain Bolt set a new 100 m record time of 9.58 seconds. Round this time to the nearest tenth of a second.

5 The total shown on the receipt for Russell's grocery shopping is $53.67. If Russell is paying in cash, how much money will he have to hand over?

6 Sir Donald Bradman's cricket batting average was 99.943 (rounded to the nearest thousandth). What would be his average rounded to the nearest:

(a) tenth **(b)** hundredth?

7 The top temperature recorded one day in Mildura was 41.6°C. What figure should the weather presenter read out in his weather report as the day's maximum if he rounds the temperature reading to the nearest whole number (i.e. no decimal places)?

8 The price of a box of chocolates is $11.97. If Beau buys the chocolates and pays with cash, how much will he pay?

9 A physics student calculates the speed of sound to be 352.153 metres per second. In case her equipment is not accurate enough for this result to be reliable, she decides to round it to the nearest tenth. What value does she write in her report, assuming she rounds off correctly?

Reasoning

10 Ruth calculates the length of shelf required for a wall unit to be 1.266 66 m. As her tape measure is only accurate enough to measure in centimetres (cm), she needs to round off this figure to the nearest cm. If 1 cm = 0.01 m, what length should she measure?

11 In a cycling sprint race at a velodrome, the following results were recorded: 12.149 s, 12.143 s, 12.138 s, 12.152 s and 12.157 s.

(a) Write these in order from fastest to slowest.

(b) It was found that there was a small error in the timing equipment and the results were to be recorded to the nearest hundredth. What were the new times?

(c) Does this affect the race result?

12 **(a)** The total cost of Rani's shopping was $20.34. Round this amount to the nearest 5 cents.

The individual costs of the items Rani bought were:

Cereal	$6.93
Yoghurt	$4.78
Bananas	$2.51
Juice	$3.63
Pasta	$2.49

(b) How much more would Rani have paid if the individual prices of the items were each rounded to the nearest 5 cents?

Open-ended

13 Write at least three different decimal numbers that, when rounded, would give the number 3.79.

14 Write down two prices that would give $100 dollars when rounded to:

(a) the nearest 5 cents

(b) the nearest dollar.

15 Taking 2.548 46 and rounding it to four decimal places gives 2.5485, then rounding this to three decimal places gives 2.549.

What answer do you get when rounding 2.548 46 to three decimal places?

Explain why this is different, and what is wrong with doing it the first way.

16 Why do you think 1 and 2 cent coins were taken out of circulation? List two or three possible reasons why a government might make this decision.

Decimals and fractions

4.3

Writing a decimal as a fraction

As we have seen, a decimal can be written in expanded fractional form. This expanded form can be simplified to describe a decimal as a single fraction or as a mixed number.

For example: $3.72 = 3 + \frac{7}{10} + \frac{2}{100}$

$= 3 + \frac{70}{100} + \frac{2}{100}$

$= 3 + \frac{72}{100}$

$= 3\frac{72}{100}$ or $\frac{372}{100}$

$= 3\frac{18}{25}$ or $\frac{93}{25}$

The place value of the last digit (hundredths) gives the denominator of the single, unsimplified fraction (100), and the digits themselves form the numerator (72). Remember that fractions should always be given in simplest form.

Worked Example 7 WE7

Convert each of the following decimals to fractions, giving your answers in simplest form.

(a) 6.28 **(b)** 0.3125

Thinking	Working
(a) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.	(a) 6.28 $= 6\frac{28}{100}$ or $\frac{628}{100}$
2 Simplify the fraction if possible. (Here, a common factor of 4 has been cancelled.)	$= 6\frac{7}{25}$ or $\frac{157}{25}$
(b) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.	(b) 0.3125 $= \frac{3125}{10\,000}$
2 Simplify the fraction if possible. (Here, we cancel common factors of 5, then 25, then 5 again.)	$= \frac{625}{2000}$ $= \frac{25}{80}$ $= \frac{5}{16}$

The decimal point was first used in the 1600s by mathematician Bartholo Maeus Pitiscus. Thanks, Bart!

Writing a fraction as a decimal

There are two methods we can use to convert a number in fraction form to decimal form:

Method 1: Write the fraction as an equivalent fraction (or mixed number) with a denominator of 10, 100, 1000 etc. Then, write the digits of the numerator of this new fraction as the decimal place values.

For example: $\frac{4}{5} = \frac{8}{10} = 0.8$

$\frac{13}{20} = \frac{65}{100} = 0.65$

This method is only convenient for fractions with denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2: The line between the numerator and the denominator is equivalent to the division symbol, ÷. To convert a fraction to a decimal, we simply perform the division. This method can be used for all fractions.

For example: $\frac{7}{8} = 7 \div 8 = 0.875$

$\frac{37}{40} = 37 \div 40 = 0.925$

Worked Example 8

WE8

Convert each of the following fractions to decimals.

(a) $\frac{7}{5}$ (b) $\frac{3}{8}$ (c) $\frac{4}{3}$

Thinking		Working
(a) 1	Is the denominator a factor of 10, 100 or 1000? (Yes)	(a) $\frac{7}{5}$
2	Write the fraction as an equivalent fraction with a denominator of 10, 100, or 1000 (10 in this case).	$= \frac{14}{10}$
3	If the fraction is improper, write it as a mixed number.	$= 1\frac{4}{10}$
4	Write any whole numbers on the left of a decimal point. (If there is no whole number part, write a zero.) Write the numerator of the fraction on the right of the decimal point.	$= 1.4$ $\frac{7}{5} = 1.4$

(b) **1**	Is the denominator a factor of 10, 100 or 1000? (No)	(b) $\frac{3}{8}$	
2	Write the fraction as a division calculation.	$3 \div 8$	$8\overline{)3}$
3	Perform the division. If you are unable to begin because the divisor is a greater number (here, $8 > 3$), add a decimal point and a zero to the dividend. Place a decimal point in the quotient and a zero in front of it.	$3.0 \div 8$	$\begin{array}{r} 0. \\ 8\overline{)3.0} \end{array}$
4	Continue to divide as though the decimal point was not there, adding extra zeroes where required.	$3.000 \div 8$	$\begin{array}{r} 0.375 \\ 8\overline{)3.0^{6}0^{4}0} \end{array}$
5	State the answer.	$\frac{3}{8} = 0.375$	
(c) **1**	Is the denominator a factor of 10, 100 or 1000? (No)	(c) $\frac{4}{3}$	
2	Write the fraction as a division calculation.	$4 \div 3$	$3\overline{)4}$
3	Perform the division. Add a decimal point and zeroes to the dividend to continue the division, placing a corresponding decimal point in the quotient.	$4.000 \div 3$	$\begin{array}{r} 1.333 \\ 3\overline{)4^{1}0^{1}0^{1}0} \end{array}$
4	If the division gives a repeating pattern, stop. Write the answer with a dot above the repeating digit.	$\frac{4}{3} = 1.\dot{3}$	

Recurring decimals

Dividing the numerator by the denominator sometimes results in a **recurring decimal**—one that has a repeating pattern of digits, such as in part **(c)** in the previous Worked Example. To write a recurring decimal, place dots or a line above the digits that form the repeating pattern. If it is a single repeating digit, place a dot or a line above that digit. If more than one digit forms the repeating pattern, place a line above the repeating digits, or a dot above the first and last digit in the repeating pattern.

For example: $\frac{2}{3} = 0.\dot{6}$, $\frac{1}{12} = 0.08\dot{3}$, $\frac{4}{7} = 0.\overline{571\,428}$ or $0.\dot{5}71\,42\dot{8}$

A recurring decimal can also be rounded to give an approximate value; for example, $0.\dot{6}$ rounded to three decimal places is 0.667.

A decimal that is not recurring, but has a finite number of digits (such as 1.4 or 0.375 in the previous Worked Example) is called a **terminating decimal**.

Fractions and decimals on a calculator

On a scientific calculator, the $a\frac{b}{c}$ or S⇔D key can be used to convert between the fraction and the decimal forms of a number. Pressing the key again will convert it from decimal form back to the fraction.

4.3 Decimals and fractions

Navigator

Answers page 642

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q12 (a)	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12 (b), Q13

Equipment required: A calculator may be used for Question **2 (i)–(p)**

Fluency

WE7

1 Convert each of the following decimals to fractions, giving your answers in simplest form.

(a) 4.1	**(b)** 6.5	**(c)** 8.6	**(d)** 9.4
(e) 4.71	**(f)** 2.37	**(g)** 0.35	**(h)** 2.48
(i) 5.009	**(j)** 0.884	**(k)** 6.128	**(l)** 3.172
(m) 7.045	**(n)** 0.088	**(o)** 6.0015	**(p)** 1.0075

WE8

2 Convert each of the following fractions to decimals.

(a) $\frac{7}{10}$	**(b)** $\frac{9}{100}$	**(c)** $\frac{17}{100}$	**(d)** $\frac{123}{1000}$
(e) $\frac{1}{5}$	**(f)** $\frac{3}{25}$	**(g)** $\frac{13}{50}$	**(h)** $\frac{7}{4}$
(i) $\frac{7}{8}$	**(j)** $\frac{11}{16}$	**(k)** $\frac{23}{40}$	**(l)** $\frac{47}{60}$
(m) $\frac{1}{9}$	**(n)** $\frac{7}{12}$	**(o)** $\frac{17}{6}$	**(p)** $\frac{16}{15}$

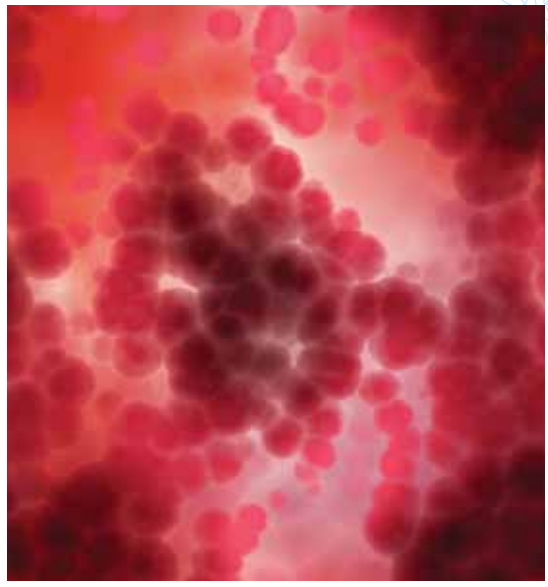
3 0.55 expressed as a fraction in simplest form is:

A $\frac{55}{1000}$ **B** $\frac{11}{200}$ **C** $\frac{11}{25}$ **D** $\frac{11}{20}$

4 2.000 47 is equal to:

A $2\frac{47}{100}$ **B** $2\frac{47}{1000}$ **C** $2\frac{47}{10\,000}$ **D** $2\frac{47}{100\,000}$

Understanding



5 As part of a science experiment, Galina's reaction time was measured to be 0.48 seconds. What fraction of a second is this?

6 One inch is approximately equal to 2.54 centimetres. What is this when expressed as a fraction in simplest form?

7 Hair grows at an average rate of 1.35 cm per month. Write this as a mixed number in simplest form.

8 A virus particle has a diameter of 0.000 024 mm. Write this as a fraction in simplest form.

9 A red blood cell's diameter is $\frac{1}{200\,000}$ of a metre. Write this measurement in decimal form.

Reasoning

10 A bank offers its customers an interest rate of 5.85 per cent. Write this rate in single fraction form. Is this rate higher than the $5\frac{3}{4}$ per cent offered by another bank?

11 Jeremy has five different containers and is trying to place them in ascending order of size. The volumes of all five containers are measured in litres but some are given using decimals and others using fractions. The tubs have the following volumes: $4\frac{1}{5}$, 4.375, 4.865, $4\frac{9}{10}$, $4\frac{1}{2}$. Help Jeremy put them in order from the smallest to the largest.

Open-ended

12 **(a)** Write three different fractions that have decimal values between 3.4 and 3.5 and have a denominator of 100.

(b) Write three different fractions that have decimal values between 3.4 and 3.5 where each fraction has a different denominator that is not a power of 10 (such as 100 or 1000).

13 Two numbers are rounded to one decimal place to give 5.6. What could those two numbers be? Give your answers in fraction form.

Outside the Square Puzzle

What are the neighbours having for dinner?

Four children from the same class in school live on the same street. Find out in which house each child lives, and what they had for dinner yesterday evening.

- Steve didn't have pizza for dinner, which was the meal eaten at house number 15.
- The fish and chips were eaten at house number 1, whereas Joseph lives at house number 11.
- The salad was eaten in a lower house number than the lasagne.
- One of Rowena's classmates lives in house number 5.
- Harriet lives in the highest number house of the four classmates.

4.4 Decimal addition and subtraction

Decimal numbers are added and subtracted in the same way as whole numbers. As with whole numbers, we must be sure that we add or subtract digits of the same place value. The best way to ensure this is to line up the decimal points underneath each other.

To add or subtract decimals, line up the decimal points so that digits with the same place value are underneath each other.

If the numbers have different numbers of decimal places, zeroes can be written in the 'empty' place value columns to help them line up correctly.

Worked Example 9 WE9

Calculate: 12.45 + 6 + 0.3678

Thinking	Working
1 Place the numbers underneath each other with the decimal points lined up. Fill empty place value columns with zeroes if necessary. **2** Add as though the values are whole numbers. **3** Place a decimal point in the answer so it lines up with the other decimal points.	$\begin{array}{rr} & 12.^{1}4500 \\ & 6.0000 \\ + & 0.3678 \\ \hline & 18.8178 \end{array}$

Worked Example 10 WE10

Calculate: 15 – 2.147

Thinking	Working
1 Write the second number underneath the first with the decimal points lined up. (Write 15 as 15.000) **2** Fill empty place value columns with zeroes. **3** Subtract, lining up the decimal point in the answer with those in the question.	$\begin{array}{rr} & {}^{4}1{}^{9}5.{}^{9}0{}^{1}0{}^{1}0 \\ - & 02.147 \\ \hline & 12.853 \end{array}$

4.4 Decimal addition and subtraction

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q13, Q14	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q15, Q17	Q1 Column 3, Q2 Column 3, Q4, Q5, Q6, Q8, Q11, Q12, Q13, Q15, Q16, Q17

Answers page 642

Fluency

WE9

1 Calculate:

(a) 8.3 + 7.9 **(b)** 4.85 + 8.09 **(c)** 0.237 + 0.677

(d) 25.61 + 0.038 **(e)** 5.098 + 21.32 **(f)** 0.025 + 39.786

(g) 9.703 + 5.624 + 7.5 **(h)** 7.35 + 0.609 + 2.3 **(i)** 0.648 + 7.31 + 0.9

(j) 6 + 5.017 + 12.9 **(k)** 2.59 + 15 + 0.005 **(l)** 0.0004 + 3.583 + 8

WE10

2 Calculate:

(a) 5.7 – 3.8 **(b)** 7.05 – 2.93 **(c)** 23.982 – 11.735

(d) 9.663 – 2.7 **(e)** 4.984 – 1.8 **(f)** 7.238 – 3.4

(g) 9.5 – 2.24 **(h)** 16.2 – 8.75 **(i)** 3.7 – 0.931

(j) 8 – 7.44 **(k)** 3 – 2.92 **(l)** 93 – 0.698

3 Which of the following shows the correct way to set out the addition of 0.56, 15.092 and 2.7?

A
$$\begin{array}{r} 0.5\,6 \\ 1\,5.092 \\ +\ 2.7 \\ \hline \\ \hline \end{array}$$

B
$$\begin{array}{r} 0.\,56 \\ 15.092 \\ +\ 2.7 \\ \hline \\ \hline \end{array}$$

C
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\ \ 2.7 \\ \hline \\ \hline \end{array}$$

D
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\ \ 2.7 \\ \hline \\ \hline \end{array}$$

Understanding

4 Daily rainfall totals for three days over a long weekend were 3.78, 2.5 and 6.42 millimetres. What was the total rainfall over this three-day period?

5 John loads his shopping trolley with several items priced as shown.

1 frozen lasagna	$5.71
2 L orange juice	$4.25
1 tub of yoghurt	$3.77
1 packet of cereal	$6.14

(a) Find the total cost of John's purchases.

(b) Round the total to the nearest 5 cents.

6 Cristina's bank account balance was $335.96 just before she withdrew $40.45 to pay a bill. How much did she have left in her account after the withdrawal?

7 In a gymnastics competition, Adelia scores the following from the five judges: 7.5, 8.5, 7.9, 8 and 8.6. Find her total score.

8 A family drives to a holiday resort and records the car's odometer readings as shown.

Departure: 234.8 km Arrival: 502.7 km

How far did they travel to get to the resort?

9 During a car tour of Tasmania, Helena travelled distances of 25.64 km, 165.35 km and 5.97 km all in one day. How many kilometres did she travel altogether that day?

10 Georgina pays for $36.35 worth of groceries with a $100 note. How much change should she receive?

11 Nico pours 1.625 litres of milk from a full 2-litre container. How much milk is left in the container?

Reasoning

12 Amanthi's credit card bill shows the following purchase amounts: $23.56, $40.15, $7.89, $18.48, $45.50, $21.73 and $8.59. The total at the bottom of her statement is $184.38, which she is sure is wrong.

(a) What should the total be?

(b) How do you think the error was made?

13 Tina sells flowers at a market stall. One day, she takes in $473.50 from the sale of flowers. If the flowers cost Tina $128.35, and she had to pay a fee of $14.50 to rent the stall for 1 day, what profit did she make for the day?

Open-ended

14 **(a)** Write two numbers that add up to 2.871.

(b) Write three numbers, each with three decimal places, that add up to 3.86.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

15
$$\begin{array}{r} 4.\square \\ +\ \square.8\square \\ \hline 7.\square 5 \end{array}$$

(a) Use one digit from 0 to 9 to fill each box and make a correct sum. Find at least three different combinations.

(b) Was your choice of digits restricted? Explain how.

16 (a) Write two decimal numbers, each with three decimal places, that have a difference of 8.712.

(b) Write two decimal numbers, each with three decimal places, that have a difference of 8.71.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

17 Two Australian swimmers finish first and second in the women's 100 m freestyle at the Olympics. The difference in their times is 0.04 seconds. The Olympic record was 53.52 seconds. Assuming that they both break this record by less than $\frac{2}{10}$ of a second, what could their times be?

Outside the Square Game

1 Closer than you

Equipment required: 2–4 brains, 2 dice

How to win:
The aim is to be the person to get a score as close to 10 as possible.

How to play:

1. Each player draws up a table like the one on the right.
2. Take turns to roll both dice, and enter the numbers, one each side of the decimal point, in whatever order you like. Add as you go. You can choose to stop at any point in the table.
3. Play the best of three.

	●
+	●
=	●
+	●
=	●
+	●
=	●
+	●
	●

2 Decimal zilch

Equipment required:
2–4 brains, 3 dice

How to win:
The aim is to be the person to have the closest score to zero (but not less than zero).

How to play:

1. Each player draws up a table like the one on the right.
2. Take turns to roll the three dice. Enter the numbers in the boxes in the next empty line in any order you choose. Subtract from the above total. You can choose to stop at any point in the table.
3. Play the best of three.

10	0	0
−		
=		
−		
=		
−		
=		

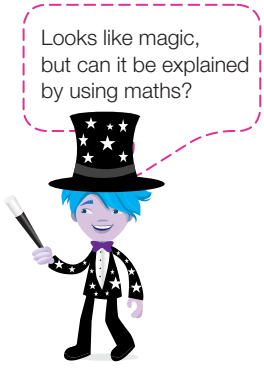
Investigation

Ten dollars and eighty-nine cents

Equipment required: 2 brains may be used for the 'Explore' section

The Big Question

Why is the answer to a seemingly random collection of calculations always the same?



Engage

1 **(a)** Choose an amount of money less than $10.00 (e.g. $7.25) and write it down.

(b) Reverse the digits and write down the amount formed, keeping the decimal point after the first digit (e.g. $5.27).

(c) Subtract the smaller number from the larger one (e.g. $7.25 – $5.27 = $1.98).

(d) Reverse the digits in your answer to part **(c)** and add the resulting number to your answer to part **(c)** (e.g. $1.98 + $8.91 = $10.89).

(e) Repeat parts **(a)** to **(d)** with a different starting amount. Do you get the same final answer?

Explore

2 To investigate further, write down at least 10 different amounts (all less than $10.00) using different combinations of digits. Perform the same steps from Question **1** for each of them. Keep a record of which amounts produced the same final answer and which didn't.

3 Can you see a pattern in the calculations that can explain why the same answer keeps appearing? If you can, try to describe it. If you can't, write down some new amounts and test them. Remember that the three digits you use do not have to all be different —try making some the same and see what happens to the result.

Strategy options

- Look for a pattern.
- Seek an exception.
- Break problem into manageable parts.

Explain

4 For your calculations that gave $10.89 as the final answer, explain the following.

(a) Look at the subtractions of the digits in the hundredths column. What do they all have in common? Explain why this is the case, if you can.

(b) Look at the middle digit that results after the subtraction step. What do you notice? Again, explain why this is the case, if you can.

(c) Did you find amounts for which the answer was not $10.89? What did the digits of these numbers have in common?

Elaborate

5 **(a)** For amounts that add to $10.89, what is the sum of the first and last digits of the answer to the subtraction? Explain why.

(b) Explain why the final addition step produces $10.89. Why does the addition of the tenths digits always result in an 8?

6 Summarise what you have found in this investigation by answering the Big Question.

Evaluate

7 **(a)** Did you find the results to your first few calculations surprising? Do you have more of an understanding now of how the 'magic' works?

(b) How many calculations did it take for you to see patterns emerging? What did you first start to notice?

(c) Did you feel frustrated at any time during this investigation? If so, how did you deal with the frustration?

Extend

8 Try this.

(a) Select any three single-digit decimal numbers (e.g. 0.4, 0.6, 0.1).

(b) Use these numbers to make six two-digit numbers with one number the units and the other number the tenths (e.g. 4.6, 4.1, 6.1, 6.4, 1.4, 1.6).

(c) Add these numbers (e.g. 24.2).

(d) Add your original numbers (e.g. 1.1).

(e) Divide this into the sum you obtained in part **(c)** (e.g. 24.2 ÷ 1.1).

Did you get 22?

Decimal multiplication

4.5

Multiplying a decimal by a whole number

To multiply a decimal by a whole number:

Multiply as though both values are whole numbers. Then, place a decimal point in the answer so that it has the same number of decimal places as the decimal number in the question.

To check that your decimal point is in the correct place, it is a good idea to estimate your answer before calculating it.

Worked Example 11 WE11

Calculate:

(a) 4.172×3 **(b)** 52.63×14 **(c)** 0.0003×2

Thinking		Working
(a) **1**	Use rounding to estimate the answer.	(a) $4.172 \times 3 \approx 4 \times 3$ ≈ 12
2	Multiply as though both values are whole numbers.	$\begin{array}{r} 4.172 \\ \times \quad {}^{2}3 \\ \hline 12.516 \end{array}$
3	Make sure that there are the same number of decimal places in the answer as in the question (in this case, 3).	$4.172 \times 3 = 12.516$
4	Check your answer against your estimate to see whether it is reasonable.	Reasonable
(b) **1**	Use rounding to estimate the answer.	(b) $52.63 \times 14 \approx 50 \times 14$ $\approx 14 \times 5 \times 10$ ≈ 700
2	Multiply as though both values are whole numbers.	$\begin{array}{r} 52.63 \\ \times \quad {}^{1}{}^{2}{}^{1}14 \\ \hline 21\,052 \\ 52\,630 \\ \hline 736.82 \end{array}$
3	Make sure that there are the same number of decimal places in the answer as in the question (in this case, 2).	$52.63 \times 14 = 736.82$

	4	Check your answer against your estimate to see whether it is reasonable.	*Reasonable*
(c)	**1**	An estimate is not needed here, as we can see that the only non-zero digit will be a 6 (3×2).	*(c)*
	2	Multiply as though both values are whole numbers. Place a decimal point in the answer so that there are the same number of decimal places as in the decimal number in the question (in this case, 4).	*0.0003* *× 2* *0.0006* *0.0003 × 2 = 0.0006*

Multiplying and dividing decimals by powers of 10

When we multiply a number by 10, we are making each digit in the number 10 times its previous value.

In the place value table, each digit is moved up one place value column to the left to show this. Hundredths become tenths, tenths become ones, ones become tens etc.

For example, $43.25 \times 10 = 432.5$.

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ 0.01		Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)
	4	3	•	2	5	× 10 =	4	3	2	•	5

× 10 × 10 × 10 × 10

When we divide by 10, we are making each digit one-tenth of its previous value. In the place value table, each digit is moved down one place value column to the right to show this. Tenths become hundredths, ones become tenths, tens become ones etc.

For example, $65.1 \div 10 = 6.51$ as shown below.

Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)		Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)
6	5	•	1		÷ 10 =		6	•	5	1

÷ 10 ÷ 10 ÷ 10

It is convenient to think of this as moving the decimal point one place to the right (for multiplication) or left (for division). This produces the same answer; however, we should remember it is the digits themselves that actually move as their values increase or decrease.

Multiplying or dividing by 100 is like multiplying or dividing by 10 twice (as $100 = 10 \times 10$). Each digit moves two place value columns. Again, we can think of this as moving the decimal point two places to the right or left.

To *multiply* a decimal by a power of 10:

Move the decimal point to the *right* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$0.45 \times 10\ (10^1) = 4.5$

$0.45 \times 100\ (10^2) = 45$

$0.45 \times 1000\ (10^3) = 450$

To *divide* a decimal by a power of 10:

Move the decimal point to the *left* the same number of place values as zeroes in the power of 10. Fill in any empty place values with zeroes.

$78.9 \div 10\ (10^1) = 7.89$

$78.9 \div 100\ (10^2) = 0.789$

$78.9 \div 1000\ (10^3) = 0.0789$

Decimal 'shorthand'

Each of the powers of 10 has its own 'name', as shown below with the first nine powers:

10^1 = ten	10^4 = ten thousand	10^7 = ten million
10^2 = hundred	10^5 = hundred thousand	10^8 = hundred million
10^3 = thousand	10^6 = million	10^9 = billion

To write very large numbers, we sometimes write a smaller decimal number and the name of the power. For example, instead of writing 2 300 000, we could write 2.3 million, or 2.3×10^6. This is equivalent to writing $2.3 \times 1\,000\,000$, or 2 300 000.

Multiplying decimals by other multiples of 10

Multiples of 10, such as 20, 190, 1700, 8000 etc., can be written as a number multiplied by a power of 10: $20 = 2 \times 10$, $190 = 19 \times 10$, $1700 = 17 \times 100$, $8000 = 8 \times 1000$.

Writing them in this way means we can multiply by the number first, then the power of 10.

Worked Example 12 WE12

Calculate:

(a) 0.6295×7000 **(b)** $9.81 \times 340\,000$

Thinking		Working
(a) **1**	Rewrite the whole number as a product of a number and a power of 10.	(a) $7000 = 7 \times 1000$
2	Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied.	0.6295 $\times\ 7$ 4.4065
3	Multiply the answer by the power of 10 by moving the decimal point to the right. (In this case, 3 places.)	4.4065×1000 $= 4406.5$

(b) 1	Rewrite the whole number as a product of a number and a power of 10.	(b) $340\,000 = 34 \times 10\,000$
2	Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied.	$\begin{array}{r} 9.81 \\ \times\ {}^{2}{}^{3}\ 34 \\ \hline 3924 \\ {}_{1}2_{1}9430 \\ \hline 333.54 \end{array}$
3	Multiply by the power of 10 by moving the decimal point to the right. (In this case, 4 places.) Write zeroes into the empty place value columns.	$333.54 \times 10\,000$ $= 3\,335\,400$

Multiplying a decimal by another decimal

To develop a method for multiplying one decimal by another, it is useful to look at some number patterns. We have seen that when we multiply by powers of 10, each digit in the number moves up the same number of place values as the power number. We show this by moving the decimal point. For example:

$0.23 \times 10 = 2.3$ $0.23 \times 100 = 23$ $0.23 \times 1000 = 230$

We can reverse the above calculations by dividing by the powers of 10:

$2.3 \div 10 = 0.23$ $23 \div 100 = 0.23$ $230 \div 1000 = 0.23$

We can see that dividing by powers of 10 corresponds to moving the decimal point to the left the same number of places as zeroes in the power of 10. This is used in the Worked Example below.

Worked Example 13 WE13

Calculate:

(a) 0.63×0.4 **(b)** 0.014×0.002

Thinking		Working
(a) 1	Write both numbers as the division of a whole number by a power of 10.	(a) $0.63 = 63 \div 100$ $0.4 = 4 \div 10$
2	Multiply as though both numbers are whole numbers. (We do not need to multiply by the zero in 0.4, as it does not affect the outcome.) Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide by 100 and 10, which is the same as dividing by 1000.) If necessary, place a zero in the units place to clearly show the position of the decimal point.	$\begin{array}{r} 0.63 \\ {}^{2}{}^{1}\ \times\ 0.4 \\ \hline 0.252 \end{array}$ $0.63 \times 0.4 = 0.252$

(b) 1 Write both numbers as the division of a whole number by a power of 10.

(b) $0.014 = 14 \div 1000$
$0.002 = 2 \div 1000$

2 Multiply as though both numbers are whole numbers (ignoring the zeroes). Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide twice by 1000, which is the same as dividing by 1 000 000.) Place zeroes in the empty place values and a zero in the units place.

$$\begin{array}{r} 0.014 \\ \times\ 0.002 \\ \hline 0.000\,028 \end{array}$$

$0.014 \times 0.002 = 0.000\,028$

To multiply one decimal by another:

Step 1 Write both numbers as the division of a whole number and a power of 10.

Step 2 Multiply as though both numbers are whole numbers.

Step 3 Divide the product by the powers of 10 identified in **Step 1**. Show this by moving the decimal place to the left.

You may be able to spot a useful shortcut from the Worked Example above: The number of decimal places in the answer to the multiplication is equal to the total number of decimal places in the numbers being multiplied.

4.5 Decimal multiplication

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q7, Q9, Q11, Q12, Q13, Q14, Q17, Q21, Q23, Q24	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q17, Q19, Q21, Q22, Q23, Q24, Q25	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q18, Q19, Q20, Q22, Q23, Q24, Q25

Answers page 643

Fluency

1 Calculate:

WE11

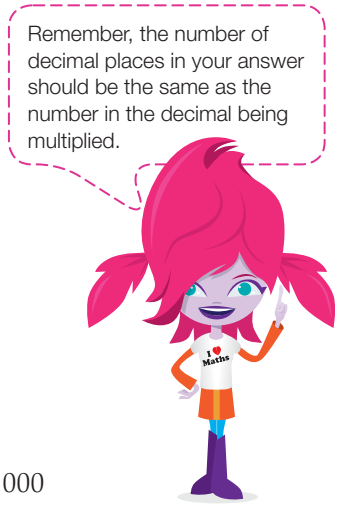
(a) 7.6×4 **(b)** 5.3×7 **(c)** 6.5×9

(d) 5×4.62 **(e)** 0.54×4 **(f)** 6×12.33

(g) 8.047×8 **(h)** 0.619×7 **(i)** 3×18.309

(j) 22.08×13 **(k)** 48.76×25 **(l)** 37.09×32

(m) 0.0004×2 **(n)** 0.0005×7 **(o)** 1.0006×3



2 Calculate:

WE12

(a) 2.4×60 **(b)** 3.27×70 **(c)** 2.991×40

(d) 0.57×200 **(e)** 4.34×500 **(f)** 6.625×400

(g) 4.591×6000 **(h)** 6.38×4300 **(i)** 3.14×9300

(j) $0.571 \times 68\,000$ **(k)** $0.9465 \times 71\,000$ **(l)** $7.4832 \times 55\,000$

WE13

3 Calculate:

(a) 0.6×0.8 **(b)** 0.4×0.6 **(c)** 0.9×0.4

(d) 9.32×0.7 **(e)** 4.12×0.3 **(f)** 5.26×0.8

(g) 7.74×0.31 **(h)** 9.46×0.22 **(i)** 3.76×0.18

(j) 0.004×0.6 **(k)** 0.0308×0.3 **(l)** 0.072×0.05

4 A reasonable estimate for the answer to 1.8×15 is:

A 3 **B** 16.8 **C** 30 **D** 300

5 How many decimal places will there be in the answer to 12.789×41?

A 1 **B** 2 **C** 3 **D** 4

6 How many decimal places will there be in the answer to 0.36×1.04?

A 2 **B** 3 **C** 4 **D** 5

Understanding

7 Renée buys 52 components from an electronics shop for $0.35 each. What is the total cost of the components?

8 A pill bottle contains 37 pills, each of mass 0.287 grams. What is the total mass of the bottle's contents?

9 Write the following as whole numbers.

(a) 3.5 million **(b)** 4.26 billion **(c)** 1.3 trillion (1 trillion = 1000 billion)

(d) 57.08 million **(e)** 10.2 trillion **(f)** 20.045 billion

10 A snail crawls 0.041 metres in an hour. If it kept up this pace, how far could it move in a day?

11 Amy orders 27.5 metres of timber priced at $4.29 per metre from her local hardware store. How much will the timber cost her in total? (Round your answer to the nearest cent.)

12 Gavin sells 300 punnets of berries to a supermarket. If he is paid $1.72 for each punnet, how much money does he receive?

13 In 2010, Mexican businessman Carlos Slim was declared the world's richest man, with a fortune worth 53.5 billion US dollars. Write this amount as a whole number.

14 **(a)** Marita fills her car's fuel tank with petrol at a cost of $1.35 per litre. If she adds 70 litres, how much must she pay?

(b) Ali pumps 9.5 litres of petrol into his car's tank. How much did the fuel cost him if he paid the same price per litre as Marita? Round your answer to the nearest cent.

15 The instructions on a bag of fertiliser say to spread 0.75 kilograms per square metre of area to be fertilised. Raisa wants to fertilise 2600 square metres of property using this product. How many kilograms of fertiliser should she use?

16 Bryan has calculated that he needs 3.6 square metres of tiles for his bathroom. What will he have to pay if the tiles he wants cost $22.39 per square metre? (Round your answer to the nearest cent.)

Reasoning

17 Kim works in an ice-cream parlour, and one hot day sells 500 scoops of vanilla. If there is 0.185 litres of ice-cream in each scoop and the day started with 130 litres of vanilla in the freezer, how many litres of vanilla ice-cream are left at the end of the day?

18 Yoghurt contains 167 kilojoules per 100 grams. How many kilojoules would a 175 gram tub contain?

19 In April 2010, the Australian Bureau of Statistics calculated that the population of Australia was 22 296 411.

(a) Write this number as a decimal number, with the number of millions as the whole number part, and the rest of the number rounded to one decimal place.

(b) Why would your answer to **(a)** be the number you would expect to see reported in the news, instead of the 'exact' value?

20 **(a)** A woodwork teacher needs 50 pieces of pine, each 0.135 metres long, to give to students to make a particular model. What total length (in metres) of timber does the teacher need? (Ignore any lost length due to saw cuts.)

(b) If he can buy the pine in 3.5 m lengths, how many pieces does he need, and how much does he have left over?

21 A multi-bag of mini chocolate bars costs $3.95 and contains 12 bars. Each mini chocolate bar can also be purchased individually for 40 cents.

(a) How much would 12 mini chocolate bars cost if they were purchased individually?

(b) How much do you save by purchasing the multi-bag?

(c) Charlotte wants to buy 30 mini bars for a party. Should she buy them individually, or in multi-bags? Which way is cheaper?

22 Wayne is following his grandfather's recipe for tomato sauce, which requires 3.5 pounds of tomatoes. (A pound is an old imperial unit of mass.) Because he only has scales in kilograms, Wayne decides to convert the mass to kilograms. A website tells him that 1 pound equals 0.454 kilograms. What answer should he get when he converts the mass of the tomatoes to kilograms? Round your answer to one decimal place.

23 Francesca read her electricity meter one Saturday, and again at the same time the next Saturday. She recorded the following readings.

First Saturday: 7562.3 kilowatt-hours
Second Saturday: 7638.6 kilowatt-hours

(a) How many kilowatt-hours of electricity did Francesca's household use in the week?

(b) If electricity costs $0.13 per kilowatt-hour, find the total cost of the electricity supplied in the week.

Open-ended

24 Jayapal has been asked to work out the length if every student in Year 7 lay end to end. He has been told that there are 89 students in Year 7 and that their average height is 1.63 m.

This is his working out:

$89 \times 1 = 89$
$89 \times 6 = 534$
$89 \times 3 = 267$

$$\begin{array}{r} 89 \\ 534 \\ +\ 267 \\ \hline = 880 \end{array}$$

Answer: 8.80 m.

(a) Where has Jayapal made his error? Explain how he could use estimation to tell that he was incorrect.

(b) Explain how he could modify his method to get the correct answer.

(c) Show how you would complete this problem.

25 Two numbers are multiplied together to give 68.4.

(a) Find at least two possible pairs of numbers with one number being a multiple of 10.

(b) Find at least two possible pairs of numbers with neither being a multiple of 10.

Outside the Square Game

Multiplicity

Equipment required: 2 brains, 1 die, 1 calculator

How to win:
The aim of the game is to get the target number as the whole number part of a decimal.

How to play:

1. The first player rolls the die twice to determine the two-digit target number (the first number followed by the second number).
2. The second player then rolls the die twice to determine the starting number.
3. The first player puts the starting number into a calculator and can multiply this by any number of their choice, with the aim of reaching the target number.
4. If they reach a number starting with the two-digit target number, then they're the winner; if not, the other person takes their answer (leave it in the calculator display) and then chooses a number to multiply this by to try to reach the target number.

For example, the first player rolls a 4 and a 3, so the target number is 43.

The second player then rolls two 1s, so the starting number is 11.

The first player chooses to multiply the 11 by 4, which gives an answer of 44—too big.

The second player chooses to multiply the 44 by 0.9, which gives an answer of 39.6—too small.

This continues until a player reaches an answer of 43 point something.

The game can be made more difficult by making the target number a three-digit number, or a two-digit number with one decimal point.

Half-time 4

1 Convert the following decimals into fractions (or mixed numbers where appropriate) in simplest form. Ex.4.3

(a) 0.25 **(b)** 0.3 **(c)** 3.54 **(d)** 9.345

2 Put the following decimals in order from smallest to largest. Ex.4.1

9.01, 19.12, 0.03, 0.321, 9.003 24

3 Calculate: Ex.4.4

(a) 12.5 + 7.9 **(b)** 124.54 + 8.379 **(c)** 0.32 + 2.5

(d) 15 – 8.98 **(e)** 127.061 – 43.2 **(f)** 6 – 4.9

4 Round the following numbers to three decimal places. Ex.4.2

(a) 4.8765 **(b)** 13.0037 **(c)** 2.3498 **(d)** 23.032 49

5 Calculate the following. Ex.4.5

(a) 7.6×4 **(b)** 28.3×5 **(c)** 1.37×9

(d) 45.2×30 **(e)** 0.26×2000 **(f)** 6.78×500

(g) 4.3×1.7 **(h)** 39.82×15.4 **(i)** 1.2×0.0005

6 Franji is saving up to purchase a games console. She has saved the following amounts over the last four weeks: \$25, \$47.50, \$18.70, \$9.35. Ex.4.4

(a) How much has she saved in total?

(b) If the console is \$149, how much more does Franji have to save?

7 Write the following in decimal form. Ex.4.1

(a) $23 + \frac{4}{10} + \frac{9}{100} + \frac{7}{10\,000}$ **(b)** $14 + \frac{235}{1000}$

(c) 2 hundreds, 9 tens, 3 ones, 4 tenths, 1 hundredth and 5 thousandths

(d) 9 tens, 0 ones, 6 tenths, 2 hundredths and 7 ten-thousandths

8 Round the following amounts of money to the nearest 5 cents. Ex.4.2

(a) \$1.93 **(b)** \$35.08 **(c)** \$57.44 **(d)** \$189.98

9 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. Ex.4.3

(a) $\frac{4}{5}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{7}{12}$

10 The times recorded by the finalists in a track cycling sprint race were: 59.023 s, 59.105 s, 59.23 s, 59.125 s. Write the times in order from fastest to slowest. Ex.4.1

11 Melinda is out buying some ingredients for a cooking class. She buys: Ex.4.4, 4.5

12 packets of pasta at \$2.49 each
18 tins of tomatoes at \$1.35 each
6 kg of mince at \$8.99 per kg
3 packets of cheese at \$4.78 per packet

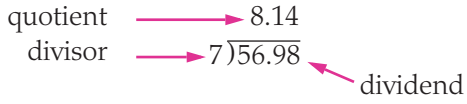
Find the total cost of Melinda's shopping.

4.6 Decimal division

When studying division, the following mathematical terms are useful.

- The dividend is the number that is being divided.
- The divisor is the number we are dividing by.
- The quotient is the result of the division (the answer).

These are shown in the example below.



When using long or short division to divide a decimal by a whole number:

Line up the decimal point in the quotient with the decimal point in the dividend.

Worked Example 14 WE14

Calculate the following. Round your answers to three decimal places if necessary.

(a) 5.28 ÷ 3 **(b)** 0.41 ÷ 7

Thinking	Working
(a) 1 Write the question with the setting out shown.	(a) 3)5.28
2 Perform the division, and place a decimal point in the quotient so it lines up with the one in the dividend.	1. 7 6 3)5.^{2}2^{1}8
(b) 1 Write the question with the setting out shown.	(b) 7)0.41
2 Perform the division, remembering to place a zero in the quotient if the divisor goes zero times into the dividend. Add zeroes to the divisor to continue the division, until there are four decimal places in the answer.	0.0 5 8 5... 7)0.4^{4}1^{6}0^{4}0
3 Round off the answer to three decimal places.	0.059 (to 3 d.p.)

Dividing decimals by multiples of 10

As we did for multiplying decimals, we can write other multiples of 10 as the product of a number and a power of 10, and do two separate division steps. Remember that we can show division by a power of 10 by moving the decimal point to the left the same number of places as the zeroes in the power of 10.

Worked Example 15 WE15

Calculate the following. Round your answers to three decimal places where necessary.

(a) 9.76 ÷ 400 **(b)** 14.7 ÷ 80

Thinking			Working
(a)	**1**	Break the divisor down into a whole number and a power of 10.	(a) 9.76 ÷ 400 = 9.76 ÷ 4 ÷ 100
	2	Divide by the whole number first (in this case, by the 4).	$4\overline{)9.^{1}7^{1}6}$ → 2.44
	3	Now, divide the quotient of the first division by the power of 10. Show this by moving the decimal point. (Here, it is moved 2 places left.)	2.44 ÷ 100 = 0.0244
	4	Round your answer to the specified number of decimal places.	= 0.024 (to 3 d.p.)
(b)	**1**	Break the divisor down into a whole number and a power of 10.	(b) 14.7 ÷ 80 = 14.7 ÷ 8 ÷ 10
	2	Divide by the whole number first (in this case, by the 8). Add zeroes to the divisor to continue the division until there are more decimal places in the answer than the rounded answer requires.	$8\overline{)14.^{6}7^{3}0^{6}0^{4}0}$ → 1.8375
	3	Now, divide the answer by the power of 10. Show this by moving the decimal point. (Here, it is moved 1 place left.)	1.8375 ÷ 10 = 0.183 75
	4	Round your answer to the specified number of decimal places.	= 0.184 (to 3 d.p.)

Dividing a decimal by another decimal

In order to divide a decimal by another decimal we need to convert it to a problem where we can divide a decimal by a whole number.

Recall that when working with fractions we can multiply or divide the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

For example, $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{10}{20}$. We can apply the same principle to dividends and divisors.

For example, 1 ÷ 2 gives the same answer as 2 ÷ 4 and 10 ÷ 20.

13.52 ÷ 1.2
= 135.2 ÷ 12 (multiplying both numbers by 10)
= 1352 ÷ 120 (multiplying by 10 again)

7.5 ÷ 2.5
= 15 ÷ 5 (multiplying by 2)

To divide one decimal by another, multiply both numbers so that the divisor is no longer a decimal. Then, divide the new dividend by the new, whole number divisor.

Multiplying both numbers by a power of 10 is often the most convenient. Multiply by the smallest power of 10 that gives a whole number divisor.

Worked Example 16

WE16

Calculate the following. Round your answer to three decimal places if necessary.

(a) 3.35 ÷ 0.005 **(b)** 10.52 ÷ 0.9

Thinking			Working
(a)	**1**	What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 1000.)	*(a)* $3.35 \div 0.005$ $= (3.35 \times 1000) \div (0.005 \times 1000)$
	2	Rewrite the question using the new divisor and new dividend.	$= 3350 \div 5$
	3	Calculate the quotient.	$5\overline{)33^{3}50}$ 6 70
	4	State the answer.	$3.35 \div 0.005 = 670$
(b)	**1**	What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 10.)	*(b)* $10.52 \div 0.9$ $= (10.52 \times 10) \div (0.9 \times 10)$
	2	Rewrite the question using the new divisor and new dividend.	$= 105.2 \div 9$
	3	Calculate the quotient to one more decimal place than the rounded number required.	$9\overline{)10^{1}5.^{6}2^{8}0^{8}0^{8}0}$ 1 1. 6 8 8 8
	4	Round off the quotient to the required number of decimal places.	$= 11.689$ (to 3 d.p.)

4.6 Decimal division

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17, Q19, Q23, Q24	Q1 Columns 2 & 3, Q2 Column 2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20, Q23, Q24	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24

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Fluency

1 Calculate the following. Round your answers to three decimal places if necessary.

WE14

(a) 8.48 ÷ 4 **(b)** 12.96 ÷ 3 **(c)** 14.22 ÷ 9

(d) 3.3 ÷ 6 **(e)** 2.4 ÷ 7 **(f)** 5.1 ÷ 9

(g) 39.42 ÷ 6 **(h)** 10.6 ÷ 7 **(i)** 0.47 ÷ 9

(j) 0.42 ÷ 6 **(k)** 1.22 ÷ 3 **(l)** 18.46 ÷ 7

(m) 154.33 ÷ 5 **(n)** 84.365 ÷ 8 **(o)** 347.507 ÷ 7

2 Calculate the following. Round your answers to three decimal places if necessary.

WE15

(a) 3.6 ÷ 40 **(b)** 8.1 ÷ 90 **(c)** 36.216 ÷ 60

(d) 96.58 ÷ 200 **(e)** 2.296 ÷ 400 **(f)** 23.4 ÷ 600

(g) 233.4 ÷ 3000 **(h)** 40.6 ÷ 8000 **(i)** 18.9 ÷ 7000

(j) 68.4 ÷ 120 **(k)** 3.19 ÷ 110 **(l)** 8.97 ÷ 1300

3 Calculate the following. Round your answers to three decimal places if necessary.

WE16

(a) 5.14 ÷ 0.2 **(b)** 4.12 ÷ 0.4 **(c)** 5.1 ÷ 0.6

(d) 15.48 ÷ 0.04 **(e)** 9.018 ÷ 0.09 **(f)** 1.736 ÷ 0.07

(g) 2.382 ÷ 0.006 **(h)** 0.3996 ÷ 0.009 **(i)** 0.196 ÷ 0.008

(j) 7.7 ÷ 0.011 **(k)** 3.78 ÷ 0.012 **(l)** 1.16 ÷ 0.008

(m) 1.74 ÷ 0.0002 **(n)** 8.05 ÷ 0.0007 **(o)** 13.36 ÷ 0.0008

4 How many decimal places will the answer to 298.12 ÷ 100 have?

A 1 **B** 2 **C** 3 **D** 4

5 Evaluating which of the following will give the answer to 3.2 ÷ 0.005?

A 0.0032 ÷ 5 **B** 320.0 ÷ 5 **C** 3200.0 ÷ 5 **D** 32 000.0 ÷ 5

Understanding

6 A block of metal of mass 2.312 kg is to be divided equally into 8 pieces. What will the mass of each piece be?

7 A company makes a profit of 7.86 million dollars. If the profit is to be divided equally among the four company owners, how much would each owner get? (Answer in millions of dollars.)

8 The total bill for a wedding reception for 100 guests is $9398.55. How much does this work out to per guest? (Round your answer appropriately.)

9 Anthony needs to mix 0.07 litres of environmentally friendly pest repellent with water each time he fills the tank of his garden sprayer. If he has 0.84 litres of repellent left, how many sprayer tankfuls can he mix?

10 A jeweller calculated that a popular style of necklace contains 0.009 g of gold. How many necklaces could she make using the 2.332 g of gold in her workshop?

11 Eleanor's car travels 37.29 km on three litres of fuel. How far would it travel on one litre of fuel?

12 A 4000 millilitre container of orange juice costs $8.90. Determine the cost per 100 mL. (Round your answer appropriately.)

13 Simon buys 7 kg of spicy sausages for $32.95. How much is this per kilogram?

14 Regina has a tub containing 5.8 kilograms of worms which she uses for fishing. If the average worm has a mass of 0.003 kilograms, how many are in her tub?

15 Hai is making a long-distance call on his mobile. The call costs $0.90 per minute. For how many minutes can he continue his call if he has $5.85 credit left on his phone?

16 A 1.2 kilometre taxi trip costs Joan $5.70. How much is this per kilometre?

Reasoning

17 The captains of two netball teams were discussing which of their teams was taller. The heights of Angie's team were 1.49 m, 1.54 m, 1.65 m, 1.67 m, 1.58 m, 1.73 m and 1.56 m. The heights of Jessica's team were 1.73 m, 1.58 m, 1.59 m, 1.62 m, 1.45 m, 1.51 m and 1.62 m. Add the seven heights and divide the result by seven to find the average (mean) height of each team. Which of the two teams was the taller, on average? Answer correct to two decimal places.

18 Monica and Celeste were trying to determine the best bargain of three different jars of coffee. The 200 g jar of BestCafe cost $5.95, the 500 g jar of Maxine House was $14.85 and the 300 g tin of Brazilian Roast was $6.75. Which is the best buy in terms of price per 100 grams?

19 Karen was counting the coins in her cash register drawer and found that she had $1.65 in 5-cent coins, $2.30 in 10-cent coins, $5.20 in 20-cent coins and $7.50 in 50-cent coins. How many of each type of coin did she have?

20 Jacob received a bill from a rent-a-car firm that gave the total cost as $310. This included the daily cost of $24.80 and $1.20 per kilometre travelled. If Jacob had hired the car for 2 days, how many kilometres did he travel?

Open-ended

21 Write three numbers, each with three decimal places, that when divided by 4 give an answer between 0.5 and 1.

22 For each of the following, write two equivalent divisions by multiplying the divisor by a number that is not a power of 10.

(a) $10.05 \div 0.5$

(b) $36.04 \div 0.02$

23 The area of a rectangle is calculated by multiplying the length and the width. If the area of a rectangle is 8.45 m^2, what might be the length and width of the sides? For each of the side dimensions, work out the related perimeter (distance along the boundary of the rectangle).

24 Tran divided 5.25 by 5 and wrote down an answer of 1.5.

(a) What error did Tran make?

(b) Explain to Tran what he needs to do to calculate decimal divisions accurately.

Outside the Square Puzzle

Magic square

Copy this 6 × 6 table into your book.

Fill in the blank squares. All the rows and columns must add up to 11.1.

2.1		2.5	2.7		
	2.5	2.8		0.5	
0.1	0.3		1.9	3.3	
0.4	0.2	1.7	1.8	3.6	
2.9	3.1	0.9	1.1	1.6	
3.2	3.0	1.2		1.3	0.8

Maths 4 Real

PERPLEXING PRICES

Imagine you are in your local supermarket, shopping for supplies for a party. You need 7 to 8 litres of soft drink and some lollies, so you move to the soft drink aisle first. A vast array of products and prices confronts you. You see that your favourite brand has three different-sized bottles:

1.25 L for $2.25

1.5 L $2.78

2 L for $3.56

"Hmmm," you think. "The 2 L bottle gives me half a litre more than the 1.5 L, for only 78 cents extra. I could get two 1.5 L bottles, that's 3 L, for $5.56. Maybe I should get a couple of 1.25 L bottles instead. Which do I choose?"

1 At first glance, which bottle size looks to be the best value for money to you?

You remember that soft drink also comes in 375 mL cans. You move further down the aisle and see the following:

6-pack: $7.00 12-pack: $13.59 18-pack: $19.34 30-pack: $26.34

You begin to wonder: "Should I buy cans instead? How many cans are equivalent to one bottle? Are they better value for money? Help!"

2 What is it that makes your 'value for money' decision a difficult one? What information would be useful to help you decide?

In December 2009, the Australian government introduced a new, compulsory pricing system in supermarkets, designed to make decisions easier for shoppers. The system is called 'unit pricing'. As well as the regular price, supermarkets are required to display the price for a particular 'unit' of the product, such as 1 L, 1 kg, 100 g or 100 mL. Comparing the prices of different products is much easier when you are comparing the same amount.

We can calculate a unit price for the bottles of soft drink by calculating the price *per litre*. To do this, we divide the price by the capacity of the bottle, in litres:

For example, the unit price for the 1.25 L bottle is: $2.25 ÷ 1.25 L = $1.80 per L.

3 **(a)** Calculate the price per litre of the other two bottle sizes.

(b) Which size bottle is the best value for money?

(c) Would it always be better to purchase this size? Explain your answer.

(d) A bit further down the aisle, you notice that a 1.5 L bottle of a rival brand of cola is on special for $2.39. Calculate the price per litre of this discounted bottle.

(e) You also notice that you can buy a 'multipack' of twelve 300 mL bottles for $11.86. Calculate the price per litre of the multipack. (300 mL is 0.3 L)

(f) Why is the unit price of the multipack so different to the unit prices for individual bottles?

4 **(a)** Calculate the price per can of each of the packs.

(b) Which pack represents the best value for money? Did you find anything surprising in your results?

(c) The price of a single can of soft drink is $1.35. Why is there such a difference between the price of a single can and the price per can in the can packs?

5 **(a)** Calculate the price per litre of each of the can packs, remembering that each can contains 375 mL (0.375 L).

(b) Suggest a possible explanation for the difference in the price per litre between the cans and the bottles.

(c) When might people prefer to buy multipacks of cans instead of bottles?

You move down to the confectionery aisle, and spot a 190 g 'Party Mix' bag of lollies for $2.90, and a 'Jumbo' size 570 g bag for $5.82.

6 The unit price for products like bags of lollies is the price per 100 g.

(a) Calculate the unit price for the 190 g bag and the 570 g bag. (Hint: Work out what you need to divide 190 and 570 by to get 100, and divide the prices by this same amount.)

(b) Which is the better buy in this case?

RESEARCH

- Visit the supermarket, use catalogues or go shopping online to investigate the cost of goods that come in different sizes, such as coffee, Vegemite™, washing powder and pet food. Use a suitable unit, such as 100 g or 1 kg, to calculate the unit prices.
- Comment on your findings. Is there a general rule or conclusion you can make? Discuss, with examples, why different people might buy different-sized products.

4.7 Percentages, fractions and decimals

Percentages are used in many everyday situations. Shopping discounts, survey results, business statistics and sporting league ladders all use percentages to communicate information. Percentages make it easier for us to visualise and compare quantities, especially when comparing parts from different-sized wholes.

The word **'per cent'** literally means 'for every hundred' or 'out of one hundred'. The percentage symbol is %. Percentages can be thought of and written as fractions with a denominator of 100. Percentages can also be thought of as division by 100.

For example, 25% means 25 out of 100, and 9% means 9 out of 100. We can write percentages as fractions, some of which can be simplified, and some which can't.

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 9\% = \frac{9}{100}$$

If we perform the division, we get the percentage in decimal form.

$$\frac{25}{100} = 25 \div 100 = 0.25 \qquad \frac{9}{100} = 9 \div 100 = 0.09$$

Writing percentages as fractions and decimals

To write a percentage as a fraction …

- Write the percentage value as the numerator of a fraction with a denominator of 100.
- Simplify the fraction if possible.

To write a percentage as a decimal …

- Divide by 100.

Worked Example 17 WE17

Write 12% as a fraction in simplest form.

Thinking	Working
1 Write the value of the percentage as the numerator over a denominator of 100.	$12\% = \frac{12}{100}$
2 Simplify the fraction if possible. (Here, we have cancelled a common factor of 4.)	$= \frac{3}{25}$

Worked Example 18 WE18

Write 45% as a decimal.

Thinking	Working
Divide the value of the percentage by 100. Write your answer without the percentage symbol.	$45\% = 45 \div 100$ $= 0.45$

Writing fractions as percentages

To write a fraction as a percentage, we can use one of two methods:

Method 1: Write the fraction as an equivalent fraction with a denominator of 100. Use the numerator of this new fraction as the percentage.

Method 2: Multiply the fraction by 100%.

Method 1 is convenient, but only works with fractions that have denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2 can be used with any fraction.

Note: As Worked Example 19 shows, not all percentages have whole number values. These percentage numbers are usually written as decimals. If it is necessary to round them, one decimal place usually gives an answer of acceptable accuracy. These percentages can also be written in fractional form; for example, $16.7\% = 16\frac{7}{10}\%$, but decimal form is usually more convenient to work with. Remember that you can use the $a\frac{b}{c}$ or S⇔D key on your calculator to convert answers from decimal to fraction form, and vice versa.

100% is equivalent to 1 whole, $\frac{1}{1}$ and 1.0.

Worked Example 19 WE19

Write each of the following fractions as percentages.

(a) $\frac{4}{5}$ **(b)** $\frac{27}{40}$

Thinking	Working
(a) 1 Is the denominator a factor of 100? (Yes) Determine what number the denominator must be multiplied by to get 100.	(a) $\frac{4}{5} \times \frac{\quad}{20} = \frac{\quad}{100}$
2 Multiply the numerator by the same number.	$\frac{4}{5} \times \frac{20}{20} = \frac{80}{100}$
3 Write the numerator of the new fraction with the percentage symbol.	$= 80\%$
(b) 1 Is the denominator a factor of 100? (No)	(b) $\frac{27}{40} \times \frac{100}{1}\%$
2 Multiply the fraction by 100. (Remember that we can write a whole number as an improper fraction with a denominator of 1.) Cancel common factors first to simplify the multiplication. (Here, a common factor of 20 is cancelled.)	$= \frac{27}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$ $= \frac{135}{2}\%$
3 Divide the numerator of the answer by the denominator. Write your answer as a decimal (rounded if necessary).Write the answer with a percentage symbol.	$= 67.5\%$

Writing decimals as percentages

To write a decimal as a percentage, multiply it by 100.

Worked Example 20 WE20

Write each of the following decimals as percentages.

(a) 0.04 **(b)** 0.125

Thinking	Working
(a) Multiply the decimal by 100%.	(a) $0.04 = 0.04 \times 100\%$ $= 4\%$
(b) Multiply the decimal by 100%.	(b) $0.125 = 0.125 \times 100\%$ $= 12.5\%$

4.7 Percentages, fractions and decimals

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q15, Q16, Q18	Q1, Q2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18	Q1, Q2, Q3 Columns 3 & 4, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18

Answers page 644

Fluency

1 Write each of the following percentages as fractions in simplest form. WE17

(a) 15% **(b)** 24% **(c)** 18% **(d)** 64%

(e) 110% **(f)** 122% **(g)** 140% **(h)** 198%

2 Write each of the following percentages as decimals. WE18

(a) 34% **(b)** 13% **(c)** 78% **(d)** 92%

(e) 120% **(f)** 123% **(g)** 254% **(h)** 320%

3 Write each of the following fractions as percentages. WE19

(a) $\frac{2}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{20}$

(e) $\frac{11}{10}$ **(f)** $\frac{3}{2}$ **(g)** $\frac{28}{25}$ **(h)** $\frac{6}{5}$

(i) $\frac{3}{8}$ **(j)** $\frac{1}{3}$ **(k)** $\frac{5}{6}$ **(l)** $\frac{4}{9}$

(m) $\frac{13}{15}$ **(n)** $\frac{19}{30}$ **(o)** $\frac{33}{40}$ **(p)** $\frac{44}{60}$

4 Write each of the following decimals as percentages. WE20

(a) 0.87 **(b)** 0.34 **(c)** 0.65 **(d)** 0.96

(e) 1.34 **(f)** 3.58 **(g)** 5.32 **(h)** 1.21

5 Complete the following table.

Percentage	Fraction (simplest form)	Decimal
40%		
	$\frac{7}{10}$	
		0.05
	$\frac{1}{50}$	
75%		
	$\frac{5}{4}$	

Understanding

6 $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{5}$ are all common fractions.

(a) Change each of these fractions into decimals.

(b) Change each of these decimals into percentages.

7 95% of all the animal species in the world are insects. Write this as a fraction in simplest form.

8 At Valley High School, $\frac{5}{8}$ of the students are studying Chinese. Write this fraction as a percentage.

9 Place the following in ascending order (smallest to largest).

(a) 82%, 0.03, $\frac{7}{50}$, $\frac{3}{5}$, 0.8, 57%

(b) $\frac{1}{2}$, 73%, 1.05, $\frac{3}{10}$, 43%, 0.05

10 The Petronas Towers in Kuala Lumpur are approximately 0.6 times the size of the tallest building in the world, the Burj Khalifa in Dubai.

(a) Write the height of the Petronas Towers as a fraction of the height of the Burj Khalifa.

(b) Now, write the equivalent percentage.

Reasoning

11 A rugby stadium contains 43 500 people. 37% of the crowd supports England and the rest supports Australia. What percentage of the crowd supports Australia?

12 Allison is a journalist. She has surveyed 150 people and found that 6 believe that aliens exist.

(a) Write the survey result:

(i) as a fraction in simplest form

(ii) as a percentage

(iii) as a decimal.

(b) Which of the three forms of the result would you recommend that Allison uses as the headline for the article she is writing? Give a reason for your answer.

(c) Use your answer from **(b)** to write a potential headline.

13 $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, or $3 \times \frac{1}{4}$. How is this fact reflected in the percentage values for $\frac{1}{4}$ and $\frac{3}{4}$?

14 13% of a class have red hair, 62% of the class have black hair.

(a) What is the total percentage of the class that have red or black hair?

(b) What is this total as a decimal?

(c) What percentage of the class have a hair colour that is not red or black?

(d) Write your answer to **(c)** as a decimal.

Open-ended

15 Write down at least three fractions that have equivalent percentage values between 10 and 40 per cent.

16 Find at least three percentages that have decimal values between 0.34 and 0.35.

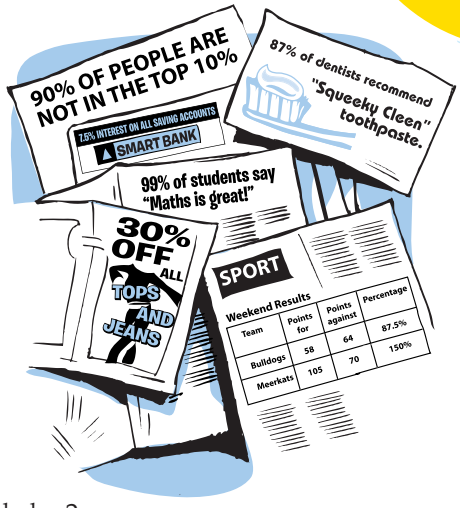
17 Estimate the percentage of one weekday that you would spend:

(a) sleeping **(b)** eating **(c)** studying **(d)** socialising.

Using percentages

4.8

In this section you will learn two important skills that will enable you to understand and work with two common uses of percentages.



Finding a percentage of a quantity

Statements involving percentages are common:

'32% of the students in the school have blue eyes.'
'48% of voters voted for a particular candidate.'
'The television is discounted by 20% of the original price.'

Knowing the fraction equivalents of some of the common percentages is very useful.

For example:

50% = $\frac{1}{2}$, so to find 50% of something, we can simply divide by 2.

25% = $\frac{1}{4}$, so we can find 25% easily by dividing by 4 (or by dividing by 2 twice).

10% = $\frac{1}{10}$, so we divide by 10 to find 10%.

20% = $\frac{1}{5}$, so we divide by 5 to find 20% (or divide by 10, then multiply by 2).

We can use these facts to find a percentage of a quantity. This is shown in Method 1 of the following Worked Example.

Another way to find a percentage of a quantity is to use the fact that in mathematics, the word 'of' means 'multiply'. This is shown in Method 2 of the following Worked Example.

Worked Example 21

WE 21

Calculate the following percentage: 15% of 70.

Method 1: Use basic percentages

Thinking	Working
1 Break the percentage up into basic percentages, such as 50%, 10%, 5% or 1%.	15% = 10% + 5%
2 Calculate each of the basic percentages.	10% of 70 = 70 ÷ 10 = 7 5% of 70 = (10% of 70) ÷ 2 = 7 ÷ 2 = 3.5
3 Use combinations of the basic percentages calculated to obtain the required percentage.	15% of 70 = 7 + 3.5 = 10.5

Method 2: Fraction or decimal multiplication

Thinking	Working
1 Replace the word 'of' with the multiplication symbol, ×.	15% of 70 $= 15\% \times 70$
2 Convert the percentage to a fraction or decimal.	$= \frac{15}{100} \times 70$ or 0.15×70
3 Perform the multiplication.	$= \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{100}} \times \frac{\cancel{70}}{1}$ $= \frac{3 \times 7}{2 \times 1}$ $= \frac{21}{2}$ $= 10.5$

Writing one amount as a percentage of another

Writing fractions of a whole as percentages makes them easier to visualise, compare and order. The results of surveys, tests and sports teams are often reported using percentages.

To write one amount as a percentage of another:

1. Write the amount we are interested in as the numerator of a fraction, with the other amount as the denominator.
2. Convert this fraction to a percentage. If the percentage is not a whole number, then rounding to one decimal place gives an accurate enough answer in most cases.

Worked Example 22 WE22

Write the following as percentages. Round your answers to one decimal place, if necessary.

(a) Melissa got 21 out of 25 questions on her grammar test correct.

(b) 13 of the 24 students in class 7C have brown hair.

Thinking	Working
(a) 1 Write the result as a fraction. Is the denominator a factor of 100? (Yes)	(a) $\frac{21}{25}$
2 Determine what number the denominator must be multiplied by to get 100.	$\frac{21}{25} \times \frac{\_}{4} = \frac{\_\_}{100}$
3 Multiply the numerator by the same number.	$\frac{21}{25} \times \frac{4}{4} = \frac{84}{100}$
4 Write the numerator of the new fraction with a percentage symbol.	84%
5 Write your answer in a sentence.	Melissa got 84% of the test questions correct.

(b) **1** Write the first amount as the numerator, and the second amount as the denominator of a fraction. Is the denominator a factor of 100? (No)

(b) $\frac{13}{24}$

2 Convert the fraction to a percentage by multiplying by 100. Cancel common factors first to simplify the multiplication. (Here, a common factor of 4 has been cancelled.)

$\frac{13}{\cancel{24}_{6}} \times \frac{\cancel{100}^{25}}{1}\%$

$= \frac{325}{6}\%$

3 Divide the numerator of the answer by the denominator. Write your answer as a decimal, rounded to one decimal place.

54.2%

4 Write your answer in a sentence.

54.2% of the students in 7C have brown hair.

4.8 Using percentages

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q18, Q20	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q20	Q1 Columns 2 & 3, Q2 (d)–(i), Q3, Q4, Q5, Q8, Q9, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

Answers page 644

Equipment required: A calculator may be used for Questions **2**, **7–12**, **15–17**

Fluency

1 Calculate each of the following percentages.

WE21

(a) 50% of $36
(b) 10% of 450 m
(c) 20% of 840 L
(d) 80% of 120 m
(e) 90% of 500 kg
(f) 2% of $8400
(g) 13% of 72 kg
(h) 27% of $354
(i) 21% of 876 m
(j) 28% of 231.25 m
(k) 34% of 210.4 L
(l) 4% of $32.65

2 Write the following as percentages. Round your answers to one decimal place, if necessary.

WE22

(a) 17 out of 20 on a spelling quiz.
(b) 5 out of 22 students in the class have blond hair.
(c) There are 100 kittens at the animal shelter, 40 of which are female.
(d) 4 out of 5 students surveyed own an mp3 player.
(e) 50 pairs of shoes were sold, of which 38 pairs were black.
(f) 54 out of 70 cars counted on a freeway had only 1 person in them.
(g) 15 of the 27 cars in the staff carpark are white or silver.
(h) 3 out of every 4 people on the bus said they were travelling to work.
(i) 28 out of 34 people surveyed said they were non-smokers.

3 Calculate the following using the 'basic percentages method' or any other mental strategy.

(a) 50% of $25 **(b)** 10% of 75 kg **(c)** 25% of 60 km

(d) 20% of 40 L **(e)** 1% of 360 m **(f)** 5% of $120

(g) 60% of 350 kg **(h)** 75% of 800 mL **(i)** 35% of $400

4 Mehmet successfully shot 13 basketball goals out of 18 attempts. The calculation that converts this result to a percentage would be set out as follows.

A $\frac{13}{100} \times \frac{18}{1}$ **B** $\frac{13}{18} \times \frac{100}{1}$ **C** $\frac{18}{13} \times \frac{100}{1}$ **D** $\frac{100}{13} \times \frac{18}{1}$

5 20% of $420 is:

A $8.40 **B** $20 **C** $84 **D** $200

Understanding

6 40% of the 25 students in class 7A have black hair. How many students is this?

7 16 out of the 28 biscuits in a packet have cream centres. What percentage of the packet are cream biscuits?

8 A 2 litre bottle of orange drink contains 35% orange juice. Work out how many litres of orange juice is contained in the bottle. (It may help to work in mL.)

9 A lion can sleep for up to 20 hours a day. What percentage of a whole day is this?

10 There are 11 boys and 15 girls in class 7B. Write the number of boys and the number of girls as percentages of the total number of students in the class.

11 Damian kicked 7 goals from 10 attempts. Franco kicked 11 goals from 16 attempts.

(a) Express the boys' results as percentages.

(b) Who is the more accurate goalkicker?

12 Australia scored 381 runs in a cricket match against India. If Shane Watson scored 126 of those runs, what percentage of the team's total did he score?

13 Shops often uses percentages to advertise savings during sales. The percentage is used to calculate the amount that is removed from the price of the goods. This amount is known as the 'discount'.

(a) Calculate the dollar value of the discount on the following.

(i) $40 shirt, 20% discount **(ii)** $125 jacket, 25% discount

(iii) $65 jeans, 50% discount **(iv)** $239 suitcase, 10% discount

(v) $799 camera, 40% discount **(vi)** $1299 sofa, 5% discount

(b) Now, find the sale price of the above items by subtracting the discount amount from the original price.

Reasoning

14 8% of the apples in a crate were rotten. What percentage of the apples were not rotten?

15 Chelsea and Sonia are netballers. In one game, Chelsea scored 17 goals from 25 attempts, and Sonia scored 15 goals from 20 attempts. In the next game, Chelsea scored 19 goals from 27 attempts, and Sonia scored 17 goals from 23 attempts. Use percentages to show which one of the girls improved the accuracy of her goal shooting. Round your figures to one decimal place, if necessary.

16 The 180 students at St John's College all completed an online survey. The results showed that:

22% of the students had blue eyes

34% spoke two or more languages fluently

7% of them were left handed.

(a) Calculate the number of students in each category using the above three percentages.

(b) Considering that we are dealing with numbers of students, what is strange about your answer to **(a)**?

(c) What might be the actual numbers of students represented by each of the percentages?

17 The energy in food is measured in kilojoules (kJ). The National Heart Foundation of Australia recommends that no more than 30% of our daily energy intake should come from eating fats.

(a) A healthy, active woman should eat up to 9000 kJ of food a day. Up to how many kJ in the form of fats can she eat in 1 day?

(b) 1 gram of fat gives the body 37 kJ of energy. How many grams of fat would provide a woman with the maximum daily amount of kilojoules allowed from fat? Round your answer to the nearest whole number.

Open-ended

18 Suggest some reasons as to why some stores use percentages to advertise savings, rather than dollar amounts.

19 Mark saved $5.00 when a store had a sale. Determine at least three different percentage discounts and original prices for the item that Mark purchased.

20 Fedora and Tanya were asked to show their working to the following question. 'Calculate 15% of 80'.

Here is Tanya's working:

$$\frac{15}{80} \times \frac{100}{1}$$

$$= \frac{150}{8}$$

$$= 18.75$$

Here is Fedora's working:

$$15\% \text{ of } 80$$
$$= 0.15 \times 80$$
$$= 12$$

Which of the girls' working is correct? Give a piece of advice to the other girl to help her avoid similar mistakes in the future.

Outside the Square — Problem solving

The great half-price sale

Tricia and Carly were out shopping. They headed towards their favourite clothes shop. On the shop window was a sign that read: 'Take 50% off all marked prices!' Stuck over the top of that sign was another sign, saying: 'Take another 50% off reduced prices!'

'Eh?' said Carly. '50% is one half. If I take half off the price, then take off another half, there won't be any price left! That means everything in the shop is free!'

'I don't think so', said Tricia. 'How can you run a business like that?'

'Well', said Carly, 'If I pick up a pair of $100 jeans and take two lots of 50% off the price, what do they cost now?'

'Errr...', said Tricia.

Can you work it out? How much would the jeans cost?

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

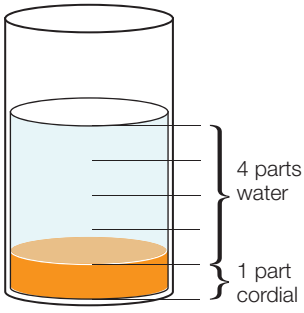
Ratio

4.9

A **ratio** is a comparison of amounts of the same type of quantity.

When we mix a cordial drink, we add different amounts of cordial and water.

The amount of cordial compared to the amount of water may be written in many ways.



1 part cordial to 4 parts water

or

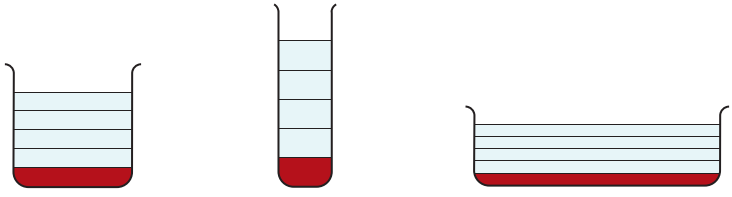
1 to 4

or using a ratio symbol,

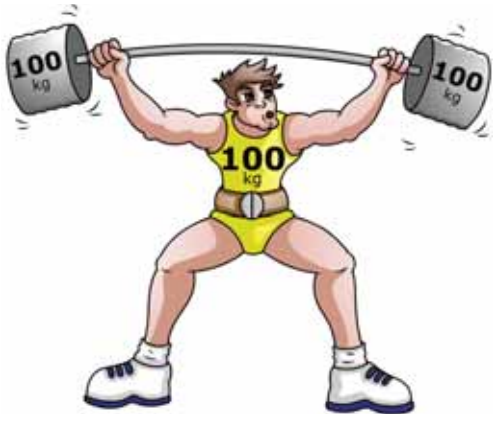
1 : 4

(The ':' can be read as 'to'.)

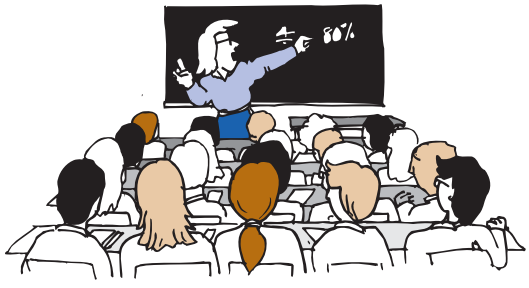
The actual volumes (the number of millilitres) of cordial and water are not given. It is the ratio, or proportion, of cordial compared to water that is important in getting the right taste. Specifying actual quantities would not be very useful, as cordial is made up in glasses and jugs of many different sizes. A sip of cordial from each of the vessels below would taste the same after the mixtures are stirred.



Some more examples of ratios are given below.



The ratio of the mass being lifted to the mass of the lifter is 2 : 1.



The teacher to student ratio in the class is 1 : 20.

A ratio is a comparison of two or more amounts, written using a ':' symbol.

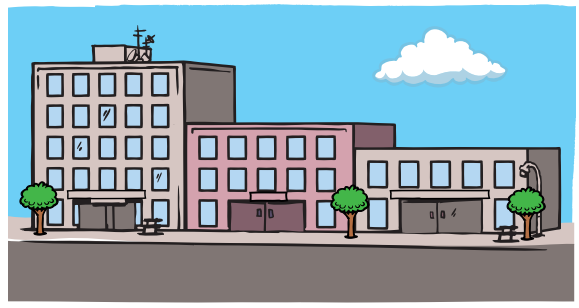
The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

Worked Example 23 WE23

Write a ratio for each of the following.

(a) the number of windows in the middle building compared to the number of windows in the smallest building

(b) the number of floors in the buildings, from left to right



Thinking			Working
(a)	**1**	Write the quantities in the correct order.	(b) Windows in middle building: 12 Windows in smallest building: 7
	2	Write as a ratio.	12 : 7
(b)	**1**	Write the quantities in the correct order.	(a) 5 floors to 3 floors to 2 floors
	2	Write as a ratio.	5 : 3 : 2

We can see from the Worked Example above that a ratio can compare more than two amounts.

Part : part and part : whole ratios

Suppose we had a class of 9 girls and 11 boys. We could write a ratio comparing the number of boys to girls as 11 : 9. This **part : part ratio** compares two parts of a whole. We can add the parts in a ratio to find the total number of parts. Here, 11 + 9 = 20 students in total. We can write another ratio that compares the number of boys to the total number of students in the class as 11 : 20. This **part : whole ratio** compares one part to the whole.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

Ratios, fractions and percentages

The numbers in part : whole ratios can be written as fractions. For example, the above boys : total students ratio of 11 : 20 could be written as the fraction $\frac{11}{20}$ or as the percentage 55%. The fraction of girls would therefore be $\frac{9}{20}$ of the class, or 45%.

Worked Example 24 WE24

A large enclosure at the zoo contains 5 giraffes and 12 zebras.

(a) Write a ratio comparing the number of zebras to the number of giraffes.

(b) Write a ratio comparing the number of giraffes to the total number of animals.

(c) Write the number of giraffes as a fraction of the total number of animals.

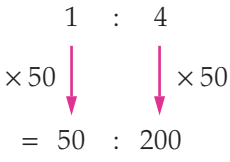
(d) Write the number of zebras as a percentage of the total number of animals.

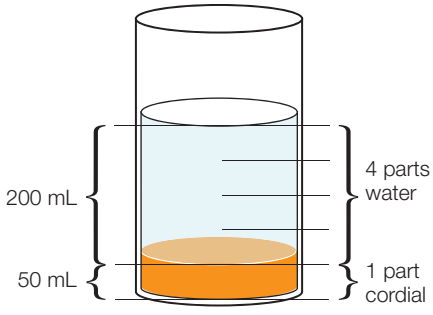
Thinking	Working
(a) Write the ratio in words, in the correct order, then write the numbers underneath.	(a) zebras : giraffes = 12 : 5
(b) **1** Find the number of parts in the whole (the total number of animals).	(b) 5 + 12 = 17
2 Write the ratio in words, in the correct order, then write the numbers underneath.	giraffes : total animals = 5 : 17
(c) Write one number in the part : whole ratio as a fraction of the other.	(c) $\frac{5}{17}$
(d) Write a new part : whole ratio as a fraction, then convert the fraction to a percentage, rounding to a sensible value.	(d) zebras : total animals = 12 : 17 $\frac{12}{17} \times \frac{100}{1}$ % = 71% (to nearest whole number)

Equivalent ratios

Imagine that you wanted to mix up a large quantity of a cordial drink using the cordial : water ratio of 1 : 4 that we considered at the beginning of this section. Mixing 1 mL of cordial and 4 mL would only give 5 mL of cordial; however, we can make 500 mL of cordial drink that would taste exactly the same if we mixed 100 mL of cordial with 400 mL of water.

1 : 4 and 100 : 400 are **equivalent ratios**. Equivalent ratios are created in the same way as equivalent fractions. We multiply each part in the ratio by the same number. Here, we have multiplied 1 : 4 by 100 to obtain the ratio 100 : 400. Multiplying by 50 would give a ratio of 50 : 200.





Simplifying ratios

Imagine that you have mixed a cordial drink using the above ratio of 50 : 200, which is equivalent to 1 : 4. Your friend Jack has made his drink by mixing 60 mL of cordial with 300 mL of water. He says his drink is stronger than yours, but it is not easy to tell by comparing the ratios of 50 : 200 and 60 : 300. It would be easier if we could compare the ratios in simplest form.

We can simplify a ratio in the same way we would simplify a fraction—by dividing each part in the ratio by a common factor. If we divide by the HCF, we will obtain the ratio in simplest form.

The HCF of the ratio 60 : 300 is 60.

$$\begin{array}{ccccc} & 60 & : & 300 & \\ \div 60 & \downarrow & & \downarrow & \div 60 \\ = & 1 & : & 5 & \end{array}$$

(we could also have obtained this ratio by doing a series of smaller divisions, such as ÷10, ÷6)

Now, instead of comparing 50 : 200 and 60 : 300, we can compare 1 : 4 with 1 : 5. The ratio 1 : 5 has one extra part of water to the same amount (1 part) of cordial, so Jack's drink is actually weaker than yours. (You may be able to tell this by looking at the colour, or by tasting it.)

Equivalent ratios are created by multiplying or dividing each part in the ratio by the same number.

Ratios can be simplified by dividing each part in the ratio by a common factor. Dividing by the highest common factor (HCF) will give the ratio in simplest form.

4.9 Ratio

Navigator

Answers page 645

Q1, Q2, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q9, Q11, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2, Q3 Column 4, Q4 Column 4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19

Fluency

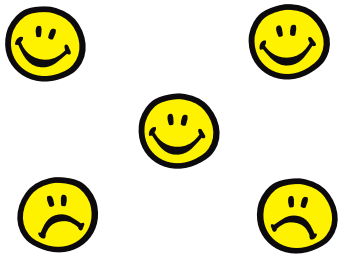
WE23

1 Write a ratio for each of the following.

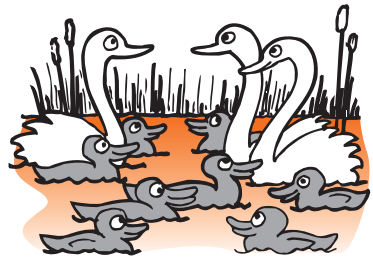
(a) circles to squares

(b) blue circles to red circles

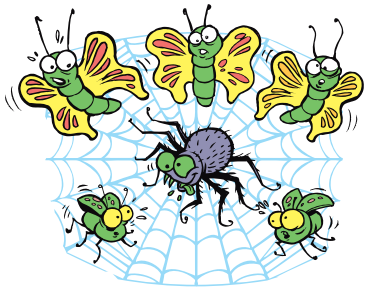
(c) happy faces to sad faces



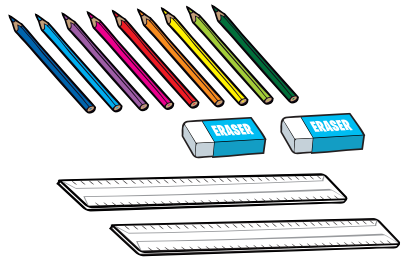
(d) ducks to swans



(e) spider to flies to butterflies

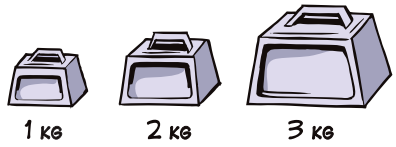


(f) rulers to pencils to erasers



(g) bees to flowers to leaves

(h) large mass to medium mass to small mass



2 Sumi has 3 canaries and 2 mice as pets.

WE24

(a) Write a ratio comparing the number of mice to the number of canaries.

(b) Write a ratio comparing the number of canaries to the total number of pets.

(c) Write the number of canaries as a fraction of the total number of pets.

(d) Write the number of mice as a percentage of the total number of pets.

3 Create pairs of equivalent ratios by multiplying or dividing each part in the ratio by the number given in brackets beside it.

(a) 1 : 3 (×2)	**(b)** 1 : 8 (×2)	**(c)** 1 : 11 (×2)	**(d)** 1 : 33 (×2)
(e) 4 : 5 (×3)	**(f)** 3 : 7 (×4)	**(g)** 6 : 9 (÷3)	**(h)** 8 : 12 (÷4)
(i) 10 : 8 (÷2)	**(j)** 18 : 9 (÷9)	**(k)** 8 : 5 (×7)	**(l)** 12 : 11 (×5)
(m) 48 : 18 (÷6)	**(n)** 20 : 6 (×5)	**(o)** 60 : 54 (÷3)	**(p)** 15 : 13 (×5)

4 Write the following ratios in simplest form, by dividing by the highest common factor (HCF).

(a) 2 : 10	**(b)** 3 : 12	**(c)** 4 : 18	**(d)** 6 : 21
(e) 21 : 14	**(f)** 24 : 15	**(g)** 32 : 20	**(h)** 48 : 36
(i) 54 : 64	**(j)** 56 : 68	**(k)** 72 : 81	**(l)** 38 : 95
(m) 75 : 50	**(n)** 120 : 80	**(o)** 105 : 60	**(p)** 240 : 66

5 There are 14 girls and 9 boys in a Year 7 maths class.

(a) The ratio of boys to girls is:

A 14 : 9 **B** 9 : 14 **C** 14 : 23 **D** 9 : 23

(b) The fraction of the whole class that are girls is:

A $\frac{9}{23}$ **B** $\frac{9}{14}$ **C** $\frac{14}{23}$ **D** $\frac{14}{9}$

Understanding

6 A packet of M&M's contains three colours: 12 red, 7 blue and 16 brown.

(a) Write the ratio of red to blue to brown M&M's.

(b) How many M&M's are in the packet?

(c) What fraction of the M&M's in the packet are red?

(d) What percentage of the M&M's are blue?

7 During a cricket season, Erika's team won 15 matches, drew 2 and lost 7.

(a) Write the won : lost : drew ratio for Erika's team for that season.

(b) What fraction of their games did Erika's team lose?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

8 The students at Hillview High School are voting for a school captain. The results show that for every 4 students who voted for Guy, 5 students voted for Dominic and 3 for Melissa.

(a) Write the ratio of the number who voted for Dominic to the number who voted for Guy, to the number who voted for Melissa.

(b) What fraction of the total vote did Dominic receive?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

9 There must be 1 instructor for every 6 students on a kayaking course.

(a) Write the comparison of students to instructors as a ratio.

(b) If 24 students want to do the course, use an equivalent ratio to determine how many instructors will be required.

(c) If 5 instructors are available, what is the maximum number of students that can do the course?

10 Tom's recipe for making concrete is to add gravel, sand and cement in the ratio 3 : 2 : 1 before adding water. Tom has 4 buckets of cement. Use an equivalent ratio to show how many buckets of sand and gravel he should add to get the correct concrete mixture.

11 'White gold' is commonly made by mixing gold and nickel in the ratio 9 : 1.

(a) Write the amount of nickel as a percentage of the total mixture.

(b) A jeweller wants to make 20 g of white gold. How much nickel will he require?

12 A survey found that 75% of Year 7 students were studying music.

(a) What percentage were not studying music?

(b) Use the two percentages to write a ratio that compares the group studying music to the group not studying music, in simplest form.

Reasoning

13

Which point divides the above line so that the ratio shorter part : longer part is:

(a) 1 : 7 **(b)** 5 : 3 **(c)** 1 : 3 **(d)** 1 : 1

14 There must be at least 1 teacher for every 20 students on a school excursion.

(a) If 68 students are going on an excursion, how many teachers are required? Give reasons for your answer.

(b) Use your answer to **(a)** to write the actual ratio of teachers : students, in simplest form.

15 Chloe made a cordial drink by mixing 30 mL of concentrate with 150 mL of water. Her friend Hannah made a cordial drink by mixing 80 mL of concentrate with 200 mL of water. Hannah said her cordial drink was 'stronger' than Chloe's. Chloe could not tell by looking.

(a) Write the ratio of concentrate : water for each girl's drink, in simplest form.

(b) Use the ratios to state whether Hannah is correct, giving a reason for your answer.

16 Mia has three cats and five dogs as pets.

(a) What fraction of her pets are dogs?

(b) What fraction of her pets are cats?

(c) If Mia adopts another cat, what fraction of her pets are cats now?

(d) After adopting the new cat, Mia moves into a share house with Shona who also has some cats and dogs. If the final ratio of cats to dogs is now 1 : 1, what is the smallest number of cats and dogs that Shona brings to the house?

Open-ended

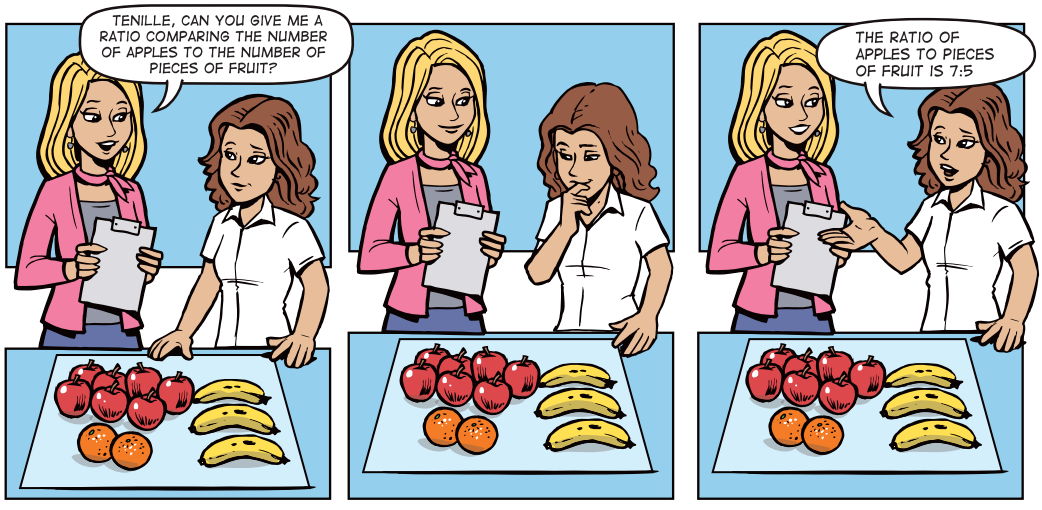
17 **(a)** What is the ratio of girls to boys in your class?

(b) Would the ratio be the same in all the Year 7 classes in your school? Explain your answer by giving examples.

(c) Would the ratio be the same for all school classes in Australia? Give some reasons for your answer.

18 Write three ratios that are equivalent to 4 : 7.

19



What mistake has Tenille made? What is the correct answer?

Mathspace

Our global village

Facts about our global village

There are over 6.9 billion people on Earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

Race

- 0.61 Asian
- $\frac{12}{100}$ European
- 0.08 North American
- 0.05 South American & Caribbean
- 0.13 African
- $\frac{1}{100}$ Oceania (this includes Australia)

Conditions

- 0.43 live without basic sanitation
- $\frac{1}{3}$ don't have access to clean, safe drinking water
- 0.13 are hungry and/or malnourished

Wealth

- 0.06 of the population own 59% of the entire wealth
- $\frac{1}{5}$ of the population have 75% of the income
- 0.53 of the population live on $2.50 or less per day

Education and technology

- 0.14 can't read
- 0.07 have a secondary education
- 0.12 have a computer
- 0.03 have the internet

How rich are you?

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage*.

$1.2 - 2.7 + 3.5 - 1.25 =$

How many people?

Currently, there are approximately 6 900 000 000 people living on Earth.

3 Using the facts you now know about our world, calculate how many people:

(a) don't have access to clean water

(b) struggle to live on $2.50 or less per day

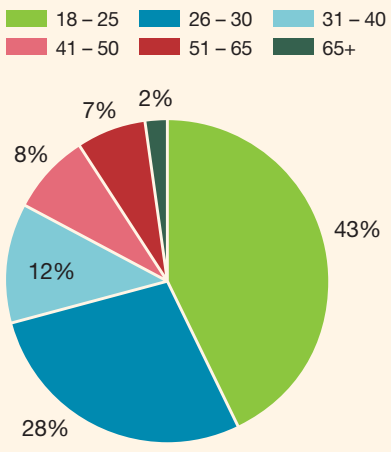
(c) have a computer

(d) don't have a secondary school education.

I want to help

There are many charities out there that are trying to create more equality in the world—Oxfam is one of these. Charities often rely on the generosity of ordinary people. Here is a pie chart of the age groups of Oxfam volunteers.

Volunteers by age group



Oxfam Annual Report 07–08

4 Using the pie chart above, find out the following.

(a) What percentage of volunteers are between 18 and 30?

(b) Why do you think most volunteers are in this age range?

(c) One of the challenges charities face is the need to pay for fundraising and administration costs. If Oxfam spends 24.4% of its income on fundraising and 8.9% on administration costs, what percentage of its income is available to go directly to community-aid projects?

In the future

5 Imagine it's the year 2055. How do you hope the 'global village' percentages have changed? Write down your predictions and ideas.

4.10 Rates

In the previous section, we used ratios to compare two or more amounts of the same type of quantity (e.g. millilitres of water and millilitres of cordial).

A **rate** is a way of comparing two amounts of different types of quantities. They usually involve the word *per* which means 'for each' and have the symbol / or the letter 'p'. A rate is written as the number of the first quantity for every 1 of the second quantity. Some examples of rates are:

If the price of petrol is $1.29/L, you will pay $1.29 for 1 litre of petrol.

Cricketers scoring at 4 runs per over will score an average of 4 runs in 1 over.

The unitary method

If we know how much 1 litre of petrol costs, we can calculate the cost of any other volume of petrol by multiplying by the cost of one litre.

If we know that 50 L of petrol costs $75, we can work out the cost of 4 L, 15 L, 37 L or any other volume of petrol by first calculating how much 1 L costs.

Solving problems by first calculating a number (such as cost) per unit is called the **unitary method**.

Worked Example 25 WE25

(a) If 8 kg of oranges costs $24, find the cost of 6 kg of oranges.

(b) A machine in a soft drink factory produces 210 bottles every hour. How many bottles would be produced in 40 minutes?

Thinking		Working
(a)	**1** Calculate the cost of one 'unit' of the amount (1 kg of oranges).	(a) $24 for 8 kg = $3 for 1 kg (÷ 8)
	2 Multiply the cost/unit by the number of units.	= $18 for 6 kg (× 6)

(b) 1	Calculate the number of items produced in one minute.	(b) 210 ÷ 60 = 3.5 bottles/min
2	Multiply the number produced in one minute by the number of minutes. Round your answer to a sensible value.	3.5 bottles/min × 40 min = 140 bottles in 40 min

Comparing prices

Many supermarket products come in different sizes, which makes it difficult to compare the prices. For example: Which is better value for money, a 500 g box of breakfast cereal for $5.99, or the larger 820 g box for $7.21?

We can use the unitary method to compare the prices. We could divide the prices by the mass of the boxes to find the price per 1 g of the cereal, but, instead, we will compare the price per 100 g, as this is how supermarkets are required to display many of their prices. The price per 100 g (or 100 mL, or 1 kg or 1 L) is called the **unit price**. Unit prices enable shoppers to easily compare sizes and brands to find products that are the best value for money.

To convert the prices to prices per 100 g, we could divide the price by the mass to find the price for 1 g, then multiply by 100. For example: 500 g box: $5.99 ÷ 500 × 100 = $1.20 per 100 g
820 g box: $7.21 ÷ 820 × 100 = $0.88 per 100 g

However, a quicker method is to mentally divide the mass by 100 to find the number of 100 g 'lots'. We can then divide the price by this number; e.g.

500 ÷ 100 = 5
$5.99 ÷ 5 = $1.20 per 100 g

820 ÷ 100 = 8.2
$7.21 ÷ 8.2 = $0.88 per 100 g

Worked Example 26 WE26

Calculate the unit price (price per 100 g) of the following pair of products, and so determine which one is better value for money: A 500 g packet of pasta for $2.42 or a 750 g packet for $3.44.

Thinking	Working
1 Divide the mass of the first product by 100.	500 g ÷ 100 g = 5
2 Divide the price of the first product by your answer to step **1**.	$2.42 ÷ 5 = $0.48/100 g
3 Repeat steps **1** and **2** for the second product.	750 g ÷ 100 g = 7.5 $3.44 ÷ 7.5 = $0.46/100 g
4 State your answer.	The 750 g packet is slightly better value for money (2c less per 100 g).

4.10 Rates

Navigator

Answers page 645

Q1, Q2, Q3, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q19	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q20	Q1 (d)–(f), Q2 (d)–(f), Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q14, Q15, Q17, Q18, Q19, Q20

Fluency

WE25

1 **(a)** If 15 kg of potatoes costs $12, find the cost of 7 kg of potatoes.

(b) A machine in a food canning factory produces 250 cans every hour. How many cans would be produced in 25 minutes?

(c) If 14 pens cost $5.60, find the cost of 3 pens.

(d) Farmer Harry buys 8 cows for $6800. How much would 10 cows have cost him?

(e) If 21 bags of rice weigh 35 kg, how much do 15 bags of rice weigh?

(f) A printing machine prints 70 pages in 8 minutes. How many pages could it print in one hour?

WE26

2 Calculate the unit price (the price per 100 g) of the following pairs of products, and so determine which one is better value for money.

(a) a 600 g jar of jam for $4.09 or a 250 g jar for $2.68

(b) a 500 g packet of pasta for $2.81 or a 750 g packet for $3.45

(c) a 375 g jar of tomato paste for $1.28 or a 500 g jar for $3.25

(d) a 200 g jar of peanut butter for $3.21 or a 375 g jar for $4.93

(e) a 500 mL bottle of soy sauce for $2.71 or a 920 mL bottle for $4.29

(f) a 450 g bag of cat food for $3.78 or a 1 kg bag for $8.79

3 Nazeh has saved $2000 in 4 months. If he continues to save at this rate, how much will he have by the end of the year?

A $6000 **B** $8000 **C** $12 000 **D** $24 000

4 An entertainer receives $450 000 for a season of five performances, each of which lasts $1\frac{1}{2}$ hours. The rate of pay per minute is closest to:

A $60 000 **B** $5000

C $1500 **D** $1000

Understanding

5 A caterer knows that 4 loaves of bread can make enough sandwiches for 25 people. How many loaves will he need to make enough sandwiches for 120 people?

6 **(a)** Which is better value for money, a 280 g jar of jam for $5.15, or a 400 g jar for $6.42?

(b) The 280 g jar goes on special for $4.49. Does this change your answer to **(a)**?

7 A mobile phone carrier charges 75c for 30 seconds. How much will a 4-minute phone call cost?

8 Rajid's cricket team required 85 runs from 12 overs to win the game. At what rate, in runs per over, must they score to win? (Answer to one decimal place.)

9 **(a)** Calculate the price per item in each pack of the following products, and so determine which pack is better value for money.

(i) a packet of 6 muesli bars for $4.83, or a packet of 10 for $7.51

(ii) a 12-pack of cans of soft drink for $14.20, an 18-pack for $20.21, or a 30-pack for $27.51

(iii) a box of 50 tea bags for $3.20, or a box of 200 for $7.99

(b) Describe another way you could compare the prices of the tea bags in part **(a) (iii)**.

10 Lucy has bought a 1 year subscription to *Australian Explorer* magazine. She paid $102 for 12 monthly magazines. The regular price at the newsagent for one copy of the magazine is $9.95. How much will Lucy save:

(a) per magazine **(b)** over the 1 year period?

11 Andrea has a leaky tap in her bathroom. She places a container underneath it and collects 1.2 litres in 8 hours. If Andrea does not fix the tap, use a mental or written method to calculate how much water would be lost in:

(a) 1 day **(b)** 1 week?

12 A 200 g serve of yoghurt contains 6.5 g of fat. Use a mental or written method to calculate how much fat there is in:

(a) a 100 g serve **(b)** a 400 g serve **(c)** a 500 g serve.

13 A 5-minute phone call costs $3.00. If the same call rate is charged, use a mental or written method to calculate the cost of:

(a) a 10-minute call **(b)** a 1-minute call **(c)** a 13-minute call.

Reasoning

14 Charlie's bakery sells hot cross buns in packets of 6. One packet costs $7.20. A customer wants to buy exactly 10 buns. Charlie is happy to take 4 buns from a second packet, but how much should he charge the customer?

15 Imagine that you are standing in a supermarket, comparing the following pairs of products. Do some mental calculations to determine which product is the better value for money. Rounding prices slightly may help to simplify some calculations. Show how you arrived at your answer.

(a) A 1 kg box of washing powder for $5.49, or a 4 kg box for $15.97.

(b) Four 250 mL cartons of orange juice for $1.25 each, or a 1 litre carton for $3.99.

(c) Buying 500 g of ham from the deli counter where the price is $13 per kg, or buying it in a packet for $5.83.

(d) A 400 g jar of honey for $7.99, or a 300 g jar for $6.54.

(e) Give some reasons why people might prefer to buy a product even though it is not the best value for money.

16 A mobile phone carrier charges a rate of 85 cents per minute for pre-paid calls. Calls made on a phone plan are charged at 75 cents per minute, plus a 35 cent connection fee. Which of the two types of charge is cheaper for a 5-minute phone call?

17 During the month of January, the water level in a Northern Territory dam increased by 78 000 kilolitres (1 kilolitre = 1000 litres).

(a) How many kilolitres per day was this? (Round your answer to two decimal places.)

(b) If the dam continued to fill at this rate, use your answer to **(a)** to calculate how much water will have flowed into the dam by the end of the year. (Assume there are 365 days in a year.)

(c) Would it be sensible to use your answer to **(b)** as a prediction for the amount of water that will flow into the dam over the year? Give some reasons for your answer.

18 'Trim and Tasty' cheese comes in three different-sized blocks:

250 g for $4.50
500 g for $7.42
750 g for $9.76

(a) How many times bigger than the 250 g block is the 500 g block?

(b) If the prices of the 500 g block and the 750 g block were in the same proportion to the price of the 250 g block as their sizes, what would you expect their prices to be?

(c) Often (but not always) larger sizes of a product have cheaper unit prices. Suggest a reason why this is the case.

Open-ended

19 List four supermarket products whose unit price would be usefully calculated per 100 mL, or per L.

20 Kumar needs 4.5 kg of rice for the students who are attending his cooking classes. It is available in the following sizes and prices.

500 g for $1.24 1 kg for $2.35 1.5 kg for $3.59 2 kg for $4.52

(a) List at least three different combinations of sizes that would give Kumar 4.5 kg of rice.

(b) Which combination is the cheapest?

Outside the Square Puzzle

Happy numbers

1 Write down your favourite single-digit number.

2 Multiply that number by 9.

3 Multiply this new number by 12 345 679. (Unless your calculator has more than 8 digits in its display you will need to do this by hand to see the full effect.)

4 If you did choose your favourite number, then the answer should make you very happy. Does it work for other single-digit numbers? Can you explain why it works?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Challenge 4

1 What percentage of whole numbers from 6 to 25 inclusive (i.e. including 6 and 25) are exact multiples of 6?

A 5 **B** 20 **C** 25 **D** 30

2 The numbers 5, 8, 7 and 3 are written on cards. Where should they be placed so that the following product has the greatest possible value?

$\square \times 0.\square\square\square$

3 Evaluate $0.1^2 - 0.1^3$.

4 Zinadene has $\frac{3}{4}$ of a dollar and Katrine has $\frac{3}{10}$ of a dollar. Together they have:

A \$0.95 **B** \$1.00 **C** \$1.05 **D** \$1.10

5 $200 \div 2.5 = 80$. Therefore, $20 \div 0.25 =$

A 0.80 **B** 8.0 **C** 80 **D** 800

6 When doing a series of additions using his calculator, Trent noticed that he added 49 095 instead of 49.95. In order to correct his error with a single entry, he should now:

A add 49.95 **B** subtract 49 045.05

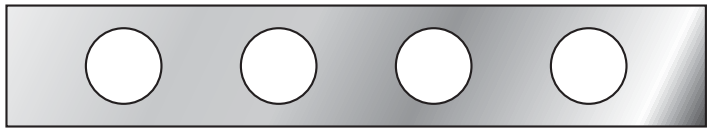
C add 49 045.05 **D** subtract 49 095

7 A decimal with 2 decimal places is multiplied by a decimal with 3 decimal places. Explain with number examples how the solution could have:

(a) 5 decimal places **(b)** 4 decimal places **(c)** 3 decimal places.

8 In Felicity's first six netball games she averaged 7.5 goals per game. If the least number of goals she scored was 4, what is the lowest possible value for the highest number of goals she scored?

9 Four holes are to be drilled along the centre line of a strip of metal so that their centres are 2.25 cm apart. The centres of the two end holes are to be 3.35 cm from their corresponding ends. What is the length of the strip of metal?



10 What is the quotient when 0.01 is divided by 0.002?

11 Mikaela had her salary reduced by 10%. She was later promoted and her salary was increased by 10%. If her original salary was \$30 000, her present salary is:

A \$24 300 **B** \$27 000 **C** \$29 700 **D** \$30 000

12 $\frac{2009}{20.09}$ equals

A 100 **B** 10 **C** 0.1 **D** 0.01

13 A town has 5000 residents, of whom 60% voted in a local council election. The result was that, of those who voted, 37% voted for *P*, 32% for *Q* and 31% for *R*. Under the voting system, *P* was elected. The number of residents who voted for *P* was:

A 1850 **B** 1110 **C** 960 **D** 930

Chapter review 4

D.I.Y. Summary

Key Words

decimal places	part : whole ratio	recurring decimal
decimal point	per cent	round
digit	place value	terminating decimal
equivalent ratios	rate	unit price
part : part ratio	ratio	unitary method

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 4.87 has three _____s and two _____ .

2 When adding or subtracting decimals, it is important to line up the _____s.

3 _____ are created by multiplying each part in the ratio by the same number.

4 _____s compare two or more parts of a whole.

5 A _____ has a finite number of decimal places, a _____ has an infinite number of decimals, in a repeating pattern.

6 _____ literally means 'for every hundred'.

7 Calculating a _____ helps to compare the prices of different sizes and brands of products.

8 Explain the difference between 'round down' and 'round up'.

Fluency

1 Write in expanded fractional form. Ex 4.1

(a) 0.968 **(b)** 5.0702 **(c)** 6.005

2 Write as decimals: Ex 4.1

(a) six tenths, five hundredths, two thousandths and four ten-thousandths

(b) eight hundredths, nine hundred-thousandths and three millionths

3 Write as decimals: Ex 4.1

(a) $4 + \frac{3}{10} + \frac{8}{100} + \frac{6}{10\,000}$ **(b)** $50 + 7 + \frac{3}{100} + \frac{2}{1000} + \frac{9}{100\,000}$

4 Write each of the following in expanded word form. Ex 4.1

(a) 1.8531 **(b)** 0.070 06 **(c)** 61.0009

5 Write the value of the 9 in each decimal as **(i)** a fraction **(ii)** in words. Ex 4.1

(a) 5.091 **(b)** 0.0659 **(c)** 25.291

6 Write < or > in each of the following pairs of numbers to make a true statement. Ex 4.1

(a) 3.0427 _____ 3.0274 **(b)** 0.009 95 _____ 0.01

7 Write in order from smallest to largest. Ex 4.1

(a) 0.6055, 0.5506, 0.607 **(b)** 0.071, 0.701, 0.71

8 Copy this number line and show the positions of the following numbers: 0.25, 1.45, 0.09, 2.05. Ex.4.1

9 Round each of the following to the number of decimal places shown in the brackets. Ex.4.2

(a) 4.398 (1) **(b)** 34.6503 (2) **(c)** 23.098 (1) **(d)** 102.4728 (2)

10 Round the following to the nearest 5 cents. Ex.4.2

(a) \$41.21 **(b)** \$10.03 **(c)** \$79.97 **(d)** \$98.98

11 Write each of the following as a single fraction or mixed number in simplest form. Ex.4.3

(a) 3.9 **(b)** 0.62 **(c)** 2.45 **(d)** 0.0018

12 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. Ex.4.3

(a) $\frac{4}{5}$ **(b)** $\frac{19}{20}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{10}{12}$

13 Calculate: Ex.4.4

(a) 23.6 + 4.79 **(b)** 3.768 + 10.9205 + 0.77 **(c)** 6 + 0.408 + 35.025

(d) 10.367 – 2.65 **(e)** 2.082 – 0.394 **(f)** 7 – 2.2198

14 Find the following products. Ex.4.5

(a) 3.376×4 **(b)** 2.99×35 **(c)** 6.09×33

(d) 23.6×2000 **(e)** 0.548×300 **(f)** 0.069×50

(g) 0.6×0.9 **(h)** 0.009×0.04 **(i)** 5.2×3.8

15 Calculate the following. Round answers to three decimal places where necessary. Ex.4.6

(a) $10.32 \div 4$ **(b)** $7.028 \div 7$ **(c)** $3.75 \div 8$

(d) $480.6 \div 2000$ **(e)** $8.23 \div 500$ **(f)** $54.63 \div 300$

(g) $0.48 \div 0.04$ **(h)** $1.2 \div 0.5$ **(i)** $11.9 \div 0.002$

16 Write the following fractions as percentages. Ex.4.7

(a) $\frac{7}{10}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{2}{3}$

17 Convert the following decimals to percentages. Ex.4.7

(a) 0.38 **(b)** 0.96 **(c)** 3.55 **(d)** 0.09 **(e)** 3.234 **(f)** 0.5432

18 Calculate the following percentages. Ex.4.8

(a) 40% of \$60 **(b)** 15% of 85 L **(c)** 83% of 200 m **(d)** 12.5% of \$600

19 Write the following results as percentages, rounding answers to one decimal place if necessary. Ex.4.8

(a) 13 out of 20 **(b)** 35 out of 70

(c) 14 out of 40 **(d)** 28 out of 30

20 There are 56 girls and 64 boys in Year 7. Ex.4.9

(a) Write the ratio boys : girls in simplest form.

(b) Write the number of girls as a fraction of the total number of students, in simplest form.

(c) Write the number of boys as a percentage of the total number of students, to the nearest whole number.

21 **(a)** A 10 kg box of tomatoes costs $23.50. Use the unitary method to calculate the cost of a 3 kg box.

Ex 4.10

(b) If a 250 g block of cheese costs $7.85, find the unit price (the price per 100 g).

Understanding

22 Write the populations of the following countries as whole numbers.

(a) China: 1.326 billion

(b) India: 1.14 billion

(c) Indonesia: 228.2 million

(d) New Zealand: 4.27 million

23 An employer wants to buy each of her staff a uniform. If each uniform costs $75.59, how much will the uniforms cost:

(a) for 40 staff

(b) for 100 staff?

24 Kayla bought the following items while out shopping: A $30 T-shirt discounted by 10%, and a $60 pair of jeans discounted by 40%.

(a) Calculate the dollar value of each of the discounts.

(b) Subtract the discount amount to find how much Kayla paid for each item.

25 **(a)** Which is better value for money: a 250 g jar of honey for $2.98, or a 400 g jar for $4.23?

(b) The 250 g jar goes on special for $1.98. Does this change your answer to **(a)**?

(c) Give one or two reasons why people might prefer to buy a certain size or product, even though it may not have the cheapest unit price.

26 A salad dressing is made by mixing oil and vinegar in the ratio 3 : 1.

(a) How much oil should be mixed with 25 mL of vinegar?

(b) Helen mixes 60 mL of oil with 40 mL of vinegar. Is this an equivalent ratio to the one given above? Demonstrate your answer by writing it in simplest form.

27 In a springboard diving competition, Claudia needed a score of 52 or more to take first place. For her final dive she chose a dive with a degree of difficulty of 2.7. The scores she received from the judges were 7.0, 8.0, 7.5, 8.5, 7.5, 8.0 and 8.0.

(a) Remove the highest and lowest scores and calculate the average of the remaining five scores (add them up and then divide by 5).

(b) Calculate the 'three-judge total' by multiplying this average by 3.

(c) Calculate her final score by multiplying the 'three-judge total' by the degree of difficulty. Did she do well enough to win?

Reasoning

28 Tim buys 6 individual cans of drink for $0.87 per can.

(a) If he pays in cash, find the total cost of the cans.

(b) A pack of 6 cans costs $4.80. How much could Tim have saved by buying the 6 pack?

(c) What is the individual cost of the cans in the 6 pack?

(d) Tim needs to buy 40 cans for a party. What is the cheapest way to buy them?

29 There must be at least one supervisor for every 12 children at a childcare facility. If there are 54 children booked in for a day, how many supervisors will be needed? Give a reason for your answer.

30 Sally is shopping for shoes. She has found the pair she wants in two different stores. Store A has them priced at $65, with a 'take 25% off' tag on them. Store B has them priced at $75, but the store is having a '30% off everything' sale. Where should Sally buy her shoes? Explain your choice by showing the relevant calculations.

NAPLAN practice 4

Numeracy: Non-calculator

1 Which number is greater than 0.06?

A 0.0069 **B** 0.007 **C** 0.05 **D** 0.1

2 Geraldine buys 400 snack bars for the school camp. If each bar costs $0.64, how much would Geraldine pay for the 400 bars?

A $25.60 **B** $256 **C** $464 **D** $25 600

3 What is the answer to 5.4 ÷ 0.9?

A 0.06 **B** 0.6 **C** 6.6 **D** 6

4 20% of a class have the flu. The class has 25 students. How many have the flu?

A 4 **B** 5 **C** 10 **D** 20

Numeracy: Calculator allowed

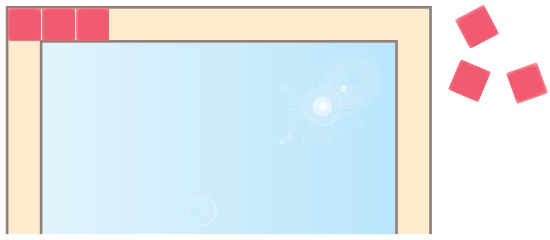
5 A pack of 10 lollipops costs $4.60.
A pack of 6 costs $3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

6 The price of grapes at the supermarket is $14 per kg. If Jack paid $3.50 for his grapes, how many grams did he buy?

A 0.25 g **B** 25 g **C** 100 g **D** 250 g

7 Lucy is adding a row of square tiles across the top of a mirror. Each tile is 4.5 cm wide. The mirror is 56 cm wide. Lucy wants to tile right across the top, with no gaps between the tiles. What calculation should she do to work out how many tiles she needs?



A 4.5 ÷ 56 **B** 5.6 ÷ 4.5 **C** 56 ÷ 4.5 **D** 4.5 × 56

8 The material for the curtains in the classroom costs $21.30 per metre. 8.4 m is needed. How much will the material cost?

Mixed review B

Fluency

1 Write the following fractions in simplest form. Ex.3.2

(a) $\frac{12}{40}$ **(b)** $\frac{15}{50}$ **(c)** $\frac{48}{64}$ **(d)** $\frac{20}{65}$

2 Write a positive or a negative integer to describe the following. Ex.2.4

(a) a withdrawal from a bank account of $74 **(b)** a weight gain of 3 kg

(c) 215 m below sea level **(d)** an increase in length of 8 m

3 For each of the following numbers: Ex.2.3

(i) use a factor tree to find the prime factors

(ii) write the number as a product of its prime factors, using index notation.

(a) 76 **(b)** 108 **(c)** 250

4 Calculate approximate answers to the following by rounding to a convenient multiple of 5 or 10. Ex.1.4

(a) 238×21 **(b)** 107×31 **(c)** $478 \div 39$ **(d)** $604 \div 153$

5 For each pair of fractions, determine which one is larger, then write a > or < sign between them. Ex.3.3

(a) $\frac{2}{3}$ _____ $\frac{2}{5}$ **(b)** $\frac{5}{12}$ _____ $\frac{5}{9}$ **(c)** $\frac{11}{15}$ _____ $\frac{3}{4}$

6 Find: Ex.4.5, 4.6

(a) 0.45×0.6 **(b)** $45.9 \div 3$ **(c)** $43.5 \div 0.1$

7 Calculate the following by first writing a single sign between the integers. Ex.2.7

(a) $-1 + (-5)$ **(b)** $3 - (-11)$ **(c)** $-6 - (+4)$

(d) $9 + (-17)$ **(e)** $-8 - (-14)$ **(f)** $21 - (+30)$

8 Write each of the following as a fraction in simplest form. Ex.4.3

(a) 0.32 **(b)** 1.24 **(c)** 3.237

9 Write the following in index form. Ex.1.2

(a) $7 \times 7 \times 7 \times 7$ **(b)** 4 squared

(c) $2 \times 2 \times 3 \times 3 \times 3$ **(d)** 11 cubed

10 Find the lowest common multiples of the following numbers. Ex.2.1

(a) 5 and 6 **(b)** 3 and 6 **(c)** 2, 3 and 5

11 Write < or > between the following pairs of numbers. Ex.2.4

(a) -5 _____ 3 **(b)** -4 _____ -11 **(c)** -7 _____ 0

12 Find the HCF of the following numbers. Ex.2.1

(a) 12 and 30 **(b)** 16 and 40 **(c)** 24 and 60

13 Calculate the following. Write your answers in simplest form. Ex.3.4

(a) $\frac{1}{5} + \frac{3}{4}$ **(b)** $1\frac{2}{3} + 2\frac{3}{8}$ **(c)** $\frac{5}{6} - \frac{3}{8}$ **(d)** $3\frac{1}{4} - 2\frac{1}{2}$

Understanding

14 Use an appropriate mental strategy to calculate the following.

(a) 29×7 **(b)** 44×11 **(c)** 13×15

(d) $260 \div 4$ **(e)** $2 \times 4 \times 7 \times 5$ **(f)** $440 \div 8$

15 Erin calculates that she needs 9 lots of 0.35 m of ribbon for her textiles project. She wants to round up to the nearest metre so that she makes sure she has enough.

(a) Calculate how many metres she should buy.

(b) If the ribbon costs \$0.75 per m, how much will she pay?

16 Write **(i)** a fraction in simplest form and **(ii)** a percentage to represent the following situations.

(a) 12 black jelly beans in a bag of 40

(b) 120 Year 7 students in a school of 520 students

17 A 250 g serve of tinned soup contains 7 g of fat. Calculate the amount of fat contained in:

(a) a 500 g serve **(b)** a 50 g serve **(c)** a 150 g serve.

18 7 pizzas were each sliced into 6 equal pieces.

(a) If $4\frac{5}{6}$ pizzas were eaten, how many slices is that?

(b) How many slices are left over?

19 Camilla is on an athlete's diet where $\frac{2}{3}$ of her daily energy must come from carbohydrates, $\frac{1}{4}$ from protein, and the rest from fats. Her daily energy allowance is 4800 kJ.

(a) Calculate how many kJ of carbohydrates she must have every day.

(b) Calculate how many kJ of protein she must have every day.

(c) What fraction of her daily kJ intake is fat?

20 Explain why $3^2 = 9$, but $3 \times 2 = 6$.

Reasoning

21 A lighting display in the city has a set of blue lights that flash every 6 minutes, and a set of red lights that flash every 8 minutes. How many times an hour will the lights flash together?

22 There is a large block of chocolate sitting in the Williams family pantry. Chloe breaks off and eats $\frac{1}{6}$, and returns the rest. Later, her brother Eamon breaks off and eats $\frac{2}{3}$ of the rest.

(a) What fraction of the original block is left?

(b) If the original block contained 54 squares of chocolate, who ate more, Chloe or Eamon?

23 Shania surveyed 130 Year 7 students to find out which charity they wanted to fundraise for. Shania reported back to the Student Council that 65% of the Year 7s wanted to support the Red Cross.

(a) What is 65% of 130?

(b) Considering that we are working with numbers of students, what is 'strange' about your answer to **(a)**?

(c) What might have been the actual number of students who said they wanted to support the Red Cross? Explain how Shania obtained her figure of 65%.

24 How many whole numbers lie on the number line between $\frac{4}{3}$ and $\frac{38}{4}$?

25 Jeffery has 46 footy cards in his collection. His little brother Joseph has half as many in his collection, whereas his friend Shahin's collection is twice the size of Jeffery's. How many more cards has Shahin than Joseph?