
5

Algebra

5

Algebra rules my kitchen! Can algebra help you cook the perfect pancake?

A mathematics lecturer at an English university has published a formula she believes will create the perfect pancake. Dr Ruth Fairclough included every detail that could affect the quality of the pancake in her formula. She says that the most important factor, apart from the batter's recipe, is the frying pan's temperature.

The formula is:

$$P = 100 - \frac{10L - 7F + C(k - C) + T(m - T)}{S - E}$$

where
P = the pancake score
L = number of lumps in the batter
C = batter consistency
F = flipping score
k = ideal batter consistency
T = temperature of the pan
m = ideal temperature of the pan
S = length of time the batter stands before cooking
E = length of time the cooked pancake sits before being eaten.

The closer you get to 100, the more delicious the pancake. Yum!

Forum

Would you use this formula to make pancakes at home?

Would a chef in a restaurant use this formula?

Where would you use a formula around the home or in the garden? Who would use formulas in their work?

Why learn this?

Algebra is a mathematical language. It uses letters and symbols to communicate general rules found from patterns and to solve problems. Algebra can help engineers to calculate stress forces on bridges, architects to design environmentally friendly buildings, nurses to calculate the correct doses of medicine and accountants to calculate how much tax needs to be paid.

After completing this chapter you will be able to:

- write and simplify algebraic expressions using pronumerals
- use a flowchart to describe algebraic rules
- generate tables of values using rules given in words or algebraic formulas
- change rules expressed in words to algebraic formulas using pronumerals
- substitute values into formulas
- use algebra to solve problems
- use tables to plot points on a Cartesian plane
- use a rule to generate linear graphs
- interpret point and line graphs.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R5.1

1 Find the value of the $\square$ in each of the following number sentences. If the $\square$ appears more than once in a number sentence, the same number must be used.

(a) $3 + 7 = \square + 5$ **(b)** $5 - 1 = 2 \times \square$ **(c)** $3 + \square + 4 = \square \times 8$

(d) $5 - 4 + \square = 5$ **(e)** $10 \div \square = 5$ **(f)** $\square \times \square = 16$

Worksheet R5.2

2 **(a)** Multiply each of the following by 11 and subtract 3 from your answer.

(i) 4 **(ii)** 10 **(iii)** 200

(b) Multiply each of the following by itself and then take 6 from your answer.

(i) 4 **(ii)** 10 **(iii)** 8

Worksheet R5.3

3 Write the following sentences using only numbers, the four operation signs (+, −, ×, ÷) and the equals sign (=).

(a) The sum of eleven and seventeen is twenty-eight.

(b) The difference between nine and seven is two.

(c) The product of three and four is twelve.

(d) The quotient of sixteen and eight is two.

Worksheet R5.4

4 Write the following sentences using only numbers, the four operation signs and brackets. Then, use the order of operations to find the answer.

(a) Take the number six, multiply it by two, and then subtract four from it.

(b) Add five to the number ten, then divide what you get by five.

(c) Multiply the sum of six and five by three.

(d) The difference between twenty-three and twenty-one is multiplied by twelve.

Worksheet R5.5

5 Decode the following by writing the letters contained in each grid square.

(a) The fastest living creature
E2 E4 A1 E4 B3 A1 D3 C2 E4
A3 A4 A2 C4 D2 C2

(b) The largest land carnivore
E3 D2 D4 D3 A4 E3 B4 E4 A4 A1

4	a	b	c	d	e
3	f	g	h	i	k
2	l	m	n	o	p
1	r	s	t	u	y
	A	B	C	D	E

Key Words

axes	define	like terms	point graph	table of values
Cartesian plane	equation	line graph	pronumeral	terms
coefficient	evaluate	linear graph	quadrant	unknown
constant	expression	ordered pair	relationship	unlike terms
coordinates	formula	origin	substitute	variable

Pronumerals and variables

5.1

The meaning of 'x' (or 'n' or …)

Algebra is a language used by mathematicians to communicate mathematical ideas and information clearly. To become a mathematician, you need to learn the language in the same way that you learn any other language. You will begin this study of algebra by learning words and algebraic conventions (rules that everyone agrees to follow) that will help you to read, speak, write and understand mathematics.

Algebra is used to write general rules to describe patterns that we find with numbers and solve problems using these rules.

The word 'algebra' comes from *al-jabr*, an Arabic word meaning 'to put back together'.

Writing algebra

- **Pronumerals** are letters or symbols we use for numbers we don't know. Pronumerals represent (stand for) a number the same way that pronouns 'stand for' a noun.
- An **unknown** is the actual number that the pronumeral represents.
- A **variable** describes the unknown number if its value can change.

Using algebra, we can use x as a pronumeral to represent an unknown number of lollies in a packet. If the number of lollies in the packet can vary, x is called a variable.

If we are given 3 extra lollies, we now have $x + 3$ lollies.

If we buy another identical packet of lollies, we have $x + x$, or $2 \times x$ lollies.

If we have 2 identical packets and 3 extra lollies, we have $2 \times x + 3$ lollies in total.

When we have different variables, we need to use different pronumerals. For example, if we represent the number of boys in a class with the pronumeral b, we need to use a different pronumeral, such as g, to represent the number of girls in the class. The total number of students in the class is the sum of the two unknowns and can be written as $b + g$. Any two letters can be used to represent the number of boys and girls, so the number of students in the class could be written as $x + y$ or $p + r$.

Algebraic conventions

Mathematicians like to write algebra as simply as possible, so they have decided that it is okay to leave out the '×' sign between a number and a pronumeral and between different pronumerals. That means we can write $2 \times n$ as $2n$ and $x \times y$ as xy. It has also been agreed that a '÷' sign can be replaced by a fraction bar, so $n \div 2$ is written as $\frac{n}{2}$. Decisions like this are called algebraic conventions. They ensure that all mathematicians use the same mathematical language. Some conventions that we use in algebra are listed below.

- Leave out the '×' between a number and a pronumeral and between different pronumerals (e.g. $2 \times n$ is written as $2n$ and $x \times y$ as xy).
- Replace ÷ with a fraction bar (e.g. $n \div 2$ is written as $\frac{n}{2}$).
- Numbers are always written in front of the pronumeral (e.g. $x \times 2$ is written as $2x$).
- When multiplying more than one pronumeral, write the letters in alphabetical order (e.g. $f \times a \times c \times e$ is written as $acef$).
- Brackets show that addition or subtraction needs to be done before multiplication or division. They change the order of operations.
- Brackets can be left out when using a fraction bar; e.g. $(a + b) \div 2$ can be written as $\frac{a+b}{2}$.
- When a pronumeral is raised to a power, we use the same notation as we do for numbers (e.g. $2 \times 2 \times 2 \times 2 = 2^4$ so we write $c \times c \times c \times c = c^4$ and p squared as p^2).
- Numbers are usually written last in an expression (e.g. $6a + 7$).

When working with variables, the same words are used to describe the four operations as when working with numbers:

+ add, sum, total	– subtract, less than, minus
× multiply, product, lots of	÷ divide, quotient, share equally

The order of operations, including the use of brackets, is also the same.

Worked Example 1

WE1

Write the following situations using algebra.

(a) Simon has x fish in his aquarium. He buys 10 more. How many fish does he have now?

(b) There are r cards in one pack. How many cards are there in three identical packs?

(c) Sonia has n dollars in her bank account. She withdraws $100. How much money is left in the account?

(d) There are d biscuits in a packet. Half of them are eaten. How many are left?

(e) A kitchen cupboard contains x plates and w bowls. What is the total number of plates and bowls in the cupboard?

Thinking	Working
(a) 'More' means add onto the unknown amount (add 10 to x).	(a) Number of fish now $= x + 10$.
(b) Multiplication is used to find 'lots of' the unknown amount (multiply r by 3).	(b) Number of cards in 3 packs $= 3r$.

(c) 'Withdraw' means subtract from the unknown amount (subtract 100 from n).	(c) Amount of money left in Sonia's account $= \$(n - 100)$.
(d) 'Half' means divide the unknown amount by 2 (divide d by 2).	(d) Number of biscuits left $= \frac{d}{2}$.
(e) 'Total' means add all the unknown amounts together (add x and w).	(e) Number of plates and bowls $= x + w$.

Worked Example 2

WE2

Write each of the following using algebra.

(a) The product of e, f and 7.

(b) c is divided by 4, then 1 is subtracted.

(c) The sum of x and 5 is multiplied by 3.

Thinking	Working
(a) 'Product' means multiply. Write the number first, then the pronumerals in alphabetical order with no multiplication signs between them.	(a) $7ef$
(b) Division is done first, then subtraction. Use a fraction bar to show division.	(b) $\frac{c}{4} - 1$
(c) Addition is done first, then multiplication. Brackets are needed to show this order of operations.	(c) $3(x + 5)$

5.1 Pronumerals and variables

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

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Fluency

1 Write the following situations using algebra.

WE1

(a) There are p books in Jacob's locker. How many are left after Jacob takes out three for class?

(b) There are k people in the line for concert tickets. Nine more people join the end of the line. How many people are lined up now?

(c) There are v chocolates in one box. How many chocolates are there in 12 identical boxes?

(d) A group of four people won a prize of a dollars, which they shared equally. How much did each person receive?

(e) Kyle has d footy cards in his collection, while Khalid has e cards in his. If they combine their collections, how many cards do they have altogether?

WE2

2 Write each of the following using algebra.

(a) The sum of x and 2.

(b) The product of 4 and y.

(c) t is subtracted from 5.

(d) The quotient when k is divided by 7.

(e) The product of h, 6 and g.

(f) d is multiplied by 10, then 8 is added.

(g) 8 is added to d, and the result is multiplied by 10.

(h) The sum of y and 9 is divided by 4.

(i) 3 less than 5 lots of e.

(j) The sum of r, s and t is divided by 3, then 11 is added.

(k) y is multiplied by itself, then 20 is added.

(l) x is squared, then multiplied by 8.

3 Nadine has 7 bags with x lollies in each bag, plus 9 extra lollies. The total number of lollies Nadine has altogether can be written using algebra as:

A $x + 7 + 9$ B $7x + 9$ C $9x + 7$ D $x^7 + 9$

4 The sum of z and 6 is divided by 2. When written using algebra it is:

A $z + 3$ B $\frac{z}{2} + 6$ C $z + 6 \div 2$ D $\frac{z + 6}{2}$

Understanding

5 Write each of the following using algebra. Use brackets where necessary.

(a) 1 is added to a number, n, and the result is multiplied by 5.

(b) 2 is added to a number, d, and the result is divided by 9.

(c) The sum of two different numbers, a and b, is multiplied by 12.

(d) 3 is subtracted from the product of two different numbers, m and n.

6 There are x people standing in line at the post office. Half of them leave, thinking they will come back later.

(a) How many people are in the line now?

(b) Three other people walk in and join the line. How many people are standing in the line now?

7 (a) What is the cost of d metres of timber, if it costs \$6 per metre?

(b) What is the cost of t kilograms of oranges, if they cost \$2.50 per kilogram?

8 What is the cost of hiring the community hall for x hours, if the rate is \$55 per hour, plus a one-off charge of \$300?

9 What is the cost of buying b movie tickets, at v dollars per ticket?

10 The cost of 1 desk is t dollars, and the cost of 1 chair is s dollars. 12 new desks and 24 new chairs are bought for a classroom.

(a) What is the total cost of the desks?

(b) What is the total cost of the chairs?

(c) What is the total cost of the furniture?

Reasoning

11 A purse contains n 5-cent coins and r 10-cent coins. There is no other money in it. Use algebra to write:

(a) the total value in cents of the 5-cent coins in the purse

(b) the total value in cents of the 10-cent coins in the purse

(c) the total value in cents of the money in the purse.

12 Emma makes and sells stationery gift packs. Her small pack has m pens, n pencils, q note pads and 1 eraser.

Her large pack has twice as many of each item. Use algebra to write:

(a) the total number of items in the small pack

(b) the total number of items in the large pack.

(c) Emma needs to make up 3 small and 2 large packs. Use algebra to write the number of pens that she will need altogether.

13 (a) The length of a rectangle is 5 cm longer than its width. If w represents the width of the rectangle:

(i) write the length of the rectangle using w as the variable

(ii) draw a rectangle and label each side using w as the variable.

(b) Write, in terms of w, the total distance around the rectangle (the perimeter).

Open-ended

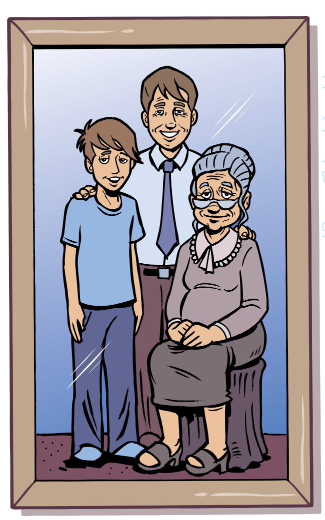
14 (a) Draw a diagram that could be represented by $3x + 5$. (Hint: Look at the 'lolly bag' diagrams at the start of this section.)

(b) What kind of diagram could you draw to show $\frac{x}{4}$? Draw a diagram, adding any necessary descriptions or labels.

15 Rory's grandma says she is twice as old as the combined ages of Rory and his dad. Rory is r years old and his dad is d years old.

(a) Write Grandma's age using algebra.

(b) Write some possible ages for Rory, his dad and his grandma, if Rory is in primary school.



5.2 Terms, expressions and equations

The language of algebra

Here are some more mathematical words that you need to know when using algebra.

Constants

A number by itself is called a **constant**.

Here are some examples of constants: 2, 9, -37, $\frac{2}{7}$, 4.6

Terms

Terms have one or more pronumerals, or may be just a number. mn means $m \times n$ and is the same as $n \times m$ or nm. The pronumerals are often multiplied by a number that is written first.

Here are some examples of terms: $5a$, $7q$, $3pr$, z, abc, 4, $\frac{3}{11}$, $-2.32x$

Coefficients

The number written in front of a pronumeral is called the **coefficient**. The sign of the number is included. For example, in $9x$, the coefficient of x is 9. In $9x - 3y$, the coefficient of y is -3.

Expressions

Expressions are made by adding terms. The expression $5a + 7q - 12$ has three terms.

Expressions can also consist of just one term, so $5a$ or 4 could each be called an expression.

In section 5.1, we wrote $x + 3$, $2x$ and $2x + 3$. These are called algebraic expressions. Here are some more examples of expressions: $5a + 7q$, $5a + 7q - 12$, $9xy - z$, $2(p - 7)$.

Equations

Equations contain an equals sign. They are made by writing two expressions equal to each other.

Equations have a left-hand side (LHS) and a right-hand side (RHS) on either side of the equals sign.

Here are some examples of equations: $9xy + z = 17$, $y = 6h$, $2(p - 7) = p + 10$

$3xy - 7z^2 + 5 = 16 + x$

- This is an equation because it has an equals sign.
- x, y and z are pronumerals that represent variables.
- 3 is the coefficient of xy and -7 is the coefficient of z^2.
- $3xy - 7z^2 + 5$ and $16 + x$ are expressions.
- $3xy$, $-7z^2$, 5, 16 and x are all terms.
- 5 and 16 are constants.

Representing situations using algebra

When we use algebra to describe practical situations, it is very important to **define** all the variables we are going to use so we know what they represent. For example, we may not know the cost per kilogram of apples and bananas, so they are the variables we need to define. We do know that we purchased 3 kg of apples and 2 kg of bananas and spent \$15.20. If we define the cost of apples/kg as x (\$) and the cost of bananas/kg as y (\$), then we can write:

- $3x$ as the cost of apples (\$)
- $2y$ as the cost of bananas (\$)
- $3x + 2y = 15.2$ as the equation
- $(3x + 2y)$ and 15.2 as the total cost of the fruit (\$).

In this situation, we have examples of:

a constant:	15.2	expressions:	$3x + 2y$, $3x$, $2y$, 15.2
pronumerals:	x, y	an equation:	$3x + 2y = 15.2$
terms:	$3x$, $2y$, 15.2		

Worked Example 3 WE3

For $3p + 4q - 8pq = 7 + 2r$

(a) state whether it is an equation or an expression

(b) identify the coefficient of pq

(c) list all the variables

(d) write down any constants

(e) list all the terms used.

Thinking	Working
(a) Is there an equals sign? If yes, then it is an equation.	(a) It is an equation.
(b) Look for the number in front of pq. Include the sign.	(b) Coefficient of pq is -8.
(c) Look for all the different letters used and list them. (Note that pq is a product of p and q.)	(c) p, q and r are the variables.
(d) Look for a number by itself.	(d) The constant is 7.
(e) Look for all the parts of the equation separated by addition or subtraction.	(e) $3p$, $4q$, $-8pq$, 7 and $2r$ are the terms.

Worked Example 4 WE4

For the following situation:

(a) identify the variables

(b) define each variable using the given pronumerals

(c) write an equation.

A farmer has a number of sheep, x, and a number of ducks, y, in a paddock. There are five times as many sheep as ducks.

Thinking	Working
(a) Identify the variables.	(a) The variables are the number of sheep and the number of ducks.
(b) Define the variables.	(b) Let $x =$ the number of sheep. Let $y =$ the number of ducks.
(c) Write the equation. (Multiply y by 5 to get x.)	(c) $x = 5y$

- An algebraic expression may have one term or it may be made up of two or more terms that are added or subtracted.
- An equation uses an equals sign to show that one expression has the same value as another expression.

5.2 Terms, expressions and equations

Navigator

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Q1, Q2, Q3, Q4 (a)–(d), Q5 (a) & (b), Q7	Q1, Q2, Q3, Q4 (b)–(e), Q5 (a)–(c), Q6, Q7	Q1, Q2, Q3, Q4 (c)–(f), Q5, Q6, Q7, Q8

Fluency

WE3

1 For each of the following:

(i) state whether it is an equation or an expression

(ii) identify the coefficient of x

(iii) list all the variables

(iv) write down any constants

(v) list all the terms used.

(a) $2x + 3y - 7 + 4x^2$

(b) $32 - 4x = 3xy + 4z$

(c) $2xy - 3x^2 = 9 - 23x$

(d) $6 - x + 8y - 4xy$

WE4

2 For each of the following situations:

(i) identify the variables

(ii) define each variable using the given pronumerals

(iii) write an equation.

(a) The number of boys, x, in the class is three times the number of girls, y.

(b) Fred is g years old. This is one less than three times James's age. James is h years old.

(c) At the market, an apple costs \$$m$ and a pear costs \$$n$. Two apples cost as much as three pears.

(d) A kilogram of potatoes costs \$$a$ and a watermelon costs \$$b$. Three kilograms of potatoes and a watermelon cost \$5.20.

(e) The distance, r, to Ranko's house is 5 km more than the distance, s, to Skye's house.

(f) You have v plants and w pavers. To make a path around a row of plants, you need to have twice as many pavers as you have plants plus 6 extra pavers.

3 Answer TRUE or FALSE for each of these statements.

(a) $6y$ is a term
(b) $7y - 9$ is an equation
(c) $y = 7x$ is a term
(d) ab is a term
(e) $r = 5t - 9$ is an expression
(f) $cd + 4$ is an expression
(g) $w = s - 5b$ is an equation
(h) $6g$ is an equation
(i) $rs = sr$
(j) $7 - 6xzy = 7 - 6yxz$

4 Which expression below matches the instruction given in each case?

(a) Choose a number and add any other number to it.

A $6 + a$ B $u + v$ C $c + 9$ D $a + b + 1$

(b) Choose a number and multiply it by any other number.

A $a - b$ B $7a$ C $4ab$ D ab

(c) Choose a number and multiply it by three, then subtract any other number.

A $3a - 1$ B $3n - y$ C $3m - 3$ D $3t$

(d) Choose a number and multiply it by two, add any other number, then subtract thirteen.

A $2f + 2d - 13$ B $2mn - 13$ C $2w + g - 13$ D $a + b - 13$

(e) Choose a number and add it to any other number, then multiply the answer by nine.

A $9xy$ B $9x + y$ C $9(x + y)$ D $9 + xy$

(f) Choose a number and multiply it by ten, choose any other number and multiply it by four, then add the two answers together.

A $w + v + 10 + 4$ B $14(v + w)$ C $10v + 4w$ D $40vw$

Understanding

5 (a) A bottle of mass 120 g contains 25 tablets. The total mass of the bottle and tablets is 195 g.

(i) If the variable is the mass of a tablet, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find the mass of a tablet.

(b) Half Mei Lee's age is the age she was 8 years ago.

(i) If the variable is Mei Lee's age, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find Mei Lee's age.

(c) In 10 years' time, Beth will be half her dad's age now.

(i) If one variable is Beth's age and the other other variable is her dad's age, use two pronumerals to represent these variables.

(ii) Write an equation you would use to find Beth's age now.

(d) In a ball game, a goal is worth 8 points and a penalty goal is worth 1 point. At a match, one team scored 100 points.

(i) Define the variables and represent them with pronumerals.

(ii) Using these pronumerals, write an equation you would use to show the team's score.

Reasoning

6 A family on holiday travelled a certain distance on the first day. The next day they were able to travel twice as far as on the first day. On the third day, they travelled 480 km further than on the first day.

(a) Define the variable.

(b) Write an expression to show the distance travelled on the second day.

(c) Write an expression to show the distance travelled on the third day.

(d) If the distance travelled on the third day was three times the distance they travelled on the first day, write an equation to show this situation.

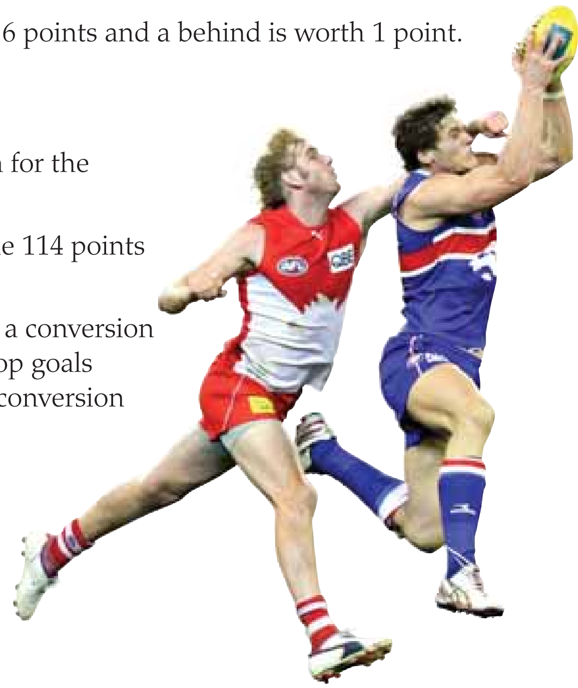
Open-ended

7 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. A team has scored 114 points.

(a) Define any variables you need to use.

(b) Using these variables, write an equation for the number of points scored.

(c) Find at least three different ways that the 114 points could have been scored.



8 In Rugby Union, a try is worth 5 points and a conversion goal is worth 2 points. Penalty goals and drop goals are each worth 3 points. To be able to get a conversion goal you must first score a try.

(a) If the Waratahs win a match against the Brumbies by 23 points to 12, write down two different ways these points could have been scored.

(b) If P is the total number of points scored, t is the number of tries, g is the number of 2-point goals and p is the number of 3-point goals, write an equation to find the total number of points scored.

Outside the Square Puzzle

We all scream for ice-cream

Summer had arrived and four industrious people decided to sell ice-creams at their local beach.

In total, there were four flavours and each person had ice-cream in 2 different flavours.

- Mr Chocolate had vanilla.
- Mr Vanilla did not have mint.
- Mr Mint had strawberry but not chocolate, whereas Mr Strawberry did not have vanilla.
- One person with mint also had chocolate.
- One person with vanilla also had strawberry.
- One of the persons with chocolate had no mint.
- Neither of the persons with vanilla had chocolate.
- No person had two ice-creams of the same flavour, no two persons had the same two flavours and none had the same flavour as their name.

Can you tell who had which flavoured ice-cream?

Using rules

5.3

Relationships

One of the main uses of algebra is to describe a **relationship** between two or more variables. A relationship means that the variables are connected in some way so that changing the value of one affects the value of the other.

Consider the following examples. Identify the two variables (values that can change) and decide whether a relationship exists between them. Does changing the value of one cause the other to change?

The number of blocks of chocolate bought, and the total cost of the chocolate.

The number of houses being built, and the number of bricks the builder will require.

The number of students sharing a packet of lollies, and the number of lollies each one receives.

The number of animals in a zoo enclosure, and the amount of space each one has to move around in.

Rules and flowcharts

Relationships are described by a rule or set of instructions that tells you how to calculate one variable if you know the other.

For example, if one block of chocolate costs $4, the rule for calculating the cost of several blocks of chocolate would be:

'Multiply the number of blocks of chocolate you are buying by 4'.

Flowcharts are step-by-step instructions for performing a task. They are used in industry to show the sequence of steps in building a product, such as a car on an assembly production line. Flowcharts are also used to write computer programs.

We can use a flowchart to represent the cost of the chocolate.

There are different ways of writing flowcharts, but, in mathematics, we usually write flowcharts this way: $\boxed{n} \xrightarrow{\times 4} \boxed{4n}$ or $\boxed{n} \xrightarrow{\times 4} \boxed{C}$

Reading from left to right, this flowchart tells us: 'Take the number of blocks (n) and multiply them by 4 to get the total cost (C)'.

Rules using algebra

Rules can be written in words, but it is much quicker to use algebra. Written in algebra, a rule for the cost of the chocolate would look like this:

$C = 4n$

where C = total cost of the chocolate
and n = the number of blocks of chocolate bought at $4 a block.

We can use a **table of values** to show the cost of buying different numbers of blocks of chocolate.

Number of blocks, n	3	7	8	5	0
Cost, C	12	28	32	20	0

Each of the numbers in the top row of the table has gone through the flowchart (been multiplied by 4) to get the number in the bottom row.

$\boxed{3} \xrightarrow{\times 4} \boxed{12}$ $\boxed{7} \xrightarrow{\times 4} \boxed{28}$ $\boxed{8} \xrightarrow{\times 4} \boxed{32}$ $\boxed{5} \xrightarrow{\times 4} \boxed{20}$ $\boxed{0} \xrightarrow{\times 4} \boxed{0}$

Creating rules

If we choose any two pronumerals, such as x and y, we can create different rules that connect them by constructing different flowcharts.

For example, the rule 'y is equal to x plus 2' would have the flowchart:

$\boxed{x} \xrightarrow{+2} \boxed{x+2}$ or $\boxed{x} \xrightarrow{+2} \boxed{y}$

Written in algebra, the rule is: $y = x + 2$.

A 'table of values' for the rule could look like this:

x	4	10	6	9	2
y	6	12	8	11	4

The numbers in the bottom row of the table (y-values) are the result of adding 2 to each of the numbers in the top row of the table (x-values).

Some rules are two-step rules; for example, the rule 'to get y, multiply x by 3, then add 5' would have a flowchart that looks like this: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+5} \boxed{3x+5}$ or $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+5} \boxed{y}$

Written in algebra, the rule is: $y = 3x + 5$.

A table of values for this rule could look like this:

x	1	4	7	2
y	8	17	26	11

$$1 \xrightarrow{\times 3} 3 \xrightarrow{+5} 8 \quad \text{or} \quad 3 \times 1 + 5 = 8$$

$$4 \xrightarrow{\times 3} 12 \xrightarrow{+5} 17 \quad \text{or} \quad 3 \times 4 + 5 = 17$$

$$7 \xrightarrow{\times 3} 21 \xrightarrow{+5} 26 \quad \text{or} \quad 3 \times 7 + 5 = 26$$

$$2 \xrightarrow{\times 3} 6 \xrightarrow{+5} 11 \quad \text{or} \quad 3 \times 2 + 5 = 11$$

Worked Example 5

WE5

For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x divided by 4.

x	20	16	4	12	10
y					

(b) 2 is added to the product of x and 5 to get y.

x	2	5	4	3	0
y					

Thinking	Working
(a) 1 Decide how many operations are performed on x, what operation(s) they are and in which order (one operation, ÷ 4). Draw the flowchart.	(a) $\boxed{x} \xrightarrow{\div 4} \boxed{y}$
2 Write y on one side of the rule. On the other side, show the operations being performed on x ($\frac{x}{4}$).	$y = \frac{x}{4}$
3 For each value of x in the table, apply the rule by passing it through the flowchart (divide it by 4).	see table (a) below
(b) 1 Decide how many operations are performed on x, what operation(s) they are and in which order (× 5, then + 2). Draw the flowchart.	(b) $\boxed{x} \xrightarrow{\times 5} \boxed{} \xrightarrow{+2} \boxed{y}$
2 Write y on one side of the rule. On the other side, show the operations being performed on x ($5x + 2$).	$y = 5x + 2$
3 For each value of x in the table, apply the rule by passing it through the flowchart (multiply by 5, then add 2).	see table (b) below

(a)

x	20	16	4	12	10
y	5	4	1	3	2.5

(b)

x	2	5	4	3	0
y	12	27	22	17	2

Using brackets

Brackets may be needed to show the correct order of operations.

For example:

If the rule is 'y is equal to twice the sum of x and 4', the flowchart would look like this:

$$\boxed{x} \xrightarrow{+4} \boxed{x+4} \xrightarrow{\times 2} \boxed{2(x+4)}$$

The rule written in algebra would be $y = 2(x + 4)$.

Brackets are needed to show that the addition is to be done before the multiplication.

5.3 Using rules

Navigator

Answers page 649

Q1 Column 1, Q2, Q3, Q4, Q5 (a) & (b), Q6, Q8, Q10, Q12	Q1 Column 1, Q2, Q3, Q4, Q5 (c) & (d), Q6, Q7, Q8, Q10, Q13, Q14	Q1 Column 2, Q2, Q3, Q4, Q5 (e) & (f), Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency

WE5

Double x is the same as saying 'two times x'.

1 For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x plus 2.

x	13	11	7	28	1
y					

(b) To find y, subtract 5 from x.

x	6	18	9	85	5
y					

(c) To find y, double x.

x	2	3.4	10	11	101
y					

(d) y is equal to x divided by 3.

x	18	12	30	9	0
y					

(e) y is equal to multiplying x by 5, then subtracting 3.

x	4	2	0	5	20
y					

(f) To find y, add 5 to x, then divide by 10.

x	5	25	45	10	33
y					

(g) To find y, divide x by 2, then subtract 1.

x	10	6	24	9	15
y					

(h) y is equal to x multiplied by itself.

x	3	11	7	6	10
y					

2 Choose the correct algebraic rule given in each case.

(a) y is equal to the sum of x and twelve.

A $y = x + 12$ **B** $y + 12 = x$ **C** $y = 12x$ **D** $y = \frac{x}{12}$

(b) To get y, subtract 50 from x.

A $y = x - 50$ B $y = 50 - x$ C $y = x \times 50$ D $y = x + 50$

(c) To get y, add thirteen to x, then multiply by nine.

A $y = (x + 9) \times 13$ B $y = (x \times 13) + 9$ C $y = x + 13 \times 9$ D $y = 9(x + 13)$

3 Write the rule shown by each of these flowcharts. Make sure you use brackets where necessary.

(a) $\boxed{x} \xrightarrow{\times 4} \boxed{} \xrightarrow{-9} \boxed{y}$

(b) $\boxed{x} \xrightarrow{+11} \boxed{} \xrightarrow{\times 5} \boxed{y}$

(c) $\boxed{a} \xrightarrow{\div 2} \boxed{} \xrightarrow{-7} \boxed{b}$

(d) $\boxed{a} \xrightarrow{+3} \boxed{} \xrightarrow{\div 6} \boxed{b}$

(e) $\boxed{p} \xrightarrow{\times p} \boxed{} \xrightarrow{+8} \boxed{q}$

(f) $\boxed{p} \xrightarrow{\sqrt{\ }} \boxed{q}$

4 Rewrite each of these rules using algebra.

(a) To find y, subtract eighteen from x.

(b) y is equal to sixty multiplied by x.

(c) Divide x by seven to find y.

(d) To find y, add forty-three to x, then multiply by twenty.

(e) Multiply x by one hundred, then subtract fifty to find y.

(f) To find y, divide x by sixteen, then add thirteen.

(g) y is equal to x multiplied by itself.

(h) Subtract twelve from x, then divide by nine to find y.

(i) Multiply x by itself, then take away thirty-seven to find y.

Understanding

5 For the following rules, draw a flowchart and describe the rule using words.

(a) $y = 5x + 11$

(b) $y = \frac{x}{7} - 5$

(c) $b = 7(a + 6)$

(d) $b = \frac{a - 7}{12}$

(e) $d = \frac{e + 9}{4} - 13$
(the flowchart for this rule should have three steps)

(f) $d = \frac{e^2 - 1}{5}$
(the flowchart for this rule should have three steps)

6 Joe has some lollies. He gives half to his little brother and then eats four of the rest.

(a) Draw a flowchart to show how many lollies he has left. Use x for the number of lollies he starts with and y for the final number.

(b) Write the rule to describe the situation.

(c) Use the rule to find how many lollies Joe has now if he had 20 to start with.

7 Anita has some money in her wallet. Her mum gives her \$20 for doing some chores and she decides to spend half of the money she now has on a birthday present for her sister. She spends another \$4 on a birthday card.

(a) Draw a flowchart you would use to find how much money she has spent. Use x for the initial amount in Anita's wallet and y for the amount she has spent.

(b) Write a rule to describe the situation.

(c) Use the rule to find how much money Anita spent if she had \$12 in her wallet initially.

8 Your Uncle Harry is twice your age, plus 5.

(a) Show this information in a flowchart. Use m to represent your age, and H to represent Uncle Harry's age.

(b) Write a rule to show this situation.

(c) If you are 12 years old, how old is Uncle Harry?

9 To produce jeans it costs \$245 to set up the machine, and then \$7 for each pair of jeans. So, for one pair of jeans to be produced, it costs \$245 + \$7 × 1 = \$252.

(a) Using this information, fill in the table.

Number of pairs of jeans, n	10	50	150	200
Cost to produce the jeans, C				

(b) Write a rule for the cost of producing the jeans. Use C for cost, and n for the number of pairs of jeans.

Reasoning

To work backwards along the flowchart, do the *opposite* operation to the one shown in the forwards direction.

10 The following tables of values have only the y-values filled in. Work *backwards* along the flowchart to determine the values of x that were used, and complete the tables of values.

(a) $\boxed{x} \xrightarrow{\times 2} \boxed{} \xrightarrow{+1} \boxed{y}$

x					
y	5	11	15	9	21

(b) $\boxed{x} \xrightarrow{+3} \boxed{} \xrightarrow{\times 4} \boxed{y}$

x					
y	52	16	20	28	84

11 For each of the following two rules:

(i) y is equal to the sum of x and 3, which is then multiplied by 2

(ii) to find y, multiply x by 2, then add 6

(a) draw a flowchart.

(b) Write the rule using algebra.

(c) Copy and complete the table of values below.

x	2	5	3	4	7
y					

(d) What do you notice about these two rules? Can you explain your observation?

Open-ended

12 Construct your own table of values and complete it for the rule $y = \frac{x}{2} + 3$. Use at least five different values, including at least one negative value.

13 **(a)** Using two pronumerals (such as x and y), at least two of the following operations ($+, -, \times, \div$) and some arrows, create at least three different flowcharts that show a rule connecting x to y. Use a different combination of operations for each flowchart.

(b) Write the rules shown by your flowcharts using algebra.

14 Kim and Jai are working on the following question: 'Draw a flowchart that represents this rule: to obtain y, divide x by 2, then add 3'.

Kim's flowchart looks like this: $\boxed{x} \xrightarrow{+3} \boxed{} \xrightarrow{\div 2} \boxed{y}$

Jai's flowchart looks like this: $\boxed{x} \xrightarrow{\div 2} \boxed{} \xrightarrow{+3} \boxed{y}$

'Hey', said Jai. 'Yours looks different to mine. 'I must be wrong.'

'No, mate', said Kim. 'It's just a different way of writing the same thing.'

Is Kim's statement correct? Are they both right? If not, who has the correct flowchart, and why?

Outside the Square

Problem solving

Cutting string

Equipment required: 1 brain, 2 lengths of string, scissors

1 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces again, how many pieces will you now have? Check your answer using some string and scissors, then complete the following table.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

2 This time, fold a piece of string in half each time before you cut it. How many pieces at each cut do you have now? *Remember*: Cut only one piece at each time.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

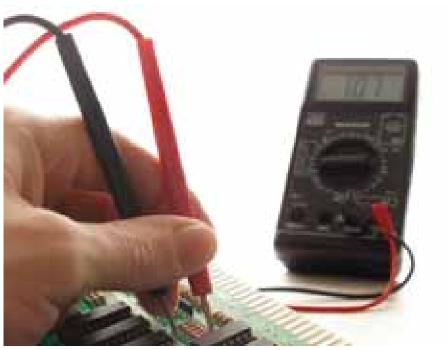
Strategy options

- Act it out.
- Look for a pattern.

5.4 Formulas and substitution

A **formula** is a mathematical rule that uses two or more variables. A formula is used to calculate the value of one variable when the value of the other variables are known. The plural of formula is formulas or formulae. The rules we have been working with so far can all be called formulas; however, we usually think of a formula as a rule we use in a practical situation. Here are some examples of practical formulas, and what they are used for:

$V = IR$



The voltage (V) of an electric circuit is found by multiplying the current (I) by the resistance (R).

$F = \frac{9C}{5} + 32$

To convert a temperature from Celsius to Fahrenheit, multiply the temperature in Celsius (C) by 9, divide by 5, and then add 32.

Substitution

In mathematics, we can **substitute** a number for a variable. This allows us to find a value for, or **evaluate**, the formula or expression.

Many team sports use substitution. How is it similar to the substitution we're doing here?

Worked Example 6 — WE6

For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = 10x - 5, \quad x = 7$

(b) $y = \frac{x}{2} + 4, \quad x = 5$

Thinking		Working
(a)	**1** Write the formula.	(a) $y = 10x - 5$
	2 Replace the variable (x) in the formula with its given value (7). Insert the × sign between the coefficient and the number. State the value you will substitute for x.	$y = 10 \times 7 - 5, x = 7$
	3 Calculate the value of the other variable (y).	$y = 70 - 5$ $= 65$
	4 State the answer.	$y = 65, x = 7.$

(b) 1 Write the formula.

(b) $y = \frac{x}{2} + 4$

2 Replace the variable (x) in the formula with its given value (5).

$y = \frac{5}{2} + 4,\ t = 5$

3 Calculate the value of the other variable (y).

$= 2\frac{1}{2} + 4$

$= 6\frac{1}{2}$ or 6.5

4 State the answer.

$y = 6.5,\ x = 5$

When substituting numbers into a formula, write in any 'hidden' multiplication signs.

e.g. $b = 4a + 3$

$= 4 \times 5 + 3,\ a = 5$

Worked Example 7

WE7

(a) The cost of hiring a community hall is calculated using the formula $C = 65t + 200$, where C = cost (in dollars) and t = the hire period (in hours). Find the cost of hiring the hall for 4 hours.

(b) The area of a rectangle (in square metres) is given by the formula $A = lw$ where l = length (in metres) and w = width (in metres). Calculate the area of a rectangular room that is 4.5 m long and 3 m wide.

Thinking	Working
(a) 1 Write the formula.	(a) $C = 65t + 200$
2 Replace the variable in the formula with its given value (replace t with 4, the number of hours). Insert a multiplication sign between the coefficients and the numbers (65 and 4).	$C = 65 \times 4 + 200,\ t = 4$
3 Evaluate the formula by performing the multiplication and the addition.	$= 260 + 200$ $= 460$
4 State the answer.	The cost of hiring the hall for 4 hours is $460.
(b) 1 Write the formula.	(b) $A = lw$
2 Replace the variables in the formula with their given values. (Replace l with 4.5 and w with 3.) Insert a multiplication sign between the numbers (4.5 and 3).	$A = 4.5 \times 3,\ l = 4.5,\ w = 3$
3 Evaluate the formula by performing the multiplication.	$= 13.5$
4 State the answer.	The area is 13.5 square metres.

5.4 Formulas and substitution

Navigator

Answers page 651

Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 (a) & (b), Q6, Q7, Q8, Q10, Q12	Q1 Column 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q9, Q10, Q11, Q12	Q1 Column 2, Q2, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE6

1 For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = x + 3$, $x = 4$ **(b)** $y = x + 5$, $x = \frac{1}{2}$

(c) $y = 2x - 7$, $x = 3$ **(d)** $y = 3x - 1$, $x = 8$

(e) $y = x - 1$, $x = 3.6$ **(f)** $y = x - 5$, $x = 30$

(g) $y = 25 - 3x$, $x = 7$ **(h)** $y = 30 - 2x$, $x = 1$

(i) $y = 4x$, $x = \frac{1}{2}$ **(j)** $y = 8x$, $x = 4$

(k) $y = 3(x + 5)$, $x = 1$ **(l)** $y = 6(x + 2)$, $x = 18$

(m) $y = \frac{x}{5}$, $x = 20$ **(n)** $y = \frac{x}{5}$, $x = 2.5$

(o) $y = 11(10 - x)$, $x = 4$ **(p)** $y = 12(10 - x)$, $x = 3$

WE7

2 **(a)** The cost of hiring power tools from the local hardware store is calculated using the formula $C = 35t + 60$, where C = cost (in dollars), and t = the hire period (in hours). Find the cost of hiring a sander for 6 hours.

(b) Body mass index (B) is one way of measuring a person's health, and is calculated by the formula $B = \frac{m}{h^2}$, where m is the person's mass in kilograms, and h is their height in metres. Calculate the body mass index of a runner who is 1.7 metres tall and weighs 64 kg. (Round your answer to two decimal places.)

(c) The average speed of a moving object (such as a car) can be calculated by the formula $s = \frac{d}{t}$, where s = speed (in kilometres per hour), d = distance (in kilometres) and t = time (in hours). Yoshi drives 210 km from Hobart to Launceston in 3 hours. Calculate her average speed.

3 Answer TRUE or FALSE for each of the following statements.

(a) If we substitute $a = 4$ into $b = 5a$ we get $b = 20$.

(b) If we substitute $a = 9$ into $b = a + 11$ we get $b = 20$.

(c) If we substitute $u = 4$ into $v = 6u + 1$ we get $v = 65$.

(d) If we substitute $q = 10$ into $k = 13q - 8$ we get $k = 122$.

(e) If we substitute $x = 6$ into $y = 4(x - 5)$ we get $y = 19$.

(f) If we substitute $x = 12$ into $y = 5(14 - x)$ we get $y = 10$.

4 Use each of the following rules to complete these tables of values.

(a) $b = 4a$

a	11	20	5	9	50
b					

(b) $y = 7x$

x	6	4	10	20	101
y					

(c) $n = 3m + 2$

m	1	2	10	6	5
n					

(d) $k = 4j + 7$

j	2	5	11	10	100
k					

(e) $q = 2p - 10$

p	11	15	10	20	100
q					

(f) $s = 8r - 2$

r	1	2	3	0	200
s					

(g) $v = 4(u - 1)$

u	5	3	1	201	6
v					

(h) $n = 3(2 - m)$

m	5	10	11	102	52
n					

5 **(a)** For the rule $m = 3x - 5$, when $x = 3$, m would be equal to:

A 1 **B** 2 **C** 3 **D** 4

(b) For the rule $y = \frac{x}{3} + 4$, when $x = 6$, y would be equal to:

A $\frac{10}{3}$ **B** $4\frac{1}{2}$ **C** 6 **D** 22

(c) For the rule $l = 3(n + 2) - 1$, when $n = 1$, l would be equal to:

A 5 **B** 6 **C** 8 **D** 9

Understanding

6 Sam's new fuel-efficient car uses 7 litres of petrol for every 100 km it travels.

(a) Write this as a formula to calculate the amount of petrol (p) that is needed to travel a distance (d) kilometres.

His father's 4WD uses 11 litres of petrol for every 100 km it travels.

(b) Write this as a formula to calculate the amount of petrol (f) that is needed to travel a distance of (d) kilometres.

(c) Use your formulas to calculate the number of litres each car uses to travel a distance of 400 km.

(d) Write a formula to calculate the cost (C), in dollars, of y litres of petrol if petrol costs \$1.30/litre.

(e) Calculate how much more Sam's father spent on petrol than Sam to travel 400 km.

7 Jabbok, a mobile phone salesman gets \$350 per week and \$10 for every phone that he sells.

(a) Write this as a formula with n representing the number of phones sold, and p representing the amount of money earned each week.

(b) If 15 phones are sold in the first week, use your formula to find how much Jabbok earns this week.

(c) If 20 phones are sold the next week, use your formula to find how much money Jabbok earns that week.

(d) If Jabbok earns \$470 in a week, how many phones were sold?

Reasoning

8 I buy three loaves of bread and a \$2.60 carton of milk. It costs me \$$D$.

(a) If l represents the cost of a loaf of bread, write a formula to find the cost of my shopping (D) in terms of l.

(b) Use the formula to evaluate D if $l = \$3.25$.

9 Jasmine is a caterer. She uses formulas to work out how much food she needs to feed different numbers of people.

(a) To calculate the number of sausages she needs at a barbeque, Jasmine uses the following formula: 'Allow 2 sausages for every person and have an extra 10 sausages'. Write Jasmine's formula using algebra using n to represent the number of people and s to represent the number of sausages.

(b) Another one of Jasmine's formulas for barbeques is: 'One bowl of salad will feed 8 people'. Write this formula using algebra. Use n to represent the number of people, and b for the number of bowls of salad.

(c) Use the formulas you have written to determine the number of sausages and bowls of salad Jasmine will need to cater for 40 people.

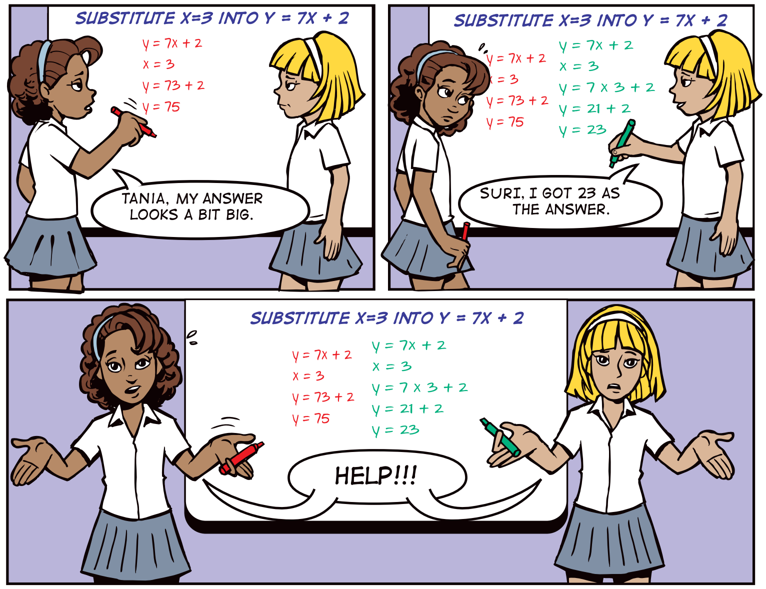
10 Show that the formulas $b = 3(a + 5)$ and $b = 3a + 5$ are different by substituting $a = 4$ into each of them. Explain how the two are different in terms of the order of operations.

Open-ended

11 Make up two different formulas connecting x and y that use addition and multiplication. Use them to complete the following table for each one.

x	7	20	13	101
y				

12



Which of the two students has substituted correctly? What mistake has the other student made? Give them some advice so they can avoid a similar mistake in the future.

Outside the Square Game

Dicey formulas

Equipment required: 2 brains, 1 die

How to win:
Be the first to 60.

How to play:

1 Each person is given a formula.

2 You roll the die in turn. After rolling the die the first time, you must substitute the value shown on the die into your formula and write down the answer.

3 On each later throw, calculate the value from the formula as before, and add this to your previous total. Your opponent checks your answer and if it is wrong you lose a turn.

4 Your game ends when you think that you can get no closer to 60.

5 The other player may keep going until they decide they can get no closer to 60.

6 The winner is the person who gets to exactly 60, or, failing that, the person with the total nearest to, but less than, 60. If you score more than 60 you have lost, so you must decide when to stop.

Three sets of formulas are given:

Game 1:
Person 1, $A = 3n + 2$;
Person 2, $B = 3n - 2$

Game 2:
Person 1, $A = 3n + 4$;
Person 2, $B = 3(n + 2)$

Game 3:
Person 1, $A = 6 - n$;
Person 2, $B = 7 - n$

As a challenge, play this game where you have to get exactly 60. You are able to not include results if you wish. Here, you need to make sure that the last number you need to total 60 is possible to get using your formula when $n = 1$, 2, 3, 4, 5, or 6.

(Use 2 dice and be the first to 150 or more.)

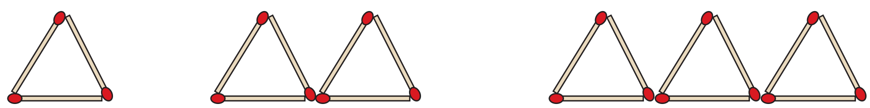
5.5 Patterns and rules

A strategy we often use in problem solving is to look for a pattern in our results. Algebra can help us describe a pattern, which can often be written as a general rule, or formula. We can then use the formula to solve problems without having to draw out endless patterns.

Worked Example 8

WE8

(a) Here is a matchstick pattern of triangles. Copy and complete this table of values by continuing the pattern.



Number of triangles (t)	1	2	3	4	5
Number of matches (m)					

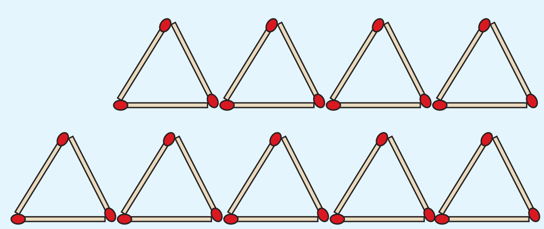
(b) Find a general rule that connects the number of triangles in the pattern (t) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 triangles.

Thinking	Working
(a) **1** Construct the table of values. Fill in any information you know from the pattern provided.	(a) see table below
2 Continue the pattern by adding matches to create more triangles.	see drawing below
3 Count the number of matches used for the continued pattern (formed with four and five triangles) and fill in the table.	see table below

(a)

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	**3**	**6**	**9**		



Number of triangles (t)	1	2	3	4	5
Number of matches (m)	3	6	9	**12**	**15**

(b) 1	Identify the number of matches that are being added on each time to make a new shape (3). (This number is also being added on to each number in the second row of the table.) This is the multiplication factor in the rule (× 3).	(b) $m = 3 \times t$
2	Write the rule that links the two variables (t and m) in the table, in words and in algebra.	The number of matches is 3 times the number of triangles. $m = 3t$
(c) 1	Substitute $t = 100$ into the rule, and evaluate.	(c) $t = 100$ $m = 3 \times 100$ $m = 300$
2	Write the answer in words.	300 matches are needed to make 100 triangles.

In the above Worked Example, we see that one 'lot' of 3 matches is added every time to make a new triangle and the next number in the sequence. Therefore, we are multiplying the number of triangles by 3 to find the number of matches.

When trying to find a pattern, look for the number that is being added or subtracted every time. This will tell us what to multiply by in our rule.

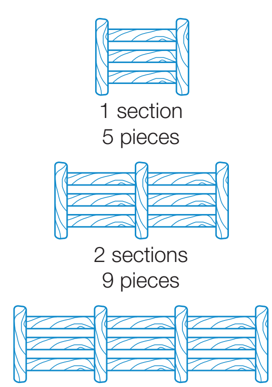
In some cases, the above procedure may not give us the complete rule. We must then look for the number that is required to complete the first pattern. We then need to add on this number to make our rule work; e.g. $m = 3t + 1$; $m = 3t + 2$; $m = 3t + 3$ etc. Worked Example 9 shows this in detail.

Always try to understand how the rule works by looking at the pattern that has been formed.

Worked Example 9

WE 9

Frederico is building a fence around Farside Farm. The fences are made up of pieces of timber as shown.



(a) Copy and complete this table of values by continuing the pattern.

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9			

(b) Find a general rule that connects the number of sections of fence (F) to the number of pieces of timber (P) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of pieces of timber required for a fence made up of 50 sections.

Thinking	Working
(a) 1 Construct the table of values. Fill in any values you know from the pattern provided. Continue the pattern to complete the table.	(a) see table below
(b) 1 Identify the number of pieces of the pattern that are needed to complete the new section (4). (This number is also being added on to each number in the second row of the table.) This number is the multiplication factor in the rule (× 4).	(b) 4 pieces are being added to make a new section. Therefore, $4F$ is in our rule.
2 When we multiply F by 4, we do not have the numbers in the second row, so we now look for a number to add or subtract. How many pieces of the pattern were needed to start the first section? This number is added on to the rule. (1 piece of timber.)	$4F+1$
3 Write the rule that links the variables, both in words and in algebra.	The number of pieces is 4 times the number of sections plus 1 extra piece. $P=4F+1$
(c) 1 Substitute the value of F into the formula ($F=50$).	(c) $P=4F+1$ $=4\times 50+1$
2 Evaluate.	$P=200+1$ $=201$
3 Write the answer in words.	To make a fence of 50 sections, 201 pieces of timber are needed.

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9	13	17	21

5.5 Patterns and rules

Navigator

Answers page 651

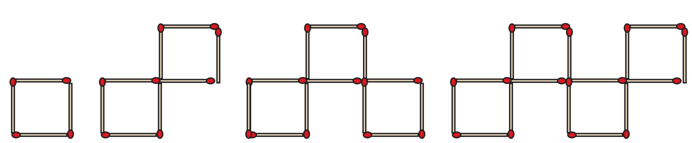
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: Centimetre grid paper for Question **12**

Fluency

WE8

1 (a) Here is a matchstick pattern of squares. Copy and complete this table of values by continuing the pattern.

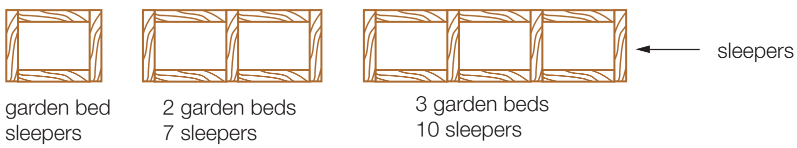


Number of squares (s)	1	2	3	4	5	6	7	8
Number of matches (m)								

(b) Find a general rule that connects the number of squares in the pattern (s) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 squares.

2 Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds as shown. WE9



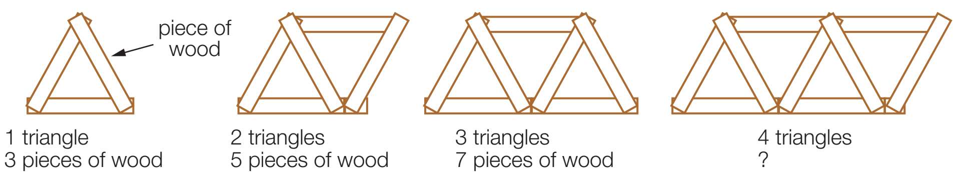
(a) Copy and complete this table of values by continuing the pattern.

Number of garden beds (B)	1	2	3	4	5
Number of sleepers (S)	4	7	10		

(b) Find a general rule that connects the number of garden beds (B) to the number of sleepers (S) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of sleepers required to divide the garden into a section made up of 21 beds.

3 Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the one shown below, running across the walls.



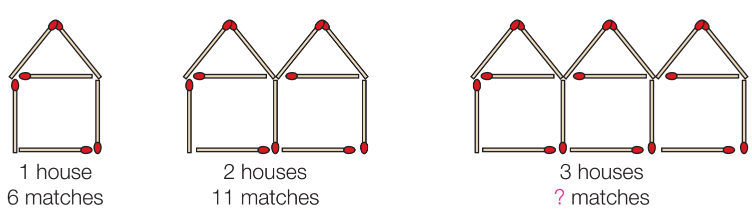
(a) Copy and complete this table of values by continuing the pattern.

Number of triangles (T)	1	2	3	4	5
Number of pieces of wood (P)	3	5	7		

(b) Find a general rule that connects the number of triangles to the number of pieces of wood required.

(c) Use your rule to find how many pieces of wood are required to make a total of 203 triangles.

4 Here is a matchstick pattern of houses.



(a) Copy and complete the table below.

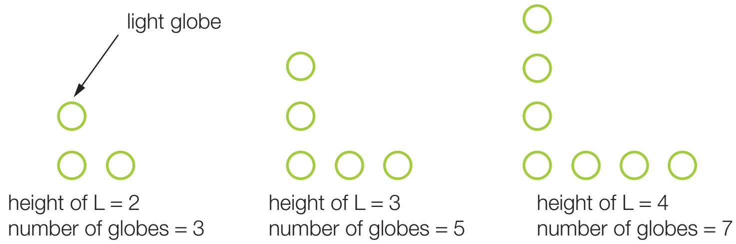
Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11						

(b) Find the general rule that connects the number of matches to the number of houses.

(c) Use your rule to find the number of matches required to build 20 houses.

Understanding

5 Lightworks International Co., designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.



(a) Construct and complete a table of values for this pattern. Use the three patterns given here to begin your table, then continue the pattern for the next two Ls in the pattern.

(b) Find the rule that connects the height of the L to the number of globes required. Let H = height of the L and G = number of globes.

(c) Use your rule to find how many globes you would need to make an 'L' with a height of 120.

6 A rule is given as $L = 3n + 1$. Which statement is not true?

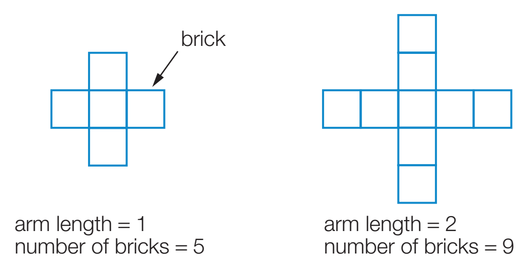
A When $n = 3$, $L = 10$.

B When $n = 4$, $L = 12$.

C When $n = 5$, $L = 16$.

D When $n = 6$, $L = 19$.

7 The White Cross charity organisation has a white cross as its emblem. Their buildings all have a white cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.



(a) Draw the next two white crosses in the pattern.

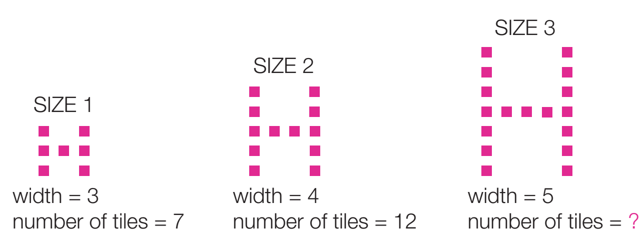
(b) Construct and complete a table of values connecting the arm length (a) and the number of white bricks (b) for these four different-sized crosses.

(c) Extend your table for arm lengths of 5 and 6.

(d) Find the rule that connects the two pronumerals.

(e) Find how many white bricks would be needed to make a cross with an arm length of 52.

8 Mr Harrison has decided that he wants to use tiles to form large letter Hs, which are built into the brickwork of the walls of his company offices. He must work out how many tiles are needed to make different-sized Hs.



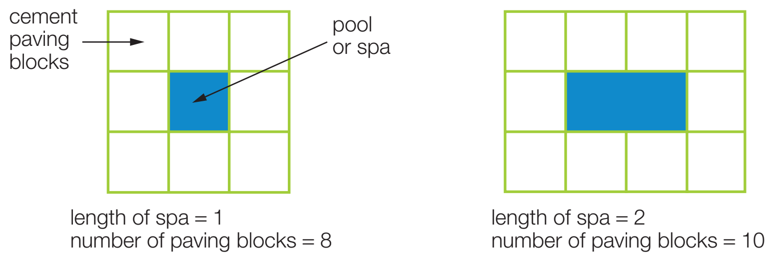
(a) Draw the next two Hs in the pattern.

(b) Construct and complete a table of values connecting the width of the H (w), and the total number of tiles (t).

(c) Find the rule for these five different sizes that connects the width of the H (w) to the total number of tiles (t).

(d) Find the number of tiles needed to make an H that is 12 tiles wide.

9 Plunge & Co., the Pool Paving People, specialises in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.



(a) Draw the next two spas and pavers in the pattern.

(b) Construct and complete a table for values of up to 4 spa lengths, connecting the spa length (l) with the number of pavers used (p) for these four different spa sizes.

(c) Find a rule without brackets connecting the two pronumerals.

(d) Explain your rule by using diagrams of spas of different lengths. Which paving blocks does the constant represent? Where does the coefficient of the pronumeral representing spa length come from?

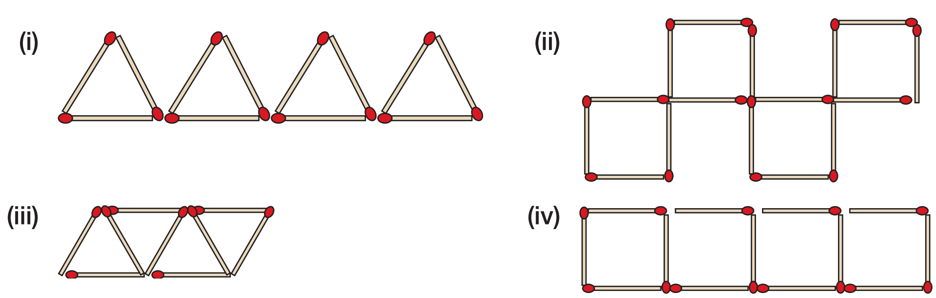
(e) Find a rule using brackets that also works.

(f) Plunge & Co. has recently been asked by a swimming club to build a single-lane lap pool to help train its long-distance swimmers. It has been worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks will be needed to pave around the whole pool.

(g) For another job, Plunge & Co. uses 40 paving blocks to surround an outdoor spa. What is the length of this spa?

Reasoning

10 In each of the following matchstick patterns, the diagrams show the pattern after the fourth set of matchsticks has been added.

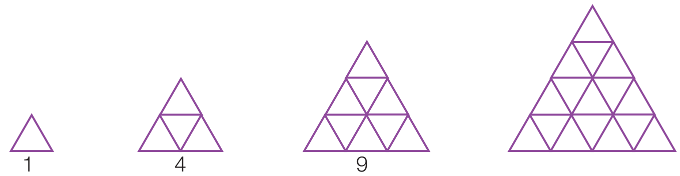


(a) Write a general rule for each of the patterns if one more set of matchsticks is added to the pattern each time. Use s = number of shapes and m = number of matchsticks as pronumerals.

(b) Patterns **(i)** and **(iii)** both add a new triangle every time. However, the rules are different. Patterns **(ii)** and **(iv)** both add a new square every time. However, the rules are also different. How are patterns **(i)** and **(ii)** different from the patterns in **(iii)** and **(iv)**? How is this difference shown in the rule for each pattern?

Not all patterns grow by a consistent amount each time. For the following question, complete the table and describe how the sequence of numbers formed in the bottom row of the table increases as extra elements of the pattern are added.

11 **(a)** Count *all* the small triangles in the fourth large triangle.



(b) How many small triangles do you think there will be in the fifth large triangle?

(c) Copy and complete the following table. Try to find the pattern to help you.

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9			

Open-ended

12 Choose one of the following letters: P, T, A, X, or Y. Create a design for the letter from small centimetre squares, and use it to create a pattern of letters that gradually increases in size. Try to find a rule that describes your pattern.

13 Shreyas has decided to make a closed matchstick shape with 6 matchsticks. He then adds matchsticks to make a pattern that has a rule $m = 5s + 1$ where m is the number of matchsticks and s is the number of repeated shapes.

(a) Construct a shape that Shreyas may have started with.

(b) Shreyas changes his shape and adds matchsticks to make a new pattern. He finds that the rule is now $m = 4s + 2$. Draw a second shape that he may have used.

Outside the Square

Puzzle

Puzzling tables

The table contains multiple patterns, across each row and down each column.

Copy the table and write the correct number in each blank space.

Describe the pattern for each row and for each column.

	Column 1	Column 2	Column 3	Column 4
Row 1	4	6	8	10
Row 2	7	10.5	14	
Row 3		15	20	25
Row 4	13		26	32.5
Row 5	16	24		40

Patterns like this often exist in tables of information. This table gives the cost to send parcels, up to 20 kg, by air.

Different parts of the table have different patterns. A pattern may only work in part of a row or column. Describe the patterns that you can find.

Weight	Zone A NZ	Zone B Asia/Pacific	Zone C USA/Canada/ Middle East	Zone D Rest of World
AIR				
Up to 250 g	\$7.20	\$8.40	\$9.55	\$11.35
Over 250 g up to 500 g	\$11.00	\$13.40	\$15.70	\$19.30
Over 500 g up to 750 g	\$14.80	\$18.40	\$21.85	\$27.25
Over 750 g up to 1000 g	\$18.60	\$23.40	\$28.00	\$35.20
Over 1000 g up to 1250 g	\$22.40	\$28.40	\$34.15	\$43.15
Over 1250 g up to 1500 g	\$26.20	\$33.40	\$40.30	\$51.10
Over 1500 g up to 1750 g	\$30.00	\$38.40	\$46.45	\$59.05
Over 1750 g up to 2000 g	\$33.80	\$43.40	\$52.60	\$67.00
Extra 500 g or part thereof	\$4.00	\$5.20	\$7.45	\$9.95

Postal Charges 2009, Australia Post website.

Half-time 5

1 Which rule describes the given information? Ex. 5.3

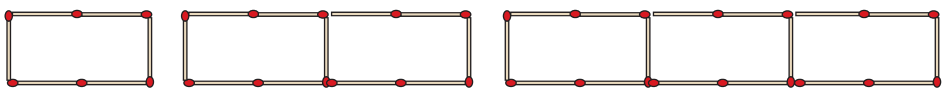
'Subtract 45 from x to get y.'

A $y = x + 45$ B $y = x - 45$ C $y = x \times 45$ D $y = x \div 45$

2 For each of the following rules, evaluate y by substituting the given values of x. Ex. 5.4

(a) $y = x - 4$ when $x = 6$ (b) $y = 2(x - 7)$ when $x = -13$ (c) $y = 54 - 3x$ when $x = -9$

3 If s represents the number of shapes formed and M is the number of matchsticks used, which rule is correct for this pattern? Ex. 5.5



A $M = 5s + 1$ B $M = 6s$ C $M = 5s - 1$ D $M = 4s + 2$

4 Write an equation to describe each of the following. Ex. 5.2

(a) Tim has p footy cards. He collects 7 more. He now has 28 cards.

(b) 20 is obtained when 4 is subtracted from the product of x and 7.

(c) A bird aviary has w parrots and g canaries. When 4 of the canaries are sold, there are 14 birds left in the aviary. How many birds are in the aviary now?

5 If y is equal to 4 plus the quotient of x divided by 2: Ex. 5.3

(a) draw a flowchart for this rule

(b) write the rule using algebra

(c) copy and complete this table of values for the rule.

x	2	8	10	5	13
y					

6 Nicola has 6 identical bags of lollies, plus an extra 11 loose lollies. If n represents the number of lollies in one bag, write an expression for the total number of lollies Nicola has. Ex. 5.1

7 Given the rule $y = 4x - 3$, which statement is not true? Ex. 5.4

A When $x = 6$, $y = 21$. B When $x = 5$, $y = 17$.

C When $x = 4$, $y = 13$. D When $x = 3$, $y = 0$.

8 The cost of an international phone call is 35 cents for every minute, plus a one-off 'flagfall' charge of 50 cents. Ex. 5.4

(a) Write a rule to calculate the total cost of an international phone call. Use the pronumeral C for the cost and m for the length of the call, in minutes.

(b) Use your rule to calculate the cost of a 6-minute phone call. Write your answer in dollars.

(c) Ravi wants to call his parents in Singapore. He has \$8. Will this be enough money for a 20-minute call?

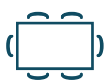
Investigation

Richie's Restaurant

Equipment required: 1 brain

Richie has just bought 30 new rectangular tables for his restaurant. One table can seat six people. He needs to arrange them according to two conditions.

- The tables must be set up so that he can seat at least one group of 6, 8, 10, 12, 14, 16 and 18 people all at the same time.
- He wants to seat the maximum number of people possible.



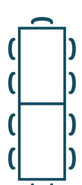
The Big Question

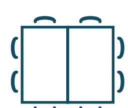
How should Richie arrange his tables to fulfil both of the conditions?

Engage

To seat more than six people, there are two ways in which Richie can join the tables together:

Lengthways or widthways





1 How many people can be seated around two tables if they are joined together:

(a) lengthways **(b)** widthways?

2 For each of the two ways of joining the tables, copy and complete the following table of values. Draw the tables and count the number of people around them if necessary.

Number of joined tables (t)	1	2	3	4	5	6	7
Number of people seated (p)							

Explore

3 If Richie had no restrictions on the way he could set up his tables, what is the maximum number of people he could seat?

4 If Richie now sets up his tables to fulfil the given conditions, draw up a seating plan of Richie's restaurant, showing your arrangement of the 30 tables. How many people can be seated using this arrangement?

Strategy options

- Draw a diagram.
- Guess and check.
- Make a table.
- Look for a pattern.
- Test all possible combinations.

Explain

5 Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient (uses fewer tables)? Is this type of arrangement always more efficient?

6 Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

7 For each of the two arrangements, write the rule that connects the number of tables (t) to the number of people that can be seated (p).

Elaborate

8 Consider the coefficient and the constant that appears in each of your rules. Explain where they come from.

9 How do the two rules show that one type of table arrangement is more efficient than the other?

10 Given what you now know about the efficiency of different table arrangements, modify your plan from Question **4**, if necessary, to seat a greater number of people, while still fulfilling Richie's condition.

11 Draw your improved seating arrangement to answer the Big Question.

Evaluate

12 Consider the way in which you worked on this problem. How did you approach it? Could you have gone about it a different way?

13 Did you use your tables of values (from Question **2**) or the algebra rules to help you solve the problem?

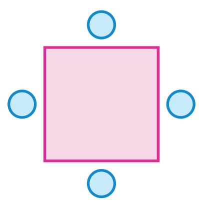
14 How confident are you that your final solution is the best solution?

15 Make a list of all the other factors you would need to consider if you were setting up tables in your own restaurant.

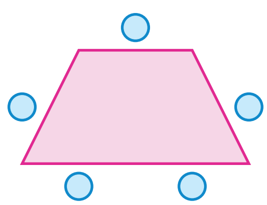
Extend

16 For his next restaurant, Richie would like to use some different-shaped tables. He could use:

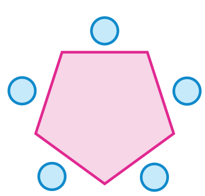
(a) square tables that sit 4 people



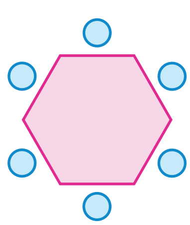
(b) trapezoidal tables that seat 5 people



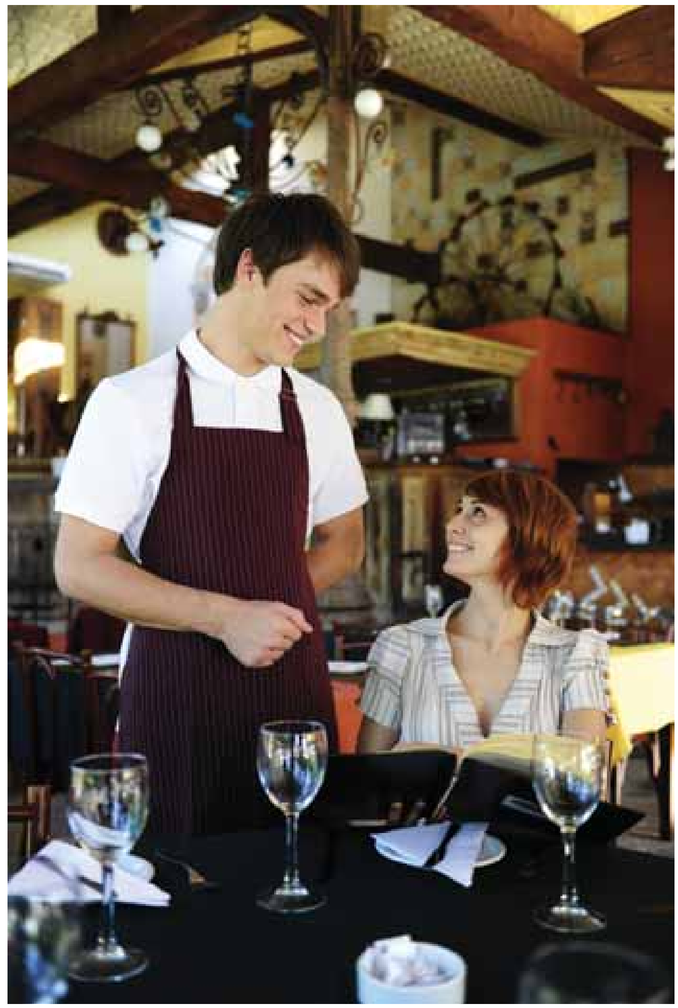
(c) pentagonal tables that seat 5 people or



(d) hexagonal tables that seat 6 people.



Write a report to explain to Richie why none of these different table shapes would be suitable if he wants to seat the maximum number of people while fulfilling the given conditions and using no more than 30 tables.



5.6 Simplifying expressions with addition and subtraction

Like terms

Like terms:

- have the same pronumeral part
- can have different coefficients; $2x$ and $5x$ are like terms
- can have pronumerals in a different order; ab and ba are like terms
- can have pronumerals raised to exactly the same power; $7x^3$ and $4x^3$ are like terms.

Unlike terms

Unlike terms:

- have different pronumeral parts; $2x$ and $2y$ are unlike terms
- can have the the same pronumerals raised to different powers; x^2 and x are unlike terms ($2^2 \neq 2$).

Because pronumerals represent numbers, we can use number laws in algebra to change the order of pronumerals.

$2 \times 3 = 3 \times 2$, so $ab = ba$ (commutative law)
$(3 \times 2) \times 5 = 3 \times (2 \times 5)$, so $(ab)c = a(bc)$ (associative law)
$abc = acb = bac = bca = cab = cba$

When terms have different pronumeral parts, we usually write the pronumerals in alphabetical order. This helps to identify like terms.

For example, $6abc$, $3bca$, $4dab$, $8cad$, $11adb$, $7cda$ can be written as $6abc$, $3abc$, $4abd$, $8acd$, $11abd$ and $7acd$. We can now see that $6abc$ and $3bca$ are like terms, $4dab$ and $11adb$ are like terms and $8cad$ and $7cda$ are also like terms.

Worked Example 10

WE10

Which of the following are like terms?

(a) $4x$ and $7x$ **(b)** $5ab$ and $9b$ **(c)** x and $12x$
(d) 8 and $8z$ **(e)** h^3 and $6h^3$ **(f)** xy and x^2y

Thinking	Working
(a) Do the terms have exactly the same pronumeral part?	(a) $4x$ and $7x$ are like terms.
(b) Do the terms have exactly the same pronumeral part?	(b) $5ab$ and $9b$ are not like terms.

(c) Do the terms have exactly the same pronumeral part?	(c) x and $12x$ are like terms.
(d) Do the terms have exactly the same pronumeral part?	(d) 8 and $8z$ are not like terms.
(e) Do the terms have exactly the same pronumeral part?	(e) h^3 and $6h^3$ are like terms.
(f) Do the terms have exactly the same pronumeral part?	(f) xy and x^2y are not like terms.

Adding and subtracting like terms

Maddie bought 6 identical packets of party balloons for her birthday party. Each packet contained x balloons, so she bought $6x$ balloons. At home, Maddie found that she had 4 more identical packets, so she added $4x$ balloons to the $6x$ balloons she has just bought. Now, she had 10 packets, all containing x balloons, so she had a total of $10x$ balloons.

$6x$ and $4x$ are like terms. When we add $6x$ and $4x$ we can simplify them to $10x$ by adding their coefficients.

$6x + 4x = 10x$

Maddie didn't need all the balloons, so she took 2 packets back to the store. She returned $2x$ balloons. She now has 8 packets of balloons, so she has $8x$ balloons for her party.

$10x$ and $2x$ are like terms. When we subtract $2x$ from $10x$ we can simplify them to $8x$ by subtracting their coefficients.

$10x - 2x = 8x$

We can simplify an expression by adding or subtracting like terms. This is called 'collecting like terms'. We add or subtract like terms by adding or subtracting their coefficients.

If Maddy buys 3 identical packets of balloons that contain x balloons and 2 different packets that each contain y balloons, we can show that she now has $3x + 2y$ balloons. $3x + 2y$ cannot be simplified any further, as $3x$ and $2y$ are unlike terms.

Examples:

$6c + 9c = 15c$ — $8x - 3x = 5x$
$12y + 3y - 4y = 11y$ — $16abc - 13abc = 3abc$
$3x + 4y + 5x = 8x + 4y$ — $6x^2 + 4x^2 + 10x^2 = 20x^2$
$7ab + 2ba = 9ab$ — $2x + 3x^2 - x = x + 3x^2$

$3ab + 4bc$ cannot be simplified, because ab and bc are not like terms.
$2x + 2x^2$ cannot be simplified because x and x^2 are not like terms.

Checking by substitution

We can replace a pronumeral with any number to check that our simplification is correct.

For example: $6x + 4x = 10x$
$10x - 2x = 8x$

for all values of x.

If $x = 20$,
$6 \times 20 + 4 \times 20 = 10 \times 20$
$10 \times 20 - 2 \times 20 = 8 \times 20$

If $x = 7$,
$6 \times 7 + 4 \times 7 = 10 \times 7$
$10 \times 7 - 2 \times 7 = 8 \times 7$

This is an application of the distributive law.

Worked Example 11

WE11

Simplify each expression by collecting like terms.

(a) $4y + 7y$ **(b)** $2mn - 4mn$ **(c)** $8a + 4 - 3a$

Thinking	Working
(a) 1 Identify like terms by looking for terms that have exactly the same pronumeral part.	(a) $4y + 7y$
2 Simplify like terms by adding the coefficients of the terms.	$= 11y$
(b) 1 Identify like terms by looking for terms that have exactly the same pronumeral part.	(b) $2mn - 4mn$
2 Simplify like terms by subtracting the coefficients of the terms.	$= -2mn$
(c) 1 Identify the like terms.	(c) $8a + 4 - 3a$
2 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($-3a$).	$= 8a - 3a + 4$
3 Simplify like terms by subtracting the coefficients of the terms.	$= 5a + 4$

Worked Example 12

WE12

Simplify each expression where possible by collecting like terms.

(a) $4x + 3y - 9x + 2y$ **(b)** $6ab - 4a + 3ab - 7$

Thinking	Working
(a) 1 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($-9x$).	(a) $4x + 3y - 9x + 2y$ $= 4x - 9x + 3y + 2y$
2 Simplify the like terms by adding or subtracting the coefficients of the terms.	$= -5x + 5y$
(b) 1 Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($+ 3ab - 4a$).	(b) $6ab - 4a + 3ab - 7$ $= 6ab + 3ab - 4a - 7$
2 Simplify the like terms by adding the coefficients of the terms.	$= 9ab - 4a - 7$

- Only like terms can be added together or subtracted.
- The operation sign (+ or –) moves with the term that follows it.

5.6 Simplifying expressions with addition and subtraction

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q11	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q11	Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11

Answers page 652

Fluency

1 Which of the following are like terms? WE10

(a) $3k$ and $5k$ (b) $4y$ and $5z$ (c) $11y$ and $12v$

(d) 17 and 8 (e) $6ab$ and $7bc$ (f) $2xy$ and $14yx$

(g) $4xyz$ and yzx (h) mnp and $3npm$ (i) 3, 5 and x

2 Simplify each expression by collecting like terms. WE11

(a) $3x + 4x$ (b) $12y + 4y$ (c) $11a + 2a$

(d) $13y - 9y$ (e) $7m - 3m$ (f) $-10x - x$

(g) $3mn + 2nm$ (h) $8xy - 3xy$ (i) $-2z + 3z + 4$

(j) $x + 4y + 6x$ (k) $10w + 3z + 4w$ (l) $17a + 2b - 5a$

3 Simplify each expression where possible by collecting like terms. WE12

(a) $3e + 4f + 7e - 6f$ (b) $12p + 8q - 4p - 12q$ (c) $8t + 15s - 16t - 3s$

(d) $6x - 4y + 3x + y^2$ (e) $12w - 2z + 3w - 3z^2$ (f) $10m + 9n - n - 5m^2$

(g) $3a + 17 + 4a - 8$ (h) $14x - 4 - 6x + 9$ (i) $21y - 36 - y - 17$

4 (a) A like term for $7x$ is:

A $12x$ B $3 + x$ C 7 D xy

(b) A like term for $6wxy$ is:

A $6w$ B $6x$ C $6y$ D $12xyw$

Understanding

5 Simplify each expression where possible.

(a) $5e + 14f + 8e + 6$ (b) $18x + 6y - 4y + 2$ (c) $4 + 6x + 5y - 3$

(d) $2x - 3y + 4$ (e) $9m - 2n + 3mn$ (f) $6a + 11b - 5c - 7$

6 Write each of the following rules using algebra, substitute the numbers 2, 3 and 4 into each rule, then answer TRUE or FALSE for each of these statements.

(a) These two rules are the same:

Rule 1: 'Take any number and multiply it by four. Take the same number and multiply it by six. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

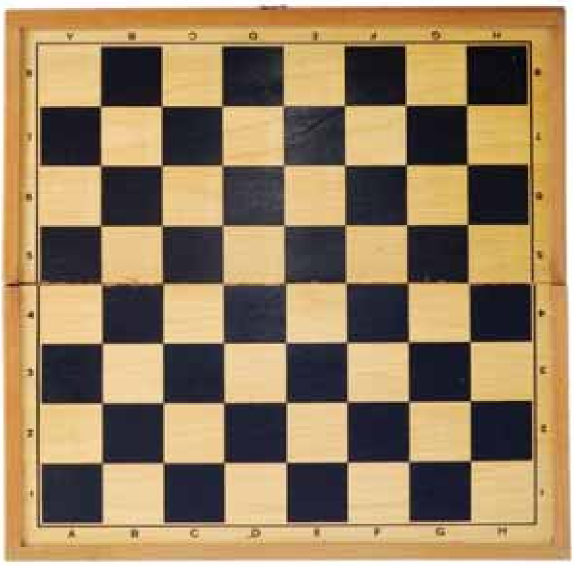
(b) These two rules are the same:

Rule 1: 'Take any number and multiply it by fifteen. Take the same number and multiply it by ten. Subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by five.'

Reasoning

7 A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.



(a) If x is the number of coins on the first square, how many are lying on the:

(i) second square

(ii) third square

(iii) sixth square?

(b) How many coins, in total, are there on the first six squares of the board?

(c) If each coin's value is $\$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins \$378.

8 On each of her birthdays, Georgia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write an expression for the amount of money Georgia received on her fourth birthday.

(b) How much money, in total, did Georgia receive for her first four birthdays? Write your answer in terms of d.

(c) How much money did Georgia receive altogether for her first four birthdays if $d = 8$?

(d) If Georgia receives \$567 on her fifth birthday, how much did she receive on her first birthday?

9 Kristian opens a bank account by depositing $\$y$. He then deposits $\$x$ every week and makes no withdrawals or other deposits.

The amount in his account after 5 weeks is:

A $x + 5y$ B $y + x + 5$ C $y + 5x$ D $5y + 5x$

Open-ended

10 This algebra pattern consists of the terms $x + y$, $2x + 3y$, $3x + 5y$, $4x + 7y$, … where the coefficient of x increases by 1 each time and the coefficient of y increases by 2 each time.

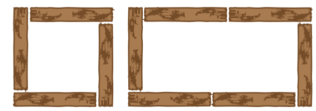
(a) Write an expression for the sixth term.

(b) Write an expression for the sum of the first five terms.

(c) Find two sets of values for x and y so that the sum of the first five terms is a multiple of 10.

(d) Find the sum for your chosen values.

11 A garden bed is made using rectangular timber planks. The planks are arranged to form square and rectangular garden beds as shown in the diagram. Let N be the total number of planks used to make the garden bed, L be the number of planks for the length of the bed and W be the number of planks for the width of the bed.



(a) Write down a rule that will find the number of planks needed for square garden beds of any side length.

(b) Write down a rule that will find the number of planks needed for a rectangular garden bed of length L and width W.

(c) Give examples of each type of garden bed using no more than 30 planks.

(d) If the length must always be double the width, rewrite the rule for the rectangular garden bed using only W as the pronumeral.

(e) Using this new rule, give examples of different types of rectangular garden beds using no more than 30 planks.

Outside the Square — Puzzle

Latin squares

a	d	b	c
c	b	d	a
d	a	c	b
b	c	a	d

The sum of each row, column and diagonal is represented by $a + b + c + d$.

If you divide the table into quarters, what is the sum of the four squares in each quarter?

Can you find other 2×2 groups of squares in the table whose sum is $a + b + c + d$?

Draw the two diagonals on the table. What can you say about the remaining rows and columns; that is, the rows and columns that are not on the diagonals?

Draw some 4×4 tables in your exercise book.

Pick four different values for a, b, c and d and make up some Latin squares, using the Latin square above to help you. Make sure that you check that your answers are actually Latin squares.

Leave out some of the values and ask your partner to find the missing numbers to complete your Latin square.

Your complete Latin square might look like:

9	8	14	3
3	14	8	9
8	9	3	14
14	3	9	8

You might then change it to this one before asking your partner to complete the Latin square.

9	8		
3		8	
8		3	
14	3		8

Age matters

Equipment required: 1 brain, 1 computer with an Excel spreadsheet program

Poh, Quentin and Rasheed are three friends who share the same birthday. They are now twelve and a half years old (150 months) and they have decided, after learning some algebra, that they may be able to negotiate a better pocket money agreement with their parents than the amount they get at present. They decide that the agreement they reach with their parents will run for 24 months.

Poh thinks that she will do quite well if she can get her parents to agree to pay her using the following formula.

$P = 2m - 285$ where m is her age in months and P is her monthly payment in dollars.

Quentin thinks he has a better plan. His formula is:

$Q = 3m - 440$ where m is his age in months and Q is his monthly payment in dollars.

Rasheed disagrees with the other two. He believes he has the winning formula.

$R = 4m - 595$ where m is his age in months and R is his monthly payment in dollars.

Versions of this Exploration are available for other technologies in Pearson Reader.

1 To see who has the best formula, open an Excel spreadsheet and set up four columns. Use P (\$) to represent Poh's pocket money, Q (\$) to represent Quentin's pocket money and R (\$) to represent Rasheed's pocket money. Enter Age in months (m) in A1, P (\$) in B1, Q (\$) in C1 and R (\$) in D1.

2 Enter 150 in A2 and 151 in A3. Highlight A2 and A3 and drag down to A13 using the small black cross on the bottom right-hand side of A3. Enter formulas in the formula bar for cells B2 (=2*A2 – 285), C2 (=3*A2 – 400) and D2 (=4*A2 – 595) as shown.

3 Total the amount of pocket money Poh receives in the first 12 months by entering the formula shown in cell B14 (=SUM(B2:B13)).

This screen shot shows the formulas you need to enter.

	A	B	C	D
1	Age in months(m)	P($)	Q($)	R($)
2	150	=2*A2-285	=3*A2-440	=4*A2-595
3	151			
4	152			
5	153			
6	154			
7	155			
8	156			
9	157			
10	158			
11	159			
12	160			
13	161			
14		=SUM(B2:B13)		

If you have entered each formula correctly your spreadsheet should look like this.

	A	B	C	D
1	Age in months(m)	P($)	Q($)	R($)
2	150	15	10	5
3	151			
4	152			
5	153			
6	154			
7	155			
8	156			
9	157			
10	158			
11	159			
12	160			
13	161			
14		15		

4 To fill the columns B, C and D, highlight B2 to D2, click on the small black cross on the right bottom corner of D2 and drag the formulas down to D13. Drag the formula shown in B14 across C14 and D14 to total the pocket money each received in 12 months.

Use your spreadsheet to answer the following questions.

5 How much would they each get paid in the first month if their parents agreed to this new arrangement?

6 Is there any time when they would all get the same amount for the month? If so, when does it happen and how much would they receive that month?

7 How much would each of them get on their 13th birthday ($m = 156$)?

8 What is the total amount each of them would receive in the first 12 months of their agreement?

9 Who has received the most pocket money after 12 months?

10 Extend your formulas for another 12 months. Who has received the most pocket money over 24 months? Is your answer the same as in Question **9**?

11 Which student has the best formula? Give reasons for your choice. What do you think each student would say to convince their parents to adopt their formula?

Taking it further

12 Poh's mum suggests a different formula. Her formula reads like this. $M = 2y + 2$ where y is the age in years and M is the pocket money in dollars. She says she will round to the nearest dollar.

(a) Add this formula to your spreadsheet, use the round function to round to the nearest dollar (=round(2*A2/12+2,0) in cell E2) and determine the total amount she would pay in the first 12 months.

	A	B	C	D	E
1	Age in mo	P($)	Q($)	R($)	M($)
2	150	15	10	5	27
3	151				
4	152				
5	153				
6	154				
7	155				
8	156				
9	157				
10	158				
11	159				
12	160				
13	161				
14		15			

(b) Compare the amount of pocket money Poh would earn in the first 12 months using this formula with the other three formulas.

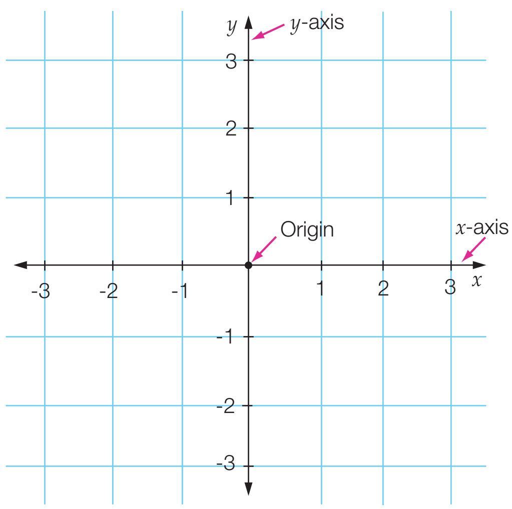
(c) Poh's older sister advises her to be careful before she decides which formula to use. Which formula would you advise her to use over a 2-year period. Justify your recommendation.

5.7 The Cartesian plane

In 1637, the French mathematician René Descartes developed a reference system that allowed any point on a plane to be located accurately. It was called the **Cartesian plane** (a plane is a flat 2-dimensional surface) in his honour, although it is also known as the number plane. This was an exciting idea as now algebra could be used to solve geometrical problems.

A Cartesian plane is constructed by drawing two lines at right angles to each other, one horizontal and the other vertical. The point at which they cross is called the **origin**. The horizontal line is called the x-axis and the vertical line is called the y-axis. Both **axes** (plural of axis) are number lines that extend infinitely in both directions. The integers on the axes are used to form a grid that allows any point to be located in reference to the origin.

The Cartesian plane is more accurate than alphanumeric grid systems used by street directories and spreadsheets, as the Cartesian plane locates points, whereas the alphanumeric system finds an area within a square or cell.



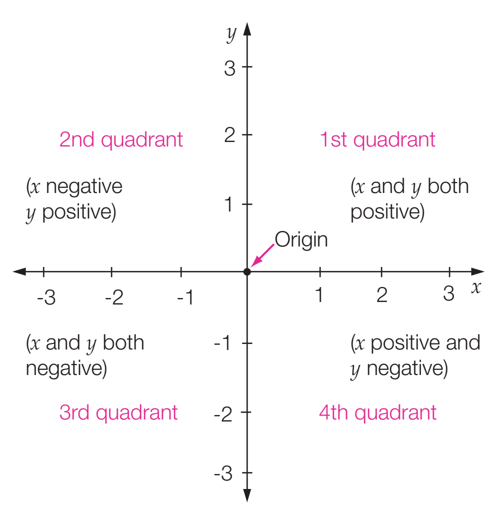
The position of any point on a number plane is described by a pair of numbers called the **coordinates** of the point. Coordinates are always written in brackets as an **ordered pair** (x, y). Locating any point on the plane involves two moves from the origin.

(x, y)

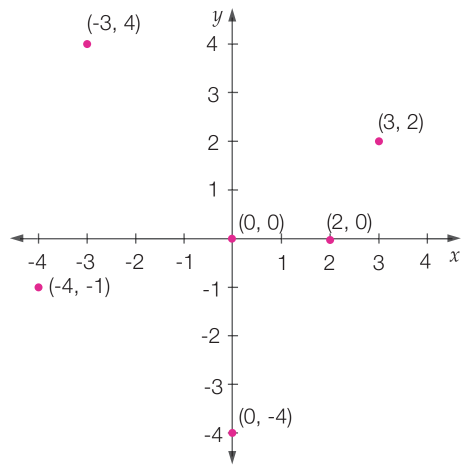
The x-coordinate shows the move to the left (–) or to the right (+) of the origin.

The y-coordinate shows the move up (+) or down (–) from the origin.

The x- and y-axes divide any plane into four **quadrants**. We number the quadrants starting with the quadrant in which both the x- and the y-coordinates have positive values. This is the 1st quadrant. We move in an anticlockwise direction to the 2nd, 3rd and 4th quadrants.



The following points and their Cartesian coordinates are shown on the Cartesian plane at right: (3, 2), (-3, 4), (-4, -1), (0, 0), (2, 0) and (0, -4).



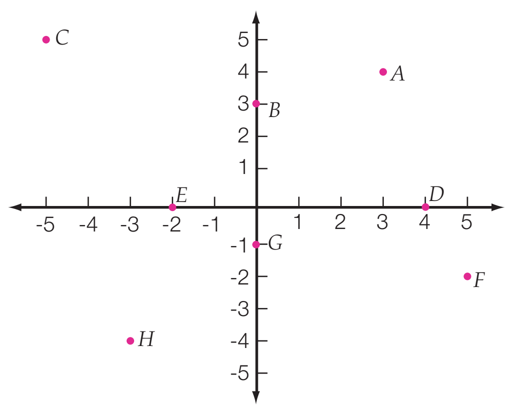
- We always move in the horizontal direction first and then in the vertical direction. This is a mathematical convention.
- The name 'ordered pair' tells you that order is important. The x-coordinate is always written first, and the y-coordinate is written second (x, y).
- The coordinates of the origin are (0, 0).

Worked Example 13

WE13

(a) Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



Thinking	Working
(a) For each point, find the number of units it is to the left or right of the origin. This is the x-coordinate of the point. Then, find the number of units it is up or down from the origin. This is the y-coordinate. The + and - signs indicate direction. (Point A is 3 units to the right of the origin and 4 units up from the origin.) Write the coordinates as an ordered pair.	(a) $A = (3, 4)$ $B = (0, 3)$ $C = (-5, 5)$ $D = (4, 0)$ $E = (-2, 0)$ $F = (5, -2)$ $G = (0, -1)$ $H = (-3, -4)$
(b) Identify the quadrant that each point is in. Points that lie on the x-axis or on the y-axis are not in a quadrant.	(b) A is in the 1st quadrant, C is in the second quadrant, H is in the 3rd quadrant and F is in the 4th quadrant. B, D, E and G lie on an axis, so they are not in any quadrant.

5.7 The Cartesian plane

Navigator

Answers page 653

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

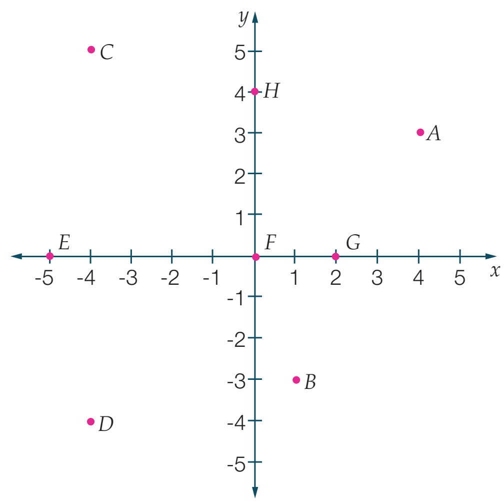
Equipment required: Graph paper for Questions **7**, **8** and **13**

Fluency

WE13

1 **(a)** Write the coordinates of each of the points *A* to *H* shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



2 **(a)** State whether each of the following coordinates is in quadrant 1, quadrant 2, quadrant 3 or quadrant 4.

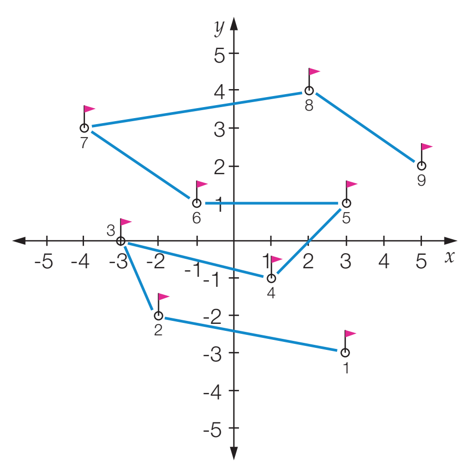
(i) (-2, 6) **(ii)** (5, -1) **(iii)** (-7, -4) **(iv)** (12, 10)

(v) (-32, 12) **(vi)** (51, -1) **(vii)** (-87, -90) **(viii)** (-2, 21)

(b) State whether each coordinate lies on the x-axis, the y-axis, or on both the x-axis and the y-axis.

(i) (0, 5) **(ii)** (0, -8) **(iii)** (0, 0) **(iv)** (-4, 0) **(v)** (15, 0)

3 The layout of a nine-hole golf course is shown on the number plane. Write in order the ordered pairs of the nine holes.



4 **(a)** A point is 1 unit right and 4 units up from the origin of a Cartesian plane. The coordinates of the point are:

A (4, 1) **B** (-4, 1)
C (1, 4) **D** (-1, 4)

(b) A point is 5 units left and 2 units up from the origin of a number plane. The coordinates of the point are:

A (-5, 2) **B** (2, -5) **C** (5, -2) **D** (-2, 5)

(c) A point is 4 units down and 3 units left of the origin of a Cartesian plane. The coordinates of the point are:

A (-4, -3) **B** (-3, -4) **C** (3, -4) **D** (-3, 4)

5 (a) The origin has the coordinates:

A (1, 0) B (0, 1) C (1, 1) D (0, 0)

(b) One coordinate that lies on the x-axis is:

A (0, 4) B (1, 1) C (2, 4) D (1, 0)

(c) One coordinate that lies on the y-axis is:

A (3, 0) B (3, 3) C (0, -3) D (-3, 0)

Understanding

6 (a) Which of the following points will give a vertical line passing through the x-axis when joined in a straight line to (10, -6)?

A (-6, -2) B (4, -6) C (10, 0) D (0, 10)

(b) Which of the following points will give a horizontal line passing through the y-axis when joined in a straight line to (8, 13)?

A (4, 2) B (8, -3) C (-5, 0) D (0, 13)

7 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -9 to 9 on the x-axis and -4 to 4 along the y-axis. Plot the following points and join them in the order given to form a picture. You may like to colour your picture when complete.

(-4, -1) (-5, -1) (-5, 0) (-9, 1) (-6, 3) (-1, 2) (-4, -1) (-4, -2) (-1, -4) (0, -3) (-1, 2) (0, 3) (1, 2) (6, 3) (9, 1) (5, 0) (5, -1) (4, -1) (1, 2) (0, -3) (1, -4) (4, -2) (4, -1) STOP

Now join (3, 4) to (0, 2.5) and join (-3, 4) to (0, 2.5).

8 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -8 to 9 along the x-axis and -11 to 11 along the y-axis. Join each of the following sets of points in the order given. When you reach the word STOP, lift your pencil and start again from the next pair of coordinates.

Join (3, 0) (3, 5) (2, 5) (2, 4) (1, 4) (1, 5) (0, 5) (0, 4) (-1, 4) (-1, 5) (-2, 5) (-2, 4) (-3, 4) (-3, 5) (-4, 5) (-4, -9) (3, -9) (3, 0) (4, 0) (4, -1) (5, -1) (5, 0) (6, 0) (6, -1) (7, -1) (7, 0) (8, 0) (8, -9) (3, -9) STOP

Join (5, 0) (5, 7) ($6\frac{1}{2}$, 10) ($6\frac{1}{2}$, 11) (9, 10) ($6\frac{1}{2}$, 10) (8, 7) (8, 0) STOP

Join (-7, -2) (-7, -9) (-4, -9) (-4, 3) (-5, 3) (-5, 2) (-6, 2) (-6, 3) (-7, 3) (-7, 6) ($-7\frac{1}{2}$, 8) (-8, 6) (-8, -1) (-7, -2) STOP

Join (-2, -6) (-1, -5) (0, -5) (1, -6) (-2, -6) (-2, -9) (-3, -11) (2, -11) (1, -9) (1, -6) STOP

Join (3, 5) (3, 7) ($3\frac{1}{2}$, 9) (4, 7) (4, 4) (3, 1) STOP Join (-4, 5) (-4, 9) (-2, 8) (-4, 8) STOP

Join (6, -5) (7, -5) (7, -2) (6, -2) (6, -5) STOP Join (6, 6) (7, 6) (7, 3) (6, 3) (6, 6) STOP

Join (2, -1) (2, 2) (1, 2) (1, -1) (2, -1) STOP Join (-3, -1) (-2, -1) (-2, -4) (-3, -4) (-3, -1) STOP

Join (-6, -3) (-5, -3) (-5, 1) (-6, 1) (-6, -3) STOP Join (3, 7) to (4, 7) STOP

Join (5, 7) to (8, 7) STOP Join (-8, 6) to (-7, 6) STOP

Join (-2, -6) to (-3, -1) STOP Join (1, -6) to (2, -11) STOP

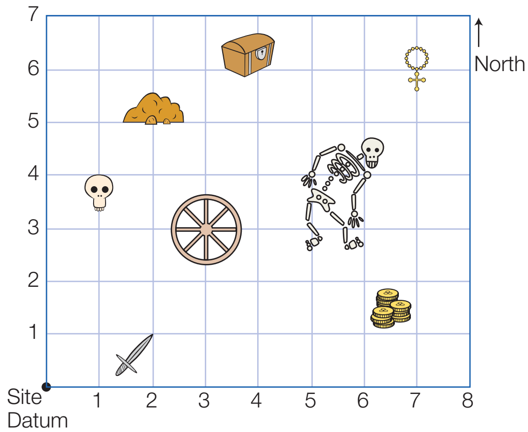
9 (a) Which of the following coordinate pairs lies furthest up on the Cartesian plane?

A (-3, 2) B (2, 3) C (3, -2) D (1, -3)

(b) Which of the following coordinate pairs is the furthest to the left on the Cartesian plane?

A (-3, 2) B (2, 3) C (3, -2) D (1, -3)

10 The grid shown allows an archaeologist to reconstruct the layout of an old tomb. (Archaeologists call the point (0, 0) the datum point.) State the point(s) at which each of the following is found.



(a) the centre of the base of the treasure chest

(b) the middle of the rock face base

(c) the tip of the dagger

(d) the centre of the chariot wheel

(e) the right wrist and the left shoulder of the skeleton

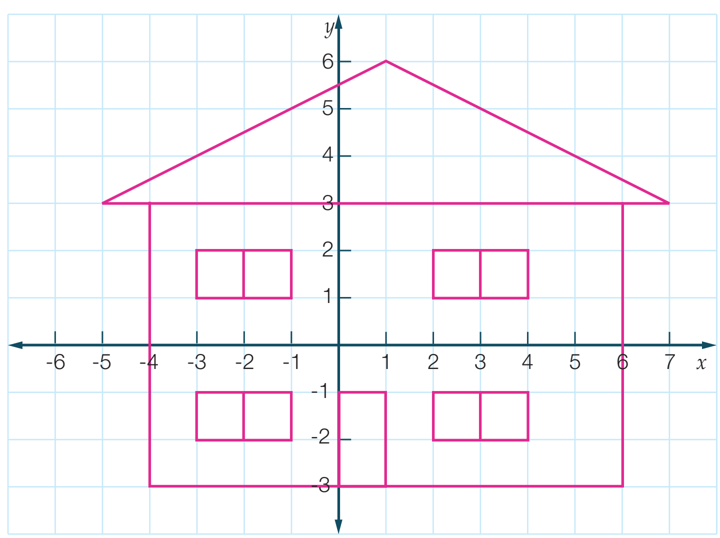
(f) the four grid points that enclose all the coins

(g) the top of the skull lying by itself

(h) the chain at the point where the cross is attached

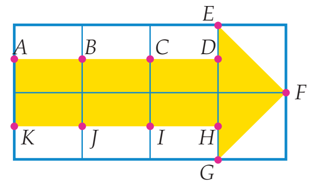
Reasoning

11 Write instructions using coordinates so that this drawing could be reproduced by another student.



Open-ended

12 Draw this arrow shape on a Cartesian grid so that it lies in at least three quadrants, and give the coordinates of each of the points *A* to *K*.



13 On a piece of grid or graph paper, rule a set of axes to form a Cartesian plane. Allow for a scale from -5 to +5 on both axes.

(a) Using more than one quadrant on your Cartesian plane, use straight lines to draw a simple design of your own.

(b) Write instructions using coordinates so that your design could be reproduced by another student.

(c) Give your instructions to another student to see if they can draw your design exactly.

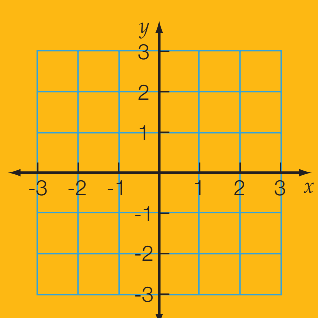
Outside the Square Game

Line up

Equipment required: 2 brains, grid paper, die

How to win:
Five games are played and the player with the greatest total of points is the winner.

How to play:



1 Copy the grid. Take turns to roll the die. The aim is to get three consecutive coordinates in a row, a column or a diagonal.

2 The number on the die is multiplied by 1 and -1 and then split into a sum or difference of two numbers used as the coordinates. For example, if a 3 is rolled, the coordinates might be (0, 3), (3, 0), (0, -3), (-3, 0), (1, 2), (2, -1), (1, -2), (-1, 2), (-1, -2), (-2, -1).

The player can mark any one of these positions.

3 Some combinations cannot be used. For example, if a 6 is rolled, the only combinations that fit on the playing board are (3, 3), (-3, -3), (-3, 3), (3, -3) but not, for example, (-5, 1) or (6, 0).

4 If a player cannot mark a coordinate because it is already marked, they miss a turn.

5 When a player has three in a row, the number of points earned is the total of the numbers, ignoring the minus sign.

For example, for the coordinates (-3, 2), (-2, 2) and (3, 2), the points awarded are
3 + 2 + 2 + 2 + 3 + 2 = 14.

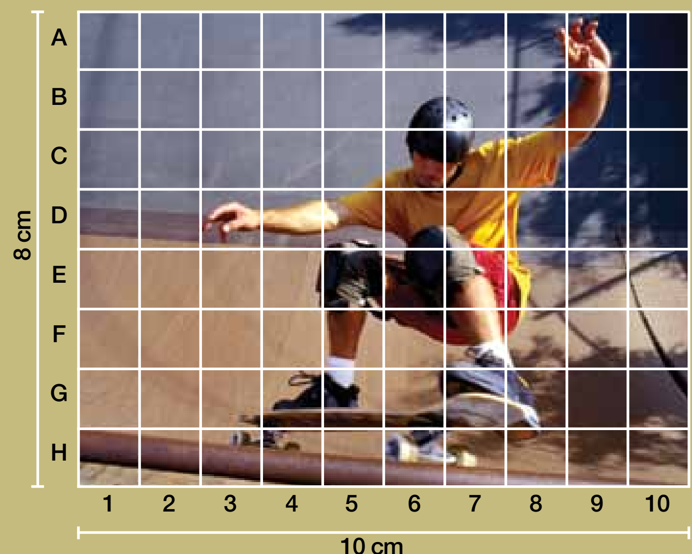
6 Then, clear the board for the next round.

7 Five games are played and the player with the greatest total of points is the winner.

Maths 4 Real

MATHS MEETS ART

Some Art schools teach students to paint using the 'grid method', a method used by Leonardo da Vinci. The method involves overlaying a grid onto the image to be painted and also placing a matching grid pattern onto the canvas. For example, if the image shown is an 8 cm × 10 cm photograph, 1 cm × 1 cm squares can be used to create a grid pattern of eighty squares. If the image is to be copied onto an 80 cm × 100 cm canvas, each square will be 10 cm × 10 cm.



Note that the image and canvas above are not shown to scale.

1 State the coordinates of each of the following.

(i) right foot **(ii)** left ear

(iii) left elbow **(iv)** left hand

2 Suppose you wish to copy the image onto a canvas that is 16 cm × 20 cm.

(a) What size square on the canvas is needed to represent one square centimetre on the image?

(b) Use centimetre grid paper or graph paper to copy the image.

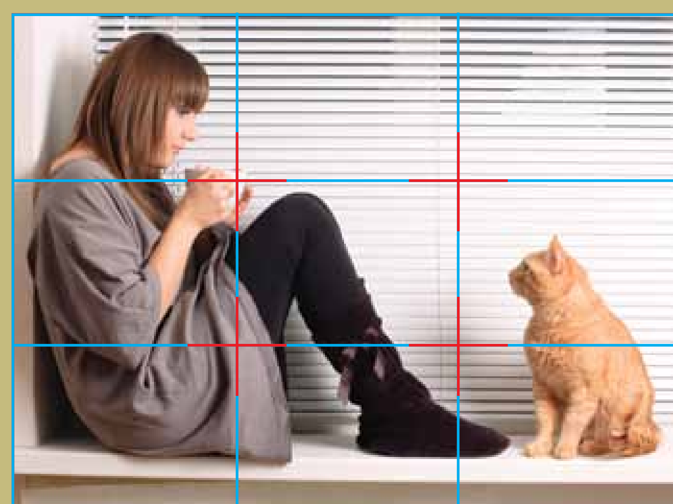
The rule of thirds

A basic principle in photography and art is 'the rule of thirds'. The image is divided into thirds horizontally and vertically to obtain nine squares or rectangles.

If points of interest are placed in any of the four marked intersections or along the lines, the image is more balanced and is pleasing to the eye.

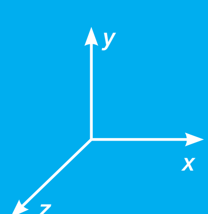
3 Do you think 'the rule of thirds' was used in the composition shown below?

4 Use graph or grid paper to create an image of your own using 'the rule of thirds'.



Research

- The rule of thirds is based on the Fibonacci spiral. Find out how the spiral is created.
- Find out more about the art of Piet Mondrian, and either present a five minute talk with illustrations or prepare a poster.
- Zedism is a painting style that gives a 3-dimensional effect. It uses a third axis (called the *z*-axis) which is at right angles to the Cartesian plane. By using geometry and perspective, the effect is an illusion of form and structure.



"Sky"

Find out how this is achieved and look for other examples of paintings where this method is used.

5.8 Patterns and plotting points

When points are plotted on a number plane they often appear to follow a pattern or shape. This shape may be a straight line or it may be a curve. If we can draw a straight line through all the points we form a **linear graph**. Any set of points with a definite shape, such as a straight line, can be described by an algebraic rule.

Worked Example 14

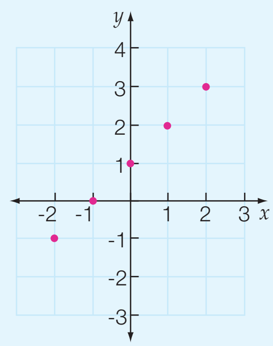
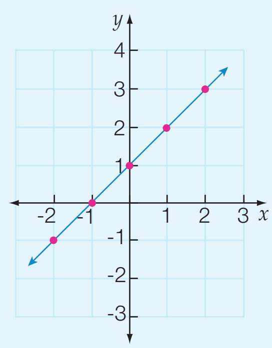
WE14

(a) Plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) on a number plane.

(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

(d) Write down the rule linking the x- and y-values.

Thinking	Working
(a) Use a ruler to draw and number the x- and y-axes on graph or grid paper. Label your axes and plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) moving left or right first, then up or down.	(a) 
(b) Join the points carefully with a ruler. You should have a straight line.	(b) 
(c) Construct a table with x-values on the top line and y-values on the second line.	(c) see table below
(d) **1** Look for a link between the x- and y-values. (Each y-value is 1 more than the x-value.)	(d)
2 State the rule.	The rule is $y = x + 1$.

(c)

x	-2	-1	0	1	2
y	-1	0	1	2	3

Worked Example 15

WE15

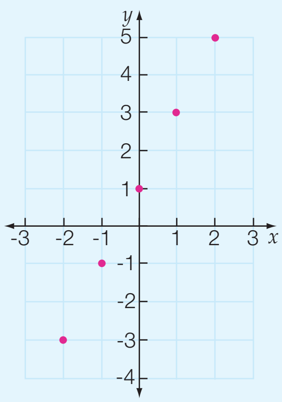
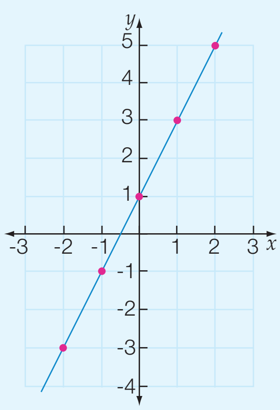
(a) List the coordinates of the points given in the table of values below.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

(d) Write down the rule linking the x- and y-values.

Thinking	Working
(a) Write the coordinates as an ordered pair with the x-coordinate first and the y-coordinate next.	(a) (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5)
(b) Use a ruler to draw and number the x- and y-axes on graph or grid paper. Label your axes and plot the points (-2, -3), (-1, -1), (0, 1), (1, 3) and (2, 5) moving left or right first, then up or down.	(b) 
(c) Join the points carefully with a ruler. You should have a straight line.	(c) 
(d) 1 Look for a link between the x- and y-values. (The y-values go up by 2, so we are multiplying by 2. Each y-value is 1 more than twice the x-value.)	(d)
2 State the rule.	The rule is $y = 2x + 1$.

5.8 Patterns and plotting points

Navigator

Answers page 653

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: Graph paper

Fluency

WE14

1 (a) Plot the points (-2, 0), (-1, 1), (0, 2), (1, 3) and (2, 4) on a number plane.

(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

(d) Write down the rule linking the x- and y-values.

WE15

2 (a) List the coordinates of the points given in the table of values below.

x	-1	0	1	2	3
y	-3	-1	1	3	5

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

(d) Write down the rule linking the x- and y-values.

3 (a) Plot the points (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1) on a number plane.

(b) Join the points in this order.

(c) Summarise the set of points in a table.

(d) Write down the rule linking the x- and y-values.

Understanding

4 Which one of the points (-1, -1), (2, 2), (3, 4) and (5, 5) will not lie on the same straight line as the three other points?

A (-1, -1) B (2, 2) C (3, 4) D (5, 5)

5 (a) Plot the points (-2, 8), (0, 6) and (5, 1) on a number plane.

(b) Draw a straight line passing through all the points.

(c) What is the y-coordinate of a point on the line if its x-coordinate is -1?

(d) What is the x-coordinate of a point on the line if its y-coordinate is 5?

6 (a) Plot the points (1, 1), (1, 5), (5, 5) and (5, 1) on a number plane and join the points in order.

(b) Name the geometrical figure drawn in part **(a)**.

Reasoning

7 (a) Plot the points (0, 0), (2, 2) and (-5, -5) on a number plane.

(b) Draw a straight line passing through all the points.

(c) Write down the coordinates of three other points through which the line passes.

(d) Summarise the set of points in a table.

(e) Write down the rule linking the x- and y-values.

(f) By substituting $x = -3$ into the rule, show that the point (-3, -3) lies on the line.

8 (a) Plot the points (-1, -5) and (5, 13) on a number plane and join them with a straight line.

(b) Write down the coordinates of three other points through which the line passes.

(c) Summarise the set of points in a table.

(d) Write down the rule linking the x- and y-values.

(e) Substitute $x = 0$ into the rule to show that (0, -3) does not lie on the line.

(f) Plot (0, -3) on the number plane to confirm it does not lie on the line.

Open-ended

9 (a) Plot the point (3, 3) on a number plane.

(b) Draw a straight line passing through this point and one other point.

(c) Write down the coordinates of two other points through which the line passes.

(d) Write down the rule linking the x- and y-values for this straight line.

(e) Repeat **(b)**, **(c)** and **(d)** using (3, 3) and a different point to find a different rule.

10 Using -4 to 4 on the x- and y-axes:

(a) choose a point and write down its coordinates

(b) choose two other points that will form a line that is not vertical or horizontal

(c) find the rule for that line.

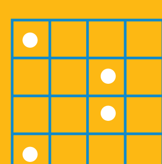
Outside the Square Puzzle

Gridlock

Equipment required: 1 brain, grid or graph paper

(a) In grid 1, mark four squares so that no two marked squares lie in the same row, column or diagonal.

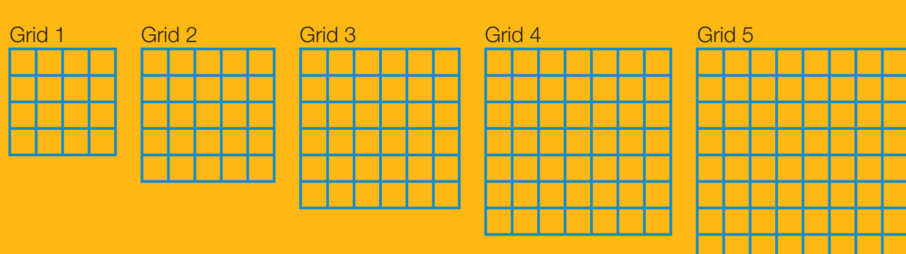
For example, the arrangement shown is not allowed.



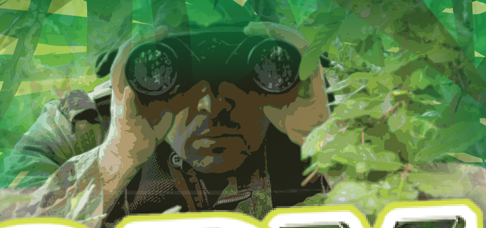
(b) In grid 2, mark five squares so that no two marked squares lie in the same row, column or diagonal.

(c) Mark six squares in Grid 3 and seven squares in Grid 4 so that no two marked squares lie in the same row, column or diagonal.

(d) The challenge: Mark eight squares in Grid 5 so that no two marked squares lie in the same row, column or diagonal.



Mathspace



SPY VS SPY

Equipment required:

For task #1: 1 brain, graph paper, ruler

For task #2: 1 brain

For task #3: 2 brains, 1 die, counters or small pieces of paper

While working on Operation Cartesian as a spy for the Confederation of Mathematicians, Freedo has been caught by the Anti-maths guerrilla forces deep in the jungle.

Task #1: Frame the assassin

Freedo's last communication from headquarters gave him a picture of 'Zero', the most deadly of the Anti-maths guerrillas.

To decipher the picture, Freedo must apply his knowledge of Cartesian graphs. On your graph paper begin by creating a set of axes, from -8 to 8 along the *x*-axis and -10 to 7 along the *y*-axis.

Use a ruler to join each of the following sets of coordinates, like a dot-to-dot drawing. Where there is a •, lift your pen and start a new line without joining to the previous lines drawn.

(0, 7), (2, 7), (4, 6), (6, 4), (6, 1), (7, 2), (8, 0), (7, -2), (6, -3), (5, -5) •

(4, -6), (3, -8), (1, -10), (0, -9) •

(0, -8), (1, -7), (3, -7), (1, -6), (0, -6) •

(3, -7), (1, -5), (0, -5) •

(0, 2), (2, 4), (5, 4), (3, 3), (1, 1), (1, -1), (2, -2), (1, -3), (0, -3) •

(0, -4), (1, -3), (3, -4), (6, -6), (4, -6), (1, -4), (0, -4) •

(6, -1), (7, -1), (7, 1), (6, 0) •

(1, 1), (2, 0), (5, 1), (4, 2), (2, 2), (2, 1), (3, 1), (3, 2) •

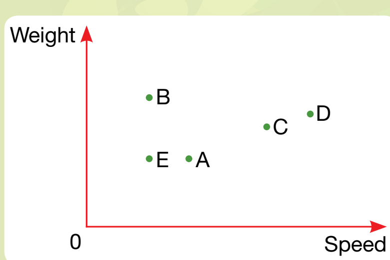
Then complete the image by drawing a reflection of the current image in the *y*-axis.

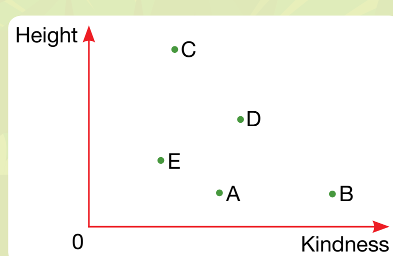
Task #2: Guard help

Freedo is locked in a cage. He was given the following information from headquarters:

'The guard who will help you is one of the two shortest guards and one of the two slowest guards.'

Looking at the following graphs can you figure out which guard will help Freedo?





Task #3: Quadrant Tic Tac Toe

Play this game to see if you are able to escape from the Anti-maths guerrilla forces.

Choose who will play the role of Zero and who will play the role of Freedo. Will Freedo escape?

Your aim is to be the first to have 3 counters in a row, column or diagonal.

Take turns in rolling. You can place counters on the intersections of the grid lines, according to the following rules.

Rolling a 1 means you must place a counter in quadrant 1.

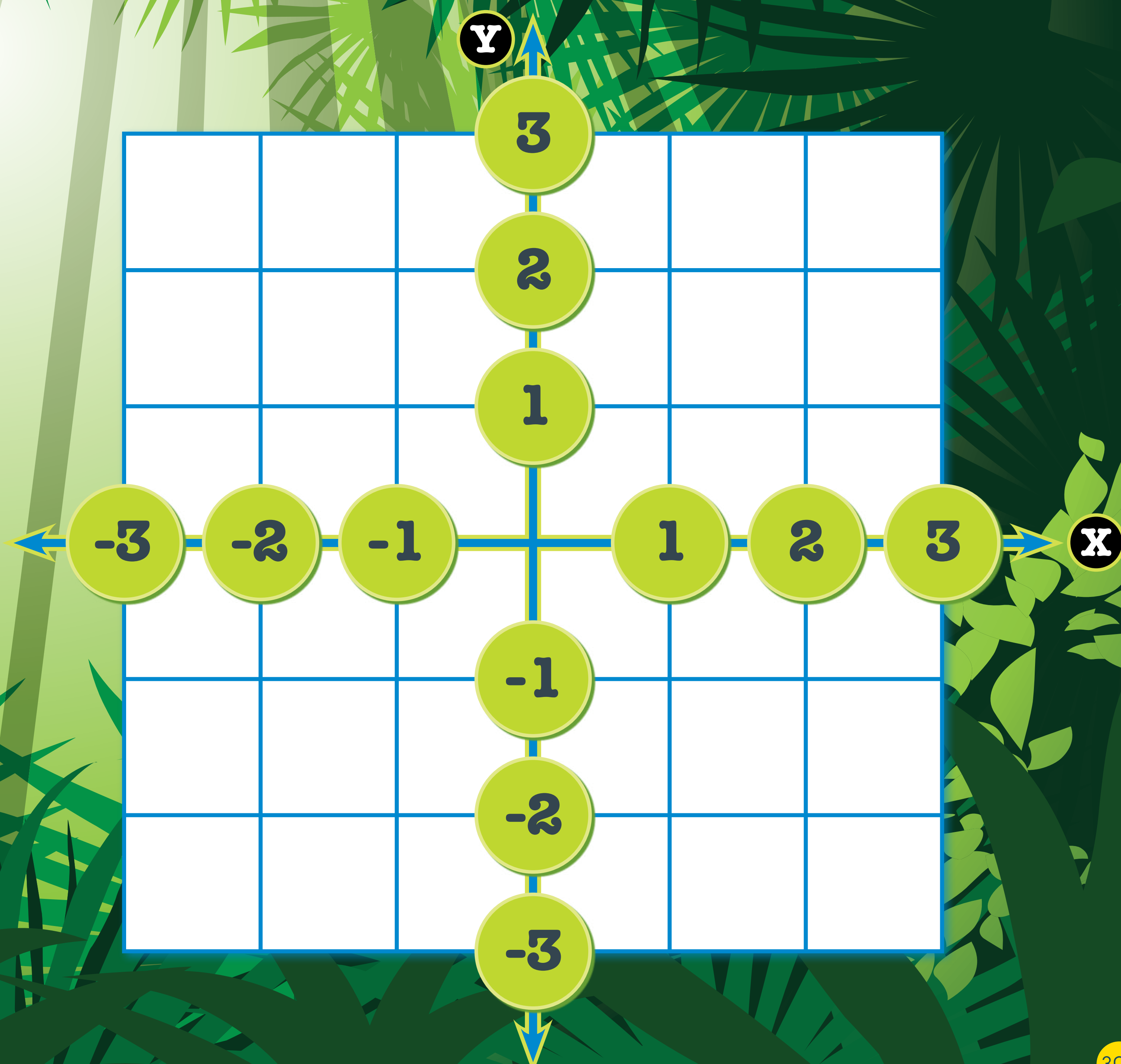
Rolling a 2 means you must place a counter in quadrant 2.

Rolling a 3 means you must place a counter in quadrant 3.

Rolling a 4 means you must place a counter in quadrant 4.

Rolling a 5 means you must place a counter on the x-axis.

Rolling a 6 means you must place a counter on the y-axis.



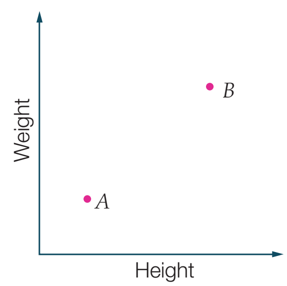
5.9 Interpreting graphs

Sometimes, when we plot points showing information relating to two variables, the points should not be joined.

This type of graph is called a **point graph**.

In the graph below, the points labelled *A* and *B* tell us about the height and weight of Oscar and Theodore.

Point *A* matches Theodore (smaller height and lower weight) and point *B* matches Oscar (taller and heavier). The graph shows that Oscar is taller than Theodore and that Theodore weighs less than Oscar.



Worked Example 16

WE16

This graph shows the age and height of Faye and Morgana.

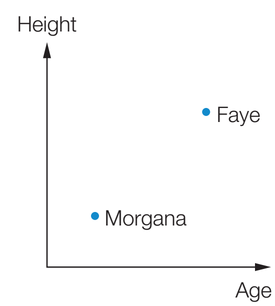
Answer TRUE or FALSE to each of these statements.

(a) Faye is taller than Morgana.

(b) Morgana is older than Faye.

(c) Faye is younger than Morgana.

(d) Morgana is shorter than Faye.



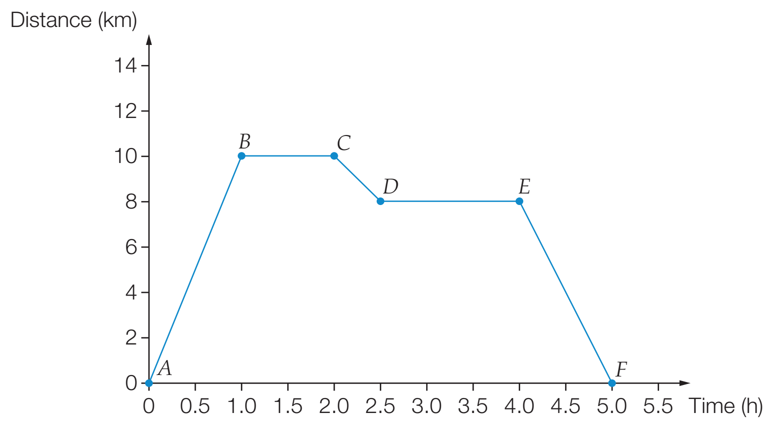
Thinking	Working
(a) Compare heights. Height increases the further you move up the vertical axis. The point for Faye is higher, so she is taller than Morgana.	(a) Faye is taller than Morgana. TRUE
(b) Compare ages. Age increases the further you move to the right. The point for Faye is further to the right, so she is older than Morgana.	(b) Morgana is older than Faye. FALSE
(c) Compare ages. Faye is older than Morgana as the point for Faye is further to the right.	(c) Faye is younger than Morgana. FALSE
(d) Compare heights. The point for Morgana is lower, so she is shorter than Faye.	(d) Morgana is shorter than Faye. TRUE

If we join points with straight lines, they may not form a linear graph, as the points may not all lie on the same straight line. These graphs are called **line graphs**. An example of line graphs are travel graphs where time is plotted on the x-axis and distance is plotted on the y-axis. When lines are horizontal, the distance is not changing. When lines are steepest, the speed is greatest.

Worked Example 17

WE17

May and Kim go jogging. They leave from Kim's house, jog to their friend Jenny's place, stay there for a swim, then jog to May's place. They stay there for a while and watch a DVD before jogging back to Kim's house.



Using the graph given above that shows their journey, answer the following questions.

(a) What are the graph coordinates of Kim's house?

(b) How far away from Kim's house does Jenny live?

(c) How long did it take to reach Jenny's house?

(d) How long did they stay at Jenny's house?

(e) How far from Kim's place does May live?

(f) How long did they stay at May's house?

(g) How long were they away from Kim's house?

(h) When were they jogging the fastest?

Thinking	Working
(a) Identify where the journey begins (A).	(a) (0, 0)
(b) Look for the distance to B on the y-axis. The y-coordinate of B gives that distance.	(b) 10 km
(c) Look for the time taken to get from A to B on the x-axis. The x-coordinate of B gives the time.	(c) 1 h
(d) The horizontal line BC shows that they stayed in the same place during this time, so the time will be the time difference between B and C.	(d) 1 h
(e) Look for the distance D on the y-axis. The y-coordinate of D gives the distance.	(e) 8 km

(f) The horizontal line *DE* shows that they stayed in the same place during this time, so the time will be the time difference between *D* and *E*.	(f) $1\frac{1}{2}$ h
(g) Look for the time to get from *A* to *F* on the *x*-axis. The *x*-coordinate of *F* gives the time.	(g) They were away for 5 hours.
(h) Look for the steepest line. This is the line from *A* to *B* at the start of the jog.	(h) They jogged fastest from Kim to Jenny's place.

5.9 Interpreting graphs

Navigator

Answers page 655

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q11, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14

Fluency

WE16

1 This graph shows the age and height of Sunjay, Zack and Brad.

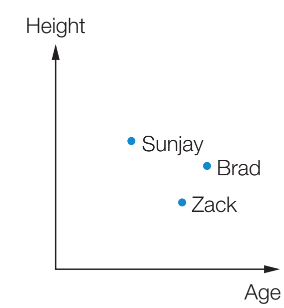
Answer TRUE or FALSE to each of these statements.

(a) Brad is taller than Sunjay.

(b) Zack is the shortest.

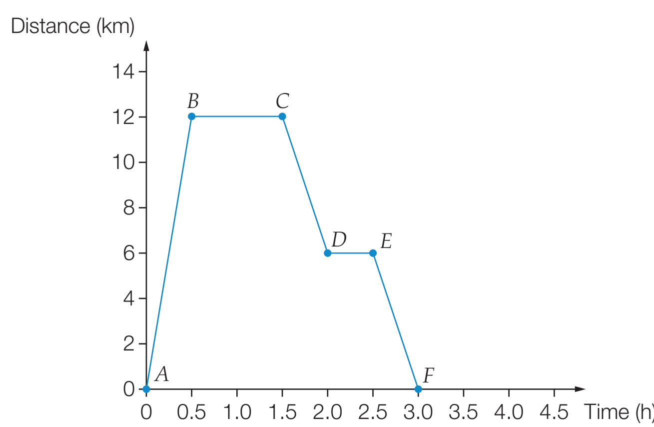
(c) Brad is older than Zack.

(d) Sunjay is the youngest.



WE17

2 Ray and Jim go for a bike ride. They leave from Jim's house, ride to their friend Terry's house, stay there for lunch, and then ride to Ray's house. After a short break, they ride back to Jim's house.



Using the graph given above that shows their journey, answer the following questions.

(a) What are the graph coordinates of Jim's house?

(b) How far away from Jim's house does Terry live?

(c) How long did it take to reach Terry's house?

(d) How long did they stay at Terry's house?

(e) How far from Jim's house does Ray live?

(f) How long did they stay at Ray's house?

(g) How long were they away from Jim's house?

(h) When were they riding the fastest?

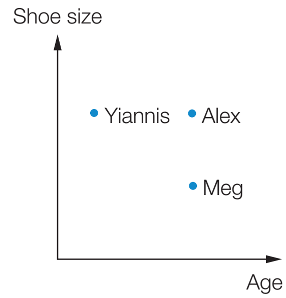
3 Use the graph opposite to answer the following questions.

(a) Who is the youngest?

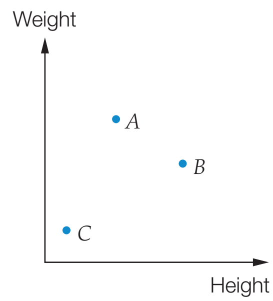
(b) Who takes the smallest shoe size?

(c) What can you say about the shoe sizes of Yiannis and Alex?

(d) What can you say about the ages of Meg and Alex?

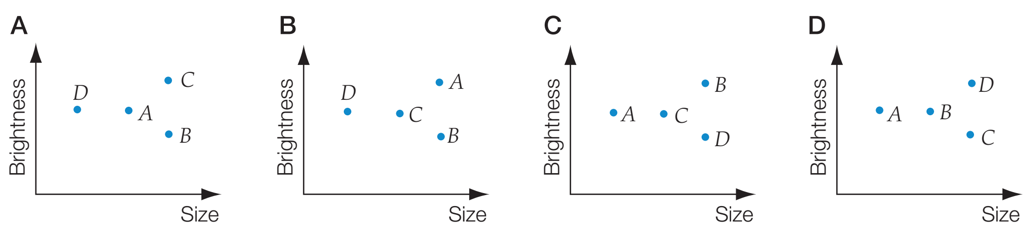


4 Consider this graph showing the weight and height of an elephant, a giraffe and a kangaroo. Match each point with the animal it represents.

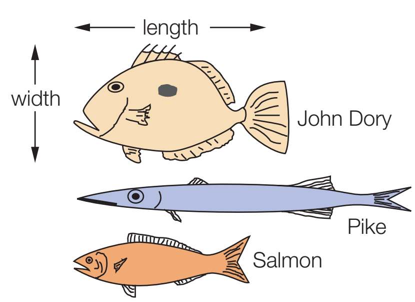


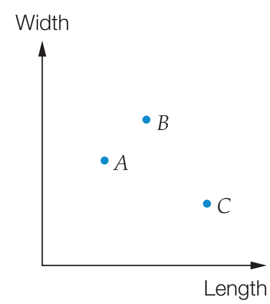
Understanding

5 Four light globes, A, B, C and D have the properties that globe A and globe B are equally bright and globe C and globe D are of equal size. Which graph can represent this situation?



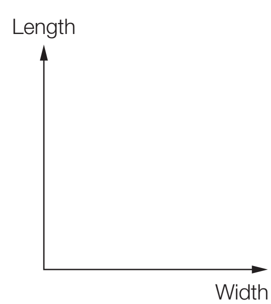
6 Consider this graph showing the widths and lengths of various fish.

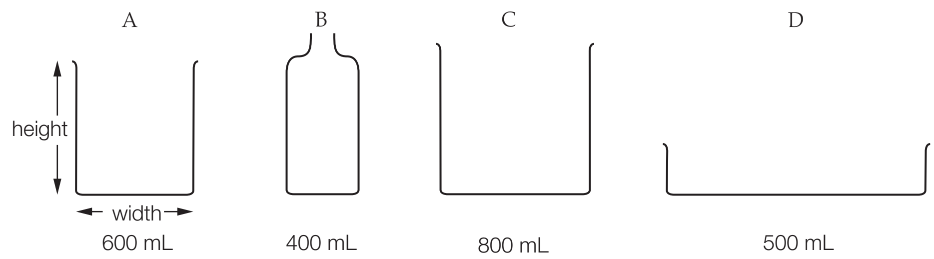




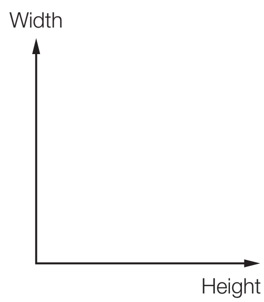
(a) Match each point with the fish it represents.

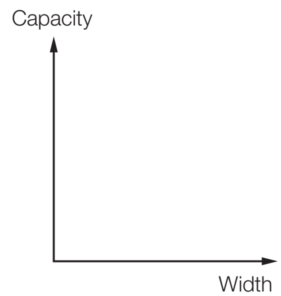
(b) Copy and complete this graph to show the positions of the points A, B and C for the three fish.

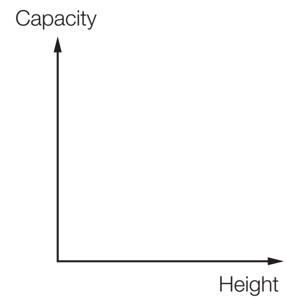


7 

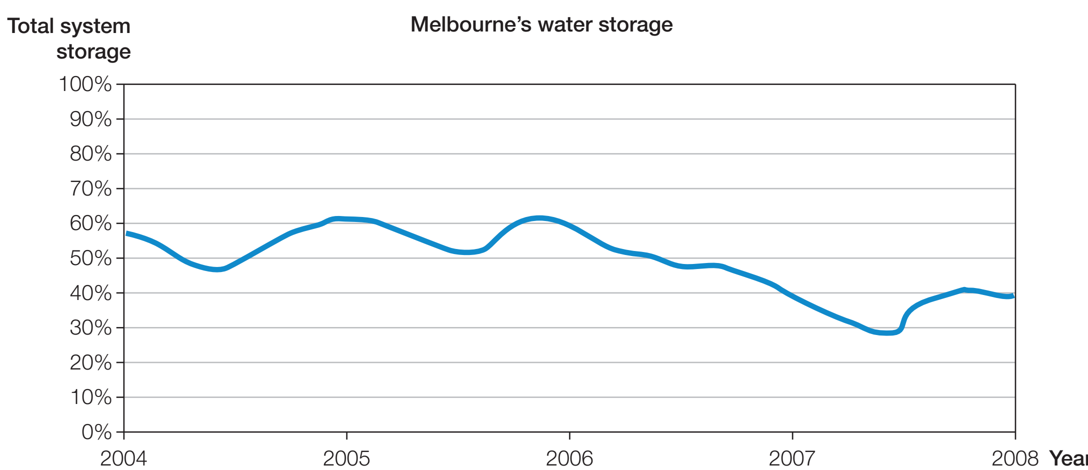
Complete the following point graphs for the containers A, B, C and D.

(a) 

(b) 

(c) 

8 Large cities are usually surrounded by dams that collect rain to supply the city with water. The total amount of water stored in the dams is called the water storage and when this amount is calculated as a percentage of the total capacity of all the dams it gives a measure of how full the dams are on average.



Above is a graph of Melbourne's water storage from 2004 to 2008. Use this graph to answer the following questions.

(a) What was the storage percentage at the start of 2004?

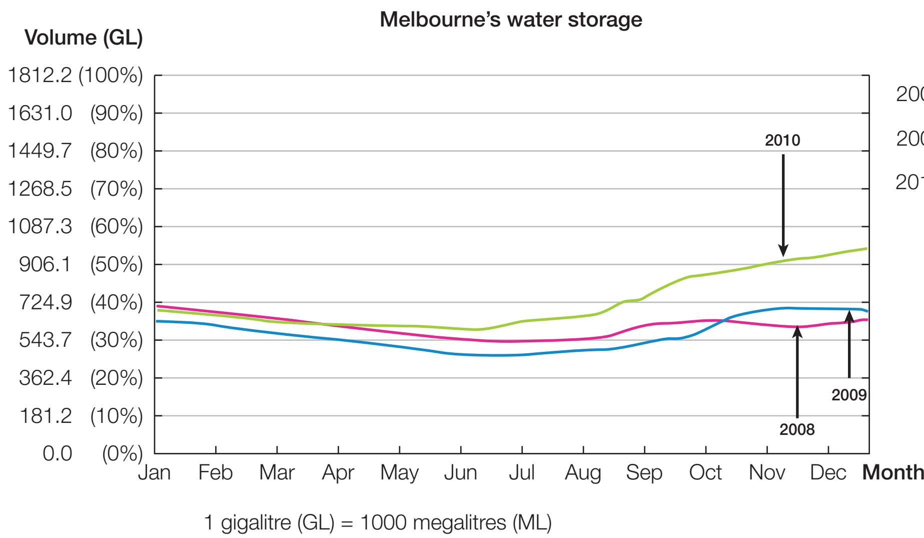
(b) Why did the water storage percentage rise in the second half of 2004?

(c) Why is there an increase in storage percentage in the second half of each year (except 2006)?

(d) What does the graph tell you about Melbourne's rainfall from the start of 2006 to mid 2007?

(e) When was the water storage the lowest in the time period 2004–2008? What was the lowest percentage level reached?

9



Above is a graph of Melbourne's water storage for the years 2008, 2009 and 2010.

Use this graph to answer the following questions.

(a) Compare the storage levels at the start of 2008, 2009 and 2010. Were the levels similar? Which year started with the lowest storage level? What was the difference between the highest and lowest storage percentage at the start of each year?

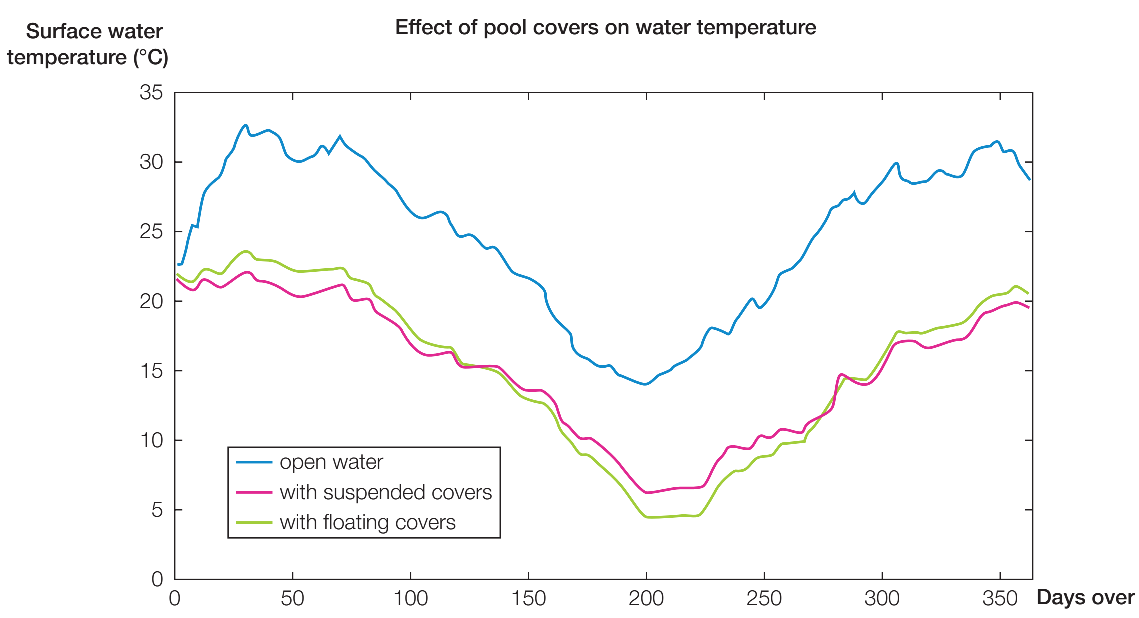
(b) Compare the storage levels at the end of 2009 and 2010. What is the difference in storage percentage?

(c) During which 3 months over the 3 year period was the storage level the highest?

(d) During which 3 months over the 3 year period was the storage level the lowest?

Reasoning

10 Evaporation is a major source of water loss from storage facilities. The rate at which water evaporates increases as temperature increases. The lower the temperature of the water, the less water will be lost to evaporation.



Above is a graph showing the effect on water temperature of using covers of different types on a pool. Use this graph to answer the following questions.

(a) Do you think covers reduce evaporation? Give reasons for your answer.

(b) Is one type of cover more effective than the other at reducing evaporation? Give reasons for your answer.

(c) When is a floating cover more effective than a suspended cover?

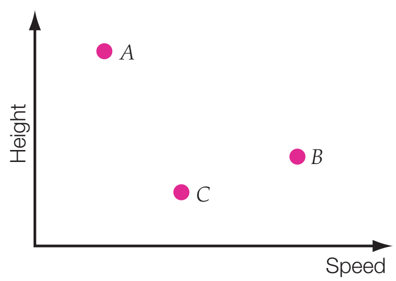
(d) What effect overall does the use of pool covers have on the water temperature?

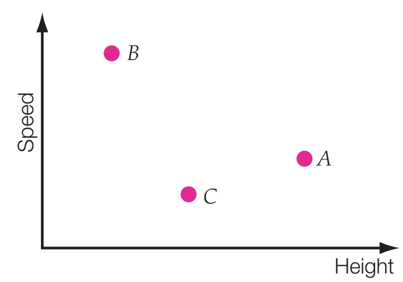
11 (a) What are the two variables shown in this graph?

(b) What happens to the shoe size as the height increases?

Open-ended

12 Agnes transferred the points on the first graph to the graph with swapped axes. She left *C* in the same place but swapped the positions of *A* and *B*.





(a) Explain why Agnes' actions are not correct for these graphs. When would it be correct to do the simple swap with *A* and *B*?

(b) What would need to be the properties of a point if it is to stay in the same place?

(c) Draw what the second graph should look like.

13 Tom, Adam, Sheena and Toula play basketball. Sheena is shorter than Tom, and Toula scores more points than Adam. One point of view is that the taller the basketball player, the more points they score. Using the variables 'Height' and 'Points scored', draw two possible graphs that show the relationship.

14 Draw a travel graph of your own and write a few sentences to explain the journey.

Outside the Square Puzzle

Solve the grid

Each letter stands for a different number, but it represents the same number each time it is used. The total for the first row and the first two columns is given.

D	D	E	D	39
E	D	D	E	
E	E	E	X	
X	E	X	X	
40	42			

1 What number does X represent?

2 Find the total of each of the other rows and columns.

Challenge 5

1 George has 15 coins, all 50c and 20c pieces. If the 20c pieces were 50c pieces and the 50c pieces were 20c pieces, he would have \$2.10 more. How many 50c pieces does he actually have?

2 If $p \ast q$ means $3(p + q)$, what is the value of $5 \ast (2 \ast 8)$?

3 Steven and Claire live next door to each other. The product of their house numbers is 483. What are their house numbers?

4 If $x > 4$, put these terms in order from smallest to largest.

$$\frac{x}{4} \qquad \frac{4}{x} \qquad \frac{4}{x+1} \qquad \frac{x+1}{4}$$

5 Each letter represents one of the digits 1, 2, 3, 4 or 5.

$$\begin{array}{r} \mathrm{K\,L} \\ \times \quad \mathrm{M} \\ \hline \mathrm{P\,N} \\ \hline \end{array}$$

The answer to this multiplication is correct.

Which statement is correct?

A K = 3 **B** L = 3 **C** M = 3 **D** N = 3

6 What are possible positive values for a and b if $(a \times b)^2 = a \times b^2$?

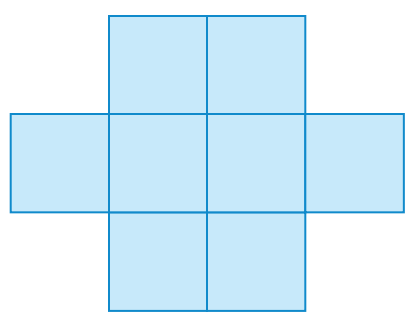
7 In a 30-question test you earn 9 marks for every correct answer and lose 5 marks for every wrong answer. You must attempt every question.

(a) How many marks do you get for 15 correct answers? (Assume you attempt all the questions.)

(b) What is the greatest number of marks you can get if you have more answers wrong than correct?

(c) Is it possible to score exactly zero on the test? Explain why or why not.

8 Place one of the numbers 1, 2, 3, 4, 5, 6, 7 and 8 in each square so that no consecutive numbers are beside each other; that is, 2 is not next to a 3 either above, below, to the left or to the right.



9 If half the number represented by x is 24, what is the value of $2x$?

10 The sum of three consecutive integers is 90. What is the smallest of the three integers?

A 28 **B** 29 **C** 30 **D** 31

11 Find two numbers which when multiplied together make 1 000 000 if neither of the numbers contains any zeroes.

Chapter review 5

D.I.Y. Summary

Key Words

axes	equation	linear graph	relationship	variable
Cartesian plane	evaluate	ordered pair	substitute	
coefficient	expression	origin	table of values	
constant	formula	point graph	terms	
coordinates	like terms	pronumeral	unknown	
define	line graph	quadrant	unlike terms	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 A ___________ is a letter or a symbol that represents a number.

2 Completing a ___________ is helpful when finding the ___________ that describes a pattern.

3 To find the value of the expression $2x + 3$, when $x = 1$, we need to ___________ 1 for x, and ___________ the expression.

4 The ___________ is the point (0, 0). It is the point where the two ___________ of the Cartesian plane intersect.

5 A pronumeral represents an ___________ or a ___________.

6 A number written next to a pronumeral is the ___________ of the pronumeral. The pronumeral is multiplied by that number.

7 To simplify the expression $3x + 4x + 5$ you collect ___________ .

8 The ___________ (3, 5) can be plotted on the ___________ by finding the point 3 across and 5 up. Draw a diagram clearly showing this point plotted in its correct location.

Fluency

1 Write the following situations using algebra. Ex. 5.1

(a) There are 12 biscuits in a packet. How many are in n packets?

(b) Tarin has a packet of p lollies. He eats half, then gives 3 to his brother. How many lollies does he have left?

(c) Multiply p by 7, then subtract 9.

(d) Subtract g from 11, then divide by 5.

2 Write an equation for each of the following situations, taking care to define all the variables you have used. Ex. 5.2

(a) Joe's age in years is four plus three times Beth's age.

(b) Three bottles of juice and five salad rolls cost $18.60.

3 For each of the following rules: **Ex. 5.3**

(i) draw a flowchart that shows how to get to y from x

(ii) write the rule using algebra

(iii) copy and complete the table of values for that rule.

(a) Add three to x to get y.

x	57	34	12	4	1.1	64
y						

(b) To find y, multiply x by three, then subtract five.

x	4	3	$2\frac{1}{2}$	9	10	12
y						

4 Choose the correct algebraic notation for each rule. **Ex. 5.3**

(a) y is equal to x plus 6.

A $y = x - 6$ **B** $y = x + 6$ **C** $y = x \times 6$ **D** $x = y + 6$

(b) Subtract five from x, then multiply by three to get y.

A $y = (x - 3) \times 5$ **B** $y = (x + 5) \times 3$ **C** $y = (x - 5) \times 3$ **D** $y = x - 5 \times 3$

5 For each of the following formulas, evaluate p by substituting the given values of a. **Ex. 5.4**

(a) $p = 6 - 20a$ **(i)** $a = 3$ **(ii)** $a = -1$

(b) $p = 2(a - 4) + 11$ **(i)** $a = 2$ **(ii)** $a = -5$

6 If you substitute $x = 3$ into the formula $y = 5x - 2$, the value of y is: **Ex. 5.4**

A $y = 3$ **B** $y = 5$ **C** $y = 13$ **D** $y = 17$

7 Use each of the following rules to complete the tables of values. **Ex. 5.4**

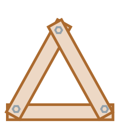
(a) $y = x - 7$

x	7	9	12	20	8.1	107
y						

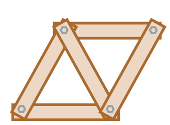
(b) $y = 10(x - 3)$

x	4	$5\frac{1}{2}$	13	7	78	54
y						

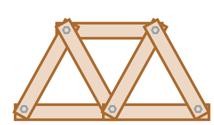
8 The frame of a bridge is made up of triangular sections. The triangles are made up of girders held together by bolts. **Ex. 5.5**



1 triangle
3 girders
3 bolts



2 triangles
5 girders
4 bolts



3 triangles
7 girders
5 bolts

(a) Complete the table of values by continuing the pattern.

Number of triangles (t)	1	2	3	4	5
Number of girders (g)	3	5			
Number of bolts (b)	3	4			

(b) Write down the rule that relates the number of triangles, t, to the number of girders, g.

(c) Write down the rule that relates the number of triangles, t, to the number of bolts, b.

(d) (i) How many girders are needed to create each extra triangular section?

(ii) How is this reflected in the table of values and the rule?

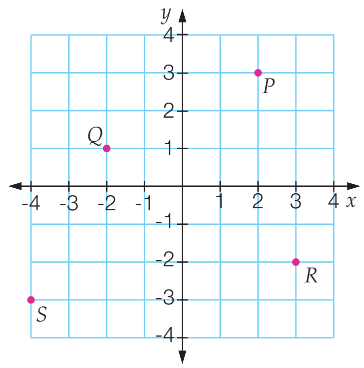
(e) (i) How many bolts are needed for each new triangular section?

(ii) How is this reflected in the table of values and the rule?

9 Simplify each expression by adding or subtracting like terms. **Ex 5.6**

(a) $12a - 7a$ (b) $6a + 12b - 7a + 11b$ (c) $x + y + 3x$

10 Write the coordinates of each of the following points shown on the Cartesian plane. **Ex 5.7**



(a) P (b) Q

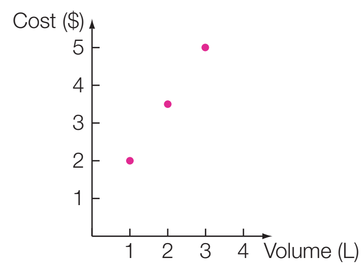
(c) R (d) S

11 (a) Plot the points (-1, -4), (1, 2), (2, 5) and (3, 8) on a number plane and draw a straight line passing through all the points. **Ex 5.8**

(b) Summarise the set of points in a table.

(c) Write down a rule linking the x- and y-values.

12 Juice is sold in three sizes: 1 L, 2 L and 3 L. Use the graph opposite to answer the following questions. **Ex 5.9**



(a) How much does a 2 L juice cost?

(b) Which size is the best value? Why?

(c) Why is this information given in a point graph?

Understanding

13 Gayle, the kindergarten teacher, has 8 packets of crayons with n crayons in each packet, plus an extra 3 loose crayons.

(a) Write an expression for the total number of crayons Gayle has.

(b) If all the crayons are to be shared equally between the 15 children in Gayle's class so that none are left over, write an expression to show how many crayons each child receives.

(c) Find the smallest possible value for n.

14 Draw a flowchart for each of the following rules.

(a) $h = 7g$ (b) $d = 9c - 2$ (c) $d = \frac{c}{10} + 13$ (d) $h = 5(g + 8)$

15 Tamsin is working out the number of bottles of soft drink she needs for a party. She is going to allow 1 bottle for every 4 people, plus 5 extra bottles.

(a) Write out Tamsin's rule using algebraic notation. Let b = number of bottles of drink required and n = number of people at the party.

(b) If Tamsin has 16 people at the party, how many bottles of drink will she need?

16 Simplify if possible:

(a) $2a - 7b + 3a + 2$ (b) $5a + 12b - 7 - 3b$ (c) $x + y + xy$

17 The points (0, 1), (2, 5) and (5, 11) are plotted on a number plane and a straight line drawn through the points. Which one of the following points also lies on the line?

A (1, 2) **B** (3, 7) **C** (3, 6) **D** (4, 10)

Reasoning

18 The following tables of values have only the y-values filled in. Work backwards along the flowchart to determine the values of x that were used, and complete the tables.

(a) $\boxed{x} \xrightarrow{\div 5} \boxed{y}$

x						
y	2	-7	20	1	0	-6

(b) $\boxed{x} \xrightarrow{\times 12} \boxed{y}$

x						
y	72	120	36	-12	0	60

19 For each of these tables, find the rule that is being used.

(a)

x	76	54	8	28	9	103
y	69	47	1	21	2	96

(b)

x	15	10	2	7.3	9	22
y	34	24	8	18.6	22	48

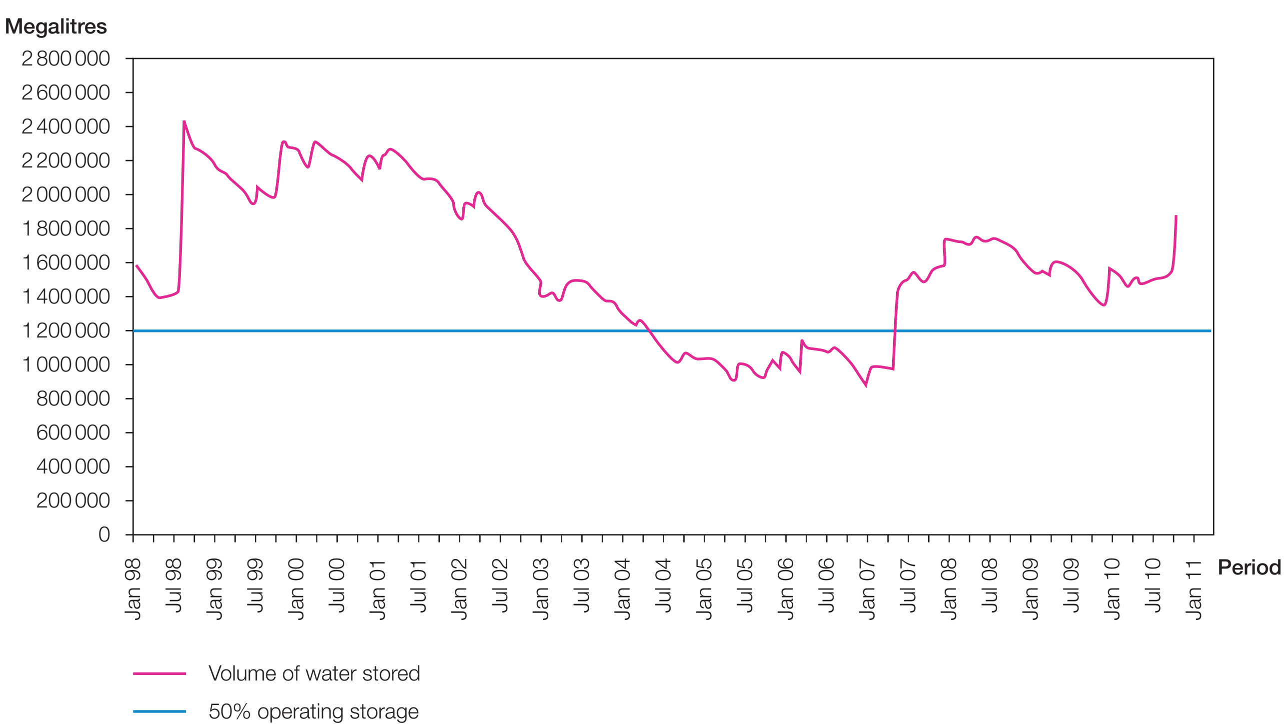
20 Jack has \$21.00 to buy some apples and oranges. An apple costs \$1.80 and an orange costs \$1.40.

(a) Using appropriate pronumerals to represent the number of apples and oranges purchased, write an equation to show the situation when Jack spends all of the \$21.00.

(b) How many apples and how many oranges could Jack buy using all the money?

(c) If Jack buys 4 apples and 5 oranges, how much change will he receive?

21



Above is a graph of Sydney's water storage from 1998 to 2010. The volume of water stored is given in megalitres (ML). 1 ML = 1 000 000 litres.

Use this graph to answer the following questions.

(a) What was the volume of water (in megalitres) at the start of 2010?

(b) During which month and year were the dams the fullest?

(c) During which month and year were the dams at their lowest capacity?

(d) What does the shape of the graph tell you about Sydney's rainfall from July 1998 to January 1999?

(e) What does the shape of the graph tell you about Sydney's rainfall from January 2000 to July 2007?

(f) During what period was the storage level below 1 200 000 megalitres (below 50%)?

NAPLAN practice 5

Numeracy: Non-calculator

1 A number is multiplied by eight and then 5 is added. The answer is 61. What is the number?

2 x and y stand for numbers. x and y are connected by a rule.

x	2	3	5	8
y	8	13	23	38

What is the rule?

A $y = 4x + 1$

B $y = 5x - 2$

C $y = 4x$

D $y = 10x - 27$

3 Ting Li followed this rule. She started with 8.

Starting number 8	add 4 →		multiply by 3 →		subtract 12 →	Final answer

(a) What is the correct final answer using Ting Li's starting number of 8?

(b) Alex followed the same rule using a different starting number.

Alex's starting number is -2. What was his final answer?

Numeracy: Calculator allowed

4 Ethan followed a different rule to Ting Li. There were three steps. Ethan started with the number 21 and ended up with a final answer of 30.

Starting number 21	Step 1 →		Step 2 →		Step 3 →	Final answer

(a) Write the numbers from 1 to 3 on the lines below to show an order of steps that Ethan followed.

Step ___________ add 8

Step ___________ divide by 3

Step ___________ double the number

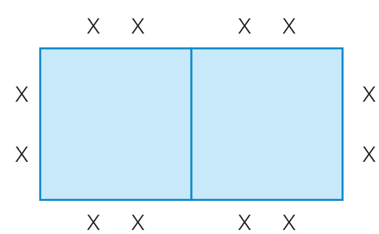
(b) Repeat the procedure in part **(a)** to get a final answer of 22.

5 A pack of 10 lollipops costs \$4.60.
A pack of 6 costs \$3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

6 Yusef investigated the seating plan for a restaurant using this table of values.

Number of tables	2	3	4	5
Number of people seated	12	16	20	?



The diagram shows the seating arrangement for 2 tables.

What rule did Yusef use to work out how many people can be seated at 5 tables?

A number of people seated = number of tables

B number of people seated = number of tables + 4

C number of people seated = number of tables × 6 – 1

D number of people seated = number of tables × 4 + 4