7

7

Linear equations

Wiis work wonders. Why do people learn physical tasks at different rates?

Why do some people seem to have 'natural talent' when others struggle to master the basics? University researchers are studying how people learn various skills. They are measuring the motions involved in tasks as simple as playing racquet sports through to more complex activities, such as flying a fighter jet, in the hope that this research will help people to learn these skills faster and more effectively.

One university research project is making use of Nintendo's popular 'Wii' video game technology to create equations that represent skills involved in many human activities from sport to surgery. To do this, a motion-capture device, such as the Wiimote, is used. Data from the Wiimote is used to measure a range of movements that is then used to create mathematical equations. The aim is then to program robots to teach people the most efficient way to learn new tasks. By bringing robotics and virtual reality together, learning will no longer be a matter of trial and error. For example, if you were learning to play tennis, a robotic device, such as a sleeve that prompted you to move your arm the correct way, would make learning easier.

Wiis are already being used in the classroom. How cool is that?

Forum

Researchers are investigating ways that different movements can be recorded by using a Wiimote. Why might the universities be interested in this? How do you think a Wii could be used in classrooms?

Why learn this?

Whether you are deciding how much material to buy to make curtains, working to a family budget, or calculating whether a business has made a profit, equations can be used to represent a variety of situations. Solving them can help us to make informed and accurate decisions.

After completing this chapter you will be able to:

- write an equation from information given in words
- check solutions to equations by substitution
- solve worded problems
- solve everyday problems using equations
- solve equations using a variety of methods.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R7.1

1 Answer TRUE or FALSE to each of the following number sentences.

(a) $3 + 7 = 5 + 6$ (b) $8 \times 3 = 20 + 4$ (c) $\frac{24}{8} = 3 \times 5 - 14$

PP Worksheet R7.2

2 (a) The term $5x$ means:

A $5 + x$ B $5 - x$ C $5 \times x$ D $x \div 5$

(b) The term $\frac{x}{7}$ means:

A $x - 7$ B $x + 7$ C $x \times 7$ D $x \div 7$

(c) $4(x + 3)$ means:

A $4 + (x + 3)$ B $4 \times (x + 3)$ C $4 \times x + 3$ D $4 - (x + 3)$

PP Worksheet R7.3

3 Rewrite each of the following sentences using mathematical symbols and numbers only.

(a) Five added to seven is equal to twelve.

(b) Subtracting two from the product of four and six is equal to twenty-two.

(c) When the sum of eight and six is divided by seven, the answer is equal to two.

PP Worksheet R7.4

4 What is the final result after applying each of the following operations to the number in the box?

(a) $\boxed{8}$ $\times 2$ then -4 then $\div 3$ then $+5$

(b) $\boxed{10}$ $\div 2$ then $\div 5$ then $+6$

PP Worksheet R7.5

5 (a) If $x = 2$, find the value of $4x - 1$.

(b) If $x = 7$, find the value of $\frac{x - 3}{2}$.

Key Words

backtracking	false number sentence	solve
balance method	guess, check and improve	solving by inspection
checking by substitution	inverse operations	true number sentence
equivalent equations	solution	

Number sentences 7.1

Vlado wants change for a \$5 note. His friend Perry gives him two \$2 coins and a \$1 coin in exchange for the note. This can be written as $5 = 2 + 2 + 1$ and is an example of a **true number sentence**.

$2 \times 4 = 5 + 3$ is another example of a true number sentence, as both the left-hand side (LHS) and the right-hand side (RHS) have the same value of 8.

left-hand side (LHS) equals sign right-hand side (RHS)

LHS = RHS

$2 \times 4 = 5 + 4$

LHS ≠ RHS

is an example of a **false number sentence**, as the LHS has a value of 8 and the RHS has a value of 9.

≠ means 'not equal to', < means less than, > means greater than

Worked Example 1 WE1

The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

(a) $5 + 8 = 15$ **(b)** $2 \times 5 - 1 = 7$

Thinking	Working
(a) **1** Find the value of the side that has the coloured number.	(a) $5 + 8 = 13$
2 Compare the answer to the other side of the number sentence.	$13 < 15$
3 Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$5 + 10 = 15$

(b)	1 Find the value of the side that has the coloured number.	(b) $2 \times 5 - 1 = 9$
	2 Compare the answer to the other side of the number sentence.	$9 > 7$
	3 Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$2 \times 4 - 1 = 7$

Worked Example 2

WE 2

Decide which of the following are true number sentences.

(a) $8 \times 3 + 4 = 28$

(b) $\dfrac{6 \times 5}{3} = 7 + 2$

Thinking		Working
(a)	1 Evaluate the LHS of the number sentence.	(a) $\text{LHS} = 8 \times 3 + 4$ $= 28$
	2 Compare the LHS and RHS to check whether they are equal.	$\text{LHS} = \text{RHS}$
	3 State your answer.	The sentence is true.
(b)	1 Evaluate the LHS of the number sentence.	(b) $\text{LHS} = \dfrac{6 \times 5}{3}$ $= 10$
	2 Evaluate the RHS of the number sentence.	$\text{RHS} = 7 + 2$ $= 9$
	3 Compare the LHS and RHS to check whether they are equal.	$\text{LHS} \neq \text{RHS}$
	4 State your answer.	The sentence is false.

7.1 Number sentences

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q10(a), Q11 (a–d), Q12, Q14, Q16	Q1, Q2 Column 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 (a–f), Q12, Q14, Q16	Q1, Q2 Column 3, Q3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Answers page 662

Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

WE1

(a) $3 + 4 = 10$ (b) $5 \times 6 = 25$ (c) $\frac{24}{4} = 4$

(d) $22 - 7 = 8 + 4$ (e) $3 \times 7 + 2 = 8 + 9$ (f) $1 + \frac{15}{5} = 6$

2 Decide which of the following are true number sentences.

WE2

(a) $\frac{32}{4} + 7 = 16$ (b) $4 + 3 \times 2 = 14$ (c) $6 + 4 \times 5 = 25$

(d) $30 \div 6 = 4 + 1$ (e) $23 - 7 = 12 + 4$ (f) $8 + 4 = 6 \times 2$

(g) $7 - 5 = 9 \div 3$ (h) $10 \times 8 = 90 - 10$ (i) $\frac{6 + 10 + 12}{7} = \frac{20 + 4}{6}$

3 (a) The number missing from the true number sentence $3 \times \_\_\_\_ = 19 + 5$ is:

A 3 **B** 6 **C** 7 **D** 8

(b) The number missing from the true number sentence $4 \times 2 = \_\_\_\_ + 4$ is:

A 0 **B** 2 **C** 4 **D** 8

(c) The number missing from the true number sentence $\frac{24}{8} + \_\_\_\_ = 10 - 2$ is:

A 4 **B** 5 **C** 6 **D** 8

Understanding

4 Rewrite each true number sentence using numbers and mathematical symbols only.

(a) Forty-three added to five is equal to forty-eight.

(b) Three multiplied by seven is equal to nineteen plus two.

(c) Fifty divided by five is equal to the product of five and two.

(d) When the sum of the numbers six and eight is subtracted from twenty, the answer is equal to twelve divided by two.

5 Write each of the following as a number sentence, then state whether it is true or false.

(a) Nineteen subtracted from thirteen is equal to six.

(b) Four divided by two is equal to eight.

(c) Eleven subtracted from eight is equal to six minus three.

(d) Twelve divided by four is equal to fifteen divided by five.

6 Jane is 5 years older than Todd. If Todd is 7 years old, then write a true number sentence to show Jane's age.

7 Six pens at 60c each cost the same as buying 4 exercise books at 90 cents each. Write this as a true number sentence.

8 The cost of a new shirt is $25 after a discount of $7 is given. Write a true number sentence to show the original price of the shirt.

9 A bus can carry 48 passengers. The bus is full, then 11 people leave while another 5 get on at the next stop. Write a true number sentence to show how many passengers are left on the bus.

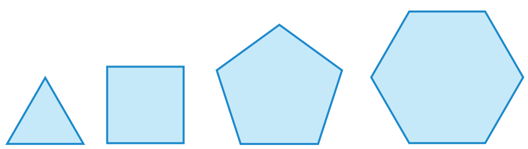
10 Numbers are consecutive if one follows the other. The numbers 8 and 9 are consecutive numbers that add to 17.

(a) Write a true number sentence that shows two consecutive numbers that add to 33.

(b) Write a true number sentence that shows three consecutive numbers whose sum is 54.

Reasoning

11 A piece of wire 60 cm long is used to make a triangle shape that has all three sides the same length. Write a true number sentence to answer each of the following.



(a) How long is each side of the triangle?

(b) The wire is now used to make a square. How long is each side?

(c) The wire is bent to make a regular pentagon. How long is each side?

(d) The wire is then used to make a regular hexagon. How long is each side?

(e) Write a number sentence you could use to determine the length of the side of a shape that has 20 sides of equal length made from the same piece of wire.

(f) Write a number sentence you could use to determine the length of the side of a shape that has 50 sides of equal length made from a wire 200 cm long.

(g) Use a 100 cm length of wire to make a shape whose sides are of equal length. Find three possible values for the number of sides the shape can have if the length of each side is to be a whole number and shorter than 25 cm.

Open-ended

12 Use the numbers 1, 2, 4, 8, the equals sign, =, and as many of the symbols +, –, ×, ÷ as you like to write three different true number sentences whose right-hand side is a number from 1 to 10.

13 A length of wire is used to make a shape that has eight sides of equal length. Find five different whole number values for the length of wire if the length of each side is to be greater than the number of sides.

14 $\square + \square \times \square - \square = 20$

What might the missing numbers be? Give three different answers.

15 Sam and Ruben have written the following number sentences. Sam wrote $3 + 4 \times 21 = 147$. Ruben wrote $3 + 4 \times 21 = 87$.

Who is correct? Why? Write a sentence so that the person who was incorrect can improve their skills.

16 Two different students were given the following number sentence.

$38 - 16 = \square - 12$

They presented the following solutions.

$38 - 16 = \square - 12$	$38 - 16 = \square - 12$
$38 - 12 - 4 = \square - 12$	$38 - (12 + 4) = \square - 12$
$38 - 4 - 12 = \square - 12$	$38 + 4 - 12 = \square - 12$
$34 - 12 = \square - 12$	$42 - 12 = \square - 12$
$\square = 34$	$\square = 42$

Which of the solutions is correct? Why? What is wrong with the incorrect solution?

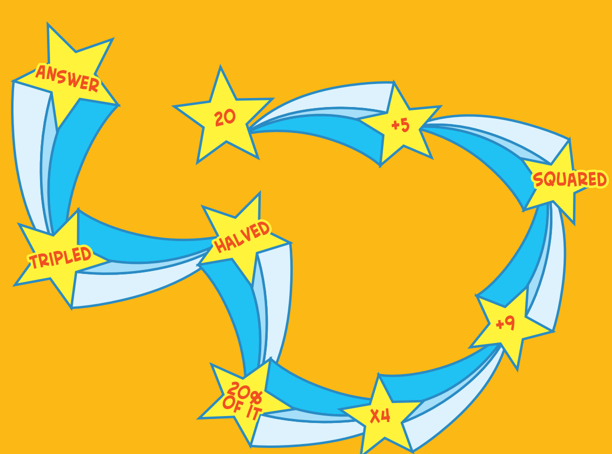
Outside the Square

Puzzle

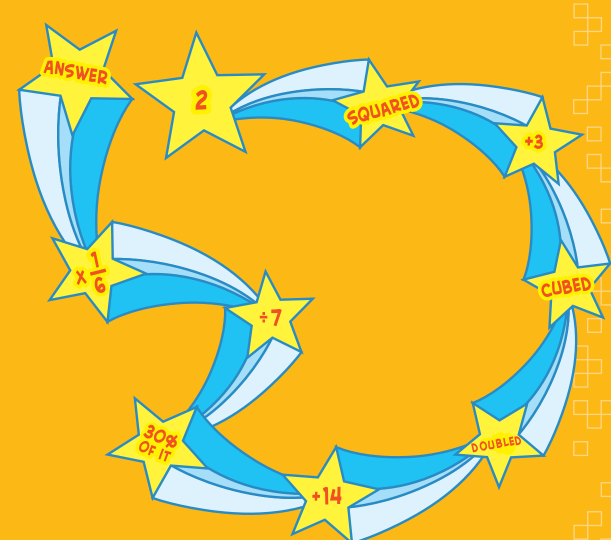
Star quest

Equipment required: 1 brain, 1 watch

(a) Start at the first star and follow the shooting stars. Time yourself to see how long it takes to complete all of the operations.



(b) What about trying something more challenging? Time yourself completing this star quest.



7.2 Introduction to equations

Often, in mathematics, we want to write information in a number sentence but some information is missing. Where a value is unknown, we can use a pronumeral to represent the unknown amount. This number sentence is now called an equation.

$2 \times 4 = 5 + 3$ is a true number sentence. $2 \times 4 = 5 + 4$ is a false number sentence.

$2 \times 4 = 5 + x$ is an equation. Equations always contain an equals sign.

We need to be able to understand and interpret the information given to us in an equation.

The equals sign was invented in 1557 by Welshman Robert Recorde.

Worked Example 3

WE3

Write each of the following equations in words.

(a) $5x - 3 = 1$

(b) $6 = \frac{x}{5} + 2$

Thinking	Working
(a) 1 Call the variable 'a number' and write in words the operation that has been performed on it first, using the correct order of operations.	(a) When a number is multiplied by five
2 Decide what happens next to the LHS and add this on to what you have already written.	and then three is subtracted
3 State the final result. This is the value shown on the RHS.	the result is equal to one.
4 Write the complete equation in words.	When a number is multiplied by five, and then three is subtracted, the result is equal to one.
(b) 1 Call the variable 'a number' and write in words the operation that has been performed on it first using the correct order of operations.	(b) When a number is divided by five
2 Decide what happens next to the RHS and add this on to what you have already written.	and then two is added
3 State the final result. This is the value shown on the LHS.	the result is equal to six.
4 Write the complete equation in words.	When a number is divided by five, and then two is added, the result is equal to six.

We **solve** an equation by finding the unknown value that makes an equation a true number sentence. By substituting $x = 3$ into $2 \times 4 = 5 + x$ we get a true number sentence, so the **solution** to $2 \times 4 = 5 + x$ is $x = 3$.

If x is any other value, we would get a false number sentence.

Equations are useful in solving everyday problems by providing a shorthand way of writing information. There are many different methods used to solve equations.

Solving by inspection

Many equations involve only one operation (+, –, ×, ÷) and are sometimes called one-step equations. For example, $3x = 9$, $\frac{x}{2} = 7$, $x + 3 = 11$, $4 - x = 1$ are one-step equations.

These equations can be solved by simply looking at the equation and guessing the value of x. When we do this, we need to check that our solution will make a true number sentence. This method is called '**solving by inspection**'.

Here are some more one-step equations and their solutions:

For $k - 4 = 5$, the solution is $k = 9$. Check: $9 - 4 = 5$ is a true number sentence.

For $\frac{a}{3} = 5$, the solution is $a = 15$. Check: $\frac{15}{3} = 5$ is a true number sentence.

Worked Example 4

WE4

For the following equation, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

$3x - 4 = 11 \quad (x = 5)$

Thinking	Working
1 Identify the side of the equation that contains the variable.	LHS $= 3x - 4$
2 Substitute the given value for the variable and simplify.	$= 3 \times 5 - 4$ $= 11$
3 Check to see whether this answer is the same as the RHS of the equation.	RHS $= 11$ Yes, $x = 5$ is the solution.

Solving equations using guess, check and improve

We can use '**guess, check and improve**' to solve equations with more than one step.

Worked Example 5

WE5

Find the solution to the following equation by using the guess, check and improve method.

$$\frac{6x - 1}{5} = 7$$

Thinking	Working
1 Guess a value for x and substitute it into the LHS of the equation to see if it makes a true number sentence.	Try $x = 5$ $\frac{6 \times 5 - 1}{5} = \frac{30 - 1}{5}$ $= \frac{29}{5}$ $\neq$ RHS

2 Try a larger value for x and substitute it into the LHS of the equation.	Try $x = 6$ $\frac{6 \times 6 - 1}{5}$ $= \frac{36 - 1}{5}$ $= \frac{35}{5}$ $= 7$ $=$ RHS
3 As this value makes a true number sentence, we have a solution. If the LHS still does not equal the RHS, try another value.	$x = 6$ is the solution to the equation.

7.2 Introduction to equations

Navigator

Answers page 662

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE3

1 Write each of the following equations in words.

(a) $n + 3 = 10$ **(b)** $4 + p = 7$ **(c)** $a + 5 = 6$

(d) $x - 2 = 4$ **(e)** $k - 7 = 2$ **(f)** $a - 8 = 6$

(g) $5k = 20$ **(h)** $7p = 14$ **(i)** $7a = 21$

(j) $\frac{x}{3} = 4$ **(k)** $\frac{f}{2} = 6$ **(l)** $\frac{y}{6} = 12$

(m) $2 + 3x = 7$ **(n)** $2 + 6x = 10$ **(o)** $1 + 3x = 8$

WE4

2 For the following equations, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

(a) $x + 6 = 11$ $(x = 5)$ **(b)** $y + 4 = 10$ $(y = 5)$ **(c)** $m + 2 = 11$ $(m = 9)$

(d) $d - 6 = 15$ $(d = 21)$ **(e)** $k - 3 = 8$ $(k = 5)$ **(f)** $a - 6 = 9$ $(a = 15)$

(g) $3x = 12$ $(x = 3)$ **(h)** $5a = 20$ $(a = 4)$ **(i)** $7m = 14$ $(m = 2)$

(j) $\frac{f}{2} = 16$ $(f = 8)$ **(k)** $\frac{m}{3} = 5$ $(m = 15)$ **(l)** $\frac{x}{7} = 14$ $(x = 2)$

(m) $5m + 1 = 9$ $(m = 2)$ **(n)** $2a - 2 = 8$ $(a = 3)$ **(o)** $5s - 4 = 6$ $(s = 2)$

(p) $\frac{x}{3} + 2 = 5$ $(x = 9)$ **(q)** $5 + \frac{b}{2} = 9$ $(b = 8)$ **(r)** $\frac{a}{15} + 2 = 5$ $(a = 30)$

WE5

3 Find the solution to each of the following equations by using the guess, check and improve method.

(a) $2x + 1 = 9$ **(b)** $6 + 5x = 41$ **(c)** $7x + 3 = 52$

(d) $\frac{x - 8}{3} = 7$ **(e)** $\frac{x - 5}{2} = 6$ **(f)** $\frac{x - 4}{5} = 12$

(g) $3(x-11)=21$ (h) $8(x-2)=32$ (i) $6(x+13)=54$

(j) $\frac{x}{2}-5 = 33$ (k) $\frac{x}{6}-8 = 14$ (l) $\frac{x}{3}+7 = 61$

4 In words, the equation $2=5-\frac{x}{6}$ can be written as:

A two is equal to five added to a number that is divided by six

B five subtracted from six times a number is equal to two

C five added to two is equal to a number that is divided by six

D a number is divided by six and the result is subtracted from five to give an answer of two.

5 (a) Which equation describes this sentence?

Four added to a number is equal to ten.

A $n+10=4$ B $n+4=10n$ C $4+10=n$ D $n+4=10$

(b) Which equation describes this sentence?

A number multiplied by three is equal to six.

A $x+3=6$ B $6x=3$ C $3x=6$ D $\frac{x}{3}=6$

(c) Which equation describes this sentence?

A number divided by four is equal to three.

A $4x=3$ B $x+4=3$ C $4-x=3$ D $\frac{x}{4}=3$

6 (a) The solution to the equation $x+3=18$ is:

A $x=6$ B $x=15$ C $x=16$ D $x=21$

(b) The solution to the equation $\frac{x}{2}=4$ is:

A $x=1$ B $x=2$ C $x=6$ D $x=8$

(c) The solution to the equation $4=x-11$ is:

A -15 B 7 C 11 D 15

Understanding

7 (a) Which equation describes this situation?

The cost, c, in dollars, of some grocery items plus a \$3 delivery charge totals \$55.

A $3c=55$ B $c-3=55$ C $c+3=55$ D $55+3=c$

(b) Which equation describes this situation?

Each of 4 people receive the same number of lollies, n, when a bag containing 36 lollies is shared between them.

A $4n=36$ B $4+n=36$ C $\frac{n}{4}=36$ D $n-4=36$

(c) Which equation describes this situation?

A school minibus seats a total of 12 passengers when full. The number of passengers, p, on the bus decreases by 3 at the next stop so that the bus is now half full.

A $p-3=12$ B $3p=6$ C $p+3=6$ D $p-3=6$

8 Use guess, check and improve to find the solution to the following equations.

(a) $\frac{x}{2} = x-5$ (b) $3x=5x-8$ (c) $\frac{3x-1}{4} = \frac{2(x+9)}{5}$

9 Daniel is three years older than his brother Christopher.

(a) Define a variable to represent Daniel's age.

(b) Use this variable to write an expression for Christopher's age.

(c) Write an expression to show how old Christopher will be in 15 years time.

(d) If Christopher will be 32 in 15 years time, write an equation using your answer to part **(c)** as the LHS of the equation and solve it to find Daniel's age now.

Reasoning

10 Cameron has some money in his wallet. Libby has $5.

(a) Define a variable to represent the amount of money Cameron has in his wallet.

(b) If Cameron spends $14, write an expression to show how much money he has left.

(c) If Libby and Cameron now have $23 between them, write an equation and solve it to find how much money Cameron had to start with.

Open-ended

11 Pete is twice as old as Chee.

(a) Choose a value that is a multiple of 3 as the sum of their ages and write the equation that you would use to find Chee's age.

(b) Choose another value that is also a multiple of 3 as the sum of their ages and write a new equation you would use to find Chee's age.

(c) What age might Chee be if the sum of their ages is less than 30?

12 Write a short story that could be represented by the following equations.

(a) $14 - x = 11$ (b) $\frac{x}{5} = 12$ (c) $6x = 108$

Outside the Square — Puzzle

The great number mystery

What number am I?

- I am a prime number.
- If you triple me, I am made up of two digits.
- If I am multiplied by four, and then divided by five, I am one smaller than my original value.
- If I am squared, I am five less than six times my value.

Solving equations using backtracking 7.3

Some equations are difficult to solve by inspection or guess, check and improve because the solution is not a whole number, or more than one operation is involved.

For example: $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3.75 = 21.8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations but their solutions are not easy to see.

$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x+6}{8} = 2$ are all two-step equations.

We need to use other methods to solve these equations.

Using flowcharts

In Chapter 5, flowcharts were used to build expressions. Flowcharts can also be used to help us solve equations. To use a flowchart correctly, we need to apply the order of operations that were used to build the expression.

For the expression $2x + 5$, x is first multiplied by 2 to give $2x$, then 5 is added to give us $2x + 5$, so our flowchart looks like this:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5}$$

If we wanted to add 5 first before multiplying by 2, the flowchart would look like this:

$$\boxed{x} \xrightarrow{+5} \boxed{x+5} \xrightarrow{\times 2} \boxed{2(x+5)}$$

This expression is $2(x + 5)$, which is quite different from $2x + 5$.

Applying the same operations in a different order produces different expressions.

The order in which we apply the operation is important in building expressions.

Inverse operations

To solve equations using flowcharts, we undo operations used to build the equation by applying **inverse operations** (opposite operations) in the reverse order. We move backwards along the flowchart.

Worked Example 6 WE6

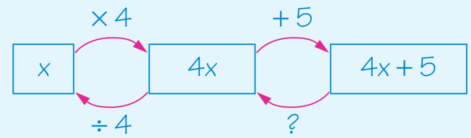
Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a) $$\boxed{x} \underset{\div 4}{\overset{\times 4}{\rightleftarrows}} \boxed{4x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{4x+5}$$

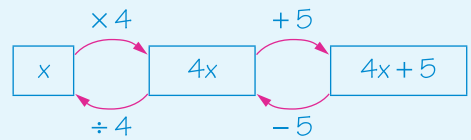
(b) $$\boxed{x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{6x} \underset{?}{\overset{?}{\rightleftarrows}} \boxed{6x-3}$$

Thinking	Working

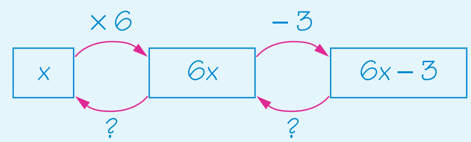
(a) 1 Above the top arrow write the missing operation needed to build the expression.



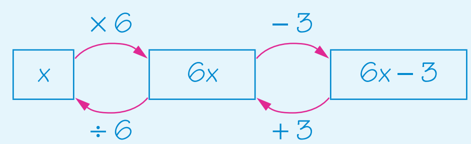
2 Below the bottom arrow write the missing operation that will undo this expression.



(b) 1 Above the top arrows write the missing operation needed to build each expression.

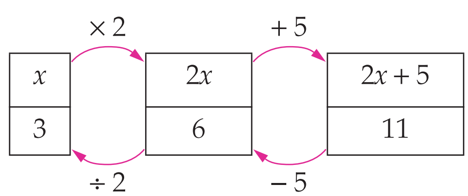


2 Below the bottom arrows write the missing operation needed to undo this expression.



Backtracking

To solve $2x + 5 = 11$, we set up the following flowchart.



Step 1 Draw a flowchart for the expression on the LHS of the equation.

Step 2 Add boxes underneath the flowchart boxes and arrow signs in the opposite direction.

Step 3 Write the inverse operations on the arrow signs.

Step 4 Write the number on the RHS of the equation in the box on the right-hand side of the flowchart underneath the expression.

Step 5 Complete the flowchart using the inverse operations.

Step 6 The number under x is the solution.

This method of solving equations is called **backtracking**. Backtracking along this flowchart gives us the solution $x = 3$ to the equation $2x + 5 = 11$.

When backtracking, we undo an operation by using the inverse (opposite) operation.

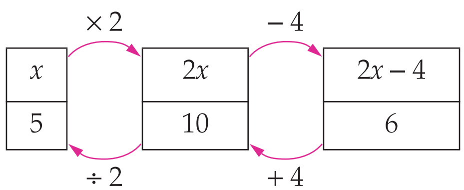
$+$ and $-$ are inverse operations

$\times$ and $\div$ are inverse operations

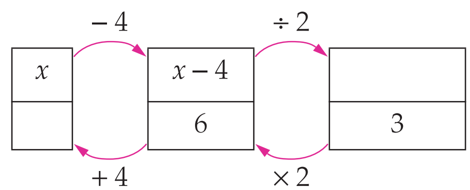
Worked Example 7 WE7

Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

(a)



(b)



Thinking / Working

(a) **(i)** The expression in the last box on the right and the number in the box underneath it form the equation.

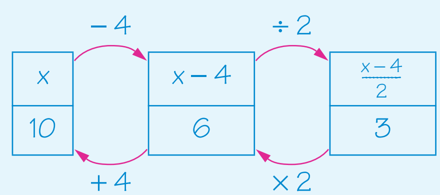
(a) (i) Equation is $2x - 4 = 6$

(ii) The solution is the number under x in the first box on the left of the flowchart.

(ii) Solution is $x = 5$

(b) **(i)** **1** Complete the missing parts of the flowchart.

(b) (i)



2 The expression in the last box on the right and the number in the box underneath it form the equation.

$$\frac{x-4}{2} = 3$$

(ii) The solution is the number under x in the first box on the left of the flowchart.

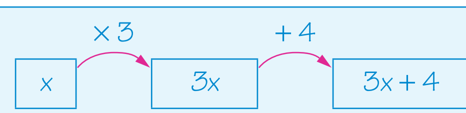
(ii) $x = 10$

Worked Example 8 WE8

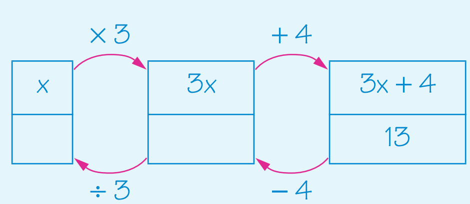
Draw a flowchart and use backtracking to solve the equation: $3x + 4 = 13$.

Thinking / Working

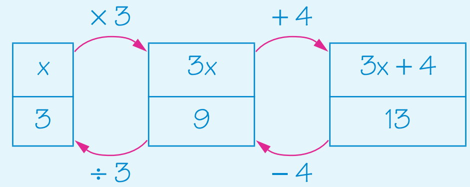
1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. (× 3, + 4)



2 Write the value from the RHS of the equation under the last box on the right of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (÷ 3, − 4)



3 Perform the required operations to obtain a value for x.



4 State the solution.

$x = 3$

Checking by substitution

We can check that the answer obtained is the solution to the equation by substituting it into the equation. We evaluate both sides of the equation. If the LHS = RHS, the answer is the solution. This process is called **checking by substitution**.

In Worked Example 8, we would check the solution by substituting $x = 3$ into the left-hand side of the equation.

Check: $\quad$ LHS $= 3x + 4$

$= 3 \times 3 + 4$

$= 9 + 4$

$= 13$

$=$ RHS

So, $x = 3$ is the solution.

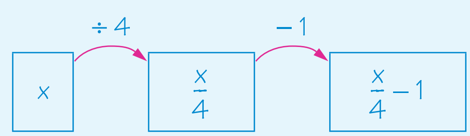
Worked Example 9

WE9

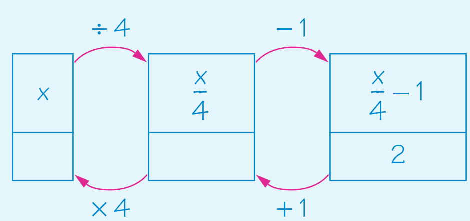
Draw a flowchart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

Thinking	Working

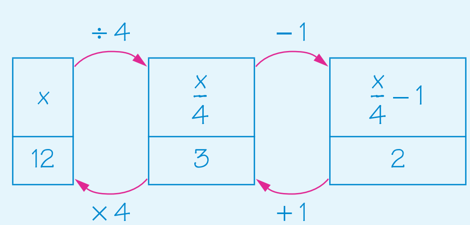
1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. (÷ 4, − 1)



2 Write the value from the RHS of the equation under the right-hand box of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (× 4, + 1)



3 Perform the required operations to obtain a value for x.



4 State the solution.

$x = 12$

5 Check the solution by substituting your answer into the left-hand side of the equation. If the left-hand side equals the right-hand side of the equation, you have found the solution.

Check:
$$\begin{aligned}\text{LHS} &= \frac{x}{4} - 1\\ &= \frac{12}{4} - 1\\ &= 3 - 1\\ &= 2\\ &= \text{RHS}\end{aligned}$$

7.3 Solving equations using backtracking

Navigator

Q1 Column 1, Q2 Column 1, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Column 1, Q6 Column 1, Q7 Column 1, Q8 Column 1, Q9, Q10 (a) & (b), Q13	Q1 Column 1, Q2 Column 1, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6 Column 2, Q7 Column 2, Q8 Column 2, Q9, Q10 (a)–(c), Q11, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7 Column 3, Q8 Column 3, Q9, Q10, Q11, Q12, Q13, Q14

Answers page 663

Fluency

WE6

1 Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a)

	$\times 4$		?	
x		$4x$		$4x + 1$
	$\div 4$		?	

(b)

	$\times 5$		?	
x		$5x$		$5x - 7$
	$\div 5$		?	

(c)

	?		?	
x				$2x + 4$
	?		?	

(d)

	?		?	
x				$7x - 3$
	?		?	

(e)

	?		?	
x				$3x - 2$
	?		?	

(f)

	?		?	
x				$8x + 1$
	?		?	

WE7

2 Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

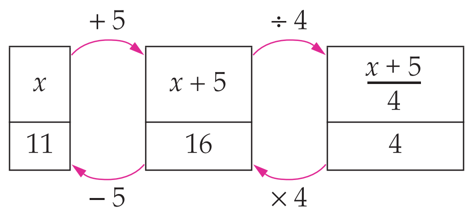
(a)

	$\times 3$		-1	
x		$3x$		$3x - 1$
4		12		11
	$\div 3$		$+1$	

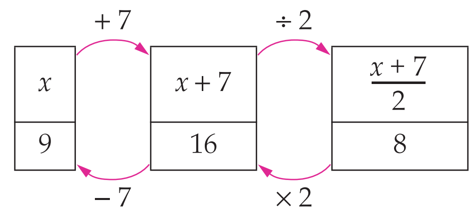
(b)

	$\times 2$		-3	
x		$2x$		$2x - 3$
5		10		7
	$\div 2$		$+3$	

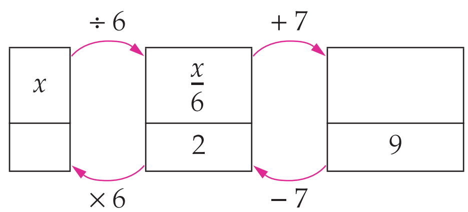
(c)



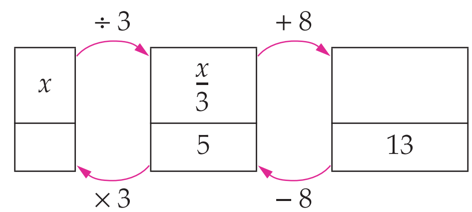
(d)



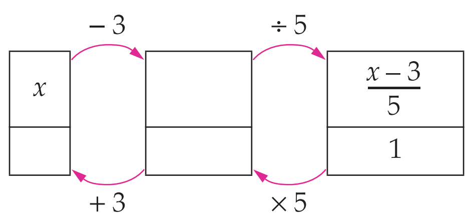
(e)



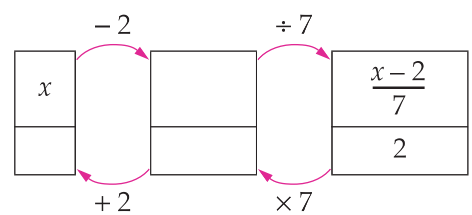
(f)



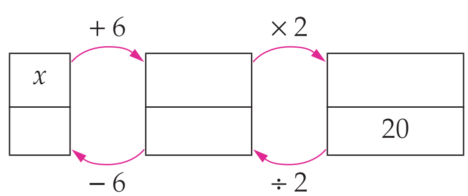
(g)



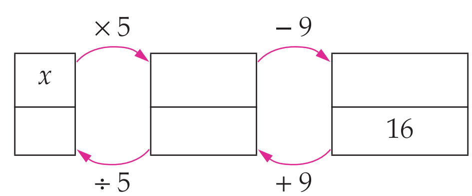
(h)



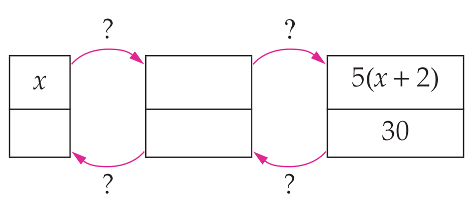
(i)



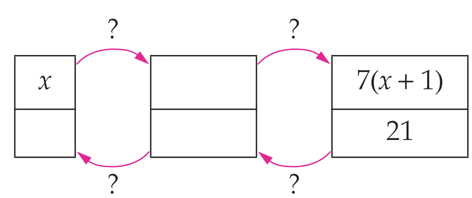
(j)



(k)



(l)



WE8

3 Draw a flowchart and use backtracking to solve each of the following equations.

(a) $5x + 3 = 8$ **(b)** $7x + 3 = 24$ **(c)** $2x + 3 = 11$

(d) $2x - 7 = 3$ **(e)** $3x - 2 = 10$ **(f)** $8x - 11 = 13$

WE9

4 Draw a flowchart and use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x}{4} + 5 = 7$ **(b)** $\frac{x}{2} + 6 = 11$ **(c)** $\frac{x}{7} + 3 = 5$

(d) $\frac{x}{2} - 2 = 1$ **(e)** $\frac{x}{5} - 1 = 2$ **(f)** $\frac{x}{2} - 4 = 2$

5 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $3(x + 1) = 12$ **(b)** $2(x + 3) = 16$ **(c)** $2(x + 5) = 14$

(d) $5(x - 6) = 10$ **(e)** $6(x - 4) = 18$ **(f)** $3(x - 5) = 3$

6 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x+8}{3} = 3$ **(b)** $\frac{x+5}{2} = 4$ **(c)** $\frac{x+10}{7} = 2$

(d) $\frac{x-2}{5} = 1$ **(e)** $\frac{x-3}{4} = 2$ **(f)** $\frac{x-5}{2} = 4$

7 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $2x + 3 = 13$ **(b)** $3x + 2 = 11$ **(c)** $5x + 6 = 11$

(d) $5x - 3 = 17$ **(e)** $4x - 1 = 7$ **(f)** $8x - 9 = 7$

(g) $2(x + 7) = 20$ **(h)** $4(x - 3) = 8$ **(i)** $6(x - 1) = 12$

(j) $\frac{x}{2} + 3 = 8$ **(k)** $\frac{x}{7} + 10 = 12$ **(l)** $\frac{x}{3} - 2 = 3$

(m) $\frac{x+2}{4} = 2$ **(n)** $\frac{x-4}{5} = 2$ **(o)** $\frac{x-3}{2} = 2$

8 Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

(a) $\frac{2x+1}{3} = 3$ **(b)** $\frac{3x+7}{2} = 11$ **(c)** $\frac{5x-2}{4} = 7$

(d) $\frac{3x}{5} + 6 = 9$ **(e)** $\frac{2x}{7} + 8 = 12$ **(f)** $\frac{7x}{2} - 10 = 4$

(g) $\frac{2(x+4)}{5} = 6$ **(h)** $\frac{3(x+2)}{4} = 3$ **(i)** $\frac{2(x-3)}{7} = 6$

9 The solution to $\frac{4x}{5} + 6 = 14$ is:

A $x = 1$ **B** $x = 10$ **C** $x = 16$ **D** $x = 25$

Understanding

10 Write an equation using the following information, then solve each equation using backtracking. Use n to represent the unknown number.

(a) A number is doubled, then one is added to the result to give an answer of forty-three.

(b) When four is added to a number and the result is divided by five, the answer is four.

(c) A number is multiplied by seven and the result is divided by three to give an answer of twenty-one.

(d) A number is doubled, then six is added. The result is divided by ten to give an answer of two.

11 Belinda buys 3 packets of pencils. She gives five pencils to her friends. She sells the rest for 10 cents each and makes $6.70. How many pencils were in each packet? Using n as the number of pencils in the packet, write an equation and use backtracking to answer this question.

Reasoning

12 Ying repairs televisions. The expression for the amount of money ($) she charges is $30x + 20$, where x is the number of hours she takes to repair a TV. Aldo also repairs televisions. The expression for the amount of money ($) he charges is $20x + 50$ where x is the number of hours he takes to repair a TV.

(a) Show the flowchart for the expression $30x + 20$.

(b) Show the flowchart for the expression $20x + 50$.

(c) Substitute the number $x = 1$ in both flowcharts and write down how much money Ying and Aldo charge for a repair that takes one hour.

(d) Ying charges a customer $95. How long did it take her to repair this TV?

(e) If Aldo charged $95, how long would he have taken?

(f) By trying different values for x, find the number of hours for which the amount charged by Ying and Aldo is the same.

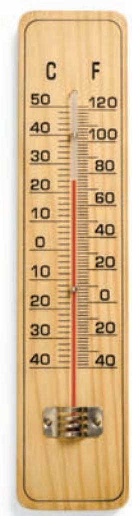
Open-ended

13 If x is the temperature in degrees Celsius, the expression $\frac{9x}{5} + 32$ gives the value of x in degrees Fahrenheit.

(a) Build and complete the flowchart for the expression.

(b) Use your flowchart to find two values for the temperature in degrees Celsius that give a whole number for degrees Fahrenheit.

14 After Ross performs four different operations on a number, he has a result of 10. Write an equation that fits this situation.



Outside the Square Puzzle

Gold digger 2

Your aim is to find the gold. The basic rule on how to solve this kind of puzzle can be found on page 63.

Here are the two maps to solve:

(a)

	2			
	4		1	
4			3	1
				1
	3			2
				1

(b)

3				1	
		4		3	
				3	
0		3		2	