8

8 Angles and shapes

Leaning Tower of Pisa stops leaning!

Sometimes, angles can cause all sorts of problems.

The building of the Leaning Tower of Pisa began in the 12th century in Pisa, Italy. The tower started tipping sideways before building had finished and, for centuries, it kept tipping further and further. It would have eventually fallen over if engineers and architects hadn't found a way to stabilise the base. In 1990, it was leaning at an angle of 5.5° from the vertical. Engineers removed soil from underneath the raised end, which straightened it a little. In 2008, even more soil was removed and now the tower leans at 3.97°. The engineers have said that it is now stabilised and will not get any worse for more than 200 years.

Forum

What problems could be caused if a building starts to lean?

At what angle do you think the Leaning Tower of Pisa would actually fall over?

Discuss how using angles could be important for engineers fixing the Leaning Tower of Pisa.

Why learn this?

You need to get things straight about angles! They are crucial in sport—whether deciding at what angle to kick a ball, choosing the best club to use for a golf shot or trying to pocket a ball on a pool table. Construction workers need to use angles to make sure that buildings and structures are secure and will not fall down. Surveyors use distances and angles to accurately determine a position on the Earth's surface. They work in construction and establish boundary lines between properties.

After completing this chapter you will be able to:

- estimate and measure angles
- construct angles and shapes using a compass or a protractor
- classify and name angles
- calculate the size of complementary and supplementary angles, vertically opposite angles and angles in a revolution
- identify alternate, corresponding and co-interior (allied) angles and calculate their size
- define and classify polygons, especially triangles and quadrilaterals, identifying their properties.

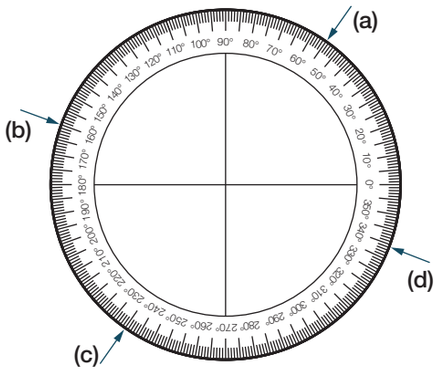
Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R8.1

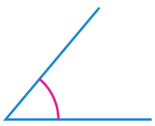
Worksheet R8.2

1 State the value indicated by each arrow in the opposite diagram.

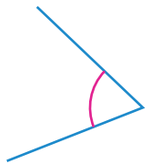


2 Arrange the following angles in order from smallest to largest.

A



B

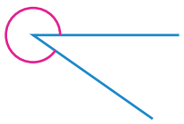


C

D

E

F



Worksheet R8.3

3 List the pairs of angles that look to be the same size.

A

B

C

D

E

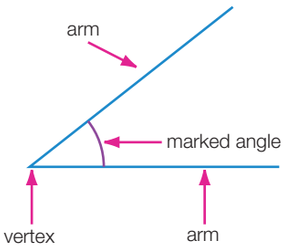
F

Key Words

acute angle	equilateral triangle	plane	right angle
acute-angled triangle	intersect	plane shapes	right-angled triangle
alternate angles	irregular	polygon	scalene triangle
angle	isosceles triangle	protractor	square
arms	kite	quadrilateral	straight angle
co-interior (allied) angles	line segment	ray	supplementary angles
complementary angles	obtuse angle	rectangle	transversal
concave	obtuse-angled triangle	reflex angle	trapezium
convex	parallel lines	regular polygon	vertex
corresponding angles	perpendicular bisector	revolution	vertically opposite angles
degree	perpendicular lines	rhombus	

8.1 Measuring, estimating and drawing angles

An **angle** is the space between two lines that start at the same point. This point is called the **vertex** of the angle and the lines form the **arms** of the angle.

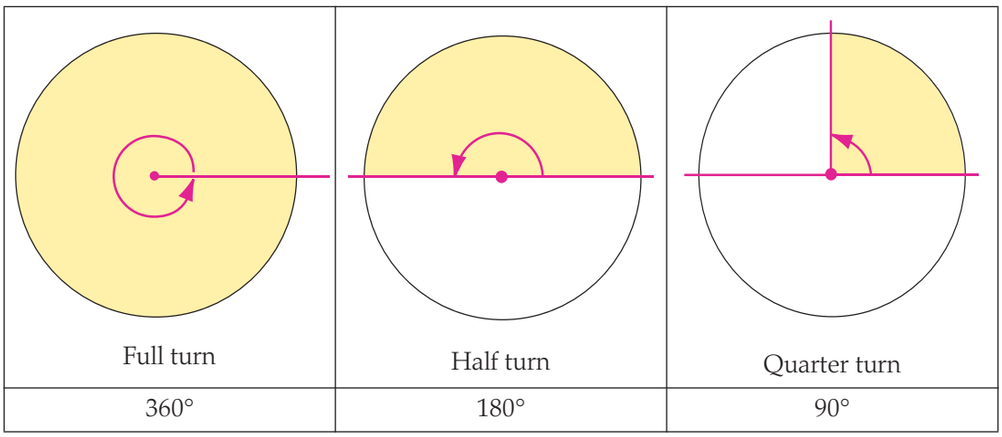


The size of an angle is the amount of turn from one arm to another. One method of measuring the amount of turn is by using **degrees** (°). When an arm is rotated completely around a circle for one full turn, we call this a **revolution** and we divide the revolution into 360 parts.

The amount of turn through one part forms a degree.

Why was a revolution divided into 360 degrees?

It is thought that the ancient Babylonians who counted in lots of 60 and who divided a year into 360 days were responsible for dividing a revolution into 360 parts. A **protractor** has a revolution divided into 360° and is used to draw and measure angles reasonably accurately.



Full turn	Half turn	Quarter turn
360°	180°	90°

Because two angles are formed between two arms, we need to mark the angle that is required.

8.1

Estimating and measuring angles

It is often important to be able to estimate angles. It may not be possible or necessary to measure the angles exactly but we need to be able to visualise how big or small an angle is.

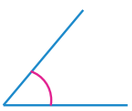
If we need to measure angles, we can use a semicircular protractor or a full-circle protractor.

Worked Example 1 WE1

Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

Estimated angle size	Measured angle size	Difference

(a)



(b)

Thinking

(a) *For angles less than 180°*

1. As the angle is small and about half of a right angle (90°), estimate the size of the angle.
2. Using a semicircular protractor, place the centre point of the protractor over the vertex of the angle and align the baseline of the protractor with an arm of the angle. Use the scale on the protractor that starts with 0°. This protractor measures the angle shown as 50°.
3. Complete the table.

Method 1: Using a semicircular protractor

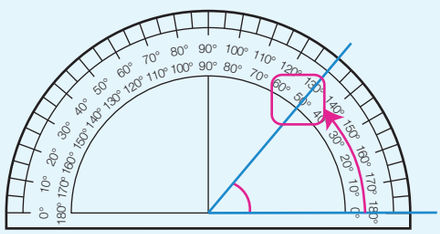
(b) *For angles larger than 180°*

1. As the angle is greater than three right angles (270°), but less than four right angles (360°), estimate the size of the angle.
2. Using a semicircular protractor, measure the smaller angle and subtract this angle from 360°.

   The small angle measures 70°, so the marked angle must be 360° – 70° = 290°.

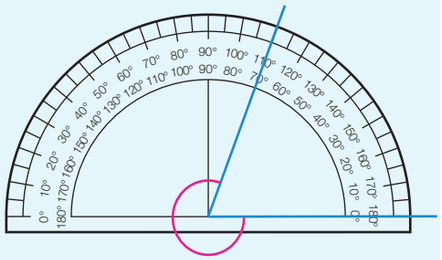
Working

(a) Estimate: 45°



Estimated angle size	Measured angle size	Difference
45°	50°	5°

(b) Estimate: 300°



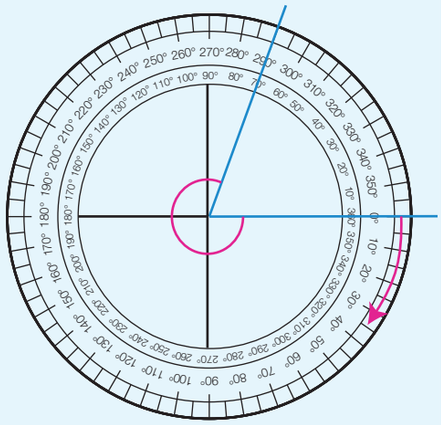
3 Complete the table.

Estimated angle size	Measured angle size	Difference
300°	290°	10°

Method 2: Using a full-circle protractor

(b) 1 Using a full-circle protractor, line up the centre point with the vertex and the 0° line along one arm, as with the semicircular protractor. Use whichever scale increases from zero. In this case, the outer scale, going clockwise.

(b)



2 Complete the table.

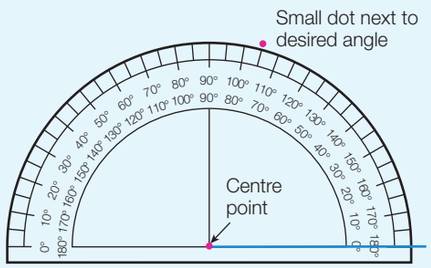
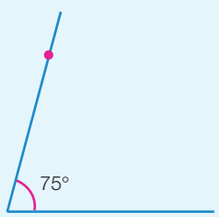
Estimated angle size	Measured angle size	Difference
300°	290°	10°

Drawing angles with a protractor

Worked Example 2 WE2

Use a protractor to draw the following angles.

(a) 75° **(b)** 200°

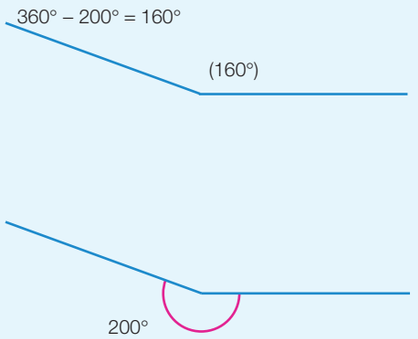
Thinking	Working
(a) *For angles less than 180°*	*(a)*
1 Draw a straight horizontal line.	
2 Place the centre point of the baseline of the protractor on one end of the straight line. Locate the desired angle and mark a small dot as shown (in this case, 75°). (The base line is not the edge of the protractor. Make sure you use the scale starting at the end of your line segment.)	
3 Remove the protractor and join the dot with the end of the line where the centre point was located. Mark the angle.	

(b) *For angles larger than 180°*

1 Subtract the angle from 360° to get the smaller angle needed to make up a full turn. Now, draw this angle. (For example, to draw 200°, we need to first draw 160°.)

2 Label the correct, larger angle.

(b)



8.1 Measuring, estimating and drawing angles

Navigator

Answers page 667

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11 Column 1, Q12, Q15	Q1, Q2 Column 2, Q3, Q4 , Q5, Q6, Q7, Q9, Q10, Q11 Column 2, Q12, Q13, Q16	Q1, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Column 3, Q12, Q14, Q16

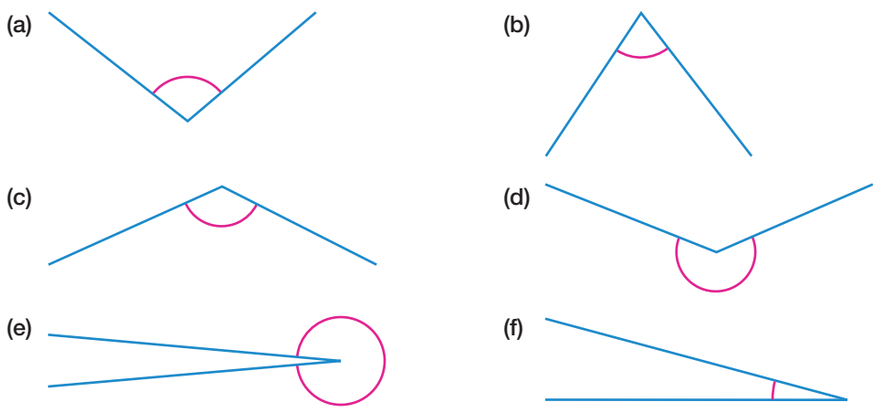
Equipment required: Protractor

Fluency

WE1

1 Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

Estimated angle size	Measured angle size	Difference



WE2

2 Use a protractor to draw the following angles.

(a) 10° **(b)** 70° **(c)** 55°

(d) 100° **(e)** 175° **(f)** 108°

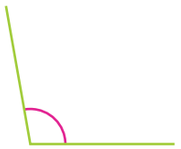
(g) 290° **(h)** 265° **(i)** 318°

3 **(a)** The angle shown here is approximately:

A 20° **B** 45°

C 60° **D** 85°

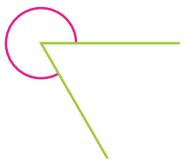
(b) The angle shown here is approximately:



A 80° **B** 100°

C 170° **D** 200°

(c) The angle shown here is approximately:



A 60° **B** 180°

C 240° **D** 300°

4 **(a)** Which of the angles below is about 170°?

A

B

C

D

(b) Which of the angles below is about 5°?

A

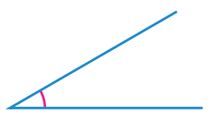
B

C

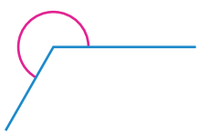
D

(c) Which of the angles below is about 330°?

A

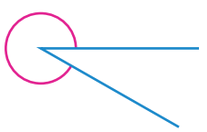


B



C

D



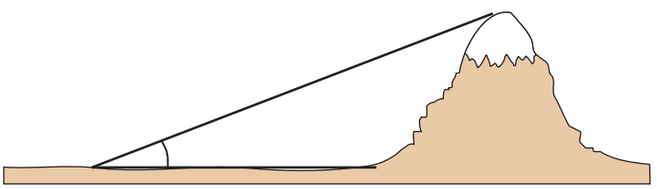
5 To draw a 250° angle with a semicircular protractor, which of the following would you do first?

A Draw a 50° angle. **B** Draw a 90° angle.

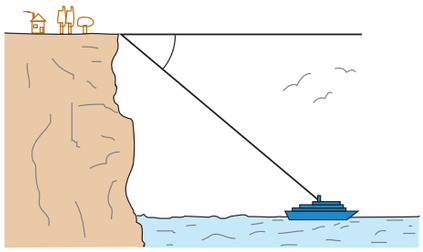
C Draw a 110° angle. **D** Draw a 200° angle.

Understanding

6 In the diagram, estimate the angle shown from ground level to the mountain top and then measure the angle with your protractor.



7 In the diagram, estimate the angle shown from the top of the cliff to the ship and then measure the angle with your protractor.

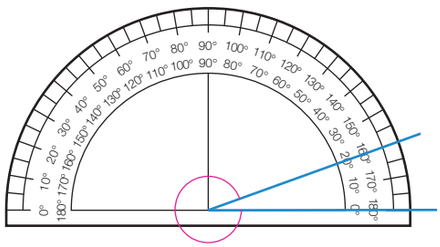


8 Explain how you would draw an angle of 320° with a semicircular protractor.

9 Liz uses a semicircular protractor to measure an angle larger than 180°.

Find the size of the actual angle (larger than 180°) if the smaller angle she has measured is:

(a) 30° **(b)** 115° **(c)** 160°



10 Draw the following using your protractor.

(a) a wire holding a vertical flagpole attached at 38° to the ground

(b) a ramp with a slope of 12°

(c) a ladder leaning against a vertical wall at 62° to the ground

(d) a round birthday cake sliced into 6 equal pieces

Reasoning

11 For each of the following angles:

(a) 35° **(b)** 65° **(c)** 83°

(d) 120° **(e)** 102° **(f)** 152°

(g) 200° **(h)** 265° **(i)** 325°

(i) draw an estimate of the angle without the use of a protractor

(ii) use your protractor to measure the angles you have drawn to find the actual size of your angles

(iii) find the error (the difference between the given angle and your measurement)

(iv) copy and complete the following table.

Estimated angle size	Measured angle size	Difference

(v) Which angles did you draw the most accurately?

(vi) Do you think it is harder to estimate angles greater than 180° than those less than 180°? Why?

Open-ended

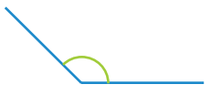
12 Draw the following angles, estimate the size of the angle you have drawn and then check the size with a protractor.

(a) any angle smaller than 90°

(b) any angle between 90° and 180°

(c) any angle between 180° and 360°

13 Your friend Nick has just answered the following question.

> Use a protractor to measure the size of the following angle. Give your answer to the nearest degree.
>
> 

Nick says "The answers in the back of the book must be wrong! The book says that the answer is 135°, but my protractor says 45°." Explain how Nick might have found his answer. Give Nick a hint, so that he can measure accurately all the time.

14 Without using a protractor, draw the following angles and estimate the size of each angle.

(a) two angles that add up to 90°

(b) three angles that add up to 180°

(c) four angles that add up to 360°

(d) Check your answers to **(a)**, **(b)** and **(c)** with a protractor.

15 **(a)** Choose any angle between 40° and 60°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle 15° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle 195° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

16 **(a)** Choose any angle less than 90°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle twice the size of the angle you chose in **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle twice the size of the angle in **(b)** and mark the size of your angle on your diagram.

Outside the Square Game

The Angle Master

Equipment required: 2 brains, ruler, protractor

How to win:
The winner is the first player to score 15 points, and is proclaimed the 'Angle Master'.

How to play:
The aim of the game is to guess the size of different angles more accurately than your opponent.

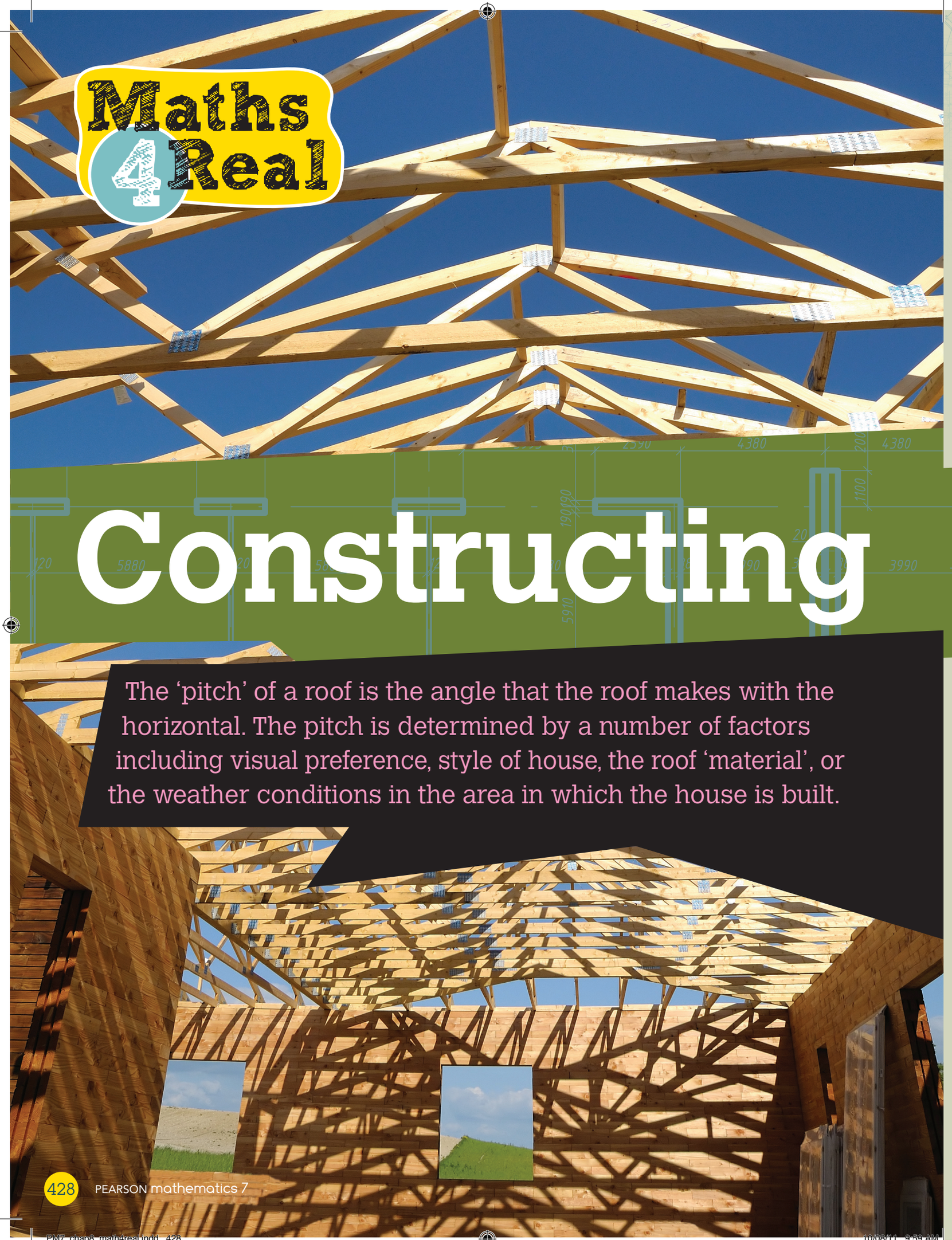
1. Take it in turns to create an angle by drawing two straight lines without using a protractor. Clearly mark the angle you have created.
2. You and your opponent must then bid to see who can get the closer guess to the size of the angle without guessing over the size of the angle. The bidding ends when one player believes the other has overestimated the angle and calls for the angle to be measured at this guess.

Scoring
0–1° under 5 points
2–5° under 4 points
6–10° under 2 points
11+° under 1 point

Correctly claiming your opponent has overestimated the angle: 3 points.

Sample game play:
Imagine player 1 draws the following angle.

Player 2 starts the bidding at 110°, player 1 ups the bidding to 118°, player 2 ups the bidding to 124°, and player 1 calls for the angle to be measured. The angle is measured at 126°, so player 2 scores 4 points.

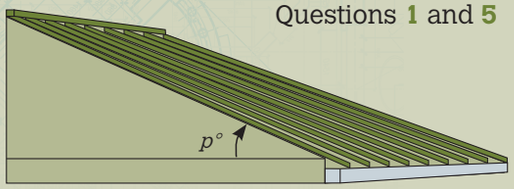


Maths 4 Real

Constructing

The ‘pitch’ of a roof is the angle that the roof makes with the horizontal. The pitch is determined by a number of factors including visual preference, style of house, the roof ‘material’, or the weather conditions in the area in which the house is built.

Equipment required: 2 rulers and a protractor for Questions 1 and 5



The main reason houses have a pitched roof is to redirect rainwater. A pitch of between 20° and 30° is common.

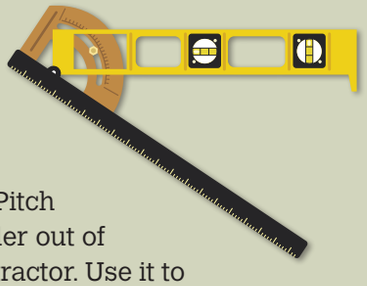
1 Measure the angle, *p*, of the pitched roof above to determine a common pitch angle for Australian house roofs.

2 Imagine you are building a house in an area that regularly receives a lot of heavy rain. What problems might this cause if the roof is:

(a) too flat? (pitch angle is too small), or

(b) too steep? (pitch angle is too big)

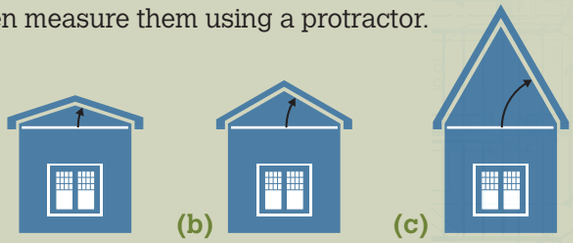
5 To measure an angle, builders use a *Pitch Angle and Level Finder* for construction.



Construct your own Pitch Angle and Level Finder out of two rulers and a protractor. Use it to measure the pitch of the roof of a building in your area, such as a school building or your house. You can do this by standing on the ground and holding your pitch angle and level finder in line with the roof of the building.

angles

3 Estimate the pitches of the roofs on these houses, then measure them using a protractor.



(a) (b) (c)

4 In parts of North America and Europe, where there is a lot of snow, the pitch of the roof must be at least 30°. Why do you think this is the case?

Research

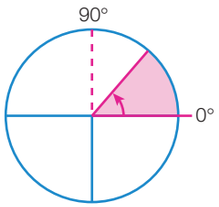
6 (a) Find out some of the common pitch angles used by builders. Does there seem to be a minimum or maximum value for the pitch angle? What problems might be created if the angle is too steep?

(b) Research the difference in pitch required by different materials, such as a thatched roof, tiled roof or corrugated iron roof.

8.2 Classifying and naming angles

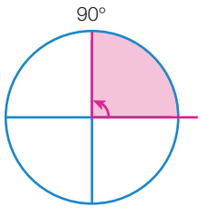
Classifying angles

Acute angle: greater than 0° but less than 90°

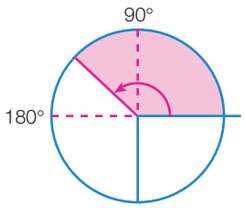


Right angle: exactly 90°, a $\frac{1}{4}$ turn.

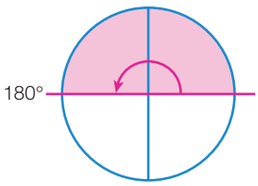
A small square drawn in the corner of an angle means it is a right angle.



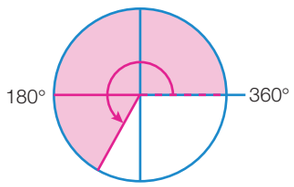
Obtuse angle: more than 90° but less than 180°.



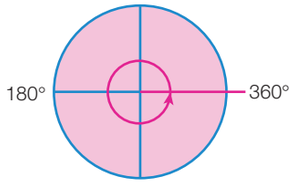
Straight angle: exactly 180°, a straight line angle, a half turn.



Reflex angle: greater than 180° but less than 360°.



Revolution: 360°, a full turn.



Points, lines and angles

A *point* defines a position in space but it has no size. It has no width or length, so a dot we can see or write is not really a point at all. We represent a point with a dot and label it with upper case letters such as A, B and P.

Similarly, a *line* we can draw or see is not really a line, because a line has no width and has no beginning or end, extending infinitely in both directions. A drawn line represents a line. A line always means a straight line and does not include curves. We can draw a line through any two points and use arrowheads on each end to show that it goes on forever in both directions.

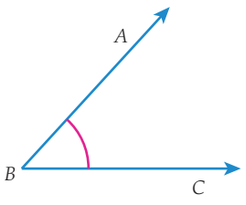
We name a line that goes through points A and B, $\overleftrightarrow{AB}$.

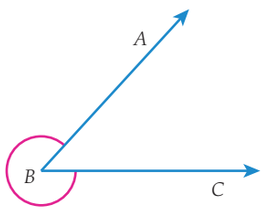
A **ray** is part of a line. Unlike a line, a ray has a beginning. It starts at a point called the vertex and extends infinitely in one direction just like a ray of light from a torch. We draw a ray by starting it at a point and using an arrowhead on the other end.

We name a ray that starts at point A and passes through B as $\overrightarrow{AB}$.

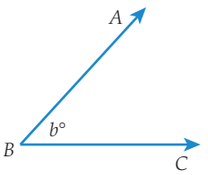
A **line segment** is part of a line that has both a beginning and an end. It can also be called an interval. We often refer to line segments as lines, but this is not strictly correct. We name a line segment that starts at A and ends at B as $\overline{AB}$.

If two rays have a common vertex, two angles are formed. When we draw an angle we have to mark clearly the angle we want.





You will see this notation in geometry software packages; however, we usually use line segments to draw angles. We can name the angles that are formed $\overline{AB}$ and $\overline{BC}$ as $\angle ABC$ or by $\angle CBA$ with the vertex B named as the middle of the three points and use the words acute, obtuse or reflex to indicate which angle we mean.



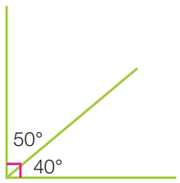
An easier and clearer way to name an angle is to use a letter with units such as $a°$ or $b°$ in the angle space. We usually use the same letter as the letter on the vertex.

Complementary and supplementary angles

Complementary angles add to 90° (a right angle).

40° and 50° are complementary angles because they add to 90°.

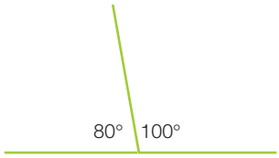
If two angles are complementary, we say one is the complement of the other. For example, 40° is the complement of 50°.



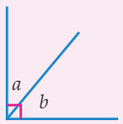
Supplementary angles add to 180° (a straight angle).

100° and 80° are supplementary angles because they add to 180°.

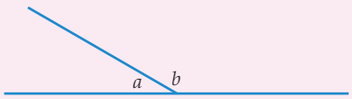
If two angles are supplementary, we say one is the supplement of the other. For example, 100° is the supplement of 80°.



Complementary angles add to 90°:
$a + b = 90$

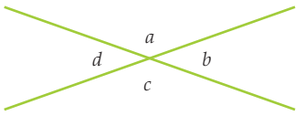


Supplementary angles add to 180°:
$a + b = 180$



Vertically opposite angles

Whenever two lines intersect, four angles are formed. In the diagram opposite, these angles have been labelled as a, b, c and d. Pairs of angles such as $a°$ and $c°$ are given a special name—they are called **vertically opposite angles**.



There is another pair of vertically opposite angles in the diagram: b and d.

If you measured a and c with your protractor you would find they are the same size. The same is true for b and d. Check both of these pairs of angles for yourself.

Vertically opposite angles are equal.

8.2 Classifying and naming angles

Navigator

Answers page 668

Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13, Q15	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Q12, Q14, Q15	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15

Equipment required: Protractor for Questions **10(a)** and **12**

Fluency

1 State the type of angle shown in each case.

(a)

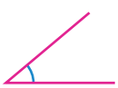
(b)

(c)

(d)

(e)

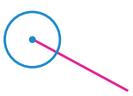
(f)



(g)

(h)

(i)

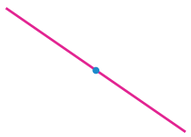


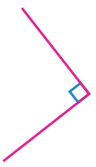
(j) 

(k) 

(l) 

(m) 

(n) 

(o) 

(p) 

(q) 

(r) 

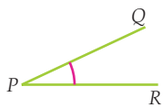
2 Classify the following angles (acute, obtuse, right etc.).

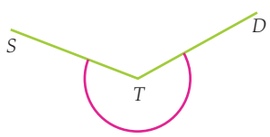
(a) 23° **(b)** 117° **(c)** 275°

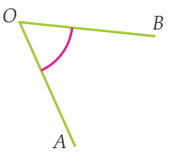
(d) 360° **(e)** 180° **(f)** 75°

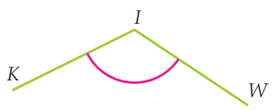
(g) 90° **(h)** 165° **(i)** 341°

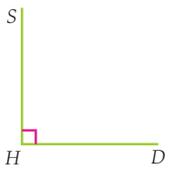
3 Name the following angles.

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

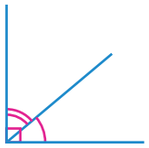
4 **(a)** Which one of the following is a pair of complementary angles?

A 20° and 40° **B** 330° and 30° **C** 30° and 150° **D** 15° and 75°

(b) Which one of the following is a pair of supplementary angles?

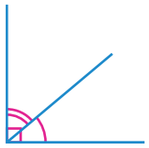
A 0° and 90° **B** 45° and 55° **C** 90° and 90° **D** 180° and 20°

(c) Which diagram shows supplementary angles?

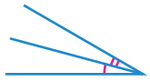
A 

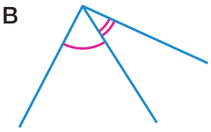
B 

C 

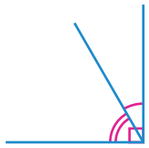
D 

(d) Which diagram shows complementary angles?

A 

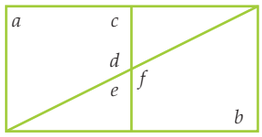
B 

C 

D 

It might help you to remember that the 'c' of complementary stands for a corner (right angle) and the 's' of supplementary stands for a straight angle.

5 Which two angles are vertically opposite?

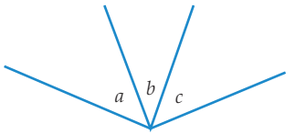


A a and b **B** a and c

C d and e **D** d and $f°$

Understanding

6 The angles a, b and c in this diagram have been drawn as adjacent angles. They have a common vertex and common arms.



For each of the following:

(a) 100°, 210°, 50° **(b)** 32°, 40°, 18° **(c)** 40°, 40°, 100°

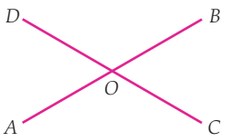
(d) 32°, 161°, 85°, 82° **(e)** 44°, 46° **(f)** 39°, 141°

(i) draw the angles as adjacent angles

(ii) find the angle sum

(iii) describe the angle you have formed.

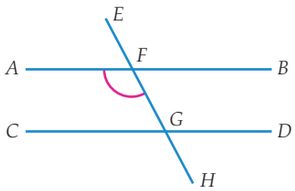
7 In the diagram opposite:



(a) name a pair of supplementary angles

(b) name a pair of vertically opposite angles.

8 In the diagram, what could the marked angle be named?



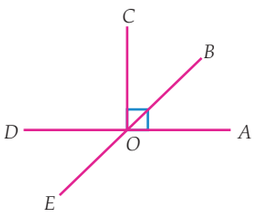
A $\angle F$

B $\angle AFE$

C $\angle AFG$

D $\angle BFG$

9 Using the diagram opposite, find, using letter names:



(a) an acute angle

(b) an obtuse angle

(c) a reflex angle

(d) a right angle

(e) a straight angle

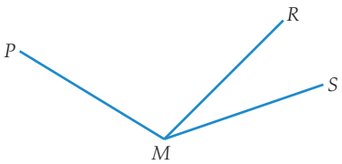
(f) one pair of complementary angles

(g) one pair of supplementary angles

(h) one pair of vertically opposite angles.

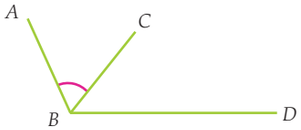
Reasoning

10 (a) Measure the reflex angle RMS in the diagram.



(b) Why is it necessary to specify 'reflex' in part **(a)**?

11 (a) Why couldn't we refer to the marked angle as $\angle B$?



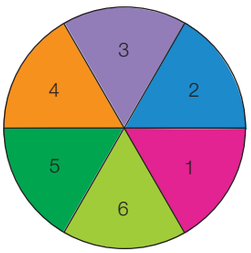
(b) Name two acute angles that have B as a vertex.

(c) Name an obtuse angle that has B as a vertex.

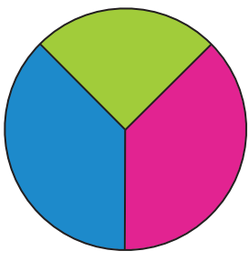
(d) Name a reflex angle that has B as the vertex.

12 Use a protractor to draw accurate copies of the following.

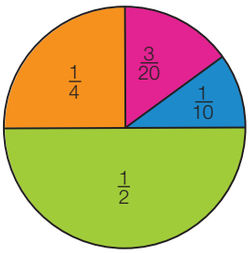
(a) A raffle wheel divided into six equal parts.



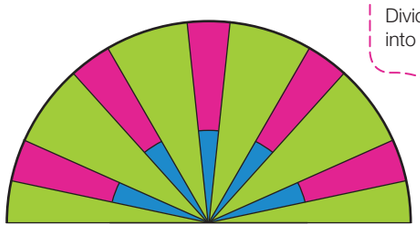
(b) This sign.



(c) This pie chart.



(d) This semicircular stained glass window.



Open-ended

13 Draw two examples of:

(a) an acute angle **(b)** a reflex angle **(c)** an obtuse angle.

14 **(a)** Write two examples of pairs of complementary angles.

(b) Write two examples of pairs of supplementary angles.

15 Write two examples of angles that add to make a revolution.

Outside the Square — Problem solving

Count the angles

Equipment required: 1 brain, 1 protractor

1 How many different acute angles can you count in the diagram? Measure each one.

2 How many different angles can you count above the horizontal line? Measure the size of each one.

3 How many different angles can you count in the diagram? Measure the size of each one.

Strategy options

- Test all possible combinations.
- Break problem into manageable parts.

8.3 Calculating angles

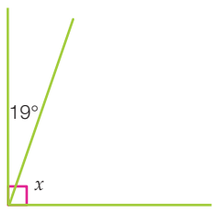
Complementary and supplementary angles

The sum of complementary angles is 90°, so we can subtract a known angle from 90° to find the unknown complementary angle. In a similar way, we can subtract a known angle from 180° to find the unknown supplementary angle.

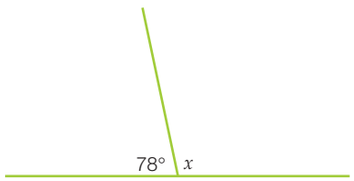
Worked Example 3 WE3

Find the size of angle x in each diagram.

(a)



(b)



Thinking		Working
(a)	1 Identify the angles as complementary angles.	(a) x and 19° are complementary angles.
	2 Write an equation with the RHS equal to 90°.	$x + 19 = 90$
	3 Solve the equation to find the angle.	$x + 19 - 19 = 90 - 19$ $x = 71°$
(b)	1 Identify the angles as supplementary angles.	(b) x and 78° are supplementary angles.
	2 Write an equation with the RHS equal to 180°.	$x + 78 = 180$
	3 Solve the equation to find the angle.	$x + 78 - 78 = 180 - 78$ $x = 102°$

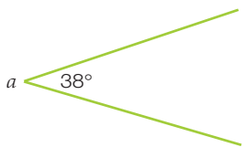
Angles in a revolution

Angles in a revolution add to 360°, so we can subtract a known angle from 360° to find the unknown angle.

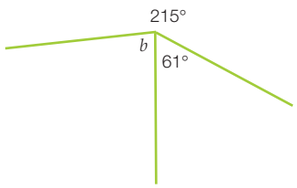
Worked Example 4 WE4

Find the value of the pronumeral in each diagram.

(a)



(b)



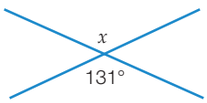
Thinking		Working	
(a)	**1** Add all the angles in the revolution, to give 360°.	(a)	$a + 38 = 360$
	2 Find the value of a.		$a + 38 - 38 = 360 - 38$ $a = 322°$
(b)	**1** Add all the angles in the revolution, to give 360°.	(b)	$215 + 61 + b = 360$ $276 + b = 360$
	2 Find the value of b.		$276 - 276 + b = 360 - 276$ $b = 84°$

Vertically opposite angles

Vertically opposite angles are equal, so if we know one angle we know that the vertically opposite angle is the same size.

Worked Example 5 WE5

Find the value of x in the diagram.



Thinking	Working
These are vertically opposite angles, which means they are equal.	$x = 131°$

8.3 Calculating angles

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7 Column 1, Q8, Q10 Column 1, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7 Column 2, Q10 Column 2, Q11, Q12, Q13	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7 Column 3, Q9, Q10 Column 3, Q11, Q12, Q13, Q14

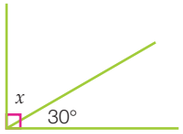
Answers page 668

Equipment required: Protractor for Questions **11(a)** and **12**

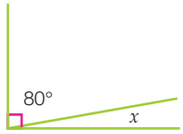
Fluency

1 Find the size of angle x in each diagram. WE3

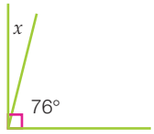
(a)



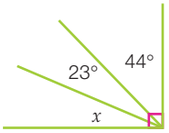
(b)



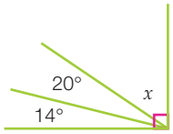
(c)



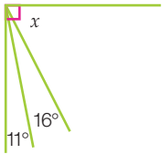
(d)

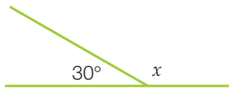


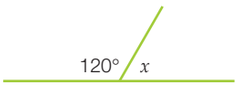
(e)



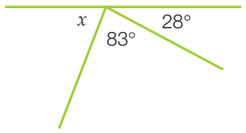
(f)

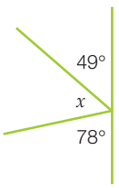


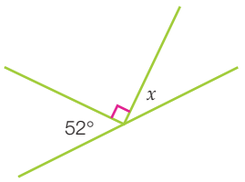
(g) 

(h) 

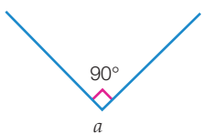
(i) 

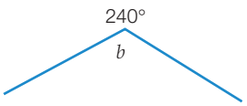
(j) 

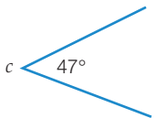
(k) 

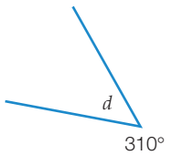
(l) 

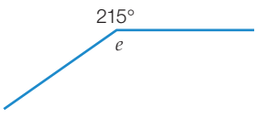
WE4 **2** Find the value of the pronumeral in each diagram.

(a) 

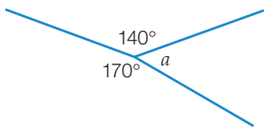
(b) 

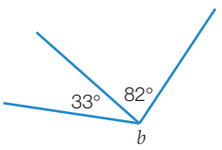
(c) 

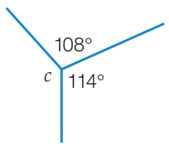
(d) 

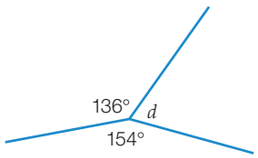
(e) 

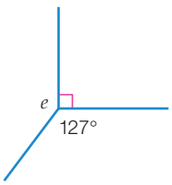
(f) 

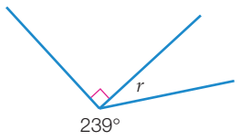
(g) 

(h) 

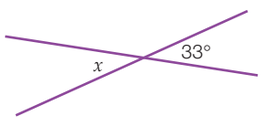
(i) 

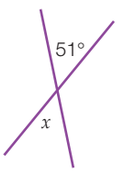
(j) 

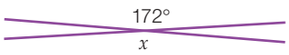
(k) 

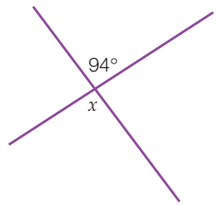
(l) 

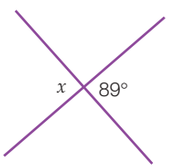
WE5 **3** Find the value of x in each diagram.

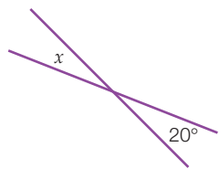
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

4 Find the complement of the following angles.

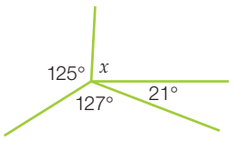
(a) 27° **(b)** 45° **(c)** 68° **(d)** 15°

5 Find the supplement of the following angles.

(a) 32° **(b)** 90° **(c)** 124° **(d)** 176°

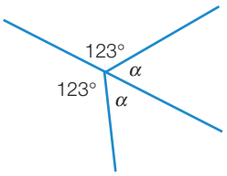
Understanding

6 Find the value of x in the diagram.

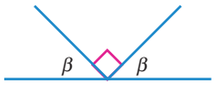


7 Find the value of the pronumerals in each diagram.

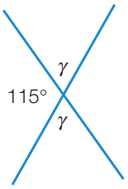
(a)



(b)



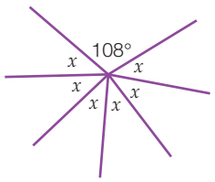
(c)



8 The value of x in the opposite diagram is:

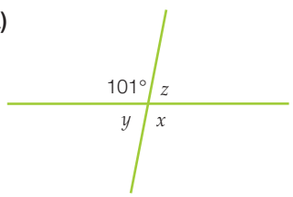
A 42° **B** 60°

C 108° **D** 360°

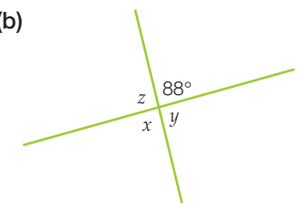


9 Find the value of the pronumerals in each diagram.

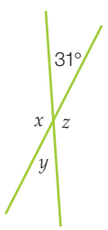
(a)



(b)



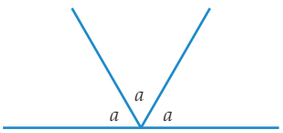
(c)



Reasoning

10 Determine the size of angle a in each diagram. Give reasons for your answer.

(a)



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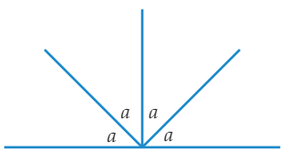
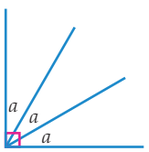
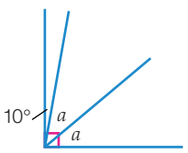


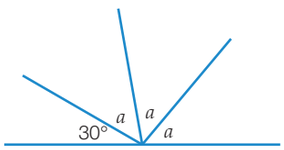
(c)



(d)



(e)



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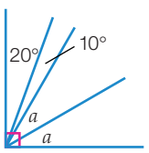
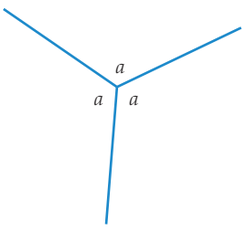
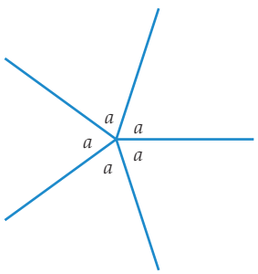


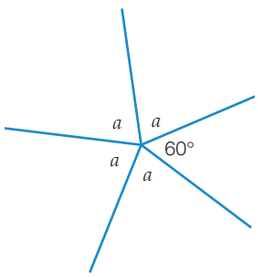
(g)



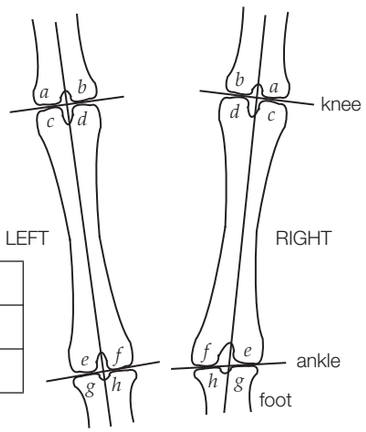
(h)



(i)



11 After breaking her leg in an accident, Kira had difficulty walking. She took her X-ray to a specialist, who marked in the following lines. He followed a procedure to decide whether she needed an operation. (You will need a protractor to measure some of the angles.)



(a) Measure all the angles and fill in the table below.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Left leg								
Right leg								

(b) All angles on Kira's knees must be approximately equal and all the angles on her ankles must also be approximately equal. Which leg do you think Kira broke?

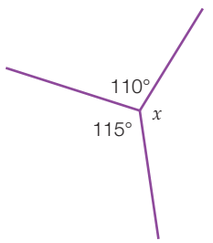
(c) The differences between the angles b and f, and c and g, must be no more than 8°. If they are more, Kira needs an operation. Does she need an operation? Why?

Open-ended

12 Sabine was answering the question on the right.

Using her protractor, Sabine measured the angle x and found that it was 140°. This was incorrect. Explain where Sabine made a mistake.

Find the value of the pronumeral.



13 **(a)** Draw two lines that intersect each other.

(b) Measure the two supplementary angles you have created.

(c) Use your answers to **(b)** to work out the two angles you did not measure.

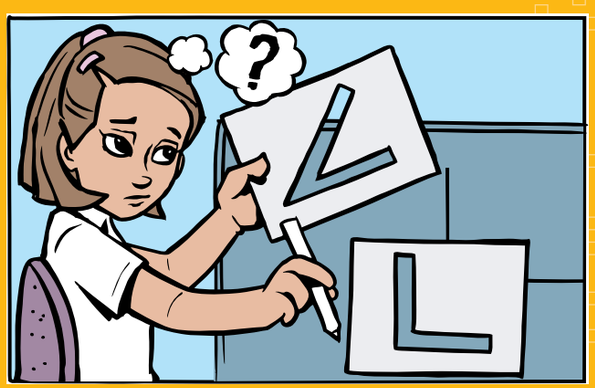
14 Draw two straight lines that intersect, forming two acute angles. What are the sizes of the other two angles?

Outside the Square — Problem solving

Ella's angles

Ella needs to draw a 60° angle, but she has lost her protractor. She has a stencil that can draw 90° and 40° angles. How can she use these two angle sizes to draw a 60° angle?

Strategy options
- Draw a diagram.
- Guess and check.



Technology Exploration GeoGebra

Equipment required: 1 brain, 1 computer with GeoGebra

Versions of this Exploration are available for other technologies in Pearson Reader.

Investigating angles on parallel lines

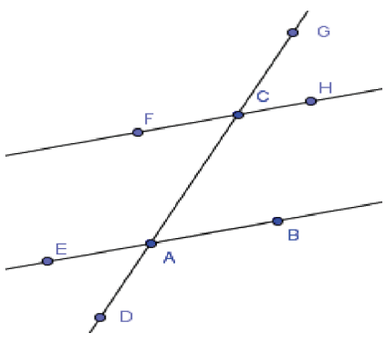
Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are eleven icons called tools. When you hover the mouse over the arrow in the bottom right-hand corner of the tool icon the arrow turns red and the name of the tool appears. By clicking on this arrow a drop-down list of more tools appears. Instructions on how to use the tool will appear in the top right-hand corner of the screen.

1. Click on the View menu. Deselect 'Axes', 'Grid' and 'Algebra View'. (Alternatively, you can right click in the space where the drawings appear, called the graphics view, to do the same for the 'Axes' and the 'Grid'.)
2. Click on the Options menu. Select 'Labelling', then 'New Points only'. Click on Options again, then select 'Rounding', and '0 Decimal Places'.
3. If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided.

Creating a pair of parallel lines and a transversal

Parallel lines are lines that are equidistant from each other (the distance between the lines is the same all the way along them). A transversal is a line that cuts two or more other lines. (The lines cut by a transversal do not have to be parallel.)

4. Construct a line using the 'Line Through two Points' tool  , then click on any two points on the page.
5. Click on the small arrow on the fourth tool from the left and select the 'Parallel Line' option  . Create a parallel line by clicking on the line and then on anywhere above the line. (You know when you are about to select the line as it becomes darker and the cursor turns to an arrow when you hover over it.)
6. Create a transversal by selecting the 'Line Through two Points' tool  , then clicking on point *A* and then on point *C*.
7. Select the 'New Point' tool  . Use this to place points on the lines as shown below. Keep the points in the same order as we will refer to them in the following steps. They are placed in a clockwise direction from under point *A*. Points will be placed on the line only if the line gets darker when your mouse hovers over it. It does not matter if your transversal or your parallel lines slope in the opposite direction, just ensure that the points are placed in the same positions.



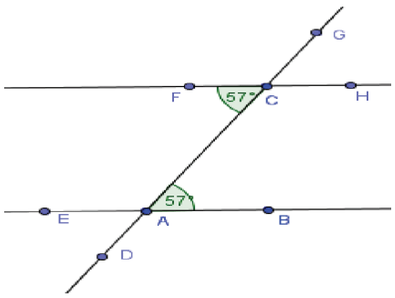
If you make a mistake, you can either:

- right click on the object and select 'Delete', or
- click the 'Edit' menu and select 'Undo', or
- press 'ctrl' + 'z'.

Alternate angles and parallel lines

8 Click on the small arrow on the 8th tool from the left.

Select the 'Angle' tool  . Click on point *B*, then *A* and then *C* (∠*BAC*). Notice that the vertex of the angle is the middle letter. Click on ∠*FCA* in the same way. Points must be selected in a clockwise order, so if you get the reflex angle, delete it and try selecting the points in the opposite order. Your diagram should look similar to the one below. You have just marked a pair of alternate angles. Alternate angles sit on opposite sides of a transversal.



9 Right click inside one of the marked angles and select 'Object Properties'. A pop-up box will appear. On the 'Colour' tab click on a different colour. With the pop-up box still open, click inside the other marked angle and select the same colour (the colour will be stored in the 'Recent' box on the right-hand side of the pop-up box). Close the pop-up box.

10 Click on the 'Angle' tool  . Mark ∠*CAE* and ∠*ACH* as described in step **8**. You have just marked another pair of alternate angles.

11 Click on the 'Select' tool  . (Pressing 'Escape' also takes you to the 'Select' tool.) Click on point *B* and drag it about.

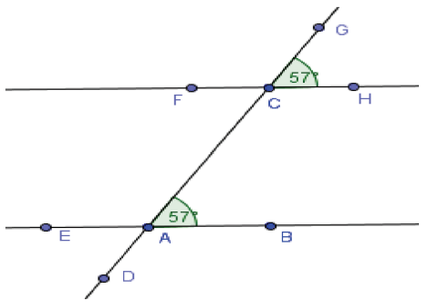
(a) You have just marked two pairs of alternate angles between a pair of parallel lines. What do you notice as you move the lines about?

(b) Copy and complete the statement: 'Alternate angles between parallel lines are _____'.

12 Right click inside a marked angle and select delete, repeat for the other marked angles. (If some of the points have disappeared, move point *B* until they reappear.)

Corresponding angles and parallel lines

13 Click on the 'Angle' tool  , then mark ∠*HCG* and then ∠*BAC* by clicking on the points. Your diagram should look similar to the one below. You have just marked a pair of corresponding angles. Corresponding angles sit on the same side of the transversal, one underneath the other.



14 Follow the process in step **9** to change the colour of both angles.

15 Repeat steps **13** and **14** for ∠*ACH* and ∠*DAB*, for ∠*EAD* and ∠*FCA*, and for ∠*CAE* and ∠*GCF*. Make sure that each pair of angles has a different colour. You have just marked three more pairs of corresponding angles.

16 Click on the 'Select' tool  (or press escape).

Click on point *B* and drag it about. Take notice of the angle sizes.

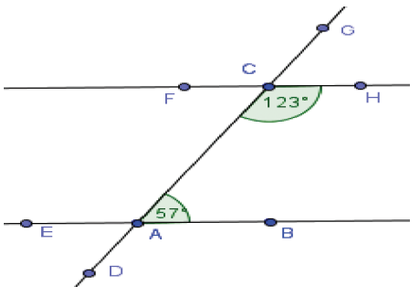
(a) What do you notice about each pair of corresponding angles as you move the lines about?

(b) Copy and complete the statement: 'Corresponding angles on parallel lines are _____.'

17 Delete the marked angles as described in step **12**.

Co-interior angles and parallel lines

18 Click on the 'Angle' tool  , then mark $\angle ACH$ and $\angle BAC$ by selecting the points. Your diagram should look similar to the one below. You have marked a pair of co-interior angles (also known as allied angles). Co-interior angles sit on the same side of the transversal and on the inside of the lines.



19 Follow the process in step **9** to change the colour of both angles.

20 Repeat steps **18** and **19** for $\angle CAE$ and $\angle FCA$ to mark another pair of co-interior angles.

21 Click on the 'Select' tool  (or press escape), then click on point *B* and drag it about. Take note of the angle sizes.

(a) What do you notice about each pair of co-interior angles as you move the line about?

(b) Copy and complete the statement: 'Co-interior angles between parallel lines sum to _____°'.

22 Clear the angles as described in step **12**.

Taking it further

23 Select the 'New Point' tool  and place a point on the interval between points *A* and *C*. This will be point *I*.

24 Click on the small arrow on the fourth tool from the left. Select the 'Parallel Line' tool  . Create another parallel line by clicking on the line *AB* and then on the new point *I*.

25 Select the 'New Point' tool  and place a point *J* on the new line to the left of point *I* and another point *K* to the right of point *I*.

26 Select the 'Angle' tool  . Click on $\angle IAE$.

27 You may use the 'Angle' tool  to assist you with the following.

Find two angles that are alternate, two angles that are corresponding, and two angles that are co-interior to $\angle IAE$. Mark each of these angles using different colours to indicate the three different angle types. Use correct terminology to name the angles.

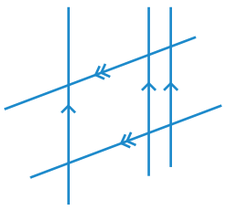
28 Using the 'Angle' tool  , mark and colour all of the angles that are the same size.

(a) Can you see any pairs of vertically opposite angles? Name as many pairs as you can.

(b) Can you see any pairs of adjacent supplementary angles? Name as many pairs as you can.

8.4 Angles and parallel lines

Parallel lines are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. The lines on a page of lined paper are parallel.



Lines that are parallel are marked with an arrow pointing in the same direction.

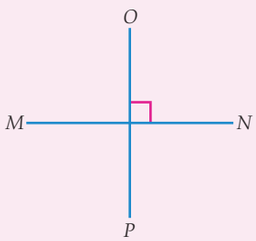
If more than one set of parallel lines appears in a diagram we use more than one arrow.

Perpendicular lines intersect at right angles.

|| means 'is parallel to'

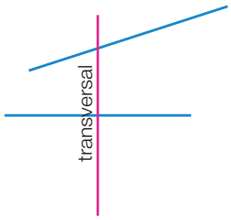
$AB \parallel CD$

$\perp$ means 'is perpendicular to'

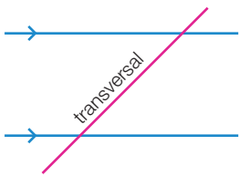


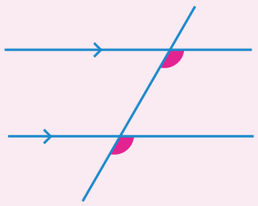
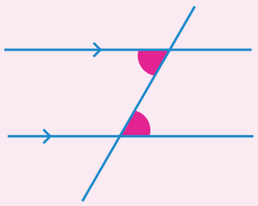
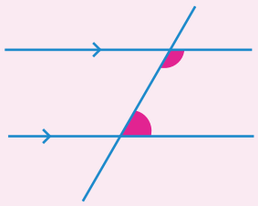
$OP \perp MN$

A **transversal** is a line that **intersects** (crosses or transverses) two or more other lines, as shown.



When parallel lines are crossed by a transversal, the pairs of angles on parallel lines described have special properties.



Corresponding angles	Alternate angles	Co-interior (allied) angles
		
Corresponding angles are *equal*. These angles both lie above or below parallel lines, and on the same side of the transversal. 'Corresponding' means 'matching', so corresponding angles are in matching positions. There are four pairs of corresponding angles formed when a transversal cuts two lines.	Alternate angles are *equal*. These angles lie between the parallel lines and on different sides of the transversal between the parallel lines. 'Alternate' means 'swap', so alternate angles swap sides of the transversal. There are two pairs of alternate angles formed when a transversal cuts two lines.	Co-interior (allied) angles are *supplementary*. They add to 180°. These angles lie between the parallel lines and on the same side of the transversal between the parallel lines. 'Co' means 'with' and 'interior' is 'inside', so co-interior angles are on the same side of the transversal and inside the parallel lines. There are two pairs of co-interior angles formed when a transversal cuts two lines. Co-interior angles are also called allied angles.

The opposite of the above is also true. If corresponding or alternate angles are equal or if co-interior angles are supplementary when two lines are cut by a transversal, the two lines are parallel.

8.4 Angles and parallel lines

Navigator

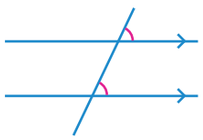
Q1, Q2, Q3, Q4, Q5, Q6 Columns 1 & 2, Q7, Q8 Column 1, Q9, Q10, Q13 Column 1, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 2, Q9, Q11, Q12, Q13 Column 2, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 3, Q11, Q12, Q13 Column 3, Q14, Q15

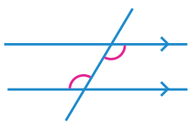
Answers page 669

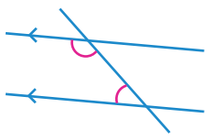
Equipment required: Protractor for Question **6 (a)**

Fluency

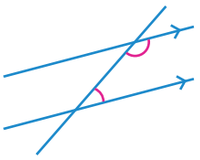
1 Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

(a) 

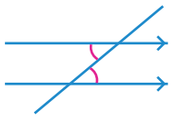
(b) 

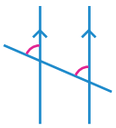
(c) 

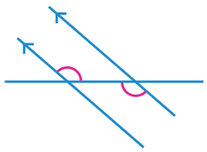
(d) 

(e) 

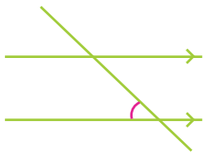
(f) 

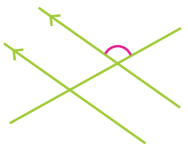
(g) 

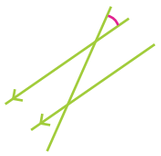
(h) 

(i) 

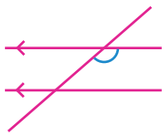
2 Copy each of the following and mark in an angle corresponding to the one shown.

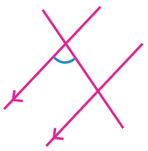
(a) 

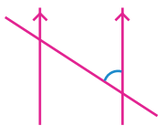
(b) 

(c) 

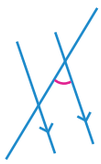
3 Copy each of the following and mark in an angle alternate to the one shown.

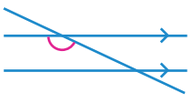
(a) 

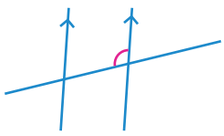
(b) 

(c) 

4 Copy each of the following and mark in an angle that is co-interior with the one shown.

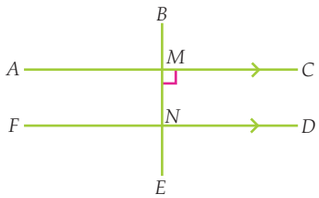
(a) 

(b) 

(c) 

Remember that ⊥ means perpendicular to and || means parallel to.

5 Which of the following statements is not true about the diagram shown?



A $\angle AMB$ is corresponding to $\angle FNB$

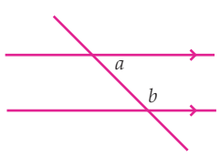
B $\angle BNF$ is alternate to $\angle EMC$

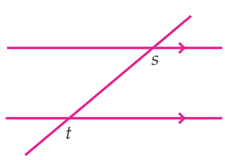
C $\angle AMN$ and $\angle CMN$ are co-interior angles

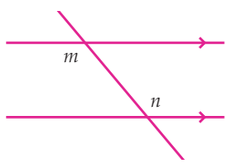
D BE is ⊥ to FD

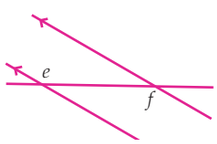
Understanding

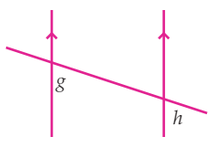
6 (a) Use a protractor to measure the labelled angles in each diagram below. State whether each pair of angles is corresponding, alternate or co-interior.

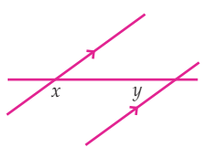
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

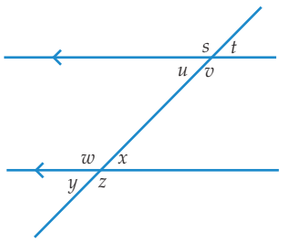
(b) State which pairs of angles in part **(a)** are equal. Which pairs of angles add to 180°?

7 State which angle in the diagram is:

(a) corresponding to u **(b)** alternate to v

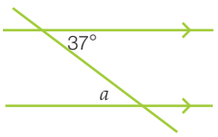
(c) co-interior with w **(d)** co-interior with x

(e) corresponding to w **(f)** alternate to u.

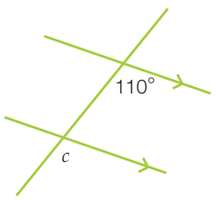


8 Find the value of the pronumerals in each case, and give a reason for your answer (e.g. corresponding angles).

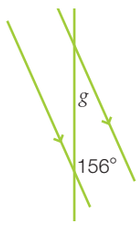
(a)



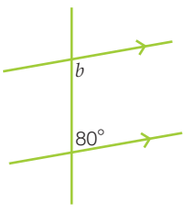
(b)



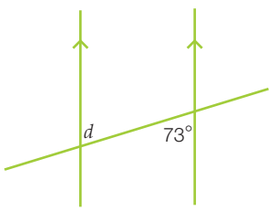
(c)



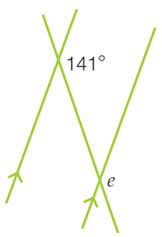
(d)



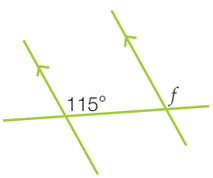
(e)



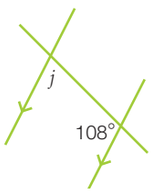
(f)



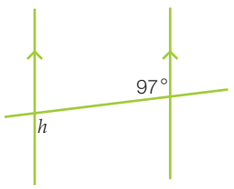
(g)



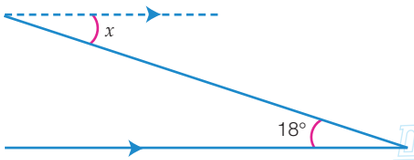
(h)



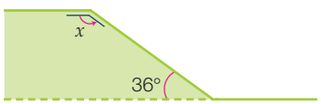
(i)



9 A pilot of a plane flying parallel to the ground can see the runway as it is coming in to land. The angle of approach to the runway is 18°. What angle must the pilot turn the plane down through to start the approach?

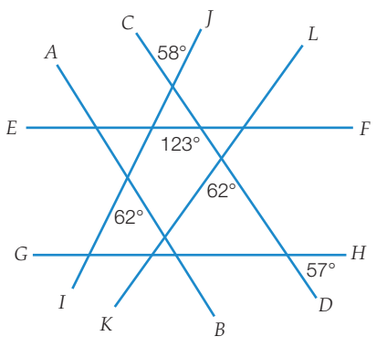


10 In an underground carpark a ramp is being constructed. The ramp will ascend at an angle of 36°. At what angle must the supports be positioned at the top of the ramp?

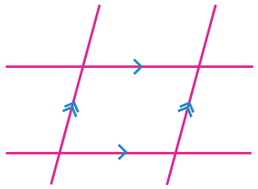


Reasoning

11 Choose the pair of lines that are parallel and explain why you choose them.



12 When a pair of parallel lines is cut by two parallel transversals, how many pairs of angles are formed that are:



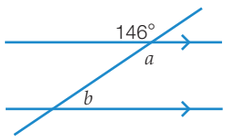
(a) corresponding

(b) alternate

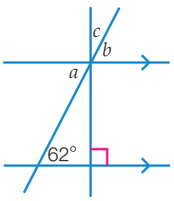
(c) co-interior?

13 Find the value of each pronumeral below, giving a reason for your answer.

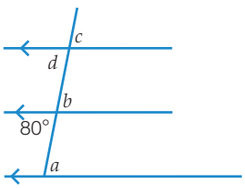
(a)



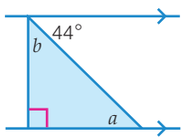
(b)



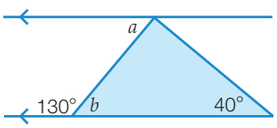
(c)



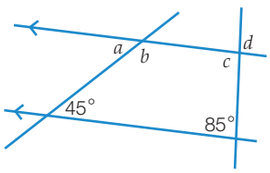
(d)



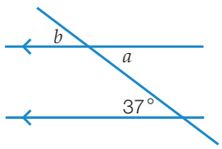
(e)



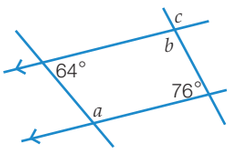
(f)



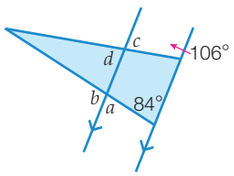
(g)



(h)

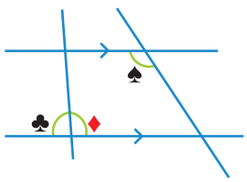


(i)



Open-ended

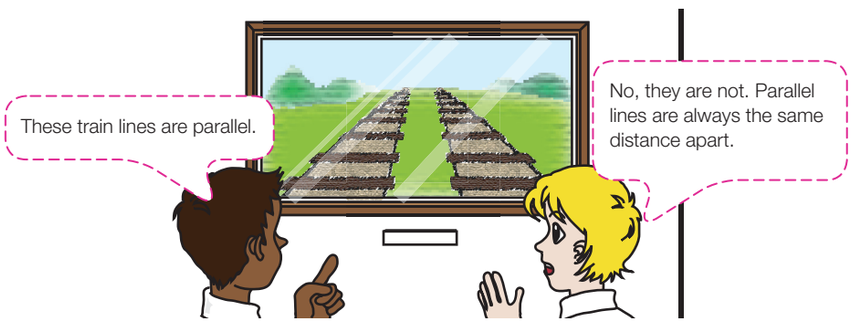
14 Aileen has drawn a pair of parallel lines with two transversals. She has marked three angles using different symbols.



(a) Copy the diagram into your book. Mark another three angles using the same symbols as Aileen so that there is a pair of corresponding angles, alternate angles and co-interior angles.

(b) In how many different ways can the three other angles be arranged, with no pair of angles sharing an angle with another pair? Draw a diagram for each combination.

15



How would you explain to the students whether the train lines are parallel or not?

Half-time 8

Equipment required: Protractor for Questions **3** and **5**

1 Draw the following angles by estimating their size. (Do not use a protractor.) Ex 8.1

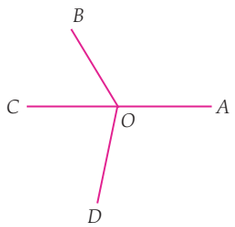
(a) 90° **(b)** 180° **(c)** 360°

(d) 34° **(e)** 75° **(f)** 120°

2 **(a)** Name the two acute angles in the diagram opposite. Ex 8.2

(b) Name the three obtuse angles in the diagram opposite.

(c) Give another name for $\angle AOB$.



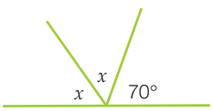
3 Draw the following angles using a protractor. Ex 8.1

(a) 90° **(b)** 180° **(c)** 360°

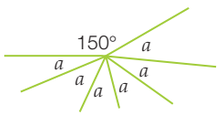
(d) 34° **(e)** 75° **(f)** 120°

4 Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.3

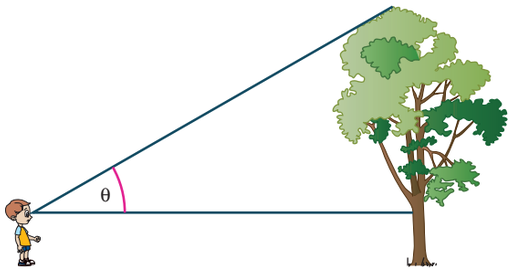
(a)



(b)

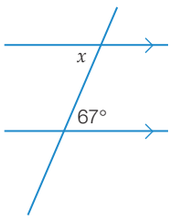


5 Measure the angle marked in the diagram opposite. Ex 8.1

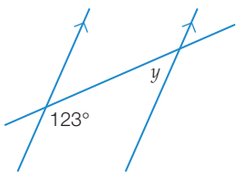


6 Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.4

(a)



(b)



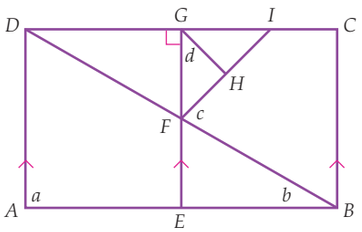
7 Using letter names such as $\angle ABC$: Ex 8.3, 8.4

(a) name an angle corresponding to a

(b) name an angle alternate to b

(c) name an angle supplementary to c

(d) name an angle complementary to d.



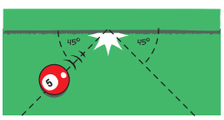
Investigation

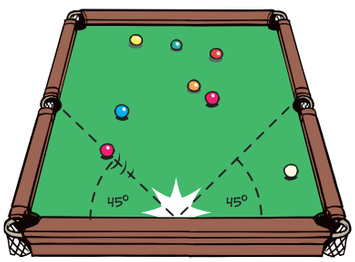
Billiard ball bounces

Equipment required: 1 or 2 brains, 1-centimetre grid paper, protractor, ruler

The game of billiards is played on a large rectangular table that is covered in felt and surrounded by a cushion along the edges. Players use a long stick, called a cue, to hit balls into pockets around the edges of the table. Snooker and pool are variations of the game that have become extremely popular over time. The size of the table can vary from 12 feet (3.7 m) along the larger side to 9 feet (2.7 m), 8 feet (2.4 m) or 7 feet (2.1 m) in length.

When planning their shots, good billiards players use the fact that a ball bounces off the side of a table at the same angle at which it hits, as shown here:





Here, the ball approaches the cushion at an angle of 45° and rebounds at an angle of 45°.

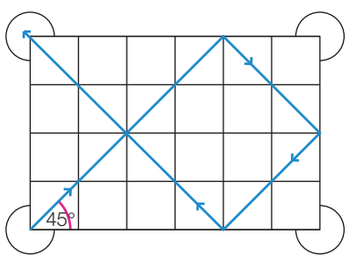
The ball may bounce off the sides several times before it either stops rolling or goes into a pocket. For this investigation, we will assume that the ball keeps rolling until it falls into a pocket.

The Big Question

Can we predict how many times a billiard ball will bounce on tables of different sizes?

Can we predict which pocket the ball will fall into?

Engage



Here, the ball is hit from the bottom left corner at an angle of 45° and bounces off the sides of the table three times before falling into the opposite pocket along the short side.

1 (a) To see what would happen if we made the table bigger, use your grid paper to draw a similar 'table' to the one above, but make it 6 cm × 8 cm. Starting in the same corner (bottom left), trace the path of a ball hit at an angle of 45°, bouncing it off the sides until the path ends in a pocket. You should find the ball bounces five times and falls into the opposite pocket on the long side.

(b) What would happen if the table was square?

Explore

2 (a) Draw up tables of the following dimensions on your grid paper (you might like to share this task with a partner).

1×2, 1×4, 2×3, 2×4, 2×8, 3×4, 3×6, 3×8, 3×12, 4×5, 4×6, 4×7, 4×8, 5×6, 5×7, 5×8, 6×7, 6×8, 6×9, 7×9, 8×10, 9×12

(b) Starting in the same corner each time, trace the path of the ball hit at an angle of 45°. Count the number of bounces, and also note which pocket the ball falls into.

Strategy options

- Make a table
- Look for a pattern.

Explain

3 (a) Collect all of the drawings you have made of the tables and their ball paths. To help you see any patterns or connections between them, group together the tables that have something in common. Some of the groups you could make are:

- tables where the pattern traced by the path of the ball is identical
- tables where the ball passes through every square on the grid.

(b) Once you have made these groups, compare the length, width and 'bounce' numbers for each table in the group. Are they connected in some way? Compare the starting and finishing pockets of the balls in each group. Is there a pattern here?

Elaborate

4 (a) Write a couple of sentences that answer the Big Question about predicting the number of bounces for tables of different sizes.

(b) Write a couple of sentences that answer the Big Question about predicting which pocket the ball will fall into.

Evaluate

5 (a) Consider how you worked on this task and the methods that you used. How did you organise or keep track of your results? Could you have done this better?

(b) Did grouping your results help you to spot patterns and connections between them? Which groups were useful?

Extend

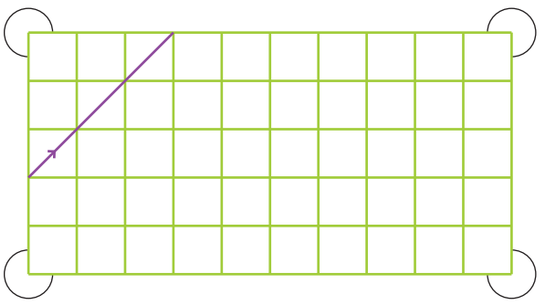
6 (a) Predict the number of bounces and which pocket the ball will fall into for the following table sizes.

(i) 18×27 **(ii)** 17×19

In which of these tables will the ball path pass through every square on the grid?

Draw these tables and check your prediction.

(b) Draw a 5×10 table on grid paper and trace the path of the ball hit from this position.



What do you notice? Try starting at different positions and comment on what you find.

(c) A billiard ball bouncing at the same angle at which it hits is an example of 'the law of reflection'. Investigate this law and other examples of where it can be seen.

8.5 Polygons

We draw many shapes on a flat surface called a plane. These are called two-dimensional or **plane shapes**. Plane shapes whose sides are all straight lines are called **polygons**.

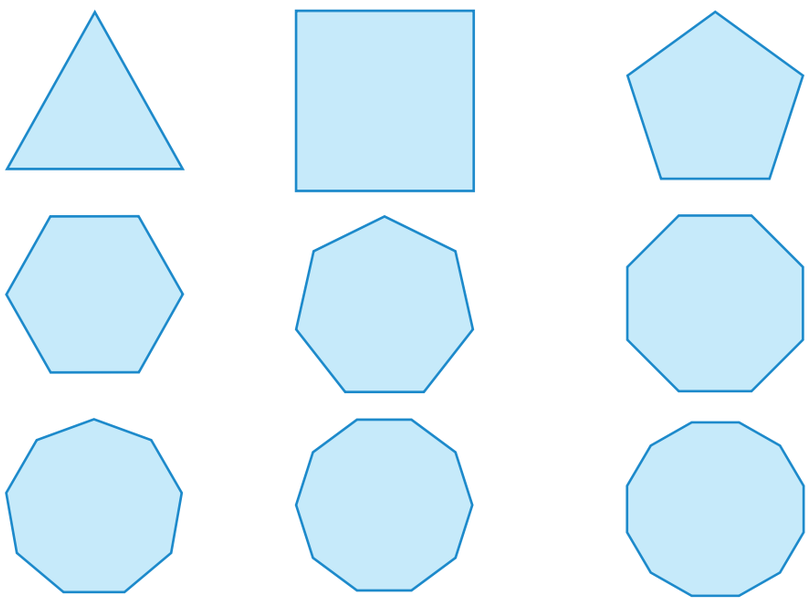
The word polygon is made up of two Greek words—*poly* (meaning many) and *gon* (meaning angle), so a polygon is a many-angled shape. As you can see in the following diagrams of polygons, the number of sides in a polygon is equal to its number of angles.

The table opposite shows the names given to the first 10 polygons. Note that an undecagon is rarely used.

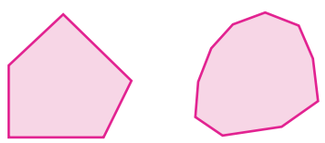
Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

A **regular polygon** has all sides of equal length and all angles of equal size. The number of sides gives the name of the polygon.

You are probably familiar with the regular polygons shown below.

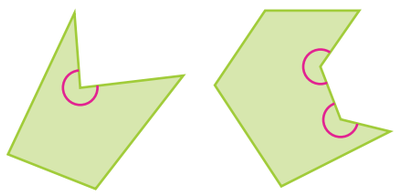


If the sides are not all equal, they are called **irregular** and the polygon is called an irregular polygon.



Concave and convex polygons

All of the polygons shown so far have been **convex** polygons, as they contain no interior angles greater than 180°. Opposite are examples of **concave** polygons which always contain at least one internal angle greater than 180°. (The sides cave in.)



8.5 Polygons

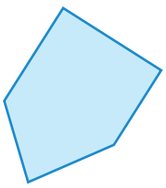
Navigator

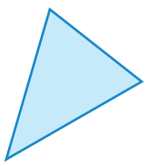
Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7 | Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7

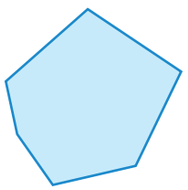
Answers page 670

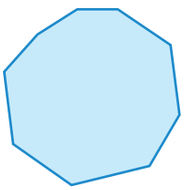
Fluency

1 Name each of the polygons below. State whether each one is concave or convex.

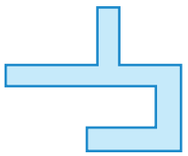
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

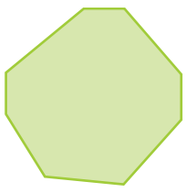
(g) 

(h) 

(i) 

2 Choose the correct answer.

(a) The shape opposite is:



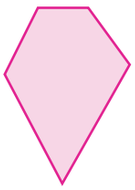
A a hexagon

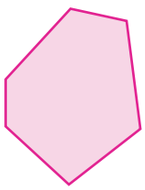
B a heptagon

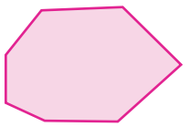
C an octagon

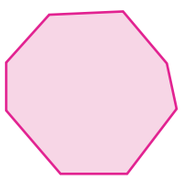
D a nonagon

(b) Which of the shapes below is a heptagon?

A 

B 

C 

D 

Understanding

3 Name the type of polygon in each photograph.

(a)

(b)

(c)

(d)

Reasoning

4 What is the greatest number of right angles that a pentagon can have? Demonstrate your answer.

5 **(a)** If a quadrilateral can have two sets of parallel sides, how many sets of parallel sides can a hexagon have?

(b) Based on your results from part **(a)**, how many pairs of parallel sides can an octagon and a decagon have? Explain any pattern you have found.

(c) Can this pattern be applied to a pentagon or to a heptagon? Explain why or why not.

Open-ended

6 Using a ruler, draw any:

(a) concave quadrilateral **(b)** concave hexagon **(c)** concave octagon.

7 Name some common objects that contain regular polygons.

Outside the Square Puzzle

The hex is gone

Equipment required: 1 brain, a pair of scissors

This is a regular hexagon.

On loose paper, draw at least six regular hexagons that are each about a quarter of an A4 page. Cut out the hexagons and use them to assist you with the following questions.

(a) How could a regular hexagon be cut into two pieces which, when put together, make a parallelogram?

(b) How could a regular hexagon be cut into three pieces which, when put together, make a rhombus?

(c) How could a regular hexagon be cut into four pieces which, when put together, make two equilateral triangles?

Investigation

Angle sum of polygons

Equipment required: 1–2 brains, two pieces of paper, ruler and protractor

The Big Question

Is there a relationship between the number of sides of a polygon and the angle sum of that polygon?

Engage

1 **(a)** On a blank sheet of paper, use a ruler to draw a triangle of reasonable size (about a quarter of a page).

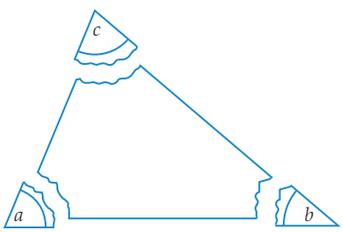
(b) Use a protractor to measure each angle.

(c) Add these angles together. Did you get 180°?

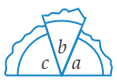
(d) Repeat this with a second triangle.

2 **(a)** Draw a third triangle on a separate piece of paper and cut it out carefully.

(b) Tear off the angles as shown.



(c) Place all the vertices together with arms touching. (Did they form a straight line?)



(d) Paste these angles together on your page.

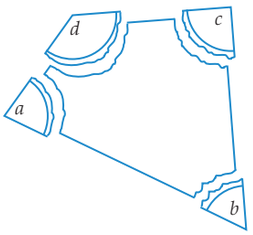
3 **(a)** On your first sheet of paper, use a ruler to draw a quadrilateral.

(b) Measure each angle.

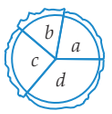
(c) Add these together. Did you get 360°?

(d) Draw another quadrilateral and cut it out carefully.

(e) Tear off the angles as shown.



(f) Place all the vertices together with arms touching. (Did they form a revolution?)



(g) Paste these angles together on your first page.

(h) Draw a second quadrilateral and divide it into two triangles.

(i) Calculate the angle sum of the quadrilateral.

Explore

4 On a blank sheet of paper, use a ruler to draw a pentagon, a hexagon, a heptagon and an octagon. Make sure that each polygon is a reasonable size (about a quarter of a page).

5 Use your protractor to measure the angles of each shape.

6 Find the angle sum for each shape. Did you get a multiple of 180° for each shape?

7 Use straight lines from vertex to vertex to divide each shape into as many triangles as you can, but do not allow any of the lines to cross. Count the number of triangles in each shape.

8 Multiply the number of triangles by 180° for each polygon.

9 Copy the following table and enter your results.

Strategy options
- Make a table.
- Make a model.
- Look for a pattern.

Polygon	Number of angles	Number of triangles	Angle sum (°)	180° × Number of triangles (°)
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				

Explain

10 Explain how you found a relationship between the number of angles (column 1) and the number of triangles (column 2) for each shape.

11 Explain how you found a relationship between the number of angles (or sides) of a polygon and the angle sum of that polygon.

Elaborate

12 Answer the Big Question by writing the rule you have found, describing carefully any of the variables you have used.

13 Use your rule to find the angle sum of a nonagon, a decagon and a dodecagon.

14 Draw each of these and measure each angle.

15 Find the sum of the angles for each shape and compare your answers with your answers in Question **13**.

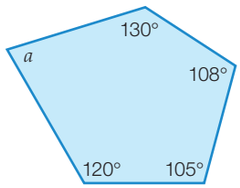
Evaluate

16 Did you get exactly the same answer when you measured your angles and added them as you did when you multiplied the number of triangles by 180°. Why could they be different? Which is more accurate?

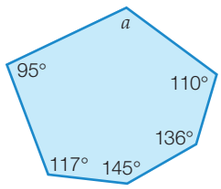
Extend

17 Calculate the size of the angle a in each of the following polygons.

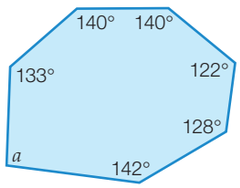
(a)



(b)



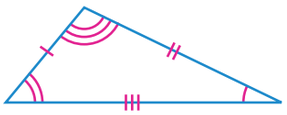
(c)



Triangles

8.6

A triangle is the simplest polygon with three straight sides and three angles.

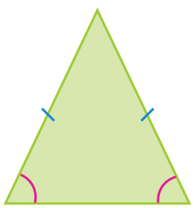
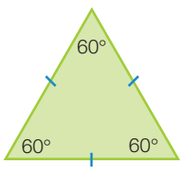


Classifying triangles

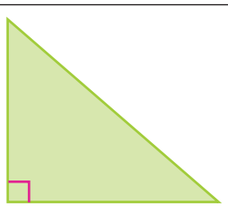
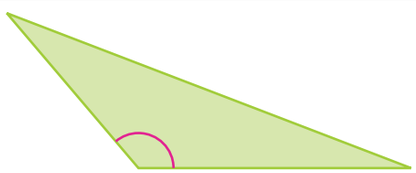
Triangles are classified according to their sides or angles.

PP Discovery Task

When using the side lengths to classify a triangle, there are three main types:

Scalene triangle—no equal sides and no equal angles.	
Isosceles triangle—two equal sides and two equal base angles, which are opposite the two equal sides.	
Equilateral triangle—three equal sides and three equal angles of 60° each.	

When using angles to classify a triangle, the three main types are:

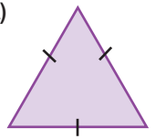
Acute-angled triangle—all three angles are acute (less than 90°).	
Right-angled triangle—one angle is a right angle (equal to 90°).	
Obtuse-angled triangle—one angle is obtuse (greater than 90° but less than 180°).	

When the angles or lines of a shape are marked in a particular way, it means that all other angles or sides with that same marking are equal in size.

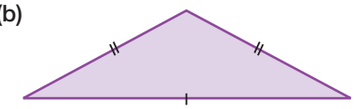
Worked Example 6 WE6

Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

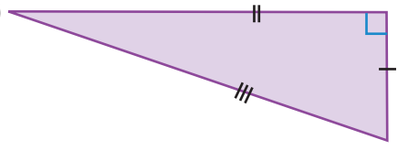
(a)



(b)



(c)



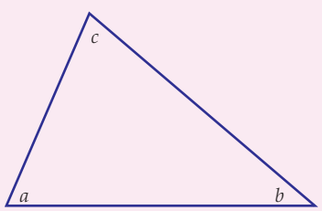
Thinking	Working
(a) (i) How many sides are the same length? (three)	(a) (i) equilateral triangle
(ii) What type of angle is the largest angle? (acute)	(ii) acute-angled triangle
(iii) Combine the side and angle name.	(iii) an acute-angled equilateral triangle
(b) (i) How many sides are the same length? (two equal sides)	(b) (i) isosceles triangle
(ii) What type of angle is the largest angle? (obtuse)	(ii) obtuse-angled triangle
(iii) Combine the side and angle name.	(iii) an obtuse-angled isosceles triangle
(c) (i) How many sides are the same length? (none)	(c) (i) scalene triangle
(ii) What type of angle is the largest angle? (one right angle)	(ii) right-angled triangle
(iii) Combine the side and angle name.	(iii) a right-angled scalene triangle

(Note: In **(a)**, it is unnecessary to describe an equilateral triangle with an angle name, as all angles must be 60° and therefore acute angles.)

Angle sum in a triangle

In any triangle, the three angles have a sum of 180°.

$a + b + c = 180$



Angles in a triangle using technology

We can explore the sum of the angles in a triangle using the GeoGebra geometry software package, or a CAS calculator. Here we will use GeoGebra.

Open a GeoGebra program. Deselect the axes found under the View menu, as they are not needed here, but select the Algebra View.

To construct a triangle, select the polygon tool  . Now, on the drawing pad, left click for the first point, move the cursor and left click again for the second point, repeat for the third point and return to the first point to complete the triangle. Move in an anticlockwise direction so that the interior angles will be measured.

To measure the angles, select the angle tool  and then 'poly' in the Algebra View (the panel on the LHS of the screen). All the interior angles will now be measured.

Using the input box at the bottom of the screen, select Sum from the drop-down Command list on the RHS. Use the drop-down variables list (next to Command) to enter the angle variables inside { } brackets; e.g. Sum[{α,β,γ}]. In Algebra View, you will see the sum displayed as $\delta = 180°$.

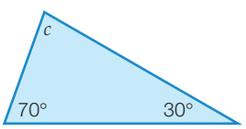
Use the select tool  to select a vertex of the triangle. Hold down the left click and move the selected point around. You will see the angle change, but the sum remains constant at 180°. Select another vertex and move it around. Repeat this procedure with the third vertex.

Worked Example 7

WE7

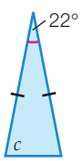
Find the size of the angle labelled *c* in each of the following triangles.

(a)



(b)

(c)

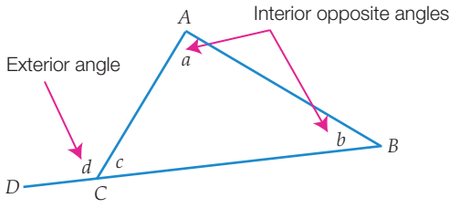


Thinking		Working
(a) 1	The three angles add to 180°. Write an equation that shows this.	(a) $c + 30 + 70 = 180$
2	Simplify the equation by adding the given angles.	$c + 100 = 180$
3	Solve the equation (subtract 100 from both sides).	$c = 80°$
(b) 1	The three angles add to 180°. Write an equation that shows this. (Remember that the small square in the triangle indicates a 90° angle.)	(b) $c + 90 + 16 = 180$
2	Simplify the equation by adding the given angles.	$c + 106 = 180$
3	Solve the equation (subtract 106 from both sides).	$c = 74°$

(c)	1	The three angles add to 180°. Write an equation that shows this. (Because this is an isosceles triangle, the two angles opposite the equal sides are equal.)	(c) $c + c + 22 = 180$
	2	Simplify the equation by adding like terms.	$2c + 22 = 180$
	3	Solve the equation (subtract 22 from both sides, then divide by 2).	$2c = 158$ $c = 79°$

Exterior angle of a triangle

An exterior angle of a triangle is formed on the outside of a triangle when one of the sides is extended.



In the diagram, $\angle d$ is the exterior angle.

There can be six different exterior angles formed on any triangle. Can you find them all? What relationship does the exterior angle have to the interior angles of a triangle?

We know that $\angle a + \angle b + \angle c = 180°$ (angles of a triangle add to 180°)

$\angle a + \angle b + \angle c - \angle c = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle a + \angle b = 180° - \angle c$

$\angle c + \angle d = 180°$ (angles on a straight line add to 180°)

$\angle c - \angle c + \angle d = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle d = 180° - \angle c$

Therefore, $\angle a + \angle b + = \angle d$ (both equal to $180° - \angle c$)

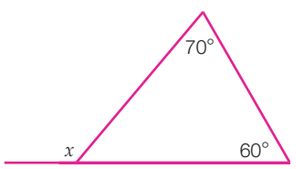
The exterior angle of a triangle is equal to the sum of the two interior opposite angles in the triangle.

Worked Example 8

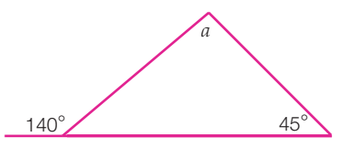
WE 8

Calculate the value of the pronumeral in each of the following.

(a)



(b)



Thinking	Working
(a) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the exterior angle.	$(a)\quad x = 60 + 70$ $x = 130°$
(b) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the unknown angle.	$(b)\quad 140 = a + 45$ $a = 140 - 45$ $a = 95°$

8.6 Triangles

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19

Answers page 671

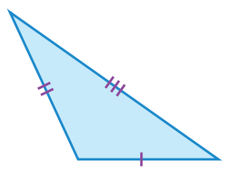
Equipment required: Protractor and ruler for Question **4**

Fluency

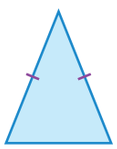
1 Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

WE6

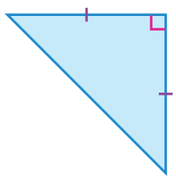
(a)



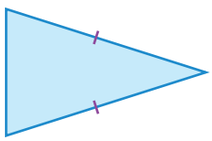
(b)



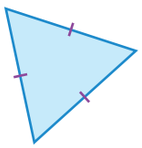
(c)



(d)



(e)



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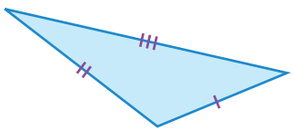
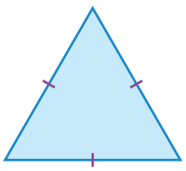


(g)



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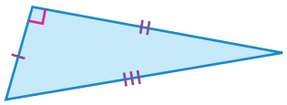
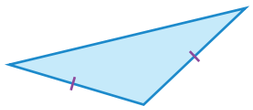


(i)



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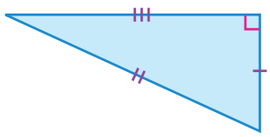
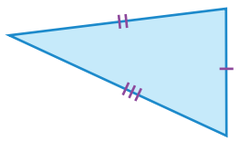


(k)

(l)

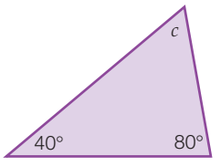


8.6

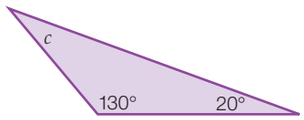
WE7

2 Find the size of the angle labelled c in each of the following triangles.

(a)



(b)



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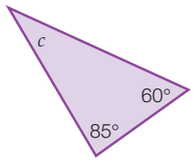
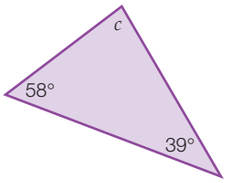
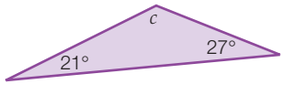


(d)



(e)



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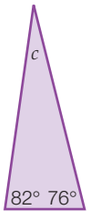
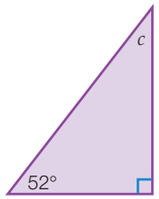
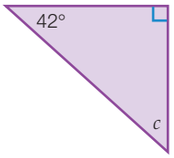


(g)

(h)



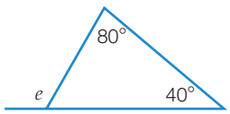
(i)



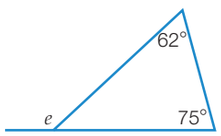
WE8

3 Calculate the value of the pronumeral in each of the following.

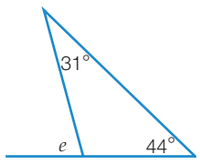
(a) (i)



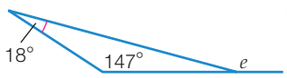
(ii)



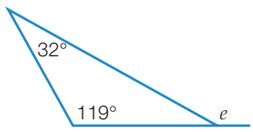
(iii)



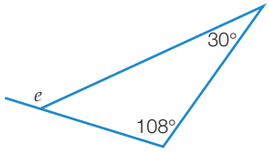
(iv)



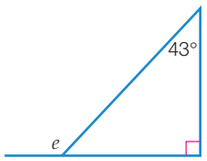
(v)



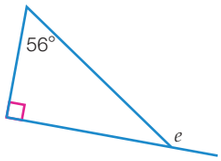
(vi)



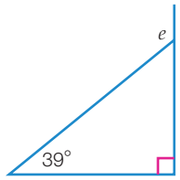
(vii)



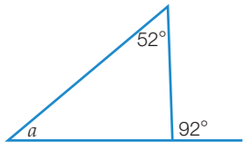
(viii)



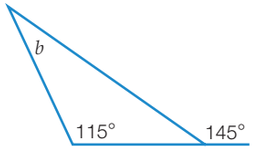
(ix)



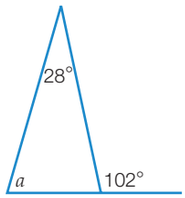
(b) (i)



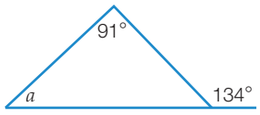
(ii)



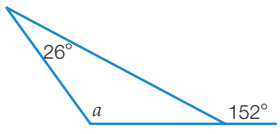
(iii)



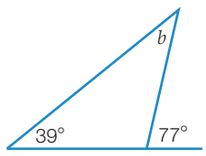
(iv)



(v)



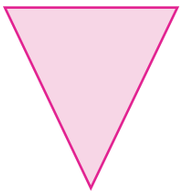
(vi)



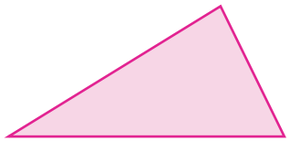
Understanding

4 Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one. Give the angle names as well, using a protractor if necessary to check the angle types.

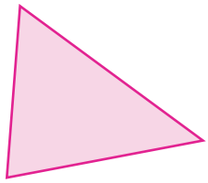
(a)



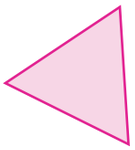
(b)



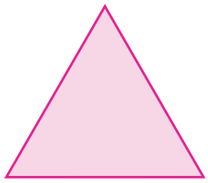
(c)



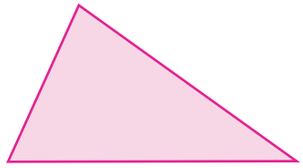
(d)



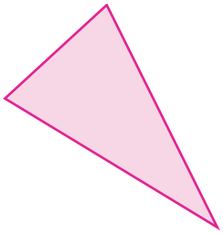
(e)



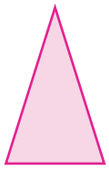
(f)



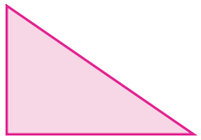
(g)



(h)



(i)



5 (a) The size of angle c is:

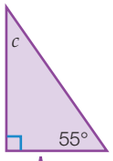
A 20° **B** 60°

C 80° **D** 100°

(b) The size of angle c is:

A 35° **B** 45°

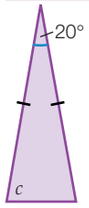
C 55° **D** 125°



(c) The size of angle c is:

A 20° **B** 80°

C 90° **D** 160°



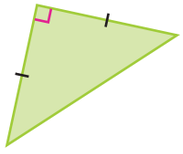
6 The triangle shown is best described as:

A isosceles, acute-angled

B isosceles, right-angled

C scalene, acute-angled

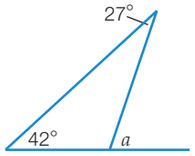
D scalene, right-angled



7 **(a)** The value of the pronumeral is:

A 15° **B** 63°

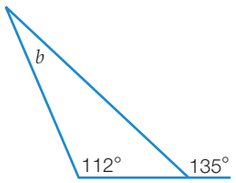
C 69° **D** 111°



(b) The value of the pronumeral is:

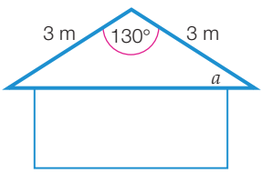
A 23° **B** 36°

C 42° **D** 68°

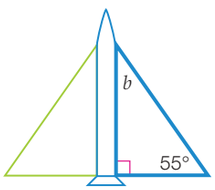


8 Name the type of triangle shown in bold in each diagram below. (Side or angle names may be used.) Find the value of any unknown angles marked with a pronumeral.

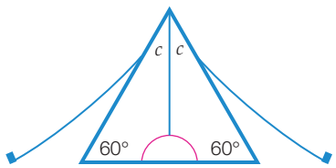
(a)



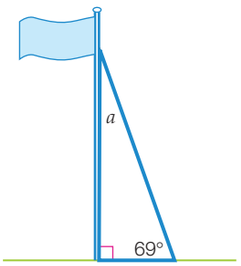
(b)



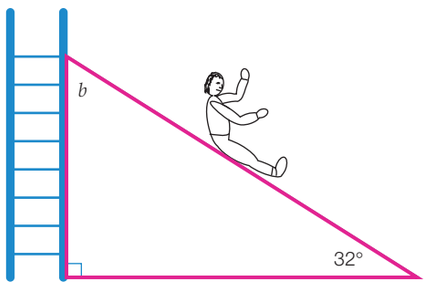
(c)



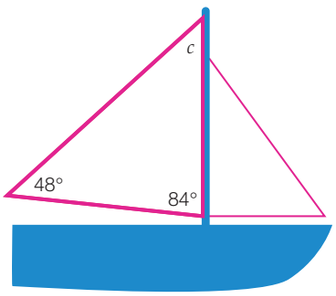
(d)



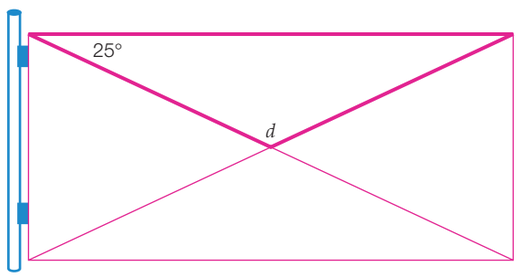
(e)



(f)



(g)



9 The diagonal of a square cuts the square into two triangles. Give the side and angle name of the triangles formed.

10 Two equilateral triangles are placed next to each other so that two sides are joined along their whole length to form a quadrilateral.

(a) What can you say about the side lengths of the quadrilateral (four-sided shape) formed?

(b) What will be the size of each of the angles in the quadrilateral?

Reasoning

11 Is it possible to have a triangle with more than one obtuse angle? Explain by drawing a diagram.

12 What is the minimum number of acute angles a triangle must have?

13 Give the side and angle names of each of the following triangles, with reasons why.

(a) a triangle with one angle measuring 36° and another angle measuring 54°

(b) a triangle with one angle measuring 28° and another angle measuring 124°

(c) a triangle with two angles measuring 60° each

(d) a triangle with one angle measuring 12° and another angle measuring 12°

14 Explain why the following triangles are not possible.

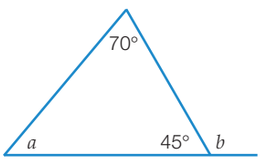
(a) a triangle with angles 55°, 65° and 70°

(b) an equilateral triangle with three 55° angles

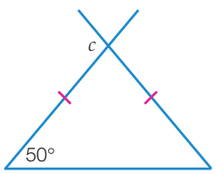
(c) isosceles triangle with 45°, 55° and 80° angles

15 Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

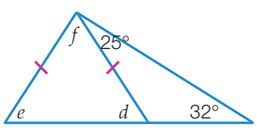
(a)



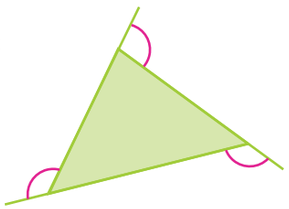
(b)



(c)



16 Find the sum of the marked angles in the diagram on the right without measuring the angles.



Open-ended

17 Draw three different obtuse-angled, scalene triangles.

18 Draw an example of:

(a) an acute-angled scalene triangle

(b) an acute-angled isosceles triangle.

19 Draw three different triangles with an exterior angle between 80° and 100°.

Outside the Square Puzzle

Eggs, spoons and turtles

1 Jane, Scotty, Jessica, Bruce and Michael all took part in their school egg-and-spoon race. Find the order in which they finished the race.

- Bruce finished the race in between the two girls.
- The winner had six letters in their name.
- Jane finished faster than Michael and slower than Jessica.

2 Four students and their turtles took part in the annual School Turtle Championships. Match the students to their turtles, and find the order in which they finished.

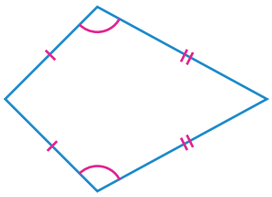
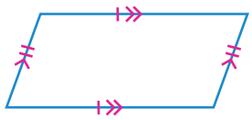
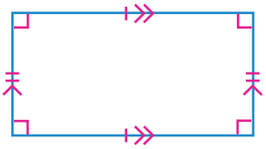
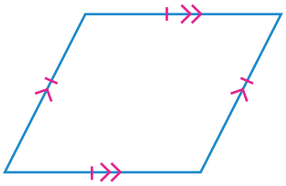
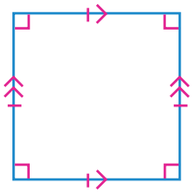
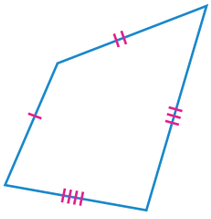
- Chloe owns Sandy, who didn't finish in last place.
- Percy finished immediately in front of Vicky's turtle.
- Myrtle finished behind Ernest, who is owned by Vicky.
- Ethan was celebrating as his turtle finished two places higher than Warren's.

8.7 Quadrilaterals

A plane shape with four straight sides is called a **quadrilateral**. We name quadrilaterals by looking at the properties of their sides and angles.

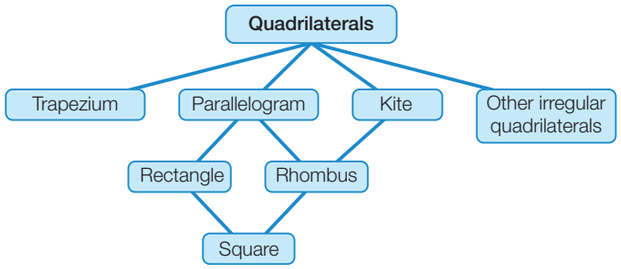
Quadrilaterals

Discovery Task

Quadrilateral	Description
Kite 	Two pairs of equal adjacent sides (sides next to each other) and one pair of equal opposite angles.
Trapezium 	Only one pair of parallel sides.
Parallelogram 	Two pairs of parallel sides. Opposite sides and angles are equal.
Rectangle 	Two pairs of opposite sides equal and all internal angles are right angles (equal to 90°).
Rhombus 	All four sides equal; opposite angles are equal. (This can be thought of as a parallelogram with all sides equal or a 'pushed over' square.)
Square 	All sides equal and all internal angles are right angles (equal to 90°).
Irregular quadrilateral 	A four-sided plane shape that has no special qualities.

Parallel side pairs are indicated by marking the side with the same number of arrows in the same direction, as shown.

A quadrilateral may have more than one correct name. For example, a square may also be called a rectangle, because it has all the angle and side properties of a rectangle (two pairs of parallel sides and 90° angles). It could also be called a rhombus, because its pairs of opposite sides are equal and its opposite angles are equal.



Angle sum of a quadrilateral

The sum of the angles in a quadrilateral is 360°.

Angles in a quadrilateral using technology

Use GeoGebra to construct a quadrilateral with the polygon tool as you did to construct a triangle (p. 459). Measure and sum the interior angles. This time, the angle sum will be 360°. Once again, use the select tool to move all four points one by one. The angles will change but the angle sum stays constant at 360°.

Angle properties of special quadrilaterals

The diagonally opposite angles of a parallelogram are equal.

The two interior opposite angles of a kite where unequal sides meet are equal.

8.7 Quadrilaterals

Navigator

Q1, Q2, Q3, Q4 Column 1, Q5, Q6 Column 1, Q7, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q17, Q20	Q1, Q2, Q3, Q4 Column 2, Q5, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20	Q1, Q2, Q3, Q4 Column 3, Q5, Q6 Column 3, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21

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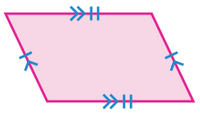
Fluency

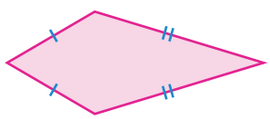
1 Give the most accurate name for each shape below.

(a)

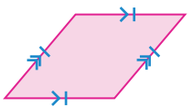
(b)



(c)

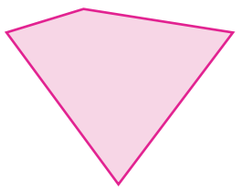


(d)



(e)

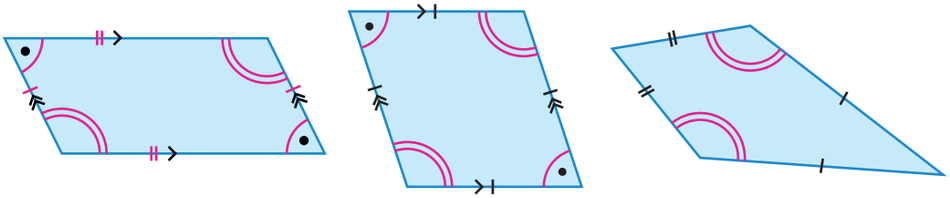
(f)



2 A quadrilateral has both pairs of opposite sides parallel and equal in length. We can be sure this quadrilateral is a:

A parallelogram **B** rhombus **C** rectangle **D** square

3 State TRUE or FALSE for each of the following, referring to the three quadrilaterals below.



(a) The opposite angles in a parallelogram are equal.

(b) All four angles in a parallelogram are equal.

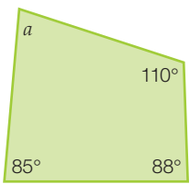
(c) The opposite angles in a rhombus are equal.

(d) All four angles in a rhombus are equal.

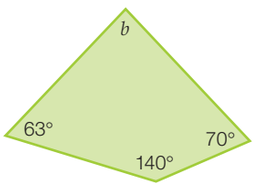
(e) One pair of opposite angles in a kite are equal.

4 Find the size of the unknown angle represented by a letter in each of the following quadrilaterals.

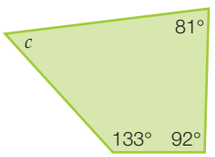
(a)



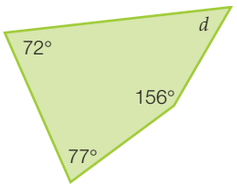
(b)



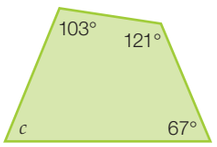
(c)



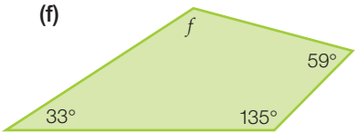
(d)



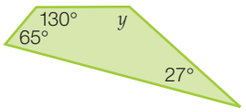
(e)



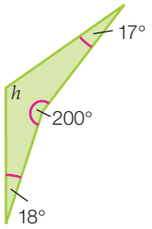
(f)



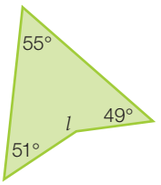
(g)



(h)



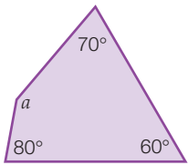
(i)



5 (a) The size of angle a shown is:

A 70° **B** 90°

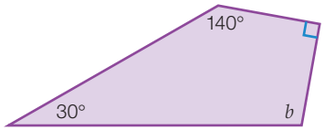
C 100° **D** 150°



(b) The size of angle b shown is:

A 10° **B** 90°

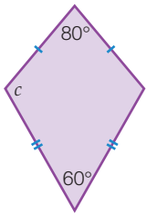
C 100° **D** 170°



(c) The size of angle c shown is:

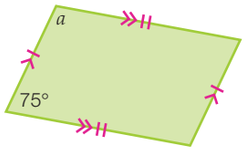
A 55° **B** 80°

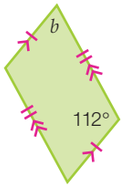
C 110° **D** 140°

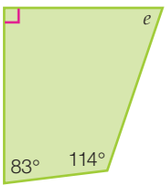


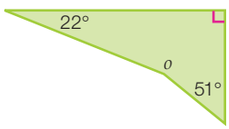
6 Find the size of the angle labelled with a letter in each quadrilateral.

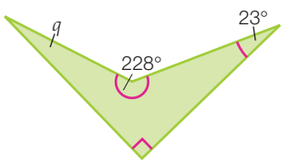
(a) 

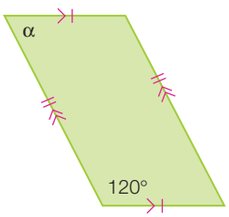
(b) 

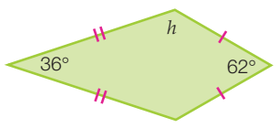
(c) 

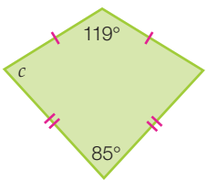
(d) 

(e) 

(f) 

(g) 

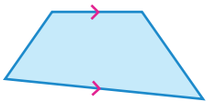
(h) 

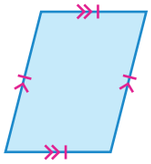
(i) 

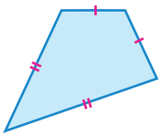
Understanding

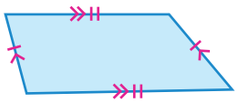
7 The diagrams below were created by a tired illustrator. What type of shape do you think he meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines are correct.

For Question **7**, make sure you do the markings for things such as equal sides, parallel sides and right angles.

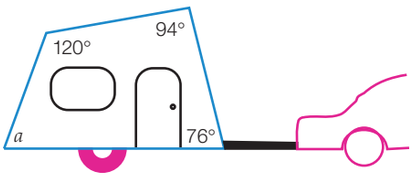
(a) 

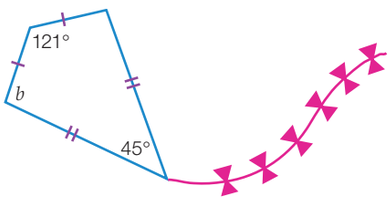
(b) 

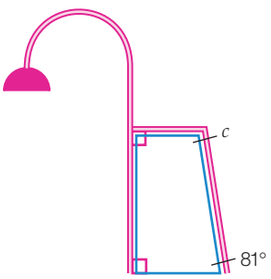
(c) 

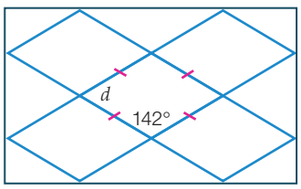
(d) 

8 Find the angle represented by the pronumeral in each quadrilateral below. Give reasons to justify your answer.

(a) 

(b) 

(c) 

(d) 

9 An angle in a parallelogram measures 40°. Draw a diagram and mark in the size of each of the other three angles.

10 A quadrilateral has one 45° angle and three unknown angles that are equal to each other. What is the size of each of the three unknown angles?

Reasoning

11 All rhombuses are parallelograms, but not all parallelograms are rhombuses. Explain.

12 Is a parallelogram a type of trapezium? Explain.

13 All rectangles are parallelograms, but not all parallelograms are rectangles. Is this true? Explain why/why not.

14 What is the maximum number of:

(a) obtuse angles a quadrilateral can have? Confirm by drawing a diagram.

(b) reflex angles a quadrilateral can have? Confirm by drawing a diagram.

15 A quadrilateral has two angles of 70° and 34°. The other two angles are equal to each other.

(a) What is the size of the two unknown angles?

(b) What type of quadrilateral could this shape be?

16 Can a quadrilateral have the angles 35°, 85°, 130° and 120°? Explain your answer.

17 What properties of a square make it also classifiable as:

(a) a rectangle **(b)** a rhombus **(c)** a parallelogram?

18 Sam is looking at a kite, which has two given angles of 50° and 120°, but it is unclear which angle is which and where they are located on the kite. He thinks that the best way to solve a problem is to draw a diagram. Help Sam by drawing three different kites that could contain the two known angles of 50° and 120°.

Open-ended

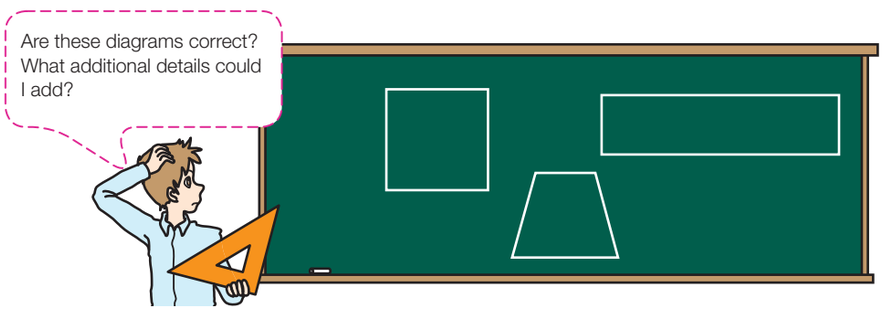
19 A quadrilateral has two angles equal to 80°.

(a) Name one type of quadrilateral that this could be and draw it, marking in the 80° angles.

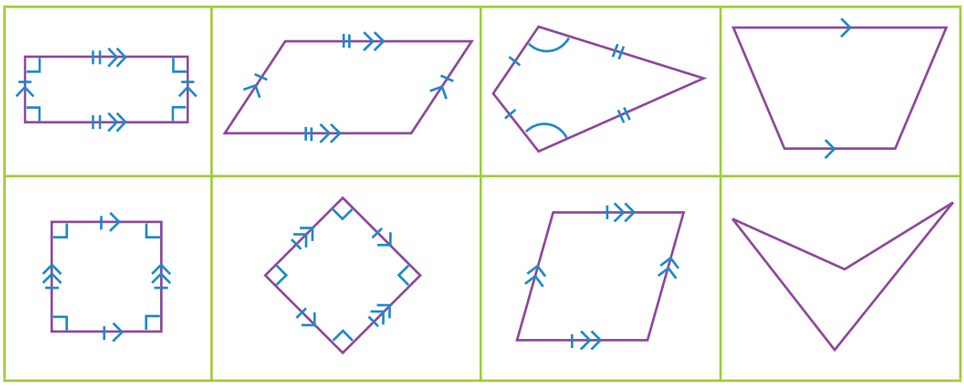
(b) How many different types of quadrilaterals can have two angles equal to 80°? List the different types.

(c) Draw an example of each, marking in the 80° angles.

20 Jack is asked by his teacher to draw a square, a rectangle and a trapezium. Do Jack's diagrams include enough information to clearly depict a square, a rectangle and a trapezium? Redraw his diagrams, adding any missing details.



21 Divide these quadrilaterals into two groups. Explain the reason(s) you used to divide them into each group.

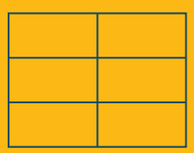


Outside the Square

Problem solving

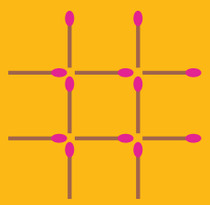
Quadrilateral quandaries

1 How many rectangles can you count here altogether? (Include rectangles tha are the same size even if they are drawn in different positions.)

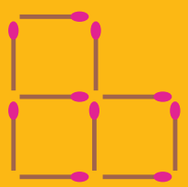


2 Find how many different size squares can be drawn on the grid shown.

3 Move three matchsticks in the arrangement shown to create three squares that touch each other.

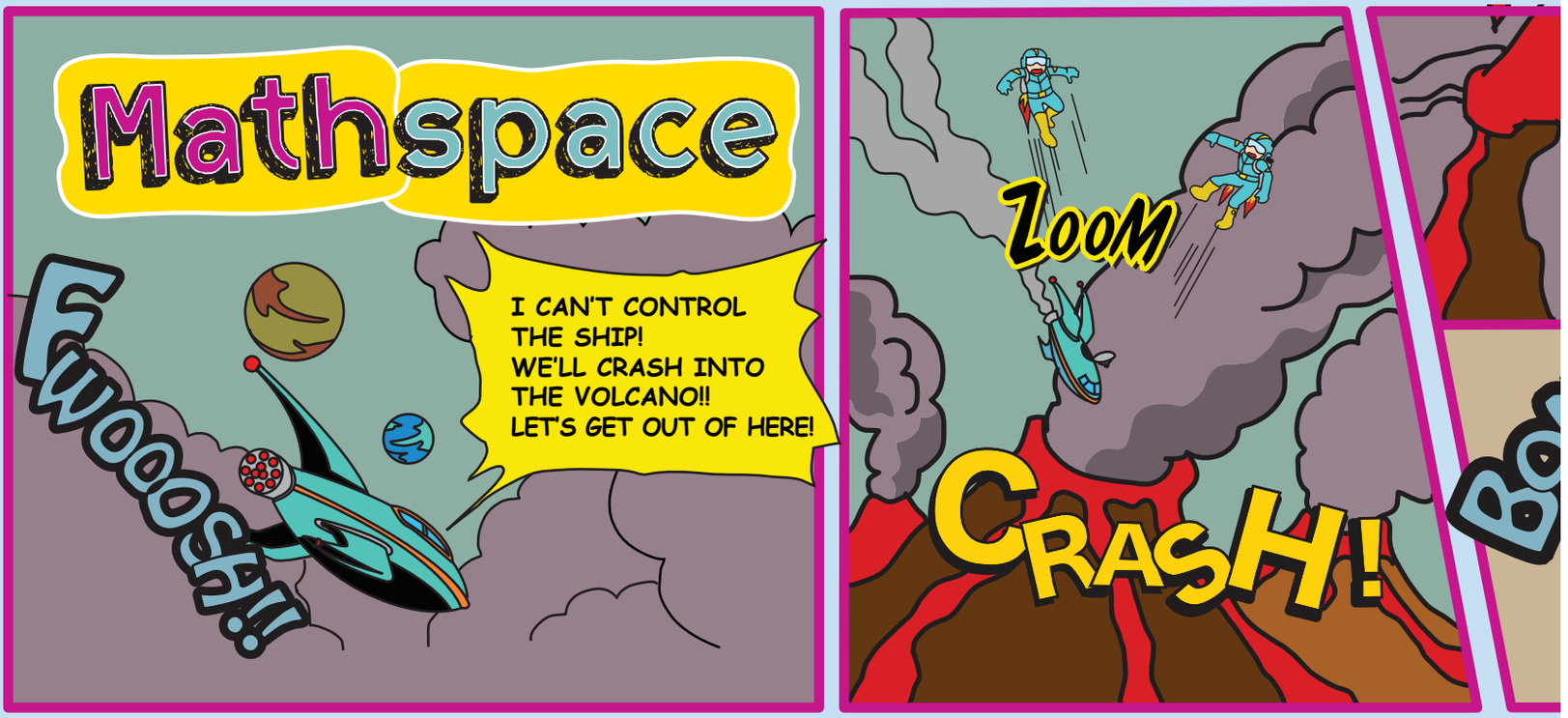


4 Remove one match and re-position three others to form one square and two parallelograms.



Strategy options

- Draw a diagram.
- Look for a pattern.



PLANET POLYGON

While exploring the universe, you and your co-pilot Violet enter the atmosphere of the strange planet Polygon. Your ship malfunctions, and to avoid certain death you both eject out of the ship. On landing you are knocked unconscious. When you wake, you look around—where is Violet and why is your spacecraft crumpled?

You find a note on your PCD (Personal Communication Device).

Message #1: 2.15 p.m. Alien code

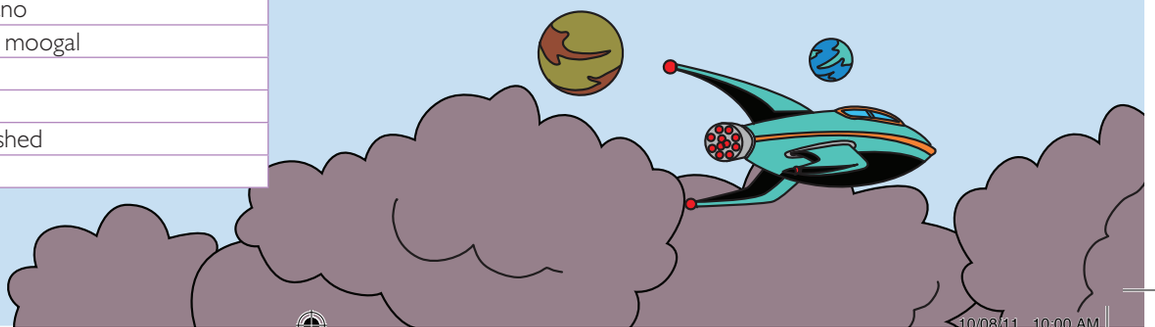
I couldn't find you when I woke. I've met a group of aliens who seem to call themselves Polygons. They don't have a verbal language but communicate instead with shapes. Through our conversation I think I have managed to decode some of their language.

Shape	Possible meaning
square	catapulted
kite	flying thing
isosceles triangle	volcano
rhombus	large moogal
rectangle	paw
trapezium	into
polygon	squashed
scalene triangle	hit

I asked them what happened to our ship, they answered, and I have described the shapes. Here is approximately what they said.

- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only
- quadrilateral with 1 pair of parallel sides only
- 3-sided shape with 2 equal angles
- special rectangle with equal side lengths
- 3-sided shape with 3 different side lengths
- parallelogram that has all equal sides, but is not a square
- 2 pairs of parallel sides, with 90 degree angles (but is not a square)
- hexagonal-looking shape
- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only

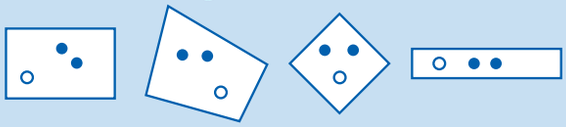
1 Decode what the aliens said happened to the ship.



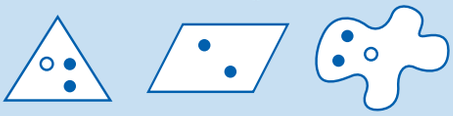
Message #2: 3.15 p.m.
Alien intelligence test

The Polygons are willing to give us the necessary materials to fix our spaceship, but we both have to pass their test. Good luck, from Violet.

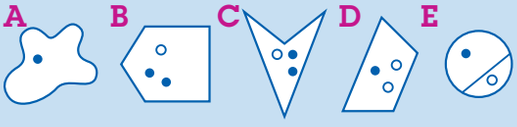
2 (a) These are Kwoogals:



These are not Kwoogals:

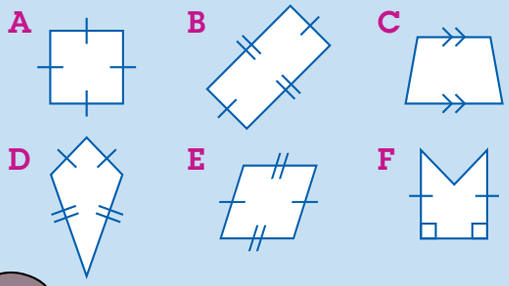


(i) Which one of these is a Kwoogal?



(ii) Write the definition of a Kwoogal.

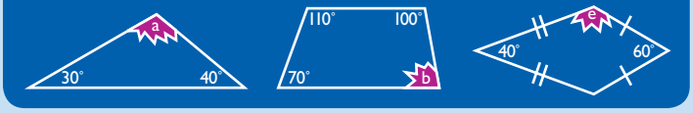
(b) Which of these are parallelograms? (More than one answer is okay.)



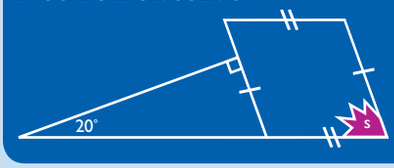
Message #3: 4.18 p.m.
Spaceship blueprints

I've found the blueprints for our spaceship but some of the angles have been burnt in the crash. Can you calculate what the missing angles should be?

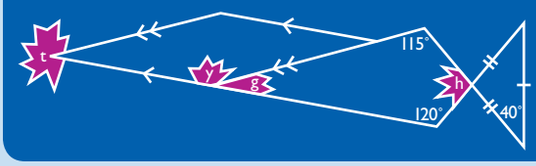
Fuel cells



Neutron absorber



Black hole combustion unit



What is a moogal?

Use your answers for the spaceship blueprint message to solve the code below.

110°		70°	100°	110°	25°	25°	155°

80°	130°	110°	70°	335°

8.8 Compass constructions

Using a ruler, a pencil and a compass, we can construct many angles and shapes without measuring any angles with a protractor. Constructions may also be made using a CAS calculator or computer geometry software.

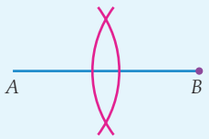
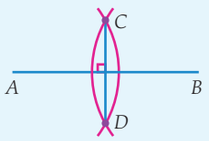
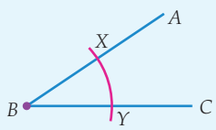
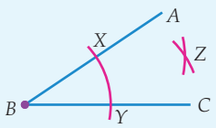
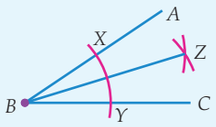
Most constructions are based on two basic techniques: bisecting a line and bisecting an angle. (Bisect means to divide into halves.) A **perpendicular bisector** bisects a line at right angles. We use a sharp pencil for drawing lines and for use in our compass, and we leave all construction lines on our diagrams.

Unless otherwise instructed, lines and compass openings should be between 5 and 8 cm long and angles between 40° and 50°.

Worked Example 9 WE9

Using only a compass, a ruler and a pencil:

(a) bisect a line **(b)** bisect an angle.

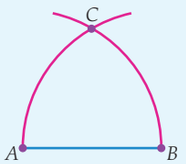
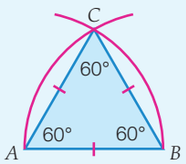
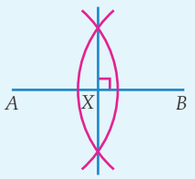
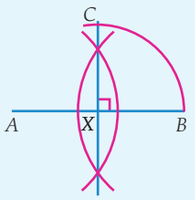
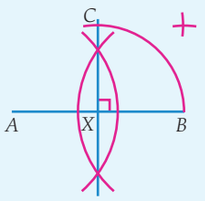
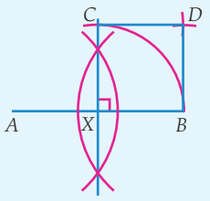
Thinking	Working
(a) 1 Use your ruler to draw a line $\overline{AB}$. Place the point of the compass on point A so that the compass opening is more than half the length of the line and draw an arc above and below the line.	*(a)* 
2 Keeping the compass at the same opening, place your compass point on point B and draw an arc to cut the first arc above and below the line.	
3 With your ruler join the points CD where the arcs cross. As this line crosses AB at right angles, it is the perpendicular bisector of the line.	
(b) 1 Draw an acute angle. Place the point of the compass on the vertex of $\angle ABC$ and draw an arc to cut both arms of your angle at X and Y.	*(b)* 
2 Keeping the compass at the same opening, place your compass point on point X and draw an arc. Repeat this using point Y so that the arcs cross.	
3 Using a ruler, draw a line connecting the vertex of $\angle ABC$ with Z, the point of intersection of the arcs. This line BZ bisects the angle (cuts the angle in half).	

An equilateral triangle not only has three equal sides but also has three equal angles of 60°. A square has four equal sides and all angles are right angles, so we can use our previous constructions to help us construct a square.

Worked Example 10 WE10

Using only a compass, a ruler and a pencil:

(a) construct an equilateral triangle **(b)** construct a square.

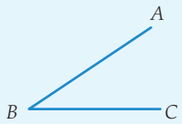
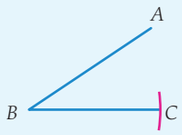
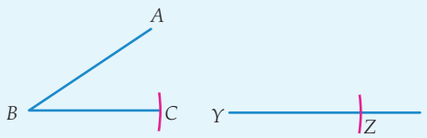
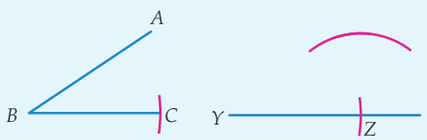
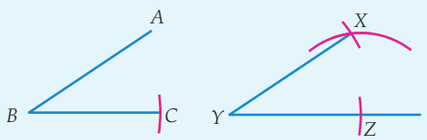
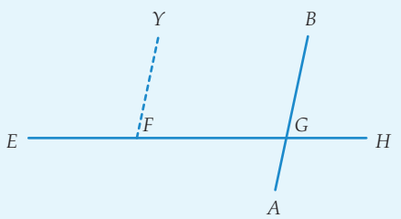
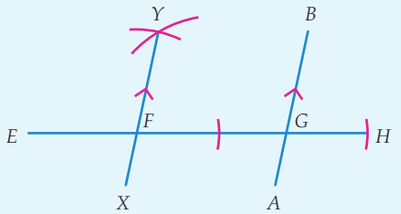
Thinking	Working
(a) 1 Use your ruler to draw a line $\overline{AB}$. Open your compass to the length of line AB. Place the point of the compass on point A and draw an arc above the line. Place the point of the compass on point B and draw another arc above the line so that the arcs cross at point C.	(a) 
2 Using a ruler, join points A and B to point C. ΔABC is an equilateral triangle. Mark all sides as equal and all angles as 60°.	
(b) 1 Construct a right angle on a horizontal line AB by constructing the perpendicular bisector of the line.	(b) 
2 Place your compass point on point X, the point of intersection of AB and the perpendicular bisector. With the compass opened to length BX, cut the perpendicular bisector at point C.	
3 Keeping the compass opening the same, place the compass point on point B and make an arc above the line. Place the compass point on point C and make an arc to cut the last arc at D.	
4 Join B to D and the point C to point D. $BXCD$ is a square.	

Worked Example 11

WE11

Using only a compass, a ruler and a pencil:

(a) construct an angle equal to another angle **(b)** construct a pair of parallel lines.

Thinking	Working
(a) 1 Draw an acute $\angle ABC$.	*(a)* 
2 Use a compass and, with the compass point on point B, draw an arc through point C.	
3 Draw a line segment and label one end point Y. Without changing the opening on your compass and, with the compass point on point Y, draw an arc to cut the line segment at Z.	
4 Place the compass point on point C and alter the compass opening so that it measures the length $\overline{CA}$. Place the compass point on point Z and draw an arc above $\overline{YZ}$.	
5 Place the compass point on point B and alter the compass opening so that it measures the length $\overline{BA}$. Place the compass point on point Y and draw an arc to cut the previous arc at X. Join $\overline{XY}$.	
(b) 1 Draw a line segment $\overline{AB}$. Draw another line $\overline{EH}$ to intersect with $\overline{AB}$. Label the point of intersection G. Choose a point F between E and G. Using the instructions in part **(a)**, construct acute $\angle YFG$ equal to acute $\angle BGH$.	*(b)* 
2 Draw a line segment through $\overline{YF}$ and extend to a point X. $\overline{XY}$ will be parallel to $\overline{AB}$, as you have constructed equal corresponding angles.	

8.8 Compass constructions

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Answers page 673

Equipment required: Compass, ruler, pencil

Fluency

1 Using only a compass, a ruler and a pencil:

(a) bisect a line **(b)** bisect an angle.

2 Using only a compass, a ruler and a pencil:

(a) construct an equilateral triangle **(b)** construct a square.

3 Using only a compass, a ruler and a pencil:

(a) construct an angle equal to another angle

(b) construct a pair of parallel lines

Understanding

4 Using only a compass, a ruler and a pencil:

(a) construct a 90° angle

(b) bisect this angle to make a 45° angle.

5 Using only a compass, a ruler and a pencil:

(a) construct a 60° angle

(b) bisect the 60° angle constructed in part **(a)** to make a 30° angle

(c) bisect a line 10 cm long.

6 Using only a compass, a ruler and a pencil construct a parallelogram.

Reasoning

7 Construct a triangle with one angle equal to 30° and another angle equal to 45° and the arm between these two angles equal to 8 cm.

8 **(a)** Construct an angle of 75°.

(b) Construct an angle of 15°.

(c) Construct an angle of 135°.

9 Construct an isosceles right-angled triangle where the two equal sides are 5 cm long.

10 Construct an equilateral triangle on each side of a square.

11 Use a compass to draw a circle. Select a point on the circle. Using the same compass opening, divide the circle perimeter into 6 equal pieces. Join the points of intersection in order to make a hexagon. Explain why this construction works.

12 Use a compass to draw a circle. Construct an octagon by dividing the perimeter of the circle into 8 equal pieces. Explain how your knowledge of constructing a 90° angle and bisecting an angle helped you.

Open-ended

13 Choose any two polygon shapes and use your construction knowledge to make a design.

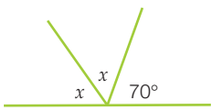
Challenge 8

1 The smaller angle between the hands of a clock at half past two is:

A 90° **B** 105° **C** 120° **D** 135°

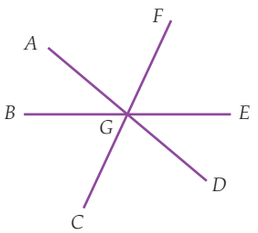
2 Two angles are complementary and one angle is 46° more than the other. What is the size of the larger angle?

3 Find the value of the pronumerals in the following.



4 In the diagram, $\angle BGF = 115°$ and $\angle AGE = 140°$.

What is the size of $\angle CGD$?



5 Two angles are supplementary and one angle is 54° more than the other. What is the size of the smaller angle?

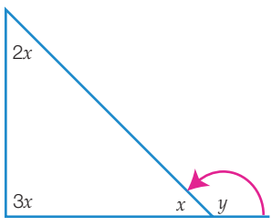
6 In the diagram, the value of y is:

A 30°

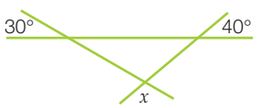
B 60°

C 90°

D 150°



7 In the diagram, the value of x is:



A 110°

B 70°

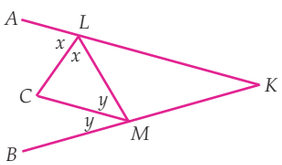
C 40°

D 30°

8 **(a)** Shortly after 12 o'clock, the hour and minute hands of a clock form an angle of 55°. What is the time now?

(b) Noor observes this and leaves, returning after 40 minutes. What is the acute angle between the hands now?

9 In the diagram, ALK and BMK are straight lines. The bisector of $\angle ALM$ meets the bisector of $\angle BML$ at C. If $\angle LCM$ is 70°, then $\angle LKM$, in degrees, is:



A 10°

B 20°

C 40°

D 110°

Chapter review 8

D.I.Y. Summary

Key Words

acute angle	equilateral triangle	plane	right angle
acute-angled triangle	intersect	plane shapes	right-angled triangle
alternate angles	irregular	polygon	scalene triangle
angle	isosceles triangle	protractor	square
arms	kite	quadrilateral	straight angle
co-interior (allied) angles	line segment	ray	supplementary angles
complementary angles	obtuse angle	rectangle	transversal
concave	obtuse-angled triangle	reflex angle	trapezium
convex	parallel lines	regular polygon	vertex
corresponding angles	perpendicular bisector	revolution	vertically opposite angles
degree	perpendicular lines	rhombus	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 To measure an angle, the centre point of the _____ must match with the _____ of the angle. The lines must match with the _____ of the angle.

2 40° and 50° are _____.

3 _____ add to 180 _____s.

4 Lines drawn at right angles to each other are called _____.

5 The angle names listed in order from largest to smallest are_____, _____, _____, _____, _____ , and _____.

6 The most general name for a four-sided figure is a _____.

7 Lines in the same plane that never meet are called _____.

8 An _____ has two sides equal in length and one unequal side.

9 A closed shape with only straight line sides is called a _____.

10 A square is a _____ with right angles.

11 A _____ has three sides all of different lengths.

12 A line crossing other lines is called a _____.

13 A _____ is a quadrilateral with one pair of parallel sides.

14 Angles that are between parallel lines and on the same side of the transversal are _____ and are _____.

Equipment required: Protractor for Questions **1**, **3** and **14**, and a compass for Question **13**

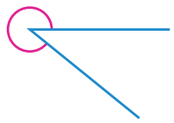
Fluency

1 Use a protractor to measure each of the following angles. Ex 8.1

(a) 

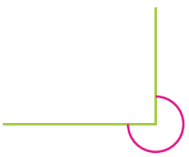
(b) 

(c) 

(d) 

2 Match the following angle sizes with the drawn angles. Do not use a protractor. Ex 8.1

A 20° **B** 55° **C** 120° **D** 210° **E** 270° **F** 310°

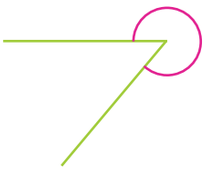
(a) 

(b) 

(c) 

(d) 

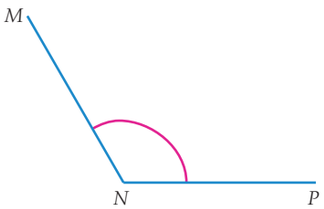
(e) 

(f) 

3 Use a protractor and a ruler to draw an angle of each size stated. Ex 8.1

(a) 56° **(b)** 162° **(c)** 257° **(d)** 304°

4 Which of the following is not true of the angle shown? Ex 8.2



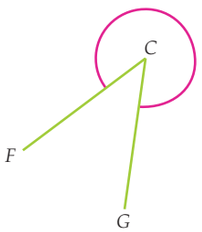
A It is named $\angle NPM$.

B It is named $\angle MNP$.

C It is named $\angle PNM$.

D It is an obtuse angle.

5 Which of the following is true of the angle shown? Ex 8.2



A It is obtuse.

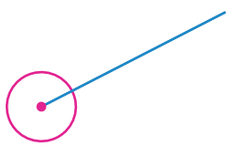
B It is reflex.

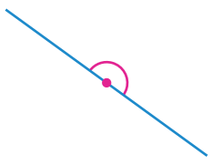
C It is less than 180°.

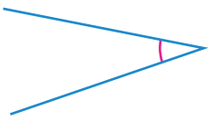
D It is called $\angle CFG$.

6 State the type of each angle below (i.e. acute etc.). Ex 8.2

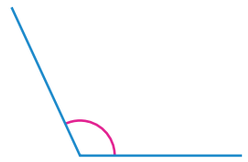
(a) 

(b) 

(c) 

(d) 

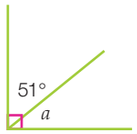
(e) 

(f) 

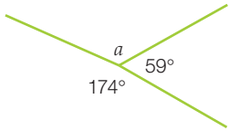
7 Find the value of a in each of the following diagrams. Ex 8.3

(a)

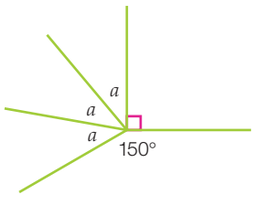
(b)



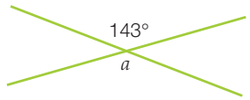
(c)



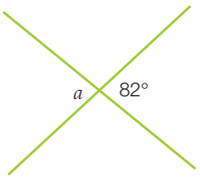
(d)



(e)

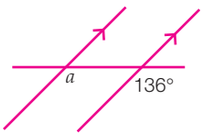


(f)

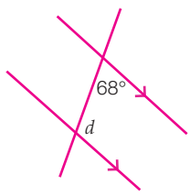


8 Find the value of the pronumeral in the following diagrams. Ex 8.4

(a)



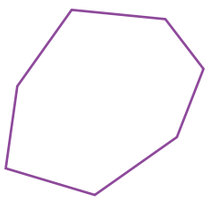
(b)



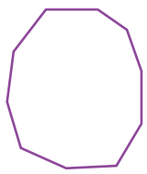
9 Name the following polygons, also stating whether they are regular or irregular, convex or concave. Ex 8.5

(a)

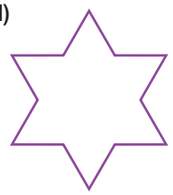
(b)



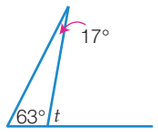
(c)



(d)

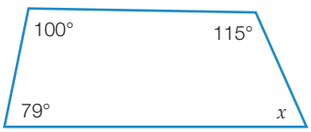


10 Find the value of the pronumeral in the diagram. Ex 8.6



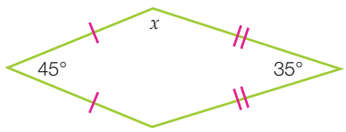
11 Find the unknown angle in each of the following quadrilaterals. Make sure you support your answer with reason(s). Ex 8.7

(a)



(b)

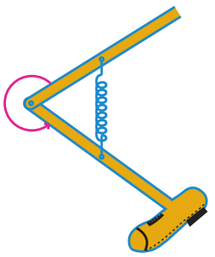
12 Find the value of x in the following quadrilateral. Make sure you support your answer with reason(s). Ex 8.7



13 Use a compass to construct a square of side length 4 cm. Ex 8.8

Understanding

14 Eloise has constructed a mechanical leg to kick a football as part of her physical education investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



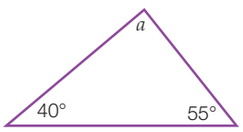
15 One angle in a right-angled triangle is 45°.

(a) What is the value of the other two angles?

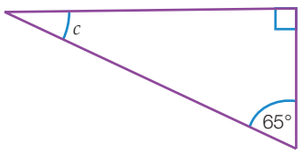
(b) The triangle described in **(a)** was named a right-angled triangle. Give another name for this triangle.

16 Find the missing angle in each of the following triangles. Make sure you support your answer with reason(s).

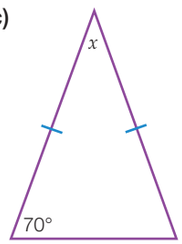
(a)



(b)

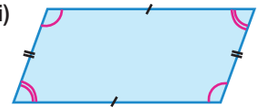


(c)

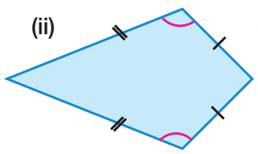


17 Using the shapes below, answer TRUE or FALSE to the following statements.

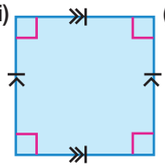
(i)



(ii)



(iii)



(iv)

(a) All four shapes are quadrilaterals.

(b) **(i)** and **(ii)** are kites.

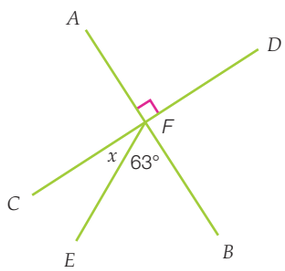
(c) **(i)**, **(iii)** and **(iv)** could be classified as parallelograms.

(d) **(i)** is a rhombus.

(e) **(iii)** is a rectangle.

Reasoning

18 Find the value of the angle x, giving reasons for each step.



19 What type of angle would result from adding an acute angle and a right angle?

20 What type of angle would result from adding a straight angle and an obtuse angle?

21 What types of angle could result from subtracting a reflex angle from a revolution?

NAPLAN practice 8

Numeracy: Non-calculator

1 Which is the largest angle?

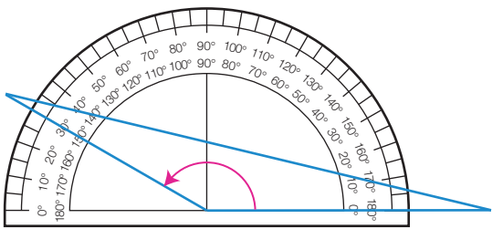
A

B

C

D

2 What is the size of the angle in the triangle marked by the arrow?



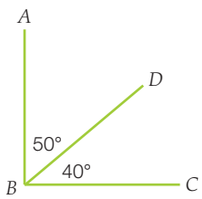
3 $\angle ABD$ and $\angle DBC$ are best described as:

A vertically opposite angles

B supplementary angles

C angles at a point

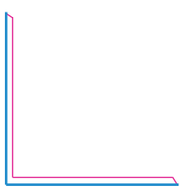
D complementary angles.



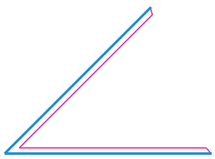
Numeracy: Calculator allowed

4 Which diagram shows a book opened to about 30°?

A

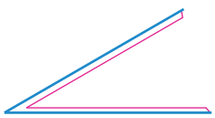


B



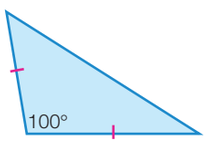
C

D

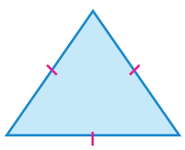


5 Which one of these is a right-angled isosceles triangle?

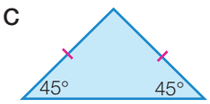
A



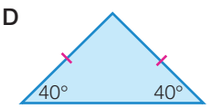
B



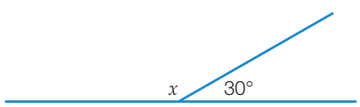
C



D



6 What is the value of x in this diagram?



A 30° **B** 60° **C** 120° **D** 150°

Mixed review D

Equipment required: Protractor for Question **15(c)**

Fluency

1 Simplify:

(a) $2a + 3b + 4a - 7b$ **(b)** $6a + 3b - 2c - b + 5c$

Ex 5.6

2 Write out factor trees for each of the following numbers, then express each number as the product of its prime factors in index form.

(a) 45 **(b)** 24 **(c)** 630

Ex 2.3

3 Write the following percentages as fractions in simplest form.

(a) 75% **(b)** 50% **(c)** 30% **(d)** 4%

Ex 4.7

4 Evaluate without using a calculator:

(a) 1.1×0.23 **(b)** $0.24 + 3.7 - 1.503$

Ex 4.4, 4.5

5 Solve the following equations using algebra.

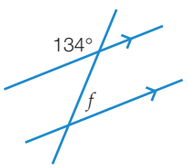
(a) $3x - 4 = -7$ **(b)** $\frac{x + 5}{3} = -12$

Ex 7.4

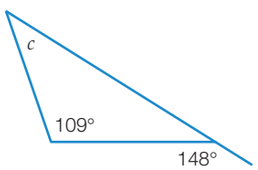
6 Find the value of the pronumeral in the following diagrams.

Ex 8.4, 8.6

(a)



(b)



7 Evaluate without a calculator:

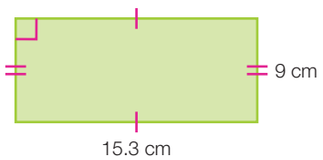
Ex 3.4–3.6

(a) $3\frac{1}{5} + 2\frac{1}{2} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} \times 1\frac{1}{3} \div 3\frac{1}{5}$

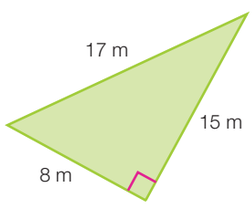
8 Find the area of the following shapes.

Ex 6.4, 6.5

(a)



(b)



Understanding

9 The table on the right shows the minimum and maximum temperatures at Frosty Hollow for 1 week in winter.

(a) On which day was the lowest maximum recorded?

(b) On which day was the highest minimum recorded?

(c) Which day had the greatest difference between minimum and maximum?

Day	Minimum (°C)	Maximum (°C)
Monday	-4	5
Tuesday	-8	0
Wednesday	-2	1
Thursday	-5	-1
Friday	-6	6
Saturday	-4	4
Sunday	-9	2

10 Which of the following is the correct answer to $(4 + 3) \times 10 - 4 \div 2$?

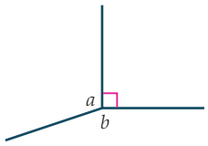
A 68 **B** 35 **C** 33 **D** 21

Reasoning

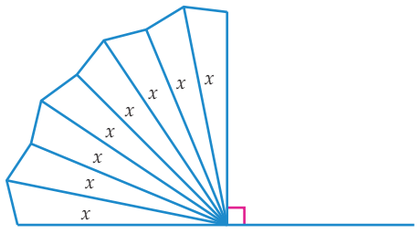
11 The angles in this diagram can be called:

A vertically opposite **B** complementary

C supplementary **D** angles at a point.

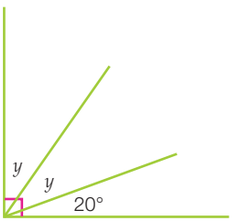


12 A fan, when opened out, looks like this. What is the size of the angle x between the folds of the fan?

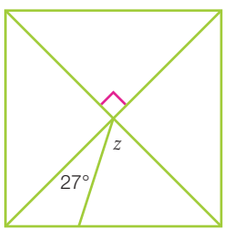


13 Find the value of the pronumeral.

(a)

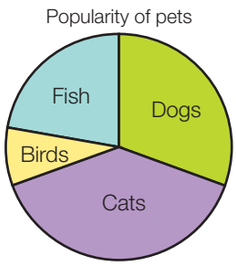


(b)



14 What two angles are formed by the two hands on a clock face at 5.00 p.m.?

15 John has drawn this pie chart, showing the popularity of pets.



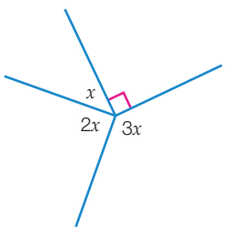
(a) Estimate the angle at the centre of each wedge.

(b) Without using a protractor, how could you check to see how reasonable the estimates you made in **(a)** are?

(c) Using a protractor, measure the size of each wedge.

16 You enter a lift on the fourth floor. Once inside, you are trapped and the lift takes you on an adventure! First, you go up five floors, then fall six floors, then fall another two floors before finally going up one floor and coming to rest. The doors open, much to your relief. What floor are you on?

17 Find the size of each angle in the diagram by first finding the value of x.



18 **(a)** Draw and label an acute angle $\angle ABC$.

(b) There is another angle in your diagram, apart from the acute angle. Classify it.

(c) Name the angle in **(b)**.

19 Name as many pairs of complementary angles, supplementary angles, vertically opposite angles and angles in a revolution as you can see in the figure.