Chapter

2 Geometry

What you will learn

2A Points, lines and angles (Consolidating)
2B Measuring angles (Consolidating)
2C Angles at a point
2D Transversal lines and parallel lines
2E Problems with parallel lines (Extending)
2F Circles and constructions
2G Dynamic geometry

Australian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Identify corresponding, alternate and cointerior angles when two parallel straight lines are crossed by a transversal (ACMMG163)

Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164) AC

ISBN 978-1-107-56882-2

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Thales, pyramids and the solar eclipse

From the Egyptian pyramids to modern architecture, points, lines and angles are everywhere. Geometry is a very visual element of mathematics where the designs of buildings and the orbits of planets can be studied using basic objects like points, lines and circles.

Thales (624–546 BCE) is known to be the founder of Greek geometry. He was an astronomer and philosopher, and records show he was the first person to use mathematical geometry to calculate the height of an Egyptian pyramid using the sun's rays and to accurately predict the timing of a solar eclipse.

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2A Points, lines and angles

CONSOLIDATING

The fundamental building blocks of geometry are the point, line and plane. They are the basic objects used to construct angles, triangles and other more complex shapes and objects. Points and lines do not actually occupy any area but can be represented on a page using drawing equipment.

Lines don't take up any area, but they still exist in nature.

Let's start: Geometry around you

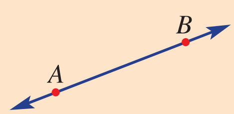
Take a look around the room you are in or consider any solid object near where you are seated (e.g. a book). Discuss what parts of the room or object could be described using:

- single points
- straight lines
- flat planes.

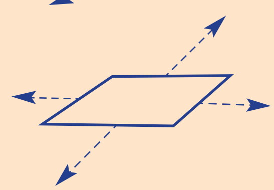
Key ideas

- A **point** is usually labelled with a capital letter.

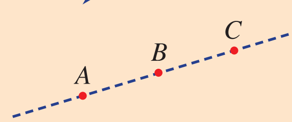
- A **line** passing through two points, A and B, can be called line AB or line BA and extends indefinitely in both directions.



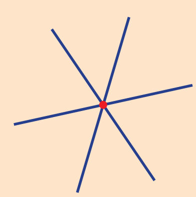
- A **plane** is a flat surface and extends indefinitely.



- Points that all lie on a single line are **collinear**.



- If two lines meet, an **intersection point** is formed.
- Three or more lines that intersect at the same point are **concurrent**.



- A line **segment** (or **interval**) is part of a line with a fixed length and end points. If the end points are A and B then it would be named segment AB or segment BA (or interval AB or interval BA).

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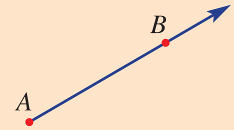
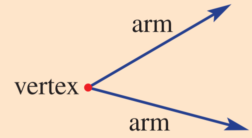
Key ideas

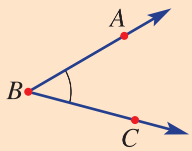
- A **ray** AB is a part of a line with one end point A and passing through point B. It extends indefinitely in one direction.



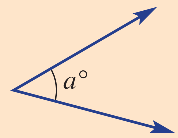
- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called **arms** of the angle.



- An **angle** is named using three points, with the vertex as the middle point. A common type of notation is $\angle ABC$ or $\angle CBA$. The measure of the angle is $a°$.



or

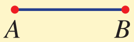



This mosaic around a fountain in Morocco is made up entirely of straight lines, even though it looks circular.

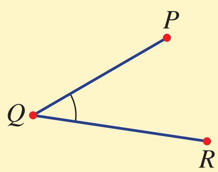
Example 1 Naming objects

Name this line segment and angle.

a



b



SOLUTION	EXPLANATION
a segment AB	Segment BA, interval AB or interval BA are also acceptable.
b $\angle PQR$	Point Q is the vertex and sits in between P and R. $\angle RQP$ is also correct.

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Exercise 2A

1–4	4	—

UNDERSTANDING

1 Draw the following objects.

- **a** a point P
- **b** a line AN
- **c** an angle $\angle ABC$
- **d** a ray ST
- **e** a plane
- **f** three collinear points A, B and C

2 Explain what it means to say:

- **a** three points D, E and F are collinear
- **b** three lines are concurrent

3 Match the words *line*, *segment* or *ray* to the correct description.

- **a** Starts from a point and extends indefinitely in one direction.
- **b** Extends indefinitely in both directions, passing through two points.
- **c** Starts and ends at two points.

4 Match the words *point*, *line* or *plane* with the following descriptions.

- **a** the edge of a sheet of paper
- **b** a flat wall
- **c** the surface of a pool of water on a calm day
- **d** where two walls and a floor meet in a room
- **e** where two walls meet in a room
- **f** one side of a cereal packet
- **g** where two sides meet on a box
- **h** where three sides meet on a box

5–9	5–9	5–6(½), 7–9

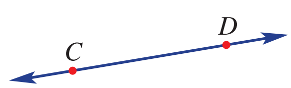
FLUENCY

Example 1

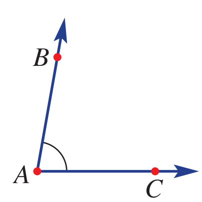
5 Name the following objects.

a

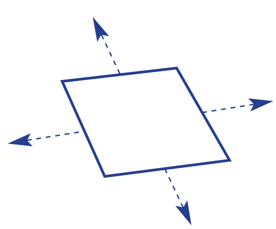
b



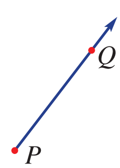
c



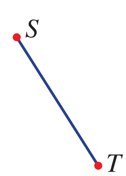
d



e



f



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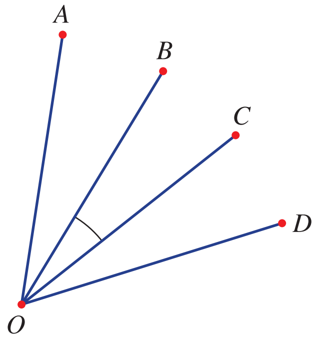
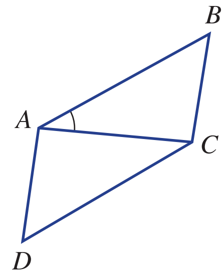
FLUENCY

6 Name the angle marked by the arc in these diagrams.

a



b



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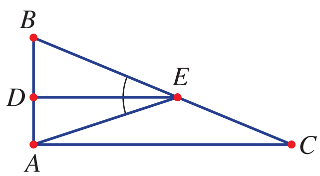
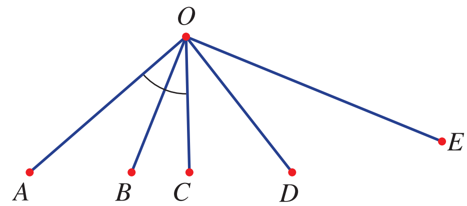
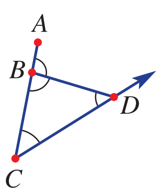


d



7 In each of these diagrams name the five line segments and the four marked angles using the given labels.

a



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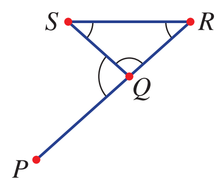
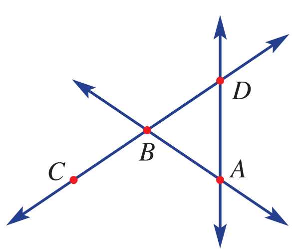
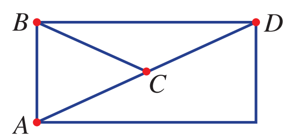


8 Name the set of three labelled points that are collinear in these diagrams.

a

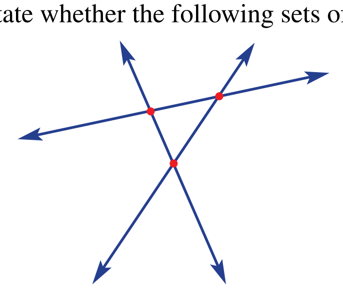


b

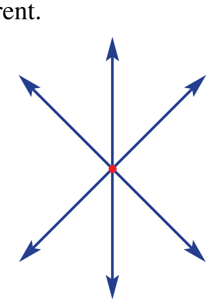


9 State whether the following sets of lines are concurrent.

a



b



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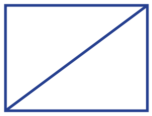
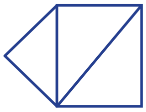
2A

10	10, 11	11, 12

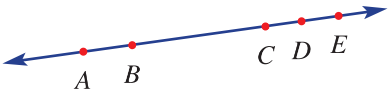
PROBLEM-SOLVING

10 Count the number of angles formed inside these shapes. Count all angles, including ones that may be the same size and those angles that are divided by another segment.

a 

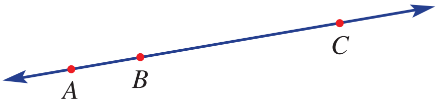
b 

11 How many line segments are there on this line? Do not count AB and BA as separate segments since they represent the same segment.



12 A line contains a certain number of labelled points. For example, this line has three points.

a Complete this table by counting the total number of segments for the given number of labelled points.



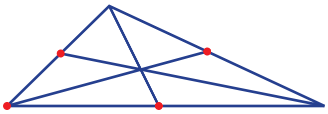
Number of points	1	2	3	4	5	6
Number of segments						

b Explain any patterns you see in the table. Is there a quick way of finding the next number in the table?

13	13	13, 14

REASONING

13 The lines joining each vertex (corner) of a triangle with the midpoint (middle point) of the opposite side are drawn here.



a Draw any triangle and use a ruler to measure and mark the midpoints of each side.

b Join each vertex with the midpoint of the opposite side.

c Are your segments from part **b** concurrent?

d Do you think your answer to part **c** will always be true for any triangle? Try one other triangle of a different size to check.

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2A

REASONING

14 **a** If points A, B and C are collinear and points A, B and D are collinear, does this mean that points B, C and D are also collinear?

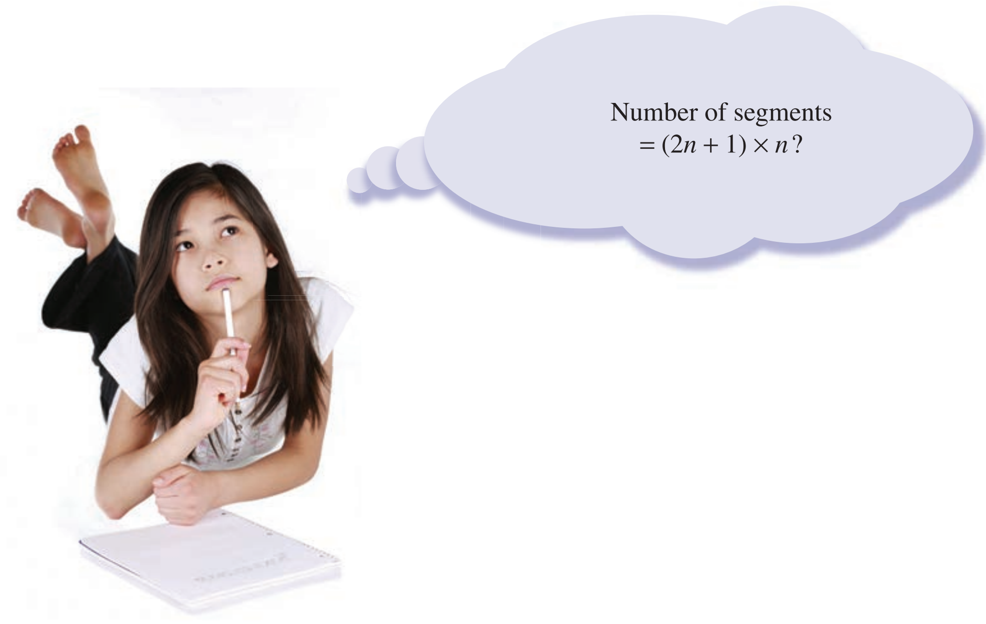
b If points A, B and C are collinear and points C, D and E are collinear, does this mean that points B, C and D are also collinear?

The general rule

—	—	15

ENRICHMENT

15 In Question **12** you may have determined a quick method of finding the number of segments for the given number of points. If n is the number of points on the line, can you find a rule (in terms of n) for the number of segments? Test your rule to see if it works for at least three cases.



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2B Measuring angles

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

Angles are usually described using the unit of measurement called the degree, where 360 degrees (360°) describes one full turn. The idea to divide a circle into 360° dates back to the Babylonians, who used a sexagesimal number system based on the number 60. Because both 60 and 360 are numbers that have a large number of factors, many fractions of these numbers are very easy to calculate.

What angle is between each spoke on this Ferris wheel?

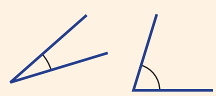
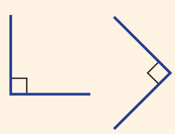
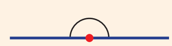
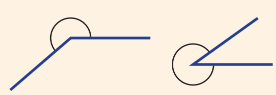
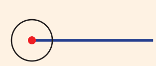
Let's start: Estimating angles

How good are you at estimating the size of angles? Estimate the size of these angles and then check with a protractor.

Alternatively, construct an angle using computer geometry. Estimate and then check your angle using the angle-measuring tool.

Key ideas

- Angles are classified according to their size.

Angle type	Size	Examples
acute	between 0° and 90°	
right	90°	
obtuse	between 90° and 180°	
straight	180°	
reflex	between 180° and 360°	
revolution	360°	

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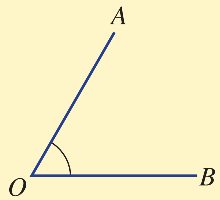
Key ideas

- A **protractor** can be used to measure angles to within an accuracy of about half a degree. Some protractors have increasing scales marked both clockwise and anticlockwise from zero. To use a protractor:
  1. Place the centre of the protractor on the vertex of the angle.
  2. Align the base line of the protractor along one arm of the angle.
  3. Measure the angle using the other arm and the scale on the protractor.
  4. A reflex angle can be measured by subtracting a measured angle from 360°.

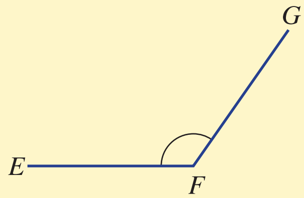
Example 2 Measuring with a protractor

For the angles shown, state the type of angle and measure its size.

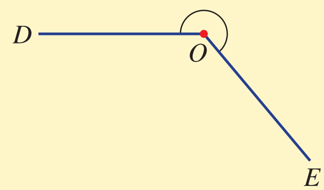
a



b



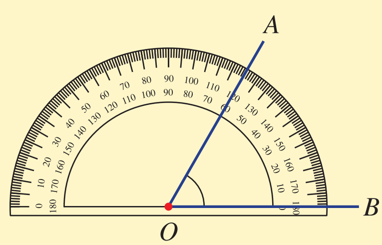
c



SOLUTION | **EXPLANATION**

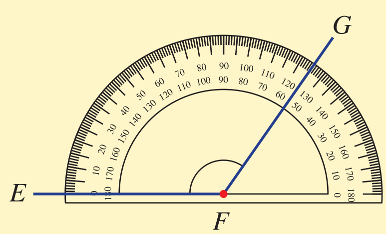
a acute

$\angle AOB = 60^\circ$



b obtuse

$\angle EFG = 125^\circ$

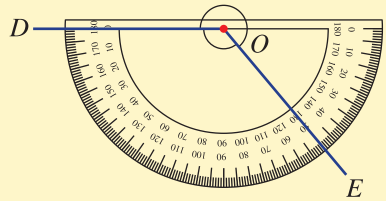


c reflex

obtuse $\angle DOE = 130^\circ$

reflex $\angle DOE = 360^\circ - 130^\circ$

$= 230^\circ$



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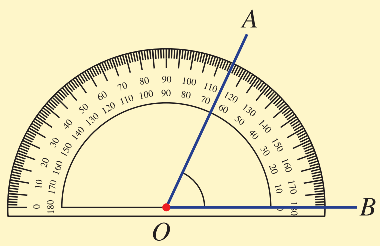
Example 3 Drawing angles

Use a protractor to draw each of the following angles.

a $\angle AOB = 65^\circ$ **b** $\angle WXY = 130^\circ$ **c** $\angle MNO = 260^\circ$

SOLUTION / **EXPLANATION**

a



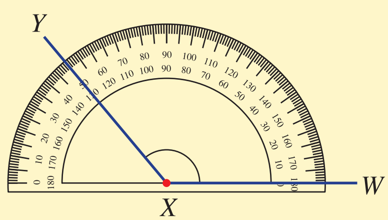
Step 1: Draw a base line OB.

Step 2: Align the protractor along the base line with the centre at point O.

Step 3: Measure 65° and mark a point, A.

Step 4: Draw the arm OA.

b



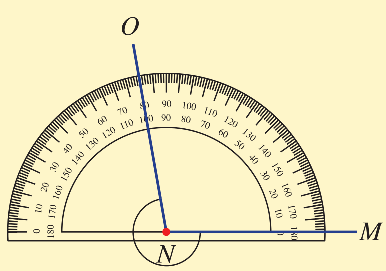
Step 1: Draw a base line XW.

Step 2: Align the protractor along the base line with the centre at point X.

Step 3: Measure 130° and mark a point, Y.

Step 4: Draw the arm XY.

c



Step 1: Draw an angle of $360^\circ - 260^\circ = 100^\circ$.

Step 2: Mark the reflex angle on the opposite side to the obtuse angle of 100°.

Alternatively, draw a 180° angle and measure an 80° angle to add to the 180° angle.

Exercise 2B

1–3	3	—

UNDERSTANDING

1 Without using a protractor, draw an example of the following types of angles.

a acute **b** right **c** obtuse

d straight **e** reflex **f** revolution

2 How many right angles (i.e. angles of 90°) make up:

a a straight angle? **b** 270°? **c** a revolution?

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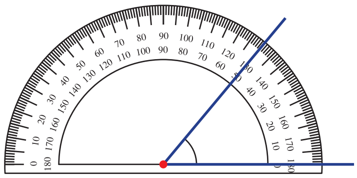
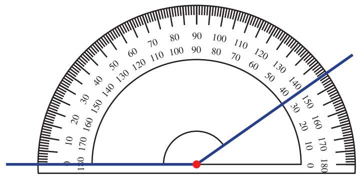
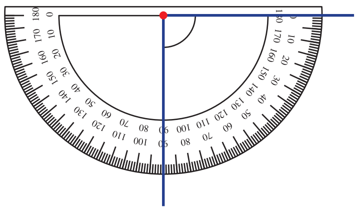
2B

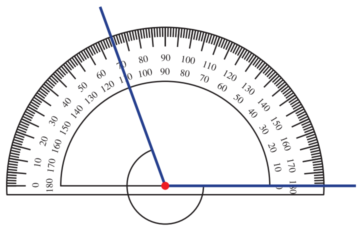
UNDERSTANDING

3 What is the size of the angle measured with these protractors?

a


b


c


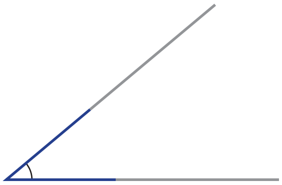
d


4–6	4(½), 5, 6(½), 7, 8	4(½), 5, 6(½), 8

FLUENCY

Example 2

4 For the angles shown, state the type of angle and measure its size.

a


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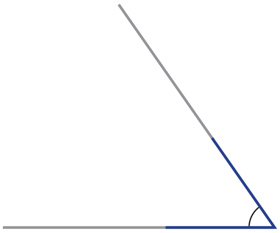

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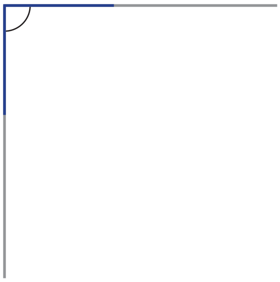

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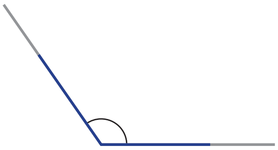

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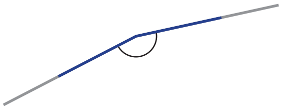

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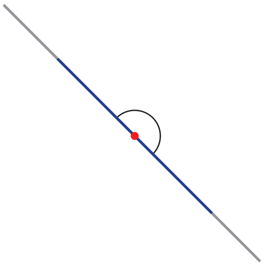
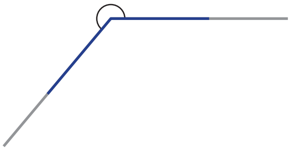


g


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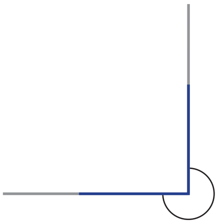
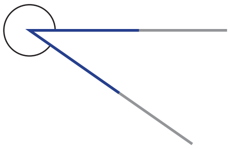


i


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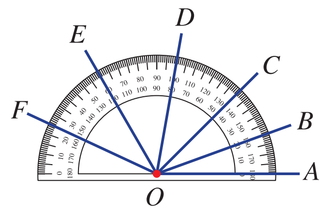
2B

FLUENCY

5 **a** Write down the size of the angles shown on this protractor.

- **i** $\angle AOB$ **ii** $\angle BOC$
- **iii** $\angle COD$ **iv** $\angle DOE$
- **v** $\angle EOF$

b Find the sum of all the angles from part **a**. Name a single angle in the diagram that equals this sum.



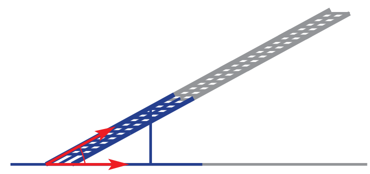
Example 3

6 Use a protractor to draw each of the following angles.

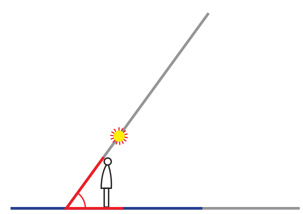
a 40°	**b** 75°	**c** 90°	**d** 135°	**e** 175°
f 205°	**g** 260°	**h** 270°	**i** 295°	**j** 352°

7 For each of the angles marked in the situations shown, measure:

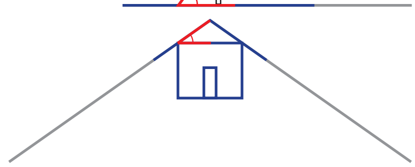
a the angle that this ramp makes with the ground



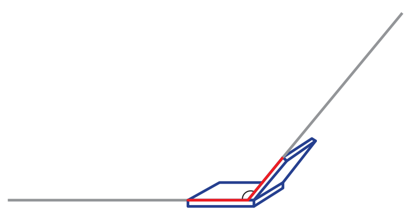
b the angle the Sun's rays make with the ground



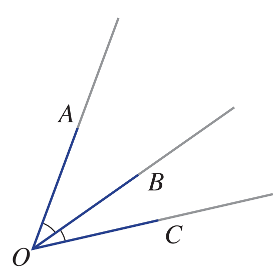
c the angle or pitch of this roof



d the angle between this laptop screen and the keyboard



8 In the diagram shown at right, there are two marked angles, $\angle AOB$ and $\angle BOC$. Measure $\angle AOB$, $\angle BOC$ and $\angle AOC$.
Does $\angle AOB + \angle BOC = \angle AOC$? Why or why not?



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PROBLEM-SOLVING	9	9(½), 10	9(½), 10

9 A clock face is numbered 1 to 12. Find the angle the minute hand turns in:

a 30 minutes **b** 1 hour **c** 15 minutes **d** 45 minutes
e 5 minutes **f** 20 minutes **g** 55 minutes **h** 1 minute
i 9 minutes **j** 10.5 minutes **k** 42.5 minutes **l** 21.5 minutes

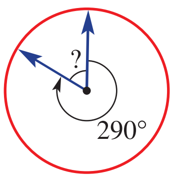
10 A clock face is numbered 1 to 12. Find the angle between the hour hand and the minute hand at:

a 6:00 pm **b** 3:00 pm **c** 4:00 pm **d** 11:00 am

REASONING	11	11	11, 12

11 The arrow on this dial starts in an upright position. It then turns by a given number of degrees clockwise or anticlockwise.

a Find the angle between the arrow in its final position with the arrow in its original position, as shown in the diagram opposite, which illustrates part **i**. Answer with an acute or obtuse angle.

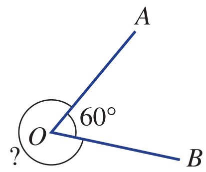


i 290° clockwise **ii** 290° anticlockwise
iii 450° clockwise **iv** 450° anticlockwise
v 1000° clockwise **vi** 1000° anticlockwise

b Did it matter to the answer if the dial was turning clockwise or anticlockwise?

c Explain how you calculated your answer for turns larger than 360°.

12 An acute angle $\angle AOB$ is equal to 60°. Why is it unnecessary to use a protractor to work out the size of the reflex angle $\angle AOB$?



ENRICHMENT — Time challenge	—	—	13

13 Find the angle between the hour hand and the minute hand of a clock at these times.

a 10:10 am **b** 4:45 am
c 11:10 pm **d** 2:25 am
e 7:16 pm **f** 9:17 pm

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2C Angles at a point

Interactive

Widgets

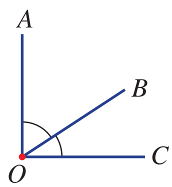
HOTsheets

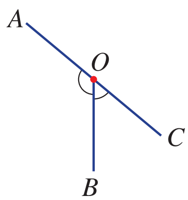
Walkthroughs

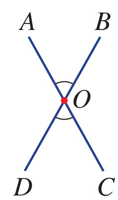
Not all angles in a diagram or construction need to be measured directly. Special relationships exist between pairs of angles at a point and this allows some angles to be calculated exactly without measurement, even if diagrams are not drawn to scale.

Let's start: Special pairs of angles

By making a drawing or using computer geometry, construct the diagrams below. Measure the two marked angles. What do you notice about the two marked angles?

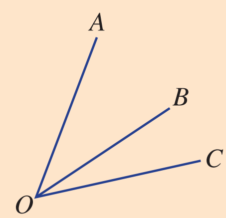




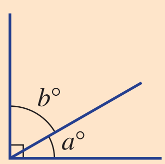


Key ideas

- **Adjacent** angles are side by side and share a vertex and an arm. $\angle AOB$ and $\angle BOC$ in this diagram at right are adjacent angles.

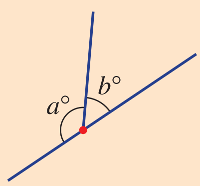


- **Complementary adjacent** angles sum to 90°.



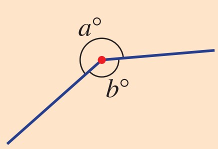
$a + b = 90$

- **Supplementary adjacent** angles sum to 180°.



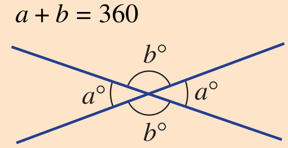
$a + b = 180$

- Angles in a **revolution** sum to 360°.

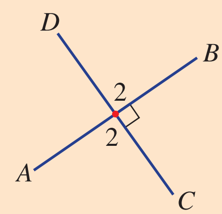


$a + b = 360$

- **Vertically opposite** angles are formed when two lines intersect. The opposite angles are equal. The name comes from the fact that the pair of angles has a common vertex and they sit in opposite positions across the vertex.



- **Perpendicular** lines meet at a right angle (90°). We write $AB \perp CD$.



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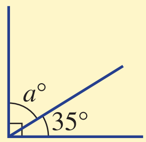
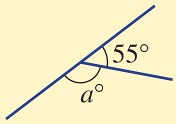
Example 4 Finding angles at a point

Without using a protractor, find the size of each angle marked with the letter a.

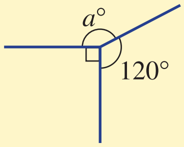
a



b



c



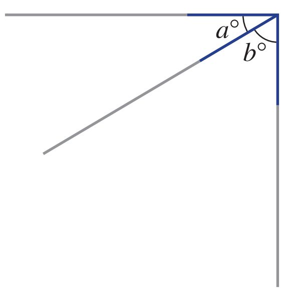
SOLUTION	EXPLANATION
a $a + 35 = 90$	Angles in a right angle add to 90°.
$a = 55$	$90 - 35 = 55$
b $a + 55 = 180$	Angles on a straight line add to 180°.
$a = 125$	$180 - 55 = 125$
c $a + 90 + 120 = 360$	The sum of angles in a revolution is 360°.
$a + 210 = 360$	Simplify by adding 90 and 120.
$a = 150$	a is the difference between 210 and 360°.

Exercise 2C

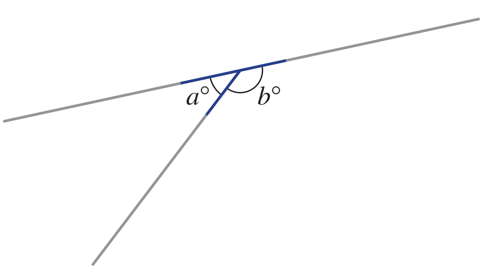
1–4	4	—

UNDERSTANDING

1 **a** Measure the angles $a°$ and $b°$ in this diagram.
b Calculate $a + b$. Is your answer 90? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are ____________ angles.



2 **a** Measure the angles $a°$ and $b°$ in this diagram.
b Calculate $a + b$. Is your answer 180? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are ____________ angles.



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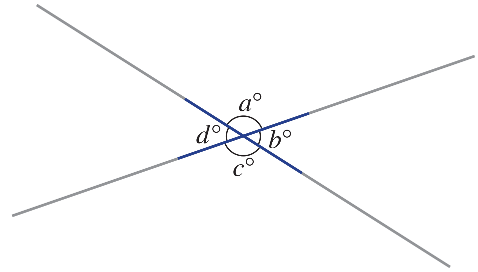
2C

UNDERSTANDING

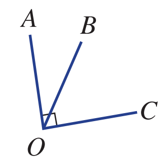
3 **a** Measure the angles a°, b°, c° and d° in this diagram.

b What do you notice about the sum of the four angles?

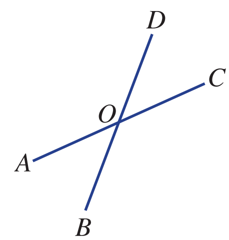
c Write the missing words: b° and d° are ______________ angles.



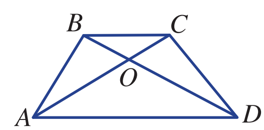
4 **a** Name the angle that is complementary to $\angle AOB$ in this diagram.



b Name the two angles that are supplementary to $\angle AOB$ in this diagram.



c Name the angle that is vertically opposite to $\angle AOB$ in this diagram.

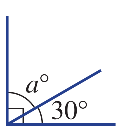


FLUENCY

5, 6	5–6(½), 7, 8(½)	5–8(½)

Example 4

5 Without using a protractor, find the value of the pronumeral a. (The diagrams shown may not be drawn to scale.)

a


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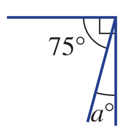

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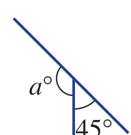

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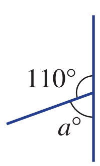

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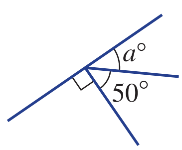

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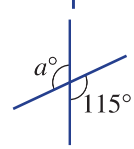

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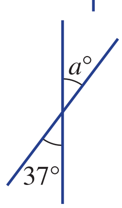

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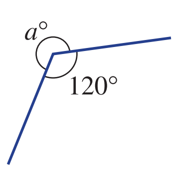

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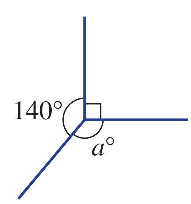

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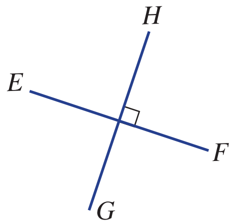
2C

FLUENCY

6 For each of the given pairs of angles, write C if they are complementary, S if they are supplementary or N if they are neither.

a $21^\circ, 79^\circ$ b $130^\circ, 60^\circ$ c $98^\circ, 82^\circ$ d $180^\circ, 90^\circ$
e $17^\circ, 73^\circ$ f $31^\circ, 59^\circ$ g $68^\circ, 22^\circ$ h $93^\circ, 87^\circ$

7 Write a statement like $AB \perp CD$ for these pairs of perpendicular line segments.

a


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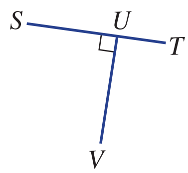
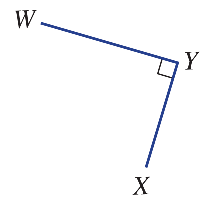
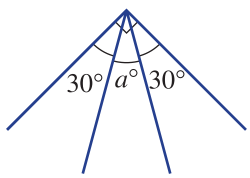


c


8 Without using a protractor, find the value of a in these diagrams.

a


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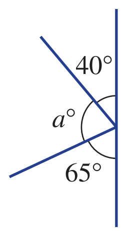

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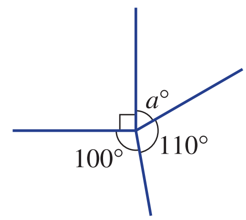

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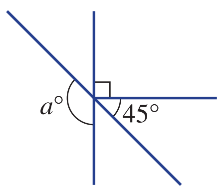

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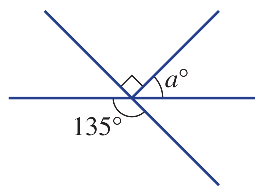

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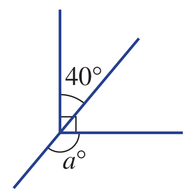
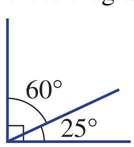


9	9, 10(½)	10, 11

PROBLEM-SOLVING

9 Do these diagrams have the correct information? Give reasons.

a


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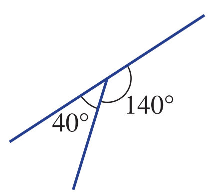

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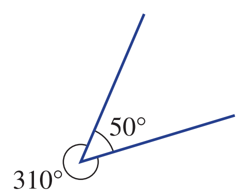

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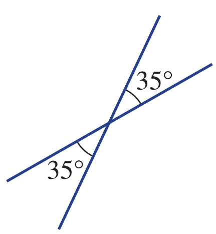

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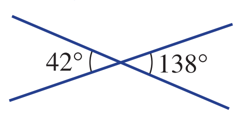

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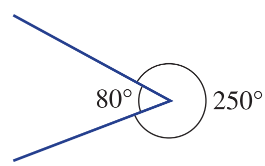

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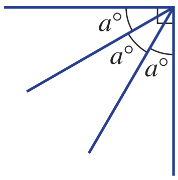
2C

PROBLEM-SOLVING

10 Find the value of a in these diagrams.

a



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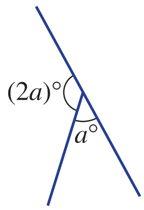
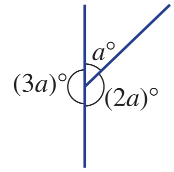


c



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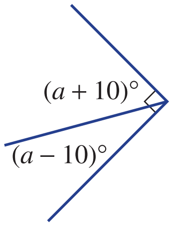
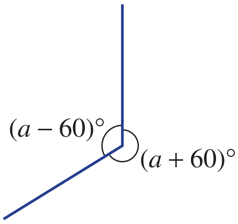


e



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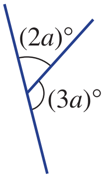


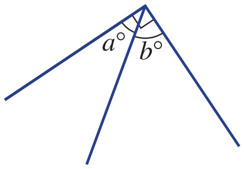
11 A pizza is divided between four people. Bella is to get twice as much as Bobo, who gets twice as much as Rick, who gets twice as much as Marie. Assuming the pizza is cut into triangular pieces, find the angle at the centre of the pizza for Marie's piece.

12	12	12, 13

REASONING

12 Write down a rule connecting the letters in these diagrams, e.g. $a + b = 180$.

a



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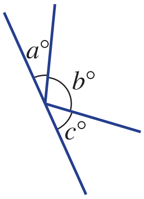

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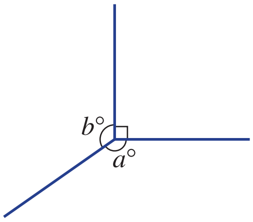
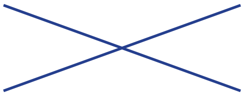


13 What is the minimum number of angles needed in this diagram to determine all other angles? Explain your answer.



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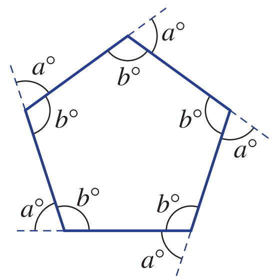
ENRICHMENT

Pentagon turns

—	—	14

14 Consider walking around a path represented by this regular pentagon. All sides have the same length and all internal angles are equal. At each corner (vertex) you turn an angle of a, as marked.



a How many degrees would you turn in total after walking around the entire shape? Assume that you face the same direction at the end as you did at the start.

b Find the value of a.

c Find the value of b.

d Explore the outside and inside angles of other regular polygons using the same idea. Complete this table to summarise your results.

Regular shape	*a*	*b*
Triangle		
Square		
Pentagon		
Hexagon		
Heptagon		
Octagon		

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Progress quiz

2A/B **1** Consider the diagram opposite and answer the following.

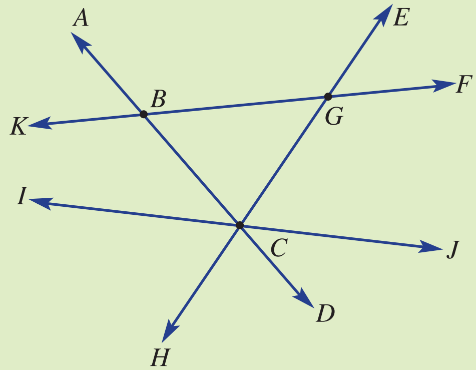
a Name the point where the line *EH* intersects *KF*.

b Name an angle which has its vertex at *G*.

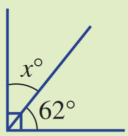
c Name an angle adjacent to ∠*FGH*.

d Name a set of three concurrent lines.

e Name an obtuse angle with its vertex at B and use your protractor to measure the size of this angle.



2C **2** Find the value of each pronumeral below and give a reason for each answer.

a


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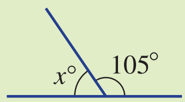

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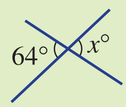

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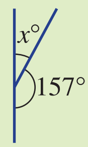

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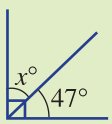

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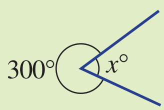
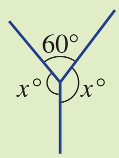


g


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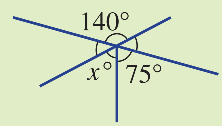
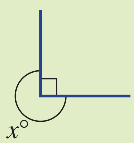


i


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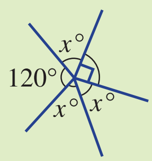

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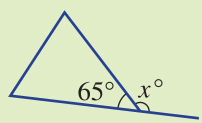
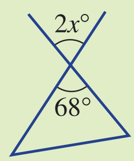


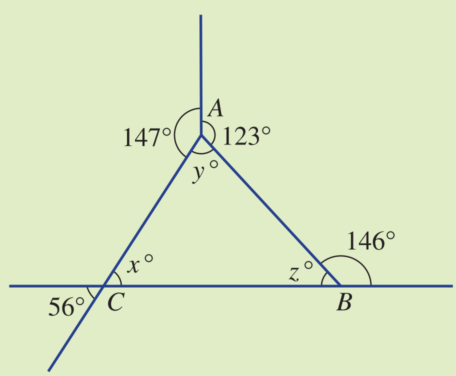
l


2B/C **3** Consider the following diagram and answer these questions.

a Explain why ∠*ACB* equals 56°.

b What is the supplement of 146°?

c Write down the value of $x + y + z$.



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2D Transversal lines and parallel lines

Interactive Widgets HOTsheets Walkthroughs

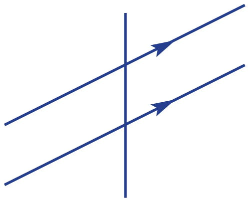
When a line, called a transversal, cuts two or more other lines a number of angles are formed. Pairs of these angles are corresponding, alternate or cointerior angles, depending on their relative position. If the transversal cuts parallel lines, then there is a relationship between the sizes of the special pairs of angles that are formed.

Multiple angles are formed when this transversal road intersects with the freeway.

Let's start: What's formed by a transversal?

Draw a pair of parallel lines using either:

- two sides of a ruler; or
- computer geometry (parallel line tool).

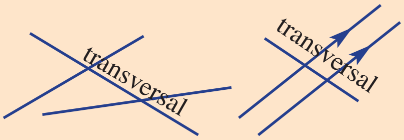


Then cross the two lines with a third line (transversal) at any angle.

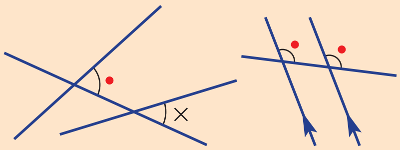
Measure each of the eight angles formed and discuss what you find. If computer geometry is used, drag the transversal and see if your observations apply to all the cases that you observe.

Key ideas

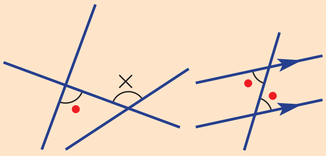
- A **transversal** is a line passing through two or more other lines that are usually, but not necessarily, parallel.



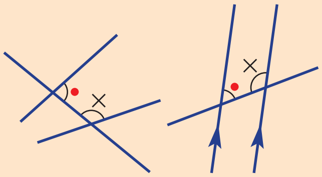
- A transversal crossing two lines will form special pairs of angles. These are:
  - **corresponding** (in corresponding positions)



  - **alternate** (on opposite sides of the transversal and inside the other two lines)



  - **cointerior** (on the same side of the transversal and inside the other two lines).



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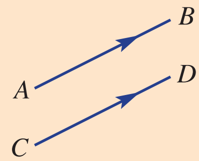
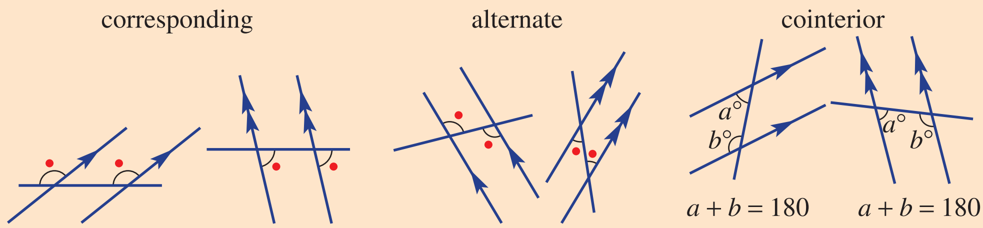
Key ideas

- **Parallel lines** are marked with the same arrow set.
  - **Skew** lines are not parallel.
  - If AB is parallel to CD, then we write $AB \parallel CD$.



- If a transversal crosses two **parallel** lines then:
  - corresponding angles are equal
  - alternate angles are equal
  - cointerior angles are supplementary (i.e. sum to 180°).

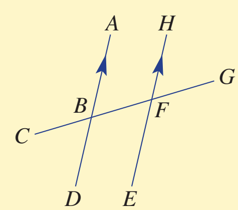


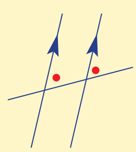
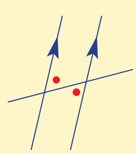
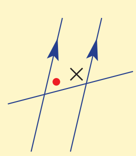
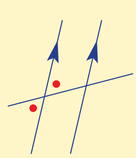
Example 5 Naming pairs of angles

Name the angle that is:

a corresponding to $\angle ABF$ **b** alternate to $\angle ABF$

c cointerior to $\angle ABF$ **d** vertically opposite to $\angle ABF$



SOLUTION	EXPLANATION
a $\angle HFG$	
b $\angle EFB$	
c $\angle HFB$	
d $\angle CBD$	

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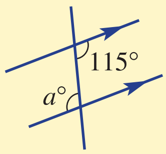
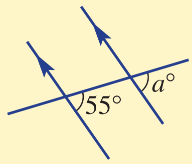
Example 6 Finding angles in parallel lines

Find the value of a in these diagrams and give a reason for each answer.

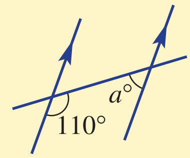
a



b



c

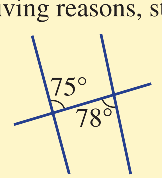


SOLUTION	EXPLANATION
a $a = 115$ alternate angles in parallel lines	Alternate angles in parallel lines are equal.
b $a = 55$ corresponding angles in parallel lines	Corresponding angles in parallel lines are equal.
c $a = 180 - 110$ $= 70$ cointerior angles in parallel lines	Cointerior angles in parallel lines sum to 180°.

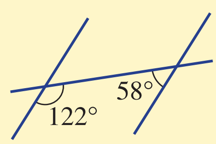
Example 7 Proving lines are parallel

Giving reasons, state whether the two lines cut by the transversal are parallel.

a



b



SOLUTION	EXPLANATION
a not parallel Alternate angles are not equal.	Parallel lines have equal alternate angles.
b parallel The cointerior angles sum to 180°.	$122° + 58° = 180°$ Cointerior angles inside parallel lines are supplementary (i.e. sum to 180°).

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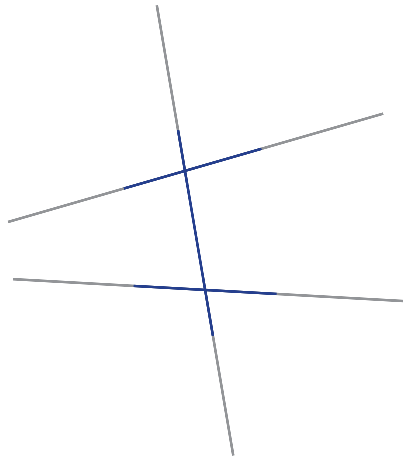
Exercise 2D

1–3	3	—

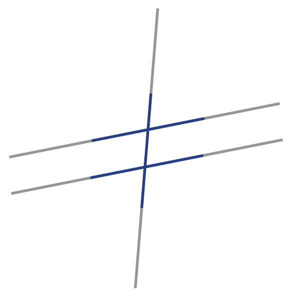
UNDERSTANDING

1 Use a protractor to measure each of the eight angles in this diagram.



- **a** How many *different* angle measurements did you find?
- **b** Do you think that the two lines cut by the transversal are parallel?

2 Use a protractor to measure each of the eight angles in this diagram.



- **a** How many *different* angle measurements did you find?
- **b** Do you think that the two lines cut by the transversal are parallel?

3 Choose the word *equal* or *supplementary* to complete these sentences.
If a transversal cuts two parallel lines, then:

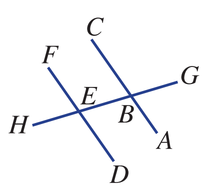
- **a** alternate angles are ______________.
- **b** cointerior angles are ______________.
- **c** corresponding angles are _________.
- **d** vertically opposite angles are ______.

4–8	4, 6, 7(½), 8, 9	4, 7–9(½)

FLUENCY

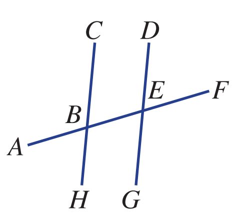
Example 5

4 Name the angle that is:



- **a** corresponding to $\angle ABE$
- **b** alternate to $\angle ABE$
- **c** cointerior to $\angle ABE$
- **d** vertically opposite to $\angle ABE$

5 Name the angle that is:



- **a** corresponding to $\angle EBH$
- **b** alternate to $\angle EBH$
- **c** cointerior to $\angle EBH$
- **d** vertically opposite to $\angle EBH$

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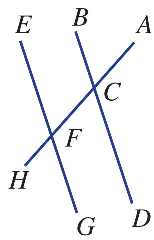
FLUENCY

6 Name an angle that is:

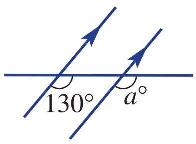
a corresponding to $\angle ACD$

b vertically opposite to $\angle ACD$



Example 6

7 Find the value of a in these diagrams, giving a reason.

a


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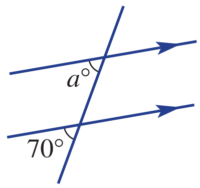

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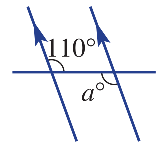

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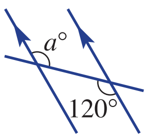

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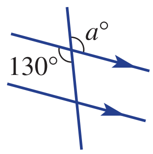

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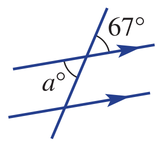
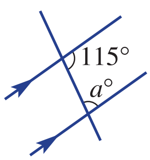


g


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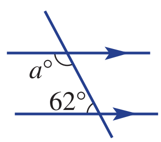
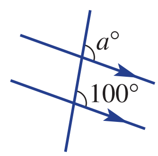


i


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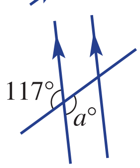

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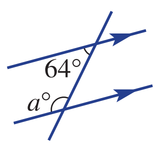
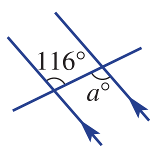
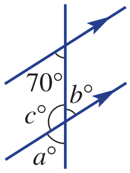
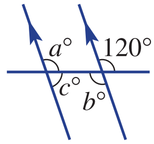


l


8 Find the value of each unknown pronumeral in the following diagrams.

a


b


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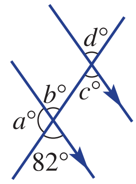

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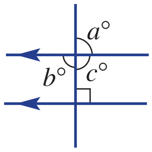

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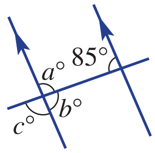

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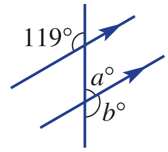

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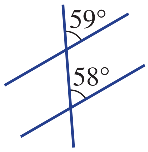
FLUENCY

Example 7

9 Giving reasons, state whether the two lines cut by the transversal are parallel.

a


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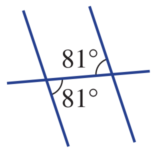

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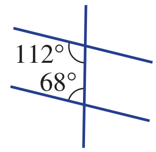

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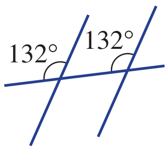

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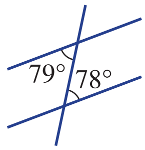

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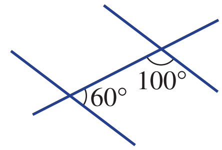
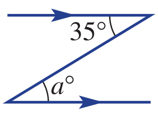


10–11(½)	10–11(½), 12	11(½), 12, 13

PROBLEM-SOLVING

10 Find the value of a in these diagrams.

a


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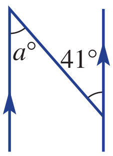

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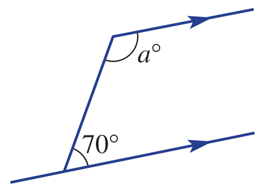

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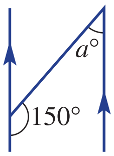

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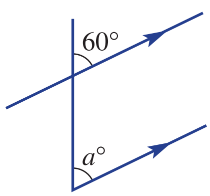

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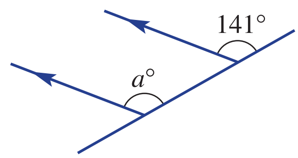
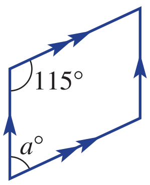


11 Find the value of a in these diagrams.

a


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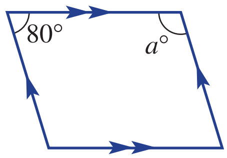

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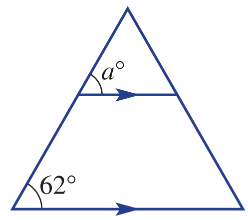

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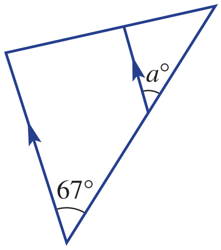

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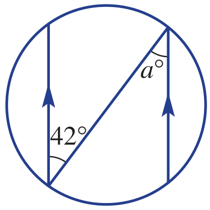

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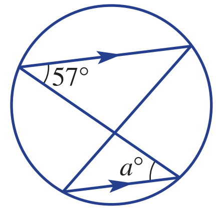

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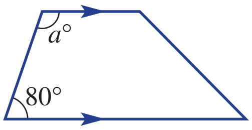
PROBLEM-SOLVING

g



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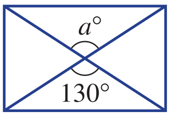
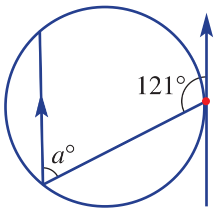


i

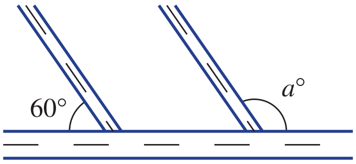


12 A transversal cuts a set of three parallel lines.

a How many angles are formed?

b How many angles of different sizes are formed if the transversal is *not* perpendicular to the three lines?

13 Two roads merge into a freeway at the same angle, as shown. Find the obtuse angle, a, between the parallel roads and the freeway.



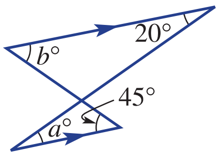
14	14, 15	14–16

REASONING

14 This diagram includes two triangles with two sides that are parallel.

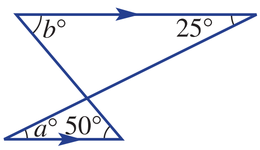
a Give a reason why:

i $a = 20$ ii $b = 45$

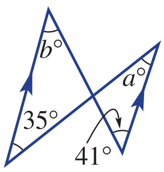


b Now find the values of a and b in the diagrams below.

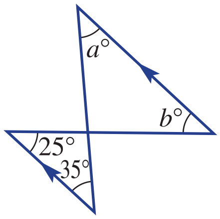
i



ii



iii



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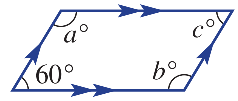
REASONING

15 This shape is a parallelogram with two pairs of parallel sides.



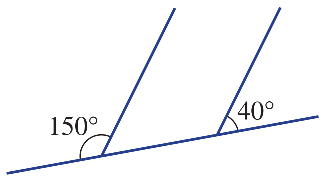
a Use the $60°$ angle to find the value of a and b.

b Find the value of c.

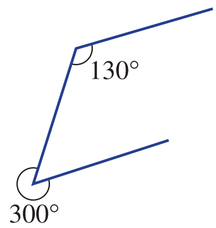
c What do you notice about the angles inside a parallelogram?

16 Explain why these diagrams do not contain a pair of parallel lines.

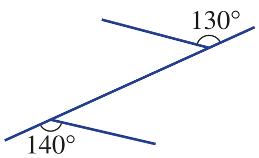
a



b



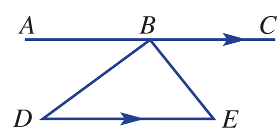
c



Adding parallel lines	—	—	17, 18

ENRICHMENT

17 Consider this triangle and parallel lines.



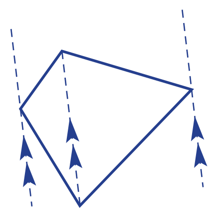
a Giving a reason for your answer, name an angle equal to:

i $\angle ABD$ **ii** $\angle CBE$

b What do you know about the three angles $\angle ABD$, $\angle DBE$ and $\angle CBE$?

c What do these results tell you about the three inside angles of the triangle BDE. Is this true for any triangle? Try a new diagram to check.

18 Use the ideas explored in Question 17 to show that the angles inside a quadrilateral (i.e. a four-sided shape) must sum to $360°$. Use this diagram to help.



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2E Problems with parallel lines EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

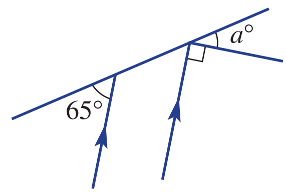
Parallel lines are at the foundation of construction in all its forms. Imagine the sorts of problems engineers and builders would face if drawings and constructions could not accurately use and apply parallel lines. Angles formed by intersecting beams would be difficult to calculate and could not be transferred to other parts of the building.


Parallel support beams in the foyer of Parliament House in Canberra

Let's start: Not so obvious

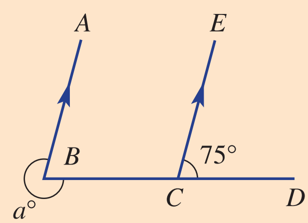
Some geometrical problems require a combination of two or more ideas before a solution can be found. This diagram includes the unknown angle $a°$.

- Discuss if it is possible to find the value of a.
- Describe the steps you would take to find the value of a. Discuss your reasons for each step.



Key ideas

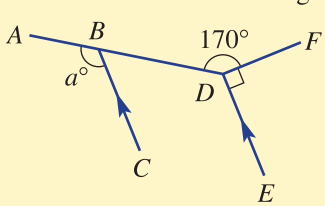
- Some geometrical problems involve more than one step.
  Step 1: $\angle ABC = 75°$ (corresponding angles on parallel lines)
  Step 2: $a = 360 - 75$ (angles in a revolution sum to 360°)
  $= 285$



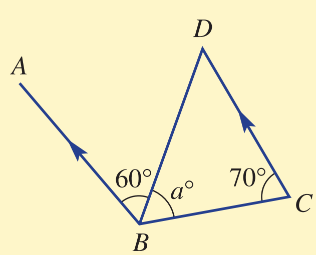
Example 8 Finding angles with two steps

Find the value of a in these diagrams.

a



b



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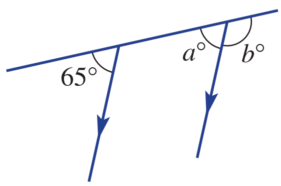
SOLUTION	**EXPLANATION**
a $\angle BDE = 360^\circ - 90^\circ - 170^\circ$ $= 100^\circ$ $\therefore\ a = 100$	Angles in a revolution add to 360°. $\angle ABC$ corresponds with $\angle BDE$, and BC and DE are parallel.
b $\angle ABC = 180^\circ - 70^\circ$ $= 110^\circ$ $\therefore\ a = 110 - 60$ $= 50$	$\angle ABC$ and $\angle BCD$ are cointerior angles, with AB and DC parallel. $\angle ABC = 110^\circ$ and $a^\circ + 60^\circ = 110^\circ$

Exercise 2E

1, 2	2	—

UNDERSTANDING

1 In these diagrams, first find the value of a and then find the value of b.

a


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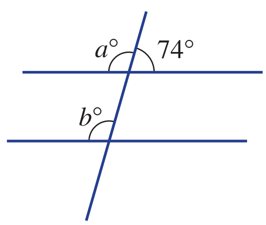
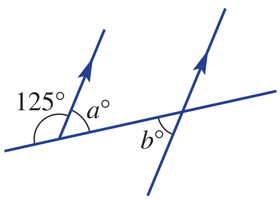
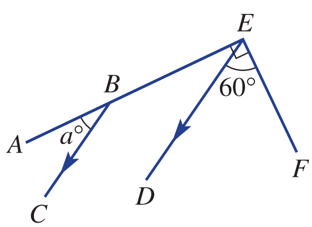


c


2 Name the angle in these diagrams (e.g. $\angle ABC$) that you would need to find first before finding the value of a. Then find the value of a.

a


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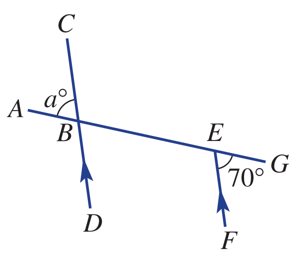

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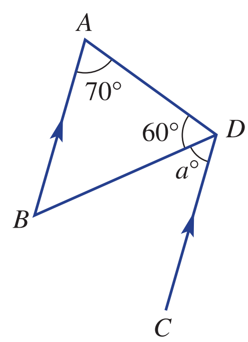

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3(½) 3(½) 3(½)

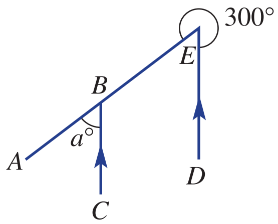
2E

FLUENCY

Example 8

3 Find the value of a in these diagrams.

a



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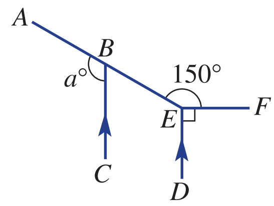

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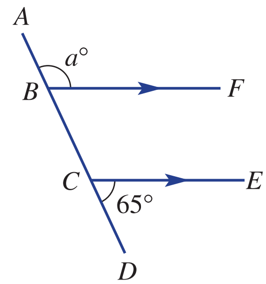

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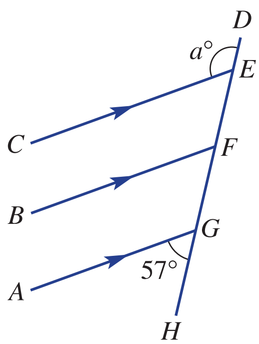

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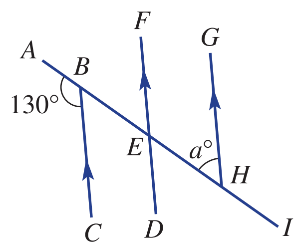

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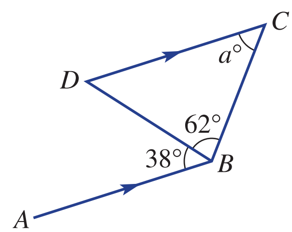
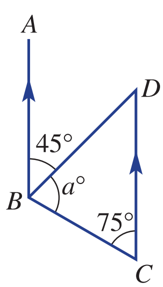
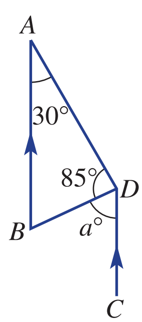


g



h



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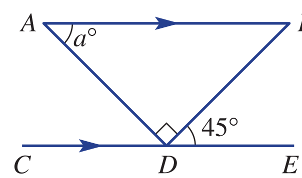

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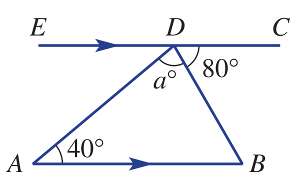

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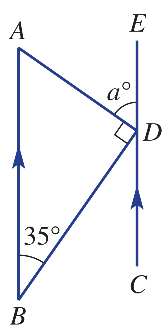

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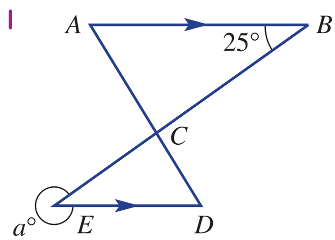

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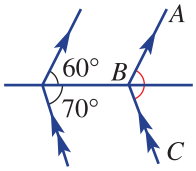
2E

4	4, 5	4, 5

PROBLEM-SOLVING

4 Find the size of $\angle ABC$ in these diagrams.

a


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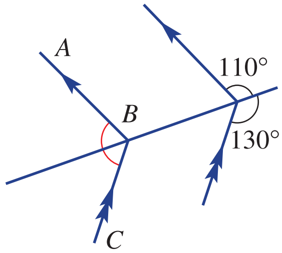

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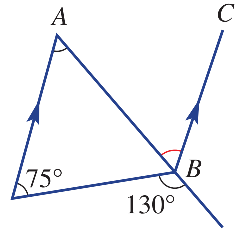

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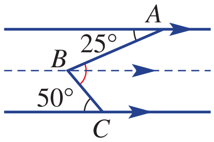

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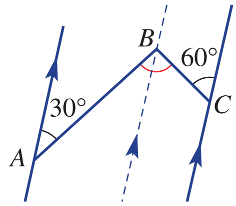

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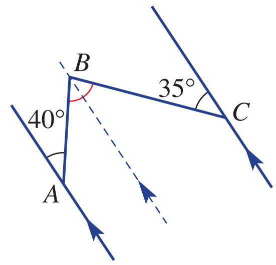
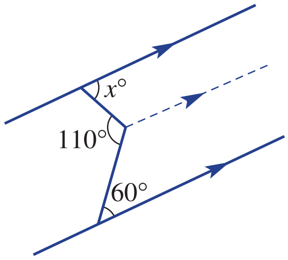
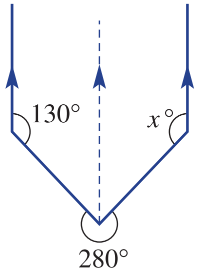
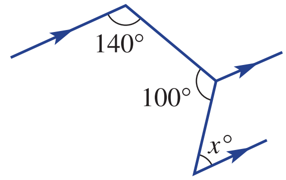


5 Find the value of x in each of these diagrams.

a


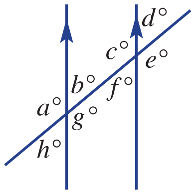
b


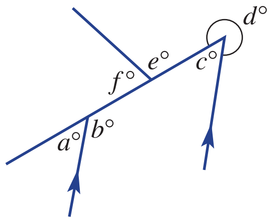
c


6	6, 7	7, 8

REASONING

6 What is the minimum number of angles you need to know to find all the angles marked in these diagrams?

a


b


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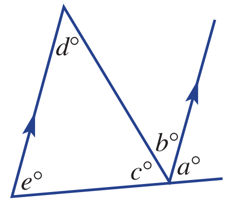

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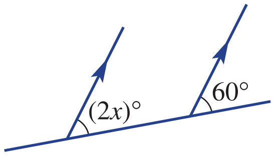
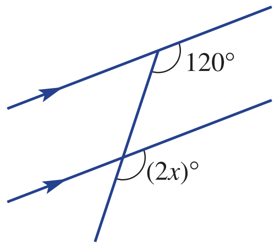
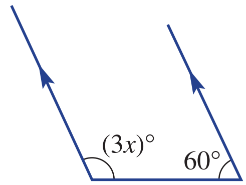
2E

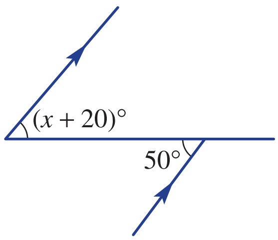
REASONING

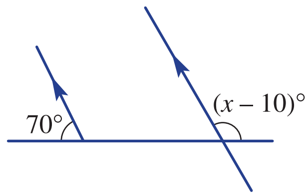
7 In these diagrams, the letter x represents a number and $2x$ means $2 \times x$. Find the value of x.

a


b


c


d


e


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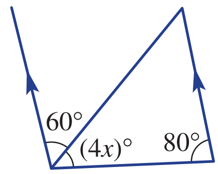
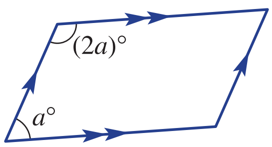


8 Find the value of a in these diagrams.

a


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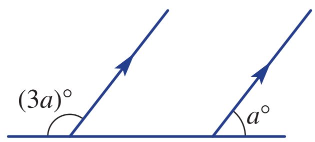

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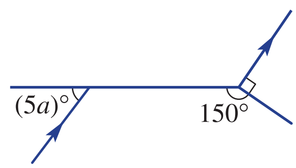
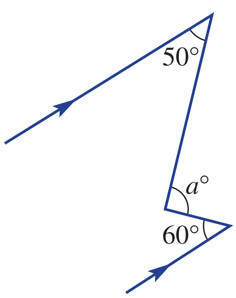


Adding parallel lines	—	—	9

ENRICHMENT

9 Find the value of a in these diagrams. You may wish to add one or more parallel lines to each diagram.

a


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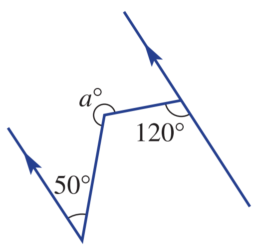

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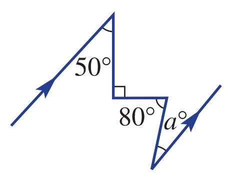
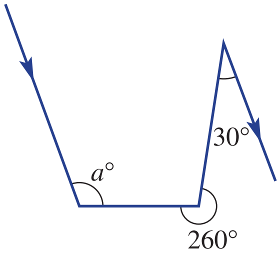
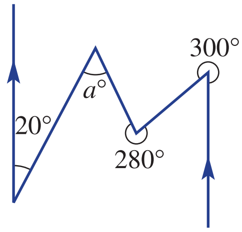


d


e


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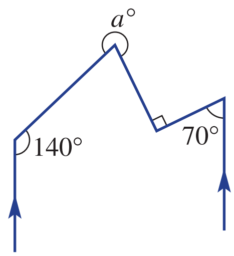

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2F Circles and constructions

Interactive
Widgets
HOTsheets
Walkthroughs

One of the most important characteristics of a circle is that the distance from the centre to the circle, called the radius, is always the same. This fact is critical in the construction of geometrical diagrams and other objects that contain circular parts like gears and wheels.


Gears in a car's gearbox must be circular.

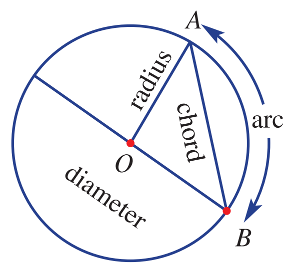
Let's start: Features of a circle

Here is a circle with some common features.

Which of the features (radius, diameter, chord or arc) would change in length if:

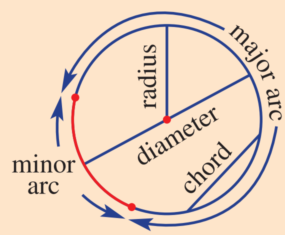
- point A is moved around the circle?
- point B is moved away from O so that the size of the circle changes?

If possible, try constructing this diagram using computer geometry. Measure lengths and drag the points to explore other possibilities.



Key ideas

- Common circle features include:
  - **centre** (point at an equal distance from all points on the circle)
  - **radius** (line interval joining the centre to a point on the circle. Plural: radii)
  - **chord** (line interval joining two points on the circle)
  - **diameter** (longest chord passing through the centre)
  - **arc** (part of a circle). It is possible for a circle to have either a minor or major arc.



- A pair of **compasses** (sometimes called a compass) and a **ruler** can be used to construct geometrical figures precisely.
- The word **bisect** means to cut in half.

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Example 9 Constructing an angle bisector

Use a pair of compasses and a ruler to bisect an angle $\angle AOB$ by following steps **a** to **e**.

a Draw and label an angle $\angle AOB$.

b Construct an arc with centre O so that it cuts OA at point C and OB at point D.

c With the same radius construct an arc with centre C and another with centre D. Ensure these arcs intersect at a point E.

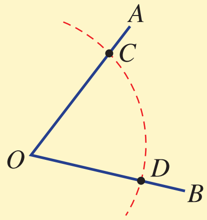
d Mark in the ray OE.

e Measure $\angle AOE$ and $\angle DOE$. What do you notice?

SOLUTION

EXPLANATION

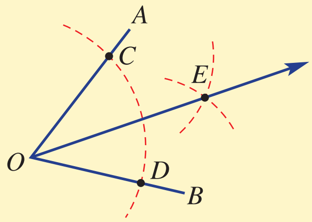
a, b



First, draw an angle $\angle AOB$. The size of the angle is not important.

Construct an arc using O as the centre to produce points C and D.

c, d



Construct E so that the intersecting arcs have the same radius.

Ray OE completes the construction.

e $\angle AOE = \angle BOE$

The angles are equal, so ray OE bisects $\angle AOB$.

Exercise 2F

1–3	3	—

UNDERSTANDING

1 Use a pair of compasses and a ruler to draw a circle with a radius of about 3 cm. Then mark and label these features.

a centre O
b two points, A and B, at any place on the circle
c radius OA
d chord AB
e minor arc AB

2 Use a ruler to draw a segment AB of length 6 cm and then complete the following.

a Construct a circle with radius 3 cm with centre A. (Use a ruler to help set the pair of compasses.)

b Construct a circle with radius 3 cm with centre B.

c Do your two circles miss, touch or overlap? Is this what you expected?

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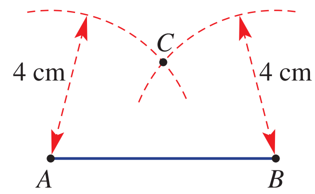
2F

UNDERSTANDING

3 Use a ruler to draw a line segment, AB, of about 5 cm in length.

- **a** Using a pair of compasses, construct arcs with radius 4 cm, as shown, using:
  - **i** centre A
  - **ii** centre B
- **b** Mark point C as shown and use a ruler to draw the segments:
  - **i** AC
  - **ii** BC
- **c** Measure the angles $\angle BAC$ and $\angle ABC$. What do you notice?



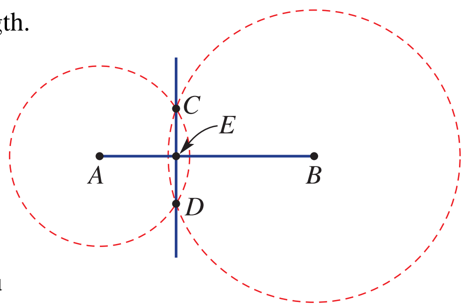
4, 5	4–6	4–6

FLUENCY

Example 9

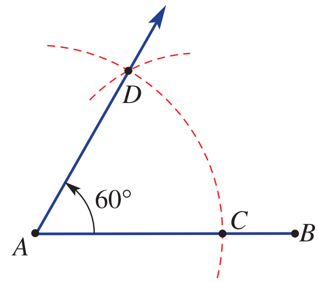
4 Follow steps **a** to **e** to construct a perpendicular line.

- **a** Draw a line segment, AB, of about 5 cm in length.
- **b** Construct overlapping circles of different sizes using the two centres A and B.
- **c** Mark the intersecting points of the circles and label these points C and D.
- **d** Draw the line CD and mark the intersection of line CD and segment AB with the point E.
- **e** Measure $\angle AEC$ with a protractor. What do you notice?



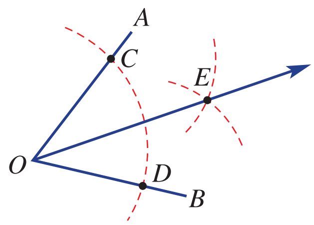
5 Follow steps **a** to **d** to construct a 60° angle.

- **a** Draw a line segment, AB, of about 5 cm in length.
- **b** Construct an arc with centre A and intersecting the segment AB at C.
- **c** With the same radius construct an arc with centre C and intersecting the first arc at D.
- **d** Draw the ray AD and measure $\angle BAD$. What do you notice?



6 Follow steps **a** to **e** to construct an angle bisector.

- **a** Draw any angle and label $\angle AOB$.
- **b** Construct an arc with centre O so that it cuts OA and OB at points C and D.
- **c** With the same radius, construct an arc with centre C and another with centre D. Ensure these arcs intersect at a point, E.
- **d** Mark in the ray OE.
- **e** Measure $\angle AOE$ and $\angle BOE$. What do you notice?



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2F

7	7, 8	8, 9

PROBLEM-SOLVING

7 Consider the construction of the perpendicular line. (See the diagram in Question 4.)

a Explain how to alter the construction so that the point E is the exact midpoint of the segment AB.

b If point E is at the centre of segment AB, then the line CD will be called the perpendicular bisector of segment AB. Complete the full construction to produce a perpendicular bisector.

8 Using the results from Questions 5 and 6, explain how you could construct the angles below. Try each construction and then check each angle with a protractor.

a $30°$ b $15°$

9 Show how you could construct these angles. After each construction, measure the angle using a protractor. (You may wish to use the results from Questions 4 and 6 for help.)

a $45°$ b $22.5°$

10	10	10, 11

REASONING

10 Consider the construction of a perpendicular line. (See the diagram in Question 4.) Do you think it is possible to construct a perpendicular line using circles with radii of any size? Explain.

11 The diagram in Question 6 shows an acute angle, $\angle AOB$.

a Do you think it is possible to bisect an obtuse angle? If so, show how.

b Do you think it is possible to bisect a reflex angle? If so, show how.

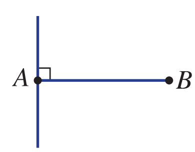
No measurement allowed

—	—	12

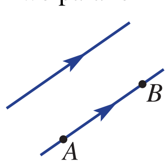
ENRICHMENT

12 Using only a pair of compasses and a ruler's edge, see if you can construct these objects. No measurement is allowed.

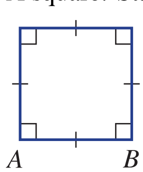
a Perpendicular line at the end point of a segment. Start with segment AB.



b Two parallel lines. Start with line AB.



c A square. Start with segment AB.



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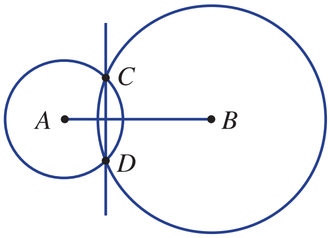
2G Dynamic geometry

Interactive
Widgets
HOTsheets
Walkthroughs

Dynamic computer geometry is an ideal tool for constructing geometrical figures. Constructing with dynamic geometry is like constructing with a ruler and a pair of compasses, but there is the added freedom to drag objects and explore different variations of the same construction. With dynamic geometry the focus is on 'construction' as opposed to 'drawing'. Although this is more of a challenge initially, the results are more precise and allow for greater exploration.

Let's start: The disappearing line



Use computer geometry to construct this figure starting with segment AB.

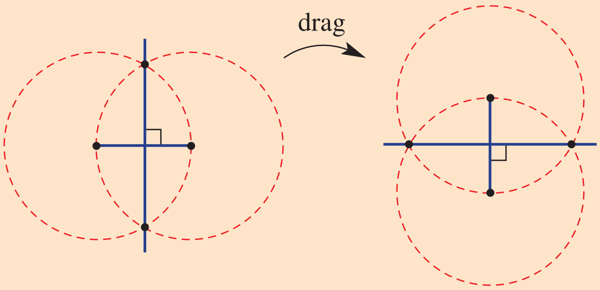
Add the line CD and check that it makes a right angle.

Drag the points A and B or increase the size of the circles.

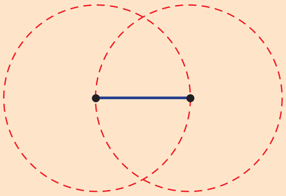
Can you drag point A or B to make the line CD disappear? Why would this happen?

Key ideas

- Using dynamic geometry is like using a pair of compasses and a ruler.
- Objects can be dragged to explore different cases.



- Upon dragging, the geometrical construction should retain the desired properties.
- The same segment can be used to ensure two circles have exactly the same radius.



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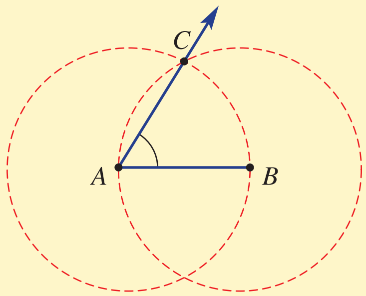
Example 10 Constructing a 60° angle

Construct an angle of 60° using computer geometry. Then drag one of the starting points to check the construction.

SOLUTION



EXPLANATION

Step 1: Construct and label a segment AB.

Step 2: Construct two circles with radius AB and centres A and B.

Step 3: Mark the intersection C and draw the ray AC.

Step 4: Measure $\angle BAC$ to check.

Exercise 2G

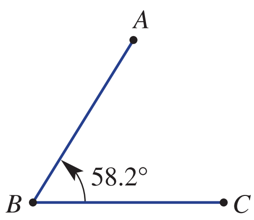
1, 2	2	—

UNDERSTANDING

1 **a** Use computer geometry to construct an angle $\angle ABC$. Any size will do.

b Mark and measure the angle using computer geometry. Drag the point A around B to enlarge the angle. See whether you can form all these types of angles.

i acute **ii** right **iii** straight
iv reflex **v** revolution



2 Look at the 60° angle construction in **Example 10**.

a Why do the two circles have exactly the same radius?

b What other common geometrical object could be easily constructed simply by adding one more segment?

3	3	3

FLUENCY

Example 10

3 Construct each of the following using dynamic geometry. If necessary, refer back to **Section 2F** and **Exercise 2F** to assist you. Check each construction by dragging one of the starting points. All desired properties should be retained.

a perpendicular line
b perpendicular bisector
c 60° angle
d angle bisector

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2G

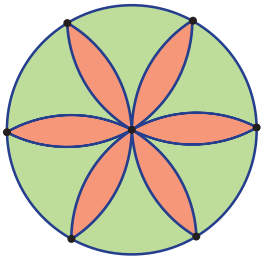
	4	4, 5	5, 6

PROBLEM-SOLVING

4 a Use the 'parallel line' tool to construct a pair of parallel lines and a transversal.
   b Measure the eight angles formed.
   c Drag the transversal to change the size of the angles. Check that:
      i alternate angles are equal
      ii corresponding angles are equal
      iii cointerior angles are always supplementary

5 Use computer geometry to construct these angles. You may wish to use the 'angle bisector' shortcut tool.
   a 30° b 15° c 45°

6 Use computer geometry to construct a six-pointed flower. Then drag one of the starting points to increase or decrease its size.



	7	7, 8	7, 8

REASONING

7 a When using computer geometry it may be necessary to use a full circle instead of an arc. Explain why.
   b When constructing a perpendicular bisector the starting segment AB is used as the radius of the circles. This is instead of two circles with different radii. Explain why.

8 Explain why geometrical construction is a precise process, whereas drawing using measurement is not.

Intricate designs

	—	—	9

ENRICHMENT

9 Sketch your own intricate design or use the internet to find a design that uses circles and lines. Use dynamic geometry to see if it is possible to precisely construct the design. Use colour to enhance your design.

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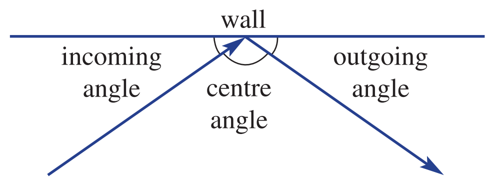
Investigation

The perfect billiard ball path

When a billiard ball bounces off a straight wall (with no side spin) we can assume that the angle at which it hits the wall (incoming angle) is the same as the angle at which it leaves the wall (outgoing angle). This is similar to how light reflects off a mirror.

Single bounce

Use a ruler and protractor to draw a diagram for each part and then answer the questions.

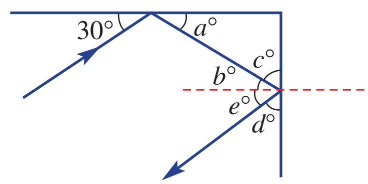


a Find the outgoing angle if:
 i the incoming angle is 30°
 ii the centre angle is 104°

b What geometrical reason did you use to calculate the answer to part **a ii** above?

Two bounces

Two bounces of a billiard ball on a rectangular table are shown here.



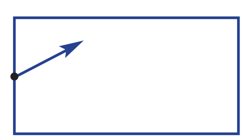
a Find the values of angles a, b, c, d and e, in that order. Give a reason for each.

b What can be said about the incoming angle on the first bounce and the outgoing angle on the second bounce? Give reasons for your answer.

c Accurately draw the path of two bounces using:
 i an initial incoming bounce of 20°
 ii an initial incoming bounce of 55°

More than two bounces

a Draw paths of billiard balls for more than two bounces starting at the midpoint of one side of a rectangular shape, using the starting incoming angles below.
 i 45°
 ii 30°



b Repeat part **a** but use different starting positions. Show accurate diagrams, using the same starting incoming angle but different starting positions.

c Summarise your findings of this investigation in a report that clearly explains what you have found. Show clear diagrams for each part of your report.

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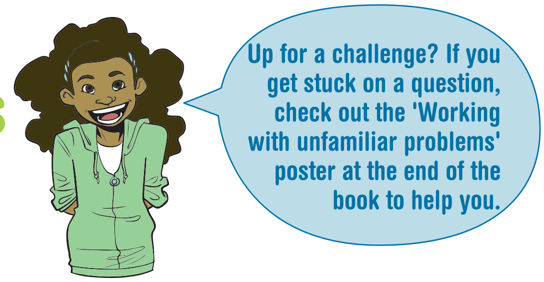
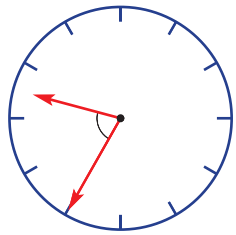
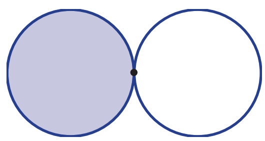
Problems and challenges



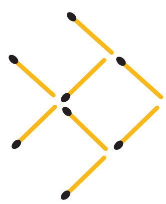
1 What is the angle between the hour hand and minute hand of a clock at 9:35 am?



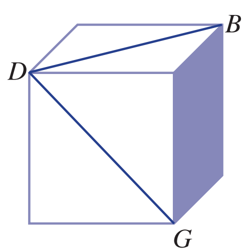
2 Two circles are the same size. The shaded circle rolls around the other circle. How many degrees will it turn before returning to its starting position?



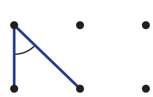
3 Move three matchsticks to turn the fish to face the opposite direction.



4 A cube is shown with diagonals *BD* and *DG* marked. What is the size of angle *BDG*?



5 How many angles of different sizes can you form from joining dots in this 2 by 3 grid? One possible angle (45°) is shown for you. Do not count the 0° or 180° angle or reflex angles outside the grid.



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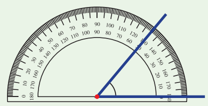
Geometry

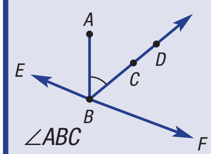
Measuring angles



Angles

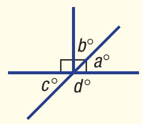
acute 0° – 90°
right 90°
obtuse 90° – 180°
straight 180°
reflex 180° – 360°
revolution 360°

Geometrical objects



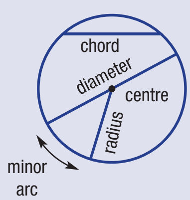
$\angle ABC$
ray *BD*
line *EF*
segment *AB*
collinear points *B*, *C*, *D*
vertex *B*

Angles at a point

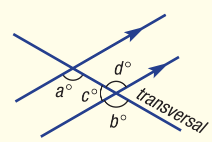


Complementary
$a + b = 90$
Supplementary
$c + d = 180$
Vertically opposite
$a = c$
Revolution
$a + b + 90 + c + d = 360$

Circle features



Parallel lines

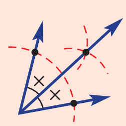


$a = b$ (corresponding)
$a = d$ (alternate)
$a + c = 180$ (cointerior)

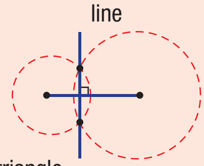
If $a = 120$, $b = 120$,
$d = 120$ and $c = 60$.

Constructions

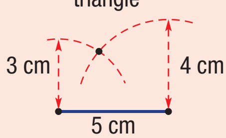
angle bisector



perpendicular line



triangle



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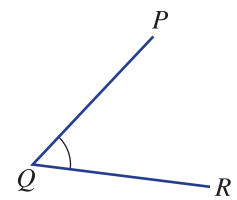
Chapter review

Multiple-choice questions

2A **1** Three points are collinear if:
- **A** they are at right angles
- **B** they form a 60° angle
- **C** they all lie in a straight line
- **D** they are all at the same point
- **E** they form an arc on a circle

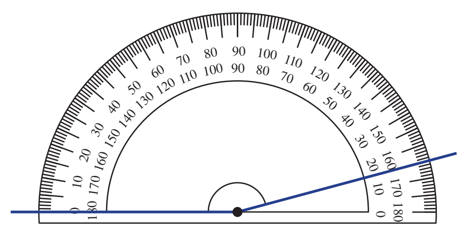
2A **2** The angle shown here can be named:
- **A** $\angle QRP$
- **B** $\angle PQR$
- **C** $\angle QPR$
- **D** $\angle QRR$
- **E** $\angle PQP$



2C **3** Complementary angles:
- **A** sum to 180°
- **B** sum to 270°
- **C** sum to 360°
- **D** sum to 90°
- **E** sum to 45°

2B **4** A reflex angle is:
- **A** 90°
- **B** 180°
- **C** between 180° and 360°
- **D** between 0° and 90°
- **E** between 90° and 180°

2B **5** What is the reading on this protractor?
- **A** 15°
- **B** 30°
- **C** 105°
- **D** 165°
- **E** 195°



2B **6** The angle a minute hand on a clock turns in 20 minutes is:
- **A** 72°
- **B** 36°
- **C** 18°
- **D** 144°
- **E** 120°

2D **7** If a transversal cuts two parallel lines, then:
- **A** cointerior angles are equal
- **B** alternate angles are supplementary
- **C** corresponding angles are equal
- **D** vertically opposite angles are supplementary
- **E** supplementary angles add to 90°

2F **8** An angle bisector:
- **A** cuts an angle in half
- **B** cuts a segment in half
- **C** cuts a line in half
- **D** makes a 90° angle
- **E** makes a 180° angle

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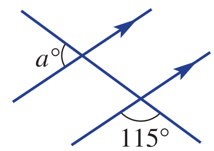
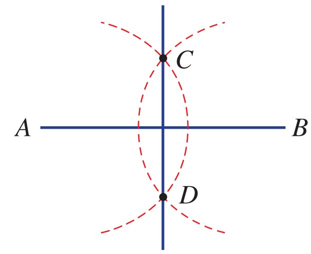
2E **9** The value of a in this diagram is

A 115 **B** 75 **C** 60 **D** 55 **E** 65



2F **10** In this diagram, if line CD is to cut segment AB in half then:

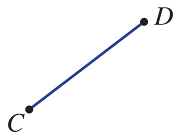
A segment AB has to be 5 cm
B the radii of the arcs must be the same
C the radii of the arcs must not be the same
D line CD should be 10 cm
E AB should be a line not a segment



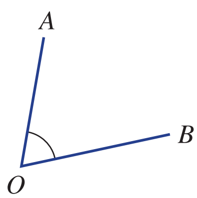
Short-answer questions

2A **1** Name each of these objects.

a

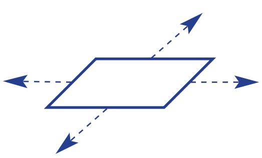


b

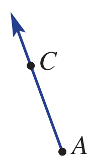


c

d



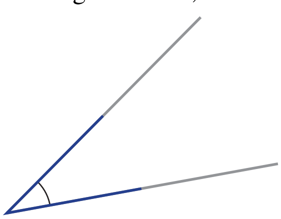
e



f

2B **2** For the angles shown, state the type of angle and measure its size using a protractor.

a



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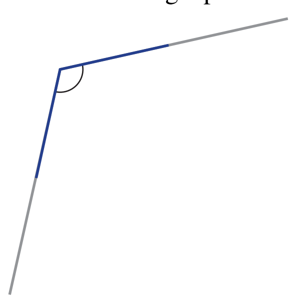

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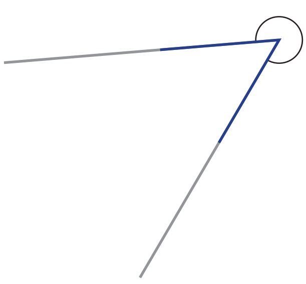

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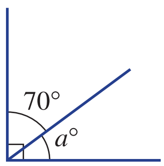
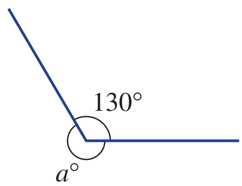
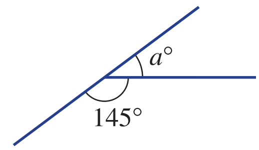
2B **3** Find the angle between the hour and minute hands on a clock at the following times.

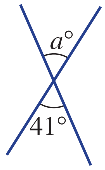
a 6:00 am **b** 9:00 pm **c** 3:00 pm **d** 5:00 am

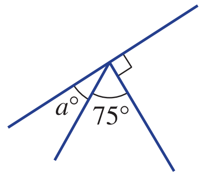
2C **4** Without using a protractor, find the value of a in these diagrams.

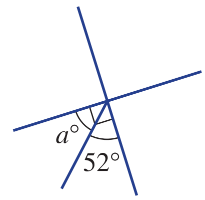
a


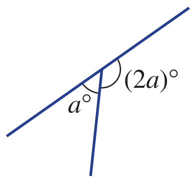
b


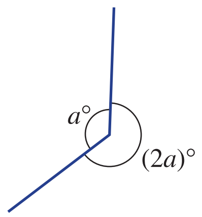
c


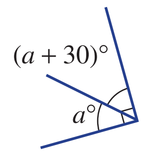
d


e


f


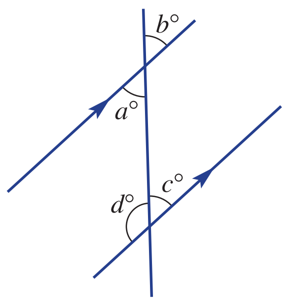
g


h


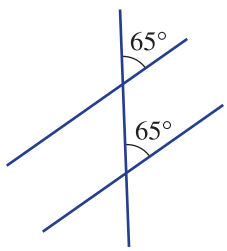
i


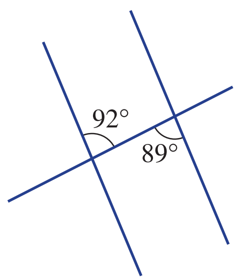
2D **5** Using the pronumerals a, b, c or d given in the diagram, write down a pair of angles that are:

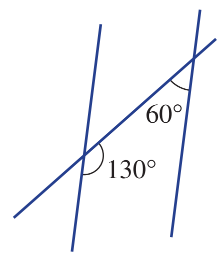
a vertically opposite
b cointerior
c alternate
d corresponding
e supplementary but not cointerior



2D **6** For each of the following, state whether the two lines cut by the transversal are parallel. Give reasons for each answer.

a


b


c


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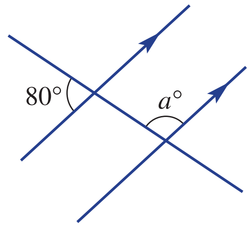
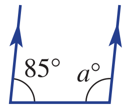
2D **7** Find the value of a in these diagrams.

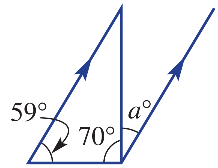
a



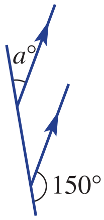
b



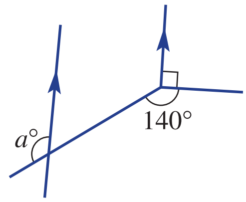
c



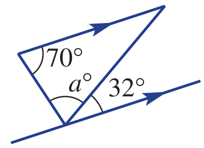
d



e

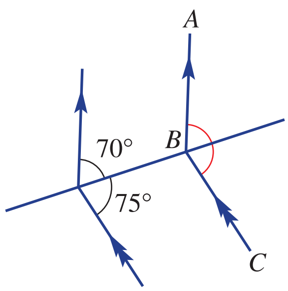


f

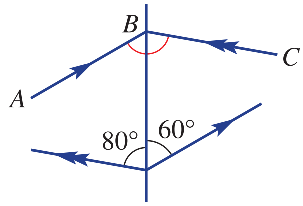


2E **8** Find the size of $\angle ABC$ in these diagrams.

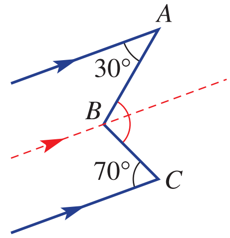
a



b

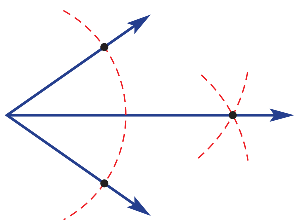


c

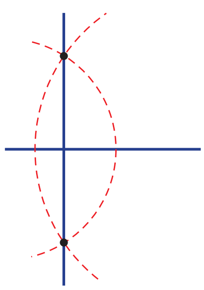


2F **9** Use the diagrams to help draw your own construction. You will need a pair of compasses and a ruler.

a angle bisector (start with any angle size).



b perpendicular line (start with a segment of about 6 cm in length).



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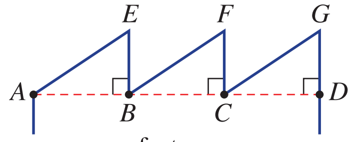
Extended-response questions

1 A factory roof is made up of three sloping sections. The sloping sections are all parallel and the upright supports are at 90° to the horizontal, as shown. Each roof section makes a 32° angle (or pitch) with the horizontal.



a State the size of each of these angles.
   i $\angle EAB$ ii $\angle GCD$ iii $\angle ABF$ iv $\angle EBF$

b Complete these sentences.
   i $\angle BAE$ is ______________________________ to $\angle CBF$.
   ii $\angle FBC$ is ______________________________ to $\angle GCB$.
   iii $\angle BCG$ is ______________________________ to $\angle GCD$.

c Solar panels are to be placed on the sloping roofs and it is decided that the angle to the horizontal is to be reduced by 11°. Find the size of these new angles.
   i $\angle FBC$ ii $\angle FBA$ iii $\angle FCG$

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2 A circular birthday cake is cut into pieces of equal size, cutting from the centre outwards. Each cut has an angle of a° at the centre.
Tanya's family takes four pieces. George's family takes three pieces.
Sienna's family takes two pieces. Anita's family takes two pieces.
Marcus takes one piece.

- **a** How many pieces were taken all together?
- **b** If there is no cake left after all the pieces are taken, find the value of a.
- **c** Find the value of a if:
  - **i** half of the cake still remains
  - **ii** one-quarter of the cake still remains
  - **iii** one-third of the cake still remains
  - **iv** one-fifth of the cake still remains

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