Computational and Numerical Methods

Group 16

Set 4 (20-08-2018): The Newton-Raphson Method

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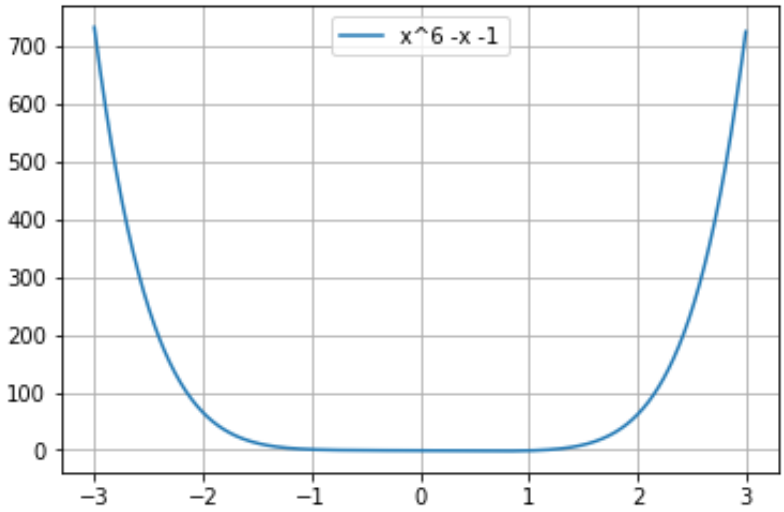
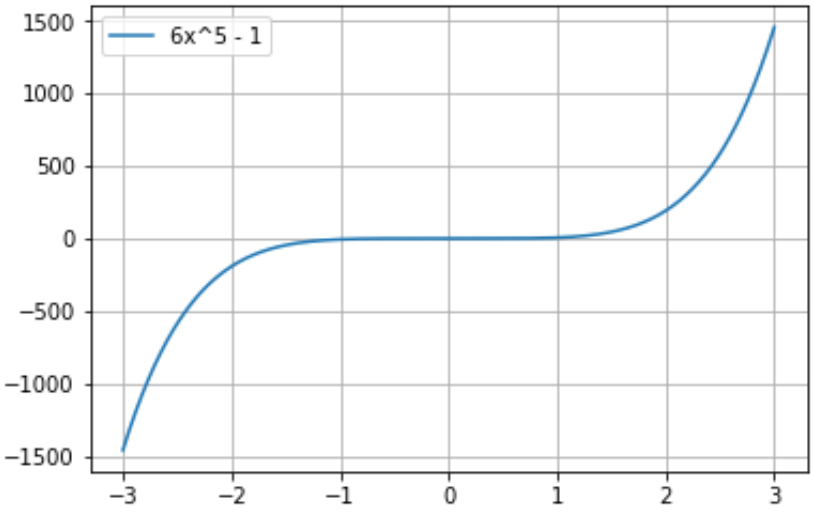
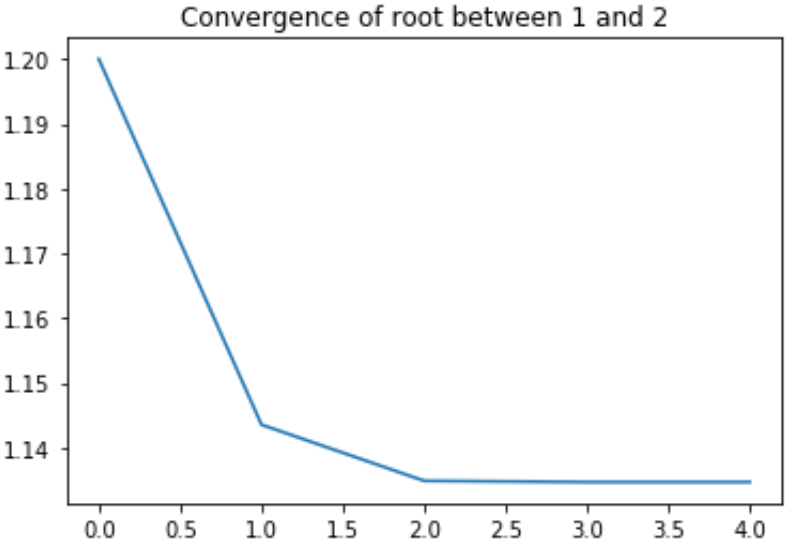
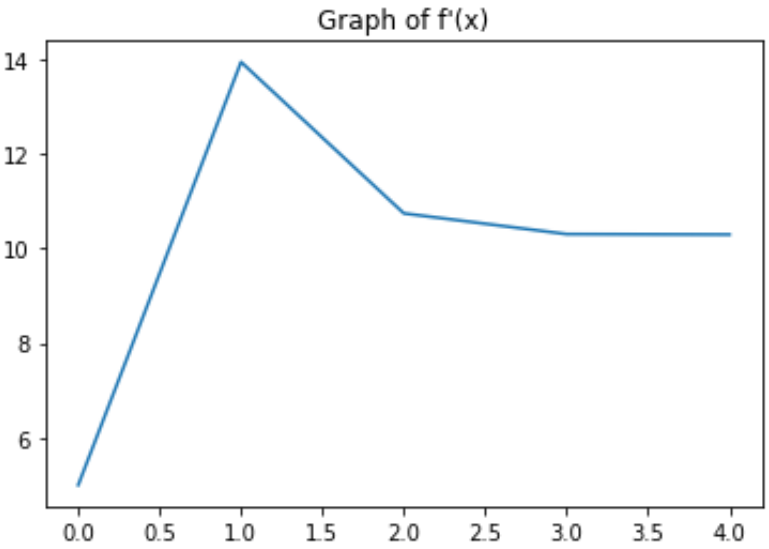
Show Code

Write a code, applying the algorithm of the Newton-Raphson method to determine both the real roots of $f(x) = x^6 - x - 1 = 0.$





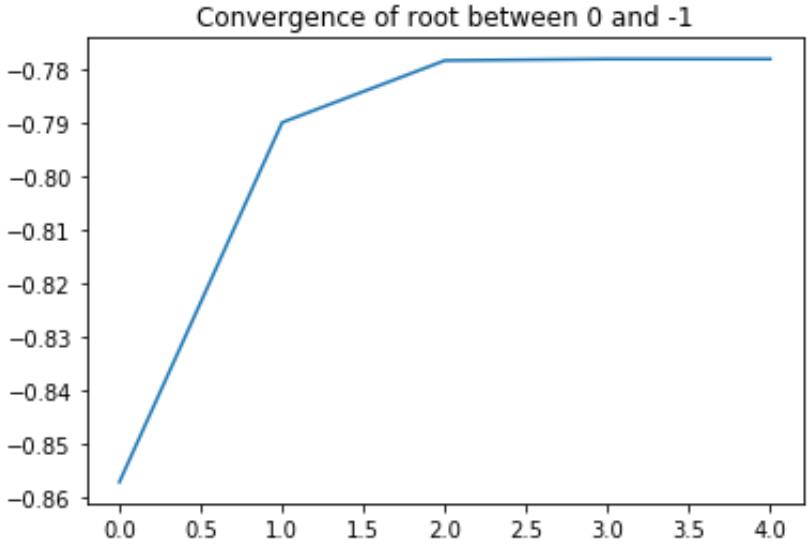


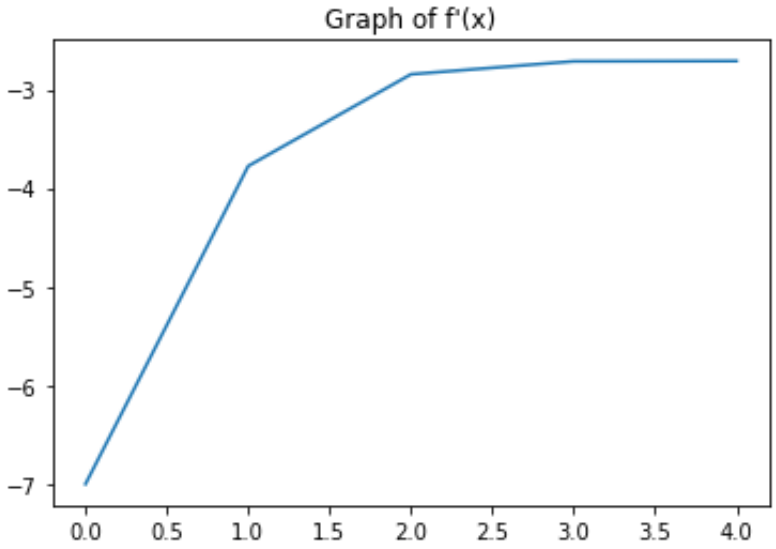



```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 6
```

Out[2]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.000000	-1.000000e+00	5.000000	0.000000	1.200000	2.000000e-01
1	1.0	1.200000	7.859840e-01	13.929920	0.200000	1.143576	-5.642416e-02
2	2.0	1.143576	9.303196e-02	10.734811	-0.056424	1.134909	-8.666380e-03
3	3.0	1.134909	1.907397e-03	10.296849	-0.008666	1.134724	-1.852409e-04
4	4.0	1.134724	8.537194e-07	10.287633	-0.000185	1.134724	-8.298502e-08





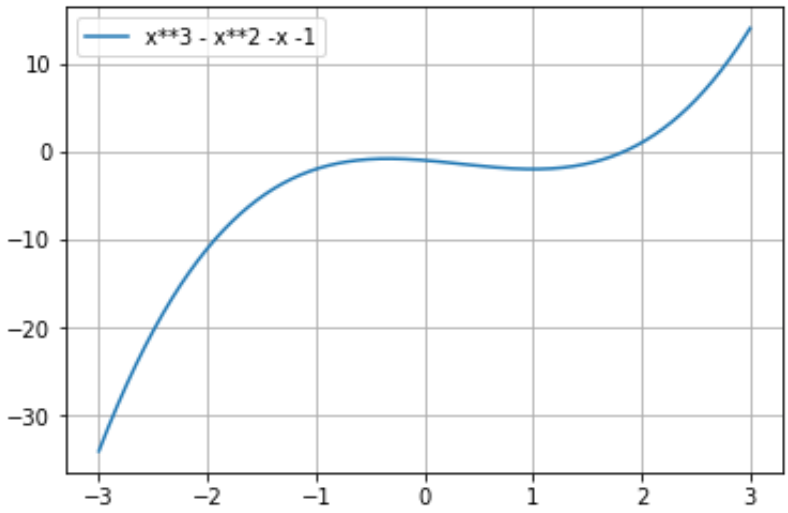

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 6
```

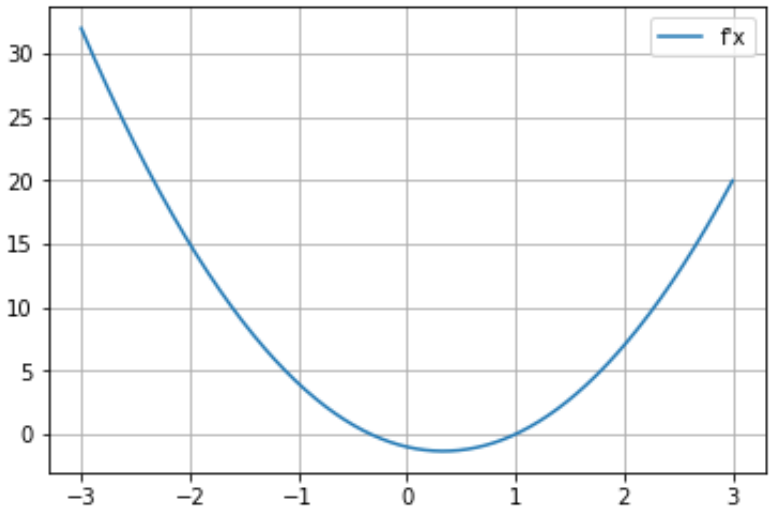
Out[3]:

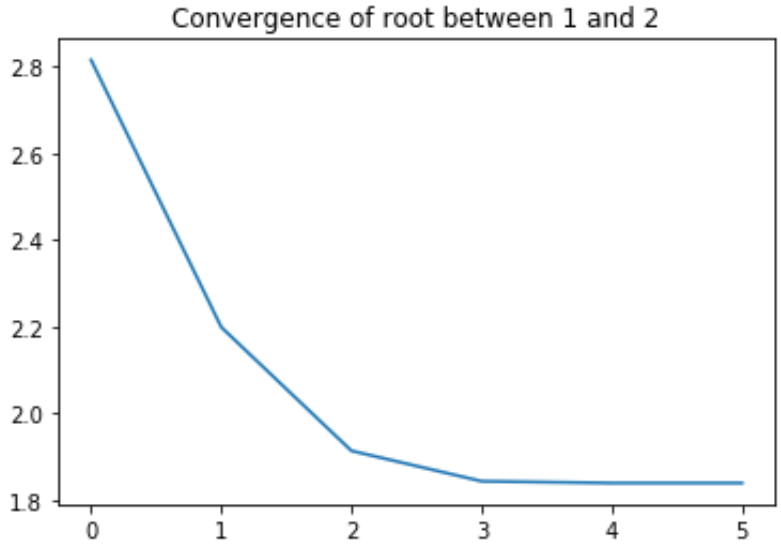
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	-1.000000	1.000000e+00	-7.000000	0.000000	-0.857143	1.428571e-01
1	1.0	-0.857143	2.537123e-01	-3.775986	0.142857	-0.789952	6.719101e-02
2	2.0	-0.789952	3.295042e-02	-2.845671	0.067191	-0.778373	1.157914e-02
3	3.0	-0.778373	7.680138e-04	-2.714310	0.011579	-0.778090	2.829499e-04
4	4.0	-0.778090	4.406060e-07	-2.711196	0.000283	-0.778090	1.625135e-07

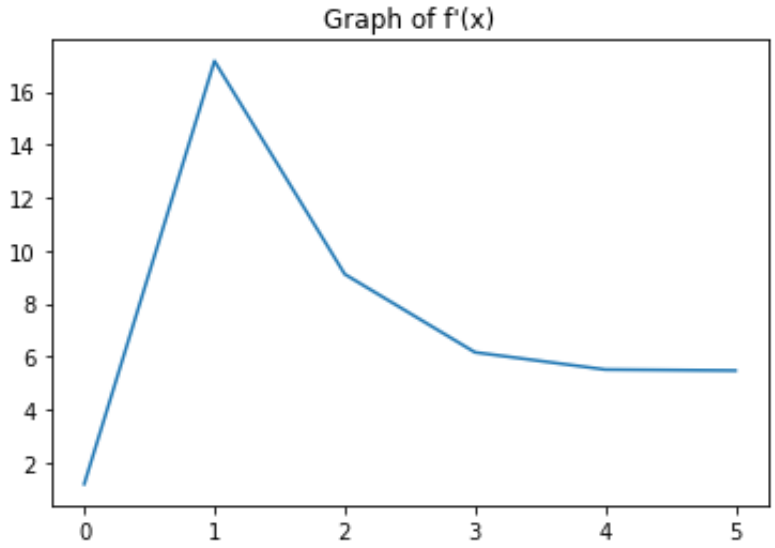
Use the Newton Raphson method to find the real roots of the following functions, using an error tolerance of $\epsilon = 0.0001$.

$A. x^3 - x^2 - x - 1 = 0$







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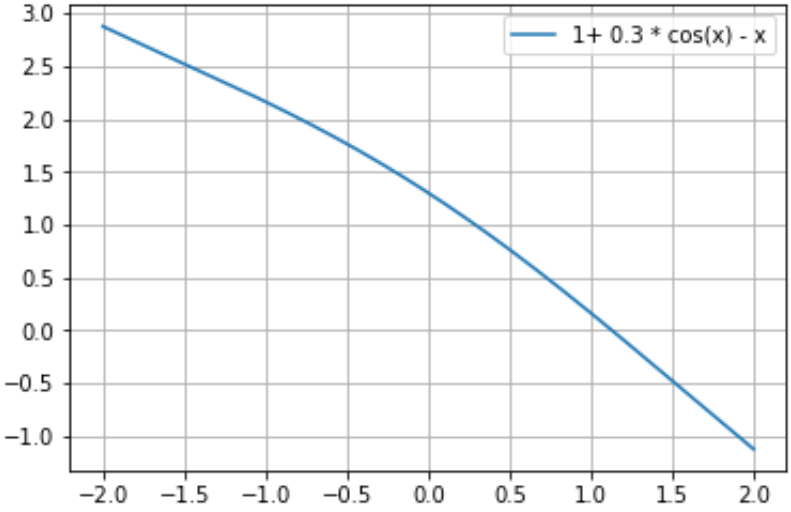
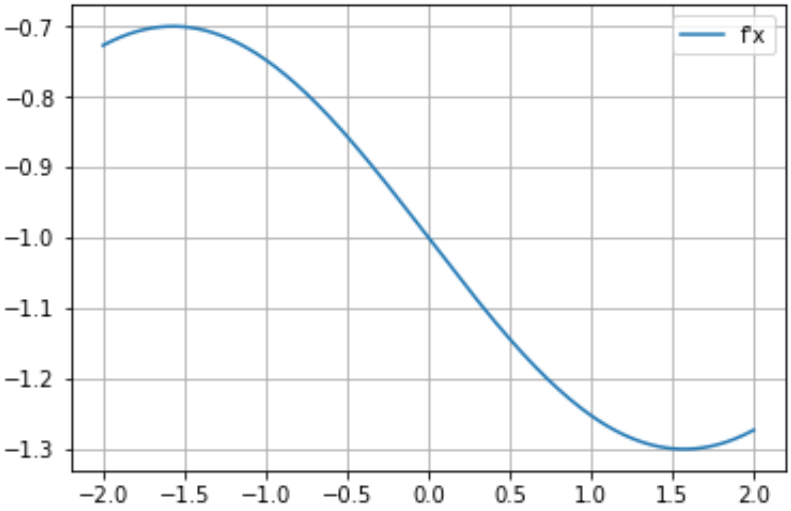
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 7
```

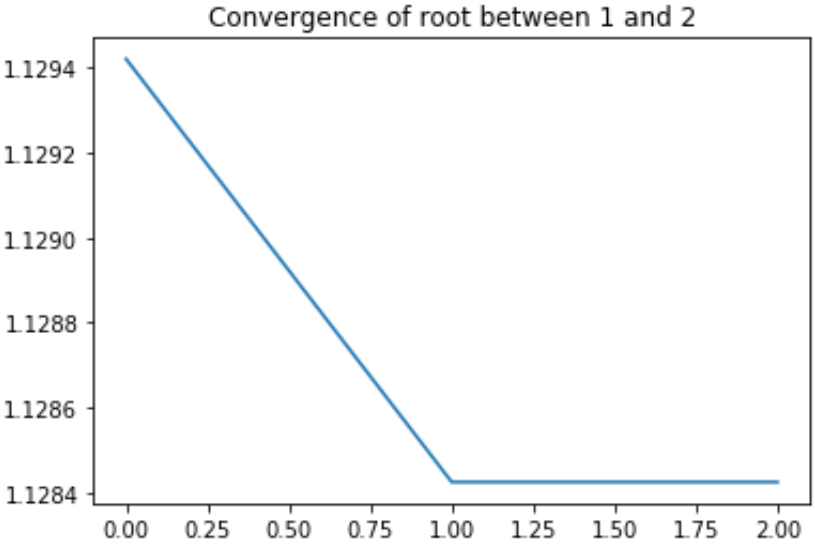
Out[4]:

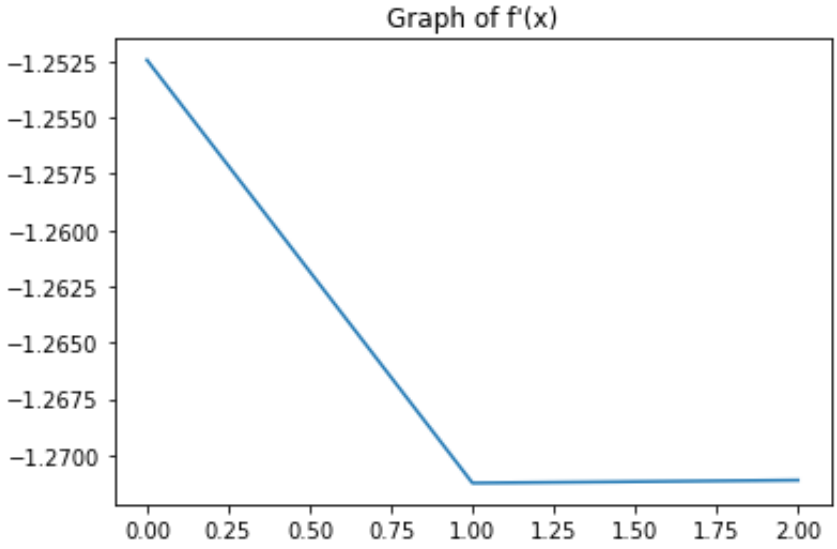
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.250000	-1.859375	1.187500	0.000000	2.815789	1.565789
1	1.0	2.815789	10.581007	17.154432	1.565789	2.198981	-0.616809
2	2.0	2.198981	2.598708	9.108585	-0.616809	1.913677	-0.285303
3	3.0	1.913677	0.432356	6.159128	-0.285303	1.843480	-0.070198
4	4.0	1.843480	0.023017	5.508293	-0.070198	1.839301	-0.004179
5	5.0	1.839301	0.000079	5.470484	-0.004179	1.839287	-0.000014

$$B.\, x = 1 + 0.3cosx$$


1+ 0.3 * cos(x) - x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


f'x
−0.7
−0.8
−0.9
−1.0
−1.1
−1.2
−1.3
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


Convergence of root between 1 and 2
1.1294
1.1292
1.1290
1.1288
1.1286
1.1284
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

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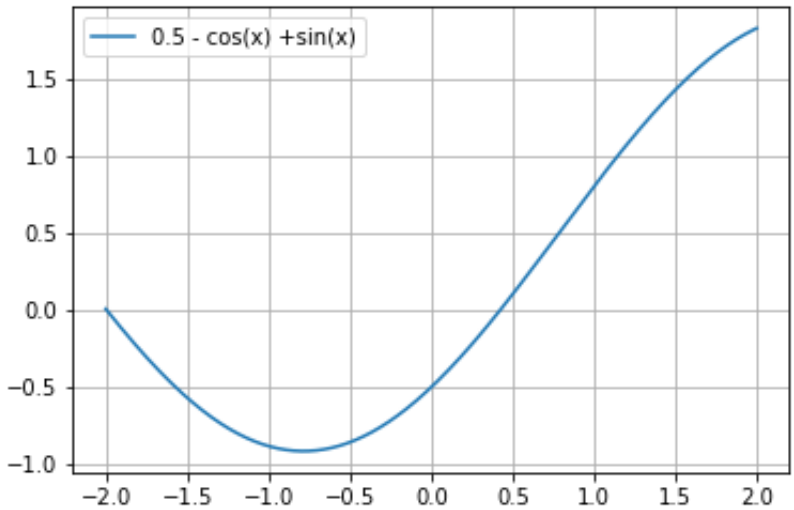
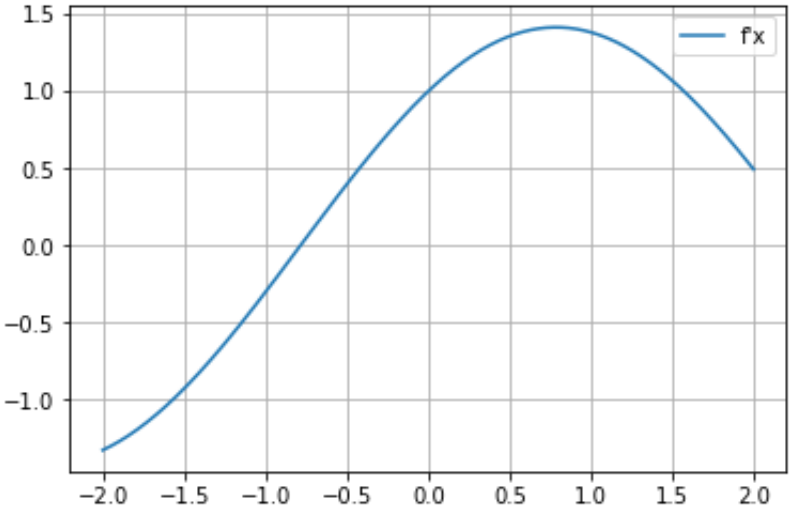
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 4
```

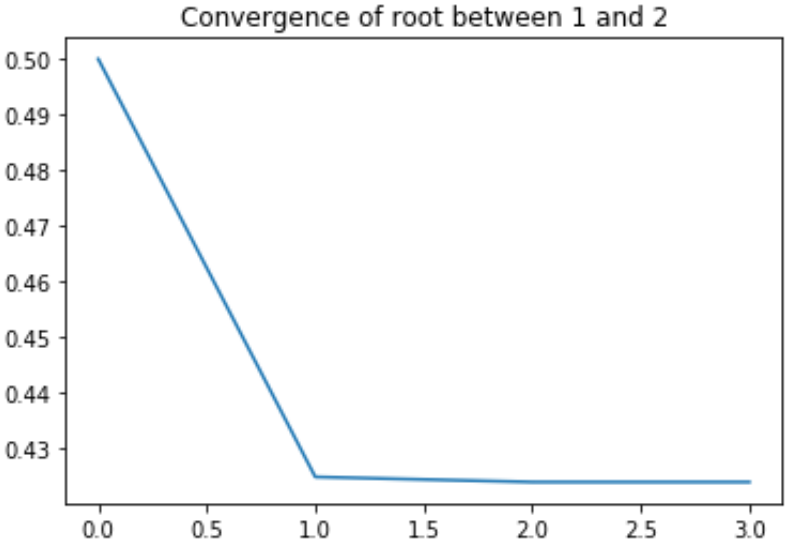
Out[5]:

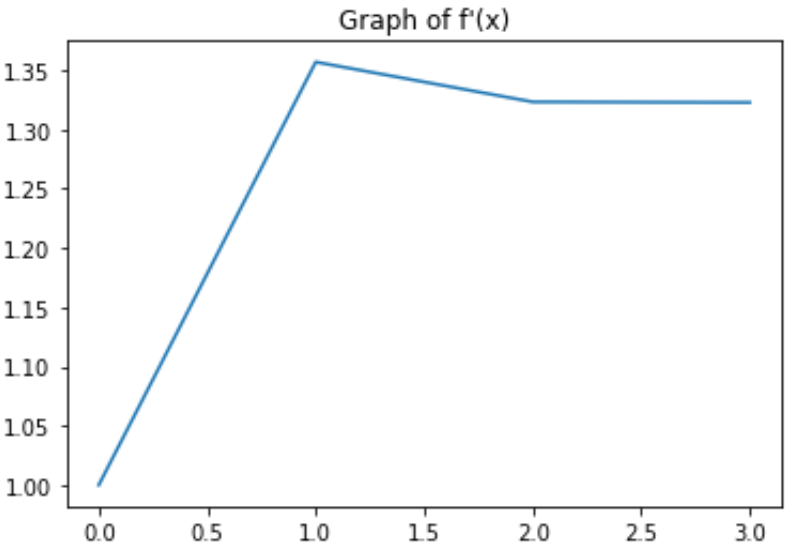
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.000000	1.620907e-01	-1.252441	0.000000	1.129420	1.294198e-01
1	1.0	1.129420	-1.264447e-03	-1.271249	0.129420	1.128425	-9.946490e-04
2	2.0	1.128425	-6.343822e-08	-1.271122	-0.000995	1.128425	-4.990728e-08

$$C.\ cosx = 0.5 + sinx$$







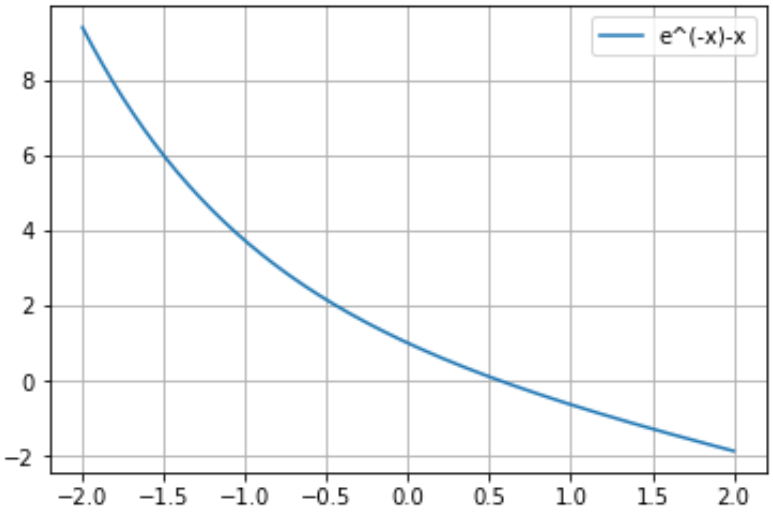


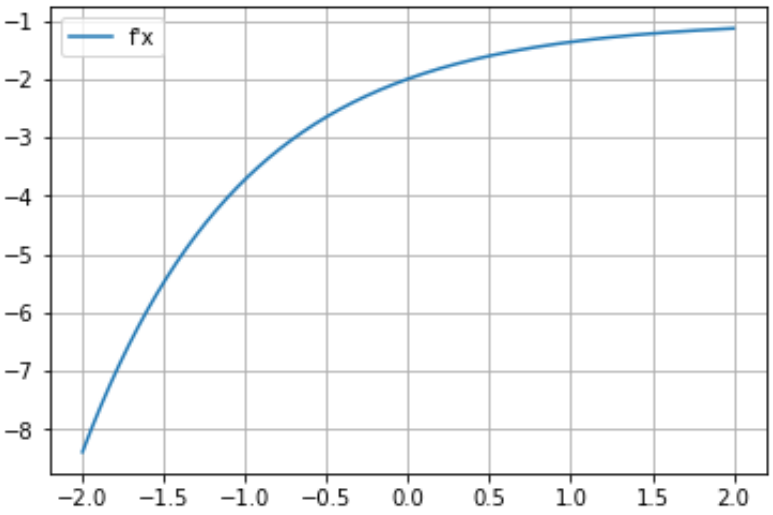

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 5
```

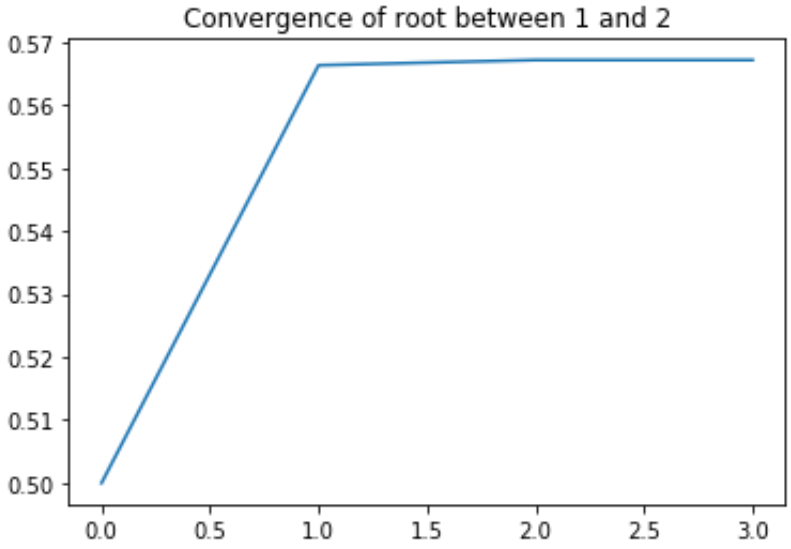
Out[6]:

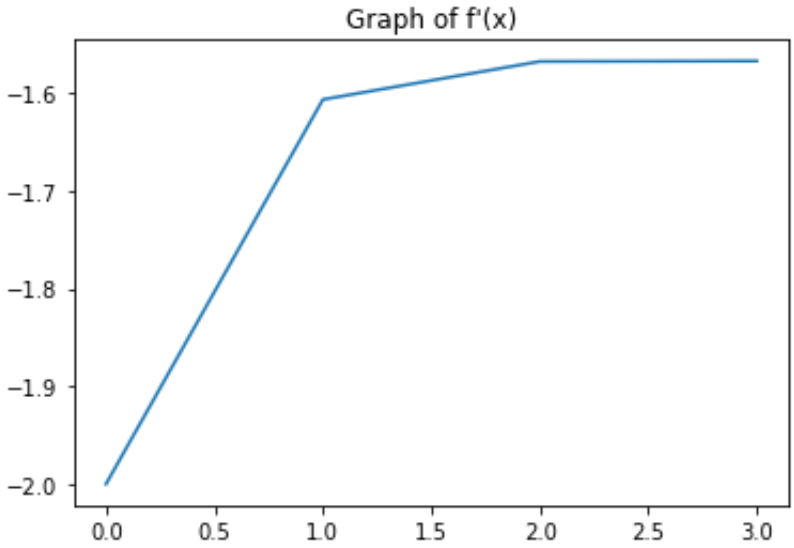
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.000000	-5.000000e-01	1.000000	0.000000	0.500000	5.000000e-01
1	1.0	0.500000	1.018430e-01	1.357008	0.500000	0.424950	-7.504965e-02
2	2.0	0.424950	1.216351e-03	1.323335	-0.075050	0.424031	-9.191557e-04
3	3.0	0.424031	2.108693e-07	1.322876	-0.000919	0.424031	-1.594022e-07

$$D.\, x = e^{-x}$$







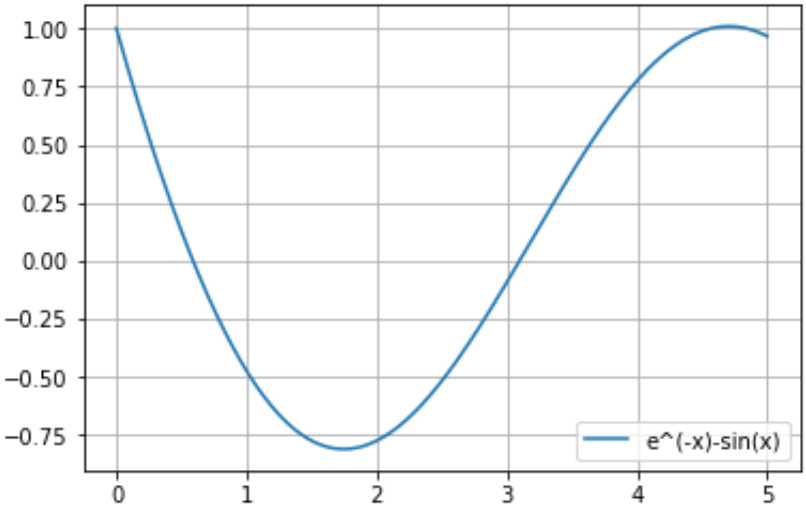


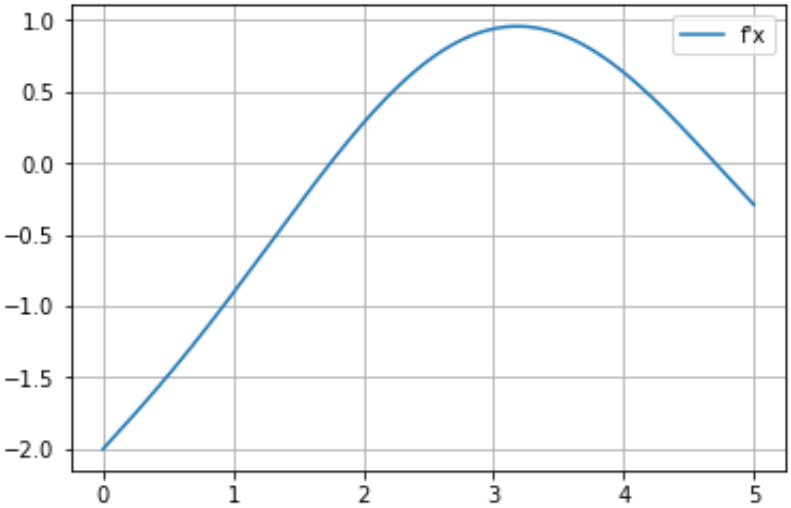

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 5
```

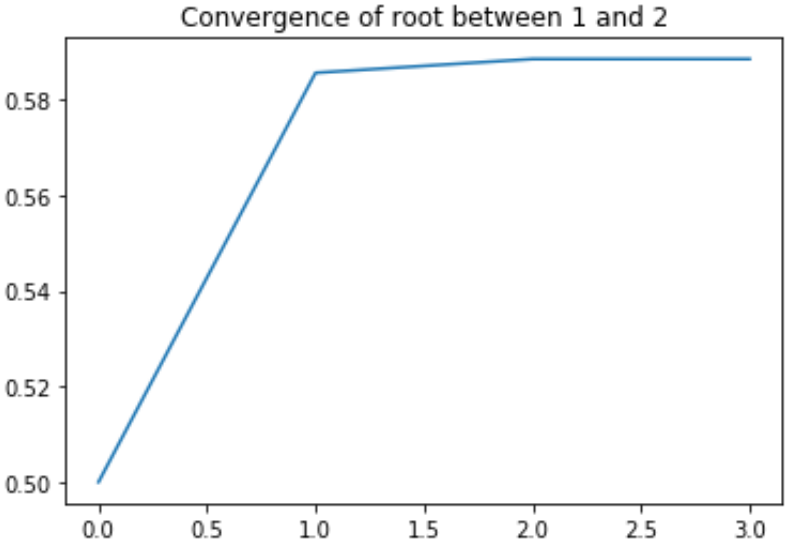
Out[7]:

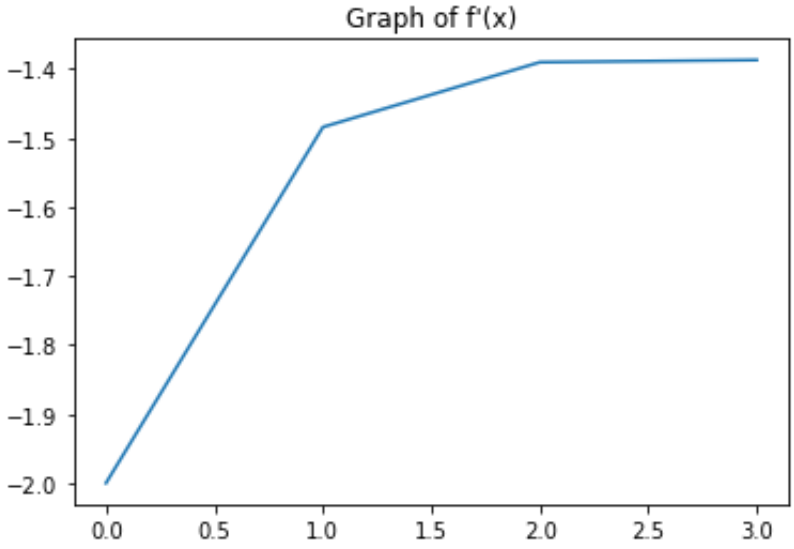
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.000000	1.000000e+00	-2.000000	0.000000	0.500000	5.000000e-01
1	1.0	0.500000	1.065307e-01	-1.606531	0.500000	0.566311	6.631100e-02
2	2.0	0.566311	1.304510e-03	-1.567616	0.066311	0.567143	8.321618e-04
3	3.0	0.567143	1.964805e-07	-1.567143	0.000832	0.567143	1.253749e-07

E. The two smallest positive roots of $e^{-x} = sinx$


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0
1
2
3
4
5
e^(-x)-sin(x)


1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
1
2
3
4
5
f'x

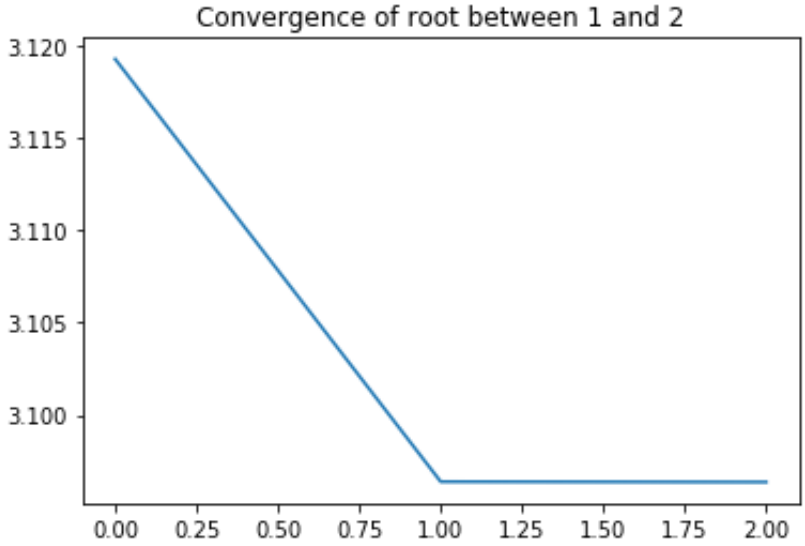

Convergence of root between 1 and 2
0.58
0.56
0.54
0.52
0.50
0.0
0.5
1.0
1.5
2.0
2.5
3.0

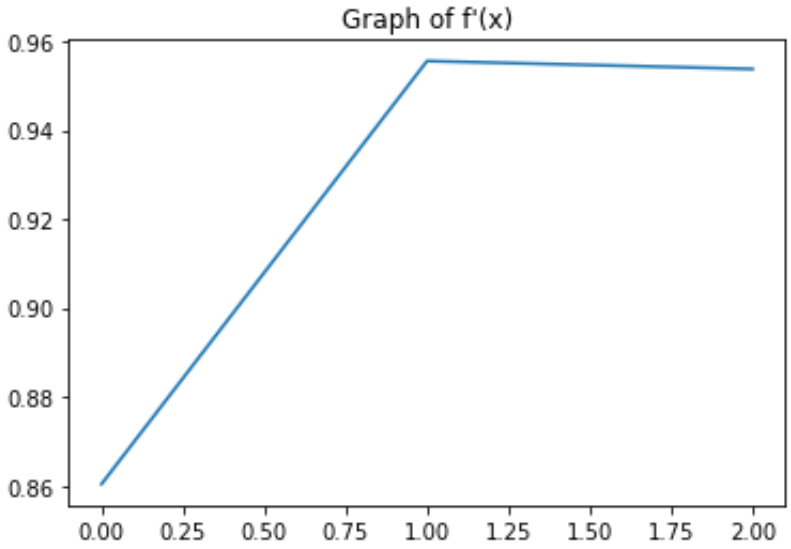



```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 5
```

Out[8]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.000000	1.000000	-2.000000	0.000000	0.500000	0.500000
1	1.0	0.500000	0.127105	-1.484113	0.500000	0.585644	0.085644
2	2.0	0.585644	0.004011	-1.390104	0.085644	0.588529	0.002886
3	3.0	0.588529	0.000005	-1.386901	0.002886	0.588533	0.000003



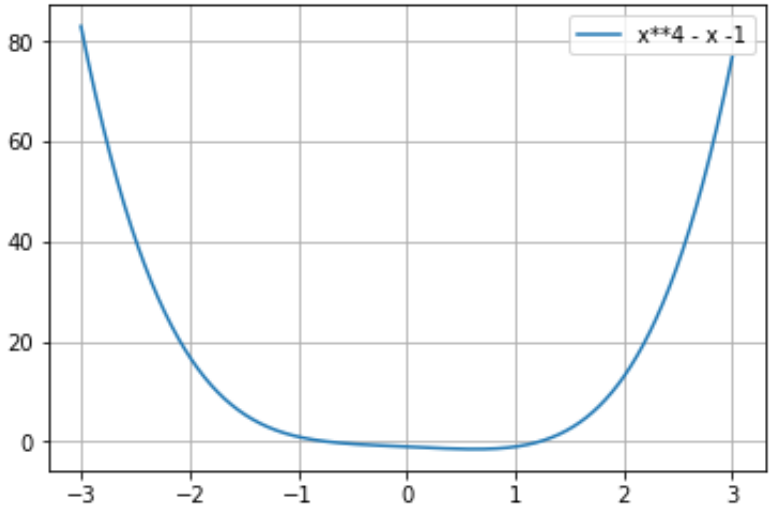


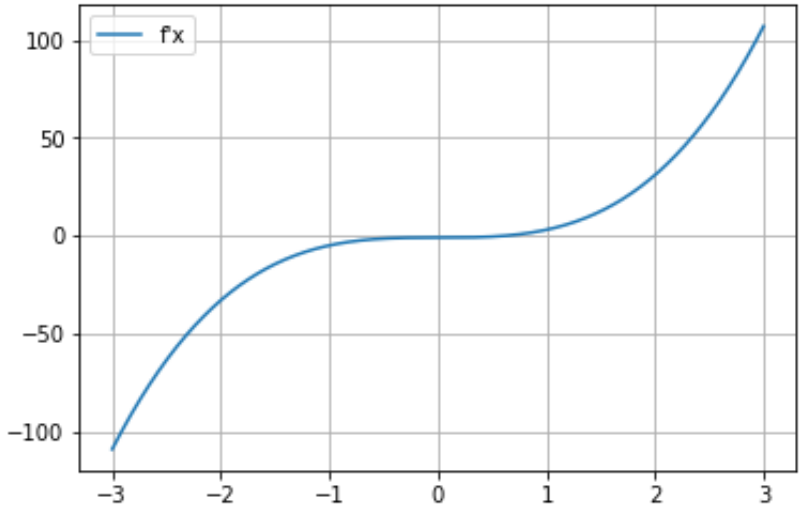

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 4
```

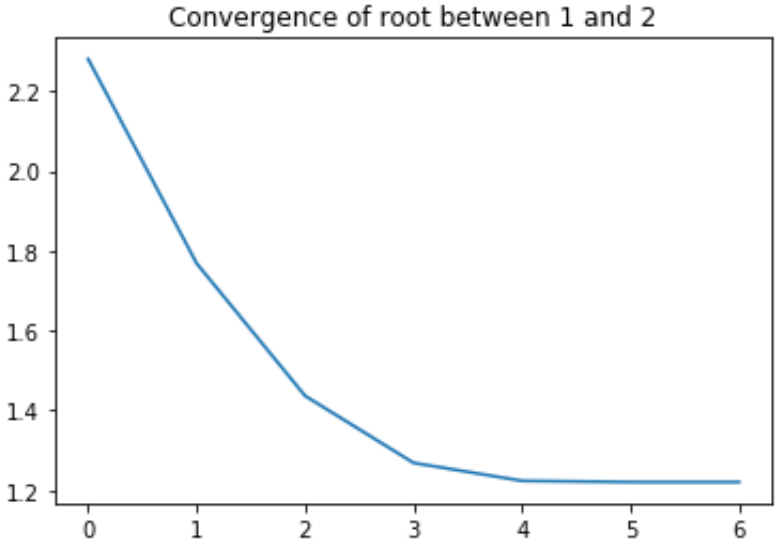
Out[9]:

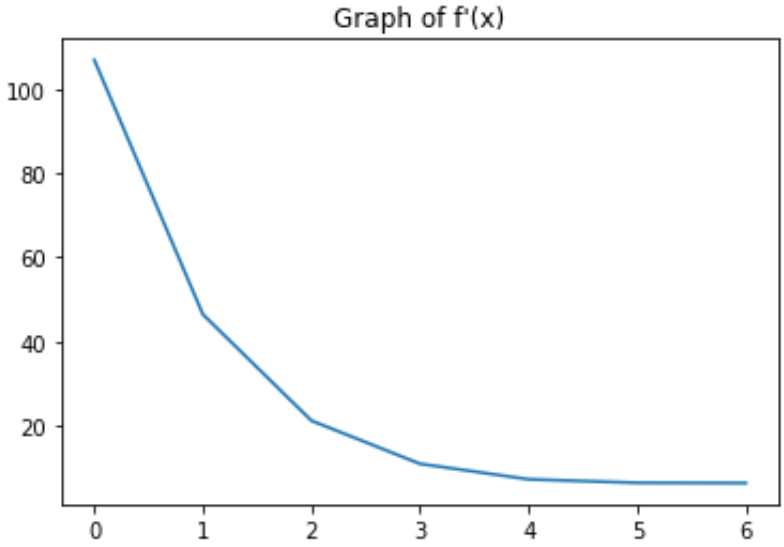
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	2.750000	-0.317733	0.860375	0.000000	3.119296	0.369296
1	1.0	3.119296	0.021894	0.955563	0.369296	3.096384	-0.022912
2	2.0	3.096384	0.000020	0.953766	-0.022912	3.096364	-0.000020

G. All real roots of $x^4 - x - 1 = 0$


x**4 - x -1
80
60
40
20
0
−3
−2
−1
0
1
2
3


f'x
100
50
0
−50
−100
−3
−2
−1
0
1
2
3


Convergence of root between 1 and 2
2.2
2.0
1.8
1.6
1.4
1.2
0
1
2
3
4
5
6

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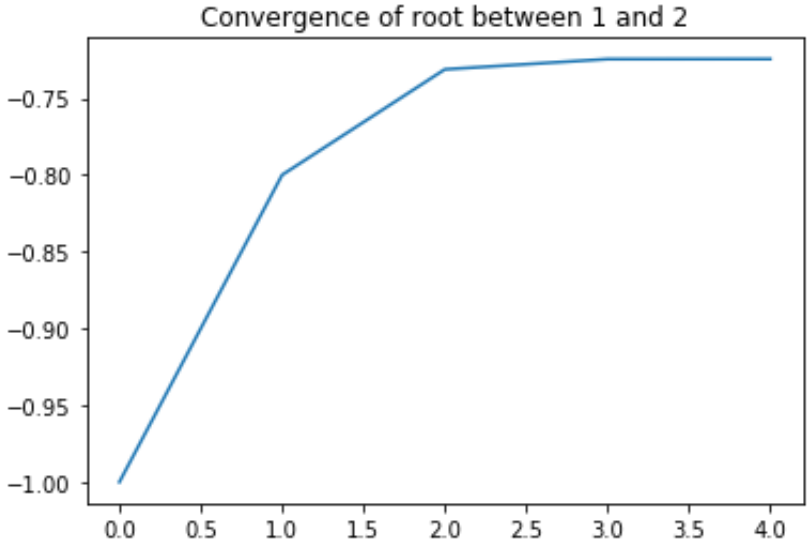
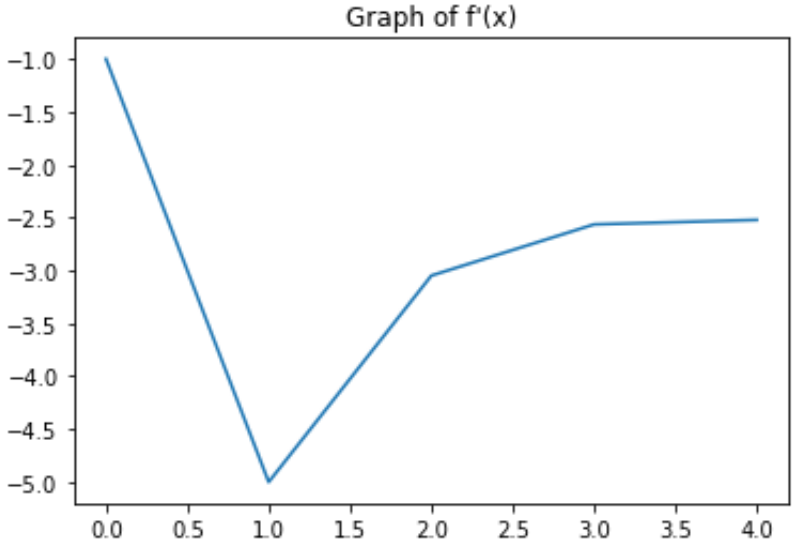
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 8
```

Out[10]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	3.000000	77.000000	107.000000	0.000000	2.280374	-0.719626
1	1.0	2.280374	23.760716	46.432732	-0.719626	1.768650	-0.511723
2	2.0	1.768650	7.016511	21.130233	-0.511723	1.436590	-0.332060
3	3.0	1.436590	1.822644	10.859288	-0.332060	1.268748	-0.167842
4	4.0	1.268748	0.322457	7.169327	-0.167842	1.223771	-0.044977
5	5.0	1.223771	0.019081	6.330952	-0.044977	1.220757	-0.003014
6	6.0	1.220757	0.000081	6.276922	-0.003014	1.220744	-0.000013



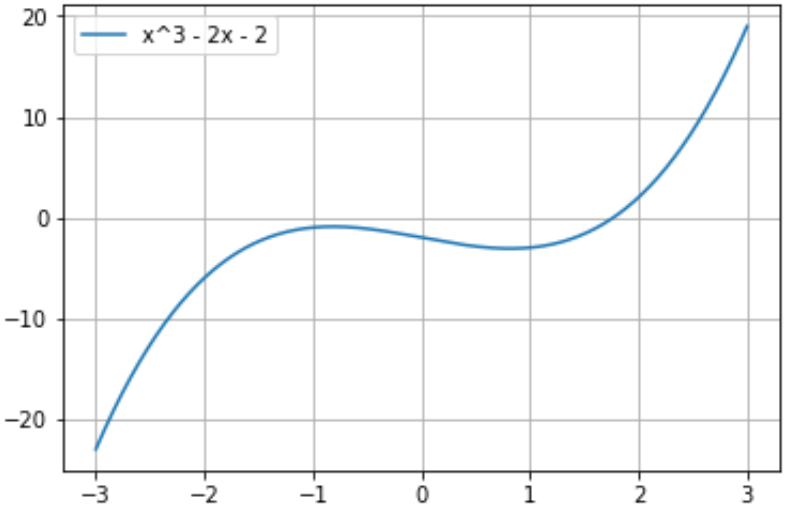


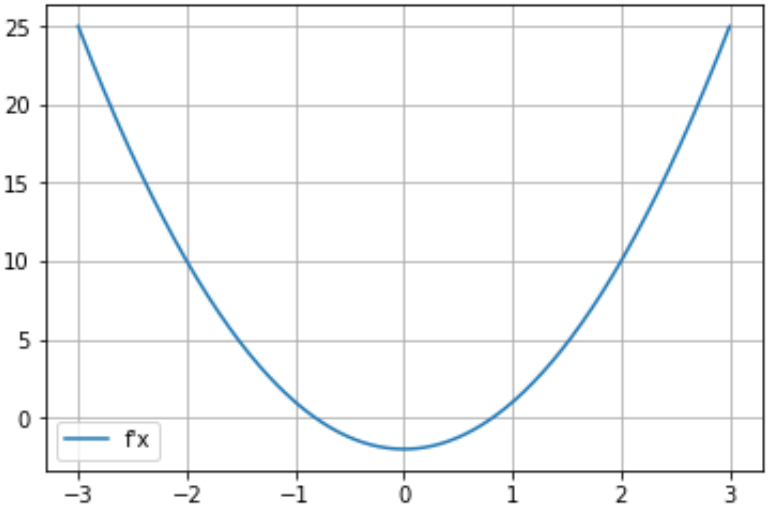

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 6
```

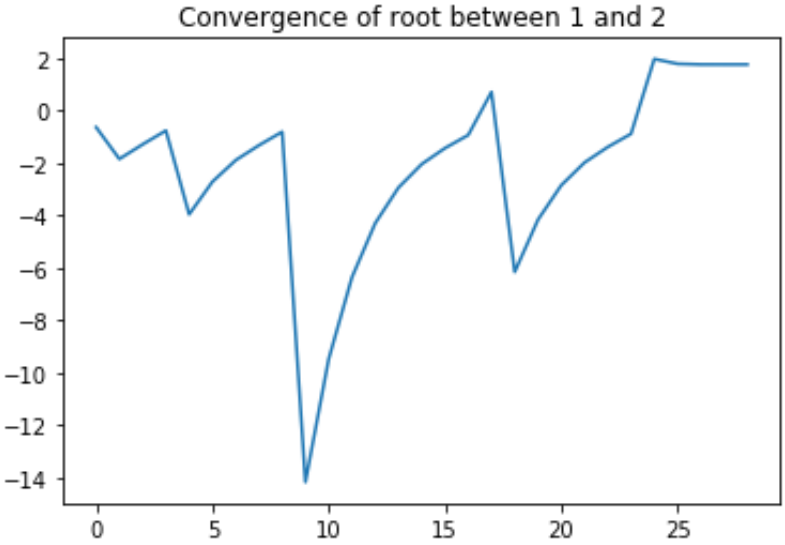
Out[11]:

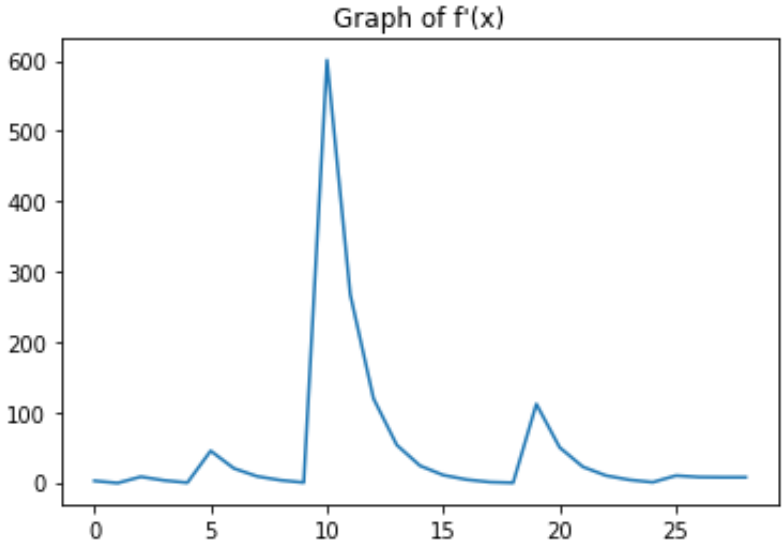
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.000000	-1.000000	-1.000000	0.000000	-1.000000	-1.000000
1	1.0	-1.000000	1.000000	-5.000000	-1.000000	-0.800000	0.200000
2	2.0	-0.800000	0.209600	-3.048000	0.200000	-0.731234	0.068766
3	3.0	-0.731234	0.017140	-2.563970	0.068766	-0.724548	0.006685
4	4.0	-0.724548	0.000143	-2.521466	0.006685	-0.724492	0.000057

$$F.x^3 - 2x - 2 = 0$$


x^3 - 2x - 2
20
10
0
−10
−20
−3
−2
−1
0
1
2
3


f'x
25
20
15
10
5
0
−3
−2
−1
0
1
2
3


Convergence of root between 1 and 2
2
0
−2
−4
−6
−8
−10
−12
−14
0
5
10
15
20
25

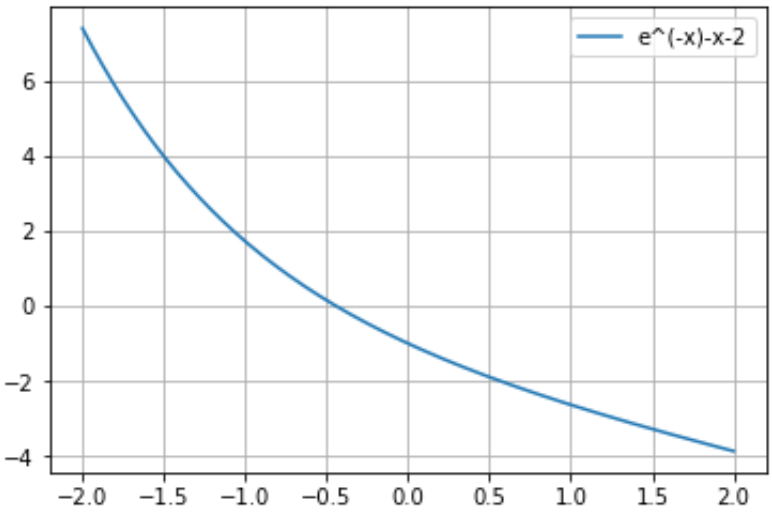


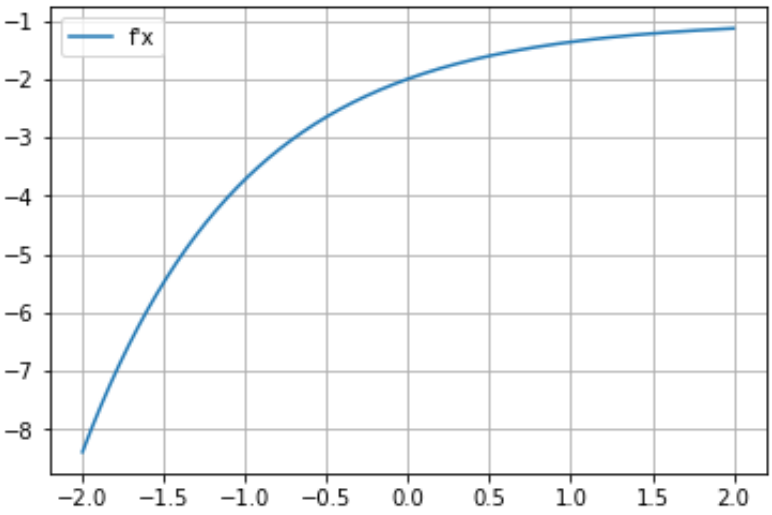

```
Steps taken to converge in Bisection Method are 17
Steps taken to converge in Newton Raphson are 5
```

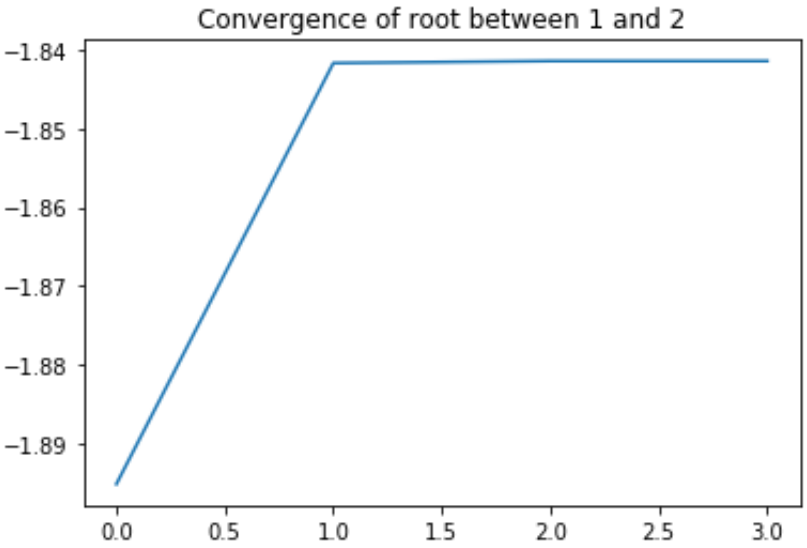
Out[12]:

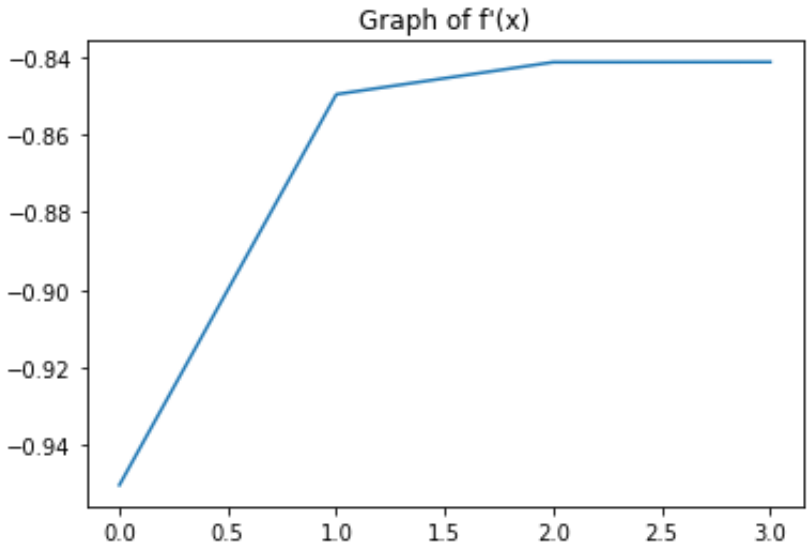
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x
0	0.0	-1.200000	-1.328000	2.320000	0.000000	-0.627586	5.724138e-0
1	1.0	-0.627586	-0.992011	-0.818407	0.572414	-1.839712	-1.212125e+
2	2.0	-1.839712	-4.547152	8.153616	-1.212125	-1.282026	5.576853e-0
3	3.0	-1.282026	-1.543075	2.930774	0.557685	-0.755519	5.265076e-0
4	4.0	-0.755519	-0.920219	-0.287574	0.526508	-3.955452	-3.199933e+
5	5.0	-3.955452	-55.974519	44.936802	-3.199933	-2.709824	1.245628e+0
6	6.0	-2.709824	-16.478995	20.029446	1.245628	-1.887086	8.227384e-0
7	7.0	-1.887086	-4.945918	8.683281	0.822738	-1.317495	5.695909e-0
8	8.0	-1.317495	-1.651909	3.207380	0.569591	-0.802461	5.150338e-0
9	9.0	-0.802461	-0.911818	-0.068167	0.515034	-14.178625	-1.337616e+
10	10.0	-14.178625	-2824.019743	601.100179	-13.376163	-9.480539	4.698085e+0
11	11.0	-9.480539	-835.155775	267.641887	4.698085	-6.360117	3.120423e+0
12	12.0	-6.360117	-246.553416	119.353263	3.120423	-4.294372	2.065745e+0
13	13.0	-4.294372	-72.606473	53.324890	2.065745	-2.932785	1.361587e+0
14	14.0	-2.932785	-21.359981	23.803683	1.361587	-2.035446	8.973393e-0
15	15.0	-2.035446	-6.362039	10.429117	0.897339	-1.425419	6.100266e-0
16	16.0	-1.425419	-2.045356	4.095458	0.610027	-0.925998	4.994206e-0
17	17.0	-0.925998	-0.942022	0.572419	0.499421	0.719687	1.645685e+0
18	18.0	0.719687	-3.066613	-0.446152	1.645685	-6.153780	-6.873467e+
19	19.0	-6.153780	-222.730035	111.607041	-6.873467	-4.158118	1.995663e+0
20	20.0	-4.158118	-65.577372	49.869824	1.995663	-2.843147	1.314971e+0
21	21.0	-2.843147	-19.296231	22.250446	1.314971	-1.975918	8.672289e-0
22	22.0	-1.975918	-5.762641	9.712751	0.867229	-1.382611	5.933068e-0
23	23.0	-1.382611	-1.877794	3.734837	0.593307	-0.879833	5.027781e-0
24	24.0	-0.879833	-0.921418	0.322316	0.502778	1.978906	2.858738e+0
25	25.0	1.978906	1.791717	9.748203	2.858738	1.795106	-1.837997e-
26	26.0	1.795106	0.194347	7.667216	-0.183800	1.769758	-2.534779e-
27	27.0	1.769758	0.003444	7.396132	-0.025348	1.769293	-4.656267e-
28	28.0	1.769293	0.000001	7.391188	-0.000466	1.769292	-1.557251e-

2. Find the largest root of $f(x) = e^x - x - 2 = 0$, with $\epsilon = 0.0001$.







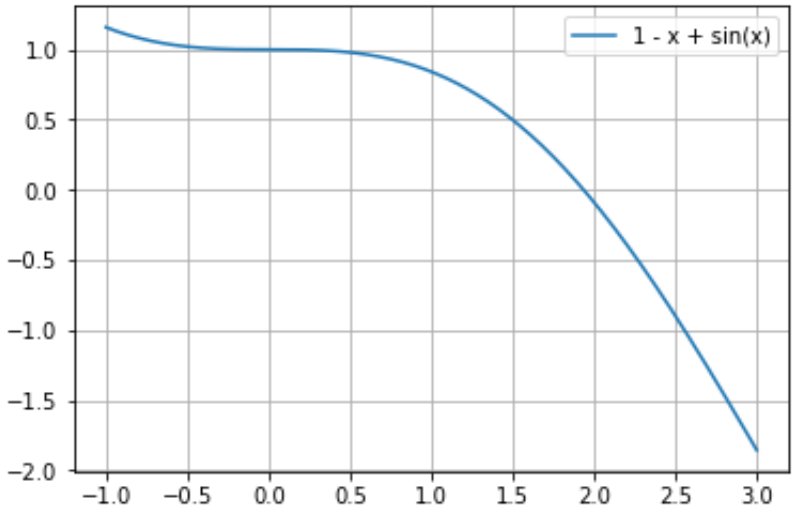


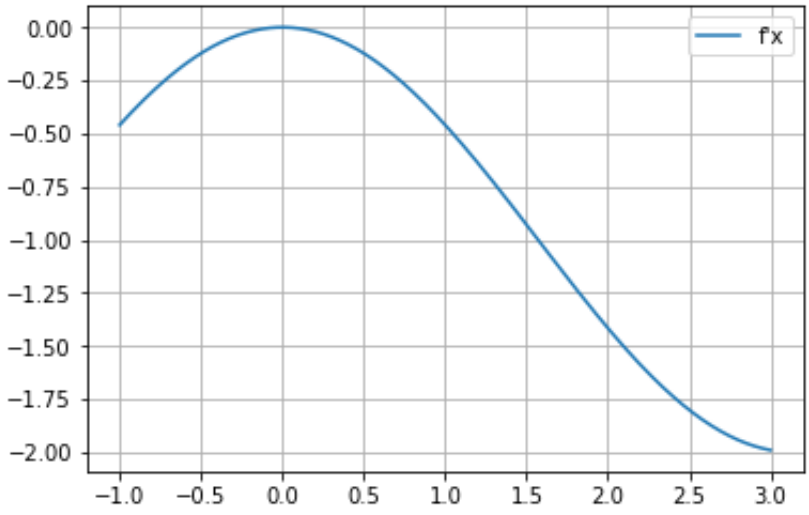

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 5
```

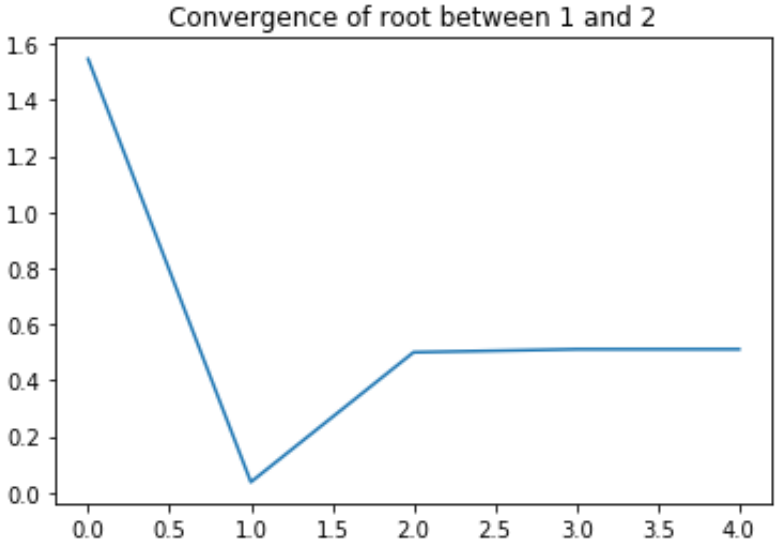
Out[13]:

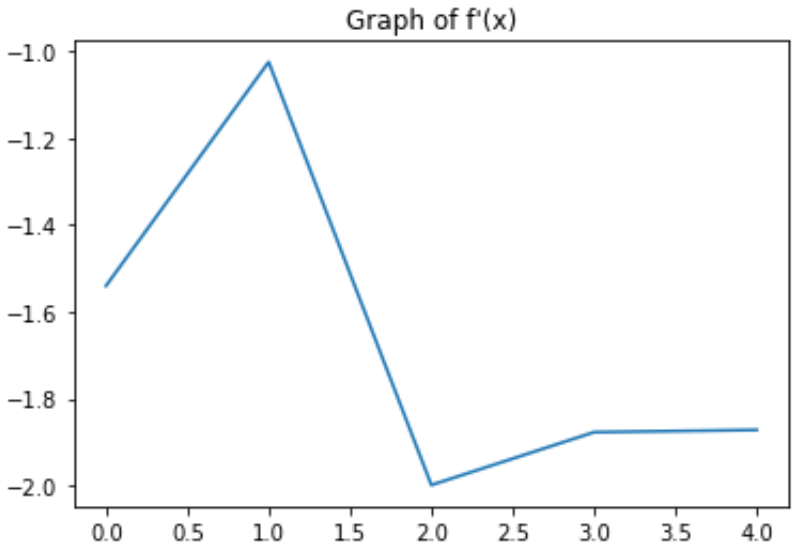
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	-3.000000	1.049787e+00	-0.950213	0.000000	-1.895209	1.104791e+00
1	1.0	-1.895209	4.549559e-02	-0.849713	1.104791	-1.841666	5.354230e-02
2	2.0	-1.841666	2.193164e-04	-0.841447	0.053542	-1.841406	2.606420e-04
3	3.0	-1.841406	5.386056e-09	-0.841406	0.000261	-1.841406	6.401260e-09

Find the smallest positive root of $f(x) = 1 - x + sinx = 0$, with $\epsilon = 0.0001$.





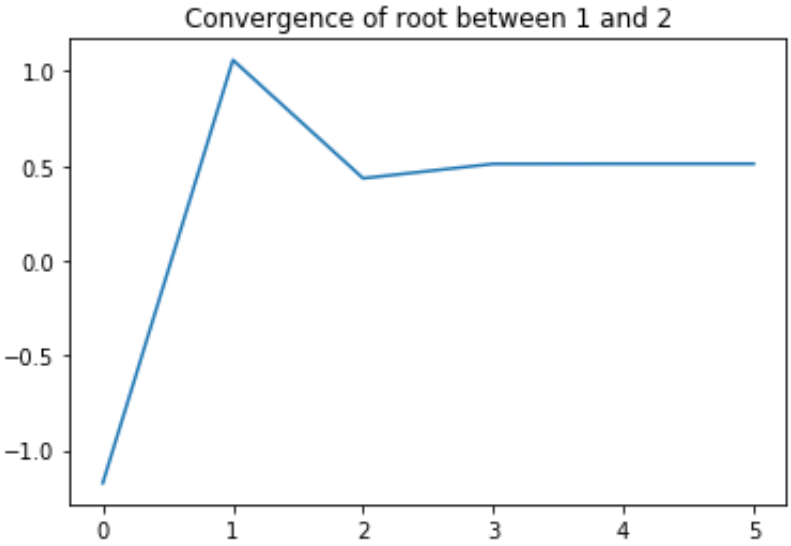


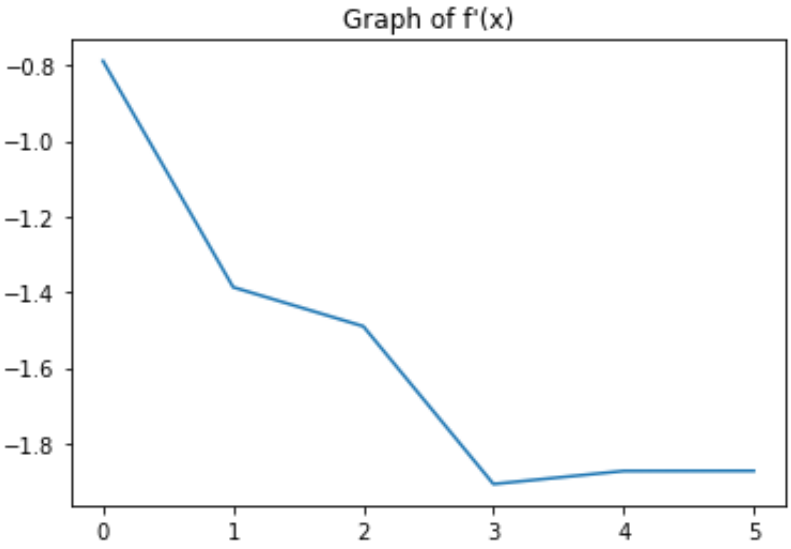



```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 6
```

Out[14]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.000000	0.841471	-1.540302	0.000000	1.546302	0.546302
1	1.0	1.546302	-1.546003	-1.024491	0.546302	0.037259	-1.509044
2	2.0	0.037259	0.925492	-1.999306	-1.509044	0.500165	0.462906
3	3.0	0.500165	0.020265	-1.877503	0.462906	0.510958	0.010793
4	4.0	0.510958	0.000028	-1.872276	0.010793	0.510973	0.000015



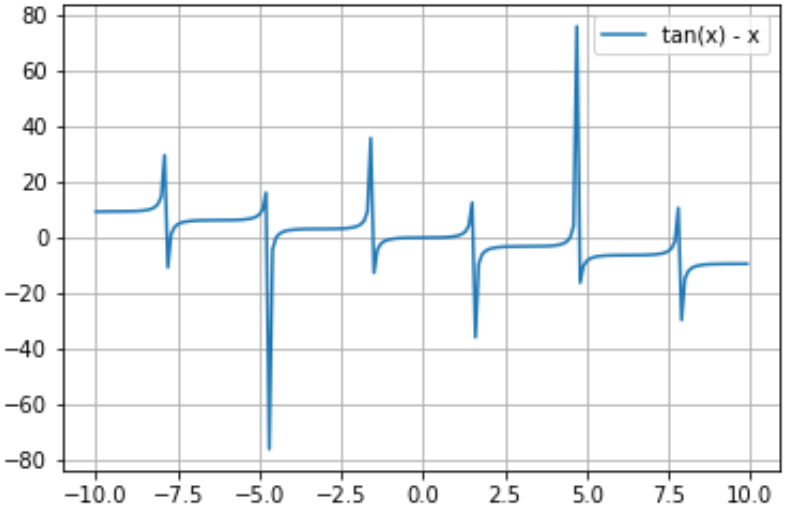


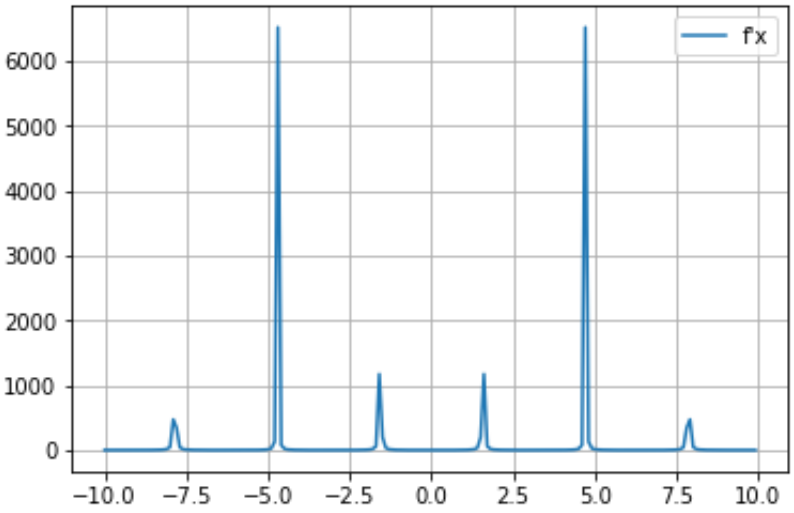

```
Steps taken to converge in Bisection Method are 14
Steps taken to converge in Newton Raphson are 7
```

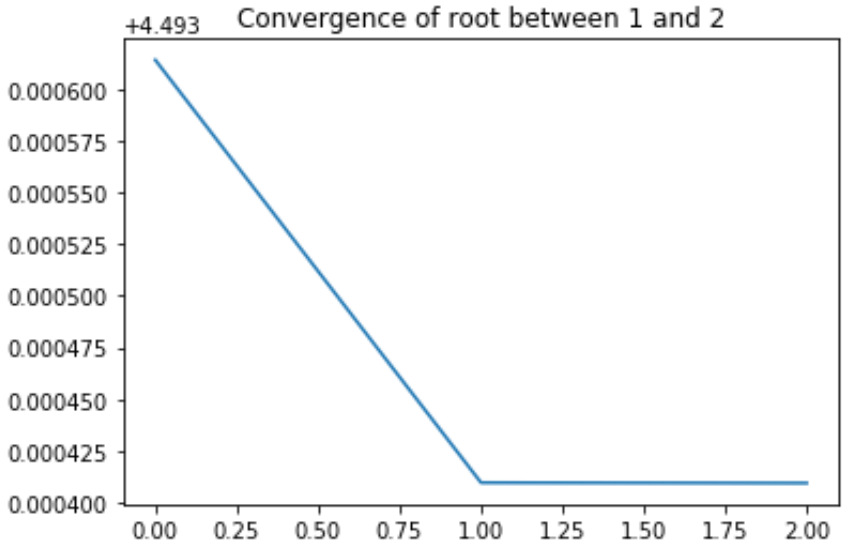
Out[15]:

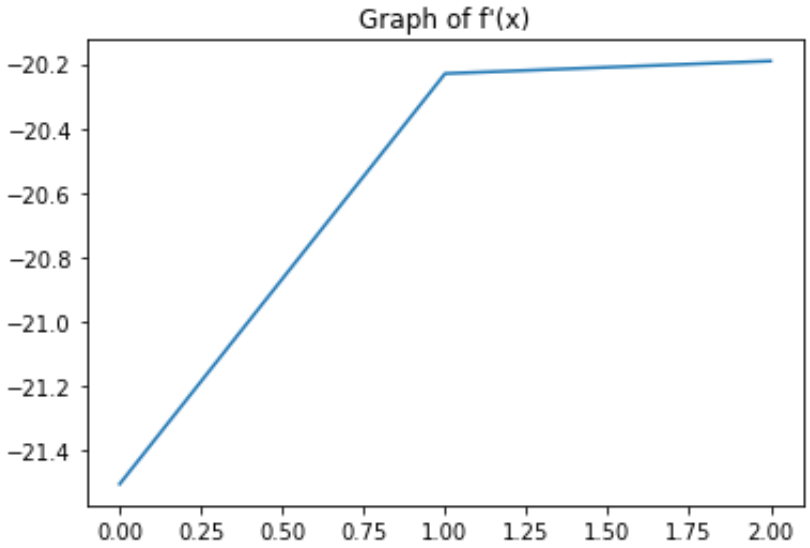
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	4.500000	-4.477530e+00	-0.789204	0.000000	-1.173475	-5.673475e+00
1	1.0	-1.173475	3.095575e+00	-1.386950	-5.673475	1.058455	2.231930e+00
2	2.0	1.058455	-9.300548e-01	-1.490219	2.231930	0.434349	-6.241062e-01
3	3.0	0.434349	1.448305e-01	-1.907144	-0.624106	0.510290	7.594105e-02
4	4.0	0.510290	1.279057e-03	-1.872603	0.075941	0.510973	6.830369e-04
5	5.0	0.510973	1.139824e-07	-1.872269	0.000683	0.510973	6.087929e-08

4. Find the smallest non-zero positive root of $x = tanx$, with an accuracy of $\epsilon = 0.0001$. Further solve for the root that is closest to $x = 100$.





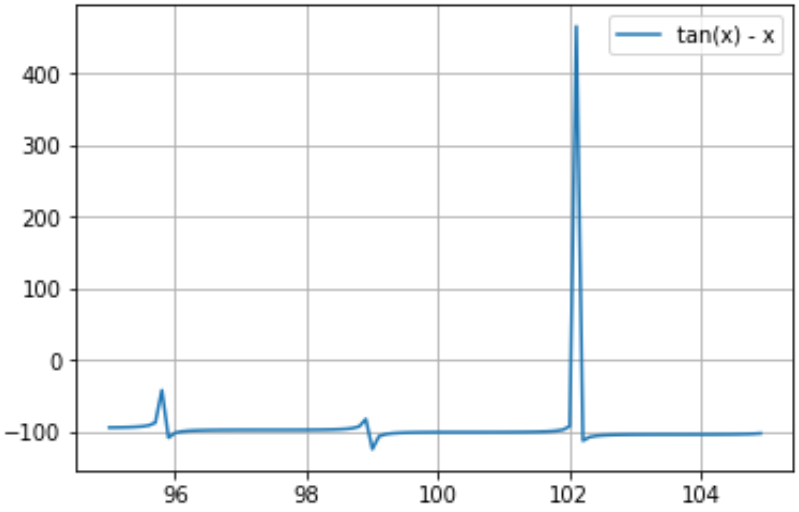


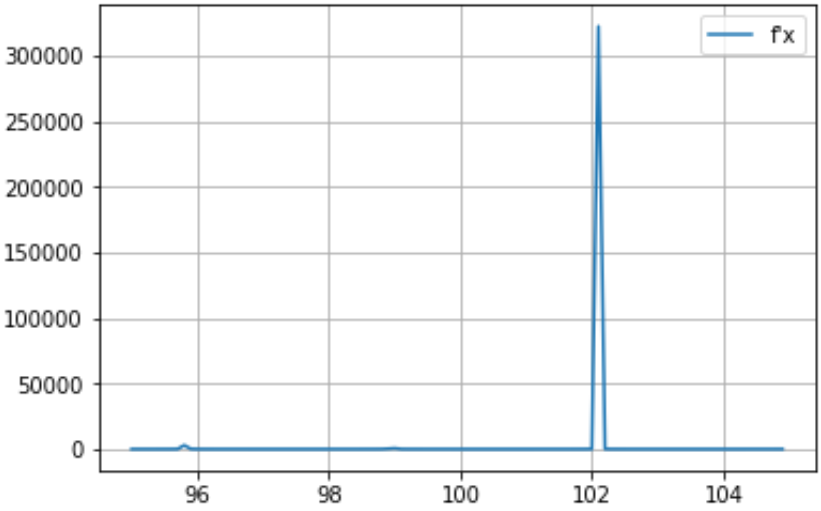


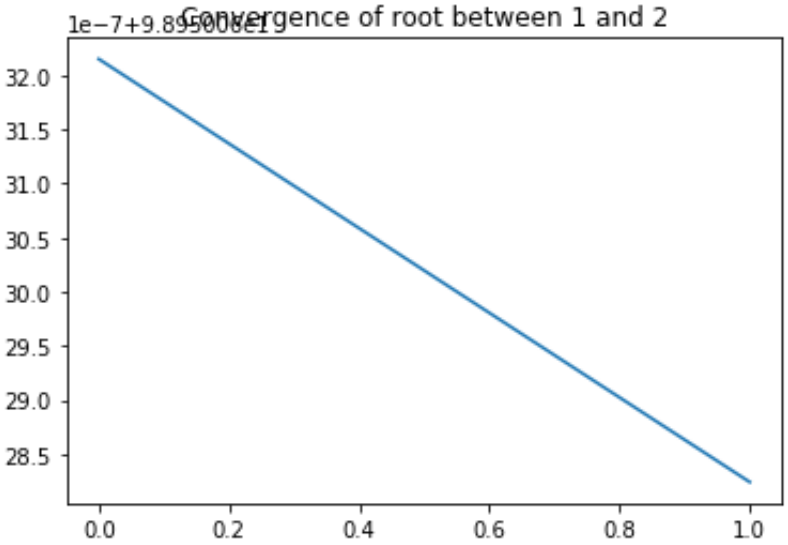

```
Steps taken to converge in Bisection Method are 19
Steps taken to converge in Newton Raphson are 4
```

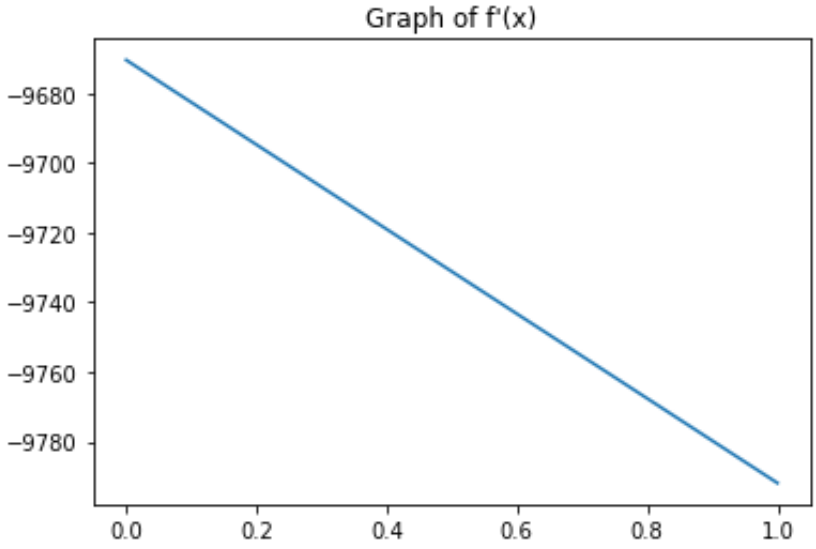
Out[16]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	4.500000	-0.137332	-21.504849	0.000000	4.493614	-6.386097e-03
1	1.0	4.493614	-0.004132	-20.229717	-0.006386	4.493410	-2.042477e-04
2	2.0	4.493410	-0.000004	-20.190766	-0.000204	4.493409	-1.971040e-07


tan(x) - x
400
300
200
100
0
−100
96
98
100
102
104


f'x
300000
250000
200000
150000
100000
50000
0
96
98
100
102
104


Convergence of root between 1 and 2
1e−7+9.8950
32.0
31.5
31.0
30.5
30.0
29.5
29.0
28.5
0.0
0.2
0.4
0.6
0.8
1.0

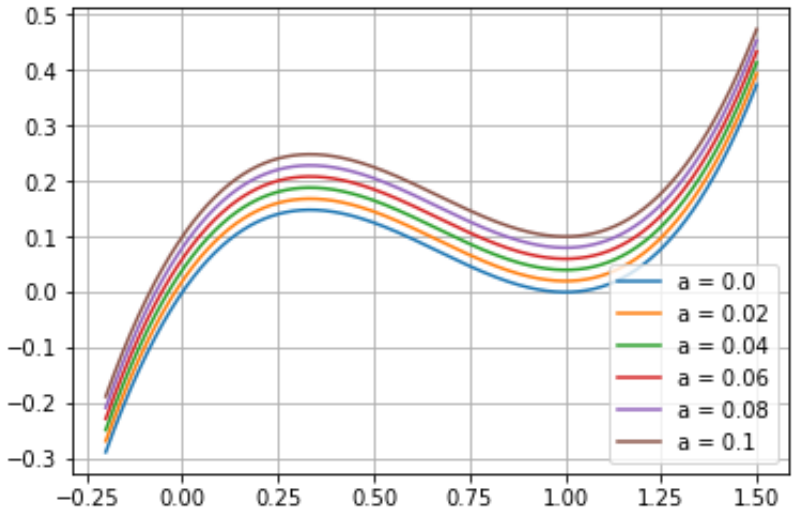



```
Steps taken to converge in Bisection Method are 17
Steps taken to converge in Newton Raphson are 2
```

Out[17]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	98.950000	0.611320	-9670.496060	0.000000	98.950063	6.321492e-05
1	1.0	98.950063	-0.003824	-9791.871878	0.000063	98.950063	-3.905708e-07

The function $y = f(x) = a + x(x-1)^2$,with $0 \leq a \leq 0.1$. When $a \neq 0$, there is only one real root of f(x) = 0, with the root being negative. Analytically check how many roots are obtained for a = 0, and what is the nature of the roots. Thereafter, using the Newton-Raphson method, test for the convergence towards the negative real root, through a series of suitably chosen a values going right down to a = 0 (the most important case). In every case your initial guess value should be slightly larger than 1, say 1.01, and slightly smaller than 1, say 0.99. For every value of a, starting from both sides of x = 1, check how quickly the convergence happens.



Out[18]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	1.010000e-04	0.020300	0.000000	1.005025	-0.004975
1	1.0	1.005025	2.537377e-05	0.010125	-0.004975	1.002519	-0.002506
2	2.0	1.002519	6.359220e-06	0.005056	-0.002506	1.001261	-0.001258
3	3.0	1.001261	1.591797e-06	0.002527	-0.001258	1.000631	-0.000630
4	4.0	1.000631	3.981995e-07	0.001263	-0.000630	1.000316	-0.000315
5	5.0	1.000316	9.958124e-08	0.000631	-0.000315	1.000158	-0.000158
6	6.0	1.000158	2.489923e-08	0.000316	-0.000158	1.000079	-0.000079

Out[19]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.020101	0.020300	0.000000	0.019803	-0.990197
1	1.0	0.019803	0.039026	0.921965	-0.990197	-0.022527	-0.042330
2	2.0	-0.022527	-0.003553	1.091629	-0.042330	-0.019272	0.003255
3	3.0	-0.019272	-0.000022	1.078202	0.003255	-0.019252	0.000020

Out[20]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.040101	0.020300	0.000000	-0.965419	-1.975419
1	1.0	-0.965419	-3.689288	7.657775	-1.975419	-0.483649	0.481770
2	2.0	-0.483649	-1.024613	3.636342	0.481770	-0.201878	0.281770
3	3.0	-0.201878	-0.251615	1.929777	0.281770	-0.071492	0.130386
4	4.0	-0.071492	-0.042080	1.301304	0.130386	-0.039155	0.032337
5	5.0	-0.039155	-0.002282	1.161221	0.032337	-0.037190	0.001965
6	6.0	-0.037190	-0.000008	1.152911	0.001965	-0.037183	0.000007

Out[21]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.060101	0.020300	0.000000	-1.950640	-2.960640
1	1.0	-1.950640	-16.922819	20.217555	-2.960640	-1.113605	0.837036
2	2.0	-1.113605	-4.914832	9.174763	0.837036	-0.577914	0.535690
3	3.0	-0.577914	-1.378898	4.313611	0.535690	-0.258252	0.319662
4	4.0	-0.258252	-0.348864	2.233090	0.319662	-0.102027	0.156225
5	5.0	-0.102027	-0.063908	1.439337	0.156225	-0.057626	0.044401
6	6.0	-0.057626	-0.004459	1.240466	0.044401	-0.054031	0.003594
7	7.0	-0.054031	-0.000028	1.224884	0.003594	-0.054009	0.000023

Out[22]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.080101	0.020300	0.000000	-2.935862	-3.945862
1	1.0	-2.935862	-45.399469	38.601307	-3.945862	-1.759750	1.176112
2	2.0	-1.759750	-13.322640	17.329158	1.176112	-0.990951	0.768799
3	3.0	-0.990951	-3.848015	7.909753	0.768799	-0.504461	0.486490
4	4.0	-0.504461	-1.061798	3.781286	0.486490	-0.223657	0.280803
5	5.0	-0.223657	-0.254891	2.044698	0.280803	-0.098998	0.124659
6	6.0	-0.098998	-0.039570	1.425394	0.124659	-0.071238	0.027760
7	7.0	-0.071238	-0.001749	1.300175	0.027760	-0.069893	0.001345
8	8.0	-0.069893	-0.000004	1.294225	0.001345	-0.069890	0.000003

Out[23]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	1.001010e-01	0.020300	0.000000	-3.921084	-4.931084e+00
1	1.0	-3.921084	-9.485714e+01	62.809028	-4.931084	-2.410837	1.510247e+00
2	2.0	-2.410837	-2.794721e+01	28.079750	1.510247	-1.415557	9.952800e-01
3	3.0	-1.415557	-8.159654e+00	12.673631	0.995280	-0.771728	6.438292e-01
4	4.0	-0.771728	-2.322468e+00	5.873601	0.643829	-0.376320	3.954078e-01
5	5.0	-0.376320	-6.128463e-01	2.930129	0.395408	-0.167167	2.091533e-01
6	6.0	-0.167167	-1.277273e-01	1.752500	0.209153	-0.094284	7.288288e-02
7	7.0	-0.094284	-1.290060e-02	1.403803	0.072883	-0.085094	9.189755e-03
8	8.0	-0.085094	-1.920143e-04	1.362099	0.009190	-0.084953	1.409695e-04
9	9.0	-0.084953	-4.481505e-08	1.361463	0.000141	-0.084953	3.291684e-08

Out[24]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.020101	0.020300	0.000000	0.019803	-0.990197
1	1.0	0.019803	0.039026	0.921965	-0.990197	-0.022527	-0.042330
2	2.0	-0.022527	-0.003553	1.091629	-0.042330	-0.019272	0.003255
3	3.0	-0.019272	-0.000022	1.078202	0.003255	-0.019252	0.000020

Out[25]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.040101	0.020300	0.000000	-0.965419	-1.975419
1	1.0	-0.965419	-3.689288	7.657775	-1.975419	-0.483649	0.481770
2	2.0	-0.483649	-1.024613	3.636342	0.481770	-0.201878	0.281770
3	3.0	-0.201878	-0.251615	1.929777	0.281770	-0.071492	0.130386
4	4.0	-0.071492	-0.042080	1.301304	0.130386	-0.039155	0.032337
5	5.0	-0.039155	-0.002282	1.161221	0.032337	-0.037190	0.001965
6	6.0	-0.037190	-0.000008	1.152911	0.001965	-0.037183	0.000007

Out[26]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.060101	0.020300	0.000000	-1.950640	-2.960640
1	1.0	-1.950640	-16.922819	20.217555	-2.960640	-1.113605	0.837036
2	2.0	-1.113605	-4.914832	9.174763	0.837036	-0.577914	0.535690
3	3.0	-0.577914	-1.378898	4.313611	0.535690	-0.258252	0.319662
4	4.0	-0.258252	-0.348864	2.233090	0.319662	-0.102027	0.156225
5	5.0	-0.102027	-0.063908	1.439337	0.156225	-0.057626	0.044401
6	6.0	-0.057626	-0.004459	1.240466	0.044401	-0.054031	0.003594
7	7.0	-0.054031	-0.000028	1.224884	0.003594	-0.054009	0.000023

Out[27]:

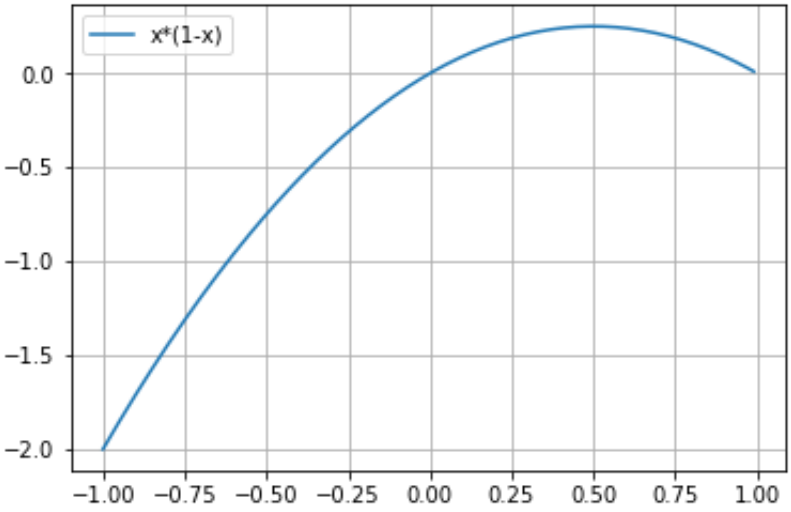
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	0.080101	0.020300	0.000000	-2.935862	-3.945862
1	1.0	-2.935862	-45.399469	38.601307	-3.945862	-1.759750	1.176112
2	2.0	-1.759750	-13.322640	17.329158	1.176112	-0.990951	0.768799
3	3.0	-0.990951	-3.848015	7.909753	0.768799	-0.504461	0.486490
4	4.0	-0.504461	-1.061798	3.781286	0.486490	-0.223657	0.280803
5	5.0	-0.223657	-0.254891	2.044698	0.280803	-0.098998	0.124659
6	6.0	-0.098998	-0.039570	1.425394	0.124659	-0.071238	0.027760
7	7.0	-0.071238	-0.001749	1.300175	0.027760	-0.069893	0.001345
8	8.0	-0.069893	-0.000004	1.294225	0.001345	-0.069890	0.000003

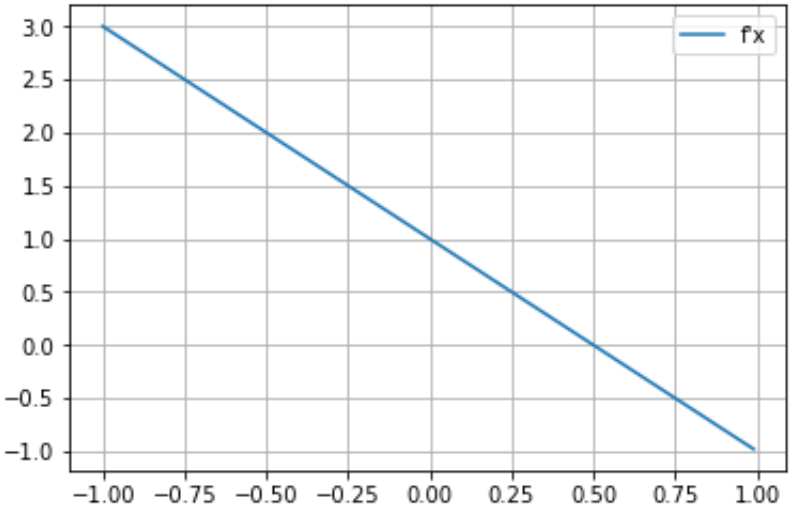
Out[28]:

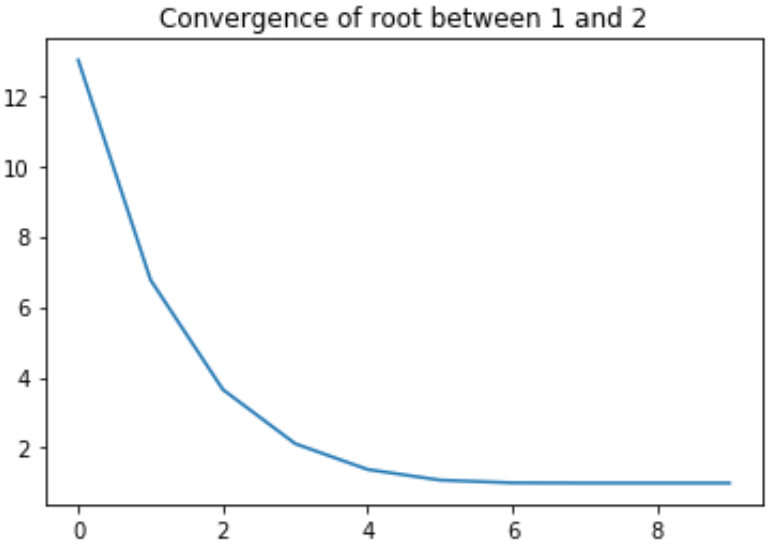
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	1.010000	1.001010e-01	0.020300	0.000000	-3.921084	-4.931084e+00
1	1.0	-3.921084	-9.485714e+01	62.809028	-4.931084	-2.410837	1.510247e+00
2	2.0	-2.410837	-2.794721e+01	28.079750	1.510247	-1.415557	9.952800e-01
3	3.0	-1.415557	-8.159654e+00	12.673631	0.995280	-0.771728	6.438292e-01
4	4.0	-0.771728	-2.322468e+00	5.873601	0.643829	-0.376320	3.954078e-01
5	5.0	-0.376320	-6.128463e-01	2.930129	0.395408	-0.167167	2.091533e-01
6	6.0	-0.167167	-1.277273e-01	1.752500	0.209153	-0.094284	7.288288e-02
7	7.0	-0.094284	-1.290060e-02	1.403803	0.072883	-0.085094	9.189755e-03
8	8.0	-0.085094	-1.920143e-04	1.362099	0.009190	-0.084953	1.409695e-04
9	9.0	-0.084953	-4.481505e-08	1.361463	0.000141	-0.084953	3.291684e-08

The function $y = f(x) = ax(1 - x)$, with $a = \pm 1$, has two roots at $x = 0, 1$ and a turning point at $x = 1/2$. Use the Newton-Raphson method to test the convergence towards $x = 0$, starting from an initial value of 0.51. Similarly test the convergence towards x = 1, starting from 0.49. Perform this exercise for both signs of a. Repeat this entire exercise for initial values of 0.501, 0.5001 and 0.499, 0.4999, respectively.

For $a = 1$:

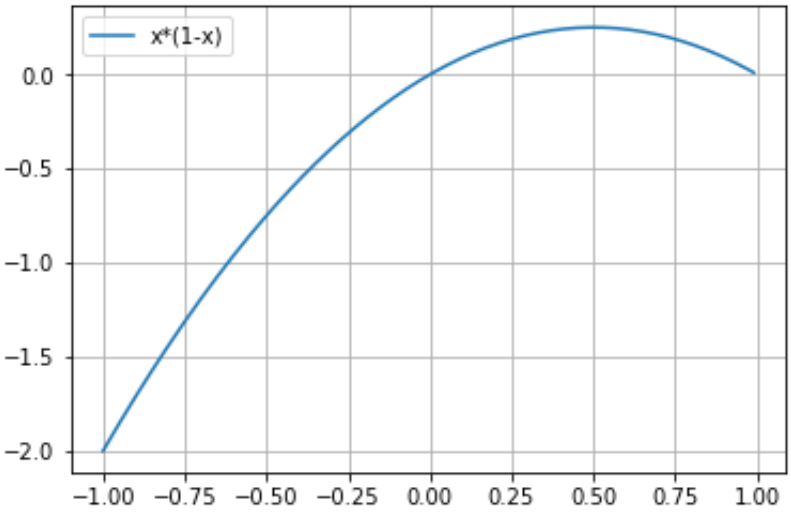

x*(1-x)
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

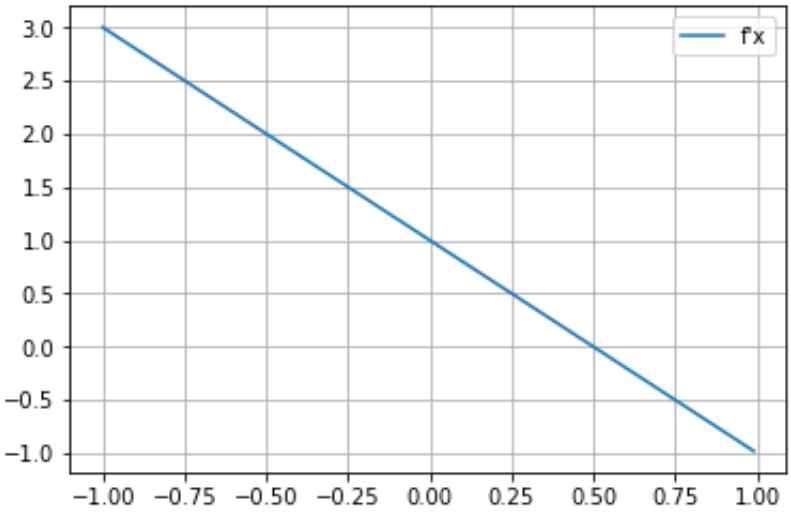

f'x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

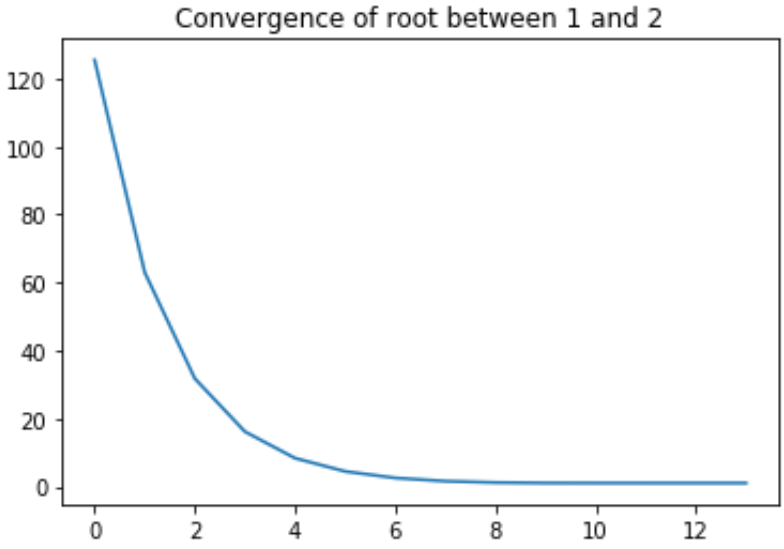

Convergence of root between 1 and 2
12
10
8
6
4
2
0
2
4
6
8

Out[29]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.510000	2.499000e-01	-0.020000	0.100000	13.005000	1.249500e+01
1	1.0	13.005000	-1.561250e+02	-25.010000	12.495000	6.762496	-6.242504e+00
2	2.0	6.762496	-3.896886e+01	-12.524992	-6.242504	3.651208	-3.111288e+00
3	3.0	3.651208	-9.680112e+00	-6.302416	-3.111288	2.115271	-1.535937e+00
4	4.0	2.115271	-2.359102e+00	-3.230543	-1.535937	1.385022	-7.302493e-01
5	5.0	1.385022	-5.332641e-01	-1.770044	-0.730249	1.083750	-3.012716e-01
6	6.0	1.083750	-9.076459e-02	-1.167501	-0.301272	1.006008	-7.774263e-02
7	7.0	1.006008	-6.043916e-03	-1.012016	-0.077743	1.000036	-5.972157e-03
8	8.0	1.000036	-3.566666e-05	-1.000071	-0.005972	1.000000	-3.566411e-05
9	9.0	1.000000	-1.271929e-09	-1.000000	-0.000036	1.000000	-1.271929e-09

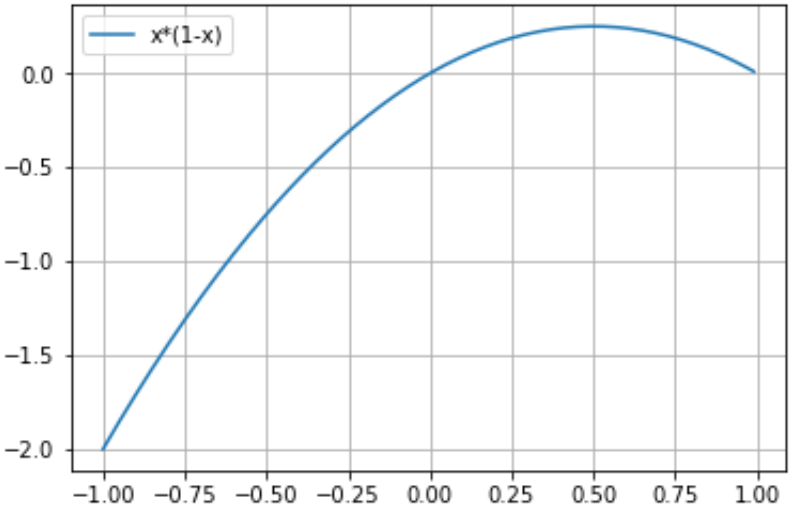


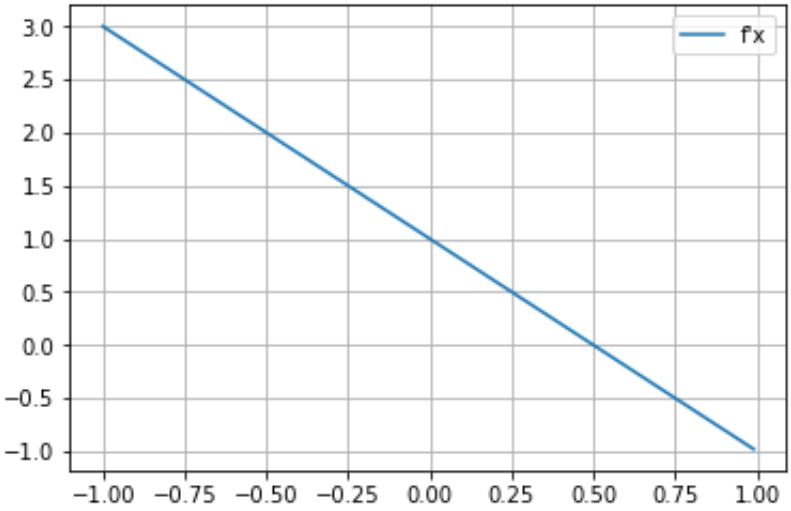


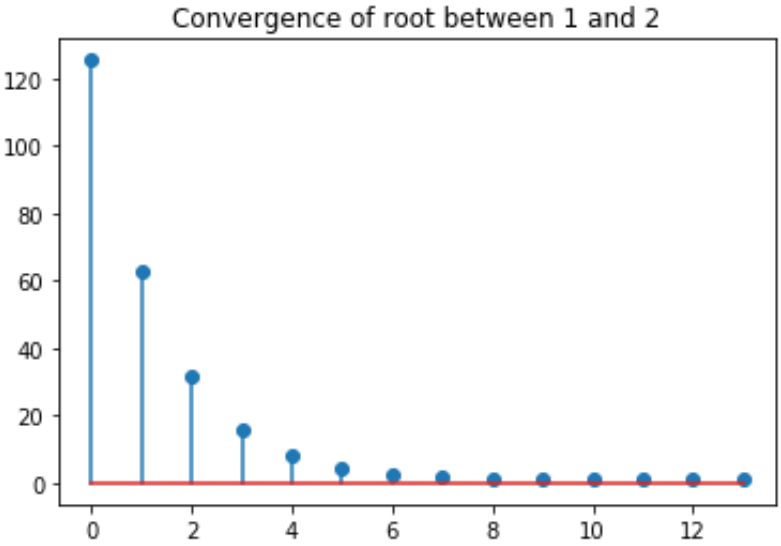


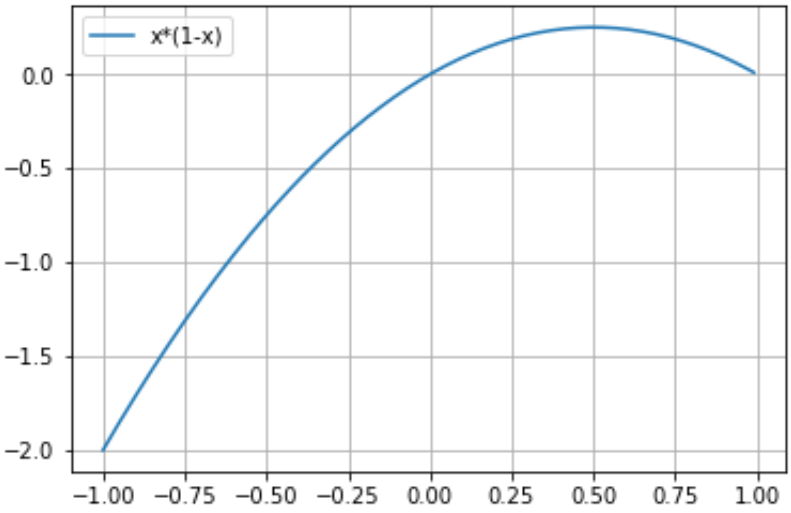
Out[30]:

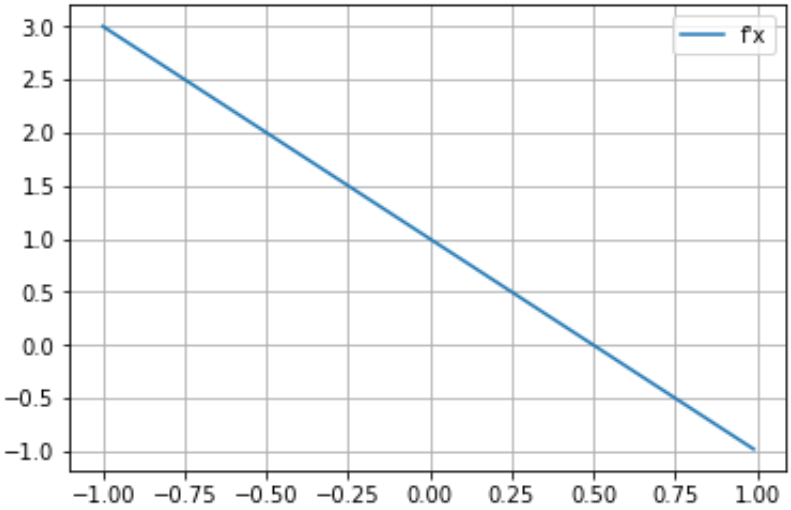
	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(r
0	0.0	0.501000	2.499990e-01	-0.002000	1.000000e-01	125.500500	1.249
1	1.0	125.500500	-1.562488e+04	-250.001000	1.249995e+02	63.001250	-6.24
2	2.0	63.001250	-3.906156e+03	-125.002500	-6.249925e+01	31.752625	-3.12
3	3.0	31.752625	-9.764766e+02	-62.505250	-3.124863e+01	16.130312	-1.56
4	4.0	16.130312	-2.440567e+02	-31.260624	-1.562231e+01	8.323153	-7.80
5	5.0	8.323153	-6.095173e+01	-15.646307	-7.807159e+00	4.427555	-3.89
6	6.0	4.427555	-1.517569e+01	-7.855110	-3.895598e+00	2.495604	-1.93
7	7.0	2.495604	-3.732435e+00	-3.991208	-1.931951e+00	1.560440	-9.35
8	8.0	1.560440	-8.745322e-01	-2.120879	-9.351642e-01	1.148095	-4.12
9	9.0	1.148095	-1.700277e-01	-1.296191	-4.123442e-01	1.016921	-1.31
10	10.0	1.016921	-1.720685e-02	-1.033841	-1.311749e-01	1.000277	-1.66
11	11.0	1.000277	-2.770099e-04	-1.000554	-1.664362e-02	1.000000	-2.76
12	12.0	1.000000	-7.664958e-08	-1.000000	-2.768566e-04	1.000000	-7.66
13	13.0	1.000000	-5.773160e-15	-1.000000	-7.664957e-08	1.000000	-5.77

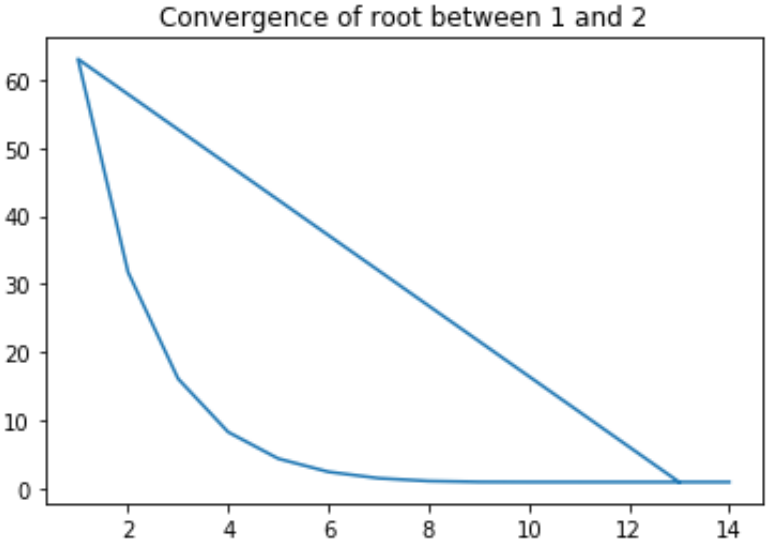

x*(1-x)
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


f'x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Convergence of root between 1 and 2
120
100
80
60
40
20
0
0
2
4
6
8
10
12

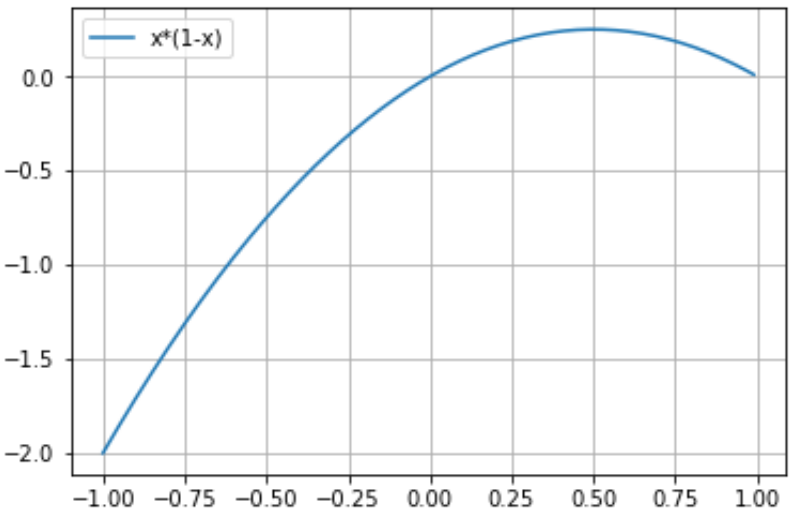

x*(1-x)
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

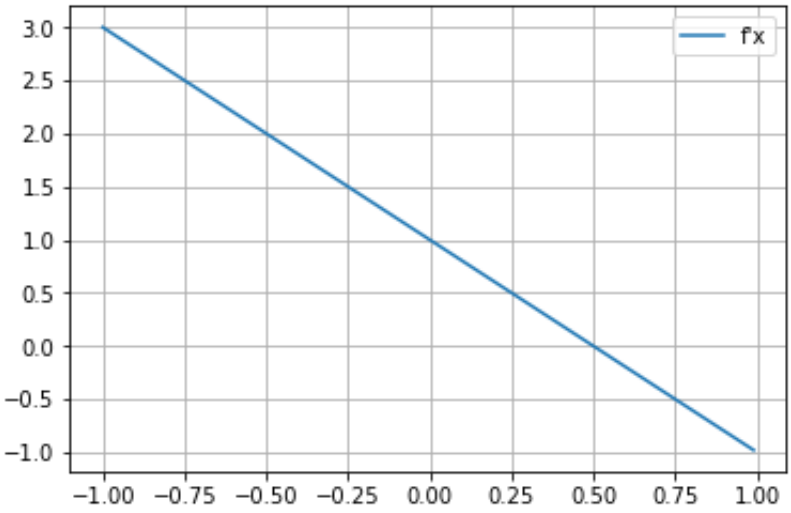

f'x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

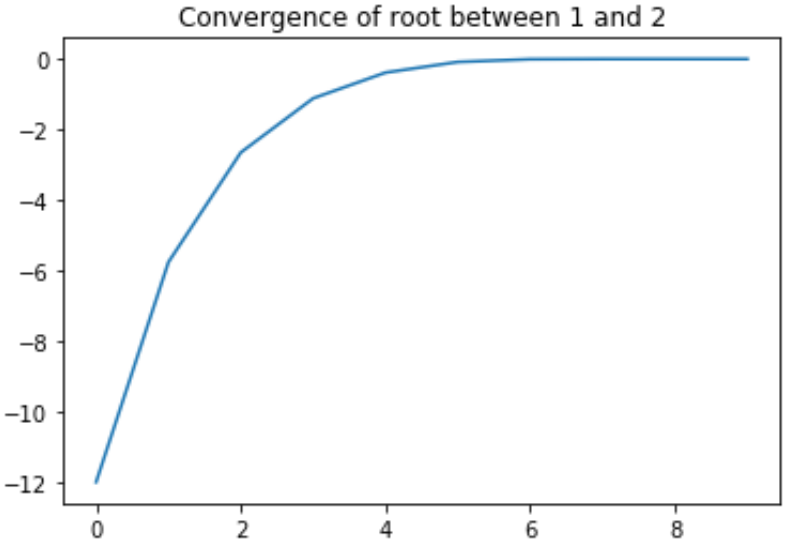

Convergence of root between 1 and 2
60
50
40
30
20
10
0
2
4
6
8
10
12
14

Out[31]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+
0	13.0	1.000000	-5.773160e-15	-1.000000	1.000000e-01	1.000000	-5.773
1	1.0	125.500500	-1.562488e+04	-250.001000	1.249995e+02	63.001250	-6.249
2	2.0	63.001250	-3.906156e+03	-125.002500	-6.249925e+01	31.752625	-3.124
3	3.0	31.752625	-9.764766e+02	-62.505250	-3.124863e+01	16.130312	-1.562
4	4.0	16.130312	-2.440567e+02	-31.260624	-1.562231e+01	8.323153	-7.807
5	5.0	8.323153	-6.095173e+01	-15.646307	-7.807159e+00	4.427555	-3.895
6	6.0	4.427555	-1.517569e+01	-7.855110	-3.895598e+00	2.495604	-1.931
7	7.0	2.495604	-3.732435e+00	-3.991208	-1.931951e+00	1.560440	-9.351
8	8.0	1.560440	-8.745322e-01	-2.120879	-9.351642e-01	1.148095	-4.123
9	9.0	1.148095	-1.700277e-01	-1.296191	-4.123442e-01	1.016921	-1.311
10	10.0	1.016921	-1.720685e-02	-1.033841	-1.311749e-01	1.000277	-1.664
11	11.0	1.000277	-2.770099e-04	-1.000554	-1.664362e-02	1.000000	-2.768
12	12.0	1.000000	-7.664958e-08	-1.000000	-2.768566e-04	1.000000	-7.664
13	13.0	1.000000	-5.773160e-15	-1.000000	-7.664957e-08	1.000000	-5.773
14	14.0	1.000000	0.000000e+00	-1.000000	-5.773160e-15	1.000000	0.0000

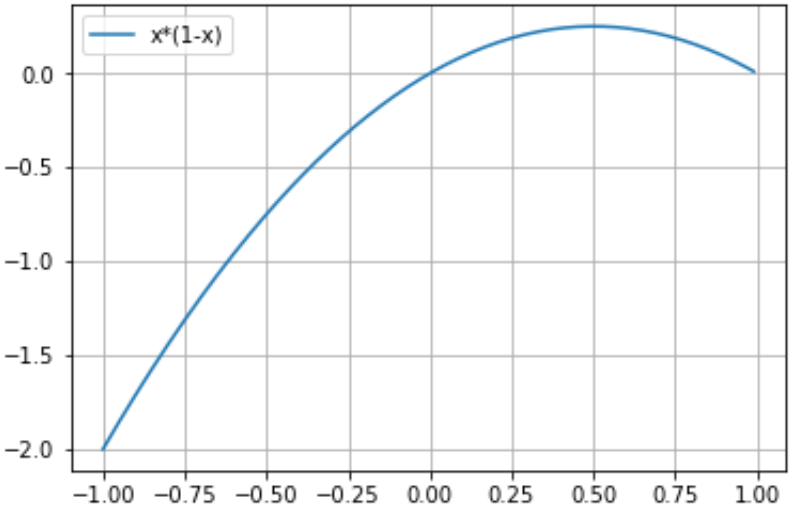


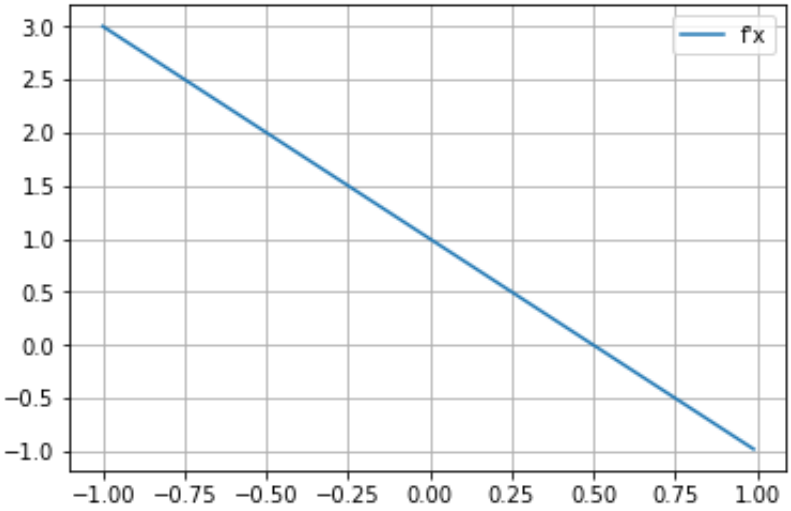


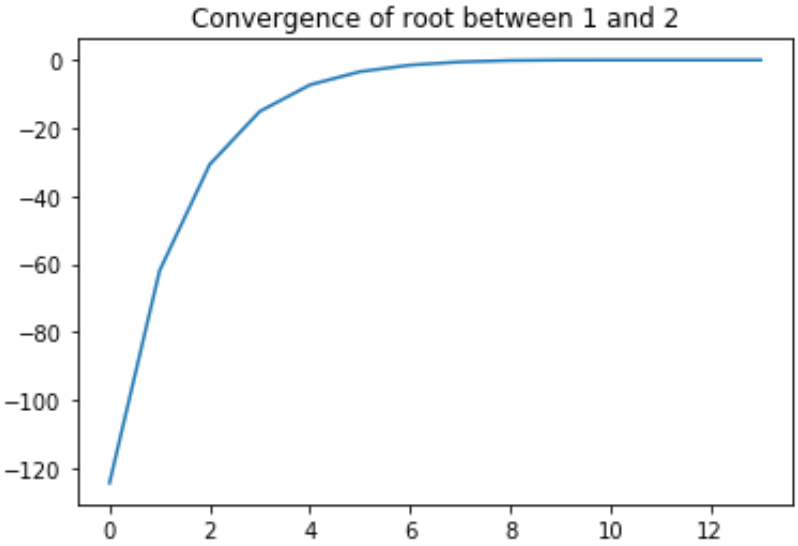


Out[32]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+
0	0.0	4.900000e-01	2.499000e-01	0.020000	0.100000	-1.200500e+01	-1.249
1	1.0	-1.200500e+01	-1.561250e+02	25.010000	-12.495000	-5.762496e+00	6.2425
2	2.0	-5.762496e+00	-3.896886e+01	12.524992	6.242504	-2.651208e+00	3.1112
3	3.0	-2.651208e+00	-9.680112e+00	6.302416	3.111288	-1.115271e+00	1.5359
4	4.0	-1.115271e+00	-2.359102e+00	3.230543	1.535937	-3.850221e-01	7.3024
5	5.0	-3.850221e-01	-5.332641e-01	1.770044	0.730249	-8.375045e-02	3.0127
6	6.0	-8.375045e-02	-9.076459e-02	1.167501	0.301272	-6.007822e-03	7.7742
7	7.0	-6.007822e-03	-6.043916e-03	1.012016	0.077743	-3.566538e-05	5.9721
8	8.0	-3.566538e-05	-3.566666e-05	1.000071	0.005972	-1.271929e-09	3.5664
9	9.0	-1.271929e-09	-1.271929e-09	1.000000	0.000036	-1.617803e-18	1.2719

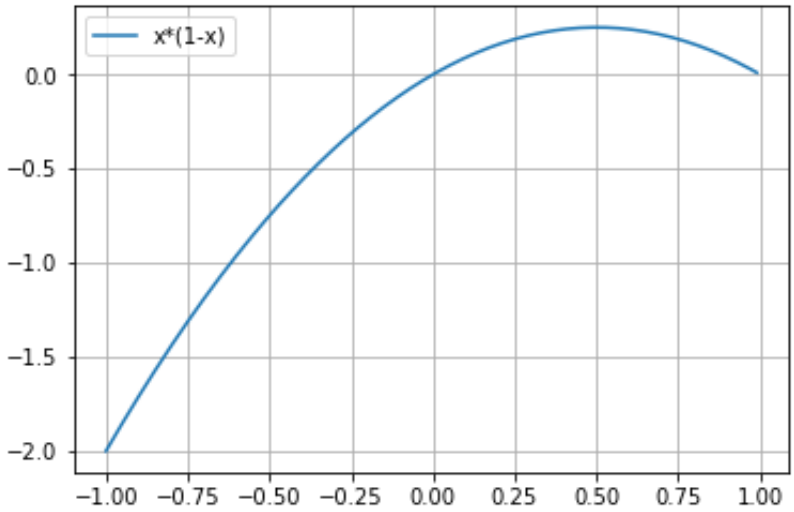

x*(1-x)
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

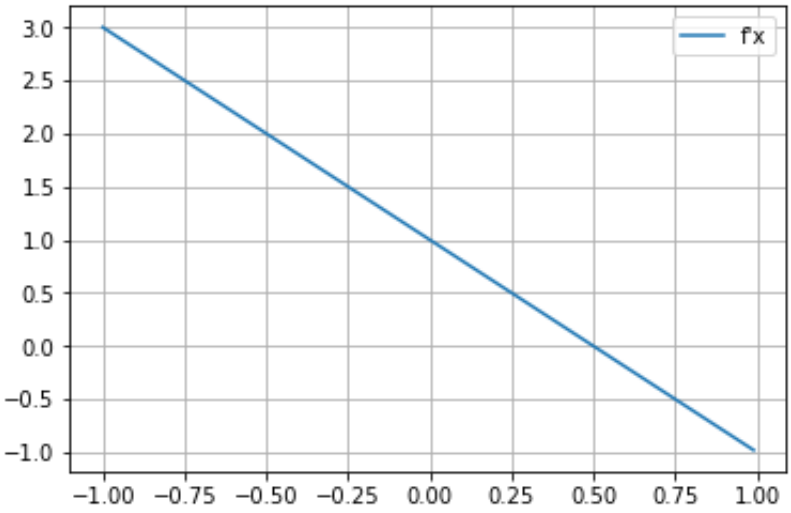

f'x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

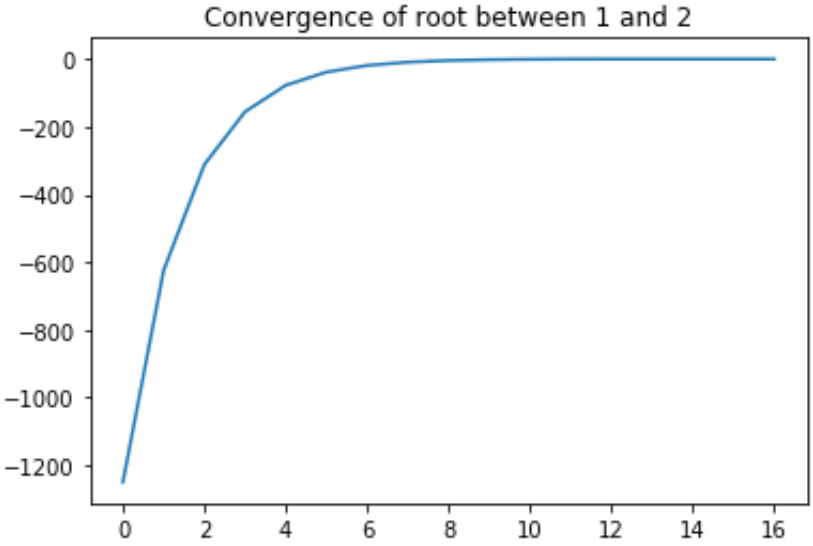

Convergence of root between 1 and 2
0
−20
−40
−60
−80
−100
−120
0
2
4
6
8
10
12

Out[33]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)
0	0.0	4.990000e-01	2.499990e-01	0.002000	1.000000e-01	-1.245005e+02
1	1.0	-1.245005e+02	-1.562488e+04	250.001000	-1.249995e+02	-6.200125e+01
2	2.0	-6.200125e+01	-3.906156e+03	125.002500	6.249925e+01	-3.075262e+01
3	3.0	-3.075262e+01	-9.764766e+02	62.505250	3.124863e+01	-1.513031e+01
4	4.0	-1.513031e+01	-2.440567e+02	31.260624	1.562231e+01	-7.323153e+00
5	5.0	-7.323153e+00	-6.095173e+01	15.646307	7.807159e+00	-3.427555e+00
6	6.0	-3.427555e+00	-1.517569e+01	7.855110	3.895598e+00	-1.495604e+00
7	7.0	-1.495604e+00	-3.732435e+00	3.991208	1.931951e+00	-5.604396e-01
8	8.0	-5.604396e-01	-8.745322e-01	2.120879	9.351642e-01	-1.480954e-01
9	9.0	-1.480954e-01	-1.700277e-01	1.296191	4.123442e-01	-1.692055e-02
10	10.0	-1.692055e-02	-1.720685e-02	1.033841	1.311749e-01	-2.769333e-04
11	11.0	-2.769333e-04	-2.770099e-04	1.000554	1.664362e-02	-7.664958e-08
12	12.0	-7.664958e-08	-7.664958e-08	1.000000	2.768566e-04	-5.875157e-15
13	13.0	-5.875157e-15	-5.875157e-15	1.000000	7.664957e-08	-3.549874e-29



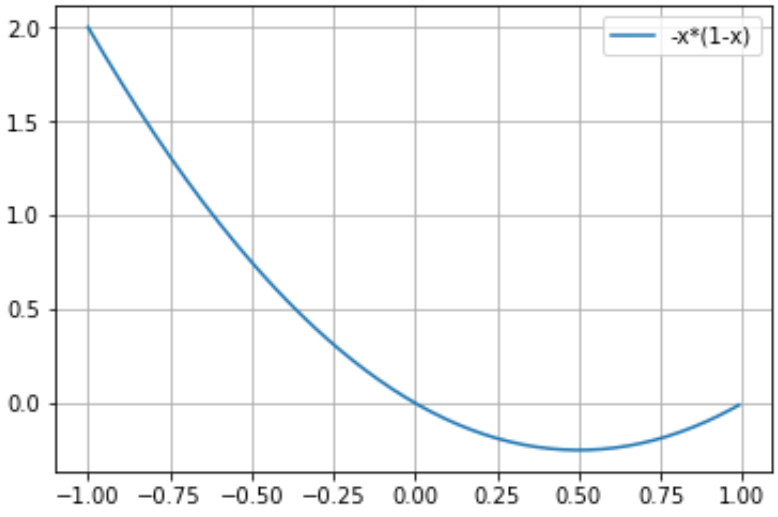

fx
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

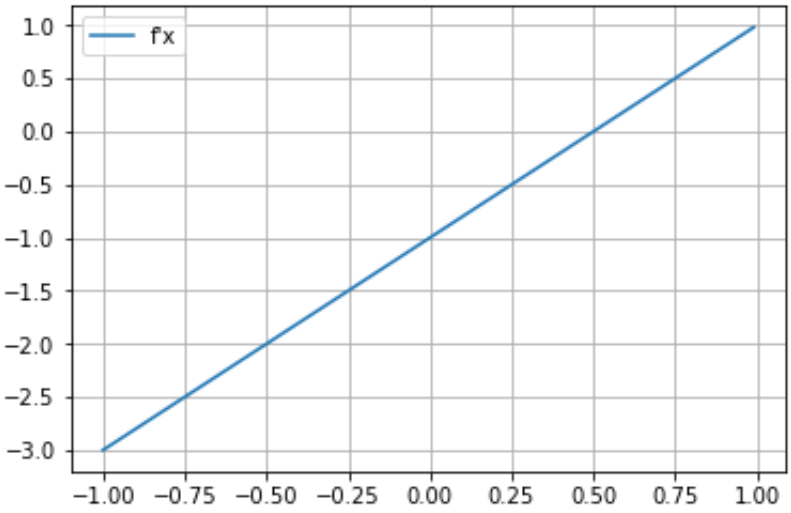

Convergence of root between 1 and 2
0
−200
−400
−600
−800
−1000
−1200
0
2
4
6
8
10
12
14
16

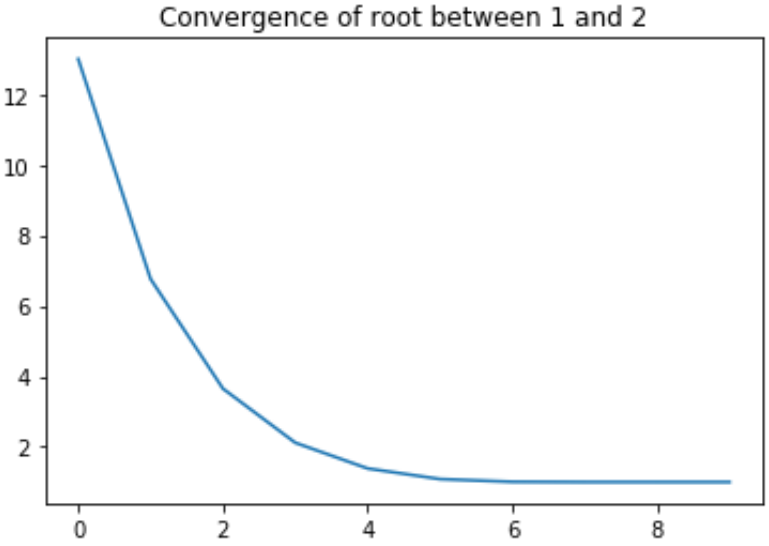
Out[34]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)
0	0.0	4.999000e-01	2.500000e-01	0.000200	0.100000	-1.249500e+03
1	1.0	-1.249500e+03	-1.562500e+06	2500.000100	-1249.999950	-6.245001e+02
2	2.0	-6.245001e+02	-3.906249e+05	1250.000250	624.999925	-3.120003e+02
3	3.0	-3.120003e+02	-9.765616e+04	625.000525	312.499863	-1.557505e+02
4	4.0	-1.557505e+02	-2.441398e+04	312.501062	156.249731	-7.762607e+01
5	5.0	-7.762607e+01	-6.103432e+03	156.252131	78.124466	-3.856463e+01
6	6.0	-3.856463e+01	-1.525796e+03	78.129266	39.061433	-1.903552e+01
7	7.0	-1.903552e+01	-3.813864e+02	39.071032	19.529117	-9.274157e+00
8	8.0	-9.274157e+00	-9.528414e+01	19.548313	9.761360	-4.399867e+00
9	9.0	-4.399867e+00	-2.375870e+01	9.799734	4.874290	-1.975444e+00
10	10.0	-1.975444e+00	-5.877825e+00	4.950889	2.424423	-7.882182e-01
11	11.0	-7.882182e-01	-1.409506e+00	2.576436	1.187226	-2.411424e-01
12	12.0	-2.411424e-01	-2.992920e-01	1.482285	0.547076	-3.922974e-02
13	13.0	-3.922974e-02	-4.076871e-02	1.078459	0.201913	-1.427010e-03
14	14.0	-1.427010e-03	-1.429046e-03	1.002854	0.037803	-2.030562e-06
15	15.0	-2.030562e-06	-2.030566e-06	1.000004	0.001425	-4.123164e-12
16	16.0	-4.123164e-12	-4.123164e-12	1.000000	0.000002	-1.700082e-23

For $a = -1$

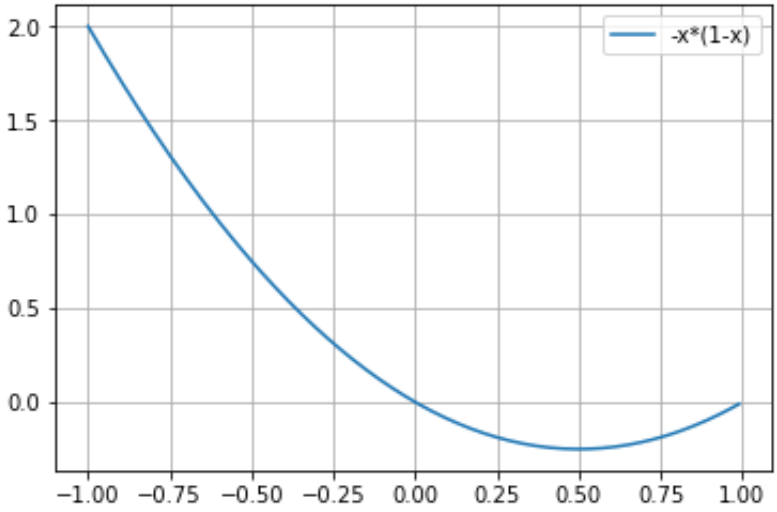


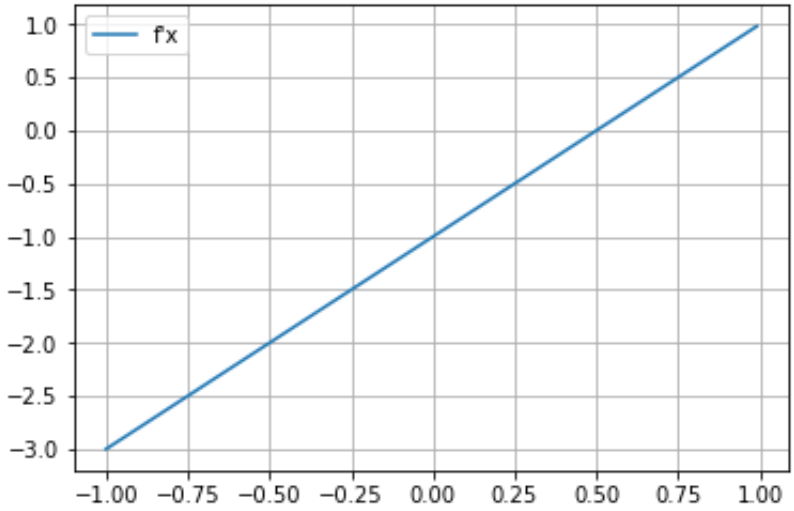


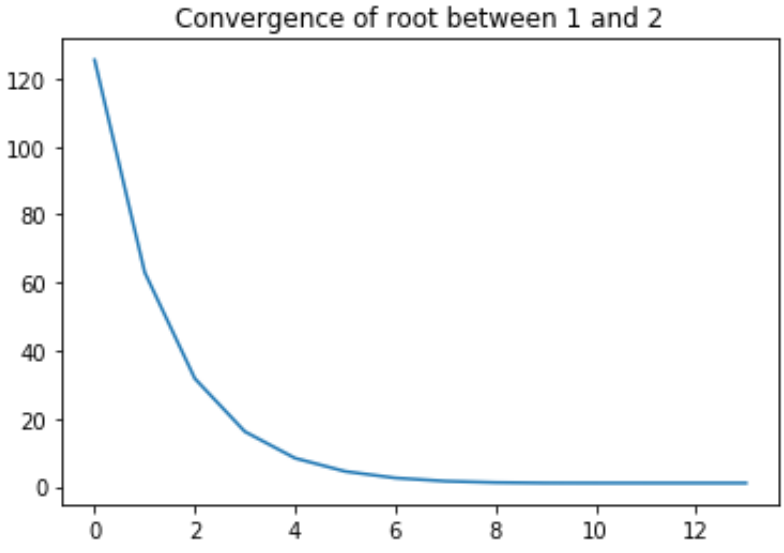


Out[35]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+1) - x(n)
0	0.0	0.510000	-2.499000e-01	0.020000	0.100000	13.005000	1.249500e+01
1	1.0	13.005000	1.561250e+02	25.010000	12.495000	6.762496	-6.242504e+00
2	2.0	6.762496	3.896886e+01	12.524992	-6.242504	3.651208	-3.111288e+00
3	3.0	3.651208	9.680112e+00	6.302416	-3.111288	2.115271	-1.535937e+00
4	4.0	2.115271	2.359102e+00	3.230543	-1.535937	1.385022	-7.302493e-01
5	5.0	1.385022	5.332641e-01	1.770044	-0.730249	1.083750	-3.012716e-01
6	6.0	1.083750	9.076459e-02	1.167501	-0.301272	1.006008	-7.774263e-02
7	7.0	1.006008	6.043916e-03	1.012016	-0.077743	1.000036	-5.972157e-03
8	8.0	1.000036	3.566666e-05	1.000071	-0.005972	1.000000	-3.566411e-05
9	9.0	1.000000	1.271929e-09	1.000000	-0.000036	1.000000	-1.271929e-09

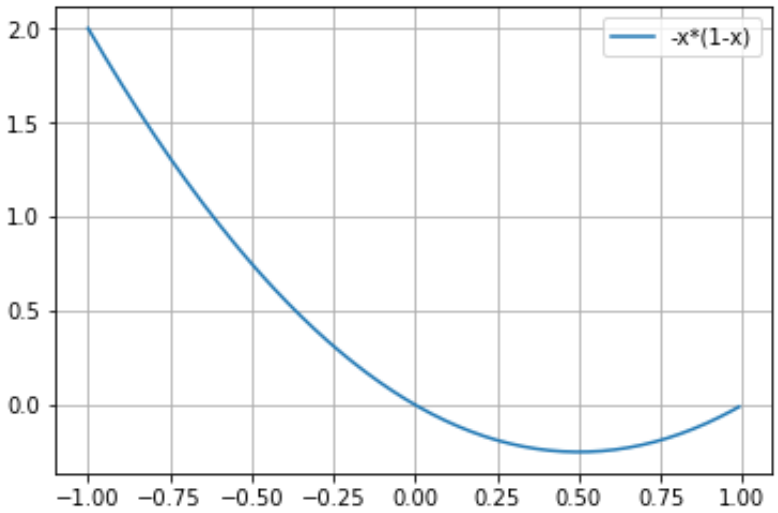

-x*(1-x)
2.0
1.5
1.0
0.5
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

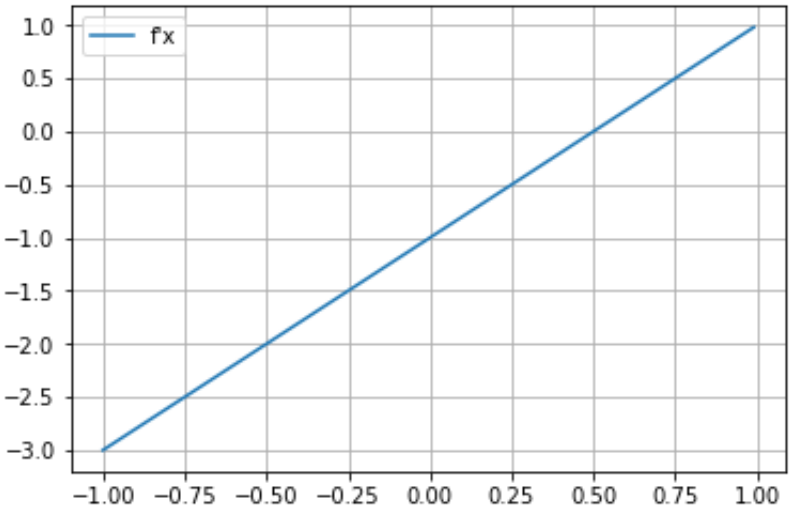

f'x
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

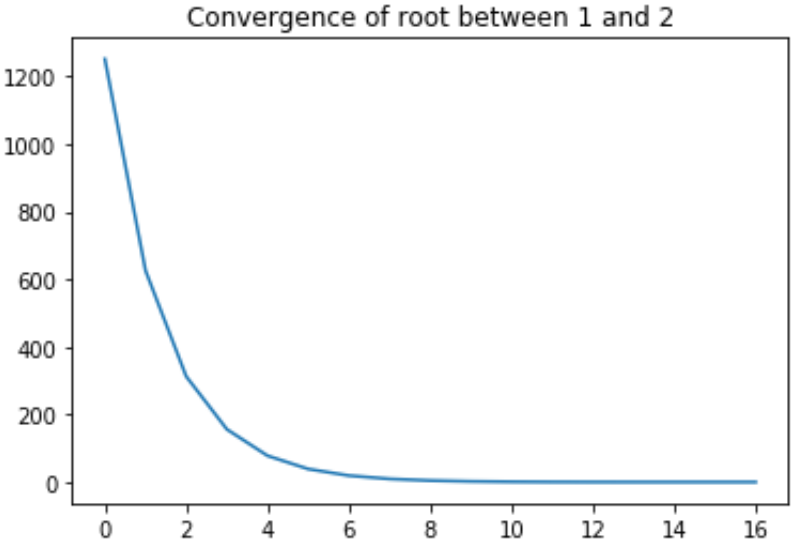

Convergence of root between 1 and 2
120
100
80
60
40
20
0
0
2
4
6
8
10
12

Out[36]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+
0	0.0	0.501000	-2.499990e-01	0.002000	1.000000e-01	125.500500	1.2499
1	1.0	125.500500	1.562488e+04	250.001000	1.249995e+02	63.001250	-6.2499
2	2.0	63.001250	3.906156e+03	125.002500	-6.249925e+01	31.752625	-3.1248
3	3.0	31.752625	9.764766e+02	62.505250	-3.124863e+01	16.130312	-1.5622
4	4.0	16.130312	2.440567e+02	31.260624	-1.562231e+01	8.323153	-7.8071
5	5.0	8.323153	6.095173e+01	15.646307	-7.807159e+00	4.427555	-3.8955
6	6.0	4.427555	1.517569e+01	7.855110	-3.895598e+00	2.495604	-1.9319
7	7.0	2.495604	3.732435e+00	3.991208	-1.931951e+00	1.560440	-9.3516
8	8.0	1.560440	8.745322e-01	2.120879	-9.351642e-01	1.148095	-4.1234
9	9.0	1.148095	1.700277e-01	1.296191	-4.123442e-01	1.016921	-1.3117
10	10.0	1.016921	1.720685e-02	1.033841	-1.311749e-01	1.000277	-1.6643
11	11.0	1.000277	2.770099e-04	1.000554	-1.664362e-02	1.000000	-2.7685
12	12.0	1.000000	7.664958e-08	1.000000	-2.768566e-04	1.000000	-7.6649
13	13.0	1.000000	5.773160e-15	1.000000	-7.664957e-08	1.000000	-5.7731

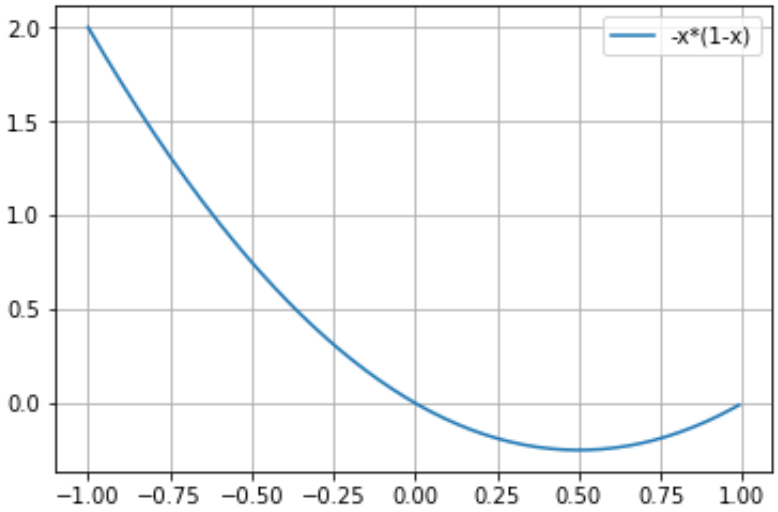


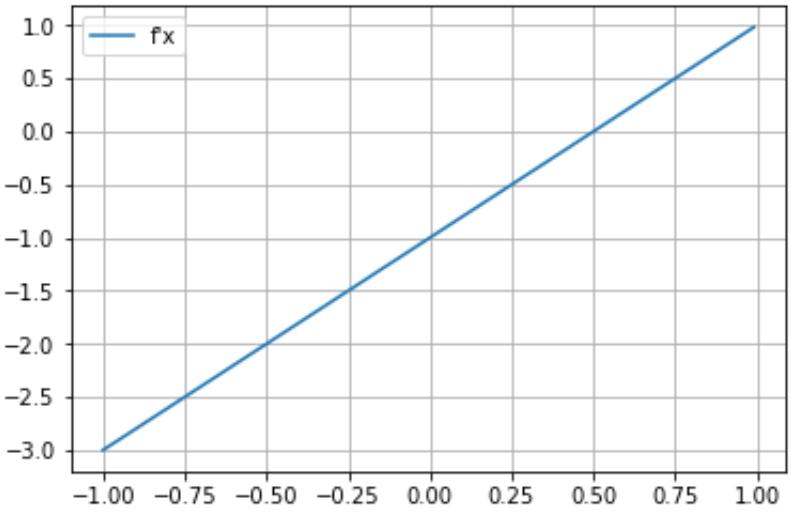

f'x
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

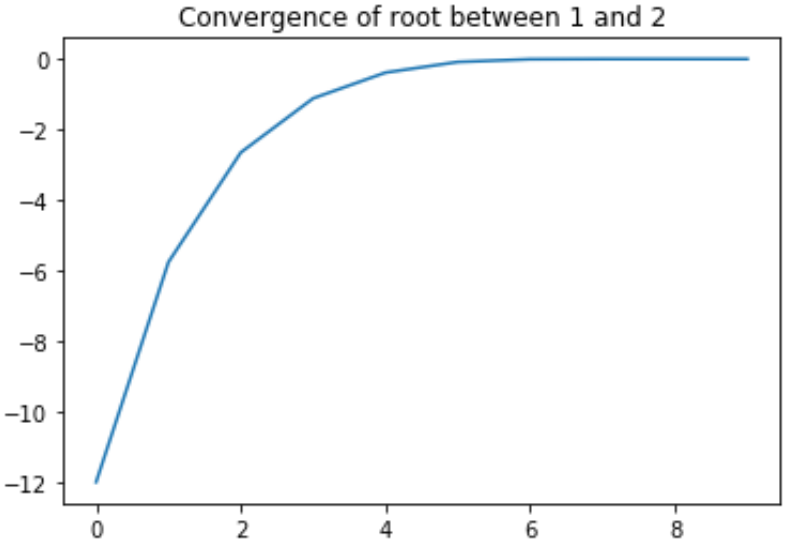

Convergence of root between 1 and 2
1200
1000
800
600
400
200
0
0
2
4
6
8
10
12
14
16

Out[37]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n
0	0.0	0.500100	-2.500000e-01	0.000200	0.100000	1250.500050	1.250
1	1.0	1250.500050	1.562500e+06	2500.000100	1249.999950	625.500125	-6.249
2	2.0	625.500125	3.906249e+05	1250.000250	-624.999925	313.000262	-3.124
3	3.0	313.000262	9.765616e+04	625.000525	-312.499863	156.750531	-1.562
4	4.0	156.750531	2.441398e+04	312.501062	-156.249731	78.626066	-7.812
5	5.0	78.626066	6.103432e+03	156.252131	-78.124466	39.564633	-3.906
6	6.0	39.564633	1.525796e+03	78.129266	-39.061433	20.035516	-1.952
7	7.0	20.035516	3.813864e+02	39.071032	-19.529117	10.274157	-9.761
8	8.0	10.274157	9.528414e+01	19.548313	-9.761360	5.399867	-4.874
9	9.0	5.399867	2.375870e+01	9.799734	-4.874290	2.975444	-2.424
10	10.0	2.975444	5.877825e+00	4.950889	-2.424423	1.788218	-1.187
11	11.0	1.788218	1.409506e+00	2.576436	-1.187226	1.241142	-5.470
12	12.0	1.241142	2.992920e-01	1.482285	-0.547076	1.039230	-2.019
13	13.0	1.039230	4.076871e-02	1.078459	-0.201913	1.001427	-3.780
14	14.0	1.001427	1.429046e-03	1.002854	-0.037803	1.000002	-1.424
15	15.0	1.000002	2.030566e-06	1.000004	-0.001425	1.000000	-2.030
16	16.0	1.000000	4.123146e-12	1.000000	-0.000002	1.000000	-4.123

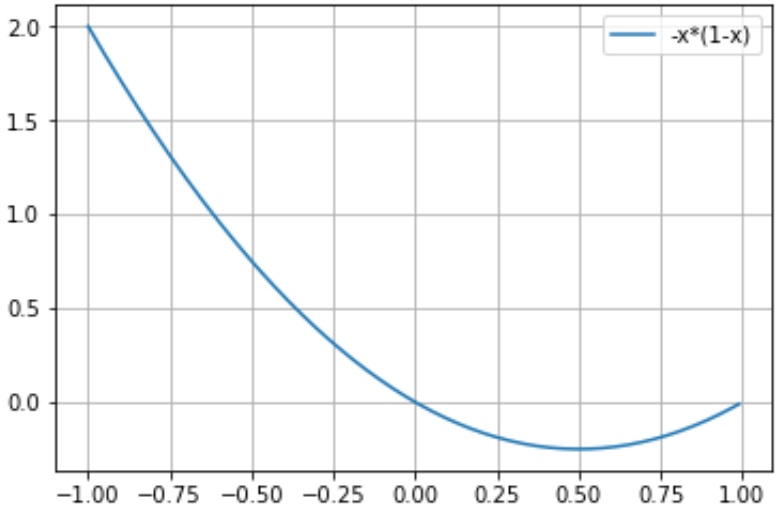


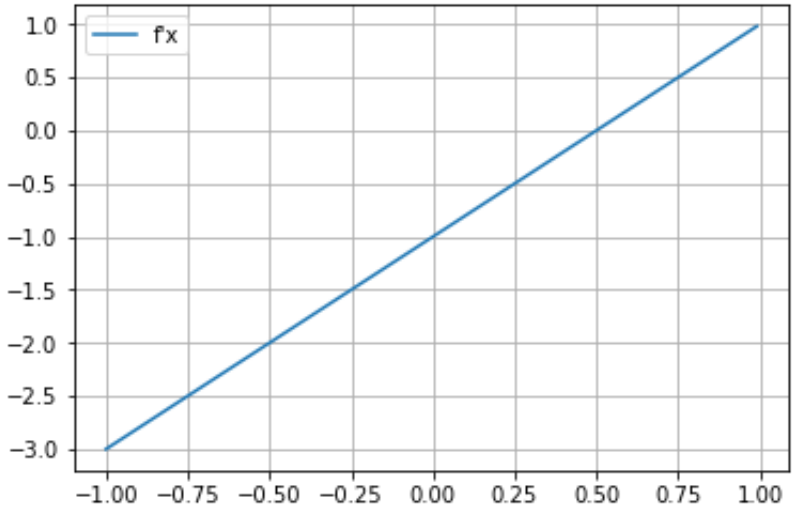


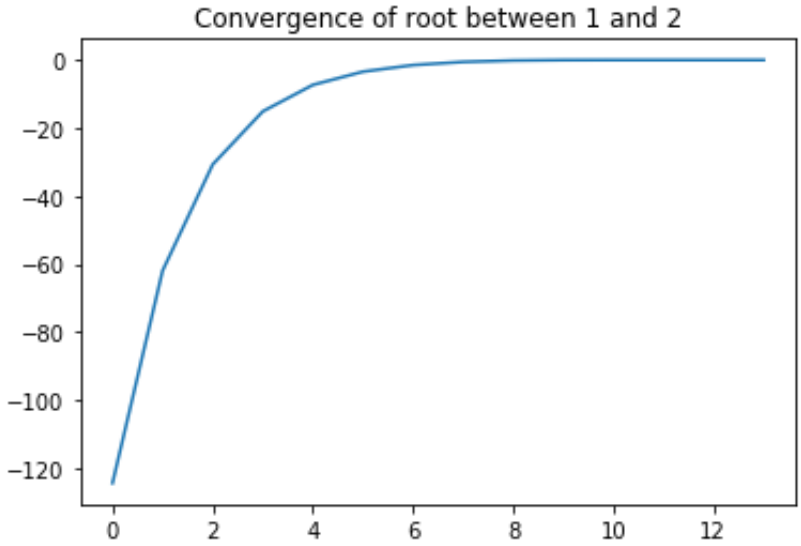


Out[38]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)	x(n+
0	0.0	4.900000e-01	-2.499000e-01	-0.020000	0.100000	-1.200500e+01	-1.249
1	1.0	-1.200500e+01	1.561250e+02	-25.010000	-12.495000	-5.762496e+00	6.2425
2	2.0	-5.762496e+00	3.896886e+01	-12.524992	6.242504	-2.651208e+00	3.1112
3	3.0	-2.651208e+00	9.680112e+00	-6.302416	3.111288	-1.115271e+00	1.5359
4	4.0	-1.115271e+00	2.359102e+00	-3.230543	1.535937	-3.850221e-01	7.3024
5	5.0	-3.850221e-01	5.332641e-01	-1.770044	0.730249	-8.375045e-02	3.0127
6	6.0	-8.375045e-02	9.076459e-02	-1.167501	0.301272	-6.007822e-03	7.7742
7	7.0	-6.007822e-03	6.043916e-03	-1.012016	0.077743	-3.566538e-05	5.9721
8	8.0	-3.566538e-05	3.566666e-05	-1.000071	0.005972	-1.271929e-09	3.5664
9	9.0	-1.271929e-09	1.271929e-09	-1.000000	0.000036	-1.617803e-18	1.2719

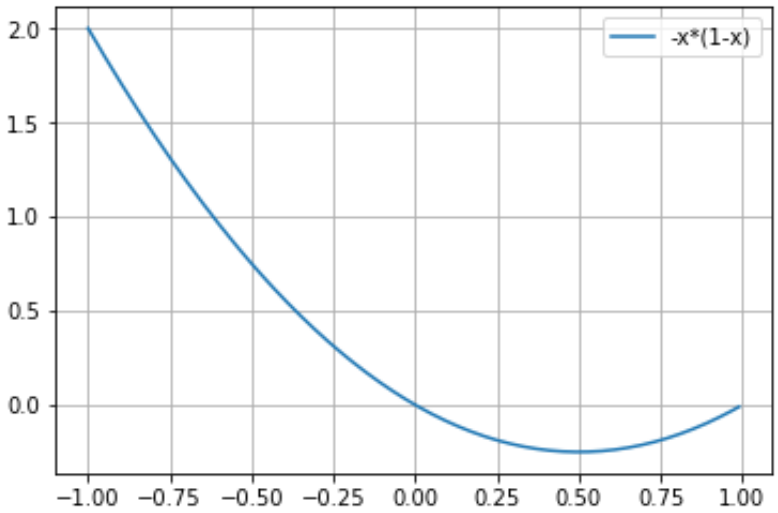

-x*(1-x)
2.0
1.5
1.0
0.5
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

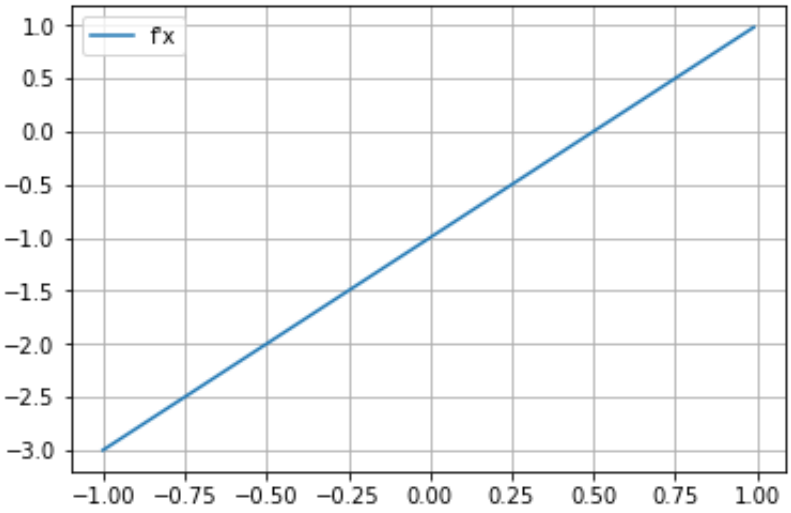

f'x
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

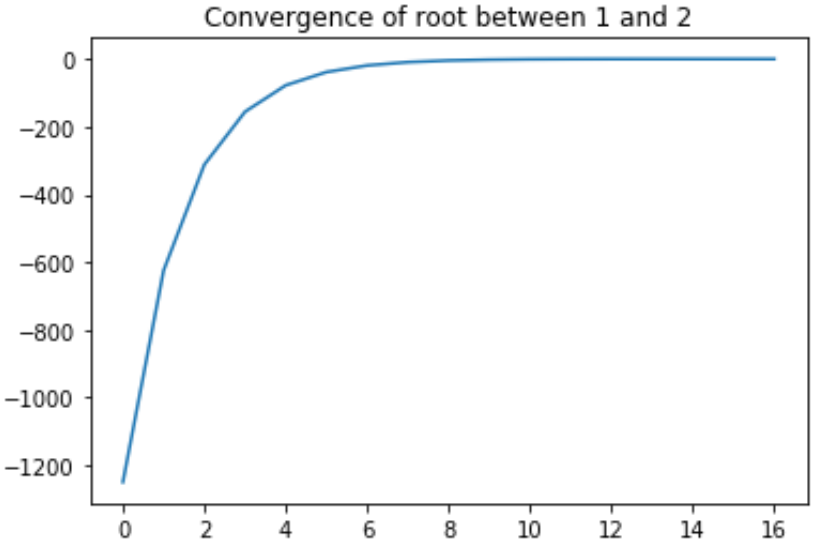

Convergence of root between 1 and 2
0
−20
−40
−60
−80
−100
−120
0
2
4
6
8
10
12

Out[39]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)
0	0.0	4.990000e-01	-2.499990e-01	-0.002000	1.000000e-01	-1.245005e+02
1	1.0	-1.245005e+02	1.562488e+04	-250.001000	-1.249995e+02	-6.200125e+01
2	2.0	-6.200125e+01	3.906156e+03	-125.002500	6.249925e+01	-3.075262e+01
3	3.0	-3.075262e+01	9.764766e+02	-62.505250	3.124863e+01	-1.513031e+01
4	4.0	-1.513031e+01	2.440567e+02	-31.260624	1.562231e+01	-7.323153e+00
5	5.0	-7.323153e+00	6.095173e+01	-15.646307	7.807159e+00	-3.427555e+00
6	6.0	-3.427555e+00	1.517569e+01	-7.855110	3.895598e+00	-1.495604e+00
7	7.0	-1.495604e+00	3.732435e+00	-3.991208	1.931951e+00	-5.604396e-01
8	8.0	-5.604396e-01	8.745322e-01	-2.120879	9.351642e-01	-1.480954e-01
9	9.0	-1.480954e-01	1.700277e-01	-1.296191	4.123442e-01	-1.692055e-02
10	10.0	-1.692055e-02	1.720685e-02	-1.033841	1.311749e-01	-2.769333e-04
11	11.0	-2.769333e-04	2.770099e-04	-1.000554	1.664362e-02	-7.664958e-08
12	12.0	-7.664958e-08	7.664958e-08	-1.000000	2.768566e-04	-5.875157e-15
13	13.0	-5.875157e-15	5.875157e-15	-1.000000	7.664957e-08	-3.549874e-29




fx
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Convergence of root between 1 and 2
0
−200
−400
−600
−800
−1000
−1200
0
2
4
6
8
10
12
14
16

Out[40]:

	n	x(n)	f(x)	f'(x)	x(n) - x(n-1)	x(n+1)
0	0.0	4.999000e-01	-2.500000e-01	-0.000200	0.100000	-1.249500e+03
1	1.0	-1.249500e+03	1.562500e+06	-2500.000100	-1249.999950	-6.245001e+02
2	2.0	-6.245001e+02	3.906249e+05	-1250.000250	624.999925	-3.120003e+02
3	3.0	-3.120003e+02	9.765616e+04	-625.000525	312.499863	-1.557505e+02
4	4.0	-1.557505e+02	2.441398e+04	-312.501062	156.249731	-7.762607e+01
5	5.0	-7.762607e+01	6.103432e+03	-156.252131	78.124466	-3.856463e+01
6	6.0	-3.856463e+01	1.525796e+03	-78.129266	39.061433	-1.903552e+01
7	7.0	-1.903552e+01	3.813864e+02	-39.071032	19.529117	-9.274157e+00
8	8.0	-9.274157e+00	9.528414e+01	-19.548313	9.761360	-4.399867e+00
9	9.0	-4.399867e+00	2.375870e+01	-9.799734	4.874290	-1.975444e+00
10	10.0	-1.975444e+00	5.877825e+00	-4.950889	2.424423	-7.882182e-01
11	11.0	-7.882182e-01	1.409506e+00	-2.576436	1.187226	-2.411424e-01
12	12.0	-2.411424e-01	2.992920e-01	-1.482285	0.547076	-3.922974e-02
13	13.0	-3.922974e-02	4.076871e-02	-1.078459	0.201913	-1.427010e-03
14	14.0	-1.427010e-03	1.429046e-03	-1.002854	0.037803	-2.030562e-06
15	15.0	-2.030562e-06	2.030566e-06	-1.000004	0.001425	-4.123164e-12
16	16.0	-4.123164e-12	4.123164e-12	-1.000000	0.000002	-1.700082e-23