Computational and Numerical Methods

Group 16

Set 8 (24-09-2018): Numerically Verify and Extend Theory Exercises

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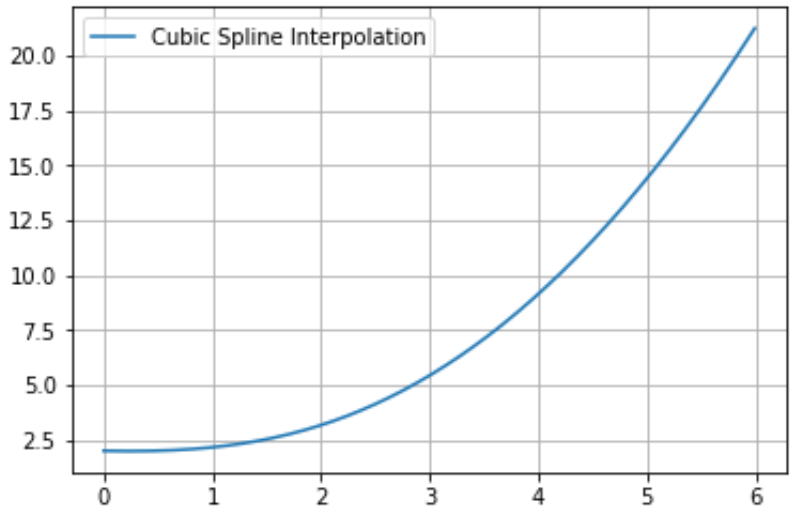
Show Code

Q1 Cubic spline interpolation

x = [0, 1, 2, 3, 4, 5, 6], y = [2, 2.1592, 3.1697, 5.4332, 9.1411, 14.406, 21.303]:



Q2 x = [-2, -1, 0] y = [-15, -8, -3]

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The Lagrangian polynomial is:
     2
-1 x + 4 x - 3
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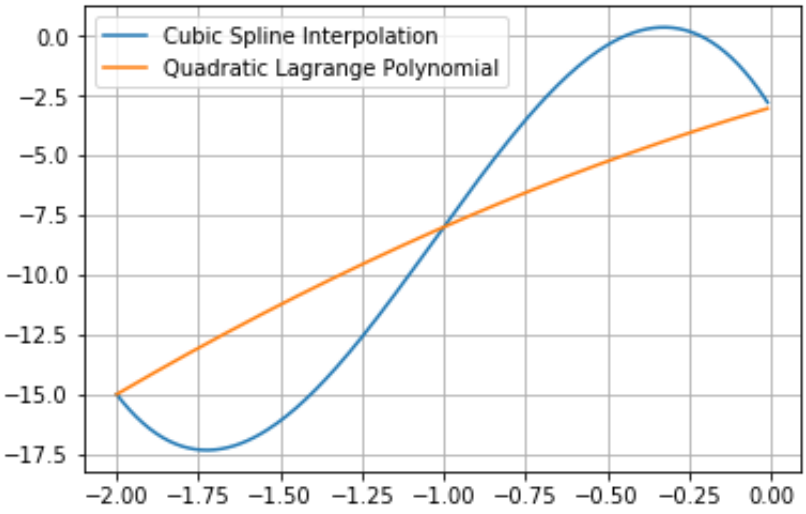

Q3 Tabulating n,T(n),S(n)

Q3.1.1 $e^x cos(4x)$ from 0 to π

Out[6]:

	n	T(n)	S(n)	TError	SError
1	2.0	26.516336	22.715077	25.213942	2.141268e+01
2	4.0	3.249050	-4.506711	1.946657	-5.809105e+00
3	8.0	1.624525	1.083017	0.322132	-2.193769e-01
4	16.0	1.375723	1.292788	0.073329	-9.605410e-03
5	32.0	1.320312	1.301842	0.017918	-5.520189e-04
6	64.0	1.306848	1.302360	0.004454	-3.379643e-05
7	128.0	1.303506	1.302392	0.001112	-2.101451e-06
8	256.0	1.302672	1.302394	0.000278	-1.311722e-07
9	512.0	1.302463	1.302394	0.000069	-8.195638e-09

Q 3.1.2 $x^{2.5}$ from 0 to 1

Out[5]:

	n	T(n)	S(n)	TError	SError
1	2.0	0.338388	0.284518	5.267406e-02	-1.196489e-03
2	4.0	0.298791	0.285593	1.307721e-02	-1.217400e-04
3	8.0	0.288975	0.285702	3.260454e-03	-1.179823e-05
4	16.0	0.286529	0.285713	8.142822e-04	-1.108410e-06
5	32.0	0.285918	0.285714	2.034940e-04	-1.020813e-07
6	64.0	0.285765	0.285714	5.086654e-05	-9.279914e-09
7	128.0	0.285727	0.285714	1.271601e-05	-8.363089e-10
8	256.0	0.285717	0.285714	3.178946e-06	-7.492451e-11
9	512.0	0.285715	0.285714	7.947314e-07	-6.685208e-12

Q 3.1.3 $\frac{1}{1+(x-\pi)^2}$ from 0 to 5

Out[6]:

	n	T(n)	S(n)	TError	SError
1	2.0	2.166655	2.625095	-0.173111	2.853292e-01
2	4.0	2.268668	2.302672	-0.071099	-3.709437e-02
3	8.0	2.332270	2.353471	-0.007496	1.370512e-02
4	16.0	2.337813	2.339660	-0.001953	-1.059309e-04
5	32.0	2.339277	2.339765	-0.000489	-1.079988e-06
6	64.0	2.339644	2.339766	-0.000122	-6.743238e-08
7	128.0	2.339736	2.339766	-0.000031	-4.216918e-09
8	256.0	2.339759	2.339766	-0.000008	-2.635945e-10
9	512.0	2.339764	2.339766	-0.000002	-1.647527e-11

Q3.2.1 e^{-x^2} from 0 to 10

Out[7]:

	n	T(n)	S(n)	TError	SError
1	2.0	2.500000	1.666667	1.613773e+00	7.804397e-01
2	4.0	1.254826	0.839768	3.685991e-01	-4.645882e-02
3	8.0	0.889428	0.767629	3.201278e-03	-1.185980e-01
4	16.0	0.886227	0.885160	-7.452838e-08	-1.067192e-03
5	32.0	0.886227	0.886227	-7.454724e-08	-7.455353e-08
6	64.0	0.886227	0.886227	-7.454724e-08	-7.454724e-08
7	128.0	0.886227	0.886227	-7.454724e-08	-7.454724e-08
8	256.0	0.886227	0.886227	-7.454724e-08	-7.454724e-08
9	512.0	0.886227	0.886227	-7.454724e-08	-7.454724e-08

Q3.2.2 $tan^{-1}(1 + x^2)$ from 0 to 2

Out[8]:

	n	T(n)	S(n)	TError	SError
1	2.0	2.186548	2.195798	0.012288	0.021538
2	4.0	2.177450	2.174418	0.003190	0.000158
3	8.0	2.175061	2.174265	0.000801	0.000005
4	16.0	2.174461	2.174261	0.000201	0.000001
5	32.0	2.174311	2.174261	0.000051	0.000001
6	64.0	2.174274	2.174261	0.000014	0.000001
7	128.0	2.174264	2.174261	0.000004	0.000001
8	256.0	2.174262	2.174261	0.000002	0.000001
9	512.0	2.174261	2.174261	0.000001	0.000001

Q4.1.1 $tan^{-1}(x^2 - x + 1)$

Out[9]:

	h	FD	CD	FDError	CDError
0	0.10000	0.520855	0.495856	0.020855	-0.004144
1	0.05000	0.511460	0.498960	0.011460	-0.001040
2	0.02500	0.505990	0.499740	0.005990	-0.000260
3	0.01250	0.503060	0.499935	0.003060	-0.000065
4	0.00625	0.501546	0.499984	0.001546	-0.000016

Analytical derivative at x = 1 is 0.5

Q4.1.2 $tan^{-1}(100x^2 - 199x + 100)$

Out[10]:

	h	FD	CD	FDError	CDError
0	0.10000	3.409790	0.200294	2.909790	-0.299706
1	0.05000	2.594051	0.390426	2.094051	-0.109574
2	0.02500	1.675666	0.469776	1.175666	-0.030224
3	0.01250	1.109328	0.492262	0.609328	-0.007738
4	0.00625	0.808388	0.498054	0.308388	-0.001946

Analytical derivative at x = 1 is 0.5