INTRO TO ROBOTICS (Lec 1)

→ Robot

* Mechanical Moving Part.
* Electrical Actuation.
* Some autonomy. [usually sensing and actuation based on the code]

TYPES OF ROBOTS

- Manipulator
  - Serial
  - Parallel.
- Mobile
- Aerial
- Higher-level Robots.

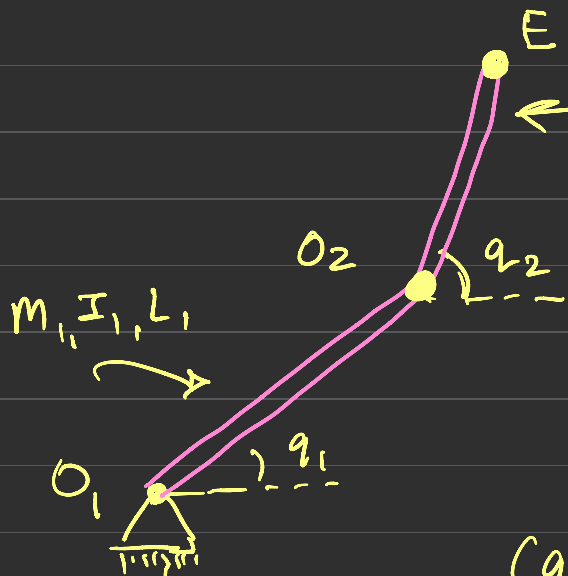
2 R Manipulator

[also called planar - elbow manipulator)

2R = 2 Revolute Joints.

Joints → Revolute (R), Prismatic (P)

2R Manipulator



E = End effector means end of the robot.

E(x, y) = end effector coordinates

(q_1, q_2) - joint angles.

Assume origin at O_1.

→ Note convention of q_2.

→ let us assume that motors are connected to both joints O_1 and O_2 and and we have the ability to control either

τ_1 and τ_2 applied on this joints or q_1, q_2 at which this arms are.

let us consider 3 tasks

Task 1 :- Given arbitary trajeetory of end effector (given x and y wrt time) make the robot follow the trajectory.

Task 2: Given a location of a wall, make the robot touch a wall and apply a constant predefined force on the wall.

Task 3: Make a robot behave like a virtual spring (that has stifness k and connects E to a given point (x_0, y_0)].

→ Now,

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2 .$$

→ Simplified notation :-

$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2 \end{aligned}\right\} - ①$$

[forward kinematics]

$\boxed{\text{Differentiating :-}}$

$$\left.\begin{aligned} \dot{x} &= -l_1 \sin q_1 \dot{q}_1 - l_2 \cos q_2 \dot{q}_2 \\ \dot{y} &= l_1 \cos q_1 \dot{q}_1 + l_2 \cos q_2 \dot{q}_2 \end{aligned}\right\} \boxed{\text{End effector Velocity.}}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - ②$$

$\boxed{\text{① and ② are combined called forward kinematics.}}$

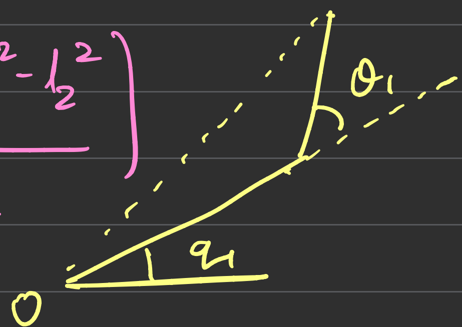
→ We will also need the reverse relationship. Given x & y, we need to able to solve for q_1 and q_2.

→ Option 1 :- Solve numerically

Option 2 :- Derive closed form expression.

- Hard in general.

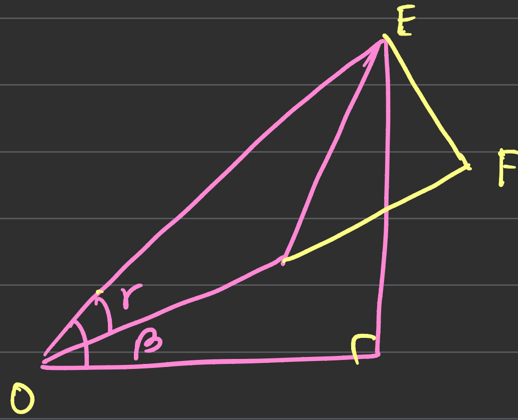
$$\theta_1 = \cos^{-1}\left(\frac{x_1^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$



$$q_1 = \beta - q_2$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta_1}{l_1+l_2\cos\theta_1}\right)$$

$$\tan^{-1}\left(\frac{EF}{OF}\right)$$

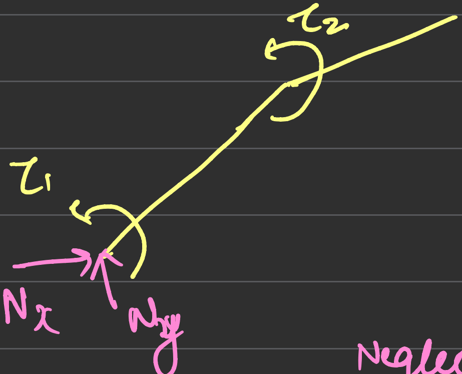


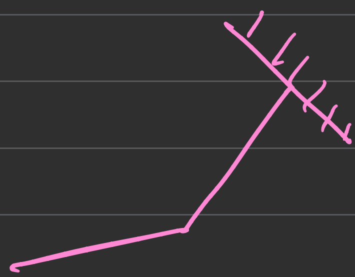
$$q_2 = q_1 + \theta \quad -(3)$$

First level answer to T1,

- Control both motors in position control mode to achieve above q_1 and q_2 at each time-step.

<u>Task 2:</u> FBD of entire robot.

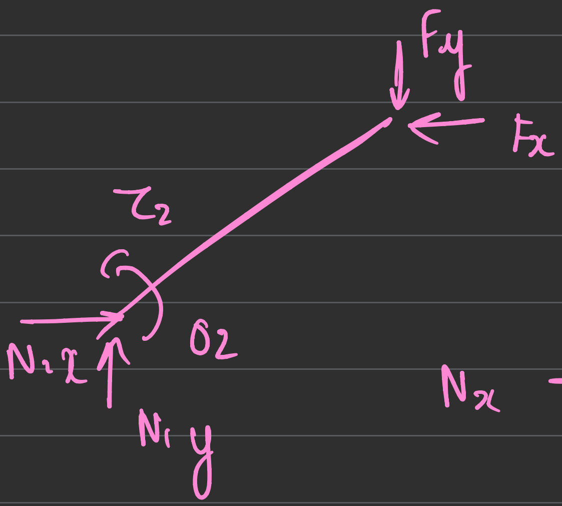


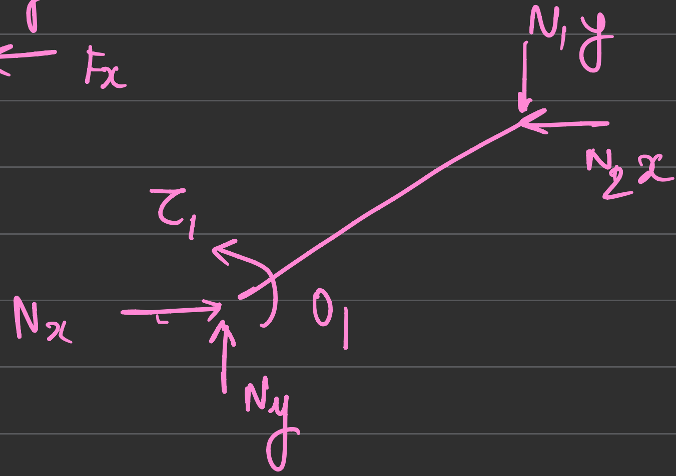


Neglect gravity.

FBD of link 2

FBD of link 1.





(*). <u>Static</u> <u>Equillibrium.</u>

→ $\Sigma M_{O_2} = 0$ & $\Sigma M_{O_1} = 0$

$$\boxed{\begin{aligned} F_y l_2 c q_2 - F_x l_2 s q_2 &= \tau_2 \\ F_y l_1 c q_1 - F_x l_1 s q_1 &= \tau_1 \end{aligned}} \quad - (4)$$

(3) and (4) answers τ_2

$$\begin{bmatrix} -l_2 s q_2 & l_2 c q_2 \\ -l_1 s q_1 & l_2 c q_1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \tau_2 \\ \tau_1 \end{bmatrix}$$

→ Dynamic Effects

→ $L = K - V$

→ $$\boxed{\frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) - \frac{dL}{dq_i} = Q_i\text{'s}.}$$

→

q_{i_s} are independent degrees of freedom generalised coordinates.

Q'_{i_s} = are generalised forces derived using principle of virtual work.

$i_s \to$ 1, 2, ... number of degree of freedom.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3}m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of } L_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12}m_2 l_2^2\right)\dot{q}_2^2}_{\text{rotation of link } L_2} + \underbrace{\frac{1}{2}m_2 v_{c_2}^2}_{\text{traslation of}}$$

$$V = m_1 g \frac{l_1}{2} sq_1 + m_2 g\left(l_1 sq_1 + \frac{l_2}{2} sq_2\right)$$

$$L = K - V$$

$$\frac{1}{3}m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 c(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} (\dot{q}_2)(\dot{q}_2 - \dot{q}_1)(s(q_2 - q_1))$$

$$+ m_1 g \frac{l_1}{2} cq_1 + m_2 g l_1 cq_2 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 l_1 l_2 \ddot{q}_1 c(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) s(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2$$

$$= \tau_2$$

→ Captures the dynamic effects

This are the equation of motion.

$$v_c^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 c(q_2 - q_1)$$

Task - 3

→ (4) is valid for any end-effector forces F_x & F_y.

$$F_x = kx \qquad \left[\text{more generally} \quad F_x = k(x - x_0) \right.$$
$$F_y = ky \qquad \left. \quad F_y = k(y - y_0) \right]$$

Then using (1) and (4)

$$k(l_1 sq_1 + l_2 sq_2) l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2 = \tau_2$$

$$k(l_1 sq_1 + l_2 sq_2) l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) l_1 sq_1 = \tau_1$$

↳ (7) This is the answer to T_3.