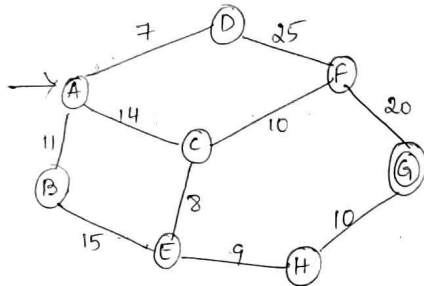


8/12/23 Heuristic Search Techniques (uninformed Search)

* Best first

* A* algorithm.

→ Best first Search Alg



States	$h(n)$
A	40
B	32
C	25
D	35
E	19
F	17
G	10
H	0

$g(n)$ → cost from start state to any node

$h(n)$ → heuristic value
↳ cost from any node to goal state

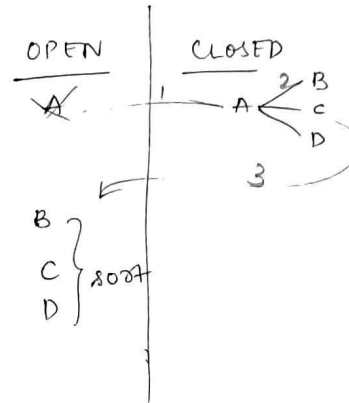
⊗ Heuristic value of goal node is always 0

1) push start state to open list assigned to NODE

2) check goal or not

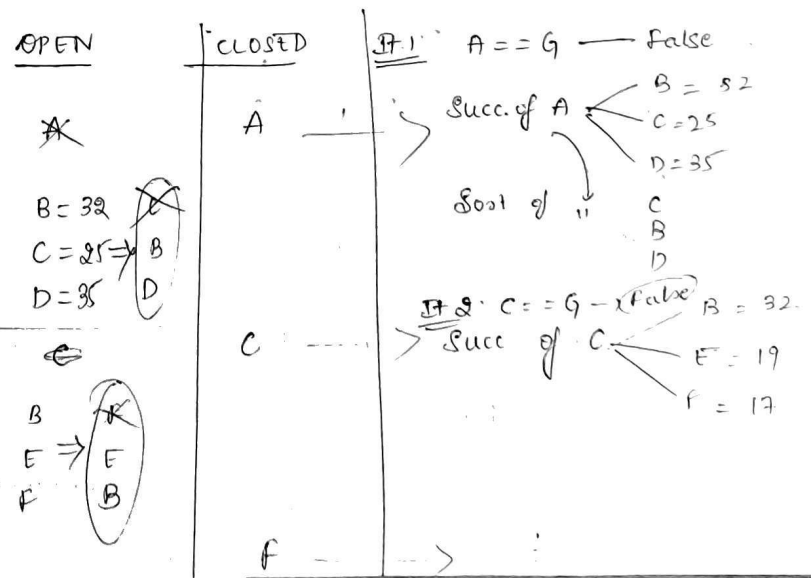
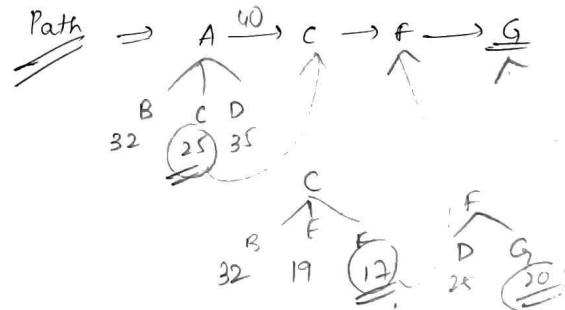
3) CLOSED & expand
B C D } sorted acc. to heuristic value, not cost

(Only diff in Best first Search as compared to Uniform Cost Search ⇒ is sorted acc. to heuristic value & not cost)



Evaluation function:

$$f(n) = g(n) + h(n)$$



A* Algorithm

(8 puzzle prob using " algo)

Type 1

3	7	6
5	1	2
4	□	8

Start

5	3	6
7	□	2
4	1	8

Goal

$h(n) = 4$
(in 1st iteration)

$g(n) \rightarrow$ level no. [for start state $\Rightarrow g(n) = 0$]
 $h(n) \rightarrow$ no. of places/positions } for ex:
 are misplaced } in 1st iteration \Rightarrow (by comparing it with goal)

$$f(n) = \underbrace{0+4}_{g(n)+h(n)}$$

\Rightarrow Soln.

0

3	7	6
5	1	2
4	□	8

$$\begin{aligned} g(n) &= 0 \\ h(n) &= 4 \\ f(n) &= 0+4 = 4 \end{aligned}$$

3	7	6
5	□	2
4	1	8

$$\begin{aligned} f(n) &= g(n) + h(n) \\ &= 1 + 3 \end{aligned}$$

$$f(n) = 4$$

least

3	7	6
5	1	2
□	4	8

$$= 1 + 5$$

$$f(n) = 6$$

3	7	6
5	1	2
4	8	□

$$= 1 + 5$$

$$f(n) = 6$$

least $f(n)$
expand that

3	□	6
5	7	2
4	1	8

$$\begin{aligned} f(n) &= g(n) + f(n) \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

3	7	6
5	1	2
4	□	8

$$\begin{aligned} f(n) &= g(n) + f(n) \\ &= 2 + 4 \\ &= 6 \end{aligned}$$

3	7	6
□	5	2
4	1	8

$$\begin{aligned} &= 2 + 3 \\ &= 5 \end{aligned}$$

3	7	6
5	2	□
4	1	8

$$\begin{aligned} &= 2 + 5 \\ &= 7 \end{aligned}$$

3	7	6
5	□	2
4	1	8

$$3 + 3 = 6$$

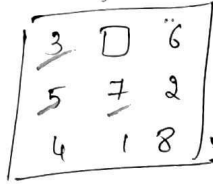
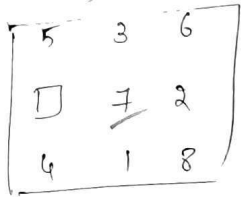
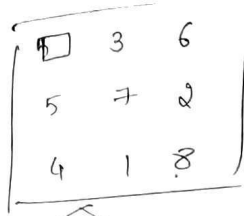
□	3	6
5	7	2
4	1	8

$$3 + 2 = 5$$

3	6	□
5	7	2
4	1	8

$$3 + 4 = 7$$

$$f(n) = 3$$



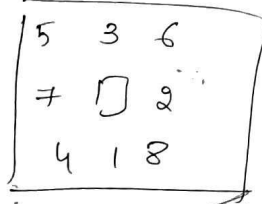
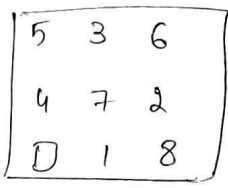
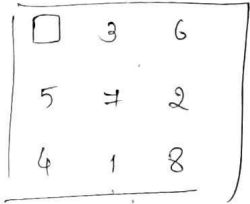
$$g(n) = 4$$

$$= 4 + 1$$

$$= 4 + 3$$

$$= 5$$

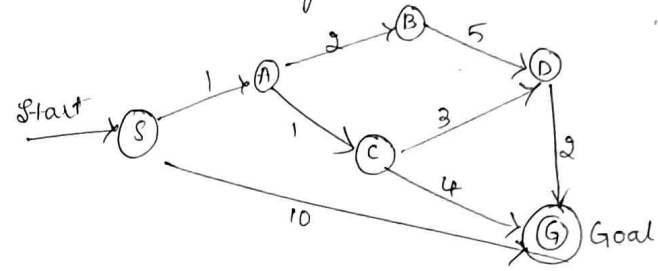
$$= 7$$



Goal state $g(n) = 5$

Type 2

here $g(n) \rightarrow$ edge cost
 $h(n) \rightarrow$ given.

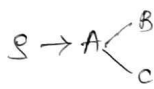


f(n)	States	h(n)
	S	5
	A	3
	B	4
	C	2
	D	6
	G	0

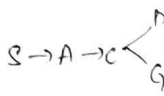
$$\Rightarrow f(n) = g(n) + h(n)$$

$$\left. \begin{aligned} S \rightarrow A &\Rightarrow f(n) = g(n) + h(n) \\ &= 1 + 3 \\ &= 4 \end{aligned} \right\} \rightarrow \text{least}$$

$$\left. \begin{aligned} S \rightarrow G &\Rightarrow f(n) = 10 + 0 \\ &= 10 \end{aligned} \right\}$$



$$\left. \begin{aligned} S \rightarrow A \rightarrow B &\Rightarrow f(n) = 2 + 4 = 6 \\ S \rightarrow A \rightarrow C &\Rightarrow f(n) = 1 + 2 = 3 \end{aligned} \right\} \rightarrow \text{least}$$



$$\left. \begin{array}{l} S \rightarrow A \rightarrow C \rightarrow D \\ S \rightarrow A \rightarrow C \rightarrow G \end{array} \right\} \Rightarrow f(n) = 3+6 = 9$$

$$\left. \begin{array}{l} S \rightarrow A \rightarrow C \rightarrow D \\ S \rightarrow A \rightarrow C \rightarrow G \end{array} \right\} \Rightarrow f(n) = 4+0 = 4 \text{ (least)}$$

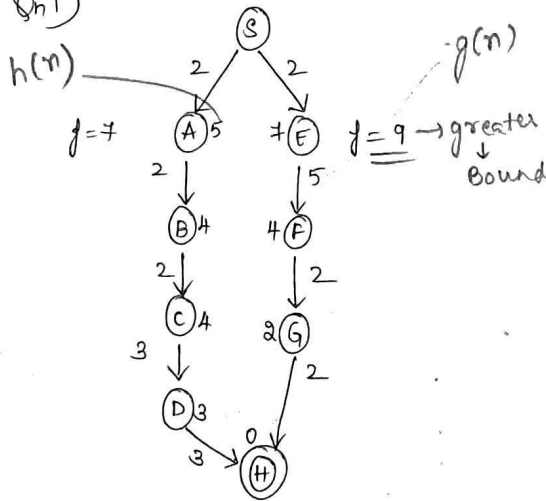
Goal state (so stop)

(if heuristic not used $\Rightarrow S \rightarrow A \rightarrow C \rightarrow D \rightarrow G$ long)

11/12/23

* Recursive Best first search (RBFS)

Qn 1)



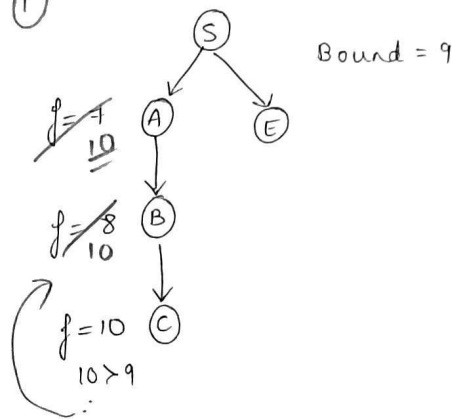
$f(n) \Rightarrow F\text{-score}$
 $f(n) = g(n) + h(n)$

A $f = 7$
 B $f = 2+2+4 = 8$
 C $f = 10$
 D $f = 2+2+2+3+3 = 12$
 E $f = 9$
 F $f = 5+2+4 = 11$
 G $f = 2+5+2+2 = 11$

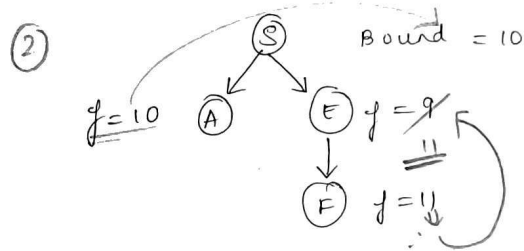
everytime expand
 ↓
 compared with
 Bound value

if $f < \text{Bound} \rightarrow \text{expand}$

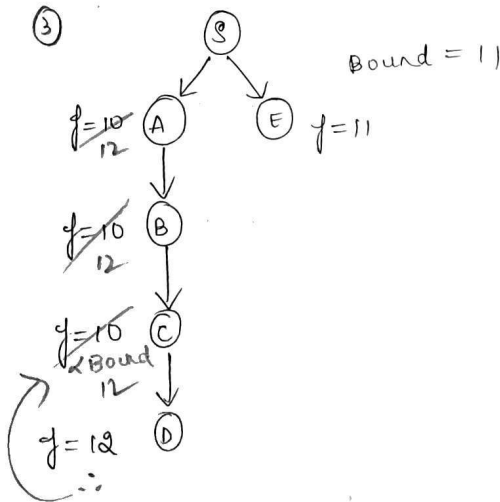
①



②

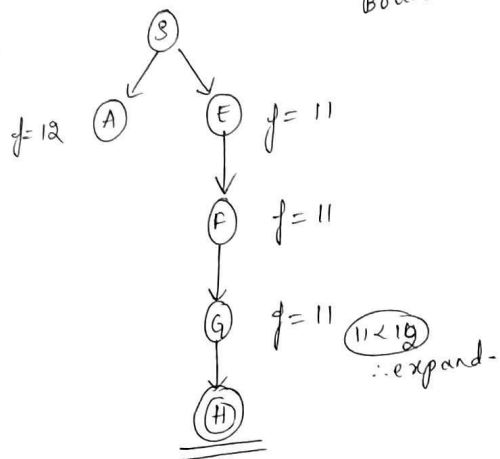


③

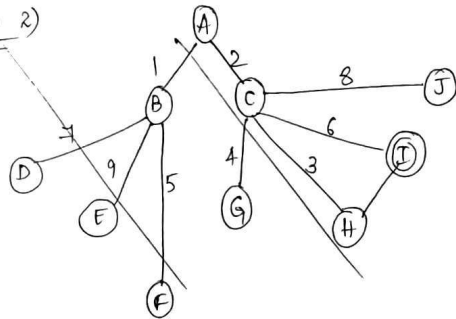


Bound = 12

(11)



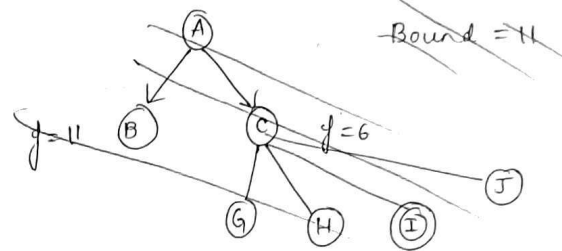
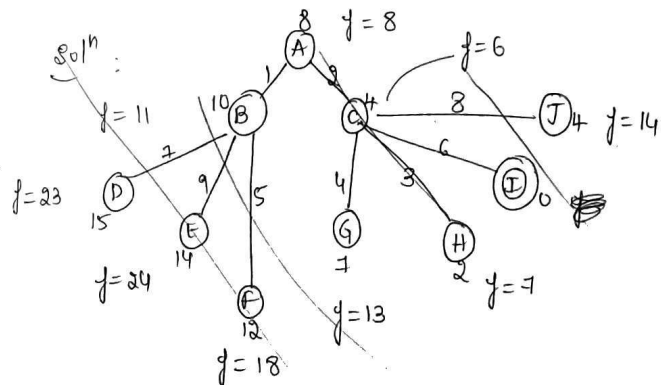
Qn 2)



$h(n)$

- A - 8
- B - 10
- C - 4
- D - 15
- E - 14
- F - 12
- G - 7
- H - 2
- I - 0
- J - 4

Soln:



Bound = 11

Qn. on A* Algo 8 puzzle exp.

(1)

7	2	4
5		6
8	3	1

	1	2
3	4	5
6	7	8

(2)

1	2	3
8		4
7	6	5

2	8	1
	4	3
4	6	5

1) Soln?

Start State:

Goal State

7	2	4
5		6
8	3	1

	1	2
3	4	5
6	7	8



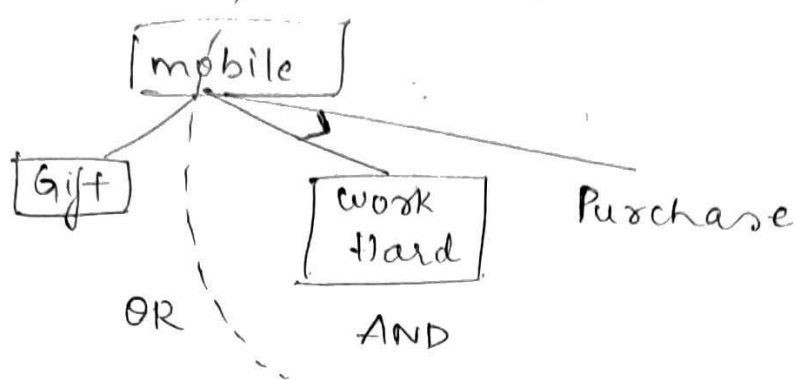
Qn^{vvv} Consider the foll. prob:

A farmer wants to get a lion, a fox & a goose & some corn across the river, there is a boat but he can only take one in addition to himself on each trip. or else both the goose & the corn ~~&~~ or both the fox & the corn. The corn cannot be left with goose, similarly as it will eat the corn. Similarly the fox can eat goose & also lion cannot be left with the fox. How does everything get across the river. ~~Assume~~ Assume animals do not wander off when left alone

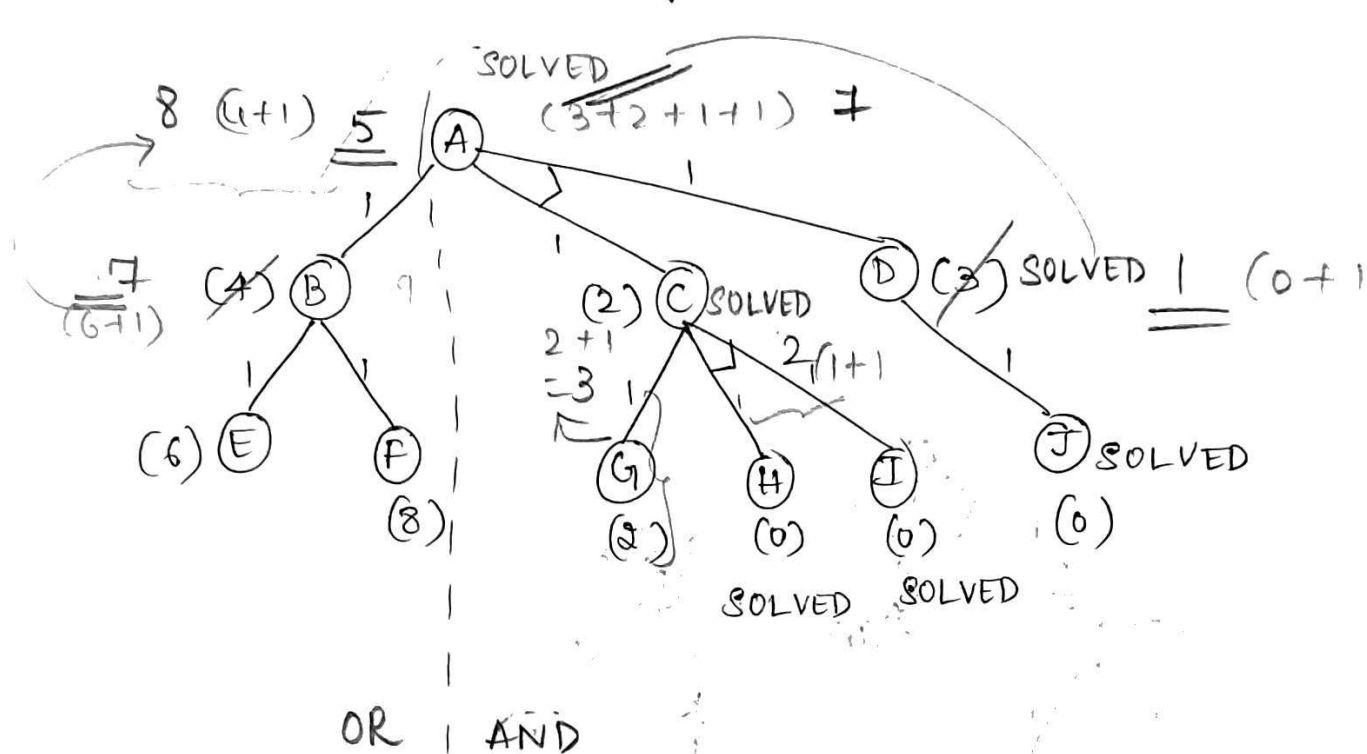
- 1) Give start & goal states along with constraints
- 2) Draw state space search trees using DFS & BFS to find first solution.

15/12/23

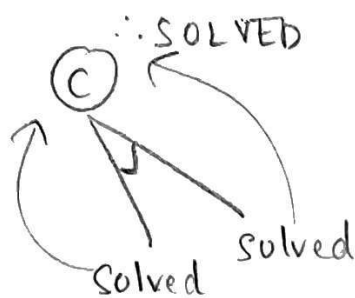
A* algorithm \rightarrow explained using AND-OR graph



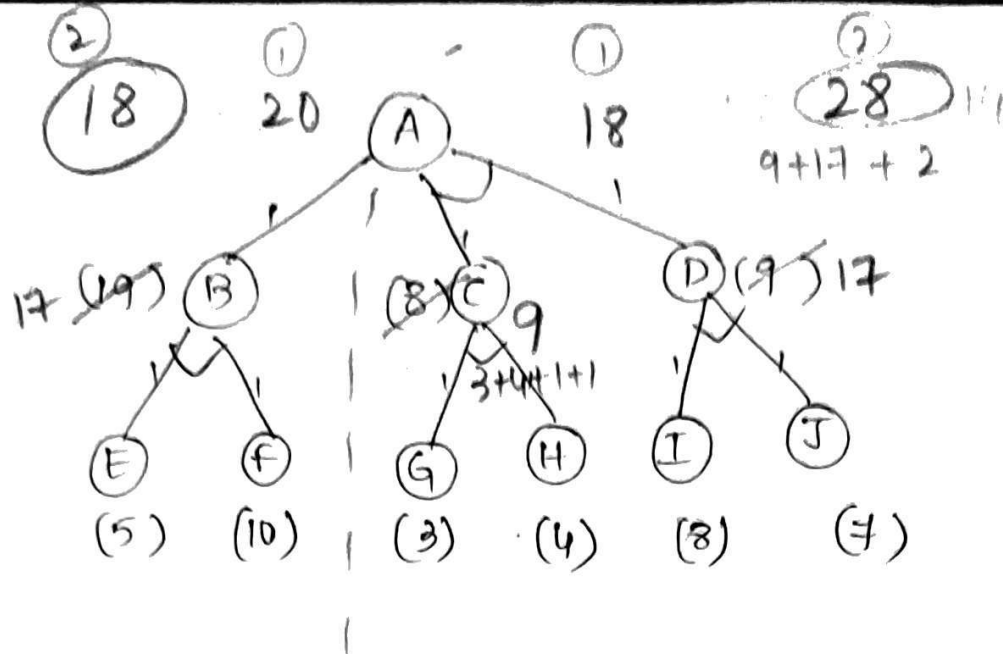
edge values $\rightarrow g(n) \rightarrow$ default taken as 1



$h(n) \rightarrow 0$
 \downarrow
Goal node
 \rightarrow label it
as SOLVED



4)



1
1
9
28

Constraint Satisfaction - Crypt - Arithmetic Puzzle

0-9 digits to be assigned
 $8+9 \text{ max} \rightarrow \text{carry}$ ✓

be always 1

$$\begin{array}{r} \text{T O} \\ + \text{G O} \\ \hline \text{O U T} \end{array}$$

$$\begin{array}{r} \square \square \\ \square \square \\ \hline \square \square \square \end{array}$$

left most \rightarrow assign 1 (carry)

$$\begin{array}{r} \downarrow \\ \begin{array}{r} \square \square \\ + \Rightarrow \square \square \\ \hline \square \square \square \end{array} \end{array}$$

if it is 9 $\rightarrow 2+9$

$$\begin{array}{r} \square \square \\ \downarrow \\ \text{OX} \end{array}$$

+1 \Rightarrow carry $\Rightarrow 10$ $\Rightarrow 9+1$

2, 3, 4

$$\begin{array}{r} \text{S E N D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

\Rightarrow

$$\begin{array}{r} \begin{array}{r} \square \square \square \square \\ \square \square \square \square \\ \hline \square \square \square \square \end{array} \end{array}$$

17 N

3)

$$\begin{array}{r} \text{B A S E} \\ + \text{B A L L} \\ \hline \text{G A M E S} \end{array}$$

\Rightarrow

$$\begin{array}{r} \square \square \square \square \\ + \square \square \square \square \\ \hline \square \square \square \square \square \end{array}$$

$$\begin{array}{r} \begin{array}{r} \square \square \square \square \\ + \square \square \square \square \\ \hline \square \square \square \square \square \end{array} \end{array}$$

444x

5
6
7
8
9

$$\begin{array}{r} \begin{array}{r} \square \square \square \square \\ + \square \square \square \square \\ \hline \square \square \square \square \square \end{array} \end{array}$$

✓

0 1 x 2 3 4 x 5 6 7 x 8 9

5/1/24.

Game playing -

Nim Game Problem

Game: $\sqrt{\quad}$

if j is a non-terminal MAX node then; WIN - taken
if any are WIN WIN

j is a non-leaf node

$status(j) = \begin{cases} \text{WIN} & \text{- if any child is WIN} \\ \text{LOSS} & \text{- if all children are LOSS} \\ \text{DRAW} & \text{- if any child is DRAW} \end{cases}$

& NONE is WIN

if j is a non-terminal MIN node then

Status(i,j) =
$$\begin{cases} \text{WIN} & \text{if all are WIN} \\ \text{LOSS} & \text{if any is LOSS} \\ \text{DRAW} & \text{if any is DRAW \& None is LOSS} \end{cases}$$

Status Labeling Procedure

[illegible]


MAX player
(computer)

MINI Player

✓ pile of match sticks

can take half of pile (maximum)

✓ Goal: MAX should win

✓ Goal: Max should win
if MIN is first player \rightarrow  \Rightarrow Max Lost it means

NOTE Irrespective who plays first \Rightarrow the root node label is MAX's status

✓ players take alternate turns


✓ labeling \rightarrow done in bottom up approach

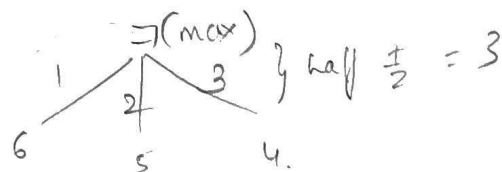
✓ whole game \rightarrow MAX's point of view \Rightarrow MAX should win

~~Level Complete~~

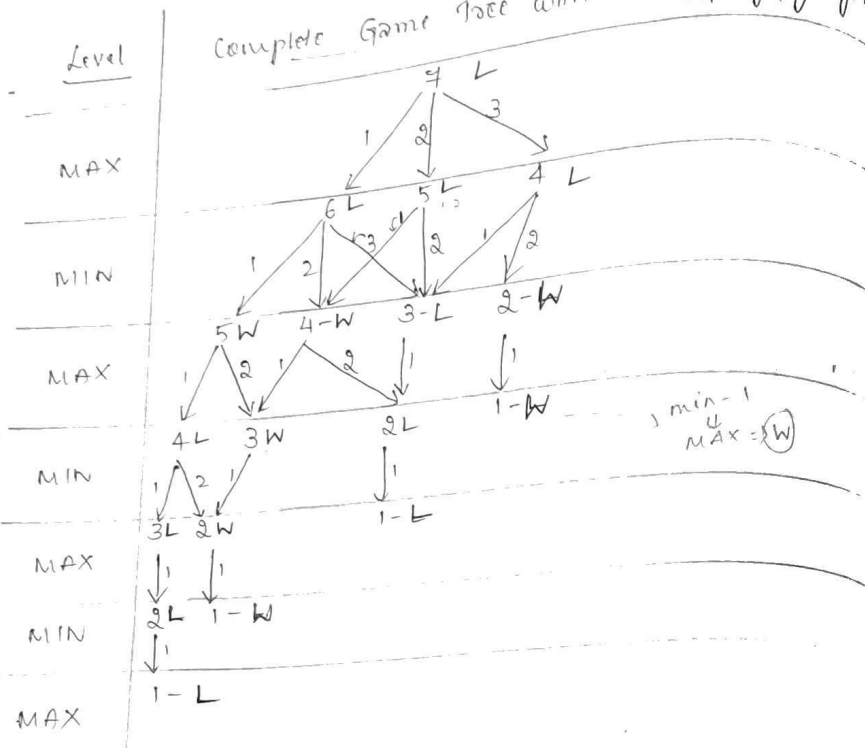
1 stick left \rightarrow loss.

$\int \text{MIN Las} \Rightarrow \textcircled{W}$

i) MAX has \Rightarrow 



Complete Game Tree with MAX playing first



Case 1: MAX is the first player &

$N \notin \{3, 7, 15, 31, 63, \dots\}$ matchsticks

Case 2: MAX is the second player &

$N \in \{3, 7, 15, 31, 63, \dots\}$ matchsticks

①

$N = \{3, 7, 15, 31, 63, \dots\}$

2 strategies

① Binary rep

②

③

Case-1

$N = 29$

$N = \{3\}$

MAX - first player

2 Cases

16 8 4 2 1
1 1 1 0 1
0 1 1 0 1
1

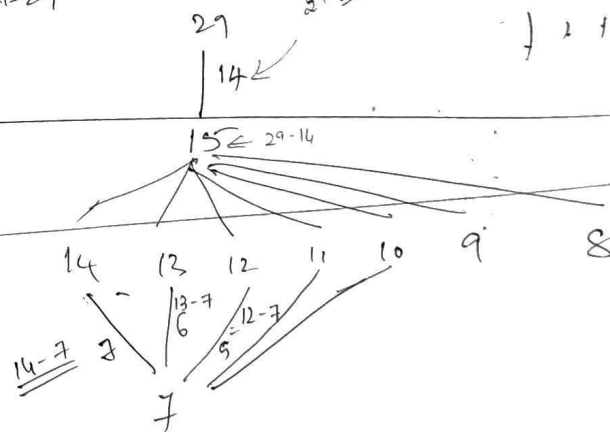
Case 1

$N = 29$

Max

Min

Max



$N = 29$

half

① Binary Rep
 $N = 13$

1101

Remove MSB & add.

0101

0001

0111

8 4 2 1

② $\Rightarrow N = \{3, 7, 15, 31, 63, \dots\}$

$N = 13 \Rightarrow$ subtract nearest

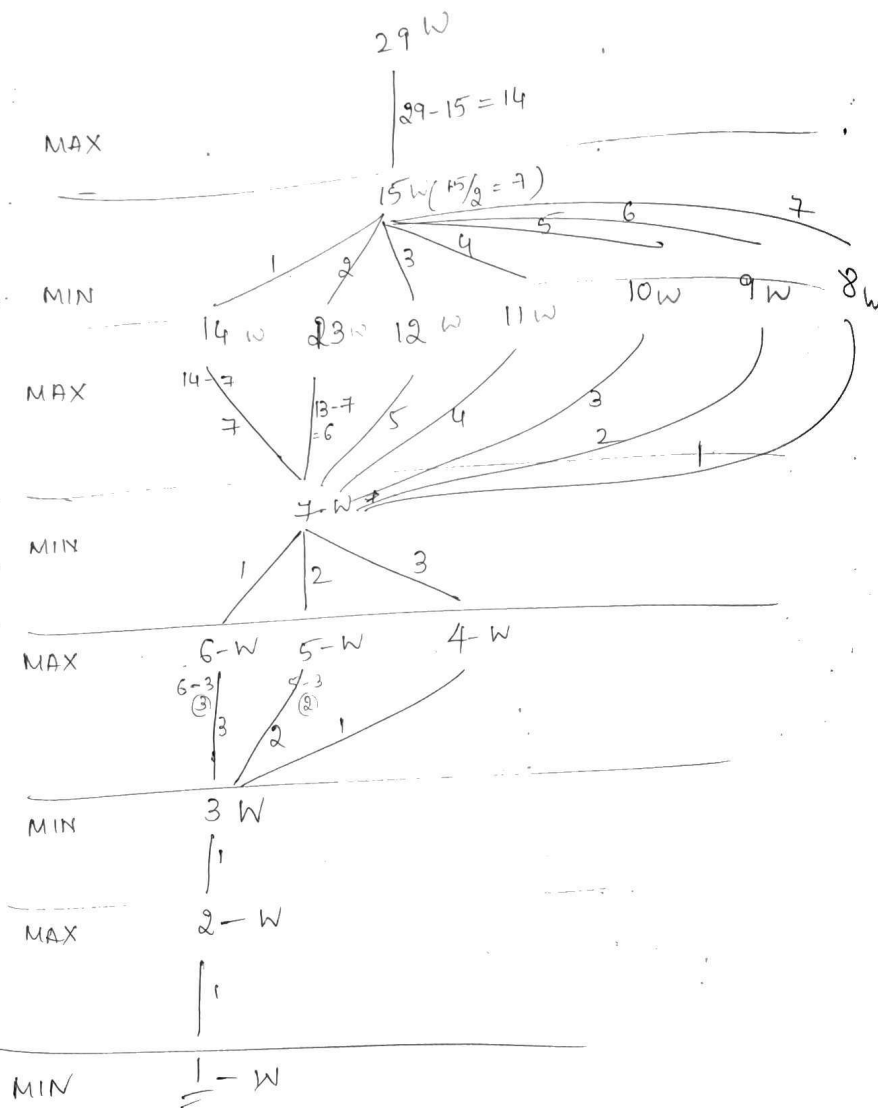
$13 - 7 = 6$

2 strategies
(do it for max)

MIN \rightarrow half & distribute

Solⁿ: Given : $N=29$
 ~~$N=13$~~ $N=1$ }

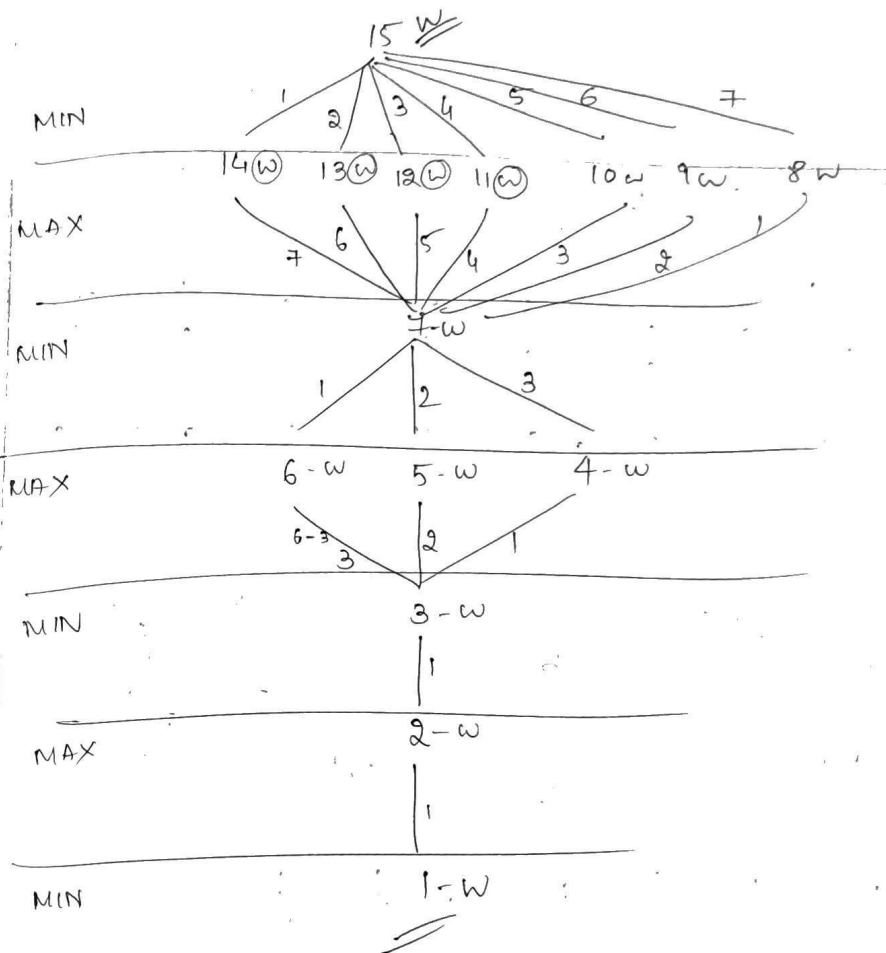
~~$N=29$~~ $\{1\}$ \rightarrow \therefore MAX is the first player.



2

$N=15$

Case 2.



$$B \rightarrow M, N$$

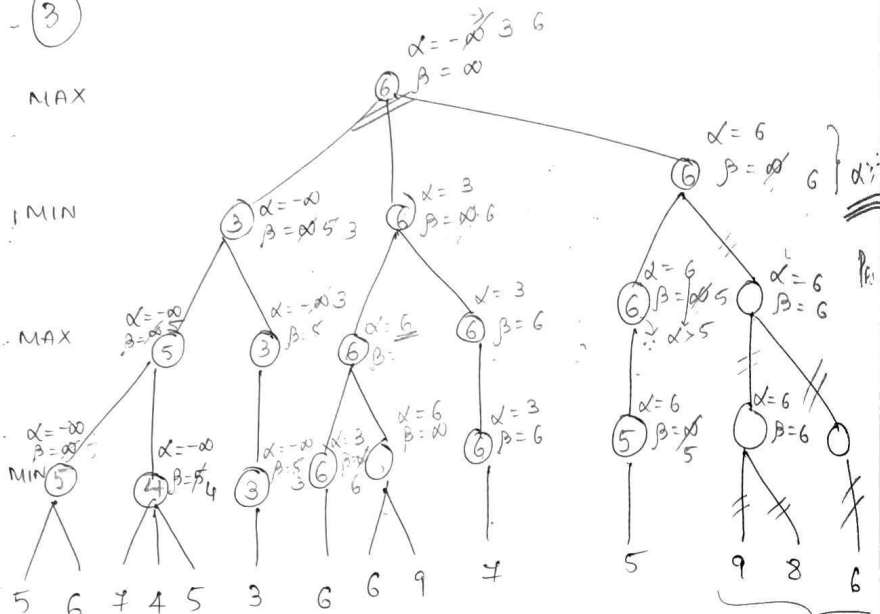
3

MAX

1 MIN

MAX

Min



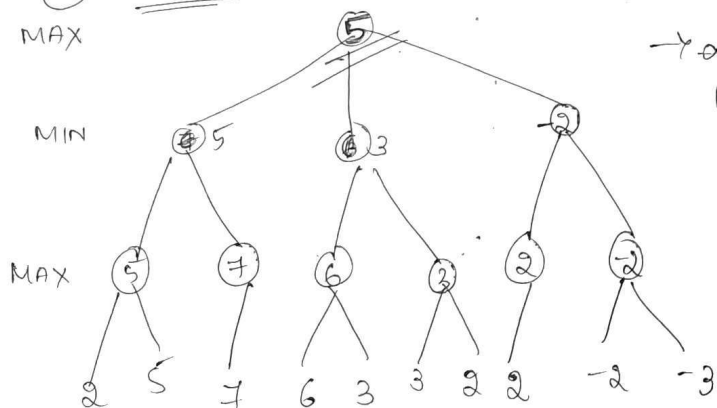
a

MINIMAX

MAX

MIN

MAX




• if $\alpha \geq \beta \rightarrow$ prime

↳ delete that branch

- MAX $\rightarrow \infty$

- MIN $\rightarrow \beta$

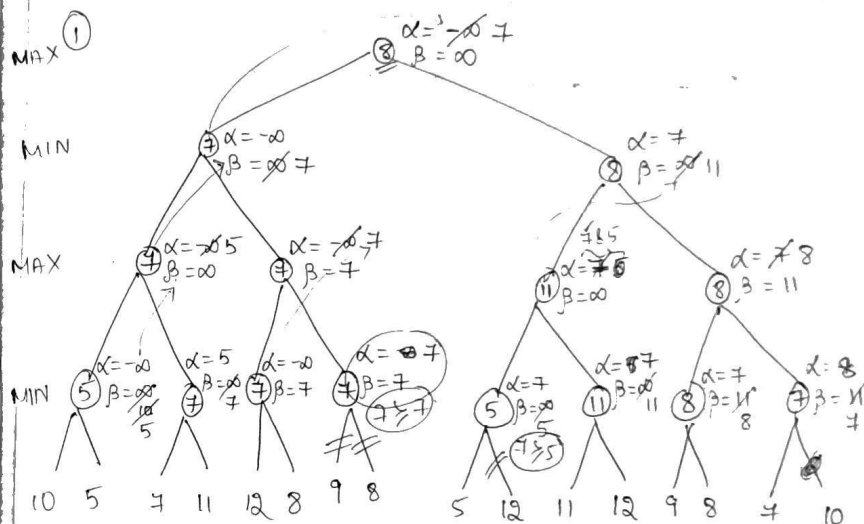
- ° while deciding parent node \Rightarrow  check no in child nodes
don't take α, β

max ①

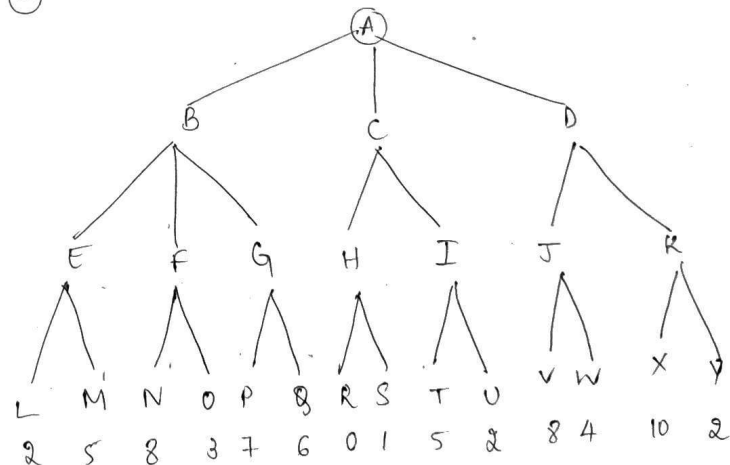
MIN

MAX

MIN

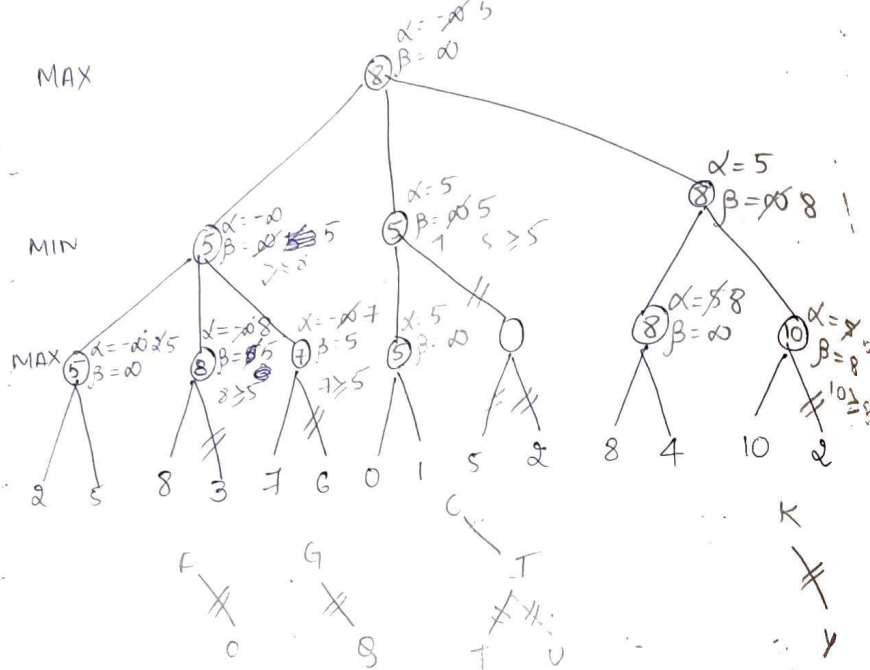


②



MAX

MIN



Constraint Satisfaction

1) R O A D S

+ C R O S S

D A N G E R

2) D O N A L D

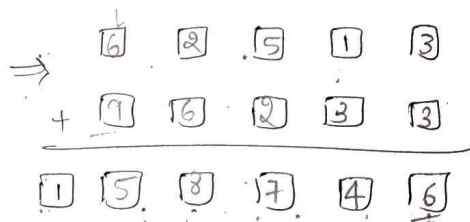
+ G E R A L D

R O B E R T

1) Solⁿ: R O A D S

+ C R O S S

D A N G E R



16/1/24.

Block World Problem

⇒

b
a c d
Start State

a b
c d
Goal state

arm can manipulate ^{one} block at a time
block 'a' ... block 'd' are all of same size
pick b → put it on top of d
pick a → put it on top of c

⇒ Operations:

UNSTACK → US(X, Y) { pick X which is on top of Y
arm → free
STACK → S(X, Y) { place X on Y
arm → should hold X
PICKUP → PU(X) { X on table
arm should pick up.
PUTDOWN → PD(X)

⇒ Predicates ^{X status} X is on Y

ON(X, Y) → O(X, Y)

ONTABLE(X) → T(X) { a, c, d (c, b)

CLEAR(X) → C(X) { b, c, d } clear ⇒ no n/kg on its top

HOLDING(X) → H(X) { robot arm is holding

ARMEMPTY → AE { empty

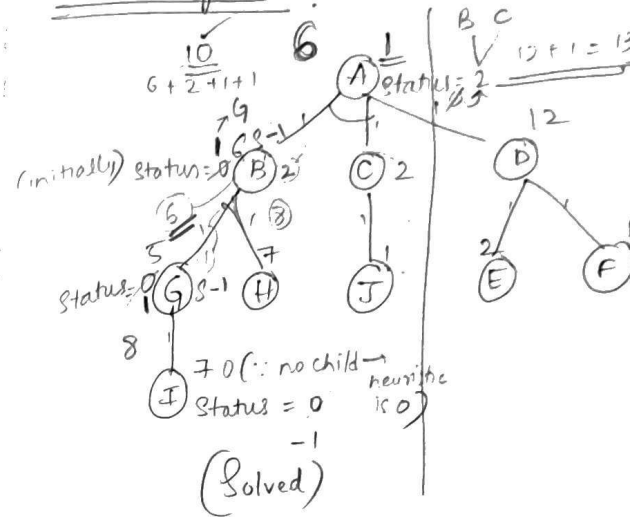
⇒ Operators with Precedence & Associativity

Operator	Pre List	Del List	Add List
ST(x, y)	$C(y) \wedge H(x)$	$C(y) \wedge H(x)$	$AE \wedge O(x, y)$
US(x, y)	$O(x, y) \wedge C(x)$	$O(x, y) \wedge AE$	$H(x) \wedge C(y)$
PU(x)	$T(x) \wedge C(x)$	$T(x) \wedge AE$	$H(x)$
PD(x)	$H(x)$	$H(x)$	$T(x) \wedge AE$

child be empty to pick it up from stack

child be clear you have to pickup x

Ao* Algo



(Solved)

status = 0 (default) for all nodes
 ↓
 no. of child nodes

goal ⇒ make $\underline{\underline{status = -1}}$ if (no child node)

(Solve in iterations)
 ↓
 write graph

