Vidisha Purohit
102103838
3CO28

FREEMIND

Date

Page

1 Let $(x_1 x_2 \dots x_n)$ be sample of size $\underline{\underline{n}}$
where
mean $\rightarrow \theta_1$
Variance $\rightarrow \theta_2$

$$L(\theta_1 \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_i)^2}{2\theta_2}}$$

Taking log both sides

$$\log(L(\theta_1\theta_2)) = -\frac{n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_1}\sum_{i=1}^{n}(x_i-\theta_i)^2$$

Differentiate log (L)
w.r.t (θ_1) & set it to zero

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_i) = 0$$

$$\boxed{\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

Ily for θ_2 diff w.r.t θ_2 & put zero

$$\boxed{\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_i)^2}$$

2) Binomial Distribution

m = number of trials
θ = (0,1) prob. of success

PMF $\quad b(x, n, \theta) = {}^{n}C_{x}\, \theta^{n} (1-\theta)^{m-n}$

$$L(\theta) = \prod_{i=1}^{n} ({}^{n}C_{x}) \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking log both sides

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{m} (n - x_i) = 0$$

$$= \frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{m} (m - x_i)$$

Multiply by $\theta(1-\theta)$

$$= (1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{m} (m - x_i)$$

$$\Rightarrow \boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{m}}$$