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Scale invariance of the dynamical rules governing neural systems

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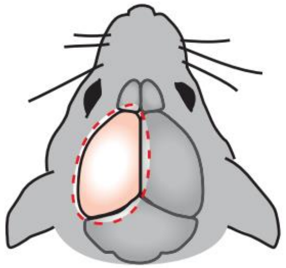
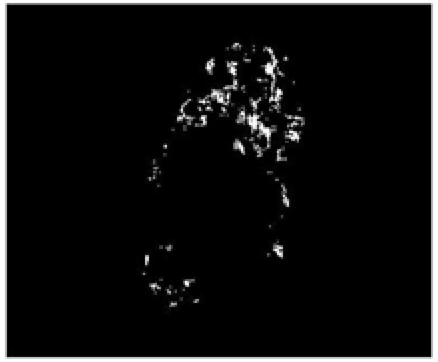
Outline

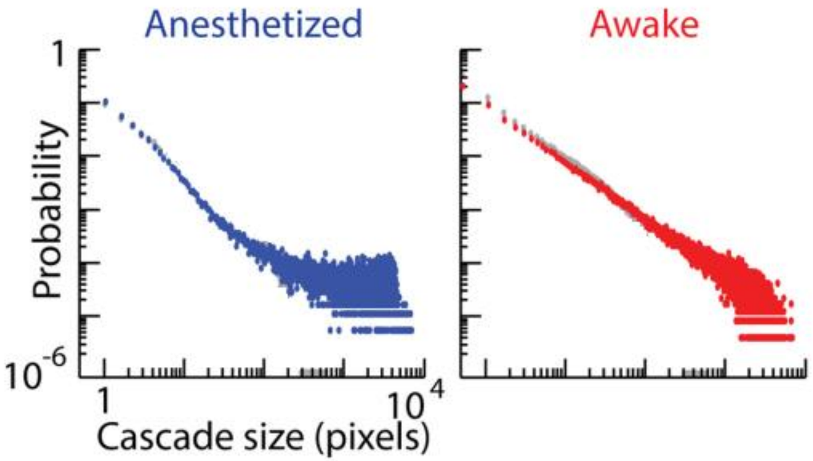
- Motivation
- Hypothesis
- Neural Network Model
- Coarse-grainning transformation scheme
- Results
- Realistic Neural Network Model
- Application on Experimental Data
- Conclusions

Motivation

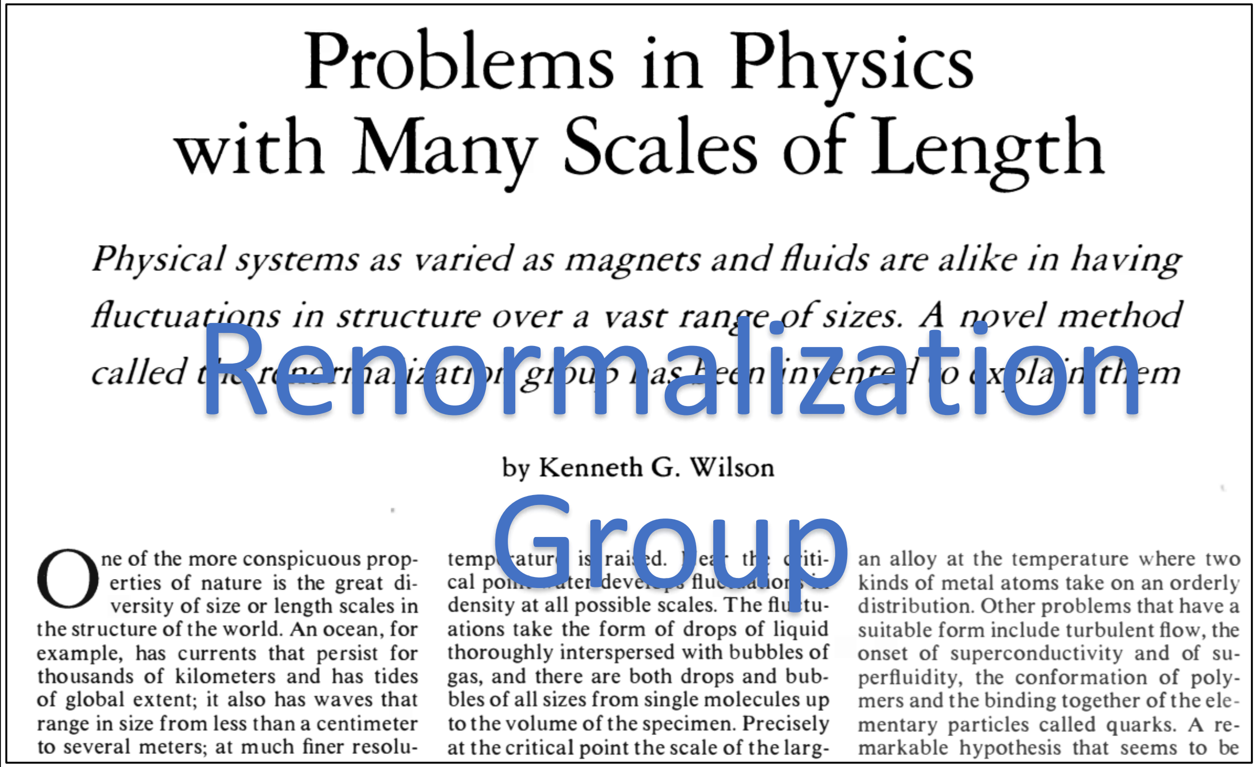
Fact 1: Experiments suggest that under certain conditions cerebral cortex operates near criticality, exhibits scale-invariant statistics[1].







Fact 2: In physical systems (e.g. Ising Model), the **rules** that govern the dynamics are also scale invariant, at criticality.



Problems in Physics with Many Scales of Length

Physical systems as varied as magnets and fluids are alike in having fluctuations in structure over a vast range of sizes. A novel method called the renormalization group has been invented to explain them

by Kenneth G. Wilson

One of the more conspicuous properties of nature is the great diversity of size or length scales in the structure of the world. An ocean, for example, has currents that persist for thousands of kilometers and has tides of global extent; it also has waves that range in size from less than a centimeter to several meters; at much finer resolu- temperature is raised. Near the critical point water develops fluctuations in density at all possible scales. The fluctuations take the form of drops of liquid thoroughly interspersed with bubbles of gas, and there are both drops and bubbles of all sizes from single molecules up to the volume of the specimen. Precisely at the critical point the scale of the larg- an alloy at the temperature where two kinds of metal atoms take on an orderly distribution. Other problems that have a suitable form include turbulent flow, the onset of superconductivity and of superfluidity, the conformation of polymers and the binding together of the elementary particles called quarks. A remarkable hypothesis that seems to be

Renormalization Group

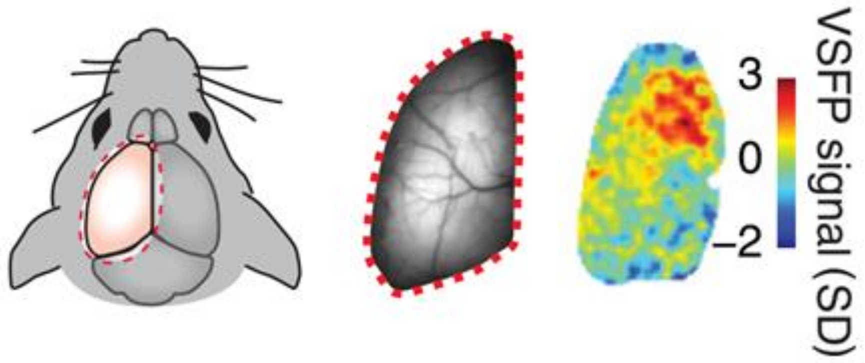
1. G. Scott, E. D. Fagerholm, H. Mutoh, R. Leech, D. J. Sharp, W. L. Shew, and T. Knöpfel, J. Neurosci. **34**, 16611 (2014).

Hypothesis

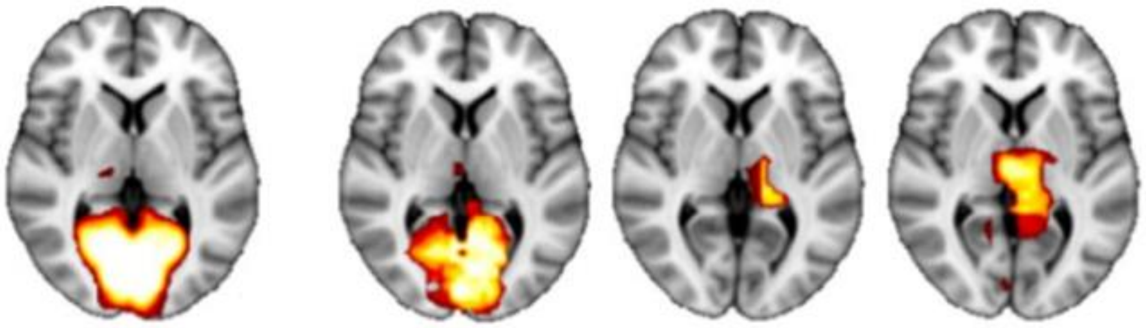
If a system is scale-invariant, then an appropriately chosen coarse-graining procedure will leave the governing laws unchanged and leave the system variables with identical statistics.

Possible implication in neuroscience

spatially-resolved measurements in animals[1]

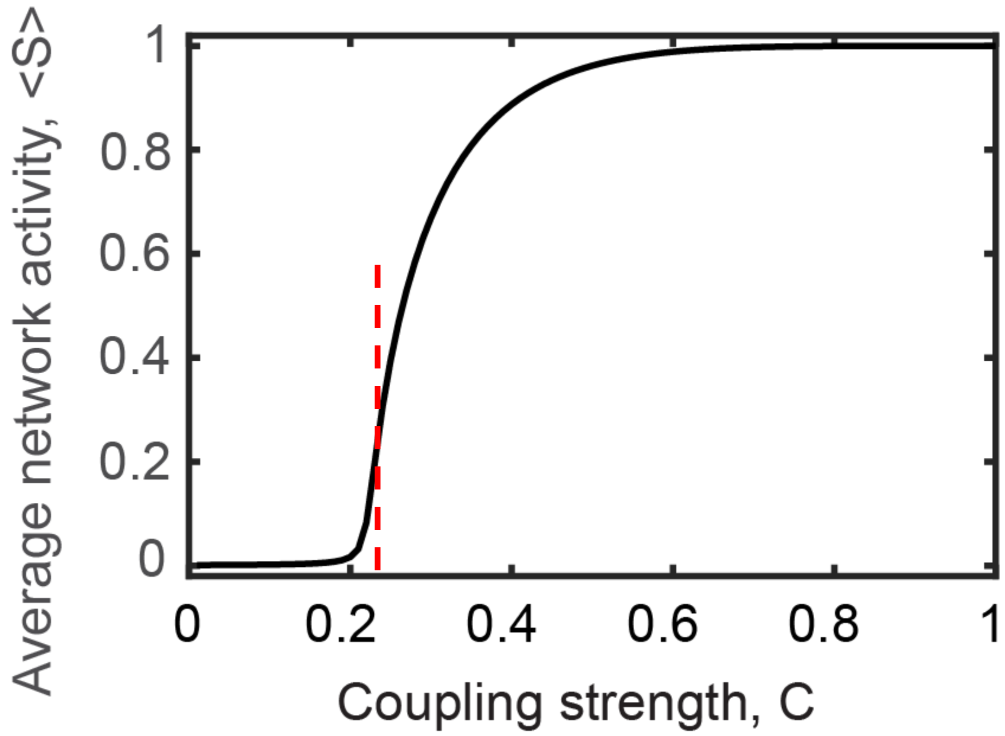


low-resolution measurements in humans[2]



1. G. Scott, E. D. Fagerholm, H. Mutoh, R. Leech, D. J. Sharp, W. L. Shew, and T. Knöpfel, J. Neurosci. **34**, 16611 (2014)
2. A. Haimovici, E. Tagliazucchi, P. Balenzuela, and D. R. Chialvo, Phys. Rev. Lett. **110**, 178101 (2013).

Neural Network Model



$$X(t+1) = \begin{cases} 1 \; with\ probability\ \ \phi_n \\ \quad 0 \quad otherwise \end{cases}$$

$$\boxed{\phi_n = 1 - (1 - C)^{n(t)}(1 - p)}$$

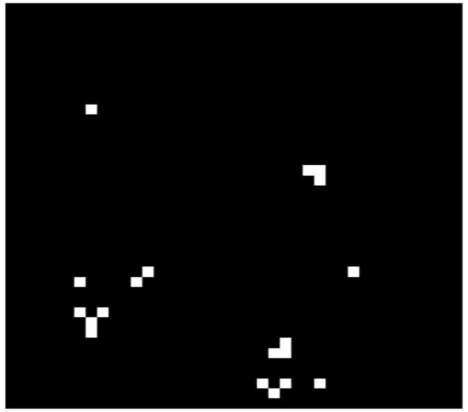
Activation probability, ϕ_n

***n(t)**: Number of active neighbor*

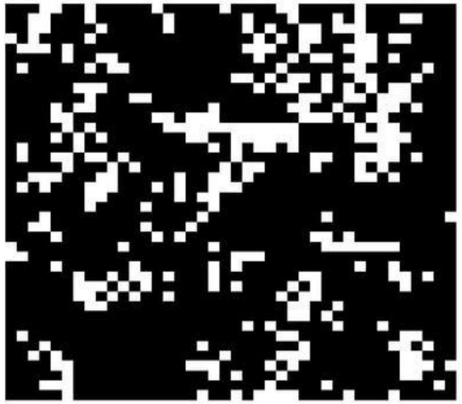
n= 0, 1, 2, 3, 4, 5.

***C**: Coupling strength*

***p**: noise*



Low firing state
Subcritical (C: 0.2)



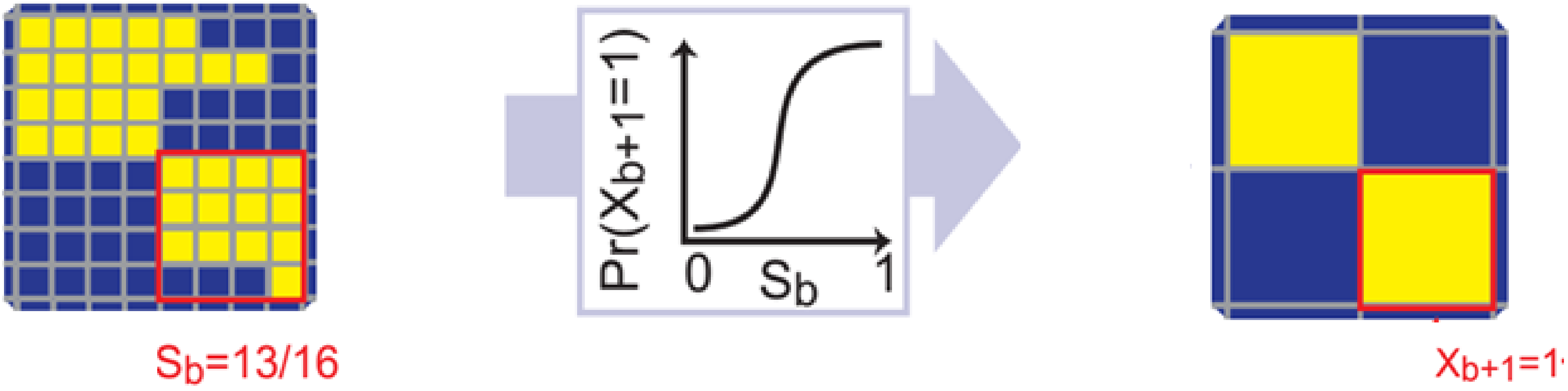
Balanced firing state
Critical (C: 0.23)

High firing state
Supercritical (C: 0.3)

Coarse graining transformation scheme

scale $(\boldsymbol{b})$

scale $(\boldsymbol{b}+\mathbf{1})$



$$\boldsymbol{r\ X\ r}\ \textbf{nodes}\ \Longrightarrow S_b = \frac{\sum_{i=1}^{r^2} X_b^i}{r^2} \Longrightarrow \Pr(X_{b+1} = 1) = \frac{1}{1+\exp(-k(S_b - x_0))} \Longrightarrow \textbf{1 node}$$

Tunable transformation parameters: $\boldsymbol{k, x_0}$

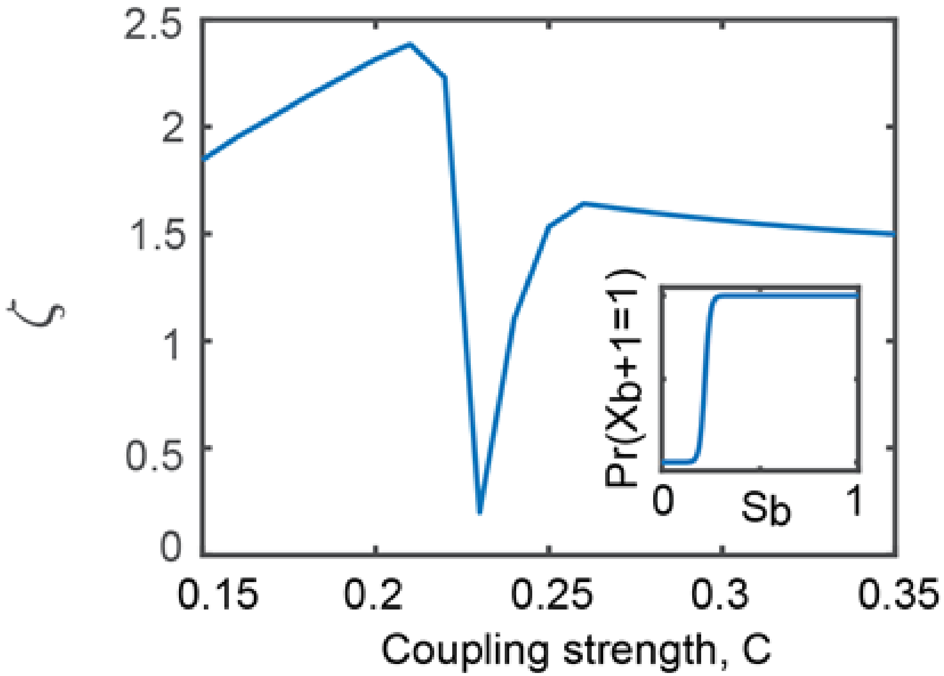
Measure of change in dynamical rules

$\phi_n = 1 - (1 - C)^{n(t)}(1 - p)$ $\quad\quad$ $data(simulation\ or\ expt.) \rightarrow n(t) \rightarrow \phi_n$

How much does the activation probabilities ϕ_ns change from one observational scale (b) to coarser scale (b+1)?

$$\zeta = \sum_{n=0}^{5} |\phi_n^b - \phi_n^{b+1}|$$

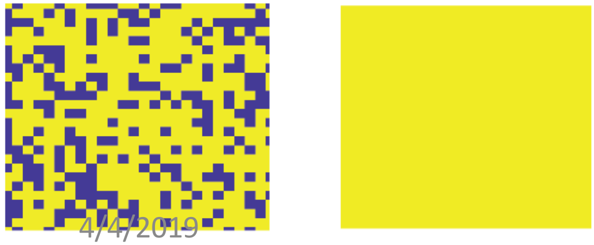
Dynamical rules are most scale-invariant near critical point C~C*



$$\zeta = \sum_{n=0}^{5} |\phi_n^b - \phi_n^{b+1}|$$

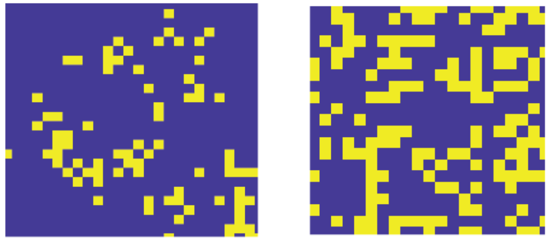
Transformation parameters:
r= 8; k= 76; x_0 = 0.22

b b+1



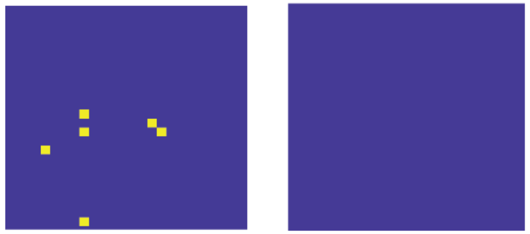
C: 0.2

b b+1



C: 0.23

b b+1

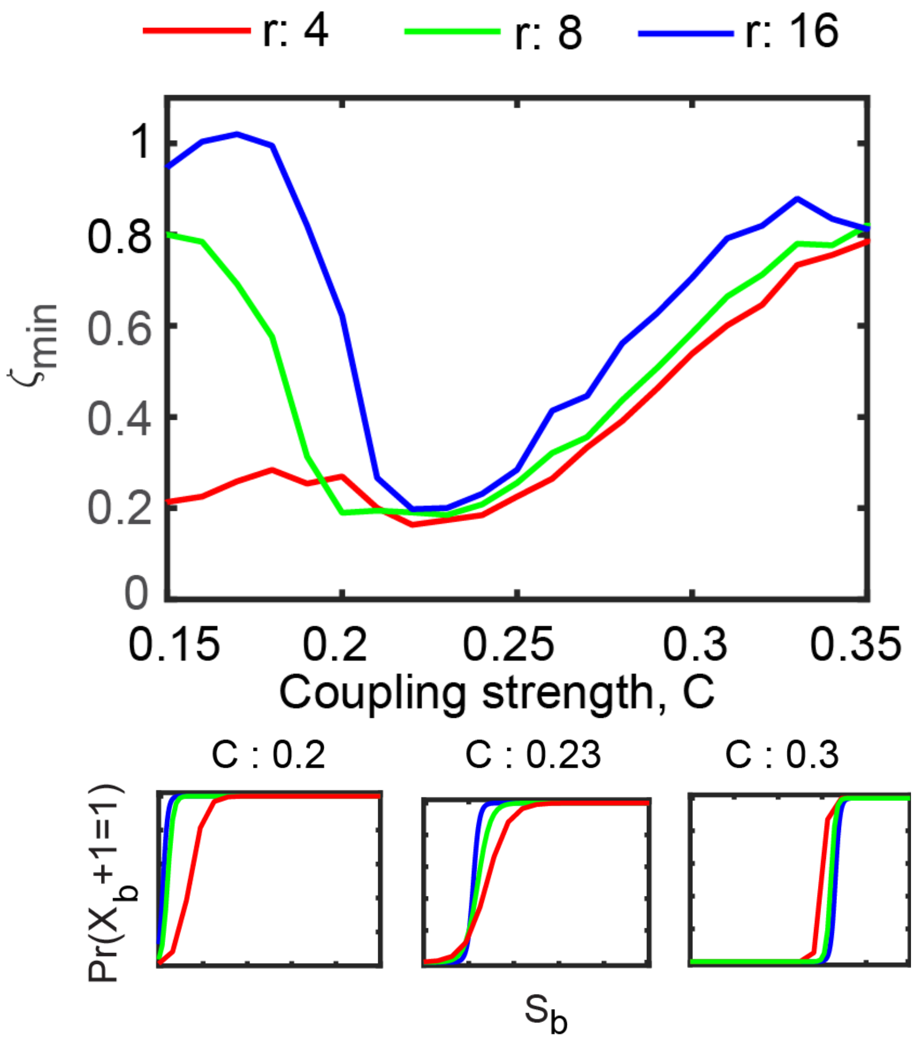


C: 0.3

Can ζ be minimized at some other C by some other transformation function?

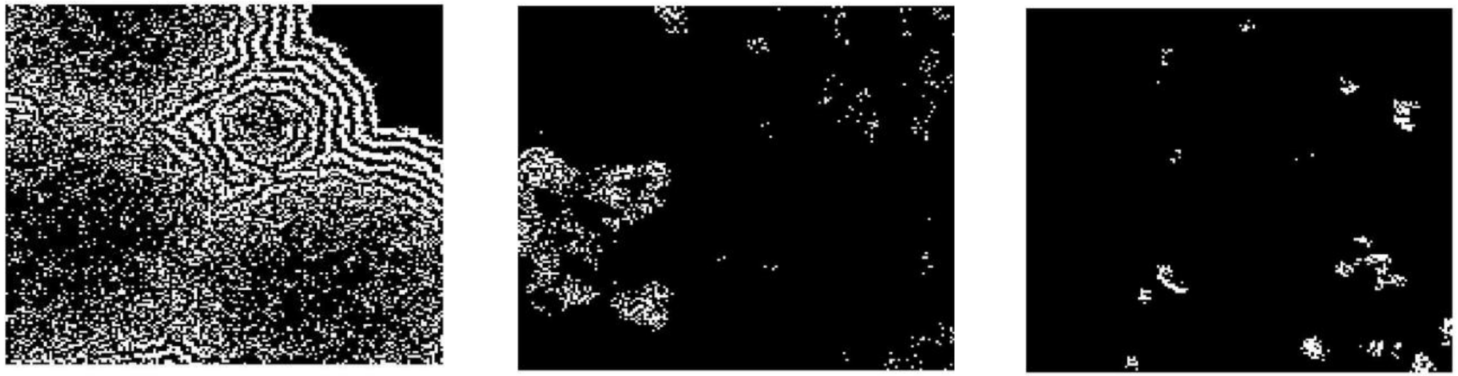
- Consider all C values independently.
- Systematically searched all possible *(k,x₀)* transformation function.
- Estimated ζ_{min}

Most Scale-invariant near C~C*

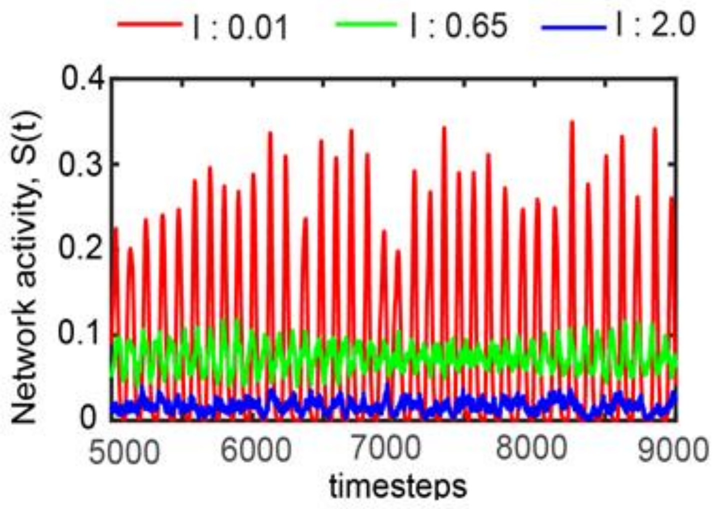


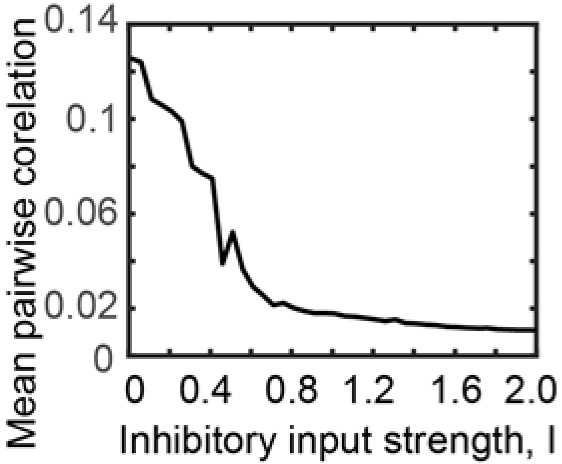
Realistic Neural Network Model

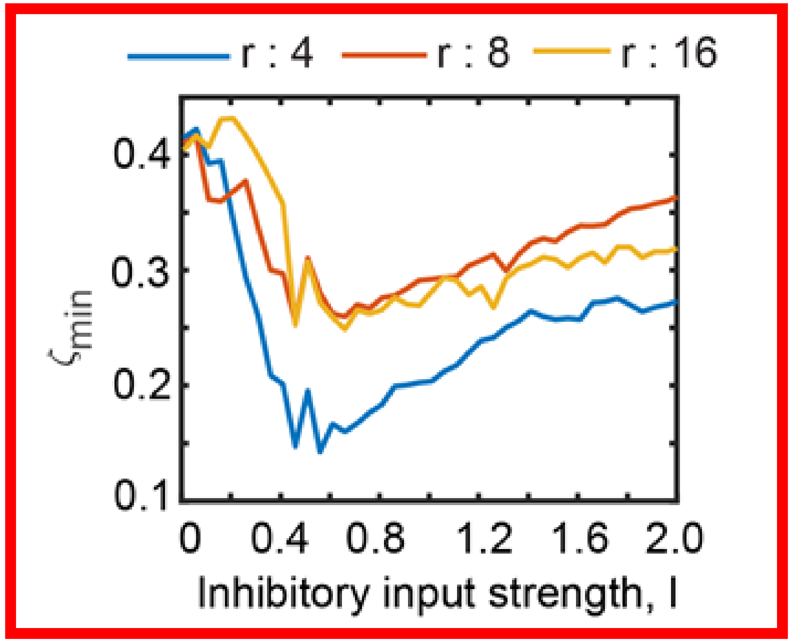
I = 0.01 | I = 0.65 | I: 2.0



- Probabilistic integrate and fire neurons
- Inhibitory-Excitatory neurons: 80%-20%
- Long range Inhibition, Short range Excitation
- Refractory period
- Spike Frequency Adaptation





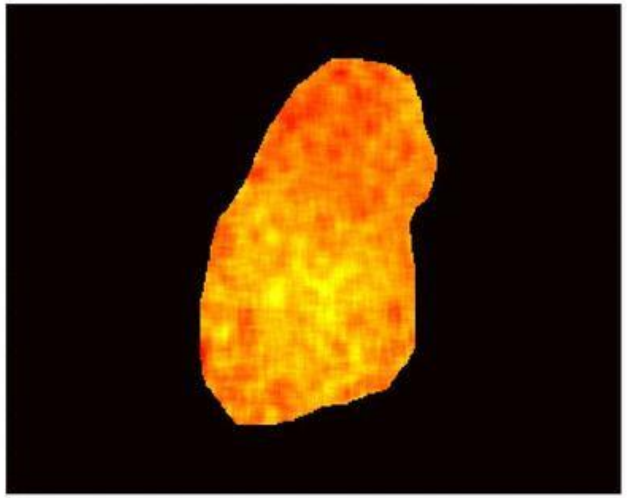


Most Scale-invariant near I ~ I*

Note: ϕ_0 *and* ϕ_5 *are excluded in estimating* ζ_{min} *in this case.*

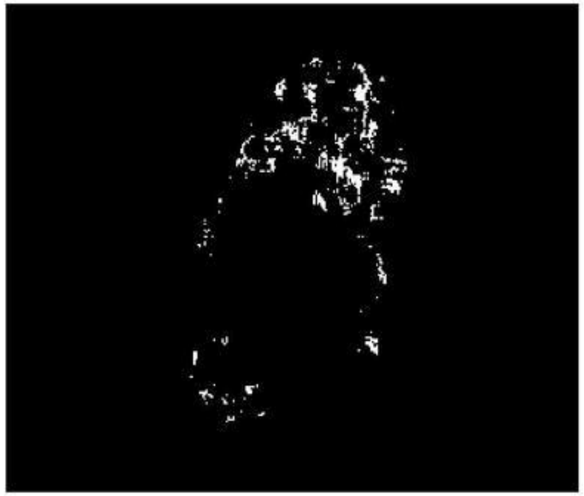
Application on Experimental data

Voltage-sensitive
fluorescence imaging signal



$$\xrightarrow[\substack{Active\ when\ voltage\ crosses\\ threshold\ from\ below}]{Point\ Process\ analysis^3}$$

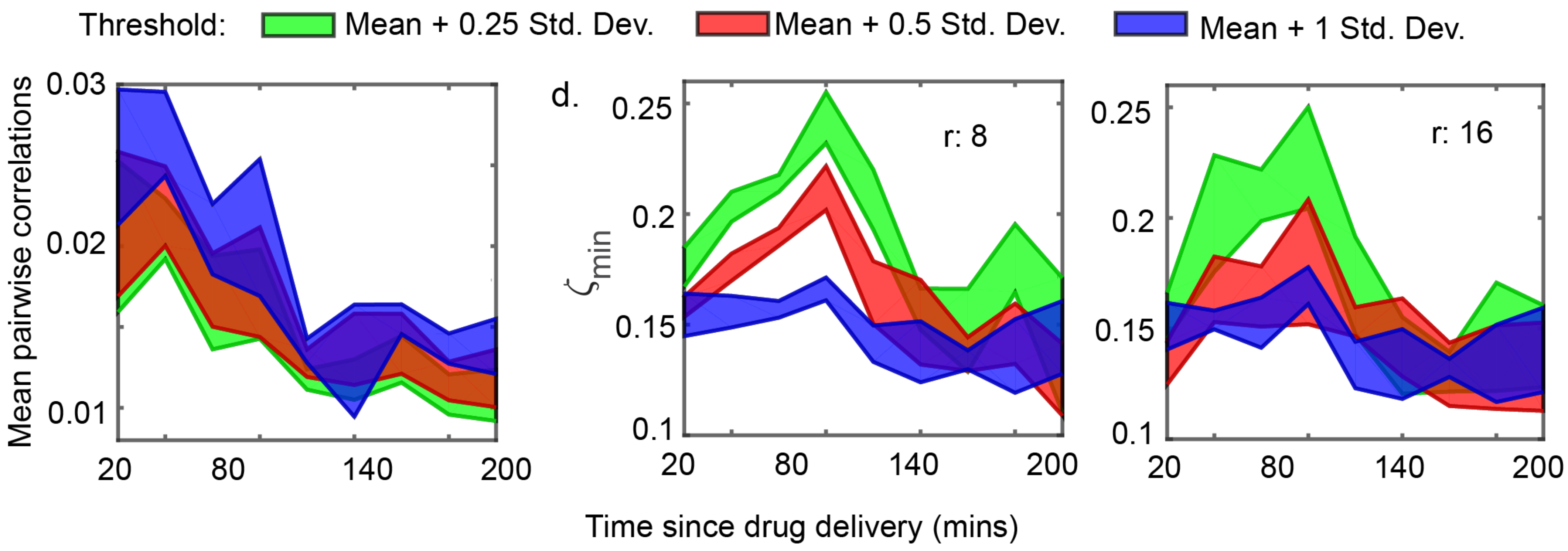
Binary data



- Cerebral cortex mouse waking up after the delivery of anesthesia[1].
- 1 pixel ($33 \times 33\ \mu m^2$) → aggregate activity of many neurons within cortical layers 2 and 3.

1. Data used here is previously reported in: G. Scott, E. D. Fagerholm, H. Mutoh, R. Leech, D. J. Sharp, W. L. Shew, and T. Knöpfel, J. Neurosci. **34**, 16611 (2014). Sample rate: 50 Hz , Imaged region: dorsal surface of 1 hemisphere of mouse cortex (320 × 240 pixels).
2. Tagliazucchi, E., Balenzuela, P., Fraiman, D. & Chialvo, D. R. Criticality in large-scale brain fmri dynamics unveiled by a novel point process analysis. *Front. Physiol.* **3 FEB,** 1–12 (2012).

As the mouse wakes up from the effect of anesthesia scale-invariance in dynamical rules increases.



Note: ζ_{min} estimated by assuming nearest neighbor connections, 6 activation probabilities, ϕ_n and excluding $\phi_0\ and\phi_5$.

Conclusions

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- In a highly non-equilibrium model of an excitable network, the **dynamics governing rules** obtain a degree of scale invariance that is most pronounced when the system operates near criticality.

- Results hold for realistic Neural Network Model with inhibition, adaptation and refractory period.

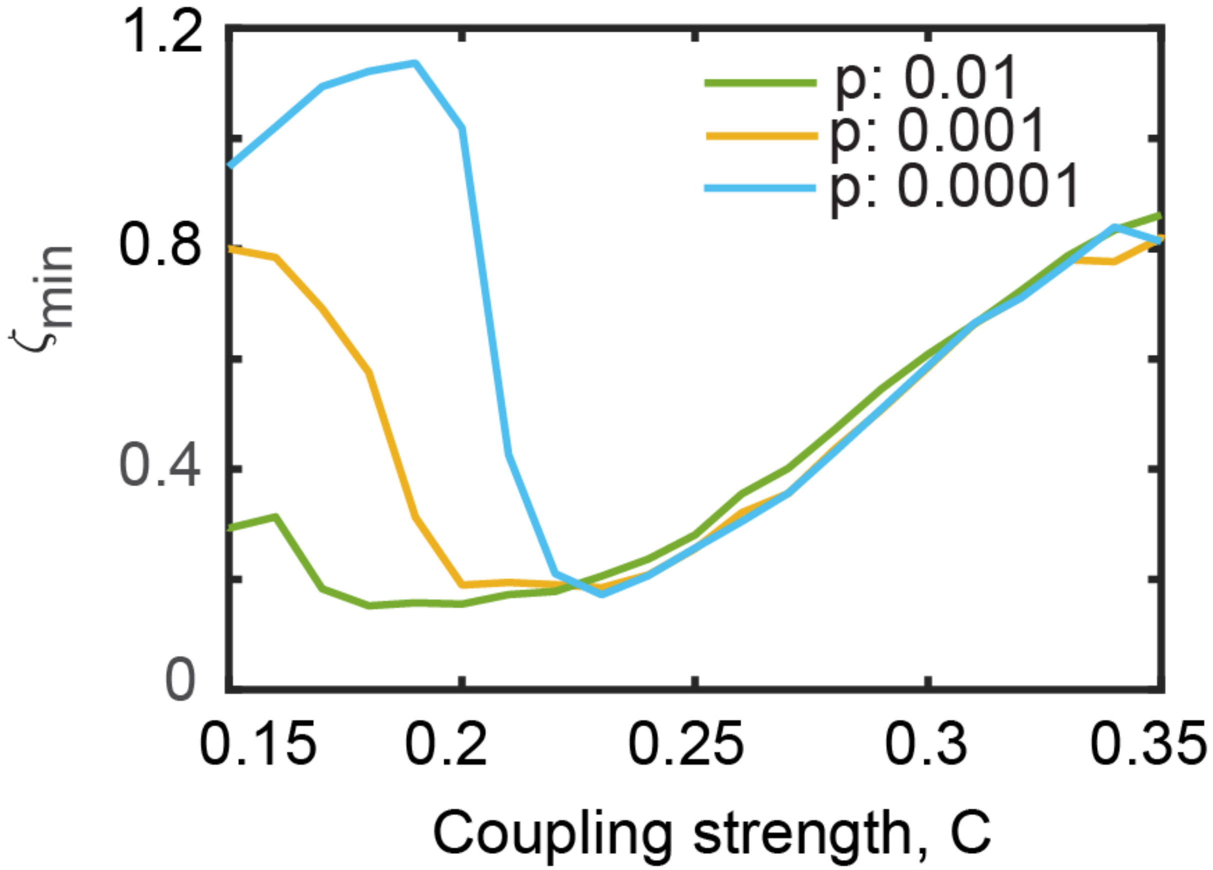
- Dynamical rules are more scale invariant in conscious mouse cortex than the unconscious mouse cortex under anesthesia.

Future Directions:

- Coarse-graining in time as many experiments have poor time resolution.

Thank you!

External input '*p*' *influences* ζ_{min} *trends*



As the external input decreases the minimum ζ near C* becomes narrower.

Ising Model Analogy,

- Completely scale-invariant at zero external magnetic field.
- External magnetic field influences the critical point.
- Cause dynamics deviated slightly from true critical dynamics.