some residual ionization. There are a number of methods by which the ionization in this layer is generated. It depends on various factors, including the altitude within the layer, the state of the sun, and the latitude. However, X-rays and ultraviolet rays produce a large amount of the ionization light, especially that with very short wavelengths.

**F layer** The F layer is the most important region for long distance HF communications. During the day it splits into two separate layers. These are called the $F_1$ and $F_2$ layers, the $F_1$ layer being the lower of the two. At night these two layers merge to give one layer called the F layer as shown in Fig. 5.9 later on. The altitudes of the layers vary considerably with the time of day, season and the state of the sun. Typically in summer the $F_1$ layer may be around 300 km with the $F_2$ layer at about 400 km or even higher. In winter these figures may be reduced to about 200 km and 300 km. Then at night the F layer is generally around 250 to 300 km. Like the D and E layers, the level of ionization falls at night, but in view of the much lower air density, the ions and electrons combine much more slowly and the F layer decays much less. Accordingly, it is able to support communications, although changes are experienced because of the lessening of the ionization levels. The figures for the altitude of the F layers are far more variable than those for the lower layers. They change greatly with the time of day, the season, and the state of the sun. As a result, the figures that are given must only be taken as an approximate guide. Most of the ionization in this region of the ionosphere is caused by ultraviolet light, both in the middle of the UV spectrum and the portions with very short wavelengths.

## 5.2 BASIC PROPAGATION MECHANISMS

Radio signals are affected in many ways by objects in their path and by the media through which they travel. The properties of the path by which the radio signals will propagate govern the level and quality of the received signal. It is, therefore, very important to know the likely radio propagation characteristics that are likely to prevail.

The distances over which radio signals may propagate vary considerably. For some applications, only a short range may be needed. For example, a Wi-Fi link may only need to be established over a distance of a few metres. On the other hand, a short-wave broadcast station or a satellite link would need the signals to travel over much greater distances. Even for these two examples of the short-wave broadcast station and the satellite link, the radio propagation characteristics would be completely different–the signals reaching their final destinations having been affected in very different ways by the media through which the signals have travelled.

### 5.2.1 Radio Propagation Categories for Long-Distance Case

There are a number of categories into which different types of long-distance radio propagation mechanisms can be placed. These relate to the effects of the media through which the signals propagate.

**Free space propagation** Here, the radio signals travel in free space and away from other objects that influence the way in which they travel. If the wavelength is fixed, it is only the distance from the source that affects the way in which the field strength reduces.

This type of radio propagation is mainly encountered with signals travelling to and from satellites. This is rather a hypothetical case.

**Ground wave propagation (below 2 MHz)** When signals travel via the ground, they are modified by the ground or terrain over which they travel. They also tend to follow the earth's curvature. Signals heard on the medium waveband during the day use this form of propagation.

**Ionospheric propagation or sky wave propagation (2 to 30 MHz)** Here the radio signals are modified and influenced by the action of the free electrons in the upper reaches of the earth's atmosphere in the ionosphere. This form of radio propagation is used by stations on the short wavebands for their signals to be heard around the globe.

**Tropospheric propagation** Here the signals are influenced by the variations of refractive index in the troposphere just above the earth's surface. Tropospheric radio propagation is often the means by which the signals at VHF and above (maybe TV signals) are heard over extended distances.

Most of the above applications are broadcast applications. The AWGN is observed in these communications as it is due to temperature variations. Even sometimes the shadowing effect is also observed. All these propagation methods are described afterwards.

A special case is line-of-sight propagation above 30 MHz as shown in Fig. 5.3.

During line-of-sight transmission, the following significant impairments are observed:

- Attenuation and attenuation distortions
- Atmospheric absorption

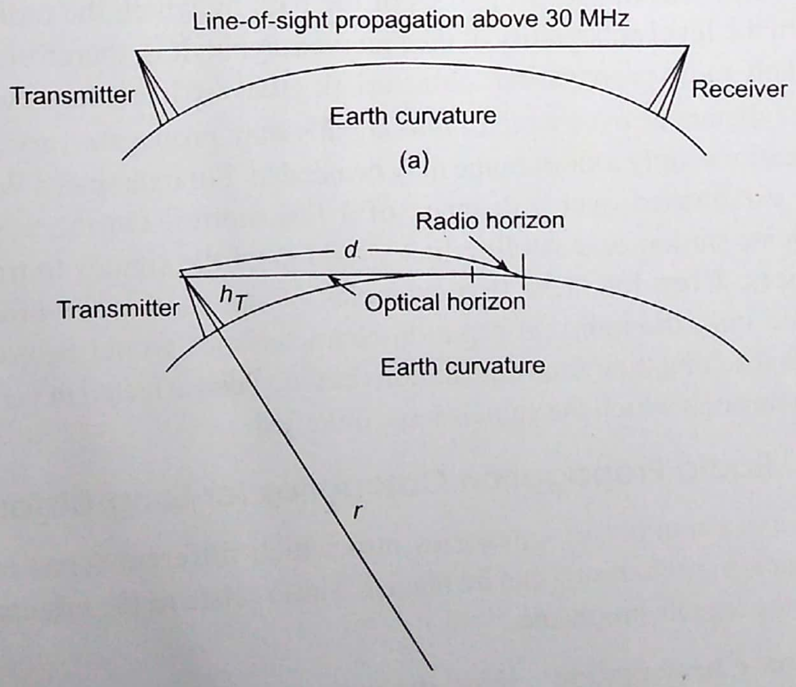


**Fig. 5.3** (a) Line-of-sight propagation mode and (b) limitations on antenna height due to optical horizon and radio horizon (due to radio horizon slightly more coverage is achieved)

- Free space loss
- Noise
- Refraction
- Mulipath

In addition to these categories, many short-range radio or wireless systems have radio propagation scenarios that do not fit neatly into these categories because they may have indoor models also. However, for outdoor models, groundwave concepts can be correlated. Wi-Fi and cellular systems, for example, need to have their radio propagation models generated for office, or urban situations like for buildings, vegetations, vehicles, etc. Under these circumstances, the analysis approach is modified by multiple reflections, refractions, and diffractions with multipath (NLOS). Despite these complications, it is still possible to generate rough guidelines and models for these radio propagation scenarios.

### 5.2.2 Short-Distance Mobile Communication Case

In an ideal radio channel condition, the received signal would consist of only a single direct path signal, which would be a perfect reconstruction of the transmitted signal. However, in a real channel, the signal is modified during transmission through the channel. For NLOS multipath, the received signal consists of a combination of attenuated, reflected, refracted, and diffracted replicas of the transmitted signal. Consider the situation depicted in Fig. 5.4, where a receiver in a moving automobile receives a signal from a single transmitter, which has propagated along two paths. One propagation path is a direct path (maybe slightly diffracted) from the transmitter to mobile. The second path is due to a reflection from a building. A majority of such systems work on above 800 MHz, up to 2–11 GHz.

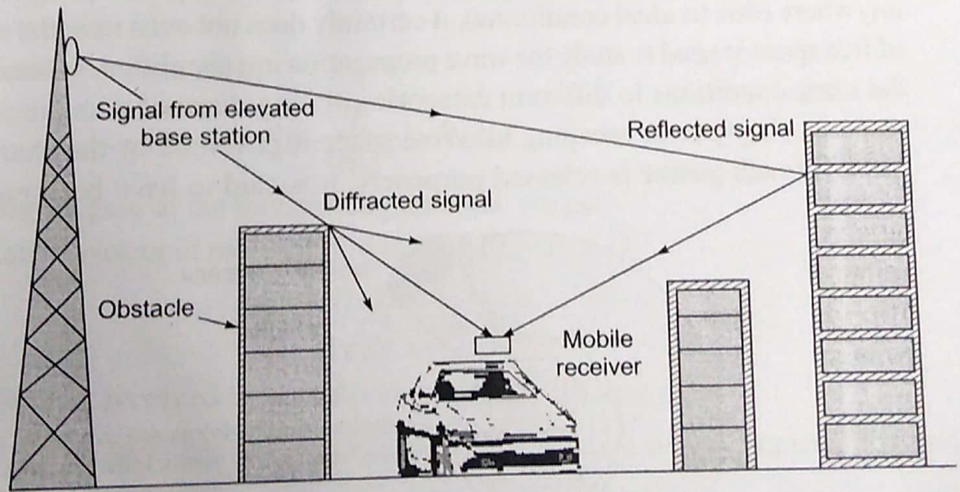


**Fig. 5.4** Diffracted and reflected rays

Now some important terms related to wireless channel effects are introduced. On top of all, the channel adds white Gaussian noise to the signal as usual. Since multiple versions of the signal are received at the receiver, there are interferences with each other due to multipath effect, intersymbol interference (ISI) or delay spread occurs, and it becomes very hard to extract the original information without complicated *equalizers* (described in the next chapter) or some modulation schemes designed to combat ISI (Chapter 9). The common representation of the multipath channel is the *channel impulse response* (CIR) of the channel, which is the response at the receiver if a single impulse is transmitted. It can be calculated by *channel estimation* procedures, which is described in the next chapter. Mobility of users and hence mobile receivers or mobility of surrounding objects can cause a shift in the received carrier frequency, which is called *Doppler effect*. Figure 5.5 gives the typical scenario of the multipath/NLOS communication.

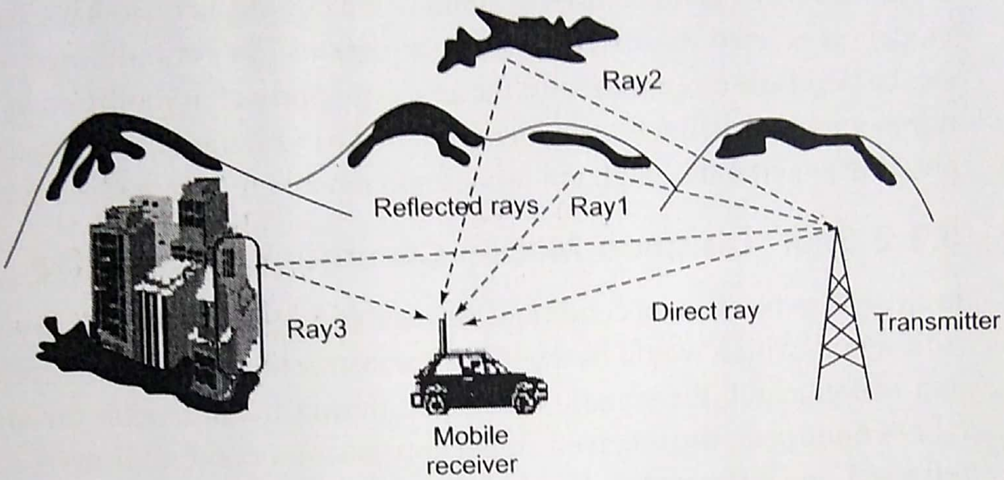


**Fig. 5.5** Multipath effect between transmitter and receiver

## 5.3 FREE SPACE PROPAGATION MODEL

Free space is the space that does not interfere with the normal radiation and propagation of the radio waves. Thus, it does not have any magnetic or gravitational fields, solid bodies, and ionized particles. Apart from the fact that free space is unlikely to exist anywhere (due to ideal conditions), it certainly does not exist near the earth. The concept of free space is used to study the wave propagation in a simplified manner and then applying the same conditions to different categories of long-distance communication with actual scenario. Any power escaping into free space is governed by the characteristics of free space. If such power is released purposely, it is said to have been radiated and it then

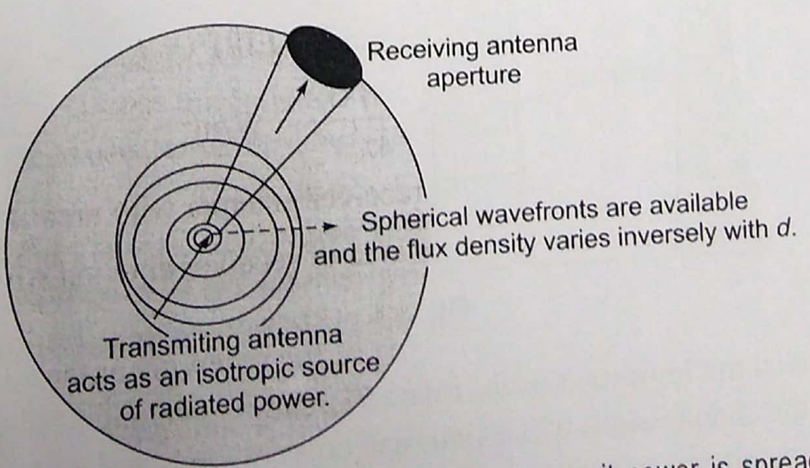


**Fig. 5.6** Transmit antenna modelled as a point source (Transmit power is spread over the surface area of a hypothetical sphere at a distance $d$. The receiver antenna has an aperture $A_r$, illustrated over area $4\pi d^2$.)

form of an electromagnetic wave. This action can also be related to the term *power density*, which is radiated power per unit area. Power density reduces with the distance.

The free space propagation model assumes a transmit antenna and a receiver antenna to be located in an empty environment. Neither absorbing obstacles nor reflecting surfaces are there. In particular, the influence of the earth's surface is assumed to be entirely absent. For propagation distances $d$ much larger than the antenna size, the far field of the electromagnetic wave dominates all other components, that is, we are allowed to model the radiating antenna as a point source with negligible physical dimensions. In such a case, the energy radiated by an omni-directional antenna is spread over the surface of a sphere. This allows us to analyse the effect of distance on the received signal power.

This Friis free space model is only valid for all the values of $d$, which are in the far-field of the transmitting antenna. The far-field is also called Fraunhofer region after the name of its inventor. It is defined as the region beyond the threshold value of far-field distance $d_f$, which relates the largest linear dimension of the transmitter antenna aperture $A_t$ and the carrier wavelength $\lambda$ as given in Eq. (5.1). The Fraunhofer distance is given by

$$d_f = \frac{2A_t^2}{\lambda} \quad \text{D} \tag{5.1}$$

Also, to be in the far-field region, $d_f >> A_t$ and $d_f >> \lambda$.

As with most large-scale radio wave propagation models, the free space model predicts that the received power decays as a function of the distance between transmitter and receiver raised to some power (a power law function). It is seen that the power density is inversely proportional to the square of distance from the source. This is the inverse square law, which applies universally to all forms of radiation in free space. Considering the isotropic source of transmitted power (which may be a half-wave dipole antenna), we get

Flux density at distance $d$,

$$F = \frac{P_t}{4\pi d^2} \quad \text{W/m}^2 \tag{5.2}$$

Considering gain of the transmitting antenna, we get

Effective isotropic radiated power (EIRP) = $P_t \times G_t$

or
$$F = \frac{P_t G_t}{4\pi d^2} \quad \text{W/m}^2 \tag{5.3}$$

The power $P_r$ received by a receiver antenna with area $A_r$

= Flux (power) density × area of receiver antenna which receives flux density

= $F \times \eta A_r$ watts (5.4)

where $\eta A_r = A_e$ = effective area of the receiving antenna

$\eta$ = antenna aperture efficiency

Substituting value of $F$ in Eq. (5.4),

$$P_r = \frac{P_t G_t A_e}{4\pi d^2} \tag{5.5}$$

Thus, the received power is proportional to EIRP of the transmitting antenna and does not depend on the frequency.

Now, the receiving antenna gain,

$$G_r = \frac{4\pi A_e}{\lambda^2} \tag{5.6}$$

Using relationship of Eq. (5.6) and substituting for $A_e$, Eq. (5.5) will become

$$P_r = \frac{\lambda^2}{(4\pi d)^2} G_t P_t G_r \tag{5.7}$$

The wavelength $\lambda$ (in metres) is

$$\frac{c}{f_c} = \frac{2\pi c}{\omega_c} \tag{5.8}$$

where $c$ = velocity of light
$f_c$ = carrier frequency in Hz
$\omega_c$ = angular frequency

Now, the path loss in general is defined as

$$P_{LdB} = 10 \log \frac{P_t}{P_r} \tag{5.9}$$

$$= 10 \log \left[\frac{(4\pi d)^2}{\lambda^2}\right] \quad \text{(for unity gain conditions)} \tag{5.10}$$

$$= -10 \log \frac{\lambda^2}{(4\pi d)^2} \tag{5.11}$$

If the wavelength is constant, loss simply depends upon the distance $d$ between isotropic transmitter and receiver. Considering the real systems, according to Friis free space equation,

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2 L} \tag{5.12}$$

where $L$ is the system loss factor, which is not related to propagation. $L$ may be greater than or equal to 1. These miscellaneous losses are usually due to transmission line attenuation, filter losses, antenna losses, etc. $L = 1$ means no losses in the system hardware.

In practice, *effective radiated power* (ERP) is used instead of EIRP to denote the maximum radiated power. Also antenna gains are given in units of dBi (dB gain with respect to an isotropic antenna) or dBd (dB gain with respect to a half-wave dipole).

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**Example 5.1** A satellite link is established between an earth station and a satellite transponder for the RF frequency of 4 GHz. For the earth station transmitter, the transmitted power is 1 kW, and the transmitter and receiver antenna gains are 0 dB. The free space distance is 30,000 m. Find out the received power at the transponder. Assume $L = 1$.

**Solution**

$f_c = 4$ GHz $\Rightarrow \lambda = c/f = (3 \times 10^8$ m/s$)/(4 \times 10^9$ Hz$) = 3/40$ metre

Now,

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2 L}$$

$$P_r = \frac{1000 \times 1 \times 1 \times (3/40)^2}{(4\pi\, 30000)^2}$$

$$= \frac{5.625}{142{,}122{,}303{,}375}$$

$$= 3.96 \times 10^{-11} \text{ W} = -104 \text{ dB}w$$

### 5.3.1 20 log *d* Path Loss Law

As the propagation distance increases, the radiated energy is spread over the surface of a sphere of radius *d*. So, the power received decreases proportional to $d^{-2}$. For the unity gain antennas, the received power expressed in dB is

$$P_{r\text{dB}} = p_{t\text{dB}} - 20 \log \frac{d}{\lambda/4\pi} \tag{5.13}$$

This is called path loss law, where $p_{t\text{dB}}$ is the transmitted power in dB. The characteristic plot is given in Fig. 5.7. Furthermore, Friis equation does not hold for $d = 0$. This will be useful to understand large-scale propagation models using a close-in distance $d_0$ as a known received power reference point. The received power at any distance $d > d_0$ may be related to $P_r$ at $d_0$ as follows:

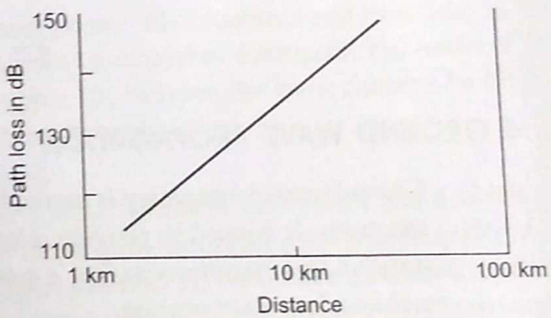



**Fig. 5.7** Typical average path loss in dB versus distance

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2 \quad \text{for } d \geq d_0 \geq d_f \tag{5.14}$$

In mobile radio systems, it is common to find that $P_r$ may have large variations of amplitude values over a typical coverage area of several square kilometers. As the dynamic range of the received power levels is large, often dBm or dBW units are used to express the received power levels. Equation (5.14) can be expressed in dBm or dBW units, simply by taking the logarithm of both sides and multiplying by 10. If $P_r$ is represented in units of dBm, the received power is given by

$$P_r(d)\text{dBm} = 10 \log\left[\frac{P_r(d_0)}{0.001\text{W}}\right] + 20 \log\left(\frac{d_0}{d}\right) \quad \text{for } d \geq d_0 \geq d_f \tag{5.15}$$

Here, $P_r(d_0)$ is in units of watts.

The reference distance ($d_0$) for practical systems using low-gain antennas in the 1–1.5 GHz region is typically chosen to be 1 m for indoor environments and 100 m or 1 km for outdoor environments.

Path loss causes attenuation between the transmitter power amplifier and receiver front end. Some other effects are listed below, which should be considered during the link design:

- Losses in the antenna feeder (0–4 dB).
- Losses in transmit filters, particularly if the antenna radiates signal of multiple transmitters (0–3 dB) and antenna directionality gain (0–12 dB).
- Losses in duplex filter.
- Fade margins to anticipate for multipath (9–19 dB) and shadow losses (~5 dB).
- Penetration losses if the receiver is indoors, typically about 10 dB for 900 MHz signals.

### 5.3.2 Field Strength

While cellular telephone operators mostly calculate received power, broadcasters plan the coverage area of their transmitters using the (CCIR-recommended) *electric field strength E* (e-field) at the location of the receiver. The conversion formula is

$$E = \sqrt{\frac{120\pi P_r}{A_e}} \tag{5.16}$$

## 5.4 GROUND WAVE PROPAGATION

Ground wave propagation is particularly important on the LF and MF portions of the radio spectrum. It is used to provide relatively local coverage, especially by radio broadcast stations that require covering a particular locality. It can also be considered for land mobile telecommunication.

Ground wave radio signal propagation is ideal for relatively short-distance propagation on these frequencies during the daytime. Sky wave ionospheric propagation is not possible during the day because of the attenuation of the signals on these frequencies caused by the D region in the ionosphere. In view of this, stations need to rely on the ground wave propagation to achieve their coverage.

A ground wave signal is made up from a number of constituents. If the antennas are in the line-of-sight, then there will be a *direct wave* as well as a *reflected wave*. As the name suggests, the direct signal is one that travels directly between two antennas and is not affected by the locality. There will also be a reflected signal as the transmission will be reflected by a number of objects, including the earth's surface and any hills or large buildings. In addition to this, there is a *surface wave*. This tends to follow the curvature of the earth and enables coverage to be achieved beyond the horizon. It is the sum of all these components that is known as the ground wave.

Beyond the horizon, the direct and reflected waves are blocked by the curvature of the earth, and the signal is purely made up from the diffracted surface wave. It is for this reason that surface wave is commonly called ground wave propagation.

cases. With the satellite moving towards the earth station, the frequency appears higher than nominal and then as it moves away, the apparent frequency falls. The degree of shift is dependent upon a number of factors, including the speed of the satellite (more correctly, its speed relative to the earth station) and the frequencies in use. Shifts of the order of 10 kHz may be experienced. As most satellites operate in a cross-mode configuration, the Doppler shift is not just applicable to the band on which the signal is received but also to the cumulative effect of the uplink and downlink transmissions. In many instances, the effects will subtract because of the way the satellite mixing process is configured. Such effects can even be observed in mobile satellite communication.

## 5.9 MULTIPATH EFFECT/FADING IN LAND MOBILE SYSTEMS

Because there are obstacles and reflectors in the wireless propagation channel, the transmitted signal arrives at the receiver from various directions over a multiplicity of paths. Such a phenomenon is called *multipath*. The multipath effect is mostly there in wireless communications but depends upon the distance and surroundings. (Some of the issues were discussed previously.) However, specifically for very short distance communication systems like indoor/outdoor communication and urban area systems, multipath is a serious problem because of the dynamic environment. It is an unpredictable set of reflections and/or direct waves, each with its own degree of attenuation and delay as shown in Fig. 5.4.

The multipath will cause amplitude and phase fluctuations and time delay in the received signals. We can use diverse schemes to combat multipath. The results of the multipath effect are also discussed in Chapter 15, where the basic concepts for MIMO are described showing that MIMO exploits multipath.

*The major effect of multipath is fading.*

The communications quality between a base station transmitter and a mobile (or stationary) receiver depends on a number of factors, including the general quality of the propagation channel through which the signal passes. In wireless applications, especially cellular communications, the propagation path is terrestrial air with a significant number of human-made and natural objects that get in the way. As the transmitted signal gets absorbed by the atmosphere and reflected off buildings and trees, it experiences fluctuations in its amplitude as well as phase. Wireless channels, in short, exhibit highly irregular amplitude or signal strength behaviour. It is the time variation of the received signal power caused by changes in the transmission medium or path. This phenomenon is generally referred to as *fading*. The fading, essentially caused by the reception of multiple reflections of the transmitted signal (illustrated in Fig. 5.19), is a key inherent problem of wireless channels, which unfortunately cannot be avoided.

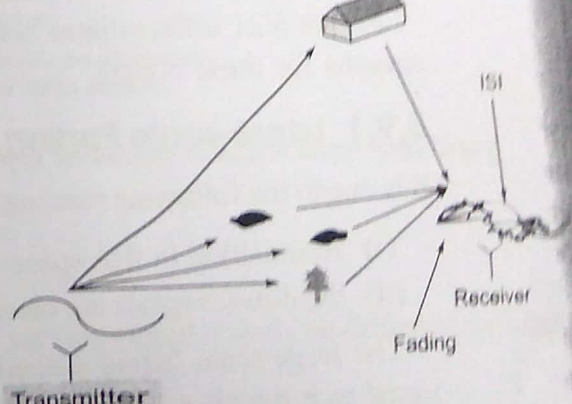



**Fig. 5.19** Multipath fading effect

As shown in Fig. 5.20, the mechanism of fading is normally broken down into two different categories based on the position of the receiver relative to the transmitter:

- Large-scale fading for radio propagation over long distances
- Small-scale fading due to time-varying reflections from the surroundings near the receive antenna

However, both the effects are overlapped and distinct identification of both at the receiver is difficult. From the plot of signal strength versus distance, one can approximate them.

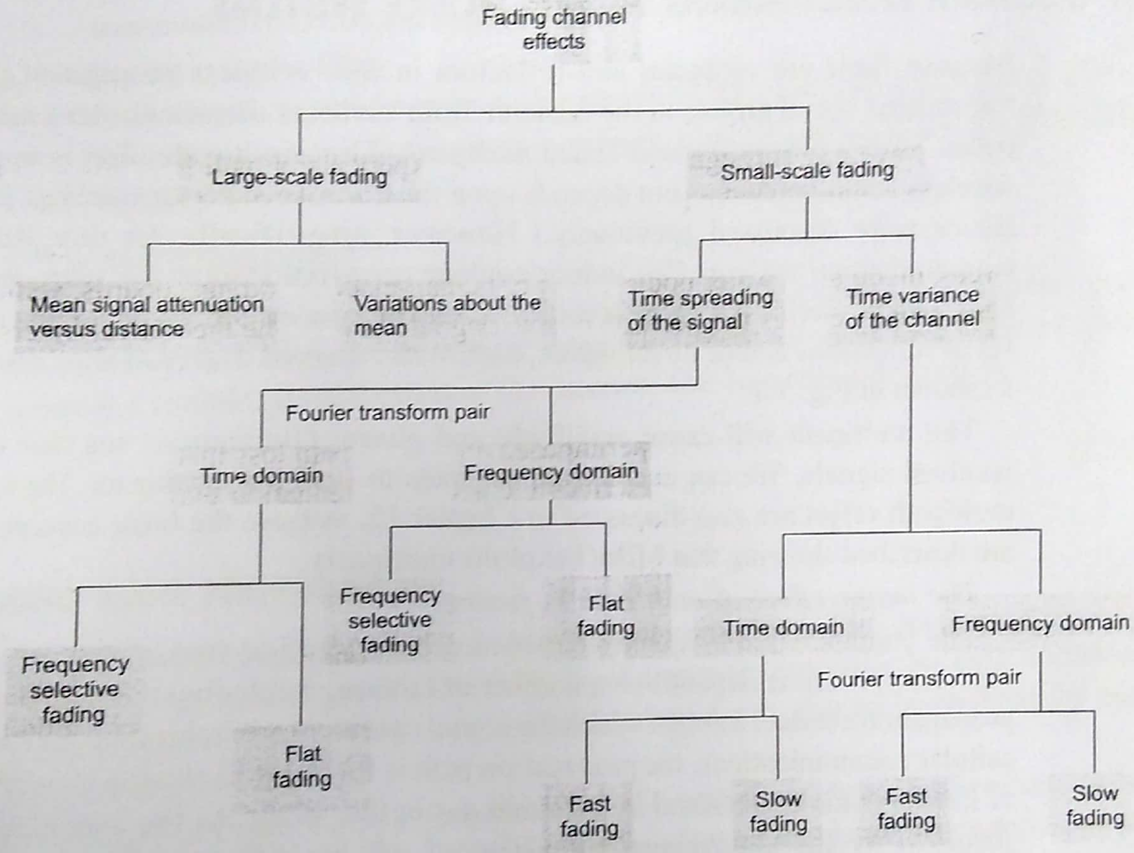



**Fig. 5.20** Fading channel manifestation

Figure 5.21 differentiates between the small- and large-scale fading effects and the reasons for these effects.

### 5.9.1 Large-scale Fading

It is due to the following reasons:

(i) Attenuation in free space: Power degrades with $1/d^2$.

(ii) Shadows: Signals are blocked by obstructing structures.

The large-scale fading essentially represents the average attenuation of a wireless signal as it travels a long distance (several hundred wavelengths or more). Degradation

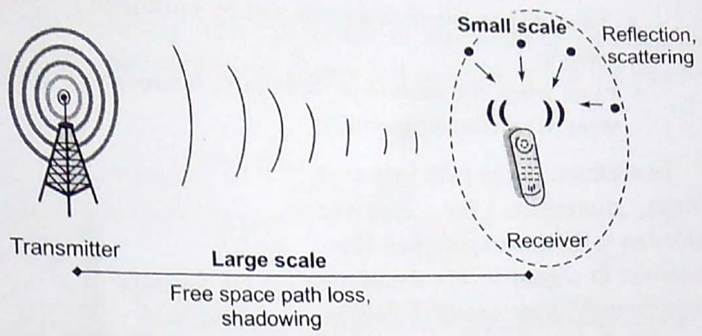



Fig. 5.21 Difference between large- and small-scale fading environment

due to the blockage by large objects is sometimes referred to as shadowing, because the fade area is very large and tends to blanket the area surrounding the antenna. An ideal signal travelling through free space would experience a path loss proportional to the distance squared. In the real world, a signal's energy is absorbed and reflected by the atmosphere, the curvature of the earth, and obstacles. The obstacles can be natural (trees, mountains, water bodies, etc.) or human-made (buildings, boards, vehicles, etc.). This usually causes the performance to degrade beyond the theoretical inverse squared free space path loss law.

Mathematically, large-scale fading can be realized by a *log-normally distributed* (see Section 5.11) fluctuation superimposed on a mean path loss that is distance dependent. The distance dependence describes the average attenuation experienced as the signal travels through the atmosphere. Figure 5.22 gives an idea about the signal strength variations due to these phenomena.

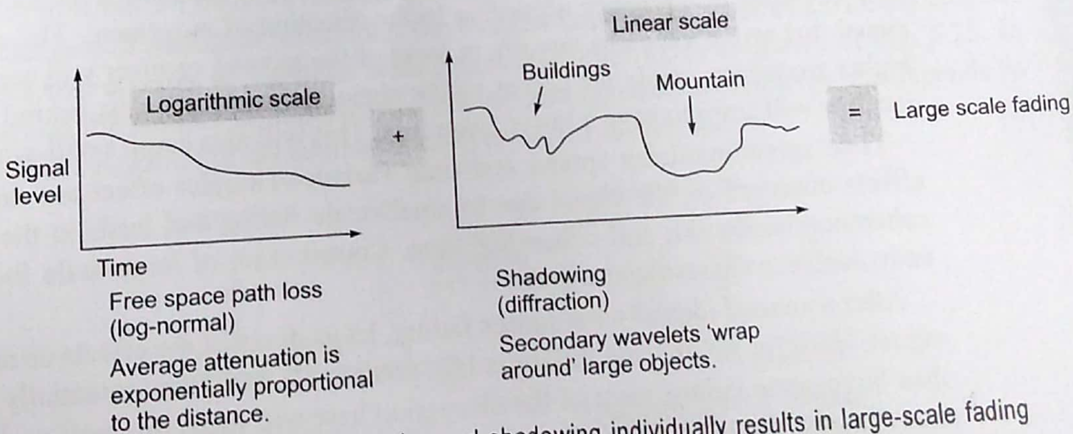



Fig. 5.22 Effect of attenuation and shadowing individually results in large-scale fading

### 5.9.2 Small-scale Fading

It is due to the following reasons:

- Random frequency modulation due to varying Doppler shifts on different multipath signals.

- Time dispersion (echoes) caused by multipath propagation delays due to nearby objects.
- Even when the mobile is stationary, the received signals may fade due to movement of surrounding objects.

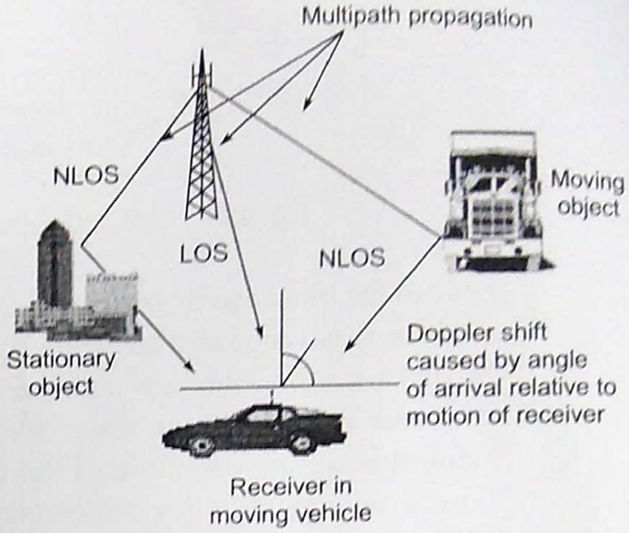



**Fig. 5.23** Various reasons of small-scale fading

In addition to the path loss over large distances, the receiver antenna will also experience fluctuations in signal levels that vary significantly over small distances (on the order of one to tens of wavelengths). The signal being transmitted from the base station can take different paths to the receiver due to reflection, diffraction, and local scattering. Different paths have different lengths associated with them, which causes the receiver to see multiple copies of the signal at different times of arrival. Also, the signal can shift in phase as it is reflected and scattered off local objects. All of these signals at different power levels and phase converge on the receiver antenna with constructive or destructive interference. As the antenna moves through space, it will experience peaks and valleys of signal strength as these interfering wavelets add and subtract at the receiver. Another adverse effect of motion is Doppler shift. As the receiver antenna moves in relation to a fixed transmitter, the incoming signal will modulate in frequency according to the direction of movement. The copies of the signal that arrive via paths directly in front of the moving receiver will seem to have a higher frequency, while the copies of the signal arriving via paths behind the moving receiver will seem to have a lower frequency. This will also cause small-scale fading.

Time spreading/delay spread and time variance/Doppler effect are the two main effects observed on the signal due to small-scale fading and leads to the concept of coherence bandwidth and coherence time. Contribution of large-scale fading is also unavoidable in the resultant signal.

After a general idea of what causes fading, let us describe the effects observed on the signal. Because small-scale fading is less predictable and more potentially destructive than large-scale fading, most of the discussion here will focus on small-scale fading. In the following discussion, it is shown how small-scale fading is observed on the signal and how delay spread and Doppler shift alter the transmitted signal and make it more difficult for the receiver to accurately detect the altered signal.

### *Delay Spread Effect*

Delay spread effect is mainly due to small-scale fading. Because multiple reflections of the transmitted signal may arrive at the receiver at different times and all get added constructively or destructively, this can result in intersymbol interference or bits 'crashing/smearing' into one another, as shown in Fig. 5.24, which the receiver cannot sort out.

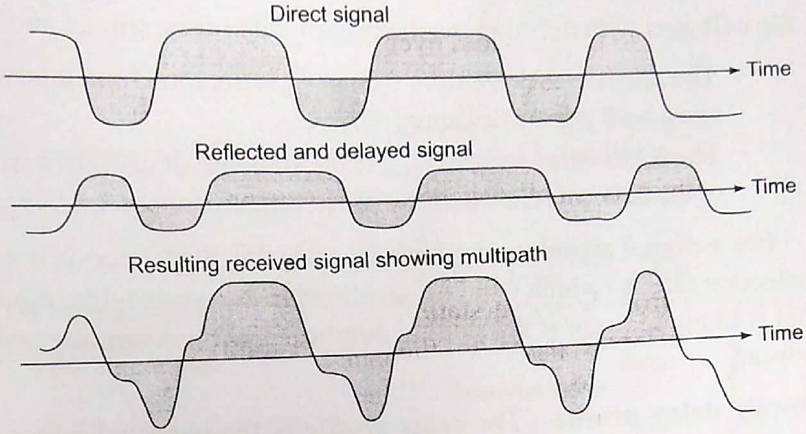



**Fig. 5.24** Effect of delay spread

This time dispersion of the channel is called *multipath delay spread*, which is an important parameter to access the performance capabilities of wireless systems. A common measure of multipath delay spread is the *root mean square* (*RMS*) *delay spread*. There is some finite delay between the time when the antenna receives the first copy of the signal on the shortest path and when it receives the last copy of the same signal on the longest path. The maximum excess delay time is represented by $T_m$. (The RMS value of delay spread, $T_{rms}$, is more commonly used in practice than the maximum delay time but is a bit more mathematically complex.)

Because of multipath reflections, the channel impulse response of a wireless channel looks like a series of impulses with decreasing amplitudes as shown in Fig. 5.25. In practice, the number of impulses that can be distinguished is very large and depends on the time resolution of the communication or measurement system.

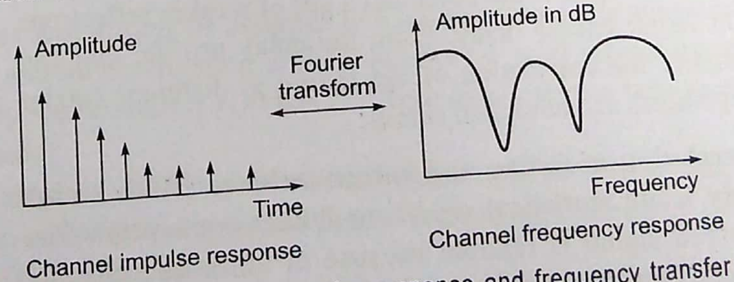



**Fig. 5.25** Example of impulse response and frequency transfer function of a multipath channel

In system evaluations, we typically prefer to address a class of channels with properties that are likely to be encountered, rather than one specific impulse response. Therefore, we define the average (local-mean) power that is received with an excess delay that falls within the interval ($T$, $T + d\tau$). Such characterization for all $T$ gives the delay profile of the channel. The delay profile determines the frequency dispersion, that is, the extent to which the channel fading at two different frequencies $f_1$ and $f_2$ is correlated.

### Important Definitions Related to Delay Spread Effect

We will start with defining the maximum delay time spread and the RMS delay spread.

- The maximum delay time spread $T_m$ is the total time interval, during which reflections with significant energy arrive.
- The RMS delay spread, $T_{rms}$, is the standard deviation (or root-mean-square) value of the delay of reflections, weighted proportional to the energy in the reflected waves.

For a digital signal with a high bit rate, this dispersion is experienced as frequency selective fading (which will be considered in the consecutive topics) and ISI. No serious ISI is likely to occur if the symbol duration is longer than, say, ten times the RMS delay spread.

**Power delay profile** The *delay profile* is the expected power variation per unit of time received with a certain excess delay. It is obtained by averaging a large set of impulse responses. In Figures 5.26, 5.27, 5.28, and 5.30, delay profiles are shown with various environmental conditions.

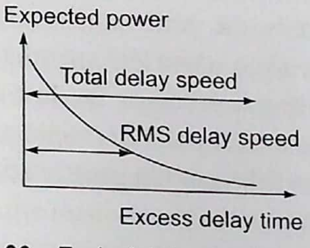



Fig. 5.26 Typical delay profile: exponential

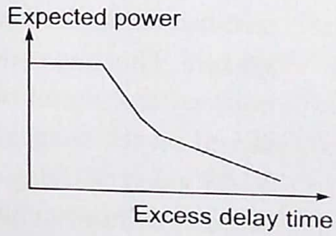



Fig. 5.27 Typical indoor delay profile

In an indoor environment (Fig. 5.27), early reflections often arrive with almost identical power. This gives a fairly flat profile up to some point and a tail of weaker reflections with larger excess delay. From the delay profile, one can compute the correlation of the fading at different carrier frequencies as mentioned earlier.

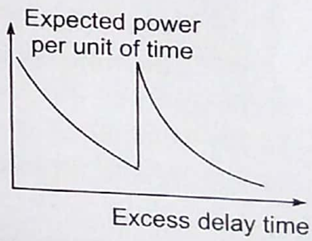



Fig. 5.28 Typical 'bad urban' delay profile with tall buildings

**Correlation of fading and autocovariance (With a case study using statistical signal modelling concepts)** The received signal is random because of noise and channel effects. Now, the discrete time signal analysis of random processes is easy. A discrete time random process is a mapping from the sample space $\Omega$ into the set of discrete time signal $x(n)$. Thus, it is a collection or *ensemble* of discrete time signals. A sequence of coefficients $x(n)$ forms the discrete time random process. It is also indexed sequence of random variables … $x(-2), x(-1), x(0), x(1), x(2)$… . Normally, each random variable in the sequence has its own PDF. In discrete time random processes, rather than probability density function, the mean and autocorrelation of a process are of interest. The same concepts are applied to wireless channel also.

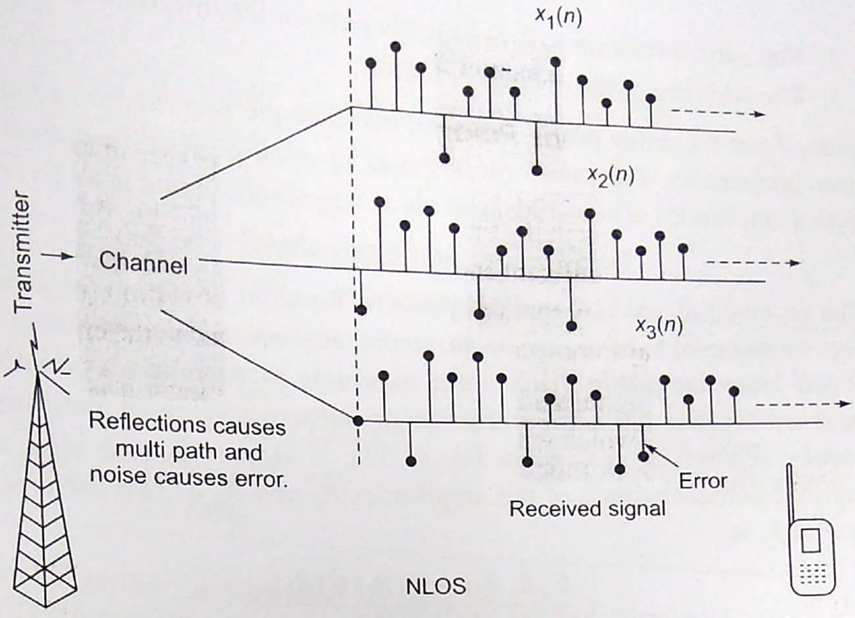



**Fig. 5.29** Autocorrelation among multiple reflected rays at the receiver

The direct and reflected rays add up but some autocorrelation exists among them. A discrete time random process $x(n)$ is said to be Gaussian if every finite collection of samples of $x(n)$ is jointly Gaussian. If the process is formed from the sequence of Gaussian random variables $x(n)$ and if they are uncorrelated, then the process is known as *white Gaussian noise*. For each $x(n)$, we can find mean and variance. Two other ensemble averages are autocovariance and autocorrelation. See below:

- Mean, $m_x(n) = E\{x(n)\}$
- Variance, $\sigma_x^2 = E\{|x(n) - m_x(n)|^2\}$
- Autocovariance, $c_x(k, l) = E\{[x(k) - m_x(k)]\ [x(l) - m_x(l)]^*\}$
- Autocorrelation, $r_x(k, l) = E\{x(k)\, x^*(l)\}$

Random processes will always be assumed to have zero mean for simplicity, so that the autocovariance and autocorrelation sequences may be used interchangeably. This assumption is acceptable because if $x(n)$ has non-zero mean, then a zero-mean process can always be formed.

As in the case of random variables, the autocorrelation and autocovariance functions provide information about the degree of linear dependence between two random variables $x(k)$ and $x(l)$. For example, if

$$c_x(k, l) = 0$$

for $k \neq 0$, then the random variables $x(k)$ and $x(l)$ are uncorrelated and knowledge of one does not help in the estimation of the other using a linear estimator.

A random process $x(n)$, due to channel impulse response $h(n)$, is said to be, in wide sense, stationary if the following three conditions are satisfied:

1. The mean of the process is a constant $m_x(n) = m_x$.

2. The autocorrelation $r_x(k, l)$ depends only on the difference $k - l$.
3. The variance of the process is finite.

Now, from the delay profile, one can compute the *correlation of fading* at different carrier frequencies. The transform of the delay profile gives the autocorrelation of the complex amplitudes of sinusoidal signals at frequencies $f_1$ and $f_2$ as given in Eq. (5.33).

$$\text{Delay profile} \leftrightarrow E[x(f_1)\, x^*(f_2)] \tag{5.33}$$

The received signal is a random process. Random processes (due to channel) will always be assumed to have zero mean, so that autocovariance and autocorrelation terms are used interchangeably. After some algebraic manipulations, relationship can be derived to express the autocorrelation or autocovariance of the amplitude versus frequency separation, $f_1 - f_2$. In Eq. (5.33), if $x(f_1) = R_1$ and $x(f_2) = R_2$, then the normalized autocovariance of the amplitudes $R_1$ and $R_2$ of two carriers, one at $f_1$ and another at $f_2$, is

$$C = \frac{E\{R_1 R_2\} - E\{R_1\} E\{R_2\}}{s(R_1)\, s(R_2)} \tag{5.34}$$

where $s(R_1)$ = standard deviation of $R_1$ and $s(R_2)$ = standard deviation of $R_2$.

**Coherence bandwidth** For a reliable communication, without using adaptive equalization or other anti-multipath techniques, the transmitted data rate may be much smaller than the inverse of the RMS delay spread, called *coherence bandwidth*.

We shall see in this chapter that narrow band transmission uses radio signals that see flat fading. The channel may be considered relatively constant over the transmit bandwidth. This criterion is found to be satisfied if the transmission bandwidth does not substantially exceed the *coherence* bandwidth $B_c$ of the channel. This is the bandwidth over which the channel transfer function remains virtually constant.

One can define *narrowband* transmission in the time domain, considering the inter-arrival times of multipath reflections and the time scale of variations in the signal caused by modulation: A signal sees a narrow band channel if the bit duration is sufficiently larger than the inter-arrival time of reflected waves. In such cases, the intersymbol interference is small.

Formally, the coherence bandwidth is the bandwidth for which the autocovariance of the signal amplitudes at two extreme frequencies reduces from 1 to 0.5. For a Rayleigh fading channel with an exponential delay profile, one finds

$$B_c = 1/(2\pi T_{\text{rms}}) \tag{5.35a}$$

where $T_{\text{rms}}$ is the rms delay spread

or sometimes $$B_c \cong 1/(5\, T_{\text{rms}}) \tag{5.35b}$$

---

**Example 5.3** In a measurement of power delay profile, the maximum excess delay is 50 ns. Assuming exponentially decaying profile and Rayleigh fading channel, find out the maximum transmission bandwidth for which the data can be transferred with minimum ISI.

***Solution***

The RMS delay spread is approximately maximum excess delay/$\sqrt{2}$. So, RMS delay spread $= 0.707 \times 50 = 35.35$ ns.

The maximum transmission bandwidth for minimum ISI is nothing but coherence bandwidth

$$\begin{aligned} B_c &= 1/(2\pi\, T_{\text{rms}}) \\ &= 1/\, 2\pi \times 35.35 \times 10^{-9} \\ &= 0.00450225 \times 10^{9} \\ &\sim 4.5 \text{ MHz} \end{aligned}$$

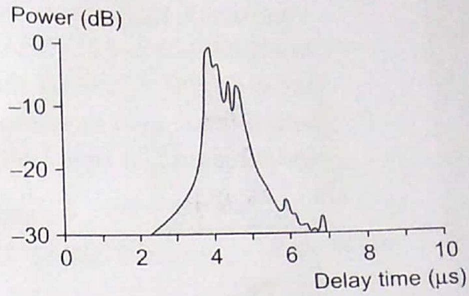



**Fig. 5.30** Typical power delay profile for delay spread = 1.2 msec and coherence bandwidth $B_c$ = 1.3 MHz

**Resolvable paths** A wideband signal with symbol duration $T_s$ [or a direct sequence (DS)-CDMA signal with chip time $t_{\text{chip}}$] can 'resolve' the time dispersion of the channel with an accuracy of about $T_s$. For DS-CDMA, the number of resolvable paths is

$$N_p = \text{round}\left(\frac{T_{\text{delay}}}{t_{\text{chip}}} + 1\right) \tag{5.36}$$

where round ($x$) is the largest integer value smaller than $x$ and $T_{\text{delay}}$ is the total length of the delay profile. A DS-CDMA rake receiver can exploit $N_p$-fold path diversity which can be.

### *Delay Spread and ISI*

Table 5.1 shows the typical delay spread for various environments. The maximum delay spread in an outdoor environment is approximately 20 μs. Thus, significant intersymbol interference can occur at bit rates as low as 25 kbps.

**Table 5.1** Typical delay spreads

| Environment or Cause | Delay Spread | Maximum Path Length Difference |
|---|---|---|
| Indoor (room) | 40 nsec–200 nsec | 12 m–60 m |
| Outdoor | 1 μ sec–20 μsec | 300 m–6 km |

As such, delay spread and ISI have similar effects. However, there is a difference. The ISI is caused by the reception of a small number of reflections from remote objects (as opposed to the large number of reflections from nearby objects that causes fading). The ISI causes the receiver to receive the original signal, overlapped by some delayed versions of the signal. Traditionally, different types of equalizers were used to reject ISI. The OFDM modulation provides a fairly strong and simple ISI rejection mechanism using cyclic prefix (Chapter 9). Intersymbol interference may be measured by eye patterns. In ASK-like modulations, it is possible to (ideally) remove the interference between different symbols using a filter satisfying the Nyquist ISI criterion. The ISI can cause significant time errors in high bit rate systems, especially while using *time division multiple access* (TDMA). Figure 5.31 shows the effect of intersymbol interference on the received signal. As the transmitted bit rate is increased, the amount of intersymbol interference also increases. The effect starts to become very significant when the delay spread is greater than ~50% of the bit time.

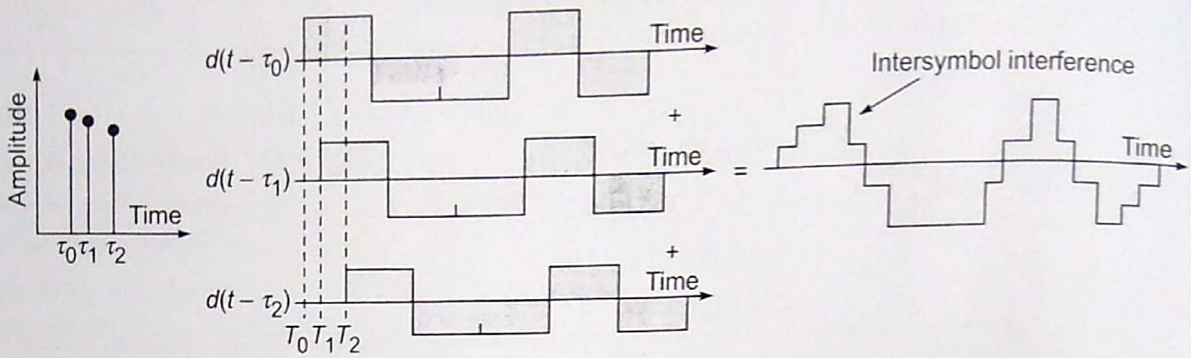



**Fig. 5.31** ISI due to delay spread effect

The delay spread restricts the maximum allowable symbol transmission rate. The intersymbol interference can be minimized in several ways. One method is to reduce the symbol rate by reducing the data rate for each channel (i.e. splitting the bandwidth into more channels using frequency division multiplexing or OFDM). Another is to use a coding scheme that is tolerant to intersymbol interference, such as CDMA.

Table 5.2 lists out some measures to eliminate delay spread.

**Table 5.2** Measures to eliminate delay spread

| System | Measure to Eliminate Delay Spread |
|---|---|
| Analog | • Narrowband transmission |
| GSM | • Adaptive channel equalization<br>• Channel estimation training sequence |
| DECT | • Use the handset only in small cells with small delay spreads<br>• Diversity and channel selection can help a little bit (pick a channel where late reflections are in a fade) |
| IS95 CDMA | • Rake receiver separately recovers signals over paths with excessive delays. CDMA array processing can further improve performance, because it also exploits angle spreads. |
| Digital audio/video broadcasting | • OFDM multicarrier modulation: The radio channel is split into many narrowband sub channels with orthogonality. There is no ISI. |
| MIMO | • Exploits multipath diversity. Uses multiple transmitting and receiving antennas. |

### *Doppler Shift/Spread*

Figures 5.32 and 5.33 show the scenario of Doppler effect. Along with the multiple reflected signals, the receiver undergoes one more effect of Doppler spread due to its own mobility, especially vehicular mobility. The Doppler spread is the width of the received spectrum when a single tone is transmitted and it is related to the rate at which fading

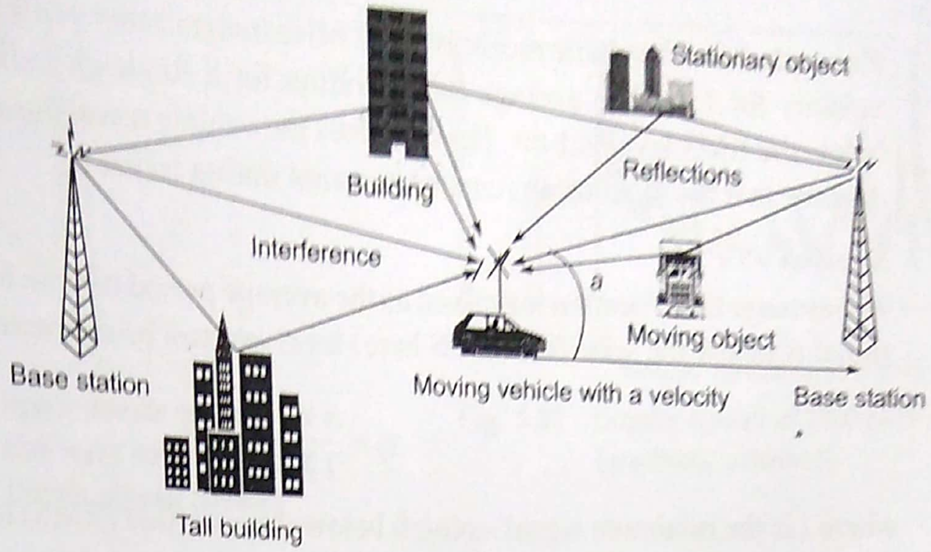



Fig. 5.32 Scenario for Doppler shift

occurs. The Doppler spread is important in determining the minimum adaptation rate for an adaptive receiver. Motion of a receiving antenna produces the Doppler shifts of incoming received waves. We consider a signal received over a multipath channel, with many incoming waves. Let the $n$th reflected wave with amplitude $c_n$ and phase $\phi_n$ arrive with an angle $\alpha_n$ relative to the direction of the motion of the antenna.

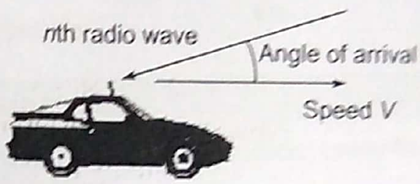



Fig. 5.33 Jake's model for Doppler effect

The Doppler shift of each wave is

$$\delta f_n = \frac{v}{\lambda} \cos \alpha_n \tag{5.37}$$

where $v$ is the speed of the receiving antenna.

The maximum Doppler shift $f_d$ occurs for a wave coming from the direction opposite to the direction the antenna is moving. It has a frequency shift, with $f_c$ being the carrier frequency and $c$ the velocity of light,

$$f_d = \frac{v}{c} f_c \tag{5.38}$$

Such motion of the antenna leads to (time-varying) phase shifts of individual reflected waves. It is not so much this minor shift that bothers radio system designers as a receiver oscillator can easily compensate for it. Rather, the problem is that many waves arrive with different shifts. Thus, their relative phases change all the time and, so, it affects the amplitude of the resulting composite signal. So, the Doppler effects determine the rate at which the amplitude of the resulting composite signal changes.

If the same effect is applied to multicarrier transmission (Chapter 9), ali the orthogonal tones placed near each other will be received with a spread. It means that all the carriers will be received with offsets. This will be the worst situation because the carriers will not remain orthogonal and as they are placed near each other, the limiting conditions will be generated. Hence, in multicarrier environment, more attention is required about the allowable Doppler spread and subcarrier spacing.

**Example 5.4** A vehicle receives a 910 MHz transmission while travelling at a constant velocity for 15 s. The average fade duration for a Rayleigh fading signal level 10 dB below the RMS level is 1 ms. How far does the vehicle travel during 15 s time duration? Assume that the local mean remains constant during travel.

***Solution***

The average fade duration is defined as the average period of time for which the received signal is below the specified (RMS here) level and can be expressed as

$$\tau_f = \frac{e^{l^2} - 1}{l f_d \sqrt{2\pi}}$$

where $l$ is the minimum signal strength below the specified (RMS) level due to fade.

We have $l = -10 \text{ dB} = 0.316$

$$\Rightarrow \quad \tau_f = \frac{e^{(0.316)^2} - 1}{0.316 \times f_d \times \sqrt{2\pi}} = 1 \text{ ms} \quad \Rightarrow \quad f_d = 132.8 \text{ Hz}$$

Now,
$$v = f_d \times \frac{c}{f_c} = 132.8 \times \frac{3 \times 10^8}{910 \times 10^6} = 43.78 \text{ m/s}$$

$\Rightarrow$ Total distance $= 15 \times 43.78 = 656.7$ m

### Doppler Power Spectrum

The models behind Rayleigh or Rician fading (Chapter 6) assume that many waves arrive each with its own random angle of arrival (thus, with its own Doppler shift), which is uniformly distributed within $[0, 2\pi]$, independently of other waves. This allows us to compute a probability density function of the frequency of incoming waves. Moreover, we can obtain the Doppler spectrum of the received signal.

This leads to the U-shaped power spectrum for isotropic scattering as shown in Figures 5.34 and 5.35.

$$S(f) = \frac{1}{4\pi f_d} \frac{1}{\sqrt{1 - \frac{(f - f_c)^2}{f_d^2}}} \tag{5.39}$$

where we assume a unity local mean power.

If a sinusoidal signal is transmitted (represented by a central line in the frequency domain in Fig. 5.34) after transmission over a fading channel, we will receive a power spectrum that is spread around the single-frequency tone. The frequency range where the power spectrum is non-zero defines the Doppler spread. The Doppler spread is relevant, for instance, to compute threshold crossing rates and average fade durations.

### Excess Delay and Doppler Spread

There are two different forms of multipath scattering according to the excess time delay of the given channel tap—small excess and large excess. Their corresponding effects on the Doppler spectrum are discussed below.

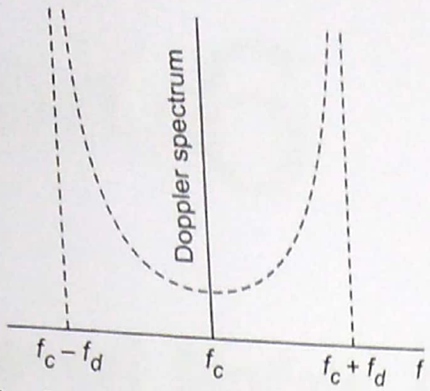



**Fig. 5.34** Power density spectrum of a sine wave suffering from a Dopple spread (ideally)

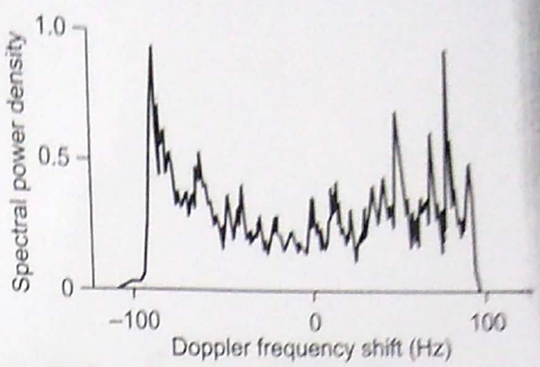



**Fig. 5.35** Doppler spread at 1800 MHz = 60.3 Hz (practically achieved)

Table 5.3 list out some measures to tackle Doppler spread.

**Table 5.3** How do systems handle Doppler spreads

| System | Countermeasure |
|---|---|
| Analog | • Doppler causes random FM modulation that may be audible; carrier frequency is low enough to avoid problems. |
| GSM | • Channel bit rate is well above Doppler spread.<br>• TDMA during each bit/burst transmission, the channel is fairly constant.<br>• Receiver training/updating during each transmission burst.<br>• Feedback frequency correction. |
| DECT | • Intended for pedestrian use:<br>o Only small Doppler spreads are to be anticipated.<br>o Bit rate is very large compared to Doppler spread. |
| IS95 Cellular CDMA | • Downlink: Pilot signal for synchronization and channel estimation.<br>• Uplink: Continuous tracking of each signal. |
| Wireless LAN's | • Mobility is slow and thus Doppler spread is only a few hertz. |

**Small excess time delays** The channel tap may be modelled as the accumulation of multipath components received from the scatterers close to the mobile. This gives rise to the classical Doppler power spectrum of the received multipath components.

**Large excess time delays** The classical Doppler model does not provide a satisfactory geometric model for this type of scattering. Instead, multipath energy is more likely to have a narrow Doppler spread, having arisen from the reflections of isolated obstacles, such as buildings or hills. The instantaneous variation of signal power in space for a channel depends on the angles of arrival of the multipath components.

## 5.10 FADING EFFECTS TO THE SIGNAL AND FREQUENCY COMPONENTS

The fading effects to the frequency components are related to bit or symbol transmission rate and time spreading of those pulses. When the transmitted data rate is much less than the coherence bandwidth, the wireless channel is referred to as the *flat channel* or chapter 6

# Channel Models, Diversity, Equalization, and Channel Estimation Techniques

**Key Topics**

- Considerations for modelling a wireless channel
- Realization of channels by digital filter
- Wideband channel model
- Phase error calculation by mathematical model
- Stochastic channel modelling
- Rayleigh fading channel model
- Rician channel model
- Nakagami channel model
- Comparison of Rayleigh, Rician, and Nakagami models
- Diversity techniques and space diversity combining
- Equalizers
- Channel estimation issues

**Chapter Outline**

After studying the various channel effects, one must be able to model a channel in order to analyse the effects of the channel on the transmitted signal and to predict the reception of the signal. Modelling of a channel is also required for the simulation purpose to check the performances in various selected conditions. Rayleigh, Rician, and Nakagami models are some popular models. Modelling a channel means correlating mathematics with the channel statistics. The earlier part of the chapter represents such models, while the latter deals with important techniques to eliminate the channel effects. Due to the channel effects, the received signal is degraded. By using some techniques, these channel effects can be reduced up to certain extent. This can improve the BER performance. There are a few techniques by which the receiver can combat the channel problems. They are diversity, equalization, and channel estimation methods. These all are described in detail in the chapter.

## 6.1 INTRODUCTION TO CHANNEL MODELLING

Perfect channel modelling is always required to study or analyse a wireless system. For that, the properties of the channels must be listed out and one or the other must be considered at a time for the modelling purpose. Based on the assumptions for the channel, the performance of the system will vary. Some important properties of a channel are as follows:

- Channels may be time varying or static. (Multipath effect makes the channel time varying and, depending upon the constructive or destructive interference, the quality of the received signal will vary.) Effect of mobility is that the channel varies with the user's location and time, which results in rapid fluctuations of received power. The slower you move, the lesser variations will be observed.

- Channels may be time dispersive or non-dispersive. (Due to dispersion, pulse spreading will be observed, which will result in ISI effect.)
- Channels may be linear or non-linear.
- All channels act as a low-pass filter under certain conditions as they show pulse spreading effect.
- Channel may be fast fading or slow fading, frequency selective or flat fading.

Depending upon the scenario or requirement of the system, mobile systems must follow the appropriate model for the analysis purpose. Two main categories of models for this purpose are as follows:

- Outdoor propagation model (e.g., Longley–Rice model, Durkin's Okumura model, and Hata model)[1]
- Indoor propagation model

For most of the channel modelling considerations as well as the equalization and channel estimation problems, the concept of digital signal processing is applied. Hence, reader must be thorough with the basics of DSP (discussed in Chapter 2) as well as the concept of digital filters, unit impulse response, etc.

The channel for mobile system applications is characterized by *multipath reception*—The signal offered to the receiver contains not only a direct line-of-sight radio wave but also a large number of reflected radio waves. Even worse in urban centres, the line of sight is often blocked by obstacles, and a collection of differently delayed waves is all what is received by a mobile antenna. The reflected waves interfere with the direct wave, which causes significant degradation of the performance of the link. If the antenna moves, the channel varies with location and time, because the relative phases of the reflected waves change. This leads to fading—time variations of the received amplitude and phase (as described in Chapter 5).

There are many constraints in modelling a channel. There may be different environment for different situation and different weather. There may be different objects in a room, or different surrounding conditions. Hence, it is very difficult to predict the number of reflected rays and whether the constructive (add-in phase) or destructive interference will occur. Because of this, the channel model is based on probability and its behaviour is represented by the probability density function (PDF).

Most of the conventional digital modulation techniques are sensitive to intersymbol interference unless the channel symbol rate is small compared to the delay spread of the channel. On the other hand, a narrowband signal with bit durations much longer than the delay spread may vanish completely in fade. A signal received at a frequency and location where reflected waves cancel each other, is heavily attenuated and may thus suffer large bit error rates.

1. These models can be self-studied.

A typical characteristic is observed on the wireless channel within fading and non-fading environment. In Fig. 6.1, the bit error probability for fading and non-fading channels is shown. In the non-fade channel, if the *C/I* ratio is increased slightly, there will be considerable drop in bit errors, i.e. the BER decreases rapidly.

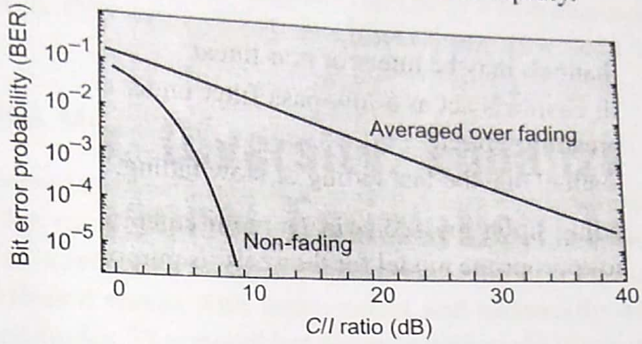



**Fig. 6.1** Channel response over fading and non-fading environment

In a fading channel, the received signal is very weak and many bit errors occur. This phenomenon remains present even if the (average) signal-to-noise ratio is large. So, the BER only improves very slowly and with a fixed slope if plotted on a log-log scale. (Diversity or error correction can help to make the slope steeper and hence improves performance.) A few effects due to multipath reception are listed out in Table 6.1 in a simplified manner.

**Table 6.1** Effects due to multipath reception

| Application | Effect |
|---|---|
| 1. Fast moving user | Rapid fluctuations of the signal amplitude and phase |
| 2. Wideband (digital) signal | Dispersion and intersymbol interference |
| 3. Analog television signal | 'Ghost' images (shifted slightly to the right) |
| 4. Multi-carrier signal | Different attenuation at different (sub)carriers and at different locations |
| 5. Stationary user of a narrowband system | Good reception at some locations and frequencies; poor reception at other locations and frequencies. |
| 6. Satellite positioning system | Strong delayed reflections may cause a severe miscalculation of the distance between user and satellite. This can result in a wrong estimate of the position. |

As shown in Fig. 6.2, the model of many randomly phased sinusoids with variations in amplitudes appears to describe the wireless radio channel appropriately and to allow calculation of outage probabilities, fade durations, and many other critical parameters of wireless links. It greatly facilitated the development systems that can reliably communicate despite the anomalies and unpredictability of the mobile communication channel. However, the number of sinusoids will be limited due to inconvenience in adding all.

Although channel fading is experienced as an unpredictable and stochastic phenomenon to the user's device or the system planner, powerful models have been of data, for further interpretation and processing, a few characteristic parameters must be extracted. This will reduce the amount of data while keeping the loss of information as less as possible. Stanford University Interim (SUI) 1 to 6 channels and ITU pedestrian A, B as well as ITU vehicular A, B channels are now used for wideband system simulations.

## 6.5 RAYLEIGH FADING MODEL

When the waves of multipath signals are out of phase, a reduction of the signal strength at the receiver can occur. This may result in deep fading. The basic model of Rayleigh fading assumes a received multipath signal to consist of a (theoretically infinite) large number of reflected waves with independent and identically distributed in-phase and quadrature amplitudes. This model has played a major role in our understanding of mobile propagation. The model was first proposed in a comment paper written by Lord Rayleigh in 1889. Signal amplitude (in dB) versus time graph for an antenna moving at constant velocity exhibiting Rayleigh fading is shown in Fig. 6.5. Note that the deep fades occur

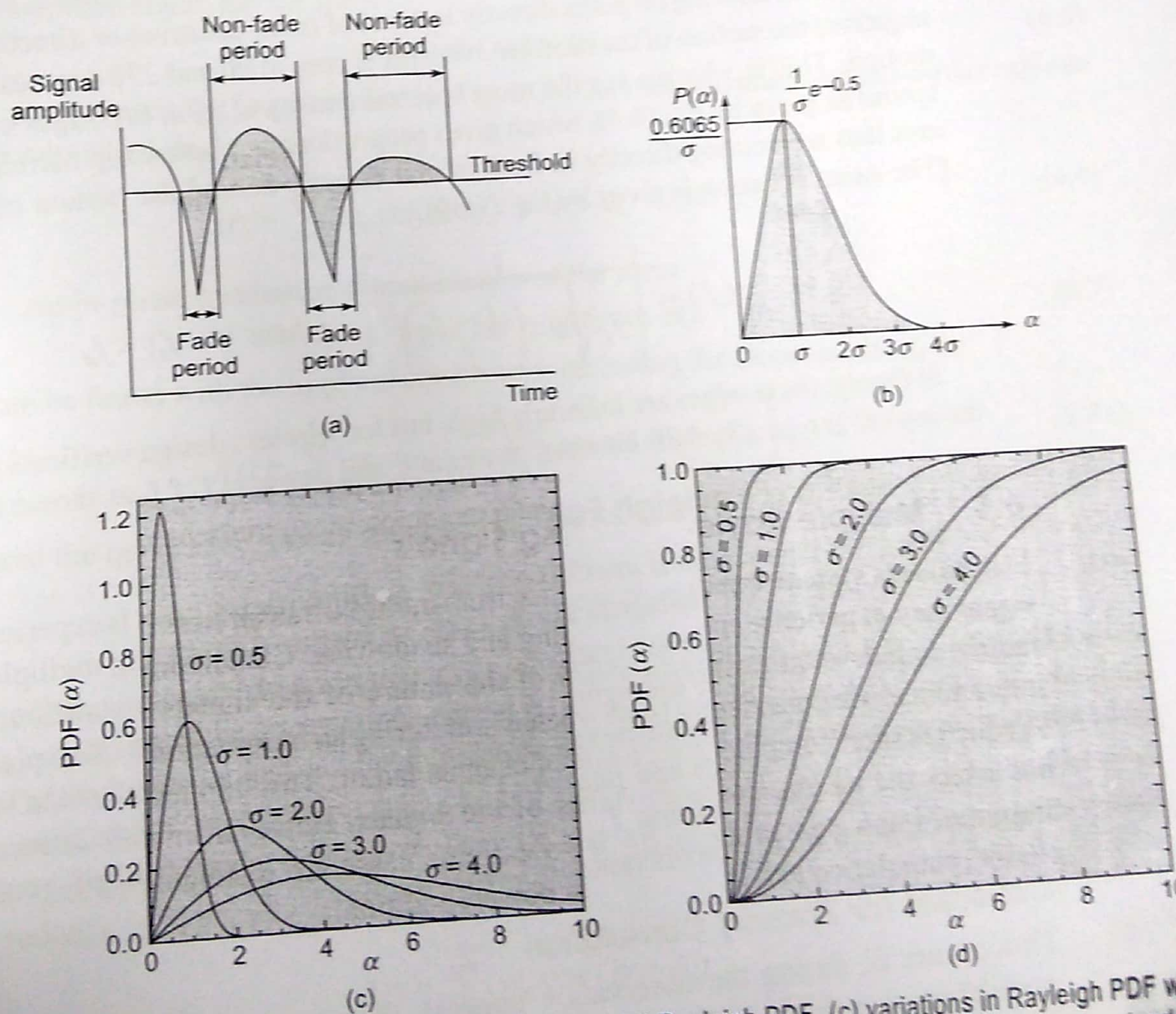



**Fig. 6.5** (a) A sample of a Rayleigh fading signal, (b) Rayleigh PDF, (c) variations in Rayleigh PDF with change in standard deviation, and (d) variations in Rayleigh CDF with change in standard deviation or mean $\mu$

occasionally. Although fading is a random process, deep fades have a tendency to occur approximately every half a wavelength of motion.

The Rayleigh distribution is a good model for channel propagation, where there is no strong line-of-sight path from the transmitter to the receiver. This can be used to represent the channel conditions seen on a busy street in a city, where the base station is hidden behind a building several blocks away and the arriving signal is bouncing off many scattering objects in the local area. In the time domain, Rayleigh fading looks like periodic peaks of 10 dB or less interspersed between deep troughs of 40 dB or more. These deep fades (nulls in signal power) will typically occur at separations of half a wavelength. The dense scatterer model, which explains cellular communications propagation, states that the amplitude of multipath rays will follow a Rayleigh distribution, while the angle of arrival (multipath phase) follows a uniform distribution. The PDF of Rayleigh distribution shown in Fig. 6.5(b) is given in Chapter 5, which is covered while explaining the nature of flat fading.

The Doppler shift of each incident ray is given by Eq. (5.37). The max spread $f_d$ is determined by the speed of the vehicle and is only experienced by the spectral components arriving on paths directly in front of (max positive) or directly behind (max negative) the motion of the receiver. No shift is seen at 90 and 270 degrees relative to the motion. This is why we see the most spectral density $s(f)$ at the edges of the Doppler spread as given by Eq. (6.4), which gives proportionate relationship correlating with the rays that are coming directly in front of and directly behind the motion of the antenna. [The exact equation is given by Eq. (5.39).]

$$s(f) \propto \left[\frac{1}{f_d\sqrt{1-\left(\dfrac{f-f_c}{f_d}\right)^2}}\right] \quad \text{where } |f-f_c| < f_d \tag{6.4}$$

In theory, these edges are infinitely high, but for wireless design verification purposes, the cut-off is typically 6 dB between power at $f_c$ and power at $f_c \pm f_d$ as shown in Fig. 5.34.

### 6.5.1 Multiple Rayleigh Fading Signals

In a wireless system, typical interference from multiple transmitters is experienced. Each signal may experience multipath fading and shadowing. Cumulating multiple Rayleigh fading signals require investigation of the nature of the signals contributing to the interference. We consider the signal behaviour during an observation interval of duration $T$, which is short compared to the rate of channel fading. This implies that the fading does not affect the amplitudes and phases of the signals. However, modulation can affect amplitudes and phases during $T$. Two extreme cases are distinguished—coherent (or phasor) cumulation and incoherent (or power) addition.

#### *Coherent (or Phasor) Cumulation*

This occurs if, during the observation interval, the phase fluctuations caused by the modulating signals are sufficiently small and the carrier frequencies of the signals are exactly equal. The joint signal behaves as a Rayleigh phasor, with Gaussian in-phase and quadrature phase components. The instantaneous power is exponentially distributed. The local-mean power is equal to the sum of the local-mean powers of the individual signals as shown in Fig. 6.6.

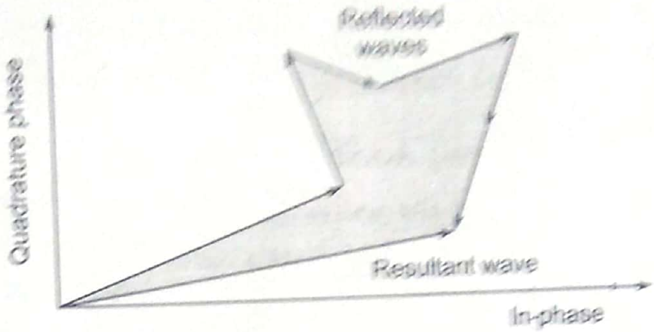



**Fig. 6.6** Phasor diagram of a set of scattered waves resulting in a Rayleigh fading envelope

To understand Fig. 6.6 in a better way, consider a case of an unmodulated carrier, the transmitted signal has the form

$$s(t) = \cos(\omega_c t + \psi) \tag{6.5}$$

Considering the Doppler effect in Fig. 5.32, the received unmodulated carrier $r(t)$ can be expressed as

$$r(t) = \sum_{n=1}^{N} \alpha_n \cos(2\pi f_c t + \psi + \phi_n + 2\pi\Delta f_n t) \tag{6.6}$$

An in-phase quadrature representation of the form

$$r(t) = I(t)\cos\omega_c t - Q(t)\cos\omega_c t \tag{6.7}$$

can be found with the in-phase component, including the Doppler effect, as

$$I(t) = \sum_{n=1}^{N} \alpha_n \cos\left(\frac{2\pi v f_c t}{c}\cos\alpha_n + \psi + \phi_n\right) \tag{6.8a}$$

and the quadrature phase component can be found as

$$Q(t) = \sum_{n=1}^{N} \alpha_n \sin\left(\frac{2\pi v f_c t}{c}\cos\alpha_n + \psi + \phi_n\right) \tag{6.8b}$$

Coherent addition can occur only if phase modulation with a very small deviation is applied, or if the observation interval is taken short with respect to the rate of modulation. This occurs, for instance, in digital systems if the joint interference signal is studied during one bit interval or during the lock-in of a carrier-recovery loop in a synchronous detector.

Let us consider a stationary user, i.e. $v = 0$. An in-phase and quadrature representation reduces to

$$I(t) = \sum_{n=1}^{N} \alpha_n \cos(\psi + \phi_n) \tag{6.9a}$$

$$Q(t) = \sum_{n=1}^{N} \alpha_n \sin(\psi + \phi_n) \tag{6.9b}$$

Thus, both the in-phase and quadrature components, $I(t)$ and $Q(t)$, can be interpreted as the sum of many (independent) small contributions. Each contribution is due to a particular reflection, with its own amplitude $\alpha_n$ and phase. For sufficiently large number of reflections (large $N$), the central limit theorem now says that the in-phase and quadrature components tend to a Gaussian distribution of their amplitude. The components $I(t)$ and $Q(t)$ appear to be independent and identically distributed.

#### Incoherent (or Power) Addition

If the phases of each of the individual signals substantially fluctuate due to mutually independent modulation, the signals add incoherently. The interference power experienced during the observation interval is the power sum of the individual signals.

With coherent addition, the joint interference signal may exhibit deep fades, caused by mutually cancelling phasors from the signals. This cannot continue for a sustained period due to the phase variations caused by angle modulation of each signal or by slightly different carrier frequencies due to Doppler shifts and free-running oscillators. With incoherent cumulation, the joint interference signal behaves as a band-limited Gaussian noise source if the number of components is sufficiently large. Moreover, any fade of one of the signals is likely to be hidden by other interfering signals. Hence, the joint interference signal tends to exhibit less multipath fluctuations per unit of time than the signal from one individual interferer.

From the above description, it can be noted that if the antenna speed is set to zero, channel fluctuations no longer occur. Fading is due to motion of the antenna. An exception occurs if the reflecting objects move. In a vehicular cellular phone system, the user is likely to move out of a fade, but in a wireless LAN, a terminal may, by accident, be placed permanently in a fade where no reliable coverage is available.

### 6.5.2 Multiple Incoherent Rayleigh Fading Signals with Equal Mean Power

If the interference is caused by the power sum of $n$ Rayleigh fading signals with identical local-mean power, the PDF of the joint interference power is the $n$th convolution of the exponential distribution of the power of an individual interfering signal. The PDF of the joint interference power caused by interfering signals with different local-mean powers can be approximated by a gamma distribution. It may not be fully appropriate to speak of the envelope of such a joint interference signal, but if one defines the amplitude to be proportional to the square root of the power, then one finds that the amplitude has a Nakagami distribution, which will be studied afterwards in the chapter.

### 6.5.3 PDF of Rayleigh Signal Amplitude

In order to obtain the PDF of the signal amplitude $r$ of a Rayleigh fading signal, we observe the random processes of the in-phase and quadrature components, $I(t)$ and $Q(t)$, respectively, at one particular instant $t_0$.

If the number of received waves $N$ becomes very large and all the waves are *independent* and identically distributed, the central limit theorem says that $I(t_0)$ and $Q(t_0)$

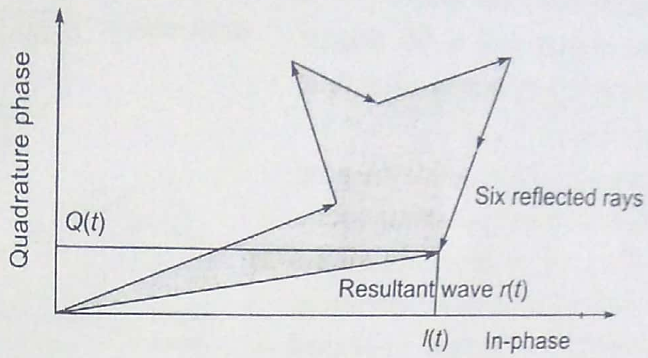



**Fig. 6.7** A received signal consisting of $N = 6$ reflected waves (The resulting signal amplitude $r$ consists of an in-phase component $I$ and a quadrature component $Q$. If the antenna moves, the relative phases of the reflected waves change over time. So, $r$, $I$, and $Q$ become functions of time $t$.)

at a particular instant $t_0$ are (zero-mean) Gaussian random variables, each with variance $\sigma^2$. Then Lord Rayleigh argued that the received signal is of the form mentioned in Eq. (5.40) and has a Rayleigh amplitude $\alpha(t)$, which is found from $\sqrt{I^2(t)+Q^2(t)}$, and a uniform phase $q(t)$ between 0 and $2\pi$. The probability density of the amplitude is described by the 'Rayleigh' PDF represented in Eq. (5.41).

A few more relationships for Rayleigh fading distribution are given below.

1. The probability that the envelope of the received signal does not exceed a specified (threshold) value $J$ is given by the corresponding CDF

$$P(J) = \int_0^J p(\alpha)d\alpha = 1 - \exp\left(-\frac{J^2}{2\sigma^2}\right) \tag{6.10}$$

2. The mean value of Rayleigh distribution is given by

$$E[\alpha] = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma \tag{6.11a}$$

3. The variance of Rayleigh distribution is given by

$$\sigma_\alpha^2 = E[\alpha^2] - E^2[\alpha] = 0.4292\sigma^2 \tag{6.11b}$$

4. The RMS value of the envelope is the square root of the mean square, i.e. $\sqrt{2}\sigma$, where $\sigma$ is the standard deviation and median value of $\alpha$ is $1.177\sigma$.

An important application of the probability density function is the calculation of outage probabilities, that is the probability that the signal strength drops below a certain threshold level.

If the set of reflected waves is dominated by one strong component, Rician fading is a more appropriate model.

### 6.5.4 Rayleigh Fading Simulator: An Example

Narrowband Rayleigh fading is modelled often as a random process that multiplies the radio signal by a complex-value Gaussian random function (for $I$ and $Q$ components). The spectrum of this random function is determined by the Doppler spread of the channel. Thus, one can generate two appropriately filtered Gaussian noise signals and use these to modulate the signal and a 90 degree phase shifted version of the signal as shown in Fig. 6.8.

It is a common practice to generate the two filtered noise components by adding a set of six or more sinusoidal signals. Their frequencies are chosen as to approximate the typical U-shaped Doppler spectrum as shown in Fig. 6.9. The $N$ frequency components are taken at

$$f_i = f_d \cos [2 \pi i / 2(2N + 1)] \quad (6.12)$$

where $i = 1, 2, \ldots, N$.

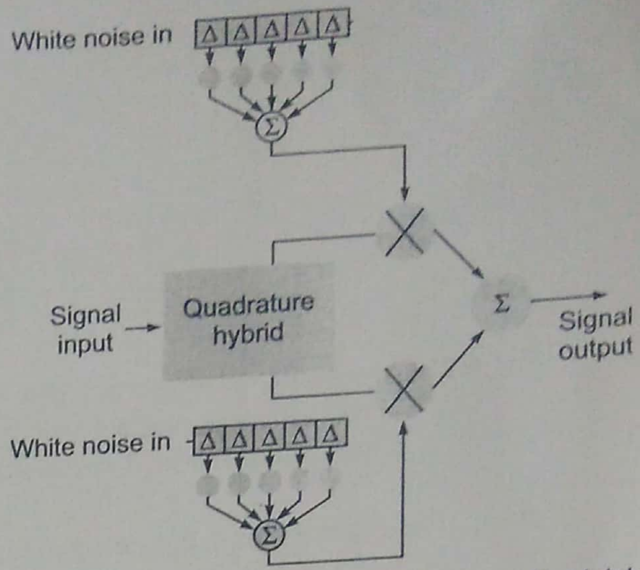



Fig. 6.8 Block diagram of a narrowband Rayleigh fading simulator (in baseband form)

This specific set of frequencies is chosen to approximate the U-shaped Doppler spectrum. All amplitudes are taken equal to unity. One component at the maximum Doppler shift is also added but at an amplitude of $1/\sqrt{2}$, i.e. at about 0.707. One more simulator is designed by Dr. Kemilo Feher and is given in his book on *Digital Communication*.

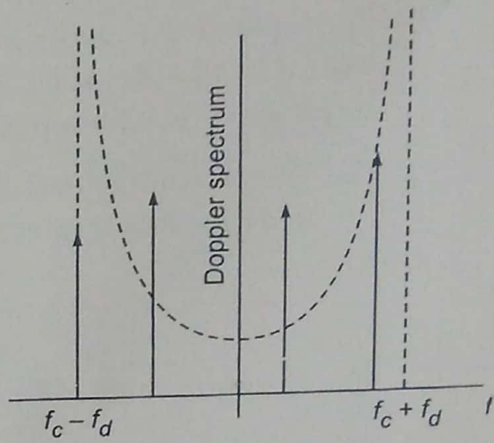



Fig. 6.9 $N$ frequency components are chosen from the Doppler spectrum for simulation

As the demand for mobile communication increases, systems have to be more efficient and cell sizes smaller and smaller as described in Chapter 11. To describe microcellular propagation, the Rayleigh model lacked the effect of a dominant line-of-sight component, and Rician model appeared to be more appropriate.

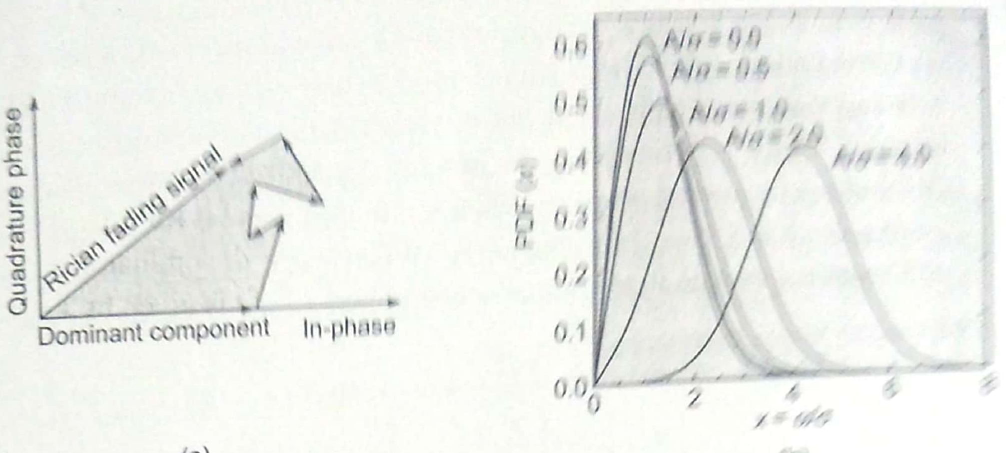
## 6.6 RICIAN FADING MODEL

The Rician fading model is similar to that for Rayleigh fading, except that in Rician fading, a strong dominant component is present. This dominant component can, for instance, be the line-of-sight wave. The refined Rician models also consider the following points:

- The dominant wave can be a phasor sum of two or more dominant signals, e.g. the line of sight and a ground reflection. This combined signal is then mostly treated as a deterministic (fully predictable) process.
- The dominant wave can also be subject to shadow attenuation. This is a popular assumption in the modelling of satellite channels.

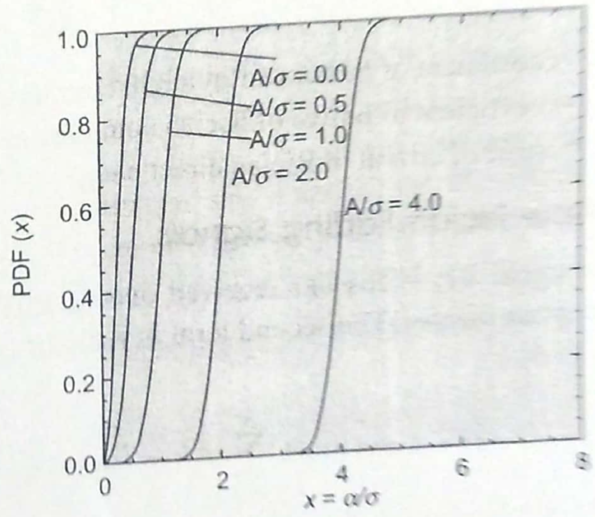
Besides the dominant component, the mobile antenna receives a large number of reflected and scattered waves as shown in Fig. 6.10.

Again consider the Doppler shift theory explained previously. The max spread $f_d$ is determined by the speed of the vehicle and is only experienced by the spectral




(a)

(b)




(c)

**Fig. 6.10** (a) Phasor diagram of Rician fading signal, (b) Rician PDF, and (c) Rician CDF with change in standard deviation

components arriving on paths directly in front of (max positive) or directly behind (max negative) the motion of the receiver because this is where the cosine of the angle of arrival is 1. At any other angle, the cosine is less than 1. No shift is seen at 90 and 270 degrees relative to the motion. This is why we see the most spectral density at the edges of the Doppler spread, which correlates with the rays that are coming in directly in front of and directly behind the motion of the antenna.

The Rayleigh fading is considered a worst-case scenario here. In rural environments, where the multipath profile includes a few reflected paths combined with a strong line-of-sight path, the spectral power follows a Rician distribution. The angle of arrival of the direct ray as well as the ratio of the powers between the direct ray and the multipath rays, determine how much effect the energy from the direct path has on the normal Rayleigh model. Observing this effect in the frequency domain, what you see is a spike in the power corresponding to the frequency shift attributed to the direct ray. As an example, in the GSM specifications, the angle of arrival of the direct path is set to 45 degrees. We take the cosine of the angle to see how much statistical weight we give to that particular path. In this case, it is 0.7. Thus, we have a spike in received signal power at 0.7 times the max frequency shift. The Rician model also allows for setting the ratio of powers between the direct path and the Rayleigh path. This ratio is called the $K$-factor. The Rician $K$-factor is defined as the ratio of signal power in dominant component over the (local-mean) scattered power. When $K = 0$, the channel is Rayleigh (i.e. the numerator is zero) and when $K = \infty$, the channel is AWGN (i.e. the denominator is zero). The variation of Doppler spectrum in terms of spectral density $s(f)$ is given by Eq. (6.13).

$$S(f) \propto \left\{ \frac{P_{Ry}}{\pi f_d \sqrt{1 - \left( \frac{f - f_c}{f_d} \right)^2}} + P_{Rc}\delta[(f - f_c) - \theta_{Rc} f_d] \right\} \tag{6.13}$$

if $|f - f_c| < f_d$

where $P_{Ry}$ = coefficient of power of Rayleigh component

$P_{Rc}$ = coefficient of power of Rician component

$\theta_{Rc}$ = angle of arrival of Rician direct path

### 6.6.1 Multiple Rician Fading Signals

A sinusoidal signal $s(t) = \cos \omega_c t$ received over a Rician multipath channel can be expressed as given below. [The second term is without Doppler effect; otherwise, it is same as Eq. (6.6).]

$$r(t) = A \cos \omega_c t + \sum_{n=1}^{N} \alpha_n \cos (\omega_c t + \phi_n) \tag{6.14}$$

where $A$ is the amplitude of the line-of-sight component (Rayleigh fading is recovered for $A = 0$), $\alpha_n$ is the amplitude of the $n$th reflected wave, $\phi_n$ is the phase of the $n$th reflected wave, and $n = 1, \ldots, N$ identify the reflected, scattered waves.

Examples of Rician fading are found in

- Microcellular channels
- Vehicle-to-vehicle communication
- Indoor propagation
- Satellite channels

Similar to the case of Rayleigh fading, the in-phase and quadrature phase components of the received signal are independent and identically distributed jointly Gaussian random variables. However, in Rician fading, the mean value of (at least) one component is non-zero due to a deterministic strong wave.

For a large fade margin $F >> 1$, the probability that the instantaneous power drops below the noise threshold $z$ tends to

$$\text{Outage probability} = \frac{(1 + K)\exp(-K)}{F} \tag{6.15}$$

where the fade margin $F$ is defined as the local mean power minus the threshold $z$.

### 6.6.2 PDF of Rician Signal Amplitude

The derivation of the probability density function of amplitude in Rician fading [Fig. 6.10(b)] is more involved than in Rayleigh fading, and a Bessel function occurs in the mathematical expression. It has been proposed to approximate this expression by the model for Nakagami fading. However, the behaviour of Nakagami and Rician fading in deep fades is essentially different. Approximations that focus on the behaviour near the mean value will divert by orders of magnitude in predicting the probability of deep fades.

The derivation is similar to the derivation for Rayleigh fading. In order to obtain the probability density of the signal amplitude $\alpha$, we observe the random processes $I(t)$ and $Q(t)$ at one particular instant $t_0$. If the number of scattered waves is sufficiently large and the waves are independent and identically distributed, the central limit theorem says that $I(t_0)$ and $Q(t_0)$ are Gaussian but, due to the deterministic dominant term, are no longer zero mean. The Rician PDF is given by Eq. (5.42). The transformation of variables shows that the amplitude and the phase have the joint PDF represented by Eq. (5.43).

In the expression for the received signal, the power in the line of sight equals $A^2/2$. In indoor channels with an unobstructed line of sight between the transmit and receive antennas, the $K$-factor is between, say, 4 and 12 dB. Rayleigh fading is recovered for $K = 0$ ($-\infty$ dB). The total local-mean power $\overline{p}$ is the sum of the power in the line of sight and the local-mean scattered power. The local-mean scattered power equals $\overline{p}/(K+1)$. The amplitude of the line of sight is $A = \sqrt{[2K\overline{p}/(K+1)]}$.

## 6.7 NAKAGAMI FADING MODEL

Besides Rayleigh and Rician fading, a refined model for the PDF of a signal amplitude exposed to mobile fading has been suggested. The distribution of the amplitude and signal power can be used to find probabilities on signal outages. The following points should be noted:

- If the envelope is Nakagami distributed, the corresponding instantaneous power is gamma distributed.
- The parameter $m$ is called the *shape factor* of the Nakagami or the gamma distribution.
- In the special case $m = 1$, Rayleigh fading is recovered, with an exponentially distributed instantaneous power.
- For $m > 1$, the fluctuations of the signal strength reduce as compared to Rayleigh fading.

The Nakagami fading model was initially proposed because it matched empirical results for short-wave ionospheric propagation. In current wireless communication, the main role of the Nakagami model can be summarized as follows.

- It describes the amplitude of received signal after maximum ratio diversity combining (described in Section 6.11.4). After $k$-branch maximum ratio combining (MRC) with Rayleigh fading signals, the resulting signal is Nakagami with $m = k$.