



Data mining - Association rules

(Ref:Data Mining Concepts by Arun K Pujari)

DATA MINING

- Data mining is the non trivial extraction of implicit, previously unknown and potentially useful information from the data

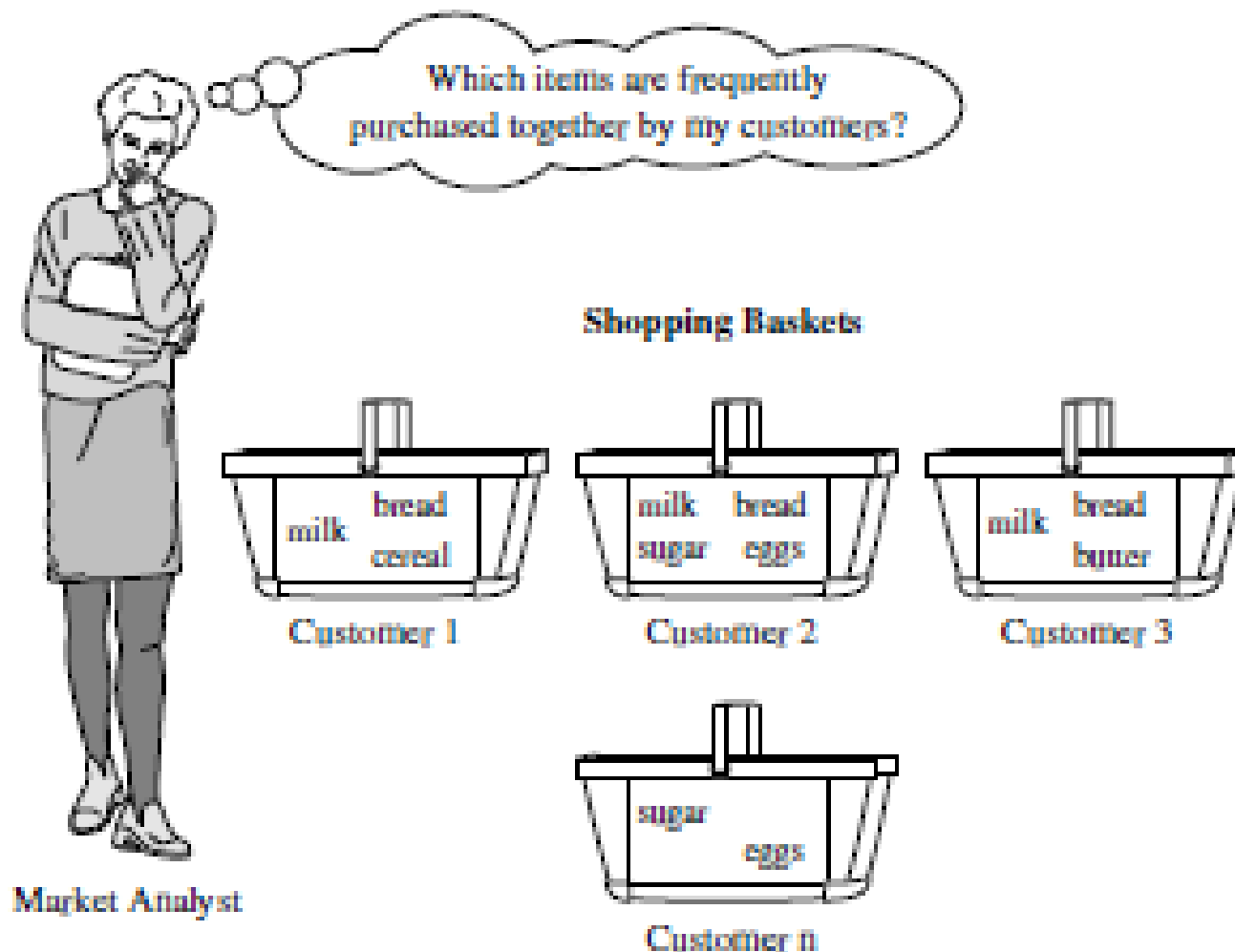
What Is Frequent Pattern Analysis?

- **Frequent pattern**: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami in the context of **frequent itemsets** and **association rule mining**
- Motivation: Finding inherent regularities in data
 - What products were often purchased together?
 - What are the subsequent purchases after buying a PC?
 - Can we automatically classify web documents?
- Applications
 - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Market basket



Develop marketing strategies, inventory management, sale promotion strategies etc.



Association Rules: Basic Concepts

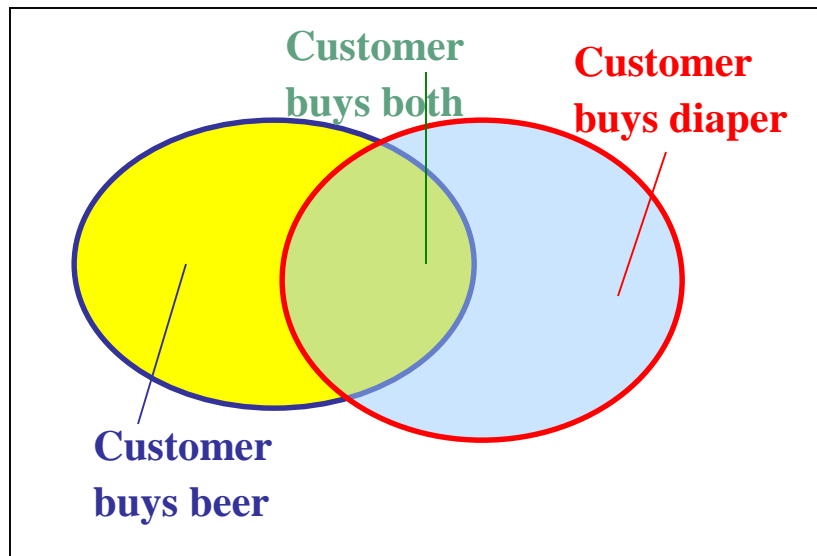
- Given: (1) database of transactions, (2) each transaction is a list of items (purchased by a customer in a visit)
- Find: all rules that correlate the presence of one set of items with that of another set of items

Problem Statement

- $I = \{\iota_1, \iota_2, \dots, \iota_m\}$: a set of literals, called items
- Transaction T : a set of items s.t. $T \subseteq I$
- Database \mathcal{D} : a set of transactions
- A transaction contains X , a set of items in I , if $X \subseteq T$
- An association rule is an implication of the form $X \Rightarrow Y$, where $X, Y \subseteq I$
- The rule $X \Rightarrow Y$ holds in the transaction set \mathcal{D} with confidence c if $c\%$ of transactions in \mathcal{D} that contain X also contain Y
- The rule $X \Rightarrow Y$ has support s in the transaction set \mathcal{D} if $s\%$ of transactions in \mathcal{D} contain $X \cup Y$
- Find all rules that have support and confidence greater than user-specified min support and min confidence

Example : Frequent Patterns and Association Rules

Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F



- Itemset $X = \{x_1, \dots, x_k\}$
- Find all the rules $X \rightarrow Y$ with minimum support and confidence
 - **support**, s , **probability** that a transaction contains $X \cup Y$
 - **confidence**, c , **conditional probability** that a transaction having X also contains Y

Let $sup_{min} = 50\%$, $conf_{min} = 50\%$
Freq. Pat.: $\{A:3, B:3, D:4, E:3, AD:3\}$

Association rules:

$A \rightarrow D$ (60%, 100%)

$D \rightarrow A$ (60%, 75%)

Example: Database with transactions (customer_# : item_a1,
item_a2, ...)

1: 1, 3, 5.

2: 1, 8, 14, 17, 12.

3: 4, 6, 8, 12, 9, 104.

4: 2, 1, 8.

support {8,12} = 2 (,or 50% ~ 2 of 4 customers)

support {1, 5} = 1 (,or 25% ~ 1 of 4 customers)

support {1} = 3 (,or 75% ~ 3 of 4 customers)

Example: Database with transactions (customer_# : item_a1, item_a2, ...)

1: 3, 5, 8.
2: 2, 6, 8.
3: 1, 4, 7, 10.
4: 3, 8, 10.
5: 2, 5, 8.
6: 1, 5, 6.
7: 4, 5, 6, 8.
8: 2, 3, 4.
9: 1, 5, 7, 8.
10: 3, 8, 9, 10.

Conf ({5} => {8}) ?

supp({5}) = 5 , supp({8}) = 7 , supp({5,8}) = 4,

then **conf({5} => {8}) = 4/5 = 0.8 or 80%**

Example: Database with transactions (customer_# : item_a1, item_a2, ...)

1: 3, 5, 8.
2: 2, 6, 8.
3: 1, 4, 7, 10.
4: 3, 8, 10.
5: 2, 5, 8.
6: 1, 5, 6.
7: 4, 5, 6, 8.
8: 2, 3, 4.
9: 1, 5, 7, 8.
10: 3, 8, 9, 10.

Conf ({5} => {8}) ? 80% Done. Conf ({8} => {5}) ?

supp({5}) = 5 , supp({8}) = 7 , supp({5,8}) = 4,
then **conf({8} => {5}) = 4/7 = 0.57 or 57%**

Conf ({5} => {8}) ? 80% Done.

Conf ({8} => {5}) ? 57% Done.

**Rule ({5} => {8}) more meaningful than
Rule ({8} => {5})**

Example: Database with transactions (customer_# : item_a1, item_a2, ...)

1: 3, 5, 8.
2: 2, 6, 8.
3: 1, 4, 7, 10.
4: 3, 8, 10.
5: 2, 5, 8.
6: 1, 5, 6.
7: 4, 5, 6, 8.
8: 2, 3, 4.
9: 1, 5, 7, 8.
10: 3, 8, 9, 10.

Conf ({9} => {3}) ?

supp({9}) = 1 , supp({3}) = 4 , supp({3,9}) = 1,
then **conf({9} => {3}) = 1/1 = 1.0 or 100%. OK?**

Conf({9} => {3}) = 100%. Done.

Notice: High Confidence, Low Support.

-> Rule ({9} => {3}) not meaningful

Frequent set

- An item set $X \subseteq I$ is said to be a **frequent item set** in T , if
- $S(X)_T \geq \text{user specified minimum support}$

The **downward closure** property of frequent sets

Any subset of a frequent itemset must be frequent

If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**

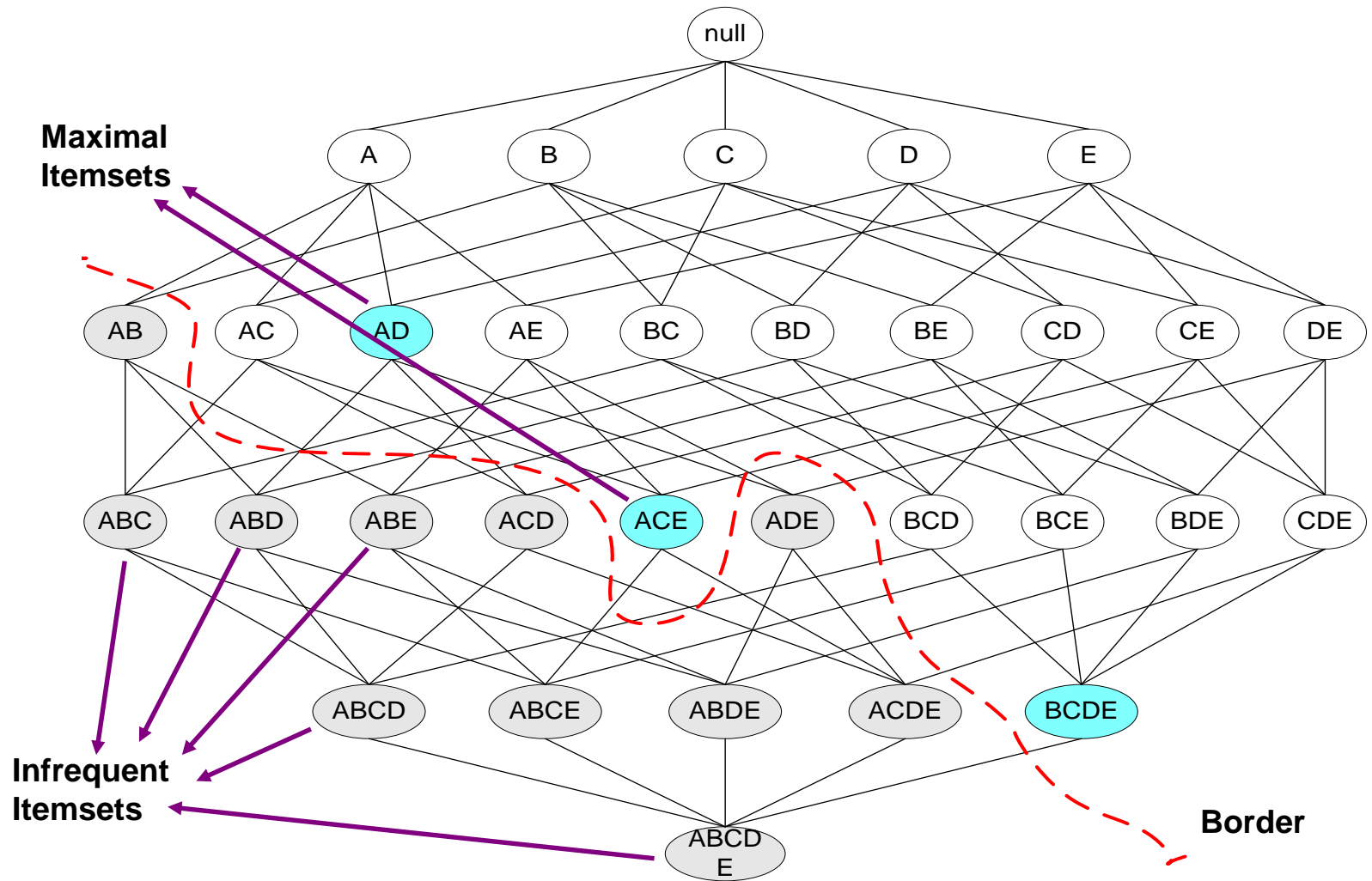
i.e., every transaction having {beer, diaper, nuts}
also contains {beer, diaper}

The **upward closure** property of frequent sets

Any superset of an infrequent set is an infrequent set

-
- **Maximal frequent set**: a frequent set is a **maximal frequent set** if it is a frequent set and no superset of this is a frequent set
 - **Border set**: an itemset is a **border set** if it is not a frequent set, but all its proper subsets are frequent sets
 - **Note**: set of all maximal frequent sets can act as a compact representation of the set of all frequent sets

Lattice of subsets



ID apples, beer, cheese, dates, eggs, fish, glue, honey, ice-cream

1	1	1	0	1	0	0	1	1	0
2	0	0	1	1	1	0	0	0	0
3	0	1	1	0	0	1	0	0	0
4	0	1	0	0	0	1	0	0	1
5	0	0	0	0	1	0	1	0	0
6	0	0	0	0	0	1	0	0	1
7	1	0	0	1	0	0	0	1	0
8	0	0	0	0	0	1	0	0	1
9	0	0	1	0	1	0	0	0	0
10	0	1	0	0	0	0	1	0	0
11	0	0	0	0	1	0	1	0	0
12	1	0	0	0	0	0	0	0	0
13	0	0	1	0	0	1	0	0	0
14	0	0	1	0	0	1	0	0	0
15	0	0	0	0	0	0	0	1	1
16	0	0	0	1	0	0	0	0	0
17	1	0	0	0	0	1	0	0	0
18	1	1	1	1	0	0	0	1	0
19	1	1	0	1	0	0	1	1	0
20	0	0	0	0	1	0	0	0	0

Numbers

Our example transaction DB has 20 records of supermarket transactions, from a supermarket that only sells 9 things

One month in a large supermarket with five stores spread around a reasonably sized city might easily yield a DB of 20,000,000 baskets, each containing a set of products from a pool of around 1,000

1GB database, 125,000 block reads for a single pass
(blocksize=8KB)

if algorithm requires 10 passes, 1,250,000 block read

assume avg. read time=12ms per page/block

time spent in I/O=1,250,000 x 12 ms= 4 hrs.

Apriori Algorithm

- the **Apriori algorithm** was proposed by Agarwal and Srikant in 1994.
- Apriori uses a "bottom up" approach, where frequent subsets are extended one item at a time (a step known as *candidate generation*), and groups of candidates are tested against the data.
- Also called level-wise algorithm
- The algorithm terminates when no further successful extensions are found.
- Apriori uses breadth-first search to count candidate item sets efficiently.
- Since, algorithm uses **prior knowledge** of frequent item set properties, it is called as apriori.

Apriori Algorithm

Uses a Level-wise search, where k -itemsets (An itemset that contains k items is a k -itemset) are used to explore $(k+1)$ -itemsets, to mine frequent itemsets from transactional database for Boolean association rules.

First, the set of frequent 1-itemsets is found. This set is denoted L_1 . L_1 is used to find L_2 , the set of frequent 2-itemsets, which is used to find L_3 , and so on, until no more frequent k -itemsets can be found.

Apriori candidate generation

- The **candidate-gen** function takes L_{k-1} and returns a **superset** (called the **candidates**) of the set of all **frequent k -itemsets**. It has two steps
 - **join step**: Generate all possible candidate itemsets C_k of length k
 - **prune step**: Remove those candidates in C_k that cannot be frequent.

gen_candidate_itemsets with the given L_{k-1} as follows:

$C_k = \emptyset$

for all itemsets $l_1 \in L_{k-1}$ *do*

for all itemsets $l_2 \in L_{k-1}$ *do*

if $l_1[1] = l_2[1] \wedge l_1[2] = l_2[2] \wedge \dots \wedge l_1[k-1] < l_2[k-1]$

then $c = l_1[1], l_1[2] \dots l_1[k-1], l_2[k-1]$

$C_k = C_k \cup \{c\}$

prune(C_k)

for all $c \in C_k$

for all $(k-1)$ -subsets d of c *do*

if $d \notin L_{k-1}$

then $C_k = C_k \setminus \{c\}$

Apriori Algorithm

Initialize: $k := 1$, C_1 = all the 1-itemsets;
read the database to count the support of C_1 to determine L_1 .
 $L_1 := \{\text{frequent 1-itemsets}\}$;
 $k := 2$; // k represents the pass number//
while ($L_{k-1} \neq \emptyset$) **do**
 begin
 $C_k := \text{gen_candidate_itemsets with the given } L_{k-1}$
 prune(C_k)
 for all transactions $t \in T$ **do**
 increment the count of all candidates in C_k that are contained in t ;
 $L_k :=$ All candidates in C_k with minimum support ;
 $k := k + 1$;
 end
Answer $:= \bigcup_k L_k$;

Min. sup. = 3

T1	{Mango, Onion, Nestle Bar, Key-chain, Eggs, Yogurt}
T2	{Doll, Onion, Nestle Bar, Key-chain, Eggs, Yogurt}
T3	{Mango, Apple, Key-chain, Eggs}
T4	{Mango, Umbrella, Corn, Key-chain, Yogurt}
T5	{Corn, Onion, Onion, Key-chain, Ice-cream, Eggs}

Transaction ID	Items Bought	Items Bought (order)
T1	{M, O, N, K, E, Y }	{E, K, M, N, O, Y}
T2	{D, O, N, K, E, Y }	{D, E, K, N, O, Y}
T3	{M, A, K, E}	{A, E, K, M}
T4	{M, U, C, K, Y }	{C, K, M, U , Y}
T5	{C, O, O, K, I, E}	{C, E, I, K, O}

TDB

Tid	Items
T1	{E, K, M, N, O, Y }
T2	{D, E, K, N, O, Y }
T3	{ A, E, K, M}
T4	{C, K, M, U, Y }
T5	{C, E, I, K, O}

C_1
1st scan

Itemset	sup
{A}	1
{C}	2
{D}	1
{E}	4
{I}	1
{K}	5
{M}	3
{N}	2
{O}	3
{U}	1
{Y}	3

L_1

Item set
{E}
{K}
{M}
{O}
{Y}

C_2

Items et
{E,K}
{E, M}
{E,O}
{E, Y}
{K,M}
{K, O}
{K, Y}
{M, O}
{M, Y}
{O, Y}

C_2
2nd scan

Items et	SUP
{E,K}	4
{E, M}	2
{E,O}	3
{E, Y}	2
{K,M}	3
{K, O}	3
{K, Y}	3
{M, O}	1
{M, Y}	2
{O, Y}	2

L_2

Itemset
{E, K}
{E, O}
{K, M}
{K, O}
{K, Y}

C_3
3rd scan

Itemset
{E, K, O}
{K, M, O}
{K, M, Y}
{K, O, Y}

C_3

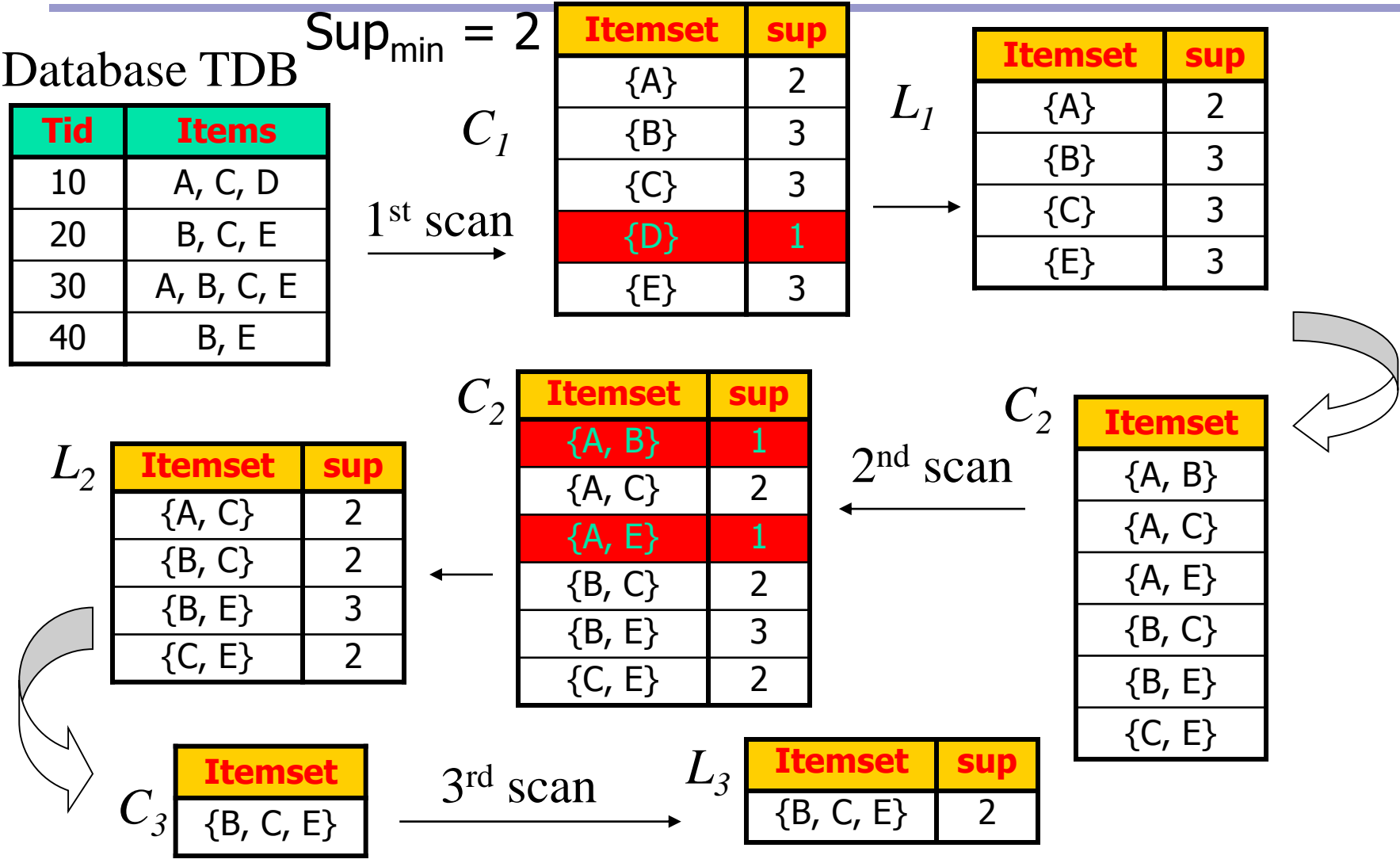
Item set	SUP
{E, K, O}	3

L_3

Itemset
{E, K, O}

Frequent items (L) = $L_1 \cup L_2 \cup L_3$

Frequent items = { {E}, {K}, {M}, {O}, {Y}, {E,K}, {E,O}, {K,M}, {K,O}, {K,Y}, {E,K,O} }



Generating association rules

- For each frequent itemset l , generate all nonempty subsets of l .
- For every nonempty subset s of l , output the rule " $s \Rightarrow l - s$ " if $(\text{support count}(l) / \text{support count}(s)) \geq \text{min conf}$, where min conf is the minimum confidence threshold.
- Because the rules are generated from frequent itemsets, each one automatically satisfies minimum support.
- Let $l = \{B, C, E\}$, given, min conf threshold = 60%
- Subsets of $l = \{\{B, C\}, \{B, E\}, \{C, E\}, \{B\}, \{C\}, \{E\}\}$
- $s = \{B, C\}$, $\{B, C\} \Rightarrow \{E\}$, $\text{conf}(\{B, C\} \Rightarrow \{E\}) = 2/2 = 100\%$
- $s = \{B, E\}$, $\{B, E\} \Rightarrow \{C\}$, $\text{conf}(\{B, E\} \Rightarrow \{C\}) = 2/3 = 66\%$
- $s = \{C, E\}$, $\{C, E\} \Rightarrow \{B\}$, $\text{conf}(\{C, E\} \Rightarrow \{B\}) = 2/2 = 100\%$
- $s = \{B\}$, $\{B\} \Rightarrow \{C, E\}$, $\text{conf}(\{B\} \Rightarrow \{C, E\}) = 2/3 = 66\%$
- $s = \{C\}$, $\{C\} \Rightarrow \{B, E\}$, $\text{conf}(\{C\} \Rightarrow \{B, E\}) = 2/3 = 66\%$
- $s = \{E\}$, $\{E\} \Rightarrow \{B, C\}$, $\text{conf}(\{E\} \Rightarrow \{B, C\}) = 2/3 = 66\%$

Apriori Advantages/Disadvantages

- Advantages
 - Uses large itemset property
 - Easily parallelized
 - Easy to implement
- Disadvantages
 - Assumes transaction database is memory resident.
 - Requires many database scans.

Database 2

1: bread, milk

2: bread, meat, orange juice, eggs

3: milk, meat, orange juice, cola

4: bread, milk, meat, orange juice

5: bread, milk, meat, cola

Database 1

Tid	Items
10	1,2,5
20	2,4
30	2,3
40	1,2,4
50	1,3
60	2,3
70	1,3
80	1,2,3,5
90	1,2,3

Itemsets

{1,2,3,4}

{1,2,4}

{1,2}

{2,3,4}

{2,3}

{3,4}

{2,4} min support
count ≥ 3

Partition Algorithm

- Frequent sets are very few compared to the set of all itemsets
- Partition set of transaction to smaller segments so that, each segment can be accommodated in main memory
- Compute set of frequent sets for each partition

Partition algorithm

- 2 scan of database
 - Generate set of all potentially frequent itemsets
 - Measure actual support
- 2 phases
 - Logically divide database into number of non overlapping partitions
 - If 'n' partitions – 'n' iterations
 - Merge frequent item sets
 - Local frequent item sets of same length from all partitions are combined to generate global candidate item sets

Partition algorithm

- Identify frequent item sets by generating actual support
- NOTE
 - A partition P refers to any subset of transaction in DB
 - Local support – fraction of transactions containing given item set
 - Local frequent item set – local support in a partition is at least minimum support
 - If an item set is not frequent in any partition, it is not frequent in whole DB.

Apriori Limitations

- Apriori algorithm can be very slow and the bottleneck is candidate generation.
- For example, if the transaction DB has 10^4 frequent 1-itemsets, they will generate 10^7 candidate 2-itemsets even after employing the downward closure.
- To compute those with support more than minimum support, the database need to be scanned at every level. It needs $(n + 1)$ scans, where n is the length of the longest pattern.

P=partition_database(T); n=number of partitions;

//Phase I

for i = 1 to n do begin

■ read_in_partition(T_i in P)

L_i =generate all frequent itemsets of T_i using apriori method in main memory

end

//Merge Phase

for ($k=1$; $L_k^i \neq \Phi$, $i=1,2,\dots,n$; $k++$) do begin

■ $C_k^G = \cup_{i=1}^n L_k^i$

End

//Phase II

for i = 1 to n do begin

 read_in_partition(T_i in P)

 for all candidates $c \in C^G$ compute $s(c)_{T_i}$

End

$L^G = \{c \in C^G \mid s(c)_{T_i} \geq \sigma\}$

Answer = L^G

- Min. sup.count ≥ 3
- Local sup. Count ≥ 1

Tid	Items
10	1,2,5
20	2,4
30	2,3
40	1,2,4
50	1,3
60	2,3
70	1,3
80	1,2,3,5
90	1,2,3

➤ L^1

- $L_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
- $L_2 = \{\{1,2\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}\}$
- $L_3 = \{\{1,2,5\}\}$

➤ L^2

- $L_1 = \{\{1\}, \{2\}, \{3\}, \{4\}\}$
- $L_2 = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}\}$
- $L_3 = \{\{1,2,4\}\}$

➤ L^3

- $L_1 = \{\{1\}, \{2\}, \{3\}, \{5\}\}$
- $L_2 = \{\{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,5\}, \{3,5\}\}$
- $L_3 = \{\{1,2,3\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\}\}$
- $L_4 = \{\{1,2,3,5\}\}$

Tid	Items
10	1,2,5
20	2,4
30	2,3
40	1,2,4
50	1,3
60	2,3
70	1,3
80	1,2,3,5
90	1,2,3

- $C^G_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
- $C^G_2 = \{\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,5\}\}$
- $C^G_3 = \{\{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\}\}$
- $C^G_4 = \{\{1,2,3,5\}\}$

$$C^G = \{C^G_1, C^G_2, C^G_3, C^G_4\}$$

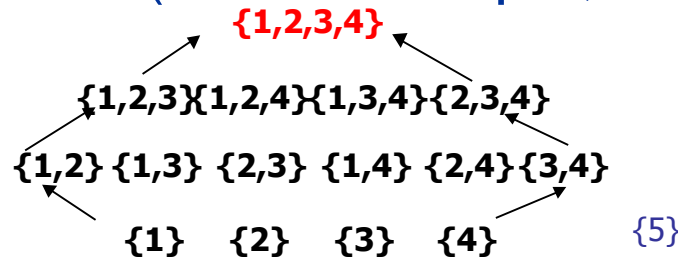
$$L^G = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$$

Pincer Search Method

- Apriori
 - Bottom-up, breadth-first search
 - # DB passes = largest size of frequent item set
 - Performance – decreases
- Solution – pincer search method
 - Bi-directional search
 - Top-down and bottom-up search
 - Find frequent item sets by bottom-up and maintain list of maximal frequent item sets
 - Count support for both

Complexity of One-Way Searches

For **bottom-up** search, every frequent itemset is explicitly examined (in the example, until $\{1,2,3,4\}$ is examined)

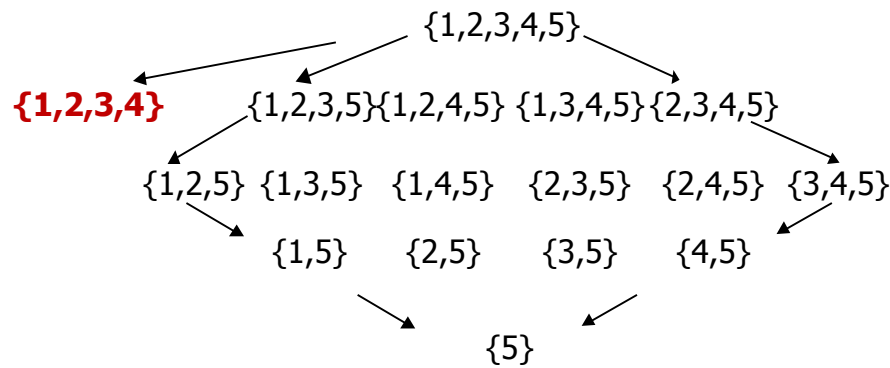


Black: frequent itemsets

Red: maximal frequent itemsets

Blue: infrequent itemsets

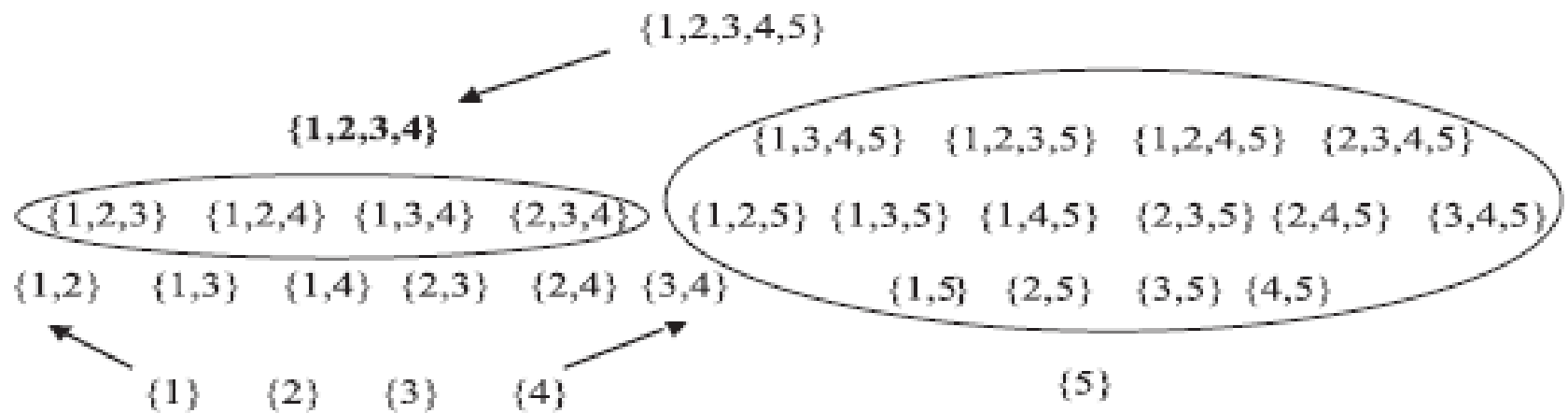
❓ For top-down search, every infrequent itemset is explicitly examined (in the example until $\{5\}$ is examined)



Red: maximal frequent itemsets

Black: infrequent itemsets

Use Property 1 to eliminate candidates in the top-down search
Use Property 2 to eliminate candidates in the bottom-up search



Algorithm: The Pincer-Search algorithm

$L_0 := \emptyset; k := 1; C_1 := \{\{i\} \mid i \in I\}; S_0 := \emptyset$

$\text{MFCS} := \{\{1, 2, \dots, n\}\}; \text{MFS} := \emptyset$

while $C_k \neq \emptyset$

 read database and count supports for C_k and MFCS

 remove frequent itemsets from MFCS and add them to MFS

$L_k := \{\text{frequent } k\text{-itemset}\}$

$S_k := \{\text{infrequent itemsets in } C_k\}$

 call the *MFCS-gen* algorithm if $S_k \neq \emptyset$ // $\text{MFS} = \text{MFS} \cup \{\text{frequent itemsets in MFCS}\}$

 call MFS-pruning procedure

 generate C_{k+1} from L_k (apriori join)

if any frequent itemset in L_k is removed in MFS-pruning procedure

 call the *recovery* procedure to recover candidates to C_{k+1}

 call MFCS *prune* procedure to prune candidates in C_{k+1}

$k := k + 1$

end-while

Answer = $\bigcup_k L_k \cup \text{MFS}$

MFCS-Gen Algorithm

for all itemset s in S_k

for all itemsets m in MFCS

if s is a subset of m

MFCS := MFCS \setminus { m }

for all items e in itemset s

if $m \setminus \{ e \}$ is not a subset of any itemset in the MFCS

MFCS := MFCS \cup { $m \setminus \{ e \}$ }

return MFCS

Recovery

for all items l in L_k

for all items m in MFS

if the first $k-1$ items in l are also in m

for i from $j+1$ to $|m|$

/*suppose $m.item_j = l.item_{k-1}$ */

$C_{k+1} = C_{k+1} \cup \{l.item_1, l.item_2, \dots, l.item_k, m.item_i\}$

MFS-Prune

for all items l in L_k
 if l is a subset of any itemset in the current MFS
 delete l from L_k

MFCS-Prune

for all items c in C_{k+1}
 if c is not a subset of any itemset in the current MFCS
 delete c from C_{k+1}

Tid	Items
10	1,2,5
20	2,4
30	2,3
40	1,2,4
50	1,3
60	2,3
70	1,3
80	1,2,3,5
90	1,2,3

$L_0 = \{ \}$ $k=1$ $C_1 = \{ \{1\}, \{2\}, \{3\}, \{4\}, \{5\} \}$ $S_0 = \{ \}$
 $MFCS = \{ \{1,2,3,4,5\} \}$, $MFS = \{ \}$

$C_1 = \{ \{1\}-6, \{2\}-7, \{3\}-6, \{4\}-2, \{5\}-2 \}$
 $MFCS = \{ \{1,2,3,4,5\}-0 \}$, $MFS = \{ \}$

$L_1 = \{ \{1\}, \{2\}, \{3\}, \{4\}, \{5\} \}$
 $S_1 = \{ \}$

Support Count ≥ 2

call the *MFCS-gen* algorithm if $S_l \neq \emptyset$
 $MFS = MFCS \cup \{ \text{frequent itemsets in MFCS} \}$
 call *MFS-pruning* procedure
 generate C_2 from L_1 (apriori)

$C_2 = \{ \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\} \}$

MFS Prune, MFCS Prune, k=2

$C_2 = \{ \{1,2\}-4, \{1,3\}-4, \{1,4\}-1, \{1,5\}-2, \{2,3\}-4, \{2,4\}-2, \{2,5\}-2, \{3,4\}-0, \{3,5\}-1, \{4,5\}-0 \}$
 $MFCs = \{ \{1,2,3,4,5\}-0 \}$ $MFS = \{ \}$

$L_2 = \{ \{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\} \}$

$S_2 = \{ \{1,4\}, \{3,4\}, \{3,5\}, \{4,5\} \}$

MFCs Gen

$s = \{1,4\}$, $m = \{1,2,3,4,5\}$, s is subset of m , $MFCs = MFCs/m = \{ \}$

$e = 1$, $m/e = \{1,2,3,4,5\}/\{1\} = \{2,3,4,5\}$ not subset of $MFCs$, $MFCs = \{2,3,4,5\}$

$e = 4$, $m/e = \{1,2,3,4,5\}/\{4\} = \{1,2,3,5\}$ not subset of $MFCs$,

$MFCs = \{ \{1,2,3,5\}, \{2,3,4,5\} \}$

$s = \{3,4\}$, $m = \{1,2,3,5\}$, s is not a subset of m

$s = \{3,4\}$, $m = \{2,3,4,5\}$, s is a subset of m , $MFCs = MFCs/m = \{ \{1,2,3,5\} \}$

$e = 3$, $m/e = \{2,3,4,5\}/\{3\} = \{2,4,5\}$ not subset of $MFCs$, $MFCs = \{ \{2,4,5\}, \{1,2,3,5\} \}$

$e = 4$, $m/e = \{2,3,4,5\}/\{4\} = \{2,3,5\}$ is subset of $MFCs$, $MFCs = \{ \{2,4,5\}, \{1,2,3,5\} \}$

$$MFCS = \{\{1,2,3,5\}, \{2,4,5\}\}$$

$s = \{3,5\}$, $m = \{1,2,3,5\}$, s is a subset of m , $MFCS = \{\{2,4,5\}\}$

$e = 3$, $m/e = \{1,2,3,5\}/\{3\} = \{1,2,5\}$ not subset of $MFCS$, $MFCS = \{\{1,2,5\}, \{2,4,5\}\}$

*$e = 5$, $m/e = \{1,2,3,5\}/\{5\} = \{1,2,3\}$ not subset of $MFCS$,
 $MFCS = \{\{1,2,3\}, \{1,2,5\}, \{2,4,5\}\}$*

$s = \{4,5\}$, $m = \{1,2,3\}$, s is not a subset of m ,

$s = \{4,5\}$, $m = \{1,2,5\}$, s is not a subset of m ,

$s = \{4,5\}$, $m = \{2,4,5\}$, s is a subset of m , $MFCS = MFCS/m = \{\{1,2,3\}, \{1,2,5\}\}$

$e = 4$, $m/e = \{2,4,5\}/\{4\} = \{2,5\}$ subset of $MFCS$, $MFCS = \{\{1,2,3\}, \{1,2,5\}\}$

$e = 5$, $m/e = \{2,4,5\}/\{5\} = \{2,4\}$ not a subset of $MFCS$, $MFCS = \{\{2,4\}, \{1,2,3\}, \{1,2,5\}\}$

$$MFS = \{\{2,4\}\}, \quad MFCS = \{\{1,2,3\}, \{1,2,5\}\}$$

MFS Prune

$L_2 = \{ \{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\} \}$ // before MFS prune

$L_2 = \{ \{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,5\} \}$ // after MFS prune

$MFS = \{ \{2,4\} \}, \quad MFCS = \{ \{1,2,3\}, \{1,2,5\} \}$

Generate C_3

$C_3 = \{ \{1,2,3\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\} \}$

Recovery

$MFS = \{2,4\}, L_2 = \{2,3\} \Rightarrow \{2,3,4\}$

$MFS = \{2,4\}, L_2 = \{2,5\} \Rightarrow \{2,4,5\}$

$C_3 = \{ \{1,2,3\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\}, \{2,3,4\}, \{2,4,5\} \}$ //after recovery

MFCFS Prune

$$MFCFS = \{\{1,2,3\}, \{1,2,5\}\}$$

$$C_3 = \{\{1,2,3\}, \{1,2,5\}, \{1,3,5\}, \{2,3,5\}, \{2,3,4\}, \{2,4,5\}\} \text{ //before prune}$$

$$C_3 = \{\{1,2,3\}, \{1,2,5\}\} \text{ //after prune}$$

$$k=3$$

$$MFCFS = \{\{1,2,3\}-2, \{1,2,5\}-2\}$$

$$C_3 = \{\{1,2,3\}-2, \{1,2,5\}-2\}$$

$$MFS = \{\{2,4\}, \{1,2,3\}, \{1,2,5\}\}, \quad MFCFS = \{ \}$$

$$L_3 = \{\{1,2,3\}, \{1,2,5\}\}, \quad S_3 = \{ \}, \quad MFCFS \text{ Gen} - \text{not called}$$

MFS Prune

$$L_3 = \{ \}, \quad C_4 = \{ \}, \quad MFCFS \text{ Prune}$$

$$k=4$$

$$\text{Frequent item sets} = L_1 \cup L_2 \cup L_3 \cup MFS$$

Database1

1: 1,5,6,8
2: 2,4,8
3: 2,3,7,8
4: 2,3,4
5: 2,6,7,9
6: 2,3,6,7,9
7: 2,4,6,7,9
8: 1,3,5,7
9: 2,3,7
10: 2,3,8,9

Database2

1: *a b c d e f*
2: *a b c g*
3: *a b d h*
4: *b c d e k*
5: *a b c*

Min supp=2

Dynamic Itemset Counting (DIC) Algorithm

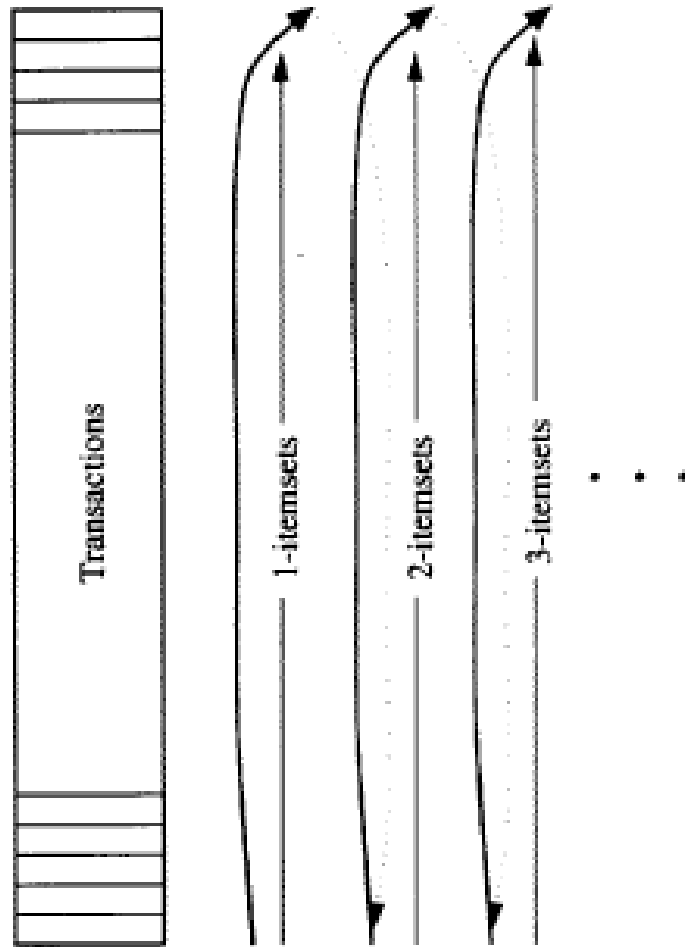
- Bin et al. -1997
- Basics: works like a train running over the data
- Stops at intervals M between transactions
- Passengers – itemsets
- Requirement – get on and off at the same stop
- Assumption – records are read sequentially

Apriori vs. DIC

- Apriori
 - level-wise
 - many passes
- DIC
 - reduce the number of passes
 - fewer candidate itemsets than sampling
- example : 40,000 transaction, $M = 10,000$

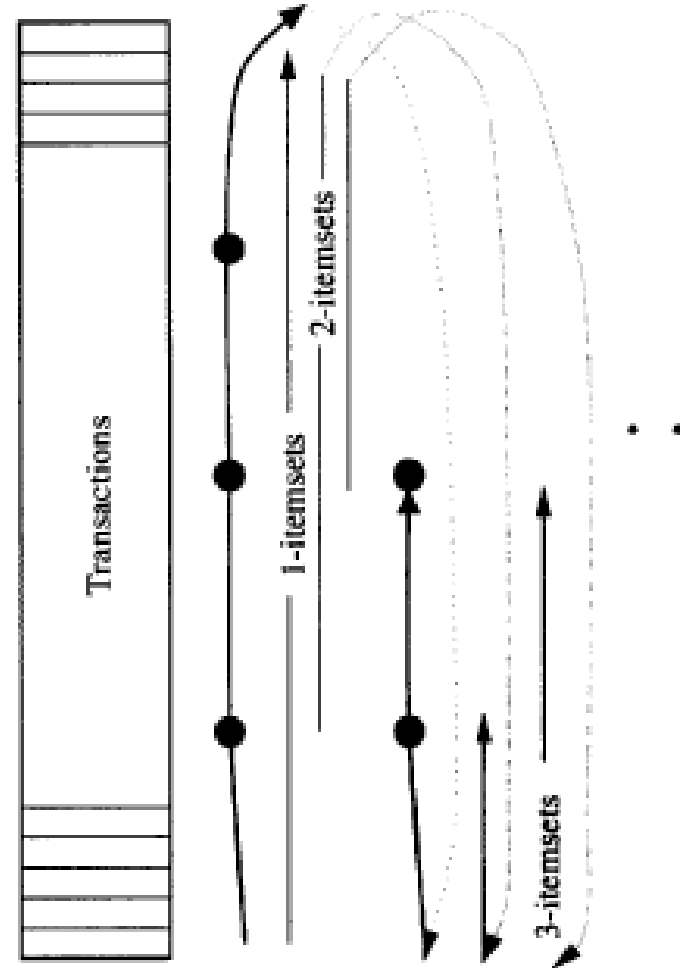


Apriori



3 passes

DIC



1.5 passes

Counting large itemsets

- Itemsets : a large lattice
- count just the minimal small itemsets
 - the itemsets that do not include any other small itemsets
- mark itemset
 - Solid box - confirmed frequent itemset
 - Solid circle - confirmed infrequent itemset
 - Dashed box – item with support $>$ min. support
 - Dashed circle – fresh items sets

DIC Algorithm

Initially,

solid-box – empty itemset;

solid-circle – empty;

dashed-box – empty;

dashed-circle – all 1-itemset with stop-number as 0;

current-stop-number = 0;

do until the dashed-circle or dashed-box is empty

read database till next stop and increase the counters for itemsets in the dashed-box & dashed-circle to reach the next stop;

increase current-stop-number by 1;

for each itemset in the dashed-circle

{ if count of itemset is greater than σ

move the itemset to dashed-box;

generate new itemsets if possible from dashed-box and solid-box, put into dashed-circle with counter value = 0 and stop-number = current-stop-number;

}

for each itemset in the dashed-circle

if stop-number = current-stop-number and itemset
is counted through all transactions then
move this itemset to solid-circle;

for each itemset in the dashed-box

if stop-number = current-stop-number
move this itemset to solid-box;

end // do

return items in solid-box

