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DSAA

Assignment-1

Q1 Given

$x[n] = \begin{cases} 1, & \text{if } 0 \le n \le 9 \\ 0, & \text{else.} \end{cases}$; $h[n] = \begin{cases} 1, & \text{if } 0 \le n \le N \\ 0, & \text{else.} \end{cases}$

$y[n] = x[n] * h[n]$ $N \le 9$, $y[4] = 5$, $y[14] = 0$

To find :- N

Since, $y[n] = x[n] * h[n] = \sum_{K=0}^{\infty} x[K]\, h[n-K]$

since $h[n] = 1$ for $n \ge 0$

So for $y[4]$

K can vary only from $0 \le K \le 4$ bcz for other values $h[n-K]$ becomes 0.

So,

$y[4] = x[0]h[4] + x[1]h[3] + \dots\ x[4]h[0]$

$5 = h[4] + h[3] + h[2] + h[1] + h[0]$

$\Rightarrow$ all of $h[K] = 1$ $0 \le K \le 4$

So, least value of N is 4

$\Rightarrow 4 \le N \le 9$

Now,

$y[14] = 0$

$= \sum_{K=0}^{\infty} x[K]h[14-K]$ here K can vary till 9 only bcz after that $x[K]$ becomes 0.

$0 = x[0]h[14] + x[1]h[13] + \dots\ x[9]h[5]$

$\Rightarrow$ all terms has to be zero.

$\Rightarrow h[5]$ has to be zero.

$\Rightarrow 4 \le N < 5$

$\Rightarrow$ $\boxed{N = 4}$ $\Leftarrow$ Answer

Q3 Given

$$x[n] = [\ldots 0\ 0\ 1\ \overset{\downarrow}{1}\ 1\ 0\ 0 \ldots]$$
$$y[n] = [\ldots 0\ 1\ 2\ \underset{\uparrow}{3}\ 2\ 1\ 0 \ldots]$$

$$z[n] = x[n] * y[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\, y[n-k]$$

for $k < -1$ and for $k > 1$ $\quad x[k] = 0$

$$\Rightarrow z[n] = \cancel{x[-1]y[n+1]}$$
$$x[-1]\,y[n+1] + x[0]\,y[n] + x[1]\,y[n-1]$$

$$= 1 \times [0\ 0\ 1\ \overset{\downarrow}{2}\ 3\ \overset{\downarrow}{2}\ 1\ 0]$$
$$+ 1 \times [0\ 0\ 1\ 2\ \overset{\downarrow}{3}\ 2\ 1\ 0]$$
$$+ 1 \times [0\ 0\ 1\ \overset{\downarrow}{2}\ 3\ 2\ 1\ 0]$$

$$= [0\ 1\ 3\ 6\ \overset{\downarrow}{7}\ 6\ 3\ 1\ 0]$$

or

	1	2	$\overset{\downarrow}{3}$	2	1
1	1	2	3	2	1
$\rightarrow$ 1	1	2	3	2	1
1	1	2	3	2	1

$$= [1\ 3\ 6\ \overset{\downarrow}{7}\ 6\ 3\ 1]$$

$$z[n] = [0\ 1\ 3\ 6\ 7\ 6\ 3\ 1\ 0 \ldots]$$

Q4

$y[n] = x[n] - y^2[n-1] + y[n-1]$

$\therefore \; x[n] = y[n] + y^2[n-1] - y[n-1]$

$\therefore \; y[n] \xrightarrow{T} x[n]$

Now if we prove that T is non linear, so it will be

$x[n] \xrightarrow{T'} y[n]$

Putting $\alpha y[n]$ in place of $y[n]$

RHS $= \alpha y[n] + \alpha^2 y^2[n-1] - \alpha y[n-1]$

LHS $= \alpha x[n]$

$= \alpha (y[n] + y^2[n-1] - y[n-1])$

$= \alpha y[n] + \alpha y^2[n-1] - \alpha y[n-1]$

$\Rightarrow$ LHS $\neq$ RHS.

$\Rightarrow$ system is non linear

Now if we put $z[n] = y[n-k]$ instead of $y[n]$, we should get $x[n-k]$ if it is time invariant

RHS $= z[n] + z^2[n-1] - z[n-1]$

$= y[n-k] + y^2[n-k-1] - y[n-k-1]$

LHS $= x[n-k] = x[l]$ $(l = n-k)$

$= y[l] + y^2[l-1] - y[l-1]$

$= y[n-k] + y^2[n-k-1] - y[n-k-1]$

$\Rightarrow$ LHS $=$ RHS

$\Rightarrow$ Time invariant.

Also, to find $y[n]$, $n \Rightarrow \infty$ if $x[n] = \alpha\, u[n]$

$$\lim_{n\to\infty} y[n] = \lim_{n\to\infty} y[n-1]$$

$$\therefore \lim_{n\to\infty} x[n] = \lim_{n\to\infty} \left(y[n] + y^2[n-1] - y[n-1] \right)$$

$$= \lim_{n\to\infty} \left(y[n] + y^2[n] - y[n] \right)$$

$$\lim_{n\to\infty} x[n] = \lim_{n\to\infty} y^2[n]$$

$$y[n] = \sqrt{x[n]}$$

But as $\lim_{n\to\infty} x[n] = \lim_{n\to\infty} \alpha u[n] = \alpha$

$$\therefore y[n] = \sqrt{\alpha} \quad (n \to \infty)$$

Q5

(a) First we will find the formula for just 1 convolution

Lets assume we are right now doing the i^{th} convolution on the output of $(i-1)^{th}$ convolution

Let w_i be the output after convolution

$$\therefore w_i = \left\lfloor \frac{w_{i-1} + 2Z^i - F}{S} \right\rfloor + 1$$

Also one thing we need to take care of is that Z is almost $(F-1)$ for this formula as otherwise the filter will go fully out of the image.

So if $Z > F-1$ we put $Z = F-1$ and then do the calculation

We also have to take care of channels as each channel will have separate convolution and so no. of channels will remain fixed - always. Similarly, for H_i

$$H_i = \left(\frac{H_{i-1} + 2Z^i - F}{S} \right) + 1$$

(b) There will be F^2 multiplications at each step of convolution and (F^2-1) additions for each channel

So, in total there would be

$$\underbrace{\sum_{i=0}^{n-1}}_{n\ \text{convolutions}} \underbrace{(W_i \times H_i)}_{\text{No. of pixels}} \times \underbrace{F^2 \times C}_{\text{Multiplications per pixel per channel multiplied by no. of channels}} \quad \text{multiplications}$$

and

$$\sum_{i=0}^{n-1} (W_i \times H_i) \times (F^2-1) \times C \quad \text{additions.}$$

Q7 In this question, first we are supposed to read a .wav file sampled originally at 44.1 KHz. So we first read 'casio.wav' (in my case) and then resampled ~~at~~ it at 24KHz, 16 KHz, 8KHz and 4KHz

So when we play the resampled sound at new frame per sec rate, the sound quality decreased with decrease in sampling frequency.

Also, if we play the sound at default frames per sec., it plays faster with decreasing sampling rate

Also, as mentioned in question, we ~~so~~ successfully simulated it by convolution in 3 diff. environments

(i) church
(ii) Hall
(iii) Basement

Now, we record sound ~~on~~ at 24KHz, 16KHz, 8 KHz and 4KHz sampling rates digitalized at 24 bits.

But, when we played the recorded sounds at default frame rate of matlab, the higher sampling frequency sounds was distorted and were playing slower than real time.

So, to play the sound correctly, we had to play it on same frame rate as it was recorded. When we did so, we got the clear sound output. Also, the sounds with higher sampling rate was better in quality than the lower sampling rate sounds.

We than tried to change the digitization bits, and the quality of recorded sound increases with the ↑ in digization bits

For the 3rd part of question, we convolved the sound with 3 impulse responses in following environment :-

1) Church
2) Hall
3) Basement

We got the corresponding convolution output successfully.

Q6

The rectangle marks ~~in Shaye~~ Shyam in the image of Kumbhanda. This is found by correlation of the original image ~~of~~ by the image of Shyam normxcorr2 works by calculating the cross correlation in the spatial or frequency domain, depending on size of the image. The implementation follows the formula.

$$\gamma = \frac{(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{(\Sigma x^2 - (\Sigma x)^2)(\Sigma y^2 - (\Sigma y)^2}}$$

Q2 Matrix Used : $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ Here, in the first image, the filter if used on a region with ~~sa~~ same color it sums it to 0 (as 1 and 0 and -1) which makes it black. So at the place in the input image where it was completely white or completely black, it is now completely black. But at the centre of the image, this is not the case as the upper part is black and the lower part is white, so it gives 1 of black, and -1 of white so it isn't black at the centre and we get the white line.

This filter helps identify the border of the input file. In case of camera man, it detects the horizontal borders. On convolving with the transpose of the filter with cameraman, it marks the vertical borders of the image.

Homework question

Ans:-

$$S^2 = \frac{1}{n-1} \Sigma(x_i - \bar{x})^2$$

is not accurate when tried with large number

Where as,

~~well fu~~ Welford's Method :-

$$SN = SN_1 + (x[N] - \mu_N) \cdot (x[N] - \mu_{N-1})$$

is accurate

The first method gives -ve answers also which is not feasible

For eg:-

Generate 10^6 samples from a uniform (0,1) distributions and add 10^9 to each sample, so that, ~~the~~ the variance in the samples would be small relative to the mean. This is the case that causes accuracy problems. welford's method was correct to 8 significant ~~figures~~ figures : 0.0832265971. The sum of squares method gave the impossible Result - 37154.734

Repeat the samples and shift it to 10^{12} instead of 10^9. This time welford's method gave 0.08326. Correct to 3 significant figures. The sum of squares method gave 1423 3 2268125959

In summary, the sum of squares method is bad but the other method is quite good.

The advantage of welford's method is that it requires only one pass.