CS101

PROJECT

FORD-FULKERSON ALGORITHM FOR MAXIMUM FLOW PROBLEM

INTRODUCTION

The Ford–Fulkerson method or Ford–Fulkerson algorithm (FFA) is a greedy algorithm that computes the maximum flow in a flow network. It is sometimes called a "method" instead of an "algorithm" as the approach to finding augmenting paths in a residual graph is not fully specified.

It was published in 1956 by L. R. Ford Jr. and D. R. +Fulkerson.

The idea behind the algorithm is as follows: as long as there is a path from the source (start node) to the sink (end node), with available capacity on all edges in the path, we send flow along one of the paths. Then we find another path, and so on. A path with available capacity is called an augmenting path.

MAXIMUM FLOW PROBLEM

We are given a directed graph which represents a flow network where every edge has a certain capacity. Also, there are two vertices source ‘s’ and sink ‘t’ in the graph.

We need to find out the maximum possible flow from s to t with the following constraints:

- Flow on an edge doesn’t exceed the given capacity of the edge.
- In flow is equal to the out flow for every vertex except s and t.
- Total flow out of the source node is equal to total flow into the sink node.

Flow network: It is defined as a directed graph involving a source and a sink and several other nodes connected with edges. Each edge has an individual capacity which is the maximum limit of flow that edge could allow.

Maximum flow: It is defined as the maximum amount of flow that the network would allow to flow from source to sink.

IMPORTANT TERMINOLOGIES

- Residual Graph: It's a graph which indicates additional possible flow. If there is such path from source to sink then there is possibility to add flow.
- Residual capacity: It's the original capacity of the edge minus flow.
- Minimal cut: Also known as bottleneck capacity, it decides maximum possible flow from source to sink through an augmented path.
- Augmenting path: Augmenting path can be done in two ways-
  i. Non-full forward edges
  ii. Non- empty backward edges

THE ALGORITHM

A SIMPLE IDEA OF THE FORD-FULKERSON ALGORITHM

INPUT: A flow network G= (V, E) with a source s, sink t, and a certain flow capacity for every edge of the network.

STEPS:

- Start with initial flow as zero.
- While there is an augmenting path from source to sink add path flow to the flow value.
- Return flow.

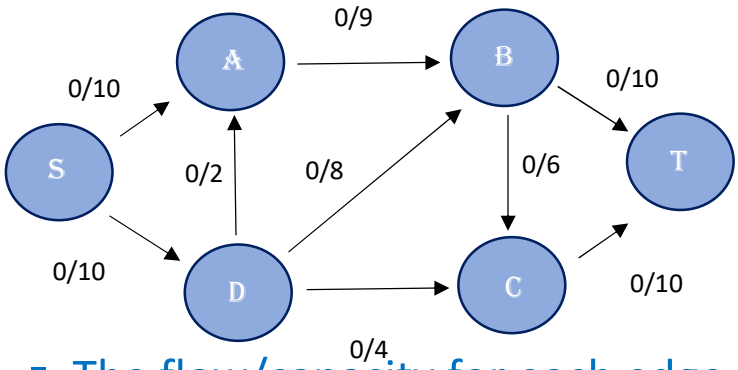
After every step in the algorithm the following is maintained:

- Capacity constraints: The flow along an edge cannot exceed its capacity.
- Flow conservation: The net flow to a node is zero, except for the sink and source.
- The flow leaving from s must be equal to flow arriving at t.

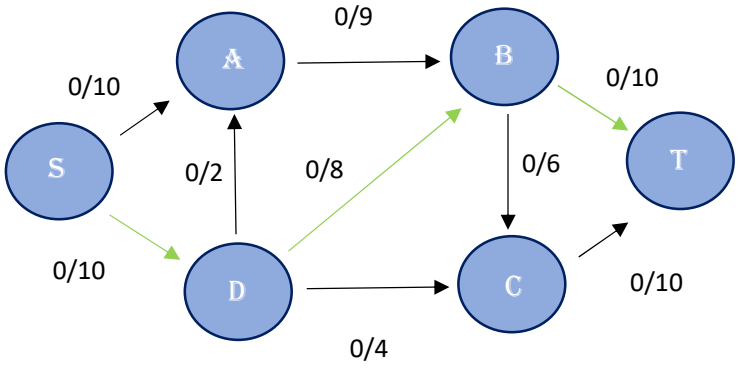
EXAMPLE

Let us take an example in order to understand the algorithm

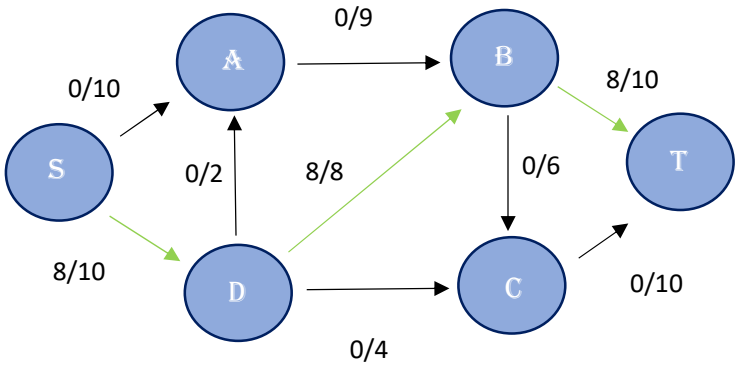
Consider the graph below:



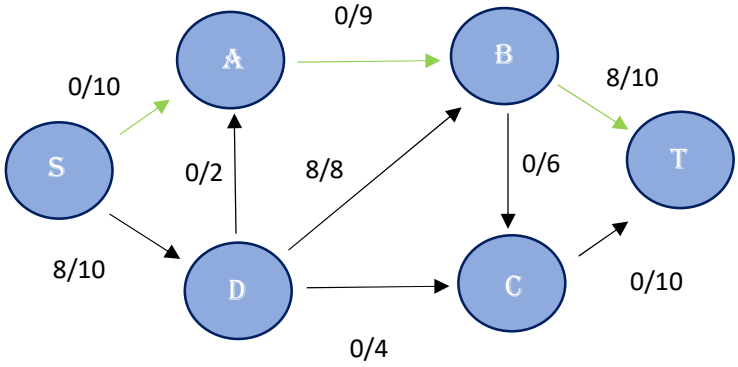
- The flow/capacity for each edge is given. Initially flow is zero for every edge. Now, select any arbitrary path from S to T. In this step we have selected path S-D-B-T.



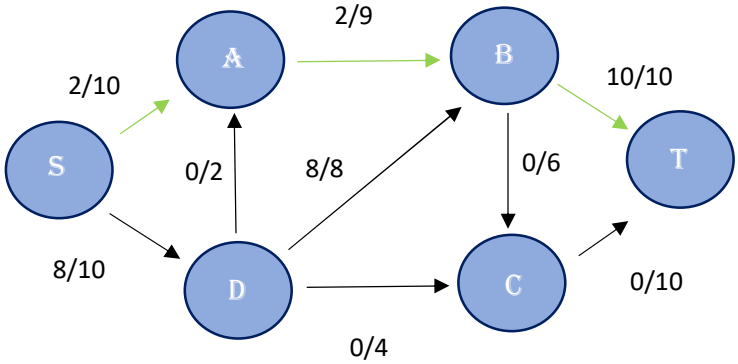
- The minimum capacity among the three edges is 8(D-B). This is our bottleneck capacity. Based on this, update the flow/capacity for each path.



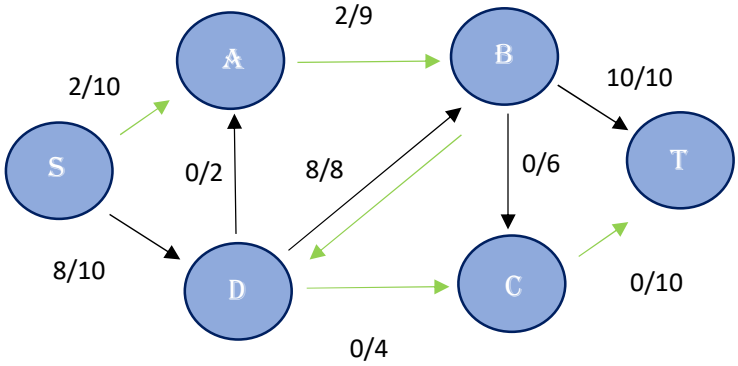
- Select another path S-A-B-T. The minimum capacity among these edges is 2 (B-T).



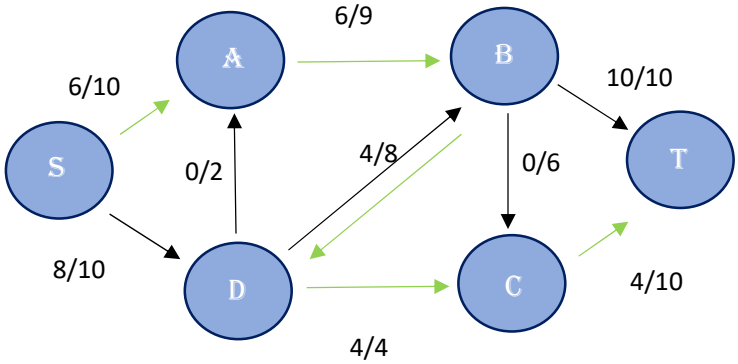
- Update the capacities according to this.



- Next, we select the path S-A-B-D-C-T. The bottleneck capacity for this augmented path is 4(D-C). Here we have considered the non-empty backward edge B-D.



- Now, we update the capacities.



- Next, we select S-D-B-C-T. The bottleneck capacity for this path is 2(S-D).