10·5. RESONANCE

If we bring a vibrating tuning fork near another stationary tuning fork of the same natural frequency as that of vibrating tuning fork, we find that stationary tuning fork also starts vibrating. This phenomenon is known as resonance.

The phenomenon of making a body vibrate with its natural frequency under the influence of another vibrating body with the same frequency is called resonance.

Consider three spring S_1, S_2 and S_3 suspended from a flexible rod AB such S_1 and S_2 are identical in all respect and carry equal masses at the ends, while S_3 has ditterent spring constant and carries different mass. Now if S_1 spring is set in vibration by pulling down the attached mass and let go, we find that springs S_2 and S_3 also start vibrating. The vibrations in S_3 die out quickly while the vibrations set in spring (S_2 which is identical to S_1) keeps on increasing in its amplitude till it is very nearly equal to the amplitude of the spring S_1. The vibrations of spring S_2 have the same frequency as that of S_1 and are called resonant vibrations and this phenomenon is called resonance.

Examples of resonance

(*a*) Tuning of a radio or transistor, when natural frequency is so adjusted, by moving the tuning knob of the receiver set that it equals the frequency of the radio ways, the resonance takes place and the incoming sound waves can be listened after being amplified.

(*b*) Musical instrument can be made to vibrate by bringing them in contact with vibrations which have the frequency equal to the natarал frequency of the instruments.

(*c*) Soldiers crossing a suspension bridge are prohibited to march in steps and are advised to march on the suspension bridges out of steps so as to avoid the resonance between the natural frequency of the bridge and the frequency of the steps of soliders which may cause the collapse of the bridge.

(*d*) When an atom or a molecule is struck by electromagnetic radiation having frequency exactly equal to the natural frequency of the atom or molecule, the radiation is absorbed more readily than at other frequencies. This is known as *'resonance abrorption of radiation'*. The phenomenon is called as *'optical resonance'*.

10·6. AMPLITUDE RESONANCE

The amplitude of forced oscillations varies with the frequency of applied force and becomes maximum at a particular frequency. This phenomenon is known as amplitude resonance.

Condition of amplitude resonance.

In case of forced vibrations, we have

$$A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \qquad ...(1)$$

and

$$\theta = \tan^{-1}\left[\frac{2bp}{(\omega^2 - p^2)}\right] \qquad ...(2)$$

The expression (1) shows that the amplitude varies with the frequency of the force p. For a particular value of p, the amplitude becomes maximum. The phenomenon is known as ***amplitude resonance.*** The amplitude is maximum when

$$\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]} \text{ is minimum}$$

or

$$\frac{d}{dp}[(\omega^2 - p^2)^2 + 4b^2p^2] = 0$$

or

$$2(\omega^2 - p^2)(-2p) + 4b^2(2p) = 0$$

or

$$\omega^2 - p^2 = 2b^2$$

or

$$p = \sqrt{(\omega^2 - 2b^2)} \qquad ...(3)$$

Thus the amplitude is maximum when the frequency $p/2\pi$ of the impressed force becomes $\sqrt{(\omega^2 - 2b^2)}/2\pi$. This is the resonant frequency. This gives frequency of the system both in presence of damping *i.e.*, $\sqrt{\{\omega^2 - 2b^2\}}/2\pi$ and in the absence of damping *i.e.*, $\omega/2\pi$.

If the damping is small then it can be neglected and the condition of maximum amplitude reduced to

$$p = \omega$$

Putting condition (3) in eq. (1), we get

$$A_{\max} = \frac{f}{\sqrt{\{(\omega^2 - \omega^2 + 2b^2)^2 + 4b^2(\omega^2 - 2b^2)\}}}$$

$$= \frac{f}{\sqrt{(4b^2\omega^2 - 4b^4)}} = \frac{f}{2b\sqrt{(\omega^2 - b^2)}}$$

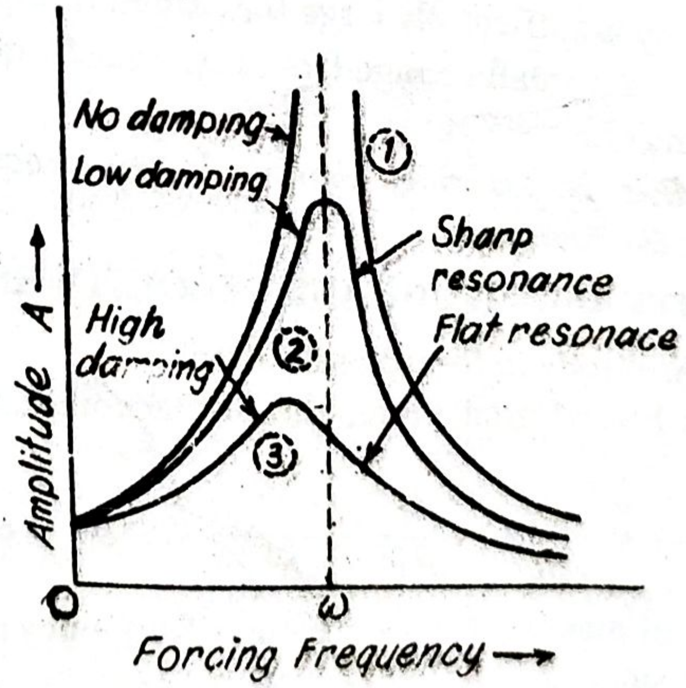
$$= \frac{f}{2b\sqrt{(p^2 + b^2)}} \qquad [\because p^2 = \omega^2 - 2b^4]$$

and for low damping it reduces to

$$A_{\max} \approx \frac{f}{2bp}$$

Showing that $A_{\max} \to \infty$ as $b \to 0$:

Fig. (3) shows the variation of amplitude with forcing frequency at different amounts of daming. Curve (1) shows the amplitude when there is no damping *i.e.*, $b = 0$, In this case the amplitude becomes infinite at



Fi.g (3)

$p = \omega$. This case is never attained in practice due to frictional resistance, as slight damping is always present Curve (2) and (3) show the effect of damping on the amplitude. It is observed that the peak of the curve moves towards the left. It is also observed that the value of A, which is different for different values of b (damping), diminishes as the value of b

increases. For smaller values of b, the fall in the curve about $\omega = p$ is steeper than for large values. This shows that the smaller is the value of damping, greater is the departure of amplitude of forced vibration from the maximum value and vice-versa.

10·7. SHARPNESS OF RESONANCE

We have seen that the amplitude of the forced oscillation is maximum when the frequency of the applied force has a value to satisfy the conditon of resonance *i.e.*, $p = \sqrt{(\omega^2 - 2b^2)}$. If the frequency changes from this value, the amplitude falls. When the fall in amplitude for a small departure from the resonance condition is very laege, the resonance is said to be *sharp*. On the other hand if the fall in amplitude is small, the resonance is termed as *flat*. ***Thus the term sharpness of resonance means the rate of fall in amplitude, with the change of forcing frequency on each side of resoance frequency.***

Fig. (3) shows the variation of amplitude with forcing frequency at different amounts of damping. It is obvious from the figure that the resonance is sharp when fall in amplitude for a small change from resonant frequency is sufficiently large (*i.e.*, small damping) and flat when fall in amplitude for a small change from resonant frequency is very small (i.e. large damping).

Hence smaller is damping, sharper is resonance or larger is the damping, flater is the resonance.

10·8. VELOCITY AMPLITUDE AND VELOCITY RESONANCE

Velocity amplitude in forced oscillations.

In case of forced oscillations, the displacement x at any instant is given by

$$x = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \sin(pt - \theta) \quad ...(1)$$

where a system of mass m, having a natural frequency ω oscillater under external force $F \sin pt$.

The velocity is given by

$$u = \frac{dx}{dt} = \frac{fp}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \cos(pt - \theta)$$

$$= \frac{fp}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \sin\{pt - (\theta - \pi/2)\}$$

$$= \frac{fp}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \sin(pt - \theta + \pi/2) \quad ...(2)$$

The maximum value of u *i.e.*, u_{max} is known as velocity amplitude.

From eq. (2), it is clear that u will be maximum when $\sin(pt - \theta + \pi/2) = 1$. Hence.

$$u_{max} = \frac{fp}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \quad ...(3)$$

Velocity resonance (Energy resonance).

At very low driving frequencies ($p \ll \omega$), the velocity amplitude is given by

$$u_{max} \simeq \frac{(F/m)p}{\omega^2} \simeq \frac{(F/m)p}{(\mu/m)} \simeq \frac{Fp}{\mu}$$

Thus the amplitude is mainly governed by the force constant μ, when ($p \gg \omega$), then

$$u_{max} = \frac{f}{p} \simeq \frac{F}{mp}$$

In this case, the amplitude is mainly governed by the mass m *i.e.*, inertia factor.

At frequencies comparable with natural frequency, the amplitude is maximum for a particular frequency. For this freuqency

$$u_{max} = \frac{f}{\sqrt{\left[\left(\frac{\omega^2 - p^2}{p}\right)^2 + 4b^2\right]}}$$

This is maximum when denominator is minimum. *i.e.*

$$\left(\frac{\omega^2 - p^2}{p}\right)^2 = 0 \quad \text{or} \quad (\omega^2 - p^2) = 0 \quad \text{or} \quad \omega = p$$

At this frequency of applied force, the velocity (or kinetic energy) of the oscillator is maximum. This phenomenon is known as velocity resonance. So when the driving frequency is equal to the natural undamped frequency of the oscillator, the velocity amplitude is maximum.

Phase of velocity.

We know that, $\tan\theta = \frac{2bp}{(\omega^2 - p^2)}$

At velocity resonance $p = \omega$, hence we have

$$\tan\theta = \infty \qquad \therefore \quad \theta = (\pi/2)$$

Again the velocity lags behind the driving force $F \sin pt$ by an angle ϕ, where $\phi = (\theta - \pi/2)$.

At resonance, $\phi = (\pi/2)$, so that $\phi = (\pi/2 - \pi/2) = 0$

So the velocity at resoncance is always in phase with the driving force.

At low driving frequencies ($p < \omega$), $\tan\theta$ is positive *i.e.*, θ lies between 0 and $\pi/2$. Now ϕ is negative *i.e.*. velocity leads the driving force. At high driving frequencies ($p > \omega$), $\tan\theta$ is negative *i.e.*, θ lies between $\pi/2$ and π. Now ϕ is positive, *i.e.*, velocity lags behind the driving force.

Fig. (4*a*) shows the variation of velocity amplitude of forced oscillations with frequency of the applied force, while fig. (4*b*) shows the variation of phase difference between the velocity of the forced oscillator and the driving force with the frequency of the driving force.

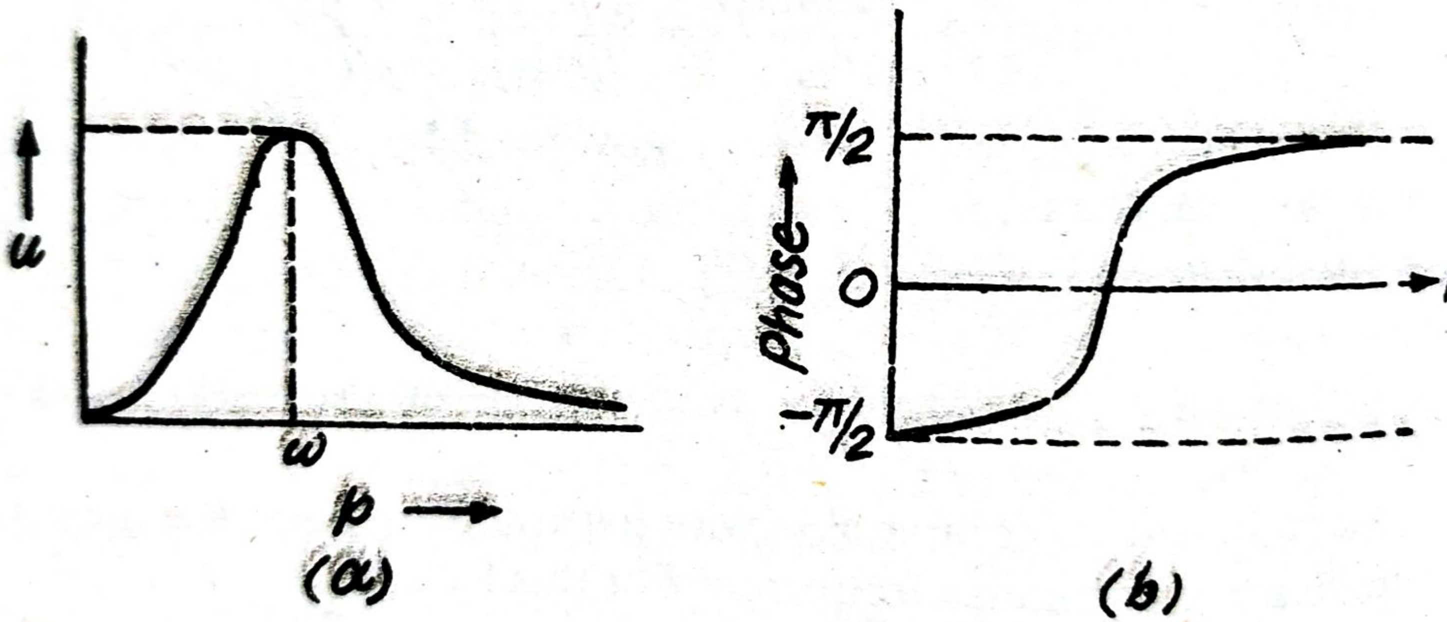


Fig. (4)

So we conclude that the velocity resonance occurs when the frequency of the applied force is equal to the natural frequency of the oscillator and the velocity is in phase with the applied force.

10·9 POWER CONSIDERATIONS