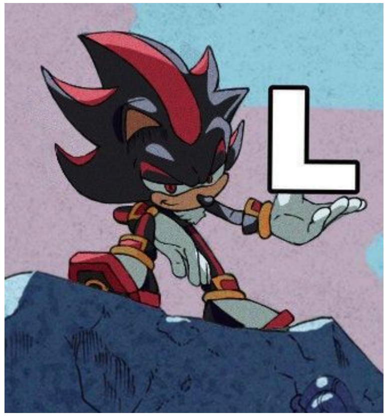
Competições de Algoritmos e Programação UFRJ



Universidade Federal do Rio de Janeiro

Meu nome é shadow. Sou aquele que deixou todo o seu passado para trás. Não faço esse contest por ninguém, não estou preso a nada

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Sumário

1 Geometria

1.1 Minkowski Sum

```
// Computa A+B = {a+b : a \in A, b \in B}, em que
// A e B sao poligonos convexos
// A+B eh um poligono convexo com no max |A|+|B| pontos
//
// O(|A|+|B|)
// Do cadeno do Brunas Maletas UFMG

vector<pt> minkowski(vector<pt> p, vector<pt> q) {
    auto fix = [](vector<pt>& P) {
        rotate(P.begin(), min_element(P.begin(), P.end()), P.end());
        P.push_back(P[0]), P.push_back(P[1]);
    };
    fix(p), fix(q);
    vector<pt> ret;
    int i = 0, j = 0;
    while (i < p.size()-2 or j < q.size()-2) {
        ret.push_back(p[i] + q[j]);
        auto c = ((p[i+1] - p[i]) ^ (q[j+1] - q[j]));
        if (c >= 0) i = min<int>(i+1, p.size()-2);
        if (c <= 0) j = min<int>(j+1, q.size()-2);
    }
    return ret;
}

ld dist_convex(vector<pt> p, vector<pt> q) {
    for (pt& i : p) i = i * -1;
    auto s = minkowski(p, q);
    if (inpol(s, pt(0, 0))) return 0;
    ld ans = DINF;
    for (int i = 0; i < s.size(); i++) ans = min(ans,
            disttoseg(pt(0, 0), line(s[(i+1)%s.size()], s[i])));
    return ans;
}
```

1.2 Primitiva Double

```
typedef double ld;
const ld DINF = 1e18;
const ld pi = acos(-1.0);
const ld eps = 1e-9;

#define sq(x) ((x)*(x))

bool eq(ld a, ld b) {
    return abs(a - b) <= eps;
}

struct pt { // ponto
    ld x, y;
    pt(ld x_ = 0, ld y_ = 0) : x(x_), y(y_) {}
    bool operator < (const pt p) const {
        if (!eq(x, p.x)) return x < p.x;
        if (!eq(y, p.y)) return y < p.y;
        return 0;
    }
    bool operator == (const pt p) const {
        return eq(x, p.x) and eq(y, p.y);
    }
    pt operator + (const pt p) const { return pt(x+p.x, y+p.y); }
    pt operator - (const pt p) const { return pt(x-p.x, y-p.y); }
    pt operator * (const ld c) const { return pt(x*c  , y*c  ); }
    pt operator / (const ld c) const { return pt(x/c  , y/c  ); }
    ld operator * (const pt p) const { return x*p.x + y*p.y; }
    ld operator ^ (const pt p) const { return x*p.y - y*p.x; }
    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y;
    }
};

struct line { // reta
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// PONTO & VETOR

ld dist(pt p, pt q) { // distancia
    return hypot(p.y - q.y, p.x - q.x);
}

ld dist2(pt p, pt q) { // quadrado da distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}

ld norm(pt v) { // norma do vetor
    return dist(pt(0, 0), v);
}

ld angle(pt v) { // angulo do vetor com o eixo x
    ld ang = atan2(v.y, v.x);
    if (ang < 0) ang += 2*pi;
    return ang;
}

ld sarea(pt p, pt q, pt r) { // area com sinal
    return ((q-p)^(r-q))/2;
}

bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
    return eq(sarea(p, q, r), 0);
}

bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
    return sarea(p, q, r) > eps;
}

pt rotate(pt p, ld th) { // rotaciona o ponto th radianos
    return pt(p.x * cos(th) - p.y * sin(th),
            p.x * sin(th) + p.y * cos(th));
}

pt rotate90(pt p) { // rotaciona 90 graus
    return pt(-p.y, p.x);
}

// RETA

bool isvert(line r) { // se r eh vertical
    return eq(r.p.x, r.q.x);
}

bool isinseg(pt p, line r) { // se p pertence ao seg de r
```

```
    pt a = r.p - p, b = r.q - p;
    return eq((a ^ b), 0) and (a * b) < eps;
}

ld get_t(pt v, line r) { // retorna t tal que t*v pertence a reta r
    return (r.p^r.q) / ((r.p-r.q)^v);
}

pt proj(pt p, line r) { // projecao do ponto p na reta r
    if (r.p == r.q) return r.p;
    r.q = r.q - r.p; p = p - r.p;
    pt proj = r.q * ((p*r.q) / (r.q*r.q));
    return proj + r.p;
}

pt inter(line r, line s) { // r inter s
    if (eq((r.p - r.q) ^ (s.p - s.q), 0)) return pt(DINF, DINF);
    r.q = r.q - r.p, s.p = s.p - r.p, s.q = s.q - r.p;
    return r.q * get_t(r.q, s) + r.p;
}

bool interseg(line r, line s) { // se o seg de r intersecta o seg de s
    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
           ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

ld disttoline(pt p, line r) { // distancia do ponto a reta
    return 2 * abs(sarea(p, r.p, r.q)) / dist(r.p, r.q);
}

ld disttoseg(pt p, line r) { // distancia do ponto ao seg
    if ((r.q - r.p)*(p - r.p) < 0) return dist(r.p, p);
    if ((r.p - r.q)*(p - r.q) < 0) return dist(r.q, p);
    return disttoline(p, r);
}

ld distseg(line a, line b) { // distancia entre seg
    if (interseg(a, b)) return 0;

    ld ret = DINF;
    ret = min(ret, disttoseg(a.p, b));
    ret = min(ret, disttoseg(a.q, b));
    ret = min(ret, disttoseg(b.p, a));
    ret = min(ret, disttoseg(b.q, a));
    return ret;
}

// POLIGONO

// corta poligono com a reta r deixando os pontos p tal que
// ccw(r.p, r.q, p)
vector<pt> cut_polygon(vector<pt> v, line r) { // O(n)
    vector<pt> ret;
    for (int j = 0; j < v.size(); j++) {
        if (ccw(r.p, r.q, v[j])) ret.push_back(v[j]);
        if (v.size() == 1) continue;
        line s(v[j], v[(j+1)%v.size()]);
        pt p = inter(r, s);
        if (isinseg(p, s)) ret.push_back(p);
    }
    ret.erase(unique(ret.begin(), ret.end()), ret.end());
    if (ret.size() > 1 and ret.back() == ret[0]) ret.pop_back();
    return ret;
}

// distancia entre os retangulos a e b (lados paralelos aos eixos)
// assume que ta representado (inferior esquerdo, superior direito)
ld dist_rect(pair<pt, pt> a, pair<pt, pt> b) {
    ld hor = 0, vert = 0;
    if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x - b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y - a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y - b.second.y;
    return dist(pt(0, 0), pt(hor, vert));
}

ld polarea(vector<pt> v) { // area do poligono
    ld ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea(pt(0, 0), v[i], v[(i + 1) % v.size()]);
    return abs(ret);
}

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i+1)%v.size();
```

```
        if (eq(p.y, v[i].y) and eq(p.y, v[j].y)) {
            if ((v[i]-p)*(v[j]-p) < eps) return 2;
            continue;
        }
        bool baixo = v[i].y+eps < p.y;
        if (baixo == (v[j].y+eps < p.y)) continue;
        auto t = (p-v[i])^(v[j]-v[i]);
        if (eq(t, 0)) return 2;
        if (baixo == (t > eps)) qt += baixo ? 1 : -1;
    }
    return qt != 0;
}

bool interpol(vector<pt> v1, vector<pt> v2) { // se dois poligonos se intersectam -
↪   O(n*m)
    int n = v1.size(), m = v2.size();
    for (int i = 0; i < n; i++) if (inpol(v2, v1[i])) return 1;
    for (int i = 0; i < n; i++) if (inpol(v1, v2[i])) return 1;
    for (int i = 0; i < n; i++) for (int j = 0; j < m; j++)
        if (interseg(line(v1[i], v1[(i+1)%n]), line(v2[j], v2[(j+1)%m]))) return 1;
    return 0;
}

ld distpol(vector<pt> v1, vector<pt> v2) { // distancia entre poligonos
    if (interpol(v1, v2)) return 0;

    ld ret = DINF;

    for (int i = 0; i < v1.size(); i++) for (int j = 0; j < v2.size(); j++)
        ret = min(ret, distseg(line(v1[i], v1[(i + 1) % v1.size()]),
                    line(v2[j], v2[(j + 1) % v2.size()])));
    return ret;
}

vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n log(n))
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l.end()[-2], l.end()[-1], v[i]))
            l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u.end()[-2], u.end()[-1], v[i]))
            u.pop_back();
            u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

struct convex_pol {
    vector<pt> pol;

        // nao pode ter ponto colinear no convex hull
    convex_pol() {}
    convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

        // se o ponto ta dentro do hull - O(log(n))
    bool is_inside(pt p) {
        if (pol.size() == 0) return false;
        if (pol.size() == 1) return p == pol[0];
        int l = 1, r = pol.size();
        while (l < r) {
            int m = (l+r)/2;
            if (ccw(p, pol[0], pol[m])) l = m+1;
            else r = m;
        }
        if (l == 1) return isinseg(p, line(pol[0], pol[1]));
        if (l == pol.size()) return false;
        return !ccw(p, pol[l], pol[l-1]);
    }
        // ponto extremo em relacao a cmp(p, q) = p mais extremo q
        // (copiado de https://github.com/gustavoM32/caderno-zika)
    int extreme(const function<bool(pt, pt)>& cmp) {
        int n = pol.size();
        auto extr = [&](int i, bool& cur_dir) {
            cur_dir = cmp(pol[(i+1)%n], pol[i]);
            return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
        };
        bool last_dir, cur_dir;
        if (extr(0, last_dir)) return 0;
        int l = 0, r = n;
        while (l+1 < r) {
            int m = (l+r)/2;
            if (extr(m, cur_dir)) return m;
            bool rel_dir = cmp(pol[m], pol[l]);
            if ((!last_dir and cur_dir) or
                    (last_dir == cur_dir and rel_dir == cur_dir)) {
                l = m;
                last_dir = cur_dir;
```

```
        } else r = m;
    }
    return l;
}
int max_dot(pt v) {
    return extreme([&](pt p, pt q) { return p*v > q*v; });
}
pair<int, int> tangents(pt p) {
    auto L = [&](pt q, pt r) { return ccw(p, r, q); };
    auto R = [&](pt q, pt r) { return ccw(p, q, r); };
    return {extreme(L), extreme(R)};
}
};

// CIRCUNFERENCIA

pt getcenter(pt a, pt b, pt c) { // centro da circunf dado 3 pontos
    b = (a + b) / 2;
    c = (a + c) / 2;
    return inter(line(b, b + rotate90(a - b)),
            line(c, c + rotate90(a - c)));
}

vector<pt> circ_line_inter(pt a, pt b, pt c, ld r) { // intersecao da circunf (c, r)
↪  e reta ab
    vector<pt> ret;
    b = b-a, a = a-c;
    ld A = b*b;
    ld B = a*b;
    ld C = a*a - r*r;
    ld D = B*B - A*C;
    if (D < -eps) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+eps))/A);
    if (D > eps) ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

vector<pt> circ_inter(pt a, pt b, ld r, ld R) { // intersecao da circunf (a, r) e
↪  (b, R)
    vector<pt> ret;
    ld d = dist(a, b);
    if (d > r+R or d+min(r, R) < max(r, R)) return ret;
    ld x = (d*d-R*R+r*r)/(2*d);
    ld y = sqrt(r*r-x*x);
    pt v = (b-a)/d;
    ret.push_back(a+v*x + rotate90(v)*y);
    if (y > 0) ret.push_back(a+v*x - rotate90(v)*y);
    return ret;
}

bool operator <(const line& a, const line& b) { // comparador pra reta
        // assume que as retas tem p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    if (!eq(angle(v1), angle(v2))) return angle(v1) < angle(v2);
    return ccw(a.p, a.q, b.p); // mesmo angulo
}
bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
}

// comparador pro set pra fazer sweep line com segmentos
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
            // assume que os segmentos tem p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps < b.p.x))
            return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador pro set pra fazer sweep angle com segmentos
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) + eps < get_t(dir, b);
    }
};
```

1.3 Primitiva Inteiro

```
#define sq(x) ((x)*(ll)(x))

struct pt { // ponto
    int x, y;
    pt(int x_ = 0, int y_ = 0) : x(x_), y(y_) {}
    bool operator < (const pt p) const {
        if (x != p.x) return x < p.x;
        return y < p.y;
    }
```

```
    bool operator == (const pt p) const {
        return x == p.x and y == p.y;
    }
    pt operator + (const pt p) const { return pt(x+p.x, y+p.y); }
    pt operator - (const pt p) const { return pt(x-p.x, y-p.y); }
    pt operator * (const int c) const { return pt(x*c, y*c); }
    ll operator * (const pt p) const { return x*(ll)p.x + y*(ll)p.y; }
    ll operator ^ (const pt p) const { return x*(ll)p.y - y*(ll)p.x; }
    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y;
    }
};

struct line { // reta
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// PONTO & VETOR

ll dist2(pt p, pt q) { // quadrado da distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}

ll sarea2(pt p, pt q, pt r) { // 2 * area com sinal
    return (q-p)^(r-q);
}

bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
    return sarea2(p, q, r) == 0;
}

bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
    return sarea2(p, q, r) > 0;
}

int quad(pt p) { // quadrante de um ponto
    return (p.x<0)^3*(p.y<0);
}

bool compare_angle(pt p, pt q) { // retorna se ang(p) < ang(q)
    if (quad(p) != quad(q)) return quad(p) < quad(q);
    return ccw(q, pt(0, 0), p);
}

pt rotate90(pt p) { // rotaciona 90 graus
    return pt(-p.y, p.x);
}

// RETA

bool isinseg(pt p, line r) { // se p pertence ao seg de r
    pt a = r.p - p, b = r.q - p;
    return (a ^ b) == 0 and (a * b) <= 0;
}

bool interseg(line r, line s) { // se o seg de r intersecta o seg de s
    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
            ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

int segpoints(line r) { // numero de pontos inteiros no segmento
    return 1 + __gcd(abs(r.p.x - r.q.x), abs(r.p.y - r.q.y));
}

double get_t(pt v, line r) { // retorna t tal que t*v pertence a reta r
    return (r.p^r.q) / (double) ((r.p-r.q)^v);
}

// POLIGONO

// quadrado da distancia entre os retangulos a e b (lados paralelos aos eixos)
// assume que ta representado (inferior esquerdo, superior direito)
ll dist2_rect(pair<pt, pt> a, pair<pt, pt> b) {
    int hor = 0, vert = 0;
    if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x - b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y - a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y - b.second.y;
    return sq(hor) + sq(vert);
}

ll polarea2(vector<pt> v) { // 2 * area do poligono
    ll ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea2(pt(0, 0), v[i], v[(i + 1) % v.size()]);
    return abs(ret);
```

```
}

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i+1)%v.size();
        if (p.y == v[i].y and p.y == v[j].y) {
            if ((v[i]-p)*(v[j]-p) <= 0) return 2;
            continue;
        }
        bool baixo = v[i].y < p.y;
        if (baixo == (v[j].y < p.y)) continue;
        auto t = (p-v[i])^(v[j]-v[i]);
        if (!t) return 2;
        if (baixo == (t > 0)) qt += baixo ? 1 : -1;
    }
    return qt != 0;
}

vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n log(n))
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l.end()[-2], l.end()[-1], v[i]))
            l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u.end()[-2], u.end()[-1], v[i]))
            u.pop_back();
        u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

ll interior_points(vector<pt> v) { // pontos inteiros dentro de um poligono simples
    ll b = 0;
    for (int i = 0; i < v.size(); i++)
        b += segpoints(line(v[i], v[(i+1)%v.size()])) - 1;
    return (polarea2(v) - b) / 2 + 1;
}

struct convex_pol {
    vector<pt> pol;

        // nao pode ter ponto colinear no convex hull
    convex_pol() {}
    convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

        // se o ponto ta dentro do hull - O(log(n))
    bool is_inside(pt p) {
        if (pol.size() == 0) return false;
        if (pol.size() == 1) return p == pol[0];
        int l = 1, r = pol.size();
        while (l < r) {
            int m = (l+r)/2;
            if (ccw(p, pol[0], pol[m])) l = m+1;
            else r = m;
        }
        if (l == 1) return isinseg(p, line(pol[0], pol[1]));
        if (l == pol.size()) return false;
        return !ccw(p, pol[l], pol[l-1]);
    }
        // ponto extremo em relacao a cmp(p, q) = p mais extremo q
        // (copiado de https://github.com/gustavoM32/caderno-zika)
    int extreme(const function<bool(pt, pt)>& cmp) {
        int n = pol.size();
        auto extr = [&](int i, bool& cur_dir) {
            cur_dir = cmp(pol[(i+1)%n], pol[i]);
            return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
        };
        bool last_dir, cur_dir;
        if (extr(0, last_dir)) return 0;
        int l = 0, r = n;
        while (l+1 < r) {
            int m = (l+r)/2;
            if (extr(m, cur_dir)) return m;
            bool rel_dir = cmp(pol[m], pol[l]);
            if ((!last_dir and cur_dir) or
                    (last_dir == cur_dir and rel_dir == cur_dir)) {
                l = m;
                last_dir = cur_dir;
            } else r = m;
        }
        return l;
    }
    int max_dot(pt v) {
```

```
        return extreme([&](pt p, pt q) { return p*v > q*v; });
    }
    pair<int, int> tangents(pt p) {
        auto L = [&](pt q, pt r) { return ccw(p, r, q); };
        auto R = [&](pt q, pt r) { return ccw(p, q, r); };
        return {extreme(L), extreme(R)};
    }
};

bool operator <(const line& a, const line& b) { // comparador pra reta
        // assume que as retas tem p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    bool b1 = compare_angle(v1, v2), b2 = compare_angle(v2, v1);
    if (b1 or b2) return b1;
    return ccw(a.p, a.q, b.p); // mesmo angulo
}
bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
}

// comparador pro set pra fazer sweep line com segmentos
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
            // assume que os segmentos tem p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (a.p.x != a.q.x and (b.p.x == b.q.x or a.p.x < b.p.x))
            return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador pro set pra fazer sweep angle com segmentos
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) < get_t(dir, b);
    }
};
```

1.4 Simple Polygon

```
// Verifica se um poligono com n pontos eh simples
//
// O(n log n)
// Direto do Caderno do Brullas Mano

bool operator < (const line& a, const line& b) { // comparador pro sweepline
    if (a.p == b.p) return ccw(a.p, a.q, b.q);
    if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps < b.p.x))
        return ccw(a.p, a.q, b.p);
    return ccw(a.p, b.q, b.p);
}

bool simple(vector<pt> v) {
    auto intersects = [&](pair<line, int> a, pair<line, int> b) {
        if ((a.second+1)%v.size() == b.second or
            (b.second+1)%v.size() == a.second) return false;
        return interseg(a.first, b.first);
    };
    vector<line> seg;
    vector<pair<pt, pair<int, int>>> w;
    for (int i = 0; i < v.size(); i++) {
        pt at = v[i], nxt = v[(i+1)%v.size()];
        if (nxt < at) swap(at, nxt);
        seg.push_back(line(at, nxt));
        w.push_back({at, {0, i}});
        w.push_back({nxt, {1, i}});
            // casos degenerados estranhos
        if (isinseg(v[(i+2)%v.size()], line(at, nxt))) return 0;
        if (isinseg(v[(i+v.size()-1)%v.size()], line(at, nxt))) return 0;
    }
    sort(w.begin(), w.end());
    set<pair<line, int>> se;
    for (auto i : w) {
        line at = seg[i.second.second];
        if (i.second.first == 0) {
            auto nxt = se.lower_bound({at, i.second.second});
            if (nxt != se.end() and intersects(*nxt, {at, i.second.second})) return
↪   0;
            if (nxt != se.begin() and intersects(*(--nxt), {at, i.second.second}))
↪   return 0;
            se.insert({at, i.second.second});
        } else {
            auto nxt = se.upper_bound({at, i.second.second}), cur = nxt, prev =
↪   --cur;
            if (nxt != se.end() and prev != se.begin()
                and intersects(*nxt, *(--prev))) return 0;
            se.erase(cur);
        }
    }
    return 1;
```

```
}
```

2 Matematica

2.1 Aritmetica Modular

```
// O mod tem q ser primo
// Creditos: bruno meleca

template<int p> struct mod_int {
    ll expo(ll b, ll e) {
        ll ret = 1;
        while (e) {
            if (e % 2) ret = ret * b % p;
            e /= 2, b = b * b % p;
        }
        return ret;
    }
    ll inv(ll b) { return expo(b, p-2); }

    using m = mod_int;
    int v;
    mod_int() : v(0) {}
    mod_int(ll v_) {
        if (v_ >= p or v_ <= -p) v_ %= p;
        if (v_ < 0) v_ += p;
        v = v_;
    }
    m& operator +=(const m& a) {
        v += a.v;
        if (v >= p) v -= p;
        return *this;
    }
    m& operator -=(const m& a) {
        v -= a.v;
        if (v < 0) v += p;
        return *this;
    }
    m& operator *=(const m& a) {
        v = v * ll(a.v) % p;
        return *this;
    }
    m& operator /=(const m& a) {
        v = v * inv(a.v) % p;
        return *this;
    }
    m operator -(){ return m(-v); }
    m& operator ^=(ll e) {
        if (e < 0) {
            v = inv(v);
            e = -e;
        }
        v = expo(v, e);
            // possivel otimizacao:
            // cuidado com 0^0
            // v = expo(v, e%(p-1));
        return *this;
    }
    bool operator ==(const m& a) { return v == a.v; }
    bool operator !=(const m& a) { return v != a.v; }

    friend istream& operator >>(istream& in, m& a) {
        ll val; in >> val;
        a = m(val);
        return in;
    }
    friend ostream& operator <<(ostream& out, m a) {
        return out << a.v;
    }
    friend m operator +(m a, m b) { return a += b; }
    friend m operator -(m a, m b) { return a -= b; }
    friend m operator *(m a, m b) { return a *= b; }
    friend m operator /(m a, m b) { return a /= b; }
    friend m operator ^(m a, ll e) { return a ^= e; }
};

typedef mod_int<(int)1e9+7> mint;
```

2.2 Combinatorics

```
const int maxn = 1e6;
vector<ll> fact(maxn+1), ifact(maxn+1);

ll fastexp(ll b, ll e){
    ll res = 1;
    while(e){
```

```
        if(e&1) res = (res * b)%mod;
        b = (b * b)%mod;
        e/=2;
    }
    return res;
}

ll inv(ll x){
    return fastexp(x, mod-2);
}

ll choose(ll a, ll b){
    if(a < b) return 0;
    return fact[a] * ifact[b] %mod * ifact[a-b] %mod;
}

void build(){

    fact[0] = 1;
    for(int i = 1; i <= maxn; i++) fact[i] = (fact[i-1] * i)%mod;
    ifact[maxn] = inv(fact[maxn]);
    for(int i = maxn-1; i >= 0; i--) ifact[i] = (ifact[i+1] * (i+1))%mod;

}
```

2.3 Crivo + Fatoração

```
struct Sieve{
    int maxn;
    vector <int> is_prime, min_div;
    Sieve(int n){
        this->maxn = n;
        is_prime.assign(n+1, 1);
        min_div.resize(n+1);

        for(int i = 0; i <= n; i++)
            min_div[i] = i;

        is_prime[0] = is_prime[1] = 0;
        for (int i = 2; i <= n; i++) {
            if (is_prime[i] && (long long)i * i <= n) {
                for (int j = i * i; j <= n; j += i){
                    if(is_prime[j]) min_div[j] = i;
                    is_prime[j] = false;
                }
            }
        }
    }

    vector<pair<int,int>> factorize(int n){
        assert(n <= maxn);
        vector <pair<int,int>> fact;
        while(n > 1){
            if(fact.empty() || fact.back().first != min_div[n]){
                fact.push_back({min_div[n], 1});
            }else{
                fact.back().second += 1;
            }
            n /= min_div[n];
        }
        return fact;
    }
};
```

2.4 Euclides Estendido

```
//Retorna o GCD de a e b, e os coeficientes x e y
//tais que ax + by = gcd(a, b).
//Complexidade: O(log(min(a, b)))

int egcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = egcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
```

2.5 Fast Subset Transform

```
 * Author: Lucian Bicsi
 * Description: Transform to a basis with fast convolutions of the form
 * $\displaystyle c[z] = \sum\nolimits_{z = x \oplus y} a[x] \cdot b[y]$,
 * where $\oplus$ is one of AND, OR, XOR. The size of $a$ must be a power of two.
 * Time: O(N \log N)
 * Também chamada de Transformada Rápida de Walsh-Hadamard
 */

void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                    // inv ? pii(v - u, u) : pii(v, u + v); // AND /// include-line
                    // inv ? pii(v, u - v) : pii(u + v, u); // OR  /// include-line
                    // pii(u + v, u - v);                   // XOR /// include-line
        }
    }
        // if (inv) for (int& x : a) x /= sz(a); // XOR only /// include-line
}
vi conv(vi a, vi b) {
    FST(a, 0); FST(b, 0);
    rep(i,0,sz(a)) a[i] *= b[i];
    FST(a, 1); return a;
}
```

2.6 FFT

```
struct FFT{
    typedef complex<double> C;
    typedef vector<double> vd;
    typedef vector<long long int> vl;
    typedef vector<int> vi;

        /*
        * Author: Ludo Pulles, chilli, Simon Lindholm
        * Date: 2019-01-09
        * License: CC0
        * Source: http://neerc.ifmo.ru/trains/toulouse/2017/fft2.pdf (do read, it's
↪   excellent)
        Accuracy bound from http://www.daemonology.net/papers/fft.pdf
        * Description: fft(a) computes $\hat f(k) = \sum_x a[x] \exp(2\pi i \cdot k
↪   x / N)$ for all $k$. N must be a power of 2.
        Useful for convolution:
        \texttt{conv(a, b) = c}, where $c[x] = \sum a[i]b[x-i]$.
        For convolution of complex numbers or more than two vectors: FFT, multiply
        pointwise, divide by n, reverse(start+1, end), FFT back.
        Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2{N} < 9\cdot10^{14}$
        (in practice $10^{16}$; higher for random inputs).
        Otherwise, use NTT/FFTMod.
        * Time: O(N \log N) with $N = |A|+|B|$ ($\tilde 1s$ for $N=2^{22}$)
        * Status: somewhat tested
        * Details: An in-depth examination of precision for both FFT and FFTMod can
↪   be found
        * here
↪   (https://github.com/simonlindholm/fft-precision/blob/master/fft-precision.md)
    */
    void fft(vector<C>& a) {
        int n = a.size(), L = 31 - __builtin_clz(n);
        static vector<complex<long double>> R(2, 1);
        static vector<C> rt(2, 1);  // (^ 10% faster if double)
        for (static int k = 2; k < n; k *= 2) {
            R.resize(n); rt.resize(n);
            auto x = polar(1.0L, acos(-1.0L) / k);
            for(int i=k; i<2*k; i++) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
        }
        vi rev(n);
        for(int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
        for(int i = 0; i < n; i++)  if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int k = 1; k < n; k *= 2)
            for (int i = 0; i < n; i += 2 * k) for(int j = 0; j < k; j++) {
                    // C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)  ///
↪   include-line
                auto x = (double *)&rt[j+k], y = (double *)&a[i+j+k];        ///
↪   exclude-line
                C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);           ///
↪   exclude-line
                a[i + j + k] = a[i + j] - z;
                a[i + j] += z;
            }
    }

    vd conv(const vd& a, const vd& b) {
        if (a.empty() || b.empty()) return {};
        vd res(a.size() + b.size() - 1);
        int L = 32 - __builtin_clz(res.size()), n = 1 << L;
        vector<C> in(n), out(n);
        copy(a.begin(), a.end(), begin(in));
```

```
        for(int i = 0; i < b.size(); i++) in[i].imag(b[i]);
        fft(in);
        for (C& x : in) x *= x;
        for(int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
        fft(out);
        for(int i = 0; i < res.size(); i++) res[i] = imag(out[i]) / (4 * n);
        return res;
    }

    vl conv(const vl& a, const vl& b) {
        if (a.empty() || b.empty()) return {};
        vd res(a.size() + b.size() - 1);
        int L = 32 - __builtin_clz(res.size()), n = 1 << L;
        vector<C> in(n), out(n);
        copy(a.begin(), a.end(), begin(in));
        for(int i = 0; i < b.size(); i++) in[i].imag(b[i]);
        fft(in);
        for (C& x : in) x *= x;
        for(int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
        fft(out);
        for(int i = 0; i < res.size(); i++) res[i] = imag(out[i]) / (4 * n);
        vl r(a.size() + b.size() - 1);
        for(int i = 0; i < res.size(); i++) r[i] = (ll)(res[i]+.5);
        return r;
    }

        /*
        * Author: chilli
        * Date: 2019-04-25
        * License: CC0
        * Source: http://neerc.ifmo.ru/trains/toulouse/2017/fft2.pdf
        * Description: Higher precision FFT, can be used for convolutions modulo
    arbitrary integers
        * as long as $N\log_2N\cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice
    $10^{16}$ or higher).
        * Inputs must be in $[0, \text{mod})$.
        * Time: O(N \log N), where $N = |A|+|B|$ (twice as slow as NTT or FFT)
        * Status: stress-tested
        * Details: An in-depth examination of precision for both FFT and FFTMod can
    be found
        * here
    (https://github.com/simonlindholm/fft-precision/blob/master/fft-precision.md)
    */
        // multiplica dois polinomios modulo algum inteiro
    template<int M> vl convMod(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {};
        vl res(a.size() + b.size() - 1);
        int B=32-__builtin_clz(res.size()), n=1<<B, cut=int(sqrt(M));
        vector<C> L(n), R(n), outs(n), outl(n);
        for(int i = 0; i < a.size(); i++) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
        for(int i = 0; i < b.size(); i++) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
        fft(L), fft(R);
        for(int i = 0; i < n; i++) {
            int j = -i & (n - 1);
            outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
            outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
        }
        fft(outl), fft(outs);
        for(int i = 0; i < res.size(); i++) {
            ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
            ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
            res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
        }
        return res;
    }


};
```

2.7 Gauss

```
//Complexidade: O(n^3), onde n é o número de variáveis

template<typename T>
pair<int, vector<T>> gauss(vector<vector<T>> a, vector<T> b) {
    const double eps = 1e-6;
    int n = a.size(), m = a[0].size();
    for (int i = 0; i < n; i++) a[i].push_back(b[i]);

    vector<int> where(m, -1);
    for (int col = 0, row = 0; col < m and row < n; col++) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs(a[i][col]) > abs(a[sel][col])) sel = i;
        if (abs(a[sel][col]) < eps) continue;
        for (int i = col; i <= m; i++)
            swap(a[sel][i], a[row][i]);
        where[col] = row;

        for (int i = 0; i < n; i++) if (i != row) {
            T c = a[i][col] / a[row][col];
```

```
        for (int j = col; j <= m; j++)
            a[i][j] -= a[row][j] * c;
    }
    row++;
}

vector<T> ans(m, 0);
for (int i = 0; i < m; i++) if (where[i] != -1)
    ans[i] = a[where[i]][m] / a[where[i]][i];
for (int i = 0; i < n; i++) {
    T sum = 0;
    for (int j = 0; j < m; j++)
        sum += ans[j] * a[i][j];
    if (abs(sum - a[i][m]) > eps)
        return pair(0, vector<T>());
}

for (int i = 0; i < m; i++) if (where[i] == -1)
    return pair(INF, ans);
return pair(1, ans);
}
```

2.8 Interpolation Pts-> Pol

```
 *  Description: Given $n$ points (x[i], y[i]), computes an n-1-degree polynomial
↪  $p$ that
 *  passes through them: $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$.
 *  For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k=0 \dots n-1$.
 * Time: O(n^2)
 */
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    for(int k = 0; k < n-1; ++k) for(int i = k+1; i < n; ++i)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1;
    for(int k = 0; k < n; ++k) for(int i = 0; i < n; ++i) {
        res[i] += y[k] * temp[i]; swap(last, temp[i]);
        temp[i] -= last * x[k];
    }
    return res;
}
```

2.9 Interpolação Pts -> Pt

```
//
// Interpolação is a numerical method to
// know the result of a function of degree n
// just by knowing  n+1 point from it
//
//
// Proof of Uniques: say we have another polynome
// of degree <=k M(x). So in M(x) - L(x) = 0 in k+1
// points, but the only function that has K+1 roots
// with degree <=k is f(x) = 0, so
// M(x) - L(X) = 0 -> M(x) = L(x)

struct Interpolation
{

        //naive implementation O(n^2)
    void interpolate(vector<pair<ll,ll>> &P, int x){

        ll ans = 0;
        for(int i = 0; i < P.size(); i++){
            ll li = 1;
            for(int j = 0; j < P.size(); j++){
                if(i == j) continue;
                li *= (x - P[j].first);
                li /= (P[i].first - P[j].first);
            }
            li *= P[i].second;
            ans += li;
        }
        return ans;

    }

};
```

2.10 Matrix

```
//Cuido, tirar o mod em caso de matrizes de double

template<typename T> struct Matriz
{
```

```
    int n, m;
    vector<vector<T>> mat;
    const int MOD = 1e9+7;

    Matriz(int n, int m) : n(n), m(m), mat(n, vector<T> (m)) {}

    T* operator[](int i){
        return mat[i].data();
    }

    Matriz<T> operator*(Matriz<T> &oth){
        assert(m == oth.n);
        Matriz<T> res(n, oth.m);
        for(int i = 0; i < n; i++) {
            for(int j = 0; j < oth.m; j++) {
                res[i][j] = 0;
                for(int k = 0; k < m; k++) {
                    res[i][j] = (res[i][j] + (mat[i][k] * 1LL * oth[k][j]) % MOD) %
↪   MOD;
                }
            }
        }
        return res;
    }

    Matriz<T> operator^(long long e){
        assert(n == m);
        Matriz<T> R(n, n), b = *this;
        for(int i = 0; i < n; i++) R[i][i] = 1;
        while (e) {
            if (e&1) R = R*b;
            b = b*b;
            e >>= 1;
        }
        return R;
    }
};

using M = Matriz<int>;
```

2.11 NTT

```
{
    typedef vector<long long int> vl;
typedef vector<int> vi;

    /*
* Author: chilli
* Date: 2019-04-16
* License: CC0
* Source: based on KACTL's FFT
* Description: ntt(a) computes $\hat f(k) = \sum_x a[x] g^{xk}$ for all $k$,
↪ where $g=\text{root}^{(mod-1)/N}$.
* N must be a power of 2.
* Useful for convolution modulo specific nice primes of the form $2^a b+1$,
* where the convolution result has size at most $2^a$. For arbitrary modulo, see
↪ FFTMod.
\texttt{conv(a, b) = c}, where $c[x] = \sum a[i]b[x-i]$.
For manual convolution: NTT the inputs, multiply
pointwise, divide by n, reverse(start+1, end), NTT back.
* Inputs must be in [0, mod).
* Time: O(N \log N)
* Status: stress-tested
*/
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
    // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
    // and 483 << 21 (same root). The last two are > 10^9.
void ntt(vl &a) {
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        for(int i = k; i < 2*k; i++) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vi rev(n);
    for(int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    for(int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) for(int j = 0; j < k; j++) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
}
vl conv_ntt(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    int s = a.size() + b.size() - 1, B = 32 - __builtin_clz(s),
        n = 1 << B;
    int inv = modpow(n, mod - 2);
    vl L(a), R(b), out(n);
```

```
        L.resize(n), R.resize(n);
        ntt(L), ntt(R);
        for(int i = 0; i < n; i++)
            out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
        ntt(out);
        return {out.begin(), out.begin() + s};
    }
    ll modpow(ll b, ll e) {
        ll ans = 1;
        for (; e; b = b * b % mod, e /= 2)
            if (e & 1) ans = ans * b % mod;
        return ans;
    }
};
```

2.12 Pollard Ho

```
//Complexidade: O(n^(1/4)) em média, O(n^(1/2)) no pior caso

ll mul(ll a, ll b, ll m) {
    ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
    return ret < 0 ? ret+m : ret;
}

ll pow(ll x, ll y, ll m) {
    if (!y) return 1;
    ll ans = pow(mul(x, x, m), y/2, m);
    return y%2 ? mul(x, ans, m) : ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 1795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) {return mul(x, x, n) + 1;};

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x==y) x = ++x0, y = f(x);
        q = mul(prd, abs(x-y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<ll> fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vector<ll> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

3 Grafos

3.1 2 Sat

```
//(a ou b) e (c ou d) e (~a ou c) ...
//Complexidade: O(n + m), onde n é o número de variáveis e m é o número de
↪  implicações
//
// (+a  ou -b) -> add_edge(1, a, 0, b)
//
//Status: tested - https://cses.fi/problemset/result/8784228/

struct SAT2{

    int n, cont;
```

```
vector<char> resp;
vector<int> marc, ord, comp;
vector<vector<int>> grafo, rgrafo,scc;

SAT2(int n) :  n(n), marc(2*n+2), grafo(2*n+2), rgrafo(2*n+2), comp(2*n+2),
↪ resp(2*n + 2){}

void add_edge(int sx, int x, int sy, int y){ // '+' = 1, '-' = 0

    grafo[y+n*(!sy)].push_back(x+n*sx); //~y -> x
    grafo[x+n*(!sx)].push_back(y+n*sy); //~x -> y

    rgrafo[x+n*sx].push_back(y+n*(!sy));
    rgrafo[y+n*sy].push_back(x+n*(!sx));

}

void dfs1(int v){
    marc[v] = 1;
    for(auto viz : grafo[v]){
        if(!marc[viz]) dfs1(viz);
    }
    ord.push_back(v);
}

void dfs2(int v, int c){
    comp[v] = c;
    for(auto viz : rgrafo[v]){
        if(!comp[viz]) dfs2(viz, c);
    }
}

void build(){

    cont = 0;

    for(int i = 1; i <=2*n ;i++){
        if(!marc[i]) dfs1(i);
    }

    reverse(ord.begin(), ord.end());

    for(int v : ord){
        if(!comp[v]){
            dfs2(v, ++cont);
        }
    }

        bool can = true;
        for(int i= 1; i <=n; i++){
            if(comp[i] == comp[i+n]) can = false;
            resp[i]=comp[i]<comp[i+n]?'+':'-';
                //positiva é comp[i+n], está escolhendo a variavel que não
                //tem um caminho de implicação que resulta em impossivel
        }

        if(can){
            for(int i = 1; i <=n; i++){
                cout << resp[i] << " ";
            }
        }else{
            cout << "IMPOSSIBLE" << endl;
        }
    }

};
```

3.2 Bellman-Ford

```
//Podemos encontrar ciclos negativos guardando os pais de cada vértice.

struct Edge{
    int v, u, cost;
    Edge(int v, int u, int cost): v(v), u(u), cost(cost) {}
};

struct Ford
{
    const ll INFL = 1e18;
    int n, m;
    vector<Edge> edges;
    vector<ll> dist;

    Ford(int n, int m) : n(n), m(m), dist(n+1, INFL) {}

    void add_edge(int v, int u, int cost){
        edges.emplace_back(v,u,cost);
    }

    ll bellman(int s, int t){
        dist[s] = 0;
```

```
        //Encontrar distancias
    for(int k=1; k < n; k++){
        for(Edge e : edges){
            int a = e.v, b = e.u, c = e.cost;
            if(dist[a] != MINFL && dist[b] > dist[a] + c){
                dist[b] = dist[a] + c;
            }
        }
    }

        //Se conseguirmos melhorar após n-1, significa que existe ciclo negativo
    for(Edge e : edges){
        int a = e.v, b =e.u, c=e.cost;
        if(dist[a] != MINFL && dist[b] > dist[a]+c){
            return -1;
        }
    }

    return dist[t];

  }

};
```

3.3 BFS

```
queue<int> q;
vector<bool> used(n);

q.push(s);
used[s] = true;
while (!q.empty()) {
    int v = q.front();
    q.pop();
    for (int u : adj[v]) {
        if (!used[u]) {
            used[u] = true;
            q.push(u);
        }
    }
}
```

3.4 Bridge Tree

```
//Complexidade: O(V + E), onde V é o número de vértices e E é o número de arestas.
//A árvore de pontes é um grafo que representa as componentes conexas de um grafo
↪ original,
//onde cada aresta é formada por uma ponte do grafo original.

struct BridgeTree{

    int n;
    int count = 0;
    vector<int> marc, tin, low, is_bridge;
    vector<vector<pair<int,int>>> grafo;
    vector<vector<int>> BT;
    vector<pair<int,int>> edge;

    vector<int> BTcomponent;

    BridgeTree(int n) : n(n), grafo(n+1), marc(n+1), tin(n+1), low(n+1),
↪   BTcomponent(n+1){}

    void add_edge(int a, int b){
        grafo[a].push_back({b, edge.size()});
        grafo[b].push_back({a, edge.size()});
        edge.push_back({a,b});
        is_bridge.push_back(0);
    }

    void dfs(int x, int p){
        marc[x] = 1;
        tin[x] = low[x] = ++count;
        int children = 0;
        for(auto [viz, e] : grafo[x]){
            if(viz == p) continue;
            if(marc[viz]){
                low[x] = min(low[x], tin[viz]);
            }else{
                dfs(viz,x);
                low[x] = min(low[x], low[viz]);
                if(low[viz] > tin[x]){
                    is_bridge[e] = 1;
                }
                children++;
            }
        }
    }
```

```
    void find_bridges(){
        for(ll i=1; i<=n; i++){
            if(!marc[i]) dfs(i,0);
        }
    }

    void BTdfs(int v, int comp){
        BTcomponent[v] = comp;
        for(auto [viz, e] : grafo[v]){
            if(BTcomponent[viz] || is_bridge[e]) continue;
            BTdfs(viz, comp);
        }
    }

    void BrigeTree(){
        int comp = 0;
        for(int i = 1; i <= n; i++){
            if(!BTcomponent[i]) BTdfs(i, ++comp);
        }

        BT.resize(comp+1);

        for(int i = 1; i <= n; i++){
            for(auto [j,e] : grafo[i]){
                if(is_bridge[e]){
                    BT[BTcomponent[i]].push_back(BTcomponent[j]);
                    BT[BTcomponent[j]].push_back(BTcomponent[i]);
                }
            }
        }
    }
};
```

3.5 Dijkstra - O(n*n + m)

```
//Algoritmo de Caminho mínimo para grafos compesos não negativos. Um para todos

struct Graph{
    int n;
    const int binf = 1e9;
    vector <vector<pair<int,int>>> adj;
    Graph(int n){
        this->n = n;
        adj.resize(n);
    }
    void add_edge(int u, int v, int w){
        adj[u].emplace_back(v, w);
        adj[v].emplace_back(u, w);
    }
    int min_dist(int s, int e){//disjktras da silva
        vector <int> d(n, binf);
        vector <bool> u(n, false);
        d[s] = 0;
        for(int i = 0; i < n; i++){
            int v = -1;
            for(int j = 0; j < n; j++){
                if(!u[j] && (v==-1 || d[j] < d[v])){
                    v = j;
                }
            }
            if(d[v] == binf){
                break;
            }
            u[v] = true;
            for(auto edge : adj[v]){
                int to = edge.first;
                int len = edge.second;
                if(d[v]+len < d[to]){
                    d[to] = d[v]+len;
                }
            }
        }
        return d[e];
    }
};
```

3.6 Dijkstra - O(nlog)

```
//Algoritmo de Caminho mínimo para grafos compesos não negativos. Um para todos
//Complexidade: O(n log n) onde n é o número de vértices do grafo.

struct Dykstra
{
    ll INF = 1e18;

    int n;
    vector<ll> dist;
```

```
    vector<vector<pair<int,int>>> g;

    Dykstra(int n) : n(n), dist(n+1,INF), g(n+1) {}

    void addEdge(ll v, ll u, ll p){
        g[v].push_back({u,p});
        g[u].push_back({v,p});
    }

    void run(ll v){

        priority_queue<pair<ll,ll>, vector<pair<ll,ll>>,
        greater<pair<ll,ll>>> fila;

        fila.push({0,v});

        while (!fila.empty())
        {
            ll vert = fila.top().second;
            ll price = fila.top().first;
            fila.pop();

            if(dist[vert] != INF) continue;

            dist[vert] = price;

            for(auto viz : g[vert]){
                ll nxt = viz.first;
                ll cost = viz.second;
                fila.push({price + cost, nxt});
            }
        }

    }

};
```

3.7 Dinic

```
// Grafo com capacidades 1: O(min(M*sqrt(M), M*N^(2/3)))
// Todo vértice tem grau de entrada ou saída 1 e a maior capacidade é 1: O(sqrt(N)*M)
template<typename T>
struct Dinic{
    struct Edge {int v, u; T cap, flow;};
    int m=0;
    vector<Edge> edges;
    vector<vector<int> > vec;
    vector<int> lv, pos;
    queue<int> fila;

    Dinic() {}

    Dinic(int n) : vec(n), lv(n), pos(n) {}

    void add_edge(int v, int u, T cap) {
        edges.push_back({v, u, cap, 0});
        edges.push_back({u, v, 0, 0});
        vec[v].push_back(m);
        vec[u].push_back(m+1);
        m+=2;
    }

    int bfs(int t){
        while(!fila.empty()){
            int v=fila.front();
            fila.pop();
            for(int i:vec[v]){
                if(edges[i].cap-edges[i].flow<1) continue;
                if(lv[edges[i].u]!=-1) continue;

                lv[edges[i].u]=lv[v]+1;
                fila.push(edges[i].u);
            }
        }
        return lv[t]!=-1;
    }

    T dfs(int v, int t, T menor) {
        if(!menor) return 0;
        if(v==t) return menor;

        for(int& j=pos[v]; j<(int)vec[v].size(); j++){
            int i=vec[v][j];
            int u=edges[i].u;

            if(lv[v]+1!=lv[u] || edges[i].cap-edges[i].flow<1) continue;

            T agr=dfs(u, t, min(menor, edges[i].cap-edges[i].flow));
            if(!agr) continue;
```

```
            edges[i].flow+=agr;
            edges[i^1].flow-=agr;

            return agr;
        }
        return 0;
    }

    T max_flow(int s, int t){
        T flow=0;
        while(1){
            fill(lv.begin(), lv.end(), -1);

            lv[s]=0;
            fila.push(s);

            if(!bfs(t)) break;

            fill(pos.begin(), pos.end(), 0);

            while(T atual=dfs(s, t, INF)) flow+=atual; //remember to change INF
        }
        return flow;
    }
    auto recap(){
        vector<pair<int, int> > resp;
        for(int i=0; i<(int)edges.size(); i+=2){
            if(lv[edges[i].v]>=0 && lv[edges[i].u]==-1) resp.push_back({edges[i].v,
↪   edges[i].u});
        }
        return resp;
    }
};
```

3.8 Euler Path

```
 * Author: Simon Lindholm
 * Date: 2019-12-31
 * License: CC0
 * Source: folklore
 * Description: Eulerian undirected/directed path/cycle algorithm.
 * Input should be a vector of (dest, global edge index), where
 * for undirected graphs, forward/backward edges have the same index.
 * Returns a list of nodes in the Eulerian path/cycle with src at both start and
↪  end, or
 * empty list if no cycle/path exists.
 * To get edge indices back, add .second to s and ret.
 * Time: O(V + E)
 * Status: stress-tested
 *
 * Condições para a existencia de um caminho/cicutio euleriano:
 *
 *           |        Direcionado            |         Não Direcionado
 * ---------+-----------------------------+--------------------------
 *           | "existem 0 ou 1 vértices      |
 * Caminho  |  com difrença 1 entre grau    | "existem 0 ou 2 vértices de grau impar"
 *           |  de entrada e saida"          |
 * ---------+-----------------------------+-----------------------------------
 *           | "Grau de entrada e saida      |
 * Circuito |  de todos os vértices          | "não existe vértice de grau impar"
 *           |  são iguais"                  |
 *

 */
vector<int> eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = gr.size();
    vector<int> D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) { ///start-hash
        int x = s.back(), y, e, &it = its[x], end = int(gr[x].size());
        if (it == end){ ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e])
            D[x]--, D[y]++, eu[e] = 1, s.push_back(y);
    } ///end-hash
    for(auto &x : D) if (x < 0 || int(ret.size()) != nedges+1) return {};
    return {ret.rbegin(), ret.rend()};
}
```

3.9 Floyd Warshall

```
//Algoritmo todos para todos de distancia mínima
//Se houver ciclos negativos, para algum vertice a  -> dist[a][a] < 0
//Complexidade: O(n^3)

struct FloydWarshall
{
```

```
    const int MAXN = 500;
    cont ll INF = 1e18;
    vector dist(maxn, vector<ll>(maxn, INF));

    void floydWarshall( ){

        for(int i = 0; i < MAXN; i++)dist[i][i] = 0;

        for(int k = 1; k < MAXN; k++)
            for(int i = 1; i < MAXN; i++)
                for(int j = 1; j < MAXN; j++){
                    if(dist[i][k] < INF && dist[k][j] < INF)
                        dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
                }

    }


};
```

3.10 Hopcroft Karp

```
 * Author: Chen Xing
 * Date: 2009-10-13
 * License: CC0
 * Source: N/A
 * Description: Fast bipartite matching algorithm. Graph $g$ should be a list
 * of neighbors of the left partition, and $btoa$ should be a vector full of
 * -1's of the same size as the right partition. Returns the size of
 * the matching. $btoa[i]$ will be the match for vertex $i$ on the right side,
 * or $-1$ if it's not matched.
 * Usage: vector<int> btoa(m, -1); hopcroftKarp(g, btoa);
 * Status: Tested on oldkattis.adkbipmatch and SPOJ:MATCHING
 * Time: O(\sqrt{V}E)
 */
struct Hop{

    using vi = vector<int>;

    int n, m;
    vector<vi> g;
    vi btoa;
    Hop(int n, int m) : n(n), m(m), g(n+1), btoa(m+1, -1) {}

    void add_edge(int a, int b){
        g[a].push_back(b);
    }

    bool dfs(int a, int L, vi &A, vi &B) { ///start-hash
        if (A[a] != L) return 0;
        A[a] = -1;
        for(auto &b : g[a]) if (B[b] == L + 1) {
            B[b] = 0;
            if (btoa[b] == -1 || dfs(btoa[b], L+1, A, B))
                return btoa[b] = a, 1;
        }
        return 0;
    } ///end-hash

    int solve() { ///start-hash
        int res = 0;
        vector<int> A(g.size()), B(int(btoa.size())), cur, next;
        for (;;) {
            fill(A.begin(), A.end(), 0), fill(B.begin(), B.end(), 0);
            cur.clear();
            for(auto &a : btoa) if (a != -1) A[a] = -1;
            for (int a = 0; a < g.size(); ++a) if (A[a] == 0) cur.push_back(a);
            for (int lay = 1;; ++lay) {
                bool islast = 0; next.clear();
                for(auto &a : cur) for(auto &b : g[a]) {
                    if (btoa[b] == -1) B[b] = lay, islast = 1;
                    else if (btoa[b] != a && !B[b])
                        B[b] = lay, next.push_back(btoa[b]);
                }
                if (islast) break;
                if (next.empty()) return res;
                for(auto &a : next) A[a] = lay;
                cur.swap(next);
            }
            for(int a = 0; a < int(g.size()); ++a)
                res += dfs(a, 0, A, B);
        }
    } ///end-hash
};
```

3.11 Hungarian Matching

```
 * Source: https://github.com/bqi343/USACO/blob/master/Implementations/content/graph
 ↪ s%20(12)/Matching/Hungarian.h
 * Description: Given a weighted bipartite graph, matches every node on
 * the left with a node on the right such that no
 * nodes are in two matchings and the sum of the edge weights is minimal. Takes
 * cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and
 * returns (min cost, match), where L[i] is matched with
 * R[match[i]]. Negate costs for max cost.
 * Time: O(N^2M)
 * Status: Tested on kattis:cordonbleu, stress-tested
*/
// o valor na posição i do vector retornado indica a coluna do elemento da linha i
 ↪ que foi escolhido

template<class cost_t> pair<cost_t, vector<int>> hungarian(const
 ↪ vector<vector<cost_t>> &a){
    int n = a.size() + 1, m = a[0].size() + 1;

    vector<int> p(m), ans(n - 1);
    vector<cost_t> u(n), v(m);
    for(int i = 1; i < n; ++i) {
        p[0] = i; int j0 = 0;
        vector<cost_t> dist(m, 1e9);
        vector<int> pre(m, -1);
        vector<bool> done(m + 1);
        do {
            done[j0] = true;
            int i0 = p[j0], j1;
            cost_t delta = 1e9;
            for(int j = 1; j < m; ++j) if (!done[j]) {
                auto cur = a[i0-1][j-1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            }
            for(int j = 0; j < m; ++j)
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            j0 = j1;
        } while (p[j0]);
        while (j0) {
            int j1 = pre[j0]; p[j0] = p[j1], j0 = j1;
        }
    }
    for(int j = 1; j < m; ++j) if (p[j]) ans[p[j]-1] = j-1;
    return {-v[0], ans};
}
```

3.12 Kosaraju

```
//Retorna também em scc as componentes em ordem topologica
//Complexidade: O(n+m)

struct Kosa{

    int n, cont;
    vector<int> marc, ord, comp;
    vector<vector<int>> grafo, rgrafo,scc;

    Kosa(int n) : n(n), marc(n+1), grafo(n+1), rgrafo(n+1), comp(n+1), scc(n+1) {}

    void add_edge(int a, int b){
        grafo[a].push_back(b);
        rgrafo[b].push_back(a);
    }

    void dfs1(int v){
        marc[v] = 1;
        for(auto viz : grafo[v]){
            if(!marc[viz]) dfs1(viz);
        }
        ord.push_back(v);
    }

    void dfs2(int v, int c){
        comp[v] = c;
        for(auto viz : rgrafo[v]){
            if(!comp[viz]) dfs2(viz, c);
        }
    }

    void build(){

        cont = 0;

        for(int i = 1; i <=n ;i++){
            if(!marc[i]) dfs1(i);
        }
```

```
        reverse(ord.begin(), ord.end());

        for(int v : ord){
            if(!comp[v]){
                dfs2(v, ++cont);
            }
        }

        for(int i = 1; i <=n; i++){
            for(int j : grafo[i]){
                if(comp[i] == comp[j]) continue;
                scc[comp[i]].push_back(comp[j]);
            }
        }

    }

};
```

3.13 Kuhn

```
struct bm_t
{
    int N, M, T;
    vector<vector<int>> grafo;
    vector<int> match, seen;
    bm_t(int a, int b) : N(a), M(a+b), T(0), grafo(M),match(M, -1), seen(M, -1) {}

    void add_edge(int a, int b){
        grafo[a].push_back(b + N);
    }

    bool dfs(int cur){
        if(seen[cur] == T) return false;
        seen[cur] = T;
        for(int nxt : grafo[cur]) if(match[nxt] == -1){
            match[nxt] = cur;
            match[cur] = nxt;
            return true;
        }
        for(int nxt : grafo[cur]) if(dfs(match[nxt])){
            match[nxt] = cur;
            match[cur] = nxt;
            return true;
        }
        return false;
    }

    int solve(){
        int res = 0;
        for(int cur = 1; cur;){
            cur = 0; ++T;
            for(int i = 0; i < N; ++i) if(match[i] == -1)
                cur += dfs(i);
            res += cur;
        }
        return res;
    }

};
```

3.14 Min-cist max-flow

```
//Time: O (F (V + E)logV ), being F the amount of flow.

template<class flow_t, class cost_t> struct min_cost {
    static constexpr flow_t FLOW_EPS = flow_t(1e-10);
    static constexpr flow_t FLOW_INF = numeric_limits<flow_t>::
        max();
    static constexpr cost_t COST_EPS = cost_t(1e-10);
    static constexpr cost_t COST_INF = numeric_limits<cost_t>::
        max();
    int n, m{}; vector<int> ptr, nxt, zu;
    vector<flow_t> capa; vector<cost_t> cost;

    min_cost(int N) : n(N),ptr(n,-1),dist(n),vis(n),pari(n) {}

    void add_edge(int u, int v, flow_t w, cost_t c) {
        nxt.push_back(ptr[u]); zu.push_back(v); capa.push_back(w);
        cost.push_back(c); ptr[u] = m++;
        nxt.push_back(ptr[v]); zu.push_back(u); capa.push_back(0);
        cost.push_back(-c); ptr[v] = m++;
    }

    vector<cost_t> pot, dist; vector<bool> vis; vector<int> pari;
    vector<flow_t> flows; vector<cost_t> slopes;
        // You can pass t = =1 to find a shortest
```

```
    void shortest(int s, int t) {//path to each vertex . // hash=1
        using E = pair<cost_t, int>;
        priority_queue<E, vector<E>, greater<E>> que;
        for(int u = 0; u < n; ++u){dist[u]=COST_INF; vis[u]=false;}
        for (que.emplace(dist[s] = 0, s); !que.empty(); ) {
            const cost_t c = que.top().first;
            const int u = que.top().second; que.pop();
            if (vis[u]) continue;
            vis[u] = true; if (u == t) return;
            for (int i = ptr[u]; ~i; i = nxt[i]) if (capa[i] > FLOW_EPS) {
                const int v = zu[i];
                const cost_t cc = c + cost[i] + pot[u] - pot[v];
                if(dist[v] > cc){que.emplace(dist[v]=cc,v);pari[v]=i;}
            }
        }
    }
     // hash=1 = 89f16a
    auto run(int s, int t, flow_t limFlow = FLOW_INF) { // hash=2
        pot.assign(n, 0); flows = {0}; slopes.clear();
        while (true) {
            bool upd = false;
            for (int i = 0; i < m; ++i) if (capa[i] > FLOW_EPS) {
                const int u = zu[i ^ 1], v = zu[i];
                const cost_t cc = pot[u] + cost[i];
                if(pot[v] > cc + COST_EPS) { pot[v] = cc; upd = true; }
            } if (!upd) break;
        }
        flow_t flow = 0; cost_t tot_cost = 0;
        while (flow < limFlow) {
            shortest(s, t); flow_t f = limFlow - flow;
            if (!vis[t]) break;
            for(int u = 0; u < n; ++u)pot[u] += min(dist[u],dist[t]);
            for (int v = t; v != s; ) { const int i = pari[v];
                if (f > capa[i]) { f = capa[i]; } v = zu[i^1];
                }
            for (int v = t; v != s; ) { const int i = pari[v];
                capa[i] -= f; capa[i^1] += f; v = zu[i^1];
                }
            flow += f; tot_cost += f * (pot[t] - pot[s]);
            flows.push_back(flow); slopes.push_back(pot[t] - pot[s]);
        } return make_pair(flow, tot_cost);
    } // hash=2 = 285527
};
```

3.15 Pontes e Articulação

```
//Complexidade: O(V + E), onde V é o número de vértices e E é o número de arestas.
struct ArticPont{

    int n;
    int count = 0;
    vector<int> marc, tin, low, artic;
    vector<vector<int>> grafo;
    vector<pair<int,int>> bridges;

    ArticPont(int n) : n(n), grafo(n+1), marc(n+1), tin(n+1), low(n+1), artic(n+1) {}

    void add_edge(int a, int b){
        grafo[a].push_back(b);
        grafo[b].push_back(a);
    }

    void dfs(ll x, ll p){
        marc[x] = 1;
        tin[x] = low[x] = ++count;
        ll children = 0;
        for(ll viz : grafo[x]){
            if(viz == p) continue;
            if(marc[viz]){
                low[x] = min(low[x], tin[viz]);
            }else{
                dfs(viz,x);
                low[x] = min(low[x], low[viz]);
                if(low[viz] > tin[x]){
                    bridges.push_back({min(viz,x), max(viz, x)});
                }
                if(low[viz] >= tin[x] && p) artic[x] = 1;
                children++;
            }
        }
        if(!p && children>1) artic[x] = 1;
    }

    void find_brig_and_artc(){
        for(ll i=1; i<=n; i++){
            if(!marc[i]) dfs(i,0);
        }
    }

};
```

3.16 Topo Sort

```
//It returns a vector with the vertices in topological order.
//Complexity: O(n + m), where n is the number of vertices and m is the number of
↪ edges.
struct TopoSort
{
    int n;
    vector<int> grau;
    vector<vector<int>> grafo;

    TopoSort(int n): n(n), grau(n+1), grafo(n+1){}

    void add_edge(int a, int b){
        grau[b]++;
        grafo[a].push_back(b);
    }

    vector<int> top_sort(){
        vector<int> resp;
        queue<int> fila;
        for(int i=1; i<=n;i++){
            if(!grau[i])fila.push(i);
        }
        while (!fila.empty())
        {
            int u = fila.front();
            resp.push_back(u);
            fila.pop();
            for(int viz : grafo[u]){
                grau[viz]--;
                if(!grau[viz])fila.push(viz);
            }
        }
        if(resp.size() < n){
            return {};
        }else{
            return resp;
        }
    }
};
```

4 DP

4.1 Digit DP

```
ll solve(string &s, int i, int tight, int last, int started){
    if(i==(int)s.size()) return 1;

    if(!tight && dp[i][last][started]!=-1) return dp[i][last][started];

    int lim=(tight?s[i]-'0':9);

    ll resp=0;
    for(int j=0; j<=lim; j++){
        if(started && j==last) continue;
        resp+=solve(s, i+1, tight&(j==lim), j, (started|j)>0);
    }

    if(!tight) return dp[i][last][started]=resp;
    return resp;
}

ll func(ll a, ll b){
    string agr1=to_string(a-1);
    memset(dp, -1, sizeof(dp));
    ll ans1 = solve(agr1, 0, 1, 10, 0);

    string agr2=to_string(b);
    memset(dp, -1, sizeof(dp));
    ll ans2 = solve(agr2, 0, 1, 10, 0);

    return ans2-ans1;
}
```

4.2 Sos DP

```
//I will be addressing the following problem: Given a fixed array A of 2N integers,
//we need to calculate forAll x function F(x) = Sum of all A[i] such that x&i = i,
↪ i.e., i is a subset of x.
//$$F[\text{mask}] = \sum_{i \subseteq \text{mask}} A[i]$$
//F[mask]=i(\subseteq)mask\sumA[i]
```

```cpp
for(int i = 0; i<(1<<N); ++i)
    F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}
```

4.3 Submask DP

```cpp
/*
Iterate for all strict subsets of mask
Complexity: O(3^n)
*/

for (int mask = 0; mask < (1 << n); mask++) {
    for (int submask = mask; submask != 0; submask = (submask - 1) & mask) {
        int subset = mask ^ submask;
            // do whatever you need to do here
    }
}
```

4.4 Subset Sum - Sqrt(n)

```cpp
//Subset sum - Implementation O(n) memory and O(S * sqrt(N)) runtime
//Uses sliding window technique to optimize the subset sum problem.

vector<pair<int,int>> sack; // {item, frequency}
vector<int> dp(S+1, 0);
dp[0] = 1;

for(int i = 0; i < sack.size(); i++){
    vector<int> ndp(S+1);
    auto [item, freq] = sack[i];
    for(int j = 0; j < item; j++){  //starting at position j
        int numTrues = 0;
        for(int k = j; k <= S; k += item){
            ndp[k] = dp[k];
            if(numTrues > 0) ndp[k] = true;
            if(k - freq*item >= 0) numTrues -= dp[k - freq*item];
            numTrues += dp[k];
        }
    }
    swap(ndp, dp);
}
```

5 Arvore

5.1 Centroid Decomposition

```cpp
struct Centroid{
    int n;
    vector<int> used, pai, sub;
    vector<vector<int >> vec;

    Centroid(int n) : n(n), used(n+1), pai(n+1), sub(n+1), vec(n+1) {}

    void add_edge(int v, int u){
        vec[v].push_back(u);
        vec[u].push_back(v);
    }

    int dfs_sz(int x, int p=0){
        sub[x]=1;
        for(int i:vec[x]){
            if(i==p || used[i]) continue;
            sub[x]+=dfs_sz(i, x);
        }
        return sub[x];
    }

    int find_c(int x, int total, int p=0){
        for(int i:vec[x]){
            if(i==p || used[i]) continue;
            if(2*sub[i]>total) return find_c(i, total, x);
        }
        return x;
    }

    void build(int x=1, int p=0){
        int c=find_c(x, dfs_sz(x));

            //do something
```

```
        used[c]=1;
        pai[c]=p;
        for(int i:vec[c]){
            if(!used[i]) build(i, c);
        }
    }
};
```

5.2 Heavy Light Decomposition

```
/**
 * Description: Heavy-Light Decomposition, add val to verts
     * and query sum in path/subtree.
 * Time: any tree path is split into $O(\log N)$ parts
 * Source: http://codeforces.com/blog/entry/22072,
↪  https://codeforces.com/blog/entry/53170
 * Verification: *
 */

#include "../../data-structures/1D Range Queries (9.2)/LazySeg (15.2).h"

template<int SZ, bool VALS_IN_EDGES> struct HLD {
    int N; vi adj[SZ];
    int par[SZ], root[SZ], depth[SZ], sz[SZ], ti;
    int pos[SZ]; vi rpos; // rpos not used but could be useful
    void ae(int x, int y) { adj[x].pb(y), adj[y].pb(x); }
    void dfsSz(int x) {
        sz[x] = 1;
        each(y,adj[x]) {
            par[y] = x; depth[y] = depth[x]+1;
            adj[y].erase(find(all(adj[y]),x)); /// remove parent from adj list
            dfsSz(y); sz[x] += sz[y];
            if (sz[y] > sz[adj[x][0]]) swap(y,adj[x][0]);
        }
    }
    void dfsHld(int x) {
        pos[x] = ti++; rpos.pb(x);
        each(y,adj[x]) {
            root[y] = (y == adj[x][0] ? root[x] : y);
            dfsHld(y); }
    }
    void init(int _N, int R = 0) { N = _N;
        par[R] = depth[R] = ti = 0; dfsSz(R);
        root[R] = R; dfsHld(R);
    }
    int lca(int x, int y) {
        for (; root[x] != root[y]; y = par[root[y]])
            if (depth[root[x]] > depth[root[y]]) swap(x,y);
        return depth[x] < depth[y] ? x : y;
    }
        /// int dist(int x, int y) { // # edges on path
        ///     return depth[x]+depth[y]-2*depth[lca(x,y)]; }
    LazySeg<ll,SZ> tree; // segtree for sum
    template <class BinaryOp>
    void processPath(int x, int y, BinaryOp op) {
        for (; root[x] != root[y]; y = par[root[y]]) {
            if (depth[root[x]] > depth[root[y]]) swap(x,y);
            op(pos[root[y]],pos[y]); }
        if (depth[x] > depth[y]) swap(x,y);
        op(pos[x]+VALS_IN_EDGES,pos[y]);
    }
    void modifyPath(int x, int y, int v) {
        processPath(x,y,[this,&v](int l, int r) {
            tree.upd(l,r,v); }); }
    ll queryPath(int x, int y) {
        ll res = 0; processPath(x,y,[this,&res](int l, int r) {
            res += tree.query(l,r); });
        return res; }
    void modifySubtree(int x, int v) {
        tree.upd(pos[x]+VALS_IN_EDGES,pos[x]+sz[x]-1,v); }
};
```

5.3 Lowest Common Ancestor

```
struct LCA{

    int n;
    const int sz = 32;
    vector<int> marc, height;
    vector<vector<int>> g, bl;

        //Trocar se a raiz nao for 1
    LCA(int n) : n(n), g(n+1), bl(sz, vector<int> (n+1, 1)), marc(n+1), height(n+1){}

    void add_edge(int a, int b){
        g[a].push_back(b);
        g[b].push_back(a);
```

```
    }

        //Trocar se a raiz nao for 1
    void build(int x = 1){
        marc[x] = 1;
        for(int i = 1; i < sz; i++){
            bl[i][x] = bl[i-1][bl[i-1][x]];
        }

        for(auto viz : g[x]){
            if(marc[viz]) continue;
            bl[0][viz] = x;
            height[viz] = height[x]+1;
            build(viz);
        }
    }

    int find_lca(int a, int b){
        if(height[a] < height[b]) swap(a,b);

        int dif = height[a] - height[b];
        for(int i = 0; i < sz; i++){
            if((1<<i) & dif){
                a = bl[i][a];
            }
        }

        assert(height[a] == height[b]);
        if(a == b) return a;

        for(int i = sz-1; i >=0; i--){
            if(bl[i][a] == bl[i][b]) continue;
            a = bl[i][a];
            b = bl[i][b];
        }

        assert(a != b);
        assert(bl[0][a] == bl[0][b]);
        return bl[0][a];
    }

    int dist(int a, int b){
        int l = find_lca(a,b);
        return height[a] + height[b] - 2*height[l];
    }

};
```

5.4 Virtual Tree

```
// O(NlogN), sendo N o tamanho do conjunto de vértices passados
/*
Dado o conjunto de vértices do vector v, uma árvore minimal com esse conjunto
é construida, a virtual tree possui no máximo 2*n-1 vértices

virt[i] guarda os vizinhos do vértice i,
com um pair que guarda o vértice e a distância até ele
*/

vector<pair<int, int> > virt[mxn];

int build(vector<int> v){
    auto cmp = [&](int a, int b) {return tempo[a]<tempo[b];};
    sort(v.begin(), v.end(), cmp);
    for(int i=(int)v.size()-1; i>0; i--) v.push_back(lca(v[i], v[i-1]));
    sort(v.begin(), v.end(), cmp);
    v.erase(unique(v.begin(), v.end()), v.end());
    for(int i=0; i<v.size(); i++) virt[v[i]].clear();
    for(int i=1; i<v.size(); i++) virt[lca(v[i], v[i-1])].clear();
    for(int i=1; i<v.size(); i++){
        int pai = lca(v[i], v[i-1]);
        int d = dist(pai, v[i]);
        virt[pai].emplace_back(v[i], d);
    }
    return v[0];
}
```

6 Strings

6.1 Hashing

```
//Cria o hashing de uma string
//ha[0] = 0
//ha[1] = s[0]
//ha[2] = p*s[0] + s[1]
```

```
//ha[3] = p^2*s[0] + p*s[1] + s[2]


template<int MOD> struct Hashing{
    ll base, n;
    vector<ll> pow, ha;

        /*
    for random base:
    mt19937 rng((uint32_t)chrono::steady_clock::now().time_since_epoch().count());
    const ll B = uniform_int_distribution<ll>(0, M - 1)(rng);
    */

    Hashing(string & s, int a) : n(s.size()), base(a) ,pow(n+1), ha(n+1){

        pow[0] = 1;
        for(int i = 0; i < n; i++){
            ha[i+1] = (ha[i] * base + s[i])%MOD;
            pow[i+1] = (pow[i] * base)%MOD;
        }
    }

        //Retorna o Hashing da substring [a, b), indexado em 0
    int getRange(int a, int b){
        assert(a <= b);
        ll hash =  (ha[b] - (ha[a] * pow[b-a])%MOD)%MOD;
        return hash < 0 ? hash + MOD : hash;
    }

};
```

6.2 KMP

```
vector<int> find_pi(string s){

    vector<int> pi(s.size());
    for(int i = 1, j = 0; i < s.size(); i++){
        while(j > 0 && s[j] != s[i]) j = pi[j-1];
        if(s[j] == s[i]) j++;
        pi[i] = j;
    }
    return pi;
};
```

```
vector<int> kmp(string t, string p){

    vector<int> pi= find_pi(p + '$'), match;
    for(int i = 0, j = 0; i < t.size(); i++){
        while(j > 0 && t[i] != p[j]) j = pi[j-1];
        if(t[i] == p[j]) j++;
        if(j == p.size()) match.push_back(i-j+1);
    }
    return match;
};

struct autKMP {
    vector<vector<int>> nxt;

    autKMP(string& s) : nxt(26, vector<int>(s.size()+1)) {
        vector<int> p = pi(s);
        nxt[s[0]-'a'][0] = 1;
        for (char c = 0; c < 26; c++)
            for (int i = 1; i <= s.size(); i++)
                nxt[c][i] = c == s[i]-'a' ? i+1 : nxt[c][p[i-1]];
    }
};
```

6.3 Manacher

```
//Complexidade: O(n), onde n é o tamanho da string

vector<int> manacher_odd(string s) {
    int n = s.size();
    s = "$" + s + "^";
    vector<int> p(n + 2);
    int l = 0, r = 1;
    for(int i = 1; i <= n; i++) {
        p[i] = min(r - i, p[l + (r - i)]);
        while(s[i - p[i]] == s[i + p[i]]) {
            p[i]++;
        }
        if(i + p[i] > r) {
            l = i - p[i], r = i + p[i];
        }
    }
    return vector<int>(begin(p) + 1, end(p) - 1);
}
```

```
pair<vector<int>, vector<int>> manacher(string s) {
    string t;
    for(auto c: s) {
        t += string("#") + c;
    }
    vector<int> res = manacher_odd(t + "#");
    vector<int> dodd(s.size()), deven(s.size());
    for(int i = 0; i < s.size(); i++){
        dodd[i] = res[2*i + 1]/2;
        deven[i] = (res[2*i]-1)/2;
    }

    return {dodd, deven};
}
```

6.4 Suffix Array - O(n log n)

```
// kasai recebe o suffix array e calcula lcp[i],
// o lcp entre s[sa[i],...,n-1] e s[sa[i+1],..,n-1]
// obs: lcp entre sufixo i e sufixo j é o minimo do lcp no intervalo apropriado
//
// Complexidades:
// suffix_array - O(n log(n))
// kasai - O(n)
// Creditos: Caderno do breno Maletas

vector<int> suffix_array(string s) {
    s += "$";
    int n = s.size(), N = max(n, 260);
    vector<int> sa(n), ra(n);
    for(int i = 0; i < n; i++) sa[i] = i, ra[i] = s[i];

    for(int k = 0; k < n; k ? k *= 2 : k++) {
        vector<int> nsa(sa), nra(n), cnt(N);

        for(int i = 0; i < n; i++) nsa[i] = (nsa[i]-k+n)%n, cnt[ra[i]]++;
        for(int i = 1; i < N; i++) cnt[i] += cnt[i-1];
        for(int i = n-1; i+1; i--) sa[--cnt[ra[nsa[i]]]] = nsa[i];

        for(int i = 1, r = 0; i < n; i++) nra[sa[i]] = r += ra[sa[i]] !=
            ra[sa[i-1]] or ra[(sa[i]+k)%n] != ra[(sa[i-1]+k)%n];
        ra = nra;
        if (ra[sa[n-1]] == n-1) break;
    }
    return vector<int>(sa.begin()+1, sa.end());
}

vector<int> kasai(string s, vector<int> sa) {
    int n = s.size(), k = 0;
    vector<int> ra(n), lcp(n);
    for (int i = 0; i < n; i++) ra[sa[i]] = i;

    for (int i = 0; i < n; i++, k -= !!k) {
        if (ra[i] == n-1) { k = 0; continue; }
        int j = sa[ra[i]+1];
        while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
        lcp[ra[i]] = k;
    }
    return lcp;
}
```

6.5 Suffix Array - O(n)

```
// Rapidao
// Computa o suffix array em 'sa', o rank em 'rnk'
// e o lcp em 'lcp'
// query(i, j) retorna o LCP entre s[i..n-1] e s[j..n-1]
//
// Complexidades
// O(n) para construir
// query - O(1)
// Creditos: Caderno do Brawn Malous

template<typename T> struct rmq {
    vector<T> v;
    int n; static const int b = 30;
    vector<int> mask, t;

    int op(int x, int y) { return v[x] <= v[y] ? x : y; }
    int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
    int small(int r, int sz = b) { return r-msb(mask[r]&((1<<sz)-1)); }
    rmq() {}
    rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at<<1)&((1<<b)-1);
            while (at and op(i-msb(at&-at), i) == i) at ^= at&-at;
        }
        for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
```

```
        for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0; i+(1<<j) <= n/b; i++)
            t[n/b*j+i] = op(t[n/b*(j-1)+i], t[n/b*(j-1)+i+(1<<(j-1))]);
    }
    int index_query(int l, int r) {
        if (r-l+1 <= b) return small(r, r-l+1);
        int x = l/b+1, y = r/b-1;
        if (x > y) return op(small(l+b-1), small(r));
        int j = msb(y-x+1);
        int ans = op(small(l+b-1), op(t[n/b*j+x], t[n/b*j+y-(1<<j)+1]));
        return op(ans, small(r));
    }
    T query(int l, int r) { return v[index_query(l, r)]; }
};

struct suffix_array {
    string s;
    int n;
    vector<int> sa, cnt, rnk, lcp;
    rmq<int> RMQ;

    bool cmp(int a1, int b1, int a2, int b2, int a3=0, int b3=0) {
        return a1 != b1 ? a1 < b1 : (a2 != b2 ? a2 < b2 : a3 < b3);
    }
    template<typename T> void radix(int* fr, int* to, T* r, int N, int k) {
        cnt = vector<int>(k+1, 0);
        for (int i = 0; i < N; i++) cnt[r[fr[i]]]++;
        for (int i = 1; i <= k; i++) cnt[i] += cnt[i-1];
        for (int i = N-1; i+1; i--) to[--cnt[r[fr[i]]]] = fr[i];
    }
    void rec(vector<int>& v, int k) {
        auto &tmp = rnk, &m0 = lcp;
        int N = v.size()-3, sz = (N+2)/3, sz2 = sz+N/3;
        vector<int> R(sz2+3);
        for (int i = 1, j = 0; j < sz2; i += i%3) R[j++] = i;

        radix(&R[0], &tmp[0], &v[0]+2, sz2, k);
        radix(&tmp[0], &R[0], &v[0]+1, sz2, k);
        radix(&R[0], &tmp[0], &v[0]+0, sz2, k);

        int dif = 0;
        int l0 = -1, l1 = -1, l2 = -1;
        for (int i = 0; i < sz2; i++) {
            if (v[tmp[i]] != l0 or v[tmp[i]+1] != l1 or v[tmp[i]+2] != l2)
                l0 = v[tmp[i]], l1 = v[tmp[i]+1], l2 = v[tmp[i]+2], dif++;
            if (tmp[i]%3 == 1) R[tmp[i]/3] = dif;
            else R[tmp[i]/3+sz] = dif;
        }

        if (dif < sz2) {
            rec(R, dif);
            for (int i = 0; i < sz2; i++) R[sa[i]] = i+1;
        } else for (int i = 0; i < sz2; i++) sa[R[i]-1] = i;

        for (int i = 0, j = 0; j < sz2; i++) if (sa[i] < sz) tmp[j++] = 3*sa[i];
        radix(&tmp[0], &m0[0], &v[0], sz, k);
        for (int i = 0; i < sz2; i++)
            sa[i] = sa[i] < sz ? 3*sa[i]+1 : 3*(sa[i]-sz)+2;

        int at = sz2+sz-1, p = sz-1, p2 = sz2-1;
        while (p >= 0 and p2 >= 0) {
            if ((sa[p2]%3==1 and cmp(v[m0[p]], v[sa[p2]], R[m0[p]/3],
                R[sa[p2]/3+sz])) or (sa[p2]%3==2 and cmp(v[m0[p]], v[sa[p2]],
                v[m0[p]+1], v[sa[p2]+1], R[m0[p]/3+sz], R[sa[p2]/3+1])))
                sa[at--] = sa[p2--];
            else sa[at--] = m0[p--];
        }
        while (p >= 0) sa[at--] = m0[p--];
        if (N%3==1) for (int i = 0; i < N; i++) sa[i] = sa[i+1];
    }

    suffix_array(const string& s_) : s(s_), n(s.size()), sa(n+3),
            cnt(n+1), rnk(n), lcp(n-1) {
        vector<int> v(n+3);
        for (int i = 0; i < n; i++) v[i] = i;
        radix(&v[0], &rnk[0], &s[0], n, 256);
        int dif = 1;
        for (int i = 0; i < n; i++)
            v[rnk[i]] = dif += (i and s[rnk[i]] != s[rnk[i-1]]);
        if (n >= 2) rec(v, dif);
        sa.resize(n);

        for (int i = 0; i < n; i++) rnk[sa[i]] = i;
        for (int i = 0, k = 0; i < n; i++, k -= !!k) {
            if (rnk[i] == n-1) {
                k = 0;
                continue;
            }
            int j = sa[rnk[i]+1];
            while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
            lcp[rnk[i]] = k;
        }
        RMQ = rmq<int>(lcp);
    }
```

```
    int query(int i, int j) {
        if (i == j) return n-i;
        i = rnk[i], j = rnk[j];
        return RMQ.query(min(i, j), max(i, j)-1);
    }
    pair<int, int> next(int L, int R, int i, char c) {
        int l = L, r = R+1;
        while (l < r) {
            int m = (l+r)/2;
            if (i+sa[m] >= n or s[i+sa[m]] < c) l = m+1;
            else r = m;
        }
        if (l == R+1 or s[i+sa[l]] > c) return {-1, -1};
        L = l;

        l = L, r = R+1;
        while (l < r) {
            int m = (l+r)/2;
            if (i+sa[m] >= n or s[i+sa[m]] <= c) l = m+1;
            else r = m;
        }
        R = l-1;
        return {L, R};
    }
        // quantas vezes 't' ocorre em 's' - O(|t| log n)
    int count_substr(string& t) {
        int L = 0, R = n-1;
        for (int i = 0; i < t.size(); i++) {
            tie(L, R) = next(L, R, i, t[i]);
            if (L == -1) return 0;
        }
        return R-L+1;
    }

        // exemplo de f que resolve o problema
        //
↪   https://codeforces.com/edu/course/2/lesson/2/5/practice/contest/269656/problem/D
    ll f(ll k) { return k*(k+1)/2; }

    ll dfs(int L, int R, int p) { // dfs na suffix tree chamado em pre ordem
        int ext = L != R ? RMQ.query(L, R-1) : n - sa[L];

            // Tem 'ext - p' substrings diferentes que ocorrem 'R-L+1' vezes
            // O LCP de todas elas eh 'ext'
        ll ans = (ext-p)*f(R-L+1);

            // L eh terminal, e folha sse L == R
        if (sa[L]+ext == n) L++;

            // se for um SA de varias strings separadas como s#t$u&, usar no lugar
↪   do if de cima
            //    (separadores < 'a', diferentes e inclusive no final)
            // while (L <= R && (sa[L]+ext == n || s[sa[L]+ext] < 'a')) {
            //   L++;
            // }

        while (L <= R) {
            int idx = L != R ? RMQ.index_query(L, R-1) : -1;
            if (idx == -1 or lcp[idx] != ext) idx = R;

            ans += dfs(L, idx, ext);
            L = idx+1;
        }
        return ans;
    }

        // sum over substrings: computa, para toda substring t distinta de s,
        // \sum f(# ocorrencias de t em s) - O (n)
    ll sos() { return dfs(0, n-1, 0); }
};
```

6.6 Trie

```
struct Vertex {
    int next[K];
    ll output = 0;

    Vertex() {
        fill(begin(next), end(next), -1);
    }
};

struct Trie{

    int n;
    const int K = 26;
    vector<Vertex> t;

    Trie() : t(1){}
```

```
    void add_string(string s){
        int p = 0;
        for(int i = 0; i < s.size(); i++){
            if(t[p].next[s[i] - 'a'] == -1){
                t[p].next[s[i] - 'a'] = t.size();
                t.push_back(Vertex());
            }
            p = t[p].next[s[i] - 'a'];
        }
        t[p].output++;
    }
};
```

6.7 Trie pro caio não chorar

```
// trie T() constroi uma trie para o alfabeto das letras minusculas
// trie T(tamanho do alfabeto, menor caracter) tambem pode ser usado
//
// T.insert(s) - O(|s|*sigma)
// T.erase(s) - O(|s|)
// T.find(s) retorna a posicao, -1 se nao achar - O(|s|)
// T.count_pref(s) numero de strings que possuem s como prefixo - O(|s|)
// Creditos: Bruno meleitas dnv, o caderno inteiro é dele já a esse ponto

struct trie {
    vector<vector<int>> to;
    vector<int> end, pref;
    int sigma; char norm;
    trie(int sigma_=26, char norm_='a') : sigma(sigma_), norm(norm_) {
        to = {vector<int>(sigma)};
        end = {0}, pref = {0};
    }
    void insert(string s) {
        int x = 0;
        for (auto c : s) {
            int &nxt = to[x][c-norm];
            if (!nxt) {
                nxt = to.size();
                to.push_back(vector<int>(sigma));
                end.push_back(0), pref.push_back(0);
            }
            x = nxt, pref[x]++;
        }
        end[x]++, pref[0]++;
    }
    void erase(string s) {
        int x = 0;
        for (char c : s) {
            int &nxt = to[x][c-norm];
            x = nxt, pref[x]--;
            if (!pref[x]) nxt = 0;
        }
        end[x]--, pref[0]--;
    }
    int find(string s) {
        int x = 0;
        for (auto c : s) {
            x = to[x][c-norm];
            if (!x) return -1;
        }
        return x;
    }
    int count_pref(string s) {
        int id = find(s);
        return id >= 0 ? pref[id] : 0;
    }
};
```

6.8 Z

```
//e é igual ao prefixo da string original.
//Complexidade: O(n), onde n é o tamanho da string

vector<int> zfunc(string s){
    int n = s.size();
    vector<int> z(n);
    for(int i = 1, l = 0, r = 0; i < n; i++){
        if(i <= r) z[i] = min(z[i-l], r-i+1);
        while(i + z[i] < n && s[i + z[i]] == s[z[i]]){
            z[i]++;
        }
        if(i+z[i]-1 > r){
            r = i+z[i]-1;
            l = i;
        }
    }
    return z;
}
```

7 DataStructures

7.1 BIT

```
//dada uma função f associativa em um sobre um
//conjunto com elemento neutro e inversos
//Querry - O(log(n))++suporta apenas querry de update singular
//Update - O(log(n))

struct FenwickTree {
    vector<int> bit;
    int n;

    FenwickTree(int n) {
        this->n = n;
        bit.assign(n, 0);
    }

    FenwickTree(vector<int> const &a) : FenwickTree(a.size()){
        for (int i = 0; i < n; i++) {
            bit[i] += a[i];
            int r = i | (i + 1);
            if (r < n) bit[r] += bit[i];
        }
    }


    int sum(int r) {
        int ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }

    int sum(int l, int r) {
        return sum(r) - sum(l - 1);
    }

    void add(int idx, int delta) {
        for (; idx < n; idx = idx | (idx + 1))
            bit[idx] += delta;
    }
};
```

7.2 BIT - Range Update

```
vector<int> bit1, bit2;
void init(int n){
    bit1.assign(n+1, 0);
    bit2.assign(n+1, 0);
}

int rsq(vector<int> &bit, int i){
    int ans = 0;
    for(; i; i-=i&-i)
        ans += bit[i];
    return ans;
}

void update(vector<int> &bit, int i, int v){
    for(; i < bit.size(); i+=i&-i)
        bit[i] += v;
}

void update(int i, int j, int v){
    update(bit1, i, v);
    update(bit1, j+1, -v);
    update(bit2, i, v*(i-1));
    update(bit2, j+1, -v*j);
}

int rsq(int i){
    return rsq(bit1, i)*i - rsq(bit2, i);
}

int rsq(int i, int j){
    return rsq(j) - rsq(i-1);
}
```

7.3 BIT 2D

```
#define pii pair<ll,ll>
#define upper(v, x) (upper_bound(begin(v), end(v), x) - begin(v))

struct BIT2D{
    vector<ll> ord;
    vector<vector<ll>> bit,coord;
    BIT2D(vector<pii> pts){
        sort(begin(pts),end(pts));

        for(auto [x,y] : pts)
            if(ord.empty() || x != ord.back())
                ord.push_back(x);

        bit.resize(ord.size() + 1);
        coord.resize(ord.size() + 1);

        sort(begin(pts),end(pts), [&](pii &a , pii& b){
            return a.second < b.second;
        });

        for(auto [x,y] : pts)
            for(int i = upper(ord,x); i < bit.size(); i += i & -i)
                if(coord[i].empty() || coord[i].back() != y)
                    coord[i].push_back(y);

        for(int i = 0; i < bit.size(); i++) bit[i].assign(coord[i].size() + 1,0);
    }

    void update(ll X, ll Y, ll v){
        for(int i = upper(ord, X); i < bit.size(); i += i & -i)
            for(int j = upper(coord[i], Y); j < bit[i].size(); j += j & -j)
                bit[i][j] += v;
    }

    ll query(ll X, ll Y){
        ll sum = 0;
        for(int i = upper(ord,X); i > 0; i -= i & -i)
            for(int j = upper(coord[i], Y); j > 0; j -= j & -j)
                sum += bit[i][j];
        return sum;
    }

    ll queryArea(ll xi , ll yi, ll xf, ll yf){
        return query(xf,yf) - query(xf, yi-1) - query(xi-1, yf) + query(xi-1, yi-1);
    }
};
```

7.4 Line Container

```
 * Author: Simon Lindholm
 * Date: 2017-04-20
 * License: CC0
 * Source: own work
 * Description: Container where you can add lines of the form kx+m, and query
↪  maximum values at points x.
 *  Useful for dynamic programming (``convex hull trick'').
 * Time: O(\log N)
 * Status: stress-tested
 */

struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {

    static const ll inf = LLONG_MAX; //for doubles 1/.0

    ll div(ll a, ll b) { //for doubles return a/b
        return a / b - ((a ^ b) < 0 && a % b); }

    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }

        //para achar o mínimo, é preciso fazer insert({-k, -m, 0}), além disso
↪   multiplicar por -1 o resultado da query
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
```

```
            isect(x, erase(y));
    }

    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

7.5 Merge Sort Tree

```
//Segtree node for Merge-Sort
struct Node{
    vector<int> vec;
    Node operator+(Node other) const{
        vector<int> novo(vec.size() + other.vec.size());
        merge(vec.begin(), vec.end(), other.vec.begin(), other.vec.end(),
 ↪  novo.begin());
        return {novo};
    }
    Node operator=(int x){
    return {this->vec = {x}};
    }
};
```

7.6 Mo

```
const int blockSize = 500;

struct Query
{
    int l, r, idx;

    bool operator<(Query other) const{
        return make_pair(l/blockSize, r) < make_pair(other.l/blockSize, other.r);
    };
};

struct Mo{
        //TODO: declare the data structures

    Mo(){

    }

    void add(int idx){
            //TODO: add an element to the data structure
    }

    void remove(int idx){
            //TODO: remove an element from the data structure
    }

    int get_answer(){
            //TODO: get answer from the data structure
    }

    vector<int> solve(vector<Query> queries) {
        vector <int> answers(queries.size());
        sort(queries.begin(), queries.end());

            //TODO: initialize data structue

        int cur_l = 0;
        int cur_r = -1;
        for (Query q : queries) {
            while (cur_l > q.l) {
                cur_l--;
                add(cur_l);
            }
            while (cur_r < q.r) {
                cur_r++;
                add(cur_r);
            }
            while (cur_l < q.l) {
                remove(cur_l);
                cur_l++;
            }
            while (cur_r > q.r) {
                remove(cur_r);
                cur_r--;
            }
            answers[q.idx] = get_answer();
        }
        return answers;
    }
};
```

7.7 Prefix Sum 2D

```
struct pref2D{
    int n, m;
    vector <vector<int>> mat, pref;

    pref2D(int n, int m, vector<vector<int>> tmp){
        this-> n = n; this->m = m;
        mat = tmp;

        pref.resize(n+1);
        for(auto& v : pref) v.resize(m+1, 0);

        for(int i = 1; i <= n; i++){
            for(int j = 1; j <= m; j++){
                pref[i][j] = pref[i-1][j] + pref[i][j-1] - pref[i-1][j-1] +
↪   mat[i-1][j-1];
            }
        }
    }

    int query(int rowl, int rowr, int coll, int colr){
            //rowl++, rowr++, coll++, colr++;
        if(rowl > rowr) swap(rowl, rowr);
        if(coll > colr) swap(colr, coll);
        return pref[rowr][colr] - pref[rowl-1][colr] - pref[rowr][coll-1] +
↪   pref[rowl-1][coll-1];
    }
};
```

7.8 SegTree

```
struct SegTree{
  int n;
  struct Node{
    int val;
    Node operator+(Node other) const{
      return {this->val + other.val};
    }
    Node operator=(int x){
      return {this->val = x};
    }
  };
  Node neutral = {0};
  vector <Node> t;

  SegTree(vector <int> a){
    n = a.size();
    t.resize(4*n);
    build(a, 1, 0, n-1);
  }

  void build(vector <int>& a, int v, int tl, int tr) {
    if (tl == tr) {
      t[v] = a[tl];
    } else {
      int tm = (tl + tr) / 2;
      build(a, v*2, tl, tm);
      build(a, v*2+1, tm+1, tr);
      t[v] = t[v*2] + t[v*2+1];
    }
  }

  Node query(int l, int r){
    return query(1, 0, n-1, l, r);
  }

  Node query(int v, int tl, int tr, int l, int r){
    if (l > r)
      return neutral;
    if (l == tl && r == tr) {
      return t[v];
    }
    int tm = (tl + tr) / 2;
    return query(v*2, tl, tm, l, min(r, tm))
      + query(v*2+1, tm+1, tr, max(l, tm+1), r);
  }

  void update(int pos, int val){
    update(1, 0, n-1, pos, val);
  }

  void update(int v, int tl, int tr, int pos, int new_val){
    if (tl == tr) {
      t[v] = new_val;
    } else {
      int tm = (tl + tr) / 2;
      if (pos <= tm)
        update(v*2, tl, tm, pos, new_val);
      else
```

```
      update(v*2+1, tm+1, tr, pos, new_val);
    t[v] = t[v*2] + t[v*2+1];
  }
 }
};
```

7.9 SegTree c/ Lazy

```
struct SegTree{
 int n;
 struct Node{
   int val;
   Node operator+(Node other) const{
     return {this->val + other.val};
   }
   Node operator=(int x){
     return {this->val = x};
   }
 };
 Node neutral = {0};
 vector <Node> t;
 vector<int> lazy;

 SegTree(vector <int> a){
   n = a.size();
   t.resize(4*n);
   lazy.resize(4*n);
   build(a, 1, 0, n-1);
 }

 void build(vector <int>& a, int v, int tl, int tr) {
   if (tl == tr) {
     t[v] = a[tl];
   } else {
     int tm = (tl + tr) / 2;
     build(a, v*2, tl, tm);
     build(a, v*2+1, tm+1, tr);
     t[v] = t[v*2] + t[v*2+1];
   }
 }

 void unlazy(int v, int tl, int tr){
   if(lazy[v] == 0) return;
       //Update current range
   t[v].val += (tr-tl+1) * lazy[v];

       //Pass lazy to child if any
   if(tl != tr){
       lazy[2*v] += lazy[v];
       lazy[2*v+1] += lazy[v];
   }

       //Reset lazy
   lazy[v] = 0;
 }

 Node query(int l, int r){
   return query(1, 0, n-1, l, r);
 }

 Node query(int v, int tl, int tr, int l, int r){
   unlazy(v, tl, tr);
   if (l > r)
     return neutral;
   if (l == tl && r == tr) {
     return t[v];
   }
   int tm = (tl + tr) / 2;
   return query(v*2, tl, tm, l, min(r, tm))
     + query(v*2+1, tm+1, tr, max(l, tm+1), r);
 }

 void RangeUpdate(int l, int r, int new_val){
   RangeUpdate(1,0,n-1,l,r,new_val);
 }

 void RangeUpdate(int v, int tl, int tr, int l, int r, int new_val){
   unlazy(v, tl, tr);
   if (l > r)
     return;
   if (l == tl && r == tr) {
       lazy[v] += new_val; //Change here
       unlazy(v, tl, tr);
       return;
   }
   int tm = (tl + tr) / 2;
   RangeUpdate(v*2, tl, tm, l, min(r, tm), new_val);
   RangeUpdate(v*2+1, tm+1, tr, max(l, tm+1), r, new_val);
   t[v] = t[2*v] + t[2*v+1];
 }
```

```
  void PointUpdate(int pos, int val){
    PointUpdate(1, 0, n-1, pos, val);
  }

  void PointUpdate(int v, int tl, int tr, int pos, int new_val){
    unlazy(v, tl, tr);
    if (tl == tr) {
      t[v] = new_val;
    } else {
      int tm = (tl + tr) / 2;
      if (pos <= tm)
        PointUpdate(v*2, tl, tm, pos, new_val);
      else
        PointUpdate(v*2+1, tm+1, tr, pos, new_val);
      t[v] = t[v*2] + t[v*2+1];
    }
  }
};
```

7.10 SegTree Sparse

```
struct Node {
    int left, right;
    int sum = 0;
    Node *left_child = nullptr, *right_child = nullptr;

    Node(int lb, int rb) {
        left = lb;
        right = rb;
    }

    void extend() {
        if (!left_child && left + 1 < right) {
            int t = (left + right) / 2;
            left_child = new Node(left, t);
            right_child = new Node(t, right);
        }
    }

    void add(int k, int x) {
        extend();
        sum += x;
        if (left_child) {
            if (k < left_child->right)
                left_child->add(k, x);
            else
                right_child->add(k, x);
        }
    }

    int get_sum(int lq, int rq) {
        if (lq <= left && right <= rq)
            return sum;
        if (max(left, lq) >= min(right, rq))
            return 0;
        extend();
        return left_child->get_sum(lq, rq) + right_child->get_sum(lq, rq);
    }
};
```

7.11 Sparse Table

```
struct SparseTable{
    int K = 25, n;
    vector <vector<int>> st; //st[i][j] = min on range [j, j + 2^i-1]
    vector <int> lg2; //lg2[i] = floor(log2(i))

    SparseTable(vector <int> arr){
        n = arr.size();
        st.resize(K+1);
        for(auto& v : st) v.resize(n);

        st[0] = arr;
        for(int i = 1; i <= K; i++){
            for(int j = 0; j + (1 << i) - 1 < n; j++){
                st[i][j] = min(st[i-1][j], st[i-1][j + (1 << (i - 1))]);
            }
        }

        lg2.resize(n+1);
        lg2[1] = 0;
        for(int i = 2; i <= n; i++){
            lg2[i] = lg2[i/2] + 1;
        }
    }

    int query(int l, int r){
```

```
        int i = lg2[r-l+1];
        return min(st[i][l], st[i][r-(1<<i)+1]);
    }

        //Query [l,r]
    int querylog(int l , int r){

        int neutral = 1e9; //elemento neutro
        int dif = r-l+1;

        for(int i = 0; i <= K; i++){
            if((1<<i) & dif){
                neutral = min(neutral,st[i][l]);
                l = l + (1<<i);
            }
        }

        return neutral;
    }

};
```

7.12 Union Find

```
//Complexidade: O($\alpha$(n)), onde $\alpha$ é a função de Ackermann inversa
struct DSU
{
    int n;
    vector<int> pai, rank;

    DSU(int n) : n(n), pai(n+1), rank(n+1,1){
        for(int i = 1; i <=n; i++){
            pai[i] = i;
        }
    }

    int find(int a){
        if(pai[a] == a) return a;
        return pai[a] = find(pai[a]);
    }

    void uu(int a, int b){
        a = find(a), b = find(b);
        if(a == b) return;
        if(rank[a] > rank[b]) swap(a,b);
        rank[b] += rank[a];
        pai[a] = b;
    }

};
```

8 Extra

8.1 BitOperators.h

```
Pra fazer funcionar com long long é só colocar ll no final da função, por exemplo
↪  "_builtin_clz(x)" vira "_builtin_clzll(x)"
__builtin_clz(x): Count leading zeros. Retorna o número de zeros no início da
↪  representação binária de x (incio == bits mais significativos)
__builtin_ctz(x): Count trailing zeros. Retorna o número de zeros no final da
↪  representação binária de x
__builtin_popcount(x): Population count. Retorna o número de bits definidos como 1
↪  na representação binária de x
__builtin_parity(x): Parity. Retorna 1 se o número de bits definidos como 1 for
↪  ímpar e 0 se for par.
int L = 32 - __builtin_clz(n), n = 1 << L; : "n" se torna a primeira potencia de 2
↪  estritamente maior que n
```

8.2 XorBasis.h

```
//Xor Basis

struct Basis{
  vector <int> basis;
  Basis(){

  }
  Basis(int x){
    add(x);
  }
  Basis operator+(Basis other) const{
    Basis res;
    for(int x : basis){
```

```
      res.add(x);
    }
    for(int x : other.basis){
      res.add(x);
    }
    return res;
  }
  void add(int x){
    for(auto& i : basis){
      x = min(x, x^i);
    }
    if(x){
      basis.push_back(x);
    }
  }
};
```

8.3 TernarySearch.h

```
//Ternary Search

double ternary(double l, double r){
    // < for maximum and > for minimum value
    int cont = 300;
    while (cont --)
    {
        double m1 = l + (r-l)/3;
        double m2 = r - (r-l)/3;
        double f1 = f(m1);
        double f2 = f(m2);
        if(f1>f2){
            l = m1;
        }else{
            r = m2;
        }
    }
    return l;
}

/**
 * Author: Simon Lindholm
 * Date: 2015-05-12
 * License: CC0
 * Source: own work
 * Description:
 * Find the smallest i in $[a,b]$ that maximizes $f(i)$, assuming that $f(a) < \dots
↪  < f(i) \ge \dots \ge f(b)$.
 * To reverse which of the sides allows non-strict inequalities, change the < marked
↪  with (A) to <=, and reverse
 * the loop at (B).
 * To minimize $f$, change it to >, also at (B).
 * If you are dealing with real numbers, you'll need to pick $m_1 = (2a + b)/3.0$
↪  and $m_2 = (a + 2b)/3.0$.
 * Consider setting a constant number of iterations for the search, usually
↪  $[200,300]$ iterations are sufficient
 * for problems with error limit as $10^{-6}$.
 * Status: tested
 * Usage: int ind = ternSearch(0,n-1,[\&](int i){return a[i];});
 * Time: O(\log(b-a))
 */

int ternSearch(int a, int b) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    for(int i = a+1; i <= b; ++i)
        if (f(a) < f(i)) a = i; // (B)
    return a;
}
```

8.4 Makefile.h

```
WARNINGS := -Wshift-overflow=2 -Wfloat-equal -Wconversion
SANITIZERS := -fsanitize=address -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
↪  -fsanitize=undefined
a.out: a.cpp
    g++ -Dlocal $(WARNINGS) $(SANITIZERS) a.cpp
```

8.5 Brute.h

```
//Brute
set -e
```

```
g++ code.cpp -o code
g++ brute.cpp -o brute
g++ gen.cpp -o gen
for((i = 1; ; ++i)) do
    echo "Test: " $i
    ./gen $i > input_file
    cat input_file
    ./code < input_file > myAnswer
    ./brute < input_file > correctAnswer
    diff -Z myAnswer correctAnswer > /dev/null || break
    cat input_file
    cat myAnswer
    cat correctAnswer
    echo "Passed test" $i
done
echo "WA:"
cat input_file
echo "My:"
cat myAnswer
echo "correct:"
cat correctAnswer
```

8.6 2pointer.h

```
x = 0, L = 0
for R = 0..n-1
    x += a[R]
    while x > s:
        x -= a[L]
        L++
    res = max(res, R - L + 1)

x = 0, L = 0
for R = 0..n-1
    x += a[R]
    while x - a[L] >= s:
        x -= a[L]
        L++
    if x >= s:
       res = min(res, R - L + 1)

Suppose you received a problem at the contest in which you need to do something similar: find the longest (shortest) good segment or count the number of good segments.
How do you know if you can apply the two-pointer method to it?
First, one of the following two properties must be met:

    if the segment [L,R] is good, then any segment nested in it is also good (in this case, you can apply the code from the first problem);
    if the segment [L,R] is good, then any segment that contains it is also good (in this case, you can apply the code from the second problem).

Secondly, you should be able to recalculate your function (check if current segment is good or bad), while moving the left or right border by one to the right.
In such tasks, the code almost always looks like this:

L = 0
for R = 0..n-1
    add(a[R])
    while not good():
        remove(a[L])
        L++
```