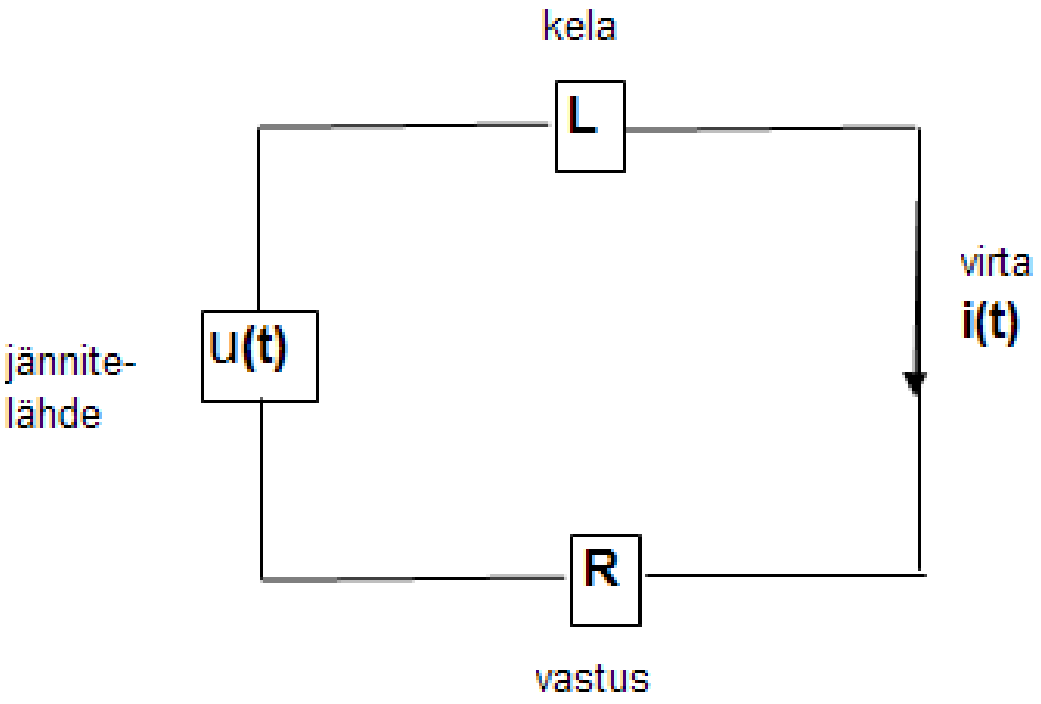
1. (RL-circuit) Given R, L and U, solve the current $i(t)$ from the equation

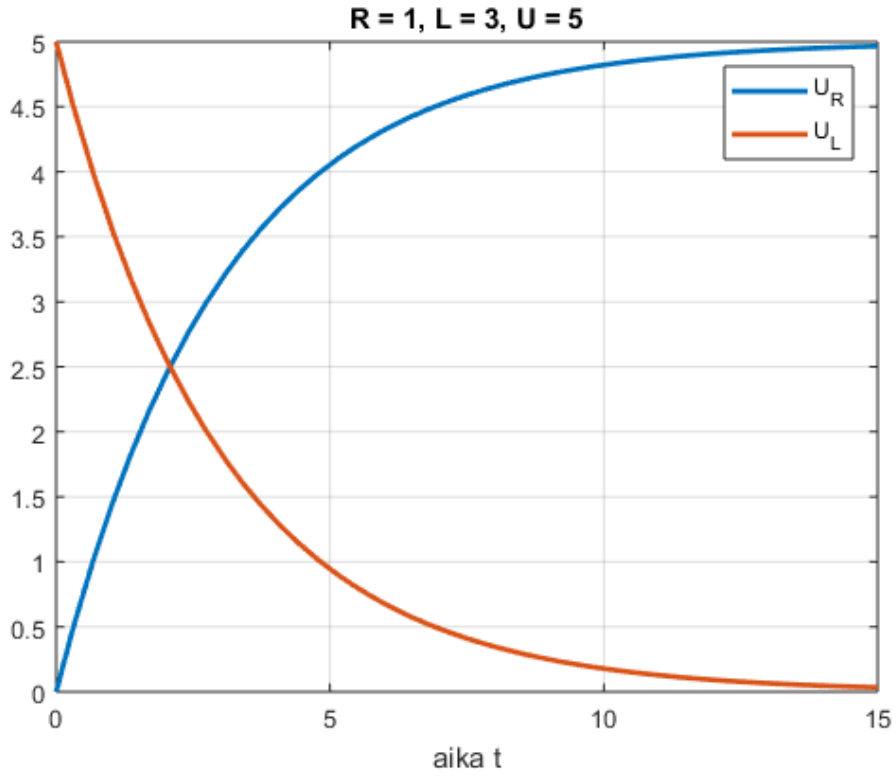
$$L \cdot i'(t) + R \cdot i(t) = u(t)$$

when $u(t) = U$ and $i(0) = 0$, and draw the graphs of the voltages

$$U_R = R \cdot i(t) \text{ and } U_L = L \cdot i'(t)$$

on the interval $t = 0 \ldots 5\tau$, where time constant $\tau = L/R$.

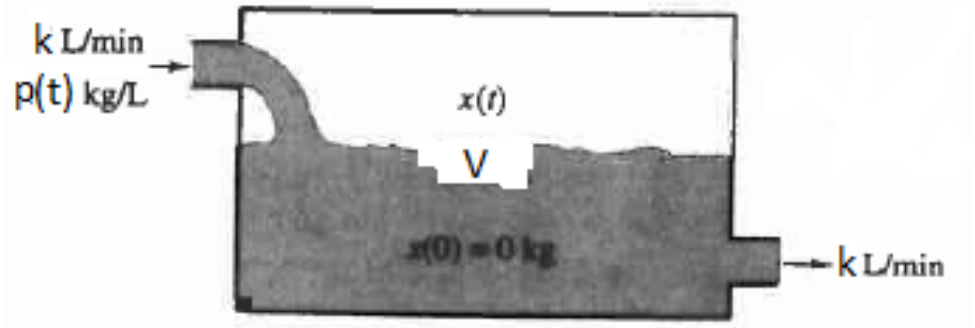


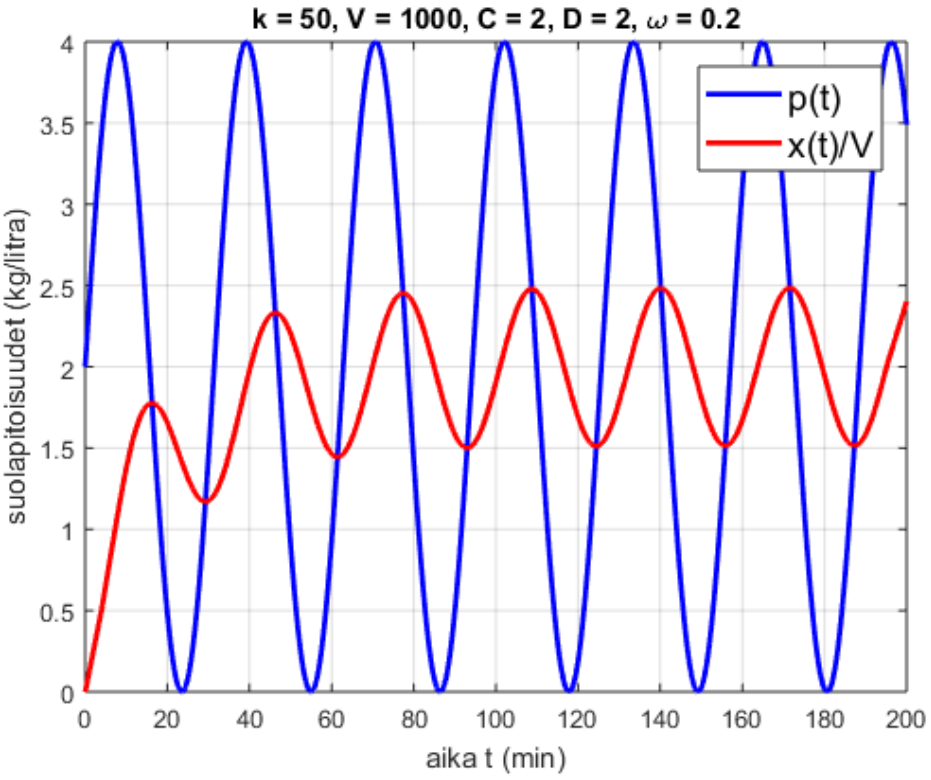

R = 1, L = 3, U = 5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
aika t
U_R
U_L

2. Given the volume V (liters) of the tank, flow rate k (liters/min), amplitude C, constant D and angular frequency ω of the incoming salt content (kg/liter)

$$p(t) = C\sin(\omega t) + D$$

solve the mass $x(t)$ (kg) of the salt in the tank and draw the graphs of salt contents $x(t)/V$ and $p(t)$ on the interval $t = 0 \ldots 10\tau$, where $\tau = V/k$

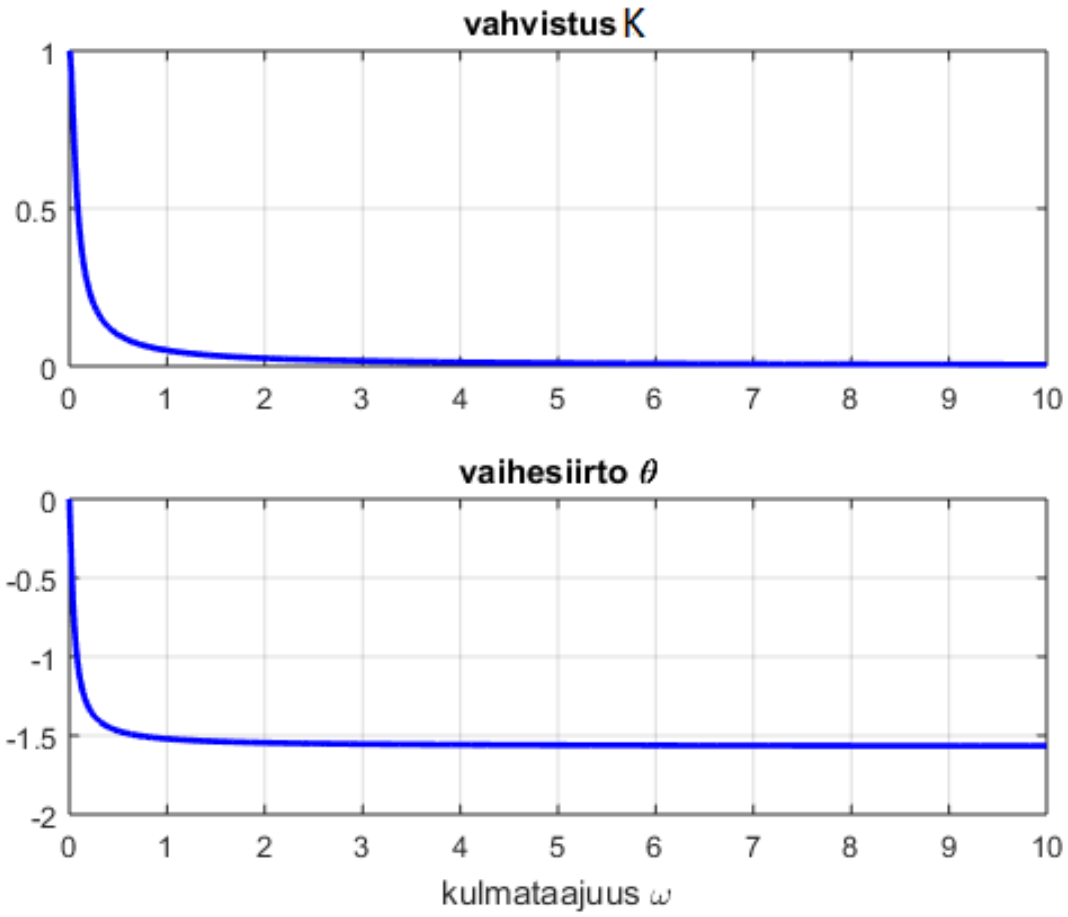




Deduce from the solution formulas that the salt content

$$\frac{x(t)}{V} \approx K \cdot C \sin(\omega t + \theta) + D, \text{ as } t \text{ increases}$$

and draw the graphs of the amplification K and phase shift θ on the interval $\omega = 0 \ldots 100$



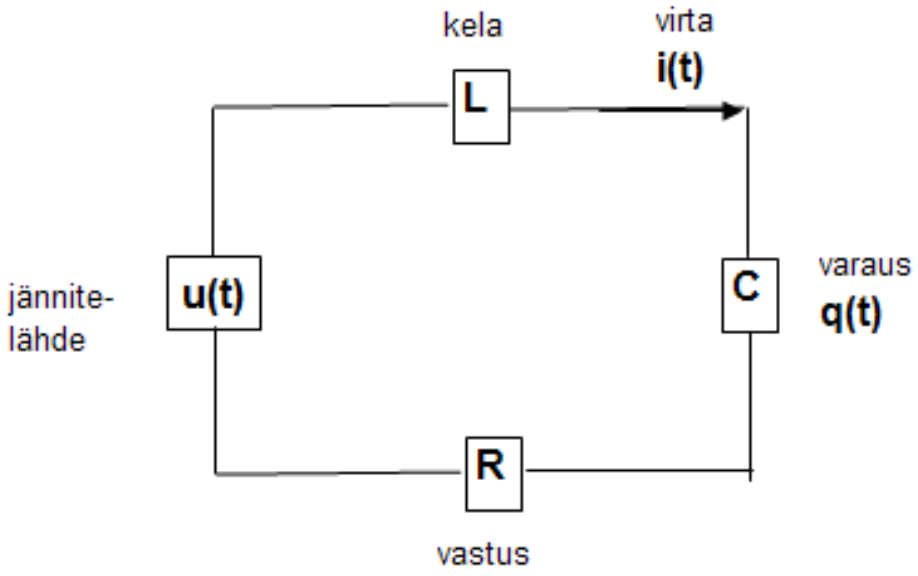
3. (RLC-circuit) Given R, L, C, U and T, solve $q(t)$ from the equation

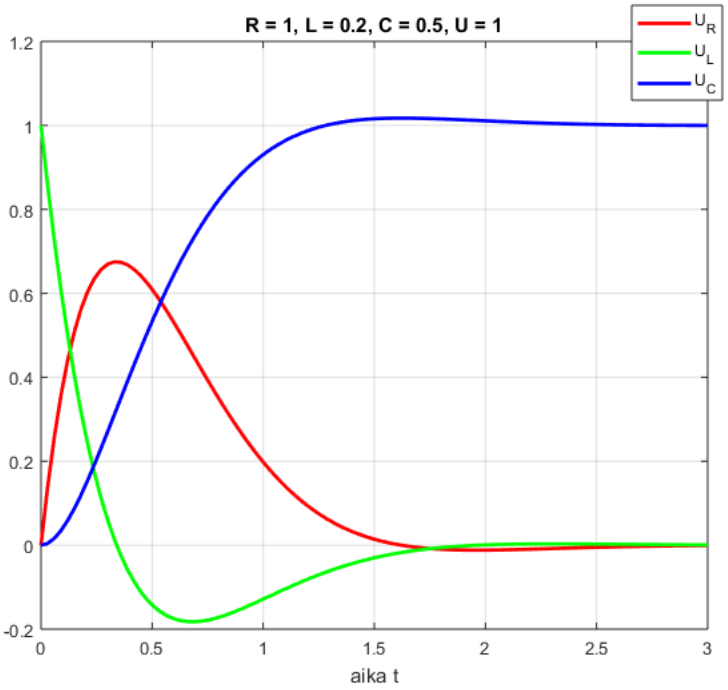
$$L \cdot q''(t) + R \cdot q'(t) + \frac{1}{C} \cdot q(t) = U, \quad q(0) = 0,\ q'(0) = 0$$

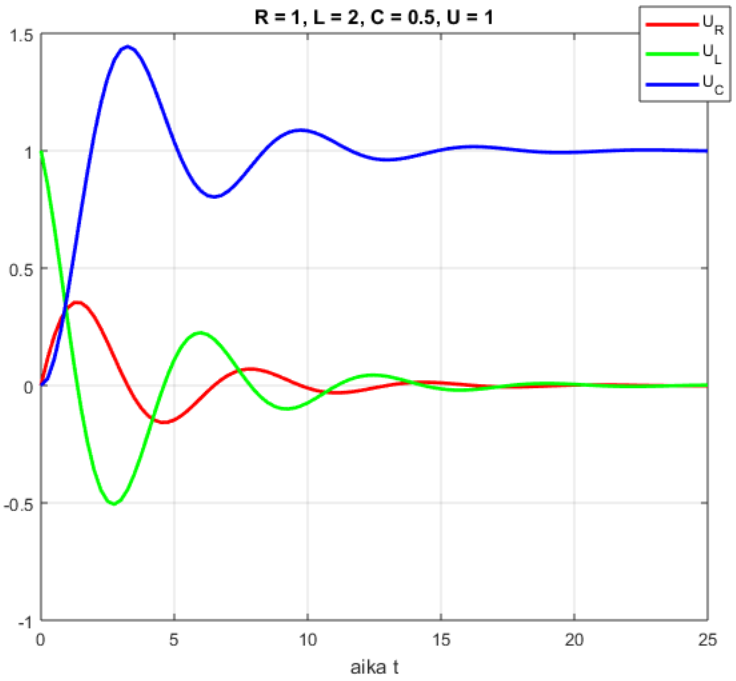
and draw the graphs of the voltages

$$U_R = R\, i(t),\ U_L = L\, i'(t) \text{ and } U_C = q(t)/C$$

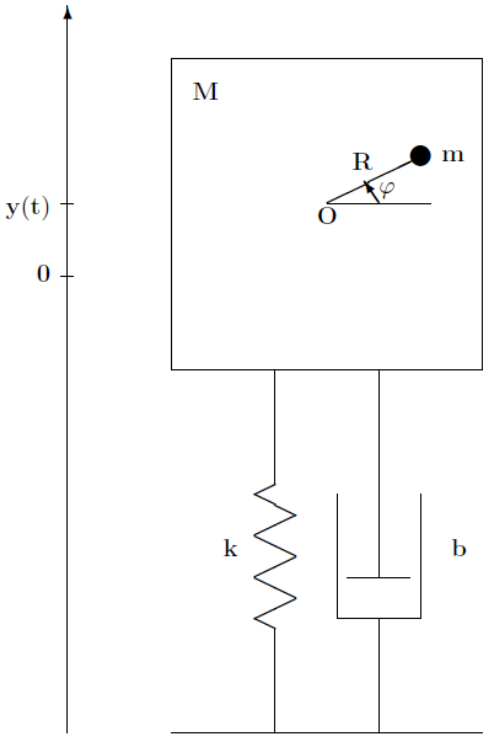
on the interval $t = 0 \ldots T$.




R = 1, L = 0.2, C = 0.5, U = 1
U_R
U_L
U_C
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3
aika t


R = 1, L = 2, C = 0.5, U = 1
U_R
U_L
U_C
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20
25
aika t

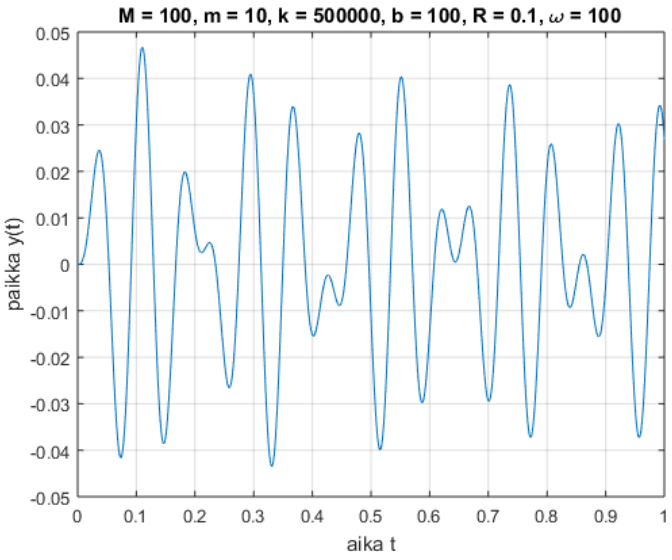
4. Mass m is rotating at angular speed ω (rad/sec), M is the total mass, $y(t)$ is the height of the center point O

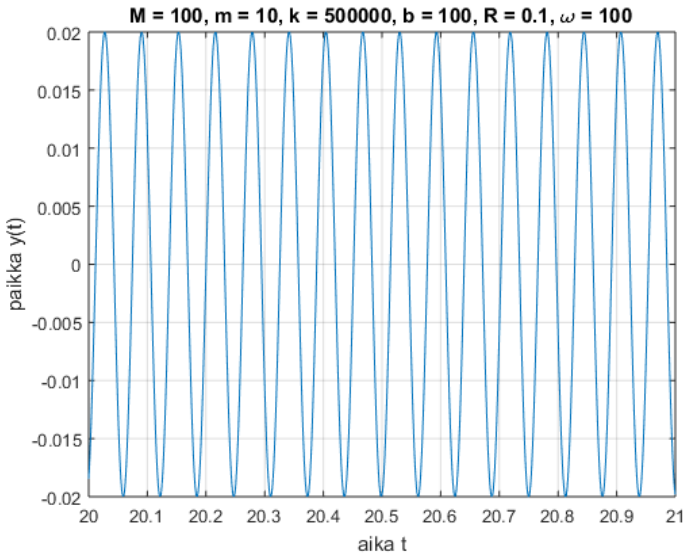


Given M, m, k, b, R and ω, solve $y(t)$ from the equation

$$M\,y''(t) + b\,y'(t) + k\,y(t) = m\,R\omega^2 \sin(\omega\,t), \quad y(0) = 0,\ y'(0) = 0$$

and draw it's graph (on a suitable interval of t).

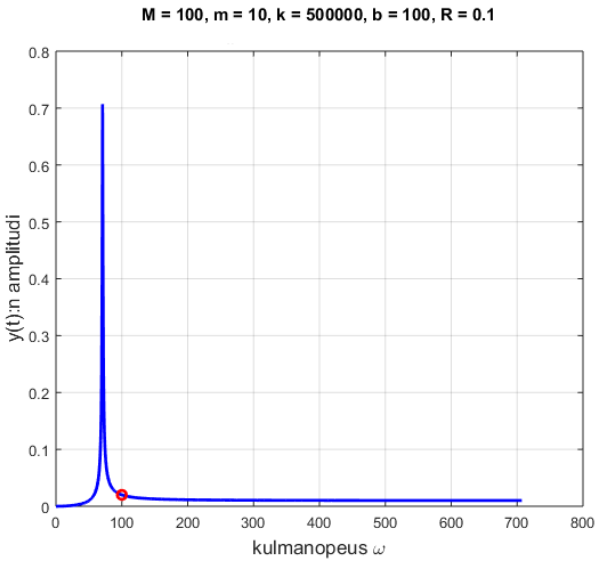




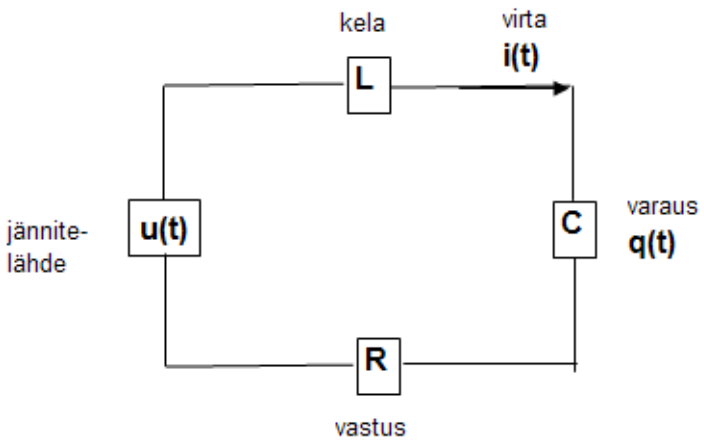
Deduce from the solution formulas, that

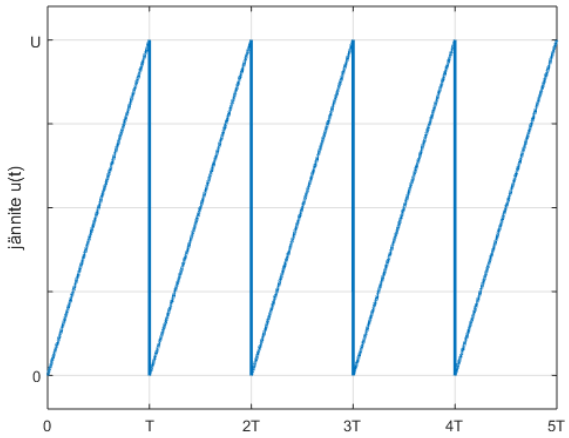
$$y(t) \approx KA\sin(\omega t + \theta), \text{ as } t \text{ increases}$$

and draw the graph of the amplitude KA on the interval $\omega = 0 \ldots 10\sqrt{k/M}$



5. RLC-circuit, input voltage $u(t)$ the saw tooth below





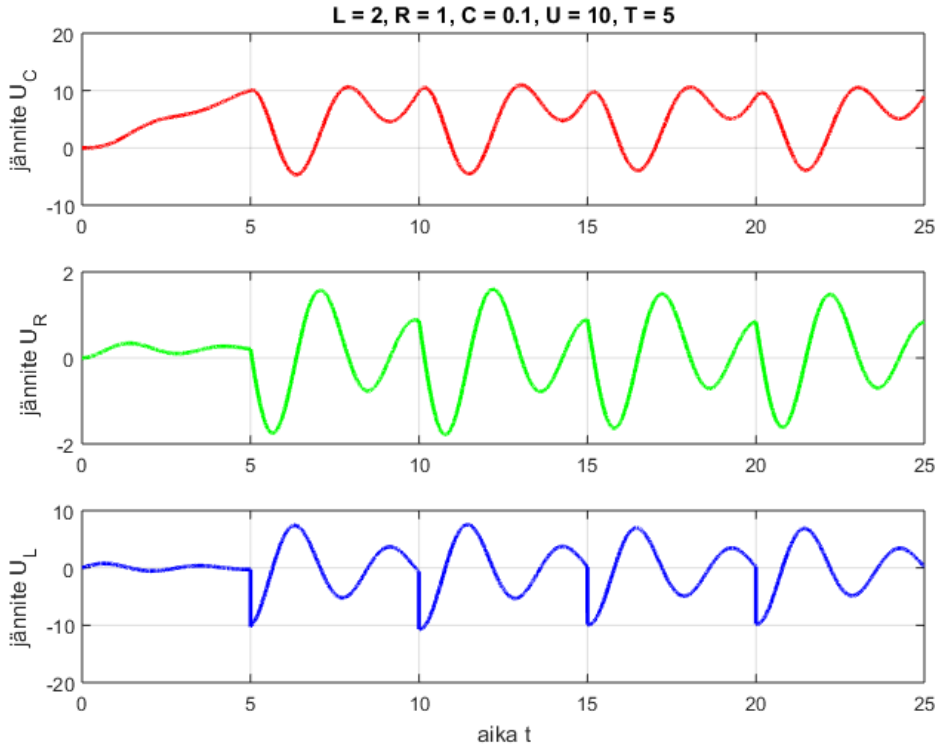
Given R, L, C, U and T, calculate the values of $q(t)$, $i(t) = q'(t)$ and $i'(t) = q''(t)$ numerically using the equation

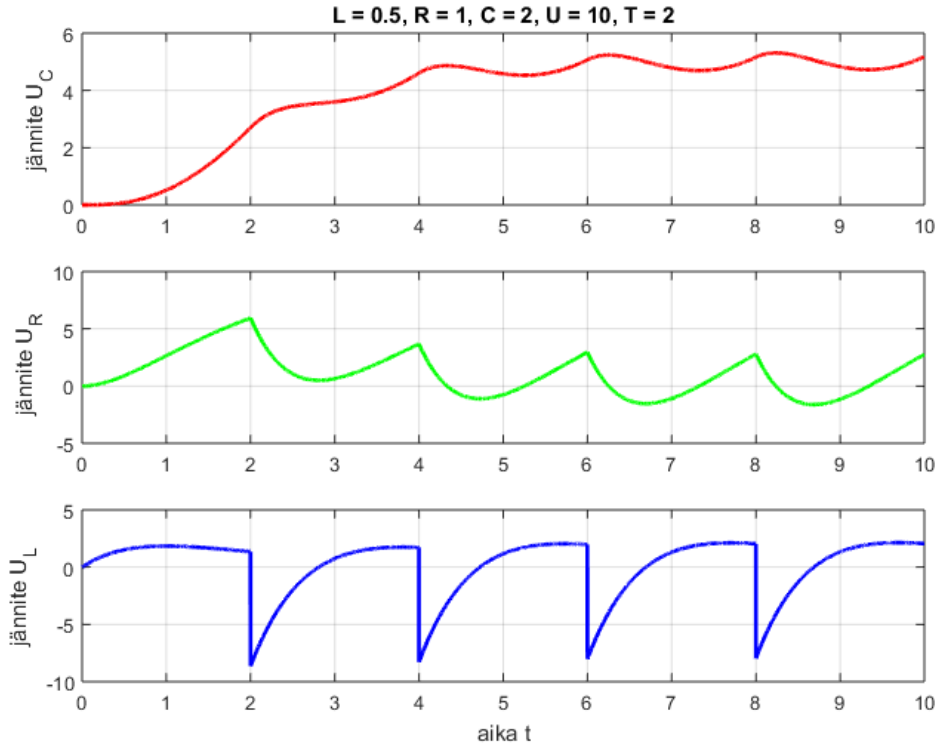
$$L \cdot q''(t) + R \cdot q'(t) + \frac{1}{C} \cdot q(t) = u(t), \quad q(0) = 0,\ q'(0) = 0$$

and draw the graphs of the voltages

$$U_R = R\, i(t),\ U_L = L\, i'(t) \ \text{ja} \ U_C = q(t)/C$$

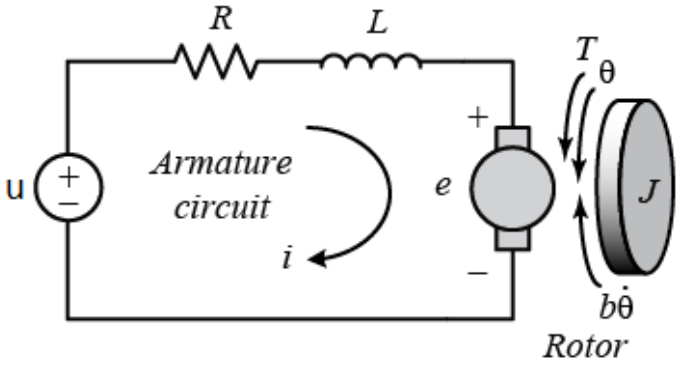
on the interval $t = 0 \ldots 5T$

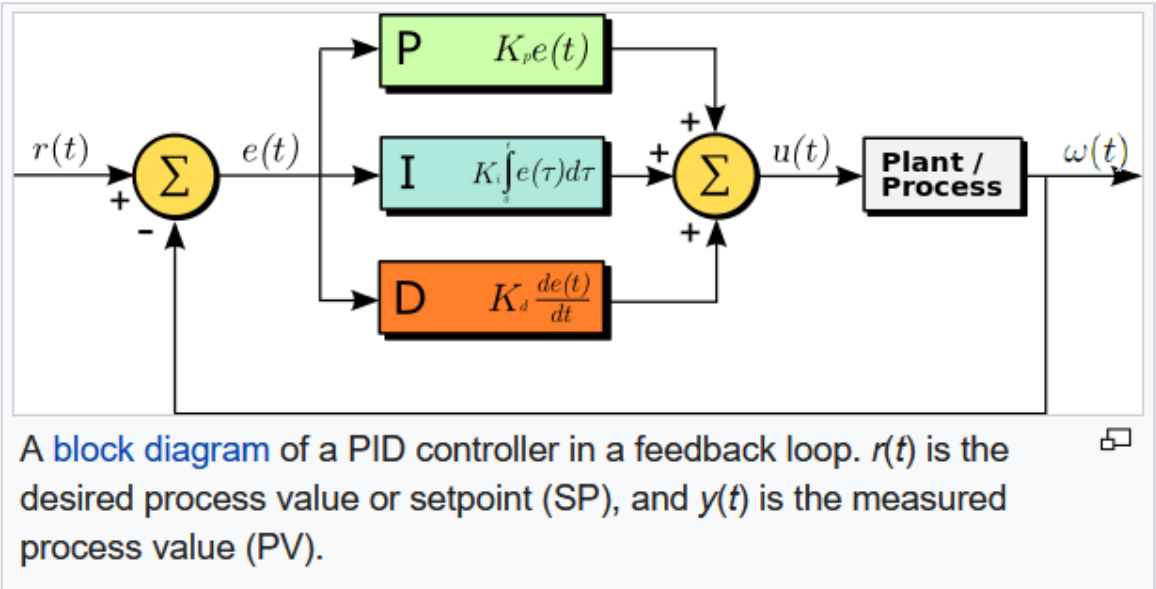

L = 2, R = 1, C = 0.1, U = 10, T = 5
jännite U_C
jännite U_R
jännite U_L
aika t


L = 0.5, R = 1, C = 2, U = 10, T = 2
jännite U_C
jännite U_R
jännite U_L
aika t

hint: ex 8

6. DC-motor





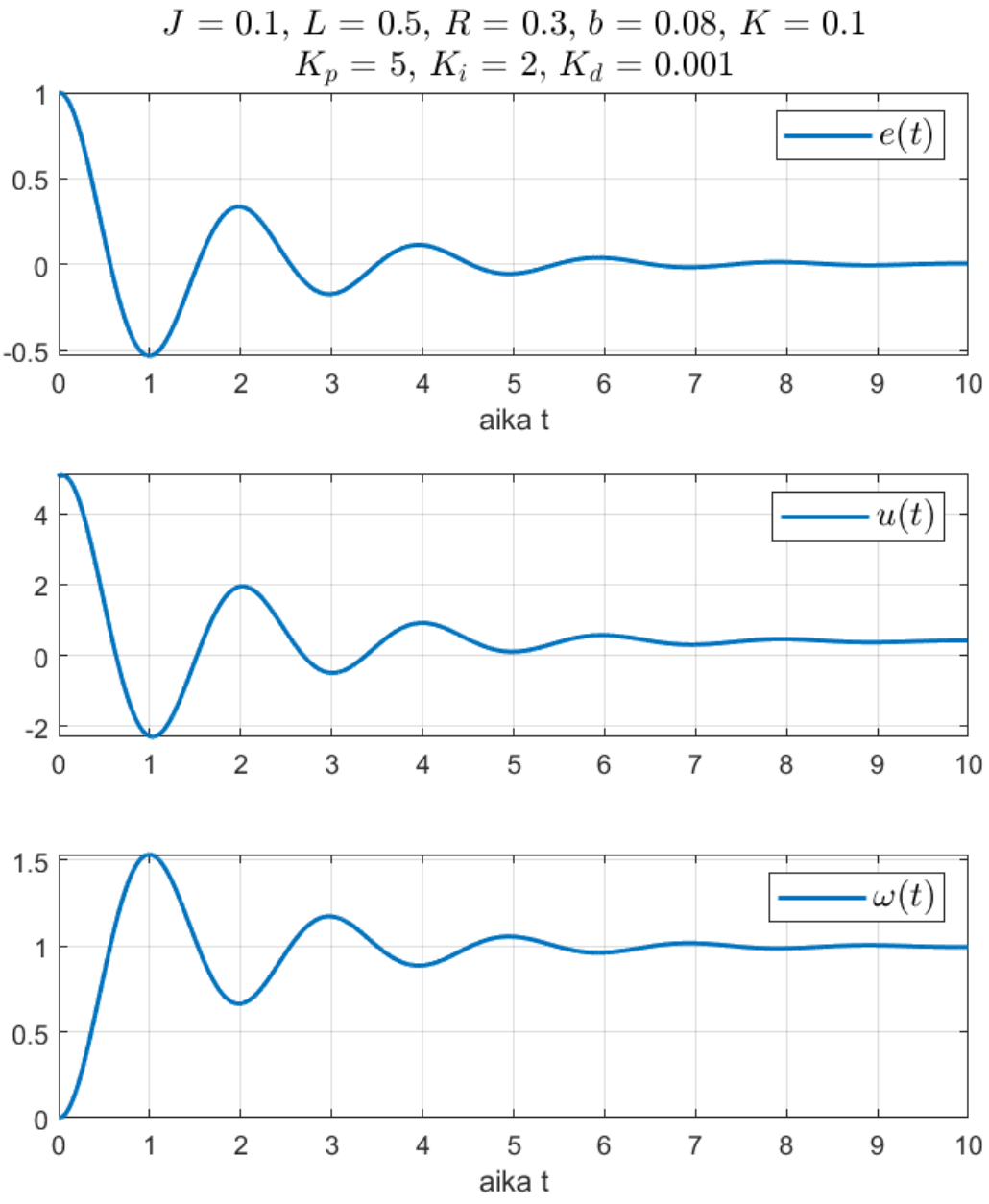
A block diagram of a PID controller in a feedback loop. *r*(*t*) is the desired process value or setpoint (SP), and *y*(*t*) is the measured process value (PV).

Given $J, L, R, b, K, K_p, K_i, K_d$, calculate the values of the PID-controlled angular speed $\omega(t)$ using the equation

$$JL\,\omega''(t) + (RJ + bL)\,\omega'(t) + (bR + K^2)\,\omega(t) = K\,u(t)$$

$$\omega(0) = 0,\ \omega'(0) = 0$$

when setpoint $r(t) = 1$, and draw the graphs of $\omega(t)$, error $e(t) = r(t) - \omega(t)$ and the output $u(t)$ of the PID-controller


$J = 0.1,\ L = 0.5,\ R = 0.3,\ b = 0.08,\ K = 0.1$
$K_p = 5,\ K_i = 2,\ K_d = 0.001$
$e(t)$
$u(t)$
$\omega(t)$
aika t
aika t