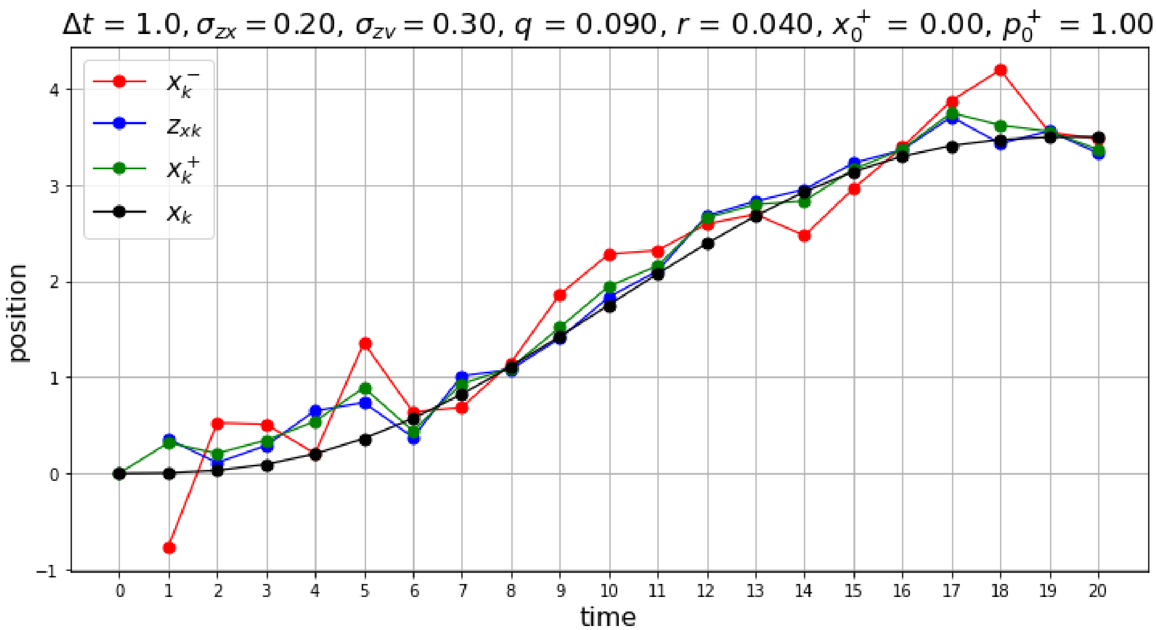
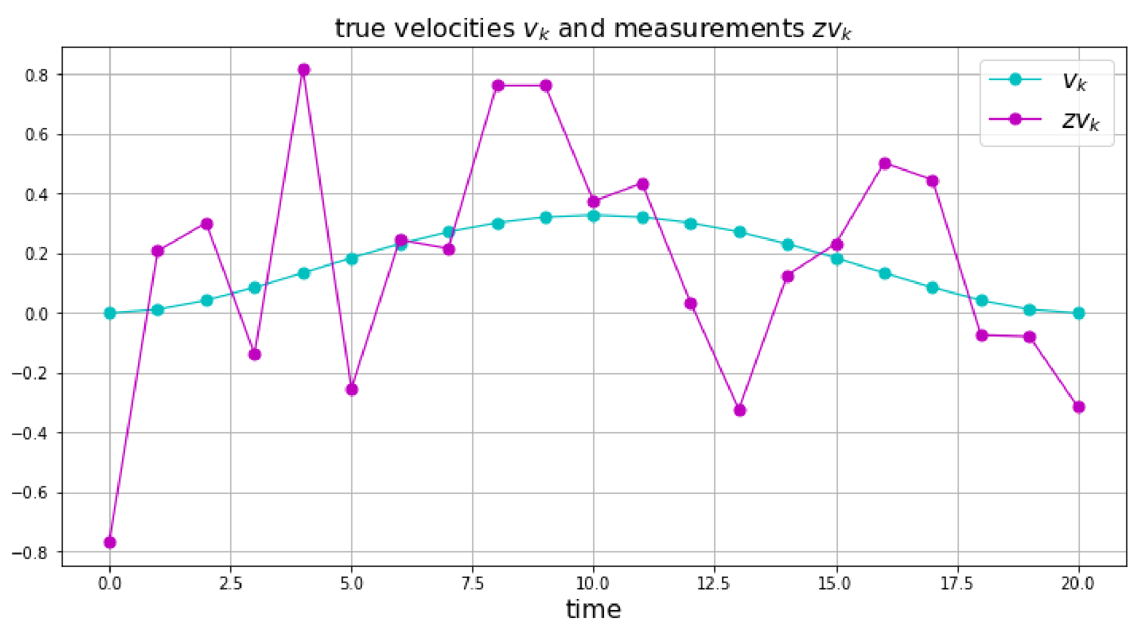
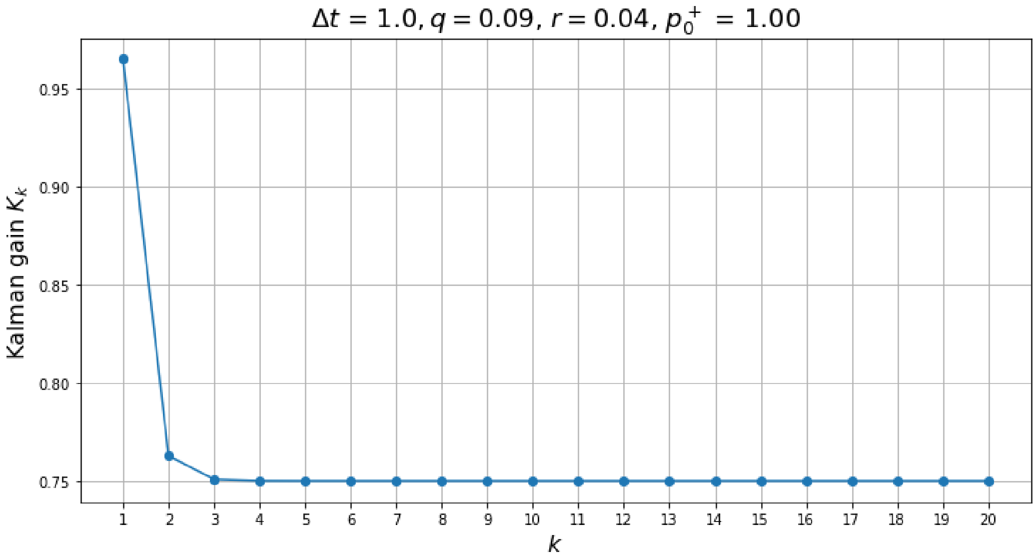
np.random.seed(0)

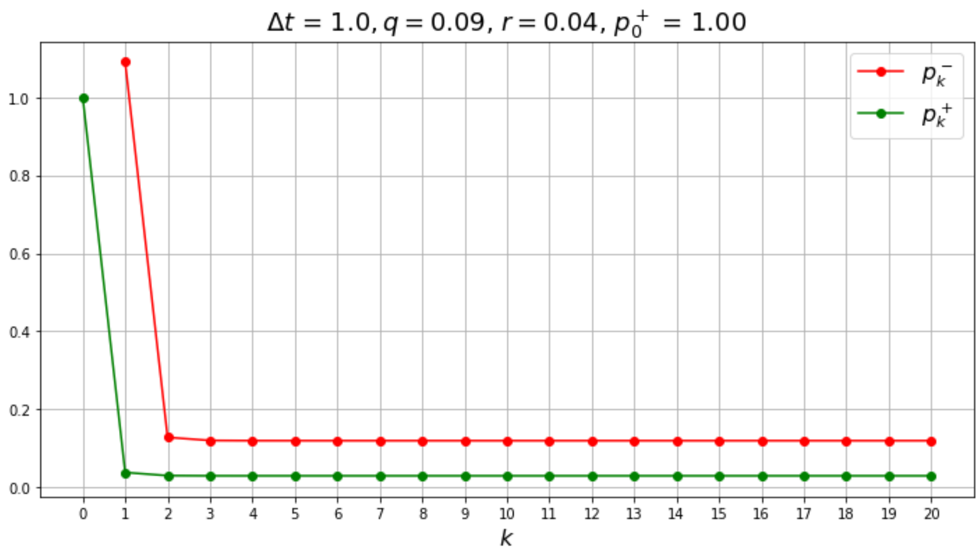
1. Simulate example 2 and draw pictures like below



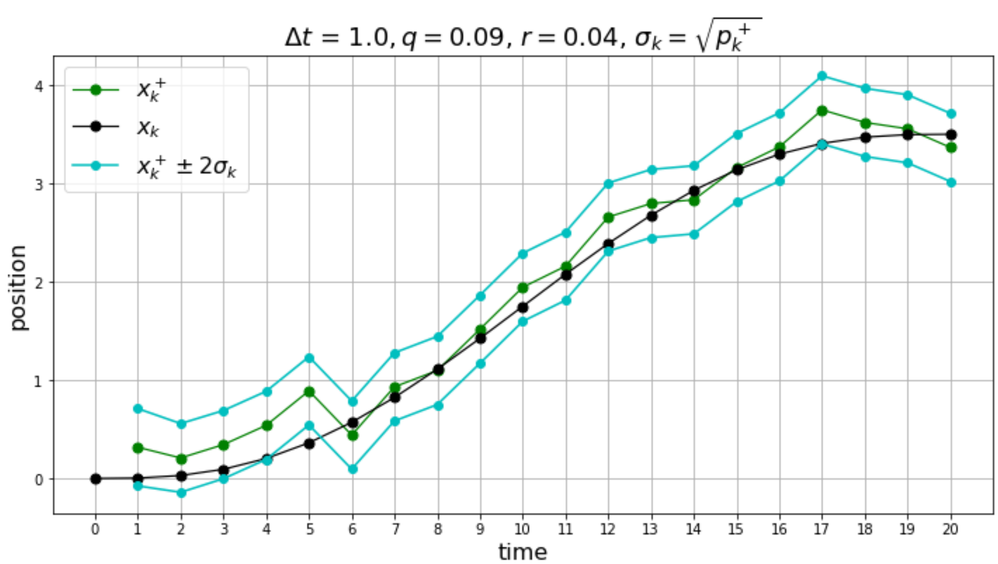




$\Delta t = 1.0, q = 0.09, r = 0.04, p_0^+ = 1.00$
Kalman gain K_k
0.95
0.90
0.85
0.80
0.75
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
k

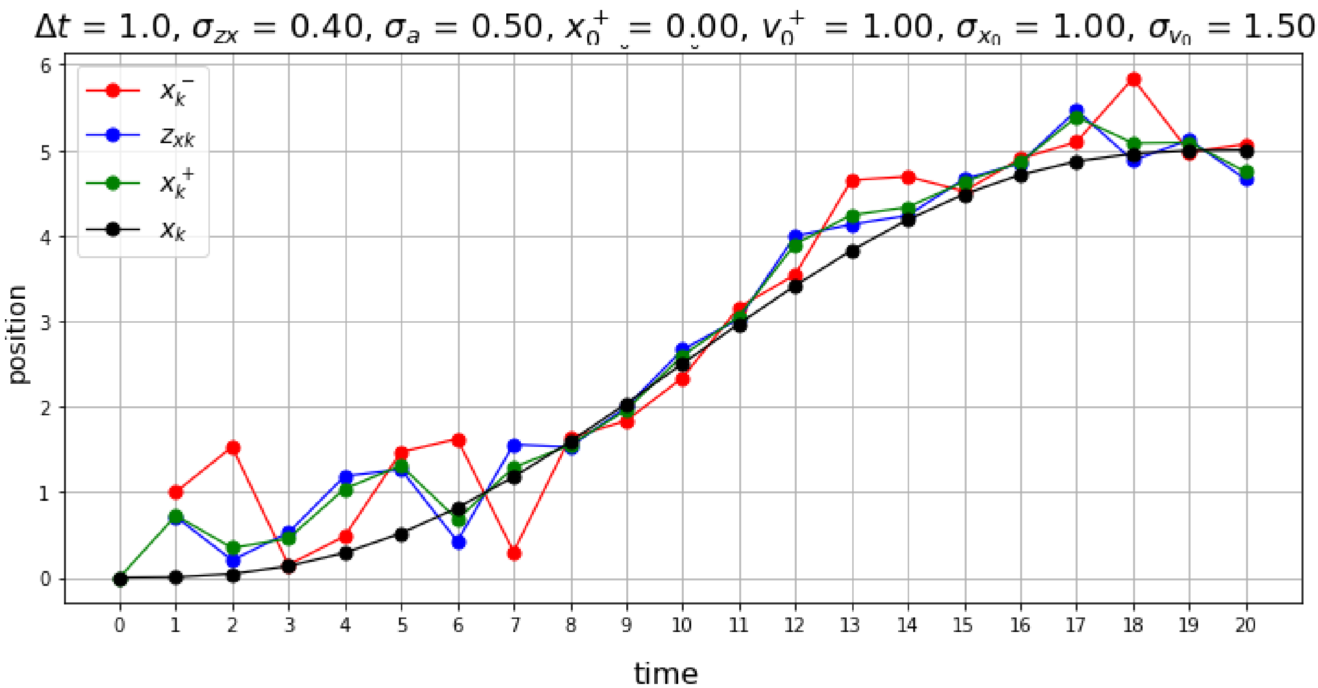


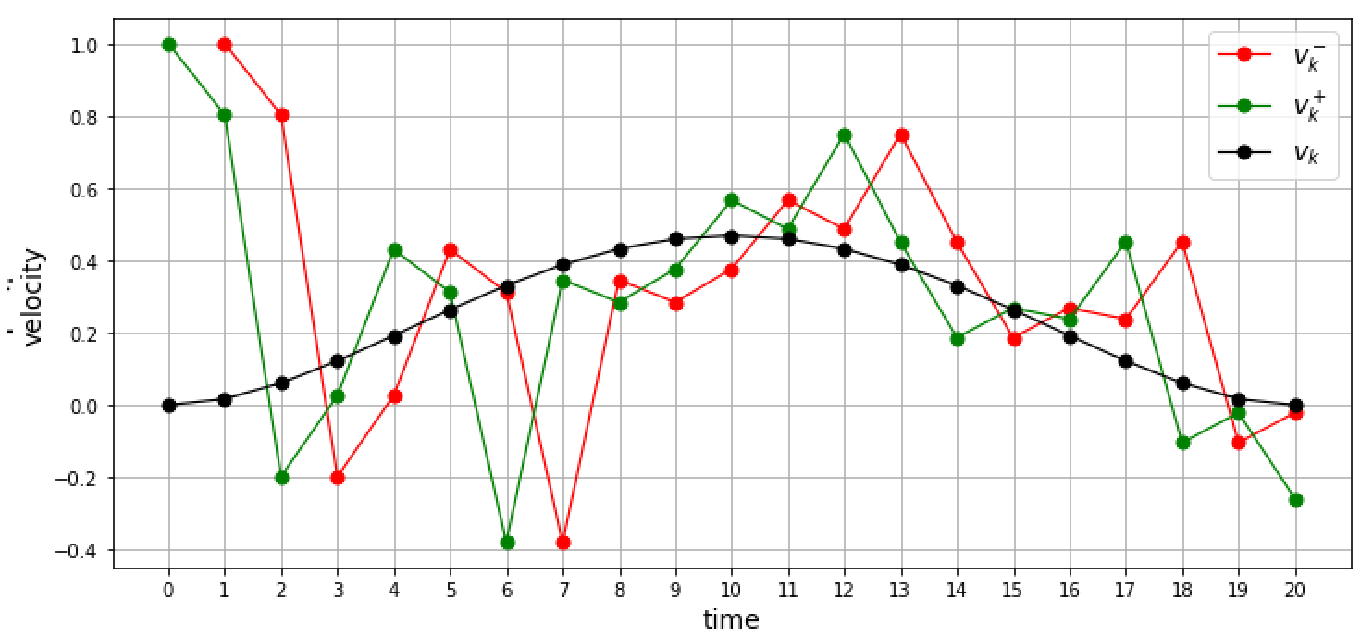
$\Delta t = 1.0, q = 0.09, r = 0.04, p_0^+ = 1.00$
p_k^-
p_k^+
1.0
0.8
0.6
0.4
0.2
0.0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
k

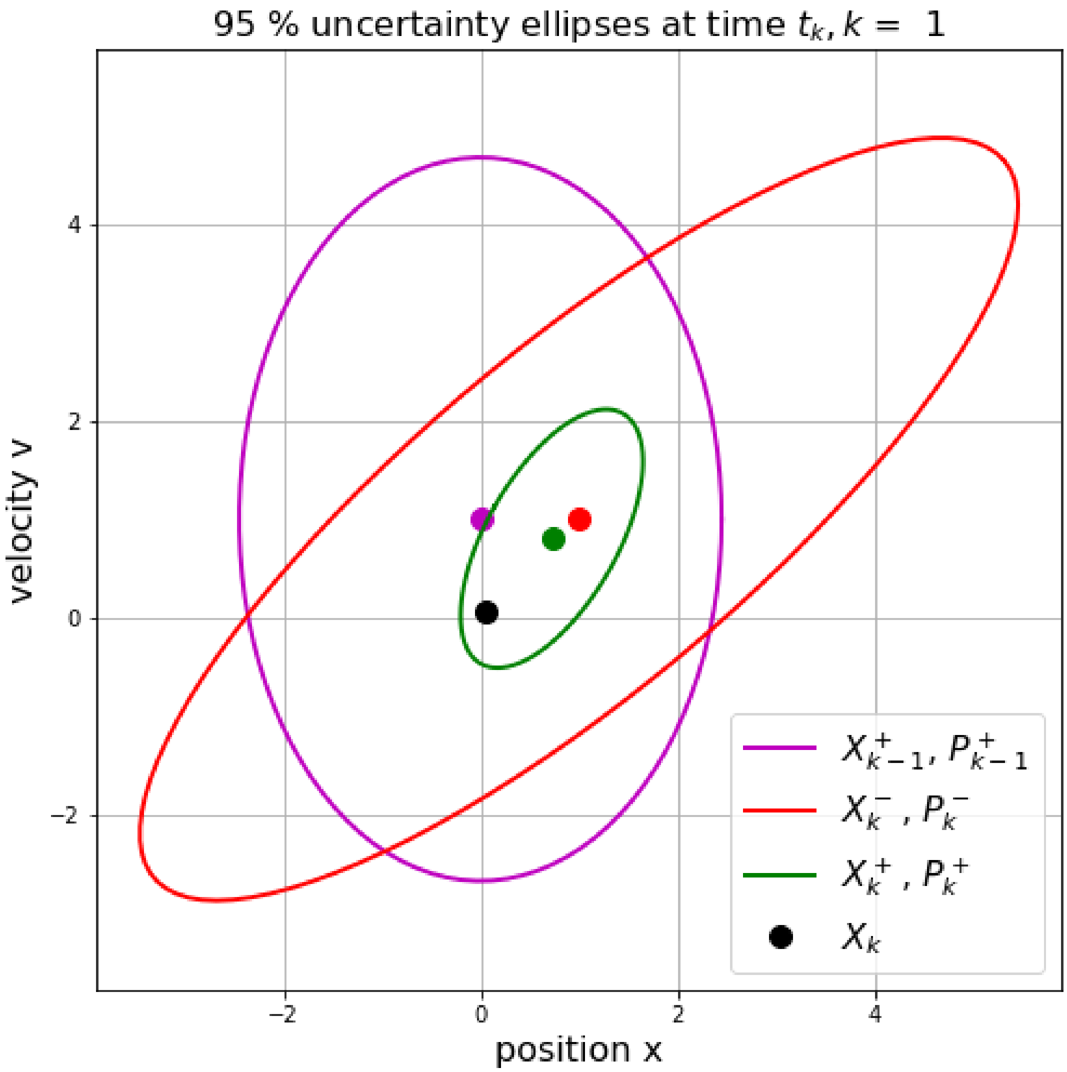


$\Delta t = 1.0, q = 0.09, r = 0.04, \sigma_k = \sqrt{p_k^+}$
x_k^+
x_k
$x_k^+ \pm 2\sigma_k$
position
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
time

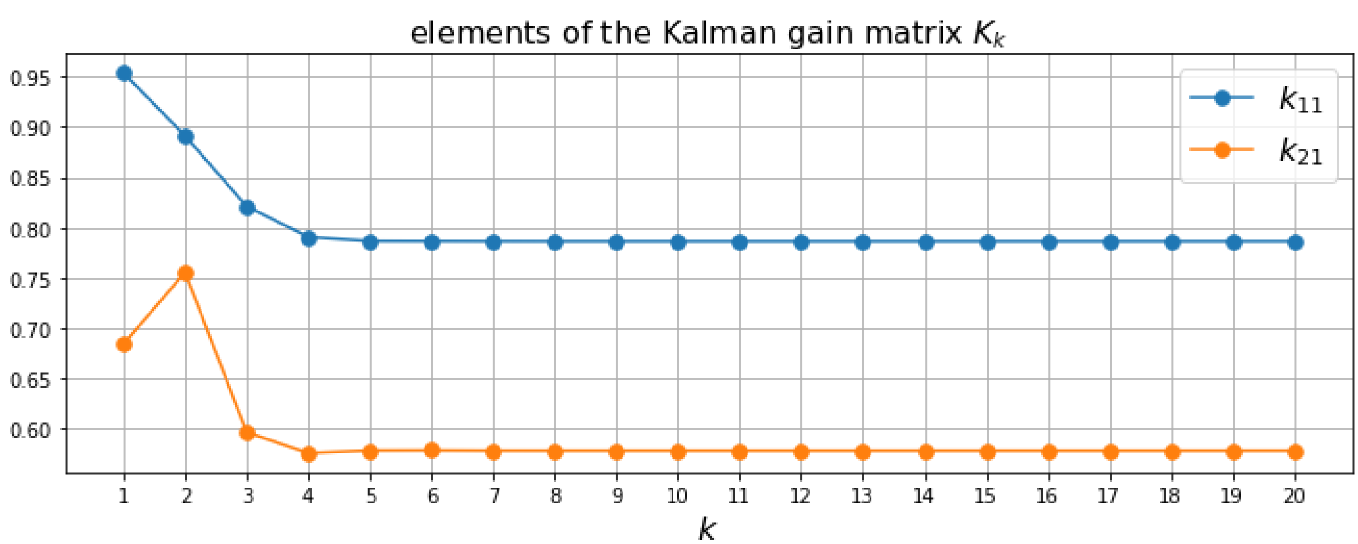
2. Simulate example 3, when only position is measured, and draw pictures like below





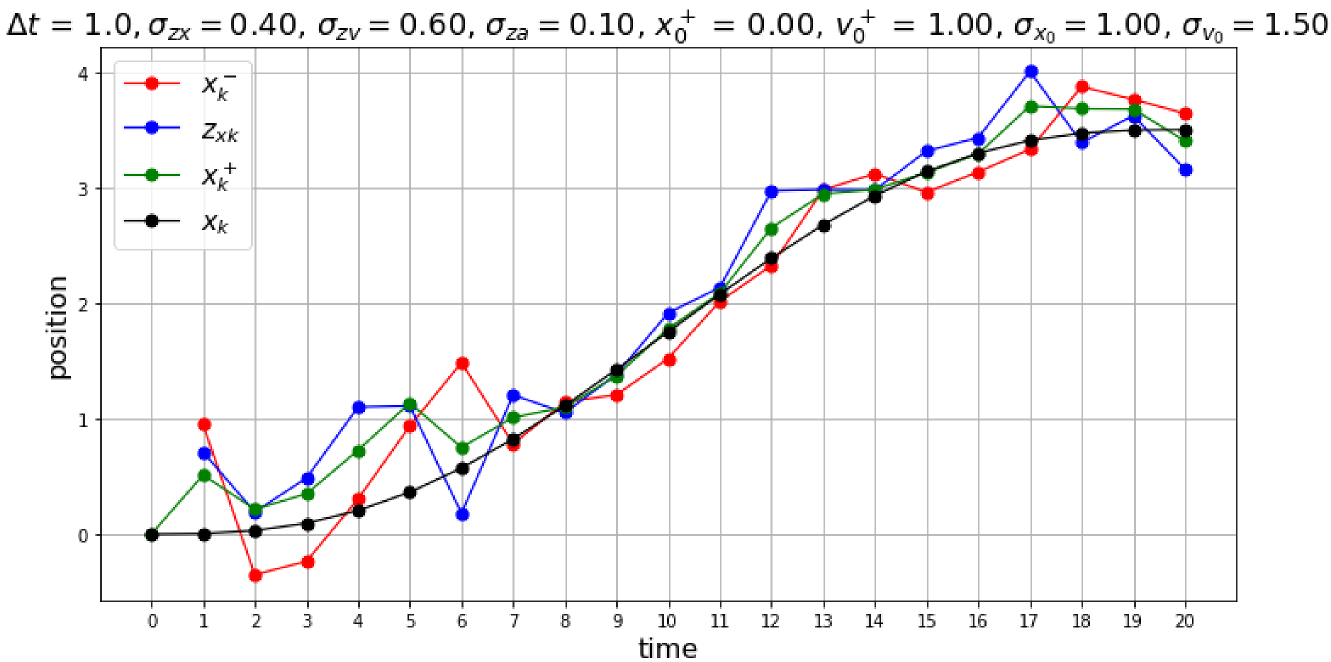


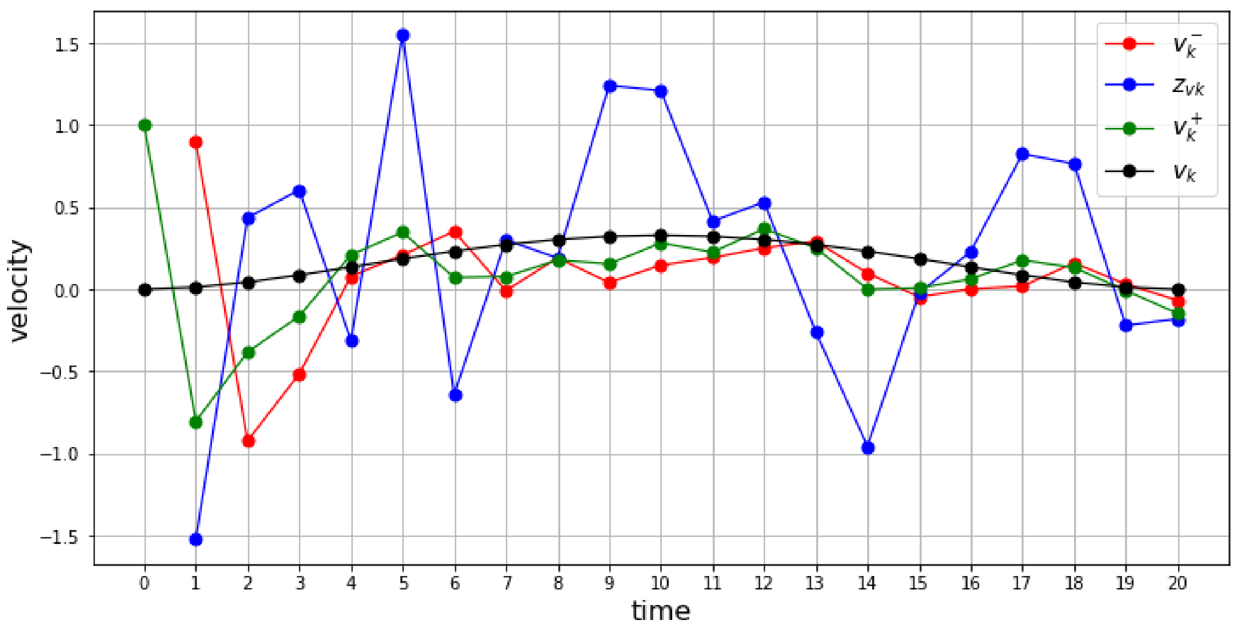
95 % uncertainty ellipses at time $t_k, k = 1$
velocity v
position x
X^+_{k-1}, P^+_{k-1}
X^-_k , P^-_k
X^+_k , P^+_k
X_k

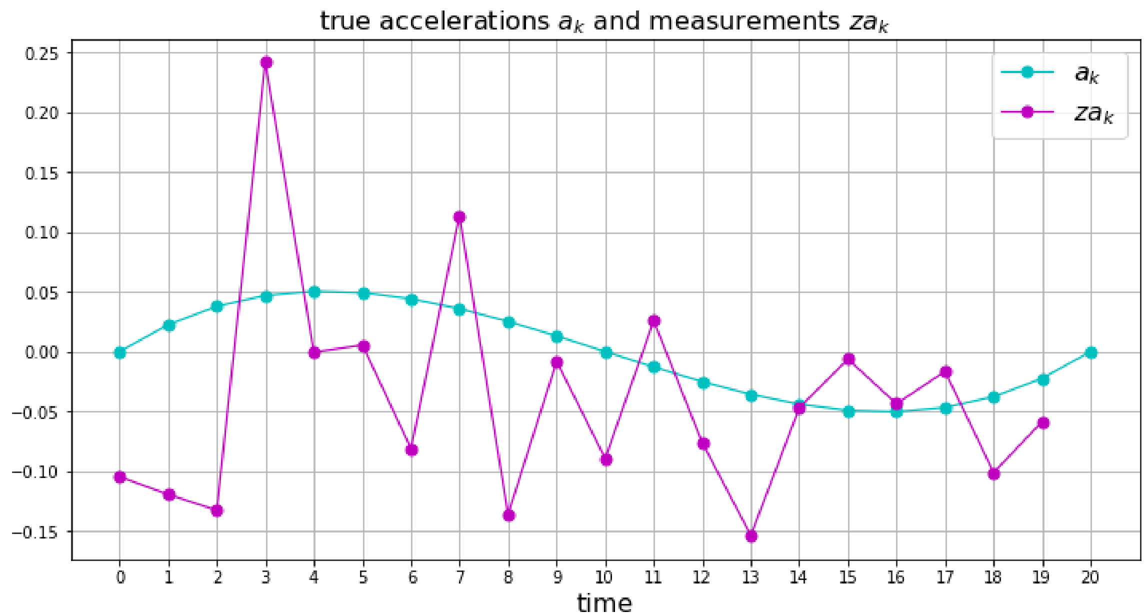


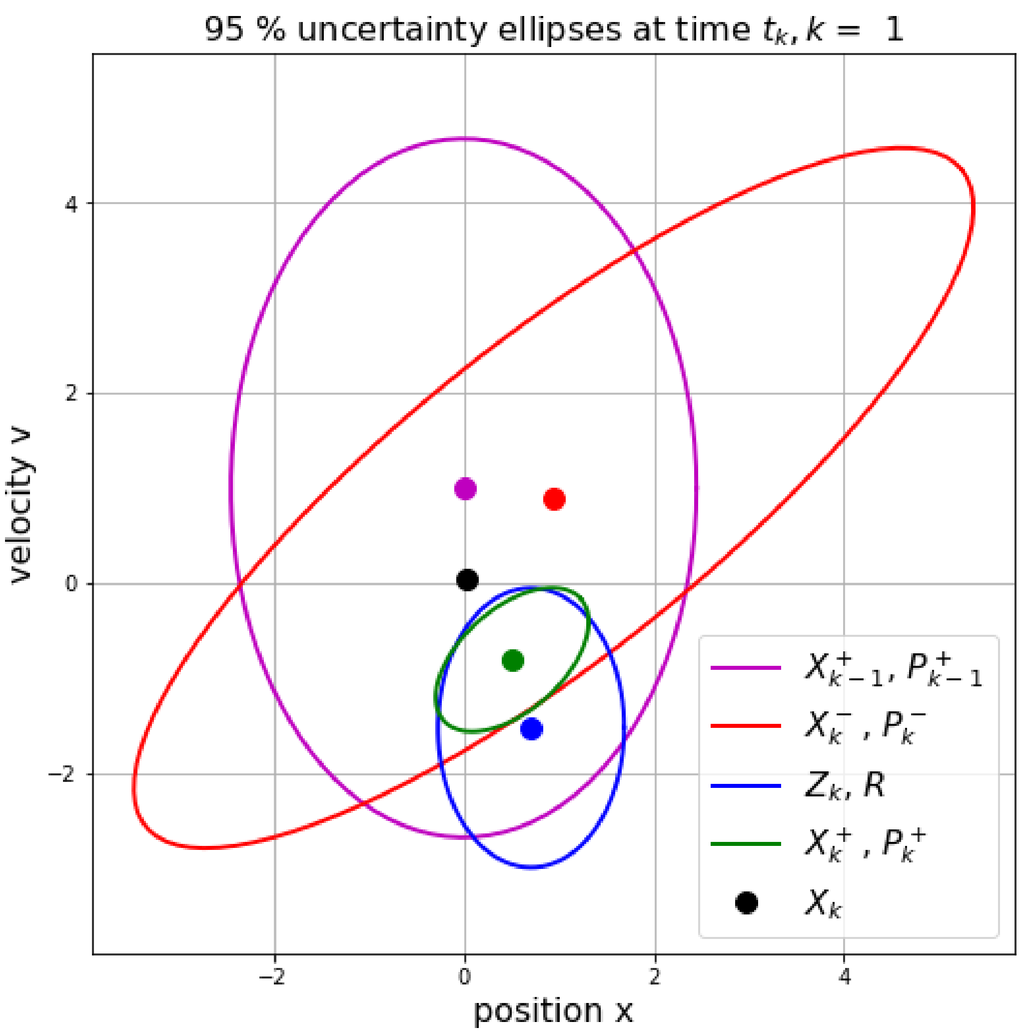
elements of the Kalman gain matrix K_k
k_{11}
k_{21}
k

3. Simulate example 4, when position, velocity and acceleration are measured, and draw pictures like below

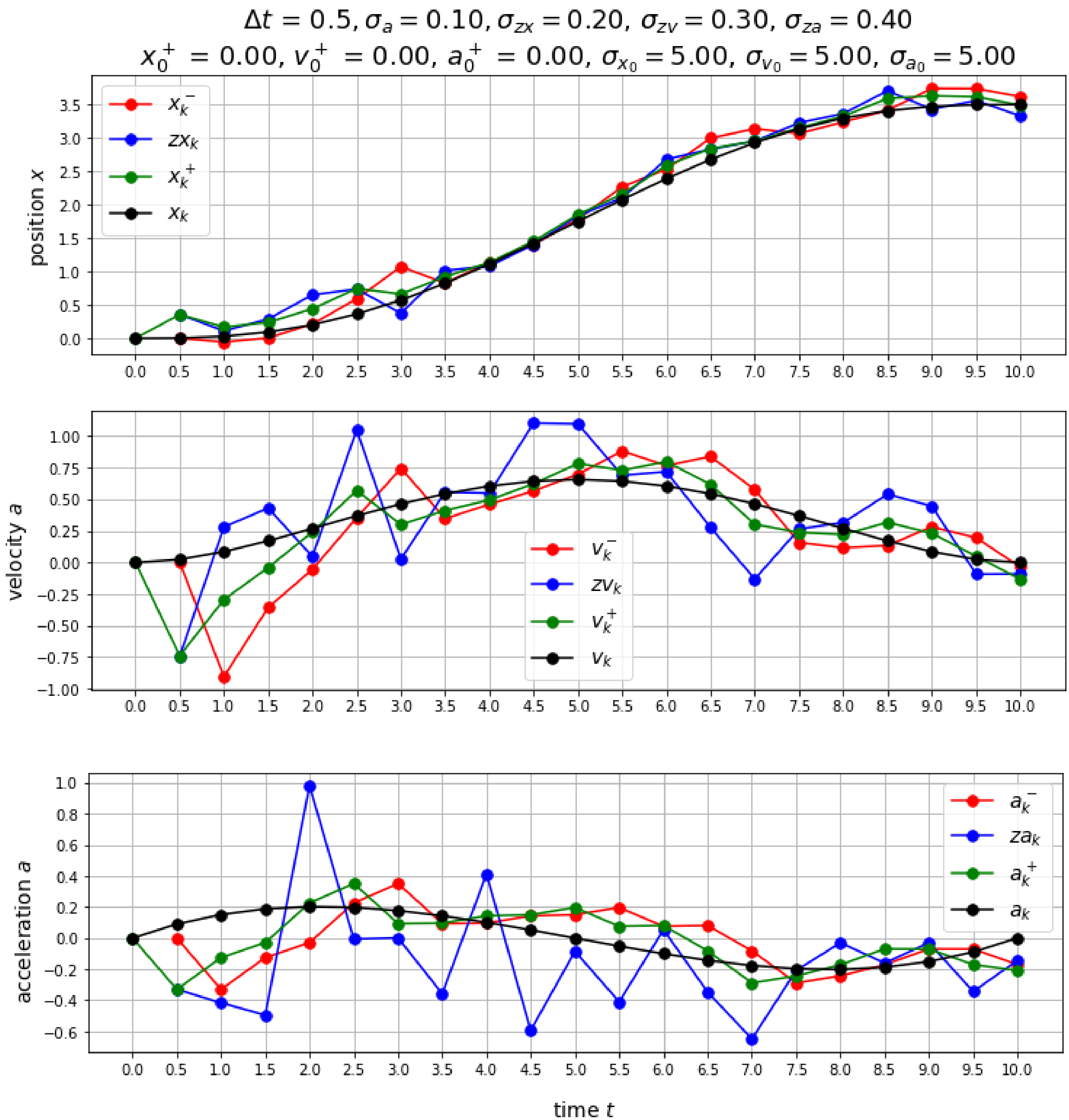


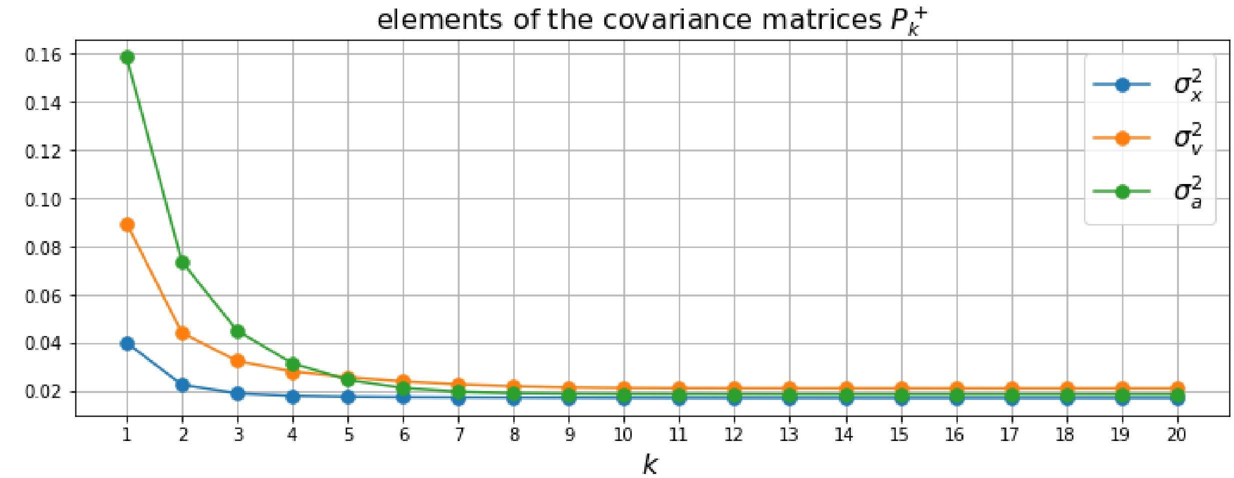


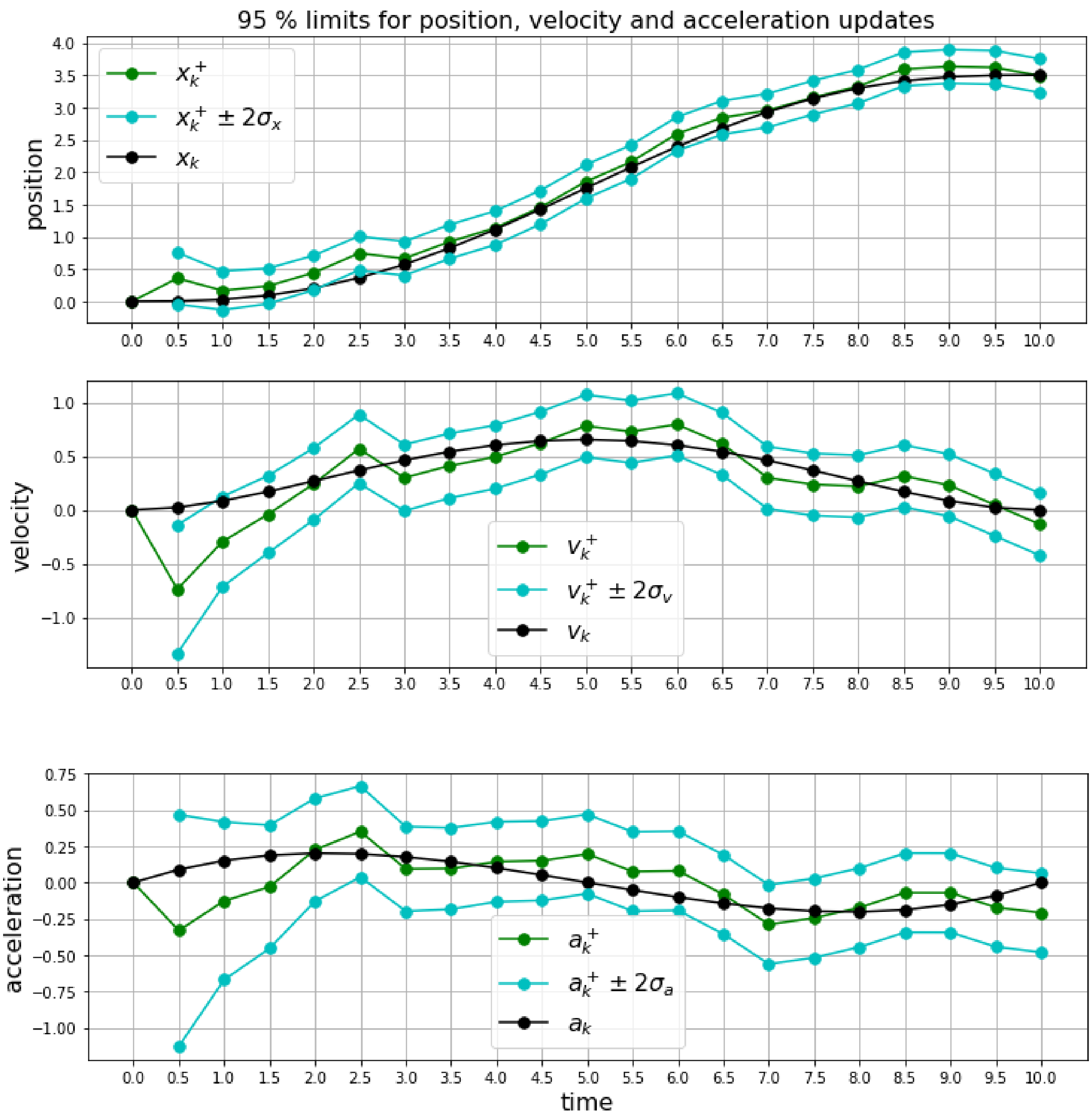

true accelerations a_k and measurements za_k
a_k
za_k
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
time


95 % uncertainty ellipses at time $t_k, k =$ 1
velocity v
position x
4
2
0
−2
−2
0
2
4
X_{k-1}^+, P_{k-1}^+
X_k^-, P_k^-
Z_k, R
X_k^+, P_k^+
X_k

4. Simulate example 5 when position, velocity and acceleration are measured, and draw pictures like below




elements of the covariance matrices P_k^+
σ_x^2
σ_v^2
σ_a^2
k


95 % limits for position, velocity and acceleration updates
position
x_k^+
$x_k^+ \pm 2\sigma_x$
x_k
velocity
v_k^+
$v_k^+ \pm 2\sigma_v$
v_k
acceleration
a_k^+
$a_k^+ \pm 2\sigma_a$
a_k
time