Implement the 0/1 knapsack problem using a VQE #39

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Introduction

Input

$V_i \in \mathbf{V}$

$W_i \in \mathbf{W}$

$W_{\max}$

Output

$x_i \in \mathbf{X}$

Condition

max $\quad \sum_i x_i V_i$

subject to $\quad \sum_i x_i W_i \le W_{\max}$

$$\mathbf{V} = [V_0, V_1, \ldots, V_{n-1}]$$

$$\mathbf{W} = [W_0, W_1, \ldots, W_{n-1}]$$

$$\mathbf{X} = [x_0, x_1, \ldots, x_{n-1}]$$

$$x_i \in \{0,1\}$$

How to build Hamiltonian in the VQE algorithm ???

Cost Function

$$C\left(\mathbf{X}'\right) = M\left(W_{\max} - \sum_{i=0}^{n-1} x_i W_i - S\right)^2 - \sum_{i=0}^{n-1} x_i V_i$$

- $\mathbf{X}' = [\mathbf{X}, y_0, y_1, \ldots, y_{m-1}]$
- $S = \sum_{j=0}^{m-1} 2^j y_j$

$$m = \log_2\left(W_{\max} + 1\right)$$

- $M = 2 \times 10^6$

Achievement – Run with exact solver

```
values = [680, 120, 590, 178]
weights = [13, 6, 15, 9]
w_max = 32
M = 2000000
qubitOp, offset = get_knapsack_qubitops(values, weights, w_max, M)

algo_input = EnergyInput(qubitOp)
ee = ExactEigensolver(qubitOp, k=1)
result = ee.run()

most_lightly = result['eigvecs'][0]
x = sample_most_likely(most_lightly)

print('result=' + str(x[:len(values)]))
```

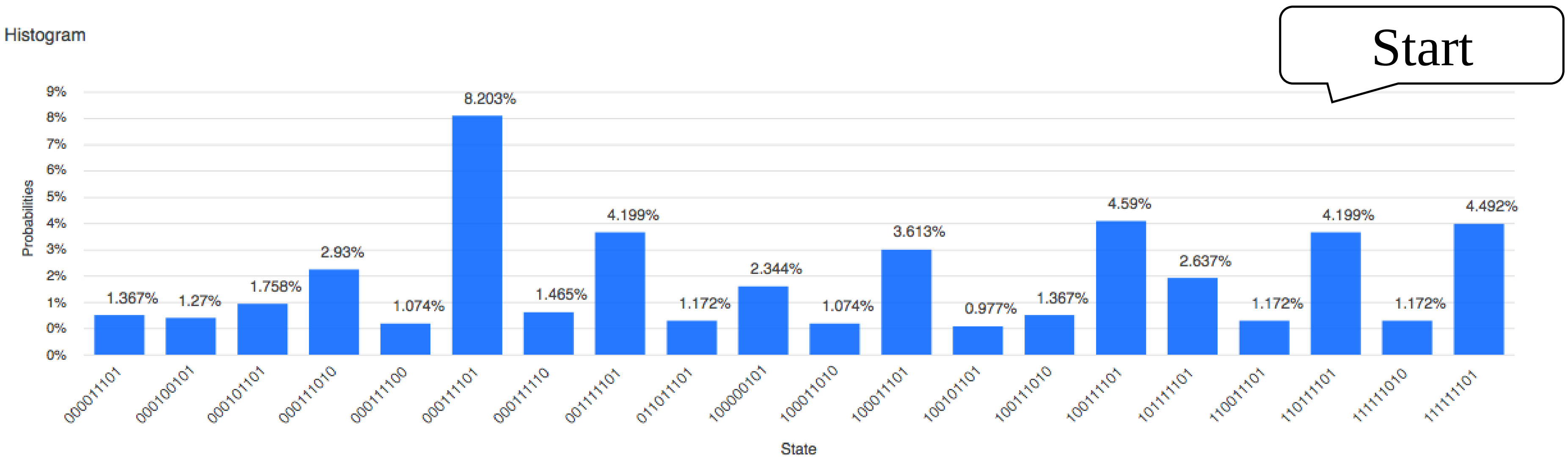
```
result=[1. 0. 1. 0.]
```

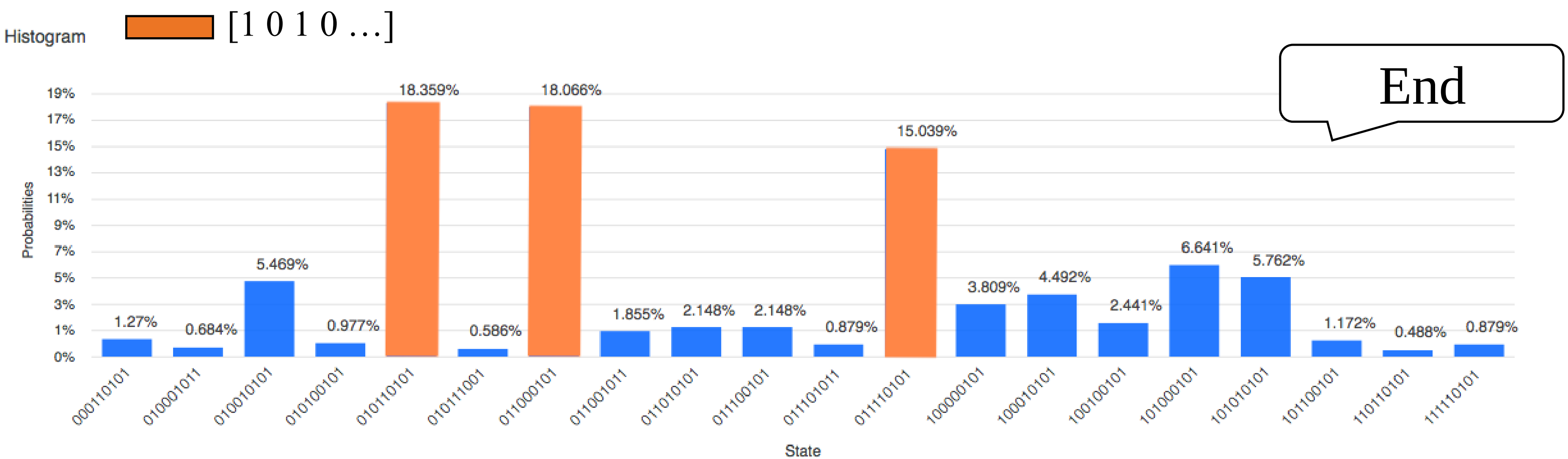
Achievement – Run with simulator

```
seed = 10598

spsa = SPSA(max_trials=300)
ry = RY(qubitOp.num_qubits, depth=5, entanglement='linear')
vqe = VQE(qubitOp, ry, spsa)

backend = BasicAer.get_backend('statevector_simulator')
quantum_instance = QuantumInstance(backend, seed_simulator=seed, seed_transpiler=seed)

result_statevector = vqe.run(quantum_instance)

most_lightly_sv = result_statevector['eigvecs'][0]
x_statevector = sample_most_likely(most_lightly_sv)

print('result usig statevector_simulator =' + str(x_statevector[:len(values)]))
```

```
result usig statevector_simulator =[1. 0. 1. 0.]
```

Achievement – Run with QASM





Thanks for your attention!

Appendix – Cost Function

$$C\left(\mathbf{X}'\right) = M\left(W_{\max} - \sum_{i=0}^{n-1} x_i W_i - S\right)^2 - \sum_{i=0}^{n-1} x_i V_i$$

$$= M\left(\begin{array}{l} W_{\max}^2 - \left(\sum_{i=0}^{n-1}\left(\dfrac{\mathrm{I}-Z_i}{2}\right)W_i\right)^2 \\ +S^2 - 2W_{\max}\sum_{i=0}^{n-1}\left(\dfrac{\mathrm{I}-Z_i}{2}\right)W_i \\ +2W_{\max}S - 2S\sum_{i=0}^{n-1}\left(\dfrac{\mathrm{I}-Z_i}{2}\right)W_i \end{array}\right) - \sum_{i=0}^{n-1}\left(\dfrac{\mathrm{I}-Z_i}{2}\right)V_i$$

- $\mathbf{X}' = [\mathbf{X}, y_0, y_1, \ldots, y_{m-1}]$
- $M = 2\times 10^6$
- $S = \sum_{j=0}^{m-1} 2^j y_j$

$m = \log_2\left(W_{\max} + 1\right)$