MATH3714 Coursework

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Contents

1 Introduction

We have been given a data frame $\mathbf{A}_{393x9}$ which is the table of different cars with mpg, cylinders, displacement, horsepower, weight, acceleration, year, origin and name for a given car. Our goals is to be make a model that is capable of predicting mpg from our data given. Now we plit up $\mathbf{A}_{393x9}$ into $\mathbf{Y}_{393x1}$ which contains only mpg and $\mathbf{Y}_{393x8}$ which contains everything in $\mathbf{A}_{393x9}$ apart from mpg. This sets up our responds and explainatory variable.

2 Initial Data Analysis and Error Correcting

In this prelimatory stage we want to investigate outliers and possible missing data in our dataframe $\mathbf{A}_{393x9}$. The summary of our data is a useful point to start from. From this there are several problems with the data:

2.1 Error in Years

First there is a problem with a data in the year. the summary says the earliest car made was in 18, is it 1918 or 2018?. On further inspection using View(dat) command we can see the name of that car is 'vw golf estate S 1.4 TSI' clearly from 2018 rather than 1918. This need to be changed from 18 to 118. Using the code in section 'R-code' we have fixed the year for the anomalies.

2.2 Problems with Names

The second problem is the name of the cars. This problem lies in the make of the car and the name of the cars are in the same string hence we are not able to 'encode' this properly i.e. amc hornet, amc gremlin are almost identical but if we were to fit these values under the model it would be treated as different. From here, the name can be plit into two more groups, which is make of the car and name of the car. From there the make of the car can be encoded, similar to the origin of the car.
Solution
The first thing to notice in the 'name' header is that the first word is the 'make' of the car and the rest is the 'model' of the car. Now take the first word of the string and add it to make while for name remove the first word of the string.
This should produce a new table with 'make' and 'name'.
NOTE: when importing the table the 'stringAsFactors=F' is a must else this wouldn't work.

2.3 Duplication of Car Makers

Another problem lies in the fact that the data use several acronyms for the name make i.e. chevrolet and chevy, vw and volkswagen etc... This is a problem since it adds unwated complexity to our data. Therefore the data needs to be changed.

Solution
From this we need to change all the maker so that the name is the same i.e. ‘vw‘, ‘vokswagen‘ and ‘volkswagen‘ should be ‘volkswagen‘ etc...

2.4 Encoding Car Makers

A problem that arise from spliting the ‘name‘ column into ‘name‘ and ‘make‘ is the fact that the ‘make‘ is a catergorical data and this need to be encoded i.e. convert catergory into integers, similarly to the origin which is a catergorical data but represented by 1-3.
Solution
We encode the car makers and the origin using r inbuilt factor function.

2.5 Overfitting Caused by Uniquiness of Names

The name of the vehicle is also a problem. This is beacause the vehical name is very unique and dependent on the maker of that car i.e. ‘100ls’ is dependent on audi since only ‘audi’ make cars with those names. This also poses the problem of that the name is so unique that it can cause over fitting.
Solution
The solution is to delete the name column and only include the brand as one of our explainatory variable.

2.6 Multicollinearity

As the R-code section shows the matrix $\boldsymbol{X}$ form from cylinders, displacement, horsepower, weight, acceleration, year (NOTE: it's doesn't matter about the constant column or the factors since they are linearly independent from the rest).
Eigenvector is:

$$3791200361.7, 1365742.4, 130921.6, 68541.8, 1553.3, 111.5$$

The conditional indicies form from this is:

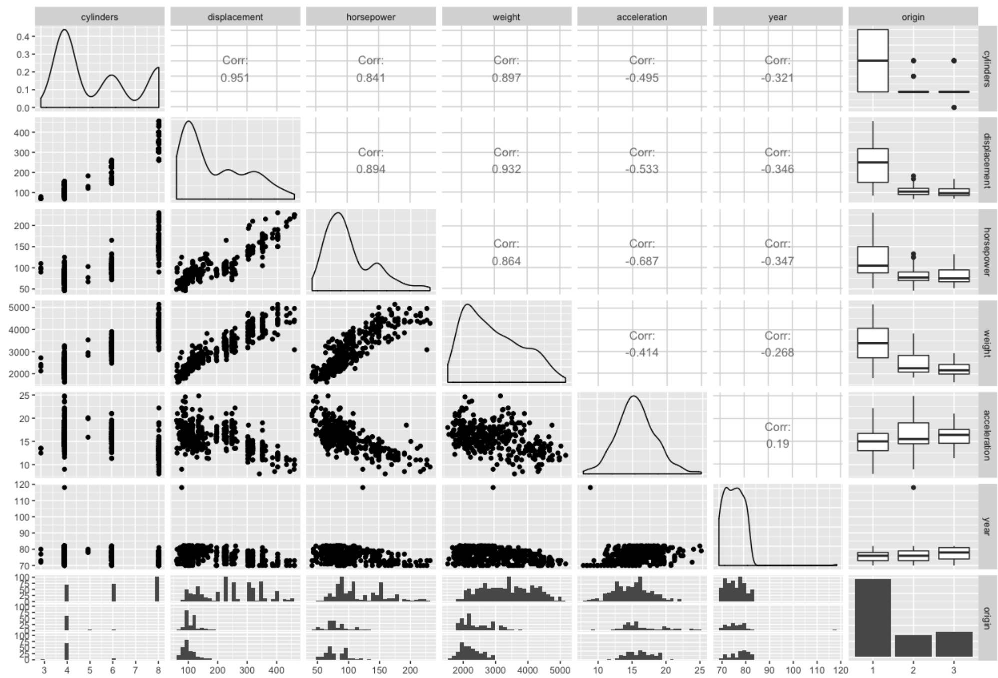
$$1, 2776, 28958, 55312, 2440751, 34013500$$

Clearly all $\lambda_i > 1000$ apart from the first value, hence this is a sign of severe collinearity. From this the eigen vectors of $\boldsymbol{X}$ is:

$$\begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1 \\ -0.1 & -0.9 & 0.0 & -0.3 & 0.0 & 0 \\ 0.0 & -0.2 & 0.9 & 0.4 & -0.1 & 0 \\ -1.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0 \\ 0.0 & 0.1 & 0.0 & -0.2 & -1.0 & 0 \\ 0.0 & 0.3 & 0.4 & -0.8 & 0.2 & 0 \end{pmatrix}$$

Clearly the years are independent but it's shows all the other variables are collinear on at leats one other variables. Now take the smallest eigenvalue and the eigenvector

corresponding to it then shows cylinder is independent from any other variables (very weird, since it's the smallest eigenvalue but yet the eigenvector is linearly independent. Although this could be because cylinders are catergorical data). From the eigenvector corresponding to the $2^{nd}, 3^{rd}$ and 4^{th} smallest eigenvalue implies there is multicollinearity between displacement, horsepower and year. Unfortunately the data set gave roughly the same output when the offending variable are removed.
Another method to observe which values are correlated is to use pairwise correlation:



From this it shows the offending variable with high correlation is cylinder, displacement, horsepower and weight. This makes sense since if you have heavy car you will need more horsepower, more horsepower means more or biggest cylinders which mean higher displacement. Now consider that cylinder, displacement, horsepower and weight is combined into one in the form of:

$$x' = \frac{cylinder + weight}{horsepower + displacement}$$

The reason for this is that the output of this need to be reasonal, x' could have been the product of all the variable but that will give it a large value and therefore when calculating the variance the inverse will be small and hence give large conditional indicies. Therefore the conditional indicies is:

$$1, 680, 3522$$

And Eigenvector is:

$$\begin{pmatrix} -0.1 & -0.5 & 0.9 \\ -0.2 & -0.8 & -0.5 \\ -1.0 & 0.2 & 0.0 \end{pmatrix}$$

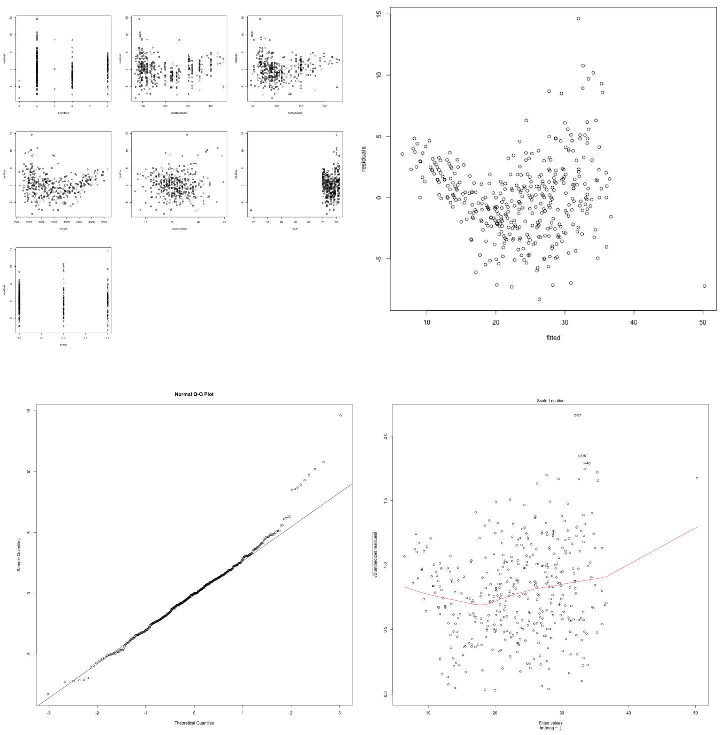
Additionaly, the VIF (variance inflation factor) shows:

```
  Variables      VIF
1     prime 3.211425
2   acceler 2.060063
3      year 1.184508
4    Origin 1.626321
```

Even though one of the conditional indicies > 1000 we can ignore since all the VIF < 4. Therefore we can conclude the data is much less collinear than before.

2.7 Base Model

This model is fitted with no transformation, interaction and with every variable and factors. Lets see the diagnostics plot to observe whether or not this have violated our assumption.



From the residual vs fitted (top right), the plot show a none linear trends this implies a transformation is needed to make the responds variable(mpg) to be linear.
The top right graph which is residual vs fitted we can observe a none-horizontal line across zero. Therefore this may also implies that a stabilizing variance transformation of the mpg is needed.
The multiple plots(top left) suggest some of the variable used is not linear. Specifically this suggest displacement, horsepower and weight are not linear. There could also be

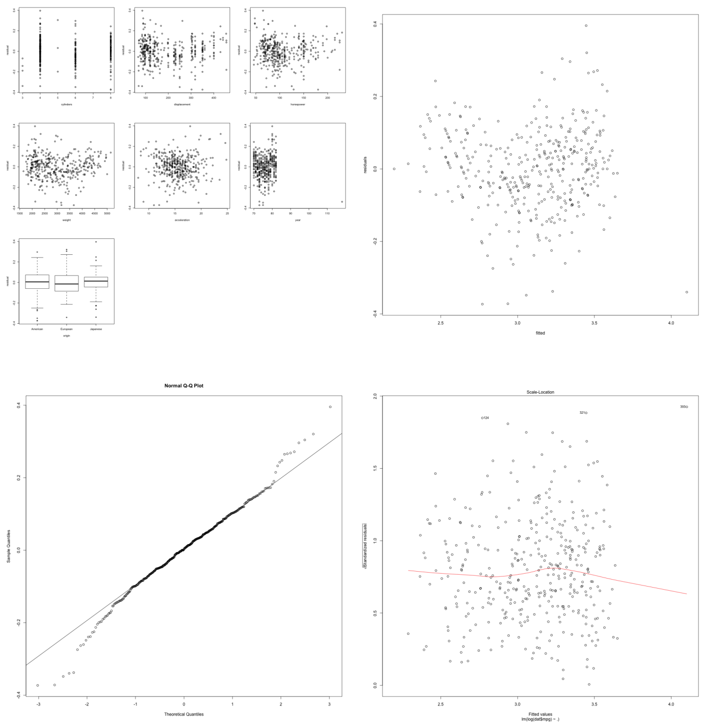
more.
The Q-Q plots(bottom left) seems mostly fine apart from the upper quartile where values diviate from the line drastically. This may pose a problem later on.

2.8 Logarithmic Transformation

This model is in the form of:

$$log(mpg) = \beta_0 + ...$$

The model in this have been fitted such that it's the logarithmic of the responds variable. The diagnostic plots shows:



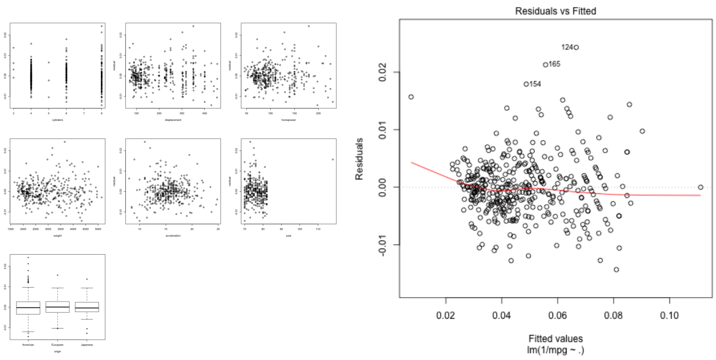
This is a relatively good transformation. The residual vs explainatory variable(top left) tells us the model now is more linear than it was before.

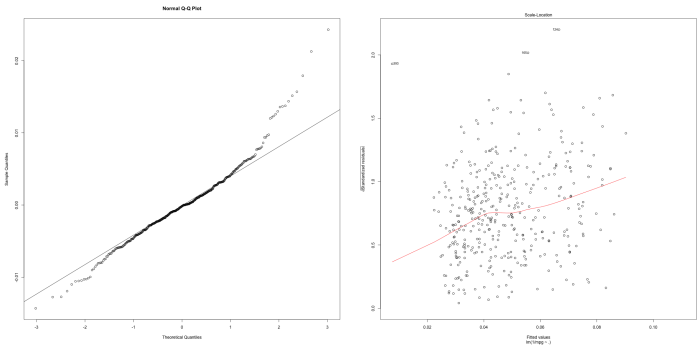
2.9 Reciprocal Transformation

This model is in the form of:

$$\frac{1}{mpg} = \beta_0 + ...$$

The model in this have been fitted such that it's the reciprocal of the responds variable. The diagnostic plots shows:

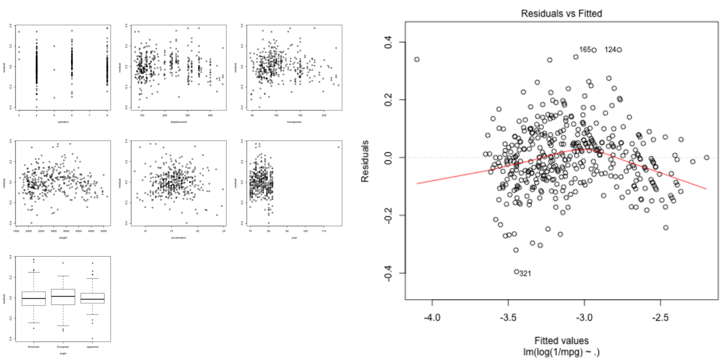


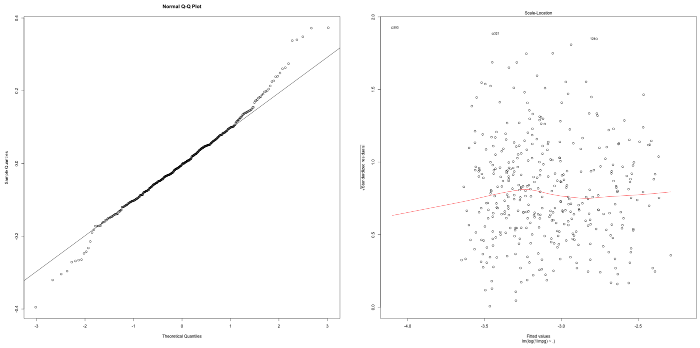


This model seems to be good for linearity of the model as shown by the explainatory variable(top left) to be more spread out and doesn't have a funnel shape as previous plots. Similarly the same can be said for the residual vs fitted plot. The problem is that this model have made the homoscedacity worse and normality of this model is slightly worse than our original model.

2.10 Log-Reciprocal model

This model we perform a transformation of mpg to reciprocal of that then transform it again into a logarithmics reciprocal of the mpg.





From this we can observe that the variance is more stablized, from bottom right plot, from the logarithmic tranformation compared to the reciprocal model. The linearity of the model (top right plot) is roughly the same of the reciprocal model, it's not a straight line but it fairs better than the square root transformation or the base model. The normality has improved from the reciprocal model (q-q plot, bottom left). Therefore this model is a good model compared to all the previous model.

2.11 Model Analysis

An attemp was made to split the data file into two catergories, one training and one testing. Unfortunately, the to run the test data to obtain how well the models perform much exceeded the stack limit of R and the amount of Ram in my computers and therefore unable to attain how well each of the model perform. The only other criterior is the how well the model conform to the assumption and the residual of the model.
From the analysis of the model assumption analysis above the logarithmic **Residual** running the R-code provide gives the value of:

```
         base square root      log reciprocal   model1 log reciprocal Ridge
[1,] 3841.958     32.73854 4.862929 0.01028882 3328.877        4.862929 33.53
```

From this the worse models are the base model, model1 while log and reciprocal model perform better than the rest. Therefore the model that will be chosen is the reciprocal and closely followed the log model. The summary of these model also shows that the log and reciprocal model have a Adjusted-R of about 0.65 which is good considering there AIC isn't used yet.
NOTE: lots more model was produce and analysed but due to space contraight was not included in the report. The model1 correspond to a navie attempt where $mpg = \beta_0 \frac{\beta_1}{cylinder} + \frac{\beta_2}{displacement} + \frac{\beta_3}{horsepower} + ...$ while Ridge regression was not included due to the fact that it produce very small k values suggesting a lot of the parameters should be zero.

3 Influential Values (Outliers)

By checking the Cook's distance the outlier will be showns and subsequently remove. Using the R-code from the Diagnostics R-code section the two model shows that the value:

```
29, 183, 333, 375
```

Both of the model agress which observation should be remove. Clearly these model affect the paramters so much that these values should be omitted.
After applying the correction to collinearity there seems to be an extra outlier namely 393 which correspond to the 2018 car. Therefore remove that data too.

4 Variables Selection for Models

Using AIC(Akaike information criterion) the model can be reduce down including all of it's interactions. Unfortunately due to the solution of solving the multicollineaity, it has seem to made it so the AIC can't perform the variable reduction if interaction is included. Therefore these model will not include interaction.
When AIC is perform on both of these model without interaction the model shows that acceleration is not needed. Therefore the final coefficient for the model is:

```
#Logarithmic model

# Call:
#   lm(formula = log(mpg) ~ prime + year + origin, data = dat3)
#
# Coefficients:
#   (Intercept)        prime          year        origin
# 0.11322      0.08052      0.02581      0.09420

#Reciprocal Model

#   lm(formula = 1/mpg ~ prime + year + origin, data = dat3)
#
# Coefficients:
#   (Intercept)        prime          year        origin
# 0.185501    -0.004119    -0.001142    -0.003869
```

The AIC for the logarithmic model is −1276.9 while reciprocal is −3609.66. Which shows the logarithmic model is much better, but this is false as shown in the next section.

5 Interpretation of Model

The model that is going to be use is the exponential model in the form of:

$$mpg = e^{0.08052 * \frac{cylinder+weight}{horsepower+displacement}} * e^{0.02581 * year} * e^{0.0942 * origin}$$

The intuition behind this model is that the increase in horsepower and displacement decrease the mpg exponentially while keeping the other variables constant. The year also factor into this model in a intuitive way, the newer the car the higher the mpg;

which is a reasonable trend. Similary for the origin, since Japan is numerical equivalent to 3 then under our mdel japanese car would have the higher mpg while American car would have the lower mpg.

6 Predicting Values

We are given Japanese manufactured (in 2000) car, which has weight 2734 lbs, engine displace- ment 81 ins3, horsepower 101 hp, 4 cylinders, and acceleration (0–60) is 12.6 seconds. The prediction is:

```
# fit       lwr       upr
# 1 4.187991 4.074648 4.301334
```

Therefore the mpg for this car would be $e^{4.074} = 58.8 \geq mpg \geq 73.8 = e^{4.30134}$ with the fitted value landing round about 65.89 mpg.

7 Summary and Reflection

There was a few problems such as fitting the model before solving the multicollinearity problem to check the assumption then revisiting to re-check the model assumptions. The second problem is that I should have used $\frac{1}{cylinder+weight+horspower+displacement}$ as the new prime variable since this would make a better explaination to why the mpg decrease and those variable increase. For the prediction section, unfortunately the reason why the reciprocal modle perform so well, in the sense of having small residual, is because it produce a large range of confidence interval for instance if the reciprocal model was use th predict the value above it would yield:

```
# fit            lwr           upr
# 1 -0.002286896 -0.008114051 0.003540259
```

Which implies that our mpg is between : $282.27 \geq mpg$ which is very large and not right. Which means this model is not good for prediction.

8 R-Code

8.1 Cleaning data

```
> dat = read.table("http://www1.maths.leeds.ac.uk/~charles/math3714/Auto.csv",
 header = T)
> View(dat)
> summary(dat)
mpg             cylinders       displacement     horsepower
Min.    : 9.0   Min.   :3.000   Min.   : 68.0   Min.   : 46.0
1st Qu.:17.0    1st Qu.:4.000   1st Qu.:105.0   1st Qu.: 75.0
Median :23.0    Median:4.000    Median :151.0   Median : 94.0
Mean   :23.5    Mean   :5.468   Mean   :194.1   Mean   :104.5
3rd Qu.:29.0    3rd Qu.:8.000   3rd Qu.:267.0   3rd Qu.:125.0
Max.   :46.6    Max.   :8.000   Max.   :455.0   Max.   :230.0

weight          acceleration    year            origin
Min.   :1613    Min.   : 8.00   Min.   :18.00   Min.   :1.000
1st Qu.:2226    1st Qu.:13.70   1st Qu.:73.00   1st Qu.:1.000
Median :2807    Median :15.50   Median :76.00   Median :1.000
Mean   :2978    Mean   :15.52   Mean   :75.83   Mean   :1.578
3rd Qu.:3613    3rd Qu.:17.00   3rd Qu.:79.00   3rd Qu.:2.000
Max.   :5140    Max.   :24.80   Max.   :82.00   Max.   :3.000

name
amc matador        :  5
ford pinto         :  5
toyota corolla     :  5
amc gremlin        :  4
amc hornet         :  4
chevrolet chevette:  4
(Other)            :366
```

Error in Year

```
> dat$year[dat$name=='vw golf estate S 1.4 TSI'] = 118
> View(dat)
```

Problems with Names

```
dat = read.table("http://www1.maths.leeds.ac.uk/~charles/math3714/Auto.csv", head
#---Addressing 2nd problem
#In order to achieved this I need to add an extra tag into the
#dataframe which is "stringAsFactors=F".
#adding a extra entry called make which stands for the maker of the car.
dat$make = dat$name

#changing the string into the first word of the sring.
#Then attaching the first word of the string to make table.
for(string in dat$make){
  substring = strsplit(string, " ")[[1]]
```

```
  maker = substring[1]
  print(maker)
  dat$make[dat$make==string]=maker
}

#changing the string into every word apart from the first word.
for(string in dat$name){
  substring = strsplit(string, " ")[[1]]
  print(paste(substring[-1], collapse=' ' ))
  dat$name[dat$name==string]=paste(substring[-1], collapse=' ' )
}
```

Duplication of Car Makers

```
>table(dat$make)
amc      audi     bmw      buick    cadillac          capri    chevroelt
27       7        2        17       2                 1        1
chevrolet         chevy    chrysler          datsun
43                3        6                 23
dodge    fiat     ford     hi       honda    maxda    mazda
28       8        48       1        13       2        10
mercedes  mercedes-benz    mercury  nissan
1         2                11       1
oldsmobile        opel     peugeot  plymouth          pontiac  renault
10                4        8        31                16       3
saab     subaru   toyota   toyouta  triumph
4        4        25       1        1
vokswagen         volkswagen        volvo    vw
1                 15                6        7

>dat$make = factor(dat$make)
>dat$origin[dat$origin==1]='American'
>dat$origin[dat$origin==2]='European'
>dat$origin[dat$origin==3]='Japanese'
>dat$origin = factor(dat$origin)
```

Uniqueness of Name

```
#---Problem 5
dat$name=NULL
```

Multicollinearity

```
> X=as.matrix(cbind(dat$cylinders,dat$displacement,dat$horsepower,
dat$weight,dat$acceleration,dat$year))
> round(diag(solve(t(X)%*%X)),3)
[1] 0.009 0.000 0.000 0.000 0.001 0.000
> v=eigen(t(X)%*%X)
> round(v$values,1)
[1] 3791200361.7    1365742.4     130921.6      68541.8        1553.3
111.5
> round(max(v$values)/v$values,0)
```

```
[1]         1      2776     28958     55312   2440751 34013500
> round(v$vectors,1)
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]  0.0  0.0  0.0  0.0  0.0    1
[2,] -0.1 -0.9  0.0 -0.3  0.0    0
[3,]  0.0 -0.2  0.9  0.4 -0.1    0
[4,] -1.0  0.1  0.0  0.0  0.0    0
[5,]  0.0  0.1  0.0 -0.2 -1.0    0
[6,]  0.0  0.3  0.4 -0.8  0.2    0
> S=svd(X)
> S$d
[1] 61572.72417  1168.64981   361.83087   261.80486    39.41183
10.55754
> max(S$d)/S$d
[1]    1.00000   52.68706  170.16990  235.18557 1562.29040 5832.10936
#Solving MultiCollinearity
prime = (dat$cylinders+dat$weight)/(dat$horsepower+dat$displacement)
check_multicollinearity(as.data.frame(cbind(prime,dat$acceleration,dat$year)))
# [1] "Eigenvalues"
# [1] 2423800.9    3567.0     688.2
# [1] "Max Eigenvalue/Eigenvalues"
# [1]    1  680 3522
# [1] "Eigenvectors"
# [,1] [,2] [,3]
# [1,] -0.1 -0.5  0.9
# [2,] -0.2 -0.8 -0.5
# [3,] -1.0  0.2  0.0
# [1] 1556.85611   59.72403   26.23276
# [1]  1.00000 26.06750 59.34779

vif(as.data.frame(cbind(prime,dat$acceleration,dat$year)))
# Variables      VIF
# 1     prime 2.103025
# 2        V2 1.873807
# 3        V3 1.175262
```

8.2 Models

First Model

```
#MODELS-------------------------------------------------------------------
#First function with no transformation or interactions
base_model = function(dat){
  return(lm(mpg~.,data = dat))
}

#square root model
sqrt_model = function(dat){
  return(lm(sqrt(mpg)~., data=dat))
}
```

```
#logarithmic model
log_model = function(dat){
  return(lm(log(mpg)~., data = dat))
}

recip_y = function(dat){
  return(lm(1/mpg ~., data = dat))
}

#reciprocal of certain variable
model2 = function(dat){
  dattemp=dat
  dattemp$displacement=1/dattemp$displacement
  dattemp$weight=1/dattemp$weight
  return(lm(mpg~.,data = dattemp))
}

#Ridge's Regresson
ridge=function(dat){
  library(MASS)
  return(lm.ridge(mpg~.,data = dat, lambda = seq(0, 0.000001, 0.0000001)))
}
#Box-Cox transformation
box_cox = function(dat){
  base = lm(mpg~., data = dat)
  boxcox.lm = boxcox(base)
  return(boxcox.lm)
}
```

8.3 Diagnostics

```
#BENCHMARK-------------------------------------------------------------------
#Testing for the residual of the model
res_bench=function(dat){
  base = base_model(dat)
  square_root = sqrt_model(dat)
  Log_model = log_model(dat)
  rec_y = recip_y(dat)
  model1 = model2(dat)
  log_recip_model = lm(log(1/mpg)~., data = dat)
  LinRidge = ridge(dat)
  res_summary=cbind(sum(base$residuals^2), sum(square_root$residuals^2),sum(Log_m
                    sum(rec_y$residuals^2),sum(model1$residuals^2),sum(log_recip_
  colnames(res_summary)=c("base","square␣root","log","reciprocal","model1", "log␣
  print(res_summary)
}
```

```
#Diagnostics-of-Model--------------------------------------------------

#Initializing the models.
base = base_model(dat)
square_root = sqrt_model(dat)
Log_model = log_model(dat)
rec_y = recip_y(dat)
model1 = model2(dat)
log_recip_model = lm(log(1/mpg)~., data = dat)
LinRidge = ridge(dat)

#Checking the assumption of models.
check_assumption("base",base,dat)
check_assumption("model1",model1,dat)
check_assumption("sqrt",square_root,dat)
check_assumption("log",log_model,dat)
check_assumption("reciprocal",rec_y,dat)
check_assumption("log_reciprocal",log_recip_model,dat)

#Checking which model has the best residual
res_bench(dat)
# base square root     log reciprocal   model1 log reciprocal Ridge
# [1,] 8129.847    83.09975 5.60372 0.03921142 3328.877       14.86362 33.53
```

8.4 Removing Outliers

```
dat = dat[-c(29,183,333,375),]
dat = dat[-393]
```

8.5 AIC

```
dat3=dat2[-6]
final_log = lm(log(mpg) ~ .^2, data = dat3)
final_rec = lm(1/mpg ~ .^2, data = dat3)

#No interactions.
final_log = lm(log(mpg)~., data = dat3)
final_rec = lm(1/mpg~., data = dat3)
step(final_log, direction="both")
step(final_rec,direction = "both")
```

8.6 Prediction

```
#Predicting the new value
prime = (dat$cylinders+dat$weight)/(dat$horsepower+dat$displacement)
dat2=as.data.frame(cbind(dat$mpg,prime,dat$acceleration,dat$year,dat$origin,dat$m
colnames(dat2)=c("mpg","prime","acceleration","year","origin","make")
dat3=dat2[-6]
final_log = lm(log(mpg) ~ ., data = dat3)
final_rec = lm(1/mpg ~ ., data = dat3)
```

```
final_rec = step(final_rec,direction = "both")
summary(final_rec)
new = as.data.frame(cbind(15.043956,100,3))
colnames(new)=c("prime","year","origin")
predict.lm(final_rec,newdata=new,interval="confidence",level=0.95)
predict.lm(final_log,newdata=new, interval="confidence",level=0.95)
# fit           lwr          upr
# 1 -0.002286896 -0.008114051 0.003540259
# fit       lwr      upr
# 1 4.187991 4.074648 4.301334
```