myproblemset

September 10, 2017

2016 A6 Find the smallest constant C such that for every real polynomial $P(x)$ of degree 3 that has a root in the interval $[0,1]$,

$$\int_0^1 |P(x)|\,dx \le C \max_{x\in[0,1]} |P(x)|.$$

$(1.07, 0.0, 0.02)$

2016 A5 Suppose that G is a finite group generated by the two elements g and h, where the order of g is odd. Show that every element of G can be written in the form

$$g^{m_1}h^{n_1}g^{m_2}h^{n_2}\cdots g^{m_r}h^{n_r}$$

with $1 \le r \le |G|$ and $m_1, n_1, m_2, n_2, \dots, m_r, n_r \in \{-1, 1\}$. (Here $|G|$ is the number of elements of G.)

$(17.65, 0.18, 0.18)$

2016 B5 Find all functions f from the interval $(1,\infty)$ to $(1,\infty)$ with the following property: if $x, y \in (1,\infty)$ and $x^2 \le y \le x^3$, then $(f(x))^2 \le f(y) \le (f(x))^3$.

$(20.32, 0.58, 0.11)$

2015 B6 For each positive integer k, let $A(k)$ be the number of odd divisors of k in the interval $[1,\sqrt{2k})$. Evaluate

$$\sum_{k=1}^{\infty}(-1)^{k-1}\frac{A(k)}{k}.$$

$(1.51, 2.0, 0.01)$

2007 A–1 Find all values of α for which the curves $y = \alpha x^2 + \alpha x + \frac{1}{24}$ and $x = \alpha y^2 + \alpha y + \frac{1}{24}$ are tangent to each other.

$(31.55, 1.02, 1.45)$

2009 B–2 A game involves jumping to the right on the real number line. If a and b are real numbers and $b > a$, the cost of jumping from a to b is $b^3 - ab^2$. For what real numbers c can one travel from 0 to 1 in a finite number of jumps with total cost exactly c?

$(32.5, 0.05, 1.03)$

2000 A–1 Let A be a positive real number. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$, given that $x_0, x_1, \dots$ are positive numbers for which $\sum_{j=0}^{\infty} x_j = A$?

$(38.46, 0.25, 0.68)$

2002 A–5 Define a sequence by $a_0 = 1$, together with the rules $a_{2n+1} = a_n$ and $a_{2n+2} = a_n + a_{n+1}$ for each integer $n \ge 0$. Prove that every positive rational number appears in the set

$$\left\{\frac{a_{n-1}}{a_n} : n \ge 1\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{2}, \dots\right\}.$$

$(39.05, 1.1, 0.4)$

2003 B–1 Do there exist polynomials $a(x), b(x), c(y), d(y)$ such that

$$1 + xy + x^2y^2 = a(x)c(y) + b(x)d(y)$$

holds identically?

$(35.82, 0.19, 0.26)$