Department of Mechanical Engineering
Indian Institute of Technology Bombay

ME704: Computational Methods in Thermal & Fluid Engineering **Autumn 2023**

Assignment # 5: FDM-based Solution for 2D Conduction **Instructor**: Prof. Atul Sharma

Date Posted: 06th Oct. (Friday) Due Date: 11th Oct. (Wednesday, Early Morning 2 AM)

VIVA on: 11th October, Wednesday, 04:30-06:30 PM, Room No. F-24, Mech. Engg. Dept.

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Submit **separate** files for the (a) filled-in answer sheet (this doc file—converted into a pdf file **rollno.pdf**), and (b) all the computer programs (name of the files should be **rollno-p1.m** and **rollno-p2.m**).

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1. **Explicit Method-based Solution of 2D unsteady state heat conduction:**

   Consider 2D conduction in a square shaped (L_1=1m and L_2=1m) long stainless-steel plate (density ρ: **7750 kg/m3**, specific-heat c_p: **500 J/Kg K**, thermal-conductivity k: **16.2 W/m-K**). The plate is initially at a uniform temperature of ***30°C*** and is suddenly subjected to a constant temperature of T_{wb} = ***100°C*** on the west boundary, T_{sb} = ***200°C*** on the south boundary, T_{eb} = ***300°C*** on the east boundary, and T_{nb} = ***400°C*** on north boundary.

   i. Develop a computer program for *explicit methods on a uniform* 2-D Cartesian grid. Use the steady state stopping criterion for non-dimensional temperature as

   $$\left(\frac{\partial \theta}{\partial \tau}\right)_{i,j} = \frac{l_c^2}{\alpha \Delta T_c}\left(\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t}\right)_{\max \text{ for } i,j} \le \varepsilon_{st} \left\{ l_c = L_1 \; \& \; \Delta T_c = T_{nb} - T_{wb} \right\}$$

   ii. Present a CFD application of the code for a volumetric heat generation of **0** and **100 kW/m³**. Consider maximum number of grid points as ***imax×jmax=11×11*** and the steady state convergence tolerance as $\varepsilon_{st}=10^{-4}$. Plot the steady state temperature contours with and without volumetric heat generation.

2. **Implicit Method-based Solution of 2D unsteady state heat conduction:**

   Consider 2D conduction in a square shaped (L_1=1m and L_2=1m) long stainless-steel plate. The plate is initially at a uniform temperature of ***30°C*** and is suddenly subjected to a constant temperature of T_{wb} = ***100°C*** on the west boundary, insulated on the south boundary, constant incident heat flux of q_W = ***10 kW/m²*** on the east boundary, and ***h=100 W/m².K*** and T_∞ = ***30°C*** on north boundary.

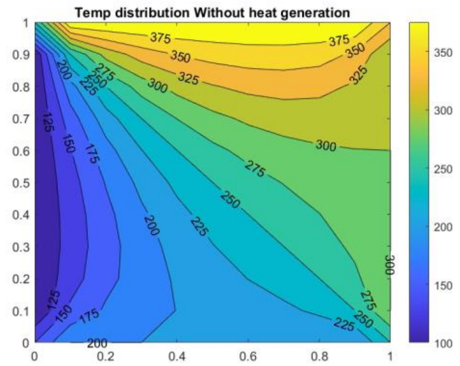
   i. Develop a Gauss-Seidel method-based computer program for the *implicit method on a uniform* 2-D Cartesian grid. Use the stopping criterion presented in the previous problem, with $\Delta T_c = T_{wb} - T_\infty$.

   ii. Using the program, present a CFD application of the code for a volumetric heat generation of **0** and ***50 kW/m³***. Consider the convergence tolerance as $\varepsilon_{st}=10^{-4}$ for the steady state, and $\varepsilon=10^{-4}$ for iterative solution by the Gauss-Seidel method. Plot the steady state temperature contours with and without volumetric heat generation.

Answer Sheet

Problem # 1: Explicit Method-based Solution of 2D unsteady state heat conduction:

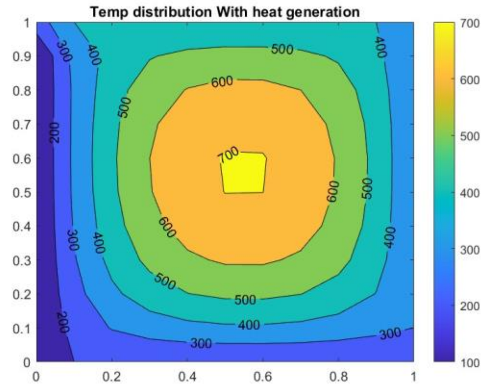
Plot the steady state temperature contours with and without volumetric heat generation. (2 figures).



(a)

(b)

Fig. od base ithout a



y

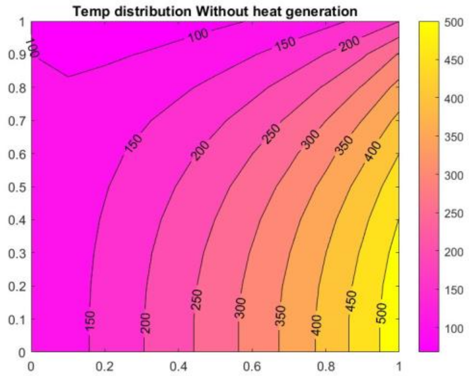
Discussion on the Fig. 1.1: Write your answer, limited inside this text box only

As expected without heat generation, the maximum temperature is seen at the north boundary. With heat generation, we have maximum temperature in middle, more than boundary values. Isothermal lines tend to be parallel to walls at it approaches walls due to dirchlett boundary condition

Problem # 2: Implicit Method-based Solution of 2D unsteady state heat conduction:

Plot the steady state temperature contours with and without volumetric heat generation. (2 figures).

Fig. 1.

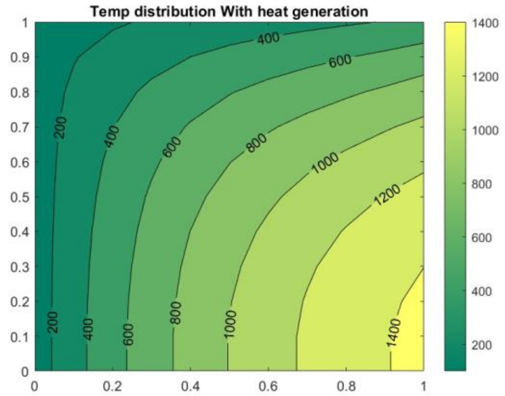


(a)

(b)

ba

(a) v



Discussion on the Fig. 1.2: Write your answer, limited inside this text box only

We used an inner loop and outerloop architecture of code to solve this problem in implicit fashion. We can see that iso thermal lines are parallel at east boundary as we have constant heat flux and they are perpendicular to the boundary at south wall because of insulation(no heat flux). Isothermal lines tend to be parallel towards west wall as its at constant temperature meanwhile heat flux at north boundary ,matches convection losses. Initial condition was taken to be 30 deg C