DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

ME704: Computational Methods in Thermal & Fluid Engineering **Autumn 2023**

Assignment # 2: Numerical Differentiation and Integration **Instructor**: Prof. Atul Sharma

Date Posted: September 7 Due Date: Sept. 13 (Wed., Early Morning 2 AM); midnight Sept. 12.

VIVA on: 13th September, Wednesday, 4:30-6:30 PM, Computation Lab, Mech. Engg. Dept.

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Create a single zipped file consisting of (a) filled-in answer sheet (this doc file—converted into a pdf file), and (b) all the computer programs. The name of the zipped file should be **rollnumber_A2**

---

1. **Numerical Differentiation**:

Consider 2D heat conduction in a **aluminum** plate (thermal conductivity ***k=237 W/m.K***) of dimension ***1m×1m*** subjected to a non-dimensional temperature of $\theta=1$ on the top surface and $\theta=0$ on all the other surfaces; shown in slide#9 of the lecture slide for the Topic#4. Grid is generated by the intersection of 10 uniformly spaced horizontal and vertical lines, with a grid size of 11×11 and the width in-between the lines $\Delta x=\Delta y=0.1m$. The temperature distribution at the grid points, near the top surface, is shown in slide#10. Store this tabular data in a data file and develop a computer program to compute local heat flux $q_y(x)$ at the top wall by I-, II-, and III-order one-sided differentiation formula (Slide#8).

For the various order of differentiation, report the results as follow:

a) Fill-in the table below
b) Plot the variation of the local heat flux $q_y(x)$ with increasing ***x (as in slide#12) and*** discuss the variation of the results in the figure.

2. **Numerical Integration**:

For the previous problem, store the results for the II-order numerical differentiation (in a data file) and develop a program to obtain the total rate of heat transfer from the top surface $Q=\int_{0.1}^{0.9} q_y(x)\,dx$ using (a) trapezoidal rule and (b) Simpson's 1/3rd rule. Present your result in the table below and discuss the result.

Answer Sheet

Problem # 1: Numerical Differentiation:

a) Fill in the table below.

$q_y(x)$ →	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9
I-Order	24421	15410	11632	9992	9527	9997	11632	15140	24421
II-Order	31405	17931	12696	10575	9999	10584	12694	17929	31403
III-Order	34521	18396	12631	10423	9847	10439	12626	18390	34515

b) Plot the variation of the local heat flux ***$q_y(x)$***, with increasing ***x (as in slide#12)***

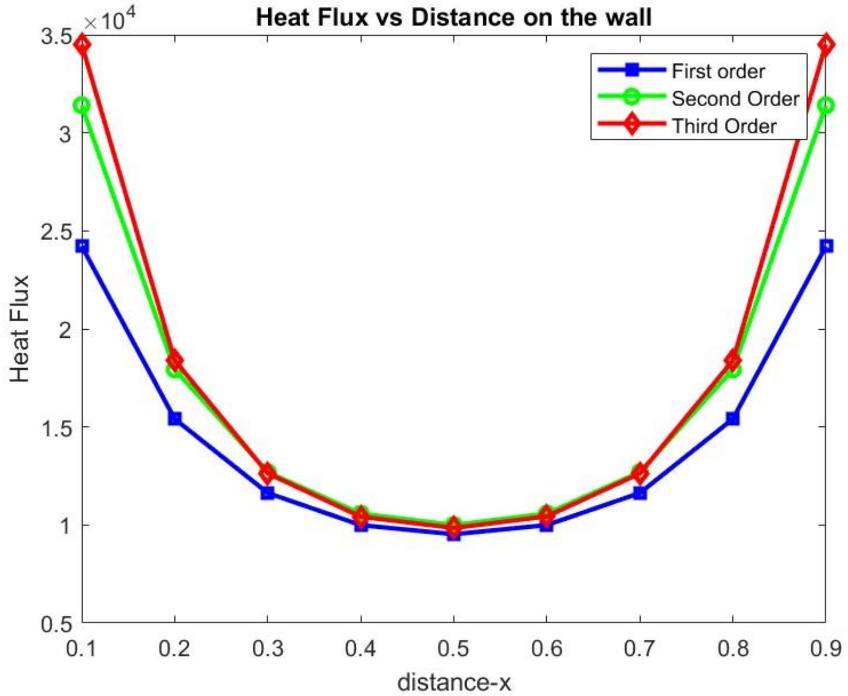

Fig 2.1: for the I-, II- and III-order numerical differentiation, variation of conduction heat flux along the top surface of a 2D plate.

Discussion on Fig 2.1: For tabulating the flux, we have assumed that the wall temperature of top wall is 10^0 C, and all other 3 walls are at room temperature 30^0 C. Since we need flux at top wall, backward methods are feasible and for I order, we took 2 nodes along y direction, linearly interpolated them and used the derivative of that at wall as our approximation. Similarly, for II and III order we took 3 and 4 nodes to interpolate the data using quadratic and cubic polynomials respectively and computed the analytical integral at top wall nodes to get the approximate. As the graph suggests II and III order are closer to each other, hinting convergence and higher order means more accurate results and more expensive computation. (All values in SI units!)

Problem # 2: Numerical Integration: Using the above results from the II-order numerical differentiation, fill-in the table below and discuss.

Rate of heat transfer at top-surface (Q_{top})		**Discussion of the table:** For computing integrals, we already knew the value of fluxes at specific points. We get the trapezoidal rule value if we do the analytical integral of piecewise (2 at a time) linear interpolation of the values at nodes. Similarly, Simpson's 1/3 rule is same process but by piecewise quadratic interpolation (3 datapoints at a time).
Trapezoidal Rule	***12381.2355***	
Simpsons 1/3 Rule	***19379.964***	