DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

ME704: Computational Methods in Thermal & Fluid Engineering **Autumn 2023**

Assignment # 4: FDM-based CFD Development for 1D Conduction **Instructor**: Prof. Atul Sharma

Date Posted: 26th September (Tuesday) Due Date: 04th Oct. (Wednesday, Early Morning 2 AM)

VIVA on: 4th October, Wednesday, 4:30-6:30 PM, Room No. F-24, Mech. Engg. Dept.

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Submit **separate** files for the (a) filled-in answer sheet (this doc file—converted into a pdf file **rollno.pdf**), and (b) all the computer programs (name of the files should be **rollno-p1.m** and **rollno-p2.m**).

1. **Steady State formulation-based 1-D Computational Heat Conduction Development, Application and Analysis**

   Consider 1D steady-state heat conduction in a long stainless-steel sheet of thickness ***L=1 cm***. The sheet is subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $T_{eb} = 100^oC$ on east boundary.

   Using the **Gauss-Seidel method** for the iterative solution along with an initial guess of 50^oC, develop a computer program *on a uniform* 1-D Cartesian grid. Present a testing of the code for a volumetric heat generation of 0 and 100, MW/m^3. Consider maximum number of grid points as ***imax=21*** and the convergence tolerance for the iterative solution as $\varepsilon=10^{-3}$. Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution.

2. **Unsteady State formulation-based 1-D Computational Heat Conduction Development, Application and Analysis:**

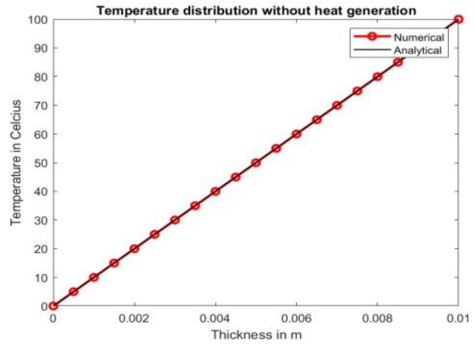
   Consider 1D conduction in a long stainless-steel sheet (density ρ: **7750 kg/m3**, specific-heat c_p: **500 J/Kg K**, thermal-conductivity k: **16.2 W/m-K**) of thickness ***L=1 cm***. The sheet is initially at a uniform temperature of **30^{o}C** and is suddenly subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $h=1000\ W/m^2.K$ and $T_\infty=100^oC$ on east boundary.

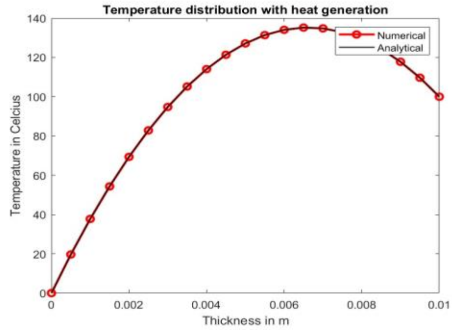
   Using the **explicit method** along with $\Delta t<0.5\Delta x^2/\alpha$, develop a computer program *on a uniform* 1-D Cartesian grid. Present a testing of the code for a volumetric heat generation of 0 and 100, MW/m^3. Consider maximum number of grid points as ***imax=21*** and the convergence tolerance as $\varepsilon_{st}=10^{-3}$ for the steady state convergence. Plot the steady state temperature profiles with and without volumetric heat generation and compare with the analytical solution.

Answer Sheet

Problem # 1: Steady State formulation-based 1-D Computational Heat Conduction Development, Application and Analysis

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the analytical solution. (2 figures).





(a)

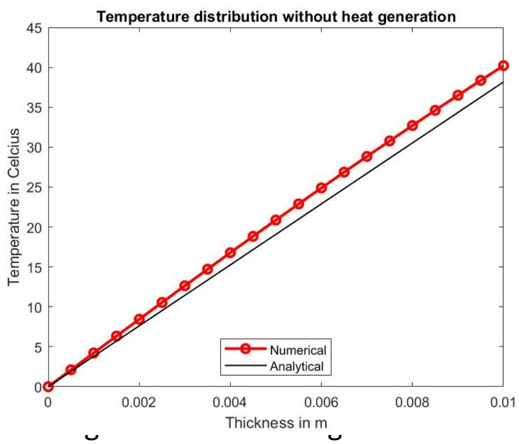
(b)

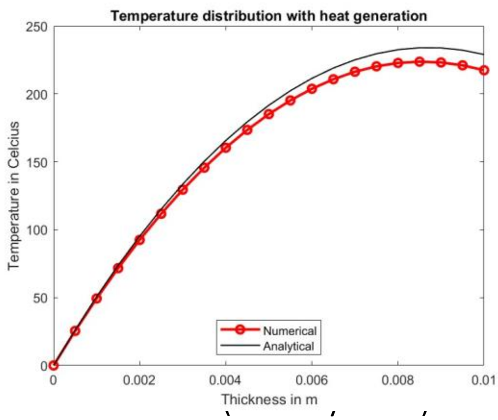
Fig. … on-based … heat conduction (a) without and (b) with heat generation. The figure shows comparison of the numerical (circle symbols) with the analytical (solid line) solution for the steady state temperature profile.

Discussion on the Fig. 1.1: Write your answer, limited inside this text box only
For conductivity, I have used the K = 16.2 W/mK , as given for stainless steel in Q2. Now without heat generation, we have a laplace equation, whose analytical solution is a linear function as shown in the black lined figure, meanwhile with a spatially constant heat generation term, the analytical solution is supposed to be quadratic polynomial. Without heat generation, maximum temperature occurs at boundary. Meanwhile with constant heat generation, the maximum temperature occurs in between in a skewed fashion.

Problem # 2: Unsteady State formulation-based 1-D Computational Heat Conduction Development, Application and Analysis:

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the analytical solution. (2 figures).





(a)

(b)

… ased 1D … an … e heat … at … parison … ne

Discussion on the Fig. 1.2: Write your answer, limited inside this text box only

The transient response shows a slow convergence to the analytical steady state. Given value of epsilon is not the best, but one must carefully tweek it with dt. As predicted, the constant head addition makes the temperature distribution parabolic and one without it tends to a linear distribution. We will observe a visbibly exact match when epsilon is 1e-6. In the explicit forward method, we had an array of temperatures at different locations which were updated at each time step dt using the linear discretised equation of heat transfer. Here dt was taken as 1/10th of maximum value.