

**DEPARTMENT OF MECHANICAL ENGINEERING**  
**INDIAN INSTITUTE OF TECHNOLOGY BOMBAY**

**ME704: Computational Methods in Thermal & Fluid Engineering** **Autumn 2023**

**Assignment # 3:** Solution of System of LAEs & Curve Fitting **Instructor:** Prof. Atul Sharma

**Date Posted:** 12<sup>th</sup> September (Tuesday) **Due Date:** 27<sup>th</sup> Sept. (Wednesday, Early Morning 2 AM)

**VIVA on:** 27<sup>th</sup> September, Wednesday, 4:30-6:30 PM, Room No. F-24, Mech. Engg. Dept.

**ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed):** Create a single zipped file consisting of (a) filled-in answer sheet (this doc file—converted into a pdf file), and (b) all the computer programs. The name of the zipped file should be **rollnumber\_A3**

1. During the numerical solution of one-dimensional heat conduction problem, with boundary condition as 0°C and 100°C and 6 grid points, a finite difference method-based algebraic formulation results in a tridiagonal-matrix form of the governing linear algebraic equations; given as

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} T_1 = 0 \\ 0 \\ 0 \\ T_6 = 100 \end{bmatrix}$$

Develop computer programs to compute the temperature at interior grid points ( $T_i$ , with  $i = 2, 3, 4, 5$ ) using (a) Tridiagonal matrix algorithm, (b) Jacobi method, and (c) Gauss-Seidel method. For (b) and (c), obtain the *temperatures* with an initial guess of 50°C and a convergence criteria  $\epsilon = 10^{-4}$ . Thereafter, for (b) and (c), compare the number of iterations for convergence with an initial guess of (i) 0°C (ii) 50°C and (iii) 100°C. Enter your answers in a table, given as

Methods	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>		No. of Iterations		
TDMA	20	40	60	80	Initial-Guess →	0°C	50°C	100°C
Jacobi	20.0000	40.0000	59.9999	79.9999	Jacobi	61	61	62
Gauss-Seidel	19.9999	39.9998	59.9999	79.9999	Gauss-Seidel	32	28	30

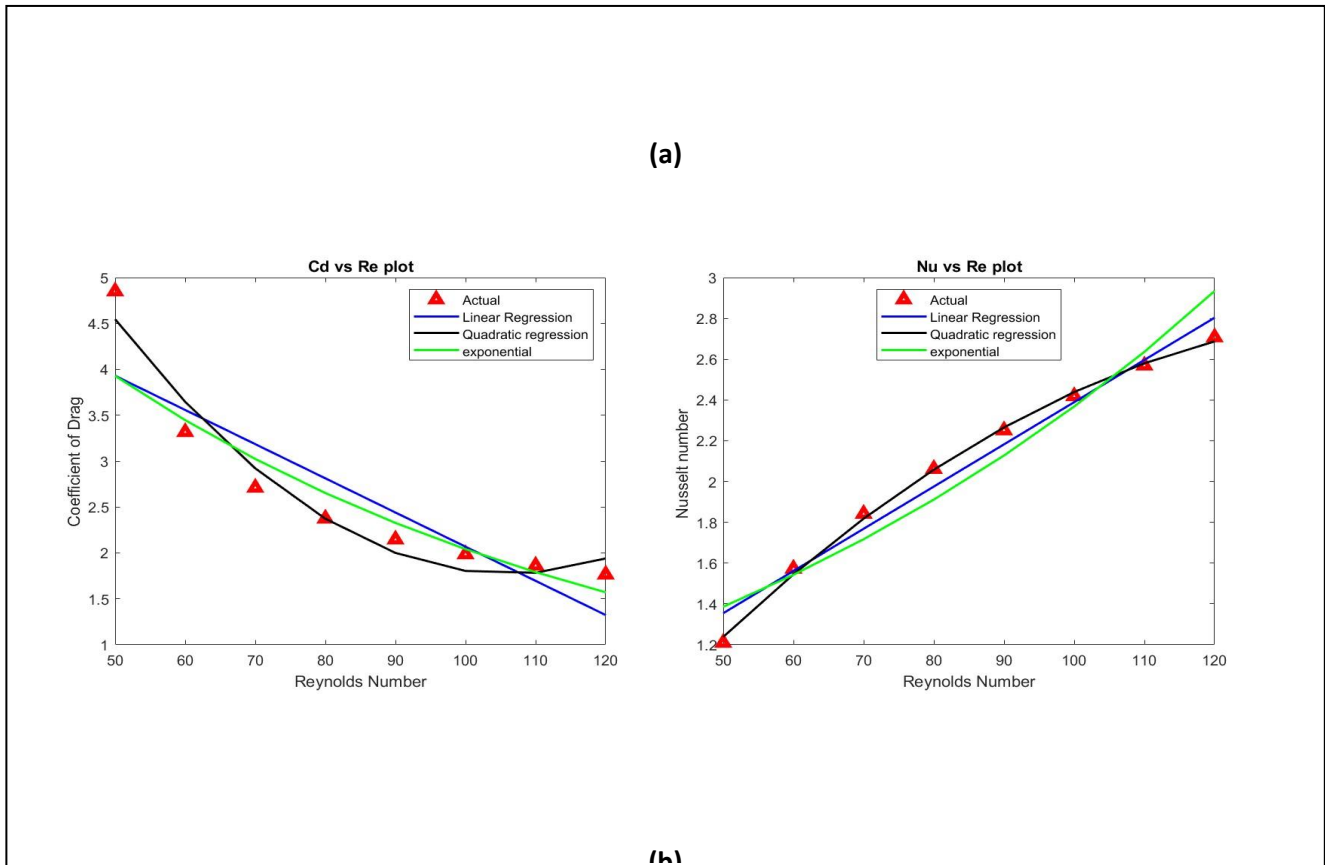
2. From a numerical simulation of flow and heat transfer across a cylinder, the variation of (a) mean drag coefficient **C<sub>D</sub>** and (b) mean Nusselt number **Nu** with Reynolds number **Re** is obtained as:

Re	50	60	70	80	90	100	110	120
C <sub>D</sub>	4.8504	3.3168	2.7111	2.3742	2.1465	1.9854	1.8618	1.7649
Nu	1.2095	1.5727	1.8417	2.0619	2.2510	2.4185	2.5689	2.7072

- (a) Obtain the expressions  $C_D = f(Re)$  and  $Nu = f(Re)$  using various least square methods: linear, exponential ( $C_D = ae^{-bRe}$  &  $Nu = ae^{+bRe}$ ), and II order polynomial. Enter your fitted expressions in a table, given as

	Linear	Exponential	Quadratic
$C_D = f(Re)$	$5.7915 - 0.0372 \cdot Re$	$7.5607e^{(-0.0131Re)}$	$11.6909 - 0.1869Re + 0.0009Re^2$
$Nu = f(Re)$	$0.3198 + 0.0207 \cdot Re$	$0.8113e^{(0.0107Re)}$	$-0.7915 + 0.0489Re - 0.0002Re^2$

- (b) Draw an overlapping plot of the three functional relationships of  $C_D = f(Re)$  and the numerical results (shown in the table above) for  $Re$  varying from 50 to 120. Draw similar overlapping plot for  $Nu = f(Re)$  and discuss the results as follows:



#### Discussion on the Fig. 3.1:

For the  $C_D$  vs  $Re$  function and  $Nu$  vs  $Re$  function, Quadratic seems to be the best fit. Moreover all 3 fits have been able to show the overall trend, increasing or decreasing of the corresponding data tabulated in above question. Linear and Exponential fit was solved by optimising the error (least squared sum) over two parameters meanwhile quadratic needed 3 parameters. More the number of parameters, better fit we get. On a special note; exponential fit problem can be converted into a linear regression problem by taking logarithm of  $y(C_D \text{ or } Nu)$ .