Universidade Estadual de Campinas
Instituto de Computação

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The Dissertation or Thesis Title in English

Título da Dissertação ou Tese em Português

CAMPINAS
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Dissertação apresentada ao Instituto de Computação da Universidade Estadual de Campinas como parte dos requisitos para a obtenção do título de Mestra em Ciência da Computação.

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Supervisor/Orientador: Prof. Dr. Ricardo Dahab

Este exemplar corresponde à versão da Dissertação entregue à banca antes da defesa.

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Resumo

O resumo deve ter no máximo 500 palavras e deve ocupar uma única página.

Abstract

The abstract must have at most 500 words and must fit in a single page.

Contents

Chapter 1

Introduction

Chapter 2

Mathematical Background

2.1 Groups

Definition 2.1.1. A **group** is a set G closed under a binary operation $\cdot$ defined on G such that:

- **Associativity:** $\forall a, b, c \in G,\ a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- **Identity element:** $\exists e \in G\ ;\ \forall a \in G,\ a \cdot e = e \cdot a = a$
- **Inverse element:** $\forall a \in G,\ \exists b \in G\ ;\ a \cdot b = b \cdot a = e$

And it is denoted by $\langle G, \cdot \rangle$, or simply G if the operation is implied.

Definition 2.1.2. A group is said to be **commutative** or **abelian** if $\forall a, b \in G,\ a \cdot b = b \cdot a$

A group is called **additive** or **multiplicative** if its operation is addition or multiplication, respectively.

Definition 2.1.3. A subset H of G is a **subgroup** of $\langle G, \cdot \rangle$ if it is closed under $\cdot$ induced by $\langle G, \cdot \rangle$.

Definition 2.1.4. The **order** of a group $\langle G, \cdot \rangle$ is the cardinality of the set G.

Definition 2.1.5. A subgroup H of G can be used to decompose G in uniform sized and disjoints subsets called **cosets**. Given an element $g \in G$:

- A **left coset** is defined by $gH := \{g \cdot h\ ;\ h \in H\}$
- A **right coset** is defined by $Hg := \{h \cdot g\ ;\ h \in H\}$

2.2 Rings and Fields

Definition 2.2.1. A **ring** is a set together with two binary operations, we will note by $+$ and $*$ and call it addition and multiplication, respectively, such that:

- $\langle R, + \rangle$ is an abelian group.

- $*$ is associative
- $*$ is distributive over $+$

And it is denoted by $\langle R, +, * \rangle$, or simply G if the operations are implied.

Definition 2.2.2. A ring is said to be **commutative** if its $*$ operation is commutative.

Definition 2.2.3. A ring is said to be **with unity** if $*$ has a identity element. We shall note it by 1 and it is called **unity**.

Definition 2.2.4. A **division ring** is a ring R where $\forall r \in R,\ \exists s \in R\ ;\ r * s = 1$.

Definition 2.2.5. A **field** is a commutative division ring.

2.3 Lattices

Definition 2.3.1. A Lattice $\Lambda \subset \mathbb{R}^n$ is a subgroup of the additive group $\mathbb{R}^n$

In other words, given m linear independent vectors in $\mathbb{R}^n$, the set $\{v_1, v_2, ..., v_m\}$ is called a **basis** for Λ and the Lattice may defined by:

Definition 2.3.2.

$$\Lambda := \left\{ x = \sum_{i=1}^{m} \lambda_i v_i \in \mathbb{R}^n \mid \lambda_i \in \mathbb{Z} \right\} \tag{2.1}$$

2.4 Number Fields

Definition 2.4.1. Let K and L be two fields, L is said to be a **field extension** of K if $L \subseteq K$ and we denote it by L/K

Note that in a field extension L/K, L has a structure of a vector space over K, where vector addition is in L and scalar multiplication $a \in K,\ v \in L \implies av \in L$. The dimension of L as a vector space is called **degree** and it is denoted by $[L : K]$.

Definition 2.4.2. A field extension is called **number field** when it is over $\mathbb{Q}$.

Definition 2.4.3. Let $\alpha \in L$ where L/K is a field extension. We say that α is **algebraic over** K if $\exists p \in K[X]\ ;\ p(\alpha) = 0$. p is said to be **the minimal polynomial of** α **over** K denoted by p_α. If $\alpha \in L = \mathbb{Q}[\theta]$, we simply call α an **algebraic number**.

Example 2.4.1. It is known that $\mathbb{Q}$ is a field. If we add $\sqrt{2}$ to the set, we can build a new field adding also all the powers and multiples of $\mathbb{Q}$. This new field is denoted by $\mathbb{Q}[\sqrt{2}]$, note that $\sqrt{2}$ is algebraic and its minimal polynomial $p_{\sqrt{2}} = x^2 - 2$. All elements of $\mathbb{Q}[\sqrt{2}]$ are in the form $\{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ and one of its basis is $\{1, \sqrt{2}\}$, so it has degree is 2.

Example 2.4.2. If we add $\sqrt[3]{2}$ to $\mathbb{Q}$ instead, its elements would have the form $\{a + b\sqrt[3]{2} + c\sqrt[3]{4} \mid a, b, c \in \mathbb{Q}\}$, so one of its basis is $\{1, \sqrt[3]{2}, \sqrt[3]{4}\}$, $p_\alpha = x^3 - 2$ and its degree is 3.

Theorem 2.4.1 (add font 45 p.40)**.** *If K is a number field, then $K = \mathbb{Q}[\theta]$ for some algebraic number $\theta \in K$, called primitive element.*

Then we conclude that $\{1, \theta, \theta^2, ..., \theta^{n-1}\}$ is a basis for the vector space $K = \mathbb{Q}[\theta]$ over $\mathbb{Q}$.

2.5 Twisted Embedding

Chapter 3

Hypothesis

Chapter 4

Results

Chapter 5

Conclusions