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Assignment

1) $y = A + Bx + Cx^2$

$(1,1), (2,-1), (3,1)$

$$[A\ b] = \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{array}\right] \quad R_3 \leftarrow R_3 - 2R_2$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$2C = 4$

$\therefore C = 2$

$B = -8$

$A = 7$

$\therefore y = 7 - 8x + 2x^2$

2) $$A = \left[\begin{array}{cccc} 2 & 5 & 2 & -5 \\ +4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{array}\right] \quad \begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 + 5R_1 \\ R_4 \leftarrow R_4 - 5R_1 \end{array}$$

$$A = \left[\begin{array}{cccc} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{array}\right] \quad \begin{array}{l} R_3 \rightarrow R_3 + 2R_2 \\ R_4 \rightarrow R_3 + 2R_2 \end{array}$$

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$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 4 \end{bmatrix} \quad R_4 \to R_4 - 3R_3$$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

(upper Δ)

L = coeff of operation

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3) $T : \mathbb{R}^3 \to \mathbb{R}^3$

$T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z)$

i) Standard basis of $\mathbb{R}^3$

$= (1, 0, 0), (0, 1, 0), (0, 0, 1)$

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~~Column vectors~~: Row vec

$$\begin{aligned} x + 2y + z &= 1 \quad 2 \quad 1 \\ y + z &= 0 \quad 1 \quad 1 \\ x + y - 2z &= 1 \quad 1 \quad -2 \end{aligned}$$

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\begin{aligned} T(1,0,0) &= (1,0,1) \\ T(0,1,0) &= (2,1,1) \\ T(0,0,1) &= (-1,1,-2) \end{aligned}$$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \qquad R_3 \leftarrow R_3 - R_1$$

Reducing T: $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \qquad R_3 \leftarrow R_3 + R_2$

$$T: \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $C(T) = \{(1,0,1), (2,1,1)\}$

$\therefore$ Basis for $C(T^T) = \{(1,2,-1), (0,1,1)\}$

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Reducing T further for N(T) :

$$T : \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - 2R_2$$

$$= \begin{array}{c} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} x & y & z \end{matrix} \end{array} \quad \cancel{R_2 \to R_2 + 1/3 R_1}$$

z is a free variable

$$\therefore N(T) = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

For $N(T^T)$:

$$T^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \cancel{R}$$

z is free

$$N(T^T) = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

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iii) Finding Eigen Values

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|T - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left((1-\lambda)(-2-\lambda) - 1\right)$$
$$\cancel{+2(1-\lambda)} \quad + 1(1-\lambda) \quad +2$$

$$\lambda^3 - 3\lambda = 0$$
$$\lambda(\lambda^2 - 3) = 0$$
$$\lambda = 0, \sqrt{3}, -\sqrt{3}$$

$\hookrightarrow$ Eigen values

For $\lambda = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From column space transform

$$\therefore \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{z = 0}$$

$$X_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \quad \text{(Null Space)}$$

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For $\lambda = \sqrt{3}$

$$\begin{bmatrix} 1-\sqrt3 & 2 & -1 \\ 0 & 1-\sqrt3 & 1 \\ 1 & 1 & -2-\sqrt3 \end{bmatrix} \quad R_3 \to R_3 - \frac{R_1}{1-\sqrt3}$$

$$\begin{bmatrix} 1-\sqrt3 & 2 & -1 \\ 0 & 1-\sqrt3 & 1 \\ 0 & \cancel{1 - \frac{2(1+\sqrt3)}{1-\sqrt3-2}}\ 2+\sqrt3 & -2-\sqrt3 + \frac{1+\sqrt3}{\cancel{1-\sqrt3-2}} \end{bmatrix} \quad \begin{array}{l} R_3 \to \cancel{R_3 + 1-\sqrt3} \\ R_3 \to R_3 + \cancel{\frac{1+\sqrt3}{(1-\sqrt3)^2}}\, R_1 \end{array}$$

$$= \begin{bmatrix} 1-\sqrt3 & 2 & -1 \\ 0 & 1-\sqrt3 & 1 \\ 0 & \cancel{-\frac{1-\sqrt3}{1-\sqrt3}}\ 2+\sqrt3 & -\left(\frac{5+3\sqrt3}{2}\right) \end{bmatrix} \quad \cancel{R_3}\ R_3 \to R_3 - \left(\frac{2+\sqrt3}{1-\sqrt3}\right) R_2$$

$$= \begin{bmatrix} 1-\sqrt3 & 2 & -1 \\ 0 & 1-\sqrt3 & 1 \\ 0 & 0 & -\left(\frac{5+3\sqrt3}{2}\right) + \frac{(2+\sqrt3)(1+\sqrt3)}{2} \end{bmatrix}$$

$$= 0$$

$$R_1 \to R_1 - \frac{2}{1-\sqrt3} R_2$$

$$= \begin{bmatrix} 1-\sqrt3 & 0 & -1 - \frac{2}{1-\sqrt3} \\ 0 & 1-\sqrt3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\cancel{X_2 = \begin{bmatrix} 1 \cancel{+2} \end{bmatrix}}$$

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$$= \begin{bmatrix} 1-\sqrt{3} & 0 & \cancel{-1} \;\; \frac{\cancel{2}(1+\sqrt{3})}{-\cancel{2}} \overset{\sqrt{3}}{\parallel} \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{3}-1} \\ \frac{1}{\sqrt{3}-1} \\ 1 \end{bmatrix}$$

For $\lambda = -\sqrt{3}$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix} \quad R_3 \to R_3 - \frac{1}{1+\sqrt{3}} R_1$$

$$= R_3 + \frac{1-\sqrt{3}}{2} R_2$$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & 1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 2-\sqrt{3} & -2+\sqrt{3} + \frac{1-\sqrt{3}}{\cancel{1+\sqrt{3}}\,2} \end{bmatrix}$$

$$= \begin{bmatrix} 1+\sqrt{3} & 2 & 1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 2-\sqrt{3} & \frac{+3+3\sqrt{3}}{2} \end{bmatrix} \quad R_3 \to R_3 - \frac{2-\sqrt{3}}{1+\sqrt{3}} R_2$$

$$R_3 = R_3 + \left(\frac{(2-\sqrt{3})(1-\sqrt{3})}{2}\right) R_2$$

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$$\begin{bmatrix} 1+\sqrt{3} & 2 & 1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & \frac{-\sqrt{3}+3\sqrt{3}}{2} + \frac{\sqrt{3}-3\sqrt{3}}{2} \end{bmatrix}$$

$$\begin{matrix} \| \\ 0 \end{matrix}$$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - \frac{2}{1+\sqrt{3}} R_2$$

$$= R_1 + 1-\sqrt{3}\, R_2$$

$$= \begin{bmatrix} 1+\sqrt{3} & 0 & \cancel{2}-\sqrt{3} \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad x = \frac{+\sqrt{3}}{1+\sqrt{3}} z \quad y = \frac{-1}{1+\sqrt{3}} z$$

$$\cancel{X_3 =} \quad \cancel{x = \frac{-2+\sqrt{3}}{1+\sqrt{3}} z} \quad \cancel{= \frac{(\sqrt{3}-2)(1-\sqrt{3})}{-2}}$$

$$\cancel{y = \frac{-1}{1+\sqrt{3}} z} \quad \cancel{= \frac{\sqrt{3}-3-2+2\sqrt{3}}{2}}$$

$$X_3 = \begin{bmatrix} \sqrt{3}/1+\sqrt{3} \\ -1/1+\sqrt{3} \\ 1 \end{bmatrix} \quad \cancel{z}$$

$$X_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} \sqrt{3}/\sqrt{3}-1 \\ 1/\sqrt{3}-1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}+3}{2} \\ \frac{\sqrt{3}+1}{2} \\ 1 \end{bmatrix}$$

$\lambda_1 = 0$
$\lambda_2 = \sqrt{3}$
$\lambda_3 = -\sqrt{3}$

$$X_3 = \begin{bmatrix} \sqrt{3}/1+\sqrt{3} \\ -1/1+\sqrt{3} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3-\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} \\ 1 \end{bmatrix}$$

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iv) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$ $a = (1, 0, 1)$ $b = (2, 1, 1)$ $c = (-1, 1, -2)$

$q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{2}}(1, 0, 1)$

$q_2 = \frac{B}{\|B\|}$ & $B = b - (q_1^T b) q_1$

$q_1^T b = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$b - \frac{3}{\sqrt{2}} q_1$

$= \left(\frac{1}{2}, 1, \frac{-1}{2}\right)$

$q_2 = \frac{2}{\sqrt{6}}\left(\frac{1}{2}, 1, \frac{-1}{2}\right) = \frac{1}{\sqrt{6}}(1, 2, -1)$

$q_3 = \frac{C}{\|C\|}$ where $C = c - (q_2^T c) q_2 - (q_1^T c) q_1$

$q_1^T c = \frac{-3}{\sqrt{2}}$

$q_2^T c = \frac{3}{\sqrt{6}}$

$C = (-1, 1, -2) - \frac{3}{6}(1, 2, -1) + \frac{3}{2}(1, 0, 1)$

$= \left(1 - \frac{1}{3} + \frac{3}{2}, \; 1 - 1, \; -2 + \frac{1}{2} + \frac{3}{3} \right)$

$q_3 = ([illegible], 0, 0)$

$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/6 & 0 \end{bmatrix}$$

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4)

x	-4	1	2	3
y	4	6	10	8

$$c - 4d = 4$$
$$c + d = 6$$
$$c + 2d = 10$$
$$c + 3d = 8$$

$$\underbrace{\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c \\ d \end{bmatrix}}_{\hat{x}} = \underbrace{\begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}}_{b}$$

$$\hat{x} = (A^T A)^{-1} . A^T b$$

$$A^T A = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{58} \begin{bmatrix} 15 & -1 \\ -1 & 2 \end{bmatrix}$$

$$(A^T A)^{-1} . A^T = \frac{1}{58} \begin{bmatrix} 19 & 14 & 13 & 12 \\ -9 & 1 & 3 & 5 \end{bmatrix}$$

$$(A^T A)^{-1} . A^T . b = \begin{bmatrix} \frac{193}{29} & \frac{20}{29} \end{bmatrix}$$

$$\therefore y = \frac{193}{29} + \frac{20x}{29}$$

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5) $x_1 + x_2 + 3x_3 + 4x_5 = 0$

$x_1 = -x_2 - 3x_3 - \cancel{4x_5} - 0x_4 - 4x_5$

~~$A = -1 \quad -3 \quad 0 \quad -4$~~

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$+ x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow$$ Not considering as it isn't in the equation

$$A = \begin{bmatrix} -1 & -3 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \cancel{0} & \cancel{0} & & \cancel{0} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = A(A^T A)^{-1} A^T \qquad (A^T A)^{-1} = \frac{1}{27} \begin{bmatrix} 26 & -3 & -4 \\ -3 & 18 & -12 \\ -4 & -12 & 11 \end{bmatrix}$$

$$= \frac{1}{27} \begin{bmatrix} 26 & -1 & -3 & -4 \\ -1 & 26 & -3 & -4 \\ -3 & -3 & 18 & -12 \\ -4 & -4 & -12 & 11 \end{bmatrix}$$

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$$Q = I - P = \frac{1}{27}\begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix}$$

6) $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$ $R_2 \leftarrow R_2 - \frac{2R_1}{a}$

$R_3 \leftarrow R_3 - \frac{2R_1}{a}$

$A = \begin{bmatrix} a & 2 & 2 \\ 0 & a - 4/a & 2 - 4/a \\ 0 & 2 - \frac{4}{a} & a - \frac{4}{a} \end{bmatrix}$ $R_3 \leftarrow R_3 - \left(\frac{\cancel{2a-4}}{\cancel{a^2-4}}\right) R_2$

$A = \begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & 2a - \cancel{4}/a \\ 0 & 0 & K \end{bmatrix}$

$K = \frac{a^2-4}{a} - \frac{(2a - 4/a)^2}{a^2-4} a$

$a > 0$, $\frac{a^2-4}{a} > 0$

$a \in (-\infty, -2) \cup (2, \infty)$

$\frac{a^2-4}{a} - \frac{\cancel{a}^2 \cdot (2a - 4/\cancel{a})^2 \cancel{a}}{a(a^2-4)} > 0$

$\frac{4a^2 + 16 - 16a}{a(a^2-4)} > 0$

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$(a^2-4+2a-4)(a^2-4-2a+4) > 0$

$(a-2)(a+4) > 0$ & $a(a-2) > 0$

$\therefore a > 0, a > 2$

$\therefore a > 2$

$a \in (2, \infty)$

ii) $f = x^T A x$

$f = 2(x_1^2 + x_2^2 + x_3^2 - x_1 x_1 - x_2 x_3)$

$x = (x_1 \; x_2 \; x_3)$

$$(x_1 \quad x_2 \quad x_3) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$= a_{11} x_1^2 + a_{22} x_2^2 + a_{33} x_3^2 + 2a_{12} x_1 x_2 + 2a_{13} x_1 x_3 + 2a_{23} x_2 x_3$

$a_{11} = 2$

$a_{22} = 2$

$a_{33} = 2$

$a_{12} = -1$

$a_{13} = 0$

$a_{23} = -1$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

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7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$A_{3\times2} = U_{3\times3} \; \Sigma_{3\times2} \; V^T_{2\times2}$

$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

Eigen Value $= \begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$

$\lambda(\lambda - 90) = 0$

$\lambda = 0, 90$

For $\lambda = 0$

$\begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 81 & -27 \\ 0 & 0 \end{bmatrix}$

$x_1 = \begin{bmatrix} +1/3 \\ 1 \end{bmatrix}$

For $\lambda = 90$

$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \rightarrow \begin{bmatrix} -9 & -27 \\ 0 & 0 \end{bmatrix}$

$x_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

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$$V_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} \quad V_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

↑ Normalized eigen vectors

$$\cancel{V = \begin{bmatrix} 1/\sqrt{10} & +3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}} \quad \cancel{\sigma_1 = \sqrt{90} \quad \sigma_2 = 0}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} \quad \sigma_1 = \sqrt{90} \quad \sigma_2 = 0$$

$$\Sigma = \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Eigen values of AA^T = 90, 0, 0

$$u_1 = \frac{A v_1}{\sigma_1} = \frac{\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}}{\sqrt{90}} = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$$

U_2, U_3 are orthogonal to u_1

$\therefore$ They are soln of $u_1^T x = 0$

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$$\begin{bmatrix} 1/3 & -2/3 & -2/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$= \begin{bmatrix} +2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$\quad a_2 \quad a_3$

$u_2 = \frac{a_2}{||a_2||} = \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0$

$u_3 = a_2 - (u_2{}^t a_2) u_2$

$$= \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \frac{4}{\sqrt{5}} \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix} = \begin{bmatrix} 2/5 \\ -4/5 \\ 1 \end{bmatrix}$$

$$= \frac{\sqrt{5}}{3} \begin{bmatrix} 2/5 \\ -4/5 \\ 1 \end{bmatrix} = \left(\frac{2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}} \right)$$

$$\underset{A}{\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & 5/3\sqrt{5} \end{bmatrix}} \underset{\Sigma}{\begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \underset{V^T}{\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}$$