CSE5311 – 011 DESIGN AND ANALYSIS OF ALGORITHMS PROJECT 1 REPORT

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## **Introduction:**

For this project, we have implemented Insertion Sort, Merge Sort and Quick sort algorithms to sort the integers from four text files which has different number of random integers. The integers from the file will be read and sorted using the algorithms and sorted array will be displayed as the output and the time it takes to sort the array from different files.

## **1.1 Individual Contribution:**

Vignesh Manikandan: Worked on Quicksort, Merge sort and project report.

Ruban Eswaravelu: Worked on Insertion sort, Merge sort and project report.

Both members of the group gave equal contributions on merge sort and report and individually worked on quicksort and insertion sort.

# **2. Time complexities of the algorithms:**

## **2.1 Insertion Sort:**

Worst Case : O(n^2)

Average Case : O(n^2)

Best Case : O(n)

## **2.2 Merge Sort:**

Worst Case : O(n log n)

Average Case : O(n log n)

Best Case : O(n log n)

## **2.3 Quick Sort:**

Worst Case : O(n^2)

Average Case : O(n log n)

Best Case : O(n log n)

# **3. Output**

## **3.1 InsertionSort.java**

Time taken to sort array of size 4000: 13762000ns

Time taken to sort array of size 20: 30000ns

Time taken to sort array of size 100: 108200ns

Time taken to sort array of size 1000: 2901800ns

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## **3.2 MergeSort.java**

Time taken to sort array of size 4000: 2340200ns

Time taken to sort array of size 20: 40300ns

Time taken to sort array of size 100: 65000ns

Time taken to sort array of size 1000: 1330800ns

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## **3.3 QuickSort.java**

Time taken to sort array of size 4000: 1444200ns

Time taken to sort array of size 20: 44900ns

Time taken to sort array of size 100: 91200ns

Time taken to sort array of size 1000: 673400ns

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# **4. Results:**

We are measuring the time taken by the algorithm for sorting the integers by nanoseconds. The time taken by each of the algorithms to sort different sizes of arrays are in the below table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **20** | **100** | **1000** | **4000** |
| **Insertion Sort** | 30000ns | 108200ns | 2901800ns | 13762000ns |
| **Merge Sort** | 40300ns | 65000ns | 1330800ns | 2340200ns |
| **Quick Sort** | 44900ns | 91200ns | 673400ns | 1444200ns |

From the table we can see that for small size array insertion sort has performed better than the other algorithms. For the large sizes of array quick sort algorithm have taken less time to sort them when compared with other algorithms. In the case of array size 100, the merge sort algorithm has performed better than the other two because of the size of the array.

# **5. Unsorted arrays used for sorting:**

## **5.1 arr20.txt:**

913,749,4929,784,2909,5370,104,3781,1128,3068,3897,1447,1441,2730,3082,3760,2787,3160,3288,5086.

## **5.2 arr100.txt**

2018,1388,4865,2335,1009,1195,4227,5535,2531,5547,2080,2272,3123,1940,2296,3158,5668,5264,1221,2367,3473,2465,2936,3382,1510,5301,1397,3353,952,5503,5508,1938,2337,1060,1895,136,594,730,2737,1454,5708,655,4046,5779,2055,1748,1743,3538,3426,1617,2348,4106,3320,24,2360,22,5461,274,3295,5357,1483,4595,1970,492,1406,5327,3887,2770,3924,1762,1558,2769,5588,4632,1279,1491,4722,602,1190,476,3365,3735,4525,4717,2635,1525,4642,3360,1759,4161,2882,5465,506,2672,364,5485,4953,991,2632,3875.

## **5.3 arr1000.txt**

5391,4433,1533,5979,769,2751,5891,5428,2284,3837,1349,2462,3723,130,4347,4994,1753,2105,1344,4811,3699,3563,2420,5100,5326,2139,1867,4406,879,1184,1768,4321,3968,4203,1482,3508,2984,2191,623,1184,483,3593,5491,923,412,2135,2920,497,174,1556,3270,5593,2381,2158,3658,4488,5600,231,1660,1253,2638,5709,2485,2658,4762,1887,5099,2838,2830,1748,2939,5757,3564,4240,2716,4741,1135,4991,4321,2750,4873,3105,101,605,5213,3171,662,3221,16,3178,2959,3788,4085,4894,2738,5055,4063,3197,5346,5138,3017,5067,4597,4852,5704,1282,731,4248,4998,4450,3559,1927,4095,1702,5556,5819,3891,5346,4155,2328,475,5862,2527,291,1853,2944,3951,742,5900,1769,352,3392,5104,5361,2738,3714,1939,3772,116,2735,1625,2880,29,3053,2665,3808,2023,3143,3896,423,1346,1846,1759,2982,1704,3821,1216,786,1233,4959,4093,5712,4810,36,3739,1643,2474,1436,599,4223,98,1464,1779,4857,477,3690,4788,682,720,820,1481,2116,1269,5882,720,1720,921,1963,5450,4423,5080,1393,1806,5632,2869,2086,404,4444,2968,2785,1225,4022,5822,4165,3468,2269,5455,3138,4348,3171,1333,5297,4165,4583,3611,5187,4951,5898,1727,4094,702,4779,4588,127,2598,4183,4537,5766,649,1040,3018,1512,4005,4339,1117,5045,1569,2916,3278,1122,717,5990,1161,607,4715,2399,5051,3221,961,1637,4309,130,2356,1050,4161,1079,2979,5094,1571,641,836,179,2140,1178,2695,4544,780,2297,4939,606,1491,3991,4056,1515,5771,1258,4647,36,1091,1725,1865,4530,509,4792,895,1559,1720,947,4458,3709,2880,5252,379,300,3158,1469,3053,2113,2336,1101,1027,759,5822,2404,112,1070,5135,4605,3208,4837,1393,5785,546,292,4354,1347,5269,2097,4413,1655,3100,3027,5415,2239,3483,1263,4278,4015,764,45,5315,135,5999,5502,636,4530,531,3713,5284,4732,5870,3484,4611,401,2345,932,69,880,3756,1777,3821,2290,4578,1234,2569,980,4383,5572,4682,1706,2868,3772,2020,3577,2342,272,2910,4741,4651,2518,1449,1839,4319,5981,4720,560,5233,410,5605,1041,4335,2387,670,4553,4724,4745,75,5715,4089,3096,5606,5515,2684,1571,5803,4610,3988,1916,3673,1798,4809,4372,711,1117,5947,1122,4863,800,5488,3798,5657,299,4542,3231,1040,1694,1581,736,3399,3179,4620,5302,924,810,4247,476,2831,430,2923,288,978,556,5146,3253,4147,1129,4838,3473,2670,4279,4877,3945,184,1705,4844,3487,2071,4966,2961,3188,638,5736,4752,2968,5238,1181,223,3693,1364,2755,1306,3351,4357,3147,5905,3025,2473,1696,2264,5838,1104,5343,1330,1639,436,141,5498,52,3174,5556,5492,4417,918,6,3248,3419,5451,1751,5353,1506,4523,1824,3085,5773,4779,1677,1237,5720,3196,1272,4671,5238,2788,3595,5122,3269,2472,1083,4916,4894,741,4039,5172,114,2286,4999,3609,3245,3864,5998,1360,5697,2556,3909,1106,1030,110,2424,2503,1321,5257,1826,5170,388,5841,1467,2542,1983,4180,4848,5537,3897,2354,5315,3424,3886,3113,5741,2306,4526,2749,67,3398,1450,2622,4315,1737,1007,4073,3524,3075,1063,5177,3228,2530,2667,1211,3282,5082,2357,5800,4358,3184,1217,488,3055,2648,3285,2983,2409,2525,2558,2440,602,1965,2772,51,3892,2977,3094,2703,4166,4643,206,4706,5121,3663,595,1512,3824,4106,4552,5947,2390,1935,5130,1732,1644,3272,1351,5098,964,5406,5081,912,2050,2337,1394,5122,3037,4072,786,2312,3291,3011,2329,1967,4311,3619,3809,5355,2410,5032,1978,5131,338,485,3376,3484,3623,2304,5649,2165,5276,2779,4995,871,3488,5494,1194,5330,3200,4633,1783,943,5779,5408,3114,2105,5642,3075,4532,5183,456,3010,4457,4778,462,4014,15,2171,4635,5392,803,5802,4756,1414,2591,331,935,2547,103,1162,3893,5249,5593,3332,142,1283,5639,5433,4377,3323,553,2603,544,2097,2882,1752,3648,1549,5777,2952,1864,4365,537,5427,5491,751,5277,2704,5180,669,3519,1367,1189,3602,626,1363,3727,613,1155,3073,82,2764,1703,1226,993,5937,1382,2378,2663,4832,5903,929,3986,4295,5208,657,2872,2146,4797,4379,1088,2520,3217,1724,5738,4084,4798,4034,1395,1703,5385,4113,7,894,1822,994,3924,3935,2983,1960,910,4683,1505,5858,601,4047,2845,5208,3724,1611,3959,1809,4430,5174,5765,1320,2263,4911,5627,2909,5727,2490,301,4244,5962,3799,242,17,832,943,2771,1808,2688,867,4412,2350,5189,2332,1688,4285,1940,2050,2858,96,192,549,2430,2314,571,4739,3837,5080,4708,1345,3417,3910,2701,1139,755,3909,1653,429,553,3752,4940,3460,3516,1593,3543,5257,4221,3484,3607,3006,4606,5074,2400,1334,599,5216,4061,4777,4454,1625,926,5377,574,3957,1804,4646,93,615,5331,1158,3019,169,2515,5710,4627,4550,5719,5381,713,1184,5643,3894,4546,4886,1435,2416,2684,5652,3722,406,3407,3570,4266,4234,228,2774,3058,4107,1897,2756,1888,1371,3405,1506,924,405,5818,2670,3476,4268,253,4449,5921,2747,4988,1082,2852,3808,4534,4639,3646,4291,5627,1344,2097,5406,5236,944,5335,2860,2199,1452,5993,4461,4654,5294,593,3744,4277,4102,1219,535,3290,4419,2906,2454,1426,5193,1699,5079,1352,2355,32,4998,2647,3741,2973,5030,4143,1391,971,778,387,5664,2026,2151,2081,2675,619,2689,4657,1398,447,3414,5662,1207,2783,498,265,814,334,1276,1104,5416,3692,5107,5703,375,3820,2189,5970,5022,1657,2767,5690,5022,4653,1174,3530,4426,4027,5842,4926,728,3692,5793,4144,303,4087,4201,2048,3196,3745.

## **5.4 arr4000.txt**

4359,3430,5635,4054,831,2329,3645,635,2719,4503,1874,5809,4750,3143,163,889,3447,3042,2472,527,5806,1713,4582,1959,777,3478,4460,1836,5462,4175,321,4368,122,4060,2307,670,5130,4464,2187,674,507,4075,5768,3453,5157,5070,4566,1669,5750,4011,4917,4768,3047,3164,1984,5680,5739,913,1094,5162,4993,5134,3494,4402,2629,720,1973,225,320,2045,5841,150,2299,4859,445,605,5923,2052,5880,4916,906,1852,557,92,1267,1282,2200,5147,3871,2240,5188,2437,2066,4220,3420,3636,2130,552,1693,3039,200,2110,2100,5424,5998,1649,1221,3158,2967,5407,762,2107,4457,4160,2909,3798,4262,5788,3948,4461,1237,3330,1675,5913,3753,2763,5794,755,4501,5021,3818,3070,5658,3395,4081,1695,4468,2153,4128,2250,2193,1157,1214,4347,4840,4371,5727,1886,2289,1461,2101,2973,1331,3174,3347,1840,5658,853,2379,2171,161,2324,2261,1428,3787,2436,3973,1203,2839,4778,2561,3157,971,1515,412,3929,5067,4927,23,5322,1936,2994,1905,2230,5119,414,2243,4113,4166,1562,3598,2331,511,3312,267,3375,4507,1170,1,2912,1309,1913,5430,1668,1135,1845,40,3411,5905,5935,4598,5790,3480,3341,4683,2415,602,3387,551,1388,4181,4670,607,1542,3851,892,5691,4831,671,4031,887,1710,3443,2723,2797,3969,5467,5149,3129,4178,2545,1332,2047,4950,3756,1565,1041,291,1865,5656,999,1721,4363,1890,890,1287,4813,2946,3758,4381,1687,4549,1804,5098,1982,3431,5034,17,3691,31,2383,2515,793,547,2299,3190,522,2191,1585,648,5627,5050,4758,697,1871,3594,5337,1043,5288,739,3347,4784,2632,3455,3838,3387,2139,1107,5459,4433,1963,183,21,5172,4736,3662,5622,1887,4327,4054,3123,5242,3298,203,4833,5061,4606,1030,4827,2600,2856,4833,751,3974,2864,4276,4948,1389,4408,2956,4487,2869,5860,1351,5109,4432,5924,2070,3156,1160,3869,2648,4377,3238,529,588,4961,3848,2942,2060,2316,3878,4709,4499,4961,5566,2140,4435,4421,3613,861,1104,5278,1899,4868,5811,2388,2682,4049,4647,2930,5125,4867,5976,5192,5159,2268,2055,4011,1013,2978,2371,420,248,1082,627,4424,865,4232,5854,2836,952,2957,2885,4881,2397,378,344,615,2483,5122,5404,3381,4651,4424,3901,1242,323,5307,5868,5874,1568,1269,209,2496,5350,181,4143,1323,373,4039,3047,3519,887,4839,2627,4882,4414,5175,1716,4397,4131,3150,655,4382,3619,2890,4994,4246,5793,4340,3242,865,5204,3072,600,3229,1406,4610,5917,5350,787,205,2729,3503,5171,2138,2942,5548,1565,2420,1644,5518,4733,5313,1680,1757,4676,5233,2882,3667,2809,5515,3382,2619,1425,81,3072,4715,5048,1294,2173,4969,1516,3235,2907,3988,1023,279,1511,1477,906,2151,3277,5781,4771,4644,880,1306,2487,5896,4104,1931,746,5483,3018,2482,2186,3675,2886,4044,2626,918,3410,194,4675,3106,3483,4301,1829,282,3372,2742,2317,3270,5166,5335,4172,2206,5449,4327,2500,5169,3453,3129,548,1955,5442,3415,4011,1648,5755,386,1036,843,2803,862,4602,4386,354,3350,5328,4062,4706,3539,3379,5029,4742,4517,5876,768,525,634,5191,3053,2829,3229,1671,5755,5046,1384,5636,457,2071,3576,2031,3540,124,2398,2855,2848,5015,2588,1305,3235,2125,330,5761,2022,3290,1177,2904,5987,3864,5084,3828,623,1589,5785,4533,365,3359,4263,3129,2962,2630,2192,2042,517,3149,5091,723,5587,5647,4473,4794,5420,3461,5497,5093,4683,423,1521,2929,5076,1991,5591,1724,2895,398,3142,2448,4990,2585,4224,4531,5662,3855,5146,5497,5746,570,4864,5234,4631,1729,419,755,2039,4047,2479,355,1261,2056,2437,337,996,1871,2653,991,5189,574,1153,5079,1101,637,5637,3981,3435,2522,761,3638,589,2414,3546,648,58,4136,2266,4287,3628,513,4426,4755,5328,5882,5334,3808,123,66,2375,4801,4999,4942,3381,3376,4492,5681,466,5347,1785,5985,1365,2841,231,3945,2645,5484,617,730,3228,2302,4628,64,340,4104,2753,343,1472,5369,1428,1398,4985,3134,3351,2180,2811,2019,830,4161,5168,5359,577,1897,5364,434,1602,5082,4617,5004,1834,1468,360,4733,3696,1275,4728,706,2203,1114,802,4975,5159,3112,857,2042,1677,672,5056,5364,2713,2349,3082,2238,3276,1440,2833,4015,4809,1311,3636,5234,1633,999,515,1219,1504,372,740,4735,3130,5066,2294,571,5011,4537,4038,401,172,3858,5071,4883,215,3376,2334,5828,3690,45,596,4268,56,97,1871,4464,5449,2764,3802,3304,47,5023,4825,3112,1258,5745,1775,3663,5811,250,3259,4166,3665,5079,1358,4112,2948,3768,5229,1226,4210,5622,816,3285,4088,2794,3253,2254,2843,2591,542,4126,3439,1622,868,1893,1579,4351,1259,2383,692,4737,4469,127,4982,1928,5677,3090,5604,3485,3147,3916,5551,2276,5070,2384,2097,58,2748,3941,5622,3805,5736,5610,2216,2080,1844,4144,2840,4207,1786,4063,3773,553,2844,3171,5809,106,253,2304,3018,5460,2942,2392,3988,5542,1940,3425,5169,4762,1128,2644,3446,3815,5011,5166,1146,2109,4962,1325,1532,735,3752,1296,334,1989,4320,3025,3061,762,4488,2134,713,4630,837,4300,2101,5099,4096,5629,1200,2403,3243,4769,4952,225,4093,1879,525,1010,2306,2171,3384,2933,2743,5512,496,228,604,2526,747,1351,5166,3794,2960,2481,5823,57,3015,5004,2876,3898,3329,5018,5794,3018,2360,4427,1754,4203,2983,4961,4995,3070,349,2347,4874,4039,4397,12,5707,3368,5335,3976,2164,5775,2322,5081,4548,5995,136,5413,2375,633,3863,4594,4598,4893,3598,2699,281,3153,3783,1801,2812,1947,3098,3545,1614,2679,4871,3805,3950,5360,4611,1243,1665,1790,714,4614,5724,976,4397,3801,1026,1050,1708,5982,3369,2699,5910,2813,5426,1001,627,3323,3365,1642,3008,2095,130,3770,3624,3448,1502,1182,2054,5611,4860,4219,1954,4423,2015,2540,990,3695,4685,4003,1291,1308,5984,5057,972,4125,3871,5469,4961,5054,727,2796,1652,37,1115,3758,4916,3327,1392,2643,4697,2236,4159,3047,5812,3545,5439,521,3618,893,5215,296,996,5886,4252,3899,5512,5181,2184,1417,770,1547,5585,4225,5792,4859,2122,4261,4733,3036,4032,2433,488,5329,4904,4313,3566,747,258,187,2222,1189,2568,2507,1084,5628,3192,2988,3421,813,1281,2048,890,3169,4880,4804,5865,5967,2399,645,5469,5534,3237,3382,3164,1994,572,4050,5057,63,5387,869,2235,3601,4373,4303,1124,1956,5801,5911,3368,4745,2382,5589,2803,381,961,849,3976,668,3225,2086,2102,2812,1751,1212,2607,5592,3492,5729,1237,5834,271,89,5309,3090,3016,1260,4923,4219,5323,5591,1917,1679,2470,1488,5944,1535,4233,3000,1389,3639,3742,5139,5872,5053,4265,4800,718,2577,5044,3204,225,1239,2303,5194,3563,790,2227,1182,2947,5601,3025,5292,2145,2913,3253,2114,502,152,3292,592,5516,3901,4287,4809,3457,5293,5941,3924,774,2856,3374,1152,4858,340,1702,4484,5325,3731,3162,5917,4259,2530,3468,5126,3750,1424,3619,3943,5727,3716,2585,5550,2405,1297,5230,1116,1656,5585,427,4529,4301,1632,2605,1020,3742,5615,5925,5847,2363,2062,3987,674,3936,3598,5275,2057,670,478,5600,4251,1722,3957,1547,1755,2736,2464,1306,1169,1560,2659,1390,5476,3431,2758,1540,467,5811,2993,1549,4885,4637,1776,964,4852,2,3969,5404,3407,4203,1724,3824,1314,760,2638,5252,3939,5303,308,1449,4728,4235,5090,5528,798,4775,1580,1874,306,4372,393,1679,2549,4530,4938,3033,4341,5860,3287,4923,5862,1613,799,480,2095,75,2847,558,4056,4013,2982,2125,3274,5896,3950,5090,1462,783,5297,5907,948,2665,1767,4841,267,2590,3403,5896,5355,4207,1097,4276,5979,253,2422,3778,1440,2173,1778,5754,3165,2370,4353,4170,4581,586,4979,1908,4618,2612,799,3742,348,4909,625,3372,2578,4776,4048,3602,5745,4478,5750,57,191,1175,996,3899,4772,627,3021,1706,2815,5469,3985,1117,4780,2358,2243,4911,355,2491,2028,4008,5446,2860,5083,5328,3259,3190,1490,1630,3814,5760,2817,1284,4313,4250,4709,4464,4281,3388,350,2504,3920,1482,4608,5038,3929,2944,1297,5146,3967,3584,2819,56,3106,5862,4394,3870,2956,324,318,1476,2642,3226,3002,5167,2547,2776,1130,1488,4622,3579,5008,1609,1144,4655,5097,1870,5084,2941,1849,299,5692,3357,3251,3679,4569,396,2987,2170,4321,4798,2809,4732,3901,2065,1555,4377,5990,3605,2314,5840,2105,5806,2493,3498,2780,3788,940,4257,2861,2484,5917,2727,5984,2444,1500,2313,1390,1142,2005,3797,2634,4595,4160,672,3172,2904,5181,1826,4636,744,1396,1200,1390,5955,852,2810,4132,2238,4222,3878,3511,5424,3495,962,4257,3011,5189,1749,883,1519,340,4623,1941,1811,1206,3340,4766,739,453,2931,4888,4482,2695,5524,3681,2498,2460,995,4393,5398,5090,3282,4485,1008,1716,748,1150,2620,2231,4343,3062,3751,3175,4213,2682,887,2654,126,2040,3362,1728,3389,2557,6000,5310,1994,2414,3294,1310,461,1032,3692,733,1226,5501,1057,1512,1552,2277,1598,5186,721,3088,2408,2011,725,2821,4356,4796,3859,1217,3289,4555,2446,5373,3483,221,3486,4460,3892,5802,215,3667,5069,1858,3503,3116,640,5777,4593,5484,5899,1105,4997,1210,5468,4217,5307,477,4076,4741,4382,4705,5189,586,5671,2987,5745,4893,4522,4809,3333,3028,5016,1453,2984,4659,5281,4793,255,5026,4358,1604,1468,4888,1468,2210,2863,4055,3989,4059,3974,3140,5643,1913,807,5994,5225,4482,1334,3364,733,4767,3456,2560,17,3957,5898,295,4933,2467,5505,562,5627,718,2112,119,836,4345,2537,3227,5083,1890,2289,2408,4277,3571,5399,1376,3636,5372,1153,1860,3818,1653,3697,3736,4915,5542,2260,3869,1374,1618,4276,5138,5992,5906,2861,1433,1912,1789,1994,2907,4080,5343,5307,446,5688,5549,5653,5178,3068,4114,4574,1882,3999,3987,3393,448,3593,5249,5004,891,4896,3835,5418,3113,3879,498,832,4351,2821,581,3356,3277,4046,3471,603,3875,5634,2172,5121,4593,2091,2110,3052,4812,5984,183,5585,5431,1270,4174,1533,3266,1495,2885,916,2121,2073,2846,5597,2639,1718,64,5909,4567,1515,3135,3786,746,3313,1042,5146,1982,5958,5216,3243,10,425,750,4321,4045,5150,4354,1215,4758,2714,284,1576,2230,734,2561,3047,1153,4993,5832,162,3593,3892,4639,4729,782,3945,3358,5286,2785,2366,2743,3615,2417,2879,3976,3458,5568,559,3141,371,3970,971,4366,2194,2602,3923,283,4760,190,979,2165,5397,3790,342,2858,5320,5089,4855,3726,5248,4931,4743,1970,2568,1648,4259,5126,964,5978,603,1091,2764,4819,4397,842,2243,3661,3619,928,2859,5338,1165,3986,720,2753,194,1870,4695,1383,1060,3486,2311,1751,1003,4143,2045,2599,804,1135,4726,244,1306,5338,5588,507,1163,2884,4041,4749,1244,2450,4388,2824,4903,2294,1619,5271,831,1087,3982,2722,4350,1527,2255,4390,5833,5984,652,1544,1531,2802,1843,2112,4260,4476,4963,999,3325,679,435,3291,686,4091,1165,2365,1613,1108,11,1867,5091,884,539,3722,3874,4606,1791,5720,448,2696,267,5731,1194,481,1437,5392,4772,4946,2922,1995,2147,2506,5982,1458,2815,4141,1160,3520,4500,1917,1798,2833,4160,3734,1697,4287,223,3163,2484,5751,5880,4472,2161,486,2748,3277,955,740,5508,2909,3902,3408,2942,5300,5614,2329,1508,5196,1349,5532,1756,1616,5919,1767,3971,4832,3997,4831,5015,2646,2185,2542,1732,495,4611,1813,5302,1739,1965,5957,1641,5630,734,5335,1403,2721,830,1549,5171,2906,4086,686,5273,3954,2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