(6) $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = \dfrac{a\cos at - b\sin bt}{(a^2-b^2)}$

$\Rightarrow L^{-1}\left[\dfrac{s}{(s^2+a^2)}\right] * L^{-1}\left[\dfrac{s}{s^2+b^2}\right]$

$\Rightarrow \cos at * \cos bt$

$\Rightarrow \int_0^t \cos au \cdot \cos b(t-u)\, du$

$\boxed{\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)}$

$\Rightarrow \frac{1}{2}\int_0^t \{\cos(au+bt-bu) + \cos(au-bt+bu)\}\, du$

$\Rightarrow \frac{1}{2}\int_0^t \{\cos[(a-b)u+bt] + \cos[(a+b)u-bt]\}\, du$

$\Rightarrow \frac{1}{2}\left[\dfrac{\sin[(a-b)u+bt]}{a-b} + \dfrac{\sin[(a+b)u-bt]}{a+b}\right]_0^t$

$\Rightarrow \frac{1}{2}\left[\dfrac{\sin(at-\cancel{bt}+\cancel{bt}) - \sin bt}{a-b} + \dfrac{\sin(at+\cancel{bt}-\cancel{bt}) + \sin bt}{a+b}\right]$

$\Rightarrow \frac{1}{2}\left[\dfrac{\sin at - \sin bt}{a-b} + \dfrac{\sin at + \sin bt}{a+b}\right]$

$\Rightarrow \dfrac{1}{2(a^2-b^2)}\left[(a+b)(\sin at - \sin bt) + (a-b)(\sin at + \sin bt)\right]$

$\Rightarrow \dfrac{1}{2(a^2-b^2)}\left[a\sin at - a\sin bt + b\sin at - b\sin bt + a\sin at + a\sin bt - b\sin at - b\sin bt\right]$

$\Rightarrow \dfrac{1}{2(a^2-b^2)}\left[2a\sin at - 2\sin bt\right]$

$= \dfrac{a\sin at - b\sin bt}{a^2-b^2}$ //

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(6) $L^{-1}\left[\dfrac{1}{(s^2+a^2)(s^2+b^2)}\right]$

$\Rightarrow L^{-1}\left(\dfrac{1}{s^2+a^2}\right) * L^{-1}\left(\dfrac{1}{s^2+b^2}\right)$

$\Rightarrow \dfrac{1}{a}\sin at * \dfrac{1}{b}\sin bt$

$t \to u \qquad t \to t-u$

$\Rightarrow \dfrac{1}{ab}\displaystyle\int_0^t \sin(au) \cdot \sin(bt-bu)\, du$

$A \qquad B$

$(A+B) = au+bt-bu = (a-b)u+bt$

$(A-B) = au-bt+bu = (a+b)u-bt$

$\sin A \cdot \sin B = \dfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$= \dfrac{1}{2ab}\displaystyle\int_0^t \cos\left[(a+b)u-bt\right] - \cos\left[(a-b)u+bt\right] du$

$= \dfrac{1}{2ab}\left[\dfrac{\sin[(a+b)u-bt]}{(a+b)} - \dfrac{\sin[(a-b)u+bt]}{(a-b)}\right]_{u=0}^{u=t}$

$u=t$

$= \dfrac{1}{2ab}\left\{\left[\dfrac{\sin(\cancel{(a+b)t}-\cancel{bt})}{(a+b)} - \dfrac{\sin[(a-\cancel{b})t+\cancel{bt}]}{(a-b)}\right] - \left[\dfrac{\sin(-bt)}{(a+b)} - \dfrac{\sin bt}{(a-b)}\right]\right\}$

$u=0$, $-\sin bt$

$\Rightarrow \dfrac{1}{2ab}\left\{\sin at\left[\dfrac{1}{(a+b)} - \dfrac{1}{(a-b)}\right] - \sin bt\left[\dfrac{1}{(a+b)} + \dfrac{1}{(a-b)}\right]\right\}$

$\Rightarrow \dfrac{1}{2ab}\left\{\sin at\,\dfrac{\cancel{(a-b)-(a+b)}\;-2b}{a^2-b^2} - \sin bt\left[\dfrac{(a-b)+(a+b)}{a^2-b^2}\right]\right\}$ $\quad 2a$

$\Rightarrow \dfrac{-1}{ab(a^2-b^2)}\left[b\sin at - a\sin bt\right]$

(3) $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{t\sin at}{2a}$ (4)

$$\Rightarrow L^{-1}\left[\frac{s}{s^2+a^2}\right] * L^{-1}\left[\frac{1}{s^2+a^2}\right]$$

$$\Rightarrow \cos at * \frac{1}{a}\sin at$$

$t \to u \quad t \to t-u$

$$\Rightarrow \frac{1}{a}\int_0^t \cos au * \sin a(t-u) \cdot du$$

$\because 2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$$\Rightarrow \frac{1}{2a}\int_0^t 2\cos au \sin(at-au) \cdot du$$

$$\Rightarrow \frac{1}{2a}\int_0^t \left\{\left[\sin(au+at-au) - \sin(au-at+au)\right]\right\} du$$

$$\Rightarrow \frac{1}{2a}\int_0^t \left\{\left[\sin at - \sin[2au-at]\right]\right\} \cdot du$$

$$\Rightarrow \frac{1}{2a}\left[[\sin at \cdot u] + \frac{\cos(2au-at)}{2a}\right]_0^t$$

$$\Rightarrow \frac{1}{2a}\left[t\sin at + \frac{\cos at}{2a} - \frac{\cos at}{2a}\right]$$

$$\Rightarrow \frac{t\sin at}{2a}$$ //

4) $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$ Using (convolution) ②

$$= L^{-1}\left[\frac{1}{(s^2+a^2)}\right] * L^{-1}\left[\frac{1}{s^2+a^2}\right]$$

$$= \frac{1}{a}\sin at * \frac{1}{a}\sin at$$

$$= \frac{1}{a^2}\int_0^t \sin au \sin a(t-u)\,du$$

$$= \frac{1}{2a^2}\int_0^t [\cos(2au-at) - \cos at]\,du$$

$$= \frac{1}{2a^2}\left[\frac{\sin(2au-at)}{2a} - (\cos at)u\right]_0^t$$

$$= \frac{1}{2a^2}\left(\left[\frac{\sin at}{2a} - t\cos at\right] - \left[\frac{-\sin at}{2a}\right]\right)$$

$$= \frac{1}{2a^2}\left[2\,\frac{\sin at}{2a} - t\cos at\right]$$

$$= \frac{1}{2a^3}\left[\sin at - at\cos at\right]$$

$L^{-1}\left[\frac{1}{\ldots}\right]$

$$\frac{}{a-b}\Big]\left[\cos at - \cos bt\right]$$

$$= \frac{1}{2b}\left[\frac{a-b-a+b}{a^2-b^2}\right]\left[\cos at - \cos bt\right]$$

$$= \frac{\cos at - \cos bt}{b^2-a^2}$$

3) $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$ Using Convolution (I)

$$= L^{-1}\left[\frac{s}{s^2+a^2}\right] * L^{-1}\left[\frac{s}{(s^2+a^2)}\right]$$

$$= \cos at * \cos at$$

$$= \int_0^t \cos au * \cos(t-u)\,du$$

$$= \frac{1}{2}\int_0^t \cos(au+at-au) * \cos(au-at+au)\,du$$

$$= \frac{1}{2}\int_0^t \cos at + \cos 2au - at \cdot du$$

$$= \frac{1}{2}\left[(\cos at)\,a + \sin\frac{(2au-at)}{2a}\right]_0^t$$

$$= \frac{1}{2}\left[t\cos at + \frac{\sin at}{a}\right]$$

$$= \frac{1}{2a}\left[\sin at + at\cos at\right]$$

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1)

4) $L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right] = \frac{\cos at - \cos bt}{b^2-a^2}$

$\Rightarrow L^{-1}\left[\frac{s}{(s^2+a^2)}\right] * L^{-1}\left[\frac{1}{(s^2+b^2)}\right]$

$\Rightarrow \cos at * \frac{1}{b}\sin bt$

$\Rightarrow \frac{1}{b}\int_0^t \cos au * \sin(bt-bu).du$

$\Rightarrow \frac{1}{2b}\int_0^t 2\cos au \sin(bt-bu).du$

$\Rightarrow \frac{1}{2b}\int_0^t [\sin(au+bt-bu) - \sin(au-bt+bu)].du$

$\Rightarrow \frac{1}{2b}\int_0^t [\sin[(a-b)u+bt] - \sin[(a+b)u-bt]].du$

$\Rightarrow \frac{1}{2b}\left[-\frac{\cos[(a-b)u+bt]}{a-b} + \frac{\cos[(a+b)u-bt]}{a+b}\right]_0^t$

$\Rightarrow \frac{1}{2b}\left[-\frac{\cos[at-\cancel{bt}+\cancel{bt}]}{a-b} + \frac{\cos bt}{a-b} + \frac{\cos[at+\cancel{bt}-\cancel{bt}]}{a+b} - \frac{\cos[-bt]}{a+b}\right]$

$\Rightarrow \frac{1}{2b}\left[-\frac{\cos at}{a-b} + \frac{\cos bt}{a-b} + \frac{\cos at}{a+b} - \frac{\cos bt}{a+b}\right]$

$\cos(-\theta) = +\cos\theta$

$\Rightarrow \frac{1}{2b}\left[\frac{-\cos at + \cos bt}{a-b} + \frac{\cos at - \cos bt}{a+b}\right]$

$$\Rightarrow \frac{1}{2b}\left[[-\cos at + \cos bt]\left(\frac{1}{a-b}\right) + [\cos at - \cos bt]\left(\frac{1}{a+b}\right)\right]$$

$x \Rightarrow -1$ for taking common

$$\Rightarrow \frac{1}{2b}\left[\frac{1}{a-b} - \frac{1}{a+b}\right]\left[-\cos at + \cos bt\right]$$

$$\Rightarrow \frac{1}{2b}\left[\frac{\cancel{a}+b-\cancel{a}+b}{a^2-b^2}\right]\left[-\cos at + \cos bt\right]$$

$$\Rightarrow \frac{1}{\cancel{2b}}\frac{[\cancel{2b}]}{a^2-b^2}\left[-\cos at + \cos bt\right]$$

$$\Rightarrow \frac{-\cos at + \cos bt}{a^2-b^2}$$

$$x - \Rightarrow \frac{\cos at - \cos bt}{b^2-a^2} \quad //$$