MISCELANEOUS PROB.

① $(D^3+D)x = 2$, $x=3$

$Dx=1$

$D^2x=-2$ at $t=0$

Sol.

$y'''(t)+y'(t)=2$

$L[y'''(t)]+L[y'(t)]=L[2]$

$s^3L[y(t)]-s^2\underset{3}{y(0)}-s\underset{1}{y'(0)}-\underset{-2}{y''(0)}+sL[y(t)]-\underset{3}{y(0)}=\frac{2}{s}$

$(s^3+s)L[y(t)]-3s^2-s+(2-3)=\frac{2}{s}$ (with -1 written above $2-3$)

$s(s^2+1)L[y(t)]=\frac{2}{s}+3s^2+s+1$

$L[y(t)]=\frac{2+3s^3+s^2+s}{s(s^2+1)}$

$\frac{2+3s^3+s^2+s}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{(s^2+1)}$

$3s^3+s^2+s+2 = A(s^2+1)+(s)(Bs+C)$

$= As^2+A+Bs^2+Cs$

$s=0$,

$2=A$

s^2 term,

$A+B=1$

$B=1$

s term

$C=1$

$\therefore y(t)=L^{-1}\left[\frac{2}{s}\right]+L^{-1}\left[\frac{s}{s^2+1}\right]+L^{-1}\left[\frac{1}{s^2+1}\right]$

$\boxed{y(t)=2+\cos t+\sin t}$

2. $(D^4 - D^3)y = 0$, $\quad y(0) = y'(0) = 2$

$\qquad\qquad\qquad\qquad y''(0) = y'''(0) = 1$

Sol. $L[y''''(t)] - L[y'''(t)] = 0$

$[s^4 L[y(t)] - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0)]$

$-[s^3 L[y(t)] - s^2 y(0) - s y'(0) - y''(0)] = 0$

$L[y(t)]\{s^4 - s^3\} - 2s^3 - \cancel{2s^2} - \cancel{s} - \cancel{1} + \cancel{2s^2} + \overset{s}{\cancel{2s}} + \cancel{1} = 0$

$s^3(s-1)L[y(t)] = 2s^3 - s$

$= \dfrac{s[2s^2 - 1]}{s^3(s-1)}$

$L[y(t)] = \dfrac{2s^2}{s^2(s-1)} - \dfrac{1}{s^2(s-1)}$

$= \dfrac{2}{(s-1)} - \dfrac{1}{s^2(s-1)}$

$\dfrac{1}{s^2(s-1)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{(s-1)}$

$1 = A(s)(s-1) + B(s-1) + Cs^2$

put $s=0$, $B = -1$ | put $s=1$, $\boxed{C = 1}$

s^2 term, $A + C = 0$

$A = -1$

$\therefore y(t) = L^{-1}\left[\dfrac{2}{s-1}\right] + L^{-1}\left[\dfrac{1}{s}\right] + L^{-1}\left[\dfrac{1}{s^2}\right] - L^{-1}\left[\dfrac{1}{s-1}\right]$

$y(t) = 2e^t + 1 + t - e^t = e^t + 1 + t$

3. Solve: $Dx + Dy = t \longrightarrow x' + y' = t \rightarrow (1)$

$D^2x - y = e^{-t} \longrightarrow x'' - y = e^{-t} \rightarrow (2)$

$x = 3,\ Dx = -2,\ y = 0 \ @\ t = 0$

Sol:

(1) $\Rightarrow L[x'(t)] + L[y'(t)] = L[t]$

$sL[x(t)] - \underset{3}{x(0)} + sL[y(t)] - \underset{0}{y(0)} = \frac{1}{s^2}$

$L[x(t)]\{s\} + L[y(t)]\{s\} = 3 + \frac{1}{s^2}$

$= \frac{3s^2+1}{s^2} \rightarrow (3)$

(2) $\Rightarrow L[x''(t)] - L[y(t)] = L[e^{-t}]$

$s^2L[x(t)] - s\,\underset{3}{x(0)} - \underset{-2}{x'(0)} - L[y(t)] = \frac{1}{s+1}$

$L[x(t)]\{s^2\} + L[y(t)]\{-1\} = \frac{1}{(s+1)} + 2 + 3s$

$= \frac{1 + 2s + 3s^2 - 2 + 3s}{(s+1)}$

$= \frac{-1 + s + 3s^2}{(s+1)} \rightarrow (4)$

By Cramer's rule,

$$\Delta = \begin{vmatrix} s & s \\ s^2 & -1 \end{vmatrix} = -s - s^3$$

$= -s(1+s^2)$

$$\Delta x = \begin{vmatrix} \frac{3s^2+1}{s^2} & s \\ \frac{3s^2+s-1}{s+1} & -1 \end{vmatrix} = \frac{-3s^2-1}{s^2} - \left(\frac{3s^3+s^2-s}{s+1}\right)$$

$= \frac{-3s^3 - s - 3s^2 - 1 - 3s^5 - s^4 + s^3}{s^2(s+1)}$

$$= \frac{-[3s^5+s^4+2s^3+3s^2+s+1]}{s^2(s+1)}$$

$$\therefore L[x(t)] = \frac{\Delta x}{\Delta}$$

$$= \frac{-[3s^5+s^4+2s^3+3s^2+s+1]}{-s^3(s+1)(1+s^2)}$$

$$= \frac{3s^2[s^3+1]+s[s^3+1]+(s^3+1)+s^3}{s^3(s+1)(1+s^2)}$$

$$= \frac{(s^3+1)(3s^2+s+1)}{s^3(s+1)(1+s^2)} + \frac{\cancel{s^3}}{\cancel{s^3}(s+1)(1+s^2)}$$

$$= \frac{\cancel{(s^3)}(3s^2+s+1)}{\cancel{s^3}(s+1)(1+s^2)} + \frac{(3s^2+s+1)}{(s^3)(s+1)(1+s^2)} + \frac{1}{(s+1)(1+s^2)}$$

$$x(t) = L^{-1}\left\{\frac{(3s^2+s+2)}{(s+1)(1+s^2)}\right\} + L^{-1}\left\{\frac{(3s^2+s+1)}{(s^3)(s+1)(s^2+1)}\right\}$$

I

$$\frac{3s^2+s+2}{(s+1)(1+s^2)} = \frac{A}{(s+1)} + \frac{Bs+C}{(1+s^2)}$$

$$3s^2+s+2 = A(1+s^2)+(s+1)(Bs+C)$$

put $s=-1$,

$3-1+2 = 2A$

$4 = 2A$

$A=2$

put $s=0$

$2 = A+C$

$C=0$

s^2 term, $3 = A+B$

$B=1$

I $\Rightarrow L^{-1}\left[\frac{2}{s+1}\right] + L^{-1}\left[\frac{s}{(1+s^2)}\right]$

$\Rightarrow 2e^{-t}+\cos t$

$L^{-1}\left\{\frac{3s^2+s+1}{\cancel{s^3}(s+1)(s^2+1)}\right\} = ?$

$\frac{3s^2+s+1}{(s^3)(s+1)(s^2+1)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{(s+1)}+\frac{Es+F}{s^2+1}$

$3s^2+s+1 = (A)\underbrace{(s^2)(s+1)}_{(s^3+s^2)}(s^2+1) + B\underbrace{(s)(s+1)}_{(s^2+s)}(s^2+1) + \underbrace{C(s+1)(s^2+1)}_{(s^3+s+s^2+1)} + D(s^3)(s^2+1) + (Es+F)\underbrace{(s^3)(s+1)}_{(s^4+s^3)}$

put $s=-1$,

$3-\cancel{1}+\cancel{1} = D(-1)(2)$

$\boxed{D=\frac{-3}{2}}$

put $s=0$,

$\boxed{1=C}$

s^5 term, $0 = A + D + E \Rightarrow A+E = \frac{3}{2}$

s^4 term, $0 = A + B + F$

s^3 term, $0 = A + B + C + D$

$0 = A+B+1-3/2$

$-1/2$

$\boxed{A+B=\frac{1}{2}}$

$\therefore \boxed{F=-1/2}$

s^2 term, $3 = A+B+C+E$

$\boxed{B=1/2}$

$3 = \frac{1}{2}+1+E$

$3-\frac{3}{2} = E$

$\boxed{E=\frac{3}{2}}$

$\therefore \boxed{A=0}$

(II) $\Rightarrow L^{-1}\left[\frac{1}{2s^2}\right] + L^{-1}\left[\frac{-1}{s^3}\right] + L^{-1}\left[\frac{-3}{2(s+1)}\right]$

$+ L^{-1}\left[\frac{3s}{2(s^2+1)}\right] - \frac{1}{2} L^{-1}\left[\frac{1}{(s^2+1)}\right]$

(II) $\Rightarrow \frac{1}{2}t + \frac{t^2}{2} - \frac{3}{2}e^{-t} + \frac{3}{2}\cos t - \frac{1}{2}\sin t$

Sol. $\Rightarrow$ (I) (II)

$x(t)$ $\quad 2e^{-t} + \cos t + \frac{1}{2}t + \frac{t^2}{2} - 3e^{-t} + \frac{3}{2}\cos t - \frac{1}{2}\sin t$

$\boxed{x(t) \Rightarrow e^{-t} + \frac{5}{2}\cos t + \frac{1}{2}t + \frac{t^2}{2} - \frac{1}{2}\sin t}$

(2) $y = x'''(t) - e^{-t}$

$x''(t) = +e^{-t} - \frac{5}{2}\sin t + \frac{1}{2} + \cancel{\frac{2t}{2}} - \frac{1}{2}\cos t$

$x'''(t) = -e^{-t} - \frac{5}{2}\cos t + 1 + \frac{1}{2}\sin t$

$\therefore \boxed{y(t) = \frac{1}{2} - \frac{1}{2}\cos t - \frac{5}{2}\sin t}$

A. Solve the Simultaneous D.E

$D^2x - Dy = \cos t \Rightarrow x''(t) - y'(t) = \cos t \to (1)$

$Dx + D^2 y = -\sin t \Rightarrow x'(t) + y''(t) = -\sin t \to (2)$

$x=1,\ Dx=0,\ Dy=1$ @ $t=0$.
$y=0$

Sol. $(1) \Rightarrow L[x''(t)] - L[y'(t)] = L[\cos t]$

$\Rightarrow s^2 L[x(t)] - \cancel{s x(0)} - x'(0) - \{sL[y(t)] - y(0)\} = \frac{s}{s^2+1}$

$\Rightarrow \{s^2\} L[x(t)] + \{-s\} L[y(t)] = \frac{s}{s^2+1} + s = \frac{s+s^3+s}{s^2+1} = \frac{2s+s^3}{s^2+1} \to (3)$

$(2) \Rightarrow L[x'(t)] + L[y''(t)] = -L[\sin t]$

$\Rightarrow sL[x(t)] - x(0) + s^2 L[y(t)] - \cancel{s y(0)} - y'(0) = \frac{-1}{s^2+1}$

$\Rightarrow \{s\} L[x(t)] + \{s^2\} L[y(t)] = 2 - \frac{1}{s^2+1} = \frac{2s^2+2-1}{s^2+1} = \frac{2s^2+1}{s^2+1} \to (4)$

By Cramer's rule,

$$\Delta = \begin{vmatrix} s^2 & -s \\ s & s^2 \end{vmatrix} = (s^4+s^2) = s^2(s^2+1)$$

$$\Delta x = \begin{vmatrix} \frac{2s+s^3}{s^2+1} & -s \\ \frac{2s^2+1}{s^2+1} & s^2 \end{vmatrix} = \frac{2s^3+s^5}{(s^2+1)} + \frac{2s^3+s}{(s^2+1)} = \frac{s^5+4s^3+s}{(s^2+1)}$$

$$\therefore L[x(t)] = \frac{(s^5+4s^3+s)}{s^2(s^2+1)^2} = \frac{s(s^4+4s^2+1)}{s^2(s^2+1)^2}$$

$$= \frac{(s^4+2s^2+1) + 2s^2}{s(s^2+1)^2}$$

$L[x(t)] = \frac{(s^2+1)^2}{s(s^2+1)^2}$ ...

$= \frac{1}{s} + \frac{2}{(s^2 ...}$

$x(t) = L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}[$ ...

$= 1 + 2 \times \frac{t}{2}$ ...

$\boxed{x(t) = 1 + t\sin t}$

$(1) \Rightarrow x''(t$ ...

$\Rightarrow 2\cos t -$ ...

$\Rightarrow \cos t$ ...

Taking $\int$ ...

$\int \cos t\, dt$ ...

$\Rightarrow \sin t -$ ...

$\Rightarrow \boxed{y(t)}$ ...

$$L[x(t)] = \frac{\cancel{(s^2+1)^2}}{s\,\cancel{(s^2+1)^2}} + \frac{2s\cancel{s}}{\cancel{s}(s^2+1)^2}$$

$$= \frac{1}{s} + \frac{2s}{(s^2+1)^2}$$

$$x(t) = L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}\left[\frac{s}{(s^2+1)^2}\right]$$

By convolution formula, $\because L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{t}{2a}\sin at$

$$= 1 + \cancel{2} \times \frac{t}{\cancel{2}\times 1}[\sin t]$$

$$\boxed{x(t) = 1 + t\sin t}$$

$x'(t) = t\cos t + \sin t$

$x''(t) = -t\sin t + \cos t + \cos t = 2\cos t - t\sin t$

(1) $\Rightarrow x''(t) - \cos t = y'(t)$

$\Rightarrow 2\cos t - t\sin t - \cos t = \frac{dy}{dt}$

$\Rightarrow \cos t - t\sin t = \frac{dy}{dt}$

Taking $\int$ on both sides,

$$\int \cos t\,dt - \int \underset{u}{t}\,\underset{v}{\sin t}\,dt = y(t)$$

$\int uv\,dt = uv_1 - u'v_2 + \ldots$

$\Rightarrow \sin t - [-t\cos t + \sin t] = y(t)$

$$\Rightarrow \boxed{y(t) = t\cos t}$$

5. Solve the Simultaneous eqns. (1 ODE, 1 I.E)

① ← $3x' + 2y' + 6x = 0$ & (ODE)

② ← $y' + y + 3\int_0^t x\,dt = \cos t + 3\sin t$ (Integral eqn)

$x(0) = 2, \quad y(0) = -3.$

Sol.

① ⇒ $3L[x'(t)] + 2L[y'(t)] + 6L[x(t)] = 0$

$\Rightarrow 3\{sL[x(t)] - x(0)\} + 2\{sL[y(t)] - y(0)\} + 6L[x(t)] = 0$

$\Rightarrow L[x(t)]\{3s+6\} + L[y(t)]\{2s\} = 0 \quad$ ③

$$\boxed{L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}L[f(t)]}$$

② ⇒ $L[y'(t)] + L[y(t)] + 3L\left[\int_0^t x\,dt\right] = L[\cos t] + 3L[\sin t]$

$\Rightarrow sL[y(t)] - y(0) + L[y(t)] + \frac{3}{s}L[x(t)] = \frac{s}{s^2+1} + \frac{3}{s^2+1}$

$\Rightarrow L[x(t)]\left\{\frac{3}{s}\right\} + L[y(t)]\{s+1\} + 3 = \frac{(s+3)}{s^2+1}$

$\Rightarrow L[x(t)]\left\{\frac{3}{s}\right\} + L[y(t)]\{s+1\} = \frac{s+3}{s^2+1} - 3$

$= \frac{s + \cancel{3} - 3s^2 - \cancel{3}}{(s^2+1)}$

$= \frac{s(1-3s)}{(s^2+1)} \quad$ ④

By cramer's rule,

$$\Delta = \begin{vmatrix} 3s+6 & 2s \\ \frac{3}{s} & s+1 \end{vmatrix} = (3s^2 + 3s + 6s + 6) - 6$$

$= 3s^2 + 9s$

$= 3s(s+3)$

$$\Delta x = \begin{vmatrix} 0 & 2s \\ \frac{s(1-3s)}{(s+1)} & (s+1) \end{vmatrix}$$

$$= -\frac{2s^2(1-3s)}{(s+1)}$$

$$L[x(t)] = \frac{-2s(1-3s)}{(s^2+1)(3s)(s+3)}$$

$$= \frac{(6s-2)}{3(s+3)(s^2+1)}$$

$$\frac{6s-2}{(s+3)(s^2+1)} = \frac{A}{(s+3)} + \frac{Bs+C}{s^2+1}$$

$$6s-2 = A(s^2+1) + (Bs+C)(s+3)$$

put $s=-3$, put $s=0$

$-20 = A(10)$, $-2 = A+3C$

$A=-2$ $C=0$

s^2 term, $0 = A+B$

$B=2$

$$\therefore x(t) = \frac{1}{3}\left\{-2L^{-1}\left[\frac{1}{s+3}\right] + 2L^{-1}\left[\frac{s}{s^2+1}\right]\right\}$$

$$\boxed{x(t) = -\frac{2}{3}e^{-3t} + \frac{2}{3}\cos t}$$

$x'(t) = +\frac{2}{3}(-3)e^{-3t} + \frac{2}{3}\sin t$

$(1) \Rightarrow 2y' = -3x' - 6x$

$$= -3\left(-2e^{-3t} + \frac{2}{3}\sin t\right) - 6\left\{-\frac{2}{3}e^{-3t} + \frac{2}{3}\cos t\right\}$$

$$= 6e^{-3t} - 2\sin t + 4e^{-3t} - 4\cos t$$

$$\frac{dy}{dt} = \frac{10e^{-3t} - 2\sin t - 4\cos t}{2}$$

$$\int dy/dt = \int 5e^{-3t} - \sin t - 2\cos t$$

6. $L^{-1}\left[\dfrac{s^2}{s^2+4a^4}\right]$ → Not in control.

Sol:

s^2+4a^4
$\downarrow \quad \downarrow$
$A^2 \quad B^2$

$A^2 = s^4 \Rightarrow A = s^2 \qquad 2AB = 4a^2s^2$

$B^2 = 4a^4 \Rightarrow B = 2a^2$

$s^4 \; 4a^4 = s^4 + 4a^4 + 4a^2s^2 - 4a^2s^2$

$= (s^2+2a^2)^2 - (2as)^2$
$\quad A^2 - B^2$

$= (s^2+2a^2-2as)\,(s^2+2a^2+2as)$

$\therefore \dfrac{s^2}{s^4+4a^4} = \dfrac{(As+B)}{(s^2+2a^2-2as)} + \dfrac{(Cs+D)}{(s^2+2a^2+2as)}$

$s^2 = (As+B)(s^2+2a^2+2as) + (Cs+D)(s^2+2a^2-2as)$

$s^2 = As^3 + 2a^2As + 2Aas^2 + Bs^2 + 2a^2B + 2aBs$
$+ Cs^3 + 2a^2Cs - 2aCs^2 + Ds^2 + 2a^2D - 2aDs$

s^3 term, $A + C = 0 \Rightarrow A = -C$

s^2 term, $1 = 2aA + \cancel{B} - 2aC + \cancel{D}$

s term, $0 = 2a^2A + 2aB + 2a^2C - 2aD$

con. term, $0 = 2a^2B + 2a^2D$

$\Rightarrow 0 = 2a^2(B+D)$

$\Rightarrow B+D = 0$

$2aA - 2aC = 1$

$\Rightarrow 2aA + 2aA \Rightarrow$

$\Rightarrow 4aA = 1$

$\boxed{A = 1/4a}$

$\boxed{C = -\dfrac{1}{4a}}$

$0 = \frac{2a/^2}{4/a} + 2aB - \frac{2a^2}{4a} - 2aD$

$2/aB - 2/aD = 0$

$\left.\begin{array}{l} B - D = 0 \\ B + D = 0 \end{array}\right\} \begin{array}{l} 2B = 0 \\ B = 0 \Rightarrow D = 0 \end{array}$

$\therefore \Rightarrow L^{-1}\left[\frac{s - a + a}{4a(s^2 + 2a^2 - 2as)}\right] - L^{-1}\left[\frac{s + a - a}{4a(s^2 + 2a^2 + 2as)}\right]$

$\downarrow (s - a)^2 + a^2 \qquad \downarrow (s + a)^2 + a^2$

$\Rightarrow \frac{1}{4a} L^{-1}\left[\frac{(s-a)}{(s-a)^2 + a^2} + \frac{a}{(s-a)^2 + a^2}\right] - \frac{1}{4a} L^{-1}\left[\frac{s+a}{(s+a)^2 + a^2}\right]$

$- L^{-1}\left[\frac{a}{(s+a)^2 + a^2}\right]$

$\Rightarrow \frac{1}{4a}\left[e^{at}\cos at + e^{at}\sin at\right] - \frac{1}{4a}\left[e^{-at}\cos at - e^{-at}\sin at\right]$

$\Rightarrow \frac{1}{4a}\cos at\left[e^{at} - e^{-at}\right] - \frac{1}{4a}\sin at\left[e^{at} + e^{-at}\right]$ //

7. $\ddot{x} + n^2 x = A \sin pt \qquad x(0) = x'(0) = 0$

Sol.

$y''(t) + n^2 y(t) = A \sin pt$

$L[y''(t)] + n^2 L[y(t)] = A\, L[\sin pt]$

$s^2 L[y(t)] - \cancel{s\, y(0)} - \cancel{y'(0)} + n^2 L[y(t)] = \dfrac{AP}{s^2+p^2}$

$L[y(t)][s^2+n^2] = \dfrac{AP}{(s^2+p^2)}$

$L[y(t)] = \dfrac{AP}{(s^2+p^2)(s^2+n^2)}$

Case (i) $n \neq p$ $\qquad y(t) = AP\, L^{-1}\left[\dfrac{1}{(s^2+p^2)(s^2+n^2)}\right]$

By convolution thm. formula,

$$L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right] =$$

$$\left\{ L^{-1}\left[\frac{1}{s^2+a^2}\right] * L^{-1}\left[\frac{1}{s^2+b^2}\right]\right.$$

$$\Rightarrow \frac{1}{a}\sin at * \frac{1}{b}\sin bt$$

$$= \frac{1}{ab}\sin at * \sin bt$$

$$= \frac{1}{ab}\int_0^t \sin(au)\,\sin(bt-bu)\,du$$

$(A-B) = au - bt + bu$
$= (a+b)u - bt$

$$\Rightarrow \frac{1}{2ab}\left[\int_0^t \left\{\cos[(a+b)u - bt] - \cos[(a-b)u + bt]\right\} du\right]$$

$$\Rightarrow \frac{1}{2ab}\left[\frac{\sin[(a+b)u-bt]}{(a+b)} - \frac{\sin[(a-b)u+bt]}{(a-b)}\right]_{u=0}^{u=t}$$

$$\Rightarrow \frac{1}{2ab}\left[\left\{\frac{\sin(at)}{(a+b)} - \frac{\sin at}{(a-b)}\right\} - \left\{-\frac{\sin bt}{(a+b)} - \frac{\sin bt}{(a-b)}\right\}\right]$$

$$\Rightarrow \frac{1}{2ab}\left\{\sin at\left[\frac{1}{a+b} - \frac{1}{a-b}\right] + \sin bt\left[\frac{1}{a+b} + \frac{1}{a-b}\right]\right\}$$

$$\Rightarrow \frac{1}{2ab}\left(\frac{-2b}{(a^2-b^2)}\sin at + \frac{2a\sin bt}{(a^2-b^2)}\right)$$

$$\Rightarrow \frac{1}{ab}\left[\frac{-b\sin at + a\sin bt}{a^2-b^2}\right]$$

Here, $a=p$, $b=n$.

$$\therefore y(t) = A\cancel{p} \times \frac{1}{pn} \times \frac{1}{(p^2-n^2)}\left[p\sin nt - n\sin pt\right]$$

Case (ii) $n=p$, $y(t) = Ap\, L^{-1}\left[\frac{1}{(s^2+p^2)^2}\right]$

By st. convol. formula,

$$L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right] = \frac{1}{2a^3}\left[\sin at - at\cos at\right]$$

Here, $a=p$

$$\therefore y(t) = \frac{Ap}{2p^3}\left[\sin pt - pt\cos pt\right] //.$$

$$L^{-1}\left[\frac{1}{(s-2)^4(s+3)}\right]$$

$$= L^{-1}\left[\frac{1}{(s-2)^4}\right] * L^{-1}\left[\frac{1}{s+3}\right]$$

$$= e^{2t}\frac{t^3}{3!} * e^{-3t}$$

$$= \frac{1}{6}\int_0^t e^{2u} u^3 e^{-3(t-u)}\,du$$

$$= \frac{1}{6}\int_0^t e^{2u} u^3 e^{-3t} \cdot e^{3u}\,du$$

$$= \frac{e^{-3t}}{6}\int_0^t e^{5u} u^3\,du$$

$u = u^3$ $\qquad v_1 = \frac{e^{5u}}{5}$

$u' = 3u^2$ $\qquad v_2 = \frac{e^{5u}}{25}$

$u'' = 6u$ $\qquad v_3 = \frac{e^{5u}}{125}$

$u''' = 6$ $\qquad v_4 = \frac{e^{5u}}{625}$

$$= \frac{e^{-3t}}{6}\left[\frac{u^3 e^{5u}}{5} - \frac{3u^2 e^{5u}}{25} + \frac{6u e^{5u}}{125} - \frac{6e^{5u}}{625}\right]_{u=0}^{u=t}$$

$$= \frac{e^{-3t}}{6}\left[\left(\frac{t^3 e^{5t}}{5} - \frac{3t^2 e^{5t}}{25} + \frac{6t\, e^{5t}}{125} - \frac{6e^{5t}}{625}\right) + \frac{6}{625}\right]$$

$$\frac{1}{(s^3-a^3)} = \frac{A}{(s-a)} + \frac{(Bs+C)}{(s^2+as+a^2)}$$

$$1 = A(s^2+as+a^2) + (s-a)(Bs+C)$$

Put $s=a$, $= As^2 + Aas + a^2A + Bs^2 + Cs$
$- aBs - aC$

$$1 = A(a^2+a^2+a^2)$$

$$\boxed{A = 1/3a^2}$$

Comparing s^2 term,

$$0 = A+B$$

$$\boxed{B = -1/3a^2}$$

Comparing s term,

$$0 = Aa + C - aB$$

$$= \frac{a}{3a^2} + C + \frac{a}{3a^2}$$

$$0 = \frac{2}{3a} + C$$

$$\boxed{C = -\frac{2}{3a}}$$

$$\therefore L^{-1}\left[\frac{1}{3a^2(s-a)}\right] + L^{-1}\left[\frac{-\frac{s}{3a^2} - \frac{2 \times a}{3a \cdot a}}{s^2+as+a^2}\right]$$

$$\Rightarrow \frac{1}{3a^2}e^{at} - \frac{1}{3a^2} L^{-1}\left[\frac{s+2a}{s^2+as+a^2}\right] \to (ii)$$

(i)

A^2 $2AB$

$\boxed{A=s}$ ~~B = 1~~

$$2AB = as$$

$$\boxed{B = \frac{a}{2}}$$

$$\frac{-1}{3a^2} L^{-1}\left[\frac{s+2a}{\left(s+\frac{a}{2}\right)^2-\frac{a}{4}+a^2}\right]$$

$$\frac{-1}{3a^2} L^{-1}\left[\frac{s+2a+\frac{a}{2}-\frac{a}{2}}{\left(s+\frac{a}{2}\right)^2+\frac{3a^2}{4}}\right]$$

$$2a-\frac{a}{2} \Rightarrow \frac{3a}{2}$$

$$\frac{-1}{3a^2} L^{-1}\left[\frac{\left(s+\frac{a}{2}\right)+\frac{3a}{2}}{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]$$

$$=\frac{-1}{3a^2} e^{-\frac{at}{2}} L^{-1}\left[\frac{s+\frac{3a}{2}}{s^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]$$ (shifting prop)

$$=\frac{-1}{3a^2} e^{-at/2}\left\{L^{-1}\left[\frac{s}{s^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]+\frac{3a}{2}L^{-1}\left[\frac{1}{s^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]\right\}$$

$$=\frac{-1}{3a^2} e^{-at/2}\left\{\cos\left(\frac{\sqrt{3}at}{2}\right)+\frac{\cancel{3}^{\sqrt{3}}\cancel{a}}{\cancel{2}}\times\frac{\cancel{2}}{\sqrt{\cancel{3}}\cancel{a}}\times\sin\left(\frac{\sqrt{3}at}{2}\right)\right\}$$

$\therefore$ Sol. is $\frac{1}{3a^2}e^{at}-\frac{1}{3a^2}e^{-at/2}\left\{\cos\left(\frac{\sqrt{3}at}{2}\right)+\sqrt{3}\sin\left(\frac{\sqrt{3}at}{2}\right)\right\}$

Prob: using log / $\cot^{-1}$ / $\tan^{-1}$

Solve. Include in Assignment by Ref. book

Formula: $L^{-1}[F(s)] = \frac{-1}{t} L^{-1}[F'(s)]$

1. Solve: $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$
2. Solve: $L^{-1}\left[\cot^{-1}(1+s)\right]$ Hint: $\frac{d}{dt}(\cot^{-1} t) = \frac{-1}{1+t^2}$
3. Solve: $L^{-1}\left[\tan^{-1}\left(\frac{a}{s}\right) + \cot^{-1}\left(\frac{s}{b}\right)\right]$ $\frac{d}{dt}(\tan^{-1} t) = \frac{1}{1+t^2}$
4. P.T $L^{-1}\left[\log\left(1+\frac{w^2}{s^2}\right)\right] = \frac{2}{t}\left[1-\cos wt\right]$