PIN details - IC 74155

$$\boxed{\Gamma n = \int_0^\infty e^{-y} y^{n-1}\, dy}$$

HW S.that $f(x) = \frac{1}{\sqrt{x}}$ is reciprocal under fourier sine or cosine transformation.

Powers of x

Use Gamma integral

$$F_c(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx; \quad F_s(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty x^{-1/2} \cdot R.p \text{ of } e^{-isx} dx$$

$$= R.p \sqrt{\frac{2}{\pi}} \int_0^\infty x^{-1/2+1-1} e^{-isx} dx$$

$$= R.p \sqrt{\frac{2}{\pi}} \int_0^\infty x^{\frac{1}{2}-1} e^{-isx} dx$$

$y = isx$

$dy = is\, dx$

$$= R.p \sqrt{\frac{2}{\pi}} \int_0^\infty \left(\frac{y}{is}\right)^{1/2-1} \times e^{-y} \frac{dy}{(is)}$$

$$= R.p \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{y^{1/2-1}}{(is)^{-1/2}} \times e^{-y} \times \frac{dy}{(is)^1}$$

$$= R.p \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{y^{1/2-1}}{\sqrt{(is)}} e^{-y} dy$$

$$= R.p \sqrt{\frac{2}{\cancel{\pi}}} \times \frac{\Gamma 1/2 \Rightarrow \cancel{\sqrt{\pi}}}{(i)^{1/2} \sqrt{s}}$$

$$= R.p \frac{\sqrt{2}}{\sqrt{s}} \times (i)^{-1/2}$$

$$= R.p \frac{\sqrt{2}}{\sqrt{s}} \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{-1/2}$$

$$= R.p \frac{\sqrt{2}}{\sqrt{s}} \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{\sqrt{s}} \times \frac{1}{\sqrt{2}}$$

IIly for $F_s(s) =$

$$= -I.p \sin\frac{\pi}{4} \times \frac{\sqrt{2}}{\sqrt{s}}$$

Show that $e^{-x^2/2}$ is self resiprocal under FCT.

(15m)

$$FCT = f(s) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty e^{-x^2/2} \cos sx \, ds.$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \cos sx \, ds.$$

$$\cos sx = RP \left\{ e^{(isx)} \right\}$$

$$= R.P. \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{x^2/2} e^{isx} \, dx.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\left(\frac{x^2}{2} + isx\right)} dx$$

$$= R.P\left(e^{-s^2/2}\right) \text{ (reffer Q 1).}$$

$$= \left\{ e^{-s^2/2} \right\}$$

NOTE: under FST its not possi because the IP $\left\{ e^{-s^2/2} \right\}$ does not exist.

$\therefore$ 0.

Therefore not self resiprocal under sine transform.

(15m)

Q

(15 mark)

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$f(x) = e^{-a^2x^2}$ is self resiprocal under FCT.

(5m)

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty e^{-a^2x^2} \cos sx \, dx$$

$$R.P \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} e^{isx} dx \right\}$$

$$R.P \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(e^{-a^2x^2 + isx} \right) dx \right.$$

$$= RP \left\{ \frac{1}{a\sqrt{2}} \times e^{-s^2/4a^2} \right\}$$

$$= \boxed{\frac{1}{a\sqrt{2}} \times e^{-s^2/4a^2}}$$

under sine transform. no. self resiprocal.

Q. Show that $f(x) = e^{-x^2/2}$ is SR under FCT and hence evaluate $F_s(x \cdot e^{-a^2x^2})$.

(15 mark)

$$F_C(s) = \frac{1}{a\sqrt{2}} e^{-s^2/4a^2}$$

$a^2 = 1/2 \quad = \quad a = 1/\sqrt{2} \qquad \frac{1}{\ldots} \times e^{-st\, s^2/4a^2}$

Q VQ 3

find F.T of $f(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$

and hence deduce that.

a. $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$. (F.I.T)

b. $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \pi/3$. (P.I) → high powers

(i) F.T → F(S).

(ii) IFT → f(x)

Parseval's Identity

$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{+isx}\, dx$.

$|x|<1 = -1<x<1$

$\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} [1-|x|]$

$\int_{-a}^{a} f(x) = \begin{cases} 2\int_0^a f(x)\,dx & \text{even} \\ 0 & \to \text{odd} \end{cases}$

$\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} [1-|x|] (\cos sx + i \sin sx)\, dx$

$= \frac{1}{\sqrt{2\pi}} \left(1-|x| \cos sx + i \int_{-1}^{1} (1-|x|) \sin sx\, dx \right)$

odd fun

$\frac{1}{\sqrt{2\pi}} \times 2 \int_0^{+1} (1-x) \cos sx\, dx$

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$$\frac{\sqrt{2}}{\sqrt{\pi}} \int_0^1 \underset{u}{(1-x)} \underset{v}{\cos sx}\, dx$$

u	v
$1-x$	$\cos sx$
-1	$\sin sx/s$
0	$-\cos x/s^2$
	$-\sin sx/s^3$

$$\frac{\sqrt{2}}{\sqrt{\pi}} \left[(1-x)\frac{\sin sx}{s} - \frac{\cos sx}{s^2}\right]_0^1$$

$$\left(0 - \frac{\cos s}{s^2}\right) - \left\{0 - 1/s^2\right\}$$

$$\sqrt{\frac{2}{\pi}} \times \left\{\frac{1-\cos(s)}{s^2}\right\}$$

$$\sqrt{2/\pi} \times \frac{2\sin^2(s/2)}{s^2} \qquad \boxed{-\infty < s < \infty}$$

(ii) IFT: $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds.$

$$\frac{\times 2\sqrt{2}/\sqrt{\pi}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\sin^2(s/2)}{s^2}\left[\cos sx - i\underbrace{\sin sx}_{\text{odd fun}=0}\right) ds$$

$$= \frac{4}{\pi} \int_0^{\infty} \frac{\sin^2(s/2)}{s^2} \cos sx\, ds$$

take $x = 0$

$\therefore \boxed{\cos sx = 1}$

$$f(0) = \frac{4}{\pi} \int_0^{\infty} \frac{\sin^2(s/2)}{s^2}\, ds$$

s	0	∞
t	0	∞

$t = s/2 \quad dt = \frac{ds}{2}$

$t^2 = s^2/4$

$$\frac{\pi}{4} = \int_0^{\infty} \frac{\sin^2(s/2)\, ds}{s^2}$$

$$\frac{\pi}{4} = \int_0 \frac{\sin^2 t \, 2dt}{4t^2}$$

$$\boxed{\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2}$$

(iii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \pi/3$ parsewals.

parsevals:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds.$$

$$\int_{-1}^{1} |1-|x||^2 dx$$

$$2\int_0^1 |[1-x]|^2 dx.$$

$$= 2\left[\frac{(1-x)^3}{(-1)\times 3}\right]_0^1$$

$$\frac{2}{-3}(0-1) = \boxed{\frac{2}{3}} = LHS$$

$$\int_{-\infty}^{\infty} |F(s)|^2 = 4 \times \frac{2}{\pi} \times \frac{\sin^4(s/2)}{s^4}$$

$$\frac{8}{\pi}\int_{-\infty}^{\infty} \underbrace{\frac{\sin^4(s/2)}{s^4}}_{\text{even.}} ds$$

$$\frac{8\times 2}{\cancel{2}\pi}\int_0^{\infty} \frac{\sin^4(s/2)}{s^4}$$

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Now we k.

$t = s/2 \Rightarrow dt = ds/2$. $s = 2t$

$$\frac{\cancel{2}}{3} = \frac{16}{\pi}\int_0^\infty \frac{\sin^4 t}{(2t)^4}\,\cancel{2}\,dt.$$

$$\Rightarrow \boxed{\frac{\pi}{3} = \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt}$$ Hence proved.

4Q. Find F.T of $f(x) = \begin{cases} a-|x| & |x|<a \\ 0 & |x|>a \end{cases}$ $-a<x<a$

and hence deduce

(i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$.

(ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \pi/3$

1. Find the fourier transform.

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx.$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-a}^{a} (a-|x|)(\cos sx + \cancel{i\sin sx})\, dx$$ $\rightarrow$ odd $= 0$

u	v
$a-x$	$\cos sx$
-1	$\frac{\sin sx}{s}$
	$-\frac{\cos sx}{s^2}$

$$= \frac{\cancel{2}\sqrt{2}}{\sqrt{\cancel{2}\pi}}\int_0^a (a-x)(\cos sx)\, dx$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}}\left[(a-x)\left(\frac{\sin sx}{s}\right) - \left(\frac{\cos sx}{s^2}\right)\right]_0^a$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \left(1/s^2 - \frac{\cos as}{s^2} \right)$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{\pi}} \left(\frac{1-\cos as}{s^2} \right)$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{\pi}} \left(\frac{2\sin^2(as/2)}{s^2} \right)$$

(ii) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(s)\, e^{-isx}\, ds.$

$$\Rightarrow \frac{\cancel{\sqrt{2}}}{\sqrt{\pi}} \times 2 \times \frac{1}{\cancel{\sqrt{2}}\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\sin^2(as/2)}{s^2}\, ds$$

$$\Rightarrow \frac{4}{\sqrt{\pi}} \int_0^{\infty} \frac{\sin^2(as/2)}{s^2}\, ds$$

$\frac{as}{2} = \theta$.

$$\frac{4}{\sqrt{\pi}} \int_0^{\infty} \frac{\sin^2(as/2)}{s^2} \times \cos sx\, ds.$$

Now let $x=0$

$\therefore \cos sx = 1$

$f(0) = a$

$$a = \frac{4}{\pi} \int_0^{\infty} \frac{\sin^2(as/2)}{s^2}\, ds.$$

$a = 2$ and put $s = t \Rightarrow s^2 = t^2$

$ds = dt$

$$2 = \frac{4}{\pi} \int_0^{\infty} \frac{\sin^2 t}{t}\, dt$$

$$\Rightarrow \boxed{\frac{\pi}{2} = \int_0^{\infty} \frac{\sin^2 t}{t}\, dt}$$

class : II ...

1. To prove that

$$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2} \times \frac{\pi}{ab(a+b)}.$$

$f(x) = e^{-ax}.$

$$Fc(f(x)) = \sqrt{2/\pi} \int_0^\infty e^{-ax} \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{a}{s^2+a^2}$$

$g(x) = e^{-bx}$

$$Fc(g(x)) = \sqrt{2/\pi} \int_0^\infty e^{-bx} \cos sx \, dx$$

$$= \sqrt{2/\pi} \times \frac{b}{s^2+b^2}$$

$\therefore$

$$\int Fc(f(x)) \times Fc(g(x)) = \frac{2}{\pi} \int_0^\infty \frac{ab}{(a^2+s^2)(s^2+b^2)}$$

and LHS $\int_0^\infty f(x) g(x) \, dx$

$$= \int_0^\infty e^{-ax} e^{-bx} \, dx$$

$$= \int_0^\infty e^{-(a+b)x} \, dx$$

$$= \left[\frac{e^{-(a+b)x}}{-(a+b)} \right]_0^\infty = 1/a+b.$$

$$= \frac{\pi}{2ab(a+b)} = \int \frac{1}{(a^2+s^2)(s^2+b^2)}$$

LHS = RHS. $\boxed{\text{Put } s = x}$

$$\frac{\pi}{2ab(a+b)} = \int \frac{1}{(x^2+a^2)(x^2+b^2)}$$

2. TO prove that

$$\int_0^\infty \frac{x^2 \, dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2} \times \frac{\pi}{(a+b)}$$

$f(x) = e^{-ax}$ $g(x) = e^{-bx}$

$$Fs(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin sx = \sqrt{2/\pi}\left(\frac{s}{s^2+a^2}\right)$$

$$Fs(g(x)) = \sqrt{2/\pi} \int_0^\infty e^{-bx} \sin sx = \sqrt{2/\pi}\left(\frac{s}{s^2+b^2}\right)$$

$$RHS = \frac{2}{\pi} \int_0^\infty \frac{s^2}{(s^2+a^2)(s^2+b^2)}.$$

LHS

$$\int_0^\infty f(x) g(x) \, dx = \int_0^\infty e^{-(a+b)x} dx$$

$$= \left[\frac{e^{-(a+b)x}}{-(a+b)}\right]_0^\infty$$

$$= (1/a+b)$$

$$= \frac{\pi}{2(a+b)} = \int_0^\infty \frac{s^2}{(s^2+a^2)(s^2+b^2)}$$

$\boxed{\text{Put } s = x}$

$$= \frac{\pi}{2(a+b)} = \int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)}.$$

3. To prove that:

$$\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}.$$

Take $f(x) = g(x) = e^{-ax}$

$$F_c(g(x)) = F_c(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos sx.$$

$$= \sqrt{\frac{2}{\pi}} \left(a/a^2+s^2 \right).$$

Now $RHS = \frac{2}{\pi}\left(\left(\frac{a^2}{(a^2+s^2)}\right)^2\right)$

LHS

$$\int_0^\infty e^{-2ax} \Rightarrow \left[\frac{e^{-2ax}}{-2a}\right]_0^\infty = \frac{1}{2a}.$$

$$\therefore \int \frac{1}{(a^2+s^2)^2} = \frac{\pi}{4a^3}$$

$\boxed{\text{Put } s = x}$

$$\int_0^\infty \frac{1}{(a^2+x^2)^2} = \frac{\pi}{4a^3}$$

Hence Proved:

4. To prove that.

$$\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)^2} = \pi/4a.$$

$$f(x) = g(x) = e^{-ax}$$

$$F_s(f(x)) = F_s(g(x)) = \sqrt{2/\pi}\int_0^\infty e^{-ax}\sin sx$$

$$= \sqrt{2/\pi}\left(\frac{s}{s^2+a^2}\right).$$

$$\therefore\ RHS = \frac{2}{\pi}\int_0^\infty \frac{s^2}{(s^2+a^2)^2}.$$

$$LHS = \int_0^\infty f(x)g(x)\,dx = \int_0^\infty e^{-(a+\cancel{a})x}\,dx$$

$$= \left[\frac{e^{-(a+\cancel{a})x}}{-(\cancel{a}+\cancel{a})}\right]_0^\infty$$

$$= \frac{1}{2a}$$

$$\therefore\ \frac{\pi}{4a} = \int_0^\infty \frac{s^2}{(s^2+a^2)^2}$$

Put $s = x$ ∴

$$\boxed{\frac{x^2}{(x^2+a^2)^2} = \frac{\pi}{4a}}$$

Hence proved.