Technische Universität Braunschweig

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Assignment 6

- Programming exercises -

Upload your solution until Mo, 05 June 2023, 03:00 pm.

Programming exercise 6.1 *Composite quadrature in* $\mathbb{R}^2$ **(10 points)**

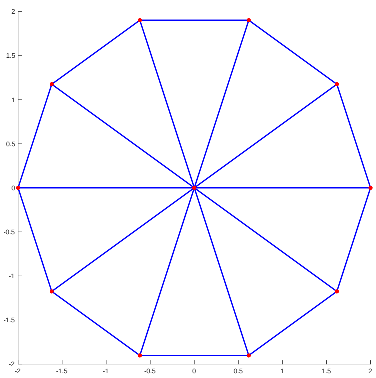
In this exercise, we built upon the results from Theoretical exercise 6.1 in which you should have derived a quadrature $Q(f)$ which computes an approximate value of

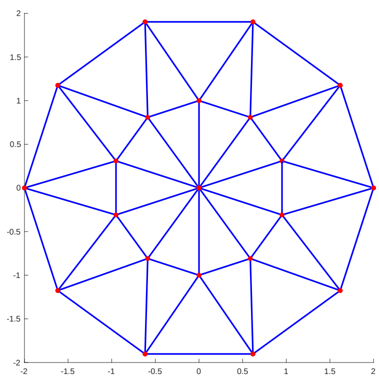
$$I(f) = \int_\Omega f(x,y)\,\mathrm{d}a,$$

where Ω is a triangle in $\mathbb{R}^2$. The fact that Ω is a triangle may be annoying if you aim to compute the integral over a more complex geometrical shape. But there is a natural way to extend the quadrature from a triangle any other geometrical form: Pave it with triangles and apply the quadrature to each triangle.

(This procedure is similar to the composite quadrature which we used in one dimension where we applied the quadrature to a number of intervals.)

For our purposes, we want to integrate the function $f(x,y) = 1 - x^2 - y^2$ over $B_2(0)$, which is the circle centered at 0 with radius 2. Therefore, subdivide the circle into triangles in two different ways:





Write three Matlab files `mesh_1.m`, `mesh_2.m` and `assignment_6.m`. The first two files are function files which contain the functions `T = mesh_1(N)` and `T = mesh_2(N)`. These functions are supposed to return an array with all triangles for the first/second subdivision technique, represented by the coordinates of their corners. The parameter `N` denotes the number of triangle corners which hit the boundary of $B_2(0)$.

Use these functions to compute triangle meshes for `N=10,100,1000,...` and compute a composite quadrature $Q_N(f)$ by applying your quadrature rule from the theoretical exercise to each triangle. Compare the results with the exact value $I(f)$ (computed by hand).

Remarks:

- There are also more advanced techniques to subdivide an arbitrary shape into triangles. The technique which we chose is in comparison relatively easy to compute.
- Don't forget to return the triangles for the second subdivision which look like they are smaller than the other triangles in the outer circle.
- If you weren't able to find a quadrature in the theoretical exercise, that is not a problem. Implementing everything up to the point where you apply the quadrature already gives you the majority of points for this exercise. Make a comment in the script if this apples for you.