Prof. Dr. Dirk Langemann & Jannis Marquardt, M.Sc.
Institute for Partial Differential Equations

Assignment 7

- Programming exercises -

Upload your solution until Mo, 12 June 2023, 03:00 pm.

Programming exercise 7.1 *Numerical differentiation* **(10 points)**

(a) Write a function file (a ".m"-file) `diff_quot`, which calculates the difference quotient of a given function $f(x)$ for a certain granularity h. Your function should take four inputs: The function `f`, the position `x` where the derivative shall be computed, a granularity `h` and a string `method`. The `method`-string may assume the values "`forward`", "`backward`" or "`central`" and `diff_quot` should return the value of the forward, backward or central difference quotient accordingly.

(b) Put a grid with $N = 2, 3, 4, \ldots$ nodes on the interval $[0, 2\pi]$. Consider the function $f(x) = \sin(x)$. Compute the forward, backward and central difference quotient for each of the grid points. Hereby, the granularity `h` shall be the distance between two nodes. Plot the results in a coordinate system and connect the dots of the forward/backward/central difference quotient by a line (Hence, you should get three polygonal chains for each N).
Create an animation, where each frame depicts the plot described above for a certain value of N. Start with $N = 1$ in the first frame and continue till you reach $N = 100$.
Hint: You may utilise the `Matlab` *command* `pause` *to create such a plot.*

(c) Consider $f(x) = \sin(x)$ again. Use the function `diff_quot` from (a) in order to compute (affine) linear functions

$$T_i(x) = m_i x + b_i,$$

such that $T_i(x)$ touches $f(x)$ tangential at the points $x_1 = 0, x_2 = \pi$ and $x_3 = \frac{3\pi}{2}$. Use central differential quotients. Plot the three tangents $T_i(x)$ in a common plot with $f(x)$.