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Assignment 9

- Programming exercises -

Upload your solution until Mo, 26 June 2023, 03:00 pm.

Programming exercise 9.1 **(2.5 + 2.5 + 2.5 + 2.5 = 10 points)**

Consider the initial value problem

$$y' = f(t,y), \quad y(0) = y_0.$$

(a) Write a `Matlab` function file `explicitEuler` which takes as input the right-hand side f of the initial value problem, the initial value y_0, the end time t_{end} and step size h and computes a numerical solution of the IVP at time t_{end} using explicit Euler method. The output of the `Matlab` function should not only be the solution of the problem but should also include intermediate values for each step (path to the solution at t_{end} from the initial time $t_0 = 0$).

(b) Write a `Matlab` function file `improvedEuler` which meets the same requirements as `explicitEuler` in (a), but uses the improved Euler method instead of the explicit Euler method.

(c) Write a `Matlab` function file `EulerHeun` which meets the same requirements as `explicitEuler` in (a), but uses the Euler-Heun method instead of the explicit Euler method.

For the rest of this exercise, consider the initial value problem

$$y' = 2t(1+y), \quad y(0) = 0,$$

which has the analytical solution

$$y = e^{(t^2)} - 1.$$

(d) Use the function files from (a)-(c) to compute numerical solutions for the given IVP at $t_{\text{end}} = 2$ as well as the paths which lead to the solution. Therefore, consider step sizes $h \in \{1, 0.5, 0.1, 0.01\}$. Plot for each choice of h the paths together with the analytic solution from (d) in a common plot over the interval $[0, 2]$.