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Assignment 4

- Programming exercises -

Upload your solution until Mo, 15 May 2023, 03:00 pm.

Programming exercise 4.1 *Summed Kepler's barrel rule* **(4 + 4 + 2 = 10 points)**

Consider the integral $\int_a^b f(x)\,dx$.

(a) Write a `Matlab` function which subdivides $[a,b]$ into equidistant subintervals and applies Kepler's barrel rule to each subinterval. The function should take four inputs: `f`, `a`, `b` and `J`, where `J` denotes the number of subintervals.

(b) We now aim to iteratively call the function from (a) in order to compute the integral of the function $f(x) = \sin(x)$ for the interval $[a,b] = [0,\pi]$. Each time, the number of used intervals J should be increased by one. The number of intervals should be increased as long as the value of the summed quadratures keeps adjusting. More precisely: Repeat this process as long as the computed values vary more than 10^{-4} between each repetition. Stop the iteration if the difference between the computed values for the J-th step and the $(J+1)$-th step is smaller than 10^{-4} for more than 10 repetitions.
Also build in a safeguard: Let your program terminate after a maximal number of steps with an error message.

(c) Extend your program in the following way: Store the value of the integral for each number of intervals J in a suitable way. Create a plot with the number of intervals on the x-axis and the distance between the value in the J-th step and the last computed value (for the largest number of intervals) on the y-axis.