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Assignment 5

- Programming exercises -

Upload your solution until Mo, 22 May 2023, 03:00 pm.

Programming exercise 5.1 *Romberg extrapolation* **(10 points)**

Write a `Matlab` function file which takes a function f, boundary values $a < b$ and an integer number n as input arguments and calculates the result of $n-1$ steps of the Romberg extrapolation starting with the step size $h = a - b$.

Use this function file in order to write a `Matlab` script which computes the approximate value of

$$I(f) = \int_0^1 \mathrm{e}^x \cos x \, \mathrm{d}x$$

executing 2 steps of the Romberg extrapolation. Compare the resulting approximation with the exact value $I(f)$. (You may consider the result from the `Matlab` function `integral` as exact). Now, find in both cases (using your `Matlab` script) a number of the extrapolation steps required so that the absolute value of an error is less than 10^{-6}.

Hint: Look at the Romberg extrapolation schemes from the lecture. The entries in the column on the very left are created with the summed trapezoid rule for $1, 2, 4, 8, 16, \ldots$ intervals. You do not need to apply the summed quadrature rules outside of this column. The other columns may be computed with the stored values from the columns to their left. Think about how to extend the given schemes to an arbitrary number of extrapolation steps.