CSE 201: Data Structures

Lecture 10: Graphs

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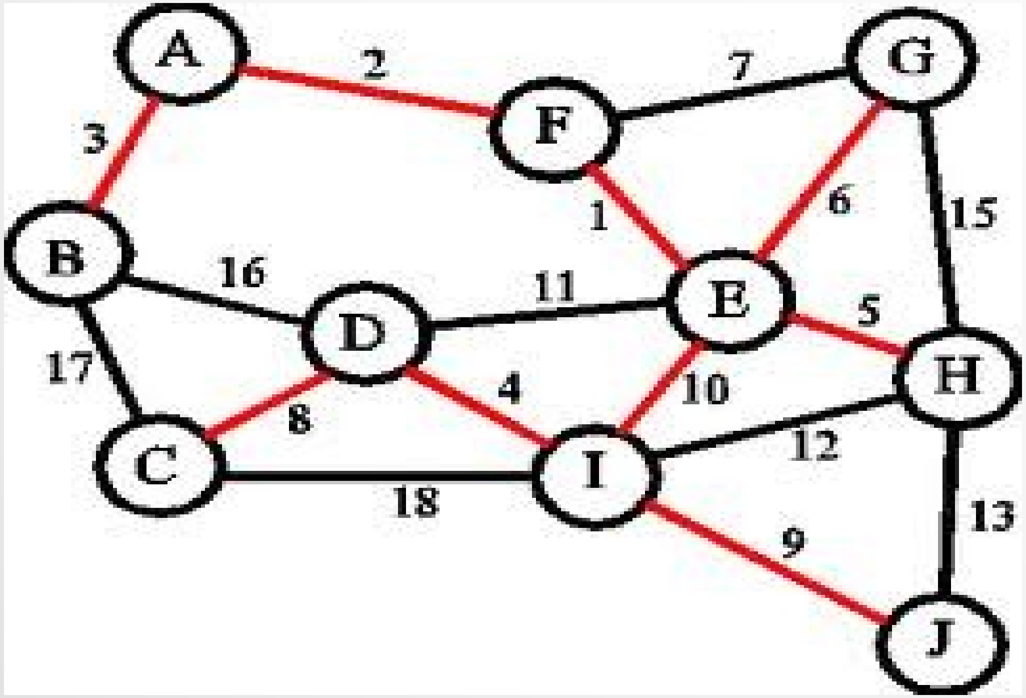
Minimum Spanning Tree

- Given a weighted undirected graph G, goal is to find a tree T such that
  - T contains all vertices in G
  - Sum of weights of edges in T is minimum



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Algorithms

- First algorithm to find MST was by a Czech scientist Baruvka
  - Baruvka's algorithm
  - Uses greedy approach
- Other greedy approaches
  - Prim's algorithm
  - Kruskal's algorithm

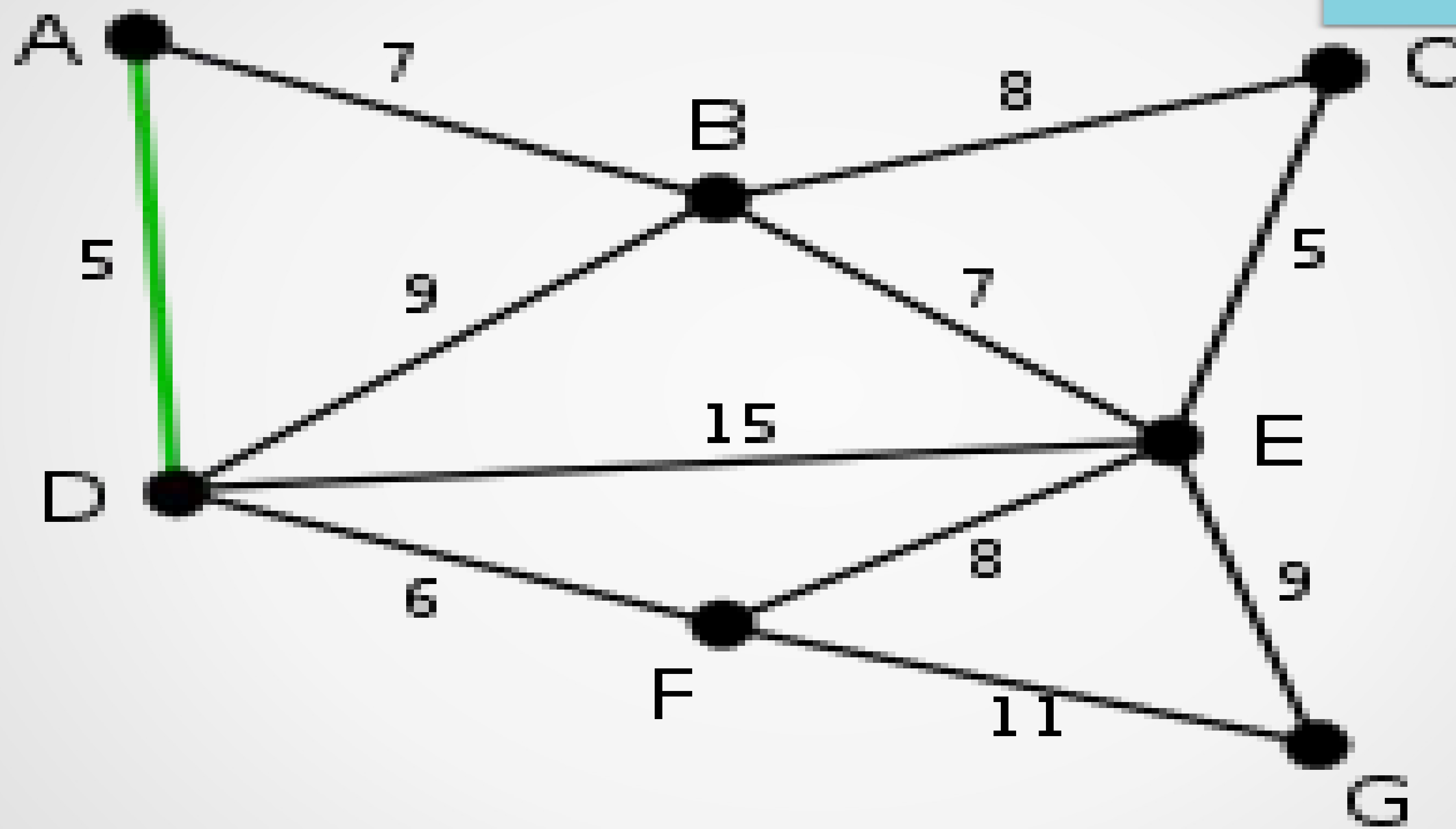
Kruskal's Algorithm

- Greedy Algorithm
- The algorithm maintains a forest of trees
- An edge is accepted it if connects distinct trees
  - The edge is chosen such that its weight is minimum amongst the edges connecting the two trees
  - The trees are also known as clusters or clouds

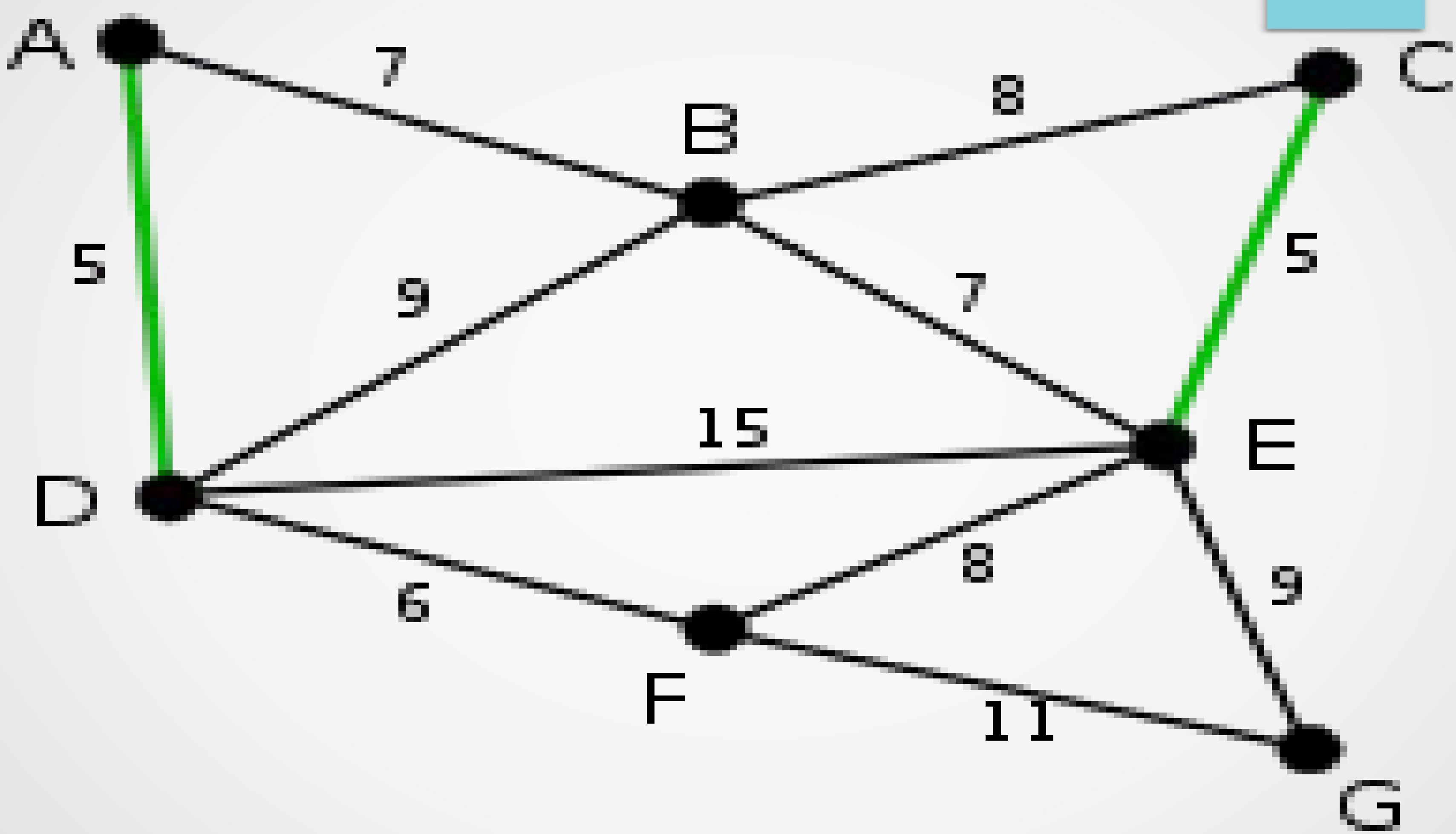
Kruskal's Algorithm

- Let every node in G be a cluster C(v)
- Initialize a priority queue Q with all edges in G using weights as keys
- Take the minimum weight edge in Q and if C(u) <> C(v)
  - add the edge to MST T
  - Merge the clusters C(u) and C(v)
- Repeat till there are no more clusters to merge

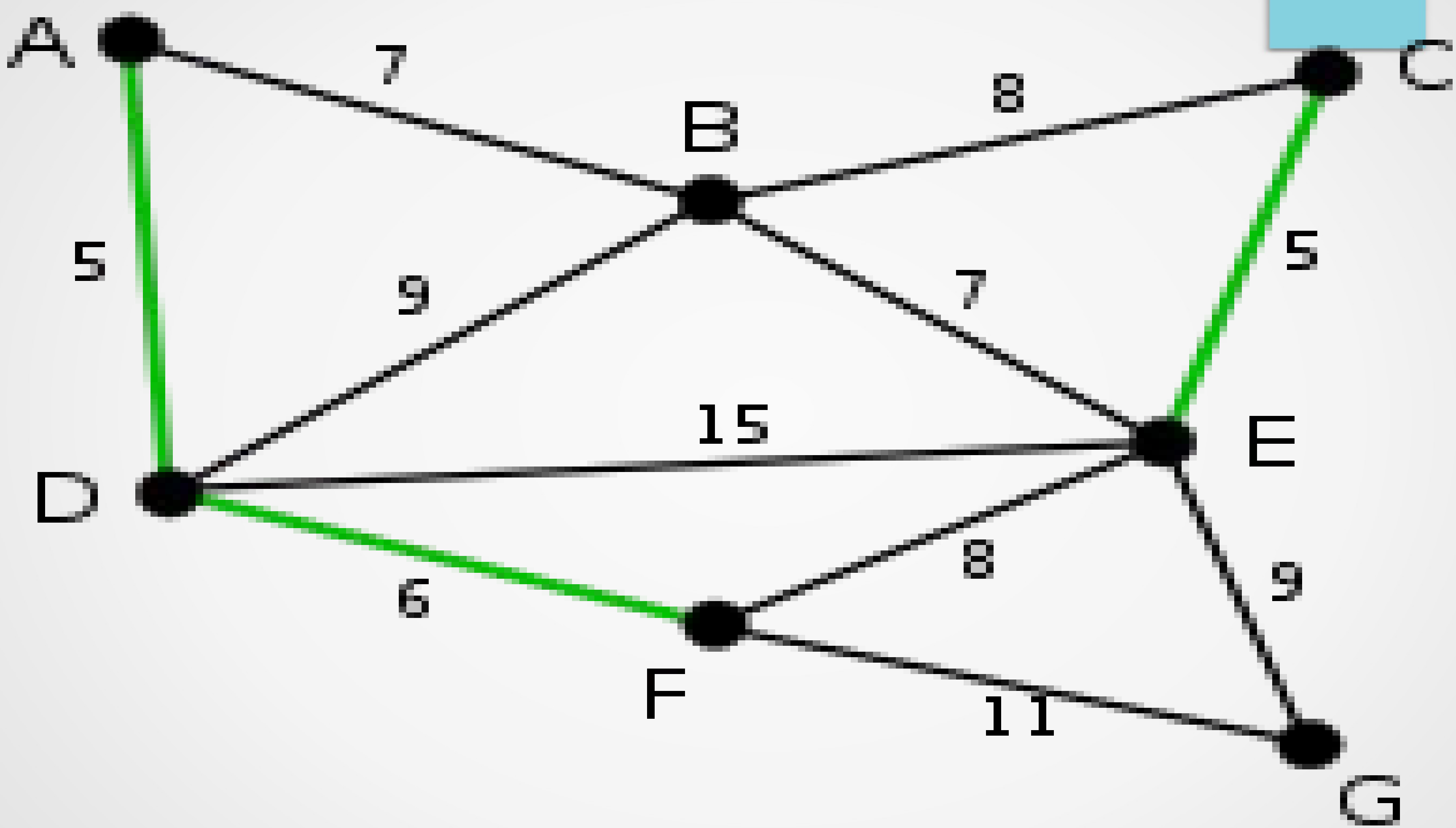
Kruskal's Algorithm: Example



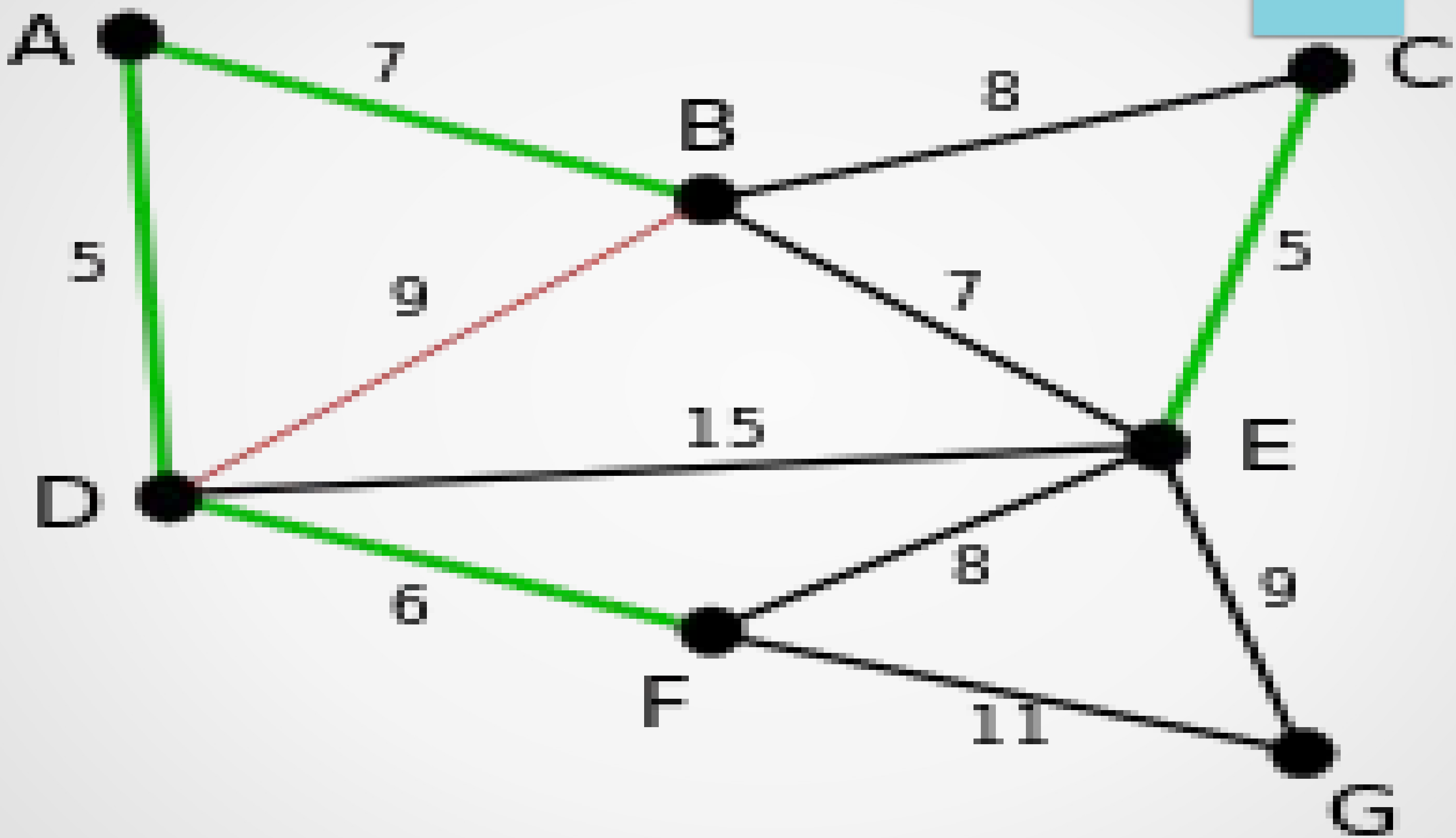
Kruskal's Algorithm: Example



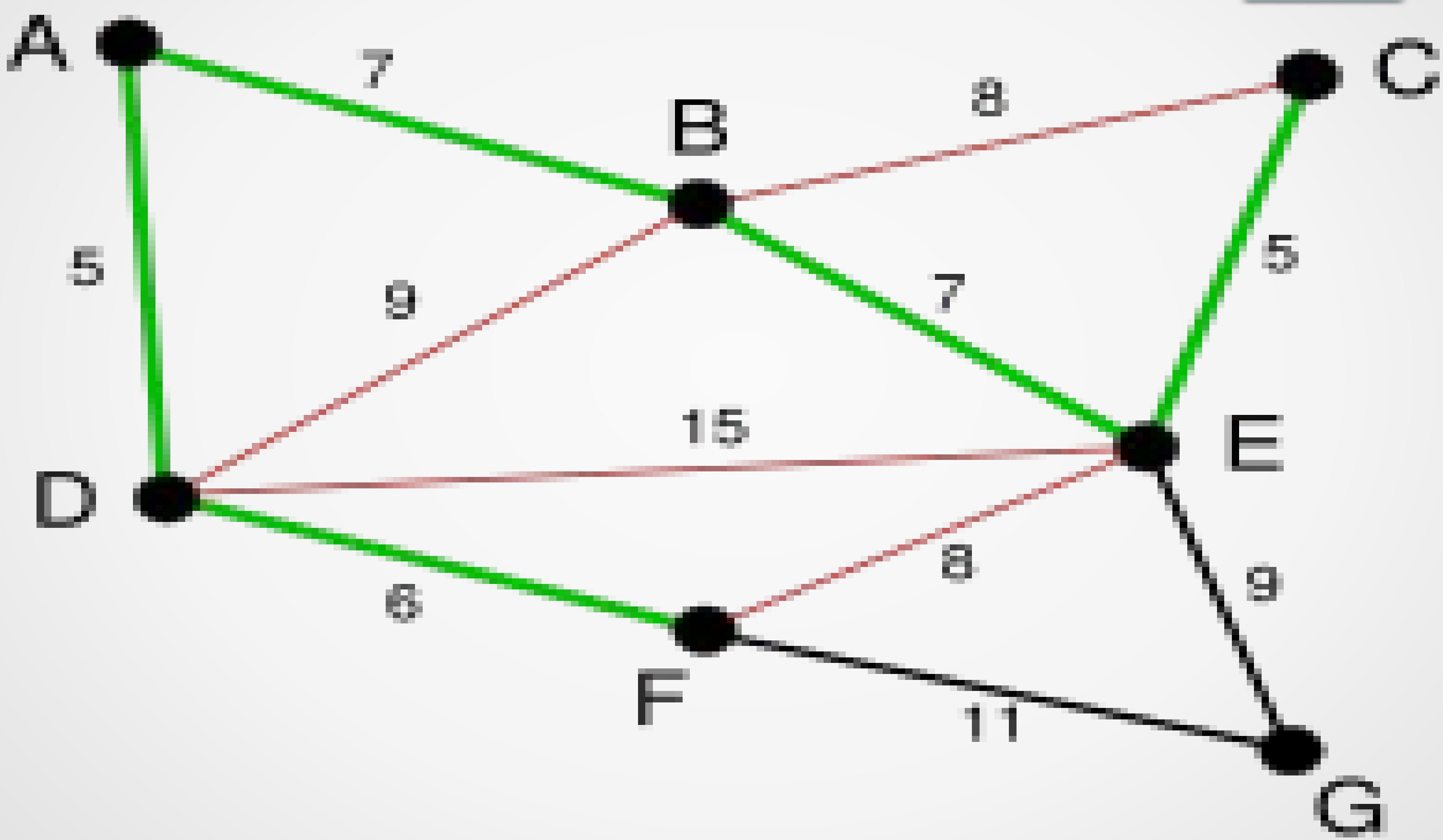
Kruskal's Algorithm: Example



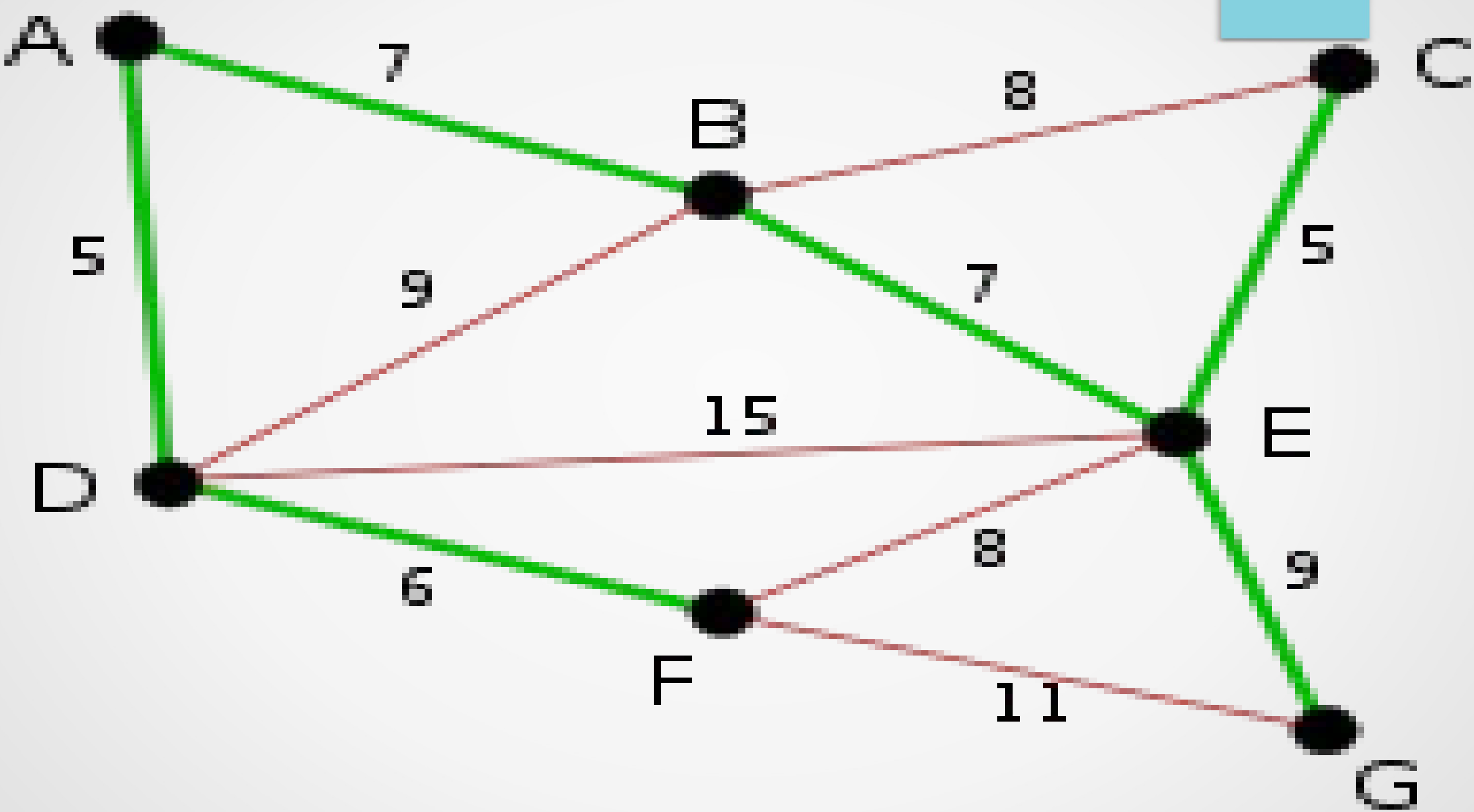
Kruskal's Algorithm: Example



Kruskal's Algorithm: Example

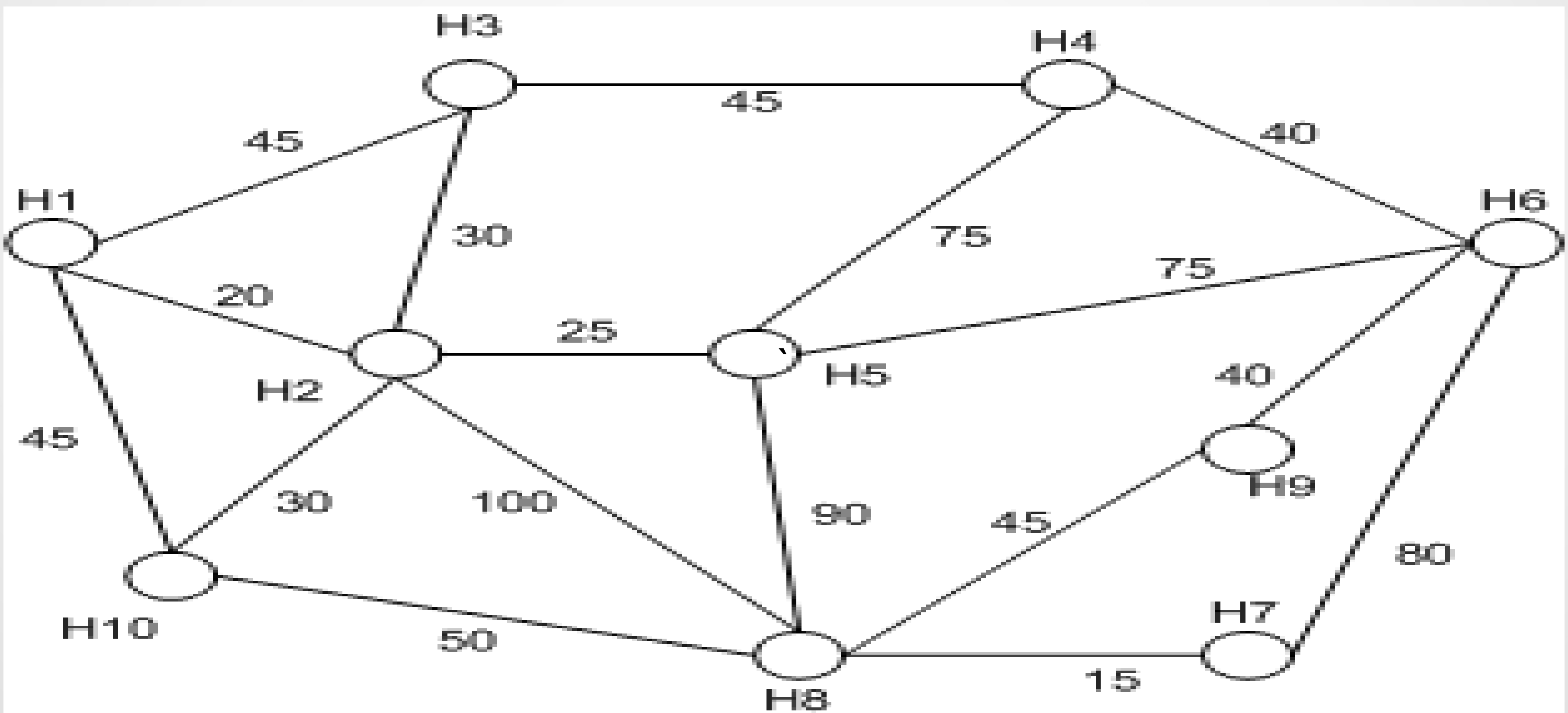


Kruskal's Algorithm: Example



Exercise

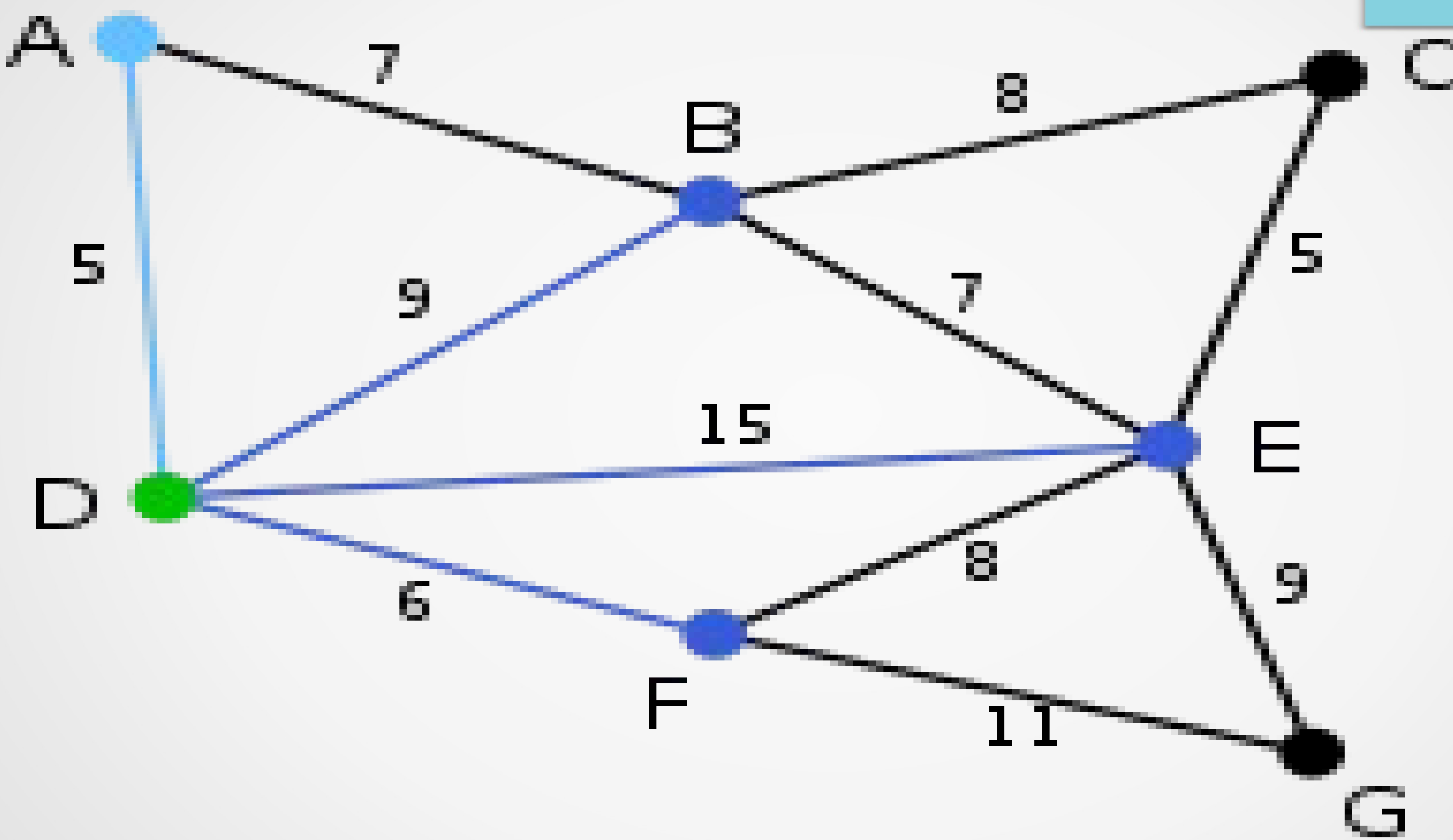
- Find the MST for the following graph using Kruskal's algorithm



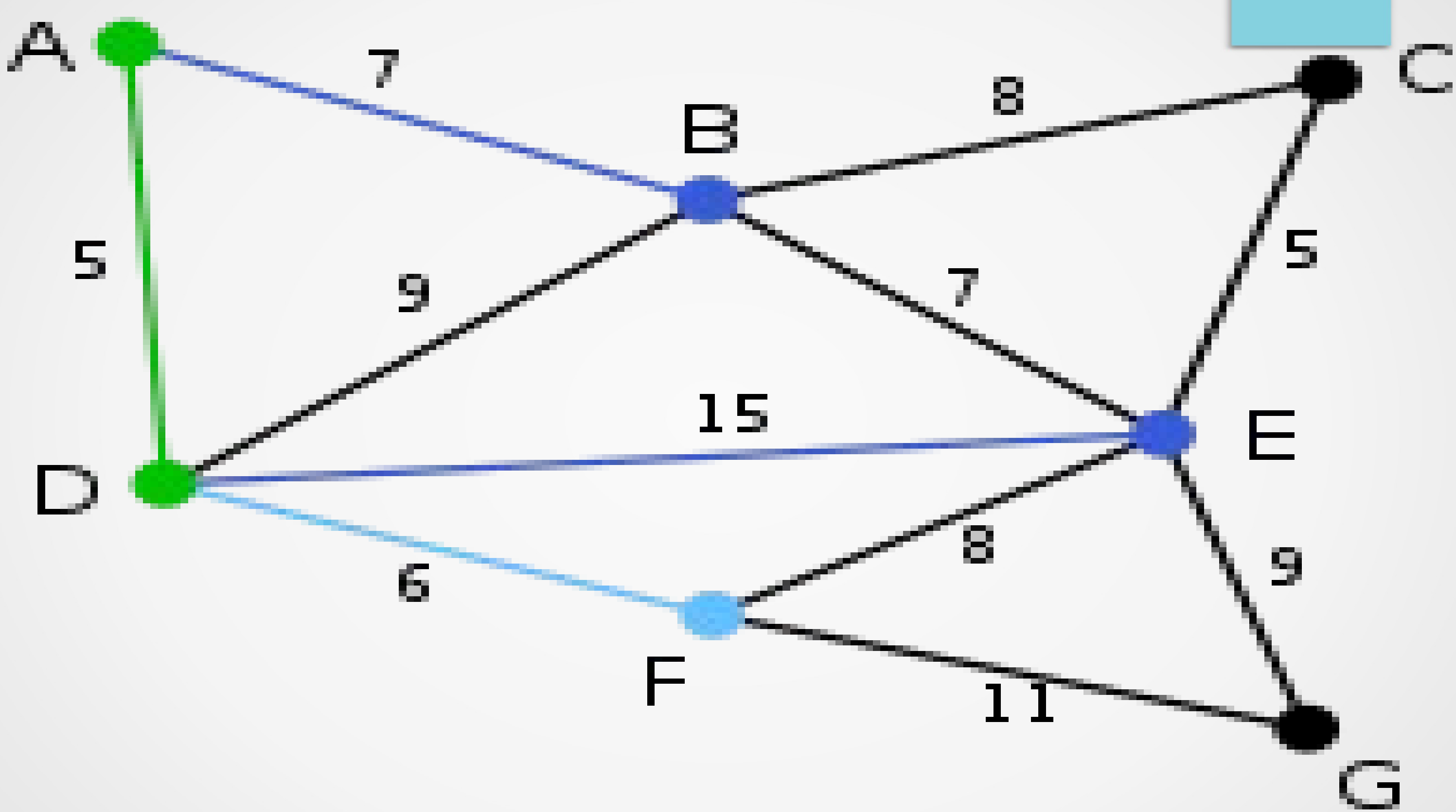
Prim's Algorithm

- Similar to Dijkstra
- Pick any vertex v of G
- Initialize for all u not v, d[u] to infinity and d[v]=0
- Remove from Q u with minimum d[u]
  - Add u and edge (u,v) to T
  - For all neighbors z of u, do relaxation by finding d[z] = w(u,z), and update Q
    - If d[z] > d(u,z), d[z] = d(u,z)

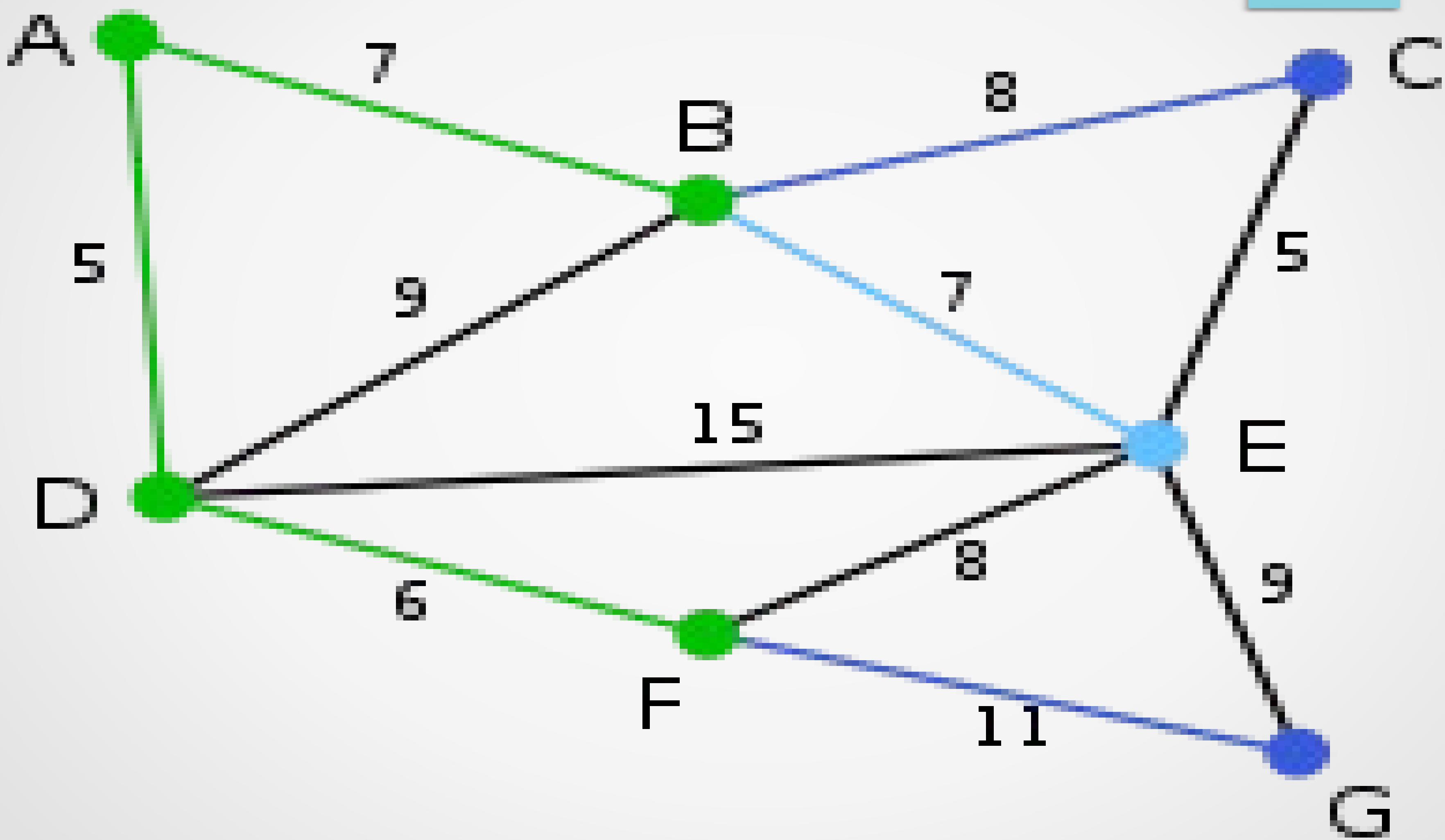
Prim's Algorithm: Example



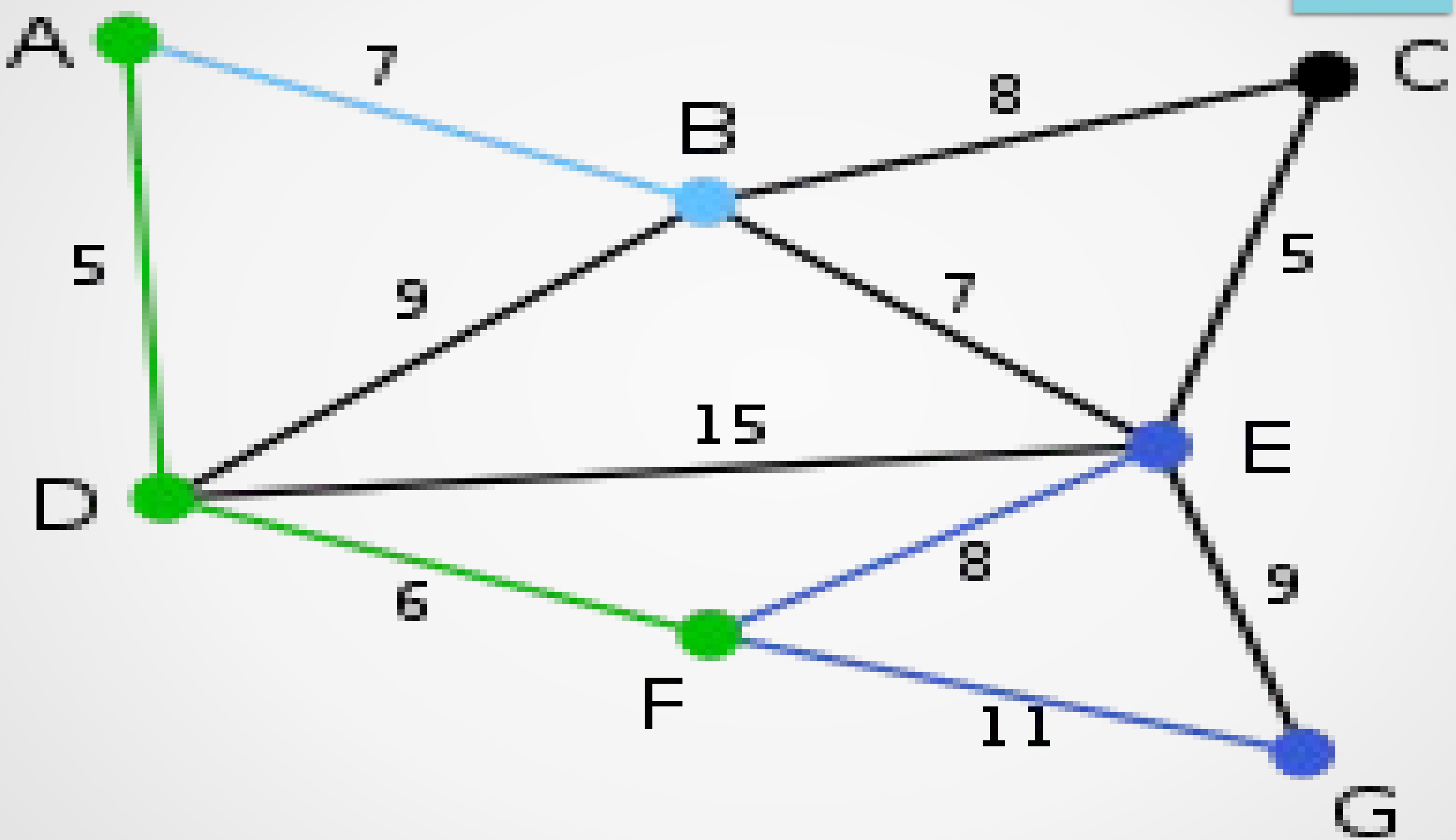
Prim's Algorithm: Example



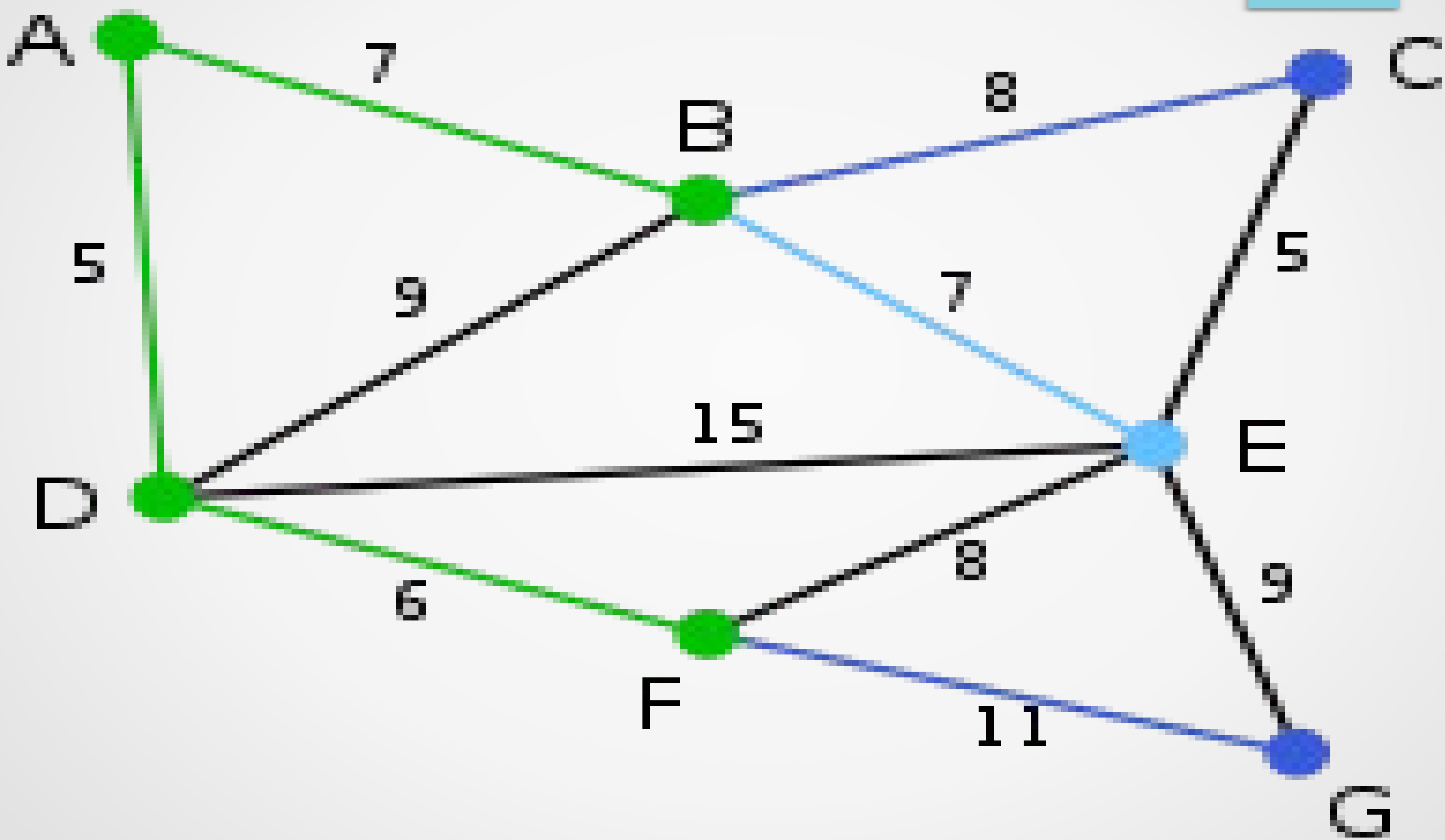
Prim's Algorithm: Example



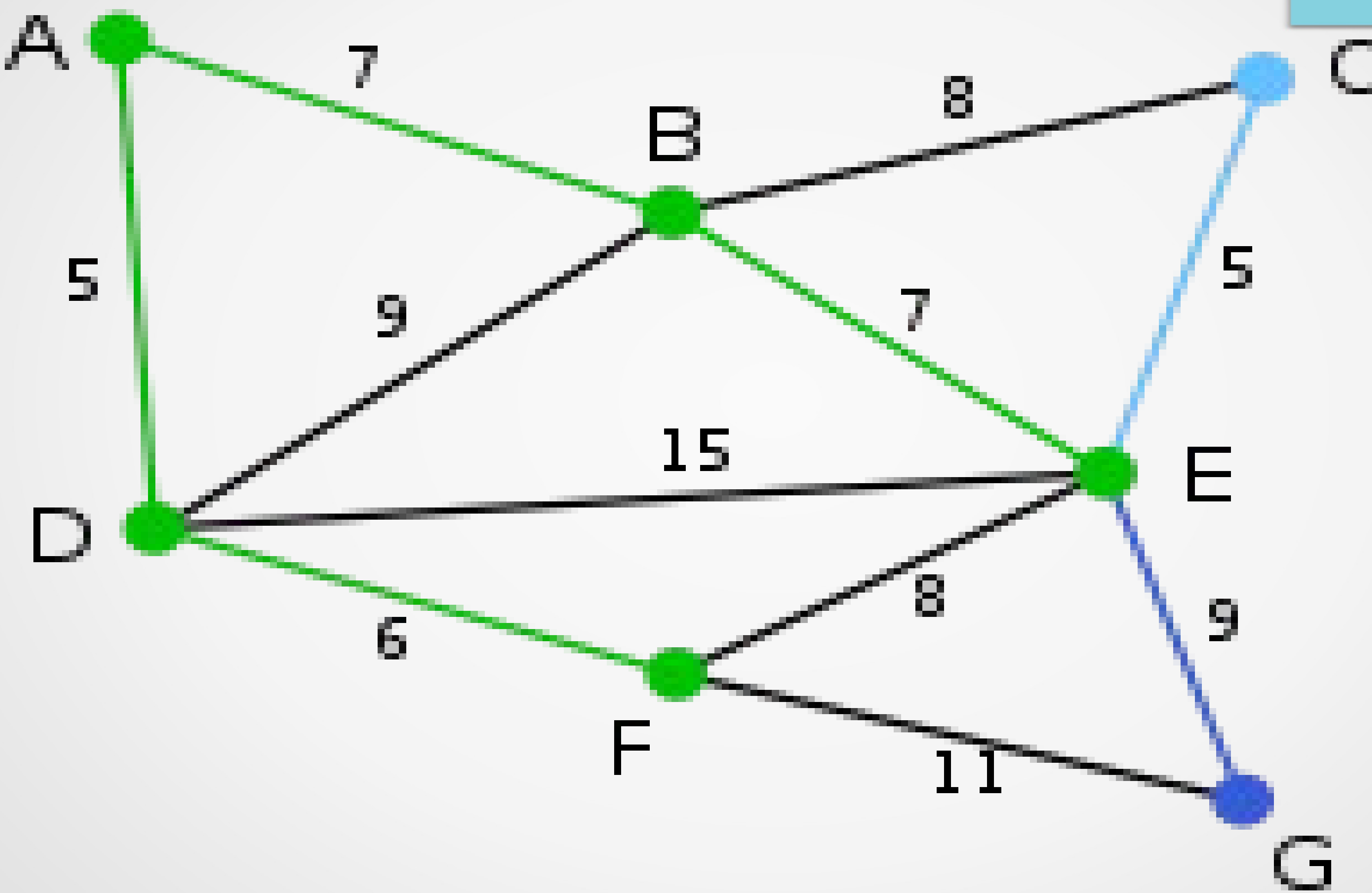
Prim's Algorithm: Example



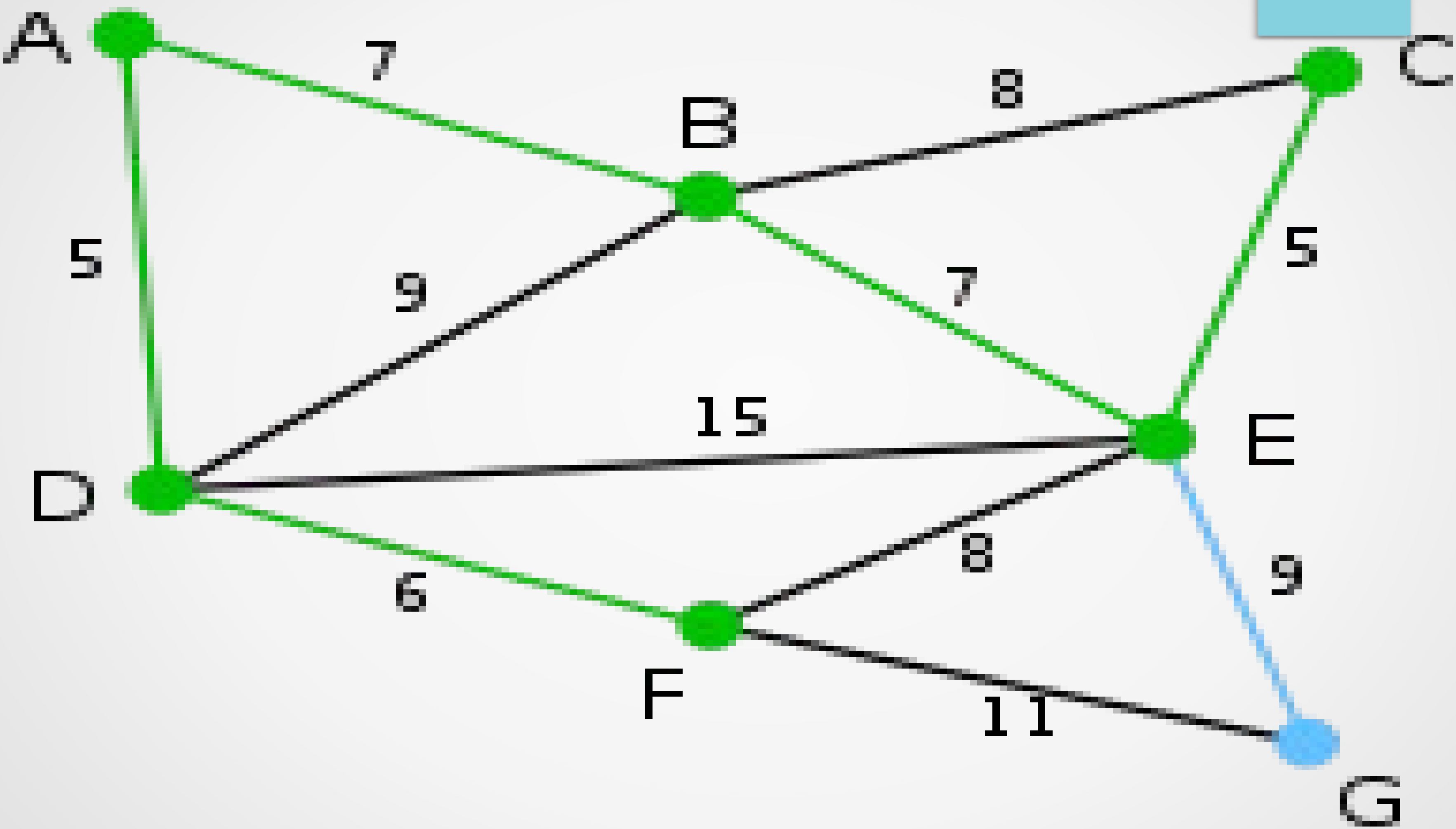
Prim's Algorithm: Example



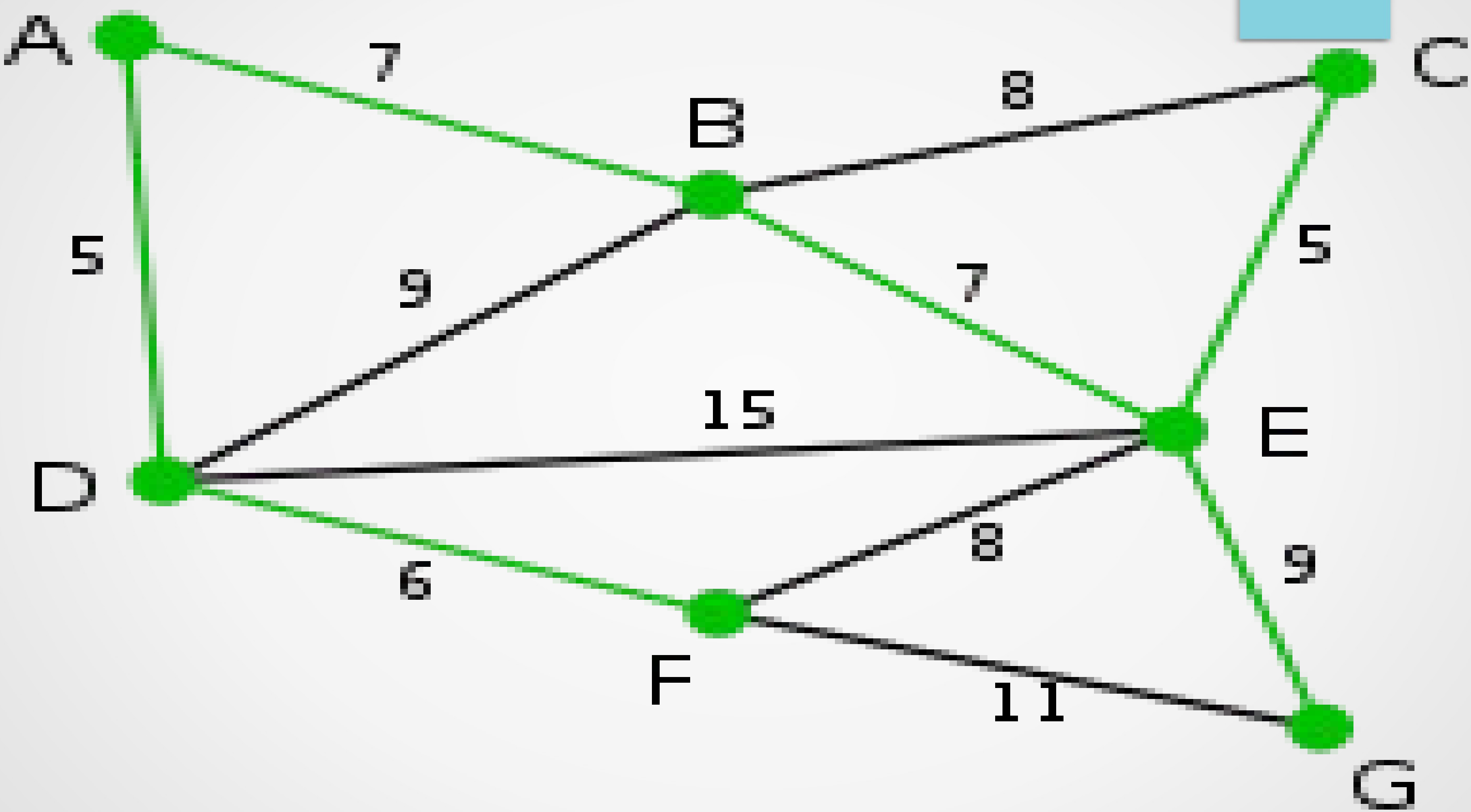
Prim's Algorithm: Example



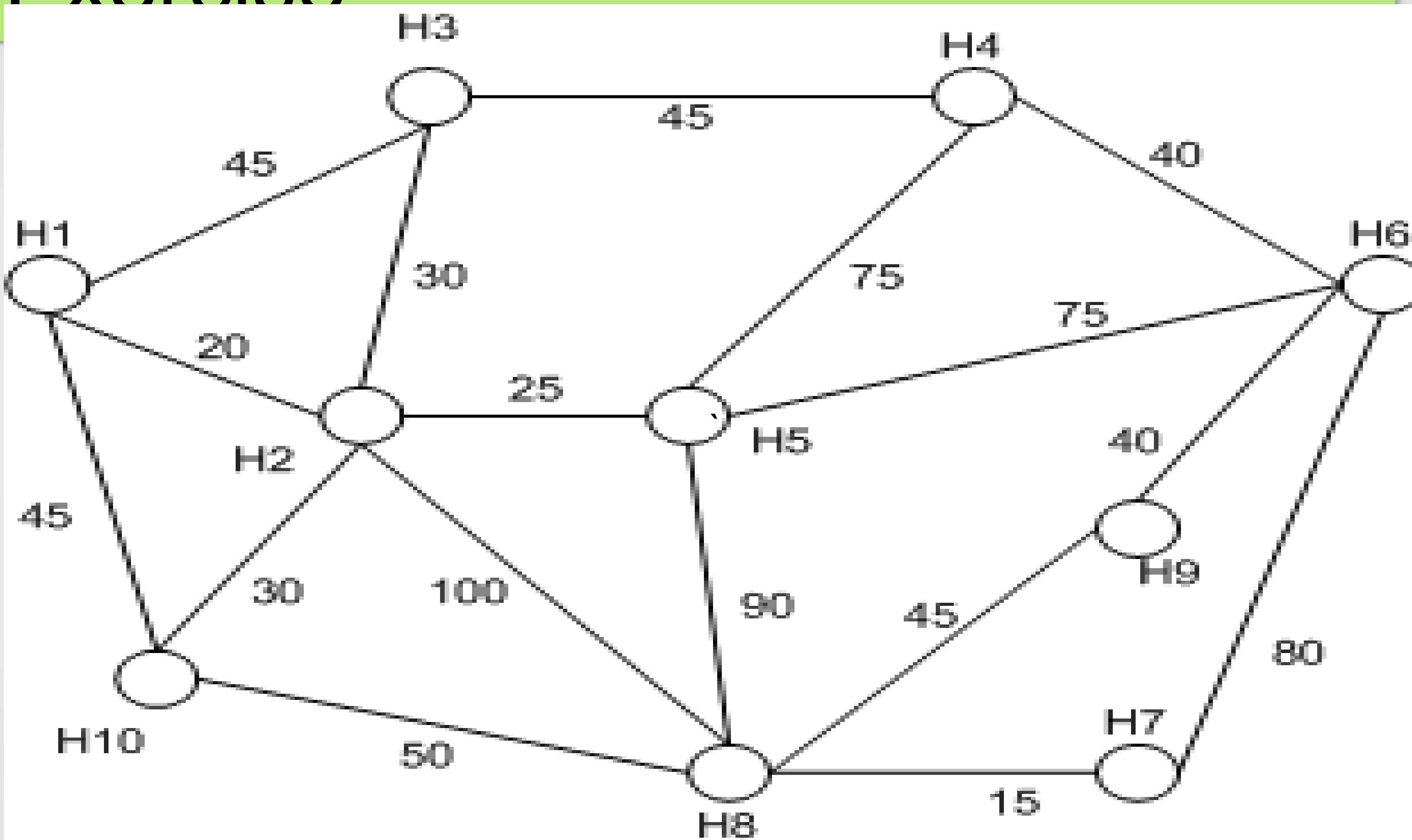
Prim's Algorithm: Example



Prim's Algorithm: Example



Exercise



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H8

Baruvka's Algorithm

- Like Kruskal's this grows many clouds at once
- Initially every vertex is a component C_i
- For each Ci find the smallest weight edge (v,u) in E and add to C_i such that
  - v is in C_i, u is not in C_i
  - Add e to T
- Each iteration reduces the number of components by half

Baruvka's Algorithm

- Algorithm BaruvkaMST(G)

  T ← V {just the vertices of G}

  while T has fewer than n-1 edges do

  for each connected component C in T do

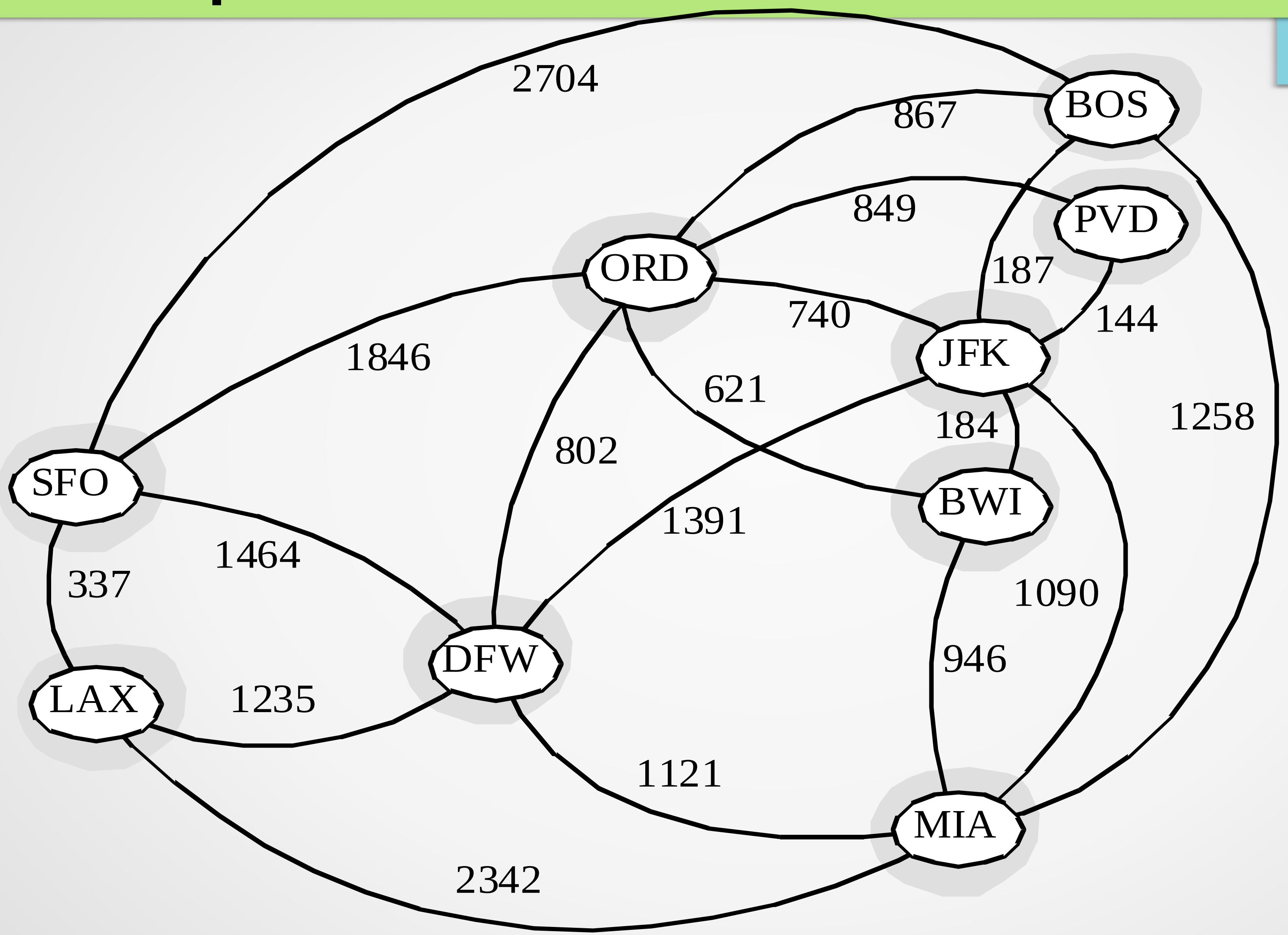
  Let edge e be the smallest-weight edge from C to another component in T.

  if e is not already in T then

  Add edge e to T

  return T

Example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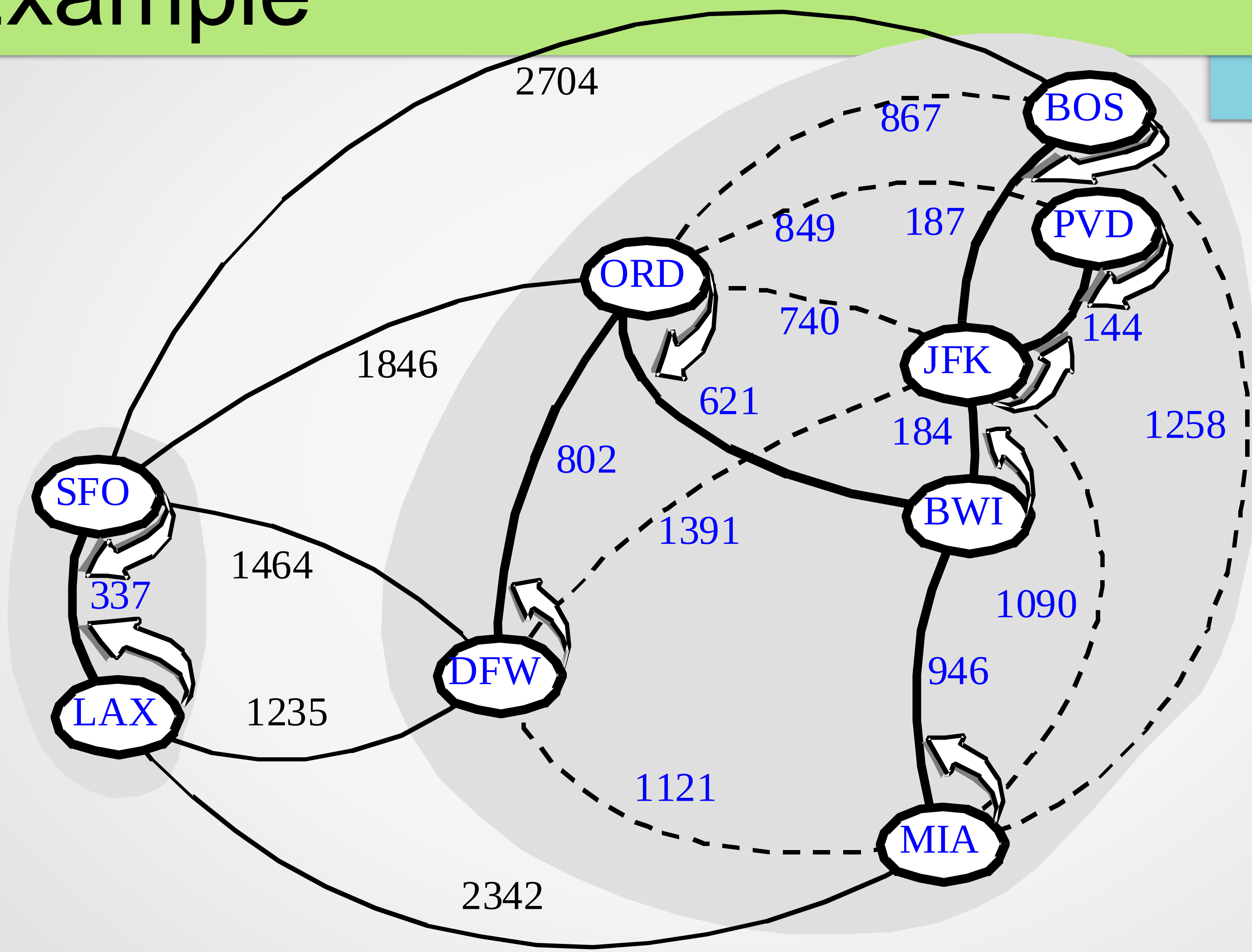
Example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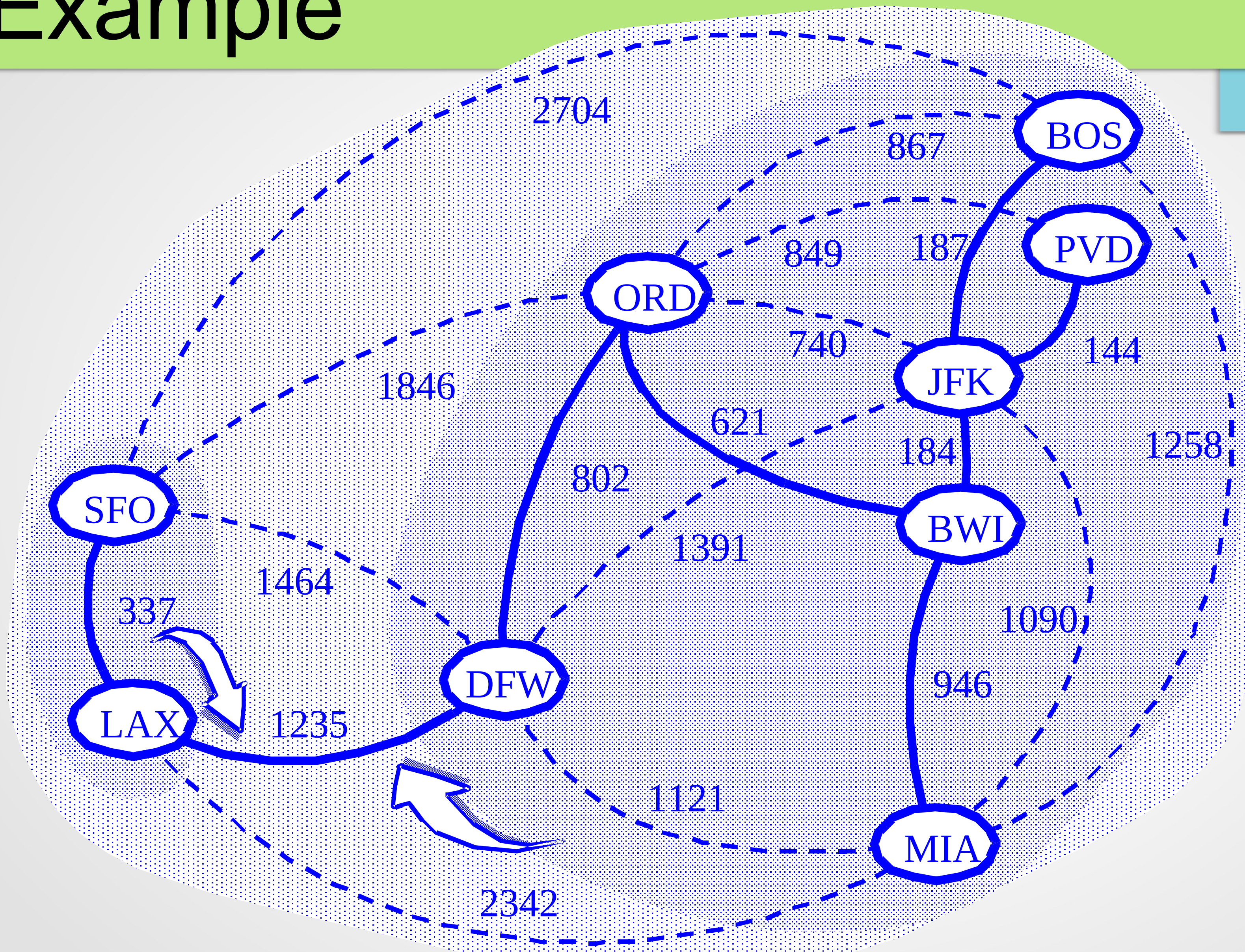

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Properties of MSTs

- There may be multiple minimum spanning trees of same weight having minimum number of edges
- If each edge has a distinct weight then there will be only one, unique minimum spanning tree.
- For any cycle C in the graph, if the weight of an edge e of C is larger than the weights of all other edges of C, then this edge cannot belong to an MST.

Shortest Paths

- Single Source Single Destination Shortest Path
  - Given a source and destination find the path between source and destination with minimum cost
- Single Source Shortest Path
  - Find the shortest paths from a given source to all other nodes

Dijkstra's Algorithm

- Similar to Prim’s
- Let v be the source node
- Initialize for all u not v, d[u] to infinity and d[v]=0
- Remove from Q u with minimum d[u]
  - Add u and edge (u,v) to T
  - For all neighbors z of u, do relaxation by finding d[z] = d[u]+w(u,z), and update Q
    - d[z] = min(d[z],d(u,z)+d[u])
- Terminate when Q empty

Shortest Paths

- Single Source Single Destination Shortest Path
  - Given a source and destination find the path between source and destination with minimum cost
- Single Source Shortest Path
  - Find the shortest paths from a given source to all other nodes

Dijkstra’s Algorithm

- Complexity
  - $O(|E|+|V|\log|V|)$
  - Cost of maintaining priority queue plays a major role
- Does not work if there are negative weighted cycles

Exercise

- Find the shortest path from A to all other nodes