COUNTING

The Product Rule : Suppose that a procedure can be broken into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

1. A new company with just two employees, Santhosh and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

2. The chairs of an auditorium are to be labelled with a letter and a positive integer not exceeding 100. What is the largest number of chairs that can be labelled differently?

3. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports are there to a microcomputer in the center?

4. How many different bit strings of length 7 are there?

5. How many different license plates are available if each plate contains a sequence of three letters followed by three digits, if (i) repetition is allowed for both letters and digits? (ii) repetition is not allowed for both? (iii) repetition is allowed only for letters? (iv) repetition is allowed only for digits? (v) repetition is allowed for either one of them?

6. How many functions are there from a set with m elements to a set with n elements?

7. How many one – to – one functions are there from a set with m elements to a set with n elements?

8. Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$.

The Sum Rule : If a task can be done either in one of n_1 ways or in one of n_2 ways, where there is no common way in n_1 ways and n_2 ways, then there are $n_1 + n_2$ ways to do the task.

9. Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative, if there are 37 members of the mathematics faculty and 83 mathematics major?

10. A student can choose a computer project from one of the three lists. The three lists contain 23, 15 and 19 possible projects respectively. No project is in more than one list. How many possible projects are there to choose from?

11. Each user on a computer system has a password, which is 6 to 8 characters long, where each character is an upper case letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

The Pigeon Hole Principle : If k is a positive integer and $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Proof : Suppose that none of the k boxes contains more than one object. Then the total number of objects would be at most k. This is a contradiction, since there are at least $k+1$ objects. Hence, there is at least one box containing two or more of the objects.

Cor : A function f from a set with $k+1$ elements or more elements to a set with k elements is not 1 – 1.

Proof : Take k+1 elements in the first set as pigeons and k elements in the second set as pigeon holes. Then clearly there will be at least one hole with more than one pigeon. i.e., there will be at least two elements (say, a, b) with the same image under any function $f: A \to B$ with $|A| = k+1$ and $|B| = k$. Here, $f(a) = f(b)$, but $a \neq b$. Hence there can not be a one – one function.

12. Show that among any group of 367 people, there must be at least two with the same birthday.

13. In any group of 27 English words, there must be at least two that begin with the same letter.

14. How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

15. What is the minimum number of elements that must be selected from the set {1,2,3,4,5,6} to guarantee that at least one pair of these numbers add up to 7 ? Explain.

Generalized Pigeon Hole Principle : If N objects are placed into k boxes, then there is at least one box containing at least $\left\lceil \frac{N}{k} \right\rceil$ objects.

16. Show that among 100 people there are at least 9 who were born in the same month.

17. What is the minimum number of students required in a discrete mathematics class to be sure that at least 6 will receive the same grade, if there are 5 possible grades A, B, C, D, F ?

18. How many cards must be selected from a standard deck of 52 cards to guarantee that (i) at least three cards of the same suit are chosen? (ii) at least three hearts are selected?