PERMUTATIONS AND COMBINATIONS

Permutation : A permutation of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of r elements of a set is called an r – permutation. The number of r – permutations of a set with n elements is denoted as $P(n,r)$ or nP_r.

Theorem : If n is a positive integer and r is an integer with $1 \leq r \leq n$, then

$$\boldsymbol{P(n,r) = nP_r = n(n-1)(n-2)\ldots(n-\overline{r-1}) = n(n-1)(n-2)\ldots(n-r+1)}$$

Cor : If n and r are integers with $0 \leq r \leq n$, then $\boldsymbol{P(n,r) = \frac{n!}{(n-r)!}}$.

Combination : A combination is an unordered selection of objects.

Theorem : The number of r – combinations $C(n,r)$ or nC_r of a set with n elements, where n is a non – negative integer and r is an integer with $0 \leq r \leq n$, is given by $\boldsymbol{C(n,r) = nC_r = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}}$.

Cor : Let n and r be non – negative integers with $r \leq n$. Then $\boldsymbol{C(n,r) = C(n,n-r)}$.

Remark : $\boldsymbol{P(n,n) = n!}$, $\boldsymbol{C(n,n) = C(n,0) = 1}$ and $\boldsymbol{C(n,1) = n}$.

Theorem : The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, … and n_k indistinguishable objects of type k, is $\boldsymbol{\frac{n!}{n_1!n_2!\ldots n_k!}}$.

Binomial Expansion : $\boldsymbol{(a+b)^n = a^n + nC_1a^{n-1}b + nC_2a^{n-2}b^2 + nC_3a^{n-3}b^3 + \cdots + nC_{n-1}ab^{n-1} + b^n}$

PRACTICE PROBLEMS

1. In how many ways can we select 3 students from a group of 5 students to stand in a line for a picture? In how many ways can we arrange all 5 of these students in a line for a picture?

2. How many ways are there to select a first – prize winner, a second – prize winner and a third - prize winner from 100 different people who have entered a contest?

3. Suppose that there are 8 runners in a race. The winner receives a gold medal, the second place finisher receives a silver medal and the third place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

4. Suppose that a saleswoman has to visit 8 different cities. She must begin her trip in a specified city, but she can visit the other 7 cities in any order she wishes. How many possible orders can the saleswoman use while visiting these cities?

5. How many permutations of the letters ABCDEFGH contain the string ABC?

6. How many permutations of the letters of the word COMPUTERS contain the string MTR ?

7. How many different committees of 3 students can be formed from a group of 4 students?

8. How many poker hands of 5 cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

9. How many ways are there to select 5 players from a 10 – member tennis team to make a trip to a match at another school?

10. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of 6 people to go on this mission (assuming that all crew members have the same job) ?

11. How many bit strings of length n contain exactly r 1s?

12. Suppose that there are 9 faculty members in the Mathematics department and 11 in the Computer Science department. How many ways are there to select a committee to develop a Discrete Mathematics course at a school, if the committee is to consists of 3 faculty members from the Mathematics department and 4 from the Computer Science department?

13. How many bit strings of length 10 contain (i) Exactly four 1s? (ii) At most four 1s? (iii) At least four 1s? (iv) An equal number of 0s and 1s?

14. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women are alternate?

15. How many subsets with an odd number of elements does a set with 10 elements have?

16. How many subsets with more than 2 elements does a set with 100 elements have?

17. A coin is flipped 10 times where each flip comes up with either heads or tails. How many possible outcomes (i) Are there in total? (ii) Contain exactly 2 heads? (iii) Contain at most 3 tails? (iv) Contain the same number of heads and tails?

18. In a group of 6 boys and 4 girls, 4 children are to be selected. In how many different ways they can be selected such that (i) At least one boy should be there? (ii) At most 2 boys should be there? (iii) Equal number of girls and boys should be there? (iv) Exactly 2 girls are there? (v) At least 2 girls are there? (vi) More number of girls than boys?

19. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

20. How many words can be formed by using the letters of the word MELVIN ?

21. How many words can be formed by using the letters of the word CONSIDER ?

22. How many different strings / words can be made from the letters of MISSISSIPPI, using all the letters?

23. How many different strings / words can be made from the letters in ABRACADABRA, using all the letters?

24. How many ways are there to deal hands of 7 cards to each of 5 players from a standard deck of 52 cards?

25. A box contains 6 black balls, 5 white balls and 4 red balls. A person wants to select 12 balls. In how many ways he can select? In how many ways he can select 12 balls if the selection must contain (i) At least 4 black balls, 2 white balls and 2 red balls? (ii) Exactly 2 black balls? (iii) At most 4 black balls? (iv) At most 4 black balls and at least 2 white balls? (v) Exactly 3 white balls? (vi) At least 3 black balls, at most 4 white balls and exactly 4 red balls?