$$\theta = \left\{ \theta_1, \theta_2, \theta_k, P_1 - P_k \right\}$$

$$P(x; 10j) = \frac{n!}{m!} \prod_{j=1}^{m} p_{j}^{n_{j}}$$

The likelihood function is

The dog like ishood function
$$\int \Phi(X, \theta_j, d_1 - - d_K, \Lambda) = \sum_{i=1}^{N} \log \frac{E}{j-1} P(X_i | \theta_j) + \lambda_i P_i + \lambda_2 P_2 + - - + d_K P_k + A(1 - E P_i)$$

$$\Lambda(1-\xi_{i}^{p_{i}})$$

log likelihood, differentiate W.r.t To derive the maximum

all parameters & equate to zero.

(3)
$$\frac{\partial \phi}{\partial \lambda_i}$$
 (x₁, θ_i , $\lambda_1, \dots, \lambda_k$, λ) =0

$$\frac{\partial \phi}{\partial \Lambda}(x, \theta_j, A, \lambda_k, \Lambda) = 0$$

As it contradicts constraint

$$\frac{\partial \phi}{\partial \Lambda} (n, \theta, \lambda, ---\lambda_{k}, \Lambda) = 1 - \frac{R}{2} P_{j} = 0$$
 $\frac{R}{2} P_{j} = 1$
 $\frac{R}{2} P_{j} = 1$

Jahe (1)
$$\frac{\partial \phi}{\partial P_{j}}(n,\theta,\Lambda,-\Lambda_{k},\Lambda) = \frac{N}{1-1} \frac{1}{EP_{j}\cdot P(x_{j}|\theta_{j})} \frac{\partial}{\partial P_{j}} \left(\frac{E}{j-1}P_{j}\cdot P(x_{j}|\theta_{j})\right) - \Lambda$$

$$\Rightarrow \frac{\sum_{i=1}^{N} P(x_i | \theta_i)}{\sum_{j=1}^{N} P(x_j | \theta_j)} = 0$$

Multiplying & dividing by
$$P_j$$

$$\frac{1}{P_j} \begin{cases} \sum_{i=1}^{N} P_j \cdot P(x_i | \theta_j) \\ \sum_{i=1}^{N} P_i \cdot P(x_i | \theta_j) \end{cases} - \Lambda = 0$$
By Bayes sule
$$\frac{1}{P_j} = \sum_{i=1}^{N} P(j | x_i) - \Lambda = 0$$

$$P_j = \frac{1}{N} \sum_{i=1}^{N} P(j | x_i) \longrightarrow \emptyset$$
Sub in G

$$\frac{k}{k} = \frac{1}{N} \sum_{i=1}^{N} P(j | x_i) = 1$$

$$\frac{k}{k} = \frac{1}{N} \sum_{i=1}^{N} P(j | x_i) = 1$$

Sub in 6

$$k = 1$$
 $j=1 \land N \Rightarrow 0$
 $j=1 \land N \Rightarrow 0$

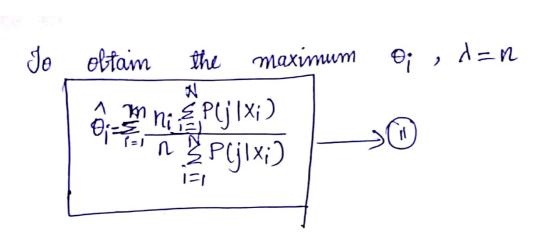
Sub in (b)
$$\hat{P}_{i} = \frac{1}{N} \sum_{i=1}^{N} P(j|x_{i}) \longrightarrow (8)$$

Solve (2)
$$\frac{\partial \Phi}{\partial \theta_{j}}(X,\theta,\lambda_{1},...,\lambda_{K},\Lambda) = \frac{\sum_{i=1}^{N} P_{i} \frac{\partial}{\partial \theta_{j}} \frac{E_{i}}{\partial \theta_{j}} P(x_{i}|\theta_{j})}{\sum_{j=1}^{N} P_{i}(X_{i}|\theta_{j})} + \frac{\partial}{\partial \theta_{j}} \Lambda(1-\frac{E_{i}}{E_{i}}P_{j}^{*})$$

$$= \underbrace{\sum_{i=1}^{N} P(x_{i}|\theta_{j})}_{E_{i}} \underbrace{\frac{\partial}{\partial \theta_{j}} \underbrace{\sum_{j=1}^{N} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})}$$

$$= \underbrace{\sum_{i=1}^{N} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})} \underbrace{\frac{\partial}{\partial \theta_{j}} \log_{p} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})} \xrightarrow{\mathcal{P}} \underbrace{\frac{\partial}{\partial \theta_{j}} \log_{p} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})} \xrightarrow{\mathcal{P}} \underbrace{\frac{\partial}{\partial \theta_{j}} \log_{p} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})} \xrightarrow{\mathcal{P}} \underbrace{\frac{\partial}{\partial \theta_{j}} \log_{p} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})}$$

$$= \underbrace{\frac{\partial}{\partial \theta_{j}} \log_{p} P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j})}}_{P(x_{i}|\theta_{j})}_{P(x_{i}|\theta_{j$$



Soln: (1) and (8)