Mixture Model - Binomial distribution

Likelihood function for $P(\vec{X_i} | P)$

$$P(\vec{X_i} | P) = \prod_{i=1}^{N} \binom{n}{K} P^K (1-P)^{n-K}$$

n - no of trials

K - no of outcomes

$$\binom{n}{K} = \frac{n!}{K!\,(n-K)!}$$

COPY everything till eqn 4

$$\log P(\vec{X_i} | P) = \log \prod_{i=1}^{N} \binom{n}{K} P^K (1-P)^{n-K}$$

$$= \sum_{i=1}^{N} \log \left[\frac{n!}{K!\,(n-K)!} P^K (1-P)^{(n-K)} \right]$$

$$= \sum_{i=1}^{N} \left[\log \frac{n!}{K!\,(n-K)!} + \log P^K + \log (1-P)^{n-K} \right]$$

$$\frac{\partial \log P(\vec{x_i} \mid P)}{\partial P} = \sum_{i=1}^{N} \frac{\partial}{\partial P} \log P^{K} + \sum_{i=1}^{N} \log (1-P)^{n-K}$$

$$= \sum_{i=1}^{N} \frac{\partial}{\partial P} K \log P + \sum_{i=1}^{N} (n-K) \frac{\partial}{\partial P} (1-P)$$

$$= \sum_{i=1}^{N} \left[\frac{K}{P} - \frac{(n-K)}{(1-P)} \right]$$

Substituting in

$$\sum_{i=1}^{N} P(j \mid \vec{x_i}) \frac{\partial}{\partial P} \log P(\vec{x_i} \mid P) = 0$$

$$\sum_{i=1}^{N} P(j \mid \vec{x_i}) \left[\sum_{i=1}^{N} \left[\frac{K}{P} - \frac{(n-K)}{(1-P)} \right] \right] = 0$$

$$\sum_{i=1}^{N} P(j \mid \vec{x_i}) \left[\sum_{i=1}^{N} (K - nP) \right] = 0$$

$$\sum_{i=1}^{N} P(j \mid \vec{x_i}) \sum_{i=1}^{N} K = \sum_{i=1}^{N} P(j \mid \vec{x_i}) \sum_{i=1}^{N} nP$$

$$\boxed{\hat{P} = \frac{\sum_{i=1}^{N} P(j \mid \vec{x_i}) \sum_{i=1}^{N} K}{\sum_{i=1}^{N} P(j \mid \vec{x_i}) \, N \cdot n}}$$