Multivariate gaussian Mixture Model (Without z_{ij})

All eqns. will remain same except $\frac{\partial \phi}{\partial \theta_j}$

$$P(\vec{x_i} \mid \theta_j) = \frac{1}{(2\pi)^{D/2} |\Sigma_j|^{1/2}} e^{\left[-1/2 (\vec{x_i} - \vec{\mu_j})^T \Sigma_j^{-1} (\vec{x_i} - \vec{\mu_j})\right]}$$

$$\frac{\partial \phi(\quad)}{\partial \theta_j} = \sum_{i=1}^{N} P(j \mid \vec{x_i}) \sum_{j=1}^{k} \frac{\partial}{\partial \theta_j} \log P(\vec{x_i} \mid \theta_j) \longrightarrow (4)$$

$$\log P(\vec{x_i} \mid \theta_j) = \log \left\{ \frac{1}{(2\pi)^{D/2} |\Sigma_j|^{1/2}} e^{\left[-1/2 (\vec{x_i} - \vec{\mu_j})^T \Sigma_j^{-1} (\vec{x_i} - \vec{\mu_j})\right]} \right\}$$

$$= \log \left\{ e^{-1/2 (\vec{x_i} - \vec{\mu_j})^T \Sigma^{-1} (\vec{x_i} - \vec{\mu_j})} \right\} - \log \left[(2\pi)^{D/2} |\Sigma_j|^{1/2} \right]$$

$$= -\frac{1}{2} (\vec{x_i} - \vec{\mu_j})^T \Sigma_j^{-1} (\vec{x_i} - \vec{\mu_j}) - \left[\log (2\pi)^{D/2} + \log |\Sigma_j|^{1/2} \right]$$

$$= -\frac{1}{2} (\vec{x_i} - \vec{\mu_j})^T \Sigma_j^{-1} (\vec{x_i} - \vec{\mu_j}) - \frac{D}{2} \log (2\pi) - \frac{1}{2} \log |\Sigma_j|$$

$$\frac{\partial \log P(\vec{x_i} \mid \theta_j)}{\partial \mu_j} = -\frac{1}{2} (\vec{x_i} - \vec{\mu_j})^T \Sigma_j^{-1} - \frac{D}{2} \log (2\pi) - \frac{1}{2} \log |\Sigma_j|$$

$$\frac{\partial}{\partial \mu_j} \log P(\vec{x_i} \mid \theta_j) = -\frac{1}{2} \left[2 (\vec{x_i} - \vec{\mu_j}) (0 - 1) \right] \Sigma_j^{-1}$$

$$\frac{\partial}{\partial \mu_j} \log P(\vec{x_i} \mid \theta_j) = (\vec{x_i} - \vec{\mu_j}) \Sigma_j^{-1} \longrightarrow (5)$$

$$\frac{\partial}{\partial \Sigma_j} \log P(\vec{x_i} \mid \theta_j) = -\frac{1}{2} \left[-(\vec{x_i} - \vec{\mu_j})^2 \Sigma_j^{-2} \right] - \frac{1}{2} \Sigma_j^{-1}$$

$$= \frac{1}{2}\left[\vec{x_i} - \vec{\mu_j}\right]^2 \Sigma_j^{-2} - \frac{1}{2}\Sigma_j^{-1} \longrightarrow (6)$$

Sub (5) in (4)

$$\sum_{i=1}^{N} P(j|\vec{x_i})\left\{(\vec{x_i} - \vec{\mu_j})\Sigma_j^{-1}\right\} = 0$$

$$\sum_{i=1}^{N} P(j|\vec{x_i})\vec{x_i}\Sigma_j^{-1} = \sum_{i=1}^{N} P(j|\vec{x_i})\vec{\mu_j}\,\Sigma_j^{-1}$$

$$\boxed{\hat{\mu}_j = \frac{\sum_{i=1}^{N} P(j|x_i)\vec{x_i}}{\sum_{i=1}^{N} P(j|x_i)}}$$

Sub (6) in (4)

$$\sum_{i=1}^{N} P(j|\vec{x_i})\left\{\frac{1}{2}(\vec{x_i} - \vec{\mu_j})\Sigma_j^{-2}(\vec{x_i} - \vec{\mu_j}) - \frac{1}{2}\Sigma_j^{-1}\right\} = 0$$

$$\sum_{i=1}^{N} P(j|\vec{x_i})(x_i - \mu_j)^T \Sigma_j^{-2}(\vec{x_i} - \vec{\mu_j}) = \sum_{i=1}^{N} P(j|\vec{x_i})\,\Sigma_j^{-1}$$

$$\boxed{\hat{\Sigma}_j = \frac{\sum_{i=1}^{N} P(j|\vec{x_i})(\vec{x_i} - \vec{\mu_j})(x_i - \mu_j)}{\sum_{i=1}^{N} P(j|\vec{x_i})}}$$