Ex

Dado um canal

$h(t) = r\exp(j\theta) = x(t) + j\,y(t)$ onde x e y são variavél aleatória independente e uniformemente distribuindo entre $-B$ e B

* A pdf de uma uniforme entre a, b é

$$f = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{outro} \end{cases}$$

Conhecendo a pdf, generico, então a gente pode escreve a pdf de x e y da seguinte forma.

$$f(x) = \begin{cases} \frac{1}{2B} & -B < x < +B \\ 0 & \text{outre} \end{cases} \qquad f_Y = \begin{cases} \frac{1}{2B} & -B < y < B \\ 0 & \text{outro} \end{cases}$$

$$re^{-j\theta} = x + jy \Rightarrow r^2 = x^2 + y^2 \quad e \quad \theta = tg^{-1}\left(\frac{y}{x}\right)$$

$$x^2 = r^2 - y^2 \quad e \quad y = x\,tg\,\theta$$

$$x^2 = r^2 - x^2 tg\,\theta^2$$

$$r^2 = x^2 + x^2 tg^2\theta$$

$$= x^2\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$= x^2\left(\frac{1}{\cos^2\theta}\right) \Rightarrow x^2 = r^2\cos^2\theta \Rightarrow$$

$$x = r\cos\theta$$

$$y = r\cos\theta \cdot tg\,\theta$$

$$= r\cancel{\cos\theta}\,\frac{\sin\theta}{\cancel{\cos\theta}}$$

$$y = r\sin\theta$$

$$p(r,\theta) = p(x)\cdot p(y)\cdot J\begin{vmatrix} x & y \\ r & \theta \end{vmatrix}$$

$$J\begin{vmatrix} x & y \\ r & \theta \end{vmatrix} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial x}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial y}{\partial r} = \sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J\begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

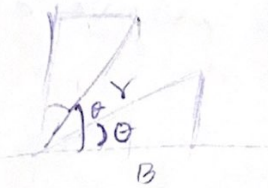
$$p(r,\theta) = r\,p(x)\cdot p(y)$$

Sabendo que $f_x = \begin{cases} \frac{1}{2B} & -B < x < B \\ 0 & \text{outro} \end{cases}$ e $f_y = \begin{cases} \frac{1}{2B} & -B < y < B \\ 0 & \text{outro} \end{cases}$

$$p(r,\theta) = r \cdot \frac{1}{2B} \cdot \frac{1}{2B} = \frac{r}{4B^2}$$

Sendo $-\pi \leq \theta \leq \pi$ e $r \leq B$ para o circulo Azul

e $B \leq r \leq B \cdot \sqrt{2}$ e $\sin^{-1}\frac{B}{r} \leq \theta \leq \cos^{-1}\left(\frac{B}{r}\right) \Rightarrow$  4 area



$\cos\theta = \frac{B}{r}$: $r = \frac{B}{\cos\theta}$ $\theta = \cos^{-1}\left(\frac{B}{r}\right)$

$\sin\theta = \frac{B}{r}$ $r = \frac{B}{\sin\theta}$ $\theta = \sin^{-1}\left(\frac{B}{r}\right)$

Para $\theta = 45$ r_{max}

$\sin 45 = \cos 45 = \frac{1}{\sqrt{2}} = \frac{B}{r}$

$\underline{B < r \leq \sqrt{2}B}$

$$\cos^{-1}\left(\frac{B}{r}\right) \leq \theta < \sin^{-1}\left(\frac{B}{r}\right)$$

$$p(r,\theta) = \begin{cases} \frac{r}{4B^2} & 0 < r \leq B \ \wedge \ -\pi < \theta < \pi \\ 4\left(\frac{r}{4B^2}\right) & B \leq r < \sqrt{2}B \ \wedge \ \cos^{-1}\left(\frac{B}{r}\right) < \theta < \sin^{-1}\left(\frac{B}{r}\right) \\ 0 & \text{outro} \end{cases}$$

$$p(r) = \int_{-\infty}^{\infty} p(r,\theta)\, d\theta = \int_{-\pi}^{\pi} \frac{r}{4B^2} d\theta + 4\int_{\cos^{-1}\left(\frac{B}{r}\right)}^{\sin^{-1}\left(\frac{B}{r}\right)} \frac{r}{4B^2}\, d\theta$$

$$p(r) = \begin{cases} \dfrac{\pi r}{2B^2} & 0 \le r \le B \\ \dfrac{r}{B^2}\left[\sin^{-1}\left[\dfrac{B}{r}\right] - \cos\left(\dfrac{B}{r}\right)\right] & B \le r \le \sqrt{2}\,B \\ 0 & \text{outro} \end{cases}$$