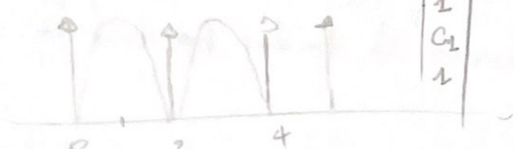
①

1) Considerando OFDM 5 portadoras

- Pilotos $(0, 2, \alpha)$ $\quad h = [1, 0{,}1]^T$
- $n \approx 0$
- Sinal $(1, 3)$

$$c = \begin{vmatrix} 1 \\ c_0 \\ 1 \\ c_1 \\ 1 \end{vmatrix}$$



$S = F^H c$

- En dominio de tiempo

$$r = h \circledast c + n$$

- En dominio de frecuencia

$$R = FHF^H c + N \approx 0$$

Portanto

$$R = FHF^H c$$

$$F = \frac{1}{\sqrt{K}} \left[e^{-j\frac{2\pi}{K}\cdot(i\cdot j)} \right]$$

Para $i = j = 0 \approx e^{-j\frac{2\pi}{K}0} = 1$

Portanto

$$F = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{5}} & e^{-j\frac{4\pi}{5}} & e^{-j\frac{6\pi}{5}} & e^{-j\frac{8\pi}{5}} \\ 1 & e^{-j\frac{4\pi}{5}} & e^{-j\frac{8\pi}{5}} & e^{-j\frac{12\pi}{5}} & e^{-j\frac{16\pi}{5}} \\ 1 & e^{-j\frac{6\pi}{5}} & e^{-j\frac{12\pi}{5}} & e^{-j\frac{18\pi}{5}} & e^{-j\frac{24\pi}{5}} \\ 1 & e^{-j\frac{8\pi}{5}} & e^{-j\frac{16\pi}{5}} & e^{-j\frac{24\pi}{5}} & e^{-j\frac{32\pi}{5}} \end{vmatrix}$$

$$H = \begin{vmatrix} 1 & 0 & 0 & 0 & 0{,}1 \\ 0{,}1 & 1 & 0 & 0 & 0 \\ 0 & 0{,}1 & 1 & 0 & 0 \\ 0 & 0 & 0{,}1 & 1 & 0 \\ 0 & 0 & 0 & 0{,}1 & 1 \end{vmatrix}$$

②

$$R = FHF^H$$

$$= \begin{bmatrix} 0{,}44 & 0{,}44 & 0{,}44 & 0{,}44 \\ 0{,}44 & 0{,}13-0{,}42i & -0{,}36-0{,}26i & -0{,}36+0{,}26 \\ 0{,}44 & -0{,}32-0{,}26i & 0{,}13+0{,}42i & 0{,}13-0{,}42i \\ 0{,}44 & -0{,}36+0{,}26i & 0{,}13-0{,}42i & 0{,}13+0{,}42 \\ 0{,}44 & 0{,}13+0{,}42i & -0{,}16+0{,}26 & -0{,}46-0{,}26 \end{bmatrix}$$

$$\begin{bmatrix} 0{,}44 \\ 0{,}13+0{,}42i \\ -0{,}36+0{,}26i \\ -0{,}36-0{,}26i \\ 0{,}13-0{,}42i \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0{,}1 \\ 0{,}1 & 1 & 0 & 0 & 0 \\ 0 & 0{,}1 & 1 & 0 & 0 \\ 0 & 0 & 0{,}1 & 1 & 0 \\ 0 & 0 & 0 & 0{,}1 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} 0{,}44 & 0{,}44 & 0{,}44 & 0{,}44 & 0{,}44 \\ 0{,}44 & 0{,}13+0{,}42i & -0{,}36+0{,}26i & -0{,}36-0{,}26i & 0{,}13-0{,}42i \\ 0{,}44 & -0{,}32+0{,}26i & 0{,}13-0{,}42i & 0{,}13+0{,}42i & -0{,}36-0{,}26 \\ 0{,}44 & -0{,}36-0{,}26i & 0{,}13-0{,}42i & 0{,}13-0{,}42i & -0{,}36-0{,}26 \\ 0{,}44 & 0{,}13-0{,}42i & 0{,}16-0{,}26 & -0{,}16+0{,}26i & 0{,}13+0{,}42i \end{bmatrix}$$

$$R = \begin{vmatrix} 1{,}1 & 0 & 0 & 0 & 0 \\ 0 & 1{,}03-0{,}095i & 0 & 0 & 0 \\ 0 & 0 & 0{,}91-0{,}058i & & \\ 0 & 0 & 0 & 0{,}91+0{,}058i & 0 \\ 0 & 0 & 0 & 0 & 1{,}03+0{,}09 \end{vmatrix}$$

$$* \begin{bmatrix} 1 \\ c_0 \\ 1 \\ c_1 \\ 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1{,}1 \\ (1{,}03-0{,}095)c_0 \\ 0{,}91-0{,}058i \\ (0{,}91+0{,}058i)c_1 \\ (1{,}03+0{,}095) \end{bmatrix}$$

③

Escolha as recta ($\mathbb{R}$, $\mathcal{I}$) que ligam com as palavras nas posições 0,2.

* Real

$H_r = an + b$

* $n = 0 \Rightarrow \hat{H}_R = 1{,}1$

$1{,}1 = a(0) + b \Rightarrow b = 1{,}1$

* $n = 2$ $\quad H_r(\mathbb{R}) = 0{,}91$

$0{,}91 = a(2) + b \Rightarrow b =$

$0{,}91 = a(2) + 1{,}1$

$a = 0{,}91 - 1{,}1$

$a = -0{,}095 \approx -0{,}09$

$\hat{h}_R = -0{,}09n + 1{,}1$

$$\hat{h}_R = \begin{vmatrix} 1{,}1 \\ 1{,}01 \\ 0{,}92 \end{vmatrix}$$

* Imaginaria

$H_I = an + b$ (0 ... 2)

$n = 0 \Leftrightarrow H_I = 0 \Leftrightarrow a(0) + b = 0$

$b = 0$

$n = 2$ $H_I = -0{,}058$

$-0{,}058 = a(2) + 0$

$a = -0{,}029 \approx -0{,}03$

$\hat{H}_I = -0{,}03$

$$\hat{H}_I = \begin{vmatrix} 0 \\ -0{,}03 \\ -0{,}06 \end{vmatrix}$$

④

$$\hat{H} = \begin{vmatrix} 1{,}1 \\ 1{,}01 - 0{,}03j \\ 0{,}92 - 0{,}06j \end{vmatrix}$$

* na posição 2, 3, 4 ⇒ mesmo procedimento.

* Real

$H_r = an + b$

* $n = 2$ $\quad H_r = 0{,}91$

$0{,}91 = a(2) + b$ ①

$1{,}03 = a(4) + b$ ②

de ① temos

$b = 0{,}91 - 2a$ ③

③ em ② $\Rightarrow 1{,}03 = 4a + 0{,}91 - 2a$

$a = 0{,}06$

$b = 0{,}91 - 2a$

$= 0{,}91 - 0{,}06 \times 2$

$= 0{,}79 \approx 0{,}8$

$\hat{H}_R = 0{,}06n + 0{,}8$

$$\hat{H}_r = \begin{vmatrix} 0{,}92 \\ 0{,}98 \\ 1{,}04 \end{vmatrix}$$

* Imaginaria

$-0{,}052 = 2a + b$ ④

$0{,}095 = 4a + b$ ⑤

de ④ temos

$b = -0{,}052 - 2a$ ⑥

⑥ em ⑤ $\quad 0{,}095 = 4a - 0{,}052 - 2a$

$a = 0{,}073$ $\quad b = -0{,}2$

①

$$\hat{H}_f = \begin{vmatrix} 0{,}075 \quad 0{,}2 \end{vmatrix}$$

$$\hat{H}_f \begin{vmatrix} -0{,}05 \\ 0{,}025 \\ 0{,}1 \end{vmatrix}$$

$$\hat{H} = \begin{vmatrix} 0{,}92 - 0{,}05j \\ 0{,}98 + 0{,}025j \\ 1{,}04 + 0{,}1j \end{vmatrix}$$

Combinando

$$\hat{H} \approx \begin{vmatrix} 1{,}1 \\ 1{,}02 - 0{,}03j \\ 0{,}92 - 0{,}05j \\ 0{,}98 + 0{,}025j \\ 1{,}04 + 0{,}1j \end{vmatrix}$$

2) Calculo erro quadratico medio

$$e = \frac{1}{N}\sum_{n=0}^{N} \left| H(n) - \hat{H}_n \right|^2$$

$$= \frac{1}{5}\left| \underbrace{(1{,}1 - 1{,}1)^2}_{\approx 0} + \left| (1{,}03 - 0{,}095) - (1{,}2 - 0{,}03j) \right|^2 + \underbrace{\left| (0{,}92 - 0{,}05j) - (0{,}92 - 0{,}05j) \right|^2}_{\approx 0} + \left| (0{,}91 + 0{,}0058j) - (0{,}98 + 0{,}02j) \right|^2 + 1{,}03 + j0{,}1) - (1{,}04 + 0{,}1j) \right|$$

②

$$e = \left(0 + 5{,}33 \cdot 10^{-3} + 5{,}35 \cdot 10^{-3} \right) / 5$$

$$e \approx 2{,}01 \cdot 10^{-3}$$

$$e = 2 \cdot 10^{-3}$$

3) 16-QAM $\quad T_s = 1ms$

$$R_b = \frac{N - N_b}{(N + N_{cp})T_s} \log_2 M$$

$N = 5$

$N_p = 3$

$N_{cp} = 1$

$$R_b = \frac{5 - 3}{(5+1) \cdot 10^{-3}} \log_2 M$$

$$= \frac{2}{6} \cdot \frac{4}{10^{-6}}$$

$$R_b = 1{,}33 \text{ Kps}$$

4) 16 QAM e $E/N_0 = 9$ dB

$E/N_0 = 10^{9/10} = 7{,}94$

$$H = \begin{vmatrix} 1{,}1 \\ 1{,}03 - 0{,}095j \\ 0{,}92 - 0{,}052j \\ 0{,}91 + 0{,}058j \\ 1{,}03 + 0{,}095j \end{vmatrix}$$

(7)

$$H^2 = \begin{vmatrix} 1{,}21 \\ 1{,}07 \\ 0{,}84 \\ 0{,}84 \\ 1{,}07 \end{vmatrix}$$

$$P_e = \frac{\alpha}{N} \sum_{n=0}^{N-1} Q\left(\sqrt{|H_n|^2 \xi \frac{E}{N}}\right)$$

$$\alpha = \frac{4(\sqrt{M}-1)}{\sqrt{M}} = 3$$

$$\xi = 3/M-1 = \frac{1}{5}$$

$$\xi \frac{E}{N_0} = \frac{1}{5} \times 7{,}95 \approx 1{,}58 \approx 1{,}6$$

$1{,}21 \times 1{,}6 \approx 2{,}56$
$1{,}07 \times 1{,}6 \approx 1{,}71$
$0{,}84 \times 1{,}6 \approx 1{,}34$
$0{,}$

$$P_e = \frac{3}{5}\left(Q(\sqrt{2{,}66}) + 2Q(\sqrt{1{,}77}) + 2Q(\sqrt{1{,}34})\right)$$
$$= 0{,}6\left(8{,}2 \cdot 10^{-2} + 2 \times 9{,}5 \cdot 10^{-2} + 2 \cdot 1{,}22 \cdot 10^{-1}\right)$$

$$P_e = 0{,}8096$$

3) BPSK $R_b = 1\,Mbps$

$N_0 = 10^{-11}$

$S_1 = 10^{-2}\cos(wt)$

$S_2 = -10^{-2}\cos(wt)$

$$E_1 = \int_0^T 10^{-2}\cos(wt)$$
$$= 10^{-2} \int_0^T \underbrace{\cos(wt)}_{\to 1}$$

$E_1 = 10^{-2}$

$E_2 = 10^{-2}$ (de forma análoga)

$\Rightarrow 10^{-2} \Rightarrow$ pulso das amplitude do sinal

$$P_e = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right)$$

$A = 10^{-2}$

$\sigma = \frac{N_0}{2}$

$$P_e = Q\left(\sqrt{\frac{2A^2}{N_0}}\right)$$

Considerando a influência do canal

$$P_e = \frac{1}{N} Q\left(\sqrt{|H_n|^2 \cdot \frac{2A^2}{N_0}}\right)$$

$$\frac{2A^2}{N_0} = \frac{2(10^{-7})^2}{10^{-15}} = 20$$

$$B = R_b / \log_2(2) = 1\,MHz$$

$$N_0 \times B = 10^{6} \cdot 10^{-11} = 10^{-5}$$

$$P_e = \frac{1}{5} \sum_{n=0}^{N-1} Q\left(\sqrt{(h_n)^2 \cdot 20}\right)$$

$$= 0{,}2 \cdot \left[Q\left(\sqrt{20 \times 1{,}21}\right) + 2Q\left(\sqrt{20 \times 1{,}07}\right) + 2Q\left(\sqrt{20 \times 0{,}84}\right)\right]$$

$$= 0{,}2\left(4{,}31 \cdot 10^{-07} + 9{,}72 \cdot 10^{-06} + 6{,}33 \cdot 10^{-5}\right)$$

$$P_e = 1{,}3 \cdot 10^{-5}$$

6) 16-QAM

$V = 60\,km/m = 16{,}66$

$f = 1\,GHz$

$T_s = 1\,ms$

$R_1 = 1000$

$R_b = ?$

$$f_D = \frac{V}{c/f} = \frac{16{,}66}{3 \cdot 10^8 / 10^9} = \frac{500}{9} = 55{,}56$$

$$T_c = \frac{1}{2 f_D} = \frac{1}{2 \cdot 55{,}5} = \frac{9}{1000} = 9 \cdot 10^{-3}$$

Para manter a coerência do canal

1 canal de piloto é envia a cada 9 ms já que $T_s = 1\,ms$

8 dados

P	D	D	D	D	D	D	D	D	P	D
1 ms / 1	2	3	4	5	6	7	8	9		

$K_D = 8$

$$R_b = \frac{K_D}{K_D + 1} \cdot \frac{N \log_2 M}{T_s}$$

$$= \frac{8}{9} \cdot \frac{1000 \cdot \log_2 16}{1\,ms}$$

$$= 3{,}55 \cdot 10^{6}\ bps$$