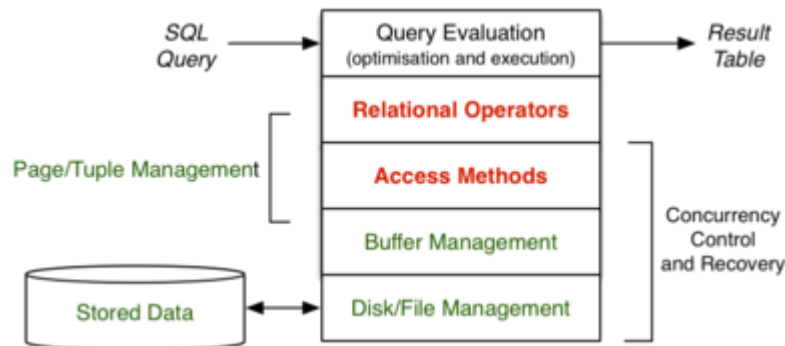


Week 06 Lectures

Implementing Relational Operators

1/72

Implementation of relational operations in DBMS:



... Implementing Relational Operators

2/72

So far, have considered file structures ...

- *heap file* ... tuples added to any page which has space
- *sorted file* ... tuples arranged in file in key order
- *hash file* ... tuples placed in pages using hash function

with relational algebra operations ...

- scanning (e.g. `select * from R`)
- sorting (e.g. `select * from R order by x`)
- projection (e.g. `select x,y from R`)
- selection (e.g. `select * from R where Cond`)

and now ...

- *indexes* ... search trees based on pages/keys
- *signatures* ... bit-strings which "summarize" tuples

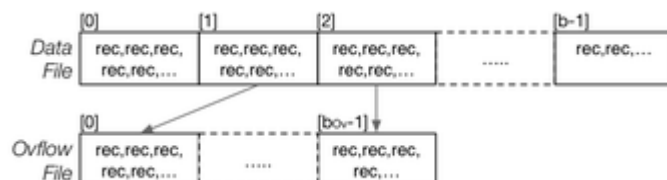
... Implementing Relational Operators

3/72

File/query Parameters ...

- r tuples of size R , b pages of size B , c tuples per page
- $Rel.k$ attribute in where clause, b_q answer pages for query q
- b_{Ov} overflow pages, average overflow chain length Ov

File structures ...



Reminder on Cost Analyses

4/72

When showing the cost of operations ...

- for queries, simply count number of pages read
- for updates, use n_r and n_w to distinguish reads/writes

When comparing two methods for same query

- ignore the cost of writing the result (same for both)

In counting reads and writes, assume minimal buffering

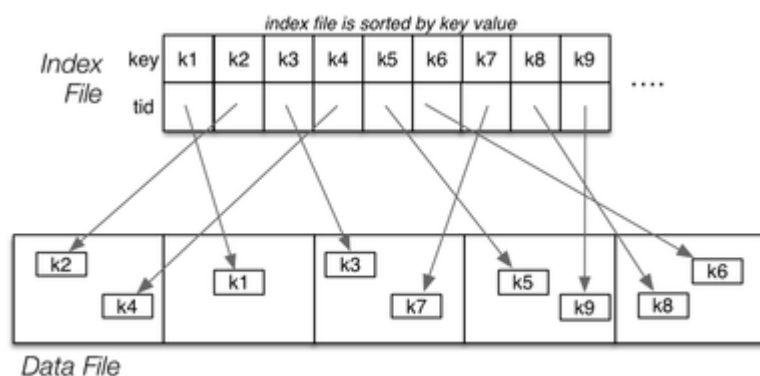
- each `request_page()` causes a read
- each `release_page()` causes a write (if page is dirty)

Indexing

Indexing

6/72

An index is a file of (keyVal,tupleID) pairs, e.g.



Indexes

7/72

A 1-d *index* is based on the value of a single attribute *A*.

Some possible properties of *A*:

- may be used to sort data file (or may be sorted on some other field)
- values may be unique (or there may be multiple instances)

Taxonomy of index types, based on properties of index attribute:

primary index on unique field, may be sorted on *A*

clustering 簇索引 index on non-unique field, file sorted on *A*

secondary file *not* sorted on *A*

A given table may have indexes on several attributes.

... Indexes

8/72

Indexes themselves may be structured in several ways:

Normally, index files are usually not managed in heap

- dense** every tuple is referenced by an entry in the index file
- sparse** only some tuples are referenced by index file entries
- single-level** tuples are accessed directly from the index file
- multi-level** may need to access several index pages to reach tuple

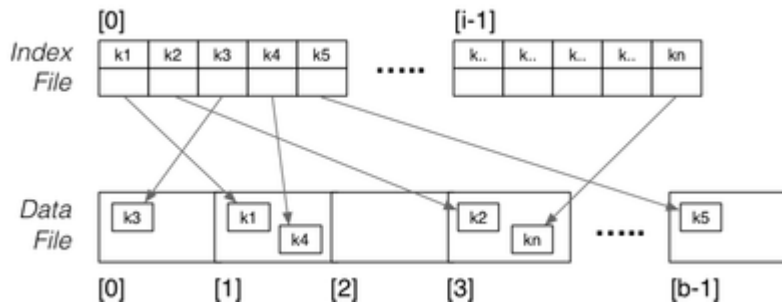
Index file has total i pages (where typically $i \ll b$)

Index file has page capacity c_i (where typically $c_i \gg c$)

Dense index: $i = \text{ceil}(r/c_i)$ Sparse index: $i = \text{ceil}(b/c_i)$

Dense Primary Index

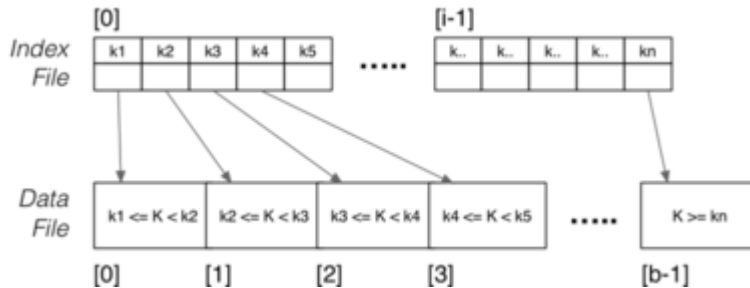
9/72



Data file unsorted; one index entry for each tuple

Sparse Primary Index

10/72



Data file sorted; one index entry for each page

Exercise 1: Index Storage Overheads

11/72

Consider a relation with the following storage parameters:

capacity of a page The size of a tuple

- $B = 8192$, $R = 128$, $r = 100000$
- header in data pages: 256 bytes
- key is integer, data file is sorted on key
- index entries (keyVal, tupleID): 8 bytes
- header in index pages: 32 bytes

Tuple per page

$$c = (B - 256) / 128$$

$$= \text{ceil}((8192 - 256) / 128)$$

$$= 62$$

$$\text{Page} = \text{ceil}(10000/62) = 1615$$

How many pages are needed to hold a dense index?

How many pages are needed to hold a sparse index?

$$c_i = (B - 32) / 8 = (8192 - 32) / 8 = 1020$$

$$\text{ceil}(r / c_i) = 100000 / 1020 = 99$$

Selection with Primary Index

$$\text{ceil}(b / c_i) = 1615 / 1020 = 2$$

12/72

For one queries:

```

ix = binary search index for entry with key K
if nothing found { return NotFound }
b = getPage(pageOf(ix.tid))
t = getTuple(b, offsetOf(ix.tid))
-- may require reading overflow pages
return t

```

Worst case: read $\log_2 i$ index pages + read $1+Ov$ data pages.

Thus, $Cost_{one,prim} = \log_2 i + 1 + Ov$

Assume: index pages are same size as data pages \Rightarrow same reading cost

... Selection with Primary Index

13/72

For *range* queries on primary key:

- use index search to find lower bound
- read index sequentially until reach upper bound
- accumulate set of buckets to be examined
- examine each bucket in turn to check for matches

For *pmr* queries involving primary key:

- search as if performing *one* query.

For queries not involving primary key, index gives no help.

... Selection with Primary Index

14/72

Method for range queries (when data file is not sorted)

```
// e.g. select * from R where a between lo and hi
pages = {} results = {}
ixPage = findIndexPage(R.ixf, lo)
while (ixTup = getNextIndexTuple(R.ixf)) {
    if (ixTup.key > hi) break;
    pages = pages U pageOf(ixTup.tid)
}
foreach pid in pages {
    // scan data page plus overflow chain
    while (buf = getPage(R.datf, pid)) {
        foreach tuple T in buf {
            if (lo <= T.a && T.a <= hi)
                results = results U T
        }
    }
}
```

Insertion with Primary Index

15/72

Overview:

tid = insert tuple into page P at position p
 find location for new entry in index file
 insert new index entry (k, tid) into index file

Problem: order of index entries must be maintained

- need to avoid overflow pages in index
- either reorganise index file or mark entries

Reorganisation requires, on average, read/write half of index file:

$$Cost_{insert,prim} = (\log_2 i)_r + i/2 \cdot (1_r + 1_w) + (1 + Ov)_r + (1 + \delta)_w$$

Deletion with Primary Index

16/72

Overview:

find tuple using index
 mark tuple as deleted
 delete index entry for tuple

If we delete index entries by marking ...

- $Cost_{delete,prim} = (\log_2 i)_r + (1 + Ov)_r + 1_w + 1_w$

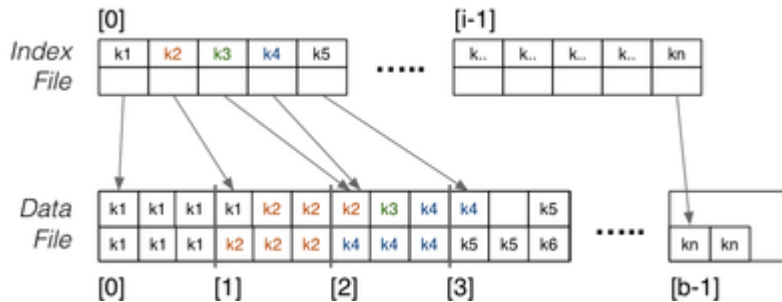
If we delete index entry by index file reorganisation ...

- $Cost_{delete,prim} = (\log_2 i)_r + (1 + Ov)_r + i/2 \cdot (1_r + 1_w) + 1_w$

Clustering Index

17/72

Data file sorted; one index entry for each key value



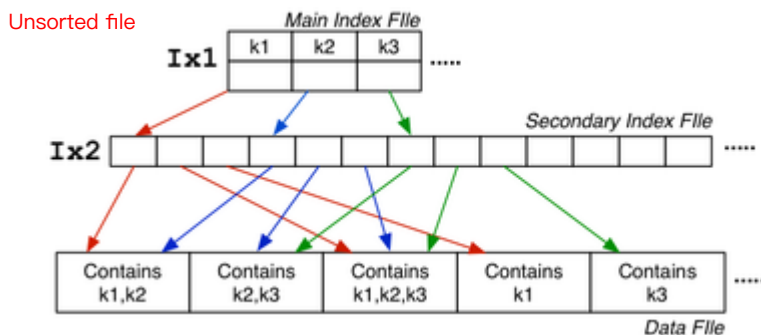
Cost penalty: maintaining both index and data file as sorted

(Note: can't mark index entry for value X until all X tuples are deleted)

Secondary Index

18/72

Data file not sorted; want one index entry for each key value



$$Cost_{pmr} = (\log_2 i_{Ix1} + a_{Ix2} + b_q \cdot (1 + Ov))$$

Multi-level Indexes

19/72

Above Secondary Index used two index files to speed up search

- by keeping the initial index search relatively quick
- I_{x1} small (depends on number of unique key values)
- I_{x2} larger (depends on amount of repetition of keys)
- typically, $b_{Ix1} \ll b_{Ix2}$

Could improve further by

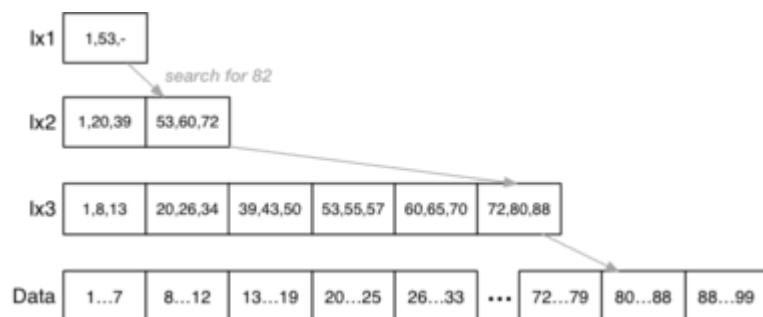
- making I_{x1} sparse, since I_{x2} is guaranteed to be ordered
- in this case, $b_{Ix1} = \text{ceil}(b_{Ix2} / c_i)$
- if I_{x1} becomes too large, add I_{x3} and make I_{x2} sparse
- if data file ordered on key, could make I_{x3} sparse

Ultimately, reduce top-level of index hierarchy to one page.

... Multi-level Indexes

20/72

Example data file with three-levels of index:



Assume: not primary key, $c = 100$, $c_i = 3$

Select with Multi-level Index

21/72

For one query on indexed key field:

```

xpid = top level index page
for level = 1 to d {
    read index entry xpid
    search index page for J'th entry
        where index[J].key <= K < index[J+1].key
    if (J == -1) { return NotFound }
    xpid = index[J].page
}
pid = xpid // pid is data page index
search page pid and its overflow pages

```

$$Cost_{one, mli} = (d + 1 + Ov)_r$$

(Note that $d = \text{ceil}(\log_{c_i} r)$ and c_i is large because index entries are small)

B-Trees

22/72

B-trees are MSTs with the properties:

- they are updated so as to remain balanced
- each node has at least $(n-1)/2$ entries in it
- each tree node occupies an entire disk page

B-tree insertion and deletion methods

- are moderately complicated to describe
- can be implemented very efficiently

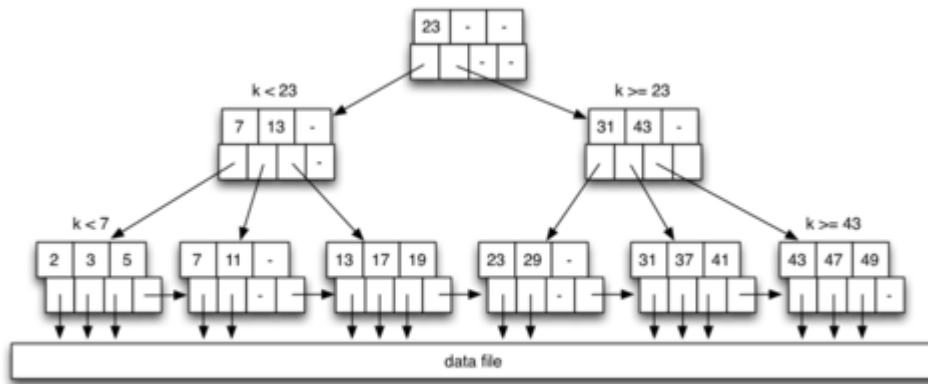
Advantages of B-trees over general MSTs

- better storage utilisation (around 2/3 full)
- better worst case performance (shallower)

... B-Trees

23/72

Example B-tree (depth=3, $n=3$) (actually B+ tree)



(Note: in DBs, nodes are pages \Rightarrow large branching factor, e.g. $n=500$)

B-Tree Depth

24/72

Depth depends on effective branching factor (i.e. how full nodes are).

Simulation studies show typical B-tree nodes are 69% full.

Gives load $L_i = 0.69 \times c_i$ and depth of tree $\sim \text{ceil}(\log_{L_i} r)$.

Example: $c_i=128$, $L_i=88$

Level	#nodes	#keys
root	1	87
1	88	7656
2	7744	673728
3	681472	59288064

Note: c_i is generally larger than 128 for a real B-tree.

Insertion into B-Trees

25/72

Overview of the method:

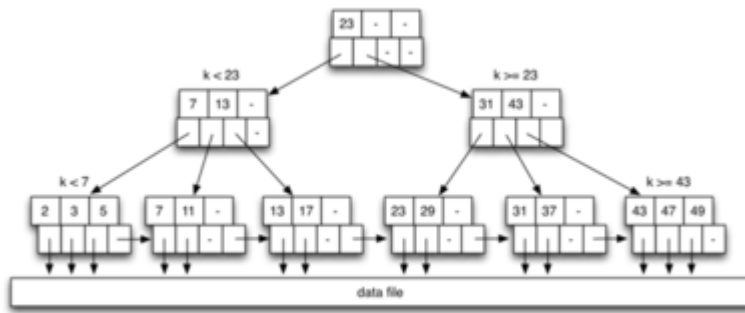
1. find leaf node and position in node **where entry would be stored**
2. if node is not full, **insert entry into appropriate spot**
3. if node is full, split node into two half-full nodes
and **promote middle element to parent**
4. if parent full, split and promote upwards
5. if reach root, and root is full, make new root upwards

Note: if duplicates not allowed and key exists, may stop after step 1.

Example: B-tree Insertion

26/72

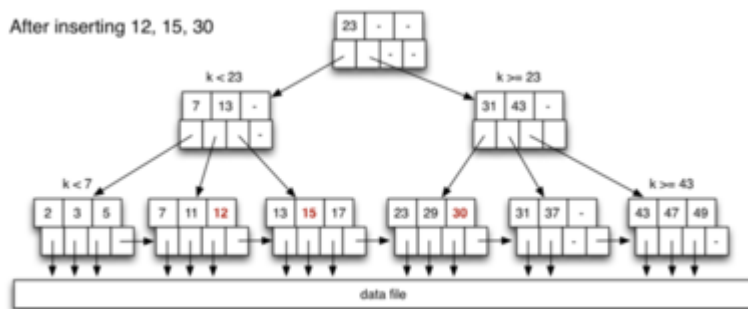
Starting from this tree:



insert the following keys in the given order 12 15 30 10

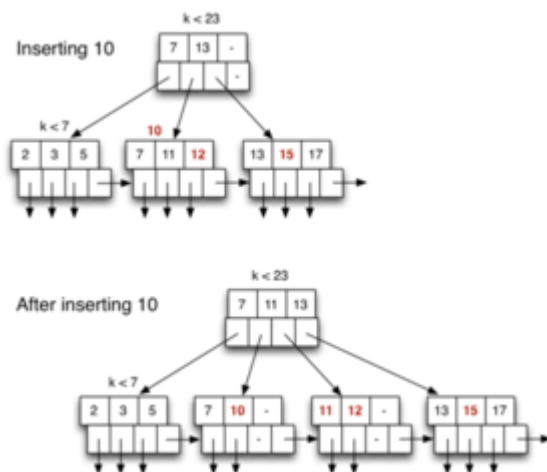
... Example: B-tree Insertion

27/72



... Example: B-tree Insertion

28/72



B-Tree Insertion Cost

29/72

$$\text{Insertion cost} = \text{Cost}_{\text{treeSearch}} + \text{Cost}_{\text{treeInsert}} + \text{Cost}_{\text{dataInsert}}$$

Best case: write one page (most of time)

- traverse from root to leaf
- read/write data page, write updated leaf

$$\text{Cost}_{\text{insert}} = D_r + 1_w + 1_r + 1_w$$

Common case: 3 node writes (rearrange 2 leaves + parent)

- traverse from root to leaf, holding nodes in buffer
- read/write data page
- update/write leaf, parent and sibling

$$Cost_{insert} = D_r + 3D_w + 1_r + 1_w$$

... B-Tree Insertion Cost

30/72

Worst case: $2D-1$ node writes (propagate to root)

- traverse from root to leaf, holding nodes in buffers
- read/write data page
- update/write leaf, parent and sibling
- repeat previous step $D-1$ times

$$Cost_{insert} = D_r + 3D_w + 1_r + 1_w \quad (\text{naive method})$$

$$Cost_{insert} = D_r + (2D-1)D_w + 1_r + 1_w \quad (\text{optimised method})$$

Selection with B-Trees

31/72

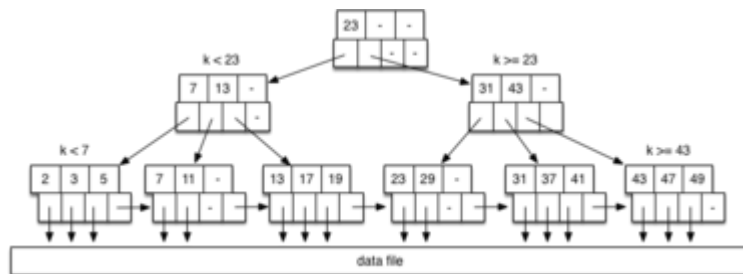
For one queries:

```

N = B-tree root node
while (N is not a leaf node)
    N = scanToFindChild(N, K)
tid = scanToFindEntry(N, K)
access tuple T using tid

```

$$Cost_{one} = (D + 1)_r$$



... Selection with B-Trees

32/72

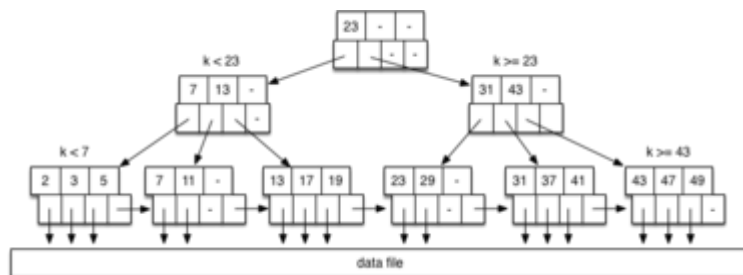
For range queries (assume sorted on index attribute):

```

search index to find leaf node for Lo
for each leaf node entry until Hi found {
    access tuple T using tid from entry
}

```

$$Cost_{range} = (D + b_i + b_q)_r$$



B-trees in PostgreSQL

33/72

PostgreSQL implements \approx Lehman/Yao-style B-trees

- variant that works effectively in high-concurrency environments.

B-tree implementation: **backend/access/nbtree**

- **README** ... comprehensive description of methods
- **nbtree.c** ... interface functions (for iterators)
- **nbtsearch.c** ... traverse index to find key value
- **nbtinsert.c** ... add new entry to B-tree index

Notes:

- stores all instances of equal keys (dense index)
- avoids splitting by scanning right if key = max(key) in page
- common insert case: new key is max(key) overall; handled efficiently

... B-trees in PostgreSQL

34/72

Changes for PostgreSQL v12

- indexes smaller
 - for composite keys, only store first attribute
 - index entries are smaller, so c_i larger, so tree shallower
- include TID in index key
 - duplicate index entries are stored in "table order"
 - makes scanning table files to collect results more efficient

To explore indexes in more detail:

- **\di+ IndexName**
- **select * from bt_page_items(IndexName,BlockNo)**

... B-trees in PostgreSQL

35/72

Interface functions for B-trees

```
// build Btree index on relation
Datum btbuild(rel,index,...)
// insert index entry into Btree
Datum btinsert(rel,key,tupleid,index,...)
// start scan on Btree index
Datum btbeginscan(rel,key,scandesc,...)
// get next tuple in a scan
Datum btgettupple(scandesc,scandir,...)
// close down a scan
Datum btendscan(scandesc)
```

N-dimensional Selection

N-dimensional Queries

37/72

Have looked at one-dimensional queries, e.g.

```
select * from R where a = K
select * from R where a between Lo and Hi
```

and heaps, hashing, indexing as ways of efficient implementation.

Now consider techniques for efficient multi-dimensional queries.

Compared to 1-d queries, multi-dimensional queries

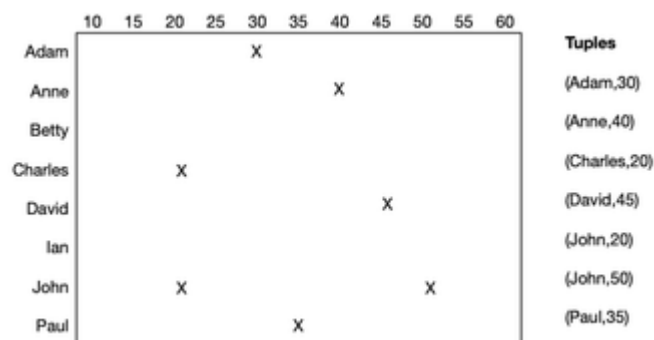
- typically produce fewer results
- require us to consider more information
- require more effort to produce results

Tuple Space

38/72

It is possible to view tuples as points in an Nd space.

Consider the relation *Person* (*name*, *age*) and its tuple-space.



Operations for Nd Select

39/72

N -dimensional select queries = condition on ≥ 1 attributes.

- pmr = partial-match retrieval (equality tests), e.g.

```
select * from Person
where name = 'John' and age = 50;
```

- $space$ = tuple-space queries (range tests), e.g.

```
select * from Person
where 20 ≤ age ≤ 50 and 'Charles' ≤ name ≤ 'John'
```

N -d Selection via Heaps

40/72

Heap files can handle pmr or $space$ using standard method:

```
// select * from R where C
r = openRelation("R", READ);
for (p = 0; p < nPages(r); p++) {
    buf = getPage(file(r), p);
    for (i = 0; i < nTuples(buf); i++) {
        t = getTuple(buf, i);
        if (matches(t, C))
            add t to result set
    }
}
```

$Cost_{pmr} = Cost_{space} = b$

N -d Selection via Multiple Indexes

41/72

DBMSs already support building multiple indexes on a table.

Which indexes to build depends on which queries are asked.

```
create table R (a int, b int, c int);
create index Rax on R (a);
create index Rbx on R (b);
create index Rcx on R (c);
create index Rabx on R (a,b);
create index Racx on R (a,c);
create index Rbcx on R (b,c);
create index Rallx on R (a,b,c);
```

But more indexes \Rightarrow space + update overheads.

N-d Queries and Indexes

42/72

Generalised view of pmr and space queries:

```
select * from R
where a1 op1 C1 and ... and an opn Cn
```

pmr : all op_i are equality tests. space : some op_i are range tests.

Possible approaches to handling such queries ...

1. use index on one a_i to reduce tuple tests
2. use indexes on all a_i, and intersect answer sets

... N-d Queries and Indexes

43/72

If using just one of several indexes, which one to use?

```
select * from R
where a1 op1 C1 and ... and an opn Cn
```

The one with best selectivity for a_i op_i C_i (i.e. fewest matches)

Factors determining selectivity of a_i op_i C_i

- assume uniform distribution of values in dom(a_i)
- equality test on primary key gives at most one match
- equality test on larger dom(a_i) ⇒ less matches
- range test over large part of dom(a_i) ⇒ many matches

... N-d Queries and Indexes

44/72

Implementing selection using one of several indices:

```
// Query: select * from R where a1op1C1 and ... and anopnCn
// choose ai with best selectivity
TupleIDs = IndexLookup(R, ai, opi, Ci)
// gives { tid1, tid2, ... } for tuples satisfying aiopiCi
PageIDs = { }
foreach tid in TupleIDs
    { PageIDs = PageIDs U {pageOf(tid)} }

// PageIDs = a set of bqix page numbers
...
```

Cost = Cost_{index} + b_{q_{ix}} (some pages do not contain answers, b_{q_{ix}} > b_q)

DBMSs typically maintain statistics to assist with determining selectivity

... N-d Queries and Indexes

45/72

Implementing selection using multiple indices:

```
// Query: select * from R where a1op1C1 and ... and anopnCn
// assumes an index on at least ai
TupleIDs = IndexLookup(R, a1, op1, C1)
foreach attribute ai with an index {
    tids = IndexLookup(R, ai, opi, Ci)
    TupleIDs = TupleIDs ∩ tids
}
PageIDs = { }
```

```
foreach tid in TupleIDs
  { PageIDs = PageIDs U {pageOf(tid)} }
// PageIDs = a set of bq page numbers
...
Cost = k.Costindex + bq (assuming indexes on k of n attrs)
```

Exercise 2: One vs Multiple Indices

46/72

Consider a relation with $r = 100,000$, $B = 4096$, defined as:

```
create table Students (
  id          integer primary key,
  name        char(10), -- simplified
  gender      char(1),  -- 'm', 'f', '?'
  birthday    char(5)   -- 'MM-DD'
);
```

Assumptions:

- data file is not ordered on any attribute
- has a dense B-tree index on each attribute
- 96 bytes of header in each data/index page

For Students(id, name, gender, birthday) ...

- calculate the size of the data file and each index
- describe the selectivity of each attribute

Now consider a query on this relation:

```
select * from Students
where name='John' and birthday='04-01'
```

- estimate the cost of answering using name index
- estimate the cost of answering using birthday index
- estimate the cost of answering using both indices

Bitmap Indexes

48/72

Alternative index structure, focussing on sets of tuples:

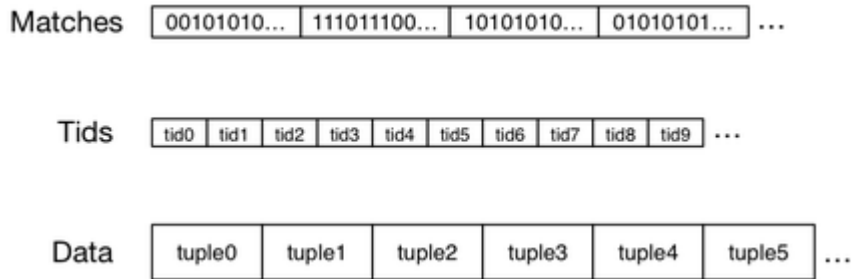
Data File				Colour Index	
	Part#	Colour	Price		
[0]	P7	red	\$2.50	red	100011...
[1]	P1	green	\$3.50	blue	001100...
[2]	P9	blue	\$4.10	green	010000...
[3]	P4	blue	\$7.00		
[4]	P5	red	\$5.20		
[5]	P5	red	\$2.50		
.....				Price Index	
				< \$4.00	110001...
				>= \$4.00	001110...

Index contains bit-strings of r bits, one for each value/range

... Bitmap Indexes

49/72

Also useful to have a file of tids, one for each tuple:



... Bitmap Indexes

50/72

Answering queries using bitmap index:

```
Matches = AllOnes(r)
foreach attribute A with index {
    // select ith bit-string for attribute A
    // based on value associated with A in WHERE
    Matches = Matches & Bitmaps[A][i]
}
// Matches contains 1-bit for each matching tuple
foreach i in 0..r-1 {
    if (Matches[i] == 0) continue;
    Pages = Pages U {pageOf(Tids[i])}
}
foreach pid in Pages {
    P = getPage(pid)
    extract matching tuples from P
}
```

Exercise 3: Bitmap Index

51/72

Using the following file structure:

Data File				Colour Index	
	Part#	Colour	Price		
[0]	P7	red	\$2.50	red	100011...
[1]	P1	green	\$3.50	blue	001100...
[2]	P9	blue	\$4.10	green	010000...
[3]	P4	blue	\$7.00		
[4]	P5	red	\$5.20		
[5]	P5	red	\$2.50		
	*****			Price Index	
				< \$4.00	110001...
				>= \$4.00	001110...

Show how the following queries would be answered:

```
select * from Parts
where colour='red' and price < 4.00
```

```
select * from Parts
where colour='green' or colour = 'blue'
```

... Bitmap Indexes

52/72

Storage costs for bitmap indexes:

- one bitmap for each value/range for each indexed attribute
- each bitmap has length $\text{ceil}(r/8)$ bytes
- e.g. with 50K records and 8KB pages, bitmap fits in one page

Query execution costs for bitmap indexes:

- read one bitmap for each indexed attribute in query
- perform bitwise AND on bitmaps (in memory)
- read pages containing matching tuples

Note: bitmaps could index pages rather than tuples (shorter bitmaps)

Hashing for N-d Selection

Hashing and pmr

54/72

For a pmr query like

`select * from R where $a_1 = C_1$ and ... and $a_n = C_n$`

- if one a_i is the hash key, query is very efficient
- if no a_i is the hash key, need to use linear scan

Can be alleviated using **multi-attribute hashing** (mah)

- form a composite hash value involving all attributes
- at query time, some components of composite hash are known
(allows us to limit the number of data pages which need to be checked)

MA.hashing works in conjunction with any dynamic hash scheme.

... Hashing and pmr

55/72

Multi-attribute hashing parameters:

- file size = $b = 2^d$ pages \Rightarrow use d -bit hash values
- relation has n attributes: a_1, a_2, \dots, a_n
- attribute a_i has hash function h_i
- attribute a_i contributes d_i bits (to the combined hash value)
- total bits $d = \sum_{i=1}^n d_i$
- a choice vector (cv) specifies for all $k \dots$
bit j from $h_i(a_i)$ contributes bit k in combined hash value

MA.Hashing Example

56/72

Consider relation *Deposit* (*branch, acctNo, name, amount*)

Assume a small data file with 8 main data pages (plus overflows).

Hash parameters: $d=3$ $d_1=1$ $d_2=1$ $d_3=1$ $d_4=0$

Note that we ignore the amount attribute ($d_4=0$)

Assumes that nobody will want to ask queries like

`select * from Deposit where amount=533`

Choice vector is designed taking expected queries into account.

... MA.Hashing Example

57/72

Choice vector:

	7	6	5	4	3	2	1	0
	$b_{2,2}$	$b_{1,2}$	$b_{3,1}$	$b_{2,1}$	$b_{1,1}$	$b_{3,0}$	$b_{2,0}$	$b_{1,0}$

This choice vector tells us:

- bit 0 in hash comes from bit 0 of hash₁(a_1) ($b_{1,0}$)
- bit 1 in hash comes from bit 0 of hash₂(a_2) ($b_{2,0}$)

- bit 2 in hash comes from bit 0 of $\text{hash}_3(a_3)$ ($b_{3,0}$)
- bit 3 in hash comes from bit 1 of $\text{hash}_1(a_1)$ ($b_{1,1}$)
- etc. etc. etc. (up to as many bits of hashing as required, e.g. 32)

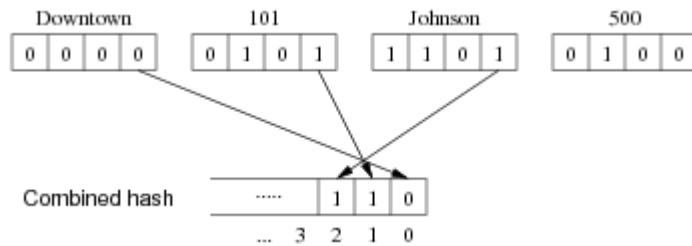
... MA.Hashing Example

58/72

Consider the tuple:

branch	acctNo	name	amount
Downtown	101	Johnston	512

Hash value (page address) is computed by:



MA.Hashing Hash Functions

59/72

Auxiliary definitions:

```
#define MaxHashSize 32
typedef unsigned int HashVal;

// extracts i'th bit from hash value
#define bit(i,h) (((h) & (1 << (i))) >> (i))

// choice vector elems
typedef struct { int attr, int bit } CVeclem;
typedef CVeclem ChoiceVec[MaxHashSize];

// hash function for individual attributes
HashVal hash_any(char *val) { ... }
```

... MA.Hashing Hash Functions

60/72

Produce combined d-bit hash value for tuple t:

```
HashVal hash(Tuple t, ChoiceVec cv, int d)
{
    HashVal h[nAttr(t)+1]; // hash for each attr
    HashVal res = 0, oneBit;
    int i, a, b;
    for (i = 1; i <= nAttr(t); i++)
        h[i] = hash_any(attrVal(t,i));
    for (i = 0; i < d; i++) {
        a = cv[i].attr;
        b = cv[i].bit;
        oneBit = bit(b, h[a]);
        res = res | (oneBit << i);
    }
    return res;
}
```

Exercise 4: Multi-attribute Hashing

61/72

Compute the hash value for the tuple

('John Smith', 'BSc(CompSci)', 1990, 99.5)

where $d=6$, $d_1=3$, $d_2=2$, $d_3=1$, and

- $cv = \langle (1,0), (1,1), (2,0), (3,0), (1,2), (2,1), (3,1), (1,3), \dots \rangle$
- $hash_1('John\ Smith') = \dots 0101010110110100$
- $hash_2('BSc(CompSci)') = \dots 1011111101101111$
- $hash_3(1990) = \dots 0001001011000000$

Queries with MA.Hashing

62/72

In a partial match query:

- values of some attributes are known
- values of other attributes are unknown

E.g.

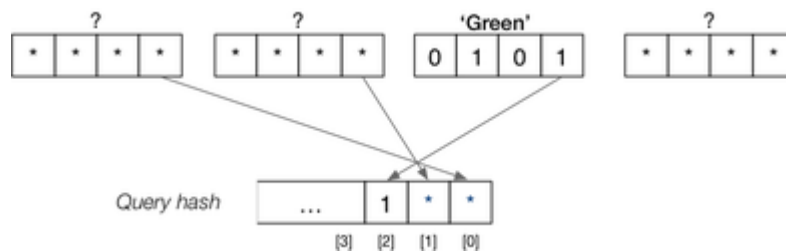
```
select amount
from Deposit
where branch = 'Brighton' and name = 'Green'
```

for which we use the shorthand (Brighton, ?, Green, ?)

... Queries with MA.Hashing

63/72

Consider query: select amount from Deposit where name='Green'



Matching tuples must be in pages: 100, 101, 110, 111.

Exercise 5: Partial hash values in MAH

64/72

Given the following:

- $d=6$, $b=2^6$, $CV = \langle (0,0), (0,1), (1,0), (2,0), (1,1), (0,2), \dots \rangle$
- $hash(a) = \dots 00101101001101$
- $hash(b) = \dots 00101101001101$
- $hash(c) = \dots 00101101001101$ 101101

What are the query hashes for each of the following:

- (a,b,c) , $(?,b,c)$, $(a,?,?)$, $(?,?,?)$

101101 *011** 1***01 *****

MA.Hashing Query Algorithm

65/72

```
// Builds the partial hash value (e.g. 10*0*1)
// Treats query like tuple with some attr values missing
nstars = 0;
for each attribute i in query Q {
    if (hasValue(Q,i)) {
        set d[i] bits in composite hash
        using choice vector and hash(Q,i)
    } else {
        set d[i] *'s in composite hash
        nstars = nstars + 1;
    }
}
```

```

        using choice vector
    nstars += d[i]
}
}
...

```

... MA.Hashing Query Algorithm

66/72

```

...
// Use the partial hash to find candidate pages

r = openRelation("R", READ);
for (i = 0; i < 2nstars; i++) {
    P = composite hash
    replace *'s in P
        using i and choice vector
    Buf = readPage(file(r), P);
    for each tuple T in Buf {
        if (T satisfies pmr query)
            add T to results
    }
}

```

Exercise 6: Representing Stars

67/72

Our hash values are bit-strings (e.g. 100101110101)

MA.Hashing introduces a third value (* = unknown)

How could we represent "bit"-strings like 1011*1*0**010?

Exercise 7: MA.Hashing Query Cost

68/72

Consider $R(x, y, z)$ using multi-attribute hashing where

$d = 9$ $d_x = 5$ $d_y = 3$ $d_z = 1$

How many buckets are accessed in answering each query?

1. select * from R where $x = 4$ and $y = 2$ and $z = 1$ 1
2. select * from R where $x = 5$ and $y = 3$ 2
3. select * from R where $y = 99$ 2^6
4. select * from R where $z = 23$ 2^8
5. select * from R where $x > 5$

Linear scan

Query Cost for MA.Hashing

69/72

Multi-attribute hashing handles a range of query types, e.g.

```

select * from R where a=1
select * from R where d=2
select * from R where b=3 and c=4
select * from R where a=5 and b=6 and c=7

```

A relation with n attributes has 2^n different query types.

Different query types have different costs (different no. of *'s)

$\text{Cost}(Q) = 2^s$ where $s = \sum_{i \in Q} d_i$ (alternatively $\text{Cost}(Q) = \prod_{i \in Q} 2^{d_i}$)

Query distribution gives probability p_Q of asking each query type Q .

... Query Cost for MA.Hashing

70/72

Min query cost occurs when all attributes are used in query

$$\text{Min Cost}_{\text{pmr}} = 1$$

Max query cost occurs when no attributes are specified

$$\text{Max Cost}_{\text{pmr}} = 2^d = b$$

Average cost is given by weighted sum over all query types:

$$\text{Avg Cost}_{\text{pmr}} = \sum_Q p_Q \prod_{i \in Q} 2^{d_i}$$

Aim to minimise the weighted average query cost over possible query types

Optimising MA.Hashing Cost

71/72

For a given application, useful to minimise Cost_{pmr} .

Can be achieved by choosing appropriate values for d_i (cv)

Heuristics:

- distribution of query types (more bits to frequently used attributes)
- size of attribute domain (\leq #bits to represent all values in domain)
- discriminatory power (more bits to highly discriminating attributes)

Trade-off: making Q_j more efficient makes Q_k less efficient.

This is a combinatorial optimisation problem
(solve via standard optimisation techniques e.g. simulated annealing)

Exercise 8: MA.Hashing Design

72/72

Consider relation *Person* (name, gender, age) ...

p_Q	Query Type Q
0.5	select name from Person where gender=X and age=Y
0.25	select age from Person where name=X
0.25	select name from Person where gender=X

Assume that all other query types have $p_Q=0$.

Design a choice vector to minimise average selection cost.

Produced: 31 Mar 2020