Problem 1:

Recall the relation composition operator; defined as:

$$\mathbf{R_1; R_2 = \{(a, c) : \text{there is a b with } (a, b) \in R_1 \text{ and } (b, c) \in R_2\}}$$

For any set S, and any binary relations $R_1, R_2, R_3 \subseteq S \times S$, prove or give a counterexample to disprove the following:

(a) $(R_1; R_2); R_3 = R_1; (R_2; R_3)$

Proof:

To prove that $(R_1; R_2); R_3 = R_1; (R_2; R_3)$ holds, we can prove statements below instead

$$(R_1; R_2); R_3 \subseteq R_1; (R_2; R_3)$$

$$R_1; (R_2; R_3) \subseteq (R_1; R_2); R_3$$

(1)

Assuming that there is an arbitrary element $(a, b) \in (R_1 ; R_2) ; R_3$

Then there must be a m_0 such that

$$(a, m_0) \in (R_1 ; R_2) \text{ and}$$

$$(m_0, b) \in R_3$$

And since $(a, m_0) \in (R_1 ; R_2)$, there must be a m_1 such that

$$(a, m_1) \in R_1 \text{ and}$$

$$(m_1, m_0) \in R_2$$

For $R_2; R_3$, since $(m_1, m_0) \in R_2$ and $(m_0, b) \in R_3$

So $$(m_1, b) \in (R_2;R_3)$$

For $R_1; (R_2; R_3)$, since $(a, m_1) \in R_1$ and $(m_1, b) \in (R_2;R_3)$:

$$(a, b) \in R_1 ; (R_2 ; R_3)$$

Thus for any element in $(a, b) \in (R_1; R_2); R_3$,it must satisfy $(a, b) \in R_1; (R_2; R_3)$

So $$(R_1; R_2); R_3 \subseteq R_1; (R_2; R_3).$$

(2)

for the versa case, for any element $(a, b) \in R_1 ; (R_2 ; R_3)$

Then there must be a m_2 such that

$$(a, m_2) \in R_1 \text{ and}$$

$$(m_2, b) \in (R_2 ; R_3)$$

And since $(m_2, b) \in (R_2 ; R_3)$, there must be a m_1 such that

$$(m_2, m_3) \in R_2 \text{ and}$$

$$(m_3, b) \in R_3$$

For $R_1; R_2$, since and $(a, m_2) \in R_1$ and $(m_2, m_3) \in R_2$

So $$(a, m_3) \in (R_1;R_2)$$

For $(R_1; R_2); R_3$, since $(a, m_3) \in (R_1;R_2)$ and $(m_3, b) \in R_3$

$$(a, b) \in (R_1;R_2); R_3$$

Thus for any element in $(a, b) \in R_1; (R_2; R_3)$,it must satisfy $(a, b) \in (R_1; R_2); R_3$

So $$R_1; (R_2; R_3) \subseteq (R_1; R_2); R_3$$

Since $$(R_1; R_2); R_3 \subseteq R_1; (R_2; R_3)$$

Thus $$R_1; (R_2; R_3) = (R_1; R_2); R_3$$

(b) $I; R_1 = R_1; I = R_1$ where $I = \{(x, x): x \in S\}$

Proof:

According to the specification:

$$R_1 \subseteq S \times S$$

Since $$I = \{(x, x): x \in S\}$$

Then for every $(r_1, r_2) \in R_1$, we can always find a $(r_1, r_1) \in I$ such that:

$$\{(r_1, r_1)\}; \{(r_1, r_2)\} = \{(r_1, r_2)\}$$

So $$I; R_1 = R_1$$

Similarly, for every $(r_1, r_2) \in R_1$, we can always find a $(r_2, r_2) \in I$ such that:

Thus $$R_1; I = R_1$$

And $$I; R_1 = R_1; I = R_1 \text{ holds}$$

(c) $(R_1; R_2)^{\leftarrow} = R_1; R_2$

Giving a counterexample:

For $$R_1 = R_1^{\leftarrow} = \{(1, 1), (2, 2)\}$$

$$R_2 = \{(1, 3), (2, 3)\}$$

$$R_2^{\leftarrow} = \{(3, 1), (3, 2)\}$$

Then $$R_1^{\leftarrow}; R_2^{\leftarrow} = \emptyset$$

$$(R_1; R_2)^{\leftarrow} = \{(3, 1), (3, 2)\}$$

So $(R_1; R_2)^{\leftarrow} = R_1; R_2$ doesn't hold.

(d) $(R_1 \cup R_2) ; R_3 = (R_1 ; R_3) \cup (R_2; R_3)$

Proof:

To prove that $(R_1 \cup R_2) ; R_3 = (R_1 ; R_3) \cup (R_2; R_3)$ hold, we can prove statements below instead

$$(R_1 \cup R_2) ; R_3 \subseteq (R_1 ; R_3) \cup (R_2; R_3) \text{ and}$$

$$(R_1 ; R_3) \cup (R_2; R_3) \subseteq (R_1 \cup R_2) ; R_3$$

(1)

Assuming that there is an arbitrary element $(a, b) \in (R_1 \cup R_2) ; R_3$

Then there must be a m_0 such that $(a, m_0) \in (R_1 \cup R_2)$ and $(m_0, b) \in R_3$

Thus $(a, m_0) \in R_1$ or $(a, m_0) \in R_2$ holds

if $\quad (a, m_0) \in R_1$

since $\quad (m_0, b) \in R_3$

so $\quad (a, b) \in R_1 ; R_3$

Since $(R_1 ; R_3) \subseteq (R_1 ; R_3) \cup (R_2; R_3)$

then for any element $(a, b) \in (R_1 \cup R_2) ; R_3$

$(a, b) \in (R_1 ; R_3) \cup (R_2; R_3)$ holds

And vice versa, when $(a, m_0) \in R_2$ holds

$(a, b) \in (R_2; R_3)$ holds

Then $\quad (a, b) \in (R_1 ; R_3) \cup (R_2; R_3)$ holds

Thus $\quad (R_1 \cup R_2) ; R_3 \subseteq (R_1 ; R_3) \cup (R_2; R_3)$

(2)

for the versa case:

Assuming that there is an arbitrary element $(a, b) \in (R_1 ; R_3) \cup (R_2; R_3)$

Then there must be a m_0 such that

$(a, m_0) \in R_1$ and $(m_0, b) \in R_3$

or $(a, m_0) \in R_2$ and $(m_0, b) \in R_3$

if $(a, m_0) \in R_1$ and $(m_0, b) \in R_3$ holds, then $(a, m_0) \in R_1 \cup R_2$ holds

Thus $\quad (a, b) \in (R_1 \cup R_2) ; R_3$

And vice versa, when $(a, m_0) \in R_2$ and $(m_0, b) \in R_3$

$(a, b) \in (R_1 \cup R_2) ; R_3$ holds

Then for any element $(a, b) \in (R_1 ; R_3) \cup (R_2; R_3)$

$(a, b) \in (R_1 \cup R_2) ; R_3$ holds

Thus $\quad (R_1 ; R_3) \cup (R_2; R_3) \subseteq (R_1 \cup R_2) ; R_3$

since $\quad (R_1 \cup R_2) ; R_3 \subseteq (R_1 ; R_3) \cup (R_2; R_3)$

So $\quad (R_1 \cup R_2) ; R_3 = (R_1 ; R_3) \cup (R_2; R_3)$

(e) $R_1; (R_2 \cap R_3) = (R_1; R_2) \cap (R_1; R_3)$

Giving a counterexample:

For $R_1 = \{(1, 2), (2, 3)\}$

$R_2 = \{(2, 2)\}$

$R_3 = \{(3, 5)\}$

Then for the left side of the equation:

$$
\begin{aligned}
R_1; (R_2 \cap R_3) &= \{(1, 2), (2, 3)\}; (\{(2, 2)\} \cap \{(3, 5)\}) \\
&= \{(1, 2), (2, 3)\}; \emptyset \\
&= \emptyset
\end{aligned}
$$

For the right side of the equation:

$$
\begin{aligned}
(R_1 ; R_2) \cap (R_1; R_3) &= (\{(1, 2), (2, 3)\}; \{(2, 2)\}) \cap (\{(1, 2), (2, 3)\}; \{(3, 5)\}) \\
&= \{(1,2)\} \cap \{(2, 5)\} \\
&= \{(1, 5)\}
\end{aligned}
$$

Problem 2:

Let $R \subseteq S \times S$ be any binary relation on a set S. Consider the sequence of relations R^0 , R^1 , R^2 , . . ., defined as follows:

$$\mathbf{R^0 := I = \{(x, x) : x \in S\}, \text{ and}}$$

$$\mathbf{R^{i+1} := R^i \cup (R; R^i) \text{ for } i \geq 0}$$

(a) Prove that if there is an i such that $R^i = R^{i+1}$, then $R^j = R^i$ for all $j \geq i$.

According to the specification:

Inductive proof: for any $j \geqslant i$, $R^j = R^i$ holds

Assume $j = i + a, a > 0$

Base case ($a = 1$):

$R^i = R^{i+1}$ holds

And $R^{i+1} = R^i = Ri \cup (R; Ri)$ holds

Inductive case: assuming that $R^{i+a} = R^i$ for any $a > 0$ holds, then:

$$
\begin{aligned}
R^{i+a+1} &= R^{i+a} \cup (R; R^{i+a}) \\
&= R^i \cup (R; R^i) = R^{i+1} = R^i
\end{aligned}
$$

Thus $R^j = R^i$ for all $j \geq i$ holds

(b) Prove that if there is an i such that $R^i = R^{i+1}$, then $R^k \subseteq R^i$ for all $k \geq 0$.

According to the specification:

Inductive proof: for any $k \geq 0$, $R^0 \subseteq R^k$ holds:

Base case ($k = 0$):

$$R^1 := R^0 \cup (R; R^0)$$

According to the set theory, $R^0 \subseteq R^1$ holds

Inductive case($k = a$ for any $0 \leq a \leq i$):

Assuming that $R^0 \subseteq R^{a-1}$

Then $$R^a = R^{a-1} \cup (R; R^{a-1})$$

According to the set theory, $R^{a-1} \subseteq R^a$ holds

Since $R^0 \subseteq R^{a-1}$ holds, according to transitivity of set, $R^0 \subseteq R^a$ holds.

Thus $R^0 \subseteq R^i$ holds

for $k > i$, according to question (a), $R^k = R^i$ for all $k \geq i$,

And $R^0 \subseteq R^i$

So $R^k \subseteq R^i$ for all $k \geq 0$ holds

(c) Let P(n) be the proposition that for all $m \in N$: $R^n ; R^m = R^{n+m}$. Prove that P(n) holds for all $n \in N$.

Inductive proof: for any $n \in N$, $R^m = R^{n+m}$ holds

Base case($n = 0$):

Since $$R^0 = I = \{(x, x): x \in S\}$$

For each relation (a, b) in R^m, from question1, we have

$$I; R^m = R^0; R^m = R^m$$

Inductive case($n > 0$) :

Assuming that for any $n-1 > 0$ $$R^{n-1}; R^m = R^{n+m+1} \text{ holds} \qquad (1)$$

Then $$R^n ; R^m = [R^{n-1} \cup (R; R^{n-1})]; R^m$$

from Problem1 (a)(d), we have proved that

$$(R_1 \cup R_2); R_3 = (R_1; R_3) \cup (R_2; R_3) \qquad (2)$$

$$(R_1; R_2); R_3 = R_1; (R_2; R_3) \qquad (3)$$

Then
$$\begin{aligned} R^n ; R^m &= [R^{n-1} \cup (R; R^{n-1})]; R^m \\ &= (R^{n-1}; R^m) \cup [(R ; R^{n-1}); R^m] && (2) \\ &= R^{n+m-1} \cup [(R ; (R^{n-1}; R^m)] && (3) \\ &= R^{n+m-1} \cup [R ; R^{n+m-1}] && (1) \\ &= R^{n+m} \end{aligned}$$

Thus for all $n \in N$, P(N) holds.

(d) If $|S| = k$, explain why $R^k = R^{k+1}$.(Hint: Show that if $(a, b) \in R^{k+1}$ then $(a, b) \in R^i$ for some $i< k + 1$)

From problem 2(a)(b), we have proved that if there is an i such that $R^i = R^{i+1}$

$$R^j = R^i \text{ for all } j \geq i$$

And $$R^k \subseteq R^{k+1} \text{ for all } k \geq 0.$$

To prove $R^k = R^{k+1}$, we can prove $R^{k+1} \subseteq R^k$ instead:

assuming that there are some relations $(a, b) \in R^{k+1}$ where $a, b \in S$

Since $$R^{k+1} = R^k \cup (R; R^k)$$

Then there are two possibilities:

1. $(a, b) \in R^k$ or
2. $(a, b) \in R; R^k$

(1) if $(a, b) \in R^k$ holds

Then for any relation $(a, b) \in R^{k+1}$, $(a, b) \in R^k$ holds.

Thus $R^{k+1} \subseteq R^k$ holds.

(2) if $(a, b) \in R; R^k$ holds

Then there must be a m_k such that

$$(a, m_k) \in R \text{ and}$$

$$(m_k, b) \in R^k$$

Inductive proof: for any p where $k \geqslant p \geqslant 0$, there must be $(a, m_{k-p}) \in R$ and $(m_{k-p}, b) \in R^{k-p}$

Base case($p = 0$) have been proven.

Inductive case: assuming that for any $k \geqslant p \geqslant 0$

$$(a, m_{k-p}) \in R \text{ and}$$

$$(m_{k-p}, b) \in R^{k-p} \text{ holds}$$

Since $R^{k-p} = R^{k-p-1} \cup (R; R^{k-p-1})$

Then there must be a m_{k-p-1} such that

$$(a, m_{k-p-1}) \in R \text{ and}$$

$$(m_{k-p-1}, b) \in R^{k-p-1}$$

Thus we have $$(m_0, b) \in R^0 \text{ and}$$

$$(m_1, b) \in R^1 \text{ and}$$

$$(m_2, b) \in R^2 \text{ and}$$

…..

$$(m_k, b) \in R^k$$

Since from problem(b) we have proved that

$$R^i \subseteq R^{i+1} \text{ for all } i \geq 0.$$

Thus $$\{(m_p, b): k \geqslant p \geqslant 0\} \subseteq R_k$$

Since $m_p \in S$, the maximum possible number of relations represented by (m, b) is $|S| = k$

Then $$(a, b) \in \{(m_p, b): k \geqslant p \geqslant 0\} \subseteq R^k \text{ holds.}$$

Thus $(a, b) \in R_k$ holds and for any relation $(a, b) \in R^{k+1}$, $(a, b) \in R^k$ holds.

So $$R^{k+1} \subseteq R^k \text{ holds.}$$

In conclusion, both $R_k \subseteq R_{k+1}$ and $R_{k+1} \subseteq R_k$ hold, then

$$R^k = R^{k+1}$$

(e) If |S| = k, show that R^k is transitive.

From (d), we have proved that if $|S| = k$, Then $R^k = R^{k+1}$.

Assuming that $(a, b) \in R^k$, $(b, c) \in R^k$, then according the specification:

$$(a, c) \in R^k; R^k$$

From Problem2 (c), we have proved that $R^n ; R^m = R^{n+m}$ holds for all $n, m \in N$

Thus $$R^k; R^k = R^{2k} \text{ holds}$$

From Problem2 (b), we have proved that if there is an k such that $R^k = R^{k+1}$, then $R^i \subseteq R^k$ for all $k \geq 0$

Thus $$R^{2k} \subseteq R^k \text{ holds}$$

Then $$(a, c) \in R^k; R^k = R^{2k} \subseteq R^k \text{ holds}$$

And $$(a, c) \in R^k$$

Thus R^k is transitive

(f) If $|S| = k$, show that $(R \cup R^{\leftarrow})^k$ is an equivalence relation.

If $(R \cup R^{\leftarrow})^k$ is an equivalence relation, it must satisfy Reflexivity(R), Symmetricity(S), Transitivity(T)

Proof:

(1) Reflexivity(R)

Suppose that $W = R \cup R^{\leftarrow}$

From Problem2 (b), we have proved that $k \geqslant 0$, then $W^0 \subseteq W^k$

Since $W^0 = I$, So for all $x \in S$, there must be

$$\{(x, x): x \in S\} \subseteq I = W^0 \subseteq W^k$$

Thus W^k is reflexive holds.

(2) Transitivity(T)

From Problem2(e), we have proved that R^k is transitive where $k = |S|$

Regard $W = R \cup R^{\leftarrow}$ as a whole

Then W^k is transitive holds

(3) Symmetricity

According to the specification:

$$W^0 = I$$

$$W^1 = W^0 \cup (W^0; W)$$

According to problem 1(b), it can be simplified as (base case):

$$W^1 = W^0 \cup W$$

Then

$$\begin{aligned} W^2 &= W^1 \cup (W^1; W) \\ &= W^1 \cup ((W^0 \cup W) ; W) \\ &= W^1 \cup ((W^0; W) \cup (W; W)) \\ &= W^1 \cup (W; W) \end{aligned}$$

Inductive case:

Assuming that

$$W^n = W^{n-1} \cup (((W; W);W)\ldots;W) \quad \text{holds}$$

(*note that W occurs n times in the latter operator, and replacing it by W_g)

Then
$$W^n = W^{n-1} \cup W_g \quad \text{holds}$$

And
$$W^n = W^{n-1} \cup (W^{n-1}; W)$$

$$W^{n-1}; W \subseteq W^n$$

Then, to prove that
$$W^{n+1} = W^n \cup (W_g ; W)$$

We have $W^{n+1} = W^n \cup (W^n; W)$

$= W^n \cup (W^{n-1}; W)$

$= W^n \cup ((W^{n-1} \cup W^g); W)$

$= W^n \cup (W^{n-1}; W) \cup (W^g; W)$

$= W^n \cup (W_g; W)$

Thus $W^{n+1} = W^n \cup (((W; W);W)\ldots;W)$ holds (W occurs (n+1) times)

Inductive proof: W^n is symmetric for all n>=0

Base case(n = 0):

Since $W^0 = I$, for all $(a, b) \in W^0$, $a = b$ holds

Then $(b, a) \in W^0$

Assuming that W^n is symmetric for all $n > 1$

Then $W^{n+1} = W^n \cup (((W; W);W)\ldots;W)$

Since $W = R \cup R^{\leftarrow}$

Then for all $x, y \in S$ such that $(x, y) \in R$, there must be $(y, x) \in R^{\leftarrow}$ and vice versa

Thus W is symmetric

Then for all $(x, y) \in W$, there must be $(y, x) \in W$ such that

$(x, x) \in W; W$

Thus $W; W \subseteq I$

Then

$$(((W;\ W);W)\ldots;W) = \begin{cases} I_0, & I_0 \in I \text{ when } (n+1) \text{ is even} \\ W, & \text{when } (n+1) \text{ is odd} \end{cases}$$

Both of them is symmetric

Thus for all n > 0, if W^n is symmetric holds

$$W^{n+1} = W^n \cup (((W; W);W)\ldots;W)$$

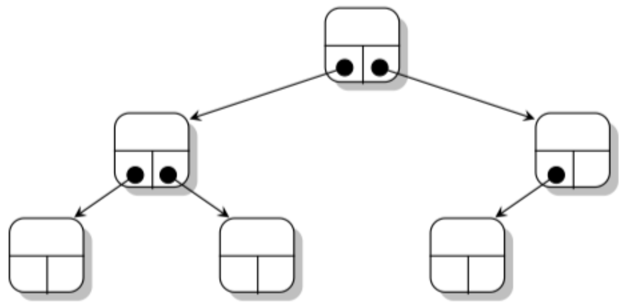
W^{n+1} is symmetric holds and W^n is symmetric for all $n \geqslant 0$

Then W^k is symmetric

In conclusion, $(R \cup R^{\leftarrow})^k$ is an equivalence relation

Problem 3

A binary tree is a data structure where each node is linked to at most two successor nodes:



If we allow empty binary trees (trees with no nodes), then we can simplify the description by saying a node has exactly two children which are binary trees.

(a) Give a recursive definition of the binary tree data structure. Hint: review the recursive definition of a Linked List

A binary tree is:

① (B) an empty tree

② (R) a node point to two binary tree children(*Left_tree, *Right_tree)

(* means a reference or a pointer(like pointer in C)

A leaf in a binary tree is a node that has no successors (i.e. it has two empty trees as children). A fully-internal node in a binary tree is a node that has two successors. The example above has 3 leaves and 2 fully-internal nodes.

(b) Based on your recursive definition above, define the function count(T) that counts the number of nodes in a binary tree T.

According the definition, empty binary trees are trees with no nodes

Then if T is empty, then T has not successor and there is no node

$$\text{count}(T) = 0, \text{ T is empty (B)}$$

Else if T is not empty, then T itself is a node

Also, there are two reference connecting to its successor

Thus, the number of nodes of its successors are:

$$\text{count}(\text{Left\_tree}) + \text{count}(\text{Right\_tree})$$

Then the sum of nodes is:

$$\text{count}(T) = \text{count}(\text{Left\_tree}) + \text{count}(\text{Right\_tree}) + 1 \text{ (R)}$$

(c) Based on your recursive definition above, define the function leaves(T) that counts the number of leaves in a binary tree T.

According to the definition, a leaf in a binary tree is a node that has no successors

If a Tree T is empty, there is no leaf

$$\text{leaves}(T) = 0,\ \ T \text{ is empty (B)}$$

Else if a Tree T has no successor, this tree itself is a leaf

$$\text{leaves}(T) = 1, \text{T.Left\_tree and T.Right\_tree are empty (B)}$$

Else if it has successor, the number of leaves in this tree is equal to the sum of the number of leaves of its two children

$$\text{leaves}(T) = \text{leaves}(\text{Left\_tree}) + \text{leaves}(\text{Right\_tree}) \text{ (R)}$$

(d) Based on your recursive definition above, define the function internal(T) that counts the number of fully-internal nodes in a binary tree T. Hint: it is acceptable to define an empty tree as having −1 fully-internal nodes.

According to the definition, a fully-internal node in a binary tree is a node that has two successors.

If a tree T is empty or T has no successor, there is no internal node

$$\text{internal}(T) = 0 \text{ (B)}$$

Else if a tree T has one successor T_1, this tree itself is not an fully-internal node and the number of internal nodes is equals to the number of internal nodes of its only successor, then:

$$\text{internal}(T) = \text{internal}(T_1)\ \ \text{(R)}$$

Else, this tree T has 2 successors, this tree itself is an internal node and the number of internal nodes is equals to the sum of the number of internal nodes of its two successor T_L and T_R plus 1

$$\text{internal}(T) = \text{internal}(T_L) + \text{internal}(T_R) + 1 \text{ (R)}$$

(e) If T is a binary tree, let P(T) be the proposition that leaves(T) = 1 + internal(T). Prove that P(T) holds for all binary trees T.

Inductive proof: for any number of nodes n, $\text{leaves}(T_n) = 1 + \text{internal}(T_n)$

Base case($n = 1$), the mother-tree T_n has no successor, then:

then the T itself is a leaf and it has no internal node

$$\text{leaves}(T_0) = 1 \text{ and}$$

$$\text{internal}(T_0) = 0$$

Thus when $n = 1$, $\text{leaves}(T) = \text{internal}(T) + 1$ holds

Inductive case: assuming that for any $n>1$, $P(T_n)$ holds, then

$$\text{leaves}(T_n) = \text{internal}(T_n) + 1$$

for the case $n +1$, there are two possibilities.

(1) if the new node connected to a leaf, then the number of leaves remain the same as the new-added node would become a new leaf connected to the previous one while this previous node turn to be a binary tree with one successor but not an internal node.

$$leaves(T_{n+1}) = leaves(T_n) + 1 - 1$$

Also, no new internal node would be generated. Then:

$$internal(T_{n+1}) = internal(T_n)$$

Thus $leaves(T_{n+1}) = internal(T_{n+1}) + 1$ holds

(2) else if the new node is added to a tree with one successor previously, then it turn it into an internal node:

$$internal(T_{n+1}) = internal(T_n) + 1$$

In the meanwhile, the new-added node is empty and become a leaf

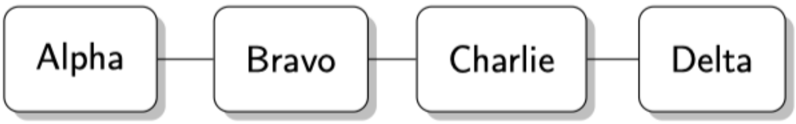
$$leaves(T_{n+1}) = leaves(T_n) + 1$$

Thus $leaves(T_{n+1}) = internal(T_{n+1}) + 1$ holds

In conclusion, the proposition $P(T_n)$ such that $leaves(T_n) = 1 + internal(T_n)$ holds for all $n > 0$

Problem 4

Four wifi networks, Alpha, Bravo, Charlie and Delta, all exist within close proximity to one another as shown below.



Networks connected with an edge in the diagram above can interfere with each other. To avoid interference networks can operate on one of two channels, hi and lo. Networks operating on different channels will not interfere; and neither will networks that are not connected with an edge.

Our goal is to determine (algorithmically) whether there is an assignment of channels to networks so that there is no interference. To do this we will transform the problem into a problem of determining if a propositional formula can be satisfied.

(a) Carefully defining the propositional variables you are using, write propositional formulas for each of the following requirements:

(i) φ_1 : Alpha uses channel hi or channel lo; and so does Bravo, Charlie and Delta.

(ii) φ_2: Alpha does not use both channel hi and lo; and the same for Bravo, Charlie and Delta.

(iii) φ_3: Alpha and Bravo do not use the same channel; and the same applies for all other pairs of networks connected with an edge.

Defining the propositional variables as below:

AH: Alpha uses channel hi;

AL: Alpha uses channel lo;

BH: Bravo uses channel hi;

BL: Bravo uses channel lo;

CH: Charlie uses channel hi;

CL: Charlie uses channel lo;

DH: Delta uses channel hi;

DL: Delta uses channel lo;

φ_1 : $(AH \vee AL) \wedge (BH \vee BL) \wedge (CH \vee CL) \wedge (DH \vee DL)$

φ_2 : $((AH \wedge \neg AL) \vee (\neg AH \wedge AL)) \wedge$

$((BH \wedge \neg BL) \vee (\neg BH \wedge BL)) \wedge$

$((CH \wedge \neg CL) \vee (\neg CH \wedge CL)) \wedge$

$((DH \wedge \neg DL) \vee (\neg DH \wedge DL)) \wedge$

φ_3 : $((AH \wedge BL) \vee (AL \wedge BH)) \wedge$

$((BH \wedge CL) \vee (BL \vee CH)) \wedge$

$((CH \wedge DL) \vee (CL \vee DH)) \wedge$

(b) (i) Show that $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ is satisfiable; So the requirements can all be met. Note that it is sufficient to give a satisfying truth assignment, you do not have to list all possible combinations.

Giving a truth assignment that all these three requirements can be met:

AH	AL	BH	BL	CH	CL	DH	DL	φ_1	φ_2	φ_3
T	F	F	T	T	F	F	T	T	T	T
F	T	T	F	F	T	T	F	T	T	T

(ii) Based on your answer to the previous question, which channels should each network use in order to avoid interference?

Based on the previous proof, to avoid interference, there are two solution:

1. Alpha uses channel hi; Bravo uses channel lo; Charlie uses channel hi; Delta uses channel lo;

2. Alpha uses channel lo; Bravo uses channel hi; Charlie uses channel lo; Delta uses channel hi;

Note that to avoid interference, each network would use either hi or lo channels but not both.