Student Name:	
Student Number:	
Signature:	

University of New South Wales
School of Computer Science and Engineering
Foundations of Computer Science (COMP9020)
FINAL EXAM — Session 1, 2017

This paper must be submitted and cannot be retained by the student

Instructions:

- **Ensure you enter your correct name and student number above!**
- This exam paper contains 10 multiple-choice questions (pages 1-3) plus 5 open questions (pages 4-8).
  Each multiple-choice question is worth 4 marks ($10 \times 4 = 40$).
  Each open question is worth 12 marks ($5 \times 12 = 60$).
  Total exam marks = 100.
- Only use a blue or black pen. **All answers must be recorded in this paper.**
- For the multiple-choice questions, tick **one** box for your answer directly (each multiple-choice question has only one correct answer).
  To make a correction, tick *all* boxes, then *circle* one box for your answer.
- For the open questions, write your answer in the space provided (if you need more space, you can write on the back of the sheet).
- A separate white booklet is provided for scratch work only. **Do not write your answers in the Examination Answer Book, it will not be marked.**
- Time allowed – 120 minutes + 10 minutes reading time.
- The exam is *closed book*. Reference materials are not allowed, apart from one A4-sized sheet (double-sided is ok) of your own notes.
- Number of pages in this exam paper: 8 (in addition to this cover sheet).

1. How many integers in the interval $[-100, 100]$ are divisible by 5 **or** 7 (or both)?

   - ☐ 64
   - ☐ 65
   - ☐ 67
   - ☐ 68

2. Consider the alphabets $\Sigma = \{s, e, a\}$ and $\Psi = \{a, r, t\}$. How many words are in the set $\{\omega \in (\Sigma \setminus \Psi)^* : \text{ length}(\omega) \leq 2\}$?

   - ☐ 2
   - ☐ 4
   - ☐ 6
   - ☐ 7

3. Which of the following is **not** a correct equivalence?

   - ☐ $\neg A \vee B \;\equiv\; \neg(B \wedge \neg A)$
   - ☐ $A \wedge \neg B \;\equiv\; \neg(B \vee \neg A)$
   - ☐ $A \Rightarrow \neg B \;\equiv\; B \Rightarrow \neg A$
   - ☐ $\neg(A \Rightarrow B) \;\equiv\; \neg B \wedge A$

4. Consider the functions $f : \mathbb{N} \longrightarrow \{0, 1, 2\}$ and $g : \{0, 1, 2\} \longrightarrow \{0, 1, 2\}$ defined by

$$\begin{aligned} f(x) &= x \bmod 3 \\ g(x) &= |x - 2| \end{aligned}$$

   Which of the following statements is true?

   - ☐ $f \circ f \neq f$
   - ☐ $g \circ g = \mathrm{Id}_{\{0,1,2\}}$
   - ☐ $f \circ g$ is **not** onto
   - ☐ $g \circ f$ is **not** onto

5. Consider the partial order $\preceq$ on $S = \{1, 2, 3, 4, 6, 12\}$ defined by

$$x \preceq y \text{ if and only if } x \mid y \qquad \text{(i.e., } x \text{ is a divisor of } y\text{)}$$

   Which of the following is **not** true?

   - ☐ $\mathrm{lub}(\{1, 4, 6\}) = 12$
   - ☐ $\mathrm{glb}(\{4, 6, 12\}) = 1$
   - ☐ $(S, \preceq)$ is a lattice
   - ☐ $1 < 3 < 2 < 6 < 4 < 12$ is a topological sort of $(S, \preceq)$

6. All connected graphs with n vertices and k edges satisfy

   - ☐ $n \geq k + 1$
   - ☐ $n \geq k$
   - ☐ $n \leq k$
   - ☐ $n \leq k + 1$

7. We would like to prove that $P(n)$ for all $n \geq 0$.
   Which of the following conditions imply this conclusion?

   - ☐ $P(0)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n + 1))$
   - ☐ $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \wedge P(n + 1) \Rightarrow P(n + 2))$
   - ☐ $P(0)$ and $P(1)$ and $\forall n \geq 0\,(P(n) \wedge P(n + 1) \Rightarrow P(n + 2))$
   - ☐ $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n + 2))$

8. Consider the recurrence given by $T(1) = 1$ and $T(n) = 4 \cdot T(\frac{n}{2}) + n$.
This has order of magnitude

- ☐ $O(n)$
- ☐ $O(n \cdot \log n)$
- ☐ $O(n^2)$
- ☐ $O(2^n)$

9. Let $S = \{1, 2, 3\}$ and $\mathbb{B} = \{0, 1\}$.
How many different *onto* functions $f : S \longrightarrow \mathbb{B}$ are there?

- ☐ 0
- ☐ 6
- ☐ 8
- ☐ 9

10. Which of the following is true for all A, B?

- ☐ $P(A \cap B|B) = P(A|B)$
- ☐ $P(A \cap B) = P(B) \cdot P(B|A)$
- ☐ $P(A \cup B) \geq P(A) + P(B)$
- ☐ $P(A|B) + P(A|\bar{B}) = 1$

11. Consider the following two formulae:

$$\begin{aligned} \phi &= \neg(A \Rightarrow (B \wedge C)) \\ \psi &= \neg A \vee C \end{aligned}$$

(a) Transform ϕ into ***disjunctive*** normal form (DNF).
(b) Prove that $\phi, \psi \models \neg B$ (i.e., $\neg B$ is a logical consequence of ϕ and ψ).
(c) Is $\phi \vee \psi$ a tautology (i.e., always true)? **Explain your answer.**

12. Prove that for all binary relations $\mathcal{R}_1 \subseteq S \times S$ and $\mathcal{R}_2 \subseteq S \times S$ the following holds:

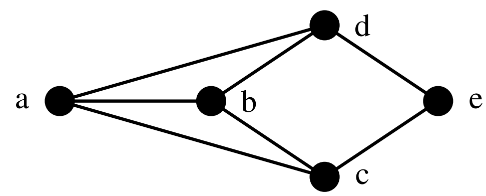
If $\mathcal{R}_1$ and $\mathcal{R}_2$ are symmetric, then $\mathcal{R}_1 \setminus \mathcal{R}_2$ is symmetric.

13. The *Fibonacci numbers* are defined as follows:

$$F_1 = 1; \quad F_2 = 1; \quad F_i = F_{i-1} + F_{i-2} \text{ for } i \geq 3$$

Write a proof by induction for the statement that every *third* Fibonacci number (that is, $F_3,\ F_6,\ F_9,\ \ldots$) is even (i.e., divisible by 2).

14. Consider the following graph G:



(a) Give all 3-cliques of G.
(b) What is the chromatic number $\chi(G)$ of G? **Explain your answer.**
(c) What is the maximal number of edges that can be added to G such that G remains planar? **Explain your answer.**

15. Consider a deck of six cards containing 2 jacks and 4 aces. One card is randomly drawn from the deck at a time. Calculate the expected number of drawing attempts until an ace is drawn:

    (a) if the cards are put back into the deck after each drawing;
    (b) if the cards are **not** put back into the deck after each drawing.

    Briefly explain your answers.

— END OF EXAM —