COMP9020

Foundations of Computer Science

COMP9020 19T3 Staff

Lecturer:	Paul Hunter
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Lectures:	Mondays and Wednesdays 4-6pm
Consults:	Wednesdays, 2-3pm, Rm204 K17
Research:	Theoretical CS: Algorithms, Formal verification

What you can expect from me

What is this course about?

What is Computer Science?

> *"Computer science no more about computers than astronomy is about telescopes"*

– E. Dijkstra

Course Aims

The actual content is taken from a list of subjects that constitute the basis of the tool box of every serious practitioner of computing:

• number theory	week 1
• sets, relations and functions	weeks 2–3
• big-O notation	week 3
• recursion	week 4
• graph theory	week 5
• logic	week 6
• induction	week 7
• combinatorics	week 8
• probability and expectation	week 9

Course Material

All course information is placed on the course website

www.cse.unsw.edu.au/∼cs9020/

Content includes:

- Lecture slides and recordings
- Quizzes and Assignments
- Course Forums
- Practice questions
- Challenge questions

Course Material

Textbook:

- KA Ross and CR Wright: Discrete Mathematics

Supplementary textbook:

- E Lehman, FT Leighton, A Meyer:
  Mathematics for Computer Science

Assessment Summary

60% exam, 30% assignments, 10% quizzes:

- 16 quizzes, worth up to 1 mark each
- 3 assignments, worth up to 10 marks each
- final exam (2 hours) worth up to 60 marks

Quizzes are available for 2 days before each lecture. Assignments due at the end of weeks 4, 7 and 10.

You must achieve 40% on the final exam to pass

Your final score will be taken from your 10 best quiz results, 3 assignments and final exam.

More information

View the course outline at:

www.cse.unsw.edu.au/$\sim$cs9020/outline.html

Particularly the sections on **Student conduct** and **Plagiarism**.

What I will expect from you

Assessments

To achieve good marks in this course you need to demonstrate:

- Your understanding of the material
- Your ability to work with the material

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Mathematical communication

Guidelines for good mathematical writing

Mathematical writing should be:

- Clear
- Logical
- Sensical

Example

Ex 1 a) 51 b) 72 c) 12

Ex 2: $(A \setminus B) \cup (B \setminus A) = (A \cap B^c) \cup (B \cap A^c) = (A \cup B) \cap (A \cup A^c) \cap (B \cup B^c) \cap (A \cup A^c)$
$= (A \cup B) \cap (A^c \cup B^c) = (A \cup B) \cap (A \cap B)^c = (A \cup B) \setminus (A \cap B)$ by DeM, Dist

Ex 3 a) Yes b) No c) Yes d) No e) Yes

Ex 4 a) True b) False

Examples

Example

Ex. 2

$$
\begin{aligned}
(A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Def.)} \\
&= ((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (B^c \cup B) \\
&\qquad \cap (A \cup A^c) \cap (B^c \cup A^c) && \text{(Dist.)} \\
&= (A \cup B) \cap (A^c \cup B^c) && \text{(Ident.)} \\
&= (A \cup B) \cap (A \cap B)^c && \text{(DeM.)} \\
&= (A \cup B) \setminus (A \cap B) && \text{(Def.)}
\end{aligned}
$$

Examples

Example

Ex. 4a

We will show that if R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.

Suppose $(a, b) \in R_1 \cap R_2$. Then $(a, b) \in R_1$ and $(a, b) \in R_2$. Because R_1 is symmetric, $(b, a) \in R_1$. Because R_2 is symmetric, $(b, a) \in R_2$. Therefore $(b, a) \in R_1 \cap R_2$. Therefore $R_1 \cap R_2$ is symmetric.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A **proposition** is a statement that is either true or false.

Example

Propositions:

- $3 + 5 = 8$
- All integers are either even or odd
- There exist a, b, c such that $1/a + 1/b + 1/c = 4$

Not propositions:

- $3 + 5$
- x is even or x is odd
- $1/a + 1/b + 1/c = 4$

Proposition structure

Common proposition structures include:

If A then B	$(A \Rightarrow B)$
A if and only if B	$(A \Leftrightarrow B)$
For all x, A	$(\forall x.A)$
There exists x such that A	$(\exists x.A)$

$\forall$ and $\exists$ are known as **quantifiers**.

Proofs

A large component of your work in this course is giving **proofs** of **propositions.**

A proof of a proposition is an argument to convince the reader/marker that the proposition is true.

A **proof** of a proposition is a finite sequence of logical steps, starting from base assumptions (**axioms** and **hypotheses**), leading to the proposition in question.

Proofs

Example

Prove: $3 \times 2 = 2 \times 3$

$$\begin{aligned}
3 \times 2 &= (2+1) \times 2 \\
&= (2 \times 2) + (1 \times 2) \\
&= (1 \times 2) + (2 \times 2) \\
&= 2 + (2 \times 2) \\
&= (2 \times 1) + (2 \times 2) \\
&= 2 \times (1+2) \\
&= 2 \times 3.
\end{aligned}$$

Proofs: pitfalls

Starting from the proposition and deriving true **is not valid**.

Example

Prove: $1 = -1$

$$\begin{array}{lrcll} & 1 & = & -1 & \\ \text{So} & (1)^2 & = & (-1)^2 & \\ \text{So} & 1 & = & 1 & \text{which is true.} \end{array}$$

Does this mean that $1 = -1$?

Proofs: pitfalls

Make sure each step is logically valid: for example, $x = y$ implies $x^2 = y^2$ but $x^2 = y^2$ does not imply $x = y$.

Example

Suppose $a = b$. Then,

$$\begin{array}{lrcl} & a^2 & = & ab \\ \text{So} & a^2 - b^2 & = & ab - b^2 \\ \text{So} & (a-b)(a+b) & = & (a-b)b \\ \text{So} & a+b & = & b \\ \text{So} & a & = & 0 \end{array}$$

This is true no matter what value a is given at the start, so does that mean everything is equal to 0?

Proofs: pitfalls

For propositions of the form $\forall x.A$ where x can have infinitely many values:

- You cannot enumerate infinitely many cases in a proof.
- Only considering a finite number of cases is not sufficient.

Example

For all n, $n^2 + n + 41$ is prime

True for $n = 0, 1, 2, \ldots, 39$. Not true for $n = 40$.

Proofs: pitfalls

The order of quantifiers matters when it comes to propositions:

Example

- For every number x, there is a number y such that y is larger than x
- There is a number y such that for every number x, y is larger than x

Proof strategies: direct proof

Proposition form	You need to do this
$A \Rightarrow B$	Assume A and prove B
$A \Leftrightarrow B$	Prove "If A then B" and "If B then A"
$\forall x.A$	Show A holds for every possible value of x
$\exists x.A$	Find a value of x that makes A true

Proof strategies: contradiction

To prove A is true, assume A is false and derive a contradiction.
That is, start from the negation of the proposition and derive false.

Example

Prove: $\sqrt{2}$ is irrational
Proof: Assume $\sqrt{2}$ is rational ...

Negating propositions

Proposition form	Its negation
A and B	Not A or not B
A or B	Not A and Not B
$A \Rightarrow B$	A and Not B
$A \Leftrightarrow B$	(A and Not B) or (not A and B)
$\forall x.A$	E x.not A
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	?
A or B	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x. A$	
$\exists x. A$	

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Negating propositions

Proposition form	Its negation
A and B	?
A or B	?
$A \Rightarrow B$	?
$A \Leftrightarrow B$	?
$\forall x . A$	
$\exists x . A$	

Negating propositions

Proposition form	Its negation
A and B	?
A or B	?
$A \Rightarrow B$	?
$A \Leftrightarrow B$	?
$\forall x.A$	?
$\exists x.A$	

Negating propositions

Proposition form	Its negation
A and B	? not a or not b
A or B	? not a and not b
$A \Rightarrow B$	? a and not b
$A \Leftrightarrow B$	? a and not b, or b and not a
$\forall x.A$	? E x not a
$\exists x.A$	? floor x not

Proof strategies: contrapositive

To prove a proposition of the form "If A then B" you can prove "If not B then not A"

Example

Prove: If $m + n \geq 73$ then $m \geq 37$ or $n \geq 37$.

Proof strategies: dealing with $\forall$

How can we check infinitely many cases?

- Choose an **arbitrary** element: an object with no assumptions about it (may have to check several cases)
- Induction (see week 7)

Example

Prove: For every integer n, n^2 will have remainder 0 or 1 when divided by 4.

Note: "Arbitrary" is not the same as "random".