COMP9020 Week 2
Binary Relations

- Textbook (R & W) - Ch. 3., Sec. 3.1, 3.4; Ch. 11, Sec. 11.1

Applications in Computer Science

Many relations that appear in CS fall into two broad categories:

Equivalence relations (generalizing "equality"):

- Programs that exhibit the same behaviour
- Logically equivalent statements
- The `.equals()` method in Java

Partial orders (generalizing "less than or equal to"):

- Object inheritance
- Simulation
- Requirement specifications
- The `.compareTo()` method in Java

Summary of topics

- Defining binary relations
- Properties of binary relations
- Equivalence relations, classes, and partitions
- Orderings

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Binary relations

A **binary relation between** S **and** T is a subset of $S \times T$: i.e. a set of ordered pairs.
Also: over S and T; from S to T; on S (if $S = T$).

Example (Special (Trivial) Relations)

Identity (diagonal, equality) $E = \{ (x, x) : x \in S \}$

Empty $\emptyset$

Universal $U = S \times S$

Defining binary relations: Set-based definitions

Defining a relation $R \subseteq S \times T$:

- Explicitly listing tuples: e.g. $\{(1, 1), (2, 3), (3, 2)\}$
- Set comprehension: $\{(x, y) \in [1, 3] \times [1, 3] : 5|xy - 1\}$
- Construction from other relations: $\{(1, 1)\} \cup \{(2, 3)\} \cup \{(2, 3)\}^{\leftarrow}$

Defining binary relations: Matrix representation

Defining a relation $R \subseteq S \times T$:

Rows enumerated by elements of S, columns by elements of T:

Examples

- The relation $\{(1,1),(2,3),(3,2)\} \subseteq [1,3] \times [1,3]$:

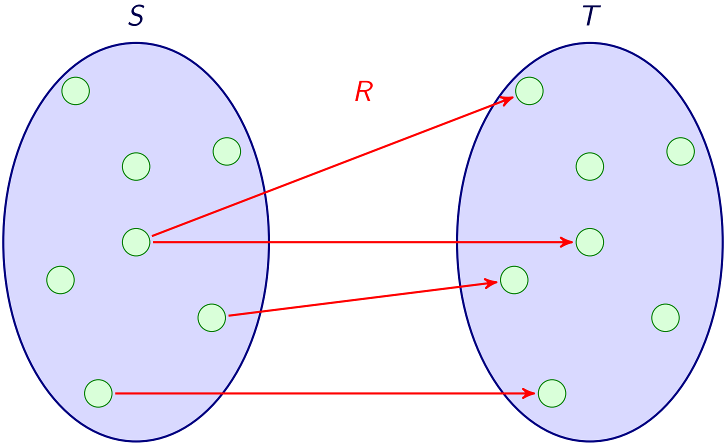
$$\begin{bmatrix} \bullet & \circ & \circ \\ \circ & \circ & \bullet \\ \circ & \bullet & \circ \end{bmatrix}$$

- The relation $\{(1,1),(1,2),(1,3),(1,4),(2,2),(3,2)\} \subseteq [1,3] \times [1,4]$:

$$\begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ \circ & \bullet & \circ & \circ \\ \circ & \bullet & \circ & \circ \end{bmatrix}$$

Defining binary relations: Graphical representation

Defining a relation $R \subseteq S \times T$:



Defining binary relations: Graphical representation

Example

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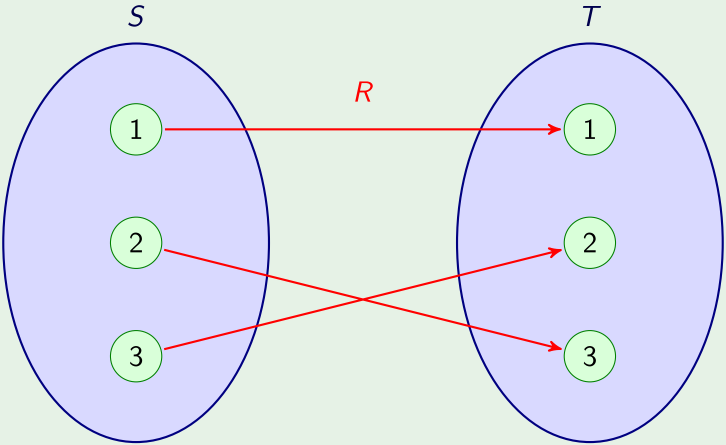
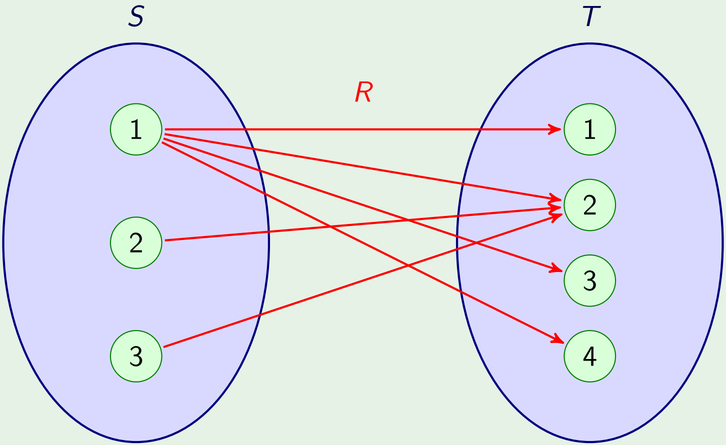


Defining binary relations: Graphical representation

Example

$\{(1,1),(1,2),(1,3),(1,4),(2,2),(3,2)\} \subseteq [1,3] \times [1,4]$:



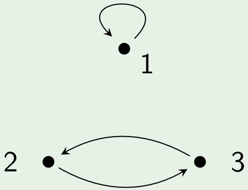
Defining binary relations: Graph representation

If $S = T$ we can define $R \subseteq S \times S$ as a **directed graph** (week 5).

- Nodes: Elements of S
- Edges: Elements of R

Example

$R = \{(1,1),(2,3),(3,2)\} \subseteq [1,3] \times [1,3]$:



Summary of topics

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Properties of Binary Relations $R \subseteq S \times S$

Definition

(R)	reflexive	For all $x \in S$: $(x,x) \in R$
(AR)	antireflexive	For all $x \in S$: $(x,x) \notin R$
(S)	symmetric	For all $x, y \in S$: If $(x,y) \in R$ then $(y,x) \in R$
(AS)	antisymmetric	For all $x, y \in S$: If (x,y) and $(y,x) \in R$ then $x = y$
(T)	transitive	For all $x, y, z \in S$: If (x,y) and $(y,z) \in R$ then $(x,z) \in R$

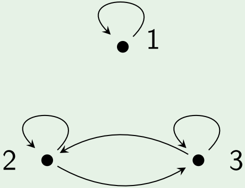
NB

- *Properties have to hold for all elements*
- *(S), (AS), (T) are conditional statements – they will hold if there is nothing which satisfies the 'if' part*

Relation properties: Examples

Examples

(R) Reflexivity: $(x, x) \in R$ for all x



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Relation properties: Examples

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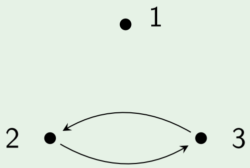

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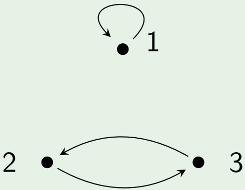
Relation properties: Examples

Examples

(R) Reflexivity: $(x, x) \in R$ for all x

(AR) Antireflexivity: $(x, x) \notin R$ for all x

(S) Symmetry: If $(x, y) \in R$ then $(y, x) \in R$ for all x, y



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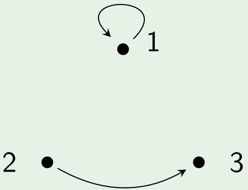
(AS) Antisymmetry: $(x, y) \in R$ and $(y, x) \in R$ implies $x = y$ for all x, y



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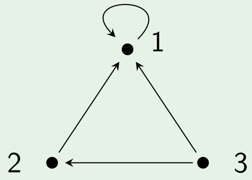
Relation properties: Examples

Examples

- **(R)** Reflexivity: $(x, x) \in R$ for all x
- **(AR)** Antireflexivity: $(x, x) \notin R$ for all x
- **(S)** Symmetry: If $(x, y) \in R$ then $(y, x) \in R$ for all x, y
- **(AS)** Antisymmetry: $(x, y) \in R$ and $(y, x) \in R$ implies $x = y$ for all x, y
- **(T)** Transitivity: $(x, y) \in R$ and $(y, z) \in R$ implies $(x, z) \in R$ for all x, y, z.



$$\begin{bmatrix} \bullet & \circ & \circ \\ \bullet & \circ & \circ \\ \bullet & \bullet & \circ \end{bmatrix}$$

Interaction of Properties

A relation *can* be both symmetric and antisymmetric. Namely, when R consists only of some pairs $(x, x), x \in S$.

A relation *cannot* be simultaneously reflexive and antireflexive (unless $S = \emptyset$).

NB

$$\left.\begin{array}{r}\textit{nonreflexive}\\ \textit{nonsymmetric}\end{array}\right\}\ \textit{is not the same as}\ \left\{\begin{array}{l}\textit{antireflexive/irreflexive}\\ \textit{antisymmetric}\end{array}\right.$$

Exercises

Exercises

$\boxed{3.1.1}$ The following relations are on $S = \{1, 2, 3\}$.
Which of the properties (R), (AR), (S), (AS), (T) does each satisfy?

(a) $(m, n) \in R$ if $m + n = 3$?

(e) $(m, n) \in R$ if $\max\{m, n\} = 3$?

$\boxed{3.1.2(b)}$ $(m, n) \in R$ if $m < n$?

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Exercises

Exercises

Complete the following table of common relations (over $\mathbb{Z}$) and their properties:

	(R)	(AR)	(S)	(AS)	(T)
$=$	t	f	t	t	t
$\leq$	T	f	f	t	t
$<$	F	t	f	t	t
$\emptyset$	F	t	t	t	t
$\mathcal{U} = \mathbb{Z} \times \mathbb{Z}$	T	f	t	f	t
$\mid$	T	f	f	f	t
$= \pmod 3$	T	f	t	f	t

Exercises

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Complete the following table of common relations (over $\mathbb{Z}$) and their properties:

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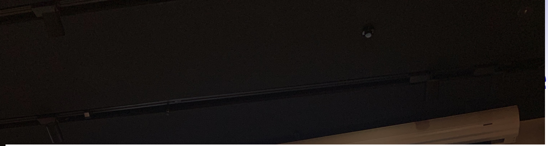

Exercises

$\boxed{3.1.10(a)}$ Give examples of relations with specified properties.

(AS), (T), not (R).

Some examples over $\mathbb{N}$, $\mathrm{Pow}(\mathbb{N})$:

- strict order of numbers $x < y$
- simple (weak) order, but with some pairs (x, x) removed from R
- being a prime divisor
  $(p, n) \in R$ iff p is prime and $p|n$
  - not reflexive: $(1, 1) \notin R, (4, 4) \notin R, (6, 6) \notin R$
  - transitivity is meaningful only for the pairs $(p, p), (p, n)$, $p|n$ for p prime

iff –> if and only if

Exercises

Exercises

$\boxed{\text{3.1.10(a)}}$ Give examples of relations with specified properties.
(AS), (T), not (R).
?

Exercises

Exercises

$\boxed{3.1.10(b)}$ Give examples of relations with specified properties.

(S), not (R), not (T).

Exercises

Exercises

$\boxed{\text{3.1.10(b)}}$ Give examples of relations with specified properties.
(S), not (R), not (T).
?

Exercises

$\boxed{\text{3.1.10(b)}}$ Give examples of relations with specified properties.
(S), not (R), not (T).
Simplest example - inequality

Exercises

Exercises

3.6.10 (supp)

R is a relation on $\mathbb{N} \times \mathbb{N}$, i.e. it is a subset of $\mathbb{N}^2 \times \mathbb{N}^2$

$(m, n)\, R\, (p, q)$ if $m = p \pmod 3$ or $n = q \pmod 5$.

(a) Is R reflexive?

Yes: $m = m \pmod 3$ (and $n = n \pmod 5$) so $(m, n)R(m, n)$.

(b) Is R symmetric?

Yes: by symmetry of $. = . \pmod n$.

(c) Is R transitive? No: Consider $(1, 1)$, $(1, 4)$ and $(2, 4)$.

Exercises

Exercises

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Exercises

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Exercises

Exercises

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?
(c) Is R transitive? ?

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- Defining binary relations
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Equivalence relations

等价关系

Equivalence relations capture a general notion of "equality". They are relations which are:

- Reflexive (R): Every object should be "equal" to itself
- Symmetric (S): If x is "equal" to y, then y should be "equal" to x
- Transitive (T): If x is "equal" to y and y is "equal" to z, then x should be "equal" to z.

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Definition

A binary relation $R \subseteq S \times S$ is *equivalence relation* if it satisfies (R), (S), (T).

等价关系

Example

Partition of $\mathbb{Z}$ into classes of numbers with the same remainder on division by p; it is particularly important for p prime

$$\mathbb{Z}(p) = \mathbb{Z}_p = \{0, 1, \ldots, p-1\}$$

One can define all four arithmetic operations (with the usual properties) on $\mathbb{Z}_p$ for a prime p; division has to be restricted when p is not prime.

NB

$(\mathbb{Z}_p, +, \cdot, 0, 1)$ *are fundamental algebraic structures known as* **rings**. *These structures are very important in coding theory and cryptography.* 密码学

Equivalence Classes and Partitions

Suppose $R \subseteq S \times S$ is an equivalence relation

The **equivalence class** $[s]$ (w.r.t. R) of an element $s \in S$ is

with reference to

$$[s] = \{t : t \in S \text{ and } sRt\}$$

Fact

$s\ R\ t$ if and only if $[s] = [t]$.

Partitions

Definition

A **partition** of a set S is a collection of sets $S_1, \ldots, S_k$ such that

- S_i and S_j are disjoint (for $i \neq j$) Don't have overlap!!
- $S = S_1 \cup S_2 \cup \cdots \cup S_k = \bigcup_{i=1}^{k} S_i$

The collection of all equivalence classes $\{[s] : s \in S\}$ forms a partition of S

In the opposite direction, a partition of a set defines the equivalence relation on that set. If $S = S_1 \cup \cdots \cup S_k$, then we can define $\sim \subseteq S \times S$ as:

$$s \sim t \text{ exactly when } s \text{ and } t \text{ belong to the same } S_i.$$

Do s and t represent element?

Exercises

Exercises

$\boxed{\text{3.6.6 (supp)}}$ Show that $m \sim n$ iff $m^2 = n^2 \pmod 5$ is an equivalence on $S = \{1, \ldots, 7\}$. Find all the equivalence classes.

Exercises

Exercises

$\boxed{\text{3.6.6 (supp)}}$ Show that $m \sim n$ iff $m^2 = n^2 \pmod 5$ is an equivalence on $S = \{1, \ldots, 7\}$. Find all the equivalence classes.
?

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Partial Order

偏序

A **partial order** $\preceq$ on S satisfies (R), (AS), (T).
We call $(S, \preceq)$ a **poset** — partially ordered set

偏序集

Examples

Posets:

What's that mean by (pow(x), ⊆)

- $(\mathbb{Z}, \leq)$
- $(\mathrm{Pow}(X), \subseteq)$ for some set X
- $(\mathbb{N}, |)$

Not posets:

- $(\mathbb{Z}, <)$
- $(\mathbb{Z}, |)$

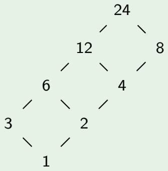
Hasse diagram

Every finite poset $(S, \preceq)$ can be represented with a **Hasse diagram**: Difference between $\preccurlyeq$ and underlined $\preccurlyeq$

- Nodes are elements of S
- An edge is drawn *upward* from x to y if $x \prec y$ and there is no z such that $x \prec z \prec y$

Example

Hasse diagram for positive divisors of 24 ordered by $|$:



Ordering Concepts

Definition

- **Minimal** and **maximal** elements (they always exist in every finite poset)
- **Minimum** and **maximum** — unique minimal and maximal element (might not exist)
- **lub** (least upper bound) and **glb** (greatest lower bound) of a subset $A \subseteq S$ of elements
  $\mathrm{lub}(A)$ — minimum of $\{x \in S : x \succeq a \text{ for all } a \in A\}$
  $\mathrm{glb}(A)$ — maximum of $\{x \in S : x \preceq a \text{ for all } a \in A\}$
- **Lattice** — poset where $\mathrm{lub}(x, y)$ and $\mathrm{glb}(x, y)$ exist for every pair of elements x, y.

Examples

Examples

- $\mathrm{Pow}(\{a,b,c\})$ with the order $\subseteq$
  $\emptyset$ is minimum; $\{a,b,c\}$ is maximum
- $\mathrm{Pow}(\{a,b,c\}) \setminus \{\{a,b,c\}\}$ (proper subsets of $\{a,b,c\}$)
  Each two-element subset $\{a,b\}, \{a,c\}, \{b,c\}$ is maximal.
  - But there is no maximum
- $\{1,2,3,4,6,8,12,24\}$ partially ordered by divisibility is a lattice
  - e.g. $\mathrm{lub}(\{4,6\}) = 12$; $\mathrm{glb}(\{4,6\}) = 2$
- $\{1,2,3\}$ partially ordered by divisibility is not a lattice
  - $\{2,3\}$ has no lub
- $\{2,3,6\}$ partially ordered by divisibility
  - $\{2,3\}$ has no glb
- $\{1,2,3,12,18,36\}$ partially ordered by divisibility
  - $\{2,3\}$ has no lub ($12, 18$ are minimal upper bounds)

NB

An infinite lattice need not have a lub (or no glb) for an arbitrary infinite subset of its elements, in particular no such bound may exist for **all** *its elements.*

Examples

- $\mathbb{Z}$ — neither lub nor glb;
- $\mathbb{F}(\mathbb{N})$ — all finite subsets, has no *arbitrary* lub property; glb exists, it is the intersection, hence always finite;
- $\mathbb{I}(\mathbb{N})$ — all infinite subsets, may not have an arbitrary glb; lub exists, it is the union, which is always infinite.

Exercises

Exercises

$\boxed{11.1.5}$ Consider poset $(\mathbb{R}, \leq)$

(a) Is this a lattice?

(b) Give an example of a non-empty subset of $\mathbb{R}$ that has no upper bound.

(c) Find $\mathrm{lub}(\{\, x \in \mathbb{R} : x < 73 \,\})$

(d) Find $\mathrm{lub}(\{\, x \in \mathbb{R} : x \leq 73 \,\})$

(e) Find $\mathrm{lub}(\{\, x : x^2 < 73 \,\})$

(f) Find $\mathrm{glb}(\{\, x : x^2 < 73 \,\})$

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(e) Find $\text{lub}(\{\ x : x^2 < 73\ \})$?

(f) Find $\text{glb}(\{\ x : x^2 < 73\ \})$?

Total orders

Definition

A **total order** is a partial order that also satisfies:

(L) *Linearity* (any two elements are comparable):

$$\text{For all } x, y \text{ either: } x \leq y \text{ or } y \leq x \text{ (or both if } x = y)$$

NB

having similar appearance but genetically different

On a finite set all total orders are "isomorphic"
On an infinite set there is quite a variety of possibilities.

Examples

Examples

- $\mathbb{Z}$ with $\leq$:
  (no minimum/maximum element)
- $\mathbb{Z}$ with $\{(x,y) : x < 0 \leq y \text{ or } |x| \leq |y|\}$:
  (no maximum element, minimum element is -1)
- $\mathbb{Z}$ with $\{(x,y) : x < 0 \leq y \text{ or } x \geq y\}$:
  (minimum element -1, maximum element 0)

? ? ? ? ? ? ?

Ordering of a Poset — Topological Sort

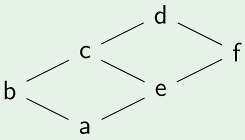
Definition 偏序集 拓扑排序

For a poset $(S, \preceq)$ any total order $\leq$ that is consistent with $\preceq$ (if $a \preceq b$ then $a \leq b$) is called a **topological sort**.

Like division relation, this is also a topological sort

Example

Consider



The following all are topological sorts:

$a \leq b \leq e \leq c \leq f \leq d$

$a \leq e \leq b \leq f \leq c \leq d$

$a \leq e \leq f \leq b \leq c \leq d$

Well-Ordered Sets

Definition

A *well-ordered set* is a poset where every subset has a least element.

NB

The greatest element is not required.

Examples

- $\mathbb{N} = \{0, 1, \ldots\}$
- $\mathbb{N}_1 \dot{\cup} \mathbb{N}_2 \dot{\cup} \mathbb{N}_3 \dot{\cup} \ldots$, where each $\mathbb{N}_i \simeq \mathbb{N}$ and $\mathbb{N}_1 < \mathbb{N}_2 < \mathbb{N}_3 \cdots$

NB

Well-ordered sets are an important mathematical tool to prove termination of programs.

Combining Orders

Product order — can combine any partial orders. In general, it is only a *partial order*, even if combining total orders.
For $s, s' \in S$ and $t, t' \in T$ define

$$(s, t) \preceq (s', t') \quad \text{if } s \preceq s' \text{ and } t \preceq t'$$

What's the result of (s, t)$\preceq$ (s' , t')
What's that mean?

Practical Orderings

They are, effectively, *total* orders on the *product* of ordered sets.

- **Lexicographic order** — defined on all of Σ^*. It extends a total order already assumed to exist on Σ.
- **Lenlex** — the order on (potentially) the entire Σ^*, where the elements are ordered first by length.
  $\Sigma^{(1)} \prec \Sigma^{(2)} \prec \Sigma^{(3)} \prec \cdots$, then lexicographically within each $\Sigma^{(k)}$. In practice it is applied only to the finite subsets of Σ^*.
- **Filing order** — lexicographic order confined to the strings of the same length.
  It defines total orders on Σ^i, separately for each i.

Example

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$\boxed{11.2.5}$ Let $\mathbb{B} = \{0, 1\}$ with the usual order $0 < 1$. List the elements $101, 010, 11, 000, 10, 0010, 1000$ of $\mathbb{B}^*$ in the

(a) Lexicographic order

(b) Lenlex order

$\boxed{11.2.8}$ When are the lexicographic order and *lenlex* on Σ^* the same?

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(a) Lexicographic order

$000, 0010, 010, 10, 1000, 101, 11$

(b) Lenlex order

$10, 11, 000, 010, 101, 0010, 1000$

$\boxed{11.2.8}$ When are the lexicographic order and *lenlex* on Σ^* the same?

Only when $|\Sigma| = 1$.

Exercises

Exercises

$\boxed{11.6.6}$ True or false?

(a) If a set Σ is totally ordered, then the corresponding lexicographic partial order on Σ^* also must be totally ordered.

(b) If a set Σ is totally ordered, then the corresponding lenlex order on Σ^* also must be totally ordered.

(c) Every finite partially ordered set has a Hasse diagram.

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? true

(c) Every finite partially ordered set has a Hasse diagram.
? true

Exercises

Exercises

$\boxed{11.6.6}$ True or false?

(d) Every finite partially ordered set has a topological sorting.

(e) Every finite partially ordered set has a minimum element.

(f) Every finite totally ordered set has a maximum element.

(g) An infinite partially ordered set cannot have a maximum element.

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true

(e) Every finite partially ordered set has a minimum element.
false

(f) Every finite totally ordered set has a maximum element.
true if it is partially, then its false

(g) An infinite partially ordered set cannot have a maximum element.
false

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