# Appendix B:  Multimodal Mathematical function to optimize with GA

Choose **<u>only one</u>** function from the two multimodal families of functions (either from *Shubert* or *Shekel* family).

## *Shekel* family functions

<u>Definition</u>

$Shekel5, Shekel7, Shekel10$ functions: $f_{4,m}(x_1,...,x_4) = -\sum_{i=1}^{m} \left( \sum_{j=1}^{4} (x_j - a_{ij})^2 + c_i \right)^{-1}$ , $m = 5, 7, 10$.

The constants $c_i$ and $a_{ij}$ are given in Table 1.

<u>Properties</u>

$$f_{4,5}(4.00004, 4.00013, 4.00004, 4.00013) = -10.1532;$$

- Global minima: $f_{4,7}(4.00057, 4.00069, 3.99949, 3.99961) = -10.4029;$

$$f_{4,10}(4.00075, 4.00059, 3.99966, 3.99951) = -10.5364;$$

<u>Search domain</u>

- $D_4 = \left\{ (x_1,...,x_4) \in R^4 : -10 \le x_i \le 10, i = 1,...,4 \right\}$

*Table 1.* Data for *Shekel 5,7,10* functions.

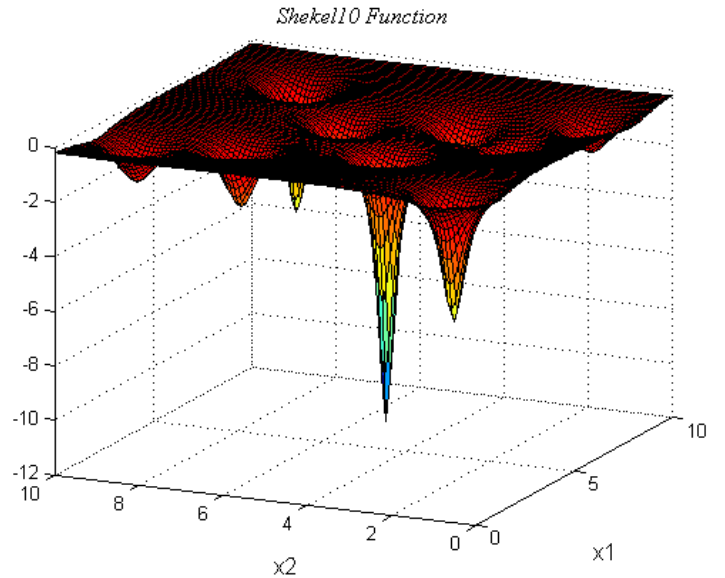| $i$ | $a_{ij}$ | | | | $c_i$ |
|---|---|---|---|---|---|
| 1 | 4.0 | 4.0 | 4.0 | 4.0 | 0.1 |
| 2 | 1.0 | 1.0 | 1.0 | 1.0 | 0.2 |
| 3 | 8.0 | 8.0 | 8.0 | 8.0 | 0.2 |
| 4 | 6.0 | 6.0 | 6.0 | 6.0 | 0.4 |
| 5 | 3.0 | 7.0 | 3.0 | 7.0 | 0.4 |
| 6 | 2.0 | 9.0 | 2.0 | 9.0 | 0.6 |
| 7 | 5.0 | 5.0 | 3.0 | 3.0 | 0.3 |
| 8 | 8.0 | 1.0 | 8.0 | 1.0 | 0.7 |
| 9 | 6.0 | 2.0 | 6.0 | 2.0 | 0.5 |
| 10 | 7.0 | 3.6 | 7.0 | 3.6 | 0.5 |



Fig.1. *Shekel10* function.

## References

Tfaili, W., Dréo, J. and Siarry, P., *"Fitting Of An Ant Colony Approach To Dynamic Optimization Through A New Set Of Test Functions"*, International Journal of Computational Intelligence Research, Vol. 3, No. 3 pp. 203-216, 2007.

Georgieva A. and I. Jordanov, *Hybrid Metaheuristics for Global Optimization Using Low-Discrepancy Sequences of Points*, Computers & Operations Research, vol. 37 (3), pp. 456-469, 2010.

Ali, M.M., Porn, C.K. and Zabinsky, Z.B., *"A Numerical Evaluation Of Several Stochastic Algorithms on Selected Continuous Global Optimization Test Problems"*, Journal of Global Optimization 31, pp. 635-672, 2005.

Chelouah, R. and Siarry, P., *"Genetic And Nelder-Mead Algorithms Hybridized For A More Accurate Global Optimization Of Continuous Multiminima Functions"*, European Journal of Operational Research 148, pp. 335-348, 2003.

## *Shubert* family functions

Definition

- *Shubert1*:
$$f_1(x_1,...,x_n) = \prod_{i=1}^{n}\left[\sum_{j=1}^{5} j\cos((j+1)x_i + j)\right];$$

- *Shubert2*:
$$f_2(x_1,...,x_n) = \prod_{i=1}^{n}\left[\sum_{j=1}^{5} j\cos((j+1)x_i + j)\right] + \frac{1}{2}\left((x_1 + 1.42513)^2 + (x_2 + 0.80032)^2\right);$$

- *Shubert3*:
$$f_3(x_1,...,x_n) = -\sum_{i=1}^{n}\left[\sum_{j=1}^{5} j\sin((j+1)x_i + j)\right].$$

Properties

- Number of local minima: 760 for $f_1$, many for $f_2$, and 400 for $f_3$.
- Number and value of the global minima:
  - ➢ 18 for *Shubert1*: $f_1^* = -186.730909$ (all 2D minimisers are given in Table 2);
  - ➢ 1 for *Shubert2*: $f_2^* = -186.730909$;
  - ➢ 9 for *Shubert3*: $f_3^* = -24.062499$.

Search domain

- $D_1 = \left\{(x_1, x_2) \in R^2 : -10 \le x_i \le 10, i = 1,2\right\}$, (n=2);

*Table 2.* Argument values for *Shubert1* global minima.

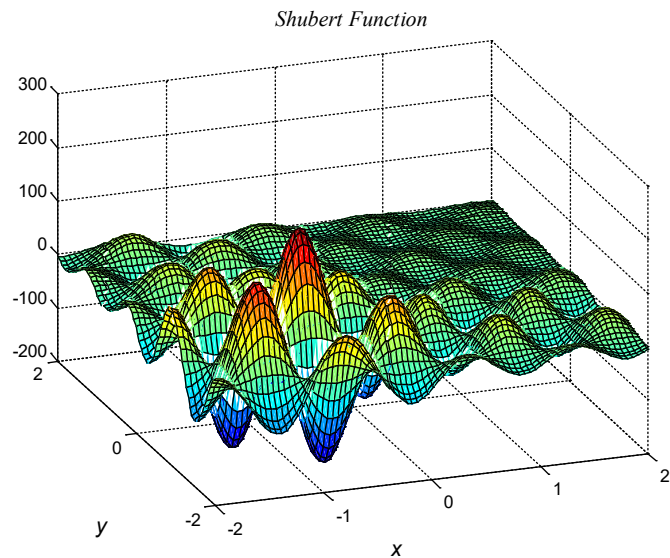| $x_1^*$ | $x_2^*$ |
|---------|---------|
| -7.0835 | 4.8580 |
| -7.0835 | -7.7083 |
| -1.4251 | -7.0835 |
| 5.4828 | 4.8580 |
| -1.4251 | -0.8003 |
| 4.8580 | 5.4828 |
| -7.7083 | -7.0835 |
| -7.0835 | -1.4251 |
| -7.7083 | -0.8003 |
| -7.7083 | 5.4828 |
| -0.8003 | -7.7083 |
| -0.8003 | -1.4251 |
| -0.8003 | 4.8580 |
| -1.4251 | 5.4828 |
| 5.4828 | -7.7083 |
| 4.8580 | -7.0835 |
| 5.4828 | -1.4251 |
| 4.8580 | -0.8003 |



Fig.2. *Shubert* function.

## References

Ali, M., Torn, C. and Zabinsky, Z., *"A Numerical Evaluation of Several Stochastic Algorithms on Selected Continuous Global Optimization Test Problems"*, Journal of Global Optimization 31, pp. 635-672, 2005.

Ali, M. and Törn, C., *"Population Set-Based Global Optimization Algorithms: Some Modifications and Numerical Studies"*, Computers & Operations Research 31, pp. 1703-1725, 2004.

Georgieva A. and I. Jordanov, *Hybrid Metaheuristics for Global Optimization Using Low-Discrepancy Sequences of Points*, Computers & Operations Research, vol. 37 (3), pp. 456-469, 2010