Gabarito P3

1) $S = \{(x,y,z,w) \in \mathbb{R}^4 \,/\, \begin{array}{l} x+2y-2z-w=0 \\ x-y+z+2w=0 \end{array} \}$

$v = (0,1,2,3)$.

$\cancel{x} + 2y - 2z - w = \cancel{x} - y + z + 2w \Rightarrow 3y = 3z + 3w$

$\boxed{y = z + w}$

$x = y - z - 2w = \cancel{z} + w - \cancel{z} - 2w = -w$

$S = \{(-w, z+w, z, w) \,/\, z, w \in \mathbb{R}\}$

~~Base~~ $S = \{ \underset{z=1,\, w=0}{(0,1,1,0)}, \underset{z=0,\, w=1}{(-1,1,0,1)} \} = \{v_1, v_2\}$

Base ortogonal de S.

$$\boxed{\{ \underbrace{(0,1,1,0)}_{w_1 = v_1}, \underbrace{(-2,1,-1,2)}_{w_2} \} = \text{Base ortog. de } S}$$

$$\tilde{w}_2 = v_2 - \frac{\langle v_1, v_2 \rangle}{\|v_1\|^2} v_1 =$$

$$= (-1,1,0,1) - \frac{1}{2}(0,1,1,0)$$

$$= (-1, 1/2, -1/2, 1).$$

$w_2 = 2\tilde{w}_2$ para simplificar a expressao.

Projeção ortogonal de $(0,1,2,3) = v$ em S

$$\text{proj}_S(v) = \frac{\langle v, w_1 \rangle}{\|w_1\|^2} w_1 + \frac{\langle v, w_2 \rangle}{\|w_2\|^2} w_2$$

$$= \frac{3}{2} w_1 + \frac{5}{10} w_2 = \frac{3}{2} w_1 + \frac{1}{2} w_2$$

$$\boxed{\text{Proj}_S(v) = (-1, 2, 1, 1)}$$

2) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ $\quad P_A(x) \overset{\det}{=} \begin{bmatrix} 1-x & 0 & 0 \\ 2 & 1-x & 0 \\ 3 & 0 & 2-x \end{bmatrix}$

A matriz é triangular o determinante é o produto da diagonal! $\boxed{P_A(x) = (1-x)^2(2-x)}$

Autovalores $+1$ com multiplicidade alg $= 2$.
$+2$ " " alg $= 1$.

Autoespaço assoc. ao autovalor 1:

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{cases} 2x = 0 \Rightarrow x = 0 \\ 3x + z = 0 \Rightarrow z = 0 \end{cases}$$

$x = 1$

$\text{Aut}(1) = \{(0, y, 0) \mid y \in \mathbb{R}\}$ $\boxed{m_g(1) = 1}$

A não é diagonalizável pois

$$\boxed{m_a(1) = 2 > m_g(1) = 1.}$$

3) $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ Autovalores -2 e 4

- $Aut(-2) = \{(2y - z, y, z) \,/\, y, z \in \mathbb{R}\}$

Base $\{\underset{y=1 \;\; z=0}{(2, 1, 0)}, \underset{y=0 \;\; z=1}{(-1, 0, 1)}\}$

- $Aut(4) = \{(x, 0, x) \,/\, x \in \mathbb{R}\}$

Base $= \{(1, 0, 1)\}$

Temos que $B = \{(2,1,0), (-1,0,1), (1,0,1)\}$

é base de $\mathbb{R}^3$ e

$$T\Big]_B^B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = D$$

$$P = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = I\Big]^B_{\underbrace{Bc}_{\text{Base canônica}}}$$

Assim $T\Big]^{Bc}_{Bc} = P \, D \, P^{-1}$

$$= I\Big]^B_{Bc} \; T\Big]^B_B \; I\Big]^{Bc}_B$$

$$T\Big]^{Bc}_{Bc} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -1/2 & 1 & 1/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}}_{P^{-1}}$$

$$\boxed{T\Big]^{Bc}_{Bc} = \begin{bmatrix} -1 & -6 & 3 \\ 0 & -2 & 0 \\ 3 & -6 & 1 \end{bmatrix}}$$

(4) Cônica $10x^2+10y^2+12xy+8\sqrt{2}\,x+24\sqrt{2}\,y-32=0$

Dividindo por 2

$$5x^2+5y^2+6xy+4\sqrt{2}\,x+12\sqrt{2}\,y-16=0.$$

Forma Quadrática $\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} = Q$

$P_Q(x) = (x-5)^2-9 = x^2-10x+16$

Autovalores 8 e 2.

$\lambda = 8$ $\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$Aut(8) = \{(x,y) \,/\, x-y=0\} = \{(x,x) \,/\, x\in\mathbb{R}\}$

Base ortonormal $\{(1/\sqrt{2},\ 1/\sqrt{2})\}$

$\lambda = 2$ $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$Aut(2) = \{(-x,\ x) \,/\ x\in\mathbb{R}\}$

Base ortonormal $\{(-1/\sqrt{2},\ +1/\sqrt{2})\}$
de $Aut(2)$

$$\boxed{P = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$$ Matriz da Rotação

$$\begin{bmatrix} 4\sqrt{2} & 12\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 16 & 8 \end{bmatrix}$$

$8x'^2 + 2y'^2 + 16x' + 8y' - 16 = 0$

$4x'^2 + y'^2 + 8x' + 4y' - 8 = 0$ ← Dividindo por 2 para simplificar

Completando quadrados:

$$4(x'+1)^2 + (y'+2)^2 = 16.$$

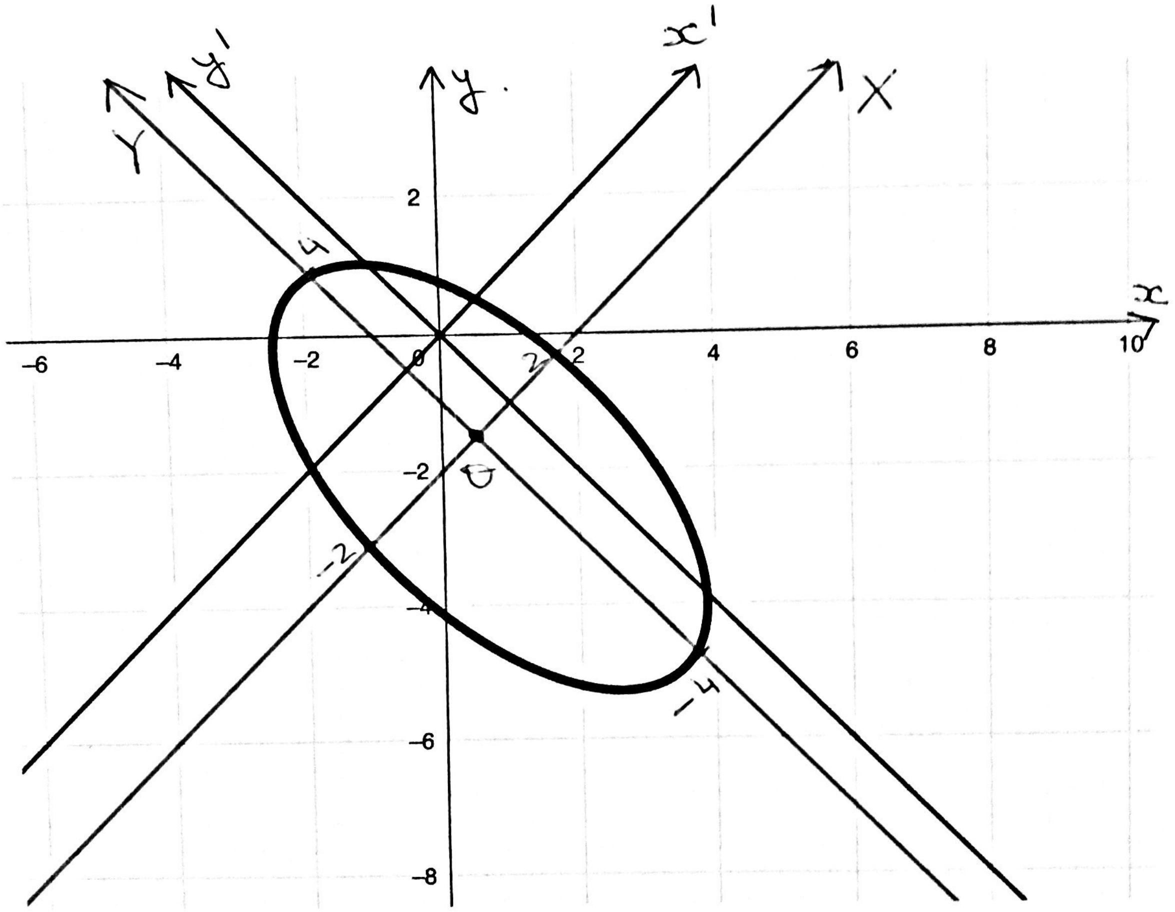
$$\frac{(x'+1)^2}{4} + \frac{(y'+2)^2}{16} = 1.$$

Nova origem $(-1, -2)$.

Equação Reduzida

$$\boxed{\frac{X^2}{2^2} + \frac{Y^2}{4^2} = 1.}$$

A cônica é uma Elipse



$$\frac{X^2}{2^2} + \frac{Y^2}{4^2} = 1.$$