1]

1.1] (a) Sample space = $\{HH, HT, TH, TT\}$

(b) Event space = $\{\phi, \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\},$
$\{HH, TT\}, \{HT, TH\}, \{HT, TT\}, \{TH, TT\}, \{HH, HT, TH\},$
$\{HH, HT, TT\}, \{HH, TH, TT\}, \{HT, TH, TT\},$
$\{HH, HT, TH, TT\}\}$

Event space is power set of sample space.

(c) (i) probability of each

$\Omega = \{HH, TH, HT, TT\}$

$P(\Omega) = 1$

$P(HH \cup TH \cup HT \cup TT) = 1$

since $\{HH\}, \{TH\}, \{HT\}, \{TT\}$ are mutually exclusive

& $P(HH) = P(TH) = P(HT) = P(TT)$

$4 \cdot P(HH) = 1$

probability of each elementary event $= 1/4$

(ii) probability of atleast one head $= P(\{HH\} \cup \{HT\} \cup \{TH\})$

~~since they~~

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$

(iii) probability of exactly one head $= P(\{HT\} \cup \{TH\})$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

2]

2.1]

$$f(k,n,p) = \frac{n!}{k!\,(n-k)!} p^{k} (1-p)^{n-k}$$

Probability of correctly recognizing a word $(p) = 0.9$

$$P(\text{recognizing } 45 \text{ out of } 50 \text{ words}) = f(45, 50, 0.9)$$

$$= \frac{50!}{45!\,5!} (0.9)^{45} (0.1)^{5}$$

2.2)

$$f(k,\lambda) = \frac{\lambda^{k} e^{-\lambda}}{k!} \quad ; \quad \lambda = 10$$

(a) probability that zero road accidents in that day ⇒

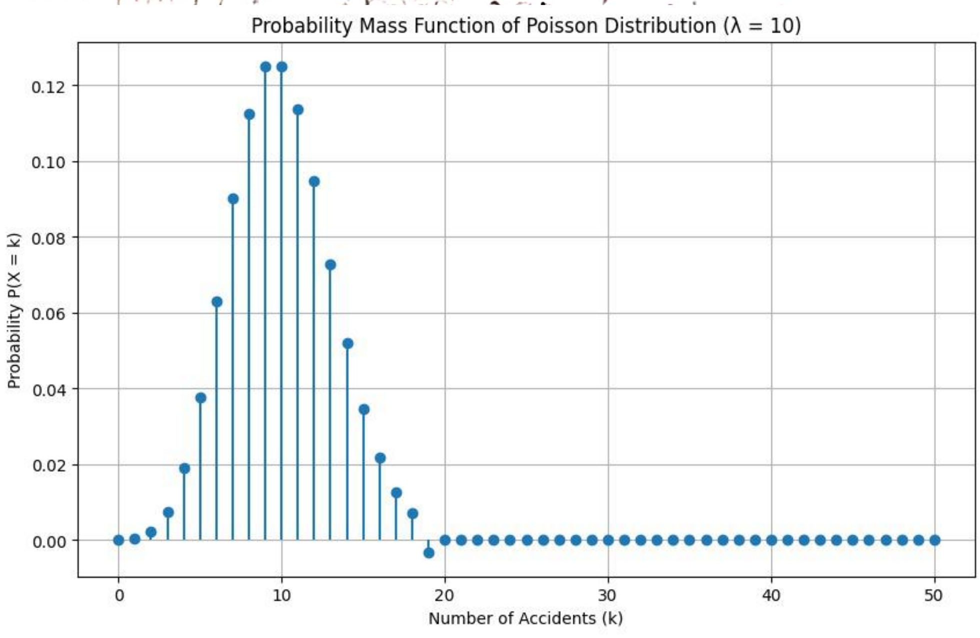
$$f(0,\lambda) = \frac{10^{0} e^{-10}}{0!} = e^{-10}$$

(b) probability of occurence of >7 & <10 road accidents in a day ⇒ $f(8,\lambda) + f(9,\lambda)$

$$= \frac{10^{8} e^{-10}}{8!} + \frac{10^{9} e^{-10}}{9!}$$

$$= e^{-10}\left[\frac{10^{8}}{8!} + \frac{10^{9}}{9!}\right]$$

[c]



3.1) P.D.F for normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

(a) P.D.F at $x=0$; $\mu=1$, $\sigma=1$

$$p = F(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$$ //

(b) P.D.F at $x=1$, $\mu=0$, $\sigma=1$

$$p = F(1) = \frac{1}{\sqrt{2\pi}} e^{-1/2}$$ //

(c) $p(x_1 \le X \le x_2) = 0.3$

$p(x_1 \le X \le x_3) = 0.45$

$p(x_2 \le X \le x_3) = ?$

$p(x_1 \le X \le x_3) = p(x_1 \le X \le x_2) + p(x_2 \le X \le x_3)$

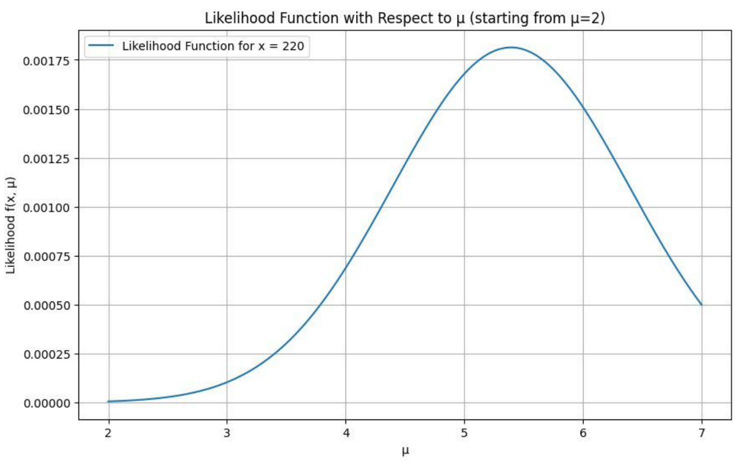
$0.45 = 0.3 + p(x_2 \le X \le x_3)$

$p(x_2 \le X \le x_3) = 0.15$ //

4)

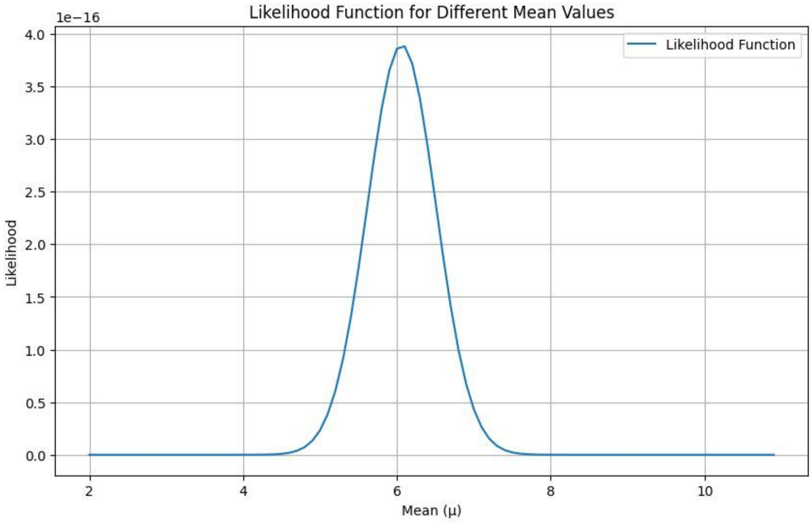
4.1) $f(x,\mu) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2}}$

(a) $L(\mu|x) = f(x,\mu) = f(220,\mu) = \frac{1}{220\sqrt{2\pi}} e^{-\frac{(\log 220 - \mu)^2}{2}}$



(b) $X = [3303.25, 443, 220, 560, 880]$

$$L(\mu \mid x) = f(x, \mu) = \cancel{\prod_{i=1}^{5}} \frac{1}{\left(\prod_{i=1}^{5} x_i\right)\left(\sqrt{2\pi}\right)^5} e^{-\frac{\left(\sum_{i=1}^{5} (\log x_i - \mu)^2\right)}{2}}$$



(c) from the above graph, Likelihood is max at $\mu = 6$