(3) Rough

$M_1: \quad Y_i | N_i \sim P(N_i \lambda_i)$

$M_2: \quad Y_i | \lambda_0 \sim P(N_i \lambda_0)$

$$BF_{2,1} = \frac{P(Y_i | M_2)}{P(Y_i | M_1)}$$

$$P(Y_i | M_1) = \int P(Y_i . \lambda_i) d\lambda_i$$

$$= \int \int \cancel{P(Y_1 | \lambda_1) d\lambda_1 . P(Y_2 | \lambda_2)} d\lambda_2$$

$$= \int \int P(Y_1 | \lambda_1) P(\lambda_1) d\lambda_1 . P(Y_2 | \lambda_2) P(\lambda_2) d\lambda_2$$

$$= \int_0^c \frac{e^{-N_1 \lambda_1} (N_1 \lambda_1)^{Y_1}}{Y_1!} \frac{1}{c} d\lambda_1 \cdot \int_0^c \frac{e^{-N_2 \lambda_2} (N_2 \lambda_2)^{Y_2}}{Y_2!} \frac{1}{c} d\lambda_2$$

$$= \frac{1}{c^2} \int_0^c \frac{e^{-N_1 \lambda_1} N_1^{Y_1} \lambda_1^{Y_1} d\lambda_1}{Y_1!} \int_0^c \frac{e^{-N_2 \lambda_2} N_2^{Y_2} \lambda_2^{Y_2} d\lambda_2}{Y_2!}$$

$$= \frac{N_1^{Y_1} N_2^{Y_2}}{c^2 \, Y_1! \, Y_2!} \int_0^c e^{-N_1 \lambda_1} \lambda_1^{Y_1} d\lambda_1 \int_0^c e^{-N_2 \lambda_2} \lambda_2^{Y_2} d\lambda_2$$

$$p(Y_i \mid M_2) = \int p(Y_i, \lambda_0)\, d\lambda_0$$

$$= \frac{1}{c} \int p(Y_1, Y_2 \mid \lambda_0)\, p(\lambda_0)\, d\lambda_0$$

$$= \frac{1}{c} \int_0^c p(Y_1 \mid \lambda_0)\, p(Y_2 \mid \lambda_0)\, \cancel{p(\lambda_0)}\, d\lambda_0$$

$$= \frac{1}{c} \int_0^c \frac{e^{-N_1\lambda_0} N_1^{Y_1} \lambda_0^{Y_1}}{Y_1!} \; \frac{e^{-N_2\lambda_0} N_2^{Y_2} \lambda_0^{Y_2}}{Y_2!}\, d\lambda_0$$

$$= \frac{1}{c} \frac{N_1^{Y_1} N_2^{Y_2}}{Y_1!\, Y_2!} \int_0^c e^{-(N_1+N_2)\lambda_0} \lambda_0^{Y_1+Y_2}\, d\lambda_0$$

$$BF_{2,1} = \frac{c \int_0^c e^{-(N_1+N_2)\lambda_0} \lambda_0^{Y_1+Y_2}\, d\lambda_0}{\int_0^c e^{-N_1\lambda_1} \lambda_1^{Y_1}\, d\lambda_1 \int_0^c e^{-N_2\lambda_2} \lambda_2^{Y_2}\, d\lambda_2}$$

$$= \frac{c \;\text{const} \int_0^c \text{Gamma}(Y_1+Y_2+1, N_1+N_2)}{\int_0^c \text{Gamma}(Y_1+1, N_1) \int_0^c \text{Gamma}(Y_2+1, N_2)}$$

$$\text{const} = \frac{\Gamma(Y_1+Y_2+1)\, N_1^{Y_1+1}\, N_2^{Y_2+1}}{(N_1+N_2)^{Y_1+Y_2+1}\, \Gamma(Y_1+1)\, \Gamma(Y_2+1)}$$