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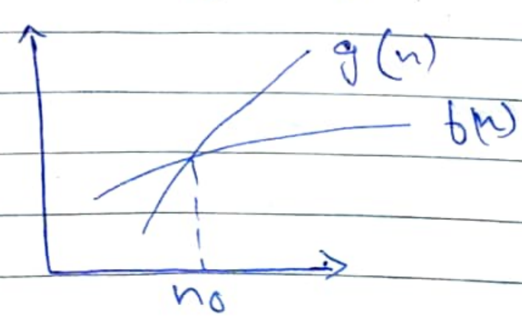
Assignment-1

Ans-1) Asymptotic notations are the mathematical notations used to describe the running time of an algo when the i/p tends towards a particular value or a limiting value.
There are mainly 3 asymptotic notation.

a) **Big-O notation** =>
It represents the upper bound of running time of an algo. This notation is called as upper bound of the algo or a worst case of an algo.
$O(g(n)) = \{ f(n)$ there exist positive constants c & n_0 such that $0 < f(n) \leq cg(n)$ for all $n \geq n_0$, where $c > 0$ & $n \geq n_0$

e.g,

$$f(n) = 3\log n + 100$$
$$g(n) = \log n.$$
$$3\log n + 100 <= c * \log(n)$$
$$c = 1 < 0 \text{ \& } n > 2$$
(undefined at $n=1$)



b) Big omega (Ω) notation =)

It represents the lower bound of the running time of an algo. This notation is known as low-bound of an algo, or best case of an algo.

$\Omega(g(n)) = \{ f(n)$: there exist (+)ve constant c & n_0 such that $0 \le c\,g(n) \le f(n)$ $\forall n, n \ge n_0$.

e.g.,

$f(n) = 3n+2$

$c\,g(n) \le f(n)$

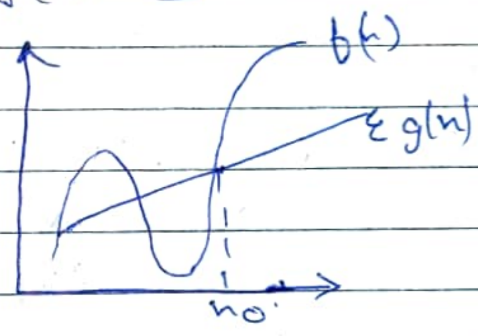
$[c = \text{constant}, g(n) = n]$

$cn \le 3n+2$

$cn - 3n \le 2$

$n(c-3) \le 2$

$n \le \frac{2}{c-3}$



if we assume $c=4$, then $n_0 = 2$, $c=4$, $n_0 = 2$.

c) Theta (Θ) notation =)

It enclose the function from above & below. Since, it represents the upper & lower bound of running time of an algo. This is known as tight bounds of an algo, or an average case of algo.

$\Theta(g(n)) = \{ f(n)$: there exid positive const c_1, c_2 & n_0 such that
$0 \le c_1 * g(n) \le f(n) \le c_2 * g(n) \; \forall n > n_0$

e.g.,

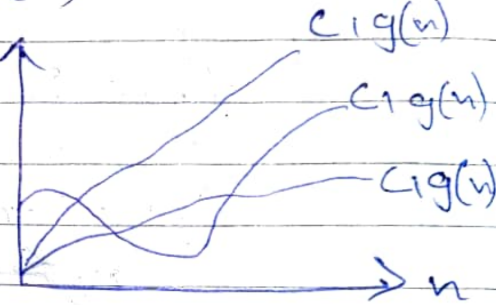
$f(n) = 5n^3 + 16n^2 + 3n + 8$

$5n^3 \le (n^3 + 16n^2 + 3n + 8$

$5n^3 \le f(n) \le 32n^3$

$c_1 = 5, \quad c_2 = 32, \quad n_0 = 1$

$f(n) \Longleftrightarrow \Theta(n^3)$



Ans) 2 $\quad i = 2, 4, 8, 16, \ldots$ k^{th} term. n

$Gn = an^{n-1}$

$Chn = 1\,(2)^{k-1}$

$n = 2^{k-1}$

$\log_2 n = (k-1)\log_2 2$

$k = \log_2 n + 1$

$O(n) = \log n$

Ans-3 $\quad T(n) = 3T(n-1)$

$T(n-1) = 3T(n-2)$

$T(n) = 3 \times 3T(n-2) \quad T(n-2) = 3T(n-3)$

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$T(n) = 3 \times 3 \times 3T(n-3)$
$T(n) = 3^3 T(n-3)$
$T(n-3) = 3T(n-4)$
$T(n) = 3^3 \times 3T(n-4)$
$T(n) = 3^4 \times T(n-4)$
⋮

General form -

$T(n) = 3T(n-i) \ldots (i)$ $[T(0)=1]$
$T(n-i) = T(0)$
$n-i = 0$
$n = i$

Putting $n=i$ in eq (i) ;
$T(n) = 3^n T(n-n)$
$T(n) = 3^n T(0)$ $[T(0)=1]$
$T(n) = 3^n$
$T(n) = O(3^n)$

Ans-4 $T(n) = 2T(n-1)-1$
$T(n-1) = 2T(n-2)-1$
$T(n) = 2 \times (2T(n-2)-1)-1$
$T(n) = 2^2 T(n-2)-2-1$
$T(n-2) = 2T(n-3)-1$

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$T(n) = 2^2(2T(n-3)-1) - 2 - 1$
$T(n) = 2^3 \; T(n-3) - 2^2 - 2 - 1$
$T(n-3) = 2T(n-4) - 1$
$T(n) = 2^3 (2T(n-4) - 1) - 2^2 - 2 - 1$
$T(n) = 2^4 \; T(n-4) - 2^3 - 2^2 - 2 - 1$

$\vdots$

General form -
$T(n) = 2^i T(n-i) - (2^{i-1} + 3^{i-2} + \ldots + 1)$

$T(n-i) = T(0)$
$n - i = 0$
$n = i$

$T(n) = 2^n \, T(0) - (1 + 2 + 2^2 + 2^3 + \ldots 2^{n-1})$

$[T(0) = 1]$

$T(n) = 2^n (1) - (1 + 2 + 2^2 + \ldots 2^{n-1})$
$T_n = 2^n - 1 \dfrac{(2^{n-1} - 1)}{2 - 1}$

$T(n) = 2^n - 2^{n-1} + 1$
$T(n) = 2^{n-1}(2-1) + 1$
$T(n) = 2^{n-1} + 1$
$T(n) = O(2^n)$

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Ans-25

No. of steps (k)	s	i
0	0	1
1	1	2
2	3	3
3	6	4
4	10	5
5	15	6
6	21	7
⋮	⋮	⋮
⋮	⋮	⋮
k step	n	

$T(n) = O(k)$

$= 0, 1, 3, 6, 10, \ldots, n$

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots + n$

$S_n \quad 1 + 3 + 6 + 10 + \ldots + (n-1) + n$

$- \quad - \quad - \quad - \quad - \quad - \quad -$

$0 = 1 + 2 + 3 + 4 + 5 + \ldots - n$

$n = 1 + 2 + 3 + 4 + \ldots k \text{ step}$

$n = \frac{k}{2}\left[2(1) + (k-1)1\right]$

$2n = k\left[2 + k - 1\right]$

$2n = k^2 + k$

$2n = \left(k + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)$

$2n + \left(\frac{1}{2}\right)^2$

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$$\left(k+\frac{1}{2}\right)^2$$

$$k+\frac{1}{2}=\sqrt{2n+(1/2)^2}$$

$$k=\sqrt{2n+(1/2)^2}$$

$$T(n)=T(k)$$
$$T(n)=T\left(\sqrt{2n+(1/2)^2}\right)-1/2)$$
$$T(n)=O\sqrt{n}$$

Ans-16 Since, i is moving from 1 to $\sqrt{n}$ with linear growth So

$$T(n)=O(\sqrt{n})$$

Ans-17 $O(n \log n \log n)$

$$O(n(\log n)^2$$

Ans-18

$$T(n)=T(n-1)+n^2$$
$$T(n)=T(n-2)+n^2+(n-1)^2$$
$$T(n)=T(n-3)+n^2+(n-1)^2+(n-2)^2$$
$$\vdots$$

General Term-

$$T(n)=T(n-i)+n^2+(n-1)^2+(n-2)^2+\ldots(n-i)^2$$
$$T(n-i)=T(1)$$
$$n=i+1$$
$$n-1=i$$

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$T(n) = T(n-(n-1)) + n^3 + (n-1)^2 + (n-2)^2 + \dots + (n-(n-1))^2$

$T(n) = T(1) = n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2$

$T(n) = 1 + 1^2 + 2^2 + 3^2 + \dots + n^2$

$T(n) = \frac{n(n+1)(2n+1)}{6}$

$T(n) = O(n^3)$

Ans-)9 $O(n\sqrt{n})$

Ans-)10 If $c > 1$ then the exponential c^n for outgrows any term, so that answer is:
n^k is $O(c^n)$

Ans-)11 $i = 0, 1, 3, 6, 10, 15, \dots$
$j = 0, 1, 2, 3, 4, 5, 6, \dots$
So, i will go on till n & general formula for k^{th} term is $n = \frac{k(k+1)}{2}$
$\therefore$ T. C $= O\sqrt{n}$

Ans-)12 $T(n) = T(n-1) + T(n-2) + c$
$T(n-2 \approx T(n-1)$
$T(n) = 2T(n-1) + c$
$T(n-1) = 2T(n-2) + c$
$T(n) = 2(2T(n-2) + c) + c$
$T(n) = 2^2T(n-2) + 2c + c$
$T(n-2) = 2T(n-3) + c$

$$T(n) = 2^3 \left(2T(n-3) + b\right) + 2c + c$$
$$T(n) = 2^3 T(n-3) + 2^2 c + 2c + c$$
$$\vdots$$

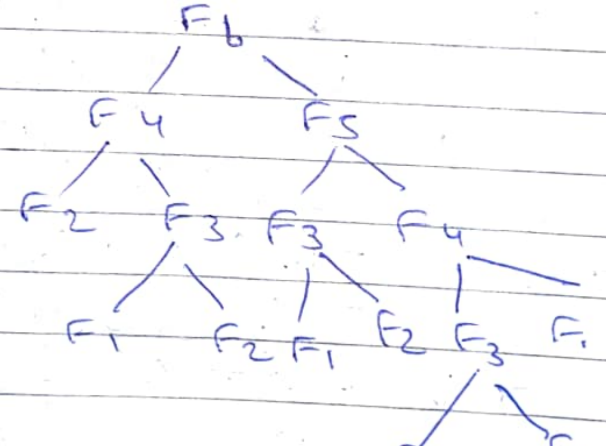
General Term :-

$$T(n) = 2^i T(n-i) + (2^0 + 2^1 + 2^2 + \dots + 2^{i-1})c$$

$$n - i = 0$$
$$n = i$$

$$T(n) = 2^n T(0) + (2^0 + 2^1 + 2^2 + \dots + 2^{n-1})c$$
$$T(n) = 2^n (1) + 2^0 \left(\frac{2^{n-1} - 1}{2 - 1}\right) c$$
$$T(n) = 2^n (1 + c) - c$$
$$T(n) = O(2^n)$$

fig(6)



The max depth is proportional to N, Hence the space complexity of fibonacci recursive is O(n).

Ans-) 13

```
void fun()
{
    int i, j;
    for(i=1 ; i<=n; i++)
    {
        for (j=0; j<=n , j= j*2)
        printf ("*");
        printf ("\n");
    }
}
```

(b)

```
void fun (int n)
{
    int i, j, k;
    for (i=0 ; i<=n; i++)
    {
      for (j=0; j<=n; j++)
      {
        for (k=0; K<=n ;k++)
        printf ("*");
      }
    }
}
```

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(c)

```
void Sieve of Endosthenes (int n)
{
    bool prime [n+1];
    memset (prime, true, size of (prime));
    for (int p=2; p*p <=n; p++)
    {
        if (prime [p] = true)
        {
            for (int i = p*p ; i<=n; i+=p)
            prime [i] = false;
        }
    }
    for (int p=2 ; p <=n; p++)
    if (prime [p])
        cout <<p << endl;
}
```

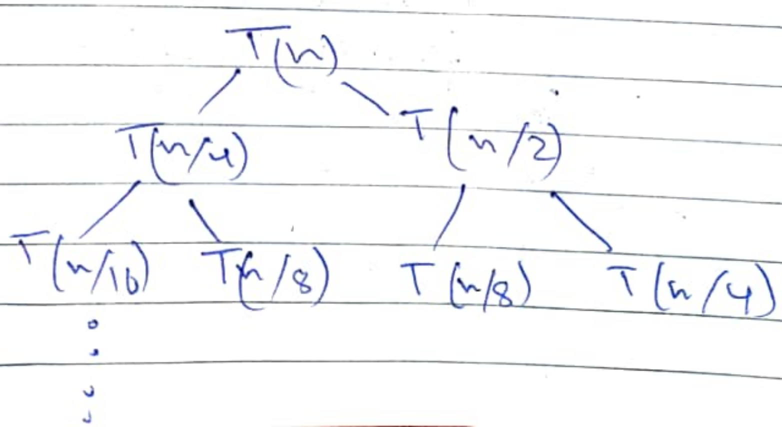
Ans-14

$T(1) = C$

$n = n/2$

$T(n/2) = T(n/8) + T(n/4) + c(n^2/4)$

$T(n) = T(n/16) + 2T(n/8) + c(n^2/16) + n^2/4 + n^2)$



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$$T(n) = c\left[n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \cdots\right]$$

$$T(n) = n^2 c\left[1 + \frac{5}{16} + \frac{5^2}{16^2} + \cdots\right]$$

$$T(n) = O(n^2)$$

Ans) 15 For $i=1$, inner loop is executed n times.

For $i=2$, inner loop is executed $n/2$ times.
For $i=3$, inner loop is executed $n/3$ times.
⋮
For $i=n$, inner loop is executed n/n times

$$\text{Total time} = n + n/2 + n/3 + \cdots n/2$$
$$= n(1 + 1/2 + 1/3 + \cdots 1/n)$$
$$= n \log n$$
$$T(n) = O(n \log n)$$

Ans) 16 $O(\log(\log n))$

Ans) 18 (a) 100, log log n, log n, root n

$n, n\log n, n^2, 2^n +, 2^{2n}, \& 4^n, n!$

(b) 1, log (log(n)), $\sqrt{\log n}$, log n, log (2n), log (n!), 2 log (n), n, 2n, 4n, n, log(n!), n^2, $2(2^n)$, n!

(c) 96, $\log_4 n$, $\log_2 n$, log (n!), 5 n, n log n, $n \log_2 n$, $8n^2$, $7n^3$. 8^{2n}. n!

Ans 19 Linear Search (A, key)

```
Linear Search (A, key)
    comp ← 0 , F ← 0
for i = 1 to A length
    comp ← comp + 1
    if A[i] = key
    print "Element found"
     F = 1
    if F == 0
    print "Element not found"
    print comp
```

Ans 20 Iterative method of insertion sort

```
INSERTION_SORT (A)
for j = 2 to A length
      key = A[j]
      i = j - 1
  while i > 0 and A[i] = key
        A[i+1] = A[i]
          i = i - 1
       A[i+1] = key.
```

Recursive Method of insertion sort.

```
INSERTION_SORT (A, n)
  if n ≤ 1
  return
  INSERTION_SORT (A, n-1)
   key = Arr [n-1];
       j = n-2;
    while j ≥ 0 and A[j] > key
     A [j+1] = A [j].
          j = j-1
        A [j+1] = key
```

Insertion sort consider one input element per iteration & prodecure a partial solution without considering future elements.

Other sorting algorithm that can be discussed in lectures all.

Bubble sorting
Selection sorting
Quick Sort
Merge sort
Heap sort
Counting sort

Ans-21

	Best Case	Average Case	Worst Case
Bubble sort	$\Omega(N)$	$\theta(N^2)$	$O(N^2)$
Selection sort	$\Omega(N^2)$	$\theta(N^2)$	$O(N^2)$
Insertion sort	$\Omega(N)$	$\theta(N^2)$	$O(N^2)$
Merge sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N\log N)$
Heap sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N\log N)$
Quick sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N^2)$
Counting sort	$\Omega(N+K)$	$\theta(N+K)$	$O(N+K)$

Ans-22

	In Place	Stable	Online
Bubble sort	Yes	Yes	Yes
Insertion sort	Yes	Yes	Yes
Selection sort	Yes	No	Yes
Merge sort	No	Yes	Yes
Quick sort	Yes	No	Yes
Heap sort	Yes	No	Yes
Count sort	No	Yes	Yes

Ans-23 Linear Search

```
Linear_SEARCH (A, key)
found ← 0
   for { i = 1 to N
   if A[i] = = key
         found ← 1
```

print "Element found"
break
if found ==0
print "Element not found"
Time complexity - O(n)
Space Complexity - O(1)

Binary Search (Iterative)

```
BINARY_SEARCH (A, beg, end, key)
  while beg <= end
     mid = beg + (end - beg)/2
     if mid == key
        return mid
     if A[mid] < key
        beg = mid+1
     if A[mid] > key
        end = mid-1
  return -1
```

Time complexity - $O(\log_2 n)$
Space complexity - $O(1)$

Binary Search (Recursion)

```
BINARY_SEARCH (A, beg, end, key)
   if end >= beg
      mid = (beg + end)/2
      if A[mid] == item
```

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```
        return mid+1
else if A[mid] < item
return BINARY_SEARCH (A, mid+1,
                      end, key)
else
    return BINARY_SEARCH (A, beg,
                      mid-1, end)
    return -1
```

Time complexity - $\Theta(\log n)$
Space complexity - $O(1)$.

Ans-24 $T(n) = T(n/2) + c$