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Linear Algebra
Assignment 1

Q(1) $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 10 & -24 & 4 \\ 0 & 25 & -61 & 22 \end{array}\right]$$

$R_2 \rightarrow \frac{R_2}{10}$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 1 & -12/5 & 2/5 \\ 0 & 25 & -61 & 22 \end{array}\right]$$

$R_3 \rightarrow R_3 - 25R_2$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 1 & -12/5 & 2/5 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

$R_3 \rightarrow \frac{R_3}{-1}$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 1 & -12/5 & 2/5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Using backward Substitution,

$z = 3$

Using $z = 3$,

$y - \frac{12}{5} \times 3 = \frac{2}{5} \Rightarrow y = \frac{42}{5}$

With the help of y, & z,

$2x - 3\left(\frac{42}{5}\right) + 7(3) = 5$

$\Rightarrow \quad x = \frac{23}{5}$

$\therefore x = \frac{23}{5}, \; y = \frac{42}{5}, \; z = 3.$

(ii) $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$\cancel{R_2 \to 2R_2 + R_1}$

$\cancel{R_3 \to R_3 + 3R_2}$

$$\cancel{\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 7 & -1 & 12 \end{array}\right]}$$

$\cancel{R_2 \to \frac{R_2}{3}}$

$$\cancel{\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 1 & 5/3 & \frac{16}{3} \\ 0 & 7 & -1 & 12 \end{array}\right]}$$

$\cancel{R_3 \to R_3 - 7R_2}$

$$\cancel{\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 1 & 5/3 & 16/3 \\ 0 & 0 & -38/3 & \end{array}\right]}$$

$R_2 \to R_2 + \frac{1}{2} R_1$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 7/2 & 12 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_3 \to R_3 - \frac{3}{2} R_1$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 7/2 & 12 \\ 0 & 5/2 & -13/2 & 0 \end{array}\right]$$

$R_2 \to \frac{2}{3} R_2$

$$\left[\begin{array}{cccc|c} 2 & -1 & 3 & 1 & 8 \\ 0 & 1 & 7/3 & 1 & 8 \\ 0 & 5/2 & -13/2 & 1 & -24 \end{array}\right]$$

$R_3 \to R_3 - \frac{5}{2} R_2$:

$$\left[\begin{array}{cccc|c} 2 & -1 & 3 & 1 & 8 \\ 0 & 1 & 7/3 & 1 & 8 \\ 0 & 0 & -58/3 & 1 & -56 \end{array}\right]$$

Since last row represent the equation $0 = -n$ which is not possible. Therefore system is inconsistent.

(3) $4x - y = 12$, $-x + 5y - 2z = 0$, $-2x + 4z = -8$

$$\left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

$R_1 \to \frac{R_1}{4}$

$$\left[\begin{array}{ccc|c} 1 & -1/4 & 0 & 3 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & -1/4 & 0 & 3 \\ 0 & 21/4 & -2 & 3 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

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$R_3 \to R_3 + 2R_2$
$$\left[\begin{array}{ccc:c} 1 & -1/4 & 0 & 3 \\ 0 & 21/4 & -2 & 3 \\ 0 & -1/2 & 4 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -1/4 & 0 & 3 \\ 0 & 1 & -8/21 & 4/7 \\ 0 & -1/2 & 4 & -2 \end{array}\right] \to R_3 \to \frac{R_3}{21/4}$$

$R_3 \to R_3 + \frac{R_2}{2}$

$$\left[\begin{array}{ccc:c} 1 & -1/4 & 0 & 3 \\ 0 & 1 & -8/21 & 4/7 \\ 0 & 0 & 76/21 & 2/7 \end{array}\right]$$

$R_3 \to \frac{R_3}{76/21}$

$$\left[\begin{array}{ccc:c} 1 & -1/4 & 0 & 3 \\ 0 & 1 & -8/21 & 4/7 \\ 0 & 0 & 1 & 14/38 \end{array}\right]$$

from the last row,

$z = \frac{14}{38}$

$y = \frac{112}{133}$

Sion, $y = \frac{112}{133}$ & $z = \frac{14}{38}$ in equ-(i)

$x = \frac{423}{133}$

$\therefore$ The system is consistent & solution is unique

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(b) for what value of λ & μ the given system of equation $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has (i) no solution (ii) a unique solution & (iii) infinite number of solution.

:-)

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

(i) $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

for Unique solution

$a-3 \neq 0 \Rightarrow a \neq 3 \quad b \in R$

for No solution,

$a-3=0$ & $b-10 \neq 0$

$\Rightarrow a=3$ and $b \neq 10$

for infinitely many solution

$a-3=0$ & $b-10=0$

$\Rightarrow a=3$ and $b=10$

(c) Find for what values of λ the given equations $x+y+z=\lambda$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution & solve them completely in each case.

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & a \\ 1 & 4 & 10 & : & a^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & a-1 \\ 1 & 4 & 10 & : & a^2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & a-1 \\ 0 & 3 & 9 & : & a^2-1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & a-1 \\ 0 & 0 & 0 & : & a^2-3(a-1)-1 \end{bmatrix}$$

$a^2 - 3(a-1) - 1 = 0 \Rightarrow (a-2)(a-1) = 0$

$\Rightarrow a = 2$ or $a = 1$

Case 1 – $a = 1$

$x + y + z = 1$

$x + 2y + 4z = 1$

$x + 4y + 10z = 1$

$x+y+z=1$

$y+3z=0$

let $z=k$

$y+3k=0$

$y=-3k$

$x-3k+k=1$

$x=1+2k$

$\therefore \quad x=1+2k$

$y=-3k$

$z=k.$

Case-2 $\quad a=2$

$x+y+z=1$

$x+2y+4z=2$

$x+4y+10z=4$

$x+y+z=1$

$y+3z=1$

let $z=k$

$y=1-3k.$

$x+1-3k+k=1$

$x=2k$

$x=2k$

$y=1-3k$

$z=k.$

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(d) Find the solution of the system of equations
$x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$

⇒
$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$$

$R_3 \to R_3 + 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

from the above

$-7y + 8z = 0 \Rightarrow y = \frac{8}{7}z$

So the system has infinitely many solution

Let $z = t$. where $t \in R$

so $x = t$, $z = t$, & $y = \frac{8}{7}t$.

(e) Find for what values of λ the given equation $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may posses non - trivial solution & solve them simplify in each case.

$$\begin{bmatrix} 3 & 1 & -\lambda & : & 0 \\ 4 & -2 & 0 & : & 0 \\ 2\lambda & 4 & \lambda & : & 0 \end{bmatrix}$$

$\Rightarrow$
$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 & : & 0 \\ 4 & -2 & 0 & : & 0 \\ 2\lambda & 4 & \lambda & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 & : & 0 \\ 0 & -14/3 & 4\lambda/3 & : & 0 \\ 2\lambda & 4 & \lambda & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 & : & 0 \\ 0 & -14/3 & 4\lambda/3 & : & 0 \\ 0 & 10/3 & 4\lambda^2/3 & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 & : & 0 \\ 0 & 1 & -2\lambda/7 & : & 0 \\ 0 & 10/3 & \frac{4\lambda^2}{3} & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 & : & 0 \\ 0 & 1 & -2\lambda/7 & : & 0 \\ 0 & 0 & \frac{20\lambda^2}{21} & : & 0 \end{bmatrix}$$

$\therefore$ $\frac{20\lambda^2}{21}=0 \Rightarrow \lambda=0$

$\therefore$ Substitute $\lambda=0$ into the equations.

$x+\frac{1}{3}y=0 \Rightarrow x=-\frac{2}{21}z$ & $z\in R$

$y-\frac{2}{7}z=0 \Rightarrow y=\frac{2}{7}z$

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Assignment-2

(1) $[1,0,0]$, $[1,1,0]$, $[1,1,1]$

$$x\begin{bmatrix}1\\0\\0\end{bmatrix} + y\begin{bmatrix}1\\1\\0\end{bmatrix} + z\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$\Rightarrow$ $x + y + z = 0$

$y + z = 0$

$z = 0$

$\therefore$ $x = 0$, $y = 0$, & $z = 0$

so, the given set of vectors is linearly independent

2. $[7, -3, 11, -6]$, $[-56, 24, -88, 48]$

$$x\begin{bmatrix}7\\-3\\11\\-6\end{bmatrix} + y\begin{bmatrix}-56\\24\\-88\\48\end{bmatrix} = 0$$

$\Rightarrow$

$7x + \cancel{}56y = 0 \Rightarrow x - 8y = 0$

$-3x + 24y = 0 \Rightarrow x - 8y = 0$

$11x - 88y = 0 \Rightarrow x - 8y = 0$

$-6x + 48y = 0 \Rightarrow x - 8y = 0$

$\Rightarrow x = 8y$

$\therefore$ $-3(8y) + 24) \Rightarrow$ & ~~y ∈~~ R 0

$\therefore$ The given set of vectors linealy independent.

(3) $$x\begin{bmatrix}-1\\5\\0\end{bmatrix} + y\begin{bmatrix}16\\8\\-3\end{bmatrix} + z\begin{bmatrix}-64\\56\\9\end{bmatrix} = 0$$

$\Rightarrow$ $-x + 16y - 64z = 0$

$5x + 8y + 56z = 0$

$-3y + 9z = 0$

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$$\begin{bmatrix} -1 & 5 & 0 \\ 16 & 8 & -3 \\ -64 & 56 & 9 \end{bmatrix}$$

$\det = -1(72+168) - 5(144+192) + 0$

$\det = (-1)(240) - 5(336) = -240 - 1680 = -1920$

Since determinant is not zero, the vectors are linealy independent

(4) $[1,-1,1]$, $[1,1,-1]$, $[-1,1,1]$, $[0,1,0]$

(5) $[2,-4]$, $[1,9]$, $[3,5]$

$$\rightarrow \begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_2$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 13 & 11 \end{bmatrix}$$

$R_2 \rightarrow \frac{1}{13} R_2$ $\rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & \frac{11}{13} \end{bmatrix}$

The reduced row echelon form of the matrix doesn't contain any zero rows, and each columns still has a leading entry. These are linealy independent.

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6. [3, -2, 0, 4], [5 0 0 1], [-6 1 0 1], [2 0 0 3]

$$\begin{bmatrix} 3 & -2 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 7 & 5 & -5 & 5 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$R_4 \to R_4 - \left(\frac{4}{3}\right) R_1$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 7 & 5 & -5 & 5 \\ 0 & -11 & 11 & 5 \end{bmatrix}$$

$R_3 \to R_3 - (7/3) R_1$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & -20 & 7 & 1 \\ 0 & -11 & 11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - \left(\frac{11}{20}\right) R_3$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & -20 & 7 & 1 \\ 0 & 0 & 11/20 & 107/20 \end{bmatrix}$$

The given vectors are linearly independent.

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(7) [3, 4, 7], [2, 0, 3], [8, 2, 3], [5, 5, 6]

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & 0 & 3 \\ 8 & 2 & 3 \\ 5 & 5 & 6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8 & -11 \\ 8 & 2 & 3 \\ 5 & 5 & 6 \end{bmatrix}$$

$R_3 \to R_3 - 8R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8 & -11 \\ 0 & -30 & -53 \\ 5 & 5 & 6 \end{bmatrix}$$

$R_4 \to R_4 - 5R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -8 & -11 \\ 0 & -30 & -53 \\ 0 & -15 & -29 \end{bmatrix}$$

$R_2 \to \frac{1}{-8} R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 11/8 \\ 0 & -30 & -53 \\ 0 & -15 & -29 \end{bmatrix}$$

$R_3 \to R_3 + 30R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 11/8 \\ 0 & 0 & 1 \\ 0 & -15 & -29 \end{bmatrix}$$

$R_4 \to R_4 + 15R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 11/8 \\ 0 & 0 & 2 \\ 0 & 0 & \end{bmatrix}$$

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The matrix has three non-zero row & three columns which means it has full rank, so the given vectors are linearly independent

(c) $[6\ 0\ 3\ 1\ 4\ 2]$, $[0, -1, 2, 7, 0, 5]$
$[12, 3, 0, -19, 8, -11]$

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 12 & 3 & 0 & -19 & 8 & -11 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 0 & 3 & -6 & -21 & 0 & -15 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 3R_2$

$$\begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \\ 0 & -1 & 2 & 7 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the 3rd row consists ~~of~~ entirely of zeros we can conclude that the vectors are linearly dependent.

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Assignment 3

Find the Eigen values & Eigen Vectors of following matrices.

(1)
$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)(-\lambda+\lambda^2) - 2(-2\lambda-6) - 3(-4-1+\lambda) = 0$$

$$\Rightarrow 2\lambda - 2\lambda^2 + \lambda^2 - \lambda^3 + 4\lambda + 12 + 12 + 3 - 3\lambda = 0$$

$$\Rightarrow \lambda^3 + \lambda^2 - 21\lambda + 3 = 0$$

$\therefore$ Eigenvalues are $\lambda = -5.4, -0.14, 4$.

$\lambda = 4$

$$\left\{\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -6 & 2 & 3 \\ 2 & -3 & -6 \\ -1 & -2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$
$$\left.\begin{aligned} -6x + 2y + 3z &= 0 \\ 2x - 3y - 6z &= 0 \\ -x - 2y - 4z &= 0 \end{aligned}\right\}$$

on solving these equation

$$y = -\frac{15}{7} z$$

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$\Rightarrow (4-\lambda)(1-\lambda)^2 + 1(-2(1-\lambda)) = 0$

$\Rightarrow (4-\lambda)(1-\lambda^2) + 2(1-\lambda) = 0$

$\Rightarrow (1-\lambda)[(4-\lambda)(1-\lambda) + 2] = 0$

$\Rightarrow (1-\lambda)(6 - 5\lambda + \lambda^2) = 0$

$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\therefore$ eigen values $= \lambda = 1, 2, 3.$

for $\lambda = 1.$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

$-2x = 0$ $\qquad$ $x = 0,\ z = 0,\ y = k$

$3x + z = 0$

Eigen vector $= k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$-2x - y = 0$ $\qquad$ $-2x - z = 0$

$2x = -y$ $\qquad$ $z \Rightarrow -2x$

$x = k,\ y = -2k,\ z = -2k$

$\therefore$ Eigen vector $= k\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

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for $\lambda = 3$ $\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$

$-2x - 2z = 0$

$x = z$

$-2x - 2y = 0$

$x = -y$

Eigen vectors $= k \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(5) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$(A - \lambda I) = 0$

$$\begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 1 & 0 & 3-\lambda \end{bmatrix} = 0$$

$\Rightarrow (5-\lambda)\left[-\lambda(3-\lambda)\right] = 0$

$\Rightarrow \lambda = 5, 0, 3$

for $\lambda = 5$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}$

$-5y = 0$

$y = 0$

$-x - 2z = 0$

$x = 2z$

$x = 2k, \ y = 0, \ z = k$

Eigen vectors $= k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda = 0$ $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$x = 0$

$-x + 3z = 0$

$z = 0$

$x = 0, \ y = k, \ z = 0$

eigen vectors $k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

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for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$y = 0$

$x = 0$

$z = k$

Eigen vectors $= k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Semester Assignment

(1) Find the rank of the matrix A by reducing in Row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 \leftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \rightarrow \frac{-R_2}{4}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 = R_4 + 4R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 = -R_3/3$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 + 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of Matrix is '3'.

(2) Let W be the vector space of all symmetric 2×2 matrices & let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

Find rank & nullity of T.

d. 5) Since the maximum degree of polynomial T = 2
So $\dim(P_2) = 3$

Kernel So a subset of kernel T is $T(A) = 0$

$(a-b) + (b-c)x + (c-a)x^2 = 0$

$\Rightarrow \quad a = b = c = t \ (let)$

new matrix - $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$.

dimension of kernel is 1 because there's only one independent parameter as 't'.
According to rank nullity theorem:

rank (T) + nullity (T) = dim(W)

rank(T) + 1 = 4

rank of T is 3 & nullity is 1.

3. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigen values & eigen vectors of A^{-1} & $A + 4I$.

$$A - \lambda I = 0$$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda) = \pm 1$$

$$\boxed{\lambda = 1, 3}$$

for $\lambda = 1$.

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x - y = 0$ — let $x = t$;

$x = y$ — $y = t$

eigen vector v_1, $t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans.

for $\lambda = 3$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0$ — let $x = t$

$x = y$ — $y = -t$

So, eigen values $v_2 = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now find A^{-1}

eigen values of A^{-1} will be $\frac{1}{\lambda}$ & $\frac{1}{\lambda^2} \Rightarrow 1, \frac{1}{3}$

and eigen vectors are same as of A.

$v_1 = t\begin{bmatrix}1\\1\end{bmatrix}$ $\quad V_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Now, for $A + 4I$

eigen values for $A + 4I$ will be $\lambda_1 + 4I$
$\lambda_2 + 4 = 5, 7$

and eigen vectors are same as of A

$v_1 = t\begin{bmatrix}2\\1\end{bmatrix}$ & $v_2 = t\begin{bmatrix}-1\\1\end{bmatrix}$

Q.4 Solve by Gauss - Siedel - Method (Take 3 iterati..)

$3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

=> $x^{k+1} \Rightarrow \dfrac{7.85 + 0.1y^k + 0.2z^k}{3}$

$y^{k+1} \Rightarrow \dfrac{-19.3 - 0.1x^{k+1} - 0.3z^k}{-7}$

$z^{k+1} \Rightarrow \dfrac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$

Since $x = y = z = 0$

(iteration 1)

$x(1) = \dfrac{7.85 + 0.1(0) + 0.2(0)}{3} = 2.6167$

$y(1) = \dfrac{19.3 - 0.1(2.6167) - 0.3(0)}{-7} = 2.7956$

$z(1) = \dfrac{71.4 - 0.3(2.6167) - 0.2(2.9756)}{10} = 7.1373$

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Iteration - 2

$x(2) = \frac{7.85 + 0.1(2.7956) + 0.2(7.1373)}{3} = 3$

$y(2) = \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$

$z(2) = \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$

Iteration 3

$x(3) = \frac{7.85 + 0.1(3) + 0.2(3)}{3} = 3$

$y(3) = \frac{-19.3 - 0.11(3) - 0.3(3)}{-7} = 3$

$z(3) = \frac{71.4 - 0.3(3) + 0.2(3)}{10} = 3$

After three iteration, $x, y, z \approx 3$

So value of $x = 3$, $y = 3$, $z = 3$.

(8) Define consistent & inconsistent of equations. Hence solve the following system of equations if consistent

$x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$

$x + 17y + 4z = 0$

Consistent
(atleast one solution)

dependent
(infinite solution)

independent
(unique solution)

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$$A = \left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ & $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$; $R_4 \to R_4 + 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A:B) = 2$

$n = 3$

$\rho(A) = \rho(A:B) \neq n$

Consistent but infinite solution.

(2) Determine whether the function $T: P_2 \to P_2$ is linear transformation or not.

where $T(a + bx + cx^2) = a + 1 + (b+1)x + (c+1)x^2$

(i) Additive :-

$T(U+V) = T(U) + T(V)$

$U = a_1 + b_1 x + c_1 x^2$

$V = a_2 + b_2 x + c_2 x^2$

$T(U+V) = T(a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2)$

$= (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

$(a_1+1) + (b_1+1)x + (c_1+1)x^2 + (c_2+1) + (b_2+1)x$
$+ (c_2+1)x^2$

$T(U) \neq T(W)$
Hence, Proved.

(5) Homogeneity →

$T(kU) \rightarrow kT(U)$
$T(k(a+bx+cx^2))$
$T(ka + kbx + kcx^2)$
$(ka+kb+kc+1) + (ka+kb+kc+1)x + (ka+kb+kc+1)x^2$
$kT(U)$
Hence Proved.

Hence it is a linear transformation.

Q.7 Determine whether set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dimension & basis of subspace spanned by

$a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$
$a+3b-2c = 0$
$2a+b+c = 0$
$3a+3c = 0$
$c = -a \quad , b = -a$

only one sol^n is possible is $a=b=c=0$
so, linearly independent

since dim = $V_3(R)$ is 3 and S also contains 3 vector & S → LI then it spans $V_3(R)$ making it a basis for $V_3(R)$

Q.9 Explain the application of matrix operation in image processing with example.

Affline Transformation

Rotation

suppose we have a 2-D image representation as grid or pixels we can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

to rotate it around centre.

(1) Translation to origin
→ Translate the image, so that it centre align with origin

(2) Rotation
Apply rotation matrix

(3) Translation Back
→ Translate it back with its original position by adding coordinates of centre.

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(10) Give a brief description of linear transformation for computer vision for rotating 2D image.

→ Linear transformation for rotating 2D images involves applying a rotation matrix to each pixel coordinate. This matrix rotates points counter clockwise by an angle θ around the origin. It preserves geometric properties like parallelism and distance. Rotation is essential in tasks like image allignment & object detection in computer vision.