

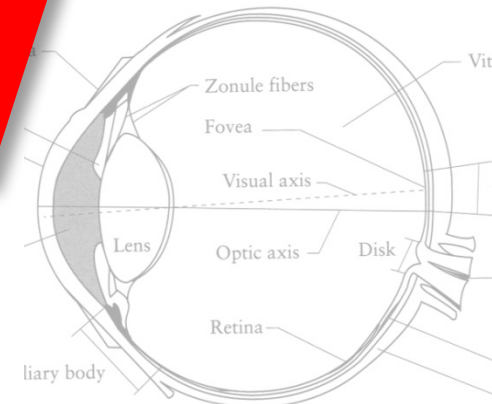
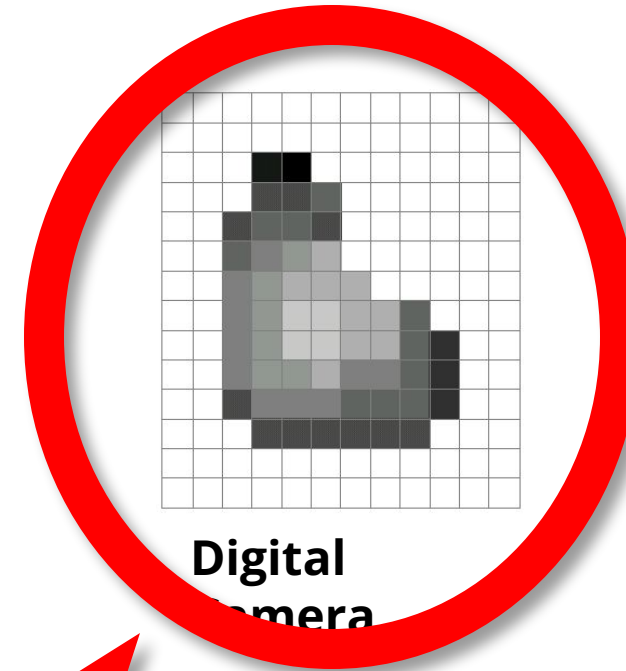
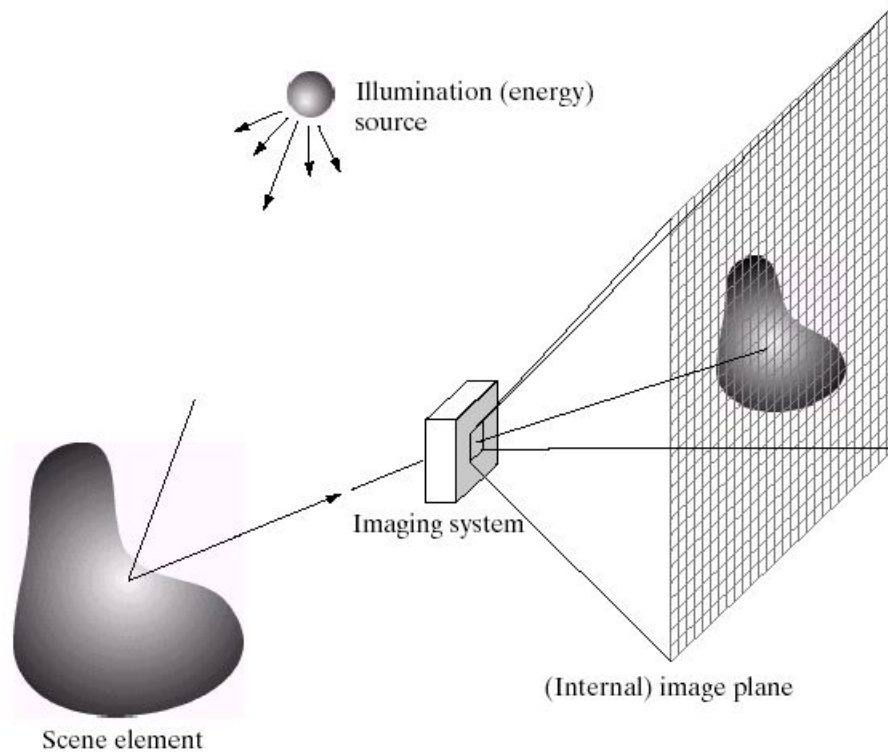
Intro to Computer Vision

Images and image filtering

What is an image?



What is an image?



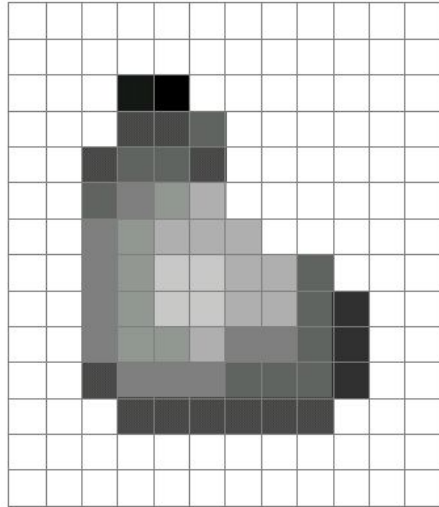
The Eye

We'll focus on these in this class

(More on this process later)

What is an image?

- A grid (matrix) of intensity values



=

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

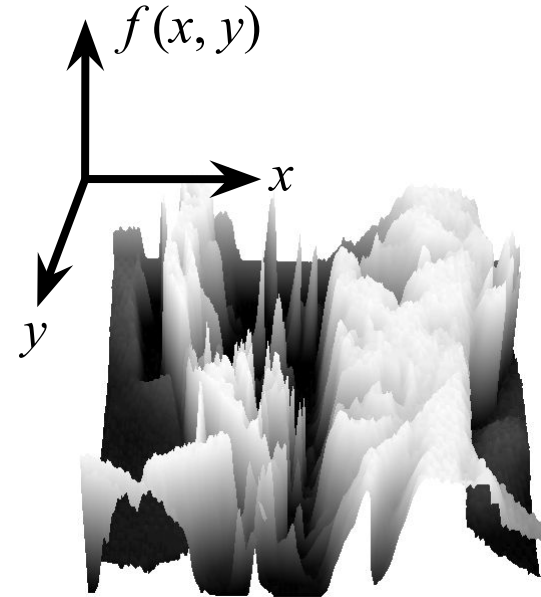
(common to use one byte per value: 0 = black, 255 = white)

What is an image?

- Can think of a (grayscale) image as a **function** f from \mathbb{R}^2 to \mathbb{R} :
 - $f(x,y)$ gives the **intensity** at position (x,y)



[snoop](#)
[p](#)



[3D](#)
[view](#)

- A **digital** image is a discrete (**sampled, quantized**) version of this function

Image transformations

- As with any function, we can apply operators to an image



$$g(x,y) = f(x,y) + 20$$



$$g(x,y) = f(-x,y)$$

- Today we'll talk about a special kind of operator, *convolution* (linear filtering)

Filters

- Filtering
 - Form a new image whose pixel values are a combination of the original pixel values
- Why?
 - To get useful information from images
 - E.g., extract edges or contours (to understand shape)
 - To enhance the image
 - E.g., to remove noise
 - E.g., to sharpen and “enhance image” a la CSI
 - A key operator in Convolutional Neural Networks

Canonical Image Processing problems

- Image Restoration
 - denoising
 - deblurring
- Image Compression
 - JPEG, HEIF, MPEG, ...
- Locating Structural Features
 - corners
 - edges

Question: Noise reduction

- Given a camera and a still scene, how can you reduce noise?



Take lots of images and average them!
What's the next best thing?

Source: S.

Image filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel

1	5	3
0	5	1
1	1	7

Local image
data

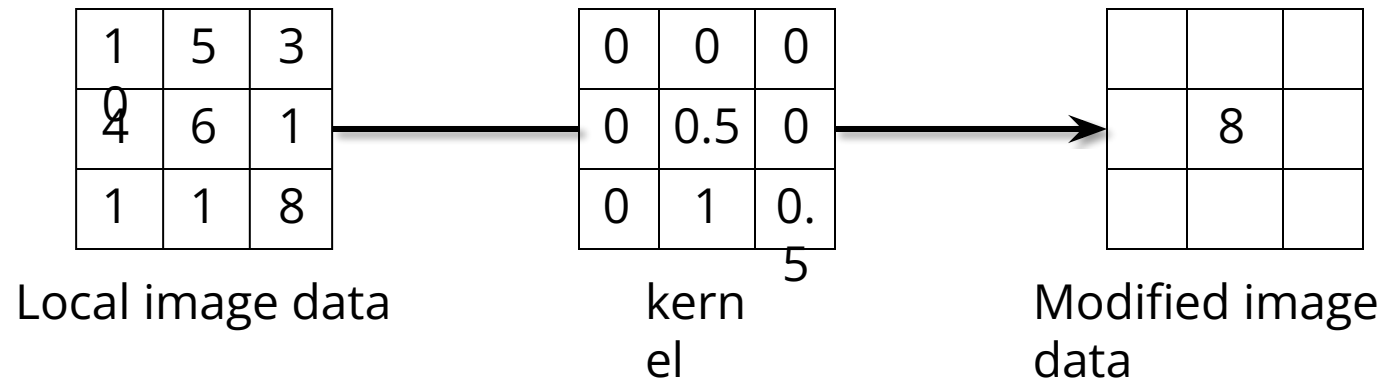


	7	

Modified image
data

Linear filtering

- One simple version of filtering: linear filtering (cross-correlation, convolution)
 - Replace each pixel by a linear combination (a weighted sum) of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)



Cross-correlation

Let F be the image, H be the kernel (of size $2k+1 \times 2k+1$), and G be the output image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

This is called a **cross-correlation** operation.

$$G = H \otimes F$$

- Can think of as a “dot product” between local neighborhood and kernel for each pixel

Convolution

- Same as cross-correlation, except that the kernel is “flipped” (horizontally and vertically)

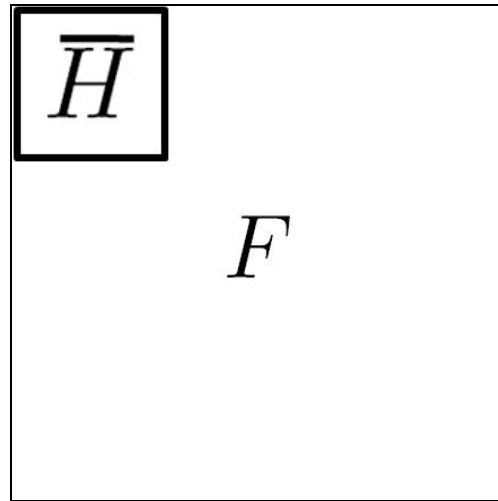
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

This is called a **convolution** operation:

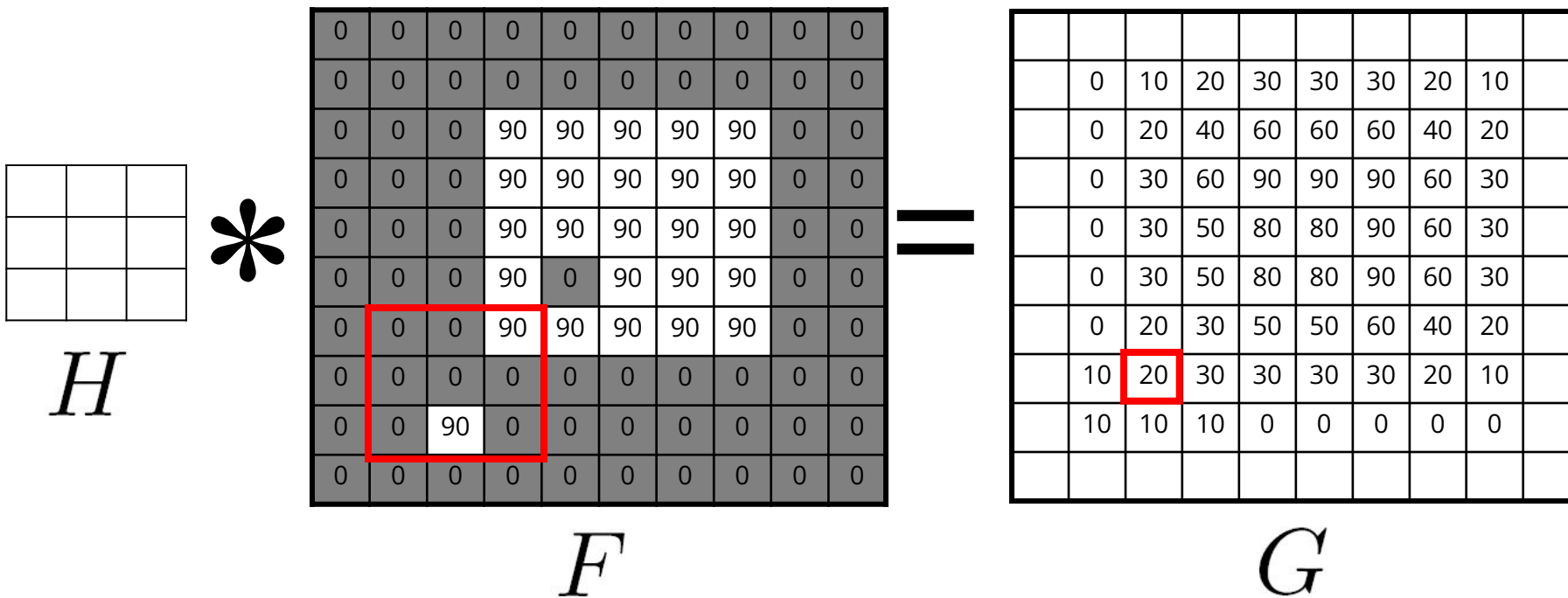
$$G = H * F$$

- Convolution is **commutative** and **associative**

Convolution



Mean filtering



Mean filtering / Moving average

$$F[x, y]$$
[illegible]
$$G[x, y]$$
A 10x10 grid with a red square in the top-left corner. The red square is located in the first row and first column, with a side length of 1 unit. The grid is composed of 10 columns and 10 rows of squares. The red square is the top-leftmost square in the grid.

Mean filtering / Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering / Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering / Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering / Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering / Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Linear filters: examples

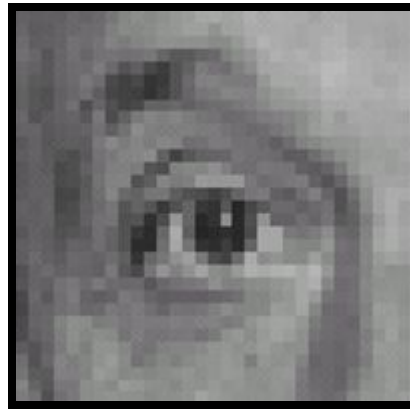


Original
|



0	0	0
0	1	0
0	0	0

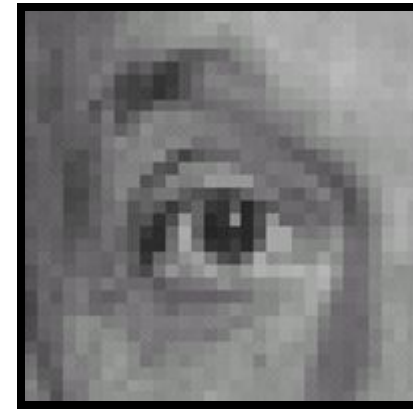
Linear filters: examples



Original
|



0	0	0
0	1	0
0	0	0



Identical image

Linear filters: examples

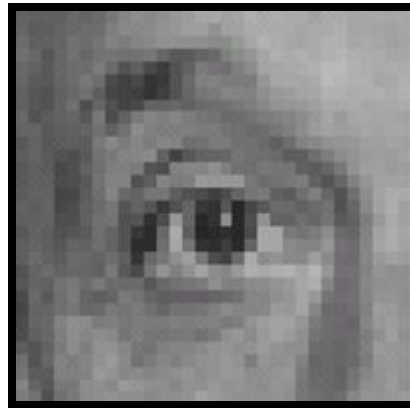


Original
|



0	0	0
1	0	0
0	0	0

Linear filters: examples



Original
|

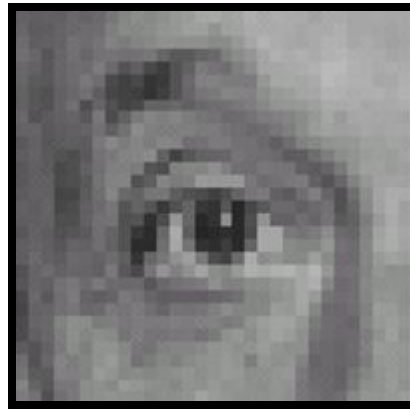


0	0	0
1	0	0
0	0	0



Shifted left by 1
pixel

Linear filters: examples

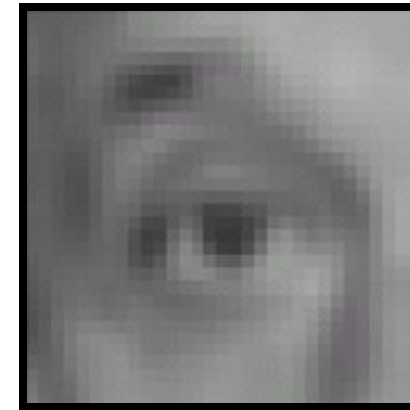


Original
|




$\frac{1}{9}$

1	1	1
1	1	1
1	1	1




Blur (with a mean filter)

Linear filters: examples

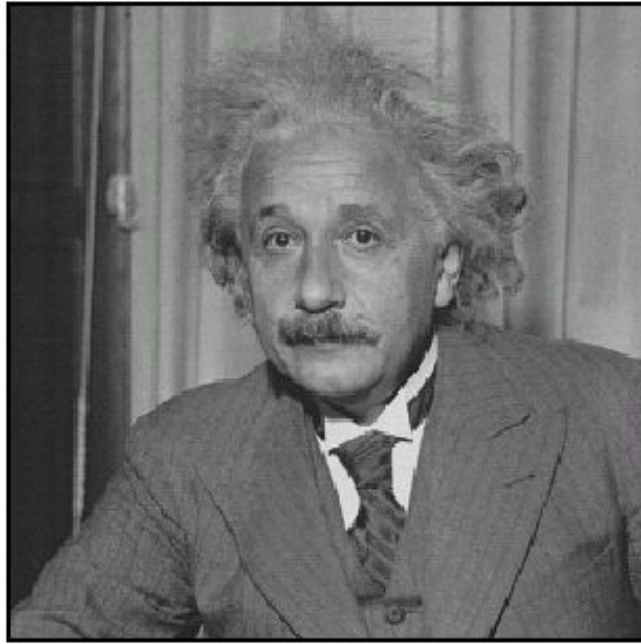


Original

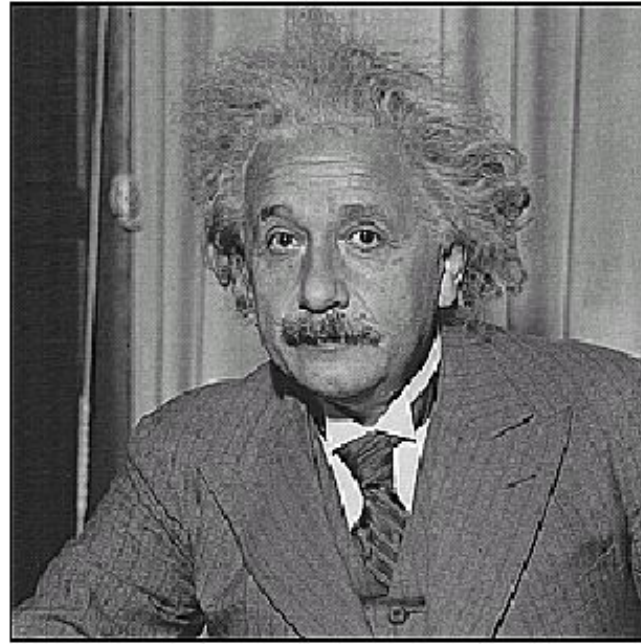
$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$


Sharpening filter
(accentuates edges)

Sharpening

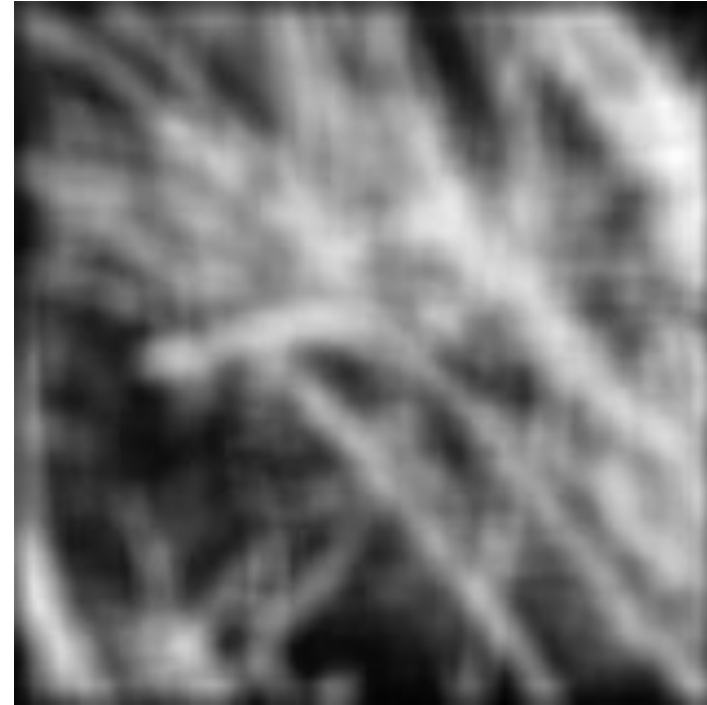


before

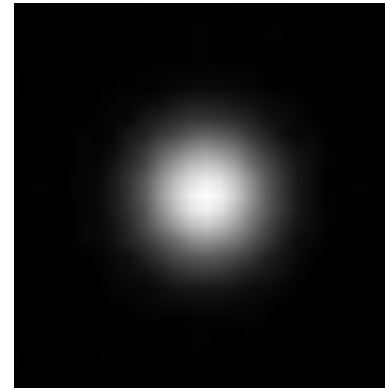
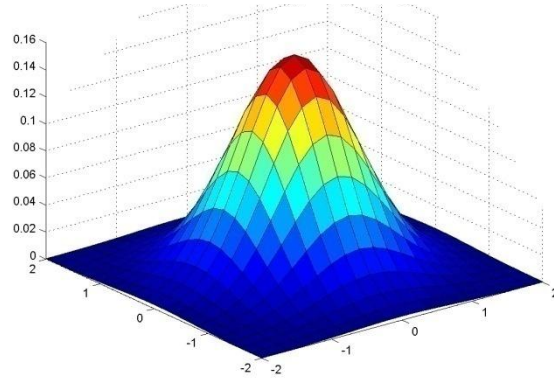


after

Smoothing with box filter revisited

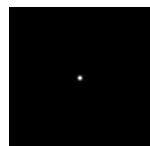
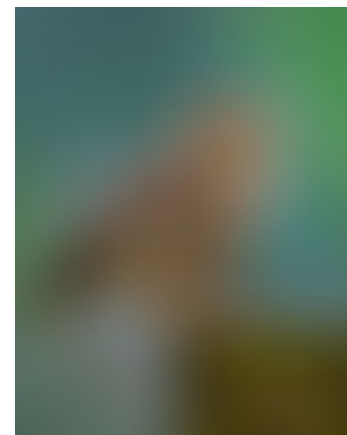
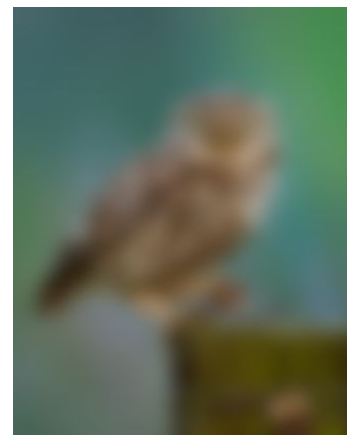
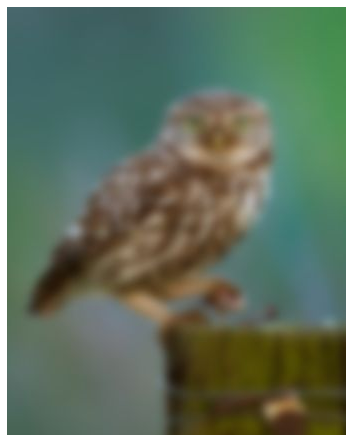


Gaussian kernel

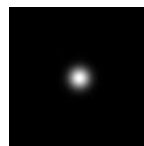


$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Gaussian filters



$\sigma = 1$
pixel



$\sigma = 5$
pixels

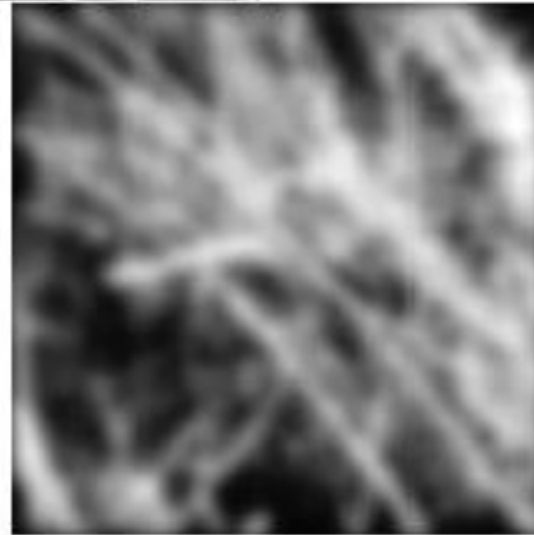
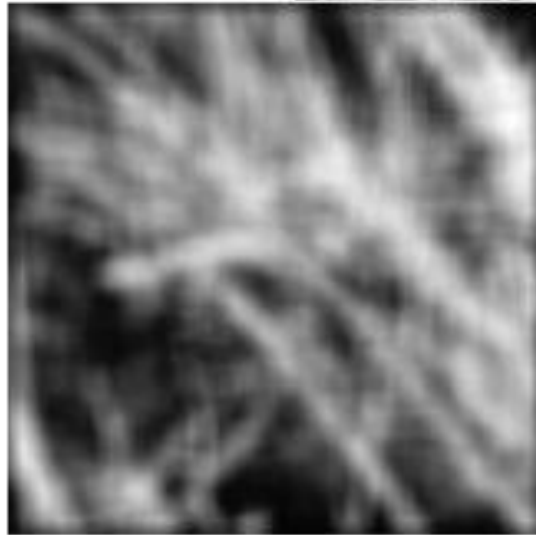


$\sigma = 10$
pixels



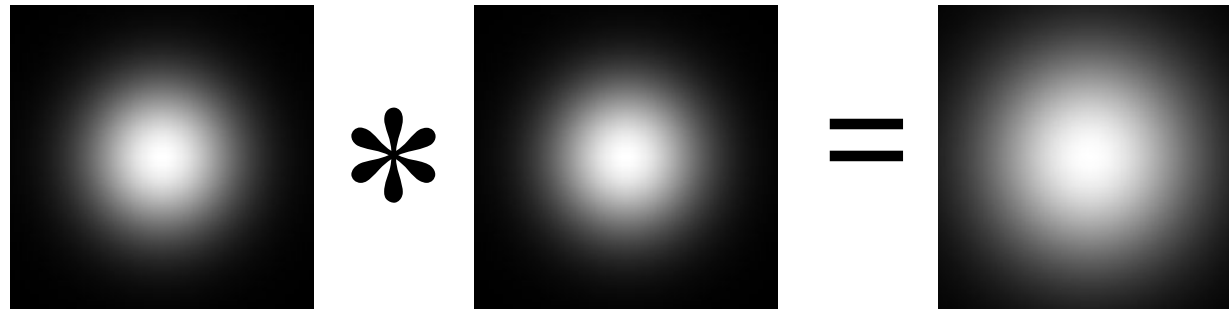
$\sigma = 30$
pixels

Mean vs. Gaussian filtering



Gaussian filter

- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian

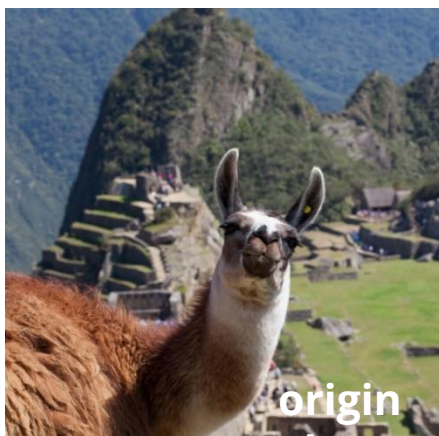


The diagram illustrates the property of Gaussian filters that the convolution of two Gaussian kernels results in another Gaussian kernel. It shows three square boxes, each containing a grayscale Gaussian blur. The first box is followed by a convolution symbol (*), the second box is followed by an equals sign (=), and the third box is the result. The resulting Gaussian in the third box is visibly wider and more spread out than the two original kernels, representing a larger standard deviation.

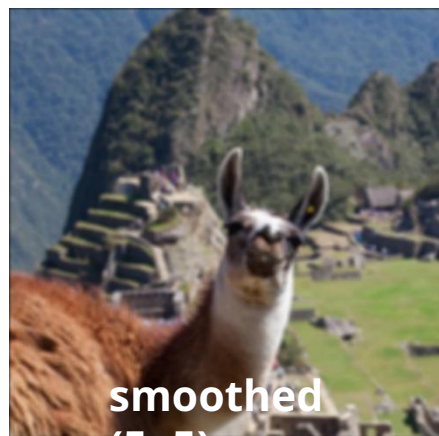
- Convolving twice with Gaussian kernel of width σ
= convolving once with kernel of width $\sigma\sqrt{2}$

Sharpening revisited

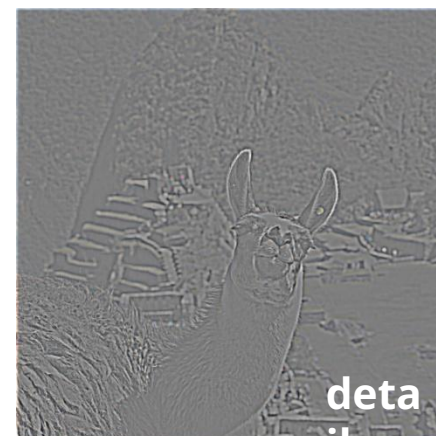
- What does blurring take away?



−



=

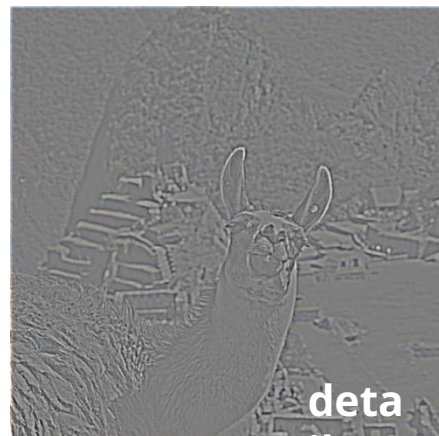


(This “detail extraction” operation is also called a **high-pass filter**)

Let's add it back:



+
 α



=

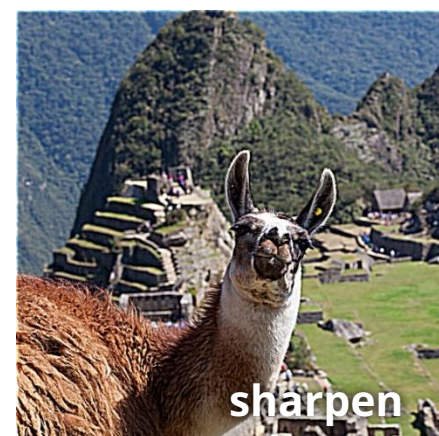
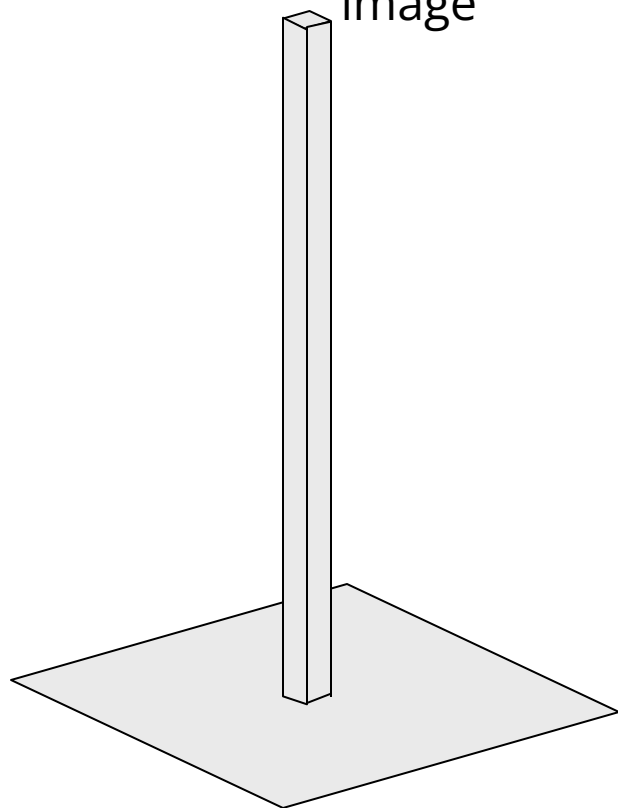


Photo credit: <https://www.flickr.com/photos/geezaweezer/16089096376/>

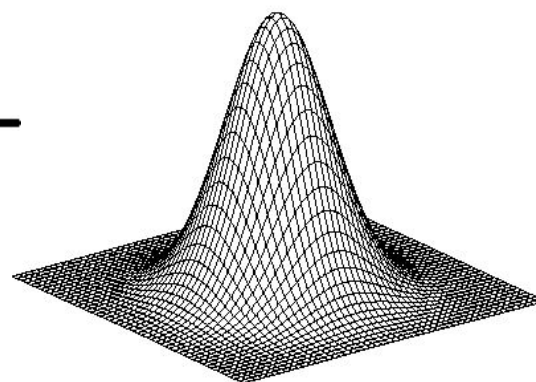
Sharpen filter

$$\underset{\substack{\uparrow \\ \text{image}}}{F} + \alpha (F - \underbrace{F * H}_{\substack{\text{blurred} \\ \text{image}}}) =$$



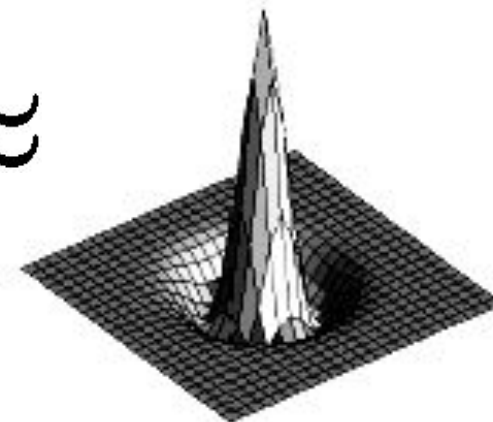
scaled
impulse

—



Gaussian

\approx



Sharpen
filter

\uparrow
unit impulse
(identity kernel
with single 1 in
center, zeros
elsewhere)

Sharpen filter



“Optical” convolution

Camera shake



Source: Fergus, *et al.* “Removing Camera Shake from a Single Photograph”, SIGGRAPH 2006

Bokeh: Blur in out-of-focus regions of an image.



Source: https://www.diyphotography.net/diy_create_your_own_bokeh/

Filters: Thresholding



$$g(m, n) = \begin{cases} 255, & f(m, n) > A \\ 0 & \text{otherwise} \end{cases}$$

Linear filters

- Can thresholding be implemented with a linear filter?