Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$(\mathbf{5},\ \overline{\mathbf{5}})$	$+1$	$(XD,\ XD^c)$	$(\mathbf{5},\ \overline{\mathbf{5}})$	-1	$(XL,\ XL^c)$
$(\mathbf{10},\ \overline{\mathbf{10}})$	$+1$	$(XU,\ XU^c),\ (XE,\ XE^c)$	$(\mathbf{10},\ \overline{\mathbf{10}})$	-1	$(XQ,\ XQ^c)$
$(\mathbf{15},\ \overline{\mathbf{15}})$	$+1$	$(XT,\ XT^c),\ (XS,\ XS^c)$	$(\mathbf{15},\ \overline{\mathbf{15}})$	-1	$(XQ,\ XQ^c)$
$\mathbf{24}$	$+1$	$XG,\ XW$	$\mathbf{24}$	-1	$(XY,\ XY^c)$

TABLE I: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SU(5)$ models.

Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$\mathbf{10}$	$+1$	Xh	$\mathbf{10}$	-1	$\overline{Xh}$
$(\mathbf{16},\ \overline{\mathbf{16}})$	$+1$	$(XF,\ \overline{XF})$	$(\mathbf{16},\ \overline{\mathbf{16}})$	-1	$(Xf,\ \overline{Xf}),\ (Xl,\ \overline{Xl})$
$\mathbf{45}$	$+1$	$XGW,\ XN$	$\mathbf{45}$	-1	$(XX,\ \overline{XX})$

TABLE II: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SO(10)$ models where the gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetries.

C. F-Theory $SU(5)$ Models with Generic Vector-Like Particles

We first briefly review the F-theory model building [20–24]. The twelve-dimensional F theory is a convenient way to describe Type IIB vacua with varying axion-dilaton $\tau = a + ie^{-\phi}$. We compactify F-theory on a Calabi-Yau fourfold, which is elliptically fibered

Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$\mathbf{10}$	$+1$	$XD\overline{D}$	$\mathbf{10}$	-1	$XL\overline{L}$
$(\mathbf{16},\ \overline{\mathbf{16}})$	$+1$	$(XFL,\ \overline{XFL})$	$(\mathbf{16},\ \overline{\mathbf{16}})$	-1	$(XFR,\ \overline{XFR})$
$\mathbf{45}$	$+1$	$XG4,\ XWL,\ XWR$	$\mathbf{45}$	-1	XZ

TABLE III: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SO(10)$ models where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries.