III. ESTIMATION OF OFF-DIAGONAL GLUON MASS IN MA GAUGE

Next, we investigate the effective gluon mass. We start from the Lagrangian of the free massive vector field A_μ with the mass $M \neq 0$ in the Proca formalism,

$$\mathcal{L} \;=\; \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}M^2 A_\mu A_\mu, \qquad (6)$$

in the Euclidean metric. The scalar combination of the propagator $G_{\mu\mu}(r;M)$ can be expressed with the modified Bessel function $K_1(z)$ as

$$\begin{aligned} G_{\mu\mu}(r;M) &= \langle A_\mu(x)A_\mu(y)\rangle \\ &= \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)} \frac{1}{k^2+M^2}\left(4+\frac{k^2}{M^2}\right) \\ &= 3\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)} \frac{1}{k^2+M^2} + \frac{1}{M^2}\delta^4(x-y) \\ &= \frac{3}{4\pi^2}\frac{M}{r}K_1(Mr) + \frac{1}{M^2}\delta^4(x-y). \end{aligned} \qquad (7)$$

In the infrared region with large Mr, Eq. (7) reduces to

$$G_{\mu\mu}(r;M) \;\simeq\; \frac{3\sqrt{M}}{2(2\pi)^{\frac{3}{2}}}\frac{e^{-Mr}}{r^{\frac{3}{2}}}, \qquad (8)$$

using the asymptotic expansion,

$$K_1(z) \simeq \sqrt{\frac{\pi}{2z}}e^{-z}\sum_{n=0}^{\infty}\frac{\Gamma(\frac{3}{2}+n)}{n!\Gamma(\frac{3}{2}-n)}\frac{1}{(2z)^n}, \qquad (9)$$

for large Re z.

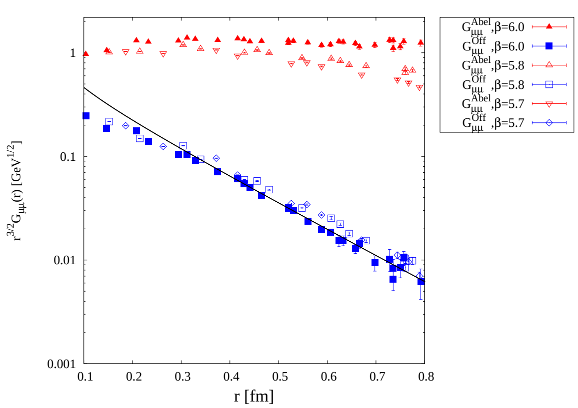

FIG. 2: The logarithmic plot of $r^{3/2}G^{\rm off}_{\mu\mu}(r)$ and $r^{3/2}G^{\rm Abel}_{\mu\mu}(r)$ as the function of the four-dimensional Euclidean distance r in the MA gauge with the U(1)$_3\times$U(1)$_8$ Landau gauge fixing, using the SU(3) lattice QCD with 16^4 at $\beta = 5.7$, 5.8 and 6.0. The solid line denotes the logarithmic plot of $r^{3/2}G_{\mu\mu}(r) \sim r^{1/2}K_1(Mr)$ in the Proca formalism.

In Fig.2, we show the logarithmic plot of $r^{3/2}G^{\rm off}_{\mu\mu}(r)$ and $r^{3/2}G^{\rm Abel}_{\mu\mu}(r)$ as the function of the four-dimensional Euclidean distance r in the MA gauge with the U(1)$_3\times$U(1)$_8$ Landau gauge fixing. From the linear slope on $r^{3/2}G^{\rm off}_{\mu\mu}(r)$, the effective off-diagonal gluon mass $M_{\rm off}$ is estimated. Note that the gluon-field renormalization does not affect the gluon mass estimate, since it gives only an overall constant factor for the propagator. We summarize in Table I the effective off-diagonal gluon mass $M_{\rm eff}$ obtained from the slope analysis in the range of $r = 0.3 - 0.8$fm at β =5.7, 5.8 and 6.0. The off-diagonal gluons seem to have a large effective mass of $M_{\rm off} \simeq 1.1 - 1.2$GeV. This result approximately coincides with SU(2) lattice calculation [18].

Also for the diagonal gluon, we try to estimate its effective mass $M_{\rm diag}$, although its propagator largely depends on β, i.e., the volume or the spacing, as is indicated in Fig.2. We estimate the diagonal gluon mass $M_{\rm diag}$ from the slope analysis in the range of $r = 0.3 - 0.8$fm at each β, and add the result in Table I. In any case, the diagonal gluon seems to have a small effective mass of $M_{\rm diag} \simeq 0.1 - 0.3$GeV. For the definite argument on $G^{\rm Abel}_{\mu\mu}(r)$ and the diagonal gluon mass, more careful analysis with a large-volume lattice would be needed.

TABLE I: Summary table of conditions and results in SU(3) lattice QCD. The off-diagonal gluon mass $M_{\rm off}$ is estimated from the slope analysis of $r^{3/2}G^{\rm off}_{\mu\mu}(r)$ for $r = 0.3 - 0.8$fm at each β. In the MA gauge, the off-diagonal gluons seem to have a large effective mass of $M_{\rm off} \simeq 1.1 - 1.2$GeV. The best-fit mass parameter $m_{\rm off}$ is also listed at each β: $G^{\rm off}_{\mu\mu}(r)$ in the range of $r = 0.1 - 0.8$fm is well described with the four-dimensional Euclidean Yukawa function $\sim e^{-m_{\rm off}r}/r$ with $m_{\rm off} \simeq 1.3 - 1.4$GeV. We add the diagonal gluon effective mass $M_{\rm diag}$ at each β, estimated in a similar manner to $M_{\rm off}$.

lattice size	β	a[fm]	$M_{\rm off}$[GeV]	$m_{\rm off}$[GeV]	$M_{\rm diag}$[GeV]
	5.7	0.186	1.2	1.3	0.3
16^4	5.8	0.152	1.1	1.3	0.2
	6.0	0.104	1.1	1.4	0.1

Finally in this section, we discuss the relation between infrared abelian dominance and the off-diagonal gluon mass. Due to the large effective mass $M_{\rm off}$, the off-diagonal gluon propagation is restricted within about $M_{\rm off}^{-1} \simeq 0.2$ fm in the MA gauge. Therefore, at the infrared scale as $r \gg 0.2$ fm, the off-diagonal gluons A^a_μ $(a \neq 3,8)$ cannot mediate the long-range force like the massive weak bosons in the Weinberg-Salam model, and only the diagonal gluons A^3_μ, A^8_μ can mediate the long-range interaction in the MA gauge. In fact, in the MA gauge, the off-diagonal gluons are expected to be inactive due to the large mass $M_{\rm off}$ in the infrared region in comparison with the diagonal gluons. Then, infrared abelian dominance holds for $r \gg M_{\rm off}^{-1}$.