can be written as [8]:

$$\mathcal{L}_{HTL} = -\frac{1}{4}F^2 + \frac{3}{4}\mathcal{M}_\gamma^2 F_{\mu\alpha}\left\langle\frac{u^\alpha u^\beta}{(\partial u)^2}\right\rangle F_\beta{}^\mu + \overline{\Psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\Psi - \mathcal{M}^2\overline{\Psi}\gamma_\mu\left\langle\frac{u^\mu}{u\cdot\Pi}\right\rangle\Psi \quad , \tag{11}$$

where the average $\langle\cdot\rangle$ is defined by

$$\langle f(u_0,\vec{u})\rangle = \int\frac{d\Omega}{4\pi}f(1,\vec{u}) \quad , \tag{12}$$

where $\vec{u}$ is a spatial unit vector. The equation of motion for Ψ that follows is

$$\left[\Pi\!\!\!/ - m - \mathcal{M}^2\gamma_\mu\left\langle\frac{u^\mu}{u\cdot\Pi}\right\rangle\right]\Psi = 0 \quad . \tag{13}$$

Equation (13) is a non-local and non-linear differential equation, which is, in general, very difficult to deal with. What makes this equation much less tractable than the thermal Dirac equation in the absence of the B-field is that the average over $\vec{u}$ is difficult to perform because $[\Pi_\mu,\Pi_\nu] = -ieF_{\mu\nu} \neq 0$. Since the spatial symmetries of the system are unaltered by the thermal heat bath, we still expect the eigenfunctions to have the same spatial form as at zero temperature. In fact, after performing the u-integral in Eq. (13) the result can only be a function of the invariants $\mathbf{\Pi}_\perp^2$, p_0^2 and p_z, and the γ-structure has to be proportional to $\gamma\mathbf{\Pi}_\perp$, $\gamma_0 p_0$ and $\gamma_z p_z$. We shall therefore compute the matrix elements

$$\langle\Phi_{\kappa'}|\left\langle\frac{u^\mu}{u\cdot\Pi}\right\rangle|\Phi_\kappa\rangle \quad , \tag{14}$$

between the vacuum eigenstates

$$\langle x|\Phi_\kappa\rangle = \exp[i(-p_0t+p_yy+p_zz)]I_{n;p_y}(x) \quad , \tag{15}$$

$$I_{n;p_y}(x) = \left(\frac{eB}{\pi}\right)^{1/4}\exp\left[-\frac{1}{2}eB\left(x-\frac{p_y}{eB}\right)^2\right] \times\frac{1}{\sqrt{n!}}H_n\left[\sqrt{2eB}\left(x-\frac{p_y}{eB}\right)\right] \quad , \tag{16}$$

where $\kappa = \{p_0, n, p_y, p_z\}$ and $H_n[x]$ are Hermite polynomials. These states form a complete set of functions in four dimensions when the energy is off shell. In the chiral representation suitable spinors can be formed from Φ_κ as $\Psi_\kappa = \mathrm{diag}[\Phi_\kappa, \Phi_{\kappa-1}, \Phi_\kappa, \Phi_{\kappa-1}]\chi$ where χ is an undetermined space-time-independent spinor. The vacuum Dirac operator in Eq. (13) obviously gives an eigenvalue when acting on Ψ_κ, but it is more difficult to determine the action of the thermal part since Φ_κ cannot be an eigenfunction to $u\cdot\Pi$ for all u. One way to calculate the matrix element in Eq. (14) is to find a basis such that $v\cdot\Pi|v_p\rangle = v\cdot p|v_p\rangle$ and insert a unit operator $\int d^4p|v_p\rangle\langle v_p|$. This unit operator is, of course, independent of v after