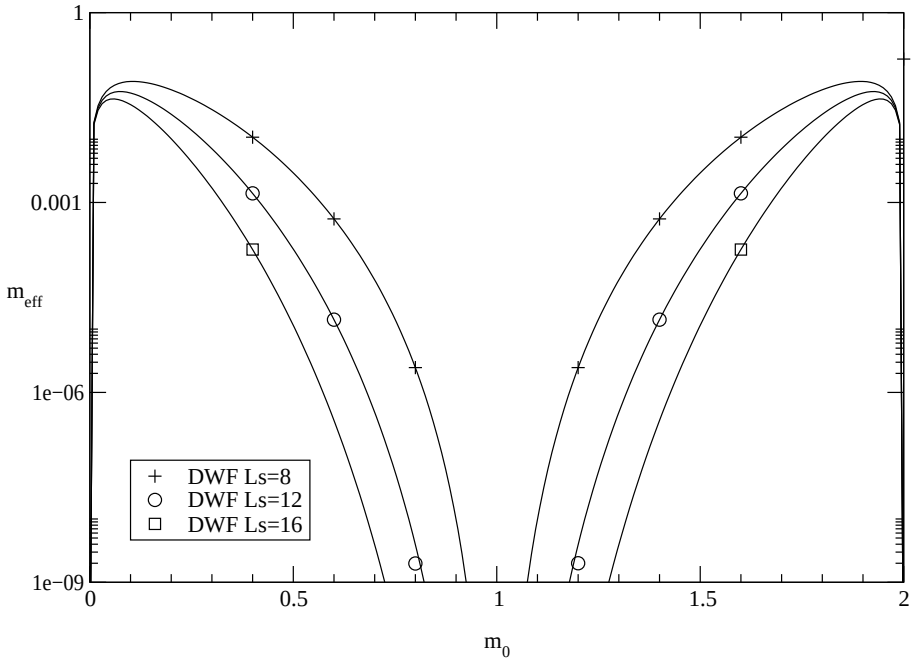

FIG. 1. Effective quark mass induced by domain-walls for the free field configuration. L_s is the number of lattice sites in the fifth direction.

In the presence of a realistic gauge potential, the effective quark mass result from the finite wall separation may depend on how it is defined. Different definitions shall yield results consistent up to a factor of order unity. One approach is to exploit the explicit quark mass dependence in chiral Ward identities such as the Gell-Mann-Oakes-Renner (GMOR) relation as done in Ref. [7]. Here we explore the effective mass in an alternative way. In continuum field theory, the Atiyah-Singer theorem [8] states that the Dirac operator has a zero eigenvalue in the presence of an external background with topological charge $|Q| = 1$. The explicit form of the solution was found by 't Hooft in 1976 [9]. On the lattice, however, the notion of topological charge is ill defined: any gauge configuration can be continuously deformed into a null gauge field. Moreover, the discretization of an instanton field can introduce finite lattice-spacing effects lifting any exact zero eigenvalue. Therefore, a test of the Atiyah-Singer theorem on lattice is usually complicated with various lattice artifacts.

There exists, however, a definition of lattice topology and fermion zero mode which largely avoids this complication. In the overlap formalism, the Dirac operator is constructed from the overlap of two many-fermion ground states [3]. According to their recipe, one starts from a four-dimensional Wilson-Dirac operator with a negative Wilson mass m_0 and calculates its eigenvalues. For m_0 small and positive, the number of positive eigenvalues is equal to that of negative ones. When m_0 increases, a level might cross from positive to negative or vice versa. When this happens, the gauge field is regarded to have a net topological charge $|Q| = 1$. Then the overlap determinant is exactly zero by construction. This definition of lattice topology and zero mode do depend on, for instance, the Wilson parameters r and m_0. However, the zero eigenvalue is exact, independent of the lattice spacing a and volume V.