spherical collapse model. There have been many attempts to model the nonlinear PDF (often called the counts-in-cells distribution) for a given initial spectrum of gaussian density fluctuations (e.g. Bernardeau 1994, Colombi 1994). Unfortunately, the models proposed so far work reasonably well only in the linear and quasilinear regimes $\delta \lesssim 1$. In testing our model, we will often use a PDF derived directly from our N-body simulations. However, for illustration, we will also show results obtained using a simple lognormal approximation to the PDF (Coles & Jones 1991):

$$p(\delta; R)d\delta = \frac{1}{(2\pi)^{1/2}\sigma_l} \exp\left[-\frac{(\ln\rho + \sigma_l^2/2)^2}{2\sigma_l^2}\right] \frac{d\delta}{\rho}, \tag{22}$$

where $\rho = (1+\delta)$, $\sigma_l^2 = \ln[1+\sigma^2(R)]$, and $\sigma(R)$ is the *rms* overdensity fluctuation in a sphere of radius R. The latter can be obtained from the initial power spectrum through the formula given by Jain, Mo & White (1995). It turns out that such a PDF works remarkably well on scales where the average mass correlation function $\bar{\xi}_{\rm m}(R) \lesssim 1$.

2.4. The autocorrelation functions of dark haloes

In analogy with the procedures of the last section we can define an average autocorrelation function at $z=0$ for haloes of mass M_1 identified at redshift z_1 by

$$\bar{\xi}_{\rm hh}(R, M_1, z_1) = \langle[\delta_{\rm h}(1|0)]^2\rangle_R = \frac{\int_{-\infty}^{\infty} [\mathcal{N}(1|0)]^2\, p(\delta; R)d\delta}{[n(M_1, z_1)V]^2} - 1, \tag{23}$$

where, as before, R_0 and δ_0 in $\mathcal{N}(1|0)$ are related to R and δ by the spherical collapse model. In the limit where $R_1 \ll R_0$ and $|\delta_0| \ll \delta_1$ this gives

$$\bar{\xi}_{\rm hh}(R, M_1, z_1) = [b(M_1, z_1)]^2\, \bar{\xi}_{\rm m}(R), \tag{24}$$

where $\bar{\xi}_{\rm m}(R)$ is the average mass correlation function. If δ_1 is large (i.e. $z_1 > 1$), equation (24) follows from eq.(23), even when $\bar{\xi}_{\rm m}(R) \gtrsim 1$.

It is important to note that the average correlation function $\bar{\xi}_{\rm hh}$ defined by equation (23) is not the same as the conventional one based on the variance of counts in randomly placed spheres (e.g. Peebles 1980, §36). This is because $\bar{\xi}_{\rm hh}(R)$ does not include the scatter of halo counts among spheres which have the same mean mass overdensity as well as the same radius. Let us denote the conventional average autocorrelation function by $\sigma_{\rm hh}^2$. Then

$$\sigma_{\rm hh}^2(R) = V^{-2} \int\int \xi_{\rm hh}(|\mathbf{x}-\mathbf{y}|) d^3x d^3y = \bar{\xi}_{\rm hh}(R) + \frac{\langle\mu(R,\delta)\rangle_R}{[n(M_1, z_1)V]^2} - \frac{1}{n(M_1, z_1)V}, \tag{25}$$