Homework 1

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Problem 1 (a)

$$\binom{10}{8} = \binom{10}{2} = \frac{10 \times 9}{2 \times 1} = 45$$

(b) The inverse of $1, 2, 3, 4, 5, 6$ in $\mathbb{Z}_7$ are $1, 4, 5, 2, 3, 6$.

(c) 8

Problem 2 For every ordered pair (A, B) which $A \cap B = \emptyset$, each element $x \in [n]$ has one of these three states: $x \in A$, $x \in B$ and $x \in [n] - A - B$.

And clearly that the map between ordered pairs and state vector of elements is bijective.

So the total number of ordered pairs is 3^n.

Problem 3

$$\begin{aligned}
(x+y)^n &= \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r} \\
\frac{d(x+y)^n}{dx} &= \sum_{r=0}^{n} r \binom{n}{r} x^{r-1} y^{n-r} && = n(x+y)^{n-1} \\
\frac{d(nx(x+y)^{n-1})}{dx} &= \sum_{r=0}^{n} r^2 \binom{n}{r} x^{r-1} y^{n-r} && = n(x+y)^{n-1} + n(n-1)x(x+y)^{n-2}
\end{aligned}$$

Let $x = y = 1$, $\sum_{r=0}^{n} r^2 \binom{n}{r} = n2^{n-1} + n(n-1)2^{n-2} = n(n+1)2^{n-2}$

Problem 4 (a) Let i denotes the size of the blue square.

$$\sum_{i=1}^{9} (9 - i + 1)^2 = 285$$

(b) TODO

Problem 5

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