Homework 12

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Problem 1 First, we have discussed in the class that the maximum size of this intersecting family is 2^{n-1} by pair each set and its complement.

Suppose that there exists a intersecting family $\mathcal{F}$ with size 2^{n-1} such that for some $A \subset B$, $A \in \mathcal{F}$ but $B \notin \mathcal{F}$. Since $A \subset B$, every set intersects A that intersects B, $\mathcal{F} \cup \{B\}$ is also an intersecting family but its size would be greater than 2^{n-1}, which is impossible. #

Problem 2 $\binom{10}{3} = 120$, take every set from $2^{[12]}$ which contains 1 and 2.

Problem 3 Take 5 sets with size 4: $\{1,2,3,4\}, \{2,3,4,5\}, \{3,4,5,1\}, \{4,5,1,2\}, \{5,1,2,3\}$. The intersection between any two of them has size 3. Then there are 6 elements remained in [11] and $5 \times 6 = 30$.

Problem 4 $f(n) = 1$. The example for $m = 1$ can be constructed easily.

Proof: by contradiction

Let $a_i = 2^i$. Assume that $m > 1$ sets are picked. Suppose p is the largest element in $\cup_{i=1}^{m} A_m$ and $p \in A_k$. Since $\sum_{i=1}^{p-1} 2^i = 2^p - 1$. In order to satisfy the condition $|S(A_i) - S(A_k)| < 1$ for some i, A_i must contain some elements larger than p, which violate the assumption. #

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