Homework 12

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Problem 1 First, we have discussed in the class that the maximum size of this intersecting family is 2^{n-1} by pair each set and its complement.

Suppose that there exists a intersecting family $\mathcal{F}$ with size 2^{n-1} such that for some $A \subset B$, $A \in \mathcal{F}$ but $B \notin \mathcal{F}$. Since $A \subset B$, every set intersects A that intersects B, $\mathcal{F} \cup \{B\}$ is also an intersecting family but its size would be greater than 2^{n-1}, which is impossible. #

Problem 2 $\binom{10}{3} = 120$, take every set from $2^{[12]}$ which contains 1 and 2.

Problem 3 Take 5 sets with size 4: $\{1,2,3,4\}, \{2,3,4,5\}, \{3,4,5,1\}, \{4,5,1,2\}, \{5,1,2,3\}$. The intersection between any two of them has size 3. Then there are 6 elements remained in [11] and $5 \times 6 = 30$. And with $\{1,2,3,4,5\}$ there are 31 sets.

Problem 4 $f(n) = \binom{n}{n/2}$

Proof:

For every subset A, denote $A_+ = \{\text{all positive elements lie in } A\}$, $A_- = A \setminus A_+$.

If there are two distinct set A and B, $A_+ \subseteq B_+$, $B_- \subseteq A_-$ infers that $|S(A) - S(B)| \geq 1$ (since every number has absolute value greater than 1).

Define $A \preceq B$ iff $A_+ \subseteq B_+$ and $B_- \subseteq A_-$. This partial order is equivalent to $\subseteq$ in $2^{[n]}$ (If A contains positive a_i then set the i-th bit as 1 otherwise 0. If A contains negative a_i then set the i-th bit as 0 otherwise 1.)

Clearly $\{A_m\}$ we choose must form an anti-chain in this poset, and we know that the maximum anti-chain size in $2^{[n]}$ is $\binom{n}{n/2}$. #

Example:

Let $a_i = 1$. Choose all sets of size $n/2$.

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