Homework 2

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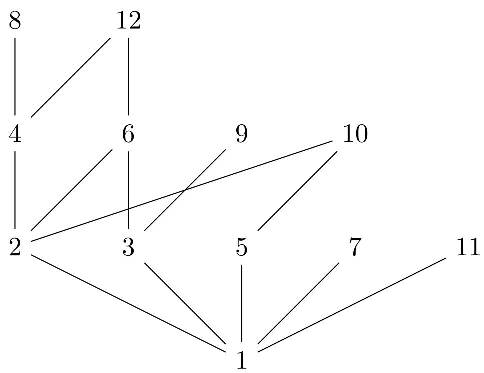
Problem 1 The probability of that number is a multiple of 2 is $1-(\frac{1}{2})^n$.

The probability of that number is a multiple of 5 is $1-(\frac{4}{5})^n$.

So the probability of that number is a multiple of 10 is $(1-(\frac{1}{2})^n)\times(1-(\frac{4}{5})^n)$.

Problem 2 The figure is drawn below:



Problem 3

$$
\begin{aligned}
(1-x)^n &= \sum_{i=0}^{n}(-1)^i x^i \binom{n}{i} \\
\frac{d(1-x)^n}{dx} &= \sum_{i=1}^{n}(-1)^i i x^{i-1}\binom{n}{i} = -n(1-x)^{n-1}
\end{aligned}
$$

Simply let $x=1$, $\sum_{i=1}^{n}(-1)^i i\binom{n}{i}=0$.

Problem 4 Assume that $|A|=a$, $|B|=b$ and $A\cap B=\emptyset$.

The left side of equation means that first choose i elements from A and add them to B. Then choose a elements from B. This is the size of $\{(X,Y)|X\subseteq A, |Y|=a, Y\subseteq A\cup B, X\cap Y=\emptyset\}$ by first choosing X and then choosing Y.

Another way to cound the size of set above is first choosing Y by enumerating $i=|Y\cap B|$, and then counting the number of X avaiable. Which is $\sum_{i=0}^{a}\binom{b}{i}\binom{a}{a-i}2^i$ and equal to the right side.

Problem 5 Assume that A and C are non-empty set.

Denote that a is the maximum of A and c is the minimum of C. Clearly that $c \not\preceq a$.

We have known that

$$
\begin{aligned}
\forall x,y, \sum_{x\preceq z\preceq y}\mu(x,z) &= \chi_{x=y} \\
\forall x,y, \sum_{x\preceq z\preceq y}\mu(z,y) &= \chi_{x=y}
\end{aligned}
$$

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And X is a lattice so that there is a maximum 1 and a minimum 0 therefore:

$$\sum_{x\in A}\sum_{y\in C}\mu(x,y) = -\sum_{x\in A}\sum_{y\in B}\mu(x,y) - \sum_{x\in A}\sum_{y\in A, x\preceq y}\mu(x,y)$$

And for every $x \in A$,

$$\sum_{y\in A, x\preceq y}\mu(x,y) = \sum_{x\preceq y\preceq a}\mu(x,y) = \chi_{x=a}$$

$$\sum_{x\in A}\sum_{y\in A, x\preceq y}\mu(x,y) = 1$$

Now i'm going to show that

$$\sum_{x\in A}\sum_{y\in B}\mu(x,y) + \sum_{x\in B}\sum_{y\in B}\mu(x,y) = \sum_{x\notin C}\sum_{y\in B}\mu(x,y) = 0$$

.

For fixed $y \in B$,

$$\sum_{x\notin C}\mu(x,y) = \sum_{0\preceq x\preceq y}\mu(x,y) = 0(\text{Since } A \neq \emptyset, y \neq 0)$$

$$\sum_{x\in A}\sum_{y\in B}\mu(x,y) + \sum_{x\in B}\sum_{y\in B}\mu(x,y) = 0$$

Therefore $\mu(A,C) = \mu(B,B) - 1$.