Q1)

(a) Essential Matrix : $E = [t]_x R$ — (A)

Here, $[t]_x = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$

Let e_r be the right null space of E

" e_l " " left " " " E

Then,

$$E e_l = 0 \quad — (1)$$

&

$$e_r^T E = 0 \quad — (2)$$

From (1) & (A)

$$[t]_x R e = 0$$

$[t]_x$ is a skew symmetric matrix of t, $t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$

If t is any vector parallel to $[t]_x$, then

$$t \times t_x = 0$$

Let $e = R^T t$

then,

$$[t]_x R e = [t]_x R (R^T t)$$

$$= [t]_x t = 0 \quad (\because R \cdot R^T = I)$$

$\therefore$

$$\boxed{\begin{array}{c} e_L = R^T t \\ \text{Left Epipole} \end{array}}$$

From (2) & (A)

$$e_R^T E = 0$$

$$e_R^T [t]_x R = 0 \quad -(3)$$

$[t]_x$ is a skew-symmetric matrix, then its own righ null space will be parallel to t.

$$\therefore \quad t^T [t]_x = 0 \quad - (4)$$

Using (3) & (4)

$$\boxed{e_r = t}$$

Righ epipole

(b)

$$R = I \quad \& \quad t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$$

$$[t]_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$E = [t]_x R = [t]_x I$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [t]_x$$

Let P be a 3D world homogeneous coordinate.

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Let P_1 to P_2 be image points by left cam & right cam.

$P_1 = P$, because there is no rotation or translation

$\therefore P_2^T E P_1 = 0$; $P_1 = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ & $P_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -t_x \\ t_x y_1 \end{bmatrix} = 0$$

$$\hookrightarrow 0 - t_x y_2 + t_x y_1 = 0$$

$$\therefore y_2 = y_1$$

$\therefore$ y coordinates are same in 2 camera image points.

So,
epipolar lines reflecting corresponding points lie on same y.