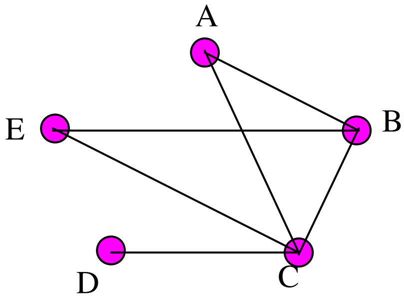
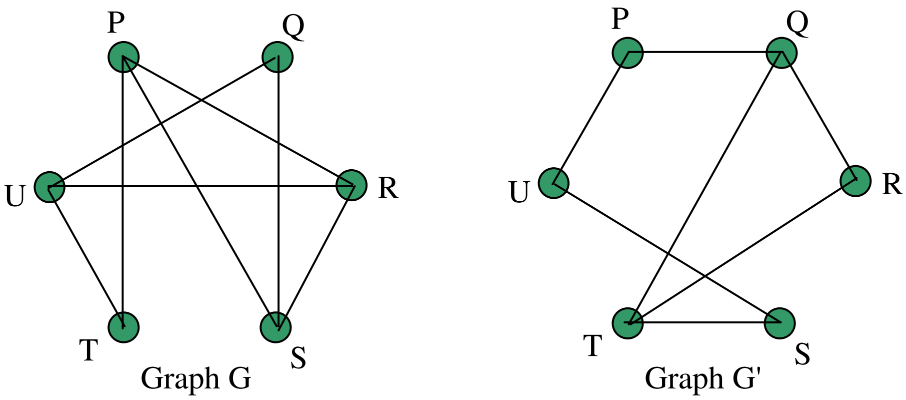
Answers to Tutorial Exercises Set 10

1.

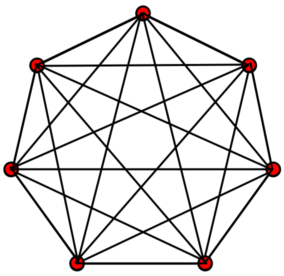


2.



Still to play: P-Q, P-U, Q-R, Q-T, R-T, S-T, S-U

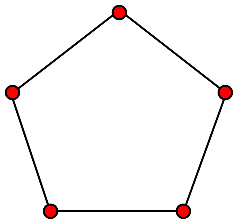
3. (i)



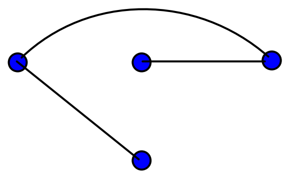
(ii) There are 21 edges in K_7.

(iii) 5 vertices: 10 edges;
6 vertices: 15 edges;
n vertices: $\frac{1}{2}$ n(n-1) edges.

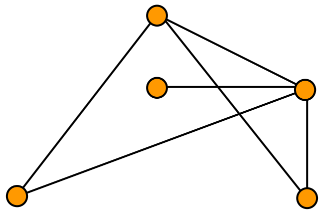
4. (i)



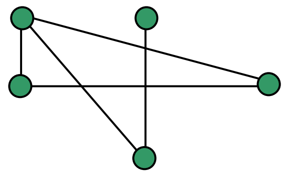
(ii)



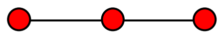
(iii)



(iv)

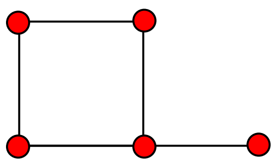


5. (i)

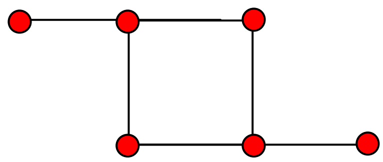


(ii) not possible because sum of degrees is odd

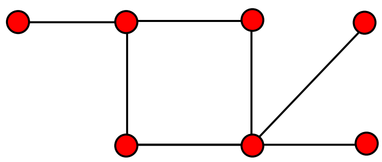
(iii)



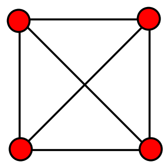
(iv)



(v)



(vi)



6. One such function is f(A) = P, f(B) = R, f(C) = T, f(D) = Q, f(E) = S.
(There are other possibilities.)

7. (i) The first graph has two vertices of degree 3 that are adjacent to each other, and the second graph does not. Alternatively, the first graph has a closed loop consisting of 3 edges, and the second does not.

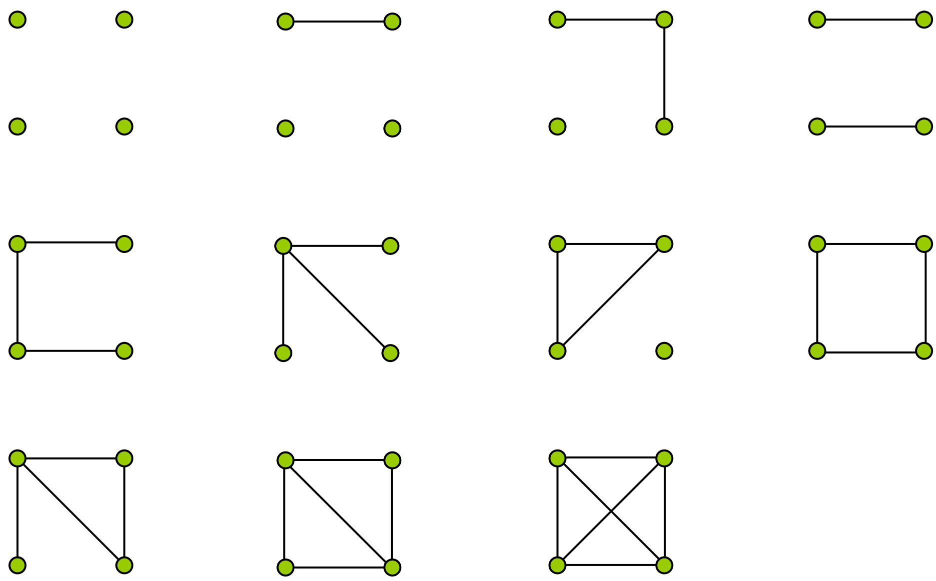
(ii) The first graph has a closed cycle of 3 edges (in fact two), but the second graph has none.

(iii) The first graph has four vertices of degree 3, but the second graph has only two vertices of degree 3. Alternatively, the second graph has a vertex of degree 4, but the first graph has no such vertex.

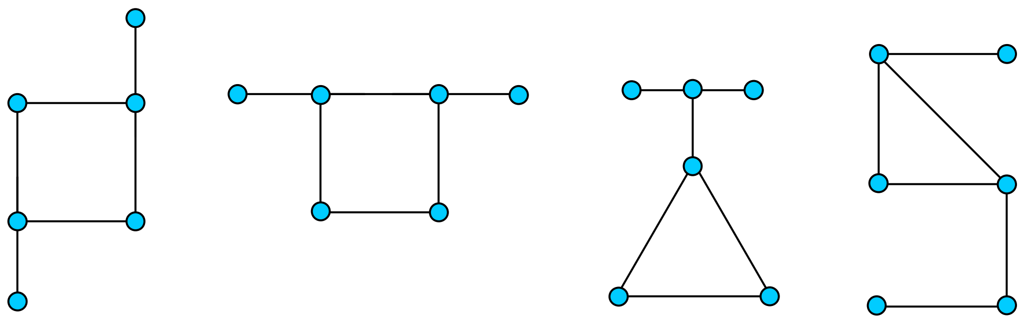
8. The graphs are isomorphic. For example consider the function f such that:
$f(A) = 2$, $f(B) = 3$, $f(C) = 6$, $f(D) = 1$, $f(E) = 4$, $f(F) = 5$.

9. The graphs are isomorphic. For example consider the mapping:
$A \to Q$, $B \to R$, $C \to U$, $D \to V$, $E \to P$, $F \to S$, $G \to W$, $H \to T$

10.



11.



12.

(i)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

(ii)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix} \end{array}$$

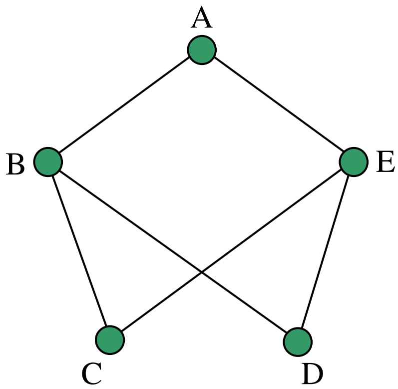
(iii)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

(iv)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

13.



14. (i) One possible mapping is: $A \to R$, $B \to T$, $C \to Q$, $D \to P$, $E \to S$
(There are many other possible mappings)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ R \\ T \\ Q \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} R & T & Q & P & S \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

The matrix entries are identical, so the graphs are isomorphic.

15. (i) One possible mapping is:

$A \to P,\quad B \to R,\quad C \to T,\quad D \to Q,\quad E \to S,\quad F \to U$

(There are many other possible mappings)

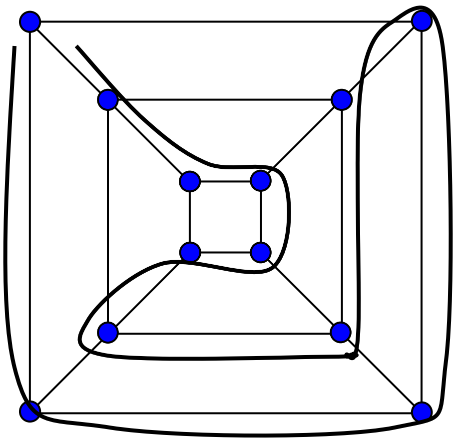
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ P \\ R \\ T \\ Q \\ S \\ U \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} P & R & T & Q & S & U \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}$$

The matrix entries are identical, so the graphs are isomorphic.

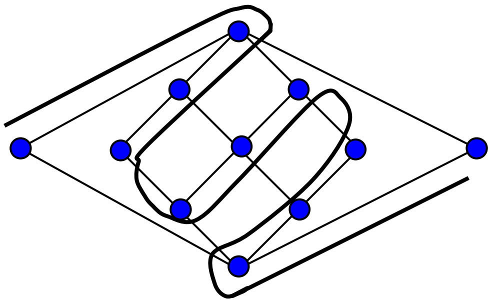
16. (a) Eulerian circuit. (Hint: Start at any vertex)
(b) No Eulerian path, because more than two odd vertices.
(c) Eulerian path (Hint: Start at a vertex of degree 3)
(d) Eulerian circuit. (Hint: Start at any vertex)
(e) No Eulerian path, because more than two odd vertices.
(f) Eulerian path (Hint: Start at a vertex of degree 3)
(g) No Eulerian path, because more than two odd vertices.
(h) Eulerian circuit. (Hint: Start at any vertex)

17. (a) Hamiltonian circuit
(b) Hamiltonian path
(c) no Hamiltonian path
(d) no Hamiltonian path
(e) Hamiltonian circuit
(f) Hamiltonian path
(g) Hamiltonian circuit
(h) Hamiltonian circuit
(j) Hamiltonian circuit

18.



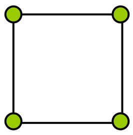
Graph A



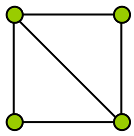
Graph B

19. (i) 3126527547843 is an Eulerian circuit
(ii) 487562134 is a Hamiltonian circuit.

20. (i)



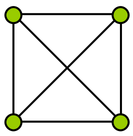
(ii)



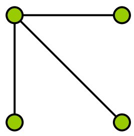
(iii) no graph

(iv) no graph

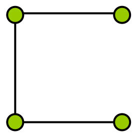
(v)



(vi)

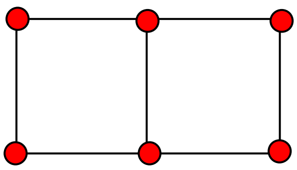


(vii)

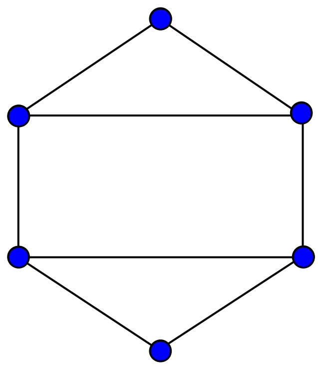


21.

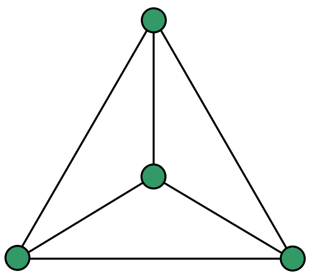
(i)



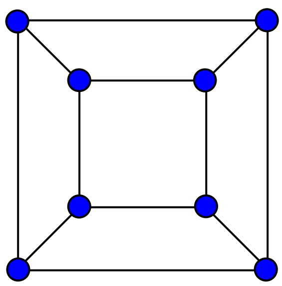
(ii)



(iii)



(iv)



22. Consider the mapping: $A \to P$, $B \to Q$, $C \to T$, $D \to U$, $E \to S$, $F \to R$

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}
\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix} \end{array}
\qquad
\begin{array}{c} \\ P \\ Q \\ T \\ U \\ S \\ R \end{array}
\begin{array}{c} \begin{array}{cccccc} P & Q & T & U & S & R \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{pmatrix} \end{array}
$$

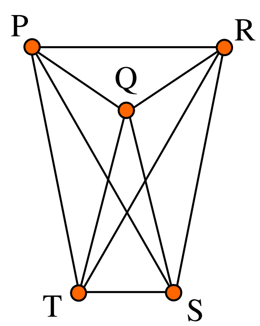
The matrix entries are identical. So graphs A and B are isomorphic. Then, because graph B is planar, so is graph A.

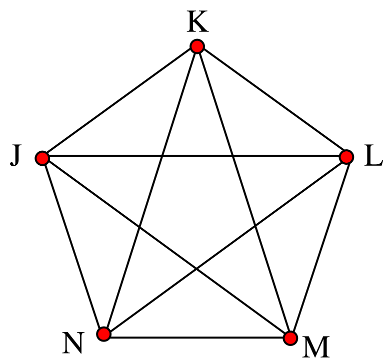
23. Consider the mapping: $P \to A$, $Q \to D$, $R \to B$ $S \to E$, $T \to C$, $U \to F$

$$
\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \\ U \end{array}
\begin{array}{c} \begin{array}{cccccc} P & Q & R & S & T & U \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix} \end{array}
\qquad
\begin{array}{c} \\ A \\ D \\ B \\ E \\ C \\ F \end{array}
\begin{array}{c} \begin{array}{cccccc} A & D & B & E & C & F \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix} \end{array}
$$

The matrix entries are identical. So graphs A and $K_{3,3}$ are isomorphic. Then, because graph $K_{3,3}$ is non-planar, so is graph A.

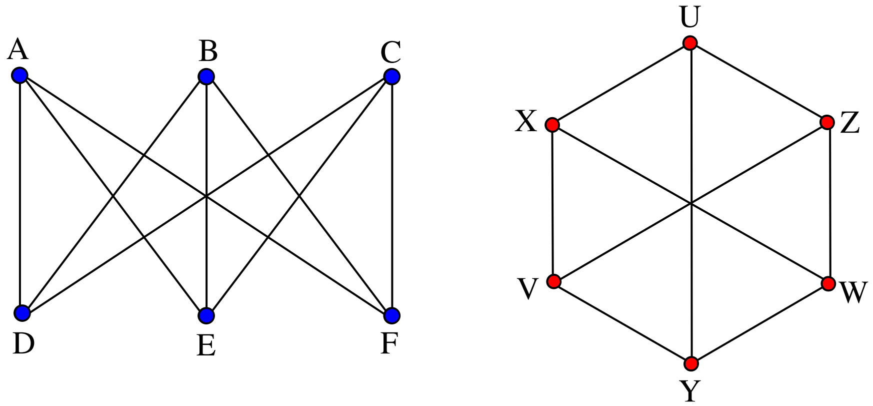
24.





Hint: Use the mapping $P \to J$, $Q \to K$, $R \to L$, $S \to M$, $T \to N$

25.



Hint: Use the mapping $A \to U$, $B \to V$, $C \to W$, $D \to X$, $E \to Y$, $F \to Z$

26. From the circuit, $P = x \oplus y$, $Q = x \oplus P$ and $R = P \oplus y$

x	**y**	**P**	**Q**	**R**
0	0	0	0	0
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

From the truth table, $Q \equiv y$ and $R \equiv x$.