ENS1161 Computer Fundamentals

ASSIGNMENT 2

Semester 2, 2014

Topics covered:

Relations, Functions, Bases and Number Systems, Computer representation of numbers, BCD addition and ASCII codes.

Source material:

Lectures 5 to 8 and tutorial exercise sheets available on the Blackboard.

Marks:

15% of total marks for the unit. The assignment will be marked out of 40 and scaled to a mark out of 15.

Due date:

Week 10 of semester (see Blackboard or lecturer for details).
A penalty for late submissions will apply as specified in the ECU handbook.

Submission instructions:

Please ensure that you submit your assignment in the correct box.
(No responsibility will be taken for assignments submitted incorrectly)
If you are unsure, check with your lecturer or tutor.

Applications for extension:

Applications for extension (on medical or other grounds) should be made in writing to the unit coordinator before the due date, and should be accompanied by appropriate evidence (e.g. medical certificate) to support your application.

Presentation:

You may use a computer package to prepare your submission, but it is not essential. In some cases it may be preferable to put in diagrams and symbolic expressions by hand if you are not competent to produce an acceptable result using a computer package. You should use one side of each sheet only. Plastic covers are not required; in fact they usually complicate the processing of submissions.

Question 1

Consider the functions f, g and h, all defined on the set {0, 1, 2, 3, …, 12}

x	0	1	2	3	4	5	6	7	8	9	10	11	12
f(x)	0	5	10	4	3	1	12	7	11	9	2	8	6

x	0	1	2	3	4	5	6	7	8	9	10	11	12
g(x)	5	4	11	0	6	10	2	7	1	12	9	8	3

x	0	1	2	3	4	5	6	7	8	9	10	11	12
h(x)	3	6	0	10	9	5	2	12	1	7	11	4	8

(i) Write down the values of: $g(f(h(7)))$ and $h^{-1}(g^{-1}(3))$

(ii) Construct a table of values (like those shown above) for $h(g^{-1}(x))$.

(iii) Construct a table for $f(f(x))$. What can you conclude about the inverse of f?

(iv) Construct a table for $h^{-1}(x)$, and draw its graph on the grid provided on the last page.

[5 marks]

Question 2

Suppose there is a set of growers G = {a, b, c, d}, a set of retailers R = {e, f, g} and a set of customers C = {m, n, p, q, r}. There are two relations A and B on $G \times R$ and $R \times C$, respectively, defined by:

aAe, aAg, bAf, cAf, cAg, dAe, and eBn, eBq, fBp, gBm, gBr

xAy means "grower x sold goods to retailer y", and
$yA^{-1}x$ means "retailer y bought goods from grower x"

xBy means "retailer x sold goods to customer y", and
$yB^{-1}x$ means "customer y bought goods from retailer x"

(i) Find the matrices M(A) and M(B) that represent the relations A and B.

(ii) Find the matrices $M(A)^T$ and $M(B)^T$ that represent the relations A^{-1} and B^{-1}

(iii) Consider the two queries:

Which customers have received goods that came from the same grower(s) as those goods received by (a) customer p? (b) customer q?

Find the logical matrix products M(A) M(B) and then $M(B)^T M(A)^T$, and finally $M(B)^T M(A)^T M(A) M(B)$, and hence answer the queries.

[Hint: To see a similar exercise, look at the "Application" on page 13 of Lecture 5.]

[1 + 1 + 3 = 5 marks]

Question 3

(a) Consider the following table:

decimal	octal	binary	hexadecimal
133 →	→	→	↓
102 →	→	→	↓
←	←	←	

(i) Convert each of the decimal numbers in the first column to octal.
(ii) Convert the two octal numbers to binary.
(iii) Convert the two binary numbers to hexadecimal.
(iv) Add the two hexadecimal numbers.
(v) Convert the hexadecimal sum to binary, then to octal and then to decimal.
(Give your answers in a table like that above.)

(b) (i) Convert the decimal fraction 0.21875 to binary;
(ii) Convert the decimal fraction 0.40625 to binary;
(iii) Add the two binary fractions from (i) and (ii);
(iv) Convert the binary fraction from (iii) to decimal.
(Show your working)

(c) Add the following, given that (i) is binary, (ii) is octal and (iii) is hexadecimal:

(i)
```
 1001011
 1100011 +
```
(ii)
```
 3375
 6743 +
```
(iii)
```
 3DE8C
 7A552 +
```

(d) Perform "BCD additions" on the following pairs of hexadecimal numbers.
(Show all your working)

(i)
```
 23267
 49684 +
```
(ii)
```
 592778
 183983 +
```

[5 + 2 + 3 + 2 = 12 marks]

Question 4

For each of the following, suppose that two 8-bit binary numbers have been added. In each case the 8-bit output is given and the values of the N, V and C flags. For each case give the correct answer as a decimal number:

(a) if the result is interpreted as the sum of **unsigned** integers;
(b) if the result is interpreted as the sum of **signed** integers.

	8-bit output	N	V	C
(i)	1100 0000	1	1	0
(ii)	0011 1111	0	1	1
(iii)	0010 1011	0	0	1
(iv)	1100 1010	1	0	0
(v)	1100 1011	1	0	1

[10 × 1 = 10 marks]

Question 5

Introduction – Part 1: Two functions

This question uses two types of function that you may not have met before, the CEILING function CLG(x) and the MOD function MOD(x, n).

Definition of the CEILING function:

CLG(x) is the smallest integer that is greater than or equal to x. Or, if you prefer, it is the smallest integer that is not smaller than x.

eg. CLG(4.3) = ?
Which integers are not smaller than 4.3?
Answer: 5, 6, 7, …, etc
Of these, which is the smallest?
Answer: 5
So CLG(4.3) = 5

eg. CLG(7.5) = ?
Which integers are not smaller than 7.5?
Answer: 8, 9, 10, …, etc
Of these, which is the smallest?
Answer: 8
So CLG(7.5) = 8

eg. CLG(6) = ?
Which integers are not smaller than 6?
Answer: 6, 7, 8, …, etc
Of these, which is the smallest?
Answer: 6
So CLG(6) = 6

Practice exercises:

(i) CLG(3.8) = …………..   (ii) CLG(12.6) = ……………

(iii) CLG(11) = …………..   (iv) CLG(13.7) = ……………

(v) CLG(5) = …………..   (vi) CLG(6.9) = ……………

(vii) CLG(5.7) = …………..   (viii) CLG(15.6) = ……………

Answers: (i) 4; (ii) 13; (iii) 11; (iv) 14; (v) 5; (vi) 7; (vii) 6; (viii) 16

Definition of the MOD function:

MOD(x, n) is the remainder when x is divided by the whole number n.

eg. MOD(38, 5) = ?
When 38 is divided by 5, what is the remainder?
Answer: 3
So, MOD(38, 5) = 3

eg. MOD(29, 8) = ?
When 29 is divided by 8, what is the remainder?
Answer: 5
So, MOD(29, 8) = 5

eg. MOD(13, 4) = ?
When 13 is divided by 4, what is the remainder?
Answer: 1
So, MOD(13, 4) = 1

eg. MOD(6, 10) = ?
When 6 is divided by 10, what is the remainder?
Answer: 6
So, MOD(6, 10) = 6

eg. MOD(47, 12) = ?
When 47 is divided by 12, what is the remainder?
Answer: 11
So, MOD(47, 12) = 11

Practice exercises:

(i) MOD(17, 5) = …………..

(ii) MOD(75, 12) = ……………

(iii) MOD(36, 9) = …………..

(iv) MOD(45, 10) = ……………

(v) MOD(3, 8) = …………..

(vi) MOD(29, 6) = ……………

(vii) MOD(24, 4) = …………..

(viii) MOD(55, 7) = ……………

(ix) MOD(21, 5) = …………..

(x) MOD(5, 7) = ……………

Answers: (i) 2; (ii) 3; (iii) 0; (iv) 5; (v) 3; (vi) 5; (vii) 0; (viii) 6; (ix) 1; (x) 5

Introduction – Part 2: Application to Computer Science

Computer scientists often use **matrices** or rectangular **arrays** of numbers. Although they may appear on a computer monitor as a rectangular array, they are usually stored in memory locations as linear **sequences** of numbers. As a very simple case, suppose we have an array with 5 columns, such as:

18	**12**	**5**	**16**	**13**
6	**7**	**21**	**20**	**11**
22	**10**	**4**	**8**	**16**
2	**19**	**11**	**21**	**12**
9	**1**	**19**	**10**	**5**
14	**9**	**17**	**11**	**19**
4	**3**	**21**	**9**	**10**

and we want to store the array in a linear sequence starting with the first row, then the second and so on. So the sequence of stored values would begin:

18 12 5 16 13 6 7 21 20 11 22 10 4 8 16 2 19 11 … etc

There are two important types of questions concerning the storage and retrieval of such a set of numbers:

Type A: Given the row number and column number of an element in the array, how to determine its position in the linear sequence?

Type B: Given the position of an element in the linear sequence, how to determine its row number and column number in the array?

There are formulae that can be used to answer such questions, and they make use of the CEILING and MOD functions. They also use the variables:

num_cols — the number of columns in the array
row_num — the row number of the element
col_num — the column number of the element
seq_pos — the position of the element in the linear sequence

Here are the formulae that provide the answers:

$$\text{seq\_pos} = (\text{row\_num} - 1) \times \text{num\_cols} + \text{col\_num} \qquad \text{(answer to Type A)}$$

$$\left.\begin{aligned} \text{row\_num} &= \text{CLG}(\text{seq\_pos} / \text{num\_cols}) \\ \text{col\_num} &= \text{MOD}(\text{seq\_pos} - 1, \text{num\_cols}) + 1 \end{aligned}\right\} \qquad \text{(answers to Type B)}$$

For example, consider the element in the 3rd row and 4th column of the array given above (which happens to be an 8). Notice also that the array has 5 columns.

The Type A question is: Given that the element is in the 3rd row and 4th column, what is its position in the linear sequence?
Answer:

$$\text{seq\_pos} = (\text{row\_num} - 1) \times \text{num\_cols} + \text{col\_num}$$
$$= (3 - 1) \times 5 + 4 = 14$$

So the element is the 14th in the linear sequence (which is obviously true.)

The Type B question is: Given that the element is in the 14th position in the linear sequence, what are its row and column numbers in the array?
Answer:

$$\text{row\_num} = \text{CLG}(\text{seq\_pos} / \text{num\_cols})$$
$$= \text{CLG}(14 / 5) = \text{CLG}(2.8) = 3$$
$$\text{col\_num} = \text{MOD}(\text{seq\_pos} - 1, \text{num\_cols}) + 1$$
$$= \text{MOD}(14 - 1, 5) + 1 = \text{MOD}(13, 5) + 1 = 3 + 1 = 4$$

So the element is in the 3rd row and 4th column.

Practice examples: Fill in the blanks in the table

	Number of columns in array	Row number	Column number	Sequence position
(i)	8	5	3	
(ii)	12	6	2	
(iii)	15			50
(iv)	30			200

Answers: (i) 35; (ii) 62; (iii) 4, 5; (iv) 7, 20

Your task for Question 5

First work through the above examples **in detail** to make sure that you understand the process.
Then calculate the missing entries for the seven exercises in the table below.
(Give your answers in a table like the one below.)

	Number of columns in array	Row number	Column number	Sequence position
(i)	14	5	11	
(ii)	36	1	27	
(iii)	9	4	8	
(iv)	28	17	11	
(v)	30			98
(vi)	45			300
(vii)	24			85

[8 marks]

Grid for graph in Question 1