ENS1161 Computer Fundamentals
ENS4103 Computer Systems and Hardware

Tutorial Exercises Set 9

Related objectives from Unit Outline:
perform simple calculations using modulo arithmetic; solve simple congruences
apply the Addition and Multiplication Principles; determine whether a given counting problem involves sequences, permutations or subsets; apply the appropriate formula and hence determine the number of r-sequences, r-permutations or r-subsets of n objects

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1. State whether the following are true or false.

   (a) $22 \equiv 40 \pmod 9$
   (b) $35 \equiv 67 \pmod 8$
   (c) $34 \equiv 44 \pmod 6$
   (d) $15 \equiv 29 \pmod 4$
   (e) $12 \equiv 60 \pmod 3$
   (f) $13 \equiv 83 \pmod 7$
   (g) $-6 \equiv 2 \pmod 8$
   (h) $-3 \equiv 11 \pmod 7$

2. Find the least residues of the following:
   (a) $73 \pmod 7$
   (b) $28 \pmod 3$
   (c) $-17 \pmod 6$
   (d) $39 \pmod 5$
   (e) $-49 \pmod 8$
   (f) $74 \pmod 4$

3. Find the least residues of the following:
   (a) $503175 \pmod{11}$
   (b) $673528 \pmod{17}$
   (c) $498236 \pmod{13}$
   (d) $45210739 \pmod{23}$
   (e) $664195 \pmod{19}$
   (f) $7499051 \pmod{29}$

4. Find the least residues of the following:
(Hint: Replace numbers by their least residues during the calculation)

(a) $5342987 \times 4420931 \pmod{17}$ (b) $6634826^5 \pmod{19}$

(c) $(4399862 \times 3398106)^6 \pmod{29}$

5. Solve the following congruences (by trial and error, if necessary)

(a) $x + 7 \equiv 4 \pmod{9}$ (b) $w + 3 \equiv 5 \pmod{11}$

(c) $y + 8 \equiv 3 \pmod{13}$ (d) $z + 6 \equiv 1 \pmod{8}$

(e) $x + 4 \equiv 3 \pmod{5}$ (f) $y + 7 \equiv 2 \pmod{10}$

6. Solve the following congruences (by trial and error, if necessary)

(a) $5x \equiv 3 \pmod{11}$ (b) $7w \equiv 5 \pmod{9}$

(c) $9y \equiv 5 \pmod{12}$ (d) $4z \equiv 1 \pmod{7}$

(e) $2x \equiv 3 \pmod{5}$ (f) $3y \equiv 1 \pmod{10}$

7. Solve the following congruences (by trial and error, if necessary)

(a) $x^2 \equiv 5 \pmod{11}$ (b) $y^2 \equiv 2 \pmod{7}$

(c) $w^2 \equiv 4 \pmod{12}$ (d) $z^2 \equiv 3 \pmod{12}$

(e) $x^2 \equiv 5 \pmod{8}$ (f) $w^2 \equiv 10 \pmod{13}$

(g) $y^2 \equiv 3 \pmod{6}$ (h) $z^2 \equiv 5 \pmod{6}$

8. Start with the seed $x_0 = 47$ and generate 10 pseudo-random numbers using the formula $x_n = 47\, x_{n-1} \pmod{100}$.

9. Start with the seed $x_0 = 19$ and generate 10 pseudo-random numbers using the formula $x_n = 19\, x_{n-1} \pmod{100}$.

10. Start with the seed $x_0 = 2345$ and generate 6 pseudo-random numbers using the formula $x_n = 2345\ x_{n-1} \pmod{65536}$.

11. Calculate $7^{134} \pmod{23}$

12. Calculate $13^{77} \pmod{19}$

13. Consider the set of 4-digit numbers from 1000 to 9999.
    (i) How many of the numbers begin with the digits 34?
    (ii) How many of the numbers end with the digits 56?
    (iii) How many of the numbers begin with 34 and end with 56?
    (iv) How many of the numbers begin with 34 or end with 56, or both?
    (v) How many of the numbers begin with 34 or end with 56, but not both?
    (vi) How many of the numbers neither begin with 34 nor end with 56?

14. Consider the set of whole numbers from 1 to 100.
    (i) How many of the numbers are multiples of 5?
    (ii) How many of the numbers are multiples of 8?
    (iii) How many of the numbers are multiples of 5 and multiples of 8?
    (iv) How many of the numbers are either multiples of 5 or else multiples of 8?
    (v) How many of the numbers are neither multiples of 5 nor multiples of 8?

15. Consider a 100-page book with pages numbered from 1 to 100.
    (i) On how many pages would the page number contain a 0?
    (ii) How many times would the digit 0 be printed?
    (iii) On how many pages would the page number contain an 8?
    (iv) How many times would the digit 8 be printed?

16. How many 8-bit binary numbers are there? (ie. from 00000000 to 11111111)

17. Suppose an assembly language uses op-codes with 3 letters (such as LDA, ADA, DEB, INX, STA, ... etc). If all 26 letters of the alphabet are used, how many different op-codes can be represented?

18. A salesperson has to visit 8 different towns exactly once. In how many ways can this be done? (Assume that each town is connected directly to every other town.)

19. Numbers are to be formed from the digits 1, 2, 3 and 4. (Suppose that repetitions are **not** permitted).
    (i) How many possible 3-digit numbers are there?
    (ii) How many possible even 3-digit numbers are there?
    (iii) How many possible 3-digit numbers are there that are not greater than 200?
    (iv) How many possible 3-digit numbers are there that are not greater than 400?

20. Repeat the previous exercise assuming that repetitions **are** permitted.

21. How many 4-permutations are there of the letters of DYNAMO?

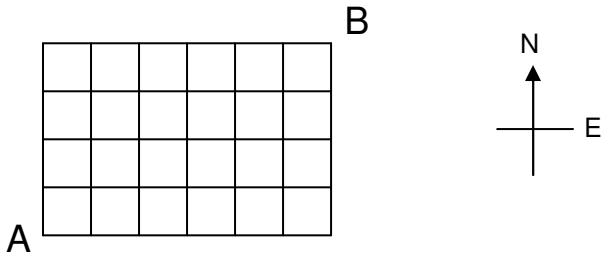
22. There are 16 teams in a competition. At the end of the season, how many different arrangements could there be for the top 6?

23. How many permutations are there of the letters of the word ORANGE?

24. How many 5-subsets are there of {a, b, c, d, e, f} ?

25. In a lottery, a player must select 6 number from 36. The winning draw is made by a mechanical selection of 6 marbles from a barrel of 36 marbles, numbered 1 to 36. If the player's selection matches the draw (in any order) then he/she wins. How many possible selections can a player make?

26. A and B are points on opposite corners of a grid, as shown:



How many shortest paths are there from A to B? (Hint: For a shortest path you can only travel North or East. So there must be 6 steps to the East and 4 steps to the North. If you choose which four steps will be to the North, there will be no choice for the 6 steps to the East.))

27. In a certain programming language, variable names must start with a letter and the subsequent characters may be upper case letters or decimal digits.
    (i) Find the number of possible 2-character variable names.
    (ii) Find the number of possible 3-character variable names.

28. A student must answer 3 out of 5 questions in a test.
(i) How many choices does she have?
(ii) How many choices does she have if she must answer the first question and two of the remaining questions?
(iii) How many choices does she have if she must answer one of the first two questions and two of the remaining questions?

29. Passwords for a certain computer system are of two types, restricted and unrestricted. The first character of a restricted password is R, and the first character of an unrestricted password is U. If restricted, the password has 3 more characters, all alphabetic. If unrestricted, the password has 4 more characters, which are decimal digits. How many possible passwords are there?

30. How many 8-bit binary numbers begin with 001?

31. How many 4-digit numbers from 1000 to 4999 are multiples of 5? (Multiples of 5 end in 5 or 0)

32. Find the number of 7-permutations of HEXAGON that start with A, or end with NX, or start with A and end with NX?

33. Find the number of 16-bit binary strings that start with 0101, or end with 001110, or start with 0101 and end with 001110.