ENS1161 Computer Fundamentals
ENS4103 Computer Systems and Hardware

Tutorial Exercises Set 10

Related objectives from Unit Outline:
identify isomorphic graphs and planar graphs; use matrix representation of graphs; identify Eulerian and Hamiltonian graphs

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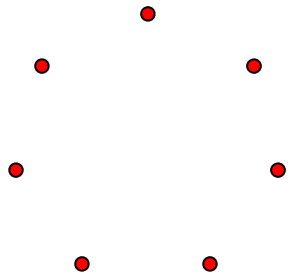
1. Draw a graph with vertices A, B, C, D and E given that: A is joined to B and C; B is joined to A, C and E; C is joined to A, B, D and E; D is joined to C; and E is joined to B and C

2. Six teams P, Q, R, S, T and U play in a competition in which each team plays every other team exactly once. So far eight matches have been played:
   P-R, P-S, P-T, Q-S, Q-U, R-S, R-U, T-U
   Use a graph and its complement to find which matches have not yet been played.

3. (i) Draw K_7, the complete graph on 7 vertices.

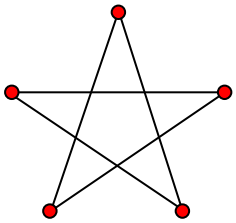
   (ii) How many edges should there be in K_7?

   (iii) How many edges will there be in a complete graph on 5 vertices? on 6 vertices? on n vertices?



4. Draw the complements of each of these graphs:

(i)



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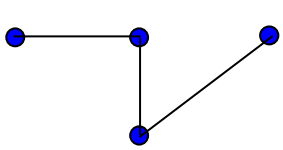

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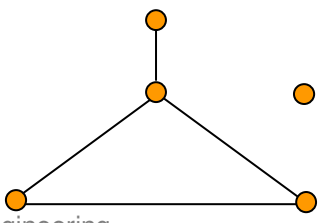

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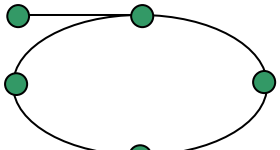
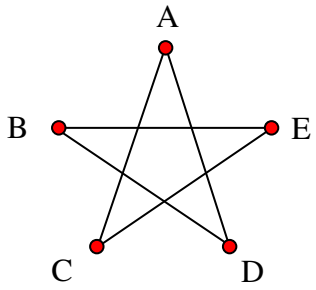
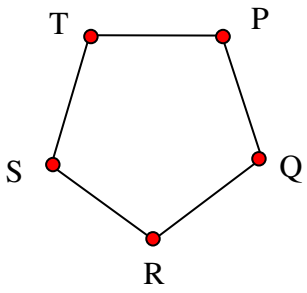


5. For each of the following, either draw a graph whose vertices have the degrees specified, or else explain why there is no such graph:

(i) 1, 2, 1
(ii) 1, 2, 3, 4, 5
(iii) 1, 2, 2, 2, 3
(iv) 1, 1, 2, 2, 3, 3
(v) 1, 1, 1, 2, 2, 3, 4
(vi) 3, 3, 3, 3

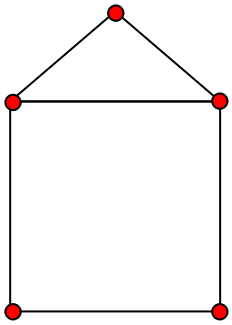
6. Find a function f: {A, B, C, D, E} → {P, Q, R, S, T} that shows that the following two graphs are isomorphic.

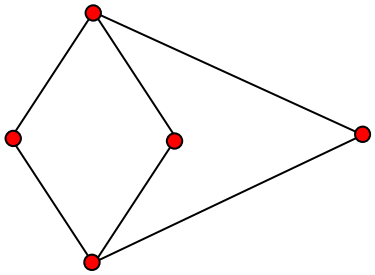




7. Explain why the following pairs of graphs are **not** isomorphic.

(i)





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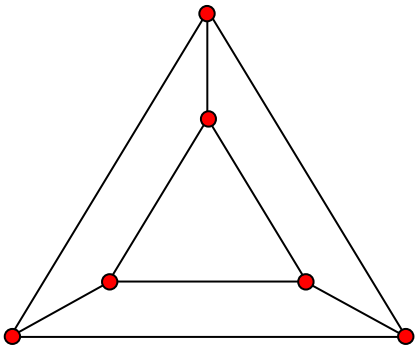

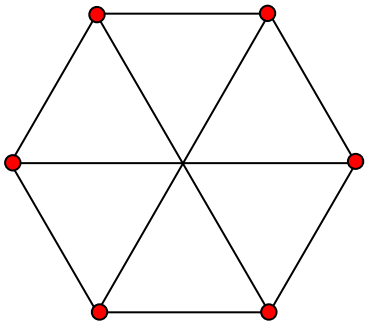

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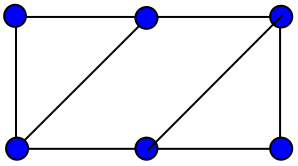
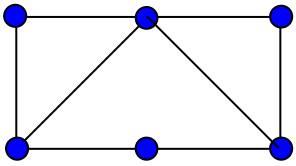
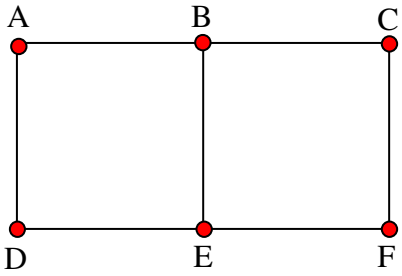
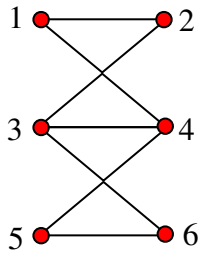


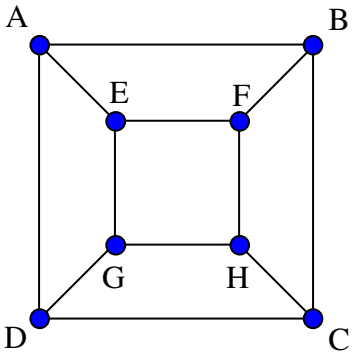


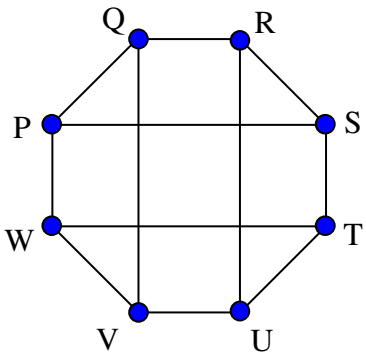
8. Determine whether the following graphs are isomorphic. If they are isomorphic, find a function f: {A, B, C, D, E, F} → {1, 2, 3, 4, 5, 6} that shows the isomorphism. If they are not isomorphic, explain why not.





9. Determine whether the following graphs are isomorphic. If they are isomorphic, find a function f: {A, B, C, D, E, F, G, H} → {P, Q, R, S, T, U, V, W} that shows the isomorphism. If they are not isomorphic, explain why not.



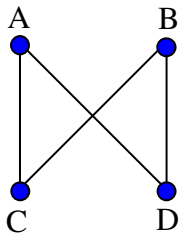


10. Draw 11 non-isomorphic graphs each with 4 vertices.

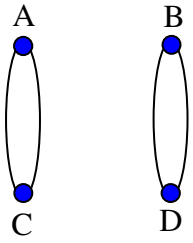
11. Draw 4 non- isomorphic graphs whose vertices have degrees 1, 1, 2, 2, 3, 3.

12. Construct adjacency matrices for the graphs shown:

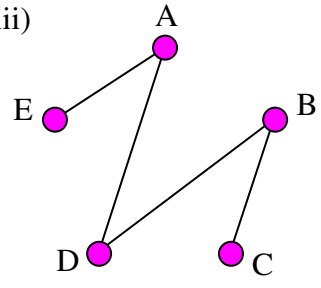
(i)



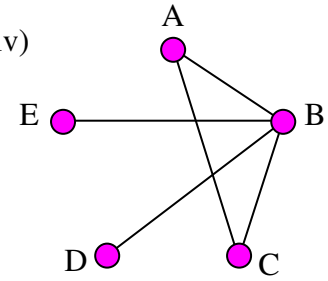
(ii)



(iii)



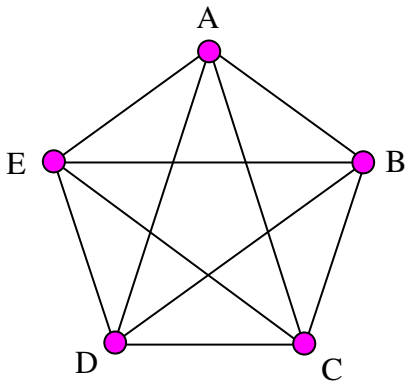
(iv)

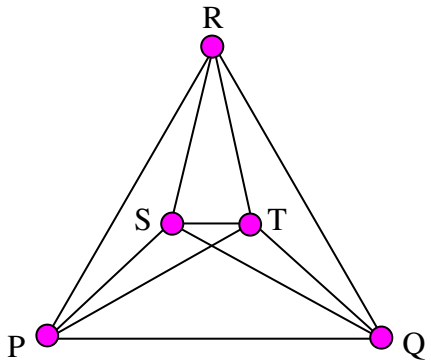


13. Draw a graph whose adjacency matrix is:

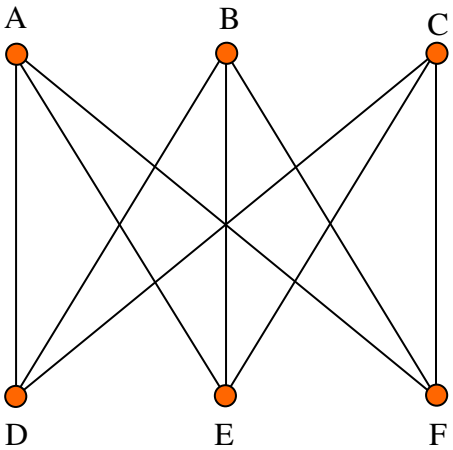
	A	B	C	D	E
A	**0**	**1**	**0**	**0**	**1**
B	**1**	**0**	**1**	**1**	**0**
C	**0**	**1**	**0**	**0**	**1**
D	**0**	**1**	**0**	**0**	**1**
E	**1**	**0**	**1**	**1**	**0**

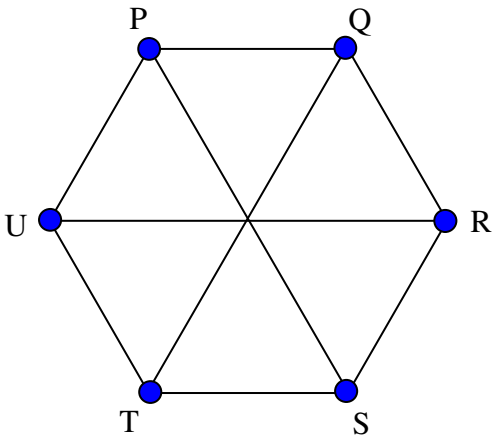
14 (i) Find a mapping that demonstrates that the two graphs below are isomorphic.
(ii) Appropriately label the rows and columns of their adjacency matrices.
(iii) Use the graphs to fill in each matrix, and check that they are identical.





15 (i) Find a mapping that demonstrates that the two graphs below are isomorphic.
(ii) Appropriately label the rows and columns of the adjacency matrices.
(iii) Use the graphs to fill in each matrix, and check that they are identical.



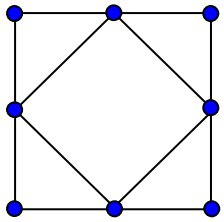


16. Determine whether each of the following graphs has:

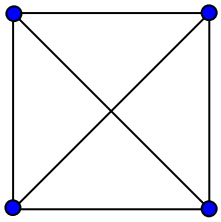
(i) no Eulerian path;
(ii) an Eulerian path, but no Eulerian circuit;
(iii) an Eulerian circuit.

Then, in the case of (i), explain why there is no Eulerian path, and in the case of (ii) or (iii), draw the path or circuit.

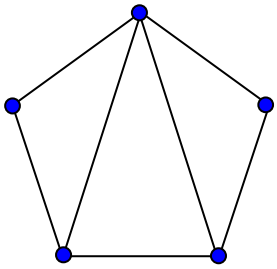
(a)



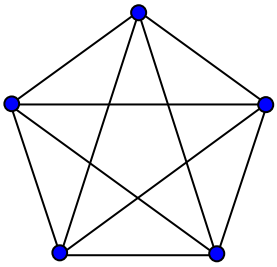
(b)



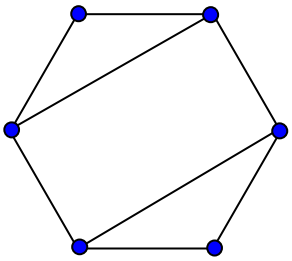
(c)



(d)



(e)



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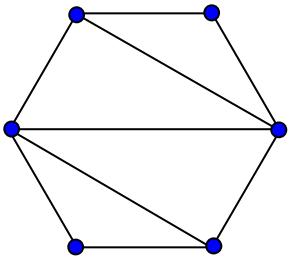

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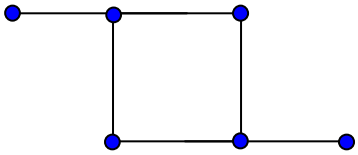

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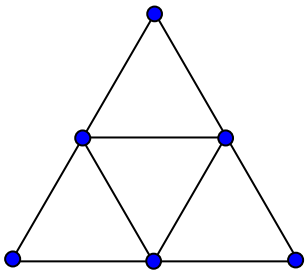


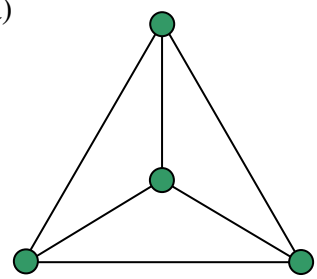
17. For each of the following graphs,

(i) find and draw a Hamiltonian circuit; or

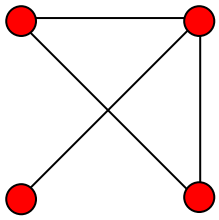
(ii) find and draw a Hamiltonian path; or

(iii) state that the graph has no Hamiltonian path.

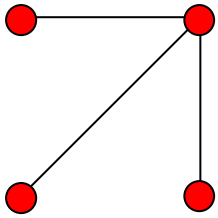
(a)



(b)



(c)



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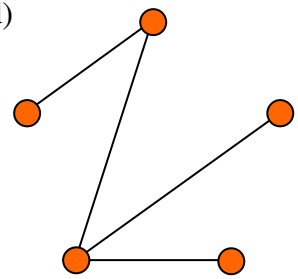

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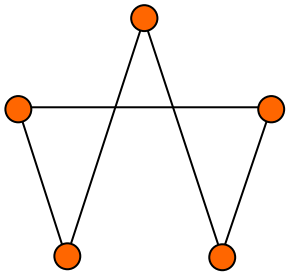

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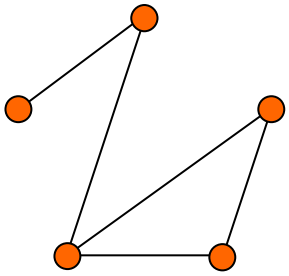

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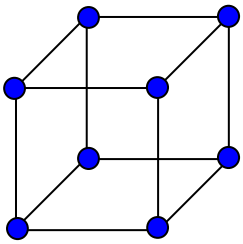

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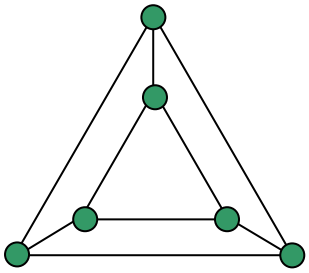

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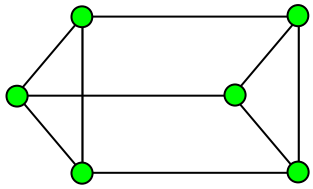
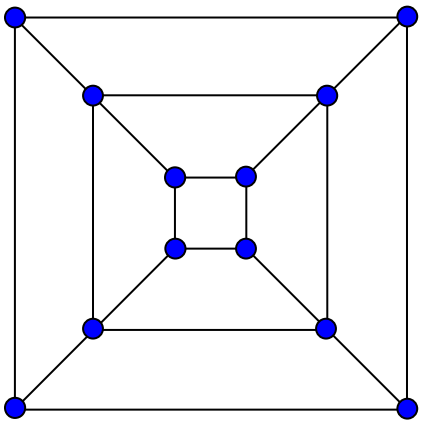
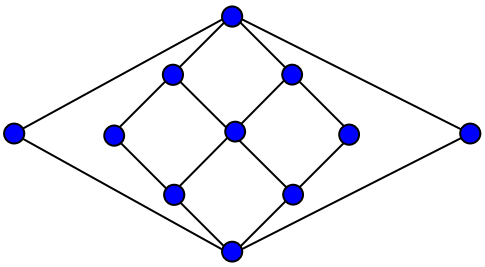


18. Find a Hamiltonian circuit in graph A and a Hamiltonian path in graph B.

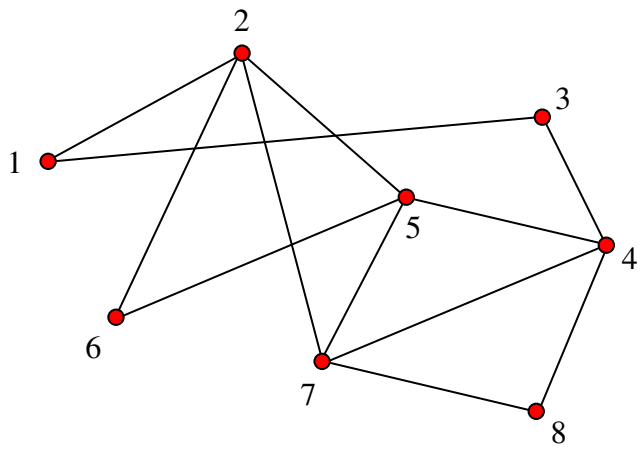


Graph A



Graph B

19. In the following graph, find (i) an Eulerian circuit; (ii) a Hamiltonian circuit.



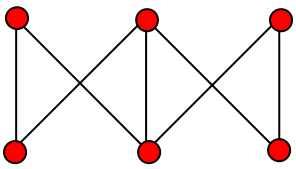
20. If possible, draw a (connected) graph with 4 vertices that has:

(i) an Eulerian circuit and a Hamiltonian circuit;
(ii) an Eulerian path and a Hamiltonian circuit;
(iii) an Eulerian circuit and a Hamiltonian path;
(iv) an Eulerian circuit but no Hamiltonian path;
(v) a Hamiltonian circuit but no Eulerian path;
(vi) neither an Eulerian path nor a Hamiltonian path;
(vii) an Eulerian path and a Hamiltonian path.

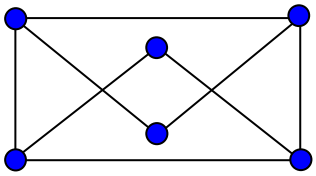
(A graph is "connected" if every pair of vertices are joined by a path)

21. For each of the following graphs, draw an isomorphic graph in which edges intersect only at vertices, and hence show that the given graph is planar.

(i)



(ii)



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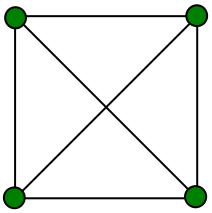
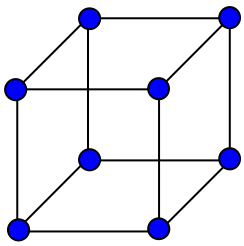
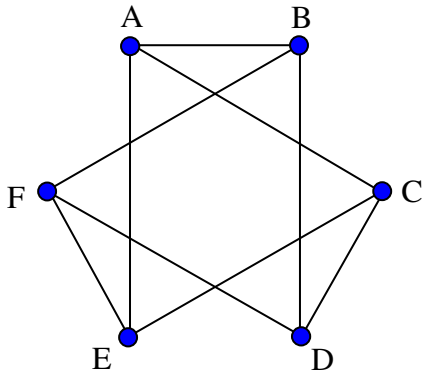


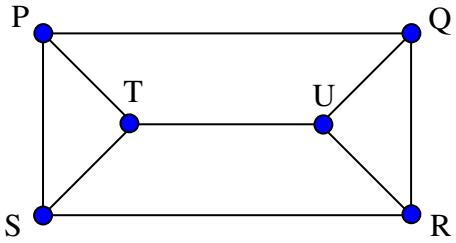
(iv)



22. Find a function that maps the vertices of graph A onto the vertices of graph B that shows that graphs A and B are isomorphic, and hence show that graph A is planar.

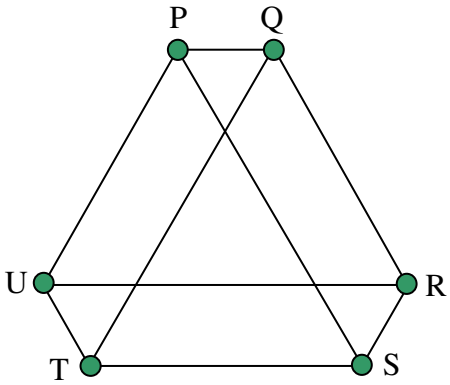


Graph A

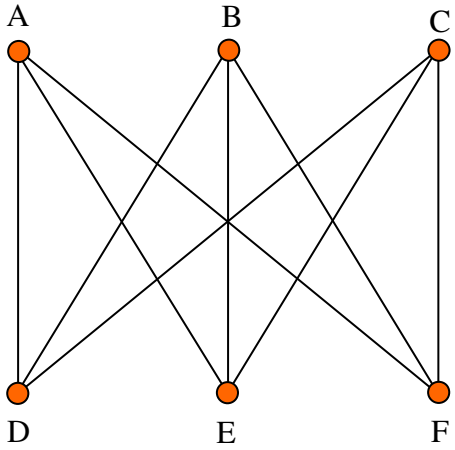


Graph B

23. Show that the graph A is isomorphic to $K_{3,3}$, and hence is graph A is non-planar.

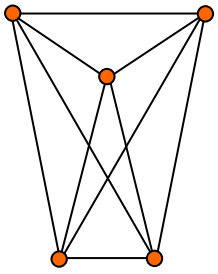


Graph A

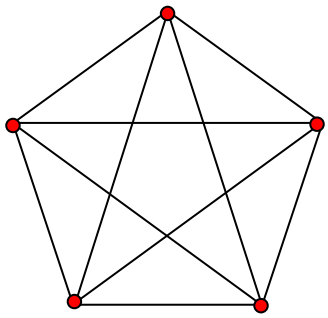


Graph $K_{3,3}$

24. Show that Graph A below is isomorphic to K_5, and hence is non-planar.

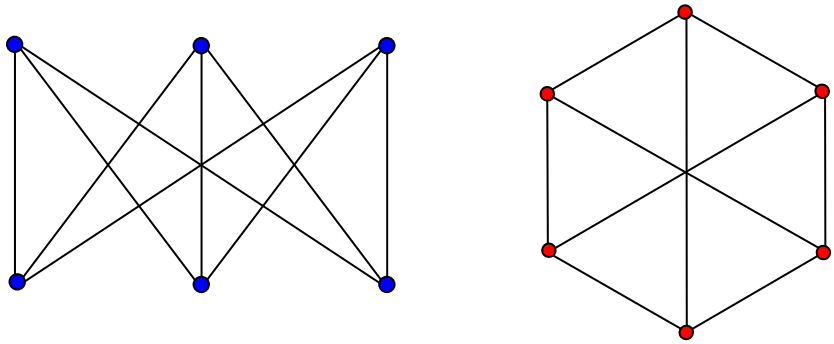


Graph A



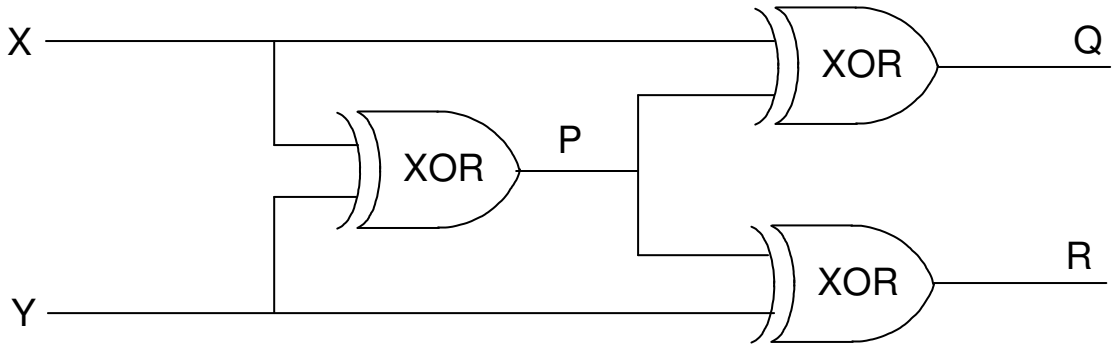
K_5

25. Show that Graph B below is isomorphic to $K_{3,3}$, and is therefore non-planar.



Graph B $K_{3,3}$

26. Construct a truth table for the outputs P, Q and R from each of the XORs in the circuit below and hence verify that Q = Y and R = X.



Challenge: Use Boolean algebra to show that:

(i) $x \oplus (x \oplus y) = y$ (ii) $y \oplus (x \oplus y) = x$

[Hint: Use the result $a \oplus b = a' b + a b'$]