ENS1161 Computer Fundamentals
ENS4103 Computer Systems and Hardware

Tutorial Exercises Set 11

Related objectives from Unit Outline:
find sums and products of matrices; apply matrices to directed graphs

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1. Suppose that $A = \begin{pmatrix} 1 & 3 & -4 & 0 \\ -2 & 2 & -1 & 5 \\ -3 & 4 & -5 & 6 \end{pmatrix}$.

   (i) What is the value of a_{13}? a_{21}? a_{32}? a_{41}?

   (ii) Which element is equal to -1?, 6? 5? -3?

2. Let $A = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 \\ 3 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 2 \\ 3 & -4 \end{pmatrix}$. Calculate the following matrices:

   (i) $A + C$ (ii) $3C$ (iii) $3B + 5C$

   (iv) $B - A$ (v) $(A + B) + C$ (vi) $A + (B + C)$

3. Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 3 & -1 \\ 0 & 2 & 0 \\ -2 & 1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 3 & -2 \\ 1 & 1 & -1 \\ 2 & 4 & -3 \end{pmatrix}$

Verify the following rules of matrix algebra by evaluating left and right sides of each rule and checking that they are equal.

(i) $A\,(B\,C) = (A\,B)\,C$

(ii) $A\,(B + C) = A\,B + A\,C$

(iii) $(B + C)\,A = B\,A + C\,A$

4. Let $L = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$, $M = \begin{pmatrix} 2 & 1 \\ 3 & -4 \end{pmatrix}$, $N = \begin{pmatrix} 1 & 4 \\ -4 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix}$

Check whether:

(i) $LM = ML$; (ii) $MN = NM$;

(iii) $LN = NL$; (iv) $LP = PL$.

5. Multiply the following pairs of matrices:

(i) $\begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & -3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & -1 & 2 & 2 & 0 \\ 1 & 2 & 1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$

(iv) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} \ldots \\ \ldots \end{pmatrix}$ (v) $\begin{pmatrix} 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \end{pmatrix}$

(vi) $\begin{pmatrix} 2 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 4 & 0 & 2 \\ 3 & 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$

6. Suppose that the orders of matrices A, B, C, D and E are as follows:

A: 3×2 B: 4×2 C: 1×3 D: 2×2 E: 2×4

Which of the following products exist, and for each that does exist, what is its order? If a product does not exist, then say so.

(i) AB (ii) CA

(iii) EB (iv) BE

(v) AD (vi) AE

(vii) DB (viii) BD

7. Define the following matrices:

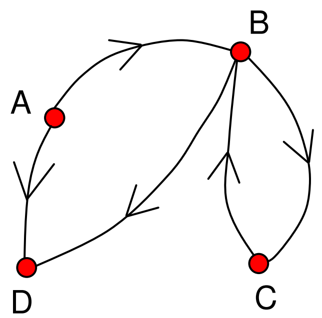
$$A = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & -3 \\ 4 & 1 \\ 1 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 3 & 3 & 0 & 1 \\ 0 & -2 & 2 & 1 \end{pmatrix} \qquad E = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

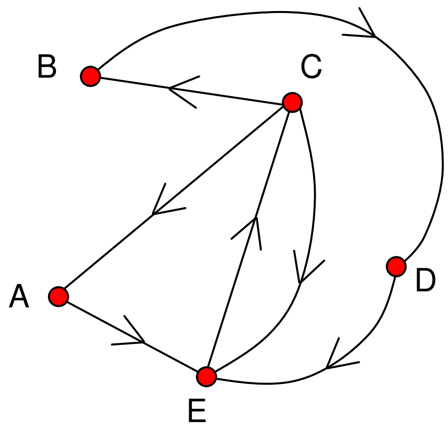
Calculate following products, if they exist. If a product does not exist, say so.

(i) AD, (ii) BD, (iii) DE, (iv) CE, (v) AC (vi) CB

8. Construct the adjacency matrix for the digraph. From the matrix find the indegree and outdegree of each vertex and check that the sum of the indegrees is equal to the sum of the outdegrees.



9. Construct the adjacency matrix for the digraph. From the matrix find the indegree and outdegree of each vertex and check that the sum of the indegrees is equal to the sum of the outdegrees.



10. The adjacency matrix M of a digraph with four vertices is M = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(i) Draw the digraph.

(ii) Calculate the matrices M^2, M^3, M^4 and hence the reachability matrix M* (using logical multiplication and addition)

(iii) Check that the matrix M* agrees with the digraph, that is that certain vertices are not reachable from others.

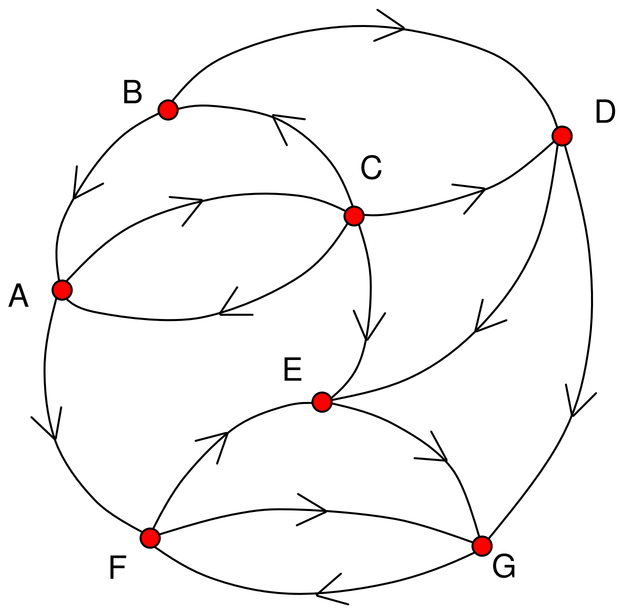
11. The adjacency matrix M of a digraph with five vertices is M = $\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$

(i) Draw the digraph.

(ii) Calculate the matrices M^2, M^3, M^4, $M^5$and hence the reachability matrix M* (using logical multiplication and addition)

(iii) Check that the matrix M* agrees with the digraph, that is that certain vertices are not reachable from others.

12. For the digraph below, work out by hand (that is by following edges from vertex to vertex in the diagram) its reachability matrix M*.



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