Bootstrap_assignment_v2.0

November 29, 2020

[1]:
```
import numpy as np # importing numpy for numerical computation
from sklearn.datasets import load_boston # here we are using sklearn's boston␣
 ↪dataset
from sklearn.metrics import mean_squared_error # importing mean_squared_error␣
 ↪metric
from sklearn.model_selection import train_test_split
from sklearn.tree import DecisionTreeRegressor

import matplotlib.pyplot as plt
plt.xkcd();
import seaborn as sns
import math
import random

random.seed(48)

import warnings
warnings.filterwarnings("ignore")
```

[2]:
```
class color:
   PURPLE = '\033[95m'
   CYAN = '\033[96m'
   DARKCYAN = '\033[36m'
   BLUE = '\033[94m'
   GREEN = '\033[92m'
   YELLOW = '\033[93m'
   RED = '\033[91m'
   BOLD = '\033[1m'
   UNDERLINE = '\033[4m'
   END = '\033[0m'
```

[3]:
```
boston = load_boston()
x=boston.data #independent variables
y=boston.target #target variable
```

[4]:
```
x.shape, y.shape
```

[4]:
```
((506, 13), (506,))
```

[5]:
```
x[505, 0:5]
```

[5]:
```
array([ 0.04741,  0.     , 11.93   ,  0.     ,  0.573  ])
```

1 TASK 1

1.1 1.1 Random Records & Features

[6]:
```
def random_split(randomness, index_range):
    randomness = randomness / 100

    def randoms():
        return randomness

    records = list(range(0,index_range))
    features = list(range(0,13))

    random.shuffle(records, randoms)
    random.shuffle(features)

    return records, features
```

1.2 1.2 Dataset Split

[7]:
```
def dataset_percentage(random_state, data, target, percentage, tot = x.
 ↪shape[0]):

    if percentage == 60:
        indices_length = math.floor(tot * percentage / 100)
        rows, cols = random_split(random_state, data.shape[0])

        train_data_rows = rows[:indices_length]
        cols = cols[:8]
        oob_rows = rows[indices_length:]

        df_data = data[:, cols]
        train_data = df_data[train_data_rows, :]
        train_target = target[train_data_rows]

        oob_data = df_data[oob_rows, :]
        oob_target = target[oob_rows]

        return train_data, train_target, cols, train_data_rows, oob_data,␣
 ↪oob_target, oob_rows

    elif percentage == 40:
        indices_length = math.ceil(tot * percentage / 100)
```

```
        rows, _ = random_split(random_state, data.shape[0])

        rows = rows[:indices_length]

        data = data[rows, :]
        target = target[rows]

        return data, target
```

1.3 1.3 BootStrap Samples

[8]:
```
def bootstrap(random_state):

    X_60, y_60, cols, rows, oob_data, oob_target, oob_rows =␣
↪dataset_percentage(random_state, x, y, 60) #60 percentage of 506 points =␣
↪303 data points

    X_60 = np.asarray(X_60)
    y_60 = np.asarray(y_60)

    X_40, y_40 = dataset_percentage(random_state, X_60, y_60, 40) #40␣
↪percentage of 303 points = 203 data points

    X_40 = np.asarray(X_40)
    y_40 = np.asarray(y_40)

    X = np.concatenate((X_60, X_40), axis=0)
    Y = np.concatenate((y_60, y_40), axis=0)

    return X, Y, cols, rows, oob_data, oob_target, oob_rows
```

1.4 1.4 Model Training

[9]:
```
models_1 = []
datasets_1 = []

oob_records_1 = []

rows_index_1 = []
feature_index_1 = []

pop_MSE_1 = []
OOB_1 = []

for i in range(30):
    X, Y, cols, rows, oob_data, oob_target, oob_rows = bootstrap(i)
```

```
    oob_records_1.append(oob_rows)

    X = np.asarray(X)
    Y = np.asarray(Y)

    feature_index_1.append(cols)
    rows_index_1.append(rows)

    datasets_1.append((X, Y))

    print(color.BOLD + "Dataset - " + color.END + str(i+1), color.RED + color.
↪BOLD \
          + "\n\tshape :", color.END + str(X.shape) + str(Y.shape) )

    model = DecisionTreeRegressor(random_state=2)
    model.fit(X, Y)
    models_1.append(model)

    mse = mean_squared_error(Y, model.predict(X)) #ith model MSE in Dataset 1
    overall_mse = mean_squared_error(y, model.predict(x[:, cols])) #ith model␣
↪MSE in overall Dataset 1

    pop_MSE_1.append(overall_mse)
    oob_pred = model.predict(oob_data)

    oob_score = np.sum(np.subtract(oob_target, oob_pred)**2)
    oob_score = oob_score // x.shape[0]

    OOB_1.append(oob_score)

    print(color.BOLD + "Model - " + color.END + str(i+1), color.RED + color.
↪BOLD +
         "\n\tsample {} MSE is : ".format(i+1) + color.END, str(mse), color.
↪RED + color.BOLD \
          + "\n\tOOB-SCORE : " + color.END, str(oob_score), color.RED + color.
↪BOLD + "\n\tpopulation MSE : " \
          + color.END, np.round(overall_mse, 3) , "\n")

print(color.BLUE + color.BOLD + "Population MSE (median pred of 30 Models) : "␣
↪+ color.END, \
      np.median(np.asarray(pop_MSE_1)))
```

```
Dataset - 1
        shape : (506, 8)(506,)
```

```
Model - 1
        sample 1 MSE is :  7.483265504072839e-32
        OOB-SCORE :  27.0
        population MSE :  27.801

Dataset - 2
        shape : (506, 8)(506,)
Model - 2
        sample 2 MSE is :  0.0
        OOB-SCORE :  21.0
        population MSE :  21.979

Dataset - 3
        shape : (506, 8)(506,)
Model - 3
        sample 3 MSE is :  0.0
        OOB-SCORE :  12.0
        population MSE :  12.138

Dataset - 4
        shape : (506, 8)(506,)
Model - 4
        sample 4 MSE is :  0.0
        OOB-SCORE :  16.0
        population MSE :  16.893

Dataset - 5
        shape : (506, 8)(506,)
Model - 5
        sample 5 MSE is :  0.0
        OOB-SCORE :  27.0
        population MSE :  27.449

Dataset - 6
        shape : (506, 8)(506,)
```

```
Model - 6
        sample 6 MSE is :  0.0
        OOB-SCORE :  34.0
        population MSE :  34.335
Dataset - 7
        shape : (506, 8)(506,)
Model - 7
        sample 7 MSE is :  0.2585210803689064
        OOB-SCORE :  34.0
        population MSE :  34.19
Dataset - 8
        shape : (506, 8)(506,)
Model - 8
        sample 8 MSE is :  0.0
        OOB-SCORE :  34.0
        population MSE :  34.22
Dataset - 9
        shape : (506, 8)(506,)
Model - 9
        sample 9 MSE is :  7.483265504072839e-32
        OOB-SCORE :  24.0
        population MSE :  24.948
Dataset - 10
        shape : (506, 8)(506,)
Model - 10
        sample 10 MSE is :  0.0
        OOB-SCORE :  12.0
        population MSE :  12.26
Dataset - 11
        shape : (506, 8)(506,)
```

```
Model - 11
        sample 11 MSE is :  0.0
        OOB-SCORE :  23.0
        population MSE :  23.273

Dataset - 12
        shape : (506, 8)(506,)
Model - 12
        sample 12 MSE is :  0.0
        OOB-SCORE :  29.0
        population MSE :  29.402

Dataset - 13
        shape : (506, 8)(506,)
Model - 13
        sample 13 MSE is :  1.4966531008145678e-31
        OOB-SCORE :  16.0
        population MSE :  16.859

Dataset - 14
        shape : (506, 8)(506,)
Model - 14
        sample 14 MSE is :  7.483265504072839e-32
        OOB-SCORE :  24.0
        population MSE :  24.19

Dataset - 15
        shape : (506, 8)(506,)
Model - 15
        sample 15 MSE is :  0.0
        OOB-SCORE :  29.0
        population MSE :  29.301

Dataset - 16
        shape : (506, 8)(506,)
```

```
Model - 16
        sample 16 MSE is :  0.0
        OOB-SCORE :  15.0
        population MSE :  15.342

Dataset - 17
        shape : (506, 8)(506,)
Model - 17
        sample 17 MSE is :  0.0
        OOB-SCORE :  14.0
        population MSE :  14.175

Dataset - 18
        shape : (506, 8)(506,)
Model - 18
        sample 18 MSE is :  0.0
        OOB-SCORE :  36.0
        population MSE :  36.446

Dataset - 19
        shape : (506, 8)(506,)
Model - 19
        sample 19 MSE is :  0.0
        OOB-SCORE :  11.0
        population MSE :  11.567

Dataset - 20
        shape : (506, 8)(506,)
Model - 20
        sample 20 MSE is :  0.0
        OOB-SCORE :  19.0
        population MSE :  19.135

Dataset - 21
        shape : (506, 8)(506,)
```

```
Model - 21

        sample 21 MSE is :  0.0

        OOB-SCORE :  11.0

        population MSE :  11.516

Dataset - 22

        shape : (506, 8)(506,)
Model - 22

        sample 22 MSE is :  0.0

        OOB-SCORE :  45.0

        population MSE :  45.469

Dataset - 23

        shape : (506, 8)(506,)
Model - 23

        sample 23 MSE is :  0.0

        OOB-SCORE :  22.0

        population MSE :  22.609

Dataset - 24

        shape : (506, 8)(506,)
Model - 24

        sample 24 MSE is :  0.0

        OOB-SCORE :  14.0

        population MSE :  14.322

Dataset - 25

        shape : (506, 8)(506,)
Model - 25

        sample 25 MSE is :  0.0

        OOB-SCORE :  25.0

        population MSE :  25.313

Dataset - 26

        shape : (506, 8)(506,)
```

```
Model - 26
        sample 26 MSE is :  0.0
        OOB-SCORE :  10.0
        population MSE :  10.563

Dataset - 27
        shape : (506, 8)(506,)
Model - 27
        sample 27 MSE is :  7.483265504072839e-32
        OOB-SCORE :  15.0
        population MSE :  15.265

Dataset - 28
        shape : (506, 8)(506,)
Model - 28
        sample 28 MSE is :  0.0
        OOB-SCORE :  8.0
        population MSE :  8.782

Dataset - 29
        shape : (506, 8)(506,)
Model - 29
        sample 29 MSE is :  7.483265504072839e-32
        OOB-SCORE :  17.0
        population MSE :  17.253

Dataset - 30
        shape : (506, 8)(506,)
Model - 30
        sample 30 MSE is :  0.0
        OOB-SCORE :  48.0
        population MSE :  48.246

Population MSE (median pred of 30 Models) :  22.294179841897233
```

1.5 1.5 OOB Score

[10]:
```
oob_pred = []

oob_score = 0

for df_row_index, data_point in enumerate(x):
    oob_model_index = []
    pred = []

    for model_index, oob_data_indices in enumerate(oob_records_1):
        if df_row_index in oob_data_indices:
            oob_model_index.append(model_index)

    for index in oob_model_index:
        model = models_1[index]
        feature_cols = feature_index_1[index]
        data = np.take(data_point, feature_cols).reshape(1, -1)
        model_pred = model.predict(data)
        pred.append(model_pred)

    if sum(pred) == 0:
        pred = 0
    else:
        pred = np.median(np.asarray(pred))

    oob_pred.append(pred)

oob_score = np.median(np.asarray(oob_pred))

print(color.BLUE + color.BOLD + "Population OOB-SCORE (Median of 30 models␣
↪pred): " + str(oob_score) + color.END)
```

```
Population OOB-SCORE (Median of 30 models pred): 15.7
```

2 2. TASK :

2.1 2.1 Model Training

[11]:
```
models_2 = []
datasets_2 = []

oob_records_2 = []

rows_index_2 = []
feature_index_2 = []

pop_MSE_2 = []
```

```
OOB_2 = []

for i in range(35):
    X, Y, cols, rows, oob_data, oob_target, oob_rows = bootstrap(i)

    oob_records_2.append(oob_rows)

    X = np.asarray(X)
    Y = np.asarray(Y)

    feature_index_2.append(cols)
    rows_index_2.append(rows)

    datasets_2.append((X, Y))

    print(color.BOLD + "Dataset - " + color.END + str(i+1), color.RED + color.
 ↪BOLD + \
          "\n\tshape :", color.END + str(X.shape) + str(Y.shape) )

    model = DecisionTreeRegressor(random_state=2)
    model.fit(X, Y)
    models_2.append(model)

    mse = mean_squared_error(Y, model.predict(X)) #ith model MSE in Dataset 1
    overall_mse = mean_squared_error(y, model.predict(x[:, cols])) #ith model␣
 ↪MSE in Population

    pop_MSE_2.append(overall_mse)
    oob_pred = model.predict(oob_data)

    oob_score = np.sum(np.subtract(oob_target, oob_pred)**2)
    oob_score = oob_score // x.shape[0]

    OOB_2.append(oob_score)

    print(color.BOLD + "Model - " + color.END + str(i+1), color.RED + color.
 ↪BOLD + \
          "\n\tsample {} MSE is : ".format(i) + color.END, str(mse), color.RED␣
 ↪+ color.BOLD \
          + "\n\tOOB-SCORE : " + color.END, str(oob_score), color.RED \
          + color.BOLD + "\n\tpopulation MSE : " + color.END, overall_mse ,␣
 ↪"\n")

print(color.BLUE + color.BOLD + "Population MSE (median pred of 35 Models) : " \
      + color.END, np.median(np.asarray(pop_MSE_2)))
```

```
Dataset - 1
        shape : (506, 8)(506,)
Model - 1
        sample 0 MSE is :  0.0
        OOB-SCORE :  26.0
        population MSE :  26.88177865612648

Dataset - 2
        shape : (506, 8)(506,)
Model - 2
        sample 1 MSE is :  0.0
        OOB-SCORE :  21.0
        population MSE :  21.47509881422925

Dataset - 3
        shape : (506, 8)(506,)
Model - 3
        sample 2 MSE is :  7.483265504072839e-32
        OOB-SCORE :  16.0
        population MSE :  16.902786561264822

Dataset - 4
        shape : (506, 8)(506,)
Model - 4
        sample 3 MSE is :  0.0
        OOB-SCORE :  16.0
        population MSE :  16.198379446640317

Dataset - 5
        shape : (506, 8)(506,)
Model - 5
        sample 4 MSE is :  0.0
        OOB-SCORE :  25.0
        population MSE :  25.013221343873518
```

```
Dataset - 6
        shape : (506, 8)(506,)
Model - 6
        sample 5 MSE is :  0.0
        OOB-SCORE :  25.0
        population MSE :  25.308102766798417

Dataset - 7
        shape : (506, 8)(506,)
Model - 7
        sample 6 MSE is :  0.0
        OOB-SCORE :  15.0
        population MSE :  15.24579051383399

Dataset - 8
        shape : (506, 8)(506,)
Model - 8
        sample 7 MSE is :  0.0
        OOB-SCORE :  16.0
        population MSE :  16.234901185770752

Dataset - 9
        shape : (506, 8)(506,)
Model - 9
        sample 8 MSE is :  7.483265504072839e-32
        OOB-SCORE :  13.0
        population MSE :  13.693498023715415

Dataset - 10
        shape : (506, 8)(506,)
Model - 10
        sample 9 MSE is :  0.0
        OOB-SCORE :  13.0
        population MSE :  13.213142292490119
```

```
Dataset - 11
        shape : (506, 8)(506,)
Model - 11
        sample 10 MSE is :  0.0
        OOB-SCORE :  14.0
        population MSE :  14.47403162055336

Dataset - 12
        shape : (506, 8)(506,)
Model - 12
        sample 11 MSE is :  0.0
        OOB-SCORE :  10.0
        population MSE :  10.098517786561265

Dataset - 13
        shape : (506, 8)(506,)
Model - 13
        sample 12 MSE is :  0.0
        OOB-SCORE :  12.0
        population MSE :  12.41092885375494

Dataset - 14
        shape : (506, 8)(506,)
Model - 14
        sample 13 MSE is :  7.483265504072839e-32
        OOB-SCORE :  31.0
        population MSE :  31.543774703557315

Dataset - 15
        shape : (506, 8)(506,)
Model - 15
        sample 14 MSE is :  7.483265504072839e-32
        OOB-SCORE :  8.0
        population MSE :  8.240810276679843
```

```
Dataset - 16
        shape : (506, 8)(506,)
Model - 16
        sample 15 MSE is :  0.0
        OOB-SCORE :  14.0
        population MSE :  14.192391304347828

Dataset - 17
        shape : (506, 8)(506,)
Model - 17
        sample 16 MSE is :  0.0
        OOB-SCORE :  13.0
        population MSE :  13.038359683794468

Dataset - 18
        shape : (506, 8)(506,)
Model - 18
        sample 17 MSE is :  0.0
        OOB-SCORE :  30.0
        population MSE :  30.11245059288537

Dataset - 19
        shape : (506, 8)(506,)
Model - 19
        sample 18 MSE is :  0.0
        OOB-SCORE :  18.0
        population MSE :  18.445513833992095

Dataset - 20
        shape : (506, 8)(506,)
Model - 20
        sample 19 MSE is :  0.0
        OOB-SCORE :  24.0
        population MSE :  24.076422924901188
```

```
Dataset - 21
        shape : (506, 8)(506,)
Model - 21
        sample 20 MSE is :  0.0
        OOB-SCORE :  12.0
        population MSE :  12.475000000000001

Dataset - 22
        shape : (506, 8)(506,)
Model - 22
        sample 21 MSE is :  0.0
        OOB-SCORE :  23.0
        population MSE :  23.162806324110672

Dataset - 23
        shape : (506, 8)(506,)
Model - 23
        sample 22 MSE is :  0.0
        OOB-SCORE :  25.0
        population MSE :  25.675573122529645

Dataset - 24
        shape : (506, 8)(506,)
Model - 24
        sample 23 MSE is :  0.0
        OOB-SCORE :  16.0
        population MSE :  16.462292490118582

Dataset - 25
        shape : (506, 8)(506,)
Model - 25
        sample 24 MSE is :  0.0
        OOB-SCORE :  28.0
        population MSE :  28.18203557312253
```

```
Dataset - 26
        shape : (506, 8)(506,)
Model - 26
        sample 25 MSE is :  0.0
        OOB-SCORE :  13.0
        population MSE :  13.958794466403162

Dataset - 27
        shape : (506, 8)(506,)
Model - 27
        sample 26 MSE is :  0.0
        OOB-SCORE :  12.0
        population MSE :  12.508537549407114

Dataset - 28
        shape : (506, 8)(506,)
Model - 28
        sample 27 MSE is :  0.0
        OOB-SCORE :  14.0
        population MSE :  14.31505928853755

Dataset - 29
        shape : (506, 8)(506,)
Model - 29
        sample 28 MSE is :  7.483265504072839e-32
        OOB-SCORE :  20.0
        population MSE :  20.258498023715415

Dataset - 30
        shape : (506, 8)(506,)
Model - 30
        sample 29 MSE is :  0.0
        OOB-SCORE :  9.0
        population MSE :  9.547134387351779
```

```
Dataset - 31

        shape : (506, 8)(506,)
Model - 31

        sample 30 MSE is :  0.0

        OOB-SCORE :  12.0

        population MSE :  12.227252964426878

Dataset - 32

        shape : (506, 8)(506,)
Model - 32

        sample 31 MSE is :  7.483265504072839e-32

        OOB-SCORE :  10.0

        population MSE :  10.719881422924901

Dataset - 33

        shape : (506, 8)(506,)
Model - 33

        sample 32 MSE is :  7.483265504072839e-32

        OOB-SCORE :  16.0

        population MSE :  16.181383399209487

Dataset - 34

        shape : (506, 8)(506,)
Model - 34

        sample 33 MSE is :  0.0

        OOB-SCORE :  7.0

        population MSE :  7.913201581027668

Dataset - 35

        shape : (506, 8)(506,)
Model - 35

        sample 34 MSE is :  0.0

        OOB-SCORE :  24.0

        population MSE :  24.573339920948623

Population MSE (median pred of 35 Models) :  16.181383399209487
```

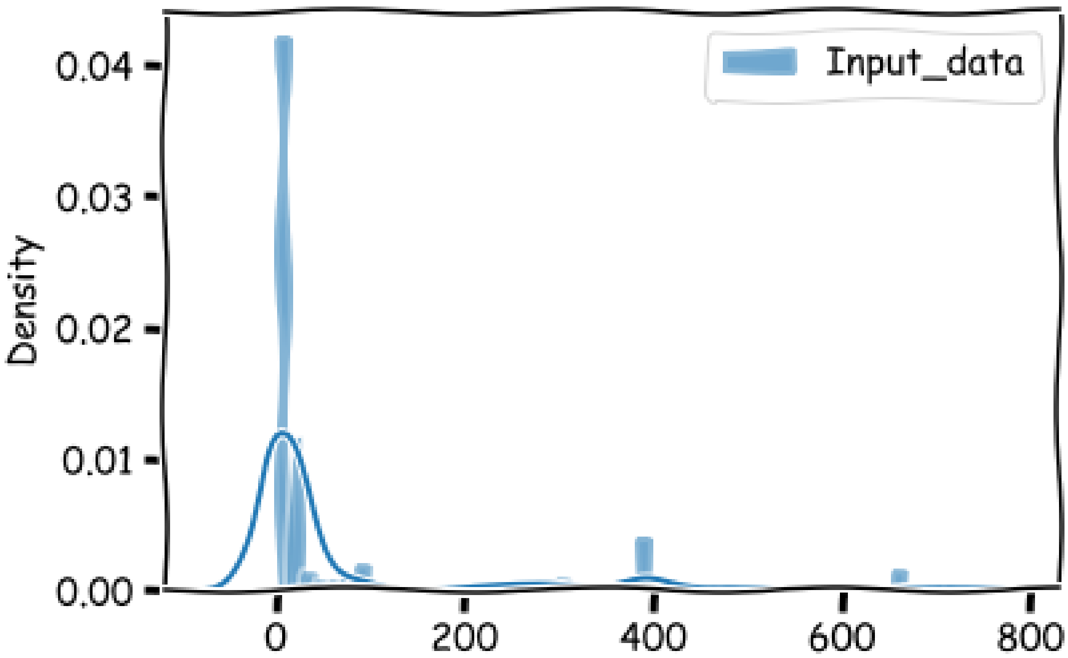
2.2 2.2 OOB SCORE:

[12]:
```
oob_pred = []

oob_score = 0

for df_row_index, data_point in enumerate(x):
    oob_model_index = []
    pred = []

    for model_index, oob_data_indices in enumerate(oob_records_2):
        if df_row_index in oob_data_indices:
            oob_model_index.append(model_index)

    for index in oob_model_index:
        model = models_2[index]
        feature_cols = feature_index_2[index]
        data = np.take(data_point, feature_cols).reshape(1, -1)
        model_pred = model.predict(data)
        pred.append(model_pred)

    if sum(pred) == 0:
        pred = 0
    else:
        pred = np.median(np.asarray(pred))

    oob_pred.append(pred)

oob_score = np.median(np.asarray(oob_pred))

print(color.BLUE + color.BOLD + "Population OOB-SCORE (Median of 35 models␣
↪pred): " \
      + str(np.round(oob_score, 2)) + color.END)
```

```
Population OOB-SCORE (Median of 35 models pred): 17.0
```

2.3 2.3 95% Confidence Intervals of 35 Samples

[13]:
```
sns.distplot(x, label="Input_data").legend();
```



[14]:
```
from prettytable import PrettyTable

T = PrettyTable()

T = PrettyTable(["#samples_name", "P_Mean", "Sample mean", "P_Std", "Sample␣
↪Std",
                 "mu_x"+u"\u2248"+"mu", "std_x"+u"\u2248"+"std/"+u"\u221A"+"n"])

for index, X in enumerate(datasets_2):

    sample_mean = np.round(np.mean(X[0]), 3)
    sample_std = np.round(np.std(X[0]), 3)
    population_std_est = np.round(x.std() / np.sqrt(X[0].shape[0]), 3)

    row = []
    row.append('sample_'+str(index+1))
    row.append(round(x.mean(), 3))
    row.append(round(X[0].mean(), 3))
    row.append(round(x.std(),3))
    row.append(round(X[0].std(), 3))
    row.append(str(round(X[0].mean(), 3))+u"\u2248"+str(round(x.mean(), 3)))
    row.append(str(sample_std)+u"\u2248"+str(population_std_est))
    T.add_row(row)

print(T)
```

#samples_name	P_Mean	Sample mean	P_Std	Sample Std	mu_x mu	std_x std/√n
sample_1	70.074	51.989	145.156	106.114	51.989 70.074	106.114 6.453
sample_2	70.074	44.513	145.156	107.433	44.513 70.074	107.433 6.453
sample_3	70.074	54.809	145.156	107.166	54.809 70.074	107.166 6.453
sample_4	70.074	53.253	145.156	108.785	53.253 70.074	108.785 6.453
sample_5	70.074	59.177	145.156	123.01	59.177 70.074	123.01 6.453
sample_6	70.074	100.746	145.156	152.09	100.746 70.074	152.09 6.453
sample_7	70.074	58.6	145.156	122.829	58.6 70.074	122.829 6.453
sample_8	70.074	93.863	145.156	156.348	93.863 70.074	156.348 6.453
sample_9	70.074	94.099	145.156	156.358	94.099 70.074	156.358 6.453
sample_10	70.074	12.922	145.156	24.682	12.922 70.074	24.682 6.453
sample_11	70.074	60.618	145.156	122.286	60.618 70.074	122.286 6.453
sample_12	70.074	53.36	145.156	122.4	53.36 70.074	122.4 6.453
sample_13	70.074	102.291	145.156	156.353	102.291 70.074	156.353 6.453
sample_14	70.074	95.02	145.156	159.665	95.02 70.074	159.665 6.453
sample_15	70.074	97.217	145.156	159.352	97.217 70.074	159.352 6.453
sample_16	70.074	95.865	145.156	159.489	95.865 70.074	159.489 6.453
sample_17	70.074	52.71	145.156	122.233	52.71 70.074	122.233 6.453
sample_18	70.074	58.651	145.156	121.274	58.651 70.074	121.274 6.453
sample_19	70.074	102.273	145.156	158.314	102.273 70.074	158.314 6.453
sample_20	70.074	58.752	145.156	124.018	58.752 70.074	124.018 6.453
sample_21	70.074	97.464	145.156	161.469	97.464 70.074	161.469 6.453

```
|    sample_22    | 70.074 |    95.033    | 145.156 |  163.396    | 95.033 70.074   |
163.396 6.453 |
|    sample_23    | 70.074 |    57.402    | 145.156 |  125.262    | 57.402 70.074   |
125.262 6.453 |
|    sample_24    | 70.074 |   104.039    | 145.156 |  160.755    | 104.039 70.074 |
160.755 6.453 |
|    sample_25    | 70.074 |    58.527    | 145.156 |  121.044    | 58.527 70.074   |
121.044 6.453 |
|    sample_26    | 70.074 |    14.472    | 145.156 |   23.18     | 14.472 70.074   |
23.18 6.453   |
|    sample_27    | 70.074 |    97.288    | 145.156 |  163.922    | 97.288 70.074   |
163.922 6.453 |
|    sample_28    | 70.074 |    15.911    | 145.156 |   24.452    | 15.911 70.074   |
24.452 6.453  |
|    sample_29    | 70.074 |    58.27     | 145.156 |  120.839    |  58.27 70.074   |
120.839 6.453 |
|    sample_30    | 70.074 |    96.553    | 145.156 |  164.079    | 96.553 70.074   |
164.079 6.453 |
|    sample_31    | 70.074 |    15.415    | 145.156 |   23.747    | 15.415 70.074   |
23.747 6.453  |
|    sample_32    | 70.074 |    60.21     | 145.156 |  121.297    |  60.21 70.074   |
121.297 6.453 |
|    sample_33    | 70.074 |    14.681    | 145.156 |   23.532    | 14.681 70.074   |
23.532 6.453  |
|    sample_34    | 70.074 |   102.259    | 145.156 |  163.416    | 102.259 70.074 |
163.416 6.453 |
|    sample_35    | 70.074 |    52.041    | 145.156 |   128.41    | 52.041 70.074   |
128.41 6.453  |
+----------------+--------+-------------+---------+------------+----------------+
---------------+
```

[15]:

```python
T = PrettyTable()
T = PrettyTable(["#samples", "Sample Size", "Sample mean", "Pop Std","Left C.
↪I","Right C.I","Pop mean","Catch"])
population_std = x.std()
population_mean= np.round(x.mean(), 3)

for index, sample in enumerate(datasets_2):
    sample = sample[0]
    sample_mean = sample.mean()
    sample_size = sample.shape[0]
    left_limit  = np.round(sample_mean - 2*(population_std/np.sqrt(x.
↪shape[0])), 3)
    right_limit = np.round(sample_mean + 2*(population_std/np.sqrt(x.
↪shape[0])), 3)
    row = []
    row.append(index+1)
```

```
    row.append(sample_size)
    row.append(sample_mean)
    row.append(population_std)
    row.append(left_limit)
    row.append(right_limit)
    row.append(population_mean)
    row.append((population_mean <= right_limit) and (population_mean >=␣
 ↪left_limit))
    T.add_row(row)
print(T)
```

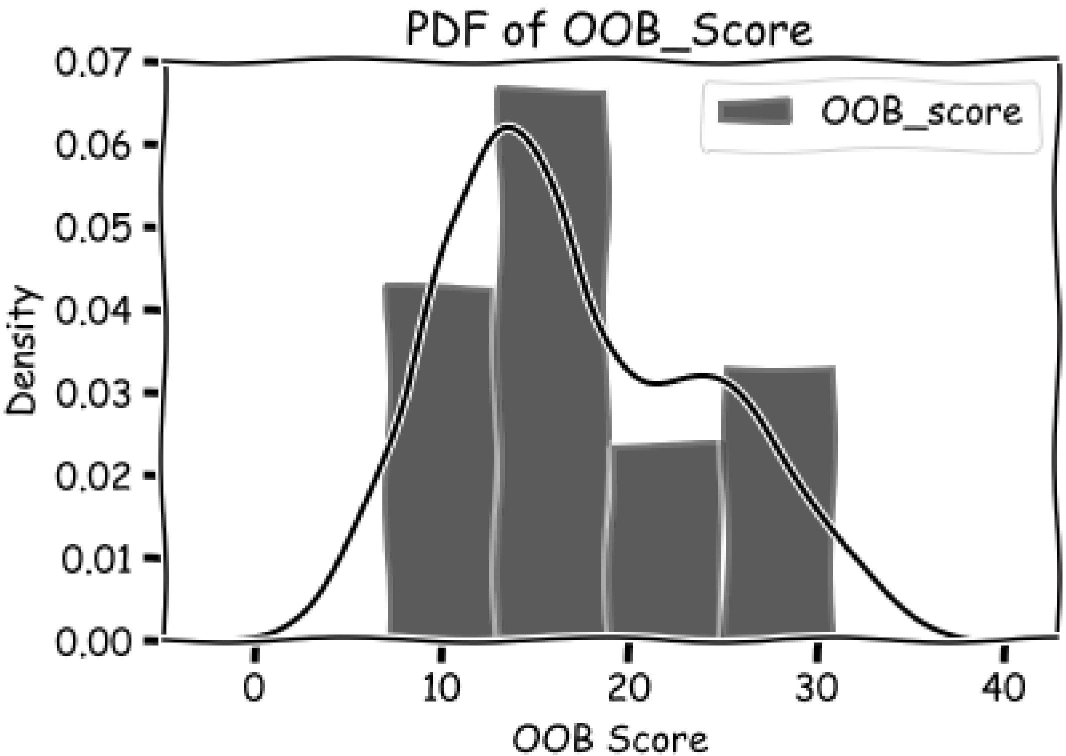
```
+----------+-------------+--------------------+-------------------+----------+--
---------+----------+-------+
| #samples | Sample Size |    Sample mean     |      Pop Std      | Left C.I |
Right C.I | Pop mean | Catch |
+----------+-------------+--------------------+-------------------+----------+--
---------+----------+-------+
|    1     |     506     | 51.989330046936765 | 145.1555388220164 |  39.083  |
64.895   |  70.074  | False |
|    2     |     506     | 44.512513216403164 | 145.1555388220164 |  31.607  |
57.418   |  70.074  | False |
|    3     |     506     |  54.8087557312253  | 145.1555388220164 |  41.903  |
67.715   |  70.074  | False |
|    4     |     506     | 53.25259990118577  | 145.1555388220164 |  40.347  |
66.159   |  70.074  | False |
|    5     |     506     |  59.1767920701581  | 145.1555388220164 |  46.271  |
72.083   |  70.074  |  True |
|    6     |     506     | 100.74628213932807 | 145.1555388220164 |  87.84   |
113.652   |  70.074  | False |
|    7     |     506     | 58.60041371047431  | 145.1555388220164 |  45.695  |
71.506   |  70.074  |  True |
|    8     |     506     |  93.8627423320158  | 145.1555388220164 |  80.957  |
106.769   |  70.074  | False |
|    9     |     506     | 94.09900213932805  | 145.1555388220164 |  81.193  |
107.005   |  70.074  | False |
|    10    |     506     | 12.922381195652175 | 145.1555388220164 |  0.016   |
25.828   |  70.074  | False |
|    11    |     506     | 60.617654001976284 | 145.1555388220164 |  47.712  |
73.524   |  70.074  |  True |
|    12    |     506     | 53.35950842391304  | 145.1555388220164 |  40.454  |
66.265   |  70.074  | False |
|    13    |     506     | 102.29137278903161 | 145.1555388220164 |  89.385  |
115.197   |  70.074  | False |
|    14    |     506     | 95.02019528656126  | 145.1555388220164 |  82.114  |
107.926   |  70.074  | False |
|    15    |     506     | 97.21680145750987  | 145.1555388220164 |  84.311  |
110.123   |  70.074  | False |
|    16    |     506     | 95.86540807806323  | 145.1555388220164 |  82.96   |
```

```
108.771  |  70.074  | False |
|    17    |     506     | 52.70982691205534  | 145.1555388220164 |  39.804  |
65.616  |  70.074  | False |
|    18    |     506     | 58.651392633399205 | 145.1555388220164 |  45.745  |
71.557  |  70.074  |  True |
|    19    |     506     | 102.27334928853756 | 145.1555388220164 |  89.367  |
115.179  |  70.074  | False |
|    20    |     506     | 58.75215155385375  | 145.1555388220164 |  45.846  |
71.658  |  70.074  |  True |
|    21    |     506     | 97.46397035573122  | 145.1555388220164 |  84.558  |
110.37  |  70.074  | False |
|    22    |     506     | 95.03343546936757  | 145.1555388220164 |  82.128  |
107.939  |  70.074  | False |
|    23    |     506     | 57.40183782855731  | 145.1555388220164 |  44.496  |
70.308  |  70.074  |  True |
|    24    |     506     | 104.03910597332015 | 145.1555388220164 |  91.133  |
116.945  |  70.074  | False |
|    25    |     506     | 58.52718850296443  | 145.1555388220164 |  45.621  |
71.433  |  70.074  |  True |
|    26    |     506     | 14.47222728507905  | 145.1555388220164 |  1.566   |
27.378  |  70.074  | False |
|    27    |     506     | 97.28761287055336  | 145.1555388220164 |  84.382  |
110.194  |  70.074  | False |
|    28    |     506     | 15.911475296442688 | 145.1555388220164 |  3.006   |
28.817  |  70.074  | False |
|    29    |     506     | 58.270105674407105 | 145.1555388220164 |  45.364  |
71.176  |  70.074  |  True |
|    30    |     506     | 96.55269174901186  | 145.1555388220164 |  83.647  |
109.459  |  70.074  | False |
|    31    |     506     | 15.414596032608694 | 145.1555388220164 |  2.509   |
28.32   |  70.074  | False |
|    32    |     506     | 60.209604755434775 | 145.1555388220164 |  47.304  |
73.116  |  70.074  |  True |
|    33    |     506     | 14.681248226284584 | 145.1555388220164 |  1.775   |
27.587  |  70.074  | False |
|    34    |     506     | 102.25939251482214 | 145.1555388220164 |  89.353  |
115.165  |  70.074  | False |
|    35    |     506     | 52.04073003952569  | 145.1555388220164 |  39.135  |
64.947  |  70.074  | False |
+----------+-------------+--------------------+-------------------+----------+--
---------+----------+-------+
```

2.4 2.4 OOB Score with Confidence Intervals of 95%

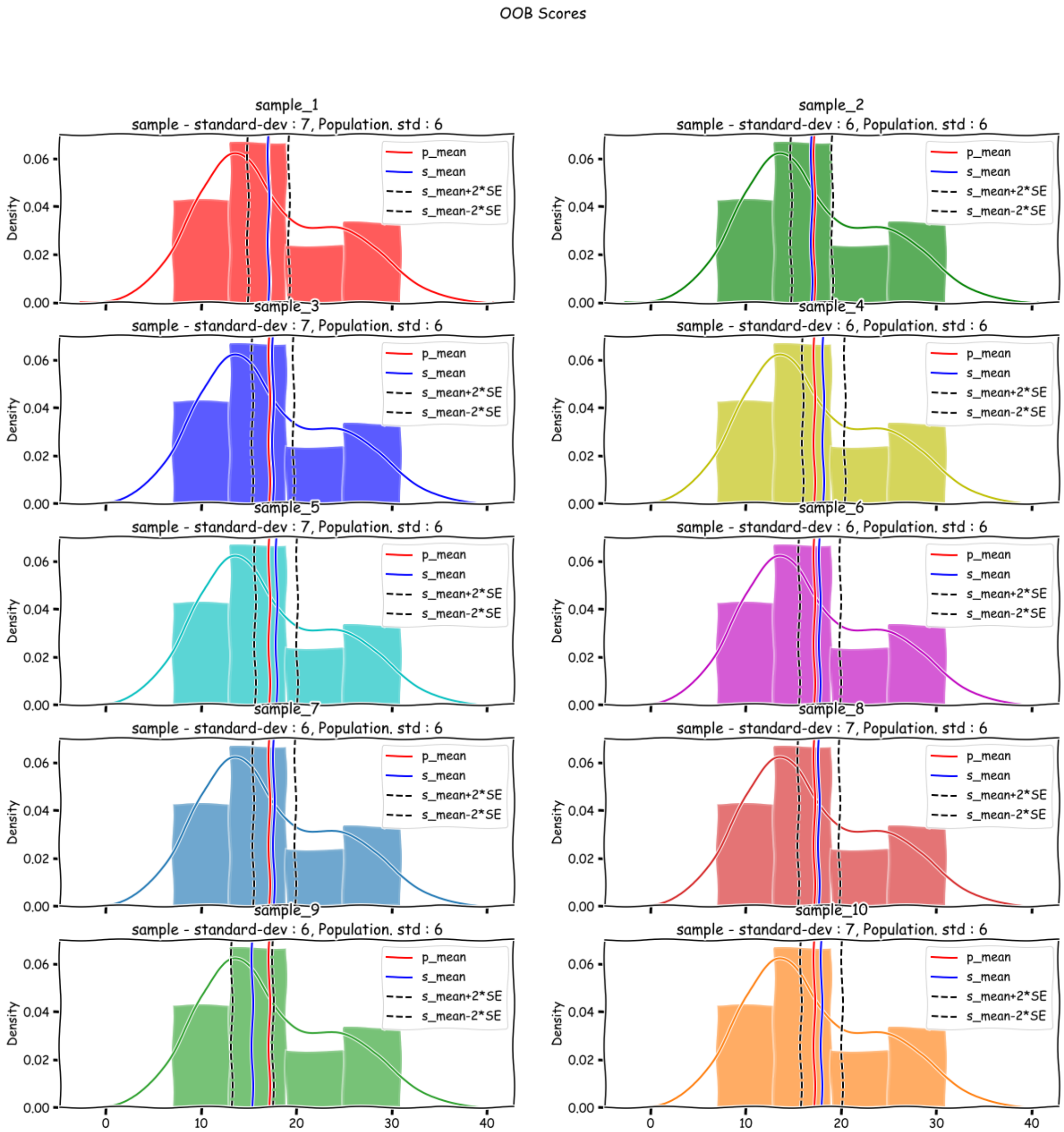
[16]:
```python
from sklearn.preprocessing import power_transform

OOB = np.asarray(OOB_2)
```

```
pop_MSE = np.asarray(pop_MSE_2)

sns.distplot(OOB, color='k', label='OOB_score').legend();
plt.xlabel("OOB Score");
plt.title("PDF of OOB_Score")
plt.show()
```



[17]:
```
colrs = ['r','g','b','y', 'c', 'm', 'tab:blue', 'tab:red', 'tab:green', 'tab:
 ↪orange', 'tab:cyan',
         'tab:purple', 'tab:brown', 'tab:gray', 'tab:pink', 'navy']

fig, axs = plt.subplots(5, 2, sharex = True, figsize=(20,20))

row_no = 0
col_no = 0
count = 0

samples = []

for index, _ in enumerate(range(10)):
```

```
    sample_60 = OOB[random.sample(range(0, OOB.shape[0]), 21)] #60% sample, 21␣
↪pts
    sample_40 = sample_60[random.sample(range(0, sample_60.shape[0]), 14)]
    sample = np.vstack((sample_60.reshape(-1,1), sample_40.reshape(-1,1)))
    samples.append(sample)

    sns.distplot(OOB, color=colrs[index], ax=axs[row_no, col_no])

    axs[row_no, col_no].axvline(OOB.mean(), linestyle="-", color='r',␣
↪label="p_mean")

    axs[row_no, col_no].axvline(sample.mean(), linestyle="-", color='b',␣
↪label="s_mean")

    axs[row_no, col_no].axvline(sample.mean()+2*(OOB.std()/np.sqrt(OOB.
↪shape[0])), linestyle="--", color='k', label="s_mean+2*SE")
    axs[row_no, col_no].axvline(sample.mean()-2*(OOB.std()/np.sqrt(OOB.
↪shape[0])), linestyle="--", color='k', label="s_mean-2*SE")

    axs[row_no, col_no].set_title('sample_'+str(index+1) + "\nsample -␣
↪standard-dev : " \
                              +str(round(sample.std())) + ", Population.␣
↪std : " + str(round(OOB.std())))

    axs[row_no, col_no].legend()

    if col_no == 1:
        col_no = -1
        row_no += 1
    col_no+=1

fig.suptitle("OOB Scores")

plt.show()
```



[18]:
```python
T = PrettyTable()

T = PrettyTable(["#samples", "Sample Size", "Sample mean", "Pop Std","Left C.
↪I","Right C.I","Pop mean","Catch"])

population_std = np.round(OOB.std(), 3)
population_mean= np.round(OOB.mean(), 3)

for index, sample in enumerate(samples):
```

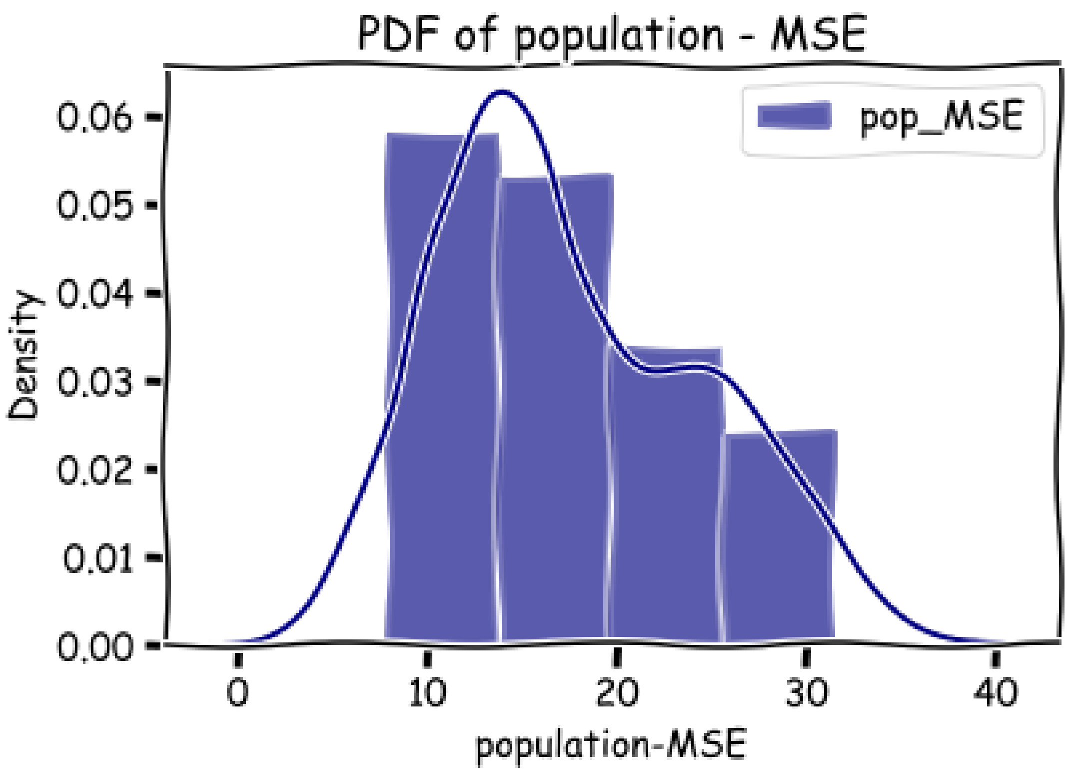
```
    sample_mean = np.round(sample.mean(), 3)
    sample_size = sample.shape[0]

    left_limit  = np.round(sample_mean - 2*(population_std/np.sqrt(OOB.
 ↪shape[0])), 3)
    right_limit = np.round(sample_mean + 2*(population_std/np.sqrt(OOB.
 ↪shape[0])), 3)

    row = []

    row.append(index+1)
    row.append(sample_size)
    row.append(sample_mean)
    row.append(population_std)
    row.append(left_limit)
    row.append(right_limit)
    row.append(population_mean)
    row.append((population_mean <= right_limit) and (population_mean >=␣
 ↪left_limit))
    T.add_row(row)

print(T)
```

```
+----------+-------------+-------------+---------+----------+-----------+-------
---+-------+
| #samples | Sample Size | Sample mean | Pop Std | Left C.I | Right C.I | Pop
mean | Catch |
+----------+-------------+-------------+---------+----------+-----------+-------
---+-------+
|    1     |      35     |    17.057   |  6.461  |  14.873  |   19.241  |
17.171   |  True |
|    2     |      35     |    16.914   |  6.461  |  14.73   |   19.098  |
17.171   |  True |
|    3     |      35     |    17.543   |  6.461  |  15.359  |   19.727  |
17.171   |  True |
|    4     |      35     |    18.171   |  6.461  |  15.987  |   20.355  |
17.171   |  True |
|    5     |      35     |    17.886   |  6.461  |  15.702  |   20.07   |
17.171   |  True |
|    6     |      35     |    17.743   |  6.461  |  15.559  |   19.927  |
17.171   |  True |
|    7     |      35     |    17.686   |  6.461  |  15.502  |   19.87   |
17.171   |  True |
|    8     |      35     |    17.657   |  6.461  |  15.473  |   19.841  |
17.171   |  True |
|    9     |      35     |    15.371   |  6.461  |  13.187  |   17.555  |
17.171   |  True |
```

```
|     10     |       35      |     17.943     |  6.461  |  15.759  |    20.127   |
17.171   |   True |
+----------+-------------+------------+---------+----------+----------+-------
---+-------+
```

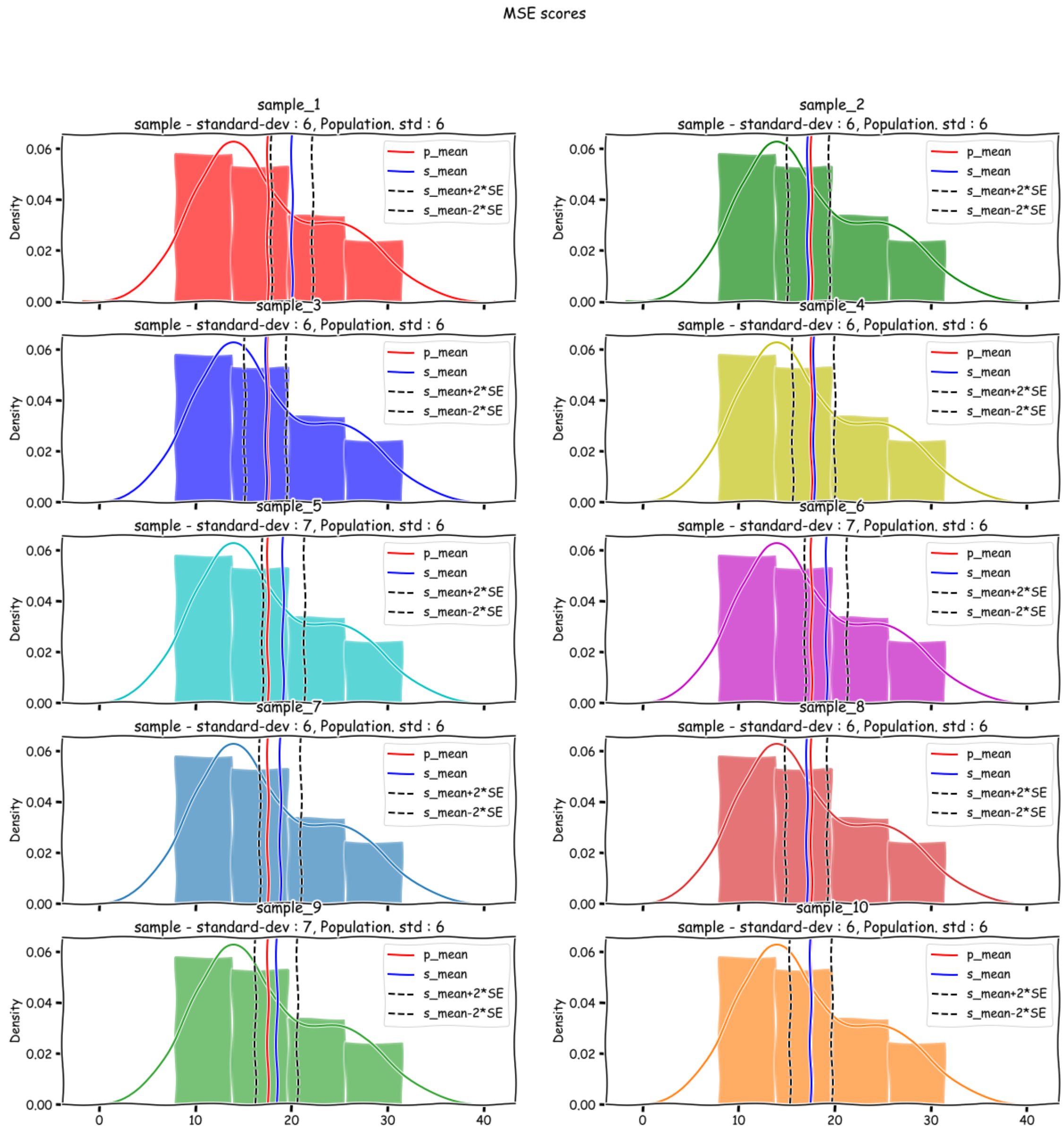
2.5 2.5 MSE with Confidence-Intervals of 95%

[19]:
```
sns.distplot(pop_MSE, color='navy', label='pop_MSE').legend();
plt.xlabel("population-MSE");
plt.title("PDF of population - MSE")
plt.show()
```



[20]:
```
colrs = ['r','g','b','y', 'c', 'm', 'tab:blue', 'tab:red', 'tab:green', 'tab:
 ↪orange', 'tab:cyan',
         'tab:purple', 'tab:brown', 'tab:gray', 'tab:pink', 'navy']

fig, axs = plt.subplots(5, 2, sharex = True, figsize=(20,20))

row_no = 0
col_no = 0
count = 0
```

```
samples = []
for index, _ in enumerate(range(10)):

    sample_60 = pop_MSE[random.sample(range(0, pop_MSE.shape[0]), 21)] #60% sample, 21 pts
    sample_40 = sample_60[random.sample(range(0, sample_60.shape[0]), 14)]
    sample = np.vstack((sample_60.reshape(-1,1), sample_40.reshape(-1,1)))
    samples.append(sample)

    sns.distplot(pop_MSE, color=colrs[index], ax=axs[row_no, col_no])

    axs[row_no, col_no].axvline(pop_MSE.mean(), linestyle="-", color='r', label="p_mean")

    axs[row_no, col_no].axvline(sample.mean(), linestyle="-", color='b', label="s_mean")

    axs[row_no, col_no].axvline(sample.mean()+2*(pop_MSE.std()/np.sqrt(pop_MSE.shape[0])), linestyle="--", color='k', label="s_mean+2*SE")
    axs[row_no, col_no].axvline(sample.mean()-2*(pop_MSE.std()/np.sqrt(pop_MSE.shape[0])), linestyle="--", color='k', label="s_mean-2*SE")

    axs[row_no, col_no].set_title('sample_'+str(index+1) + "\nsample - standard-dev : " \
                                 +str(round(sample.std())) + ", Population. std : " + str(round(pop_MSE.std())))

    axs[row_no, col_no].legend()

    if col_no == 1:
        col_no = -1
        row_no += 1
    col_no+=1

fig.suptitle("MSE scores")
plt.show()
```



[21]:
```
T = PrettyTable()

T = PrettyTable(["#samples", "Sample Size", "Sample mean", "Pop Std","Left C.
↪I","Right C.I","Pop mean","Catch"])

population_std = np.round(pop_MSE.std(), 3)
population_mean= np.round(pop_MSE.mean(), 3)

for index, sample in enumerate(samples):
```

```
    sample_mean = np.round(sample.mean(), 3)
    sample_size = sample.shape[0]

    left_limit  = np.round(sample_mean - 2*(population_std/np.sqrt(pop_MSE.
↪shape[0])), 3)
    right_limit = np.round(sample_mean + 2*(population_std/np.sqrt(pop_MSE.
↪shape[0])), 3)

    row = []

    row.append(index+1)
    row.append(sample_size)
    row.append(sample_mean)
    row.append(population_std)
    row.append(left_limit)
    row.append(right_limit)
    row.append(population_mean)
    row.append((population_mean <= right_limit) and (population_mean >=␣
↪left_limit))
    T.add_row(row)

print(T)
```

```
+-----------+-------------+-------------+---------+----------+-----------+-------
---+-------+
| #samples | Sample Size | Sample mean | Pop Std | Left C.I | Right C.I | Pop
mean | Catch |
+-----------+-------------+-------------+---------+----------+-----------+-------
---+-------+
|     1     |      35     |    20.051   |  6.425  |  17.879  |   22.223  |  17.57
| False |
|     2     |      35     |    17.231   |  6.425  |  15.059  |   19.403  |  17.57
|  True |
|     3     |      35     |    17.338   |  6.425  |  15.166  |   19.51   |  17.57
|  True |
|     4     |      35     |    17.827   |  6.425  |  15.655  |   19.999  |  17.57
|  True |
|     5     |      35     |    19.174   |  6.425  |  17.002  |   21.346  |  17.57
|  True |
|     6     |      35     |    19.137   |  6.425  |  16.965  |   21.309  |  17.57
|  True |
|     7     |      35     |    18.857   |  6.425  |  16.685  |   21.029  |  17.57
|  True |
|     8     |      35     |    17.073   |  6.425  |  14.901  |   19.245  |  17.57
|  True |
|     9     |      35     |    18.448   |  6.425  |  16.276  |   20.62   |  17.57
|  True |
```

```
|    10    |      35     |    17.515   | 6.425  | 15.343  |   19.687  |  17.57
|  True |
+----------+-------------+-------------+--------+---------+-----------+-------
---+-------+
```

3 Task 3 :

[22]:
```
pred = []

for df_row_index, data_point in enumerate([[0.18,20.0,5.00,0.0,0.421,5.60,72.
↪2,7.95,7.0,30.0,19.1,372.13,18.60]]):

    for feature_index, model in enumerate(models_1):

        feature_cols = feature_index_1[feature_index]

        data = np.take(np.array(data_point), feature_cols).reshape(1, -1)
        pred.append(model.predict(data))

pred = np.asarray(pred)
where_are_NaNs = np.isnan(pred)
pred[where_are_NaNs] = 0

print(color.BLUE + color.BOLD + "Custom-Model Test prediction : " + str(np.
↪median(pred)) + color.END)
```

```
Custom-Model Test prediction : 18.5
```

[23]:
```
query = np.array([0.18,20.0,5.00,0.0,0.421,5.60,72.2,7.95,7.0,30.0,19.1,372.
↪13,18.60])

from sklearn.ensemble import RandomForestRegressor

model = RandomForestRegressor(random_state=3).fit(x, y.reshape(-1, 1))
print(color.BLUE + color.BOLD + "SKlearn's Test prediction : " + str(model.
↪predict(query.reshape(1, -1))))
```

```
SKlearn's Test prediction : [18.747]
```

3.0.1 OBSERVATION :

TASK 1:   1. As per Central Limit theorem, When the Population distribution is non-normal (x) the sample mean approximately very near to population mean the sample distribution is Approximately Normal distribution.

TASK 2:   2. As per Central Limit theorem, When the Population distribution is normal (pop_MSE and OOB-Score) the sample mean is very near to population mean the sample distribution is Approximately Normal distribution.

TASK 3: 3. Prediction of query point is very close to scikit-learn prediction.