8E_F_LR_SVM

November 12, 2020

8E and 8F: Finding the Probability P(Y==1|X)

8E: Implementing Decision Function of SVM RBF Kernel

After we train a kernel SVM model, we will be getting support vectors and their corresponsing coefficients α_i

Check the documentation for better understanding of these attributes:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

As a part of this assignment you will be implementing the `decision_function()` of kernel SVM, here decision_function() means based on the value return by `decision_function()` model will classify the data point either as positive or negative

Ex 1: In logistic regression After traning the models with the optimal weights w we get, we will find the value $\frac{1}{1+\exp(-(wx+b))}$, if this value comes out to be < 0.5 we will mark it as negative class, else its positive class

Ex 2: In Linear SVM After traning the models with the optimal weights w we get, we will find the value of $sign(wx + b)$, if this value comes out to be -ve we will mark it as negative class, else its positive class.

Similarly in Kernel SVM After traning the models with the coefficients α_i we get, we will find the value of $sign(\sum_{i=1}^{n}(y_i\alpha_i K(x_i, x_q)) + intercept)$, here $K(x_i, x_q)$ is the RBF kernel. If this value comes out to be -ve we will mark x_q as negative class, else its positive class.

RBF kernel is defined as: $K(x_i, x_q) = exp(-\gamma||x_i - x_q||^2)$

For better understanding check this link: https://scikit-learn.org/stable/modules/svm.html#svm-mathematical-formulation

0.1 Task E

1. Split the data into $X_{train}(60)$, $X_{cv}(20)$, $X_{test}(20)$
2. Train $SVC(gamma = 0.001, C = 100.)$ on the (X_{train}, y_{train})
3. Get the decision boundry values f_{cv} on the X_{cv} data i.e. f_{cv} = `decision_function(`X_{cv}`)` you need to implement this decision_function()

[1]:
```
import numpy as np
import pandas as pd
from sklearn.datasets import make_classification
```

```
from sklearn.model_selection import GridSearchCV
import matplotlib.pyplot as plt
from sklearn.svm import SVC
from sklearn.model_selection import train_test_split
import math
```

[2]:
```
X, y = make_classification(n_samples=5000, n_features=5, n_redundant=2,
                           n_classes=2, weights=[0.7], class_sep=0.7,␣
 ↪random_state=15)
```

[3]:
```
xtrain, xtest, ytrain, ytest = train_test_split(X, y, test_size=0.3,␣
 ↪random_state=0)
xcv, xtest,ycv, ytest = train_test_split(xtest, ytest, test_size=0.3,␣
 ↪random_state=0)

print(xtrain.shape, ytrain.shape, xtest.shape, ytest.shape)
print(xtest.shape, ytest.shape, xcv.shape, ycv.shape)
```

```
(3500, 5) (3500,) (450, 5) (450,)
(450, 5) (450,) (1050, 5) (1050,)
```

[4]:
```
clf = SVC(random_state=0, decision_function_shape='ovo')

clf = GridSearchCV(clf, {'C': [0.001, 0.01, 0.1, 1, 10, 100], 'gamma' : [0.
 ↪001,0.01, 0.1, 1, 10, 100]}, n_jobs=-1, cv=5)
clf = clf.fit(xtrain, ytrain)  # set the best parameters
```

[5]:
```
clf.best_estimator_, clf.best_score_
```

[5]:
```
(SVC(C=100, decision_function_shape='ovo', gamma=1, random_state=0),
 0.9394285714285715)
```

0.1.1 Pseudo code

clf = SVC(gamma=0.001, C=100.) clf.fit(Xtrain, ytrain)

def decision_function(Xcv, ...): #use appropriate parameters for a data point x_q in Xcv: #write code to implement $(\sum_{i=1}^{\text{all the support vectors}}(y_i \alpha_i K(x_i, x_q)) + intercept)$, here the values y_i, α_i, and *intercept* can be obtained from the trained model return # the decision_function output for all the data points in the Xcv

fcv = decision_function(Xcv, ...) # based on your requirement you can pass any other parameters

Note: Make sure the values you get as fcv, should be equal to outputs of clf.decision_function(Xcv)

[6]:
```
clf = SVC(random_state=0, gamma=1, C=100, decision_function_shape='ovo')

clf.fit(xtrain, ytrain)
```

```
pred = clf.predict(xcv)

clf_dec = clf.decision_function(xcv)
```

[7]:
```
def decision_function(clf, data):
    add_intercept = []

    for x_q in data:
        add_intercept.append(np.sum(clf.dual_coef_ * np.exp(-clf._gamma*np.
 ↪sum((clf.support_vectors_ - x_q)**2, axis=1))) + clf.intercept_[0])
    return add_intercept

fcv = decision_function(clf, xcv)

print(fcv[:5], '\n', clf_dec[:5])
```

```
[-1.2615415501507021, -1.1703596219479158, -1.3159367087032825,
-1.5034977523157584, -0.6440073630763044]
 [-1.26154155 -1.17035962 -1.31593671 -1.50349775 -0.64400736]
```

8F: Implementing Platt Scaling to find P(Y==1|X)

Check this PDF

[8]:
```
unique, frequency = np.unique(ytrain, return_counts = True)
count = np.asarray((unique, frequency ))

print(count)
```

```
[[   0    1]
 [2471 1029]]
```

[9]:
```
neg, pos = frequency[0], frequency[1]

def target_calib(x):
    cal_target = []
    for i in x:
        if i == 1:
            cal_target.append((pos + 1)/(pos + 2))
        elif i == 0:
            cal_target.append(1 / (neg + 2))
    return cal_target

claibrated_target = target_calib(pred.tolist())
```

0.2 TASK F

4. Apply SGD algorithm with (f_{cv}, y_{cv}) and find the weight W intercept b `Note: here our data is of one dimensional so we will have a one dimensional weight vector i.e W.shape (1,)`

Note1: Don't forget to change the values of y_{cv} as mentioned in the above image. you will calculate y+, y- based on data points in train data

Note2: the Sklearn's SGD algorithm doesn't support the real valued outputs, you need to use the code that was done in the `'Logistic Regression with SGD and L2'` Assignment after modifying loss function, and use same parameters that used in that assignment. if Y[i] is 1, it will be replaced with y+ value else it will replaced with y- value

5. For a given data point from X_{test}, $P(Y = 1|X) = \frac{1}{1+exp(-(W*f_{test}+b))}$ where f_{test} = `decision_function(`X_{test}`)`, W and b will be learned as metioned in the above step

[10]:
```
def initialize_weights(dim):
    w = np.zeros_like((dim))
    b = np.zeros_like((1))
    print("Weights-Initialized : ", w.shape)
    return w,b

def sigmoid(z):
    sigmoid = 1/(1+math.exp(-z))
    return sigmoid

def logloss(W, b, X, Y):
    N = len(X)
    loss=[]
    for i in range(N):
        z = np.dot(X[i],W) + b
        pred = sigmoid(z)
        if pred < 0.5:
            l = (1-Y[i])*np.log10(1-pred)
            loss.append(l)
        else:
            l = Y[i]*np.log10(pred)
            loss.append(l)
    loss = (-1 * 1/len(loss) * sum(loss))
    return loss

def gradient_dw(x,y,w,b,alpha,N):
    dw =x*(y-sigmoid(np.dot(w,x)+b)) - alpha/N * w
    return dw

def gradient_db(x,y,w,b):
    db =(y-sigmoid(np.dot(w,x)+b))
    return db

def pred(w,b, X):
    N = len(X.tolist())
```

```
    predict = []
    for i in range(N):
        z = np.dot(X[i],w) + b
        predict.append(sigmoid(z))

    return np.array(predict)
```

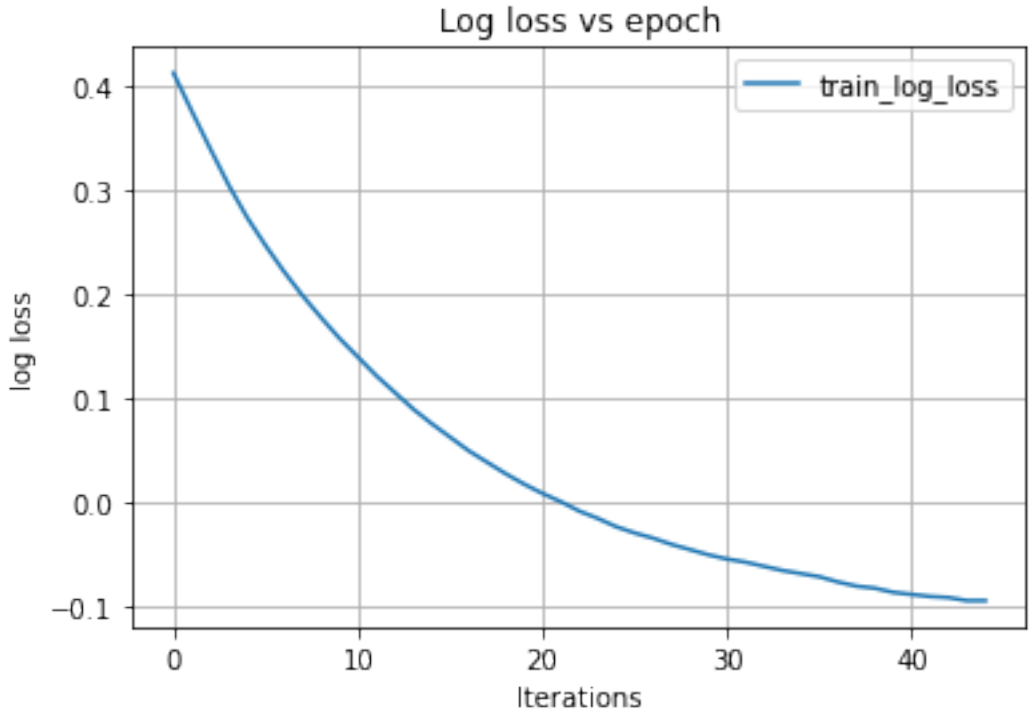
[11]:
```
def train(Y_calibrated,fcv,epochs,alpha,eta0):

    ''' In this function, we will implement logistic regression'''
    scale_down_factor = 0.0001
    epoch = 1
    w, b = initialize_weights(1)
    wl = []
    bl = []

    Lw=np.zeros_like(1)
    Lb=0

    loss = 0
    prev = 0
    train_loss = []
    test_loss = []

    while epoch <= epochs:

        y_train_pred = []
        y_test_pred = []
        np.random.RandomState(seed=2)

        for m in range(len(Y_calibrated)):

            i = np.random.choice(len(Y_calibrated))
            z = np.dot(Y_calibrated[i],w) + b

            Lw = gradient_dw(Y_calibrated[i],fcv[i],w,b,alpha,len(Y_calibrated))
            Lb = gradient_db(Y_calibrated[i],fcv[i],w,b)

            w=(1-(alpha * scale_down_factor/epochs))*w+alpha*Lw
            b=b+alpha*Lb

        train_loss.append(round(logloss(w,b,Y_calibrated, fcv), 3))

        if train_loss[-1] == prev:
            break;
        else:
            prev = train_loss[-1]
```

```
            print("Epoch: %d, train_Loss: %.3f" %(epoch, train_loss[-1]))
            epoch+=1

    %matplotlib inline
    import matplotlib.pyplot as plt
    plt.plot(train_loss, label='train_log_loss')
    plt.grid()
    plt.legend()
    plt.title('Log loss vs epoch')
    plt.xlabel('Iterations')
    plt.ylabel('log loss')
    plt.show()

    return w,b
```

[12]:
```
alpha=0.0001
eta0=0.0001
N=len(xcv)
epochs=50

w,b = train(claibrated_target,fcv,epochs,alpha,eta0)
```

```
Weights-Initialized :  ()
Epoch: 1, train_Loss: 0.413
Epoch: 2, train_Loss: 0.376
Epoch: 3, train_Loss: 0.340
Epoch: 4, train_Loss: 0.305
Epoch: 5, train_Loss: 0.274
Epoch: 6, train_Loss: 0.247
Epoch: 7, train_Loss: 0.222
Epoch: 8, train_Loss: 0.199
Epoch: 9, train_Loss: 0.178
Epoch: 10, train_Loss: 0.158
Epoch: 11, train_Loss: 0.140
Epoch: 12, train_Loss: 0.122
Epoch: 13, train_Loss: 0.106
Epoch: 14, train_Loss: 0.090
Epoch: 15, train_Loss: 0.076
Epoch: 16, train_Loss: 0.063
Epoch: 17, train_Loss: 0.050
Epoch: 18, train_Loss: 0.039
Epoch: 19, train_Loss: 0.028
Epoch: 20, train_Loss: 0.018
Epoch: 21, train_Loss: 0.009
Epoch: 22, train_Loss: 0.001
Epoch: 23, train_Loss: -0.008
Epoch: 24, train_Loss: -0.015
Epoch: 25, train_Loss: -0.023
```

```
Epoch: 26, train_Loss: -0.029
Epoch: 27, train_Loss: -0.034
Epoch: 28, train_Loss: -0.040
Epoch: 29, train_Loss: -0.045
Epoch: 30, train_Loss: -0.050
Epoch: 31, train_Loss: -0.054
Epoch: 32, train_Loss: -0.057
Epoch: 33, train_Loss: -0.061
Epoch: 34, train_Loss: -0.065
Epoch: 35, train_Loss: -0.068
Epoch: 36, train_Loss: -0.071
Epoch: 37, train_Loss: -0.076
Epoch: 38, train_Loss: -0.080
Epoch: 39, train_Loss: -0.082
Epoch: 40, train_Loss: -0.086
Epoch: 41, train_Loss: -0.088
Epoch: 42, train_Loss: -0.090
Epoch: 43, train_Loss: -0.091
Epoch: 44, train_Loss: -0.094
```



[13]:
```
w, b
```

[13]: (3.2319771056507998, -3.5624776230671333)

[14]:
```
f_test = decision_function(clf, xtest)

def calibrated_test(ftest, weight, bias):
    test_prediction = []
    for i in ftest:
        z = np.dot(i,weight) + bias
        test_prediction.append(sigmoid(z))
    return np.array(test_prediction)
test_pred = calibrated_test(f_test, w, b)

print(test_pred[:5])
```

```
[8.45382499e-01 3.89597708e-02 5.02809000e-06 5.15371386e-05
 6.28412748e-04]
```

Note: in the above algorithm, the steps 2, 4 might need hyper parameter tuning, To reduce the complexity of the assignment we are excluding the hyerparameter tuning part, but intrested students can try that

If any one wants to try other calibration algorithm istonic regression also please check these tutorials

1. http://fa.bianp.net/blog/tag/scikit-learn.html#fn:1
2. https://drive.google.com/open?id=1MzmA7QaP58RDzocB0RBmRiWfl7Co_VJ7
3. https://drive.google.com/open?id=133odBinMOIVb_rh_GQxxsyMRyW-Zts7a
4. https://stat.fandom.com/wiki/Isotonic_regression#Pool_Adjacent_Violators_Algorithm