1) if $T_1(n) \in O(g_1(n))$ and $T_2(n) \in O(g_2(n))$ then

$t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$

$O(n)$ condition $\rightarrow$ $f(n) \leq g(n) \cdot c$

$T1(n) \in O(g_1(n))$

$T_1(n) \leq g_1(n) \cdot c1$ } where $n \geq n_1$

$T_2(n) \in O(g_2(n))$

$T2(n) \leq g_2(n) \cdot c2$

$Max(g_1(n)), g_2(n)) \Rightarrow M.$

condition $t_1(n) + t_2(n) \Rightarrow c \cdot M.$

$M \geq g_1(n)$ and $M \geq g_2(n)$

$C1 \cdot g_1(n) \leq M \rightarrow$ 2 & $C2 \cdot g(n) \leq M \rightarrow$ 1

combine 1 & 2

$M \geq (C_1 \hat{} + C_2 \hat{})$ $\rightarrow$ 3, with $g_1(n)$ and $g2(n)$ written above

where $t_1(n) + t_2(n) \leq C_1 \cdot g_1(n) + C_2(g_2(n))$

$t_1(n) + t_2(n) \leq (C_1 + C_2) * M.$

2) (i) (3) $T(n) = 2T(n/2) + 1$

$T(n) = AT(n/b) + f(n)$

$A = 2$ $\quad$ $f(n) = n^k \log_a^b$.

$b = 2$ $\quad$ $n = 1$ $\quad$ $k = 1$ $\quad$ $P = 1$

$f(n) = \cancel{n} \cdot 1$

$\log_a^b \Rightarrow \log_b^a \Rightarrow \log_2^2 \Rightarrow 1$

Case 1: $\log_b^a > k$ $\quad$ $n^k \log_b^a$

Case 2: $\log_b^a = k$ $\quad$ $P > -1$ $\quad$ $n^k \log_n^{P+1}$

$P = -1$ $\quad$ $n^k \log \log n$

$P < -1$ $\quad$ (n^k)

Case 3: $\log_b^a < k$ $\quad$ $P \geq 0$ $\quad$ $n^k \log_n^P$

$P \leq 0$ $\quad$ (n^k)

$1 = k$

$\log_b^a = 1$

<u>Case 2:-</u> $\quad$ $P > -1$ $\quad$ $1 > -1$

$n^k \log_n^{1+1} \Rightarrow \log_n^2$.

$\Theta(1' \log_n^2) \Rightarrow \Theta(n \log n)$

(4)(ii) $T(n) = 2T(n-1)$

find $T(n-1) = 2T(n-1-1)$

$= 2T(n-2)$

$T(n) = 2^2 T(n-2)$

find $T(n-2) = 2T(n-2-1)$

$= 2T(n-3)$

$T(n) = 2^3 T(n-3)$

find $T(n-3) = 2T(n-3-1)$

$= 2T(n-4)$

$2^4 T(n-4)$

General form $\Rightarrow 2^k T(n-k)$

$n-k = 1$

$2^{n-1} \; T(\cancel{k}-1-\cancel{x})$

$\boxed{k = n-1}$

$2^{n-1} \Rightarrow O(2^n)$

5) Show that $n^2 + 3n + 5$ $O(n^2)$

$f(n) \leq g(n) \cdot c$

$n^2 + 3n + 5 \leq g(n^2) \cdot c$

rule 1: Replace lower term to higher terms

where $3n + 5$ is L.O.T

$\frac{3n}{\cancel{3n}} \leq \frac{3n^{\cancel{2}}}{\cancel{3n}}$ $\quad$ $5 \leq 5n^2$

$1 \leq n$ $\quad$ $1 \leq n^2$

$n^2 + 3n + 5 \leq n^2 + 3n^2 + 5n^2$ $\quad$ $n \geq 1$

$n^2 + 3n + 5 \leq 9n^2$

$c > 0$ $\quad$ n_0 = Natural $\quad$ $\forall\ n \geq n_0$

$f(n) \leq g(n) \cdot c$.

$\therefore$ $n^2 + 3n + 5 = O(n^2)$

Proved.

6) 4) $g(n) = n^3 + 2n^2 + 4n \quad \Omega(n^3)$

$f(n) \geqslant c \cdot g(n)$

$f(n) \geqslant c \cdot n^3 + 2n^2 + 4n$

$n^3 + 2n^2 + 4n \geqslant c \cdot n^3 \qquad n \geqslant n_0$

divide by n^3

$$1 + \frac{2}{n} + \frac{4}{n^2} \geqslant c \cdot 1$$

when n increases $\frac{2}{n} + \frac{4}{n^2} \Rightarrow 0.$

$1 + \frac{2}{n} + \frac{4}{n^2}$ $n = 1$ it apporaches greater than 1.

where $n_0 = 10$

$10^3 + 2 \times 10^2 + 4(10) \Rightarrow 1240$

$n^3 = (10)^3 = 1000.$

So $n \geqslant n_0 \Rightarrow 1240 \geqslant 1000$

$n^3 + 2n^2 + 4n$ grows large value of n

(10) grows as fast of n^3.

Thus proved $n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

---

3) (7) $h(n) = 4n^2 + 3n$ is $O(n^2)$

$\Theta(n^2)$ condition.

$g_1(n) \cdot c_1 \leq f(n) \leq g_2(n) \cdot c_2$

$g_1(n) \cdot c_1 \leq f(n)$

c_1 should be less than $f(n)$

$n^2 \cdot c \leq 4n^2 + 3n$

divide by n^2

c_2 should be

$c \cdot \leq 4 + \frac{3n}{n^2}$

$n=1$

$c_1 \leq 4 + 3$

$c_1 \leq 7$.

c_1 should be less than 7

$c_1 = 1$

$4n^2 + 3n \geqslant 1 \cdot n^2$

$4 + 3 \geqslant 1 n^2$

$7 \geqslant 1 n^2$

all $\geqslant 1$ $\begin{cases} n=1 \\ n=2 \\ n=3 \end{cases}$

$f(n) \le c_2 \cdot g(n)$

$4n^2 + 3n \le c_2 n^2$

divide by n^2

$4 + \frac{3}{n^2} \le c_2$

$n = 1$

$4 + 3 \le c_2$

$7 \le c_2$

c_2 is greater than 7

$c_2 = 7$.

$4n^2 + 3n \le 7n^2$

$n = 1$

~~4×... + 21~~ $\le$ ~~47~~ 7

$7 \le 7$

$n = 2$ $4(2)^2 + 3(2) \le 7(2)^2$

$16 + 6 \le 28$

$22 \le 28$

$c_1 \le f(n) \le c_2$

$1 \le f(n) \le 7$.

$\therefore 4n^2 + 3n$ is $\Theta(n^2)$

(8) $f(n) = n^3 - 2n^2 + n$

$g(n) = n^2$ $\qquad \Omega(n^2)$

$f(n) \geq g(n) \cdot c$

$n^3 - 2n^2 + n \geq n^2 c$

$n = 1$. $\quad 1 - 2 + 1 \geq -1 \cdot c$

$-1 + 1 \geq -1$

$0 \geq -1 \cdot c$

$n = 2 \quad 8 - 2 \times 4 + 2$

$2 \geq -1 \cdot c$

here seting n by all Number. here $f(n)$ higher order $= n^3$.

n^3 grows as fast as n^2 are large Number of $(n = 1)$.

$\therefore f(n)$ is $\Omega(n^2)$.

(9) 7) $h(n) = n + n\log n.$ $\Theta(n \log n)$

using Master Theorm we can

$n = n/2 + \frac{n}{2}$

$h(n) = T(n/2) + T(n/2)$

$= 2T(n/2) + n\log n.$

$a = 2$ $b = 2$

Condition $C_1 * g(n) \leq h(n) \leq C_2 \cdot g(n)$

lower bound $h(n) = \Omega(n\log n)$

* $C_1 = 1$ $g(n) = n\log n$
* $n \leq h(n)$ $n \geq n_0$
* $h(n) = n + n\log n \geq n\log n$
* $n_0 = 2$
* $n \geq 2$ n is greather than equal to n logn.

upper bound $h(n)$ is $O(n \log n)$

* $c_1 = 1$ & $g(n) = (n \log n)$

* $h(n) \leq c2 * n \log n$.

* $n \log n = 1 + 1/\log n$.

* $c_2 = 2$.

$c_1 = 1 \quad c_2 = 2$.

Thus $h(n) = n + n \log n \quad \Theta(n \log n)$

---

(10) $\cancel{8} \; T(n) = 4T(n/2) + n^2 \qquad T(1) = 1$

Master Theorm.

$\log_b^a \Rightarrow a = 4 \quad b = 2 \quad f(n) = n^2$

$f(n) = n^k \log_n^P$. $\quad K = 2$

$\log_2^4 \Rightarrow 2 \quad k = 2$

$\log_2^4 = k$.

$2 = 2$

case 2: $P \geq -1 \Rightarrow n^k \log_n^P$.

$f(n) = n^2 \log_n^{p+1} \Rightarrow n^2 \log_n^2$

$O(n^2 \log_n n) \Rightarrow O(n^2 \log n)$ //.

(11) 9) [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]

```
num = [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1,
        9, -1, 0, -6, -8, 11, -9]

P = [ ]
For  i in range(len(num)-1):  → n
     r = num[i] * num[i+1]
     p.append [r]
Print (P)
P.sort() → log n
Print(P)
Maxi = P[0]
Mini = P[-1]
Print(maxi, mini)
```

Time Complexity $O(n \log n)$ //.

12) Binary Search

$K = 23$

$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ Arr[\,] = \{ & 2, & 5, & 8, & 12, & \underline{16}, & 23, & 38, & 56, & 72, & 91\} \end{array}$$

Step 1: Middle.

$$Mid = \frac{0+9}{2} \Rightarrow 4.5$$

$Mid = 4$. $Arr[Mid] = 16$

Step 2: $Arr[Mid] < Key$.

$16 < 23$

$low = Mid + 1$. $low = 5$ //

$4 + 1 \Rightarrow 5$

Step 3: $Mid = low + high$

$$= \frac{5+9}{2} \Rightarrow \frac{14}{2} \Rightarrow 7.$$

Step 4: $Arr[Mid] = 56$.

Step 5: $Arr[Mid] > 23$ $high = Mid - 1$

$56 > 23$.

Step 6 : $Mid = \frac{low + high}{2}$

$= \frac{5 + 6}{2} \Rightarrow \frac{11}{2} \Rightarrow \cancel{6.5}\ 5.5$

Step 7 : $Mid = 5$

$arr[Mid] == Key$

$23 == 23$.

Key = 23 is found at Index 5

13) $d = [45, 67, -12, 5, 22, 30, 50, 20)$

$$\begin{array}{cccccccc} 45 & 67 & -12 & 5 & 22 & 30 & 50 & 20 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

$Mid = \frac{0 + 7}{2} \Rightarrow \frac{7}{2} \Rightarrow 3.5$

45, 67, -12, 5   22, 30, 50, 20

45, 67   -12, 5   22, 30   50, 20

45, 67   -12   5   22   30   50   20

45, 67   -12, 5   22, 30   20, 50

-12, 5, 45, 67   20, 22, 30, 50

[-12, 5, 20, 22, 30, 45, 50, 67]

Merge Sort

* Base Step = $n=1$

$T(1)=0$

* array is divided into 2 halves. $n/2$

* Recursive $2T(n/2) + cn$.

Recurren Relation $T(n) = 2T(n/2) + cn$

14) Array $S[\,] = (12, 7, 5, -2, 18, 6, 13, 4)$

12, 7, 5, -2, 18, 6, 13, 4.

min

-2, | 12, 7, 5, 18, 6, 13, 4

min

-2, 4, | 12, 7, 5, 18, 6, 13

min

-2, 4, 5 | 12, 7, 18, 6, 13

min

$-2, 4, 5, 6, | 12, 7, 18, 13$

↓ Min

$-2, 4, 5, 6, 7, | 12, 18, 13$

↓ Min

$-2, 4, 5, 6, 7, 12, | 18, 13$

↓ Min

$-2, 4, 5, 6, 7, 12, 13, | 18$

$(-2, 4, 5, 6, 7, 12, 13, 18)$

Time Complexity.

$O(n^2)$

15) arr = [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

(indices: 0 1 2 3 4 5 6 7 8 9)

Step 1: $Mid = \frac{0+9}{2} \Rightarrow 4.5 \Rightarrow 5$

Step 2: Mid = 5.

Step 3: arr [Mid] > Key.

12 > Key.

~~low = Mid + 1~~

high = Mid − 1

[4.] 5 − 1

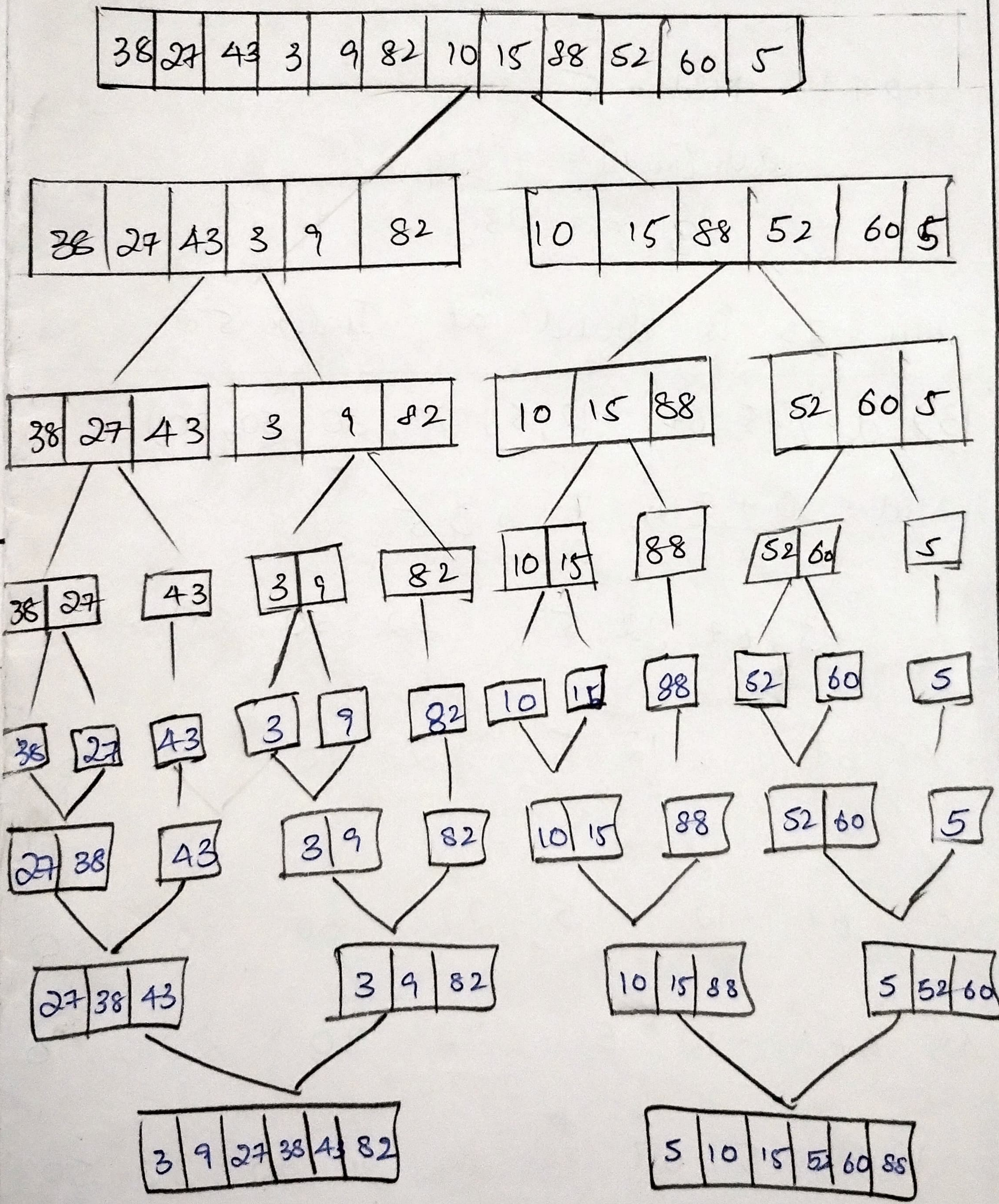
or

Step 3:-

arr [mid] == Key

10 is Found at 4

16) arr[] = [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]



17) arr[] = [64, 34, 25, 12, 22, 11, 90]

64, 34, 25, 12, 22, 11, 90

34 64 25, 12, 23, 11, 90

34, 25, 64, 12, 23, 11, 90

34, 25, 12, 64, 23, 11, 90

34, 25, 12, 23, 64, 11, 90

34, 25, 12, 23, 11, 64, 90

I

Best Case : $O(n)$

Worst Case : $O(n^2)$

Average Case : $O(n^2)$

34, 25, 12, 23, 11, 64, 90

25, 34, 12, 23, 11, 64, 90

25, 12, 34, 23, 11, 64, 90

25, 12, 23, 34, 11, 64, 90

25, 12, 23, 11, 34, 64, 90

II

25, 12, 23, ~~2~~ 11, 34, 64, 90

12, 25, 23, 11, 34, 64, 90

12, 23, 25, 11, 34, 64, 90

12, 23, ~~2~~ 11, 25, 34, 64, 90

III

12, 23, ~~2~~ 11, 25, 34, 64, 90

12, 11, 23, 25, 34, 64, 90

IV

12, 11, 23, 25, 34, 64, 90

11, 12, 23, 25, 34, 64, 90

V

11, 12, 23, 25, 34, 64, 90.

18) 64, 25, 12, 22, 11

64	25	12	22	11

min

11	64	25	12	22

min

11	12	64	25	22

min

11	12	22	64	25

min

11	12	22	25	64

min

11	12	22	25	64

Bast case : $O(n^2)$

Worst case : $O(n^2)$

Average case : $O(n^2)$

19) Insertion.

38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5

38	27	43	3	9	82	10	15	88	52	60	5
38	27	43	3	9	82	10	15	88	52	60	5
27	38	43	3	9	82	10	15	88	52	60	5
27	38	43	3	9	82	10	15	88	52	60	5
3	27	38	43	9	82	10	15	88	52	60	5
3	9	27	38	43	82	10	15	88	52	60	5
3	9	27	38	43	82	10	15	88	52	60	5
3	9	10	27	38	43	82	15	88	52	60	5
3	9	10	15	27	38	43	82	88	52	60	5
3	9	10	15	27	38	43	82	88	52	60	5
3	9	10	15	27	38	43	52	82	88	60	5
3	9	10	15	27	38	43	52	60	82	88	5
3	5	9	10	15	27	38	43	52	60	82	88

(19) Base Case : $O(n)$
Averige Case : $O(n^2)$
worst Case : $O(n^2)$

(20) 4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9.

4 | -2 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-2 4 | 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

-2, 4, 5 | 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6 -8, 11, -9

-2, 3, 4, 5 | 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6 -8, 11, -9

-2, 3, 4, 5, 10 | -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6 -8, 11, -9

-5, -2, 3, 4, 5, 10 | 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -2, 2, 3, 4, 5, 10 | 8, -3, 6, 7, -4, 1, 9, -1, 0
-6, -8, 11, -9

-5, -2, 2, 3, 4, 5, 8, 10 | -3, 6, 7, -4, 1, 9, -1, 0
-6, -8, 11, -9

-5, -3, -2, 2, 3, 4, 5, 8, 10 | 6, 7, -4, 1, 9, -1,
0, -6, -8, 11, -9

-5, -3, -2, 2, 3, 4, 5, 6, 8, 10 | 7, -4, 1, 9, -1
0, -6, -8, 11, -9

-5, -3, -2, 2, 3, 4, 5, 6, 7, 8, 10 | -4, 1, 9, -1
0, -6, -8, 11
-9

-4
-5, -3, -2, 2, 3, 4, 5, 6, 7, 8, 10 | 1, 9, -1
0, -6, -8,
11, -9

-5, -4, -3, -2, 1, 2, 3, 4, 5, 6, 7, 8, 10 | 9, -1
0, -6, -8, 11, -9

-5, -4, -3, -2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 | -1,
0, -6, -8, 11, -9

-5, -4, -3, -2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 |
0, -6, -8, 11, -9

-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8,
9, 10 | -6, -8, 11, -9

-6 | -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8
9, 10 | -8, 11, -9

-8 | -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6
7, 8, 9, 10 | 11, -9

-8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5,
6, 7, 8, 9, 10, 11 | -9

-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5,
6, 7, 8, 9, 10, 11.

Best Case : $O(n)$

Average Case : $O(n^2)$

Worst case : $O(n^2)$