Steps of Hypothesis Testing

1. Steps of hypothesis testing

Recall that hypothesis testing is a form of statistical inference. Previously, we inferred about a population by calculating a confidence interval. We estimated the true mean of a population from a sample mean and created a confidence interval provided a margin of error for our estimate. In this chapter, we also use sample data to help us make decisions about what is true about a population. When conducting a hypothesis test, we are asking ourselves whether the information in the sample is consistent, or inconsistent, with the null hypothesis about the population. We follow a series of four basic steps:

- State the null and alternative hypotheses.
- Select the appropriate significance level and check the test assumptions.
- Choose the test statistics (t-test, z-test, F-test etc)
- Interpret the result

If we reject the null hypothesis we are saying that the difference between the observed sample mean and the hypothesized population mean is too great to be attributed to chance. When we fail to reject the null hypothesis, we are saying that the difference between the observed sample mean and the hypothesized population mean is probable if the null hypothesis is true. Essentially, we are willing to attribute this difference to sampling error.

2. Symbols

STATISTICAL SYMBOLS

Population Statistical Symbols

Symbol	Meaning
N	Size of the Population
μ	Mean of the Population
σ^2	Variance of the Population
σ	Standard Deviation of the Population
f	Frequency of Occurrence
F	Cumulative Frequency

Sample Statistical Symbols

Symbol	Meaning
n	Size of the Sample
$\bar{x}$	Mean of the Sample
s^2	Variance of the Sample
s	Standard Deviation of the Sample
f	Frequency of Occurrence
F	Cumulative Frequency

3. Different Hypothesis Testing

1. Single Sample: We are testing data from one sample.
2. Multiple Sample: We are comparing two samples
3. A dependent sample is characterized by a measurement followed by an intervention of some kind and then another measurement. This could be called a "before" and "after" study.
4. Dependent sample is characterized by matching or pairing observation.
5. Sample size: Number of data items in the sample.
6. Population mean: The mean of the population from which sample is taken.
7. Population standard deviation: The standard deviation of the population from which sample is taken.
8. Test statistics: Name of the test statistics chosen for hypothesis testing
9. Example- Example number for this test statistics.

Single/ Multiple Dep/Ind	Sample size	Sample data	Hypothesis test	Population standard deviation	Test-statistics	Formula/Example
Single	Size<30	Random, Numeric, Continuous and normal distribution	Hypothesis test of population mean	Sigma is known	t-test	$t_{n-1} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \quad df = n-1$
Single	Size<30		Hypothesis test of population mean	Sigma is unknown	t-test	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Single	Size>30		Hypothesis test of population mean	Sigma is known	z-test	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
Single	Size>30		Hypothesis test of population mean	Sigma is unknown	z-test	$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Two sample	Size>30	Samples are from independent population	We compare mu1 and mu2 i.e. whether they are same or not.	Sigma1 and sigma2 is known	Paired z-test	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

Steps of hypothesis testing

Two sample	Size<30	Samples are from independent population	We compare mu1 and mu2 i.e. whether they are same or not.	Sigma1 and sigma2 is unknown but are equal	Paired t-test	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $df = n_1 + n_2 - 2$
Two sample			We compare mu1 and mu2 i.e. whether they are same or not.	Sigma1 and sigma2 is unknown and unequal	Un pooled t-test	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ $df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$
Two sample			Test Statistic for the Difference between Two Population Variances		F-test	$F = \frac{s_1^2}{s_2^2}$ $df_{numerator} = n_1 - 1$ $df_{denominator} = n_2 - 1$
Two sample			Test Statistic for the comparison of two independent (unrelated) samples		**(Mann-Whitney U-Test)**	$U = min(U_1, U_2)$ where $U_i = n_1 n_2 + \frac{n_i(n_i + 1)}{2} - \sum R_i$
Two Samples			Test Statistic for the comparison of more than two independent (unrelated) samples		**(Kruskal-Wallis H-Test)**	$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$ where $df = k - 1$ correction for ties $C_H = 1 - \frac{\sum(T^3 - T)}{N^3 - N}$

Steps of hypothesis testing

Two samples s			Test Statistics for the comparison of two dependent (related) samples		**(Wilcoxon Signed Ranks Test)**	$T = min\left(\sum R_+, \sum R_-\right)$
Two samples s			Test Statistics for the comparison of more than two dependent (related) samples		**(Friedman Test)**	$F_r = \left[\frac{12}{nk(k+1)}\sum_{i=1}^{k} R_i^2\right] - 3n(k+1)$
Two samples s			Test Statistic for the Spearman Rank Correlation Coefficient (***r**s*)			$t_{n-2} = \frac{r_s\sqrt{(n-2)}}{\sqrt{(1-r_s^2)}} \quad df = n-2$
Two samples s			Test Statistic for the Pearson Product Moment Correlation Coefficient (***r***)			$t_{n-2} = \frac{r\sqrt{(n-2)}}{\sqrt{(1-r^2)}} \quad df = n-2$
Two samples s			Test Statistic for the Population Slope in a Regression			$t_{n-2} = \frac{b_1 - \beta_1}{SE} \quad df = n-2$

Two samples s			Shapiro Wilks Test of Normality			$W = \frac{b^2}{SS}$ where $SS = \sum (x - \bar{x})^2$ $b = \sum_{i=1}^{m} a_i (x_{(n+1-i)} - x_i)$ and $\begin{cases} n\ even: & m = \frac{n}{2} \\ n\ odd: & m = \frac{(n-1)}{2} \end{cases}$

10. Example-1 (Single Sample Z-test)

Jamestown Steel Company manufactures and assembles desks. The weekly production of the Model A325 desk at the follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, new production methods have been introduced. The VP of manufacturing would like to investigate whether there has been a *change* in the weekly production of the Model A325 desk.

Soultion 1:

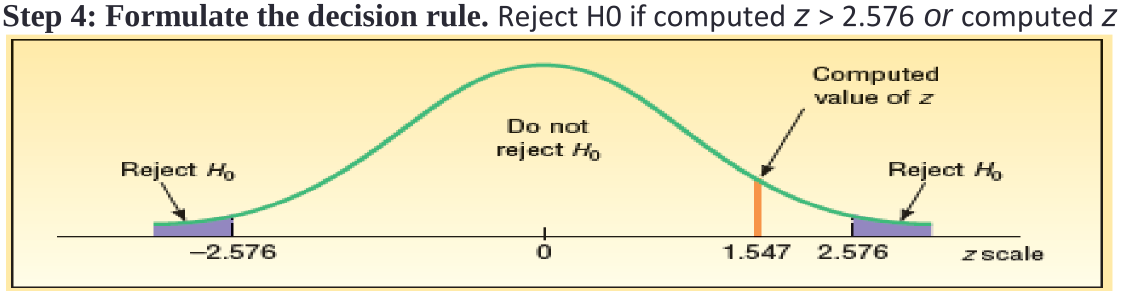
A sample yielded a *mean number of* desks produced of 203.5. Test using 0.01 significance level. σ=16, *n*=50, α=.01

Step 1: State the null hypothesis and the alternate hypothesis.
H0: μ = 200
H1: $\mu \neq$ 200

Step 2: Select the level of significance: α = 0.01 as stated in the problem.

Step 3: Select the test statistic : Use *z*-distribution since σ is known.

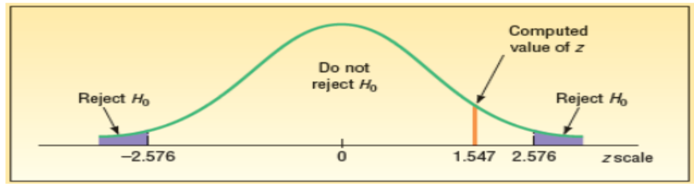
Step 4: Formulate the decision rule. Reject H0 if computed *z* > 2.576 *or* computed *z* < 2.576



Step 5: Compute the z statistic, make a decision and interpret the result.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{203.5 - 200}{16 / \sqrt{50}}$$

$$z = 1.55$$



1.55 falls within the **do not reject the null hypothesis** region. H_0 is not rejected.
We conclude that the population mean is not different from 200.

11. Example-2 (Single Sample T-test)

The McFarland Insurance Company Claims Department reports the **mean** cost to process a claim is **$60.** An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of **26** claims processed last month. The sample information is reported below (MEAN =$56.42).

At the **.01** significance level, is it reasonable a claim is **now less than $60**?

$45	$49	$62	$40	$43	$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				

Step 1: State the null hypothesis and the alternate hypothesis.

H0: $\mu \geq \$60$

H1: $\mu < \$60$

(note: keyword in the problem “now **less** than”)

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem.

Step 3: Select the test statistic.

Use t distribution (n-1 =df) since σ is unknown.

Steps of hypothesis testing

Step 4: Formulate the decision rule.

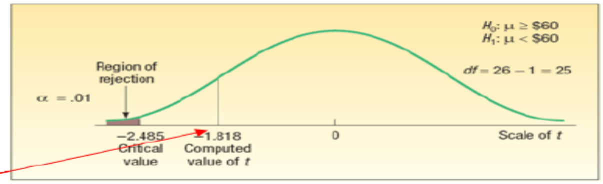
Reject H_0 if *computed t* < − *critical t*

$$t < -t_{\alpha, n-1}$$

$$\frac{\overline{X} - \mu}{s / \sqrt{n}} < -t_{\alpha, n-1}$$

$$\frac{\$56.42 - \$60}{\$10.04 / \sqrt{26}} < -t_{.01, 26-1}$$

$$-1.818 \text{ is not } < -2.485$$



Step 5: Make a decision and interpret the result.

−1.818 does not fall in the rejection region. H_0 is not rejected. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60.