

Artificial neural networks, perceptrons

Relevant Readings: Sections 4.1, 4.2, 4.3, 4.4 in Mitchell

CS495 - Machine Learning, Fall 2009

Biological motivation

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Implementation

- ▶ How about we program a perceptron?