

Advanced Algorithms
Spring 2021
IIIT Hyderabad

Homework 3, Due: Saturday February 20, 2021

1. Using the probabilistic method, show that the following bipartite graph $G = (L \cup R, E)$ exists on $2n$ vertices.

- $|L| = |R| = n$,
- Every vertex v in L has a degree $n^{3/4}$ and every vertex u in R has degree at most $3n^{3/4}$.
- Every subset of $n^{3/4}$ vertices in L has at least $n - n^{3/4}$ neighbors in R .

(5 Points)

2. Show that the function $f(x) = 1 - (1 - x/k)^k$ is concave for any $k > 0$ and $x \in [0, 1]$. (Note: x is a real number, and not just an integer). **(5 Points)**
3. Show a complete example of the two level hashing scheme with your choice of p , m , and n . **(5 Points)**
4. Write a Boolean formula on n variables such that the maximum number of satisfiable clauses is exactly half the total number of clauses. Repeat for the case when the maximum number of satisfiable clauses is exactly $3/4$ of the total number of clauses.