Solutions to Mid Exam-1 Paper IEC102

(Q) In many amplifier applications, we are concerned not only with voltage gain, but also with power gain.

$$\text{Power gain} = A_p = \frac{\text{Power delivered to the load}}{\text{Power supplied by the source}}$$

Find the power gain for the circuit shown in Fig. Q when $R_L = 60K\Omega$

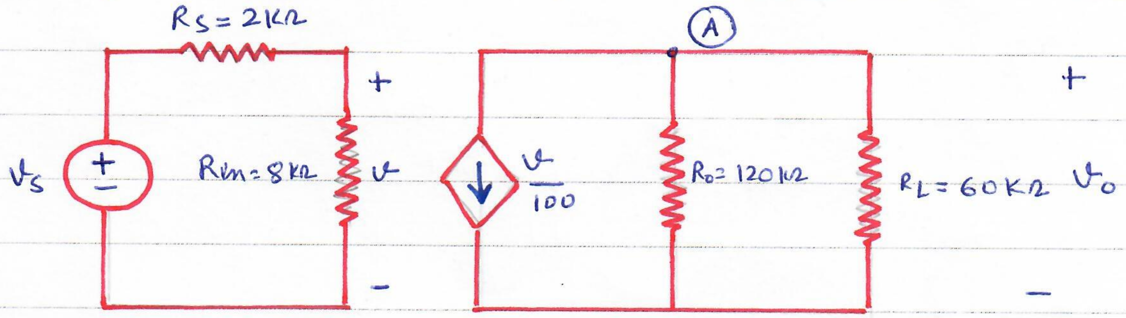



Fig. Q

Sol:

$$v = \frac{8}{2+8} \times v_s = \frac{4}{5} v_s$$

Applying KCL at node (A)

$$\frac{v}{100} + \frac{v_o}{120 \times 10^3} + \frac{v_o}{60 \times 10^3} = 0$$

$$\Rightarrow \quad v_o = -400v$$

PTO

$$\Rightarrow v_o = -400\,v = -400 \times \frac{4}{5} v_s = -320 v_s$$

$$\left(\because v = \frac{4}{5} v_s\right)$$

$$v_o = -320 v_s$$

$$i_L = \frac{v_o}{R_L} = \frac{-320 v_s}{60 \times 10^3}$$

$$i_s = \frac{v_s}{(2+8)10^3} = \frac{v_s}{10 \times 10^3}$$

Power delivered by source = $v_s i_s$

$$= \frac{v_s \times v_s}{10^4} = \frac{v_s^2}{10^4}$$

Power ~~at~~ consumed by load $(R_L)$ = $v_o \; i_L$

$$= (-320 v_s) \times \frac{(-320 v_s)}{60 \times 10^3}$$

Power gain = $A_p$ = $\dfrac{v_o i_L}{v_s i_s} = \dfrac{\dfrac{(320)^2 v_s^2}{60 \times 10^3}}{\dfrac{v_s^2}{10 \times 10^3}} = \dfrac{(320)^2 \times 10 \times 10^3}{60 \times 10^3}$

$$= \frac{(320)^2}{6}$$

$$\Rightarrow \boxed{A_p \simeq 17067}$$

Q. Find the value of $R_L$ for maximum power transfer in the circuit shown in Fig. Q. Also, find the maximum power dissipated in $R_L$.

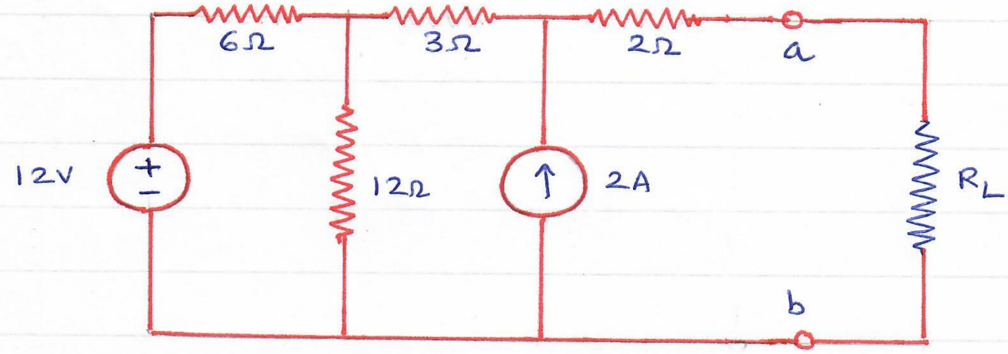
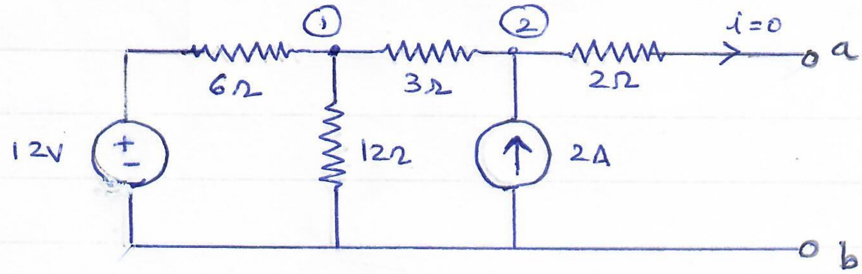



Fig. Q

Sol.

Thevenize the circuit across terminals of $R_L$.




$V_{th}$

Applying KCL at node 1

$$\frac{v_1 - 12}{6} + \frac{v_1}{12} + \frac{v_1 - v_2}{3} = 0$$

$$\Rightarrow \frac{2(v_1 - 12) + v_1 + 4(v_1 - v_2)}{12} = 0$$

$$\Rightarrow 7v_1 - 4v_2 = 24 \quad \cdots (A)$$

Applying KCL at node 2

$$-2 + \frac{v_2 - v_1}{3} = 0$$

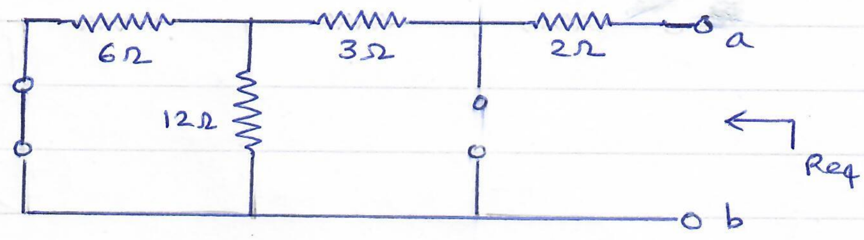
$$\Rightarrow v_2 - v_1 = 6 \quad \cdots (B)$$

Solving (A) and (B)

$$\boxed{v_2 = 22V = V_{Th}}$$

$\underline{R_{th}} = R_{eq}$

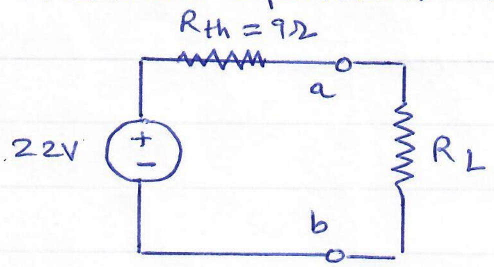
Turning off all the sources and calculating equivalent resistance across terminals a-b.




$R_{eq} = R_{Th} = 2 + 3 + 6 \parallel 12$

$= 2 + 3 + 4 = 9\Omega$

Thevenin equivalent circuit




Maximum power is transferred to $R_L$ when $R_L = R_{Th} = 9\Omega$

Maximum power transferred to $R_L$ when $R_L = R_{th} = 9\Omega$

$$P_{max \text{ in } R_L} = \left[\frac{22}{(9+9)}\right]^2 \times 9$$

$$= \left(\frac{\cancel{22}^{\,11}}{\cancel{18}_{\,9}}\right)^2 \times 9$$

$$\Rightarrow P_{max} = \left(\frac{11}{9}\right)^2 \times 9 = \frac{11 \times 11}{9} = \frac{121}{9} = 13.44\,W$$

Q. Find the Thevenin and Norton equivalent of the circuit shown in Fig. Q at terminals a-b.

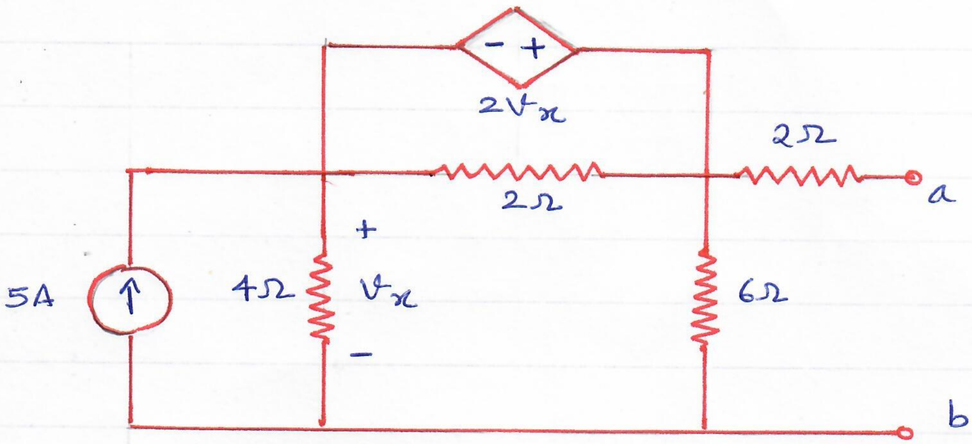
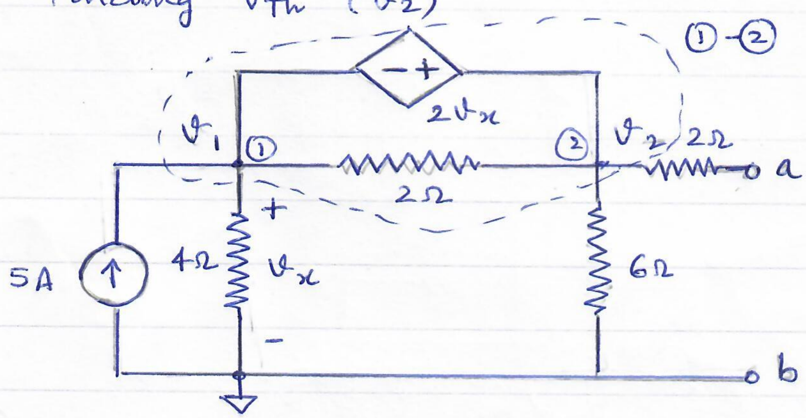



Fig. Q

Sol.

Finding $V_{Th}$ ($v_2$)




PTO

Applying KCL at supernode ① - ②

$$-5 + \frac{v_1}{4} + \frac{v_2}{6} = 0$$

$$\Rightarrow \quad -5 + \frac{3v_1 + 2v_2}{12} = 0$$

$$\Rightarrow \quad 3v_1 + 2v_2 = 60 \quad \ldots (A)$$

but $v_2 - v_1 = 2v_x$

and $\boxed{v_x = v_1}$ $\quad \therefore \quad v_2 - v_1 = 2v_1$

$$\Rightarrow \quad 3v_1 - v_2 = 0 \quad \ldots (B)$$

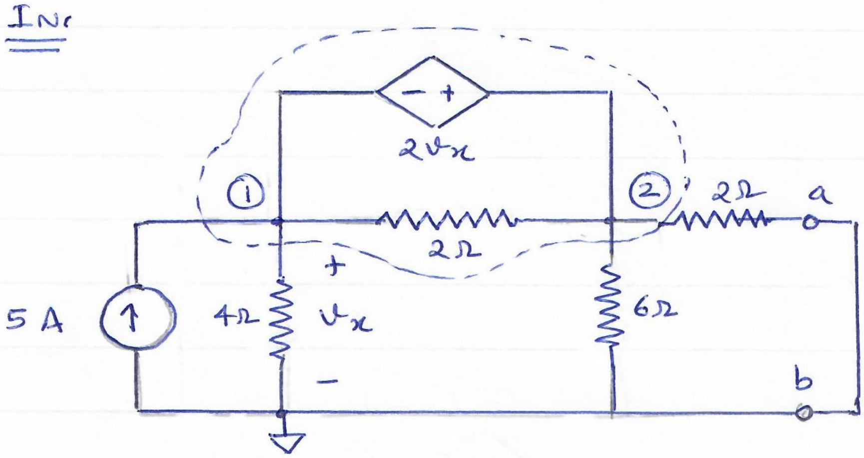
Solving (A) and (B)

$$3v_1 + 2v_2 = 60$$
$$3v_1 - v_2 = 0$$

$$\Rightarrow \quad 3v_2 = 60$$

$$\Rightarrow \quad \boxed{v_2 = 20V = V_{Th}}$$

$I_N$




Applying KCL at super-node ① - ②

$$-5 + \frac{v_1}{4} + \frac{v_2}{6} + \frac{v_2}{2} = 0$$

$$\Rightarrow \quad -5 + \frac{3v_1 + 2v_2 + 6v_2}{12} = 0$$

$$\Rightarrow \quad 3v_1 + 8v_2 = 60 \quad \ldots (C)$$

PTO

But $v_2 - v_1 = 2v_x = 2v_1 \quad (\because v_x = v_1)$

$$\Rightarrow \quad 3v_1 - v_2 = 0 \quad \ldots (D)$$

Solving (c) and (D)

$$3v_1 + 8v_2 = 60$$
$$3v_1 - v_2 = 0$$

$$\Rightarrow 9v_2 = 60$$

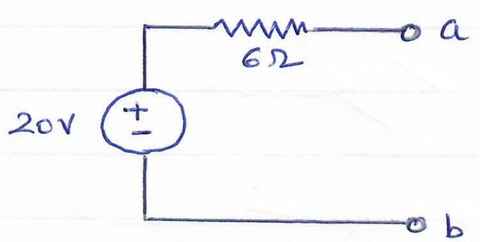
$$\Rightarrow v_2 = \frac{60}{9} = \frac{20}{3}\,V$$

$$\boxed{I_N = \frac{v_2}{2} = \frac{20/3}{2} = \frac{10}{3}\,A}$$

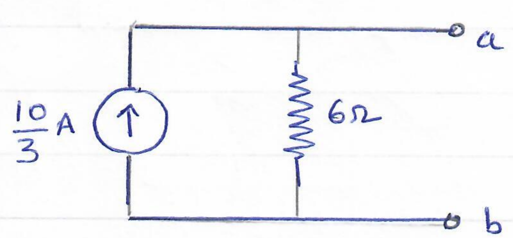
$$R_{th} = R_N = R_{eq} = \frac{V_{th}}{R_N} = \frac{20}{(10/3)} = 6\,\Omega$$

$\therefore V_{Th} = 20V$; $I_N = \frac{10}{3}A$; and $R_{Th} = R_N = 6\Omega$

Thevenin equivalent circuit is




Norton equivalent circuit is



(Q.) The switch in the circuit shown in Fig. Q has been closed for a long time (i.e., the circuit is in steady state just before the switch is operated) and it is opened at $t=0$. Find $v_c(t)$ for $t \geq 0$. Also calculate the initial energy stored in the capacitor.

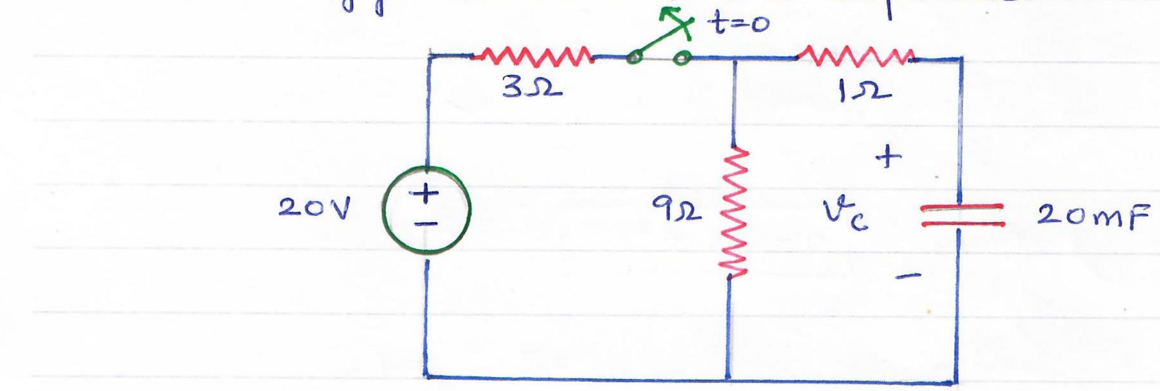
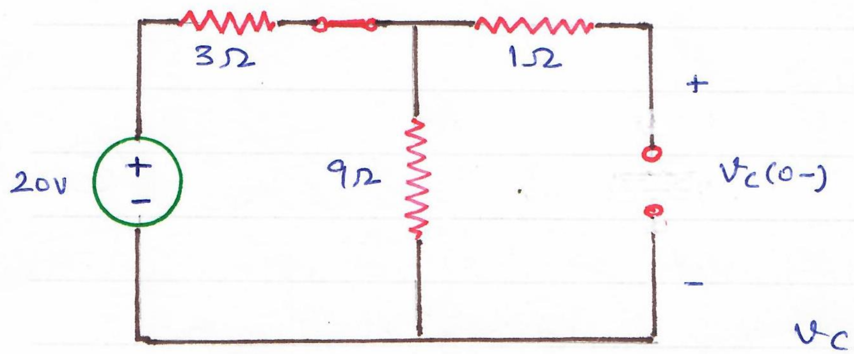



Fig. Q

Sol.

At $t=0-$, the switch is closed and capacitor acts as an open circuit since the circuit is in steady state (given).

circuit at $t=0-$



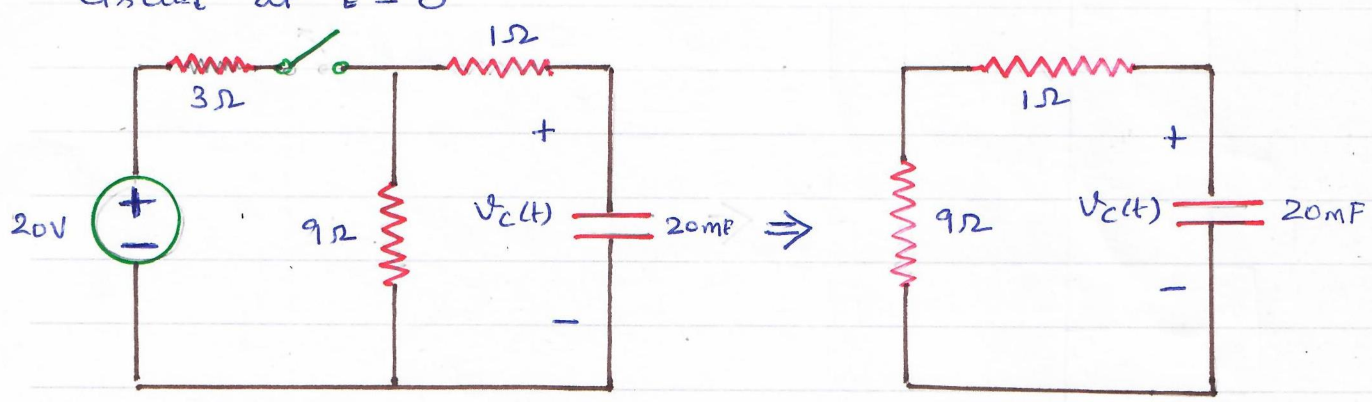

$$v_c(0-) = \frac{20 \times 9}{9+3}$$

$$= \frac{\cancel{20}^{5} \times \cancel{9}^{3}}{\cancel{12}_{\cancel{4}}} = 15V$$

$$v_c(0-) = v_c(0) = v_c(0+) = 15V$$

(Since the voltage across a capacitor has to be continuous)

Circuit at $t=0$



The response $v_c(t)$ will be of the for $Ae^{s_1 t} = v_c(0)e^{-t/\tau}$

where $v_c(0) = 15V$ (already calculated)

$\tau = R_{eq}C$

where $R_{eq} = 1+9 = 10\Omega$ ; and $C = 20mF$

$\therefore \tau = 10 \times 20 \times 10^{-3} = 0.2s$

$\therefore$ $v_c(t)$ for any time $t \geq 0$ is

$$\boxed{v_c(t) = 15e^{-\frac{t}{0.2}} = 15e^{-5t} \ V}$$

Initial energy stored in the capacitor $= \frac{1}{2}C[v_c(0)]^2$

$= \frac{1}{2} \times 20 \times 10^{-3} \times (15)^2$

$= 10 \times 10^{-3} \times 225$

$= 2.25 J$

Q) Find the expression for $i_L(t)$ for all time $t > 0$ in the circuit shown in Fig. Q. Assume that the circuit is in steady state at $t = 0-$.

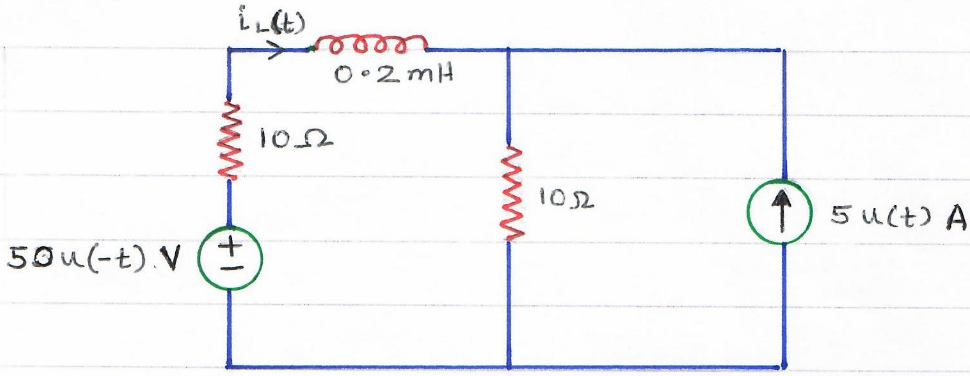
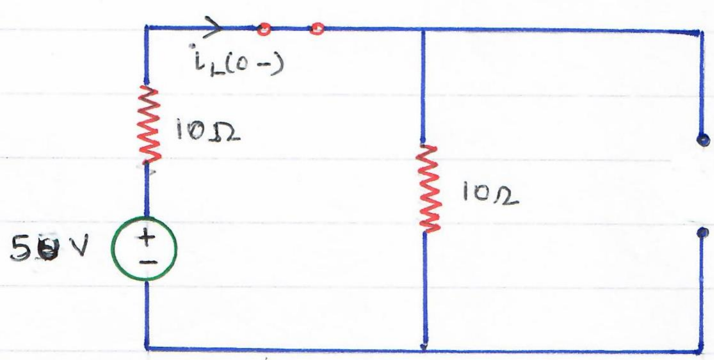



Fig. Q

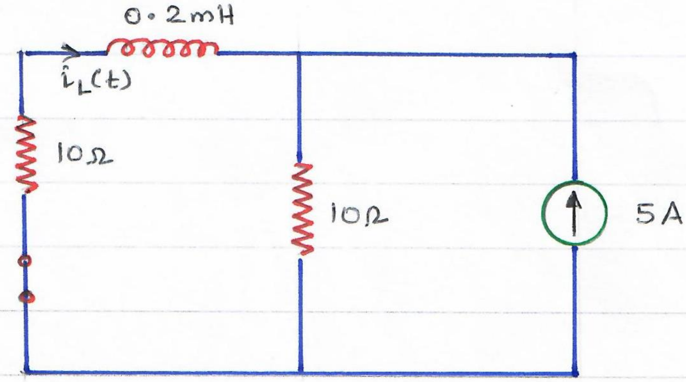
Sol.

Circuit at $t = 0-$. Inductor can be replaced by short circuit as the circuit is in steady state.



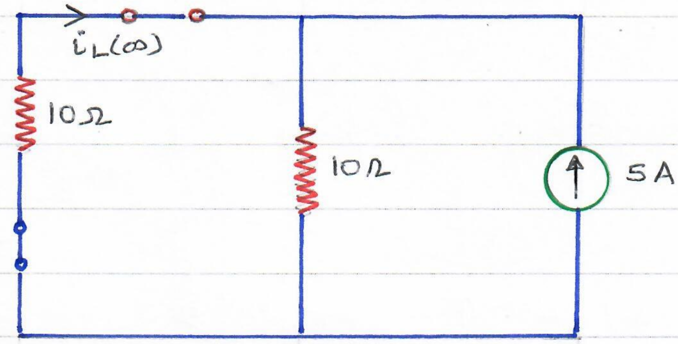

$$i_L(0-) = \frac{50}{10+10} = \frac{50}{20} = 2.5\ A = i_L(0)$$

Circuit at $t > 0$




$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))\, e^{-t/\tau}$$

Circuit at $t = \infty$. Inductor acts as short circuit.




$i_L(\infty) = -2.5\text{ A}$

Time constant $= \dfrac{L}{R_{eq}} = \dfrac{0.2\times10^{-3}}{(10+10)} = \dfrac{0.2\times10^{-3}}{20} = 10^{-5}$ sec

for $t > 0$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-t/\tau}$$

$$= -2.5 + (2.5 - (-2.5))e^{-t/10^{-5}}$$

$$\Rightarrow \boxed{i_L(t) = -2.5 + 5e^{-10^5 t} \text{ A}}$$

Q. The Thevenin equivalent at terminals a-b of the linear network shown in Fig. Q is to be determined by measurement. When a 10 KΩ resistor is connected to terminals a-b, the voltage Vab is measured as 6 V. When a 30 KΩ resistor is connected to the terminals, Vab is measured as 12 V.

Determine

a) The Thevenin equivalent at terminals a-b.

b) Vab when a 20 KΩ resistor is connected to the terminals a-b.

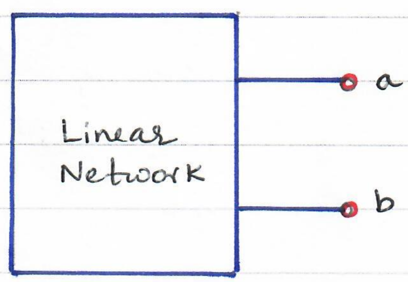
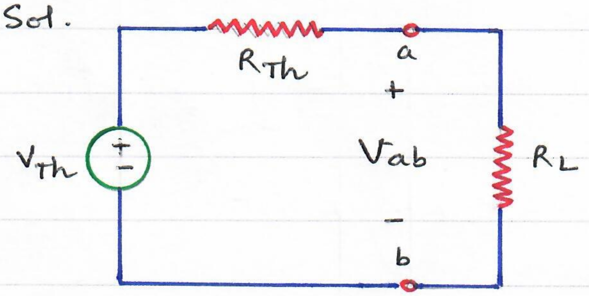



Fig. Q

Sol.




when $R_L = 10\,K\Omega$ ; $V_{ab} = 6V$

$R_L = 30\,K\Omega$ $V_{ab} = 12V$

$$V_{ab} = \frac{V_{Th}}{R_{Th} + R_L} \times R_L$$

$$\Rightarrow V_{Th} R_L = V_{ab}(R_{Th} + R_L)$$

$$6(R_{Th} + 10K) = V_{Th} \times 10K \quad \cdots (1)$$

$$12(R_{Th} + 30K) = V_{Th} \times 30K \quad \cdots (2)$$

$$\frac{30}{10} = \frac{12}{6}\,\frac{(R_{Th} + 30K)}{(R_{Th} + 10K)}$$

$$\Rightarrow \quad 3 = \frac{2\,(R_{Th} + 30K)}{(R_{Th} + 10K)}$$

$$\Rightarrow \quad 3\,(R_{Th} + 10K) = 2\,(R_{Th} + 30K)$$

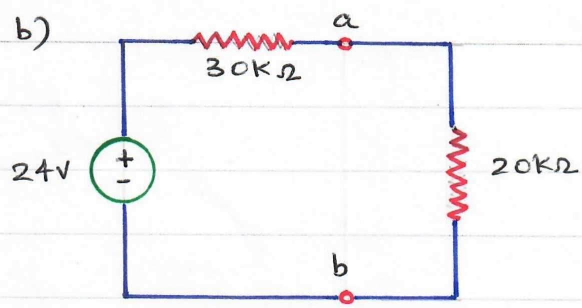
$$\boxed{R_{Th} = 60K - 30K = 30K\,\Omega}$$

$$V_{Th} \times 10K = 6\,(R_{Th} + 10K) = 6\,(30K + 10K)$$

$$V_{Th} \times 10K = 6 \times 40K$$

$$\Rightarrow \boxed{V_{Th} = 24\,V}$$

b)




$$V_{ab} = 24 \times \frac{20K}{30K + 20K}$$

$$= 9.6\,V$$