

Quiz II

Discrete Structures
IIIT Hyderabad, Monsoon 2022

October 20, 2023

Consider the following system of congruent-recurrences:

$$\begin{aligned} a_n &\equiv a_{n-1} + 3n^2 & \text{with } a_0 \equiv 4 & \pmod{5} \\ a_n &\equiv 6a_{n-2} - a_{n-1} & \text{with } a_0 \equiv -1, a_1 \equiv 8 & \pmod{11} \\ a_n &\equiv 4a_{n-1} - 3a_{n-2} - 2 & \text{with } a_0 \equiv 2, a_1 \equiv 5 & \pmod{7} \end{aligned}$$

Answer the following:

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|---|---------------------------------|
| ✓ 1. What is $a_0 \pmod{385}$? | 5 marks |
| ✓ 2. What is $a_2 \pmod{385}$? | 5 marks |
| ✓ 3. What is $(a_{100} \pmod{5})$? | 15 marks |
| ✓ 4. What is $(a_{150} \pmod{35})$? | 20 marks |
| 5. What is $(a_{200} \pmod{385})$? | 25 marks |
| 6. With the same initial/boundary conditions, how many values between 0 and 384 can $(a_3 \pmod{385})$ take, if:
<ul style="list-style-type: none">✓ all the three congruences are satisfied?• <i>none of the three</i> congruences are satisfied?• exactly <i>one of the three</i> congruences are satisfied? | 3 marks
12 marks
15 marks |