

Solutions to Mid Exam - I Paper

JEC102

**Q1** In many amplifier applications, we are concerned not only with voltage gain, but also with power gain.

$$\text{Power gain} = A_p = \frac{\text{Power delivered to the load}}{\text{Power supplied by the source}}$$

Find the power gain for the circuit shown in Fig. Q1

when  $R_L = 60\text{ k}\Omega$

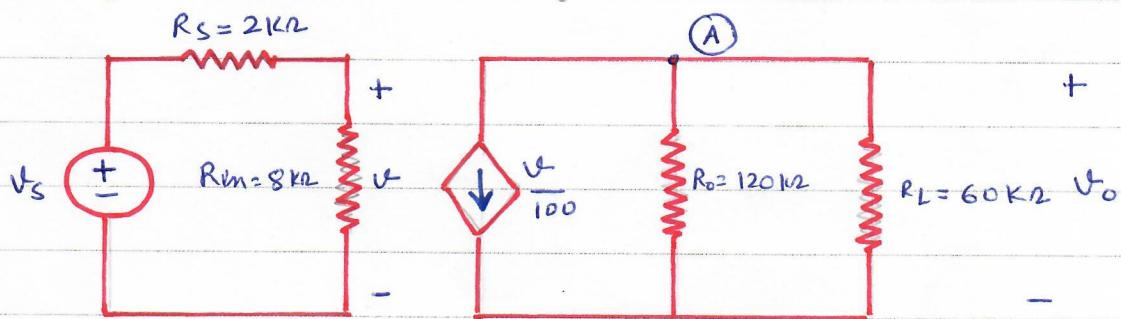


Fig. Q1

Sol:

$$V = \frac{8}{2+8} \times V_s = \frac{4}{5} V_s$$

Applying KCL at node A

$$\frac{V}{100} + \frac{V_o}{120 \times 10^3} + \frac{V_o}{60 \times 10^3} = 0$$

$$\Rightarrow V_o = -400V$$

PTO

$$\Rightarrow V_o = -400V = -400 \times \frac{4}{5} V_s = -320V_s$$

$$(\because V = \frac{4}{5} V_s)$$

$$V_o = -320V_s$$

$$i_L = \frac{V_o}{R_L} = \frac{-320V_s}{60 \times 10^3}$$

$$i_S = \frac{V_s}{(2+8)10^3} = \frac{V_s}{10 \times 10^3}$$

Power delivered by source =  $V_s i_S$

$$= V_s \times \frac{V_s}{10^4} = \frac{V_s^2}{10^4}$$

Power consumed by load ( $R_L$ ) =  $V_o i_L$

$$= (-320V_s) \times \frac{(-320V_s)}{60 \times 10^3}$$

$$\text{Power gain} = A_p = \frac{V_o i_L}{V_s i_S} = \frac{\frac{(320)^2 V_s^2}{60 \times 10^3}}{\frac{V_s^2}{10 \times 10^3}} = \frac{(320)^2 \times 10 \times 10^3}{60 \times 10^3}$$

$$= \frac{(320)^2}{6}$$

$$\Rightarrow A_p \approx 1706.7$$

(Q) Find the value of  $R_L$  for maximum power transfer in the circuit shown in Fig. Q. Also, find the maximum power dissipated in  $R_L$ .

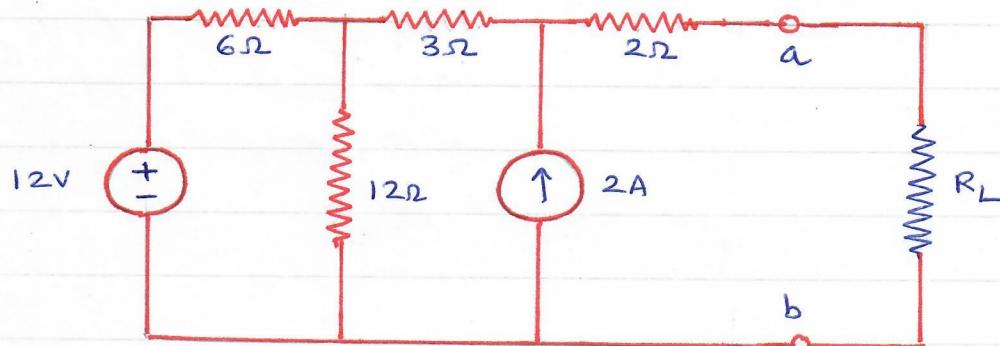
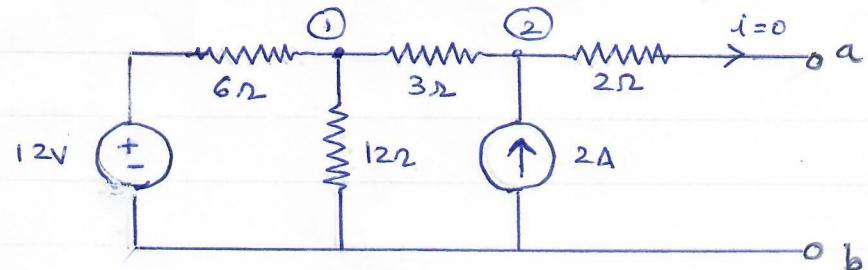


Fig. Q

Sol.

Theveninize the circuit across terminals of  $R_L$ .



$V_{th}$

Applying KCL at node (1)

$$\frac{V_1 - 12}{6} + \frac{V_1}{12} + \frac{V_1 - V_2}{3} = 0$$

$$\Rightarrow \frac{2(V_1 - 12) + V_1 + 4(V_1 - V_2)}{12} = 0$$

$$\Rightarrow 7V_1 - 4V_2 = 24 \quad \dots (A)$$

Applying KCL at node (2)

$$-2 + \frac{V_2 - V_1}{3} = 0$$

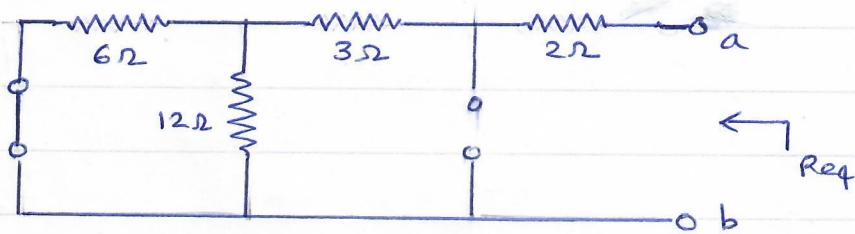
$$\Rightarrow V_2 - V_1 = 6 \quad \dots (B)$$

Solving (A) and (B)

$$V_2 = 22V = V_{th}$$

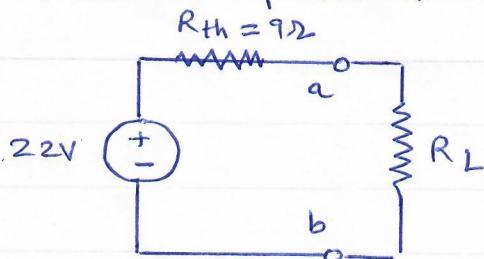
$$\underline{R_{th}} = \underline{Req}$$

Turning off all the sources and calculating equivalent resistance across terminals a-b.



$$Req = R_{th} = 2 + 3 + 6 \parallel 12 \\ = 2 + 3 + 4 = 9\Omega$$

Thevenin equivalent circuit



Maximum power is transferred to  $R_L$  when  $R_L = R_{th} = 9\Omega$

Maximum power transferred to  $R_L$  when  $R_L = R_{th} = 9\Omega$

$$P_{max} \text{ in } R_L = \left[ \frac{22}{(9+9)} \right]^2 \times 9$$

$$= \left( \frac{11}{18} \right)^2 \times 9$$

$$\Rightarrow P_{max} = \left( \frac{11}{9} \right)^2 \times 9 = \frac{11 \times 11}{9} = \frac{121}{9} = 13.44 \text{ W}$$

Q. Find the Thevenin and Norton equivalent of the circuit shown in Fig.-Q at terminals a-b.

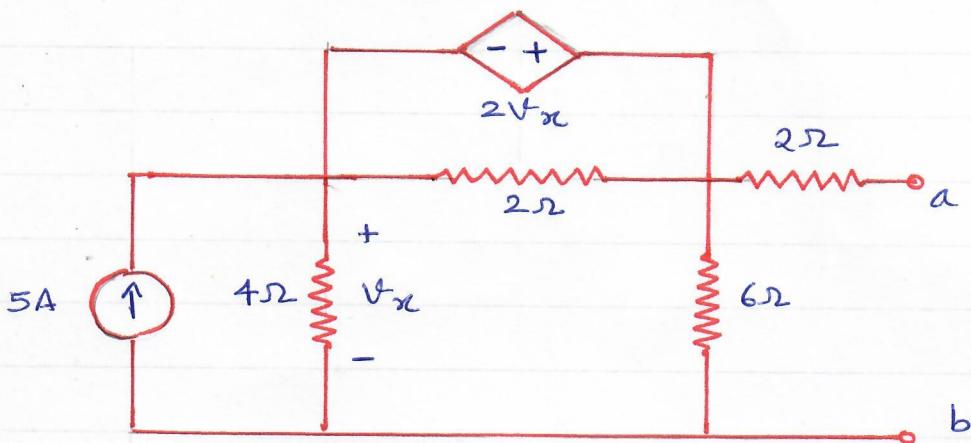
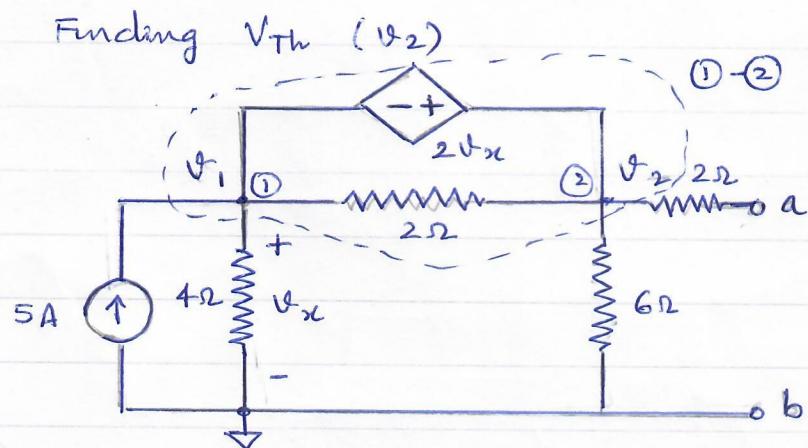


Fig.-Q

Sol.



Applying KCL at super-node ① - ②

$$-5 + \frac{v_1}{4} + \frac{v_2}{6} = 0$$

$$\Rightarrow -5 + \frac{3v_1 + 2v_2}{12} = 0$$

$$\Rightarrow 3v_1 + 2v_2 = 60 \quad \dots (A)$$

but  $v_2 - v_1 = 2v_x$

and  $v_x = v_1 \quad \therefore v_2 - v_1 = 2v_1$

$$\Rightarrow 3v_1 - v_2 = 0 \quad \dots (B)$$

Solving (A) and (B)

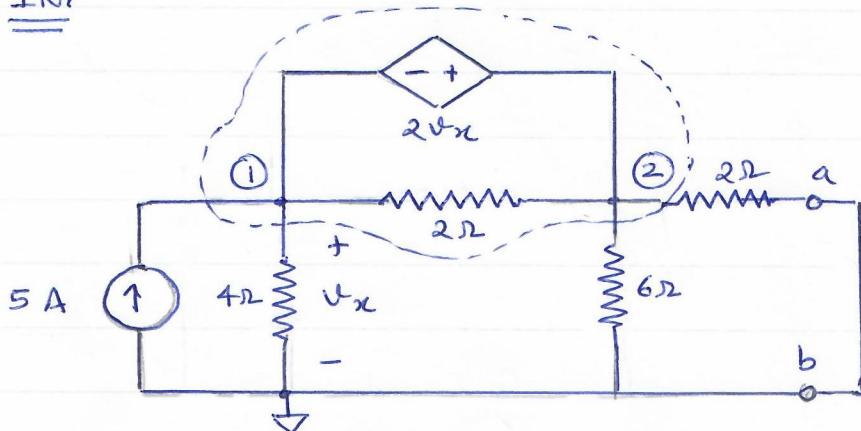
$$3v_1 + 2v_2 = 60$$

$$3v_1 - v_2 = 0$$

$$\Rightarrow 3v_2 = 60$$

$$\Rightarrow v_2 = 20V = V_{Th}$$

INr



Applying KCL at super-node ① - ②

$$-5 + \frac{v_1}{4} + \frac{v_2}{6} + \frac{v_2}{2} = 0$$

$$\Rightarrow -5 + \frac{3v_1 + 2v_2 + 6v_2}{12} = 0$$

$$\Rightarrow 3v_1 + 8v_2 = 60 \quad \dots (C)$$

$$\text{But } V_2 - V_1 = 2V_x = 2V_1 \quad (\because V_x = V_1)$$

$$\Rightarrow 3V_1 - V_2 = 0 \quad \dots (D)$$

Solving (C) and (D)

$$3V_1 + 8V_2 = 60$$

$$3V_1 - V_2 = 0$$

$$\Rightarrow 9V_2 = 60$$

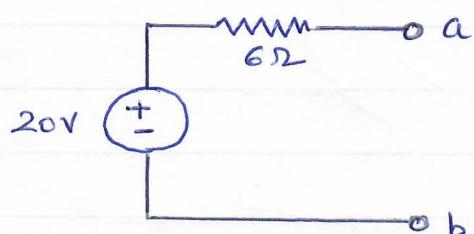
$$\Rightarrow V_2 = \frac{60}{9} = \frac{20}{3} \text{ V}$$

$$I_N = \frac{V_2}{2} = \frac{20/3}{2} = \frac{10}{3} \text{ A}$$

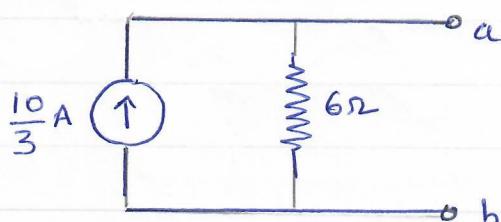
$$R_{Th} = R_N = R_{eq} = \frac{V_{Th}}{I_N} = \frac{20}{(10/3)} = 6\Omega$$

$$\therefore V_{Th} = 20 \text{ V}; \quad I_N = \frac{10}{3} \text{ A}; \quad \text{and} \quad R_{Th} = R_N = 6\Omega$$

Thevenin equivalent circuit is



Norton equivalent circuit is



(iv) The switch in the circuit shown in Fig. Q has been closed for a long time (i.e., the circuit is in steady state just before the switch is opened) and it is opened at  $t=0$ . Find  $V_C(t)$  for  $t \geq 0$ . Also calculate the initial energy stored in the capacitor.

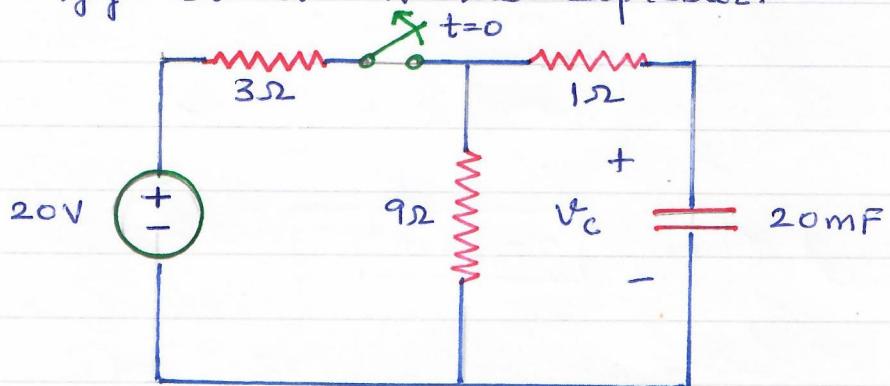
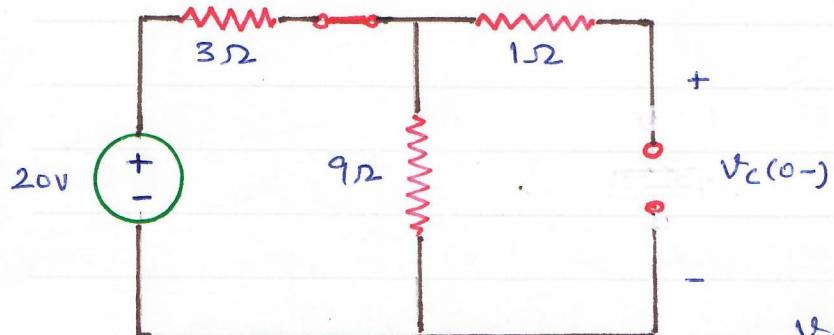


Fig. Q

Sol.

At  $t=0^-$ , the switch is closed and capacitor acts as an open circuit since the circuit is in steady state (given).

Circuit at  $t=0^-$

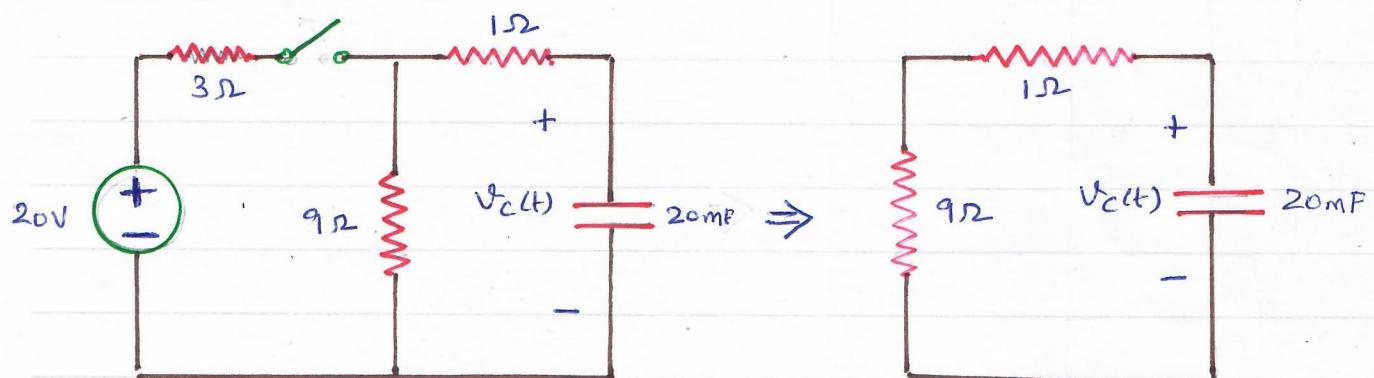


$$V_C(0^-) = \frac{20 \times 9}{9+3} \\ = \frac{20 \times 9}{12} = 15V$$

$$V_C(0^-) = V_C(0) = V_C(0^+) = 15V$$

(since the voltage across a capacitor has to be continuous)

Circuit at  $t=0$



The response  $V_C(t)$  will be of the form  $Ae^{st} = V_C(0)e^{-t/\tau}$

where  $V_C(0) = 15V$  (already calculated)

$$\tau = R_{eq} C$$

where  $R_{eq} = 1+9 = 10\Omega$ ; and  $C = 20\text{mF}$

$$\therefore \tau = 10 \times 20 \times 10^{-3} = 0.2\text{s}$$

$\therefore V_C(t)$  for any time  $t \geq 0$  is

$$V_C(t) = 15 e^{-\frac{t}{0.2}} = 15 e^{-5t} \text{ V}$$

$$\text{Initial energy stored in the capacitor} = \frac{1}{2} C [V_C(0)]^2$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times (15)^2$$

$$= 10 \times 10^{-3} \times 225$$

$$= 2.25 \text{ J}$$

(Q) Find the expression for  $i_L(t)$  for all time  $t > 0$  in the circuit shown in Fig. Q. Assume that the circuit is in steady state at  $t = 0^-$ .

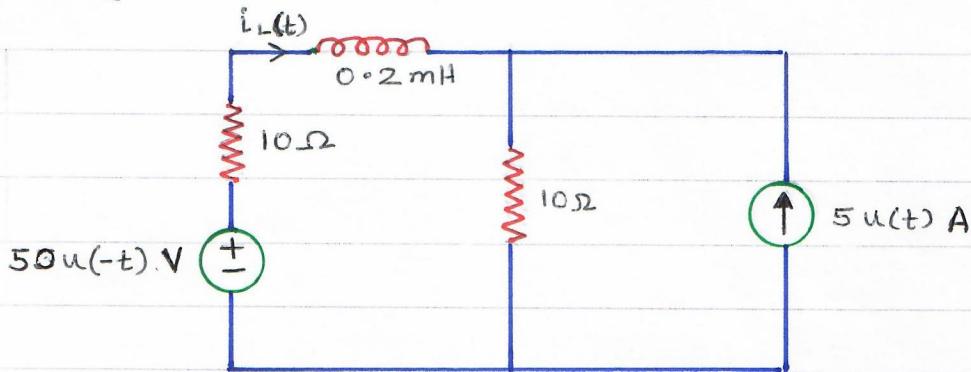
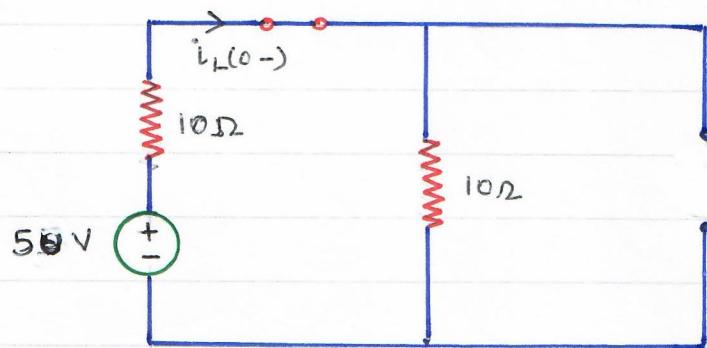


Fig. Q

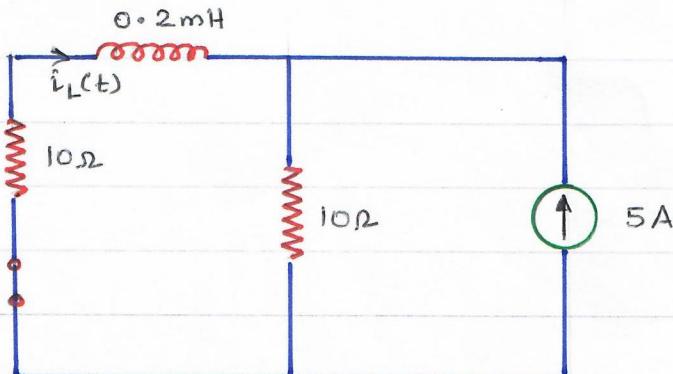
Sol.

Circuit at  $t = 0^-$ . Inductor can be replaced by short circuit as the circuit is in steady state.



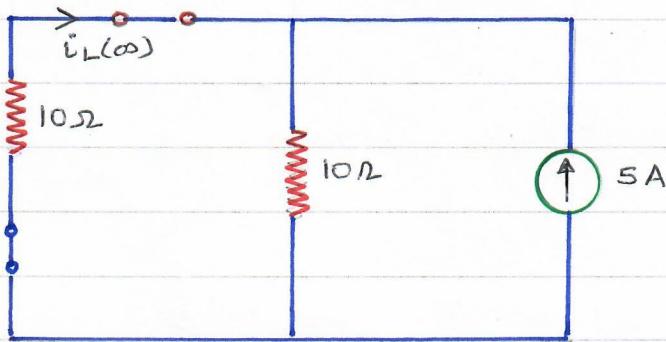
$$i_L(0^-) = \frac{50}{10+10} = \frac{50}{20} = 2.5 \text{ A} = i_L(0)$$

Circuit at  $t > 0$



$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty)) e^{-t/\tau}$$

Circuit at  $t = \infty$ . Inductor acts as short circuit.



$$i_L(\infty) = -2.5 \text{ A}$$

$$\text{Time constant} = \frac{L}{R_{\text{eq}}} = \frac{0.2 \times 10^{-3}}{(10+10)} = \frac{0.2 \times 10^{-3}}{20} = 10^{-5} \text{ sec}$$

for  $t > 0$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty)) e^{-t/\tau}$$
$$= -2.5 + (2.5 - (-2.5)) e^{-t/10^{-5}}$$

$$\Rightarrow \boxed{i_L(t) = -2.5 + 5 e^{-10^5 t} \text{ A}}$$

Q. The Thevenin equivalent at terminals a-b of the linear network shown in Fig. A is to be determined by measurement. When a  $10\text{ k}\Omega$  resistor is connected to terminals a-b, the voltage  $V_{ab}$  is measured as 6V. When a  $30\text{ k}\Omega$  resistor is connected to the terminals,  $V_{ab}$  is measured as 12V.

Determine

- The Thevenin equivalent at terminals a-b.
- $V_{ab}$  when a  $20\text{ k}\Omega$  resistor is connected to the terminals a-b.

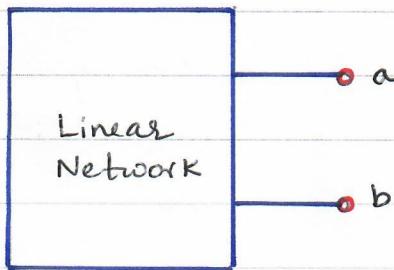
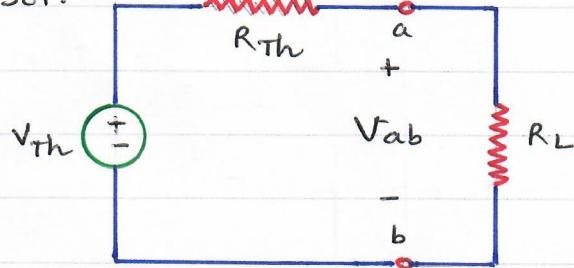


Fig. A

Sol.



$$\text{When } R_L = 10\text{ k}\Omega ; V_{ab} = 6\text{ V}$$

$$R_L = 30\text{ k}\Omega \quad V_{ab} = 12\text{ V}$$

$$\cdot V_{Th} \quad V_{ab} = \frac{V_{Th}}{R_{Th} + R_L} \times R_L$$

$$\Rightarrow V_{Th} R_L = V_{ab} (R_{Th} + R_L)$$

$$6 (R_{Th} + 10\text{ k}) = V_{Th} \times 10\text{ k} \quad \dots (1)$$

$$12 (R_{Th} + 30\text{ k}) = V_{Th} \times 30\text{ k} \quad \dots (2)$$

$$\frac{30}{10} = \frac{12}{6} \frac{(R_{Th} + 30K)}{(R_{Th} + 10K)}$$

$$\Rightarrow 3 = 2 \frac{(R_{Th} + 30K)}{(R_{Th} + 10K)}$$

$$\Rightarrow 3(R_{Th} + 10K) = 2(R_{Th} + 30K)$$

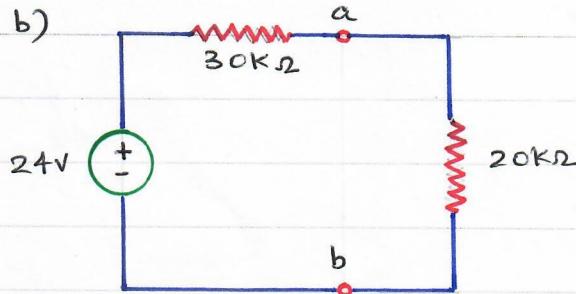
$$R_{Th} = 60K - 30K = 30K \Omega$$

$$V_{Th} \times 10K = 6(R_{Th} + 10K) = 6(30K + 10K)$$

$$V_{Th} \times 10K = 6 \times 40K$$

$$\Rightarrow V_{Th} = 24V$$

b)



$$V_{ab} = \frac{24 \times 20K}{30K + 20K}$$

$$= 9.6V$$