one cannot isolate quarks and gluons, they are

International Institute of Information Technology, Hyderabad

(Deemed to be University)

SC1.308: The Universe Across Scales

Question cum Answer Booklet

End-Semester Examination

43

Max. Time: 2 Hrs **Total Marks: 50**

Roll No: 2022101054 **Programme:** CSE **Date of Exam:** 29/4/25

Room no: H-202 **Seat No:** C5 **Invigilator's Signature:** [signature]

Special Instructions about the exam

1. You may bring **one handwritten A4 size cheat sheet** but **no printed/photocopied material is allowed.** You must submit the cheat sheet with this booklet.
2. Calculators are allowed, but no other electronic gadgets (laptops, tablets, mobiles, etc.) are permitted.
3. There are total **10** questions, attempt all of them.
4. Additional sheets for rough work are allowed, but those must be submitted with this booklet.

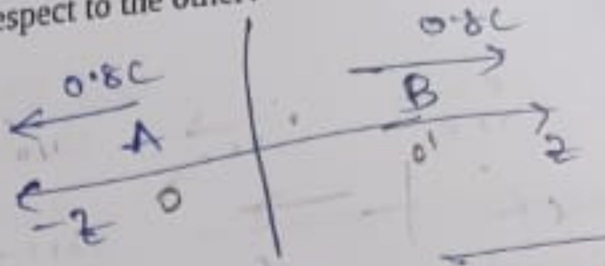
Marks Table (To be filled by the Examiner)

Question No	CO numbers							Examiner
1	2, 5	5						
2	1, 5	5						
3	1, 5	5						
4	1, 2, 5	4						
5	1, 5	5						
6	2, 5	5						
7	1, 5	5						
8	2, 5	3						
9	1, 5	5						
10	2, 5	1						

General Instructions to the students

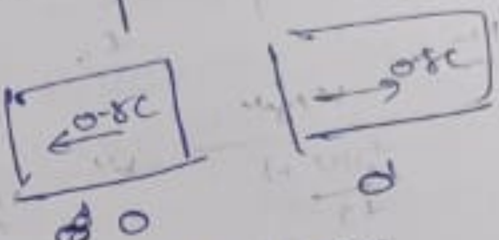
Place your Permanent / Temporary Student ID card on the desk for verification by the invigilator during the examination.
Reading materials such as books are not allowed inside the examination hall.
Borrowing writing materials or calculators from other students in the examination hall is prohibited.
If any student is found indulging in malpractice or copying in the examination hall, the student will be given 'F' grade for the course and may be debarred from writing other examinations.

Q1. Suppose you observe two muons going away from each other with velocity $\pm 0.8c\ \hat{z}$ in a lab. W... the relative velocity of the muon moving towards $-\hat{z}$ with respect to the other?



From velocity transformation relations in special relativity.

Now,



$$u_z = \frac{u_z' + v}{1 + \frac{v \cdot u_z'}{c^2}} = \frac{-0.8c - 0.8c}{1 + \frac{(-0.8\cancel{c})(-0.8\cancel{c})}{\cancel{c^2}}}$$

$v = -0.8C$

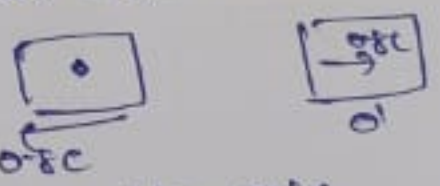
$u_z' = -0.8C$ particle speed from O' which is rest where O considered as moving at 0.8C, with a muon in it

$$= \frac{-1.6C}{1+0.64} = \frac{-1.6}{1.64}C$$

$$= \frac{-1.6}{1.64} \times 3 \times 10^8\ m/s$$

$$= -2.92 \times 10^8\ m/s$$

(5)



substituting here $\Rightarrow v = -0.8C$

$u_z' = -0.8C$

∴ The relative velocity of muon moving toward $-\hat{z}$ w.r.t other $= -2.92 \times 10^8\ m/s$

this indicates $-z$ axis direction

Q2. List the fermions in the Standard Model and specify the interactions (electromagnetic/weak/strong) that each of these interacts through. [5]

Fermions in the standard model (spin = 1/2) → leptons, → quarks

Assumptions: weak - W, electromagnetic → E, strong → S

Leptons →	Charge	Interaction
ν_e - electron neutrino	(0)	W
e - electron	(-1)	W, E
ν_μ - muon neutrino	(0)	W
μ - muon	(-1)	W, E
ν_τ - Tau neutrino	(0)	W
τ - Tau	(-1)	W, E

Quarks →	Charge	Interaction
u - up	(2/3)	S, W, E
d - down	(-1/3)	S, W, E
c - charm	(2/3)	S, W, E
s - strange	(-1/3)	S, W, E
t - top	(2/3)	S, W, E
b - bottom	(-1/3)	S, W, E

These are the fermions & their interactions in the standard Model.

(5)

one type of ...
$\mu \to \nu + L \to$ lepton ($\bar{e}$), $g \to$ ... quarks can interact ...

derabad

As $\psi(x,t)=0$ outside the box; so limits only 0 to L

Q3. Write down the kinetic energy operator. Obtain the expected kinetic energy of an electron trapped in a one-dimensional box of length L (assume $V(x) = 0$ inside the box) if it is in the 4^{th} energy state. [2+3]

Kinetic energy operator $= \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2$ $\left(\because T = \frac{p^2}{2m}\right)$ for one-dimensional box

$\langle T\rangle = \int_0^L \psi^*(x,t)\left(\frac{1}{2m}\left(\frac{\hbar}{i}\left(\frac{\partial}{\partial x}\right)\right)^2\right)\cdot\psi(x,t)\,dx$ where $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{2}{L}}\sin\left(\frac{4\pi x}{L}\right)$ $(a = L, n = 4)$ (1)

$\phi(t) = e^{-iEt/\hbar}$ where $E = \frac{n^2\pi^2\hbar^2}{2ma^2}$ $= \frac{16\pi^2\hbar^2}{2ma^2}$

$\langle T\rangle = \int_0^L \sqrt{\frac{2}{L}}\sin\left(\frac{4\pi x}{L}\right) \times e^{i\left(\frac{4\pi\hbar}{mL^2}\right)t} \times \frac{1}{2m}\times\frac{-\hbar^2}{1}\left(\frac{\partial^2}{\partial x^2}\left(\sqrt{\frac{2}{L}}\sin\left(\frac{4\pi x}{L}\right)e^{-iEt/\hbar}\right)\right)dx$

$e^{-i\left(\frac{2\pi\hbar}{mL^2}\right)t\times\frac{2\pi}{\hbar}} = e^{-i\left(\frac{4\hbar\pi t}{mL^2}\right)}$ (2) $= \frac{2\hbar^2}{ma^2} = \frac{2\hbar^2}{mL^2}$

$= \frac{1}{2m}\times -\hbar^2 \times\frac{2}{L}\int_0^L \sin\left(\frac{4\pi x}{L}\right)\cdot\frac{\partial^2}{\partial x^2}\left(\sin\left(\frac{4\pi x}{L}\right)\right)dx$

From (1) & (2)

$= \frac{-\hbar^2}{mL}\int_0^L \sin\left(\frac{4\pi x}{L}\right)\times\frac{4\pi}{L}\times -\frac{4\pi}{L}\times\sin\left(\frac{4\pi x}{L}\right)dx$ $\psi(x,t) = \psi_4(x)\cdot\phi(t) = \sqrt{\frac{2}{L}}\sin\left(\frac{4\pi x}{L}\right)\times e^{-i\left(\frac{4\hbar\pi}{mL^2}\right)t}$

$= \frac{+\hbar^2}{mL}\times\frac{16\pi^2}{L^2}\int_0^L \sin^2\left(\frac{4\pi x}{L}\right)dx = \frac{16\pi^2\hbar^2}{mL^3}\int_0^L \frac{1-\cos\left(\frac{8\pi x}{L}\right)}{2}\,dx.$

$= \frac{8\pi^2\hbar^2}{mL^3}\left(L - \left[\frac{\sin\left(\frac{8\pi x}{L}\right)}{\frac{8\pi}{L}}\right]_0^L\right) = \frac{8\pi^2\hbar^2}{mL^3}\times L = \frac{8\pi^2\hbar^2}{mL^2}$

$\boxed{\langle T\rangle = \frac{8\pi^2\hbar^2}{mL^2}} = \frac{8\pi^2}{mL^2}\times\frac{h^2}{4\pi^2} = \boxed{\frac{2h^2}{mL^2} = \langle T\rangle}$

Q4. A pion is created near the top of Earth's atmosphere, about 100 km above sea level. It travels vertically downward with a total energy of $E = 1.4\times10^5$ MeV. In its rest frame, it decays 35.0 ns after its creation. At what altitude above the sea level, as measured in the Earth's reference frame, does the decay occur? We know that the rest energy of a pion is about 140 MeV. ($1\text{ eV} = 1.6\times10^{-19}$ J, $1\text{ MeV} = 10^6$ eV) [5]

100 km above $\Rightarrow 100\times10^3$ m $= 10^5$ m

$E_{total} = 1.4\times10^5$ MeV, Rest energy $= 140$ MeV

$\Rightarrow mc^2 = 140$ MeV, $\frac{mc^2}{\sqrt{1-v^2/c^2}} = 1.4\times10^5 \Rightarrow \sqrt{1-v^2/c^2} = \frac{140\text{ MeV}}{1.4\times10^5\text{ MeV}} = 10^{-3}$

$\sqrt{1-\frac{v^2}{c^2}} = 10^{-3}$ $\qquad 1-v^2/c^2 = 10^{-6} \Rightarrow \frac{v^2}{c^2}\simeq 1$

$\gamma = 10^3$ $\qquad v\approx c$

rest frame $\Rightarrow$ decays $\Rightarrow$ 35.0 ns.

from ~~muon pion~~ earth ref frame $\Rightarrow t_0 = \dfrac{t_{\text{rest frame}}^{\text{pion frame}}}{\sqrt{1-v^2/c^2}} = \dfrac{t_{\text{pion frame}}}{\sqrt{1-v^2/c^2}}$

$= \dfrac{35\times10^{-9}\,s}{10^{-3}} \approx 35\times10^{-6}\,s$

Also as $1-v^2/c^2 = 10^{-6} \Rightarrow v \simeq c$

The altitude above the sea level, at which this decay occurs is

$= \text{Speed} \times \text{time} = c \times 35\times10^{-6}$

$= 3\times10^{8}\,m/s \times 35\times10^{-6}\,s$

$= 105\times10^{2}\,m \simeq 10.5\,km$

$\frac{35\times3}{10^5}$

Approximately $\simeq 10.5\,km$.

$(100-10.5)\,km$

4

Q5. What do we mean by stationary-state solutions of the Schrödinger equation? Explain how it is possible to get stationary-state solutions, even though the equation depends on time. [2+3]

Stationary-state solutions of the schrödinger Equation:- the solutions of S.E solve ~~where it is time independent (so we got the same stationary~~ below.

φ. The initial S.E (time-varying) $\Rightarrow i\hbar \dfrac{\partial\varphi}{\partial t} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2\varphi}{\partial x^2} + V\varphi$ — (1)

Now $\varphi(x,t) = \psi(x)\cdot\phi(t) \Rightarrow \dfrac{\partial\varphi}{\partial t} = \psi\dfrac{d\phi}{dt}$, $\dfrac{\partial^2\varphi}{\partial x^2} = \dfrac{d^2\psi}{dx^2}\phi$; $\varphi = \psi\times\phi$.

Now substituting these in eq (1)

$i\hbar\left(\psi\dfrac{d\phi}{dt}\right) = -\dfrac{\hbar^2}{2m}\left(\dfrac{d^2\psi}{dx^2}\cdot\phi\right) + V\psi\phi \Rightarrow \dfrac{1}{\phi}\, i\hbar\dfrac{d\phi}{dt} = -\dfrac{\hbar^2}{2m}\dfrac{1}{\psi}\dfrac{d^2\psi}{dx^2} + V$

$\dfrac{1}{\phi}\cdot i\hbar\dfrac{d\phi}{dt} = E \Rightarrow \boxed{\phi(t) = e^{-iEt/\hbar}}$

This Equation becomes true only when both sides are constant as LHS completely on time depends, RHS → completely on x.

$-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + V\psi = E\psi \Rightarrow \boxed{\hat{H}\psi = E\psi} \Rightarrow$ The stationary states are eigen functions of the Hamiltonian operator.

$\Rightarrow$ ~~we can~~. The solutions of these are the stationary state solutions as this equation is independent on time.

$(\varphi(x,t) = \psi(x)\cdot e^{-iEt/\hbar})$. In this way we will get the stationary-state solutions.

5

Q6. Expand the relativistic expression of energy when $v \ll c$ keeping terms up to $\mathcal{O}(v^2/c^2)$. Interpret the terms. Show, using the velocity transformation relations in special relativity, that the speed of light remains the same in all inertial frames.

[2+3]

relativistic expression of energy

Given energy $\Rightarrow E = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} = mc^2\left(1-\frac{v^2}{c^2}\right)^{-1/2} = mc^2\left(1+\frac{v^2}{2c^2}+\underbrace{(\ldots)}_{\text{negligible}}\right)$

$= \underbrace{mc^2}_{\text{rest energy}} + \underbrace{\tfrac{1}{2}mv^2}_{\text{kinetic energy}}$

kinetic energy $\Rightarrow T = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 \approx \tfrac{1}{2}mv^2$

Here we can recover newtonian expression as expected.

To prove, speed of light remains the same in all inertial frames.

velocity transformation relations in special relativity,

$u_x = x/t = u_x'+v \Big/ 1+\dfrac{v \cdot u_x'}{c^2}$, $u_y = y/t = \dfrac{u_y'\sqrt{1-v^2/c^2}}{1+\frac{v \cdot u_x'}{c^2}}$, $u_z = \dfrac{z}{t} = \dfrac{u_z'\sqrt{1-v^2/c^2}}{1+v \cdot u_x'/c^2}$

In this substituting $u_x' = c$, $u_y' = 0$, $u_z' = 0$ $\Rightarrow O'$ frame. (moving frame)

considering moving at light speed

(As $u_x', u_y', u_z' \to O'$ frame
$u_x, u_y, u_z \to O$ frame (rest frame).

$$u_x = \frac{c+v}{1+\frac{v \cdot c}{c^2}} = \frac{c+v}{1+v/c} = c$$

$\therefore$ speed of light remains the same in all inertial frames.

Hence proved.

5

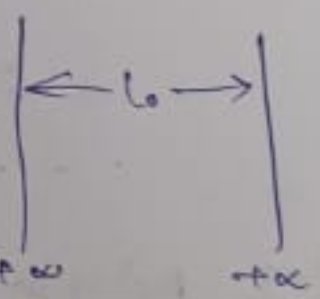
Q7. A particle of mass m is trapped inside a one-dimensional region between two infinitely high potential barriers separated by a distance L_0. Determine the minimum energy of the particle using the uncertainty principle.

[5]

From uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

5



$\Delta p \geq \dfrac{h}{4\pi \Delta x}$ As max Δx possible as the barriers are seperated by $L \Rightarrow \Delta x_{max} = L \Rightarrow \Delta x \leq L_0$

$$\frac{1}{\Delta x} \geq \frac{1}{L_0} \quad \text{— (1)}$$

Substituting (1) in above,

$$\Delta p \geq \frac{h}{4\pi \Delta x} \geq \frac{h}{4\pi (L_0)}$$

$$\therefore (\Delta P)_{min} = \frac{h}{4\pi L_0}$$

(kinetic energy)
Minimum energy of the particle $= \frac{1}{2m}(\Delta p)^2$

$$T = \frac{1}{2m}\left(\frac{h}{4\pi L_0}\right)^2$$

$$= \frac{1}{2m} \times \frac{h^2}{16\pi^2 L_0^2} = \frac{h^2}{32\pi^2 m L_0^2}$$

$\therefore$ The min energy of the particle using uncertainty principle $= \frac{h^2}{32\pi^2 m L_0^2}$

Q8. The relation $E = m_v c^2$ can also be written as $E^2 = Ac^2 + Bm^2$. Obtain A and B. [5]

$$E = m_v c^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad \left(\text{as } m_v = \frac{m}{\sqrt{1 - v^2/c^2}}\right).$$

$$\Rightarrow E^2 = \frac{m^2c^4}{1 - v^2/c^2}$$

$$= m^2c^4\left(1 - v^2/c^2\right)^{-1}$$

(From binomial expansion)

$$E^2 = m^2c^4\left(1 + v^2/c^2 + \underbrace{\cdots}_{\text{negligible}}\right)$$

3

$$= m^2c^4\left(1 + v^2/c^2\right) = m^2c^4 + m^2c^2v^2$$

$$= m^2c^4 + (mv)^2c^2 \quad (\text{As } p = mv)$$

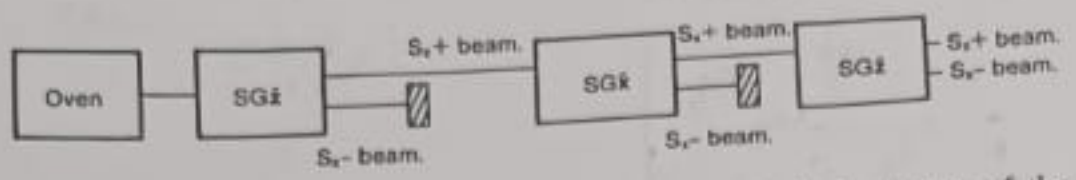
$$= m^2c^4 + p^2c^2 = p^2c^2 + c^4 \cdot m^2$$

comparing $Ac^2 + Bm^2 = p^2 \cdot c^2 + c^4 \cdot m^2$

$$\Rightarrow \boxed{A = p^2} \quad \boxed{B = c^4}$$

one type of quar...
$u \to \gamma + L \to$ lepton (e), $g \to g + g$
corresponding neutrino (quarks can interact with ...

Q9. Two persons, A and B, perform the Stern-Gerlach experiment by passing a beam of electrons through three inhomogeneous magnetic fields and, in between, allowing only the spin-up components to pass through. For A, the magnetic fields point in the $\hat{z}$, $(\hat{z}+\sqrt{3}\hat{y})/2$, and $\hat{z}$ directions sequentially; for B, they point along $\hat{z}$, $\hat{x}$, and $\hat{z}$ directions sequentially. For example, the figure below illustrates B's setup.



Estimate the ratio of the numbers of up-spin measured by A and B if the fluxes of the two beams are identical and they measure for an extended but equal amount of time. [5]

$A \rightarrow \hat{z},\ (\hat{z}+\sqrt{3}\hat{y})/2,\ \hat{z}\ ;\ B \rightarrow \hat{z}, \hat{x}, \hat{z}$

Assuming initial flux in the $\hat{z}$ direction (oven → next one) for both

(Let the no. of spins flux $= k$ for both as given identical)

For A

$\hat{z} \xrightarrow[0^\circ]{} \hat{z} \xrightarrow[60^\circ]{} (\hat{z}+\sqrt{3}\hat{y})/2 \xrightarrow[60^\circ]{} \hat{z}$

$((1)\cdot(1)\cos\theta = 1/2 \Rightarrow \theta = 60^\circ)$
dot product

number of up-spins proportional to $\cos^2(\theta/2)$

$\propto \cos^2(0/2) \times \cos^2(60/2) \times \cos^2(60/2) = 1 \times 3/4 \times 3/4 = 9/16$

$= k\left(\frac{9}{16}\right)$

For B

$\hat{z} \xrightarrow[0^\circ]{} \hat{z} \xrightarrow[90^\circ]{} \hat{x} \xrightarrow[90^\circ]{} \hat{z}$

as number of up-spins proportional to $\cos^2(\theta/2)$

$\propto \cos^2(0/2) \times \cos^2(90/2) \times \cos^2(90/2) = 1 \times 1/2 \times 1/2 = 1/4$

$= k(1/4)$

$\therefore$ ratio of no. of up-spins measured by A to B $= \dfrac{k(9/16)}{k(1/4)}$

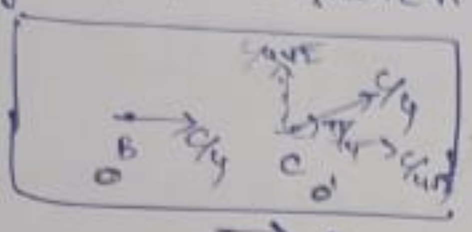
$= 9/4$

$\boxed{\therefore \text{ ratio} = 9/4}$

5

Q10. Imagine three ultra-fast jet planes, A, B, and C, moving in straight lines at constant speeds. The pilot of jet A sees jets B and C moving at the same speed, $c/4$, but while B is moving in parallel, C's trajectory makes an angle of $\pi/4$. What angle does the trajectory of C make with the trajectory of B when viewed from jet B? [5]

Considering them in the frame A.



Rough work

Seeing O' from O.

$v = \frac{c}{4\sqrt{2}}$

$u = c/4$

$u' = ?$

As B & C are in the same frame, we can neglect ~~the~~ ~~im~~: Remove A from while calculating V_{CB}

$$u = \frac{u' + v}{1 + \frac{v \cdot u'}{c^2}}$$

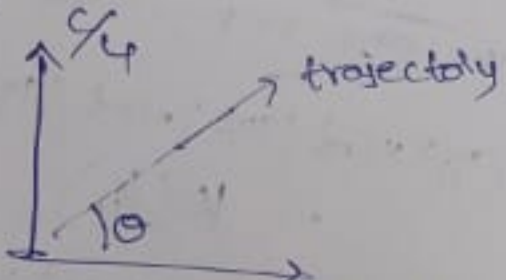
$$\frac{c}{4} = \frac{u' + \frac{c}{4\sqrt{2}}}{1 + \frac{\frac{c}{4\sqrt{2}} u'}{c^2}}$$

$$\frac{c}{4}\left(1 + \frac{u'}{4\sqrt{2}c}\right) = u' + \frac{c}{4\sqrt{2}}$$

$$\frac{c}{4} + \frac{u'}{16\sqrt{2}} = u' + \frac{c}{4\sqrt{2}}$$

$$\frac{c}{4}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = u'\left(\frac{16\sqrt{2}-1}{16\sqrt{2}}\right)$$

$$u' = \frac{4(\sqrt{2}-1)}{16\sqrt{2}-1} \times c$$



$$\tan\theta = \frac{c/4}{\frac{4(\sqrt{2}-1)c}{(16\sqrt{2}-1)}} = \frac{16\sqrt{2}-1}{16(\sqrt{2}-1)}$$

$\frac{4(\sqrt{2}-1)c}{(16\sqrt{2}-1)}$ ⇔ (Velocity of C w.r.t B)$_A$

$$\boxed{\theta = \tan^{-1}\left(\frac{16\sqrt{2}-1}{16(\sqrt{2}-1)}\right)} \Rightarrow \text{with } B$$

$\theta \approx \tan^{-1}(3.2)$

21.62

6.62

3.2

8

This one type of quark into another

$u \to \gamma + L \to$ lepton (e), $g \to g + g$

corresponding neutrino (gluons can interact with themselves...