Exercise 2: **Buckley Leverett Solution**

Objective:

To plot fractional flow curve and Frontal Displacement curve to locate the linear flood-front corresponding to the water saturation during Water flooding.

Theory:

Most of the oil and gas recovered from reservoirs are displaced immiscibly by water and/or gas. The displacement could be in the form of solution gas drive, gas cap expansion, water influx from aquifers or injection of water and/or gas. Solution-gas drive, gas cap expansion, and water influx from aquifers are essentially natural processes that supply energy to the reservoir for hydrocarbon recovery. Gas and water injection are designed and installed to artificially supply energy to the reservoir and thereby improve hydrocarbon recovery. It is important to understand the fundamental processes that occur when reservoir fluids are displaced immiscibly by gas or water. The displacement process is affected by the wettability of the rock, and the mobility ratio between the displaced and the displacing fluids. The total efficiency of the displacement process is measured in terms of the effectiveness of water or gas in displacing the reservoir fluids, and the proportion of the reservoir actually contacted by the displacing fluids.

FRACTIONAL FLOW EQUATION

The fractional flow equation is used to calculate the flow rate of a fluid as a fraction of the total fluid flow rate when only two fluids are flowing in the reservoir. The flow rate of the fluid at any point in the reservoir depends on its saturation at that point. Since relative permeability of the fluid is dependent on saturation, it follows then that the flow rate of the fluid is dependent on its relative permeability at that point in the reservoir. The fractional flow of a fluid in a reservoir is primarily dependent on its relative permeability but can be affected by capillary and gravity forces.

The development of the fractional flow equation is attributed to Buckley and Leverett (1942). For two immiscible fluids, oil and water, the fractional flow of water, fw (or any immiscible displacing fluid), is defined as the water flow rate divided by the total flow rate, or:

$$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_w + q_o} \tag{2.1}$$

Where:

fw = fraction of water in the flowing stream, i.e., water cut, bbl/bbl

qt = total flow rate, bbl/day

qw = water flow rate, bbl/day

qo = oil flow rate, bbl/day

Eq. 2.2. is the final and complete form of the fractional flow equation for water flowing linearly in an oil-water reservoir. A quick examination of Eq. (15.30) shows that it has all the factors that affect the flow of water in an oil-water reservoir. These factors are fluid properties (viscosities, densities), rock properties (effective permeabilities, saturations, capillary pressure), total flow rate, and structural inclination of the reservoir (dip angle,α).

$$f_w = \frac{1}{\left(1 + \frac{\mu_w}{k_w} \times \frac{k_o}{\mu_o}\right)} + \frac{1.127 \times 10^{-3} \frac{k_o A}{\mu_o q_t}\left[\frac{\partial p_c}{\partial l} + 0.00694(\rho_o - \rho_w)\sin\alpha\right]}{\left(1 + \frac{\mu_w}{k_w} \times \frac{k_o}{\mu_o}\right)} \tag{2.2}$$

If both capillary and gravity effects are assumed to be negligible then eq. 2.2 becomes:

$$f_w = \frac{1}{\left(1 + \frac{\mu_w}{k_w} \times \frac{k_o}{\mu_o}\right)} \tag{2.3}$$

In terms of relative permeabilities, eq. 2.3 can be written as:

$$f_w = \frac{1}{\left(1 + \frac{\mu_w}{k_{rw}} \times \frac{k_{ro}}{\mu_o}\right)} \tag{2.4}$$

If it is assumed that fluid properties and total flow rate are constant, then the fractional flow of water is only a function of water saturation in the reservoir. A plot of f_w vs S_w (Fig. 2.1) is called the ***fractional flow curve***. The fractional flow curve can be used to calculate the flow rate of water in the reservoir at any water saturation.

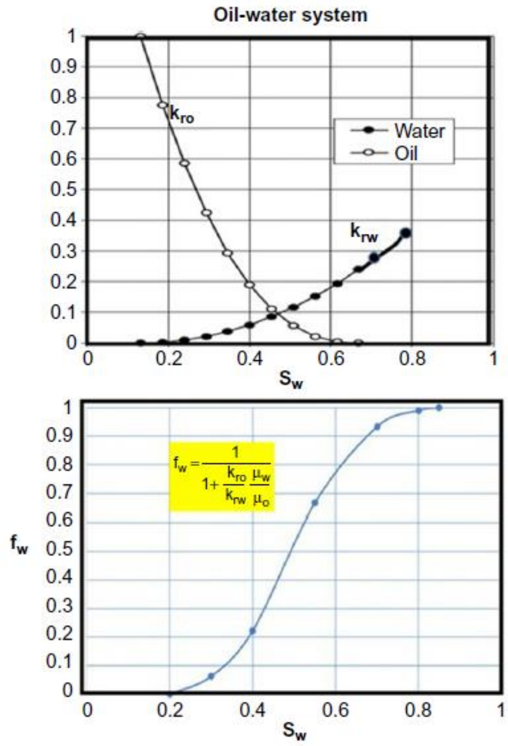

Figure 2.1: Fractional flow curve as a function of Water Saturation

BUCKLEY-LEVERETT EQUATION

The fractional flow equation, as discussed in the previous section, is used to determine the water cut f_w at any point in the reservoir, assuming that the water saturation at the point is known. The question, however, is how to determine the water saturation at this particular point. The answer is to use the frontal advance equation. The frontal advance equation is designed to determine the water saturation profile in the reservoir at any given time during water injection.

Buckley and Leverett (1942) presented what is recognized as the basic equation for describing two-phase, immiscible displacement in a linear system.

$$x = \frac{5.615 i_w t}{\phi A}\left(\frac{df_w}{dS_w}\right)_{S_w}$$

Where,

x= distance travelled by a fixed saturation in time t, feet

i_w= water injection rate, RB/D

t= time interval, days

φ=porosity, fraction

A= cross-sectional area of flow, ft^2

$(df_w/dS_w)_{Sw}$= slope of the fractional flow curve at S_w

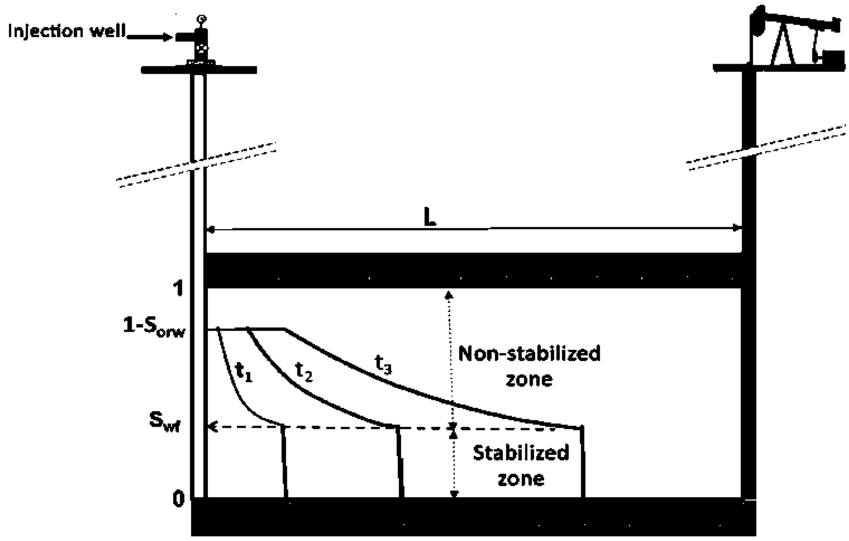

Figure 2.2: Water saturation profile as a function of time "t" and distance "x."

PROCEDURE:

1. Ignoring the capillary pressure term, construct the fractional flow curve, i.e., fw vs. Sw.
2. Draw a straight-line tangent from Swi to the curve.
3. Identify the point of tangency and read off the values of Swf and fwf.
4. Calculate graphically the slope of the tangent as $(dfw/dSw)_{Swf}$.
5. Calculate the distance of the leading edge of the water front from the injection well by using Eq. 2.5.
6. Select several values for water saturation Sw greater than Swf and determine $(dfw/dSw)_{Sw}$ by graphically drawing a straight-line tangent to the fw curve at each selected water saturation
7. Calculate the distance from the injection well to each selected saturation, using Eq. 2.5.
8. Establish the water saturation profile after t_1 days by plotting results.

9. Select a new time t_2 and repeat steps 5 through 7 to generate a family of water saturation profiles.

PROBLEM STATEMENT:

The following data are available for a linear-reservoir system:

S_w	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
k_{ro}/k_{rw}	30.23	17.00	9.56	5.38	3.02	1.70	0.96	0.54	0.30	0.17	0.10

Oil formation volume factor B_o = 1.25 bbl/STB
Water formation volume factor B_w = 1.02 bbl/STB
Formation thickness h = 20 ft
Cross-sectional area A = 26,400 ft
Porosity ϕ = 25%
Injection rate i_w = 900 bbl/day
Distance between producer and injector L = 600 ft
Oil viscosity μ_o = 2.0 cp
Water viscosity μ_w = 1.0 cp
Dip angle α = 0°
Connate water saturation S_{wc} = 20%
Initial water saturation S_{wi} = 20%
Residual water saturation S_{or} = 20%

Calculate and plot the water saturation profile after 60, 120, and 240 days.

RESULTS:

OBSERVATIONS:

CONCLUSION: