1

①

2020

(01) $A = \{x \in N \mid 1 < x \leq 5\}$ $B = \{x \in N \mid 3 \leq x \leq 8\}$.

(a)

$\cancel{x}\ A = \{2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7, 8\}$

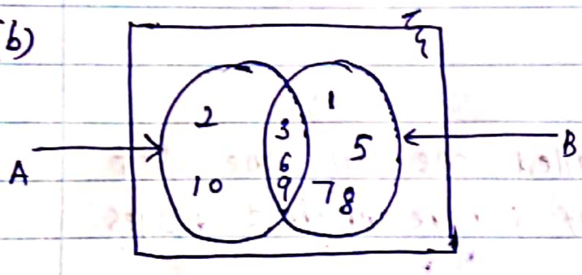
i) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

ii) $A \cap B = \{3, 4, 5\}$

iii) $A - B = \{ A \cap B' = \{2\}$

iv) $B - A = \{\cancel{4}6, 7, 8\}$

(b)



$A = \{2, 3, 6, 9, 10\}$

$B = \{1, 3, 5, 6, 7, 8, 9\}$

(c) A - candidates passed mathematics.

B - candidates passed computer science

C - cadidates who passed both subjects.

D - ~~failed~~ candi dates who failed in both subjects.

$P(A) = 70/100$ $P(C) = 64/100$

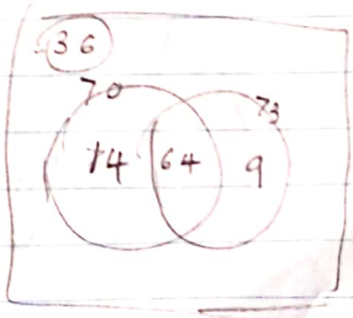
~~P(A)~~ $P(B) = 73/100$ $D = 63$

N - Number of candidates who faced to examination.

86^3

36 P. ~~Pecen~~ failed both subjects $= (100 - 64)\%$

100 $= 36\%$.



$N = \frac{63 \times 100}{36}$ $A = \frac{175 \times 70}{100}$

73
64

$= 175$

36
64
100

=

$A = \frac{70 \times 63}{36}$

=

$(A^c \cup B)^c \cap A^c$

$\equiv (A' \cup B)^c \cap A'$

$= (A')' \cap B' \cap A'$ (De morgan's law)

$= A \cap B' \cap A'$ (complementation law)

$= A \cap A' \cap B'$ (commutative law)

$= \phi \cap B$ (complement law)

$= \phi$ (Domination) law

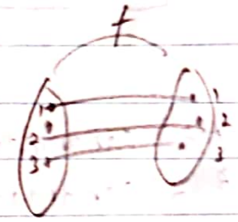
$\therefore (A^c \cup B)^c \cap A^c = \phi$

(02)

(a)(i) A function f from S to T is called one to one iff in all cases distinct elements of S have distinct images under f

One to one function (injective)

f is one to one $\Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b)$



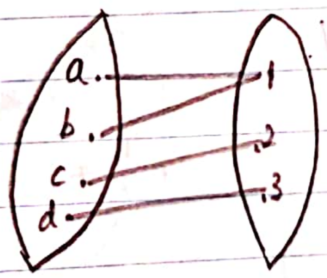
$a \neq b$ ~~$\Leftrightarrow a = b$~~

$\Leftrightarrow f(a) = f(b) \Leftrightarrow a = b$

(ii) on-to function (surjective)

let f be a function S to T said to be onto function iff $f(S) = T$. Every element in T has ~~fert~~ preimage.

$f: S \to T$ is on to $\Leftrightarrow \forall t \in T \; \exists \; s \in S$; $f(s) = T$.



on to

(iii) Bijective function

A function $f : S \to T$ is called a bijection iff f is both injective and surjective.

f is bijective $\Leftrightarrow$ f is 1-1 & onto.

(c) (1) $f(x) = 2x+1$

$f(x_1) = f(x_2)$

$2x_1+1 = 2x_2+1$

$2x_1 = 2x_2$

$x_1 = x_2$

$\therefore$ $f(x)$ is one to one

~~y~~ For onto let $y \in \mathbb{R}$, choose $y = 2x+1$.

$f(x) = f\left(\frac{y-1}{2}\right)$ $x = \frac{y-1}{2} \in \mathbb{R}$

$f(x) = \cancel{2}\left[\left(\frac{y-1}{\cancel{2}}\right)\right]+1$

$f(x) = y-1+1$

$f(y) = y$

$\therefore f$ is on to.

f is bijective function. Because,

f is both one to one function and on to function.

(ii) $f(x) = x^2+1$

$f(x_1) = f(x_2)$
$x_1^2+1 = x_2^2+1$
$x_1^2 = x_2^2$
$x_1 = \pm x_2$

$\therefore$ f(x) is not one to one ~~and on to~~ function.

let $y \in \mathbb{R}$,
Choose x; $x = \pm\sqrt{y-1}$

$f(x) = x^2+1$
$y = x^2+1$
$\pm\sqrt{y-1} = x$

$f(x) = f(y)$
$f(x) = (\pm\sqrt{y-1})^2+1$

$\therefore$ f is not on to function.

$\therefore$ f is not bijection function.

$f(x) = x^3$

$f(x_1) = f(x_2)$
$x_1^3 = x_2^3$
$x_1 = x_2$

$\therefore$ f is one to one function.

$f(x) = x^3$ $\quad \sqrt[3]{y} = x$

let $y \in \mathbb{R}$;
Choose $x = \sqrt[3]{y}$

$f(x) = f(y)$
$\cancel{f(x) = f((\sqrt[3]{y})^3)}$
$f(x) = (\sqrt[3]{y})^3$
$f(x) = y$

$\therefore$ f is on to function.

$\therefore$ f is bijection function.

(c) $f(x) = 3x+1$

let,

$$y = f(x)$$
$$y = 3x+1$$
$$y-1 = 3x$$
$$\frac{y-1}{3} = x$$
$$f^{-1}\,y = \frac{y-1}{3}$$

since the choice of the variables arbitary, we can write, this as,

$$f^{-1}\,x = \frac{x-1}{3}$$

$$f^{-1}(x) = \frac{x-1}{3}$$

(d) $f(x) = 4x-3$ $g(x) = x^2+2$

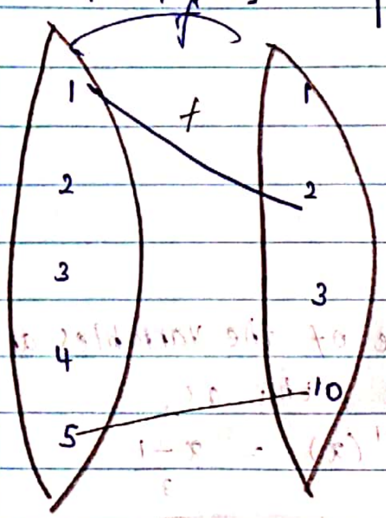
$$\cancel{fog}\ gof\ (x) = (4x-3)^2 + 2$$
$$= 16x^2 - 24x + 9 + 2$$
$$= 16x^2 - 24 + 11$$

$$fog\ (x) = 4(x^2+2) - 3$$
$$= 4x^2 + 8 - 3$$
$$= 4x^2 + 5$$

(Q3)

(i) $A = \{1,2,3,4,5\}$ | $aRb \iff 2a = b \cdot - f$

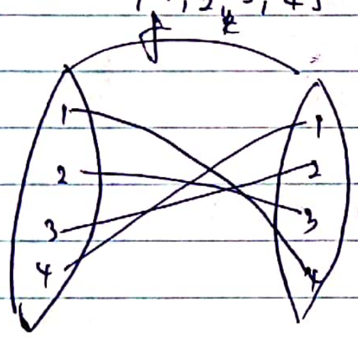
$B = \{1,2,3,10\}$



~~Range = {1, 5}~~
~~domain = {2, 10}~~

Domain = $\{1, 5\}$
Range = $\{2, 10\}$

(ii) $A = \{1,2,3,4\}$ | $aRb \iff a+b = 5 = f$
$B = \{1,2,3,4\}$



Domain = $\{1, 2, 3, 4\}$

range = $\{1, 2, 3, 4\}$

(b) Let $A = \{1,2,3\}$
$B = \{1,2,3,4\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$
$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$

(i) $R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (1,4)\}$

(ii) $R_1 \cap R_2 = \{(1,1)\}$

(iv) $R_2 - R_1 = \{(1,2), (1,3), (1,4)\}$

(iii) $R_1 - R_2 = \{(2,2), (3,3)\}$

Date

(04)

(a) $\neg(p \vee (\neg p \wedge q))$

$= \neg p \wedge \neg(\neg p \wedge q)$

$= \neg p \wedge \neg\neg p \vee \neg q$

$= \neg p \wedge p \vee \neg q$

$= \neg p \wedge \neg q$

$\therefore \neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

(b) (i) $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore (p \rightarrow q) \leftrightarrow (\neg p \vee q)$

$\therefore (p \rightarrow q) \leftrightarrow (\neg p \vee q)$ is tauotology.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$\therefore p \vee (q \wedge r) \equiv (p \vee q) \wedge p(\vee r)$

(b)

(i) $\bar{x}.z + x.0 + \bar{x}.1$

$= (\bar{x}+z).(x+1).(\bar{x}+0)$

$= (\bar{x}+z).\ 1\ .\ \bar{x}$ (OR law)

$= (\bar{x}+z).(1.\bar{x})$ (complement law)

$= (\bar{x}+z)\ \cancel{(1.\bar{x})}\ \bar{x}$ (AND law)

$= \bar{x}(\bar{x}+z)$ (Associate Law)

$= \bar{x}.\bar{x} + \bar{x}.z$ (Distributive law)

$= \bar{x} + \bar{x}.z$ (OR law)

$= \bar{x}(1+z)$ (OR law)

$= \bar{x}(1)$ (AND law)

$= \bar{x}$

(ii) $\bar{x}.y.1 + x.\bar{y}.1.(x+y.0)$

$(\bar{x}+y+0).(x+\bar{y}+0) + (x.y.1)$

(d) (i) $y + \overline{xy}$

$= y + \bar{x} + \bar{y}$ (De morgan's law)

$= (y+\bar{y}) + \bar{x}$ (Associate law)

$= 1 + \bar{x}$ (OR law)

1 (~~AN~~ OR law.

(ii) $\overline{xy}\ (\bar{x}+y)(\bar{y}+y)$

$\overline{xy}\ (\bar{x}+y).1$ (OR law)

$\cancel{(\bar{x}+\bar{y})(x}$

$\overline{xy}\ (\bar{x}+y)$ (AND law)

$(\bar{x}+\bar{y})(\bar{x}+y)$ (De morgan's law)

$\bar{x}.\bar{x} + \bar{x}.y + \bar{y}.\bar{x} + \bar{y}.y$ (Distributive law)

$1 + \bar{x}.y + \bar{y}.\bar{x} + 0$ (~~OR~~ AND law)

$1 + \bar{x}.(y+\bar{y}) + 0$ (Associative law)

$1 + \bar{x}(1) + 0$ (~~Assoc~~ OR law)

$1 + \bar{x} + 0$ (AND law)

$(1+\bar{x}) + 0$ (Associative law)

(1+0) OR law) $= \cancel{\bar{x}}\ 1$ ~~(OR law~~)

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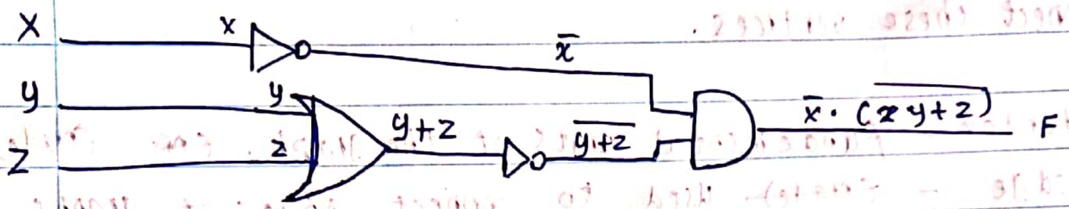
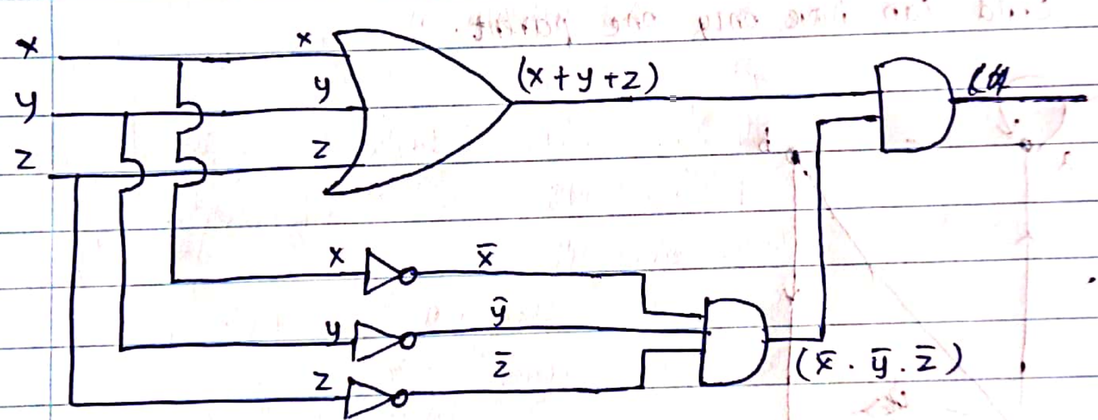
(e)

$F = \bar{x} \cdot \overline{(y+z)}$



$F = (x+y+z) \cdot (\bar{x} \cdot \bar{y} \cdot \bar{z})$.



(f)

z \ xy	$\bar{x}\bar{y}$ 00	$\bar{x}y$ 01	xy 11	$x\bar{y}$ 10
z 1	0	1	1	1
$\bar{z}$ 0	1	0	0	1

$f = yz + x\bar{y} + \bar{x}\bar{y}\bar{z}$

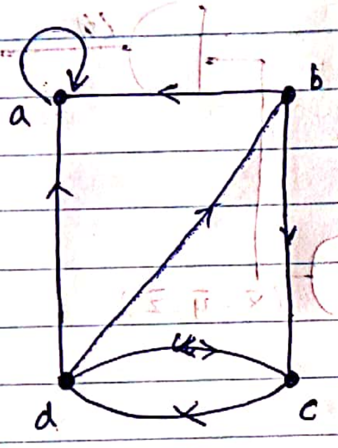
$= \cancel{yz + x\bar{y}(x+\bar{z})}$

(05)

(i) Graphs are ~~de~~ discrete structures consisting of vertices and edges that connect these vertices.

~~Tree~~ Vertex - fundemental units of the graph. ~~(A~~ (Node)
edge - ~~(Node)~~ used to connect nodes of graphs.

Tree.
There are no cycles, There is exactly one root node and every child can have only one parent.



	in degree	out degree
a	3	0
b	1	2
c	2	1
d	1	3

	a	b	c	d
a	1	0	0	0
b	1	0	1	0
c	0	0	0	1
d	1	1	1	0

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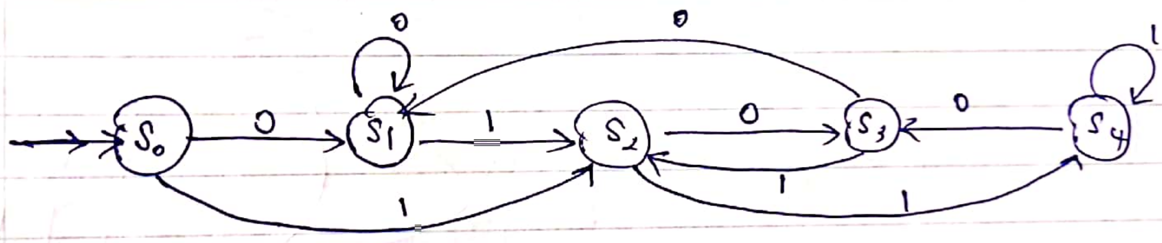
2020

(c) (i) height = 5

(ii)

(iii)

(d)



(e) characteristics of turing machine.